Multiple Representations: Tables to Equations and Graphs

Transcript

Algebra I Teacher: Thank you for making time for me today. I am trying to make sense of the subtle differences between multiple representations in 8^{th} grade, and in Algebra 1.

Eighth Grade Teacher: This is a great time to talk about it. I've been thinking about vertical alignment quite a bit lately, in terms of what comes before 8^{th} grade. Where do you want to start?

Algebra I Teacher: Well, my students are comfortable with information organized in tables and input/output relationships. Can we start by looking at how students are expected to generate equations and graphs using a table?

Eighth Grade Teacher: Yes.

Algebra I Teacher: Looking at the released STAAR test for Algebra 1, we see that tables can be oriented either vertically or horizontally, and that paired values do not necessarily have consistent whole number intervals between the inputs.

My team is making sure that our students are exposed to a variety of tables in class and on common assessments. Here are some examples. (Pulls out two sheets of paper)

Eighth Grade Teacher: OK. Can you explain a little bit further what you mean about consistent whole number intervals between inputs?

Algebra I Teacher: Sure, let's look at this first example. (Pushes away paper on left and centers paper on right)

Algebra I Teacher: If you look at the change between each of the *x*-values, it isn't the same between each set of paired values, and the intervals are not whole numbers. (Writes on problem sheet) This first interval is four and seventy-five hundredths. The second interval from negative three and twenty-five hundredths to one is four and twenty-five hundredths.

We need to compare the change in x-values to the change in y-values. We can think of the column of values for g of x as the paired y-values for the given x-values.

G of negative 8 (graphic shown) is negative twenty-two and seventy-five hundredths, and g of negative three and twenty-five hundredths is negative eight and five-tenths. (Writes on problem sheet) The different between these two values is fourteen and twenty-five hundredths.

When we move from x as negative three and twenty-five hundredths to x as one, our y-values increase by twelve and seventy-five hundredths.

Eighth Grade Teacher: When we calculate the change in the *y*-values and then calculate the ratio of the change in *y* over the change in *x*, we look to see if these ratios are equivalent for each set of paired values. (Graphic is shown, then teacher writes on problem sheet.)

The first ratio is fourteen and twenty-five hundredths to four and seventy-five hundredths, which simplifies to 3. When I simplify the ratio twelve and seventy-five hundredths to four and twenty-five hundredths, the ratio is also equivalent to three.

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Algebra I Teacher: We have a constant rate of change although the intervals between inputs are not uniform. Let's determine the ratio for the remaining pairs of values. (Writes on problem sheet) The change in *x* is one and seventy-five hundredths, and the change in *y* is five and twenty-five hundredths.

The ratio of the change in y-values to the change in x-values is five and twenty-five hundredths to one and seventy-five hundredths which simplifies to three.

For the last pair of paired values, the change in x is two and seventy-five hundredths, and the change in y is eight and twenty-five hundredths. This ratio also simplifies to three.

Right, so when a student graphs these points, they can relate that constant rate of change to the slope of the line that passes through each of these ordered pairs. A student could also plot just one of these points and use the slope to determine the other points on the line that may or may not be in the table. Let's plot our first three points from the table. (Pulls out graph and begins to plot points) Our scale is one unit for every grid line.

Eighth Grade Teacher: (Draws line through points on graph) Some students struggle with the idea that a line represents an infinite set of points, all of which satisfy a linear equation, and that a table represents only a few of these points. It appears that the line intersects the *y*-axis at zero one and twenty-five hundredths.

We know the ratio of change in *y* to change in *x* is three or three to one. We can see this when we move from (gestures to graph) zero one and twenty-five hundredths to one four and twenty-five hundredths.

Algebra I Teacher: Once students have determined the slope of the line, they have a few options to generate the equation of the line. If you'd like, we can look at other examples.

Eighth Grade Teacher: Yes, please.

Algebra I Teacher: (Removes graph and pulls out another problem sheet) If we calculate the rate of change from the table or the slope from the graphed line, the rate of change and the slope are negative two-thirds. Some students may recognize the ordered pair, zero three, as the *y*-intercept of the line. If so, they can substitute the *y*-coordinate into the equation, *y* equals *m x* plus *b*, (writes equation on blank paper) with negative two-thirds as *m* and positive three as *b*. If we substitute in an x of 4 into this equation, we would expect to have a y-value of one-third. Let's check. (Continues writing equations)

Eighth Grade Teacher: Eighth-grade students are introduced to the idea of writing an equation in the form *y* equals *m x* plus *b*, where *m* is the constant rate of change and *b* is the *y*-intercept when the points are graphed. They are introduced to the terms "slope" and "*y*-intercept" but don't use the phrase "slope-intercept form."

Students can identify and plot the *y*-intercept and then use the rate of change, or slope, to generate additional points that lie on the line.

Algebra I Teacher: When do students begin graphing points on the coordinate plane, and when do they begin to write equations from a table of values?

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Eighth Grade Teacher: (Pulls out graph paper with line) Students graph in quadrant one for the first time in the fifth grade plotting ordered pairs from an input-output table.

Algebra I Teacher: What about in sixth grade? Is that when they graph in the other quadrants?

Eighth Grade Teacher: Yes. Sixth grade students graph with rational number ordered pairs in all four quadrants. (Points to four quadrants)

Algebra I Teacher: Would we expect them to create an equation from a set of points?

Eighth Grade Teacher: Only in the simplest cases. (Writes on paper) Lines of the form *y* equals *m* x or *y* equals x plus *b.* Otherwise, they begin that in seventh grade where they work on fluency with rational number operations, and then eighth grade builds on this.

Algebra I Teacher: When do students transition from graphing points from coordinates to identifying the linear relationship represented by these sets of points? When do they begin to make a distinction between discrete and continuous data?

Eighth Grade Teacher: The difference between discrete and continuous data is not explicitly addressed in grades 5 through 8. We talk about why some contexts lead to only plotting points, while others lead to plotting points and then drawing the line that passes through them. Discrete and continuous are mentioned in Algebra 1 expectations for domain and range.

Algebra I Teacher: That makes sense. It's good to know where we need to make those connections. What about writing an equation from a table of data? When and how does that develop?

Eighth Grade Teacher: In fifth grade, students explore and develop an understanding of additive and multiplicative patterns. They use the equation and the graph to generate a table or a pattern. Then, we ask them to determine the difference between (gestures to paper) an additive and multiplicative pattern given a table or a graph.

Algebra I Teacher: So they get really close to writing the equation, but they don't do that until sixth grade?

Eighth Grade Teacher: Right. In the sixth grade, teachers help students with the additive or multiplicative relationships between the input values and the output values instead of just looking for the pattern of the output values.

Algebra I Teacher: What does that look like for students?

Eighth Grade Teacher: Here's an example from 2016 STAAR for Grade 6 Mathematics. (Teacher gestures to book and then graphic is shown)

Algebra I Teacher: . . . Hmm . . . so they needed to recognize that the same value was being added, or in this case subtracted, from the money at the beginning of each day.

Eighth Grade Teacher: Yes, students are expected to make sense of intervals that aren't as uniform or "neat" as you can see with this item. In sixth grade, students work with simple additive or multiplicative relationships. (Teacher places process column on top of open book) The teachers may use a process column to help students see the mathematical relationship between the input and output values.

Algebra I Teacher: Do you ever show the work in the process column?

Eighth Grade Teacher: Yes, that work starts in sixth grade too as students begin working with ratios and rates.

Algebra I Teacher: In sixth grade, the students start by looking at intervals of 1 for the *x*-values so that they can see the pattern. Over time, we start to look at other intervals and make the connection back to what is happening with each change between *x*-values and the related changes between the corresponding *y*-values. How is that idea expanded in seventh grade?

Eighth Grade Teacher: We continue to use a process table that has the *x*-values and the *y*-values filled in. Let's take this one for example. (Begins to write in the process column)

When *x* equals 0, *y* is six. A process has been applied to the *x*-value to produce the *y*-value. At this time, we do not know what process or rule has been applied.

When *x* equals two, *y* is fourteen. We can continue to transfer the values to the input-output table with a process column. We might have the students start by looking at how much the *y*-values are changing compared to the corresponding *x*-values.

Algebra I Teacher: So, you're really having them calculate the rate of change from the table, but not making any connection to the slope of that line that represents that relationship. Correct?

Eighth Grade Teacher: Yes. Rates and unit rates are a big emphasis for seventh grade. (Supporting Information document graphic is shown) Students are expected to calculate the unit rate from real-world contexts as well as from tables of values. The idea of slope is introduced in eighth grade as mentioned in the supporting information.

Algebra I Teacher: So how do you connect the additive part of a linear equation? When do you formally introduce the *y*-intercept?

Eighth Grade Teacher: In eighth grade, we formally introduce the term "*y*-intercept" by anchoring their understanding to the table of values and the relationship between the paired *x* and *y* values. If students determine from the table of values that 4 is added to each *y* every time we add 1 to each *x*, then they know from their work in grade 6 that they are multiplying the given *x* by 4. We have them write that in the process column. (Writes in process column)

We then ask if this is the only process that needs to take place. In any of our non-proportional relationships, the answer is no. We ask them what needs to be added to get from the multiplicative component to the final value for *y*.

Algebra I Teacher: So we want the kids to see that 4 times 2 is only 8, which means we are 6 short of 14. We want them to write in the plus 6 in the process column. (Teacher writes in process column) Correct?

Eighth Grade Teacher: Correct. Students then test the process with the other *x*-values to ensure that the process produces all the paired *y*-values in the table. In eighth grade, we connect the rate of change in the table to the slope of the graphed line. We also connect the *y*-intercept to the paired values when *x* equals zero.

Algebra I Teacher: It sounds like there are many similarities to what we do in Algebra 1.

Eighth Grade Teacher: Absolutely. Would you mind sharing with me the method you show your students for writing equations when given a table of values?

Algebra I Teacher: Sure. Let's take a look at this problem. (Pulls out problem)

Eighth Grade Teacher: Well, it certainly looks a lot like the other problem we did, but I notice that this table doesn't include the *y*-intercept value.

Algebra I Teacher: Exactly! (Problem is shown next to process column) This is where I could see the process column really helping. We leave out the *y*-intercept to help students understand that a table of values may be just a partial set of values for a linear relationship. When faced with a table that has irregular intervals and no *y*-intercept value, we first need to determine the rate of change. The rate of change then can be used to fill in some of the "in between" values to expand our table. (Writes in process column)

Eighth Grade Teacher: If I understand you correctly, (writes in process column) since the slope for the graph of this linear relationship is five halves, I can work backwards and subtract 5 from a *y*-value for every 2 that I subtract from an *x*-value.

In this case, I could subtract 2 from 2 and 5 from negative 3. This allows me to fill in some "missing information." If I needed to, I could work backwards or with smaller increments to determine the *y* value when the corresponding *x* value is zero.

Algebra I Teacher: Yes. We now have the paired values that represent the y-intercept. I know the slope is five halves (circles the 0, -8 on process column). Once the students identify the value of the slope and the *y*-intercept, they can substitute the values into *y* equals *m x* plus *b* to identify the equation of the line. In this case, the equation is y equals five halves times x minus eight (writes equation underneath process column).

Eighth Grade Teacher: Are there any other ways students can generate that equation from the table?

Algebra I Teacher: Yes, in Algebra 1, students learn the point-slope formula (begins writing formula on blank sheet of paper). This formula is *y* minus *y* one is equal to *m* times the quantity *x* minus *x* one.

Eighth Grade Teacher: (Writes on paper) If you were to divide both sides of the formula by x minus *x* one, you isolate *m*. This means that *m* is equal to the ratio of *y* minus *y* one to *x* minus *x* one. How are students able to use this formula instead?

Algebra I Teacher: Students still need to determine the slope of the line. We know that *m* is five halves. Instead of going through the process of expanding the table or working back to calculate the *y*-intercept value, we can just choose any of the points to substitute for *x* one and *y* one.

First, let me substitute the slope of five halves in for *m*. (Writes on paper) I am going to choose an ordered pair from the table. Two negative three looks like a reasonable one, because the values are integers. I am going to substitute two in for *x* one and negative three in for *y* one. Then we do some algebraic manipulation to put this into slope intercept form, and this is what we get.

Eighth Grade Teacher: This is the same equation we came up with using the earlier method. The methods used in middle school build to the more generalized forms used in Algebra 1.