Section 8: Confidence Intervals

The following maps the videos in this section to the Texas Essential Knowledge and Skills for Mathematics TAC §111.47(c).

8.01 Introduction to Confidence Intervals (this video requires advanced mathematical knowledge)

- Statistics (6)(A)
- Statistics (6)(B)

8.02 Confidence Interval for One Mean

- Statistics (1)(G)
- Statistics (6)(A)
- Statistics (6)(C)
- Statistics (6)(E)

8.03 Visualizing a Confidence Interval

- Statistics (6)(A)

8.04 Interpreting Confidence Intervals

- Statistics (1)(G)
- Statistics (6)(C)
- Statistics (6)(E)

8.05 Confidence Interval for One Proportion

- Statistics (1)(G)
- Statistics (6)(A)
- Statistics (6)(D)
- Statistics (6)(E)

8.06 Factors Affecting the Width of a Confidence Interval

- Statistics (6)(A)
- Statistics (6)(B)

8.07 Confidence Intervals in the Real World

- Statistics (6)(E)

Note: Unless stated otherwise, any sample data is fictitious and used solely for the purpose of instruction.

8.01

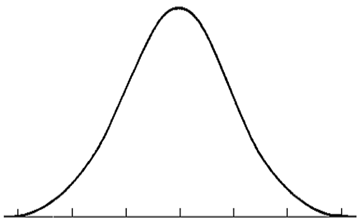
Introduction to Confidence Intervals

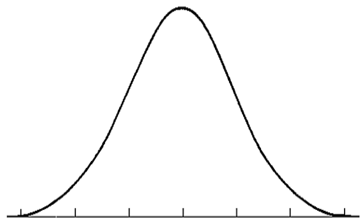
A ***confidence interval*** is a range (or interval) of possible values for an unknown population ______________________________.

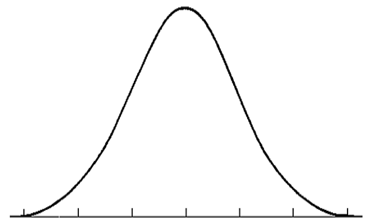
- General form of a confidence interval: *estimate ± ME,* where *ME = margin of error*
- *ME* = z × *Standard Error*
- *ME* only takes into account random sampling error.

Confidence Interval for μ	Confidence Interval for *p*
$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$ • ***The Independence Assumption:*** Your observations must be independent of each other. • ***Randomization Condition***: The sample should be obtained from randomization from the population. • ***Normal Population Assumption:*** We must assume that the sample is from a normally distributed population or $n \geq 30$.	$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where $\hat{p} = \frac{x}{n}$ • ***The Independence Assumption:*** Your observations must be independent of each other. • ***Randomization Condition:*** The sample should be obtained from randomization from the population. • ***Large Sample Condition****:* We need at least 15 successes and 15 failures in order for the sample to be large enough. $n\hat{p} \geq 15$ _____ $n(1-\hat{p}) \geq 15$

Common confidence levels and their z-critical values







$z = \pm 1.645$ $z = \pm 1.96$ $z = \pm 2.576$

Using the z-table:

z	…	.06
:		↑
–1.9	←	.0250
:		

Using the TI-83/84:

```
invNorm(.025
    -1.959963986
█
```

8.02

Confidence Interval for One Mean

Since its launch in 2012, the number of Instagram users has grown exponentially—particularly among teens. Most teens report using Instagram. Suppose a sample of 110 teens is taken, and the average number of followers per account was found to be 155. Assume the population standard deviation is 25 followers.

1. Are the required assumptions and conditions met to construct a 95% confidence interval for the true population mean number of followers per teenager, μ?

2. Construct a 95% confidence interval for the true mean number of followers per teen Instagram account.

3. Interpret the confidence interval.

8.03

Visualizing a Confidence Interval

$\overline{x} - ME$	$\overline{x}$	$\overline{x} + ME$
$155 - 4.67$	155	$155 + 4.67$
150.33	155	159.67
(	↓	)

-----|---------------|---------------|---------------|---------------|---------------|---------------|-----

150 152 154 156 158 160 162

1. Using the confidence interval shown above, work backwards to find the sample mean.

2. Using the confidence interval shown above, work backwards to find the margin of error.

8.04
Interpreting a Confidence Interval

There are two main interpretations:

1. Based on our sample, we are _______% confident that μ falls in our CI.

    Note: We are _______% confident that $\bar{x}$ falls in our CI.

2. If many confidence intervals were constructed, ______% of them would capture μ.

Determine whether each of the following statements is true or false. If the interpretation is false, identify the necessary change(s) to make it correct.

Suppose a researcher takes a random sample of 110 American teenage Instagram users and records the number of their followers. Using the sample data, a 95% confidence interval is found: 150.33, 159.67.

1. ____________ We are 95% confident that the mean number of followers for all American teenagers who use Instagram is between 150.33 and 159.67.

2. ____________ We are 95 % confident that a randomly selected Instagram user will have between 150.33 and 159.67 followers.

3. ____________ We can be 95% confident that the population mean (μ) is between 150.33 and 159.67.

4. ____________ We can be 95% confident that the sample mean ($\bar{x}$) is between 150.33 and 159.67.

5. ____________ If random samples of 110 are repeatedly constructed, in the long run 95% of the CIs formed will contain μ.

6. ____________ 95% of the Instagram users surveyed have between 150.33 and 159.67 followers.

7. ____________ 95% of all samples will have a mean number of followers between 150.33 and 159.67.

8. ____________ The probability the sample mean is between 150.33 and 159.67 is 0.95.

9. ____________ The probability the population mean is between 150.33 and 159.67 is 0.95.

8.05
Confidence Interval for One Proportion

Suppose that in a recent poll of 110 teenagers, 45 reported that they use Snapchat.

1. Are the required assumptions and conditions met to construct a 99% confidence interval for the true population proportion of teen Snapchat users, p?

2. Construct a 99% confidence interval for the true proportion of teenage Snapchat users.

3. Interpret the confidence interval.

8.06
Factors Affecting the Width of a Confidence Interval

Recall the general form of a confidence interval: *estimate* ± *ME*, where *ME* = z × *Standard Error*

When we change the standard deviation, confidence level, or sample size, the width of our confidence interval will change.

- ↑ confidence level → CI becomes _______________
- ↑ sample size → CI becomes _______________
- ↑ standard deviation → CI becomes _______________
- Ideally, we want a high degree of confidence level and a narrow confidence interval.
- The population size ______________ affects the width of the confidence interval.

1. Decreasing the confidence level while holding the sample size and standard deviation constant will cause the margin of error to

   A. increase.
   B. decrease.
   C. stay the same.
   D. It is impossible to tell.

2. Increasing the sample size while holding the confidence level and standard deviation constant will cause the width of the confidence interval to

   A. increase.
   B. decrease.
   C. stay the same.
   D. It is impossible to tell.

3. Increasing the sample size while decreasing the confidence level and holding the standard deviation constant will cause the width of the confidence interval to

   A. increase.
   B. decrease.
   C. stay the same.
   D. It is impossible to tell.

4. Decreasing the standard deviation while holding the confidence level and sample size constant will cause the width of the confidence interval to

   A. increase.
   B. decrease.
   C. stay the same.
   D. It is impossible to tell.

5. Which combination of sample size and confidence level will produce the narrowest confidence interval?

   A. 90% and $n = 1{,}000$
   B. 95% and $n = 100$
   C. 99% and $n = 100$
   D. 99% and $n = 500$

6. Which combination of sample size and confidence level will produce the widest confidence interval?

   A. 90% and $n = 1{,}000$
   B. 95% and $n = 100$
   C. 99% and $n = 100$
   D. 99% and $n = 500$

7. To cut our sampling error in half, what must we do to the sample size?

   A. Decrease it by half.
   B. Increase it by half.
   C. Increase it by two times.
   D. Increase it by four times.

8.07

Confidence Intervals in the Real World

STUDY EDGE TIP

> Be wary of confidence intervals reported in the news!

The following excerpt was taken from an article on Bloomberg.com titled “Sales of New U.S. Houses Unexpectedly Rise to Seven-Year High”:

> Purchases of new homes in the U.S. unexpectedly rose in February to a seven-year high as stronger job gains helped bolster industry activity amid severe weather. Sales climbed 7.8 percent to a 539,000 annualized pace, the most since February 2008, Commerce Department data showed Tuesday in Washington. The reading exceeded even the most optimistic forecast of economists surveyed by Bloomberg. . . . The report showed the confidence interval for last month’s reading was plus or minus 15.2 percent. That means there was a 90 percent chance the change in sales in February was between a decline of 7.4 percent and a 23 percent advance. Sales of New U.S. Homes Unexpectedly Rise to Seven-Year High. (2015, March 24). Retrieved from https://www.bloomberg.com/news/articles/2015-03-24/sales-of-new-u-s-homes-unexpectedly-climb-to-seven-year-high.

1. What problems do you notice with the report above?