Secondary Mathematics

EDITION 1

Algebra I

Scope and Sequence
165-Day Pacing

Acknowledgment
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Notice
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ALGEBRA I: SCOPE & SEQUENCE

165-Day Pacing

1 Searching for Patterns

Module Pacing: 24 Days

TOPIC 1: Quantities and Relationships

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.A, 1.B, 1.C, 1.E, 1.F, 2.C, 2.E, 2.I, 3.D, 3.E, 3.H, 4.C, 4.E, 4.G, 4.H, 5.B, 5.F

Topic Pacing: 14 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
	Introduction to the Problem-Solving Model and Learning Resources	Students reflect on learning a new skill and the variety of ways they learn. The problem-solving model, TEKS mathematical process standards, and the Academic Glossary help students complete a problem-solving activity. Students reflect on and summarize the problem-solving process. Since the intent of this lesson is to introduce the problem-solving model and review the TEKS mathematical process standards, the focus is on process, not content. Students will need access to the Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Questions to Ask, and TEKS mathematical process standards, which are located in the Course and Implementation Guide. These materials should always be available to students throughout the course.	• Create a classroom of collaboration and establish the learning process as a partnership between you and your students. • Communicate continuously with students about the objectives of the lesson to encourage self-monitoring of their learning. • The problem-solving model involves noticing patterns and formulating questions, organizing information and representing this information using appropriate mathematical notation, analyzing mathematical representations and using them to make predictions and then testing predictions, predicting, and sharing the results. • The TEKS mathematical process standards describe the ways in which students are expected to engage in content. • The Academic Glossary is a resource that helps students think, reason, and communicate their ideas.	**A.3C**	1
1	Understanding Quantities and Their Relationships	Students are presented with various scenarios and identify the independent and dependent quantities for each. They then match a graph to the appropriate scenario, label the axes using the independent and dependent quantities, and create the scale for the axes. Students make basic observations about the similarities and differences in the graphs. They then look more deeply at pairs of scenarios along with their graphs to focus on characteristics of the graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points. The lesson concludes with students creating their own scenario and a sketch of a graph to model the scenario.	• There are two quantities that change in problem situations. • When one quantity is determined by another, it is said to be the *dependent quantity*. The quantity that the dependent quantity is determined from is called the *independent quantity*. • The independent quantity is used to label the x-axis. The dependent quantity is used to label the y-axis. • Graphs can be used to model problem situations.	**A.3C** **A.7A** **A.9D**	2
2	Analyzing and Sorting Graphs	Students begin this lesson by cutting out 13 different graphs. They sort the graphs into different groups based on their own rationale, compare their groupings with their classsmates, and discuss the reasoning behind their choices. Next, four different groups of graphs are given, and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations. Students need to keep their graphs as they will be used in lessons that follow.	• A relationship between two quantities can be graphed on the coordinate plane. • Graphical behaviors can reveal important information about a relationship. • A graph of a relationship can have a minimum or maximum, or no minimum or maximum. A graph can pass through one or more quadrants. A graph can exhibit vertical or horizontal symmetry. A graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing.	**A.3C** **A.7A** **A.9D**	1

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
3	**Recognizing Functions and Function Families**	The definition of *relation*, *function*, *function notation*, *domain*, and *range* are introduced in this lesson. For the remainder of the lesson, students use graphing technology to connect equations written in function form to their graph and then identify the function family to which they belong. The terms *Vertical Line Test*, *continuous graph*, and *discrete graph* are defined, and students sort the graphs from the previous lesson into functions and non-functions. Then, the terms *increasing function*, *decreasing function*, *constant function*, *discrete function*, and *continuous function* are defined, and students sort the graphs from the previous lesson into these groups and a group labeled for functions that include a combination of increasing and decreasing intervals. The terms *function family*, *linear function*, and *exponential function* are then defined, and students sort the increasing, constant, and decreasing functions into one of these families. Next, the terms *absolute minimum* and *absolute maximum* are defined as well as the term *quadratic function*. Finally, students recall the definition of *x*-intercept and *y*-intercept. Students then complete a graphic organizer for each function family that describes the graphical behavior and displays graphical examples. In the final activity, students use their knowledge of the function families to demonstrate how the families differ with respect to their *x*- and *y*-intercepts. Graphing technology is necessary to help students connect some equations and their graphs.	• A *function* is a *relation* that assigns to each element of the domain exactly one element of the *range*. • The family of *linear functions* includes functions of the form $f(x) = ax + b$, where a and b are real numbers. • The family of *exponential functions* includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers and b is greater than 0 but is not equal to 1. • The family of *quadratic functions* includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and a is not equal to 0.	**A.2A** **A.3C** **A.6A** **A.7A** A.9A **A.9D** A.12A	3
4	**Recognizing Functions by Characteristics**	Given characteristics describing graphical behavior, students name the possible function family or families that fit each description. Using the scenarios and their graphs from the first lesson of the topic, they complete a table by naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant, or both increasing and decreasing. Students then work with a partner and write equations and sketch graphs to satisfy different lists of characteristics. They conclude the lesson by creating their own list of characteristics, providing two graphs that include those characteristics, and determining that an equation, not just a list of characteristics, is required to generate a unique graph.	• The graph of an exponential or quadratic function is a curve. • The graph of a linear function is a line. • The graph of a linear or exponential function is either increasing or decreasing. • The graph of a quadratic function has intervals where it is increasing and intervals where it is decreasing. Quadratic Functions also have an absolute maximum or an absolute minimum. • Key characteristics of graphs help to determine the function family to which it belongs.	**A.2A** **A.3C** **A.6A** **A.7A** A.9A **A.9D** A.12A	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					4

*Bold TEKS = Readiness Standard

TOPIC 2: Sequences

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.A, 1.C, 1.E, 3.D, 3.E, 3.J, 4.B, 5.A, 5.B, 5.F

Topic Pacing: 10 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	**Recognizing Patterns and Sequences**	Students begin by exploring various patterns in Pascal's triangle. *Sequences* and *term of a sequence* are defined. Given four geometric patterns or contexts, students write a numeric sequence to represent each problem. They are guided to represent each sequence as a table of values and conclude that all sequences are functions. Students then organize the sequences in a table, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences have a domain that includes only positive integers. *Infinite sequence* and *finite sequence* are defined and included as another characteristic for students to consider as they write sequences.	• A *sequence* is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. • A *term of a sequence* is an individual number, figure, or letter in the sequence. • A sequence can be written as a function. The domain includes only positive integers. • An *infinite sequence* is a sequence that continues forever, or never ends. • A *finite sequence* is a sequence that terminates, or has an end term.	A.12A	1
2	**Arithmetic and Geometric Sequences**	Given eight numeric sequences, students generate several additional terms for each sequence and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale. The terms *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio* are then defined, examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio, where appropriate. Students begin to create graphic organizers, identifying four different representations for each arithmetic and geometric sequence. In the first activity, they glue each arithmetic and geometric sequence to a separate graphic organizer and label them, and in the second activity, the corresponding graph is added. The remaining representations are completed in the following lesson. This lesson concludes with students writing sequences given a first term and a common difference or common ratio and identifying whether the sequences are arithmetic or geometric.	• An *arithmetic sequence* is a sequence of numbers in which the difference between any two consecutive terms is a positive or negative constant. This constant is called the *common difference* and is represented by the variable *d*. • A *geometric sequence* is a sequence of numbers in which the ratio between any two consecutive terms is a constant. This constant is called the *common ratio* and is represented by the variable *r*. • The graph of a sequence is a set of discrete points. • The points of an arithmetic sequence lie on a line. When the common difference is positive, the graph is increasing, and when the common difference is negative, the graph is decreasing. • The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.	A.12A A.12C	1
3	**Determining Recursive and Explicit Expressions from Contexts**	Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of terms in each sequence. The term *recursive formula* is defined and used to generate term values. As the term number increases, it becomes more time-consuming to generate the term value. This sets the stage for *explicit formulas* to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences. Students write recursive and explicit formulas for sequences and represent sequences as graphs.	• A *recursive formula* expresses each new term of a sequence based on a preceding term of the sequence. • An *explicit formula* for a sequence is a formula for calculating each term of the sequence using the term's position in the sequence. • The explicit formula for determining the *n*th term of an arithmetic sequence is $a_n = a_1 + d(n - 1)$, where n is the term number, a_1 is the first term in the sequence, a_n is the *n*th term in the sequence, and d is the common difference. • The explicit formula for determining the *n*th term of a geometric sequence is $g_n = g_1 \cdot r^{(n-1)}$, where n is the term number, g_1 is the first term in the sequence, g_n is the *n*th term in the sequence, and r is the common ratio.	A.12C A.12D	4
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					3

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2 Exploring Constant Change

Module Pacing: 36 Days

TOPIC 1: Linear Functions

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.C, 1.D, 1.E, 1.H, 2.C, 2.D, 2.E, 2.G, 3.A, 3.B, 3.C, 4.A, 4.C, 4.D, 4.F, 4.K, 5.E, 5.G

Topic Pacing: 22 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	Least Squares Regressions	Students informally determine a line of best fit by visual approximation of a hand-drawn line. They are then introduced to a formal method to determine the linear regression line of a data set using graphing technology; the mathematics behind the calculator function is explained using the related terms *Least Squares Method*, *linear regression function*, and *centroid*. Students then use the line of best fit to make predictions and distinguish between the terms *interpolation* and *extrapolation*.	• *Interpolation* is the process of using a regression function to make predictions within the data set. • *Extrapolation* is the process of using a regression function to make predictions beyond the data set. • A least squares regression line is the line of best fit that minimizes the squares of the distances of the points from the line. • You can use regression methods to build linear functions to model data.	**A.3C** A.4C A.12A	2
2	Correlation	This lesson provides several definitions related to correlations. The terms *correlation* and *correlation coefficient* are defined. The formula to compute the correlation coefficient is given; however, students are only required to use technology to determine the value of r or to estimate correlation coefficients from a list of choices. The distinction is then made between the meanings of r and r^2, the coefficient of determination. Students use these terms to make decisions regarding the model that best fits the data. The terms *causation*, *necessary condition*, and *sufficient condition* are defined. Examples are provided to help students see the difference between correlation and causation. The terms *common response* and *confounding variable* are defined as relationships often mistaken for causation.	• A *correlation* is a measure of how well a regression model fits a data set. • The *correlation coefficient*, r, is a value between -1 and 1 that indicates the type (positive or negative) of association and the strength of the relationship. Values close to 1 or -1 demonstrate a strong association, while a value of 0 signifies no association. • *Causation* is when one event causes a second event. A correlation is a *necessary condition* for causation but not a *sufficient condition* for causation. • Two relationships that are often mistaken for causation are a *common response*, when some other reason may cause the same result, and a *confounding variable*, when there are other variables that are unknown or unobserved.	A.4A A.4B A.4C	2

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165-Day Pacing

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
3	**Making Connections Between Arithmetic Sequences and Linear Functions**	Students are provided two sequences. They must identify each sequence as arithmetic or geometric, write the explicit formula for the sequence, and graph the sequence. Students then list and compare characteristics of each graphical representation. The remainder of the lesson focuses on connecting arithmetic sequences to linear functions. Students match the explicit formulas for arithmetic sequences and their graphs. A Worked Example demonstrates how to rewrite an arithmetic sequence in explicit form as a linear function in slope-intercept form. Students then use the context of stacking chairs to make connections among the terms of the explicit formula of a sequence and the linear function that models it. Students compare the terms of each equation and recognize that the common difference and the slope are constant and equal; however, the first term of the sequence is equal to $f(1)$ rather than the y-intercept of the linear function. Using tables of values for this context, *first differences* is defined as a strategy to determine whether a relationship is linear. Students move from the concrete example to generalize that the constant difference of an arithmetic sequence is equal to the slope of the corresponding linear function by completing an algebraic proof. Next, *average rate of change* is defined and presented graphically as a method to determine the unit rate using non-consecutive x-values. Students solidify these new concepts by revisiting the sequences from the start of the lesson, practicing their newly developed skills, and verifying their conclusions. The special case of a constant function is then addressed. Finally, students complete a graphic organizer to summarize the characteristics and representations of linear functions.	• The explicit formula of an arithmetic sequence can be rewritten as the slope-intercept form of a linear function using algebraic properties. • The explicit formula of an arithmetic sequence, $a_n = a_1 + d(n - 1)$, includes the first term of the sequence, $f(1)$, and the common difference. The slope-intercept form of a linear function, $f(x) = mx + b$, includes $f(0)$ and the slope. • Both the average rate of change formula and slope formula calculate the unit rate over a given interval. The average rate of change formula refers to the dependent variable as $f(x)$, while the slope formula uses y. • *First differences* is a strategy to determine whether a table of values can be modeled by a linear function. First differences are the values determined by subtracting consecutive output values when the input values have an interval of 1. • The domain of an arithmetic sequence is consecutive integers beginning with 1, while the domain of a linear function may include all real numbers.	**A.2A** A.2B **A.2C** A.3A **A.3B** A.12D	3
4	**Point-Slope Form of a Line**	Students use the slope formula to derive the point-slope form of a linear equation. They write equations in point-slope and slope-intercept form given different sets of information: a table of values, two points, a context, a slope and the y-intercept, a slope and a point, a graph with a visible y-intercept, and a graph with a non-visible y-intercept. Students explore the slopes, intercepts, and equations of horizontal and vertical lines. Finally, they match equations written in slope-intercept or point-slope form with contexts and tables.	• The *slope-intercept form* of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept of the line. • The *point-slope form* of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. • Horizontal lines have a slope of 0. The equation of a horizontal line that passes through $(0, b)$ is $y = b$. • Vertical lines have an undefined slope. The equation of a vertical line that passes through $(a, 0)$ is $x = a$.	A.2B **A.2C** A.3A **A.3C**	2
5	**Using Linear Equations**	Students use three different forms of a linear equation to graph linear relationships. First, they learn how to use the slope-intercept and point-slope forms of a line to graph. Students explore the standard form of a linear equation and connect relationships among the coefficients of the standard form with the x-intercept, y-intercept, and slope of a line. They then practice writing and graphing equations in standard form. Finally, students identify the slope and intercept of linear equations in different forms and evaluate the usefulness of each form of a linear equation.	• The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept of the line. • The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. • The *standard form* of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero. • The information contained in the equation of a line can be used to graph the line.	A.2B **A.2C** A.3A **A.3C**	3

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
6	Making Sense of Different Representa-tions of a Linear Function	Students determine whether tables of values with non-consecutive input values represent linear functions. They evaluate functions and analyze Worked Examples that demonstrate how to solve equations algebraically and graphically. For the remainder of the lesson, students deal with a context, a graph, and two translations of the graph based on additions to the context. They focus on two equivalent linear functions, one written in general form, $f(x) = ax + b$, and the other written in factored form, $f(x) = a(x - c)$. Students interpret the meaning of the terms of each function and analyze their structures. The form $f(x) = ax + b$ relates to the slope-intercept form of a line, while $f(x) = a(x - c)$ connects with the slope and zero of the function. Linear functions are placed within the wider framework of polynomial functions. The terms *polynomial*, *degree*, *leading coefficient*, and *zero of a function* are defined, setting a frame of reference for future work with other functions. Students use a graphic organizer to summarize four representations— general form, factored form, graph, and table—of a linear function.	• If a table represents a linear function, the slope, or average rate of change, is constant between all given points. • Using an equation to solve for the independent value given the dependent value always results in an exact answer. Using a graph or a table to determine the independent value sometimes results in an exact answer. • The graph of an equation plotted on the coordinate plane represents the set of all its solutions. • The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the *leading coefficient*, which describes the steepness and direction of the line. The b-value describes the y-intercept. • The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a *polynomial* is in factored form, the value of x that makes each factor equal to zero is the x-intercept. This value is called the *zero of the function*. • A linear function is a polynomial with a *degree* of one.	**A.2C** A.2D A.3A **A.3C** A.3E A.3F A.12A A.12B	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					6

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TOPIC 2: Transforming and Comparing Linear Functions

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1D, A.1E, A.1F, A.1G

ELPS: 1.D, 1.E, 2.D, 2.G, 2.H, 2.I, 3.C, 3.E, 3.F, 4.A, 4.F, 4.K

Topic Pacing: 14 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	Transforming Linear Functions	Students identify key characteristics of several linear functions. A graph and a table of values for the parent linear function $f(x) = x$ is provided, and they investigate $f(x) + d$ and $a \cdot f(x)$. Given a function $g(x)$ in terms of $f(x)$, students graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$. They prove algebraically that a line and its translation are parallel to one another and write equations of lines parallel to a given line through a given point. Finally, students use their knowledge of linear function transformations to test a video game that uses linear functions to shoot targets. They write the function transformations several ways and identify the domains, ranges, slopes, and y-intercepts of the new functions.	• For the *parent function* $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values of the function. For $d > 0$, the graph vertically shifts up. For $d < 0$, the graph vertically shifts down. The amount of shift is given by $\|d\|$. • For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ affects the output values of the function. For $\|a\| > 1$, the graph stretches vertically by a factor of a units. For $0 < \|a\| < 1$, the graph compresses vertically by a factor of a units. For $a < 0$, the graph reflects across the x-axis. • A line and its translation are parallel to one another.	**A.2A** **A.2C** A.2E **A.3C** A.3E	3

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
2	**Vertical and Horizontal Transformations of Linear Functions**	Students will identify key characteristics of several linear functions. A graph and a table of values for the parent linear function $f(x) = x$ is given, and students will translate this function horizontally and vertically to determine which transformations affect the input and output values. They will also dilate the function $f(x) = x$ horizontally and vertically. Students will generalize about equivalent translations for the parent function $f(x) = x$ and determine that these relationships do not hold true for all linear functions. Given a function $g(x)$ in terms of $f(x)$, students will graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$.	• For the parent function $f(x) = x$, a horizontal translation right n units is equivalent to a vertical translation down n units, and a horizontal translation left n units is equivalent to a vertical translation up n units. • For the parent function $f(x) = x$, the transformed function $y = f(x - c)$ affects the input values of the function. For $c > 0$, the graph horizontally shifts right c units. For $c < 0$, the graph horizontally shifts left c units. • For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values of the function. For $d > 0$, the graph vertically shifts up d units. For $d < 0$, the graph vertically shifts down d units. • For the parent function $f(x) = x$, a horizontal dilation by a factor of n units is equivalent to a vertical dilation by a factor of $\frac{1}{n}$ units. • For the parent function $f(x) = x$, the transformed function $y = af(x)$ affects the output values of the function. For $\|a\| > 1$, the graph vertically stretches by a factor of a units. For $0 < \|a\| < 1$, the graph vertically compresses by a factor of a units. For $a < 0$, the graph reflects across the x-axis. • For the parent function $f(x) = x$, the transformed function $y = f(bx)$ affects the input values of the function. For $\|b\| > 1$, the graph horizontally compresses by a factor of $\frac{1}{\|b\|}$ units. For $0 < \|b\| < 1$, the graph horizontally stretches by a factor of $\frac{1}{\|b\|}$ units. For $b < 0$, the graph reflects across the y-axis.	**A.2A** **A.2C** **A.3C** A.3E	2
3	**Determining Slopes of Perpendicular Lines**	Students rotate a line segment on the coordinate plane in increments of 90° counterclockwise and recognize patterns in the slopes and coordinates of the endpoints of the images. They analyze a proof of a theorem stating that if two lines are perpendicular, the slopes of the lines are negative reciprocals. Students then explore relationships between vertical and horizontal lines. Finally, they write the equation of a line perpendicular to a given line that passes through a given point.	• Transformations can be used to create perpendicular lines. By rotating a line 90°, the pre-image and image form perpendicular lines. • If two lines are perpendicular, their slopes are negative reciprocals. • All horizontal lines have a slope of zero, are parallel to one another, and are perpendicular to vertical lines. All vertical lines have a slope that is undefined, are parallel to one another, and are perpendicular to horizontal lines.	A.2F A.2G	2
4	**Comparing Linear Functions in Different Forms**	Students analyze functions represented as tables, graphs, equations, and verbal descriptions. They explore slope with particular attention to parallelism and perpendicularity in different representations. Students compare properties, such as slope, y-intercept, and the units for independent and dependent quantities, all in terms of the situations they represent. Students also identify the scale and origin on the graph of a function given a situation description. Finally, they generate and compare their own linear functions using tables, graphs, and equations.	• Functions can be represented using tables, equations, graphs, and with verbal descriptions. • Features of linear functions, such as y-intercepts, slope, independent quantities, and dependent quantities can be determined from different representations of functions. • Lines that are parallel have the same slope. Lines that are perpendicular have slopes that are negative reciprocals.	A.3A **A.3C** A.12B	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					4

*Bold TEKS = Readiness Standard

3 Modeling Linear Equations and Inequalities

Module Pacing: 33 Days

TOPIC 1: Linear Equations and Inequalities

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1D, A.1F, A.1G

ELPS: 1.H, 2.B, 2.C, 2.H, 3.A, 4.B, 4.D, 4.J, 5.D, 5.E

Topic Pacing: 11 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	Solving Linear Equations	Students start with a simple solution statement and create more complex equations by performing the same operation on each side of the equation. They then analyze different equations created by two students and reason about how to verify that the equations have the same solution as the original equation. The properties of equality and some basic number properties are reviewed before students practice solving linear equations and justifying their steps. They also compare the different properties two students used to solve the same equation. Next, students investigate a mathematical statement that is always true and a mathematical statement that is always false. The terms *no solution* and *infinite solutions* are defined. Finally, students play Tic-Tac-Bingo as they work together to create equations with given solution types from assigned expressions. They then summarize strategies for determining whether an equation has no solution or infinite solutions.	• A *solution* to an equation is any variable value that makes that equation true. • Solving equations requires the use of number properties and the properties of equality. • The properties of equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality. • When the properties of equality are applied to an equation, the transformed equation will have the same solution as the original equation. • Equations with *infinite solutions* are created by equating two equivalent expressions. • Equations with *no solution* are created by equating expressions of the form $mx + b$ with the same value for a and different values for b. • Equations with a solution $x = 0$ are created by equating expressions of the form $mx + b$ with different values for m and the same value for b.	**A.5A**	2
2	Literal Equations	Students begin with a perimeter problem in context to address solving formulas for different variables. They then identify the slope, x-intercept, and y-intercept of linear equations in slope-intercept, point-slope, and standard form and consider which form is most efficient in determining these characteristics. Next, the term *literal equation* is defined. The common literal equation for converting degrees Fahrenheit to degrees Celsius is provided. Students rewrite the formula to convert degrees Celsius to degrees Fahrenheit, identify errors in student work when rewriting the formula, and interpret equivalent equations written in standard form. The lesson concludes by having students solve various literal equations for specific variables.	• The slope-intercept form of a linear equation is $y = mx + b$, where m and b are real numbers; m represents the slope, and b represents the y-intercept. • The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m represents the slope, and (x_1, y_1), represents a point on the line. • The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers, and both A and $B \neq 0$. It can be rewritten in slope-intercept form as $y = -\frac{A}{B}x + \frac{C}{B}$; $-\frac{A}{B}$ represents the slope, $\frac{C}{B}$ represents the y-intercept, and $\frac{C}{A}$ represents the x-intercept. • Slope-intercept form of a linear equation is the most useful form to identify the slope and y-intercept. Point-slope form of a linear equation is the most useful form to identify the slope and a point on the line. • *Literal equations* can be rewritten to highlight a specific variable.	A.2B A.3A A.12E	2
3	Modeling Linear Inequalities	Students begin with a scenario and table that can be modeled by a linear inequality with a positive rate of change. They then analyze a graph that models the situation. Students use that graph to solve inequalities and graph the solution set on a number line. Next, the term *solve an inequality* is defined, and students write and solve inequalities algebraically, taking into account the context of the problem situation. They then analyze an inequality with a negative rate of change to make sense of how the sign of the solution to the inequality is affected. Lastly, students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation and ones that require the distributive property.	• A linear inequality context can be modeled with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement. • Solutions to linear inequalities can be determined both graphically and algebraically; they can be expressed using a number line or inequality statement. • The steps to solving a linear inequality algebraically are the same steps to solve a linear equation, except that when solving a linear inequality with a negative rate of change, the inequality sign of the solution must be reversed to accurately reflect the relationship.	**A.2C** A.5B	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					3

*Bold TEKS = Readiness Standard

TOPIC 2: Systems of Equations and Inequalities

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1C, A.1D, A.1E, A.1F

ELPS: 1.D, 2.B, 2.D, 2.H, 2.I, 3.A, 3.B, 3.C, 3.F, 4.A, 4.B, 4.G, 5.E

Topic Pacing: 22 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	**Using Graphing to Solve Systems of Equations**	Students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They graph the linear equation using intercepts and then analyze a second graph with the independent and dependent variables reversed. A new relationship between the quantities is then provided, and students write the equation expressing the relationship. Finally, they graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario creating a system of linear equations. Students solve the system both graphically and using technology, checking the solution by substituting the values back in to the original equations. Next, they are provided three related scenarios in which they write systems of equations in slope-intercept form and solve the systems graphically. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions. The related terms *consistent systems* and *inconsistent systems* are defined.	• The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero. Linear functions written in standard form can be graphed using the x- and y-intercepts. • Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane. • A *linear system of equations* is two or more linear equations that define a relationship between quantities. The *solution of a linear system* is an ordered pair that makes both equations in the system true. • Lines that do not intersect describe a system of equations in which each linear equation has the same slope and there is no solution. • Lines intersecting at a single point describe a system of equations in which each linear equation has a different slope and there is one solution. • Lines intersecting at an infinite number of points describe a system of equations in which each linear equation is the same equation and there are an infinite number of solutions. • *Consistent systems* of equations are systems that have one or many solutions. *Inconsistent systems* of equations are systems that have no solution.	**A.2A** **A.2C** **A.2I** A.3F A.3G **A.5C**	2
2	**Using Substitution to Solve Linear Systems**	Students use the substitution method to solve systems of linear equations. They use substitution to solve systems of linear equations, including those with no solution or those with infinite solutions. Students define variables, write systems of equations, solve systems, and interpret the meaning of the solution in terms of the problem context. In the last activity, they are given four systems of linear equations and solve each system using the substitution method.	• The *substitution method* is a process for solving a *system of equations*. It is an alternative method to graphing, especially when the solution is difficult to read from a graph. • To use the substitution method, it is useful when at least one equation is written in slope-intercept form. • When a system has no solution, the equation resulting from the substitution step has no solution. • When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions. • Problem situations can be expressed using systems of equations and solved for unknown quantities using substitution methods.	**A.2I** A.3F A.3G **A.5C**	3
3	**Using Linear Combinations to Solve a System of Linear Equations**	Students are given a problem scenario and use reasoning to determine the two unknowns. They then write a system of linear equations in standard form to represent a problem situation. Students analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They practice the linear combinations method with systems in which the coefficients of one variable are additive inverses. Next, Worked Examples guide students to multiply one and then both equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system, and then they solve two problems in context one with fractional coefficients. The lesson concludes with students addressing when it is appropriate to use the graphing, substitution, or linear combinations methods.	• The *linear combinations method* is a process to solve a system of linear equations by adding two equations together, resulting in an equation in one variable. • When using the linear combinations method, it is often necessary to multiply one or both equations by a constant to create two equations in which the coefficients of one of the variables are additive inverses.	**A.2I** **A.5C**	3

*Bold TEKS = Readiness Standard

ALGEBRA I: SCOPE & SEQUENCE

165-Day Pacing

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
4	**Graphing Inequalities in Two Variables**	Students explore a linear inequality in two variables through a scenario. They write an inequality, complete a table of values, graph the coordinate pairs from the table, and determine which parts of the graph are solutions to the inequality. Students then formalize the process of graphing inequalities through practice without context; they graph the corresponding equation of an inequality as a boundary line, determine whether the line should be solid or dashed, and identify which half-plane to shade by testing the point (0, 0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs. They then solve a problem in context where they use a table of values to write and graph a linear inequality and refer to the inequality and/or its graph to respond to questions. Finally, students summarize the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.	• The graph of a linear inequality is a *half-plane*, or half of a coordinate plane. • Shading is used to indicate which half-plane describes the solution to the inequality. • Dashed and solid lines are used to indicate if the line itself is included in the solution set of an inequality. • Linear inequalities and their graphs can be used to represent and solve problems in context.	A.2H **A.3D**	2
5	**Systems of Linear Inequalities**	Students represent a scenario with a system of linear inequalities and graph the system. Overlapping shaded regions identify the possible solutions to the system. Students then practice graphing several systems of inequalities and representing the solution set. A different scenario is given that students model with a system of linear inequalities. They then graph the system, determine two different solutions, and algebraically prove that the solutions satisfy both constraints defined by the system. Finally, students match systems, graphs, and possible solutions of systems that have identical terms with different inequality symbols.	• In a system of linear inequalities, the inequalities are known as *constraints* because the values of the expressions are constrained to lie within a certain region. • The *solution of a system of linear inequalities* is the intersection of the solutions to each inequality. Every point in the intersecting region satisfies all inequalities in the system.	A.2H **A.3D** A.3H	3
6	**Solving Systems of Equations and Inequalities**	Students solve problems in context requiring a system of linear equations. While most problems can be modeled by a system of two equations, they are guided through the process of solving a system of four equations, and another context can be modeled by a system of three equations. Students have the opportunity to solve the systems using any method and sometimes must respond in the format of an email or proposal. Solutions involve making a decision based upon inputs that lie before or after the point of intersection, thus requiring solutions written as inequalities.	• Contexts about choosing between two options can sometimes be modeled by a system of linear equations or inequalities. • The point of intersection of two lines separates the input values, with *x*-values less than and *x*-values greater than the *x*-value of the point of intersection. The solution to a problem in context may be dependent upon where the input values lie relative to the point of intersection. • Based upon a context, the solution of a system may be represented by inequalities rather than a single coordinate pair.	A.2H **A.2I** **A.3D** A.3H **A.5C**	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					6

*Bold TEKS = Readiness Standard

4 Investigating Growth and Decay

Module Pacing: 25 Days

TOPIC 1: Introduction to Exponential Functions

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.B, 1.E, 1.G, 2.A, 2.D, 2.H, 2.I, 3.B, 3.D, 3.F, 3.H, 4.C, 5.B, 5.C, 5.D, 5.E

Topic Pacing: 15 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	**Properties of Powers with Integer Exponents**	The terms *power, base of a power,* and *exponent of a power* are defined. Students write and evaluate expressions with positive integer exponents. They begin with a context using the power with a base of 2. Students then investigate positive and negative integer bases where the negative sign may or may not be raised to a power depending on the placement of parentheses. Some expressions also contain variables.	• Large numbers that have factors that are repeated can be written as a product of powers. • Placement of parentheses in an expression with an *exponent* determines what portion of the expression is raised to a *power.* • When a negative value is raised to a power with an exponent that is an even integer, the simplified expression is a positive value. • When a negative value is raised to a power with an exponent that is an odd integer, the simplified expression is a negative value.	**A.11B**	3
2	**Analyzing Properties of Powers**	Students use the properties of powers to justify each step when rewriting expressions with exponents. They solve additional practice problems and examine student work. Students demonstrate their understanding of the properties of powers by creating graphic organizers.	• Equivalent expressions are created by applying properties of integer exponents. • The properties of integer exponents provide mathematical justification for rewriting expression with exponents.	**A.11B**	2
3	**Geometric Sequences and Exponential Functions**	In Module 2, Topic 1, Lesson 3: *Making Connections Between Arithmetic Sequences and Linear Functions,* students rewrote the explicit form of an arithmetic sequence as a linear function and proved algebraically that the common difference in a linear function is the slope of the line. This lesson follows a similar process where they revisit geometric sequences as a launch to exponential functions. Students know that all geometric sequences are functions, and through investigation, they learn that some geometric sequences are exponential functions, while others are not. They identify the fact that the constant ratio must be positive for a geometric sequence to be an exponential function. Through a context, student work, and a Worked Example, students use properties of exponents to rewrite the explicit form of a geometric sequence as a function in the form $f(x) = ab^x$ and make connections between the two forms. Students then explore a situation modeled by an exponential function. They are guided to demonstrate that the ratio between consecutive output values of any exponential function is constant and is represented by the variable b in the function form $f(x) = ab^x$, and the y-intercept is represented by the ordered pair $(0, a)$. Students then solve a problem in context that is represented by a decreasing exponential function.	• All geometric sequences are functions; however, only geometric sequences with a positive constant ratio are exponential functions. • When a geometric sequence represents an exponential function, then the product of powers rule and the definition of negative exponents can be used to rewrite the explicit formula for the sequence as an exponential function. • The form of an exponential function is $f(x) = ab^x$, where b represents the constant ratio and $(0, a)$ represents the y-intercept. • For an exponential function in the form • $f(x) = ab^x$, the ratio $\frac{f(x+1)}{f(x)}$ is constant and equal to b	A.9B **A.9C** **A.9D** **A.11B** A.12C A.12D	2
4	**Rewriting Square Roots**	Students analyze perfect square models and models of non-perfect squares in terms of their areas and side lengths. Students understand that the areas of the squares represent radicands, and the side lengths represent the square roots of those radicands. They write the side lengths of square models both as single square roots and as multiplication expressions. This allows students to model the simplification of square roots and make observations leading to the properties of radicals. Students then use the properties to multiply, divide, and simplify square roots by extracting perfect squares and by using prime factorization. Students discuss strategies for efficiently simplifying radicals.	• A square root represents a side length of a square, with the radicand representing the area of the square. • The product property of radicals states that $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$, when a and b are greater than 0. • The quotient property of radicals states that $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, where $a \geq$ and $b > 0$. • Prime factorization can be used to simplify radicands before extracting perfect squares.	A.11A	1

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
5	Rational Exponents and Graphs of Exponential Functions	Students explore a context modeled by an exponential function, first with output values that are between two integers, then with output values that are *rational exponents*. This helps students make sense of the fact that $\sqrt{a} = a^{\frac{1}{2}}$. They then expand upon this idea to learn that the properties of powers apply to expressions with rational exponents, rewrite expressions with rational exponents as radicals and connect these two concepts to perform and justify operations involving radicals. Students explore the effects of a negative exponent, learn the meaning of a *horizontal asymptote*, and analyze the idea of end behavior on several graphs.	• When the interval between input values is the same an exponential function shows a constant ratio between output values, no matter how large or how small the gap between input values. • Multiple representations, such as tables, equations, and graphs are used to represent exponential problem situations. • Properties of powers can be used to rewrite numeric and algebraic expressions involving integer and rational exponents. • Because rational exponents can be rewritten as radicals, the properties of powers apply to radical expressions as well. • A rational expression of the form $a^{\frac{m}{n}}$ can be written as a radical expression $\sqrt[n]{a}^{m}$ or $a^{\frac{m}{n}}$. • Common bases and properties of exponents are used to solve simple exponential equations.	A.9A A.9B **A.9C** **A.9D** A.11A **A.11B** A.12B	3
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					3

*Bold TEKS = Readiness Standard

TOPIC 2: Using Exponential Equations

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1G

ELPS: 1.A, 1.C, 1.D, 2.A, 2.C, 2.G, 3.C, 4.D, 4.K

Topic Pacing: 10 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	Exponential Equations for Growth and Decay	Students begin this lesson by analyzing the structure of linear and exponential functions to sort them as either increasing or decreasing functions. Students compare linear and exponential functions in the context of simple interest and compound interest situations. They then identify the values in the exponential function equation that indicate whether it is a growth or decay function and apply this reasoning in context. Finally, for a situation modeled by an exponential decay function, students write the function, sketch its graph and then use the graph to answer a question about the problem situation.	• *Simple interest* can be represented by a linear function. *Compound interest* can be represented by an exponential function. • An *exponential growth function* is of the form $y = a(1 + r)^x$, where r is the rate of growth. • An exponential decay function is of the form $y = a(1 - r)^x$, where r is the rate of decay.	**A.3B** **A.3C** A.9B **A.9C** **A.9D** A.12B	2
2	Interpreting Parameters in Context	Students begin by using what they know about exponential functions to match four exponential equations to their graphs. Next, for a scenario based on exponential depreciation, students write the function, complete a table of values, and graph the function. They recall how to solve an equation graphically by graphing both sides of the equation and determining the point of intersection. They use this strategy to solve exponential equations and answer questions about given scenarios. Given a function that represents an annual increase in a mutual fund, students use the properties of exponents to rewrite the function to reveal approximate equivalent rates for the monthly and quarterly increases. Finally, they use what they know about the structure of exponential equations to identify equations that model a given situation and justify why others do not.	• Multiple representations, such as tables, equations, and graphs can be used to represent and compare exponential problem situations. • Graphs can be used to solve exponential equations by graphing both sides of the equation and estimating the point of intersection. • A quantity increasing exponentially eventually exceeds a quantity increasing linearly. • Properties of exponential functions can be compared using different representations. • Transforming exponential functions into equivalent forms can reveal different properties of the quantities represented.	A.9A A.9B **A.9C** **A.9D** **A.11B**	2

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
3	**Modeling Using Exponential Functions**	Students analyze data sets, create a scatterplot, write a regression function, use the function to calculate output values, and interpret the reasonableness of a prediction based on the scenario. For the first scenario, students are told to use an exponential function to model the scenario; in the second scenario, students must decide whether the scenario is best modeled by a linear or exponential function. The lesson concludes with students making a list of contexts from this module and generalizing what they have in common that identifies them as best modeled by exponential functions. They also describe the shape of a scatterplot representing an exponential function and sketch possible graphs of exponential functions.	• An exponential function and a constant function can be added to create a third function that is the sum of the two functions, resulting in a graph that is a vertical translation of the original exponential function. • Technology can be used to determine exponential regression functions to model real-world situations. The regression function can then be used to make predictions. • Sometimes, referring to the scenario or obtaining further information may be required to determine whether a scatterplot is best modeled by a linear or exponential function.	A.9A **A.9D** A.9E	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					3

*Bold TEKS = Readiness Standard

5 Maximizing and Minimizing

Module Pacing: 47 Days

TOPIC 1: Introduction to Quadratic Functions

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.A, 1.H, 2.D, 2.F, 2.G, 2.I, 3.B, 3.F, 4.G, 4.K, 5.B, 5.E, 5.F

Topic Pacing: 16 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	Exploring Quadratic Functions	Students are introduced to quadratic functions through a sequence of pennies. They are then given four different contexts that can be modeled using quadratic functions. The first context involves area and is used to compare and contrast linear and quadratic relationships, to define the term *parabola*, and to begin identifying key characteristics of the graphs of quadratic functions. The second context involves handshakes with a parabola that has a minimum. The third context involves a function written in standard form using the vertical motion formula. The final context involves revenue and demonstrates that a quadratic function can be written as the product of two linear functions. It is expected that students use technology to graph each function, allowing them to explore the key characteristics of the graphs of quadratic functions and interpret them in terms of their corresponding context. Students will revisit these same scenarios in the next lesson.	• Quadratic functions can be used to model certain real-world situations. • The graph of a quadratic function is called a *parabola*. • A parabola is a smooth curve with reflectional symmetry. • A parabola has an absolute maximum or absolute minimum point as well as an interval where it is increasing as well as an interval where it is decreasing. • A parabola has one *y*-intercept and, at most, two *x*-intercepts. • The domain of a quadratic function is the set of all real numbers. The range is a subset of the real numbers that is limited based upon the *y*-coordinate of the absolute maximum or absolute minimum point.	**A.6A** **A.7A**	3
2	Key Characteristics of Quadratic Functions	Students revisit the four scenarios from the previous lesson as a way to introduce equivalent quadratic equations with different structures to reveal different characteristics of their graphs. They learn that a table of values represents a quadratic function if its second differences are constant. The terms *standard form* and *factored form* are defined. Students analyze the effect of the leading coefficient on whether the parabola opens upward or downward. They identify the axis of symmetry and vertex for graphs using the equations in each form. Finally, students determine the *x*- and *y*-intercepts, along with intervals of increase and decrease, using a combination of technology, symmetry, and equations.	• A table of values representing a quadratic function has constant *second differences*. • A quadratic function may be written in standard form, $f(x) = ax^2 + bx + c$, where $a \neq 0$, and in factored form, $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$. • When the *a*-value of a quadratic function in standard form or *factored form* is positive, the graph opens upward and has an absolute minimum; when the leading coefficient is negative, the graph opens downward and has an absolute maximum. • The *vertex* (or maximum/minimum) *of a parabola* lies on its *axis of symmetry*. The axis of symmetry can be determined by the formula $x = \frac{r_1 + r_2}{2}$ from the factored form of the quadratic equation or by $x = -\frac{b}{2a}$ from the standard form of the quadratic equation. • In factored form, $f(x) = a(x - r_1)(x - r_2)$, the values of $(r_1, 0)$ and $(r_2, 0)$ are the *x*-intercepts of the quadratic function. In standard form, $f(x) = ax^2 + bx + c$, $(0, c)$ is the *y*-intercept of the quadratic function.	**A.6A** A.6C **A.7A**	3
3	Quadratic Function Transformations	Students are already familiar with the general shape of the graphs of quadratic functions, and they have studied transformations of linear functions. In this lesson, students experiment with the quadratic function family. They expand their understanding of transformations to include quadratic functions and interpret functions in the form $f(x) = a(x - c) + d$. They distinguish between the effects of changing values inside the argument of the function (the *c*-value) and changing values outside the function (the *a*- and *d*-values). Finally, students consider different ways to rewrite and interpret equations of function transformations.	• A function $g(x)$ of the form $g(x) = f(x) + d$ is a vertical translation of the function $f(x)$. The value $\|d\|$ describes the number of units the graph of $f(x)$ is translated up or down. When $d > 0$, the graph is translated up; when $d < 0$, the graph is translated down. • A function $g(x)$ of the form $g(x) = a \cdot f(x)$ is a vertical dilation of the function $f(x)$. For $\|a\| > 1$, the graph is vertically stretches by a factor of *a* units; for $0, \|a\| < 1$, the graph vertically compresses by a factor of *a* units. For $a < 0$, the graph also reflects across the *x*-axis.	**A.7A** **A.7C**	2

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
			• A function $g(x)$ of the form $g(x) = f(x - c)$ is a horizontal translation of the function $f(x)$. The value $\lvert c \rvert$ describes the number of units the graph of $f(x)$ is translated right or left. When $c > 0$, the graph is translated to the right; when $c < 0$, the graph is translated to the left. • A reflection across the x-axis can be expressed using the notation $(x, y) \rightarrow (x, -y)$. It affects the y-coordinate of each point on the graph. • A reflection across the y-axis can be expressed using the notation $(x, y) \rightarrow (-x, y)$. It affects the x-coordinate of each point on the graph.		
4	Horizontal Transformations and Vertex Form	Students explore horizontal dilations (b-value transformations). They sketch a graph of the transformation, compare characteristics of the transformed graph with the graph of the parent function, and represent the transformation using coordinates. Students practice writing quadratic functions in vertex form, standard form, and factored form and convert from one form to another to reveal properties of the function it defines. They identify the location of the zeros, the vertex, and the orientation of the parabola from the equation of the function. Students write quadratic equations in vertex form using the coordinates of the vertex and another point on the graph and in factored form using the zeros and another point on the graph.	• The general transformation equation is $y = a \cdot f(b(x - c)) + d$, where the a-value describes a vertical dilation or reflection, the b-value describes a horizontal dilation or reflection, the c-value describes a horizontal translation, and the d-value describes a vertical translation. • Given $f(x) = x^2$ as the parent quadratic function, reference points can be used to graph $y = a \cdot f(b(x - c)) + d$ such that any point (x, y) on $f(x)$ maps to the point $\left(\frac{1}{B}x + c, ay + d\right)$. • The *vertex form* for a quadratic function is $f(x) = a(x - h)^2 + k$, where (h, k) is the location of the vertex, and the sign of a indicates whether the parabola opens upward or downward. • A function written in equivalent forms can reveal different characteristics of the function it defines. • You can write a quadratic function in vertex form if you know the coordinates of the vertex and another point on the graph. • You can write a quadratic function in factored form if you know the zeros and another point on the graph.	A.6B A.6C **A.7A** **A.7C**	3
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					4

*Bold TEKS = Readiness Standard

TOPIC 2: Polynomial Operations

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.E, 2.A, 3.G, 3.I, 3.J, 4.J, 5.D, 5.G

Topic Pacing: 12 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	Adding and Subtracting Polynomials	Students begin the lesson with an open-ended sort of twelve mathematical expressions prior to being given the formal definitions regarding polynomials. The terms *polynomial*, *monomial*, *binomial*, *trinomial*, and *degree of a polynomial* are defined. The graphs of two functions in context provide meaning to subtraction of functions and introduce the concept of performing addition and subtraction of polynomial functions. Students add and subtract functions both graphically and algebraically within a context. They analyze polynomial expressions that have been rewritten incorrectly and write them correctly.	• A *polynomial* is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable is the sum of terms of the form ax^k, where a is any real number and k is a non-negative integer. In general, a polynomial is of the form $a_1x^k + a_2x^{k-1} + \ldots + a_nx^0$. • Polynomials with only one term are *monomials*. Polynomials with exactly two terms are *binomials*. Polynomials with exactly three terms are *trinomials*. • The algorithms to add and subtract polynomials are extensions of combining like terms.	A.10A A.10D	3

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
2	Multiplying Polynomials	Students begin the lesson analyzing a Worked Example that shows how to multiply two binomials using algebra tiles. They then use algebra tiles and multiplication tables to multiply binomials. Students compare two methods to multiply polynomials: an area model and the distributive property. They practice multiplying polynomials and apply what they learned to rewrite quadratic functions in vertex or factored form into standard form. Finally, students investigate patterns in special products to recognize perfect square trinomials and the difference of two squares expressed both in their standard form and as the linear factors that produce them.	• Algebra tiles and multiplication tables model the product of two binomials. • You can use graphing technology to verify the product of two binomials. • You can multiply polynomials using the distributive property. • The difference of two squares is an expression in the form $a^2 - b^2$ that has factors $(a - b)(a + b)$. • A perfect square trinomial is an expression in the form $a^2 + 2ab + b^2$ or the form $a^2 - 2ab + b^2$ and has the factors $(a + b)^2$ or $(a - b)^2$, respectively.	A.10B A.10D	3
3	Polynomial Division	Students analyze the graph of a quadratic function that appears to have two real zeros. The factor theorem is stated, and a Worked Example demonstrates how to determine whether a linear expression is a factor of the quadratic function. Polynomial long division is introduced, and a Worked Example is provided. Students perform polynomial long division to determine the linear function that is the other factor, and they use this information to determine the zeros and rewrite the quadratic function as a product of linear factors. The remainder theorem is stated, and students use this theorem to answer questions involving polynomial division with remainders.	• Factors of polynomials divide into a polynomial without a remainder. • *Polynomial long division* is an algorithm for dividing one polynomial by another of equal or lesser degree. • The *factor theorem* states that a polynomial function $p(x)$ has $x - r$ as a factor if and only if the value of the function at r is 0, or $p(r) = 0$. • The *remainder theorem* states that when any polynomial equation or function $f(x)$ is divided by a linear expression of the form $(x - r)$, the remainder is $R = f(r)$, or the value of the function when $x = r$.	A.10C	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					3

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TOPIC 3: Solving Quadratic Equations

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.D, 1.E, 1.H, 2.B, 2.C, 3.A, 3.B, 3.D, 4.A, 4.D, 4.F, 4.I, 4.J, 5.B

Topic Pacing: 19 Days

Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
1	Representing Solutions to Quadratic Equations	Students investigate how to solve quadratic equations using graphs and writing solutions in terms of their respective distances from the axis of symmetry. They also use their knowledge of square roots to solve parent quadratic equations algebraically. The axis of symmetry is used to express solutions to parent quadratic equations, with solutions situated symmetrically to the right and to the left of the vertical line. Students also identify double roots, estimate square roots, and extract perfect roots where possible. They show graphically that a quadratic function is the product of two linear functions and use the zero product property to explain that the zeros of a quadratic function are the same as the zeros of its linear factors. Students also generalize what they know about the difference of two squares to rewrite any quadratic in the form $f(x) = ax^2 - c$ as the product of two linear factors, including when a and c are not perfect square numbers.	• Every whole number has two square roots, a positive *principal square root* and a negative square root. • A quadratic function is a polynomial of degree 2. Thus, a quadratic function has two zeros or two solutions at $f(x) = 0$. When both solutions are the same, the quadratic function is said to have a double zero. • The x-coordinates of the x-intercepts of a graph of a quadratic function are called the *zeros* of the quadratic function. The zeros are called the *roots* of the quadratic equation. • You can represent the real solutions to a quadratic equation as the x-value of the axis of symmetry plus or minus a constant. • The *zero product property* states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero. Therefore, the zeros of a quadratic function are the same as the zeros of its linear factors. • Any quadratic function in the form $f(x) = ax^2 - c$ can be rewritten as the product of two linear factors, $(\sqrt{ax} + \sqrt{c})(\sqrt{ax} - \sqrt{c})$.	**A.7A** A.7B **A.8A** A.10F A.11A	2

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Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
2	**Solutions to Quadratic Equations in Vertex Form**	In the previous lesson, students solved quadratic functions of the form $f(x) = ax^2 - c$. In this lesson, they learn to determine the roots of any quadratic function written in vertex form. Students make sense of the solution process as they analyze increasingly complex transformations of the parent quadratic function, from $y = (x - c)^2$, to $y = a(x - c)^2$, to $y = a(x - c)^2 + d$. For each form of the function, they solve equations and generalize about the solutions. Students also learn that a quadratic function can have one unique real zero, two real zeros, or no real zeros, and how the number of real zeros relates to the graph of the function.	• The solutions to a quadratic equation can be represented as the x-value of the axis of symmetry plus or minus a constant. • A quadratic function written in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$, is in vertex form. • The solutions for a quadratic equation of the form $y = (x - c)^2$ are $x = c \pm \sqrt{y}$. • The solutions for a quadratic equation of the form $y = a(x - c)^2$ are $x = c \pm \sqrt{\frac{y}{a}}$. • The solutions for a quadratic equation of the form $y = a(x - c)^2 + d$ are $x = c \pm \sqrt{\frac{y-d}{a}}$.	**A.7A** **A.7C** **A.8A**	2
3	**Factoring and Completing the Square**	Students learn to solve quadratic equations of the form $y = ax^2 + bx + c$. First, they factor trinomials using a multiplication table and they then solve quadratic equations by factoring and using the zero product property. Students are introduced to the method of completing the square both conceptually and procedurally. They practice solving equations that are not factorable by completing the square. Students analyze a Worked Example that converts a quadratic equation in standard form, $f(x) = ax^2 + bx + c$, into vertex form, proving that the vertex of any quadratic equation is located at $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$. Students complete the square to rewrite equations in vertex form, graph the function, and identify the zeros in terms of the axis of symmetry.	• One method of solving quadratic equations in the form $y = ax^2 + bx + c$ is to set the equation equal to zero, factor the trinomial expression, and use the zero product property to determine the roots. • *Completing the square* is a method for rewriting a quadratic equation in the form $y = ax^2 + bx + c$ as a quadratic equation in vertex form. • When a quadratic equation in the form $y = ax^2 + bx + c$ is not factorable, completing the square is an alternative method of determining the roots of the equation. • Completing the square is a useful method for converting a quadratic function written as $f(x) = ax^2 + bx + c$ to vertex form for graphing purposes as well as determining the maximum or minimum in problem situations. • Given a quadratic equation in the form $y = ax^2 + bx + c$, the vertex of the function is located at (x, y) such that $x = -\frac{b}{2a}$ and $x = c - \frac{b^2}{4a}$.	**A.7A** **A.8A** **A.10E**	4
4	**The Quadratic Formula**	Students are guided through a Worked Example to derive the quadratic formula. They then use the quadratic formula to solve problems in and out of context and analyze common student errors. They connect the terms of the quadratic formula to its symmetric graph and repeat the process with numeric solutions. The term *discriminant* is defined, and students use the discriminant to identify the number of real roots for a quadratic equation.	• You can use the *quadratic formula*, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to calculate the solutions to any quadratic equation written in standard form, $f(x) = ax^2 + bx + c$, where a, b, and c represent real numbers and $a \neq 0$. • On the graph of a quadratic function, $\pm \frac{\sqrt{b^2 - 4ac}}{2a}$, is the distance from $\left(\frac{-b}{2a}, 0\right)$ to each root. • When a quadratic equation is in the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$, the *discriminant* is $b^2 - 4ac$. When $b^2 - 4ac < 0$, the quadratic equation has no real roots. When $b^2 - 4ac = 0$, the quadratic equation has one real root. When $b^2 - 4ac > 0$, the quadratic equation has two real roots.	**A.7A** **A.8A** A.11A	3
5	**Using Quadratic Functions to Model Data**	Students begin the lesson by determining a quadratic regression model for sets of data, and they use the regression models to make estimates and predictions. Throughout the lesson, students identify the independent and dependent quantities and domain and range of functions.	• You can model some data in context with a quadratic regression model. • You can use the regression model to make estimates and predictions; however, there may be limitations on the domain depending on the context.	**A.6A** **A.7A** A.8B	2
End of Topic Assessment					1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*					5

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Total Days: 165
Learning Together: 109
Learning Individually: 45
Assessments: 11

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