Section 5: Measuring Center and Spread

The following maps the videos in this section to the Texas Essential Knowledge and Skills for Mathematics TAC §111.47(c).

5.01 Measuring Center of a Distribution
- Statistics (1)(C)
- Statistics (4)(C)
- Statistics (4)(E)

5.02 Measuring Spread of a Distribution
- Statistics (1)(C)
- Statistics (4)(C)
- Statistics (4)(E)

5.03 Outliers
- Statistics (4)(C)

5.04 Visualizing and Transforming Data: Implications for Mean and Standard Deviation
- Statistics (1)(C)
- Statistics (1)(F)

5.05 Introduction to the Empirical Rule
- Statistics (1)(F)
- Statistics (4)(E)

5.06 The Normal Distribution
- Statistics (1)(F)
- Statistics (4)(E)

Note: Unless stated otherwise, any sample data is fictitious and used solely for the purpose of instruction.

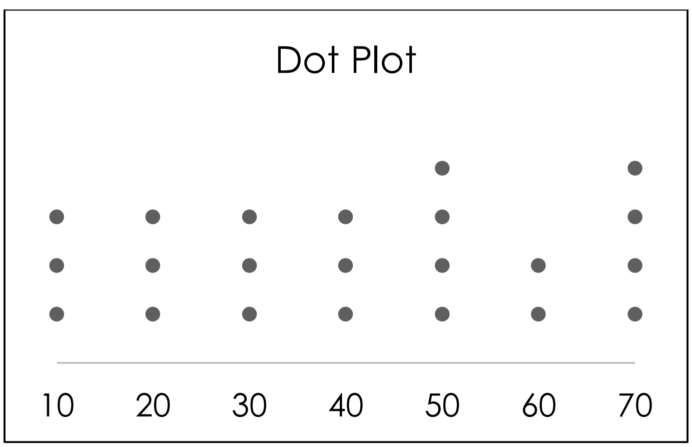
5.01

Measuring Center of a Distribution

Three common measurements of center, or central tendency, are ***mean***, ***median***, and ***mode***.

	Definition
Mean	
Median	
Mode	

1. Calculate the mean, median, and mode for the data in the following dot plot.

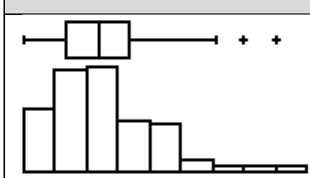
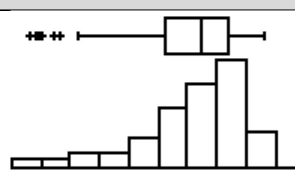
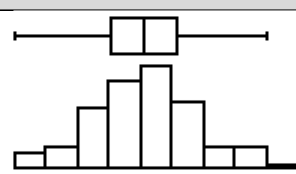
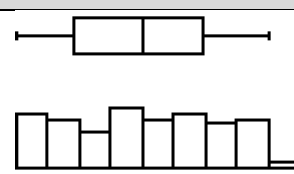


2. Calculate the mean and median for each data set. What do you notice about the relationship between the mean and median in each case?

   i. 1 2 3 3 4 5

   ii. 1 2 3 4 70 100

Based on the shape of the distribution, the mean and median are related as follows:

Right-skewed	**Left-skewed**	**Bell-shaped**	**Uniform**
 Mean Median	 Mean Median	 Mean Median	 Mean Median

5.02

Measuring Spread of a Distribution

Four common measures of spread are ***range***, ***variance***, ***standard deviation***, and ***interquartile range (IQR)***. Complete the following table for the measures of spread.

	Definition	Formula
Range	The difference between the maximum and minimum values (Note: Range is the measure of spread most affected by outliers.)	
Variance		$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$
Standard deviation	The "typical" deviation from the mean	
Interquartile range (IQR)		$IQR = Q_3 - Q_1$

1. The starting salaries (in thousands of dollars) of 9 recent college graduates are shown below, sorted in ascending order.

   29 36 40 42 42 45 46 47 96

   i. Calculate the mean.

   ii. Find the median.

   iii. Find the mode.

   iv. Calculate the range.

   v. Calculate the interquartile range.

   vi. Calculate the standard deviation.

2. Below is a stemplot of the time it took a classroom of 25 students to complete their first statistics exam to the nearest minute.

Time (to the nearest minute) to complete Exam 1

0	
1	8 9 9
2	0 0 1 1 2 2 3 3 3 4 5
3	0 1 3 5 5 7 9 9
4	1 5 5

(1 | 8 represents 18)

i. Calculate the mean.

ii. Find the median.

iii. Find the mode.

iv. Calculate the range.

v. Calculate the interquartile range.

vi. Calculate the variance.

vii. Calculate the standard deviation.

5.03

Outliers

An ***outlier*** is an extreme observation with a value far above or below the rest of the data.

1. Calculate the mean and median for each data set:

   Data Set 1: 1 2 3 3 4 5

   Data Set 2: 1 2 3 3 71 1,000

2. In data set 2, the values 71 and 1,000 are outliers. How do they affect the mean and median?

3. Which measure is more sensitive to outliers: the mean or median? Explain your reasoning.

How to Check for Outliers

Using Q_1 and Q_3 from the five-number summary, observations that fall below $Q_1 - 1.5 \times IQR$ or above $Q_3 + 1.5 \times IQR$ are considered outliers.

4. The delay in departure for each of JetBlue's first 15 flights in the year 2017 from the Orlando International Airport are shown below, where a negative value means a flight departed ahead of schedule and a positive value represents a delay in departure (Bureau of Transportation Statistics, n.d.).

   $-14 \quad -7 \quad -7 \quad -7 \quad -6 \quad -5 \quad -5 \quad -4 \quad -3 \quad 0 \quad 7 \quad 13 \quad 18 \quad 30 \quad 45$

   Use the outlier check to identify any outliers in the data set.

5.04

Visualizing and Transforming Data: Implications for Mean and Standard Deviation

Consider the numbers 5, 5, 5, and 5. What are the mean and standard deviation?

Consider the numbers 3.5, 4.5, 5.5, and 6.5. What are the mean and standard deviation?

Consider the numbers 0, 0, 10, and 10. What are the mean and standard deviation?

The ***mean*** measures where the *center* of the data values is. The ***standard deviation*** measures how *spread out* the values are *around* the mean.

1. Draw conclusions from the data sets above.

2. Consider the numbers 2, 5, and 8. Determine the mean and standard deviation of the original data set and the transformations of the data set (as shown in the table below).

Original data	Add 4	Multiply by 2
2, 5, and 8 Mean: Standard Deviation:	6, 9, and 12 Mean: Standard Deviation:	4, 10, and 16 Mean: Standard Deviation:

3. How does transforming a data set with addition and subtraction affect the mean and standard deviation?

4. How does transforming a data set with multiplication and division affect the mean and standard deviation?

5. Suppose the starting salaries (in thousands of dollars) of 12 recent college graduates are recorded, and the summary statistics are shown to the right. What would the new mean and standard deviation be if everyone were to receive a $2,000 raise?

1-Var Stats
$\bar{x}=47.58333333$
$\Sigma x=571$
$\Sigma x^2=28921$
$Sx=12.61642748$
$\sigma x=12.07930967$
$n=12$
$minX=36$
$Q_1=39.5$
$Med=43.5$
$Q_3=49.5$
$maxX=77$

6. Consider all integers from 25 to 75.

   i. Which four values have the smallest standard deviation? Explain.

   ii. Which four values have the largest standard deviation? Explain.

5.05
The Empirical Rule and an Introduction to the Normal Distribution

How can we compare the heights of a man and a woman to see who is taller, *relative* to their gender?

We often need to standardize the values of a population distribution to compare values within a distribution or between distributions.

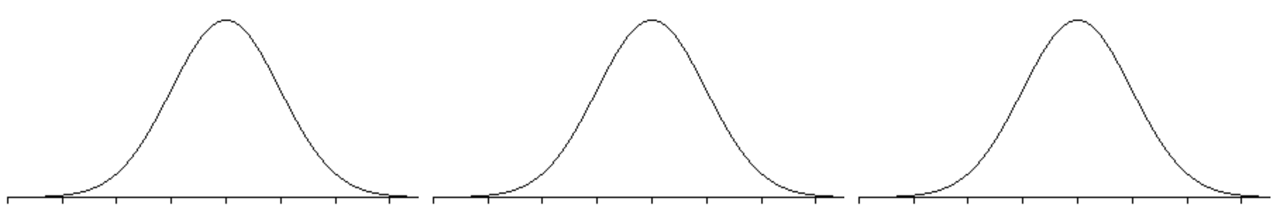
A ***normal distribution*** is a continuous distribution that is unimodal and symmetric, or approximately symmetric, with a bell shape. With this distribution, the standard deviation can be used for a more precise measurement of the percentage of data in the distribution.

A normal distribution is notated as $N(\mu, \sigma)$, where the population mean is μ and the population standard deviation is σ.

The ***empirical rule*****,** also known as the 68–95–99.7 rule, applies to the normal distribution. It states that

- 68% of observations fall within one standard deviation of the mean,
- 95% of observations fall within two standard deviations of the mean, and
- 99.7% of observations fall within three standard deviations of the mean.

Visualizing the Empirical Rule



$\mu \pm 1\sigma \ \rightarrow \cong 68\%$ $\qquad$ $\mu \pm 2\sigma \ \rightarrow \cong 95\%$ $\qquad$ $\mu \pm 3\sigma \ \rightarrow \cong 99.7\%$

1. Suppose the amount of soda in a 12-oz can of soda has a normal distribution with a mean of 12.25 oz and a standard deviation of 0.15 oz.

   i. The middle 95% of weights fall between what two values?

   ii. What percentage of cans contain between 11.95 and 12.40 oz?

   iii. What percentage of cans contain more than 12.40 oz?

   iv. What percentage of cans contain exactly 12.10 oz?

2. Suppose the lifetime of a certain brand of tire is normally distributed with a mean of 60,000 miles and a standard deviation of 5,000 miles.

    i. The central 99.7% of lifetimes are between what two mileages?

    ii. Approximately what percentage of tires last beyond 55,000 miles?

    iii. The manufacturer does not want to provide a warranty for tires once they reach the top 2.5% of their lifetime. What is the mileage limit of the warranty?

5.06

The Normal Distribution

What percentage of values are within 1.5 standard deviations from the mean? When we cannot apply the 68–95–99.7 rule, we use a ***z-table*** of values.

A ***z-table*** displays cumulative probabilities for the standard normal distribution, $z \sim N(0,1)$.

When we use normal distributions with other means and standard deviations, a ***z-score*** can determine the number of standard deviations an observation is from its mean. We can then find the corresponding cumulative probability for that observation using the z-table.

$$z = \frac{x - \mu}{\sigma}$$

1. Use the z-table to find the following values.

   i. The area below a z-score of 1.5

   ii. The probability of obtaining a z-score greater than 2.13

iii. The probability of obtaining a z-score between -1.11 and 0.57

iv. The z-score that cuts off the top 30%

v. The z-scores that capture the middle 60%

2. Suppose the average height of a male high school senior follows a normal distribution with a mean of 67 inches and a standard deviation of 3 inches.

   i. Find the probability that a randomly selected male student is less than 65 inches tall.

   ii. Find the probability that a randomly selected male student is at least 68 inches tall.

   iii. Find the probability that a randomly selected male student is exactly 67 inches tall.

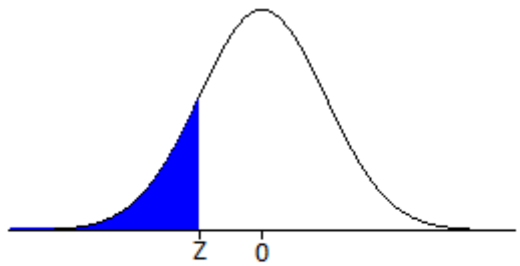
   iv. What height marks the start of the tallest 10% of males?

3. Based on the data in the previous question, if a male high school senior has a z-score of -1.55, what conclusion can be drawn?

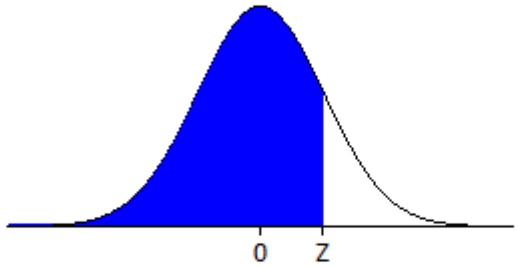
   A. He is above the average height.
   B. He is below the average height.
   C. He is the average height.
   D. Cannot be determined.

4. Suppose the height of men in North America follows a normal distribution with a mean of 68 inches and standard deviation of 4 inches, while the height of women in North America follows a normal distribution with a mean of 63 inches and standard deviation of 3 inches. Who is taller for his or her sex: a man 73 inches tall or a woman 68 inches tall?

Z table



	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

References

Bureau of Transportation Statistics. (n.d.). *Airline On-Time Statistics: Detailed Statistics Departures*. Retrieved from https://www.transtats.bts.gov/ONTIME/Departures.aspx