Section 9: Hypothesis Testing

The following maps the videos in this section to the Texas Essential Knowledge and Skills for Mathematics TAC §111.47(c).

9.01 Steps of Hypothesis Testing

- Statistics (1)(G)
- Statistics (6)(F)
- Statistics (6)(H)

9.02 Hypothesis Test for One Mean – Part 1

- Statistics (6)(G)

9.03 Hypothesis Test for One Mean – Part 2

- Statistics (1)(G)
- Statistics (6)(F)
- Statistics (6)(H)

9.04 Statistical Significance vs. Practical Significance

- Statistics (1)(G)

9.05 Hypothesis Test for One Proportion – Part 1

- Statistics (1)(G)
- Statistics (6)(F)
- Statistics (6)(G)
- Statistics (6)(H)
- Statistics (6)(I)

9.06 Hypothesis Test for One Proportion – Part 2

- Statistics (6)(G)
- Statistics (6)(H)
- Statistics (6)(I)

9.07 Type I and Type II Errors

- Statistics (1)(A)
- Statistics (6)(J)

Note: Unless stated otherwise, any sample data is fictitious and used solely for the purpose of instruction.

9.01
Steps of Hypothesis Testing

A ***hypothesis test*** is used to test a claim about an unknown population parameter (μ or p).

Test statistic for testing μ	Test statistic for testing p
$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$ • ***The Independence Assumption:*** Your observations must be independent of each other. • ***Randomization Condition:*** The sample should be obtained from randomization from the population. • ***Normal Population Assumption:*** We must assume that the sample is from a normally distributed population or $n \geq 30$.	$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$ • ***The Independence Assumption:*** Your observations must be independent of each other. • ***Randomization Condition:*** The sample should be obtained from randomization from the population. • ***Large Sample Condition:*** We need at least 15 expected successes and 15 expected failures in order for the sample to be large enough. $np_0 \geq 15$ _____ $n(1 - p_0) \geq 15$

Step #1: Check required conditions and assumptions.

Step #2: State H_0 and H_a.

H_0 = the claim assumed true until the data indicates otherwise (=)

H_a = the claim the researcher hopes to show (<, > or ≠)

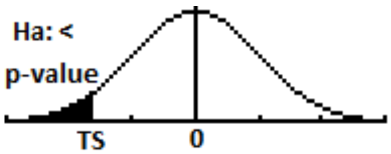
Never put $\bar{x}$ or $\hat{p}$ (or their values) in H_0 or H_a.

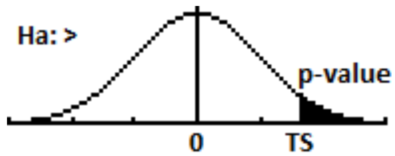
Step #3: Calculate the test statistic.

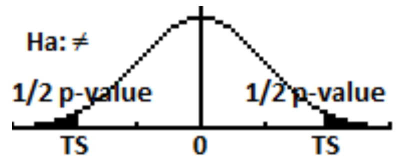
$$z = \frac{Estimate - H_0\ value\ for\ the\ paramater}{Standard\ Error}$$

The ***test statistic*** is the number of standard errors the estimate is away from the H_0 value.

Step #4: Determine the p-value: The ***p-value*** is the probability of obtaining a test statistic equal to or more extreme than the one observed *assuming H_0 is true*.







- $0 \leq$ p-value ≤ 1
- The p-value is the ___________________ associated with the test statistic in the direction of ________.

- The p-value is the smallest α-level at which we can _________________________.

Large TS $\lvert TS \rvert > \approx 2$	→	Small p-value $< .05$	→	Sample results are unusual / rare under H_0	→	Strong evidence against H_0	→	Reject H_0	→	Conclude H_a is true

- p-value > 0.10 → there is ________________________ evidence to reject H_0
- p-value is between 0.05 and 0.10 → there is _________________ evidence to reject H_0
- p-value is between 0.01 and 0.05 → there is __________________ evidence to reject H_0
- p-value < 0.01 → there is ________________________ evidence to reject H_0

Step #5: State the conclusion.

If the p-value $\leq \alpha$

- Statistically significant
- Reject H_0.
- Does not prove H_a is true! We could be rejecting a true H_0 (a Type I error).
- Based on the sample evidence, there is sufficient evidence to believe H_a is true.

If the p-value $> \alpha$

- Not statistically significant
- Fail to reject H_0 (do not accept H_0)
- Does not prove H_0 is true! We could be failing to reject a false H_0 (a Type II error).
- Based on the sample evidence, there insufficient evidence to believe H_a is true.

<u>9.02</u>
<u>Hypothesis Test for One Mean – Part 1</u>

Since its launch in 2012, the number of Instagram users has grown exponentially, particularly among teens. Most teens report using Instagram, with the typical user having 150 followers. Suppose that in a random sample of 110 teens, the average number of followers per account was found to be 155. Assume the population standard deviation is 25 followers.

1. Are the required assumptions and conditions met to perform a hypothesis test for the true population mean number of followers, μ?

2. State the null and alternative hypothesis for the following scenarios.

Do teens average fewer than 150 followers?	Do teens average more than 150 followers?	Do teens have an average other than 150 followers?

3. Calculate the test statistic.

9.03
Hypothesis Test for One Mean – Part 2

1. Determine the p-value for the following scenarios.

Do teens average fewer than 150 followers?	Do teens average more than 150 followers?	Do teens have an average other than 150 followers?

2. Suppose we wish to show that the average number of followers is greater than 150. What conclusion can be drawn using $\alpha = 0.05$?

3. What error could have been made in the previous question?

4. Suppose the p-value is 0.02. Label each of the following statements true or false.

   i. ______________ We fail to reject the null hypothesis.

   ii. ______________ On average, 98% of teens in the sample have more than 150 followers.

   iii. ______________ On average, 98% of teens in the population have more than 150 followers.

   iv. ______________ There is a 2% chance of randomly selecting a teen who has more than 150 followers.

   v. ______________ If the true average number of followers is 150, the probability of getting a test statistic as large or larger than the one we obtained is 0.02.

9.04
Statistical Significance vs. Practical Significance

Statistical Significance and Practical Significance

- An effect is said to be *statistically* significant if it _________________ happen by chance.
- An effect is said to be *practically* significant if the effect is meaningful (i.e., worth the added cost or risk).
- If an effect is significant, but not practical, you learned something but the effect is not important.
- If an effect is not significant, but practical, you have not proven it.
- If an effect is significant and practical, you performed an important study!

Suppose a test prep program advertises that its course will significantly improve your SAT score. If the course is ineffective (H_0), μ = the mean increase in SAT score = 0. If the prep course is effective (H_a), $\mu > 0$.

$$H_0: \mu = 0 \text{ vs. } H_a: \mu > 0$$

1,000 customers were sampled and their SAT increases were recorded. The applicable Minitab output is shown below.

```
Test of mu = 0 vs > 0

Variable     N     Mean   StDev   SE Mean      95.0% CI          T       P
Increase  1000      6.9    12.5     0.395   (6.12,   7.68)   17.5   0.000
```

1. Does the test prep course significantly improve SAT scores? Explain.

2. Suppose the SAT prep course costs $500. Is the effect practically significant?

STUDY EDGE TIP

Often with extremely large samples, we reject H_0 for small non-practical differences.

9.05
Hypothesis Test for One Proportion – Part 1

It is thought that the majority of teens do not use Snapchat. Suppose that in a recent poll of 110 teenagers, 45 reported that they use Snapchat.

1. Which of the following are assumptions or conditions required for a z-test for p to be valid? Check all that apply.

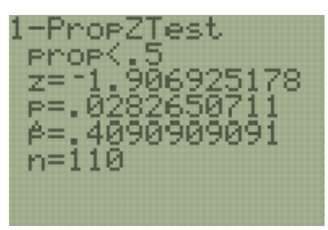
   - ☐ Data are obtained by randomization.
   - ☐ The observations are independent of one another.
   - ☐ If n is small, the population distribution is approximately normal.
   - ☐ The numbers of successes and failures in the sample are at least 15.
   - ☐ At least 15 successes and 15 failures are expected.

2. Are the required assumptions and conditions met?

3. State the null and alternative hypothesis.

4. Calculate the test statistic.

5. Determine the p-value using the calculator output below.



6. Describe in words what the p-value represents.

7. Using a significance level of 0.05, what conclusion can be made?

8. What error could we have made in the previous question?

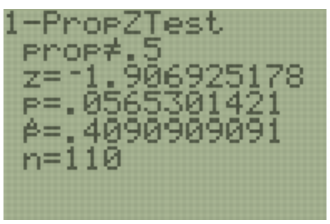
9.06
Hypothesis Test for One Proportion – Part 2

It is thought that the majority of teens do not use Snapchat. Suppose that in a recent poll of 110 teenagers, 45 reported that they use Snapchat.

Suppose that instead of performing a one-tail hypothesis test, we performed a two-tail hypothesis test.

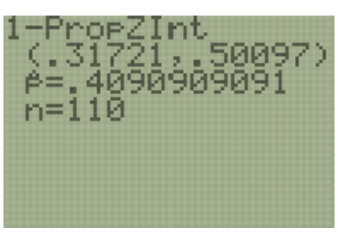
1. State the null and alternative hypothesis.

2. Identify the test statistic and the p-value using the calculator output below.



3. What conclusion can be made using a significance level of 0.05?

4. Refer to the 95% confidence interval for p shown below. What conclusion can be drawn based on the confidence interval?



9.07
Type I and Type II Errors

The Truth

Results from Hypothesis Test

	H_o True	H_o False
Reject H_o		
Fail to Reject H_o		

Type I Errors

- A Type I error occurs when we reject the null hypothesis, but it is actually ______________.
- The probability of making a Type I error = the significance level = α.

Type II Errors

- A Type II error occurs when we fail to reject the null hypothesis, but it is actually ___________.
- The probability of making a Type II error = Probability of failing to reject a false null hypothesis = β.

STUDY EDGE TIP

Which type of mistake is worse depends on the situation!

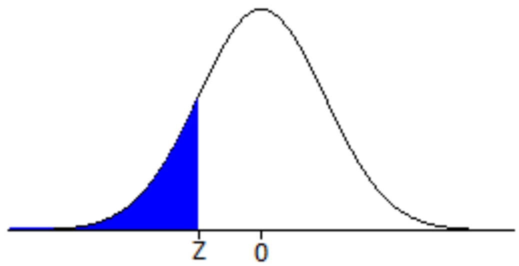
A test prep program claims that its product improves SAT scores by 100 points. Researchers conduct a survey to determine whether the increase is lower than claimed.

1. Explain the impact of a Type I error in this context.

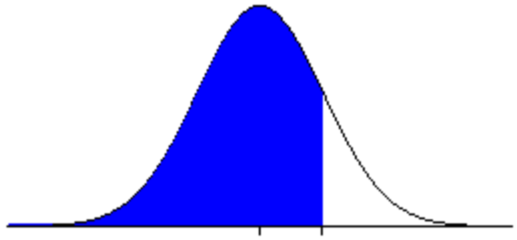
2. Explain the impact of a Type II error in this context.

3. Which error is more serious?

Z table



	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998