OER Secondary Mathematics

EDITION 1

Algebra I

Family Guides

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

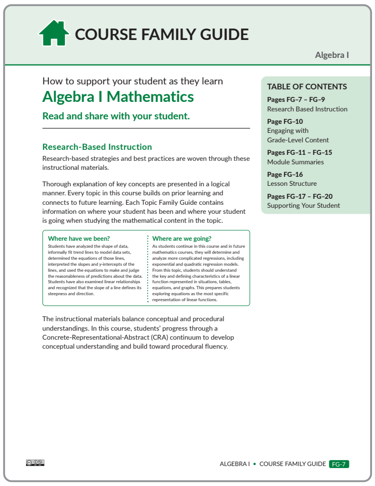
If you have further product questions or to report an error, please email openeducationresources@tea.texas.gov.

Dear Family,

We recognize that learning outside of the classroom is crucial to your student's success at school. This letter serves as an introduction to the resources designed to assist you as you talk to your student about what they are learning. Resources available to you include:

- Course Family Guide
- Topic Family Guides
- Topic Summaries
- Math Glossary

Course Family Guide



COURSE FAMILY GUIDE

Algebra I

How to support your student as they learn

Algebra I Mathematics

Read and share with your student.

Research-Based Instruction

Research-based strategies and best practices are woven through these instructional materials.

Thorough explanation of key concepts are presented in a logical manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide contains information on where your student has been and where your student is going when studying the mathematical content in the topic.

Where have we been?

Students have analyzed the shape of data, informally fit trend lines to model data sets, determined the equations of those lines, interpreted the slopes and y-intercepts of the lines, and used the equations to make and judge the reasonableness of predictions about the data. Students have also examined linear relationships and recognized that the slope of a line defines its steepness and direction.

Where are we going?

As students continue in this course and in future mathematics courses, they will determine and analyze more complicated regressions, including exponential and quadratic regression models. From this topic, students should understand the key and defining characteristics of a linear function represented in situations, tables, equations, and graphs. This prepares students exploring equations as the most specific representation of linear functions.

The instructional materials balance conceptual and procedural understandings. In this course, students' progress through a Concrete-Representational-Abstract (CRA) continuum to develop conceptual understanding and build toward procedural fluency.

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ALGEBRA I • COURSE FAMILY GUIDE FG-7

Following this letter, there is the Course Family Guide that will walk you through the research-based instructional approach, how the course is structured, how to bust math myths, using Talking Points from the Topic Family Guide, and using the TEKS mathematical process standards to initiate discussions.

Research and classroom experience guided course development, with the foundation being a scientific understanding of how people learn and a real-world understanding of how to apply that science to mathematics instructional materials. The instructional design elements presented in the Course Family Guide incorporate research-based strategies to develop conceptual understanding and creative problem solvers.

The Course Family Guide provides an overview of the structure of the course. The course consists of both a Learning Together component and a Learning Individually component. The teacher facilitates a collaborative learning experience during the Learning Together Days and uses data to target specific skills on the Learning Individually Days.

Next, the Course Family Guide includes Module Overviews of each module in the course, which include a detailed summary of what your student will be learning in each topic within the module. Below the topic summaries are facts and information that connect the concepts to the real world. Read and discuss the information below the topic summaries with your student and continue to come back to these pages as your student moves from one topic to the next within each module.

The Course Family Guide also highlights the lesson structure. Each lesson is structured the same way and includes four parts: Objectives & Essential Question, Getting Started, Activities, and the Talk the Talk.

Topic Family Guide

Each course is organized into modules. Each module consists of topics with corresponding Topic Family Guides. These guides all have the same structure. This consistency will allow you and your student to understand how to reference the content of each topic.

The Topic Family Guide begins with an overview of the content in the topic. This introduction includes a brief explanation of what your student will learn in the topic, the prior knowledge they will use to help them understand this topic, and a connection to future learning.

The next section of the Topic Family Guide is the Talking Points section. The Talking Points section provides skills you can discuss with your students and a sample question based on the math in the topic that you can talk through with them.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Functions are an important topic to know for making predictions in the sciences, creating computer programs, and college admissions tests.

HERE IS A SAMPLE QUESTION

For the function $f(x) = 2x^2 - 3x$, what is the value of $f(-5)$?

To solve this, students need to know that the input -5 is substituted for x in the equation:

$$\begin{aligned} f(-5) &= 2(-5)^2 - 3(-5) \\ &= 2(25) + 15 \\ &= 50 + 15 \\ &= 65 \end{aligned}$$

The point $(-5, 65)$ is on the graph of the function.

Next, the Topic Family Guide lists all the new key terms of the topic and details some of the math strategies students will learn in the topic. Finally, each Topic Family Guide contains a Math Myth. Busting these Math Myths helps to build confidence and explain how math is accessible to everyone.

MYTH

"I don't have the math gene."

Let's be clear about something. There isn't ***a*** gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to it.

Some believe mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without formal instruction. They can learn number sense and pattern recognition the same way.

To further nurture your student's mathematical growth, attend to the learning environment. You can support this by discussing math in the real world, offering encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and providing space for plenty of practice.

#mathmythbusted

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all new key terms of the topic and provides a summary of each lesson. Each lesson summary defines new key terms and reviews key concepts, strategies, and/or Worked Examples. The Topic Summary provides an opportunity for you and your student to discuss the key concepts from each lesson, review the examples, and do the math together.

LESSON 1

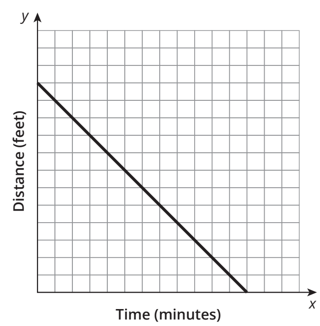
Understanding Quantities and Their Relationships

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the **dependent quantity**. The quantity that the dependent quantity depends upon is called the **independent quantity**.

Graphs relay information about data in a visual way. Connecting points on a coordinate plane with a line or smooth curve is a way to model or represent relationships. The independent quantity is graphed on the horizontal, or *x*-axis, while the dependent quantity is graphed on the vertical, or *y*-axis. Graphs can be straight lines or curves and can increase or decrease from left to right.

For example, consider the graph that models the situation where Pedro is walking home from school.

Time, in minutes, is the independent quantity and the distance Pedro is from home is the dependent quantity.

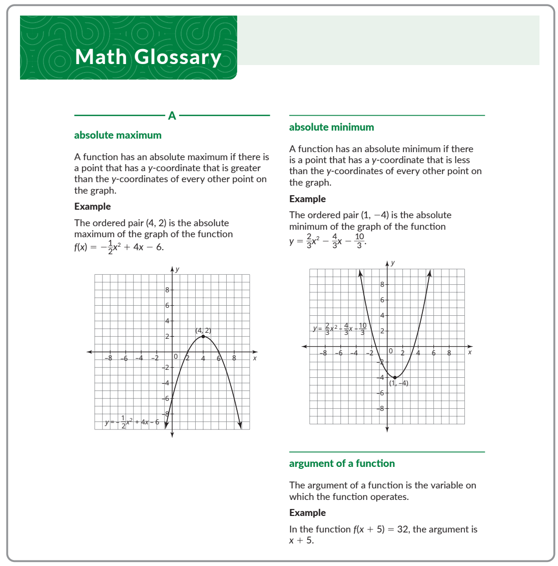


Topic Summary

Evidence of the TEKS mathematical process standards are present in the Topic Summaries. Each lesson within the topic highlights one or more of the TEKS mathematical process standards. These processes will help your student develop effective communication and collaboration skills that are essential for becoming a successful learner. Discuss with your student the “I can” statements associated with each of the TEKS mathematical process standards to help them develop their mathematical learning and understanding. The "I can" statements for each of the TEKS mathematical process standards are included in the Course Family Guide. With your help, your student can develop the habits of a productive mathematical thinker.

Math Glossary

The Math Glossary for each course is a tool for your student to utilize and reference during their learning. Along with the definition of a term, the glossary provides examples to help further their understanding.



Math Glossary

A

absolute maximum

A function has an absolute maximum if there is a point that has a y-coordinate that is greater than the y-coordinates of every other point on the graph.

Example

The ordered pair (4, 2) is the absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$.

absolute minimum

A function has an absolute minimum if there is a point that has a y-coordinate that is less than the y-coordinates of every other point on the graph.

Example

The ordered pair (1, −4) is the absolute minimum of the graph of the function $y = \frac{2}{3}x^2 - \frac{4}{3}x - \frac{10}{3}$.

argument of a function

The argument of a function is the variable on which the function operates.

Example

In the function $f(x + 5) = 32$, the argument is $x + 5$.

We all have the same goal for your student: to become a successful problem solver and use mathematics efficiently and effectively in daily life. Encourage them to use the mathematics they already know when seeing new concepts and communicate their thinking while providing a critical ear to the thinking of others.

Thank you for supporting your student.

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How to support your student as they learn

Algebra I Mathematics

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Research-Based Instruction

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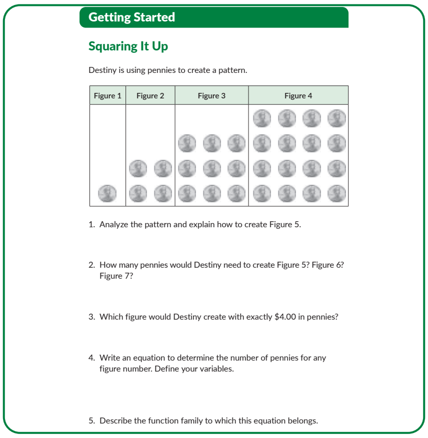
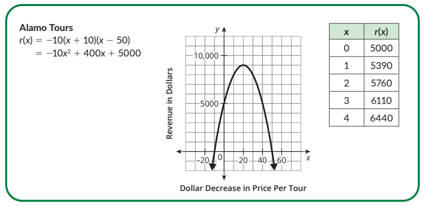
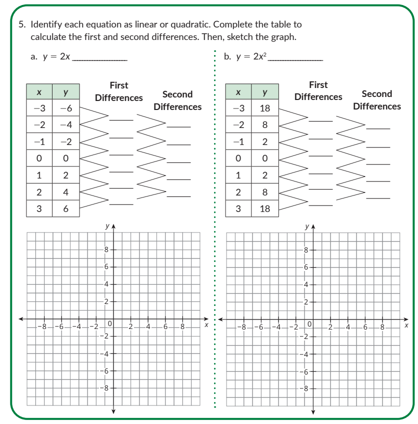
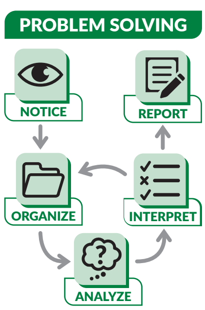
Concrete	Representational	Abstract
Students explore a real-world scenario to develop their intuitive understanding of quadratic functions. They recognize they can use the basic quadratic function, x^2, to represent the pattern.	Students learn that different forms of quadratic functions reveal specific key characteristics of the graph.	Students formalize their ability to recognize quadratic relationships represented by tables. They determine that a table represents a quadratic function when the second differences are constant.
		



Support is provided to students as they persevere in problem solving. These instructional materials include a problem-solving model, which includes questions your student can ask when productively engaging in real-world and mathematical problems. Prompts will encourage your student to use the problem-solving model throughout the course.

These instructional materials include features that support learners. Worked Examples throughout the product provide explicit instruction and provide a model your student can continually reference.

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connection between steps.

WORKED EXAMPLE

Consider this system of equations: $\begin{cases} 7x + 2y = 24 \\ 4x + y = 15 \end{cases}$.

$$7x + 2y = 24$$
$$-2(4x + y) = -2(15)$$

Multiply the second equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$\begin{aligned} 7x + 2y &= 24 \\ + \quad -8x - 2y &= -30 \\ \hline -x &= -6 \\ x &= 6 \end{aligned}$$

Now that the y-values are additive inverses, you can solve this linear system for x.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ 2y &= -18 \\ y &= -9 \end{aligned}$$

Substitute the value for x into one of the equations to determine the value for y.

The solution to the system of linear equations is $(6, -9)$.

Thumbs Up, Thumbs Down, and Who's Correct questions address your student's common misconceptions and provide opportunities for peer work analysis.

Thumbs Up Thumbs Down

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connection between steps.

Ask Yourself

- Why is this method correct?
- Have I used this method before?

Isabella

$4(3x + 2) = 8x + 4$
$12x + 8 = 8x + 4$
$4x = -4$
$x = -1$

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself

- Where is the error?
- Why is it an error?
- How can I correct it?

Ethan

$C = \frac{5}{9}(F - 32)$
$C = \frac{5}{9}F - \frac{160}{9}$
$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$
$9C = 45F - 1440$
$9C + 1440 = 45F$
$\frac{9C}{45} + \frac{1440}{45} = \frac{45F}{45}$
$\frac{1}{5}C + 32 = F$

Who's Correct

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or incorrect.

Ask Yourself

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

9. Harper and Diego each draw a line of best fit to model a set of data. They both record the vertical distances between each point and the line of best fit.

Harper
Vertical Distances: 2, 2, 2, 2, 2

Diego
Vertical Distances: 1, 1, 1, 1, 6

Both students believe they drew the least square regression line. Who's correct? Justify your choice.

Targeted Skills Practice provides support to your student as they work to gain proficiency of the course material. Spaced Practice provides a spaced retrieval of key concepts to your student. Extension opportunities provide challenges to accelerate your student's learning.

Skills Practice TOPIC 1 Quantities and Relationships

Name ______________________ Date __________

I. Understanding Quantities and Their Relationships

Topic Practice

A. Determine the independent and dependent quantities in each scenario. Be sure to include the appropriate units of measure for each quantity.

1. Selena is driving to visit her grandmother who lives 325 miles away from Selena's home. She travels an average of 60 miles per hour.
2. Benjamin works at a printing company. He is making T-shirts for a high school volleyball team. The press he runs can print 3 T-shirts per minute with the school's mascot.
3. On her way to work each morning, Sophia purchases a small cup of coffee for $4.25 from the coffee shop.
4. Phillip enjoys rock climbing on the weekends. At some of the less challenging locations, he can climb upwards of 12 feet per minute.
5. Jose prefers to walk to work when the weather is nice. He walks the 1.5 miles to work at a speed of about 3 miles per hour.
6. Gavin works for a skydiving company. Customers pay $200 per jump to skydive in tandem skydives with Gavin.

TOPIC 1 Quantities and Relationships

Extension

Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure.

1. A student performs several experiments in which he swings a pendulum for a 20-second duration. He uses a string that is 27 cm long, and he tests pendulum masses of different sizes, varying from 2 to 12 grams. He records the number of swings each pendulum makes in 20 seconds.
2. The student then decides to make a second graph showing the string length (in cm) as the independent quantity. What changes must the student make to his experiment?

Spaced Practice

1. Solve the equation $-2x + 8 = -3x + 14$.
2. Solve the equation $-3x - 6 = -5x + 8$

NEW KEY TERMS

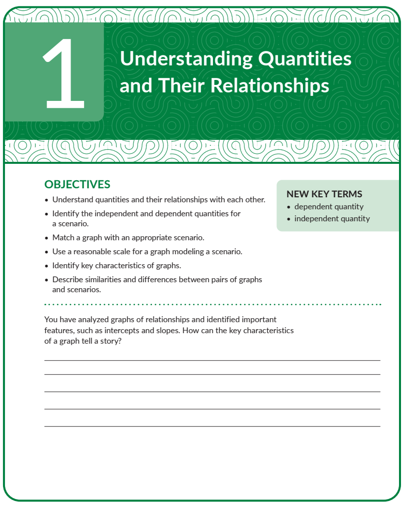
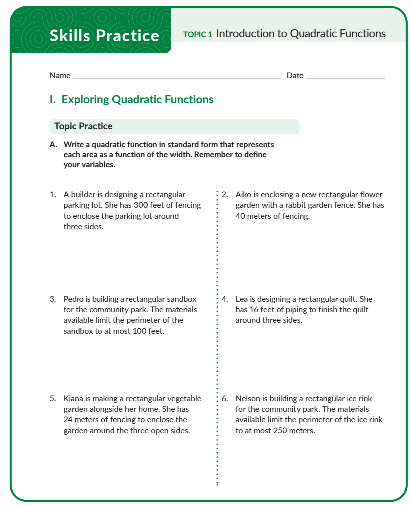
- solution [solución]
- infinite solutions [soluciones infinitas]
- no solution [sin solución]
- literal equation [ecuación literal]
- linear inequality [desigualdad lineal]

Refer to the Math Glossary for definitions of the New Key Terms.

Each lesson features one or more ELPS (English Language Proficiency Standards) and provides the teacher with implementation strategies incorporating best practices for supporting language acquisition. In addition, students are provided with cognates for New Key Terms in the Topic Summaries and Topic Family Guides.

Engaging with Grade Level Content

Your student will engage with grade-level content in multiple ways with the support of the teacher.

Learning Together	Learning Individually
The teacher facilitates active learning of lessons so that students feel confident in sharing ideas, listening to each other, and learning together. Students become creators of their mathematical knowledge.	Skills Practice provides students the opportunity to engage in additional skill building that aligns to each Learning Together lesson. The Learning Individually Days target discrete skills that may require additional practice to achieve proficiency.
	

At the end of each topic, your student will take an assessment aligned to the standards covered in the topic. This assessment consists of multiple-choice, multiselect, and open-ended questions designed for your student to demonstrate learning. Each assessment also includes a scoring guide for teachers to ensure consistent scoring. The scoring guide includes ways to support or challenge your student based on their responses to the questions on the assessment. The purpose of the assessment is for the teacher and student to reflect on the learning. Teachers will use your student's assessment results to target individual skills your student needs for proficiency or to accelerate and challenge your student.

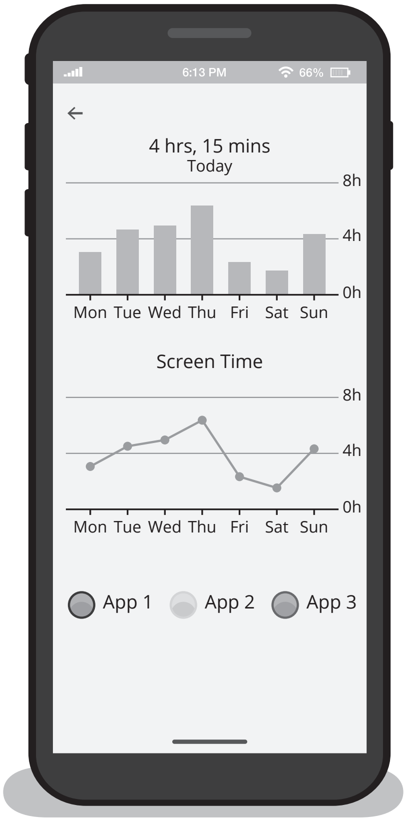
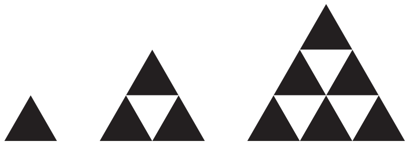
Response to Student Performance

TEKS*	Question(s)	Recommendations
A.2C	1, 6	To support students: • Use Skills Practice Set III.A for additional practice. • Review Lesson 2 Assignment Practice Question 2a and Lesson 3 Assignment Practice Questions 1a and 2a.
A.5A	2, 5	To support students: • Use Skills Practice Set I.B, for additional practice. • Review Lesson 1 Assignment Practice Question 2.
	4, 11	To support students: • Use Skills Practice Set I.A, for additional practice. • Review Lesson 1 Assignment Practice Question 1. To challenge students: • Extend student knowledge with Skills Practice Extension Set I.
A.5B	3, 9	To support students: • Use Skills Practice Sets III.B and III.C, for additional practice. • Review Lesson 3 Assignment Practice Question 3. To challenge students: • Extend student knowledge with Skills Practice Extension Set III.
A.12E	7, 8, 10	To support students: • Use Skills Practice Sets II.A, II.B, and II.C for additional practice. • Review Lesson 2 Assignment Practice Question 3. To challenge students: • Extend student knowledge with Skills Practice Extension Set II.
NOTE: Both teachers and administrators should refer to the Assessment Guidance and Analysis section of the Course & Implementation Guide for additional support in analyzing and responding to student data.		

*BOLD TEKS = Readiness Standard

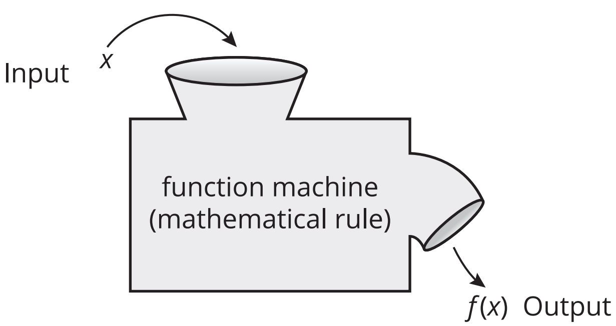
MODULE 1 **Searching for Patterns**

In this module, your student will deepen their understanding of functions to explore function families, including linear, exponential, and quadratic. There are two topics in this module: *Quantities and Relationships* and *Sequences*. Your student will use what they already know about patterns in this module.

TOPIC 1 Quantities and Relationships	TOPIC 2 Sequences
Your student will analyze scenarios and graphs representing the functions they will study in the course.	Your student will explore sequences represented as lists of numbers, tables of values, equations, and graphs.
What in the world? Graphs allow us to see data in new ways so that we can find patterns and make predictions about the things we do not know. They can even be used to track daily habits and learn more about ourselves.  	Did you know?  A sequence is a pattern of numbers, geometric figures, letters, or other objects that are placed in an exact order. What would the next figure look like in the sequence?

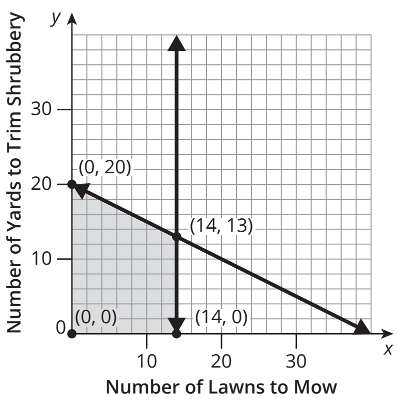
MODULE 2 Exploring Constant Change

In this module, your student will learn about linear functions. There are two topics in this module: *Linear Functions* and *Transforming Linear Functions*. Your student will use what they already know about linear equations and geometric transformations in this module.

TOPIC 1 Linear Functions	TOPIC 2 Transforming and Comparing Linear Functions
Your student will investigate what it means to be a function and learn the basic attributes of a function.	Your student will use function transformations to prove the relationships between the slopes of parallel and perpendicular lines. This work lays the foundation for students to transform any of the various function types.
It's a machine!  It's easiest to think about functions as little mathematical machines where something is input in the machine and an output is generated.	Road Map!  Parallel and perpendicular lines—and the accuracy of those lines—have great importance in construction of city streets, railroads, electrical lines, and building structures! Modern growing cities typically build their roads on a grid of parallel and perpendicular lines, which makes for simple navigation!

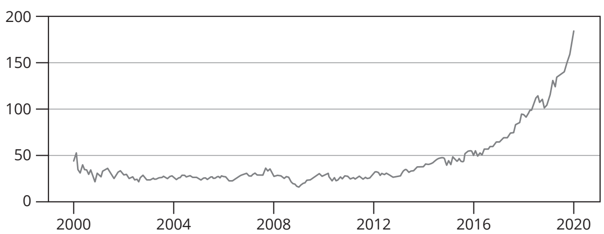
MODULE 3 Modeling Linear Equations and Inequalities

In this module, your student will explore graphing and solving linear equations and inequalities. There are two topics in this module: *Linear Equations and Inequalities* and *Systems of Linear Equations*. Your student will use what they already know about linear equations and basic inequalities in this module.

TOPIC 1 Linear Equations and Inequalities	TOPIC 2 Systems of Linear Equations and Inequalities
Your student will solve linear equations and identify the solutions to linear inequalities.	Your student will use systems to find solutions to more complex problem situations.
$E = mc^2$ Quite possibly the most famous equation ever is Albert Einstein's equation from his theory of general relativity which shows the relationship of energy to mass. The energy of an object at rest is equal to its mass (m) times the speed of light (c) squared. But this is only for objects at rest. The full equation for moving objects with momentum (p) is $E^2 = (mc^2) + (pc)^2$. You may recognize the form of this equation, which is identical to another famous equation: $a^2 + b^2 = c^2$.	Maximizing profits! Companies use systems of inequalities all the time to ensure maximum profits. Given a scenario where Tony makes \$20 per lawn mowed and \$15 per yard to trim shrubbery, which should he strive for to make the most money if he graphs his business model? 

MODULE 4 Investigating Growth and Decay

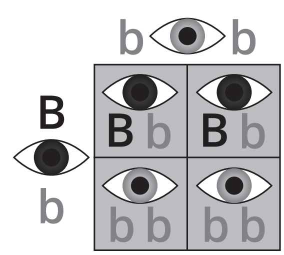
In this module, your student will deepen their understanding of power properties, exponential functions, and applications of exponential functions. There are two topics in this module: *Introduction to Exponential Functions* and *Using Exponential Equations*. Your student will use what they already know about geometric sequences and regression models in this module.

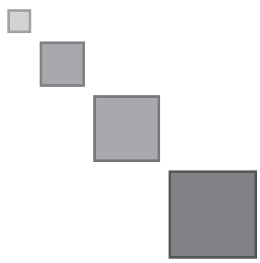
TOPIC 1 Introduction to Exponential Functions	TOPIC 2 Using Exponential Equations
Your student will expand on the idea of building from a common difference to a constant ratio to create an exponential function. They will also revisit and learn new properties that relate to exponents.	Your student will apply what they have learned about exponential functions and constant ratios to model situations of growth or decay and use the models to solve problems.
Making Money!  Many people invest their money into stocks, which tend to grow exponentially rather than in a linear fashion. If a line was drawn from the start point of this graph to the end point of this graph, would that trend be what you invest your money in?	Did you know that?  Scientists use something called radiocarbon dating to determine how old fossils are by measuring the amount of radioactive isotopes of carbon left in an object. Knowing the exact rate of decay allows them to make predictions about how long the object has been decaying based on what is left.

MODULE 5 **Maximizing and Minimizing**

In this module, your student will deepen their understanding of quadratic functions. There are three topics in this module: *Introduction to Quadratics*, *Polynomial Operations*, and *Solving Quadratic Equations*. Your student will use what they already know about the key attributes and solutions of linear and exponential functions in this module.

<table>
<tr><th>TOPIC 1
Introduction to Quadratic Functions</th><th>TOPIC 2
Polynomial Operations</th><th>Topic 3
Solving Quadratic Equations</th></tr>
<tr><td>Your student will be introduced to quadratic functions and explore the key attributes they have. Then, transformations will be applied to quadratic functions, building off of what was previously learned with transformations of linear functions.</td><td>Your student will perform polynomial operations, including using area models to multiply binomials before using the distributive property.</td><td>Your student will learn about the algebraic properties of polynomials required to solve quadratic equations and understand the meanings to their solutions.</td></tr>
<tr><td>Did you know?

The force of gravity causes an object to accelerate faster and faster towards the ground and can be modeled by a quadratic function. Rockets need to be launched into the air far enough that they escape gravity's pull, and the first rocket scientists used quadratic functions to help get to space!</td><td>Dominant or Recessive?


You can use area models to multiply multi-digit numbers and polynomials, but they also have another application you will encounter in a biology or the science class: Punnett Squares! Scientists use Punnett Square to help predict the variations and probabilities that can occur in offspring, such as eye color! The gene for brown eyes is dominant, so if that gene is passed on, the offspring will have brown eyes. However, if the recessive gene for blue eyes is passed on from both parents, then the offspring will have blue eyes! It's fun trying to figure out which of your parents or grandparents gave you your recessive or dominant traits!</td><td>It's Squared!

One of the first quadratic relationships discovered was the relationship between the length of a side of a square and the area of that square.</td></tr>
</table>

Lesson Structure

Each lesson in the course is laid out in the same way to develop deep understanding. Read through the parts of the lesson to learn more about your student's learning in their math classroom.

Objectives & Essential Question

Each lesson begins with objectives, listed to help students understand the objectives. Also included is an essential statement connecting students' learning with a question to ponder. The question is asked again at the end of each lesson to see how much your student understands.

Getting Started

The Getting Started engages your student in the learning. In the Getting Started, your student uses what they already know about the world, what they've already learned, and their intuition to get them thinking mathematically and prepare them for what's to come in the lesson.

Activities

In the Activities, students develop their mathematical knowledge and build a deep understanding of the math. These activities provide your student with the opportunity to communicate and work with others in their math classroom.

When your student is working through these activities, keep in mind:

- It's not just about answer-getting. Doing the math and talking about it is important.
- Making mistakes is an important part of learning, so take risks.
- There is often more than one way to solve a problem.

Talk the Talk

The Talk the Talk gives your student an opportunity to reflect on the main ideas of the lesson and demonstrate their learning.

Lesson Assignment

The lesson assignment provides your students with practice to develop fluency and build proficiency. The lesson assignment also includes a section to help prepare students for the next lesson.

Key Concepts of the Lesson

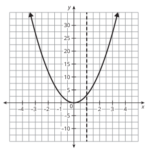
At the end of each topic, the Topic Summary provides a summary of each lesson in the topic. Encourage your student to use these as a tool to review and retrieve the key concepts of a lesson.

Supporting Your Student

Where are we now?

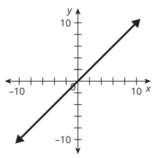
The **Vertical Line Test** is a way to determine if a relation on a graph is a function.

The equation $y = 3x^2$ is a function. The graph passes the vertical line test because there are no vertical lines you can draw that would cross the graph at more than one point.



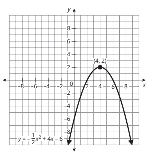
A **continuous graph** is a graph of points connected by a line or smooth curve. Continuous graphs have no breaks.

The graph shown is a continuous graph.



A function has an **absolute maximum** when there is a point that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph. It is the highest point that the curve reaches on the graph.

The absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$ is $y = 2$.



The Topic Family Guide

The Topic Family Guide provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides an example of a math model or strategy your student is learning in the topic, busting of a math myth, talking points to discuss and/or questions to ask your student, and the new key terms your student will learn. You and your student can also use the Math Glossary to check terminology and definitions. Encourage your student to reference the new key terms in the Topic Family Guide and/or Math Glossary when completing math tasks.

Learning outside of the classroom is crucial for your student's success. While we don't expect you to be a math teacher, the Topic Family Guide can assist you as you talk to your student about the mathematical content of the course. The hope is that both you and your student will read and benefit from the guides.

MYTH

"I don't have the math gene."

Let's be clear about something. There isn't ***a*** gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to it.

Some believe mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without formal instruction. They can learn number sense and pattern recognition the same way.

To further nurture your student's mathematical growth, attend to the learning environment. You can support this by discussing math in the real world, offering encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and providing space for plenty of practice.

#mathmythbusted

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Functions are an important topic to know for making predictions in the sciences, creating computer programs, and college admissions tests.

HERE IS A SAMPLE QUESTION

For the function $f(x) = 2x^2 - 3x$, what is the value of $f(-5)$?

To solve this, students need to know that the input -5 is substituted for x in the equation:

$$\begin{aligned} f(-5) &= 2(-5)^2 - 3(-5) \\ &= 2(25) + 15 \\ &= 50 + 15 \\ &= 65 \end{aligned}$$

The point $(-5, 65)$ is on the graph of the function.

Math Myths

Math myths can lead students and adults to believe that math is too difficult for them, math is an unattainable skill, or that there is only one right way to do mathematics. The Math Myths section in the Topic Family Guides busts these myths and provides research-based explanations of why math is accessible to all students (and adults).

Examples of these myths include:
Myth: Just give me the rule. If I know the rule, then I understand the math.

Memorize the following rule: *All quars are elos.* Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms.* How about now? Can you remember that? Of course you can because now, it makes sense. Learning does not take place in a vacuum. It must be connected to what you already know. Otherwise, arbitrary rules will be forgotten.

Myth: There is one right way to do math problems

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. When one road is backed up, you can always take a different route. When you know only one route, you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: "Well, that's one way to do it. Is there another way? What are the pros and cons?" That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work, or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

The TEKS Mathematical Process Standards

Each module will focus on TEKS mathematical process standards that will help your student become a mathematical thinker. The TEKS mathematical process standards are listed below. Discuss with your student the "I can" statements below the standards to help them develop their mathematical learning and understanding. With your help, your student can become a productive mathematical thinker.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Supporting Your Student

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it when necessary.
- ask useful questions to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions when trying to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Supporting Your Student

Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Supporting Your Student

Reflecting on Learning and Progress

You can support your student by encouraging your student to reflect on the learning process. The instructional resources include a student Topic Self-Reflection for each topic. Encourage your student to accurately and frequently reflect on learning and progress throughout each topic. Talk about the specific concepts in the Topic Self-Reflection with your student and celebrate the progress from the beginning to the end of the topic. Remind your student to refer to the Topic Self-Reflection on Learning Individual Days after targeting specific skills and concepts. You can have your student explain concepts from the self-reflection using the topic summaries or lesson assignments to demonstrate understanding.

TOPIC 1 SELF-REFLECTION

Name: ____________________

Quantities and Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Quantities and Relationships* topic by:

TOPIC 1: *Quantities and Relationships*	Beginning of Topic	Middle of Topic	End of Topic
choosing appropriate scale and origin for graphs.	◯	◯	◯
identifying the appropriate unit of measure for each variable or quantity.	◯	◯	◯
analyzing a graph and stating the key characteristics of the graph.	◯	◯	◯
using a problem situation to explain what the key features of a graph mean in real-world context.	◯	◯	◯
deciding whether relations represented verbally, tabularly, graphically, and symbolically define a function.	◯	◯	◯
recognizing a linear, exponential, or quadratic function by its equation or graph.	◯	◯	◯
evaluating functions, expressed in function notation, given one or more elements in their domain.	◯	◯	◯
determining the domain and range and the independent and dependent quantities in a relationship.	◯	◯	◯

continued on the next page

Thanks!

Enjoy the fun mathematical adventure that is ahead for you and your student! Remember the supports available to you, and thank you for supporting your student's learning.

MODULE
1

Searching for Patterns

TOPIC 1 Quantities and Relationships

In this topic, students explore a variety of different functions. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study in this course, they will formalize their understanding of the defining characteristics of each type of function.

Where have we been?

In previous grades, students defined a function and used linear functions to model the relationship between two quantities. They have written linear functions in slope-intercept form and should be able to identify the slope and y-intercept in the equation. Students have also characterized graphs as functions using the terms *increasing*, *decreasing*, *constant*, *linear*, and *nonlinear*.

Where are we going?

The study of functions is a main focus of Algebra I and future math courses. This topic builds the foundation for future, more in-depth study by familiarizing students with the concept of a function. Students will continue to use formal function notation throughout this course and in higher-level math courses.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Functions are an important topic to know for making predictions in the sciences, creating computer programs, and college admissions tests.

HERE IS A SAMPLE QUESTION

For the function $f(x) = 2x^2 - 3x$, what is the value of $f(-5)$?

To solve this, students need to know that the input −5 is substituted for x in the equation:

$$\begin{aligned} f(-5) &= 2(-5)^2 - 3(-5) \\ &= 2(25) + 15 \\ &= 50 + 15 \\ &= 65 \end{aligned}$$

The point (−5, 65) is on the graph of the function.

NEW KEY TERMS

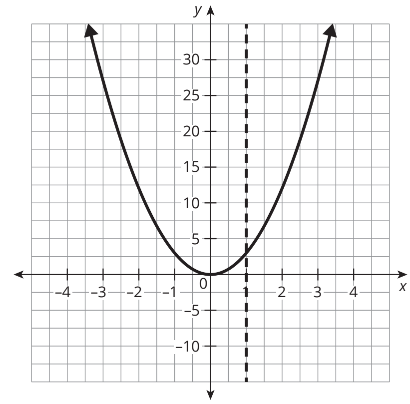
- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- relation [relación]
- domain [dominio]
- range [rango]
- function [función]
- function notation [notación de función]
- Vertical Line Test [Prueba de la línea vertical]
- discrete graph [gráfica discreta/discontinua]
- continuous graph [gráfica continua]
- increasing function [función creciente]
- decreasing function [función decreciente]
- constant function [función constante]
- function family [familia de funciones]
- linear functions [funciones lineales]
- exponential functions [funciones exponenciales]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]
- quadratic functions [funciones cuadráticas]
- *x*-intercept [intersección con el eje *x*]
- *y*-intercept [intersección con el eje *y*]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

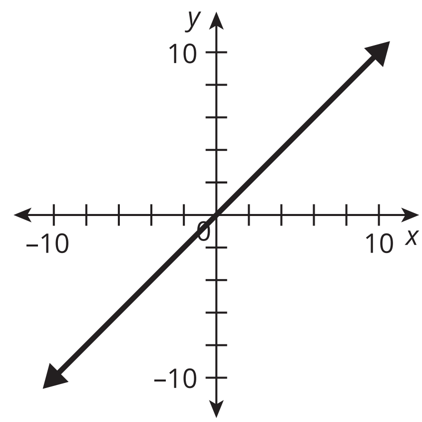
The **Vertical Line Test** is a way to determine if a relation on a graph is a function.

The equation $y = 3x^2$ is a function. The graph passes the vertical line test because there are no vertical lines you can draw that would cross the graph at more than one point.



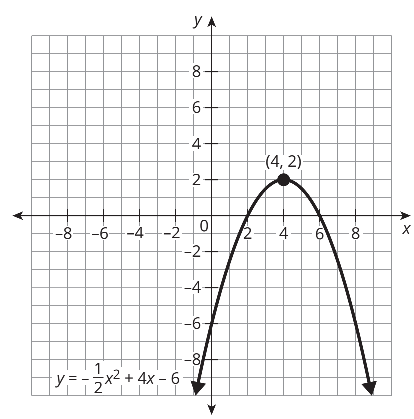
A **continuous graph** is a graph of points connected by a line or smooth curve. Continuous graphs have no breaks.

The graph shown is a continuous graph.



A function has an **absolute maximum** when there is a point that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph. It is the highest point that the curve reaches on the graph.

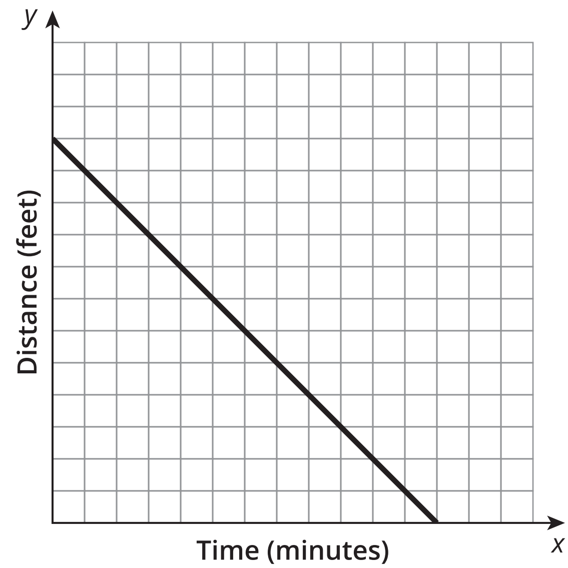
The absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$ is $y = 2$.



In **Lesson 1: Understanding Quantities and Their Relationships**, students read descriptions of relationships between two quantities and identify which is independent and which is dependent.

Dependent and Independent Quantities

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the **dependent quantity**, which is typically represented by the variable *y*. The quantity that changes the other quantity is called the **independent quantity**, which is typically represented by the variable *x*.



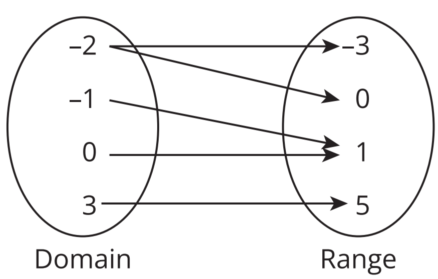
For example, consider the graph that models the situation where Pedro is walking home from school at a constant rate. The time, in minutes, that Pedro walks is the independent quantity. The distance away from home is the dependent quantity.

In **Lesson 3, Recognizing Functions and Function Families**, students investigate relations, functions, and function notation.

Functions and Relations

A **relation** is a mapping between a set of input values called the **domain** and a set of output values called the **range**. A **function** is a relation between a given set of elements, where each element in the domain is grouped with exactly one element in the range. If each value in the domain has one and only one range value, like Figure 2, then the relation is a function. If any value in the domain has more than one range value, like Figure 1, then the relation is not a function.

Figure 1



The value –2 in the domain has more than one range value. The mapping does not represent a function.

Figure 2

Domain	Range
2	1
6	3
10	5
14	7

Each element in the domain has exactly one element in the range. The table represents a function.

MYTH

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#mathmythbusted

Function Notation

Functions can be represented in a number of ways. An equation representing a function can be written using **function notation**. Function notation is a way of representing functions with algebra. This form allows you to more easily identify the independent and dependent quantities. The function $f(x)$ is read as "f of x" and shows that x is the independent variable.

$$f(x) = 8x + 15$$

name of function

independent variable

The linear equation $y = 8x + 15$ can be written to represent a relationship between the variables x and y. You can write this linear equation as a function with the name f to represent it as a mathematical object that has a specific set of inputs (the domain of the function) and a specific set of outputs (the range of the function).

Function Families

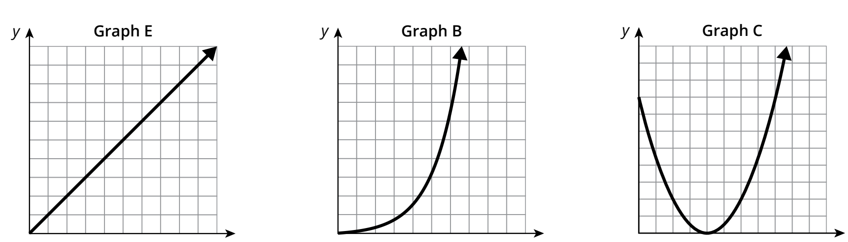
In **Lesson 4: Recognizing Functions by Characteristics**, students associate function families with specific sets of characteristics.

A **function family** is a group of functions that share certain properties. Function families have key properties that are common among all functions in the family. Knowing these key properties is useful when sketching a graph of the function.

The graph of a **linear function** is represented by a straight line which can be vertical, horizontal and diagonal.

The graph of an **exponential function** is represented by a smooth curve.

The graph of a **quadratic function** is represented by a parabola.



Family Guide

MODULE 1 Searching for Patterns

Algebra I

TOPIC 2 Sequences

In this topic, students explore sequences represented as lists of numbers, in tables of values, by equations, and as graphs on the coordinate plane. Students move from an intuitive understanding of patterns to a more formal approach of representing sequences as functions.

Where have we been?

Students have been analyzing and extending numeric patterns since elementary school. They have discovered and explained features of patterns. They have formed ordered pairs with terms of two sequences and compared the terms. In previous grades, students have connected term numbers and term values as the inputs and outputs of a function.

Where are we going?

As students deepen their understanding of functions throughout this course and beyond, recognizing that all sequences are functions is an important building block. A rich understanding of arithmetic sequences is the foundation for linear functions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Sequences are an important topic to know about for college admissions tests.

HERE IS A SAMPLE QUESTION

What is the second term in this geometric sequence?

$\frac{1}{3}, \_\_\_, \frac{1}{48}, \frac{1}{192}, \ldots$

To solve this, students need to know that each term in a geometric sequence is calculated by using the same multiplier, or constant ratio. The multiplier can be determined by dividing a term by the term before it.

In this case, $192 \div 48 = 4$. Therefore, $\frac{1}{192} \div \frac{1}{48} = \frac{1}{4}$. This means the multiplier is $\frac{1}{4}$. The second term can be calculated by multiplying the first term by $\frac{1}{4}$.

Because $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$, the second term is $\frac{1}{12}$.

NEW KEY TERMS

- sequence [secuencia/ sucesión]
- term of a sequence [término de una secuencia]
- infinite sequence [secuencia infinita]
- finite sequence [secuencia finita]
- arithmetic sequence [secuencia aritmética]
- common difference [diferencia común]
- geometric sequence [secuencia geométrica]
- common ratio [razón común]
- recursive formula [fórmula recursiva]
- explicit formula [fórmula explícita]
- mathematical modeling [modelado matemático]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

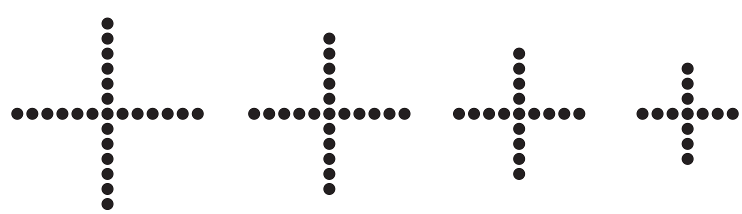
A **sequence** is a pattern that has an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term in a sequence** is an individual number, figure, or letter in the sequence.
A **recursive formula** expresses each new term of a sequence based on the term that comes before it in the sequence. The recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$. The recursive formula for a geometric sequence is $g_n = g_{n-1} \cdot r$. The formula $a_n = a_{n-1} + 2$ is an example of a recursive formula. Each term that comes next is calculated by adding 2 to the previous term. If $a_1 = 1$, then $a_2 = 1 + 2 = 3$.

In **Lesson 1, Recognizing Patterns and Sequences**, students write numeric sequences to represent geometric patterns or contexts.

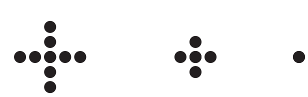
Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term of a sequence** is an individual number, figure, or letter in the sequence.

Consider the pattern shown:



In this pattern, each figure has four fewer dots than the image before. To extend this pattern, draw the following images:



The sequence representing these figures is: 25, 21, 17, 13, 9, 5, 1.

In **Lesson 2, Arithmetic and Geometric Sequences**, students will categorize sequences based on characteristics in the expression.

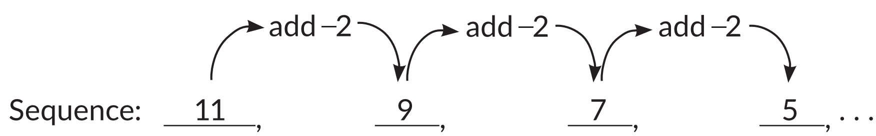
Arithmetic Sequence

An **arithmetic sequence** is a sequence of numbers where the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers where a constant is added to each term to produce the next term. This constant is called the **common difference**. The common difference is typically represented by the variable, *d*.

Consider the sequence shown.

11, 9, 7, 5, ...

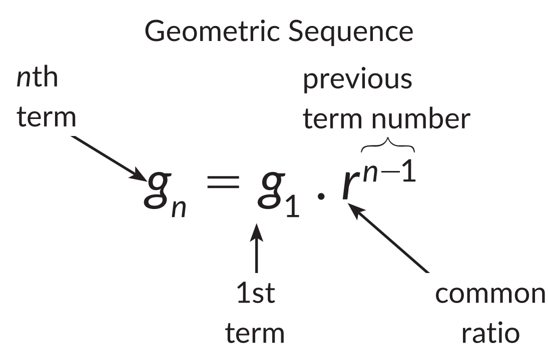
The pattern is to add the same negative number, −2, to each term to define the next term.



The sequence is arithmetic and the common difference, *d*, is −2.

Geometric Sequences

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. The constant, which is either an integer or a fraction, is called the **common ratio** and is represented by the variable *r*. For example, in the sequence 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$, the pattern is to multiply each term by the same number, $\frac{1}{3}$, to determine the next term. For that reason, this sequence is geometric and the common ratio, *r*, is $\frac{1}{3}$.

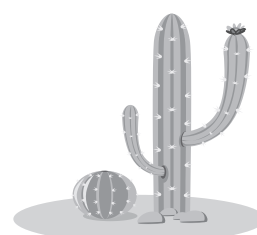


In **Lesson 3, Determining Recursive and Explicit Expressions from Contexts**, students will use and define explicit formulas.

Explicit Formulas

An **explicit formula** for a sequence is a formula for calculating each term of the sequence using the index, which is a term's position in the sequence. The explicit formula to determine the term for any number that you put in the place of n (also known as the "nth term") of an arithmetic sequence is $a_n = a_1 + d(n - 1)$. You can use the distributive property to rewrite the formula for an arithmetic sequence. The explicit formula to determine the nth term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$.

For example, consider the situation of a cactus that is 3 inches tall and will grow $\frac{1}{4}$ inch every month. The explicit formula for arithmetic sequences can be used to determine how tall the cactus will be in 12 months.

$a_n = 3 + \frac{1}{4}(n - 1)$ $a_n = 3 + \frac{1}{4}n - \frac{1}{4}$ $a_n = \frac{1}{4}n + 2.75$ $a_{12} = \frac{1}{4}(12) + 2.75$ $a_{12} = 3 + 2.75$ $a_{12} = 5.75$ or $5\frac{3}{4}$		In 12 months, the cactus will be $5\frac{3}{4}$ inches tall.

MYTH

Asking questions means you don't understand.

It is universally true that, for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. But there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more about a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, everyone should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

MODULE

2

Exploring Constant Change

TOPIC 1 Linear Functions

In this topic, students focus on the patterns that are evident in certain data sets and use linear functions to model those patterns. Using the informal knowledge of lines of best fit that was built in previous grades, students advance their statistical methods to make predictions about real-world phenomena.

Students prove that the common difference of an arithmetic sequence and the slope of the corresponding linear function are both constant and equal. They write equations and graph lines presented in slope-intercept, point-slope, and standard forms.

Where have we been?

Students have analyzed the shape of data, informally fit trend lines to model data sets, determined the equations of those lines, interpreted the slopes and *y*-intercepts of the lines, and used the equations to make and judge the reasonableness of predictions about the data. Students have also examined linear relationships and recognized that the slope of a line defines its steepness and direction.

Where are we going?

As students continue in this course and in future mathematics courses, they will determine and analyze more complicated regressions, including exponential and quadratic regression models. From this topic, students should understand the key and defining characteristics of a linear function represented in situations, tables, equations, and graphs. This prepares students exploring equations as the most specific representation of linear functions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Regressions can be an important topic to know about for modeling real-world data and for college admissions tests.

HERE IS A SAMPLE QUESTION

The data in the table show test scores after certain amounts of study time. Use a linear function to estimate the score associated with a study time of 20 minutes.

Score	86	70	90	78
Time (min)	45	15	40	35

Time is the independent variable and score is the dependent variable. Use graphing technology to determine a linear regression function.

This yields a linear regression function of $f(x) = 0.61x + 60.51$. A study time of 20 minutes would yield an estimated score of $f(20) = 0.61(20) + 60.51$, or 72.71.

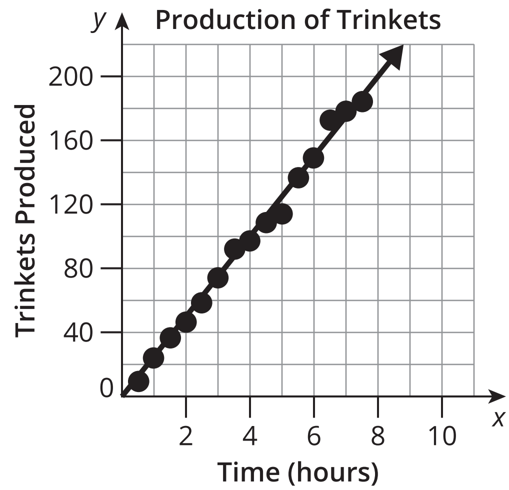
NEW KEY TERMS

- Least Squares Method
- centroid [centroide]
- linear regression function [función de regresión lineal]
- interpolation [interpolación]
- extrapolation [extrapolación]
- correlation [correlación]
- correlation coefficient [coeficiente de correlación]
- coefficient of determination [coeficiente de determinación]
- causation [causalidad]
- necessary condition [condición necesaria]
- sufficient condition [condición suficiente]
- common response [respuesta común]
- confounding variable [variable de confusión]
- conjecture [conjetura]
- first differences
- average rate of change
- point-slope form
- standard form [forma estándar/general]
- polynomial [polinomio]
- degree
- leading coefficient
- zero of a function [cero de una función]

Refer to the Math Glossary for definitions of the New Key Terms.

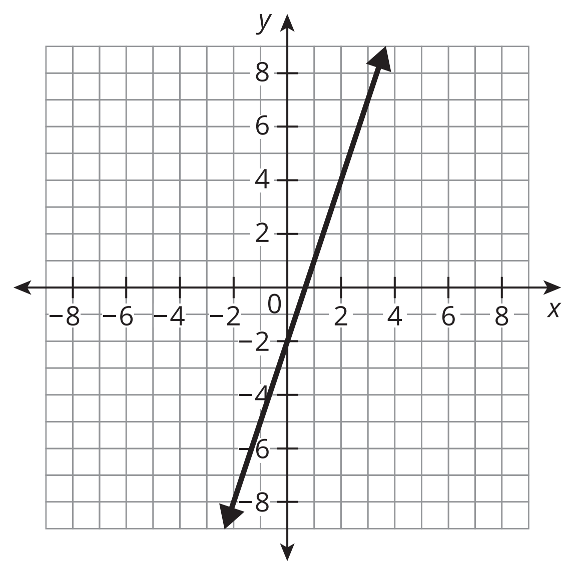
Where are we Now?

The **Least Squares Method** is a method that creates a line that is closest to the points of data, known as a **linear regression function**, for a scatterplot that has two basic requirements: (1) the line must contain the centroid of the data set, and (2) the sum of the squares of the vertical distances from each given data point is smallest with the line.



Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is $\frac{f(t) - f(s)}{t - s}$.

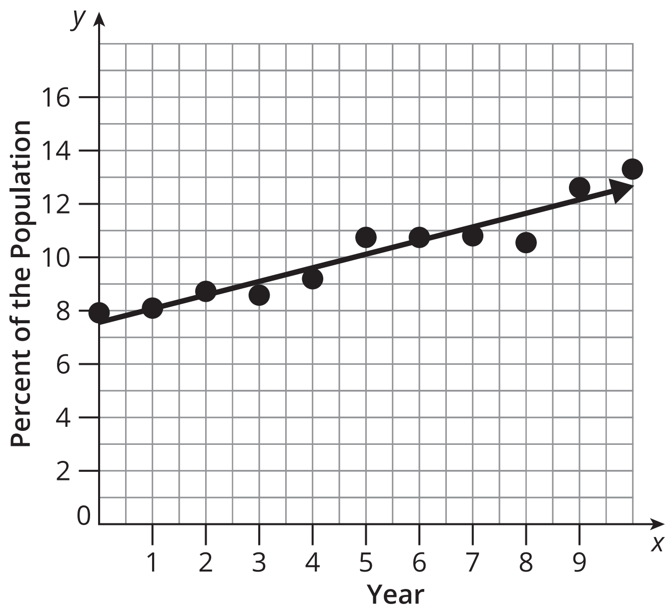
The average rate of change of the function shown is 3.



In **Lesson 1: Least Squares Regressions**, students informally determine a line of best fit before using a formal method to determine a linear regression function. They then use linear regression functions to make predictions.

Regression Lines

Real-world data points usually won't fit neatly on a line. But you can model the data points using a line, which represents a linear function. There are an infinite number of lines that can pass through the data points. But there is just one line that models the data with the minimum distances between the data points and the line. The **linear regression function** has the smallest possible vertical distance from each given data point to the line. By measuring these distances and squaring them, you will see that the squared distances add up to the least value with the linear regression function.



In **Lesson 2: Correlation**, students explore the difference between correlation and causation.

Correlation versus Causation

Consider an experiment conducted by a group of college students that found that more class absences correlated to rainy days. The group concluded that rain causes students to be sick. However, this correlation does not imply **causation**. Rain is neither a **necessary condition** (because students can get sick on days it does not rain) nor a **sufficient condition** (because not every student who is absent is necessarily sick) for students being sick.

In **Lesson 3: Making Connections between Arithmetic Sequences and Linear Functions**, students explore the relationship between the constant difference, slope, and average rate of change.

Slope and Average Rate of Change

The slope, *a*, of a linear function is equal to the constant difference of an arithmetic sequence. Another name for the slope of a linear function is **average rate of change**. The expression for the average rate of change is $\frac{f(t) - f(s)}{t - s}$. This represents the change in the output as the input changes from *s* to *t*.

MYTH

There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. If one road is backed up, then you can always take a different route. If you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: Well, that's one way to do it. Is there another way? What are the pros and cons? That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

#mathmythbusted

Forms of Linear Equations

In **Lesson 4: Point-Slope Form of a Line**, students use the slope formula to derive the point-slope form of a linear equation.

Students are already familiar with slope-intercept form from previous courses. Other forms they will encounter are **point-slope form** and **standard form**. Each of these has its own special use depending on the given information.

- You can write an equation in point-slope form when given a table.

x	y
2	6
4	5
6	2

- First, use any two points from the table to calculate the slope. For example, use (2, 6) and (4, 5).

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 6}{4 - 2}$$
$$= \frac{-1}{2} = -\frac{1}{2}$$

- Next, choose any point from the table. Let's use (2, 6).

- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, (2, 6), and the unknown point (x, y).

$$m \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$

- Finally, rewrite the equation with no variables in a denominator.

$$-\frac{1}{2} = \frac{y - 6}{x - 2}$$
$$-\frac{1}{2}(x - 2) = y - 6$$

The equation in point-slope form is $y - 6 = -\frac{1}{2}(x - 2)$.

In **Lesson 5: Using Linear Equations** students use three different forms of a linear equation to graph linear relationships.

Family Guide

MODULE 2 Exploring Constant Change

Algebra I

TOPIC 2 Transforming and Comparing Linear Functions

In this topic, students are introduced to function transformations, using vertical and horizontal dilations, and horizontal and vertical translations. Students use translations to prove that the slopes of parallel lines are the same; they use rotations to prove that the slopes of perpendicular lines are negative reciprocals. Understanding the rules of transformations for linear functions lays the groundwork for students to transform any function type.

Where have we been?

Over the last few years, students have had extensive experience with linear relationships. They have represented relationships using tables, graphs, and equations. They understand slope as a unit rate of change, and as the steepness and direction of a graph.

Where are we going?

From this topic, students should understand how to transform linear functions. This prepares students to transform other function types in this course and future courses.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Linear functions can be an important topic to know about for representing real-world situations and for admissions tests.

HERE IS A SAMPLE QUESTION

x	0	1	2	3
$f(x)$	−2	3	8	13

What equation could represent $f(x)$?

Students may recognize that since the x-values increase by 1s, they can use *first differences* to determine the slope.

$3 - (-2) = 5$ $8 - 3 = 5$ $13 - 8 = 5$

The constant difference is 5, so the slope is 5. The y-intercept is $(0, -2)$, so the equation that represents the function is $y = 5x - 2$.

NEW KEY TERM

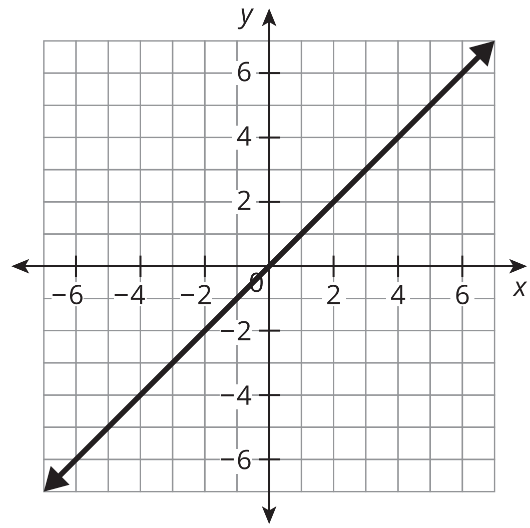
- parent function

Refer to the Math Glossary for definitions of the New Key Terms.

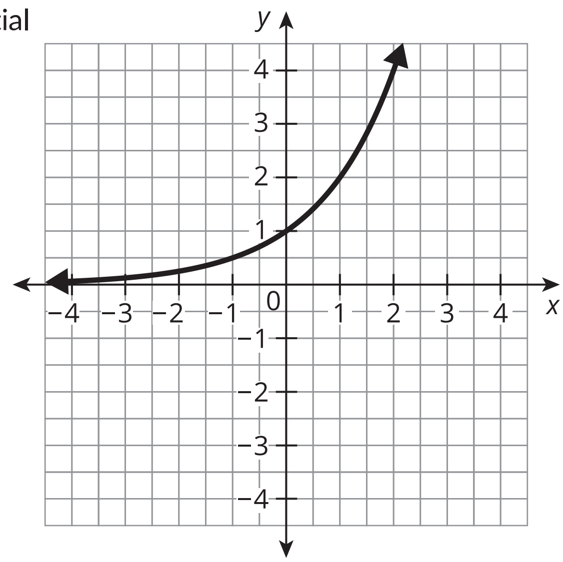
Where are we now?

A **parent function** is the simplest function of its type.

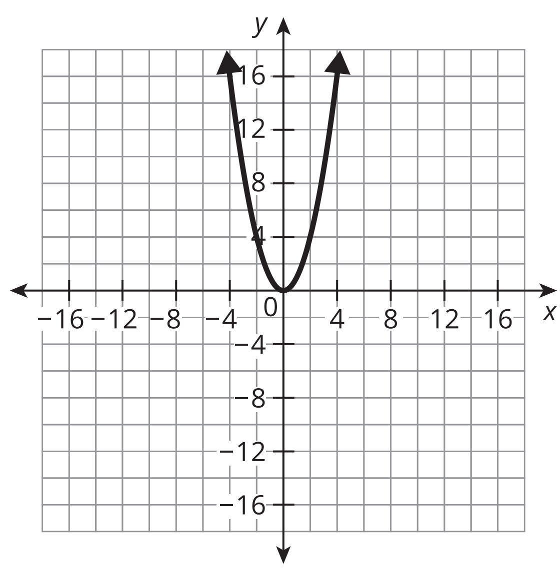
The parent linear function is $f(x) = x$.



The parent exponential function is $g(x) = 2^x$.



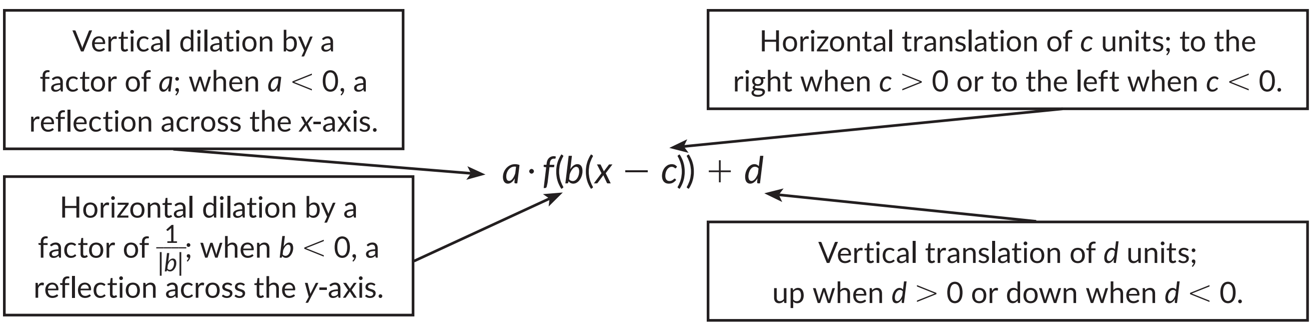
The parent quadratic function is $h(x) = x^2$.



In **Lesson 1: Transforming Linear Functions**, and **Lesson 2: Vertical and Horizontal Transformations of Linear Functions**, students are introduced to transformation form.

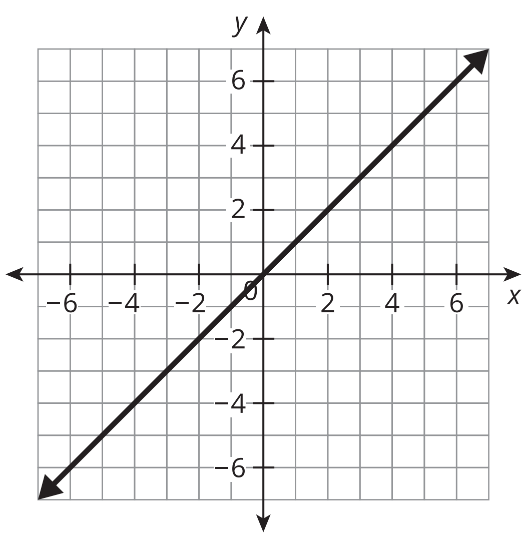
Transformation Notation

The graph of the function $f(x) = x$ is a straight diagonal line that passes through the origin. When a constant d is added, $g(x) = f(x) + d$, the graph shifts up or down vertically. When the function is multiplied by a constant a, $g(x) = a \cdot f(x)$, the graph is stretched or compressed vertically. When a constant c is added to the input of the function, $g(x) = f(x - c)$, the function shifts left or right horizontally. When the input of the function is multiplied by a constant b, $g(x) = f(bx)$, the graph is stretched or compressed horizontally.

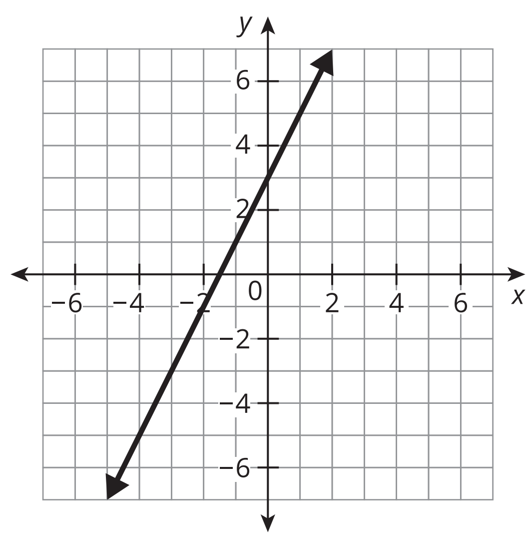


For example, the graph of $y = 2x + 3$ represents both a vertical translation of 3 units and vertical dilation by a factor of 2.

$y = x$



$y = 2x + 3$



MYTH

Students only use 10% of their brains.

Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of some science fiction movies that ask the audience: Imagine what you could accomplish if you could use 100% of your brain!

Well, this isn't Hollywood. The good news is that you do use 100% of your brain. As you look around the room, your visual cortex is busy assembling images, your motor cortex is busy moving your neck, and all of the associative areas recognize the objects that you see. Meanwhile, the corpus callosum, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts... like mathematics!

#mathmythbusted

MODULE

3

Modeling Linear Equations and Inequalities

Family Guide

MODULE 3 Modeling Linear Equations and Inequalities **Algebra I**

TOPIC 1 Linear Equations and Inequalities

In this topic, students analyze linear functions and the key characteristics that define linear functions. They solve equations in one variable, examining the structure of each equation to predict whether the equation has one solution, no solution, or infinite solutions. Students use the properties of equality and basic number properties to construct a viable argument to justify a solution method. They generalize their knowledge of solving equations in one variable to solve literal equations for given variables. Students then graph linear inequalities and explore solving an inequality with a negative slope, which affects the sign of the inequality.

Where have we been?

In previous courses, students gained proficiency in solving increasingly complex linear equations and they solved two-step inequalities and graphed the solutions on a number line. Entering this course, students have solved two-step equations with variables on both sides. They understand the underpinnings of solving equations by maintaining equality. From this intuitive understanding, students use properties to justify each step in the equation-solving process.

Where are we going?

Students will use their knowledge of equations and inequalities to solve linear absolute value equations in future courses. By recognizing the connections between algebraic and graphical solutions to an equation or inequality, students are developing the foundation to later solve linear absolute value equations and inequalities, exponential equations, and quadratic equations and inequalities.

HERE IS A SAMPLE QUESTION

Solve for x in the inequality $\frac{x}{2} - 3 < 2y$.

To solve for x, isolate the variable x.

$$\frac{x}{2} - 3 < 2y$$
$$\frac{x}{2} - 3 + 3 < 2y + 3$$
$$\frac{x}{2} < 2y + 3$$
$$\cancel{2}\left(\frac{x}{\cancel{2}}\right) < 2(2y + 3)$$
$$x < 4y + 6$$

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Inequalities can be an important topic to know about for college admissions tests.

NEW KEY TERMS

- solution [solución]
- infinite solutions [soluciones infinitas]
- no solution [sin solución]
- literal equation [ecuación literal]
- linear inequality [desigualdad lineal]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

$5x - 7 = 13$

$5x - 7 + 7 = 13 + 7$

$5x = 20$

$x = 4$

One solution

$5x - 7 = 3 + 5x - 10$

$5x - 7 = 5x - 7$

$5x - 7 - 5x = 5x - 7 - 5x$

$-7 = -7$

Infinite solutions

$5x - 7 = 5x - 10$

$5x - 7 - 5x = 5x - 10 - 5x$

$-7 = -10$

$-7 \neq -10$

No solution

Generally, there is only one **solution** to an equation. However, students will discover that there are special cases where an equation will have either **no solution** or **infinite solutions** and identify what those cases look like.

In **Lesson 1: Solving Linear Equations**, students review and use the properties of equality and basic number properties to solve linear equations and justify their steps. Justifying steps with the properties of equality will better prepare students for writing geometrical proofs in future courses.

Properties of Equality

The properties of equality state that when an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality.

Properties of Equality	For all numbers a, b, and c
addition property of equality	If $a = b$, then $a + c = b + c$.
subtraction property of equality	If $a = b$, then $a - c = b - c$.
multiplication property of equality	If $a = b$, then $ac = bc$.
division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

There are also basic number properties that can be used to justify steps when solving equations.

Number Properties	For all numbers *a*, *b*, and *c*
commutative property	$a + b = b + a$ $ab = ba$
associative property	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$
distributive property	$a(b + c) = ab + ac$

In **Lesson 2: Literal Equations**, students solve literal equations for specified variables. This connects back to students' prior knowledge of area, volume, and surface area as they solve for unknown values in formulas.

Literal Equations

Literal equations are like solving equations, but instead of having a singular number answer, it is generally a manipulation of an equation to solve for a single variable. For example, students already know how to determine the area of a triangle. They can use the formula for area of a triangle, $A = \frac{1}{2}bh$, to solve for the base or the height of a triangle. Manipulating the known equation can make calculations easier.

$$A = \frac{1}{2}bh$$
$$2(A) = \cancel{2}\left(\frac{1}{\cancel{2}}bh\right)$$
$$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$$
$$\frac{2A}{b} = h$$

In **Lesson 3: Modeling Linear Inequalities**, students model **linear inequalities** with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement.

Solutions to Inequalities

Sometimes, it is necessary to express that more than one answer could satisfy a situation. Students will learn that phrases like *at least*, *at most*, *no more than*, and *less than* will create situations where many answers may be correct.

$>$ greater than	$<$ less than
$\geq$ greater than or equal to	$\leq$ less than or equal to

MYTH

Just watch a video, and you will understand it.

Has this ever happened to you? Someone explains something, and it all makes sense at the time. You feel like you get it. But then, a day later when you try to do it on your own, you suddenly feel like something's missing? If that feeling is familiar, don't worry. It happens to us all. It's called the illusion of explanatory depth, and it frequently happens after watching a video.

How do you break this illusion? The first step is to try to make the video interactive. Don't treat it like a TV show. Instead, pause the video and try to explain it to yourself or to a friend. Alternatively, attempt the steps in the video on your own and rewatch it if you hit a wall. Remember, it's easy to confuse familiarity with understanding.

#mathmythbusted

Solving Inequalities

To solve an inequality, first write a function to represent the problem situation. Then, write the function as an inequality based on the independent quantity. To determine the solution, identify the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: isolate the variable on one side of the inequality symbol. Finally, interpret the meaning of the solution.

For example, consider the situation in which Diego has $25 in his gift fund that he is going to use to buy graduation gifts. Graduation is 9 weeks away. If he would like to have at least $70 to buy gifts, how much should he save each week?

The function is $f(x) = 25 + 9x$, so the inequality is $25 + 9x \geq 70$.

$$25 + 9x \geq 70$$
$$25 + 9x - 25 \geq 70 - 25$$
$$9x \geq 45$$
$$\frac{9x}{9} \geq \frac{45}{9}$$
$$x \geq 5$$

Diego needs to save at least $5 each week to meet his goal.

When you multiply or divide each side of an inequality by a negative number, the inequality sign reverses. Consider the division example in the inequality $250 - 9.25x < 398$.

$$250 - 9.25x < 398$$
$$250 - 9.25x - 250 < 398 - 250$$
$$-9.25x < 148$$
$$\frac{-9.25x}{-9.25} > \frac{148}{-9.25}$$
$$x > -16$$

Family Guide

MODULE 3 Modeling Linear Equations and Inequalities Algebra I

TOPIC 2 Systems of Equations and Inequalities

In this topic, students begin with writing systems of linear equations and solving them graphically and algebraically using substitution. They then move on to solve systems of linear equations using the linear combinations method. Students consider linear inequalities in two variables and learn that their solutions are represented as half-planes on a coordinate plane. They then graph two linear inequalities on the same plane and identify the solution set as the intersection of the corresponding half-planes. Finally, students synthesize their understanding of systems by encountering problems that can be solved by using either a system of equations or a system of inequalities.

Where have we been?

Coming into this topic, students know that every point on the graph of an equation represents a value that makes the equation true. They have learned that the point of intersection of two graphs provides x- and y-values that make both equations true. Students have written systems of linear equations and have solved them graphically.

Where are we going?

Knowing how to solve systems of linear equations prepares students to solve systems that include nonlinear equations. In future courses, students may encounter more advanced methods, such as matrices or Cramer's Rule, to solve systems of equations with more than two variables and more than two equations.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Systems of equations is an important topic to know for modeling real-world situations such as supply and demand or profit and cost.

HERE IS A SAMPLE QUESTION

If (x, y) is a solution to the system of equations, what is the value of $x - y$?

$$2x - 3y = -14$$
$$3x - 2y = -6$$

Multiplying the first equation by 3 and the second equation by −2 gives

$$6x - 9y = -42$$
$$-6x + 4y = 12$$

Then, adding the equations gives

$$-5y = -30$$
$$y = 6$$

The value of y can be substituted in one of the equations to get the value of x.
The solution is (4, 6), so $x - y = -2$.

NEW KEY TERMS

- system of linear equations [sistema de ecuaciones lineales]
- consistent systems [sistemas consistentes]
- inconsistent systems [sistemas inconsistentes]
- standard form of a linear equation [forma estándar/genera de una ecuación lineal]
- substitution method [método de sustitución]
- linear combinations method [método de combinaciones lineales]
- half-plane
- boundary line
- constraints
- solution of a system of linear inequalities [solución de un sistema de desigualdades lineales]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

The **standard form of a linear equation** is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

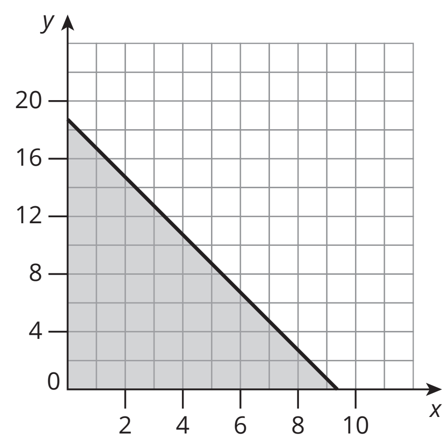
When two or more linear equations define a relationship between quantities, they form a **system of linear equations.**

The equations $y = 3x + 7$ and $y = -4x$ are a **system of linear equations**.

$$\begin{cases} y = 3x + 7 \\ y = -4x \end{cases}$$

The graph of a linear inequality is a **half-plane**, or half of a coordinate plane.

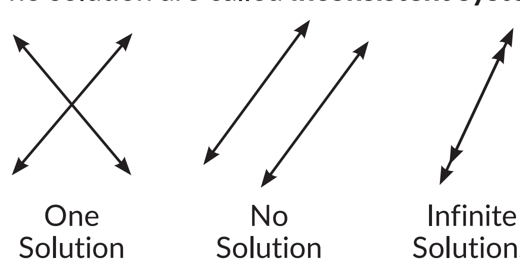
The shaded portion of the graph is a half-plane.



In **Lesson 1: Using Graphing to Solve Systems of Equations**, students write systems of linear equations and solve them graphically.

Solving Systems of Linear Equations by Graphing

The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. One way to predict the solution to a system is to graph both equations and identify the point at which the two graphs intersect. A system of equations may have no solution, one unique solution, or infinite solutions. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.



In **Lesson 2: Using Substitution to Solve Linear Systems**, students explore one of the two algebraic methods to solve a system.

Solving Systems of Linear Equations by Substitution

In many systems, it is difficult to determine the solution from the graph. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

Let's consider this system:

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Step 1: To use the substitution method, begin by choosing one equation and isolating one variable. This will be considered the first equation.

Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation.

Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$, and write the new equation.

$$1.25x + 1.05(8x) = 30$$

You have just created a new equation with only one unknown.

Step 3: Solve the new equation.

$$1.25x + 8.40x = 30$$

$$9.65x = 30$$

$$x \approx 3.1$$

Now, substitute the value for x into $y = 8x$ to determine the value of y.

$$y = 8(3.1) = 24.8$$

The solution to the system is about (3.1, 24.8).

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

MYTH

"Just give me the rule. If I know the rule, then I understand the math."

Memorize the following rule: *All quars are elos*. Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms*. How about now? Can you remember that? Of course you can, because now it makes sense.

Learning does not take place in a vacuum. It ***must be*** connected to what you already know. Otherwise, arbitrary rules will be forgotten.

#mathmythbusted

In **Lesson 3: Using Linear Combinations to Solve a System of Linear Equations**, students explore another method to solve a system algebraically.

Solving Systems of Linear Equations by Linear Combinations

The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

Consider this system of equations: $\begin{cases} 7x + 2y = 24 \\ 4x + y = 15 \end{cases}$

$$\begin{aligned} 7x + 2y &= 24 \\ -2(4x + y) &= -2(15) \end{aligned}$$

Multiply the second equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$\begin{aligned} 7x + 2y &= 24 \\ + \quad -8x - 2y &= -30 \\ \hline -x &= -6 \\ x &= 6 \end{aligned}$$

Now that the y-values are additive inverses, you can add the equations and solve this linear system for x.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ 2y &= -18 \\ y &= -9 \end{aligned}$$

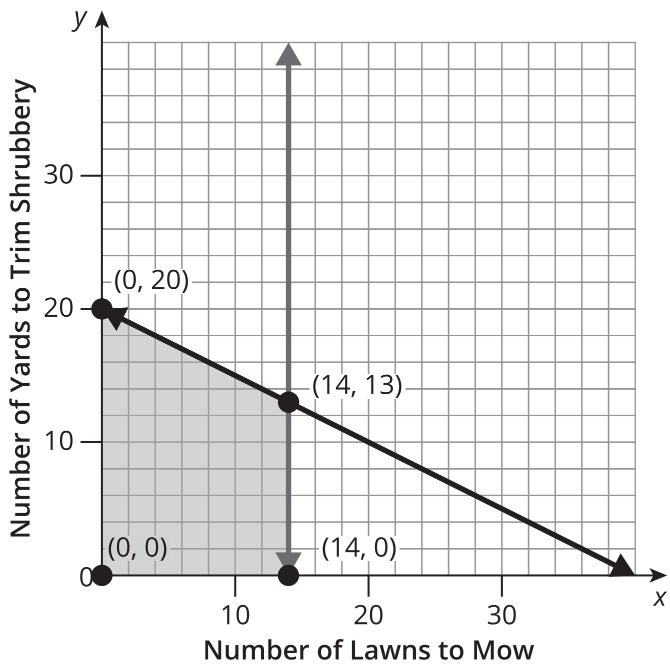
Substitute the value for x into one of the equations to determine the value for y.

The solution to the system of linear equations is $(6, -9)$.

In **Lesson 5: Systems of Linear Inequalities**, students graph a system of linear inequalities to determine possible solutions to the system.

Systems of Linear Inequalities

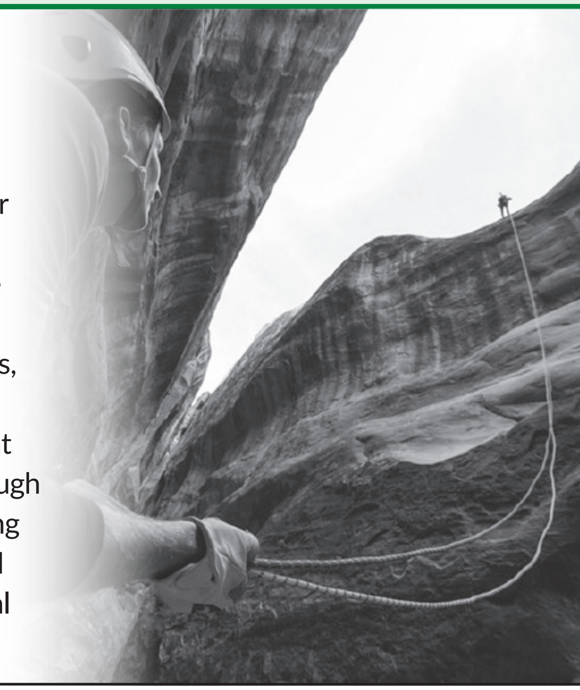
Often problem situations offer more than one solution to them. Students are going to use graphs of these problem situations that are expressed as systems of linear inequalities to identify solutions and non-solutions by recognizing key areas of the graphs.



MODULE 4

Investigating Growth and Decay

TOPIC 1 Introduction to Exponential Functions



At the start of this topic, students learn and apply properties of integer exponents to simplify expressions. Next, students build upon their previous understanding of sequences and common ratios to recognize that some geometric sequences are exponential functions and others are not. Then, students examine the structure of exponential functions, connecting the common ratio of a geometric sequence with the base of the power in an exponential function. Students explore the constant ratio between intervals of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ for the function $f(x) = 2^x$. Through this exploration, they conclude that $2^{\frac{1}{2}} = \sqrt{2}$ and $2^{\frac{1}{3}} = \sqrt[3]{2}$. By graphing these values, the misconception that $f\left(\frac{1}{2}\right)$ is halfway between $f(0)$ and $f(1)$ is addressed. Finally, students learn and apply properties of rational exponents to simplify expressions.

Where have we been?

Students have learned to write and evaluate numerical and algebraic expressions with whole number exponents in middle school. This topic expands on that knowledge. Earlier in this course, students were introduced to geometric sequences. Students can already write a recursive and an explicit formula for a given geometric sequence.

Where are we going?

Throughout this topic, students apply what they know about the key characteristics of functions (e.g., intercepts, intervals of increase or decrease, and domain and range) to include exponential functions. This prepares them for the work they will do with quadratic functions, both in this course and with more complex functions in future courses.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Exponential functions are an important topic to know about for post-secondary readiness, including college admission tests.

HERE IS A SAMPLE QUESTION

A biology class predicted that a population of animals will double in size every year. The population at the start of 2024 was about 500 animals. If *P* represents the population *n* years after 2024, what equation represents the model of the population over time?

To solve this, students should know that this represents an exponential function because the population doubles each year. This can be written as:

$$\textit{initial value} \cdot (\textit{growth rate})^{\textit{time}}$$

The initial value is 500 animals, and the growth rate is 2, for doubling. So, the function $P(n) = 500 \cdot 2^n$ models the population over time in years, n.

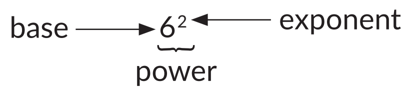
NEW KEY TERMS

- power [potencia]
- base [base]
- exponent [exponente]
- horizontal asymptote [asíntota horizontal]
- perfect square
- square root
- radical [radical]
- radicand [radicando]
- product property of radicals [propiedad del producto de radicales]
- extracting perfect squares
- index [índice]
- quotient property of radicals [propiedad del cociente de radicales]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **power** has two elements: the base and the exponent.



The **base** of a power is the factor that is multiplied repeatedly in the power.

$2^3 = 2 \cdot 2 \cdot 2 = 8$

base

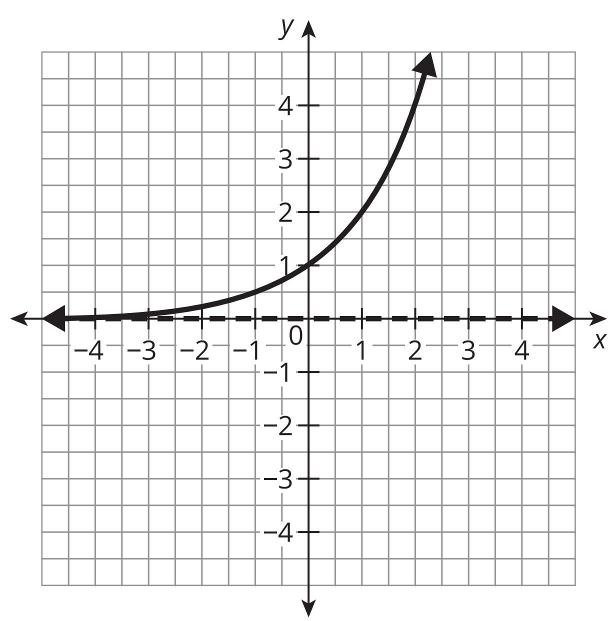
The **exponent** of the power is the number of times the base is used as a factor.

$2^3 = 2 \cdot 2 \cdot 2 = 8$

exponent

A **horizontal asymptote** is a horizontal line that a function gets closer and closer to, but never intersects.

The graph shows a horizontal asymptote at $y = 0$.



In **Lesson 1: Properties of Powers with Integer Exponents,** students write and evaluate expressions with positive integer exponents.

Power Properties

Powers have properties that can be used to expand, simplify, or just rewrite them differently. These properties are important in later math classes and students must be able to remember them because they are useful tools in algebra.

Properties of Powers	Words	Rule
Product of powers	To multiply powers with the same base, keep the base and add the exponents.	$a^m a^n = a^{m+n}$
Power of a power rule	To simplify a power of a power, keep the base and multiply the exponents.	$(a^m)^n = a^{mn}$
Quotient of powers rule	To divide powers with the same base, keep the base and subtract the exponents.	$\frac{a^m}{a^n} = a^{m-n}$, if $a \neq 0$
Zero power	The zero power of any number, except for 0, is 1.	$a^0 = 1$, if $a \neq 0$
Negative exponents in the numerator	An expression with a negative exponent in the numerator and a 1 in the denominator equals 1 divided by the power with its opposite exponent placed in the denominator.	$a^{-m} = \frac{1}{a^m}$, if $a \neq 0$ and $m > 0$
Negative exponents in the denominator	An expression with a negative exponent in the denominator and a 1 in the numerator equals the power with its opposite exponent.	$\frac{1}{a^{-m}} = a^m$, if $a \neq 0$ and $m > 0$

In **Lesson 3: Geometric Sequences and Exponential Functions**, students revisit geometric sequences as a launch to exponential functions.

Constant Ratios

Students make the connection between a geometric sequence and an exponential function of the form $f(x) = ab^x$. They prove with algebra that there is always a constant ratio from one output value to the next of an exponential function.

The sequence 3, 6, 12, 24 is a geometric sequence with a common ratio of 2 because in each step, the value is multiplied by 2.

MYTH

Cramming for a test is just as good as spaced practice for long-term retention.

Everyone has been there. You have a big test tomorrow, but you've been so busy that you haven't had time to study. So, you had to learn it all in one night. You may have received a decent grade on the test. However, did you remember the material a week, a month, or a year later?

The honest answer is, "probably not." That's because long-term memory is designed to retain useful information. How does your brain know if a memory is "useful" or not? One way is the frequency in which you encounter a piece of information. If you see something only once (like during cramming), then your brain doesn't deem those memories as important. However, if you sporadically come across the same information over time, your brain is more likely to recognize it as important. To optimize retention, encourage your student to periodically study the same information over expanding intervals of time.

#mathmythbusted

In **Lesson 4: Rewriting Square Roots**, students use the Properties of Radicals to simplify square root expressions.

In **Lesson 5: Rational Exponents and Graphs of Exponential Functions**, students learn that the Properties of Powers apply to expressions with rational exponents, rewrite expressions with rational exponents as radicals, and connect these two concepts to perform and justify operations involving radicals.

Extracting Perfect Squares

Students will be able to rewrite radical expressions by **extracting perfect squares**. This is the process of removing perfect squares from under the radical symbol. In the example below, the number 450 is broken down into its factors, which are the numbers that can be multiplied to equal 450. This is done using division. When 450 has been divided to form its prime factorization, the numbers 3 and 5 appear two times, which means that they can be combined into perfect squares and removed from under the radical symbol. This is because $\sqrt{3 \cdot 3} = \sqrt{3^2} = 3$. After removing the 3 and 5, they are multiplied to equal 15, and the factor 2 stays under the radical symbol.

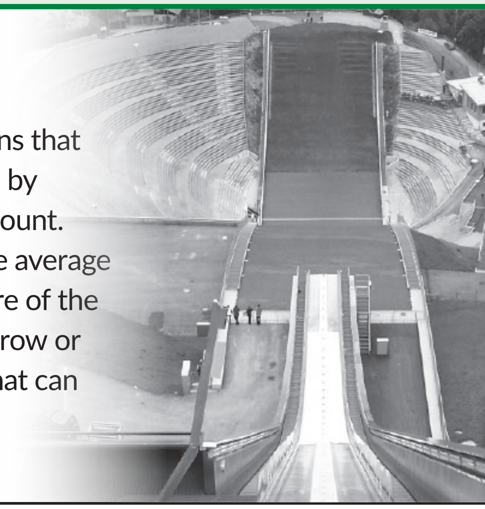
$$
\begin{aligned}
\sqrt{450} &= \sqrt{2 \cdot 225} \\
&= \sqrt{2 \cdot 3 \cdot 75} \\
&= \sqrt{2 \cdot 3 \cdot 3 \cdot 25} \\
&= \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5} \\
&= \sqrt{2 \cdot 3^2 \cdot 5^2} \\
&= 3 \cdot 5\sqrt{2} \\
&= 15\sqrt{2}
\end{aligned}
$$

Family Guide

MODULE 4 Investigating Growth and Decay

Algebra I

TOPIC 2 Using Exponential Equations



In this topic, students explore strategies for distinguishing exponential functions that represent growth scenarios versus those that represent decay. Students begin by comparing the value of a simple interest account and a compound interest account. They graph and write equations for these two scenarios and then compare the average rate of change of each for a given interval. Students then examine the structure of the exponential equations to recognize scenarios in which exponential functions grow or decay. Throughout the rest of the topic, students solve real-world problems that can be modeled by exponential functions. Students use technology to generate exponential regressions and use those functions to make predictions.

Where have we been?

Students know the rules of exponents and are familiar with the structure of exponential functions from their work in the previous topic. This topic is an opportunity to reinforce their understanding and build upon their skills by recognizing and solving problems that can be modeled by exponential functions. In previous courses, students analyzed bivariate data looking for linear, nonlinear, or no association and used trend lines to make predictions. Previously in this course, they used technology to generate linear regressions and analyzed the correlation coefficient for fit to the data set. In this topic, students use technology to generate exponential regressions to introduce a new function to fit the data.

Where are we going?

This topic represents students' first deep dive into solving equations that represent nonlinear functions. As students gain proficiency in solving increasingly complex equations, they are able to model more interesting and complex real-world phenomena.

TALKING POINTS

Discuss with your student

Exponential functions represent real-world situations such as compound interest and bacterial growth. They are also important to know for college admissions tests.

HERE IS A SAMPLE QUESTION

A car valued at $21,000 depreciates at a rate of 17% per year. What is the value of the car after 5 years?

To solve, students should know to use the model for exponential decay, $y = a(1 - r)^x$, where a represents the initial value, r represents the rate of decrease, and x represents time.

$$y = a(1 - r)^x$$
$$y = a(1 - 0.17)^x$$
$$y = 21{,}000(0.83)^5$$
$$y = 8271.99$$

In 5 years, the car will be worth $8271.99.

NEW KEY TERMS

- simple interest [interés simple]
- compound interest [interés compuesto]
- exponential growth function
- exponential decay function [función de decaimiento exponencial]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

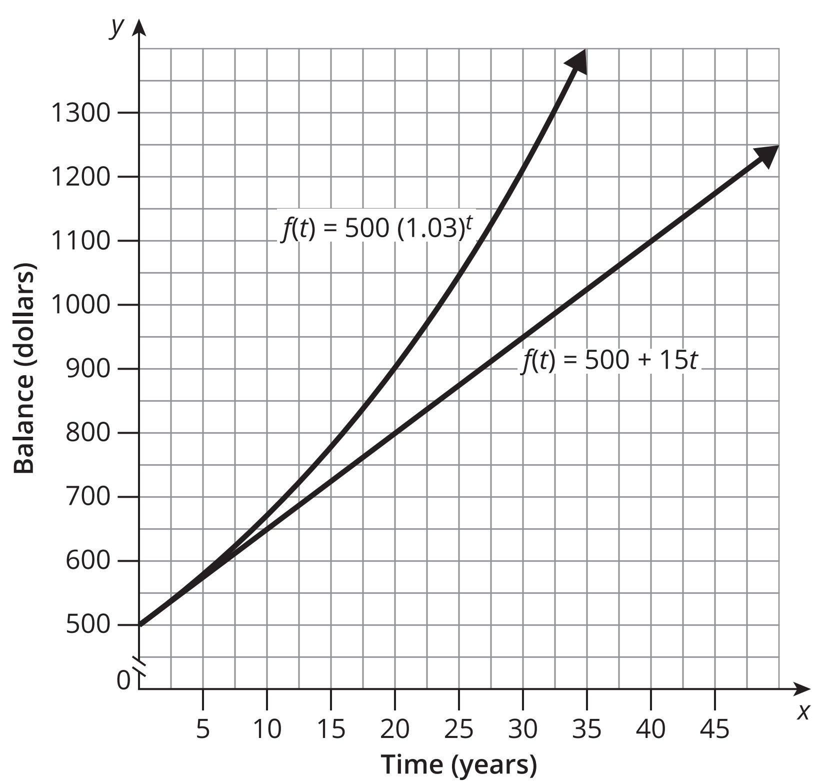
An **exponential growth function** is an exponential function with a b-value greater than 1 and is of the form $y = a(1 + r)^x$, where r is the rate of growth.
An **exponential decay function** is an exponential function with a b-value greater than 0 and less than 1 and is of the form $y = a(1 - r)^x$, where r is the rate of decay.

In **Lesson 1: Exponential Equations for Growth and Decay**, students compare linear and exponential functions in the context of simple interest and compound interest situations. They then identify the values in the exponential function equation that indicate whether it is a growth or decay function and apply this reasoning in context.

Simple and Compound Interest

Learning about simple and compound interest teaches students about saving and borrowing money. With **simple interest**, money grows at the same rate over time, while with **compound interest** money grows at a faster rate as time passes. Knowing how percentage growth makes money grow faster and faster is the foundation for building wealth.

Time (years)	Simple Interest Balance (dollars)	Compound Interest Balance (dollars)
0	500	500
1	515	515
2	530	530.45
10	650	671.96
100	2000	9609.32

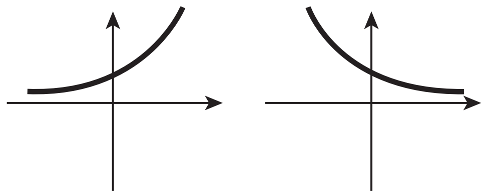


$f(t) = 500(1.03)^t$

- $a = 500$
- $b = 1.03$
- $r = 0.03$

Exponential Growth and Decay

Some things in nature, like bacteria, grow faster as time passes. This is called *exponential growth*. In the same way, some things like a healing wound will become smaller or shrink faster as time passes. This is called *exponential decay*. You can use the exponential function $f(x) = ab^x$ to model exponential growth and exponential decay. An exponential growth function is in the form $f(x) = a(1 + r)^x$, where r is the rate of growth. An exponential decay function is in the form $f(x) = a(1 - r)^x$, where r is the rate of decay.

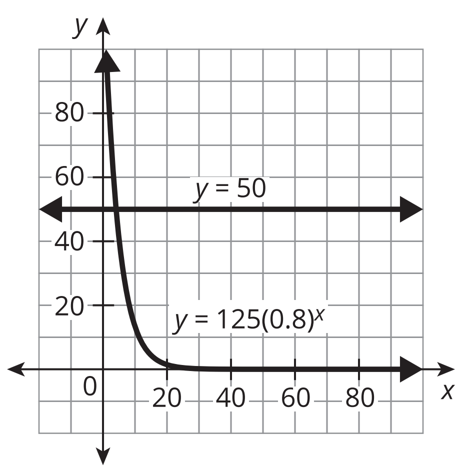


In **Lesson 2: Interpreting Parameters in Context**, students recall how to solve an equation graphically by graphing both sides of the equation and determining the point of intersection.

Solving Exponential Equations by Graphing

Graphs can be used to solve exponential equations by estimating the intersection point of the graph of an exponential function with a constant function.

For example, to determine the solution to $125(0.8)^x = 50$, graph each side of the equation as separate functions on the same coordinate plane.



The solution to the equation is $x \approx 4$.

In **Lesson 3: Modeling Using Exponential Functions**, students create a scatterplot, write a regression equation, use the function to calculate output values, and interpret the reasonableness of a prediction based on the scenario.

MYTH

"I'm not smart."

The word *smart* is tricky because it means different things to different people. For example, would you say a baby is "smart"? On one hand, a baby is helpless and doesn't know anything. But on the other hand, a baby is insanely smart because they are constantly learning new things every day!

This example is meant to demonstrate that *smart* can have two meanings. It can mean *the knowledge that you have*, or it can mean *the capacity to learn from experience*. When someone says they are "not smart," are they saying they do not have much knowledge, or are they saying that they lack the capacity to learn? If it's the first definition, then none of us are smart until we acquire information. If it's the second definition, then we know this is completely untrue because everyone has the capacity to grow as a result of new experiences.

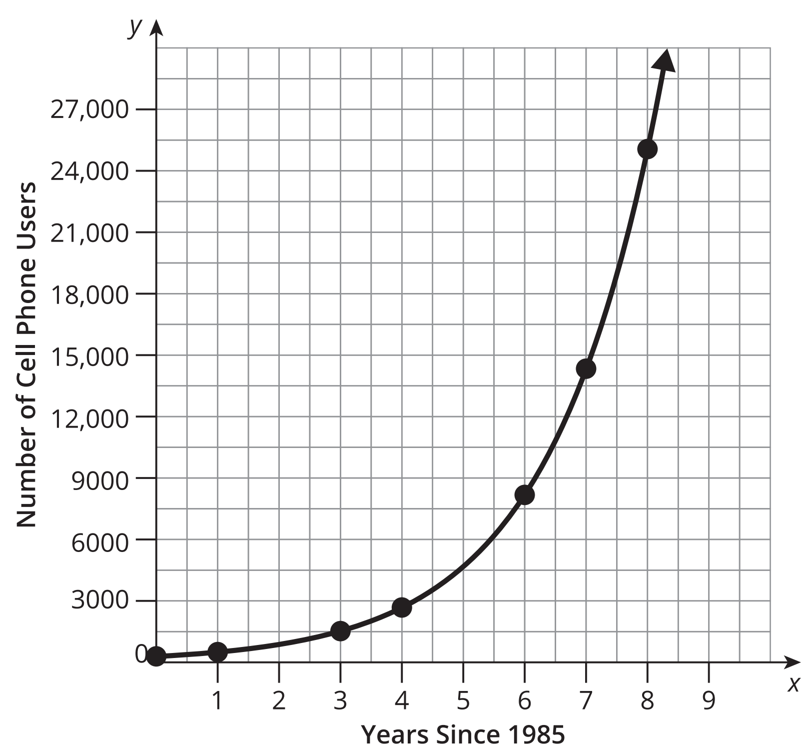
So, if your student doesn't think that they are smart, encourage them to be patient. They have the capacity to learn new facts and skills. It might not be easy, and it will take some time and effort, but the brain is automatically wired to learn. Smart should not refer only to how much knowledge you currently have.

#mathmythbusted

Exponential Regression

Students learn that just as some data points almost match the shape of a straight line, others match more closely to a curve that is made by an exponential function. This makes it possible to make predictions about things that grow exponentially in the same way that we can with linear data.

Year Since 1985	Number of Cell Phone Users
0	285
1	498
3	1527
4	2672
6	8186
7	14,325
8	25,069



MODULE 5

Maximizing and Minimizing

Family Guide

MODULE 5 Maximizing and Minimizing

Algebra I

TOPIC 1 Introduction to Quadratic Functions

In *Introduction to Quadratic Functions*, students begin by exploring four situations that can be represented with quadratic functions. Students then represent each situation with an equation, a graph, and a table of values and explore the characteristics of the functions represented by each situation and different forms of a quadratic function. They use what they have learned about function transformations and apply this knowledge to transforming quadratic functions.

Where have we been?

In *Linear Functions*, students learned that linear functions are polynomials of degree 1. They have written linear functions in general form as $f(x) = ax + b$ and in factored form as $y = a(x - c)$. Students have learned that the zeros of a function are the places where the function crosses the x-axis, and they can identify zeros on a graph. *Introduction to Quadratic Functions* is a direct extension of these concepts; students learn that quadratic functions are polynomials of degree 2 and have characteristics similar to linear functions.

Where are we going?

In this topic, students will solidify their knowledge of function transformations. Understanding how to sketch a quadratic function is the underpinning for sketching more complicated polynomials in higher levels of mathematics.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Recognizing functions from a table of values is an important topic to know about for college admissions tests.

HERE IS A SAMPLE QUESTION

What type of function models this table of values?

x	$f(x)$
0	1
1	–2
2	–1
3	4
4	13

Because the x-values are consecutive, analyze consecutive $f(x)$-values. First differences are (–3), 1, 5, and 9. *Second differences*, the differences between first differences, are 4, 4, and 4. Because second differences are equal, the function is quadratic.

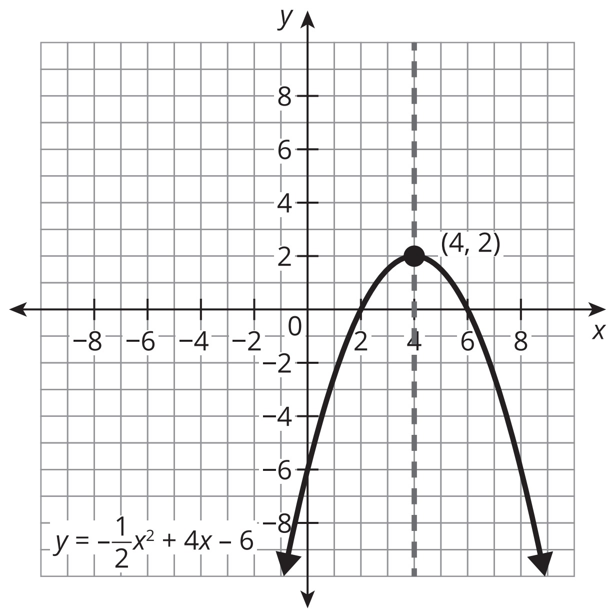
NEW KEY TERMS

- parabola [parábola]
- vertical motion model [modelo de movimiento vertical]
- roots [raíces]
- second differences [segundas diferencias]
- standard form of a quadratic function [forma estándar de una función cuadrática]
- factored form [forma factorizada]
- concave down
- concave up
- vertex [vértice]
- axis of symmetry [eje de simetría]
- argument of a function [argumento de una función]
- reflection [reflexión]
- line of reflection [línea de reflexión]
- vertex form [forma de vértice]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

The shape that a quadratic function forms when graphed is called a *parabola*. A **parabola** is a smooth curve with reflectional symmetry.



Second differences are the differences between consecutive values of the first differences.

x	y
−3	−5
−2	0
−1	3
0	4
1	3
2	0
3	−5

First Differences

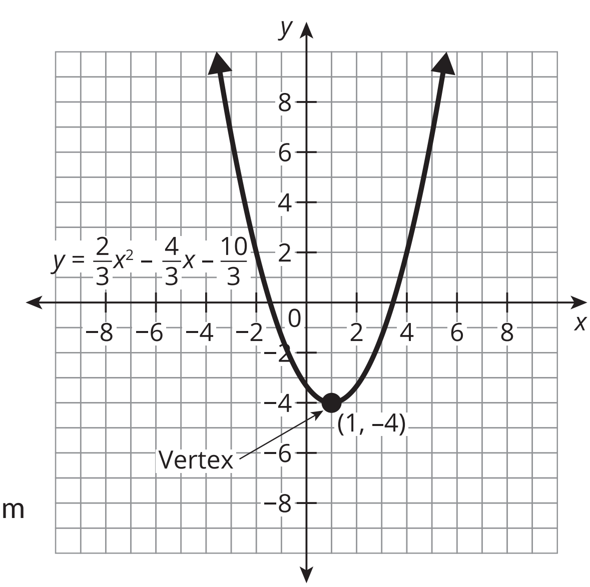
- $0 - (-5) = 5$
- $3 - 0 = 3$
- $4 - 3 = 1$
- $3 - 4 = -1$
- $0 - 3 = -3$
- $-5 - 0 = -5$

Second Differences

- $3 - 5 = -2$
- $1 - 3 = -2$
- $-1 - 1 = -2$
- $-3 - (-1) = -2$
- $-5 - (-3) = -2$

The **vertex of a parabola** is the lowest or highest point on the graph of the quadratic function.

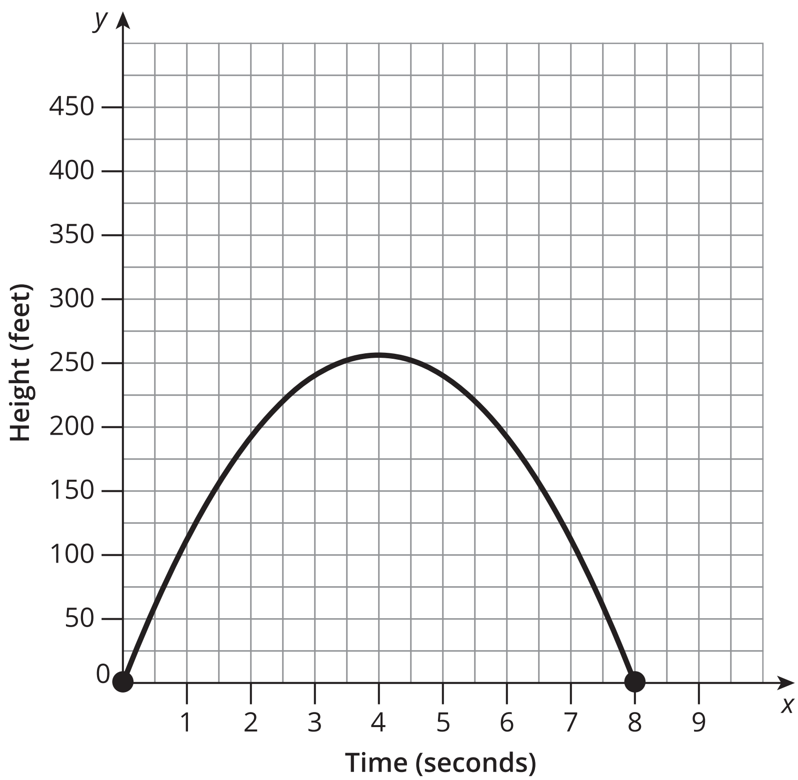
The vertex of the graph of $y = \frac{2}{3}x^2 - \frac{4}{3}x - \frac{10}{3}$ is the point $(1, -4)$, the absolute minimum of the parabola.



In **Lesson 1: Exploring Quadratic Functions**, students are introduced to quadratic functions.

Parabolas

Vertical motion models are a basic example of how parabolas are used every day. Any time an object is tossed into the air, gravity has an effect on it, the object reaches a maximum height, and then falls back down to the ground.



In **Lesson 2: Key Characteristics of Quadratic Functions**, students are introduced to the various forms of quadratic functions.

Forms of Quadratics

There are three forms of quadratic functions that students will encounter and use in different ways. The standard form, which is also called the general form, is $f(x) = ax^2 + bx + c$.

Factored form is $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$ and r_1 and r_2 represent the roots.

Vertex form is $f(x) = a(x - h)^2 + k$, where (h, k) represents the vertex of the function.

MYTH

Some students are "right-brain" learners, while other students are "left-brain" learners.

As you probably know, the brain is divided into two hemispheres: the right and the left. Some categorize people by their preferred or dominant mode of thinking. "Right-brain" thinkers are considered to be more intuitive, creative, and imaginative. "Left-brain" thinkers are said to be more logical, verbal, and mathematical.

The brain can also be broken down into lobes. The *occipital lobe* can be found in back of the brain, and it is responsible for processing visual information. The *temporal lobes*, which sit above your ears, process language and sensory information. The band across the top of your head is the *parietal lobe*, and it controls movement. Finally, the *frontal lobe* is where planning and learning occurs. Another way to think about the brain is from the back to the front, where information goes from highly concrete to abstract.

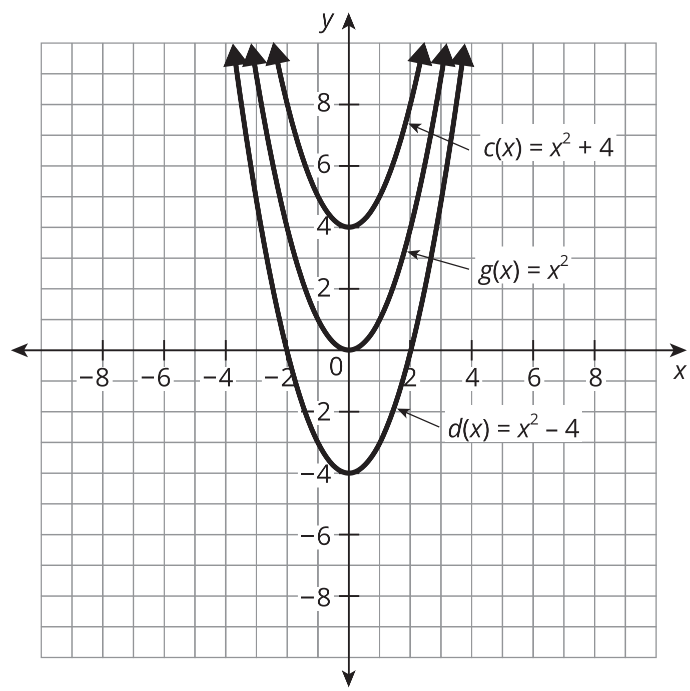
Why don't we claim that some people are "back of the brain" thinkers, who are highly concrete; whereas, others are "frontal" thinkers, who are more abstract? The reason is that the brain is a highly interconnected organ. Each lobe hands off information to be processed by other lobes, and they are constantly talking to each other. All of us are *whole-brain* thinkers!

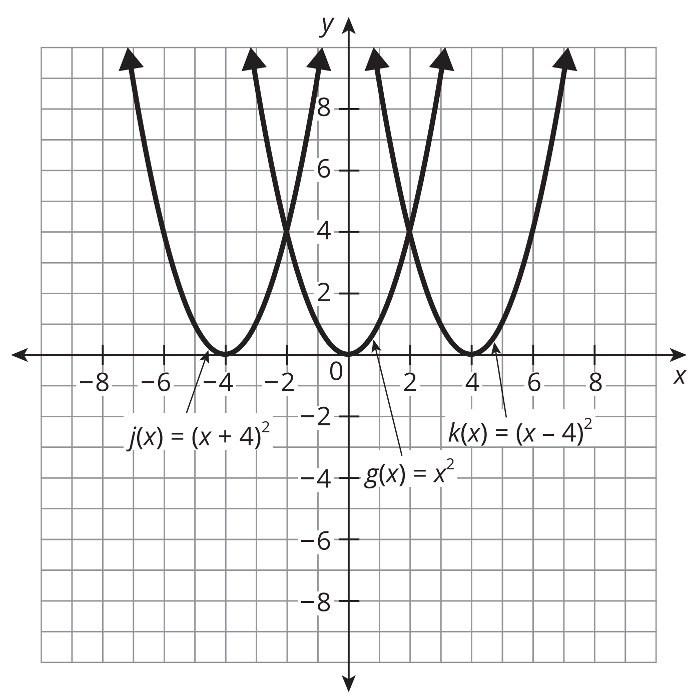
#mathmythbusted

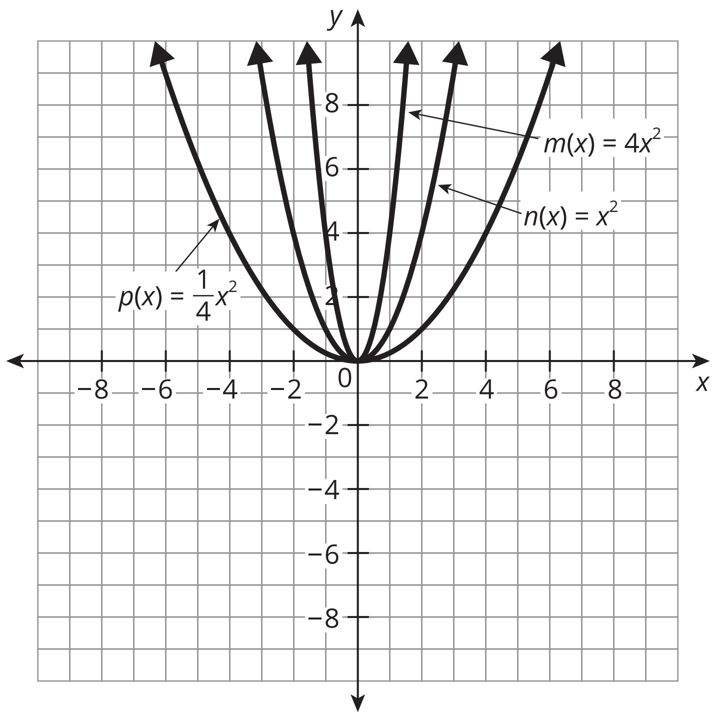
In **Lesson 3: Quadratic Function Transformations** and **Lesson 4: Horizontal Transformations and Vertex Form**, students expand their understanding of transformations to include quadratic functions.

Transformations

Transformations were first introduced with linear functions and are now applied to quadratic functions. Students will apply vertical and horizontal translations, vertical and horizontal dilations (stretches or compressions), and reflections across both axes.






y
x
m(x) = 4x^2
n(x) = x^2
p(x) = 1/4 x^2
8
6
4
2
0
−2
−4
−6
−8
−8
−6
−4
−2
2
4
6
8

Family Guide

MODULE 5 Maximizing and Minimizing

Algebra I

TOPIC 2 Polynomial Operations

Polynomial Operations begins with a review of polynomials. Students sort various polynomial expressions on whatever basis makes sense to them—they can use what they already know about terms, powers, and leading coefficients, or they may sort using other prior knowledge. Students add and subtract polynomials, first with a graphical representation of a context, and then using algebra. They build upon their knowledge of algebra tiles and an area model to multiply binomials before using the distributive property. Using the area model to multiply binomials also prepares students to factor trinomials, which they will do in the next topic. Next, students learn to divide polynomials by using polynomial long division and they interpret what the quotient and remainder mean. They learn about the factor theorem and the remainder theorem, specifically that if a polynomial is divided by a linear expression, $x - r$, and the remainder is 0, then the expression $x - r$ is a factor of the polynomial. If the remainder is not 0, then $x - r$ is not a factor of the polynomial.

Where have we been?

Beginning in elementary school, students used area models to multiply. This tool is helpful in multiplying whole numbers, fractions, and mixed numbers, and in learning the distributive property. This prior knowledge helps students to multiply a monomial by a binomial and two binomials. It also serves as the foundation for factoring of a trinomial. In previous courses, students divided multi-digit numbers using standard algorithms, such as long division, and interpreted the remainders. Students will use this prior knowledge as they divide polynomials of degree two by polynomials of degree one using polynomial long division.

Where are we going?

In future courses, students use the distributive property to multiply complex numbers. A complex number is of the form $(a + bi)$; it has a real number term a and an imaginary term bi. They will multiply and then use the fact that $i^2 = -1$ to rewrite the expression.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Special products when multiplying binomials is an important topic to know about for college admissions tests.

HERE IS A SAMPLE QUESTION

What patterns do you notice between the factors and the products?

$(x - 4)(x + 4) = x^2 - 16$ $\quad$ $(x - 3)(x + 3) = x^2 - 9$

$(x + 4)(x + 4) = x^2 + 8x + 16$ $\quad$ $(x + 3)(x + 3) = x^2 + 6x + 9$

$(x - 4)(x - 4) = x^2 - 8x + 16$ $\quad$ $(x - 3)(x - 3) = x^2 - 6x + 9$

The *difference of two squares* is an expression in the form $a^2 - b^2$ that has factors $(a - b)(a + b)$.

A *perfect square trinomial* is an expression in the form $a^2 + 2ab + b^2$ or the form $a^2 - 2ab + b^2$.

NEW KEY TERMS

- polynomial [polinomio]
- monomial [monomio]
- binomial [binomio]
- trinomial [trinomio]
- degree of a polynomial [grado de un polinomio]
- difference of two squares
- perfect square trinomial
- factor theorem [teorema del factor]
- polynomial long division [división (larga) de polinomios]
- remainder theorem

Where are we now?

Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree.

$$
\begin{array}{r}
4x^2 - 6x + 3 \\
2x + 3 \,\overline{)\, 8x^3 + 0x^2 - 12x - 7} \\
-(8x^3 + 12x^2) \\
\hline
-12x^2 - 12x \\
-(-12x^2 - 18x) \\
\hline
6x - 7 \\
-(6x + 9) \\
\hline
\text{Remainder } \; \boxed{-16}
\end{array}
$$

In **Lesson 1: Adding and Subtracting Polynomials**, students are introduced to the formal definitions regarding polynomials. They also add and subtract functions both graphically and algebraically within a context.

Polynomials

A **polynomial** is an expression involving the sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable is the sum of the form ax^k, where a, called the *coefficient*, is a real number and k is a non-negative integer. A polynomial is written in standard form when the terms are in descending order, starting with the term with the greatest degree and ending with the term with the least degree.

$$a_1x^k + a_2x^{k-1} + \ldots a_nx^0$$

Each product in a polynomial is called a *term*. Polynomials are named according to the number of terms: a **monomial** has exactly 1 term, a **binomial** has exactly 2 terms, and a **trinomial** has exactly 3 terms. The exponent of a term is the degree of the term, and the greatest exponent in a polynomial is the **degree of the polynomial**.

The characteristics of the polynomial $15x^3 + 7x + 3$ are shown in the chart.

	1st term	2nd term	3rd term
Term	$15x^3$	$7x$	3
Coefficient	15	7	3
Power	x^3	x^1	x^0
Exponent	3	1	0

This trinomial has a degree of 3 because 3 is the greatest degree of the terms in the trinomial.

Adding and Subtracting Polynomial expressions

Polynomials can be added or subtracted by identifying the like terms of the polynomial functions, using the associative property to group the like terms together, and combining the like terms to simplify the expression.

Example 1

Expression: $(7x^2 - 2x + 12) + (8x^3 + 2x^2 - 3x)$

The like terms are $7x^2$ and $2x^2$, and $-2x$ and $-3x$. The terms $8x^3$ and 12 are not like terms.

$(7x^2 - 2x + 12) + (8x^3 + 2x^2 - 3x)$
$8x^3 + (7x^2 + 2x^2) + (-2x - 3x) + 12$
$8x^3 + 9x^2 - 5x + 12$

Example 2

Expression: $(4x^4 + 7x^2 - 3) - (2x^2 - 5)$

The like terms are $7x^2$ and $2x^2$, and -3 and -5. The term $4x^4$ does not have a like term.
$(4x^4 + 7x^2 - 3) - (2x^2 - 5)$
$4x^4 + (7x^2 - 2x^2) + (-3 + 5)$
$4x^4 + 5x^2 + 2$

In **Lesson 2: Multiplying Polynomials**, students use algebra tiles and multiplication tables to multiply binomials.

Multiplying Binomials Using Algebra Tiles

Students can use algebra tiles to model two binomials and determine their product.

Getting Started

Modeling Binomials

Represent each binomial with algebra tiles.

x	1

$x + 1$

x	1	1

$x + 2$

Create an area model using each binomial

•	x	1	1
x	x^2	x	x
1	x	1	1

The product of $(x + 1)(x + 2)$ is $x^2 + x + x + x + 1 + 1$. This simplifies to $x^2 + 3x + 2$.

MYTH

"If I can get the right answer, then I should not have to explain why."

Sometimes, you get the right answer for the wrong reasons. Suppose a student is asked, "What is 4 divided by 2?" and she confidently answers "2!" If she does not explain any further, then it might be assumed that she understands how to divide whole numbers. But, what if she used the following rule to solve that problem? "Subtract 2 from 4 one time." Even though she gave the right answer, she has an incomplete understanding of division.

However, if she is asked to explain her reasoning, either by drawing a picture, creating a model, or giving a different example, the teacher has a chance to remediate her flawed understanding. If teachers aren't exposed to their students' reasoning for both right and wrong answers, then they won't know about or be able to address common misconceptions. This is important because mathematics is cumulative in the sense that new lessons build upon previous understandings.

You should ask your student to explain their thinking, when possible, even if you don't know whether the explanation is correct. When children (and adults!) explain something to someone else, it helps them learn. Just the process of trying to explain is helpful.

#mathmythbusted

Multiplying Binomials Using Multiplication Tables

Students can use multiplication tables to multiply binomials and determine the product.

$$(9x - 1)(5x + 7)$$

$\cdot$	$9x$	-1
$5x$	$45x^2$	$-5x$
7	$63x$	-7

$$\begin{aligned}(9x - 1)(5x + 7) &= 45x^2 - 5x + 63x - 7\\ &= 45x^2 + 58x - 7\end{aligned}$$

In **Lesson 3: Polynomial Division**, students are introduced to polynomial long division. They perform polynomial long division to determine the linear function that is the other factor, and students use this information to determine the zeros and rewrite the quadratic function as a product of linear factors.

Factor and Remainder Theorem

Students will use the factor theorem to show that a linear expression is a factor of a polynomial. After determining if a linear expression is a factor, they may use polynomial long division to determine the other factor of the polynomial.

If $f(x) = 3x^2 + 10x + 1$, to determine if $(x + 4)$ is a factor of $f(x)$, we must determine the value of $f(-4)$.

$f(-4) = 3(-4)^2 + 10(-4) + 1$
$f(-4) = 9$

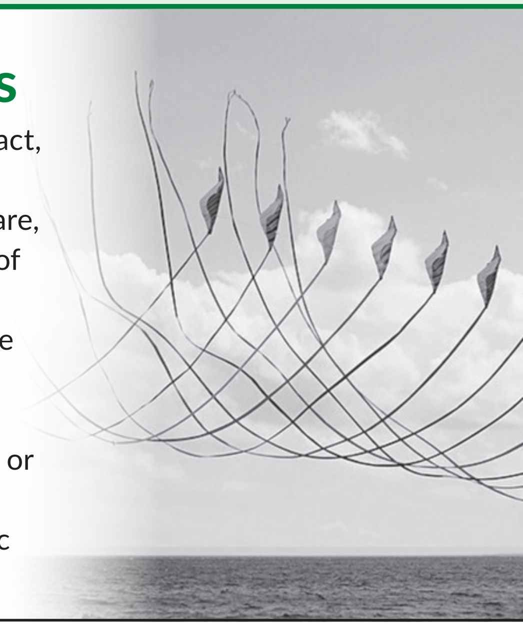
This means the remainder when $f(x)$ is divided by $(x + 4)$ is 9, not 0; therefore, $(x + 4)$ is not a factor of $f(x) = 3x^2 + 10x + 1$.

Polynomial Long Division

Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree. The process is similar to integer long division.

Integer Long Division	Polynomial Long Division
$3660 \div 12$ or $\frac{3660}{12}$ 305 12)3660 −36 6 −0 60 −60 0	$(2x^2 + 5x - 12) \div (x + 4)$ or $\frac{2x^2 + 5x - 12}{x + 4}$ (A) $2x$ (D) -3 $x + 4 \overline{)2x^2 + 5x - 12}$ (B) $-(2x^2 + 8x)$ ↓ (C) $-3x - 12$ $-(-3x - 12)$ Remainder 0 A. Divide $\frac{-x^2}{x} = 2x$. B. Multiply $2x(x + 4)$ and then subtract. C. Bring down −12. D. Divide $\frac{23x}{x} = -3$. E. Multiply $-3(x + 4)$ and then subtract.

TOPIC 3 Solving Quadratic Equations



Students review polynomials. They use different methods to add, subtract, multiply, and divide polynomials. In this topic, students solve quadratic equations using the traditional methods—factoring, completing the square, and the quadratic formula—they use what they know about properties of equality, square roots, and parabolas to solve equations of the forms $x^2 = n$ and $ax^2 - c = n$. They then learn to factor or complete the square to solve quadratic equations and real-world problems. Next, students derive the quadratic formula. They see the structure of solutions to quadratic equations in the quadratic formula: the axis of symmetry plus or minus the distance to the parabola. Finally, students are presented with a real-world situation and use familiar strategies to complete a quadratic regression to determine the curve of best fit.

Where have we been?

Students know the characteristics that define a quadratic function. They have explored zeros of functions and have interpreted their meaning in contexts. Students know that the factored form of a quadratic equation gives the zeros of the function. They can sketch quadratic equations using key characteristics from an equation written in different forms. Students have extensive experience with locating solutions to equations using a graphical representation.

Where are we going?

The techniques for solving quadratics will be applicable as students solve higher-order polynomials in future math courses. Understanding the structure and symmetry of a quadratic equation allows students to solve quadratics with complex roots as well as higher-order polynomials.

HERE IS A SAMPLE QUESTION

The graph of $y = (x - 8)(x + 2)$ is a parabola in the xy-plane. Rewrite the equation in an equivalent form so that the x- and y-coordinates of the vertex of the parabola appear as constants.

To solve this, students might use the process of completing the square.

$y = (x - 8)(x + 2)$
$y = x^2 - 6x - 16$
$y + 16 = x^2 - 6x$
$y + 16 + 9 = x^2 - 6x + 9$
$y + 25 = (x - 3)^2$

$y = (x - 3)^2 - 25$ is the vertex form of the equation of the parabola with vertex at $(3, -25)$.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Equivalent forms of quadratic equations is an important topic to know about for college admissions tests.

NEW KEY TERMS

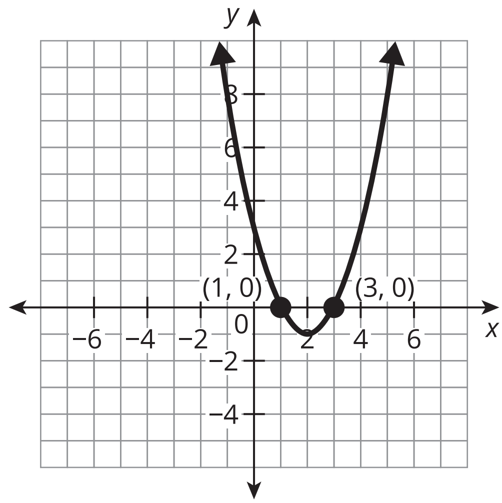
- principal square root
- roots [raíces]
- double root [raíz doble]
- zero product property [propiedad del producto cero]
- completing the square
- quadratic formula [fórmula cuadrática]
- discriminant [discriminante]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **root** of an equation indicates where the graph of the equation crosses the x-axis.

The roots of the quadratic equation $x^2 - 4x + 3 = 0$ are $x = 3$ and $x = 1$.



The **quadratic formula** is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and can be used to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$.

The **discriminant** is the radicand expression in the quadratic formula which "discriminates" the number of real roots of a quadratic equation.

The discriminant in the quadratic formula is the expression $b^2 - 4ac$.

In **Lesson 2: Solutions to Quadratic Equations in Vertex Form,** students learn to solve quadratics equations of the form $y = a(x - h)^2 + k$.

For example, consider the equation $20 = 2(x - 1)^2 + 2$

$$1 \pm \sqrt{\frac{20-2}{2}} = x$$

$$1 \pm \sqrt{9} =$$

$$1 \pm 3 =$$

The solutions to the equation are 3 units away from the axis of symmetry, $x = 1$ The solutions are $x = -2$ and $x = 4$.

In **Lesson 3: Factoring and Completing the Square**, students learn to solve quadratic equations of the form $y = ax^2 + bx + c$.

Solving Quadratic Equation by Factoring

Students can factor trinomials by rewriting them as the product of two linear expressions. They can use factoring and the zero product property to solve quadratics in the form $y = ax^2 + bx + c$.

First, the equation is set equal to zero by adding 3 to each side. Next, the left side of the equation is divided into two factors, $(x - 3)$ and $(x - 1)$. Because the two factors multiply to equal zero, at least one of them must be equal to zero. This means that solving the equations $x - 1 = 0$ and $x - 3 = 0$ will result in at least one solution to the quadratic equation.

$$x^2 - 4x = -3$$
$$x^2 - 4x + 3 = -3 + 3$$
$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$

$$(x - 3) = 0 \quad \text{or} \quad (x - 1) = 0$$
$$x - 3 + 3 = 0 + 3 \quad \text{or} \quad x - 1 + 1 = 0 + 1$$
$$x = 3 \quad \text{or} \quad x = 1$$

Completing the Square to Determine Roots

Students can use the completing the square method to determine the roots of a quadratic equation that cannot be factored.

Determine the roots of the equation $x^2 + 10x + 12 = 0$.

Isolate $x^2 + 10x$. You can complete the square and rewrite this as a perfect square trinomial.

$$x^2 + 10x + 12 - 12 = 0 - 12$$
$$x^2 + 10x = -12$$

Determine the constant term that would complete the square. Add this term to both sides of the equation.

$$x^2 + 10x + \underline{\quad\quad} = -12 + \underline{\quad\quad}$$
$$x^2 + 10x + 25 = -12 + 25$$
$$x^2 + 10x + 25 = 13$$

Factor the left side of the equation.

$$(x + 5)^2 = 13$$

Determine the square root of each side of the equation.

$$\sqrt{(x + 5)^2} = \pm\sqrt{13}$$
$$x + 5 = \pm\sqrt{13}$$

Set the factor of the perfect square trinomial equal to each square root of the constant. Solve for x.

$$x + 5 = \sqrt{13} \quad \text{and} \quad x + 5 = -\sqrt{13}$$
$$x = -5 + \sqrt{13} \quad \text{and} \quad x = -5 - \sqrt{13}$$
$$x \approx -1.39 \quad \text{and} \quad x \approx -8.61$$

The roots are approximately -1.39 and -8.61.

In **Lesson 4: The Quadratic Formula**, students derive the quadratic formula. They then use the quadratic formula to solve problems in and out of context.

Quadratic Formula

You can use the **quadratic formula**, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$.

For example, given the function $f(x) = 2x^2 - 4x - 3$ we can identify the values of a, b, and c.

$$a = 2; b = -4; c = -3$$

Then, we use the quadratic formula to solve.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{4 \pm \sqrt{16 + 24}}{4}$$

$$x = \frac{4 \pm \sqrt{40}}{4}$$

$$x \approx \frac{4 + 6.325}{4} \approx 2.581 \quad \text{or} \quad x \approx \frac{4 - 6.325}{4} \approx -0.581$$

The roots are approximately 2.581 and (−0.581).

A quadratic function can have one real zero, two real zeros, or at times, no real zeros.

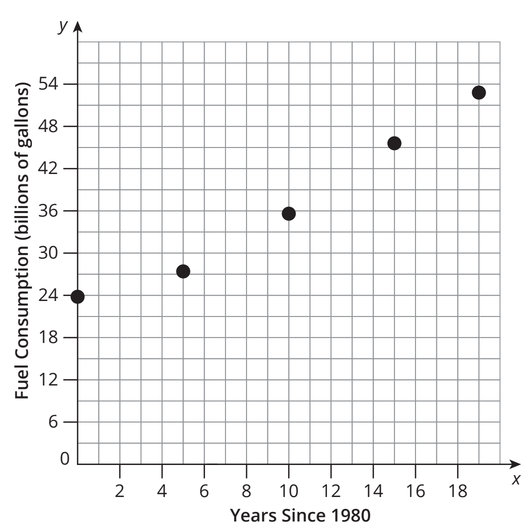
In **Lesson 5: Using Quadratic Functions to Model Data**, students determine a quadratic regression model for of data, and they use the regression models to make estimates and predictions.

Quadratic Regression

Similar to the regression models used with linear and exponential data, some situations are best modeled with quadratic regression models. After choosing the type of regression model to best model the situation, it can then be used to make predictions for the data.

The regression model for this data is $y = 0.0407x^2 + 0.809x + 23.3$.

Years Since 1980	Fuel Consumption (billions of gallons)
0	23.8
5	27.4
10	35.6
15	45.6
19	52.8



MYTH

"Once I understand something, it has been learned."

Learning is tricky for three reasons. First, even when we learn something, we don't always recognize when that knowledge is useful. For example, you know there are four quarters in a dollar. But if someone asks you, "What is 75 times 2?" you might not immediately recognize that is the same thing as having six quarters.

Second, when you learn something new, it's not as if the old way of thinking goes away. For example, some children think of north as straight ahead. But have you ever been following directions on your phone and made a wrong turn, only to catch yourself and think, "I know better than that!"

The final reason that learning is tricky is that it is balanced by a different mental process: forgetting. Even when you learn something (for example, your locker combination), when you stop using it (for example, you get a new locker), it becomes extremely hard to remember.

There should always be an asterisk next to the word when we say we learned* something.

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