Activity 3-1 – Explore a CTE Lesson Plan

Note: This lesson plan is for educational purposes only and is intended to be used in the context of an exercise as part of a TEA professional development course for CTE teachers.

Math-in-CTE Lesson Plan

Lesson Title:	**Compound Interest – A Millionaire's Best Friend**	*Lesson 02*
Unit:	Investing	
Occupational Area:	Business education	
CTE Concept(s):	Compound interest, types of investments, opportunity cost, diversification	
TEKS	Functions,C.1.c, Connections, B.2.c	
Lesson Objective:	The effects of compound interest over time by comparing various investments, calculating compound interest using various rates of return	
Supplies Needed:	Mini whiteboards, dry erase markers, student calculators with exponent button, accompanying worksheet, color pencils/markers, 45 - 60 min	

Outline of Lesson	**TEACHER NOTES (and answer key)**
Introduce this lesson by working through this traditional math problem: 1. Exponential function problem and graph If $y = a(1 + r)^t$, and *y* represents the total value after a principal amount, *a* is compounded for *t* years at an annual rate, *r* expresses as a decimal, what is the value of *y* when *a* = 500, *r* = 3.5% and *t* =7? Be sure to graph this equation on the whiteboard.	**Say:** *You may have learned about "exponential functions" and "exponential growth" in your math class. Today we're going to talk about one example of exponential growth—compound interest.* **Write** the following problem on the board: $\mathbf{y = a(1 + r)^t}$ "r" and "t" must be in the same units; i.e., if "r" is the annual growth rate then "t" must be the number of years OR if "r" is the monthly growth rate then "t" must be the number of months. 1. $y = a(1 + r)^t$ $= 500\ (1 + 0.035)^7$ $= 500\ (1.035)^7$ $= 636.14$ **Say:** *Compound interest is only one example of "exponential growth;" another is bacterial*

	growth: how quickly bacteria grow. **Write** the formula again, replacing the plus sign (+) with a minus sign (-). $\mathbf{y = a(1 - r)^t}$ **Say:** *By taking the formula and replacing the "+" sign with a "-" sign, we have created an example of exponential decay, which is applicable to half-lives of elements, such as how long it takes for uranium or asbestos to become depleted.*
Assess Students' Prior Knowledge of Concepts.	**Say:** *Let's see if you can work these problems.*
1. Jason invests $500 in a savings account at a rate of 1.04% for one year. How much is in the account at the end of the year?	1. I = Prt I = 500 • 0.0104 = 5.20 The total amount in the account will be $505.20.
2. Evaluate 3x + 5t, when x = 4 and t = 7	2. 3 • 4 + 5 • 7 = 12 + 35 = 47
3. Convert 3.6% into a decimal.	3. To convert from percent to decimal, we must divide by 100, which simply moves the decimal point two places to the left. (3.6% = 0.036)
Teacher Demonstration 1. Savings account problem & graph Alicia Martin's savings account principal is $1000. The 2% interest is compounded annually. How much is in the account at the end of the year? At the end of 3 years? 5 years? 10 years? 20 years? Represent your answers via the nearest penny and visually via a line graph.	**Note:** There are other ways of calculating compound interest, such as using an Excel spreadsheet or an online investment calculator, but this method gives students a hands-on/"see how it works" approach. **Hand out** the worksheet *bus_bus_charts_01 Element 3* to the students and demonstrate the math problem on the board. a. Start by solving 1 year and 3 years in the same manner as a repeated simple interest problem. (I = Prt) *Step 1* – Figure year 1 interest I = 1000 • 0.02 • 1 = 20

	Total value after 1 year is $1020 Add interest to principal for year 2 calculations *Step 2 – Figure year 2 interest* I = 1020 • 0.02 • 1 = 20.40 Add interest to principal for year 3 calculations *Step 3 – Figure year 3 interest* I = 1040 • 0.02 • 1 = 20.81 *Step 4 – Figure total interest after 3 years* I = 20 + 20.40 + 20.81 = $61.21 Total value after 3 years is $1061.21 **Say:** *This process is time-consuming and inefficient for anything more than a few compounding periods. There is a better, more efficient way called the "compound interest formula."* b. Progress into modeling the compound interest formula for 5 years, 10 years, and 20 years. $FV = PV (1 + r/m)^{mt}$, where: • *FV* is the future value, • *PV* is the present value (the principal you start with), • *R* is the annual rate of interest as a decimal, • *m* is the number of times per year the interest is compounded (monthly, annually, etc.), and *t* is the number of years you leave it invested. *Step 5* – Calculate total value of investment after 5 years. $FV = 1000 \cdot (1 + 0.02 / 1)^{1 \cdot 5}$ = 1104.08 The total value of $1000 invested for 5 years at 2% compounded annually is $1104.08

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Step 6 – Calculate total value of investment after 10 years.
FV = 1000 • (1 + 0.02 / 1) $^{1 \cdot 10}$
= 1218.99
The total value of $1000 invested for 10 years at 2% compounded annually is $1218.99
Step 7 – Calculate total value of investment after 20 years.
FV = 1000 • (1 + 0.02 / 1) $^{1 \cdot 20}$
= 1485.95
The total value of $1000 invested for 20 years at 2% compounded annually is $1485.95
Step 8 – Graph the results on the provided worksheet. Be sure to connect the data points.
FV = 1000 • (1 + 0.02 / 1) $^{1 \cdot 20}$
= 1485.95
The total value of $1000 invested for 20 years at 2% compounded annually is $1485.95
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Guided Practice
1. Mutual fund problem & graph
Alex Smith’s Growth Stock Mutual fund principal is $1000. The annual rate of 10.5% interest is compounded quarterly. How much is in the account at the end of the year? At the end of 3 years? 5 years? 10 years? 20 years? Represent your answers via the nearest penny and visually via a line graph.
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Say: Let’s do one more together.
Hand out the worksheet bus_bus_charts_01 Element 4 to the students and guide them through the math problem.
Say: This time we will use the compound interest formula for 1 year, 3 years, 5 years, 10 years, and 20 years:
FV = PV (1 + r/m) mt, where:

FV is the future value,
PV is the present value (the principal you start with),
r is the annual rate of interest as a decimal,
m is the number of times per year the interest is compounded (monthly, annually, etc.), and
t is the number of years you leave it invested.

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	Step 1 – Calculate the total value of the investment after 1 year. **Say:** *Remember we are compounding quarterly or 4 times per year not just one!* FV = 1000 • (1 + 0.105 / **4**) $^{4 \cdot 1}$ = 1109.21 The total value of $1000 invested for 1 year at 10.5 % compounded quarterly is $1109.21. *Step 2* – Calculate total value of investment after 3 years. FV = 1000 • (1 + 0.105 / **4**) $^{4 \cdot 3}$ = 1364.70 The total value of $1000 invested for 3 years at 10.5 % compounded quarterly is $1364.70. *Step 3 – Calculate total value of investment after 5 years.* FV = 1000 • (1 + 0.105 / **4**) $^{4 \cdot 5}$ = 1679.05 The total value of $1000 invested for 5 years at 10.5% compounded quarterly is $1679.05. *Step 4* – Calculate total value of investment after 10 years. FV = 1000 • (1 + 0.105 / **4**) $^{4 \cdot 10}$ = 2819.21 The total value of $1000 invested for 10 years at 10.5 % compounded quarterly is $2819.21. *Step 5* – Calculate total value of investment after 20 years. FV = 1000 • (1 + 0.105 / **4**) $^{4 \cdot 20}$ = 7947.92 The total value of $1000 invested for 20 years at 10.5 %

	compounded quarterly is $7947.92. *Step 6* – Ask for a volunteer to graph the results on the whiteboard. Be sure to connect the data points.
Assessment Questions Which investment will be worth the most at the date of maturity? 1. $14,000 @ 2% for 3 years compounded quarterly 2. $5,000 @ 7% for 15 years compounded annually 3. $10,500 @ 6% for 6 years compounded monthly	**Answers:** 1. $14,863.49 2. $13,795.16 3. $15,036.46 → Correct answer