OER Secondary Mathematics

EDITION 1

Algebra I

Skills Practice

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

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MODULE
1

Searching for Patterns

TOPIC 1: Quantities and Relationships

TOPIC 2: Sequences

Name ______________________________ Date ______________

I. Understanding Quantities and Their Relationships

Topic Practice

A. Determine the independent and dependent quantities in each scenario. Be sure to include the appropriate units of measure for each quantity.

1. Ashley is driving to visit her grandmother who lives 325 miles away from Ashley's home. She travels an average of 60 miles per hour.

2. Jamal works at a printing company. He is making T-shirts for a high school volleyball team. The press he runs can print three T-shirts per minute with the school's mascot.

3. On her way to work each morning, Tiara purchases a small cup of coffee for $4.25 from the coffee shop.

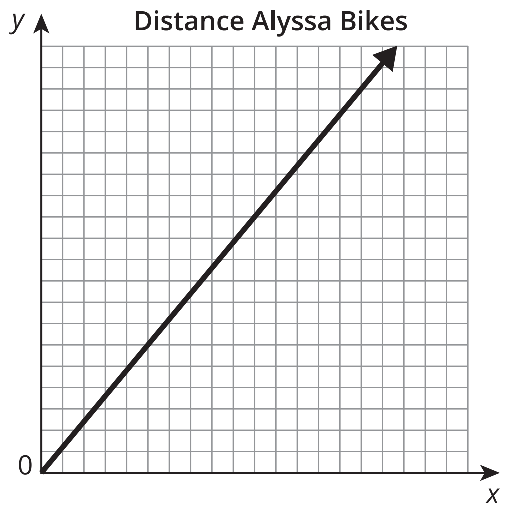
4. Matthew enjoys rock climbing on the weekends. At some of the less challenging locations, he can climb upwards of 12 feet per minute.

5. Josh prefers to walk to work when the weather is nice. He walks the 1.5 miles to work at a speed of about 3 miles per hour.

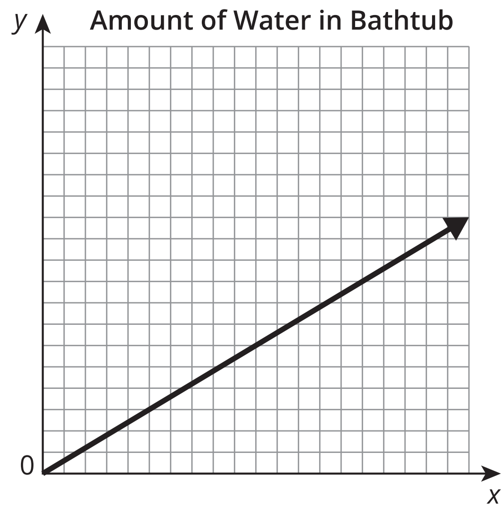
6. Lizzie works for a skydiving company. Customers pay $200 per jump to skydive in tandem skydives with Lizzie.

B. Label the axes of each graph with the independent and dependent quantities. Include appropriate intervals and units of measure.

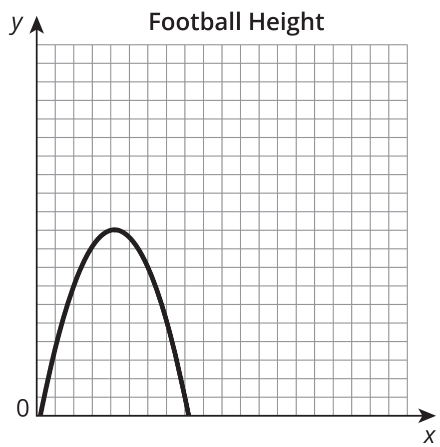
1. Alyssa enjoys bicycling for exercise. Each Saturday she bikes a course she has mapped out around her town. She averages a speed of 12 miles per hour on her journey.



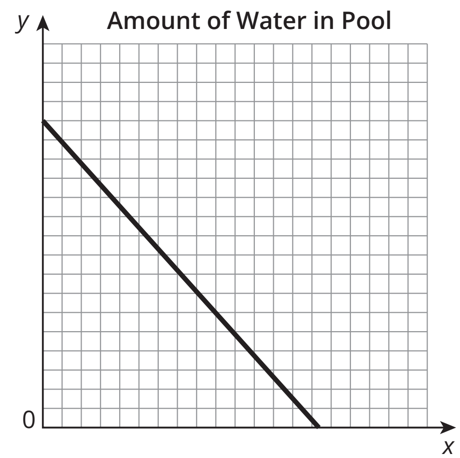
2. Natalia is filling the bathtub with water in order to give her dog Buster a bath. The faucet fills the tub at an average rate of 12 gallons per minute.



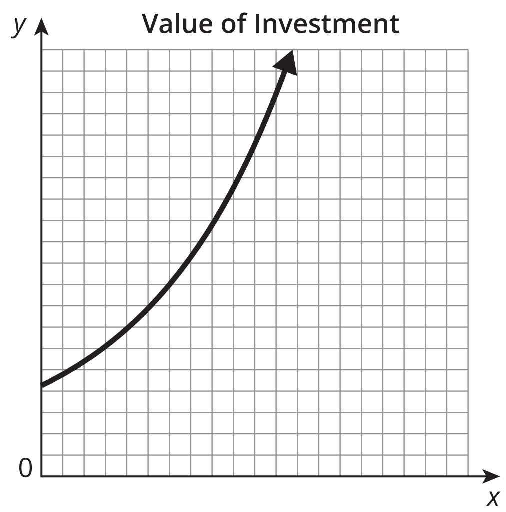
3. Mario throws a football straight up into the air. After the football reaches its maximum height of 20 feet, it descends back to the ground.



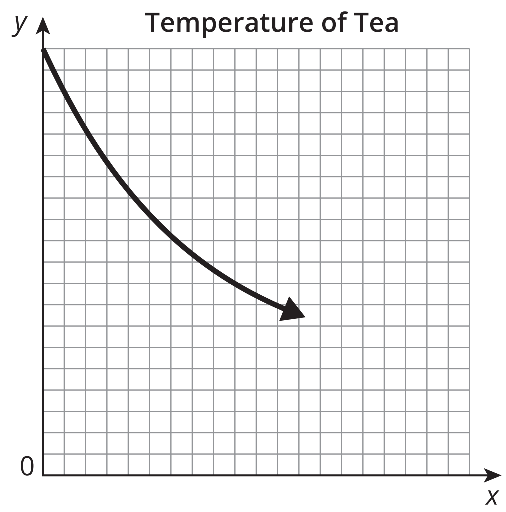
4. Kaya is using a pump to drain her backyard pool to get ready for winter. The pump removes the water at an average rate of 15 gallons per minute.



5. Andrew is saving money to purchase a used car. He places $850 dollars in a savings account that earns 1.65% interest annually.



6. A cup of hot tea is placed on a counter and begins to cool. The initial temperature of the tea was 200°F, and it cooled to room temperature.



Extension

Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure.

1. A student performs several experiments in which he swings a pendulum for a 20-second duration. He uses a string that is 27 cm long, and he tests pendulum masses of different sizes, varying from 2 to 12 grams. He records the number of swings each pendulum makes in 20 seconds.

2. The student then decides to make a second graph showing the string length (in cm) as the independent quantity. What changes must the student make to his experiment?

Spaced Practice

1. Solve the equation $-2x + 8 = -3x + 14$.

2. Solve the equation $-3x - 6 = -5x + 8$.

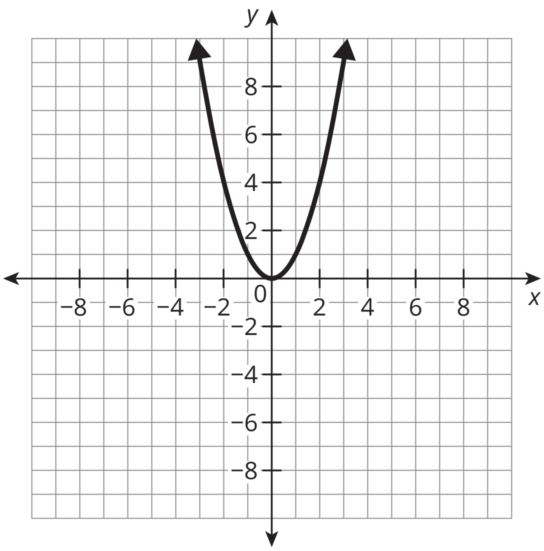
II. Analyzing and Sorting Graphs

Topic Practice

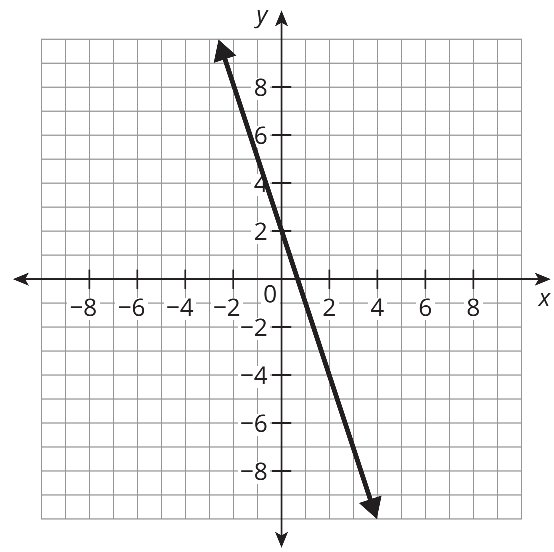
A. Sort the graphs into groups. Write the letter of the graph in the group box. (Some graphs belong to more than one group.)

Function	Maximum or Minimum	Increasing Only	Decreasing Only

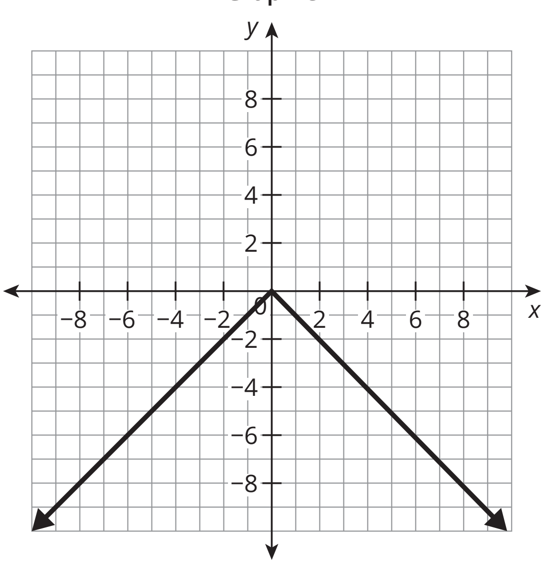
Graph A



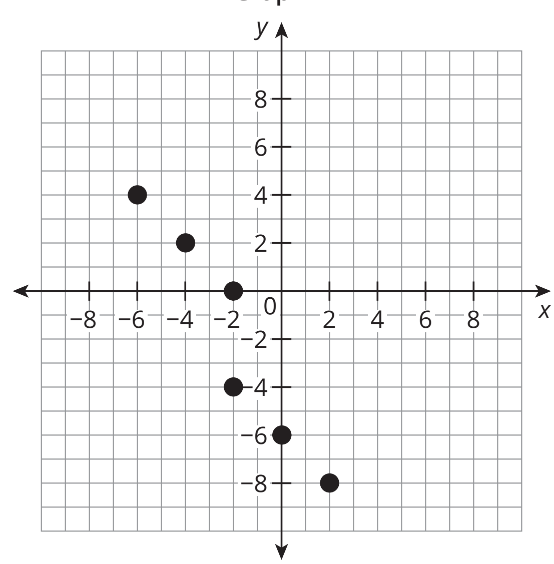
Graph B



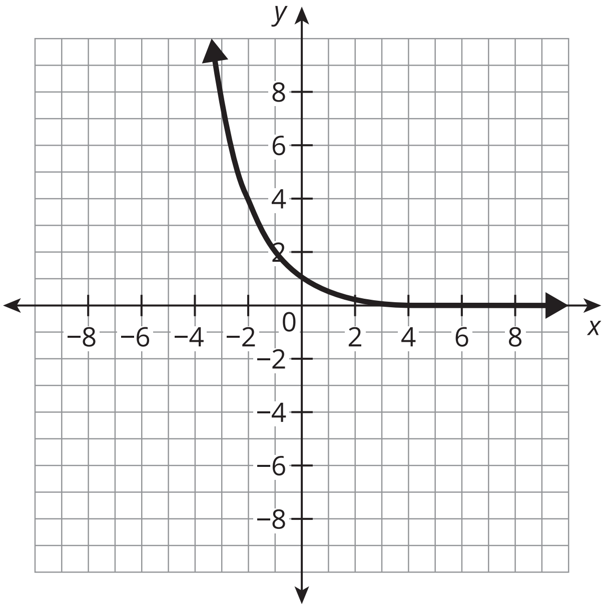
Graph C



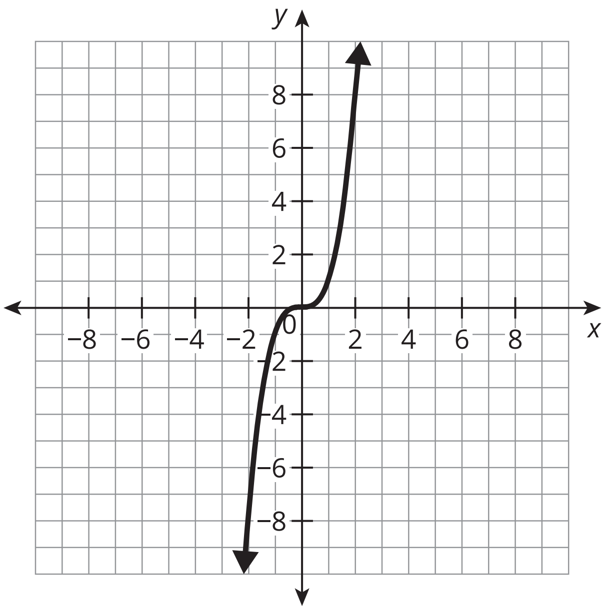
Graph D



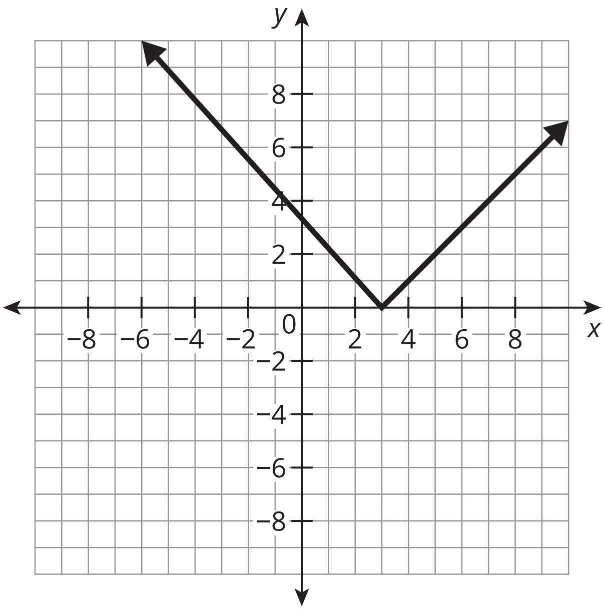
Graph E



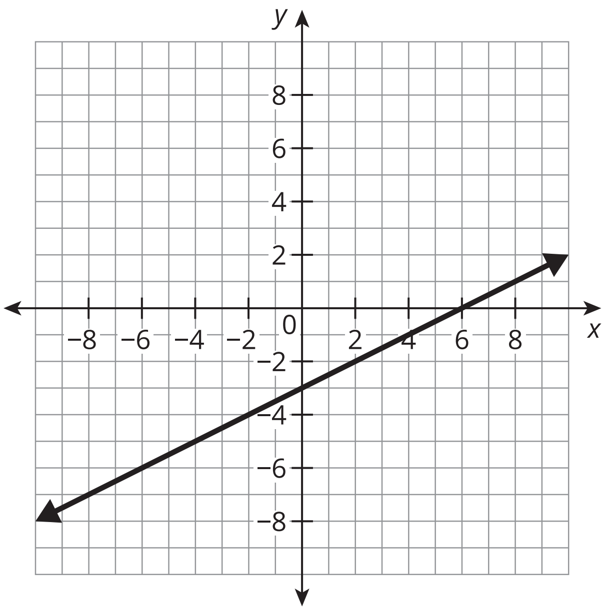
Graph F



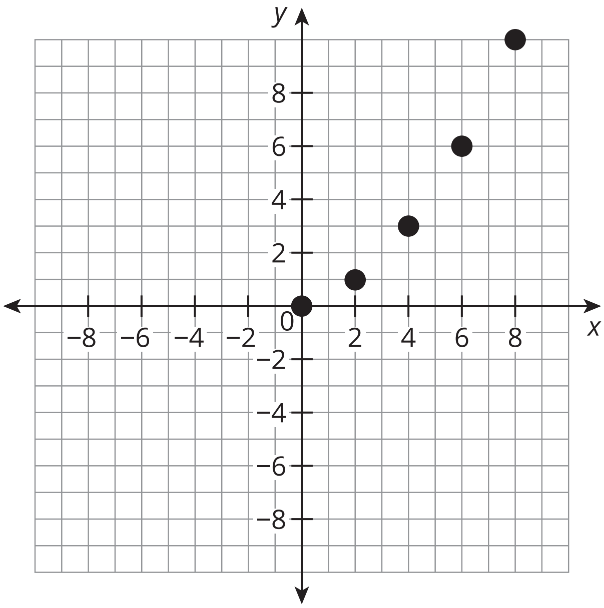
Graph G



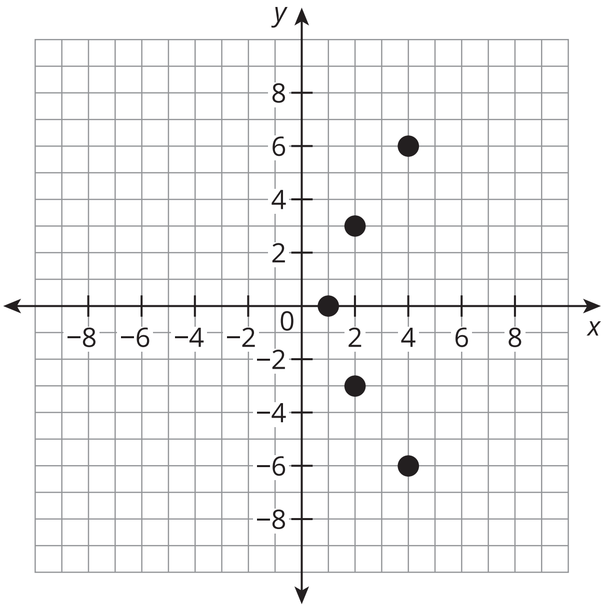
Graph H



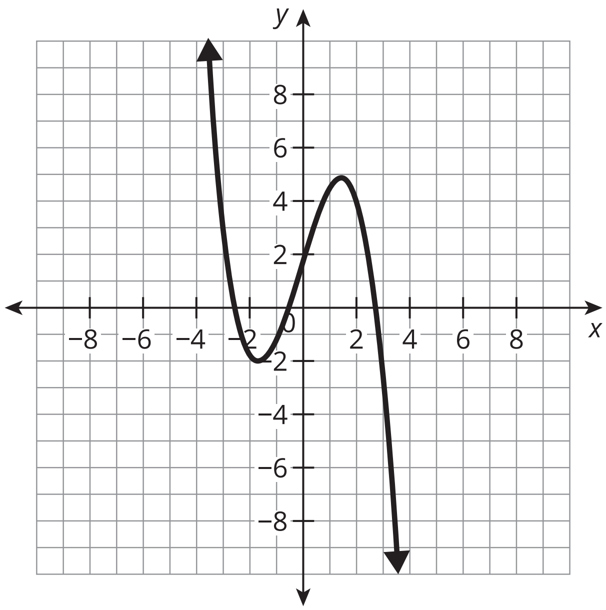
Graph I



Graph J



Graph K



Extension

Describe characteristics of each graph, including whether or not it has a vertical or horizontal axis of symmetry and the number of quadrants it passes through.

1. Diagonal line through the origin that increases from left to right

2. Diagonal line through the origin that decreases from left to right

3. Diagonal line that does not pass through the origin

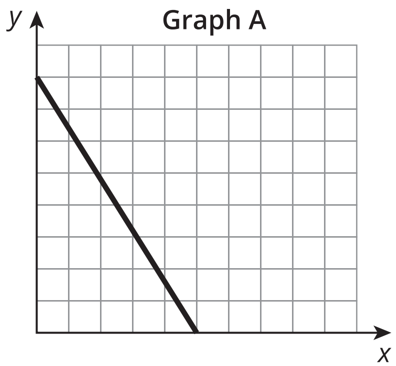
4. Horizontal line below the origin

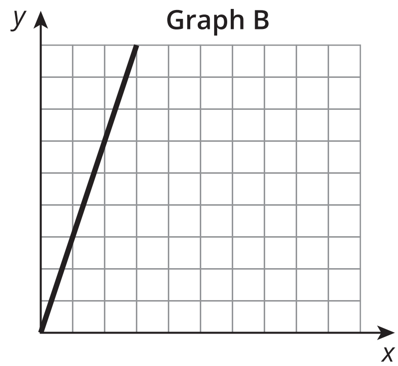
5. Vertical line to the right of the origin

Spaced Practice

1. Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. Then determine which graph models the scenario.

   Henry is cooking a turkey for his family. His recipe says to cook the turkey for 15 minutes per pound.





2. Solve the equation $8y + 13 = 29 - 3y$.

3. Rewrite the expression $6z + 5(-2z - 7)$.

III. Recognizing Functions and Function Families

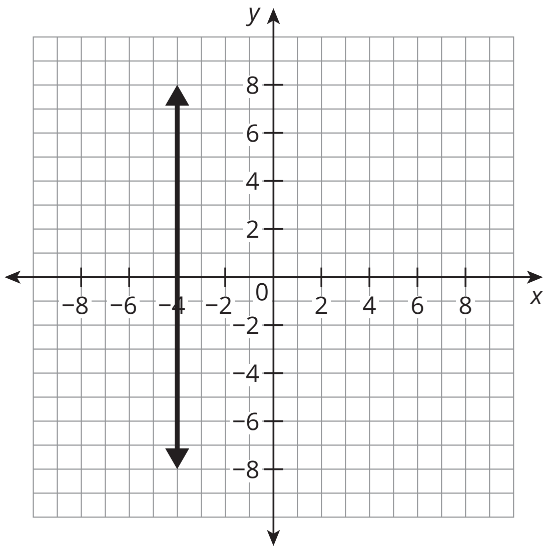
Topic Practice

A. Determine which relations represent functions. If the relation is not a function, state why not.

1.

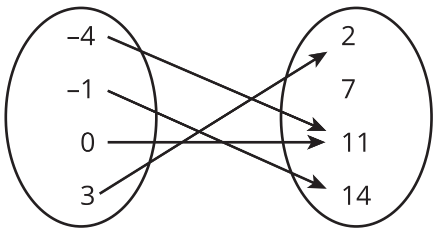
Domain	Range
−5	8
−2	10
0	8
6	15

2.



3. $y = x^2 - 4$

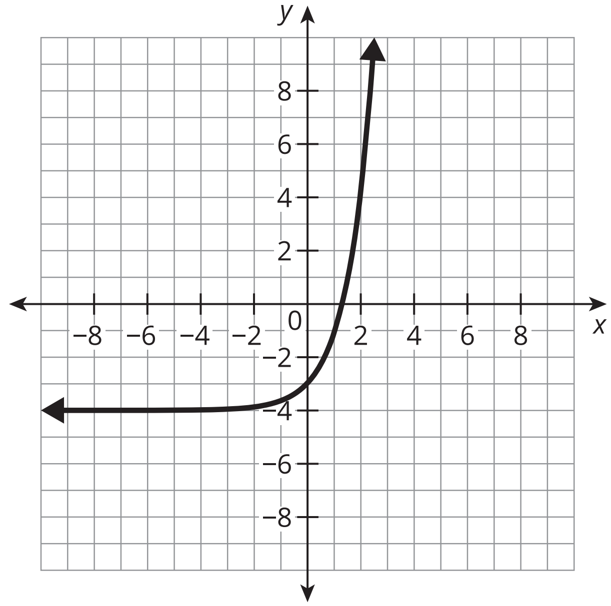
4.



5.

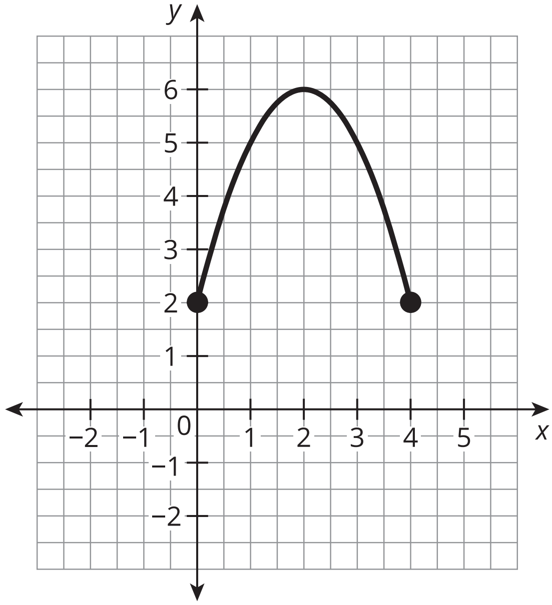
Domain	Range
2	4
7	3
2	0
3	7

6.



B. Write the domain and the range of each function in words and using inequalities notation.

1.



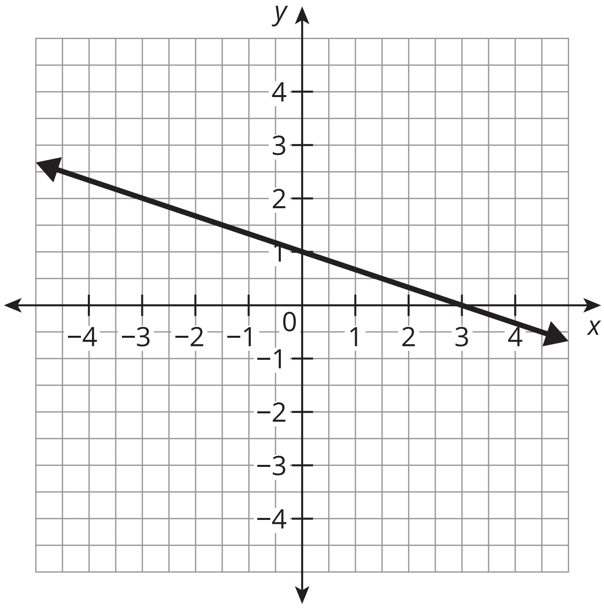
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

2.



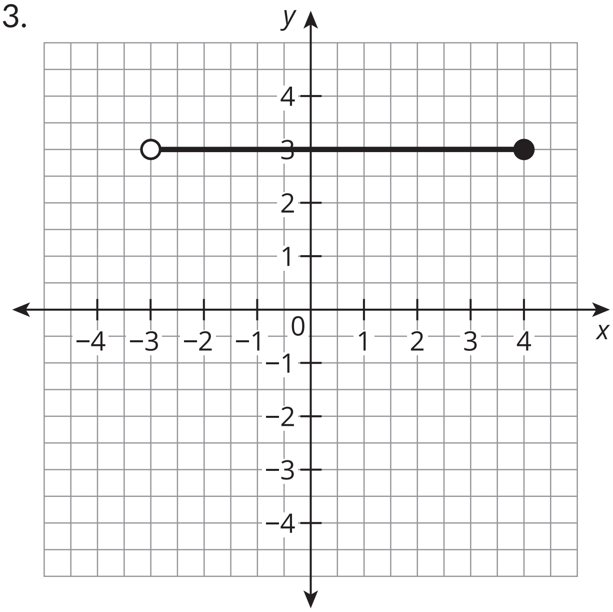
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

3.



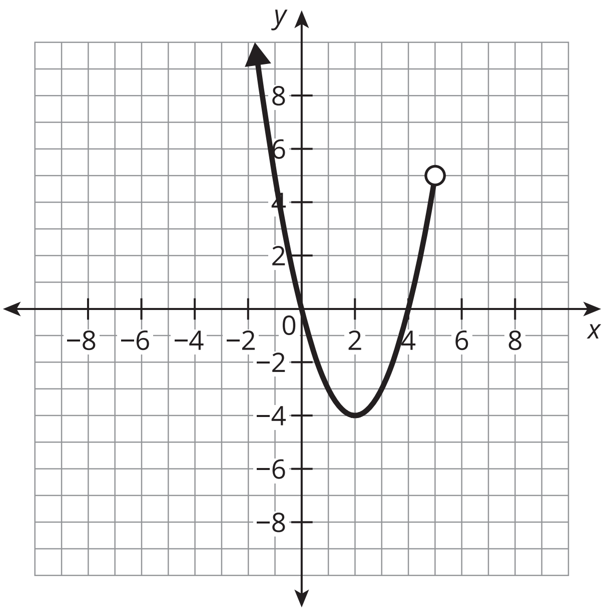
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

4.



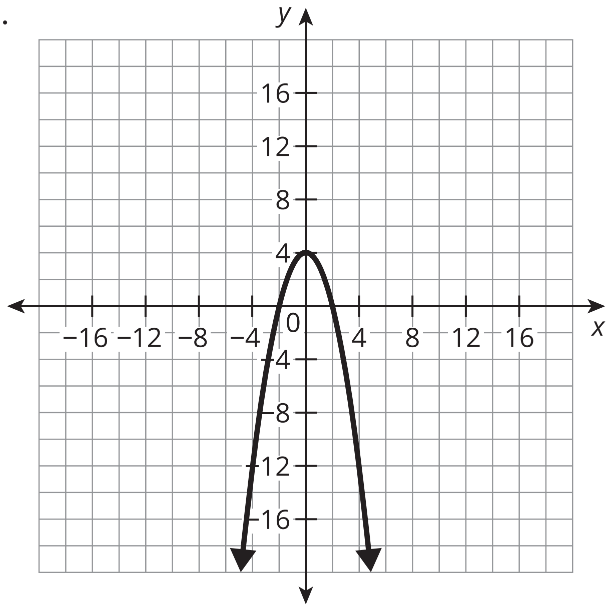
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

5.



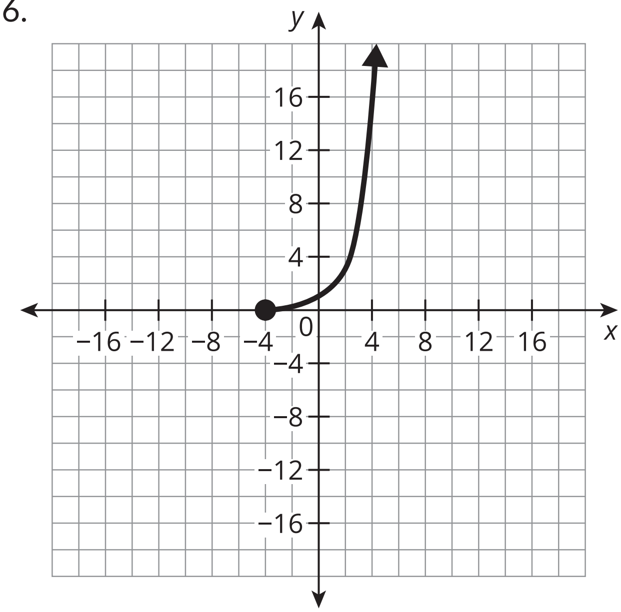
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

6.



Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

C. Identify the domain and range of each situation using words and inequalities.

1. Andrew is at an amusement park. While waiting in line, he reads the statistics on the roller coaster he is about to board. The coaster reaches a maximum speed of 75 miles per hour, and the ride lasts three minutes.

2. Alejandro uses a hot water bottle on an injury to his back he incurred playing basketball. He fills the bottle with water that is a temperature of 100°F. After 25 minutes, Alejandro finds that the bottle has cooled and he stops using it.

3. A township buys a new asphalt road paver. The paver costs $50,500. The value of the paver decreases each year after purchase. Eventually, it decreases so much in value that it can only be sold for $2,500 for parts.

4. Trung and his friends are going to a local peach festival. They plan to go on rides all day. The tickets for the rides are $0.50 each.

5. Samantha is kayaking on a river. The cost to rent the kayak is $25 per hour or $200 for the entire day, from 8 a.m. to 8 p.m.

6. Eduardo has a thermos that he takes to volleyball practice. The thermos can hold up to 18 ounces of water. The thermos develops a leak and starts losing water at a rate of 1 ounce every 2 minutes.

Extension

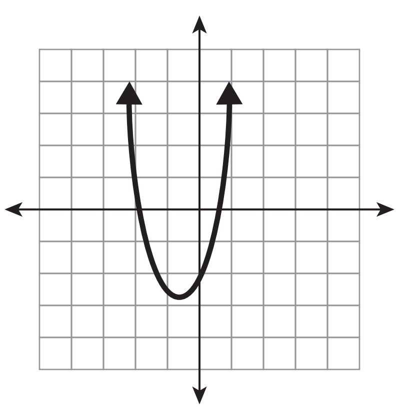
Graph both functions on the same screen using graphing technology. Use reasoning to classify the second function as a new family. Then describe the similarities and differences between the shapes of the graphs in terms of intervals of increase and decrease, maximums or minimums, and whether they are curves or lines.

$$h(x) = x^2 + 9x + 14 \qquad p(x) = |x^2 + 9x| + 14$$

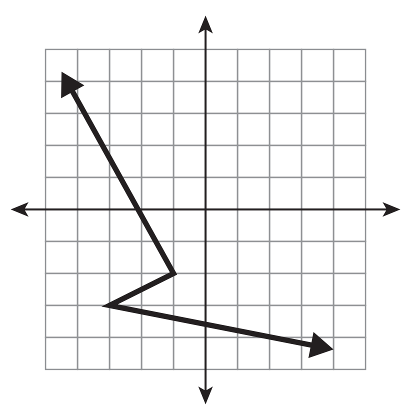
Spaced Practice

1. Identify the axis of symmetry each graph has, if any, and identify the number of quadrants it passes through.

a.



b.



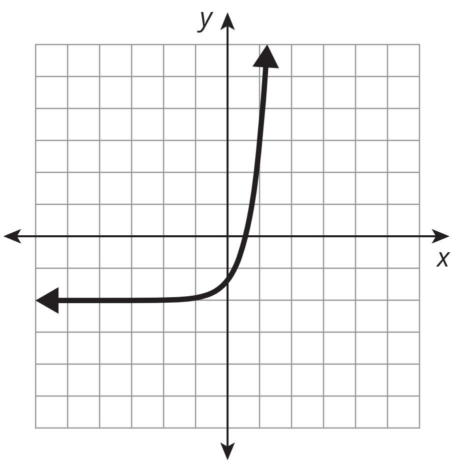
2. Solve the equation $18n + 40 = 14n + 16$.

IV. Recognizing Functions by Characteristics

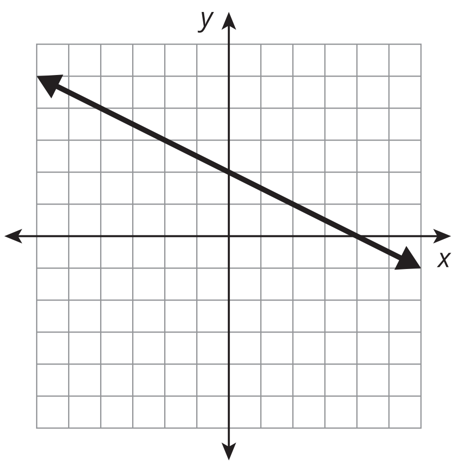
Topic Practice

A. Determine whether each graph represents a linear function, a quadratic function, or an exponential function. Then, identify if the function is increasing, decreasing, or both, and if the graph is continuous or discrete.

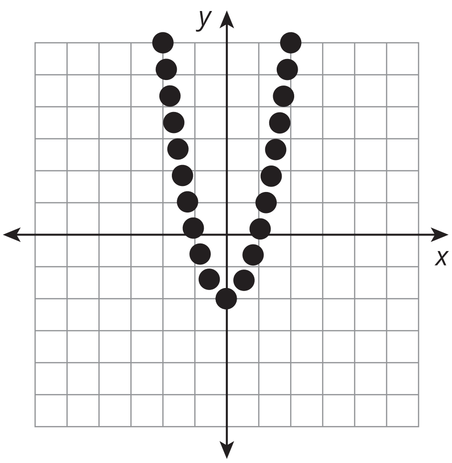
1.



2.



3.



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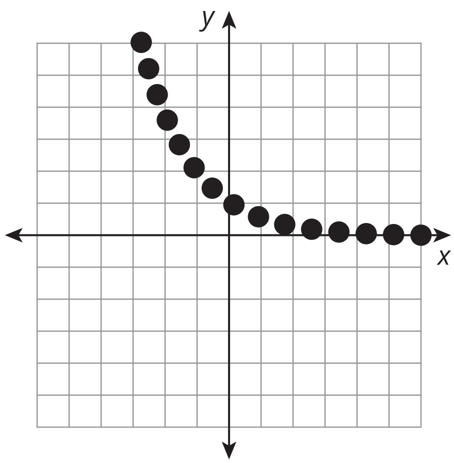
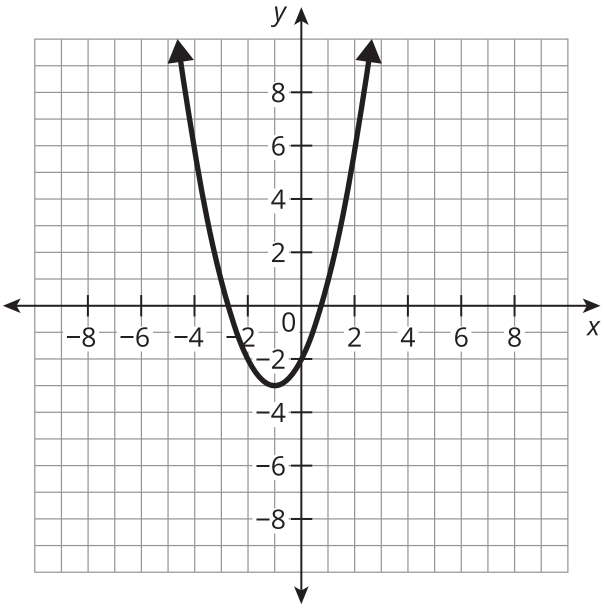


5.


y
x
8
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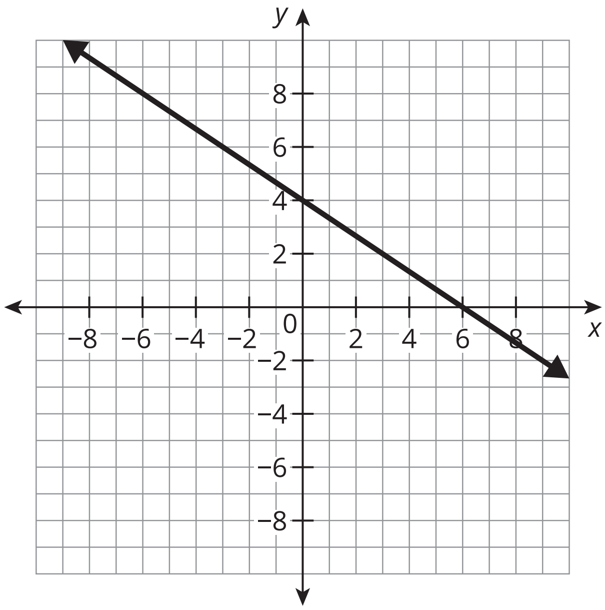
6.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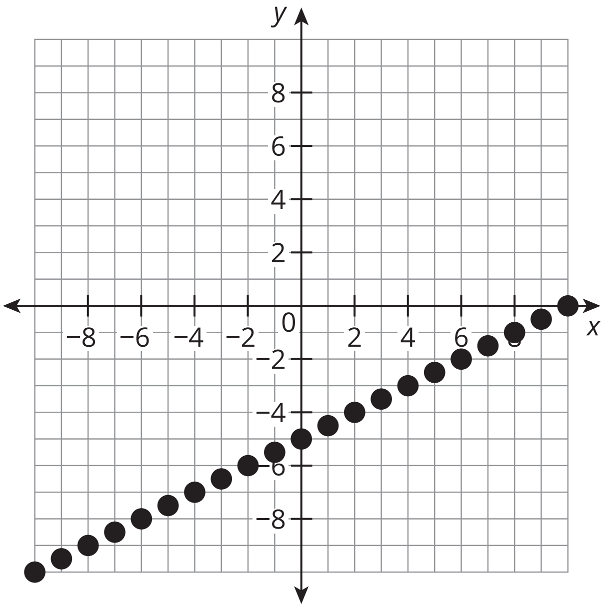
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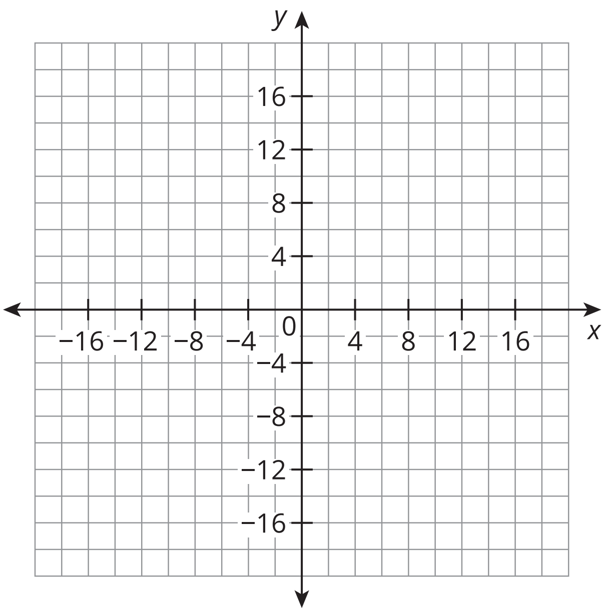
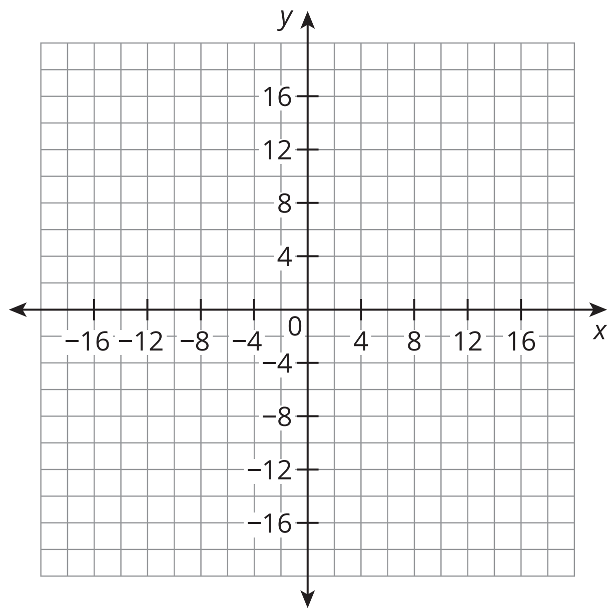
−6
−4
−2
2
4
6
8

B. Create an equation and sketch a graph for a function with each set of given characteristics. Use values that are any real numbers between − 10 and 10.

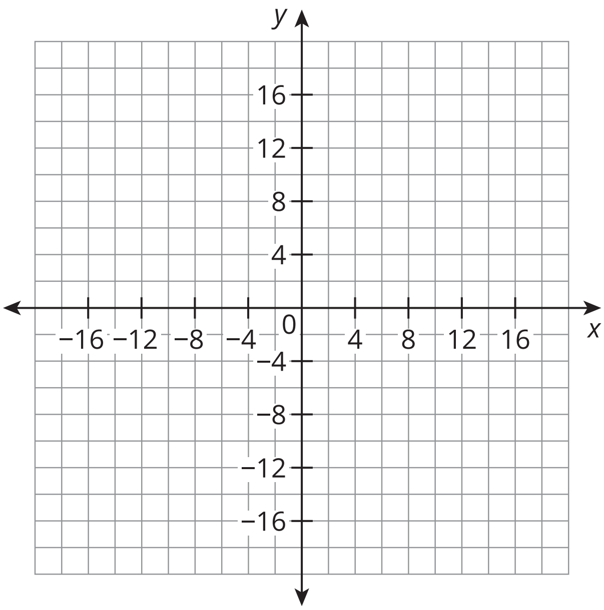
1. Create an equation and sketch a graph that:
   - is a smooth curve,
   - is continuous,
   - has a minimum, and
   - is quadratic.



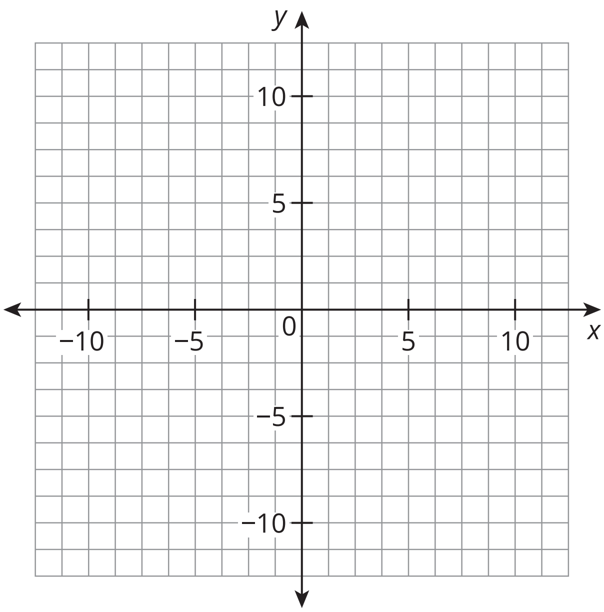
2. Create an equation and sketch a graph that is:
   - linear,
   - discrete, and
   - decreasing across the entire domain.



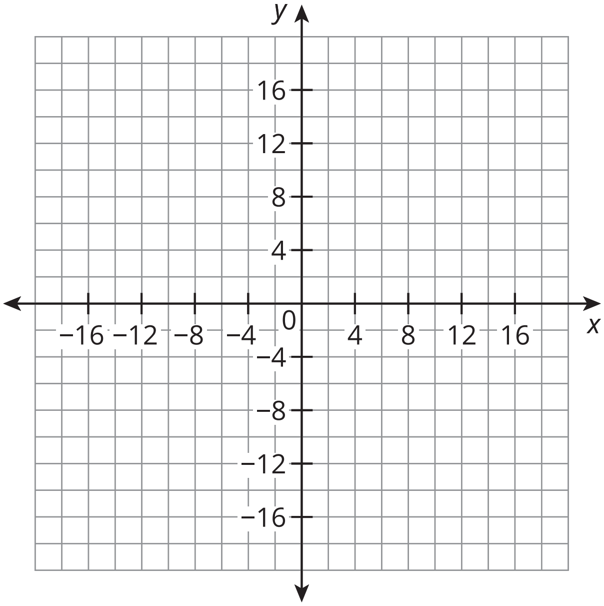
3. Create an equation and sketch a graph that is:
   - a smooth curve,
   - increasing across the entire domain,
   - continuous, and
   - exponential.



4. Create an equation and sketch a graph that:
   - is exponential,
   - is discrete,
   - is decreasing across the entire domain, and
   - does not cross the line $y = -2$



5. Create an equation and sketch a graph that:
   - is discrete,
   - has a maximum,
   - does not pass through the origin, and
   - is quadratic.



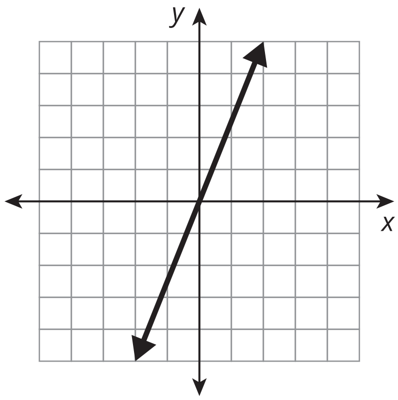
Extension

Write an equation and sketch a graph that has a minimum in Quadrant IV, is continuous, and is a linear absolute value function.

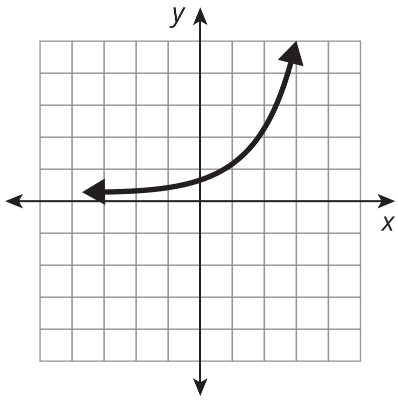
Spaced Practice

1. Choose the graph that represents the function $f(x) = -x^2 + 3x$.

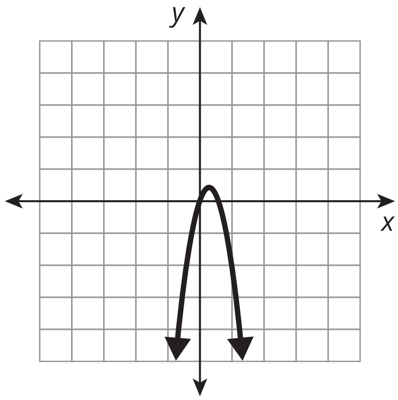
Graph A



Graph B

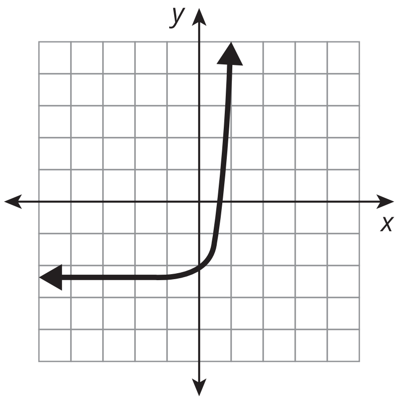


Graph C

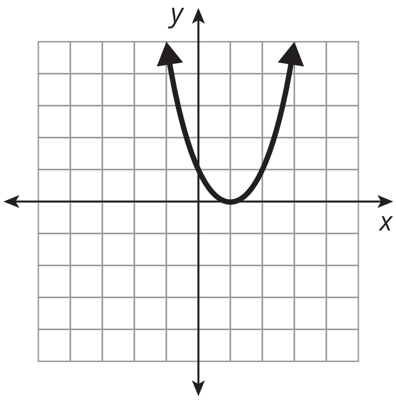


2. Which graph represents an exponential function?

Graph A



Graph B



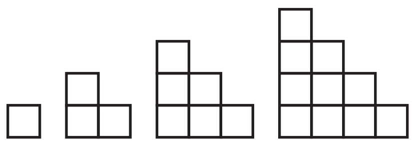
3. Solve the equation $68 = -7 - 15b$.

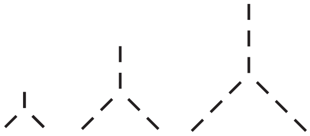
Name ____ Date ________________

I. Recognizing Patterns and Sequences

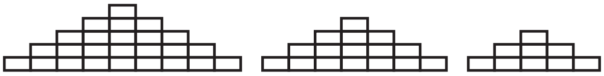
Topic Practice

A. Describe each pattern. Draw the next two figures in the pattern.

1. 

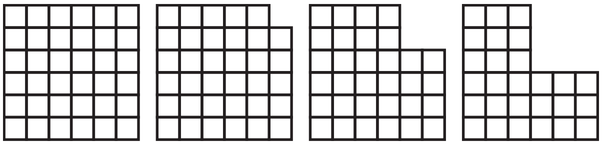
2. 

3. 

4. 

5.

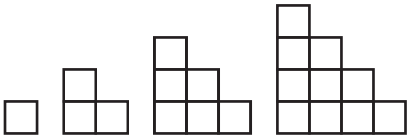
6.



B. Represent each pattern or situation with a numeric sequence.

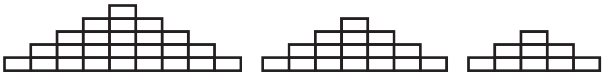
1. The school cafeteria begins the day with a supply of 1000 chicken nuggets. Each student that passes through the lunch line is given 5 chicken nuggets. Represent the total number of chicken nuggets remaining in the cafeteria's supply after each of the first 6 students pass through the line with a numeric sequence. Include the number of chicken nuggets the cafeteria started with.

2. Represent the number of squares in each of the first 7 figures of the pattern with a numeric sequence.



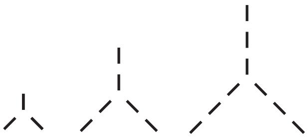
3. Alyssa starts a job at a restaurant. She deposits $40 from each paycheck into her savings account. There was no money in the account prior to her first deposit. Represent the amount of money in the savings account after Alyssa receives each of her first 6 paychecks with a numeric sequence.

4. Represent the number of blocks in each of the first 5 figures of the pattern with a numeric sequence.



5. Matthew is collecting canned goods for a food drive. On the first day he collects 1 can. On the second day he collects 2 cans. On the third day he collects 4 cans. On each successive day, he collects twice as many cans as he collected the previous day. Represent the total number of cans Matthew has collected by the end of each of the first 7 days of the food drive with a numeric sequence.

6. Represent the number of line segments in each of the first 7 figures of the pattern with a numeric sequence.



Extension

Kaya's Kitchen

Kaya is opening a restaurant and tells her staff they have to go above and beyond to please their customers, especially on opening day. She reasons that if one customer is pleased with the restaurant, that person is likely to tell 4 people about it. Then each of those people is likely to tell 4 people about it, and so on.

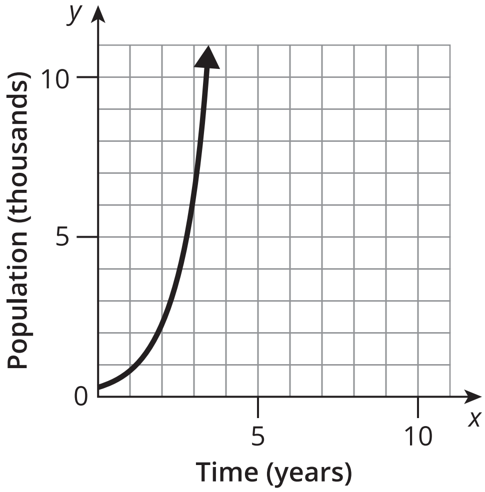
- Describe the pattern for the number of customers Kaya's Kitchen will reach with each telling.
- Determine how many customers are reached after the 5th, 6th, and 7th tellings.
- Represent the number of customers reached with each telling as a numeric sequence. Then represent the sequence using a table of values.
- Identify the appropriate function family for the function. Then describe whether the function is continuous or discrete.

Term Number	Term

Spaced Practice

1. For the scenario and graph:
   - Identify the appropriate function family.
   - Describe the domain based on the problem situation.
   - Identify the graphical behavior of the function as increasing, decreasing, or a combination.

   A city discovers that its population has been tripling every year. The function graphed models the population (in thousands) after x years.



II. Arithmetic and Geometric Sequences

Topic Practice

A. Determine the common difference and the next 3 terms in each arithmetic sequence.

1. 8, 14, 20, 26, ____, ____, ____, ...

2. −24, −14, −4, 6, ____, ____, ____, ...

3. $\frac{3}{5}, \frac{4}{5}, 1, \frac{6}{5}$, ____, ____, ____, ...

4. 12, 16.5, 21, 25.5, ____, ____, ____, ...

5. −101, −112, −123, −134, ____, ____, ____, ...

6. 3.8, 5.1, 6.4, 7.7, ____, ____, ____, ...

B. Determine the common ratio and the next 3 terms in each geometric sequence.

1. 3, 9, 27, 81, ____, ____, ____, ...

2. 512, 256, 128, 64, ____, ____, ____, ...

3. 5, −10, 20, −40, ____, ____, ____, ...

4. 3000, 300, 30, 3, ____, ____, ____, ...

5. 2, −2, 2, −2, ____, ____, ____, ...

6. 0.2, 1.2, 7.2, 43.2, ____, ____, ____, ...

C. Determine whether each given sequence is arithmetic or geometric. Write the next 3 terms of each sequence.

1. $a_n = a_{n-1} + 4$ where $a_1 = 4$ and n is a whole number greater than 1.

2. $m_n = -0.8 + m_{n-1}$, where $m_1 = -7.2$ and n is a whole number greater than 1.

3. $p_n = 4 \cdot p_{n-1}$, where $p_1 = 3$ and n is a whole number greater than 1.

4. $k_n = -2 \cdot k_{n-1}$, where $k_1 = 9$ and n is a whole number greater than 1.

5. $f(n) = \frac{1}{2} f(n - 1)$, where $f(1) = 1.5$ and n is a whole number greater than 1.

6. $h(n) = 3 + h(n - 1)$, where $h(1) = 15$ and n is a whole number greater than 1.

Extension

Consider the first 2 terms of the sequence $-6, 18, \ldots$

1. Determine the next 5 terms in the sequence if the sequence is arithmetic. Then, write a function to represent the arithmetic sequence.

2. Determine the next 5 terms in the sequence if the sequence is geometric. Then, write a function to represent the geometric sequence.

Spaced Practice

1. Josh updates his blog regularly with trivia questions for readers to answer. The month he started this, there were 8 trivia questions on his blog. The next month, there were 19 trivia questions on his blog. The month after that, there were 30 trivia questions on his blog.

   a. Think about the number of trivia questions on Josh's blog each month. Describe the pattern.

   b. Determine how many trivia questions will be on Josh's blog during months 4, 5, and 6.

c. Represent the number of trivia questions on Josh's blog for the first 6 months as a numeric sequence. Then, represent the sequence using a table of values.

Term Number	Term

2. Contestants on a popular game show have an opportunity to randomly select a cash prize in 6 hidden containers. The highest possible cash prize is $25,000. The next highest prize is $5000, and the one after that is $1000.

   a. Think about how the value of the prize changes from one container to the next. Describe the pattern.

   b. Determine the prize values in the remaining containers.

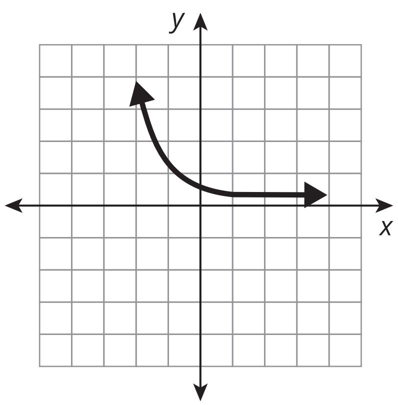
c. Represent the prize values in all six containers as a numeric sequence. Then, represent the sequence using a table of values.

Term Number	Term

3. Enter each function into your graphing calculator to determine the shape of its graph. Then, complete the table based on the characteristics of the function family.

Function	Function Family	Increasing/ Decreasing	Absolute Maximum/ Minimum	Curve/ Line
$h(x) = 5x^2 - 2.8x + 40$				
$g(x) = 30x - 550$				

4. Identify the function family.



III. Determining Recursive and Explicit Expressions from Contexts

Topic Practice

A. Determine whether each sequence is arithmetic or geometric. Then, write a recursive and the simplified explicit formula for the sequence.

1. 4, 8, 16, 32, ...

2. $a_1 = 16, a_2 = 30, a_3 = 44, a_4 = 58, \ldots$

3. $2, -6, 18, -54, \ldots$

4. $a_3 = 7.3, a_4 = 9.4, a_5 = 11.5,$
$a_6 = 13.6, a_7 = 15.7, \ldots$

5. 320, 410, 500, ...

6. $a_3 = 63, a_4 = 189, a_5 = 567, a_6 = 1701, \ldots$

B. Determine the specified terms of each sequence.

1. What is the 10th term of the sequence $a_n = 3 \cdot 2^{n-1}$?

2. What is the 50th term of the sequence $a_n = 100 + (-8)(n - 1)$?

3. What is the 6th term of the sequence $a_n = a_{n-1} \cdot (-3)$, if $a_5 = 162$?

4. What is the 12th term of the sequence $a_n = a_{n-1} + \frac{1}{3}$, if the 11th term is $\frac{17}{3}$?

5. What is the 23rd term of the sequence $a_n = a_{n-1} + 2.3$, if the 21st term is 95.8?

6. What is the 19th term of the sequence $a_n = a_{n-1} \cdot \left(-\frac{1}{2}\right)$, if $a_{17} = 162$?

C. Write a recursive and an explicit formula to represent the sequence that models each scenario.

1. The population of a city started at 378,000. Every year thereafter it decreased by 23,000 individuals.

2. A test showed that after the first hour of receiving a medication, 100 milligrams remained in the body. Continued tests showed that the dose found in the body halved every hour after the first hour.

3. A new pet bakery sold 20 dog cakes during Week 1 of operation, 27 dog cakes during Week 2, and 34 dog cakes during Week 3.

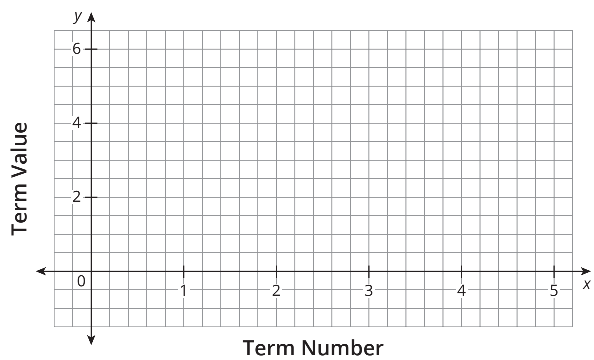
4. A barrel starts with 6 cups of water in it. During a heavy rainstorm, the amount of water in the barrel doubles every minute.

5. A petri dish starts with a sample of 3 bacteria. The number of bacteria triples every minute.

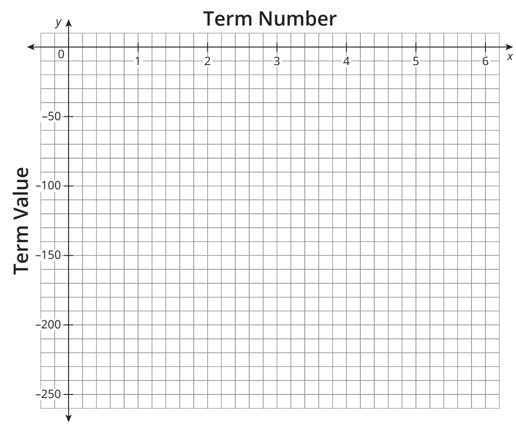
6. Natalia is selling cupcakes at a bake sale. She starts with 100 cupcakes and sells 10 per hour.

D. Write the explicit formula you can use to represent the terms of the sequence, then represent the sequence on the coordinate plane.

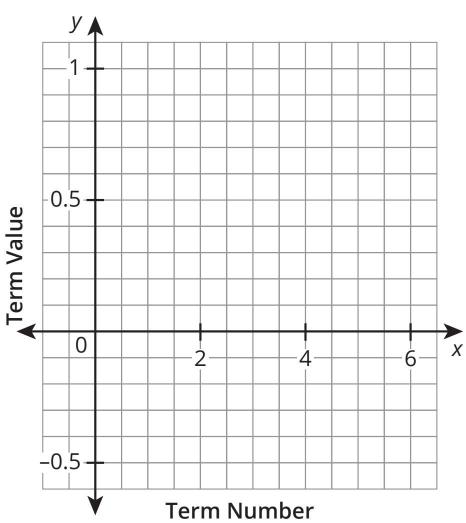
1. $a_4 = \frac{3}{4}, a_5 = 0, a_6 = -\frac{3}{4}, a_7 = -1\frac{1}{2}$



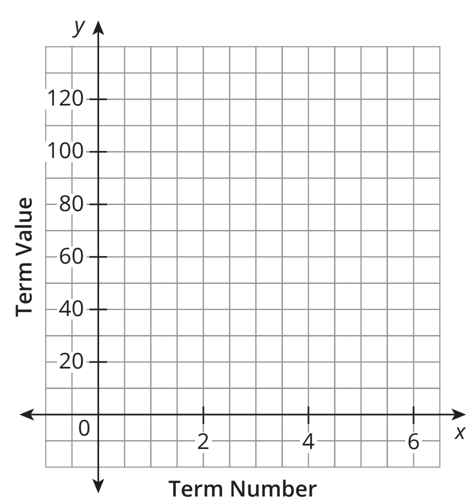
2. $k_8 = 15, k_9 = 50, k_{10} = 85, k_{11} = 120$



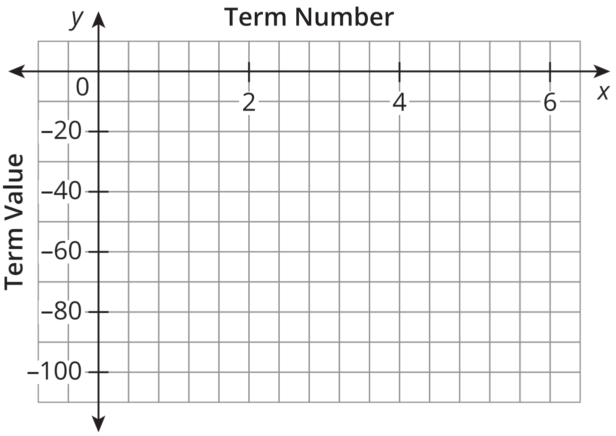
3. The first term of the sequence is $m_1 = \frac{1}{8}$ and the common ratio is -1.5.



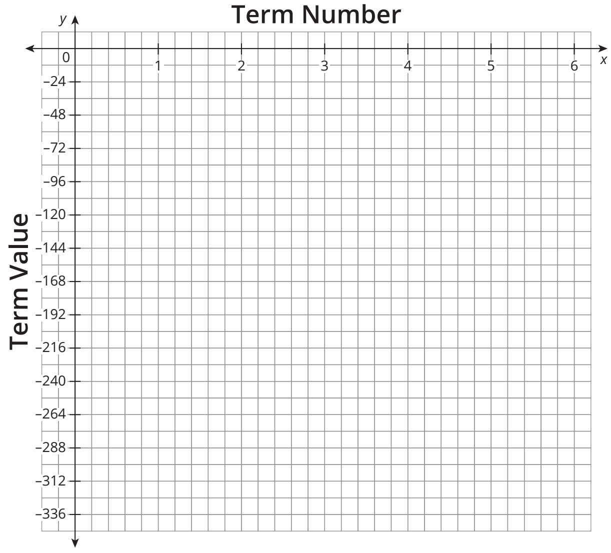
4. $p_2 = 0.6, p_3 = 3.6, p_4 = 21.6, p_5 = 129.6$



5. The first term of the sequence is $m_1 = \frac{1}{2}$ and the common difference is -20.



6. $p_4 = -12, p_5 = -4, p_6 = -\frac{4}{3}, p_7 = -\frac{4}{9}$



Extension

Consider the first two terms of this sequence $\frac{1}{16}, -\frac{3}{16}, \ldots$

1. Determine the 63rd term if this is an arithmetic sequence. Write your answer as an improper fraction in lowest terms.

2. Determine the 63rd term if this is a geometric sequence. Write your answer in scientific notation.

Spaced Practice

1. Determine whether each given sequence is arithmetic or geometric. Then write the next 3 terms of the sequence.

   a. 3, −12, 48, −192, ...

   b. 2.45, 3.86, 5.27, 6.68, ...

2. Determine the independent and dependent quantities in each scenario. Include units when possible.

   a. A lamp manufacturing company produces 750 lamps per shift.

   b. A grocery store sells pears by the pound. A customer purchases 3 pounds for $5.07.

3. Determine the function family for each equation.

   a. $g(x) = -15x^2 + 60x + 370$

   b. $h(x) = 3 \cdot (-5)^x$

MODULE
2

Exploring Constant Change

TOPIC 1: Linear Functions

TOPIC 2: Transforming and Comparing Linear Functions

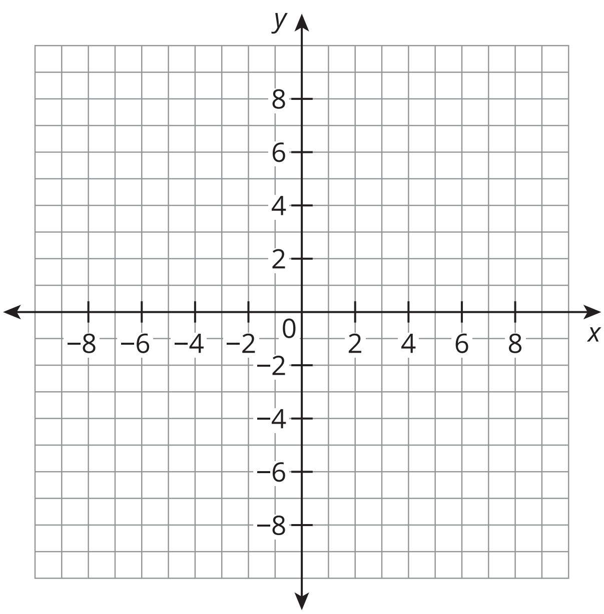
Name ______________________________ Date ______________

I. Least Square Regressions

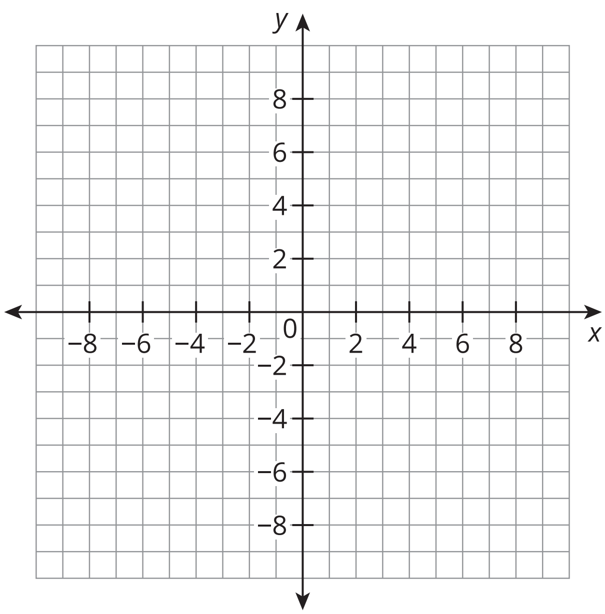
Topic Practice

A. Graph the estimated line of best fit for each set of points. Determine the estimated linear regression function for the line.

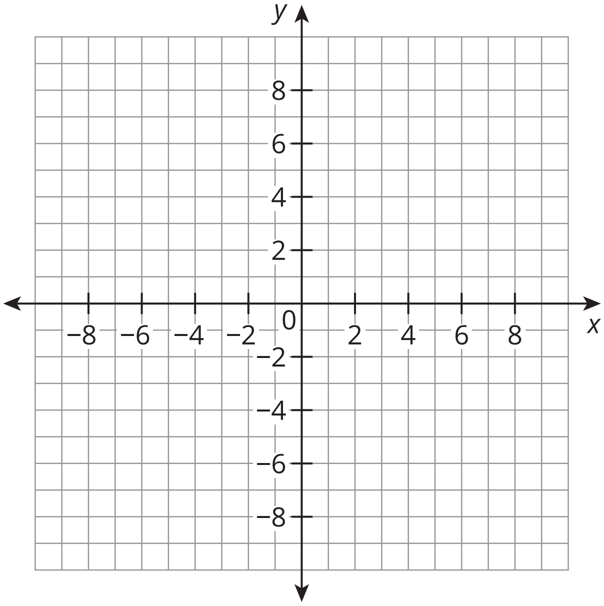
1. (3, 4), (7, 6), and (−2, −4)



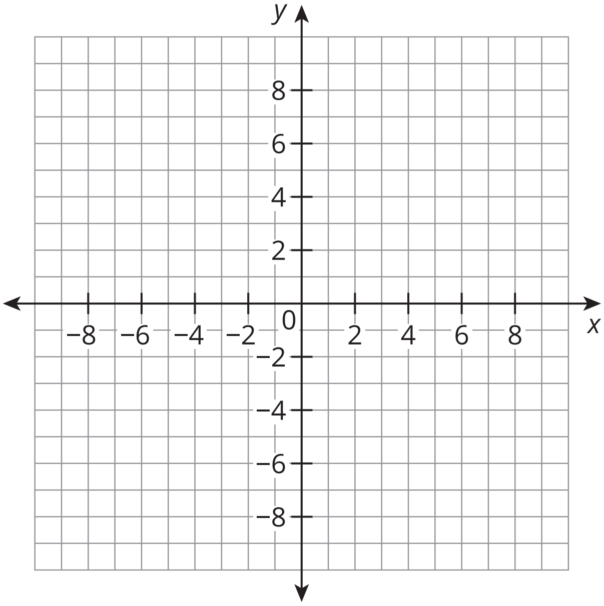
2. (−7, 1), (3, 8), and (9, 7)



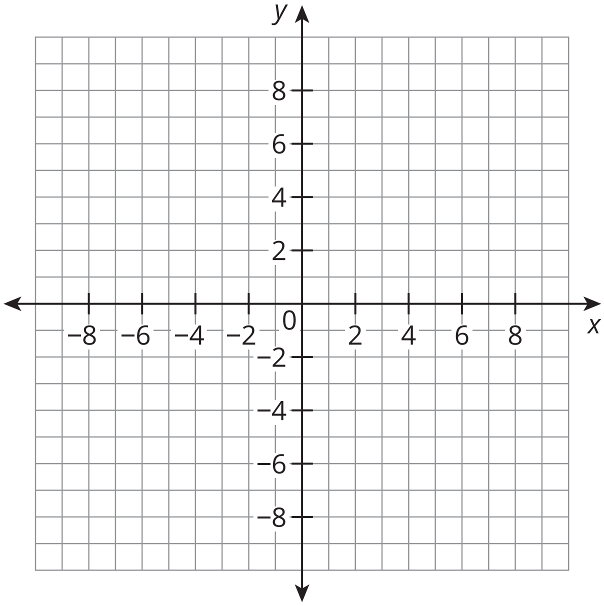
3. $(-3, 6)$, $(-2, -1)$, and $(6, -4)$



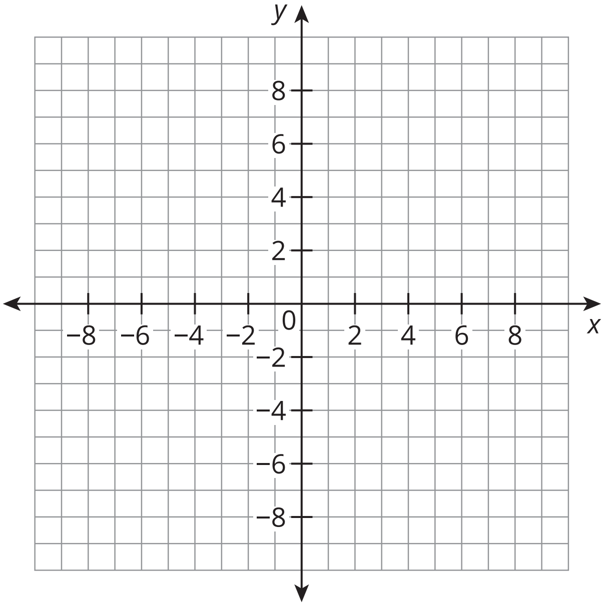
4. $(-8, 7)$, $(-5, 3)$, $(3, 6)$, and $(9, 0)$



5. $(-7, -1)$, $(-5, -9)$, $(3, 3)$, and $(6, 9)$



6. $(-8, 6)$, $(-8, -2)$, $(-6, -9)$, and $(-5, -4)$



B. Use the linear regression function to analyze each scenario.

1. While in high school, Xavier started his own T-shirt printing business. The table shows the number of T-shirts Xavier has sold each year since starting his business in 2010.

Years	2010	2011	2012	2013	2014	2015	2016
Numbers of T-shirts	50	75	175	125	250	350	375

The linear regression function representing the data shown in the table is $y = 57.14x + 28.57$, where x represents the number of years since 2010 and y represents the number of T-shirts sold.

a. Complete the table for the linear regression function, which represents the data from Xavier's T-shirt printing business.

		What it Means	
Expression	Unit	Contextual Meaning	Mathematical Meaning
y			
57.14			
x			
28.57			

Use the linear regression function to predict the number of T-shirts Xavier sold during each given year. Then, compare the prediction to the actual number of T-shirts to determine whether the prediction is reasonable based on the problem situation.

b. 2012

c. 2014

d. 2016

e. 2018

f. 2024

2. The number of students in a school chorus has increased since the school first opened 6 years ago. The table of values represents the change of students in the school chorus, where *x* represents the year and *y* represents the number of students.

Year	0	1	2	3	4	5
Number of Students in the Choir	22	36	40	59	78	83

a. Determine the linear regression function based on the given table of values. Round the values for slope and *y*-intercept to the nearest tenth.

Use your linear regression function to predict the total number students in the choir in each given year. Then, compare the prediction to the actual number of students to determine whether the prediction is reasonable based on the problem situation.

b. The year the school opened

c. Year 2

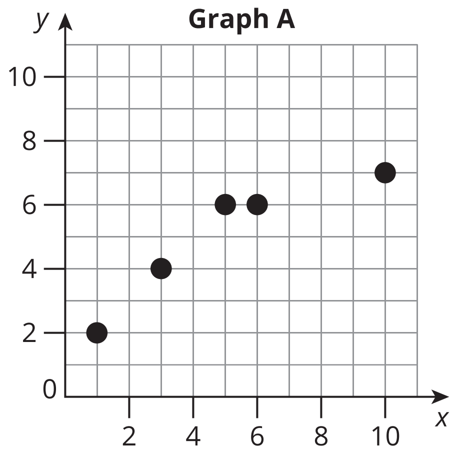
d. Year 4

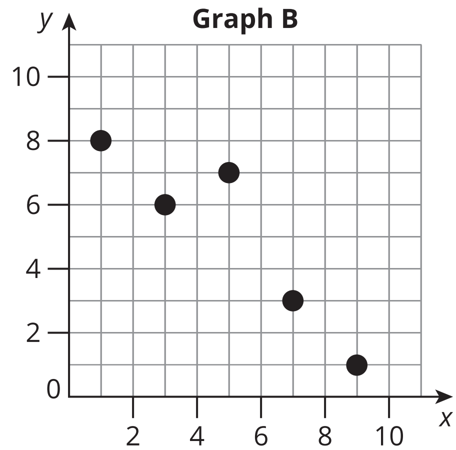
e. Year 6

f. Year 8

Extension

Consider the two sets of data shown in the graphs.





1. Calculate the mean of the *x*-values, $\bar{x}$, and the mean of the *y*-values, $\bar{y}$, for each graph.

2. Complete the tables for each graph.

Graph A				
x	$x - \bar{x}$	y	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
1	−4	2	−3	12
3		4		
5		6		
6		6		
10		7		
				SUM =

Graph B				
x	$x - \bar{x}$	y	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
1	−4	8	3	−12
3		6		
5		7		
7		3		
9		1		
				SUM =

3. Compare the two sums in the last column of each table. Determine if there seems to be a connection between the sums and the graphs of the data set.

Spaced Practice

1. A maintenance worker in a factory notices that a water tank is leaking. She records the amount of water in the tank each day in a table.

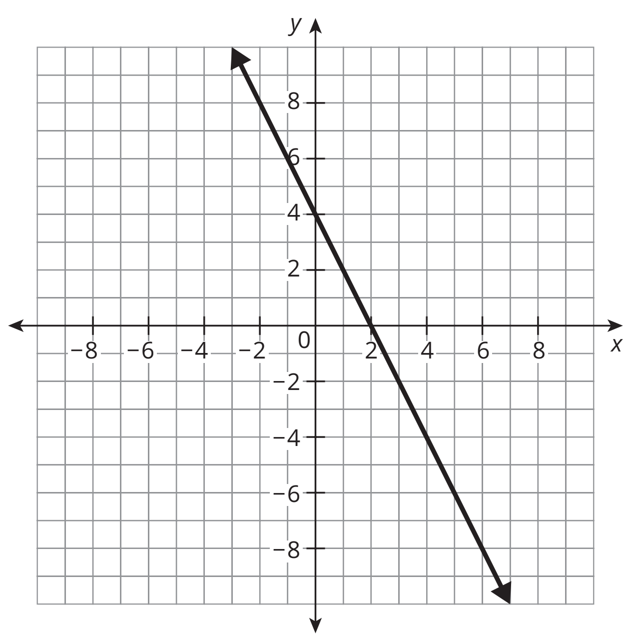
Day	Volume of Water (L)
1	16,000
2	12,000
3	9000
4	6750

   a. Write a recursive formula to represent the pattern shown in the table. What predictions does this formula make for the amount of water in the tank on the 5th day?

   b. Write an explicit formula to represent the pattern shown in the the table. What predictions does this formula make for the amount of water in the tank on the 10th day?

2. The graph represents a linear relationship between x and y.

   a. Describe whether the graph is increasing or decreasing. Justify your reasoning.



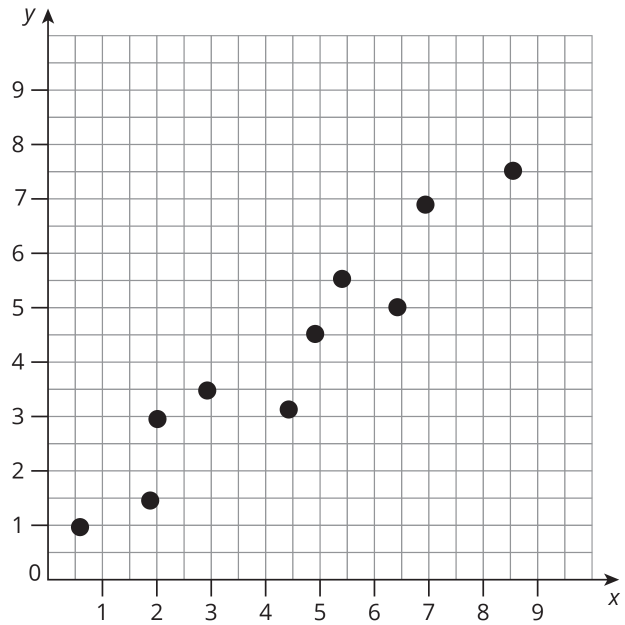
   b. Determine the x- and y-intercept.

II. Correlation

Topic Practice

A. Determine whether the points in each scatterplot have a positive correlation, a negative correlation, or no correlation. Then, determine which *r*-value is most accurate.

1.



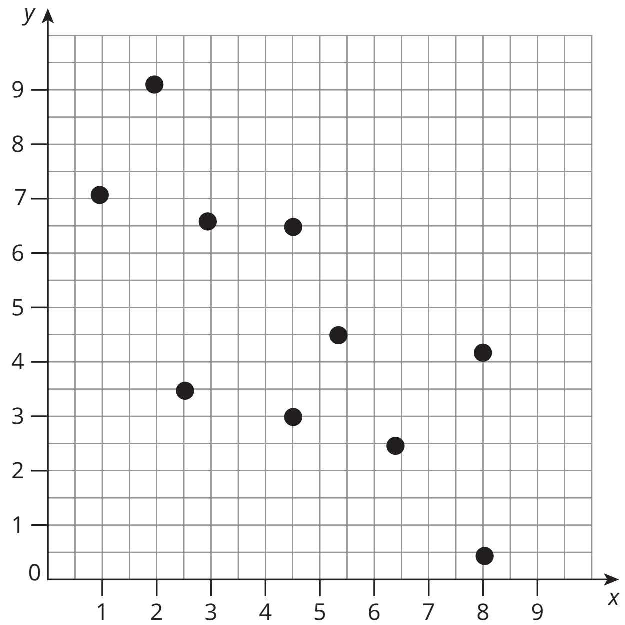
A. $r = 0.097$

B. $r = -0.57$

C. $r = 0.83$

D. $r = 1$

2.



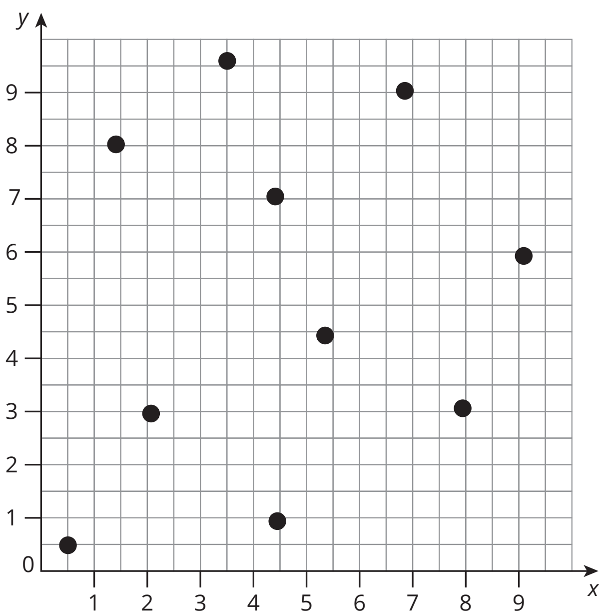
A. $r = 0.9$

B. $r = -0.6$

C. $r = 0.02$

D. $r = -0.006$

3.



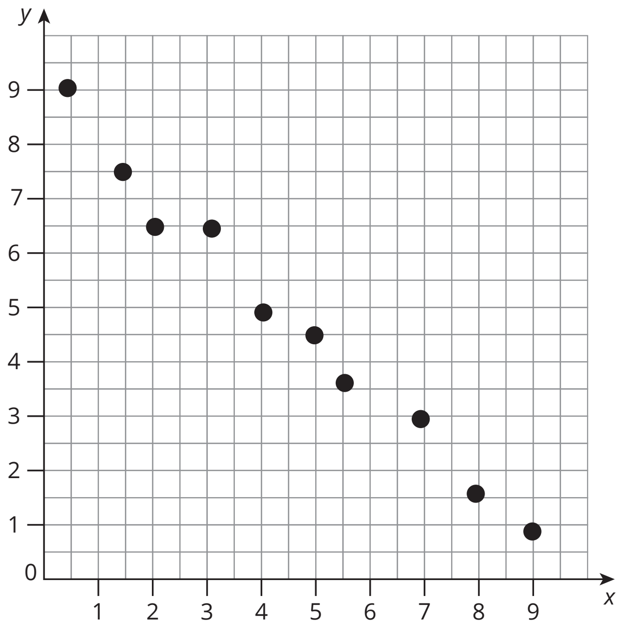
A. $r = 0.01$

B. $r = 0.8$

C. $r = -0.5$

D. $r = 0.08$

4.



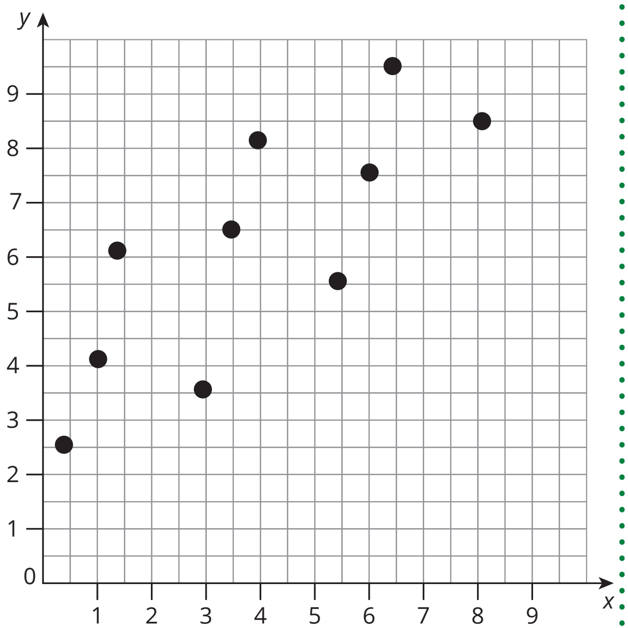
A. $r = -0.009$

B. $r = 0.8$

C. $r = -0.09$

D. $r = 0.2$

5.



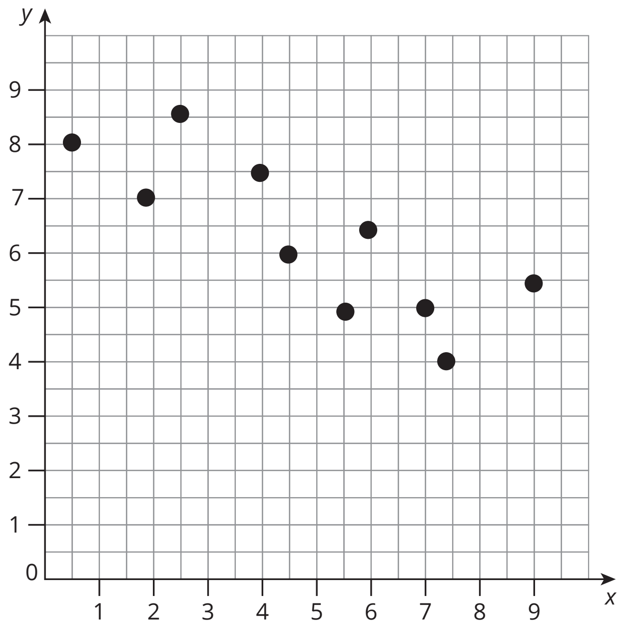
A. $r = -0.003$

B. $r = -0.6$

C. $r = 0.004$

D. $r = 0.7$

6.



A. $r = 0.01$

B. $r = -0.8$

C. $r = -0.01$

D. $r = 0.9$

B. Determine the linear regression function and correlation coefficient for each data set. Round your correlation coefficient to the nearest ten-thousandth. State whether the correlation is strong or weak and positive or negative. Explain your reasoning.

1.

Years since 2011	0	1	2	3	4	5
Profit (dollars)	50,000	75,000	150,000	125,000	195,000	225,000

2.

Month	1	2	3	4	5	6
Profit (dollars)	100,000	85,000	91,000	82,000	79,500	74,000

3.

Time (seconds)	0	1	2	3	4	5
Height (feet)	5	21	34	31	18	3

4.

Month	1	2	3	4	5	6
Sales (dollars)	1480	14,105	8925	18,750	5250	2650

5.

Years since 2014	0	1	2	3	4	5
Units Sold	5245	7840	7075	9130	10,620	12,635

6.

Time (hours)	0	1	2	3	4	5
Height (centimeters)	63	56	42	36	28	12

C. For each situation, decide whether the correlation implies causation. List reasons why or why not.

1. Since 2005, the average salary of an NBA basketball player has increased every year. The average height of an NBA player has also increased yearly since 2005.

2. Scientists have found that the number of people wearing sunscreeen has increased. The number of people getting severe sunburns has decreased.

3. The number of smartphones in the United States has increased yearly since 2005. The number of paper calendars sold yearly has decreased during the same period.

4. The number of shark attacks on a certain beach increases on days when more ice cream is purchased.

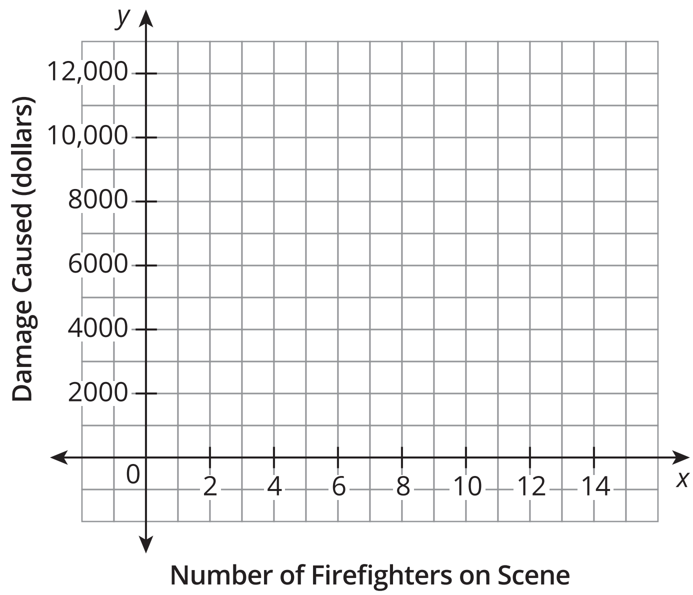
5. The number of smartphones in the United States has increased yearly since 2005. The average weight of an adult in the United States has also increased during this time.

6. A scientific study found a strong correlation between the amount someone exercises and their chance of getting skin cancer.

D. Use the data in the table to answer each question.

A city records the number of firefighters that arrive on a scene and the amount of monetary damage caused by the fire. The data for five fires that firefighters responded to are shown in the table.

Number of Firefighters	Damage Caused (dollars)
5	1000
8	5000
12	10,000
10	7000
4	800



1. Create a scatterplot of the data.

2. Describe the relationship between the number of firefighters that arrive on scene and the amount of monetary damage caused by the fire.

3. Use technology to determine the equation for the line of best fit. Sketch the line on your scatterplot.

4. Determine the correlation coefficient for the line of best fit.

5. Describe any confounding variables.

6. Does the correlation imply causation? Explain your reasoning.

Extension

Consider the points: (1, 2), (2, 3), (3, 2), (4, 5), (5, 2.5), (6, 6), (7, 3), (8, 7). The line of best fit for the graph of the points is $y = 0.5x + 1.4$.

1. Complete the table to determine the predicted values of y for each value of x using the line of best fit, and the values of the differences between the observed y-values from the points and the predicted y-values from the line of best fit.

x	Observed y-Value	Predicted y-Value	Observed y-Value − Predicted y-Value
1	2	1.9	0.1
2	3		
3	2		
4	5		
5	2.5		
6	6		
7	3		
8	7		

2. Determine whether there is a pattern in the differences between the y-values from the completed table. Explain what this might indicate about using the line of best fit to make predictions.

Spaced Practice

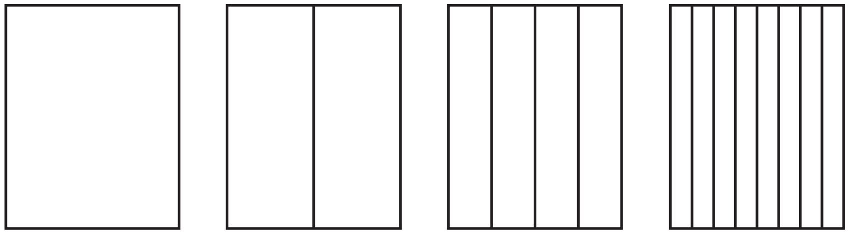
1. The table shows the highest maximum temperature for the month of October in Philadelphia, Pennsylvania, over ten years.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Highest Maximum Temperature (°F)	64.9	53.1	61	54	63	68	61	57.9	64.9	66.9

a. Identify the independent and dependent quantities and their units of measure.

b. Use the data table and graphing technology to generate a linear regression function. What is the slope and *y*-intercept of the line and what do they represent?

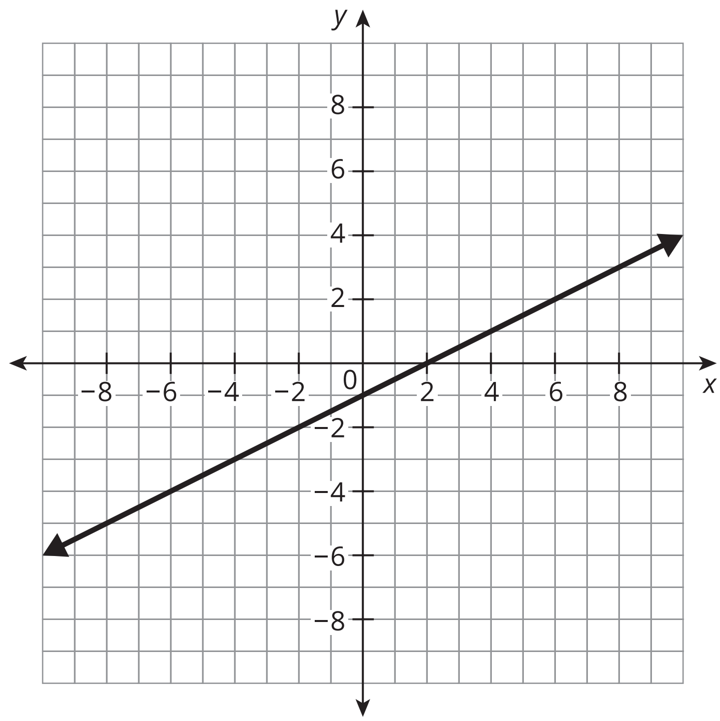
2. Miguel draws a rectangle, and then in each successive figure he splits the rectangles into two rectangles as shown.



a. Analyze the number of rectangles in each figure. Describe the pattern.

b. Write the number of rectangles in each of the first six figures as a numeric sequence.

3. Determine the slope, x-intercept, and y-intercept of the graph.

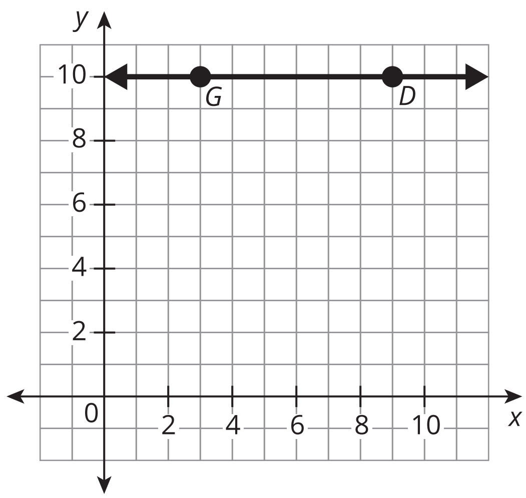


III. Making Connections Between Arithmetic Sequences and Linear Functions

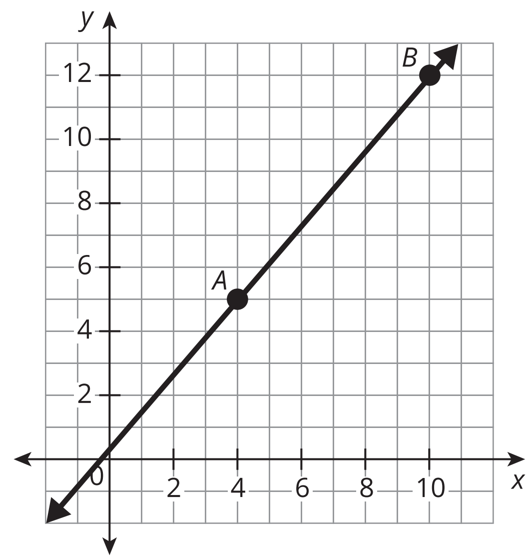
Topic Practice

A. Use the graph or the slope formula to determine the rate of change of each linear function.

1.



2.



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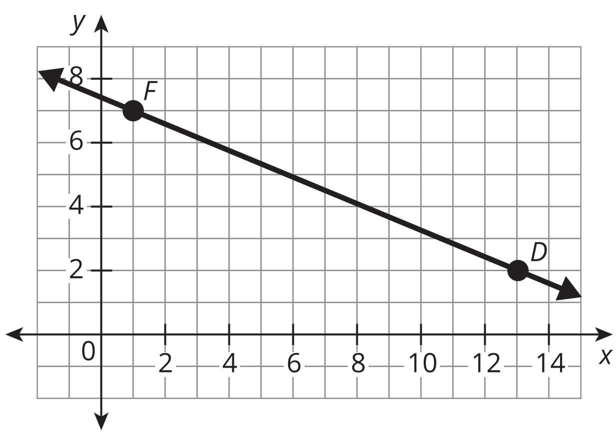
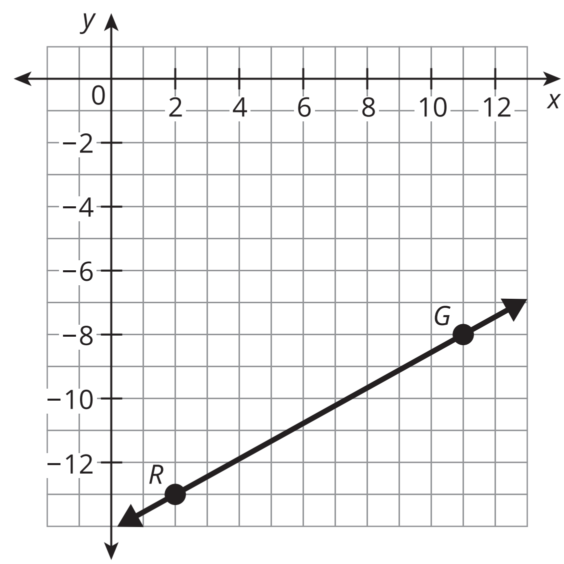


4.



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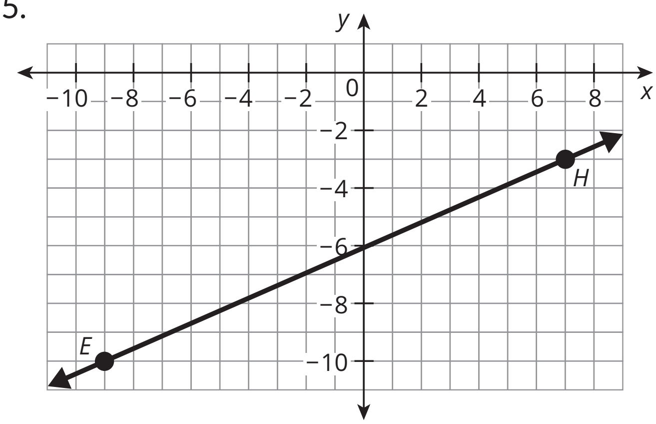

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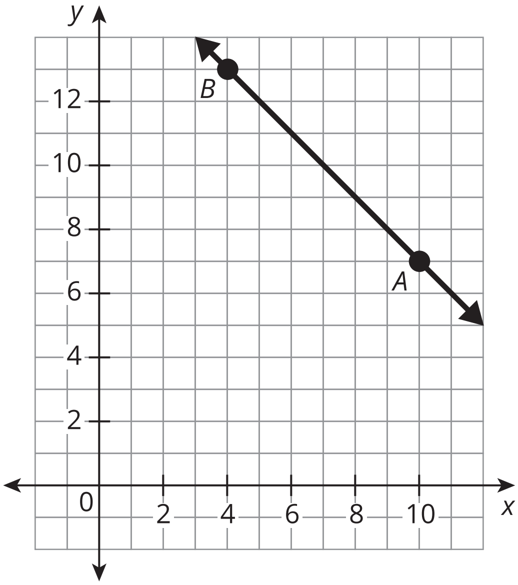
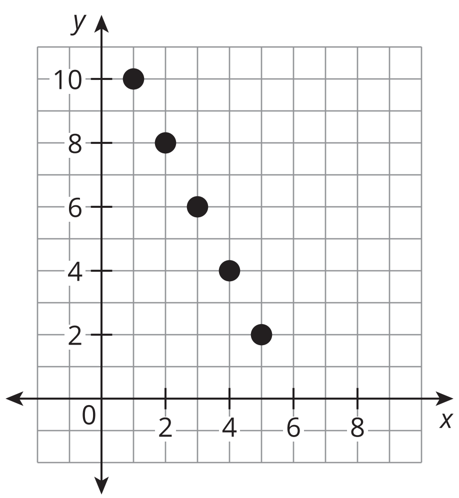
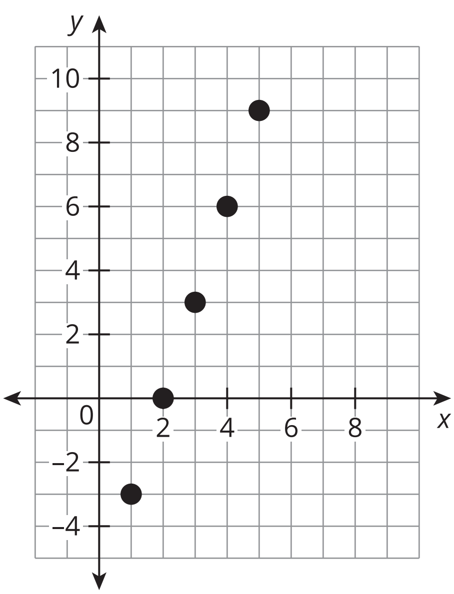


B. Write an equation using the explicit formula to represent each graph or scenario. Then, rewrite the formula in function form.

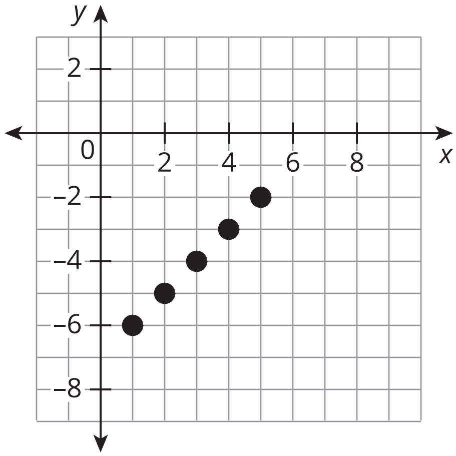
1.



2.



3.



4. Sequence: 19, 19.5, 20, 20.5, ...

5. A scooter rental company charges $3.25 for the first minute of a rental. Thereafter, the company charges an additional $0.25/minute.

6. Joey uses 4 cups of flour in each cake he bakes. On day 1, he starts the day with 60 cups of flour.

Extension

Nicky left his house at noon and drove 50 miles per hour until 3 P.M. Then, he drove the next 5 hours at 70 miles per hour. Graph Nicky's driving trip and calculate the average rate of change for the entire trip.

Spaced Practice

Evaluate each function for the given values.

1. $f(x) = 3x - 10$
   a. $f(0)$
   b. $f(5)$

2. $f(x) = 6$
   a. $f(0)$
   b. $f(-2)$

3. $f(x) = 9x + 7 - 3x$
   a. $f(0)$
   b. $f(0.5)$

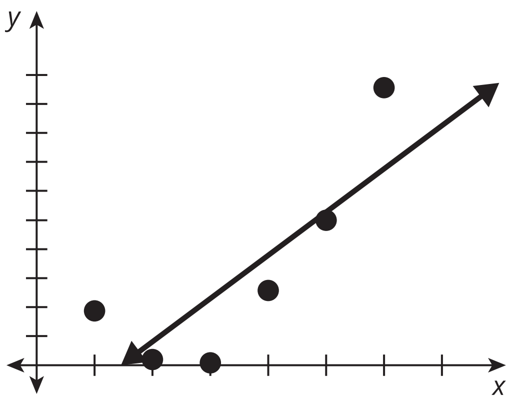
4. The linear regression function for the given data is $y = -x + 19.7$. Complete the table for the linear regression function, rounding your answers to the nearest tenth.

x	y	Predicted Value
2	17	
4	16	
6	15	
8	12	
10	9	
12	8	

5. The linear regression function for the given data is $y = 3.93x - 11.33$, $r = 0.8241$. Consider the scatterplot and the correlation coefficient. State whether a linear model is appropriate for the data.

x	2	4	6	8	10	12
y	9	2	1	12	25	48

Scatterplot and Line of Best Fit



IV. Point-Slope Form of a Line

Topic Practice

A. Use point-slope form to write an equation for a line with the given information. Then, rewrite the equation in slope-intercept form.

1. The slope is 3. The point (1, 2) lies on the line.

2. The slope is 4. The point $(0, -6)$ lies on the line.

3. The slope is $\frac{3}{4}$. The point $(-4, -5)$ lies on the line.

4. The slope is $-\frac{1}{5}$. The point $(-10, 3)$ lies on the line.

5. The slope is 5. The point (3, 4) lies on the line.

6. The slope is $-\frac{1}{3}$. The point (6, 0) lies on the line.

B. Use point-slope form to write an equation for the line that passes through each pair of points. Then, rewrite the equation in slope-intercept form.

1. Line passing through $(-4, -3)$ and $(0, -15)$

2. Line passing through (10, 5) and (4, 0)

3. Line passing through $(2, -7)$ and $(-3, -9)$

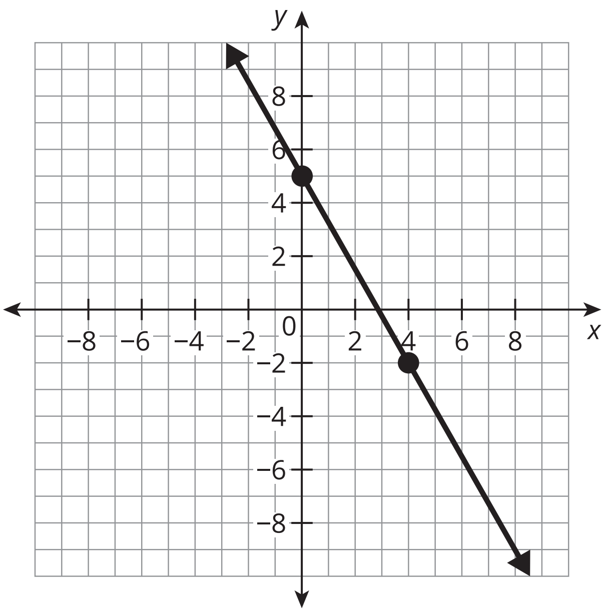
4. Line passing through $(0, 5)$ and $(3, -3)$

5. Line passing through $(0, -8)$ and (2, 6)

6. Line passing through $(-1, 1)$ and $(-3, -5)$

C. Write a linear equation in point-slope and slope-intercept form to represent each line.

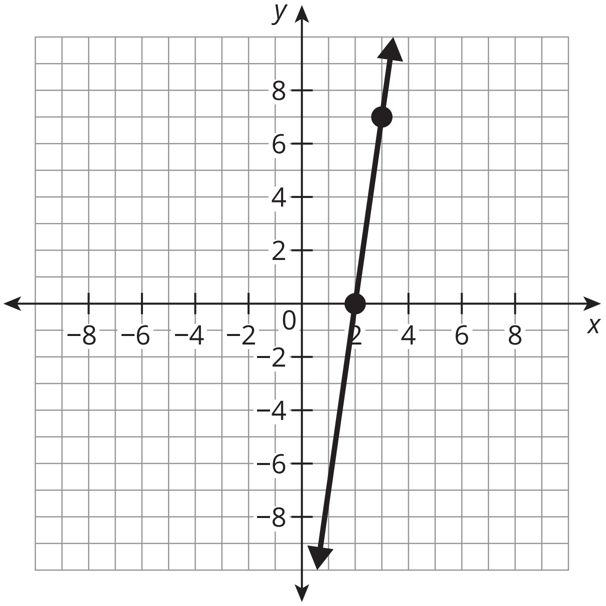
1. Line whose graph is shown



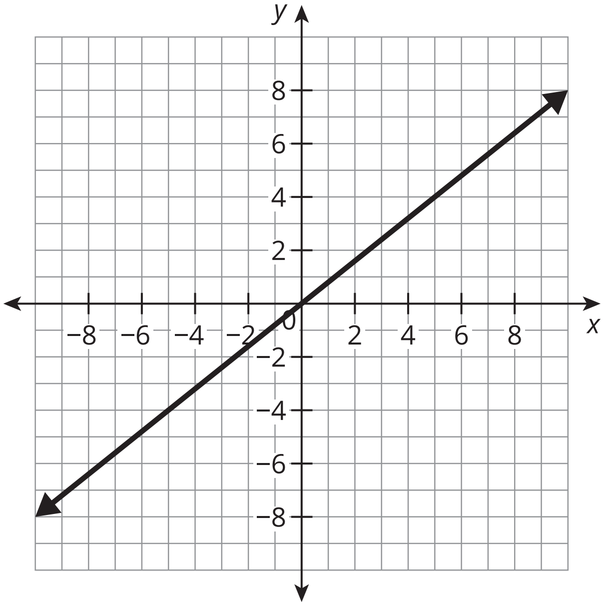
2. Line represented by the table of values

x	y
3	−8
6	−7
9	−6
12	−5

3. Line whose graph is shown



4. Line whose graph is shown



5. Line represented by the table of values

x	y
−1	42
0	30
1	18
2	6

6. Line represented by the table of values

x	y
−8	−4
−16	−5
−24	−6
−32	−7

Extension

To convert an equation from point-slope to slope-intercept form, you can solve the equation for *y*. How do you convert from slope-intercept to point-slope form? Rewrite each equation in point-slope form using only algebraic properties. What is special about the ordered pair now visible in the equation?

1. $y = 2x - 7$

2. $y = -5x + 15$

Spaced Practice

1. Write an equation in slope-intercept form with the given characteristics.

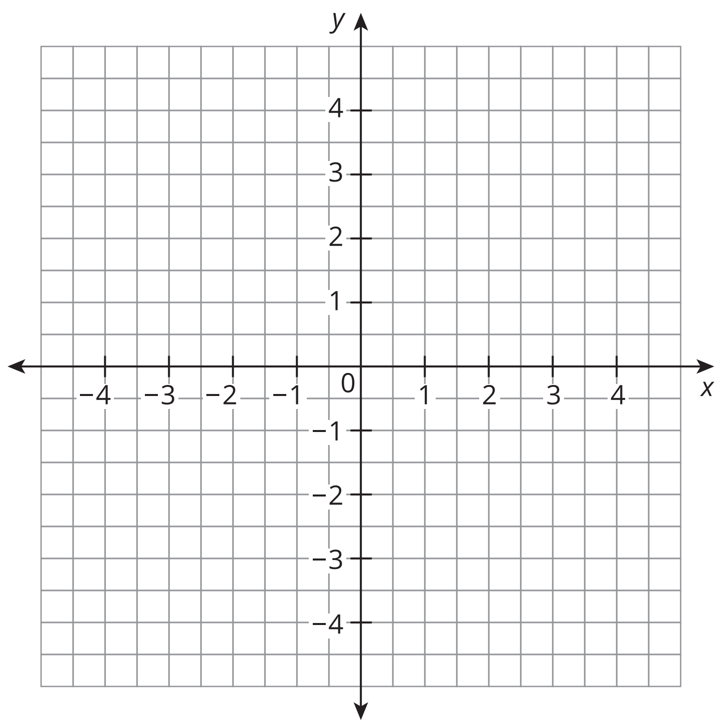
   a. The line is increasing and passes through the point (0, −10). The slope of the line is less steep than the slope of the line represented by the equation $y = x + 8$.

   b. The line is decreasing and passes through the point (0, 5). The slope of the line is steeper than the slope of the line represented by the equation $y = -\frac{1}{4}x - 4$.

2. For the linear equation $x = 4y - 5$, complete each task.

a. Use a table of values to graph the linear equation.

b. Use the points on the graph to sketch similar triangles that may be used to show that the slope of a non-vertical line is the same between any two points on the line.



c. Verify that the slopes are the same.

3. Answer each question.

a. What is a 15% tip for a restaurant bill of $24?

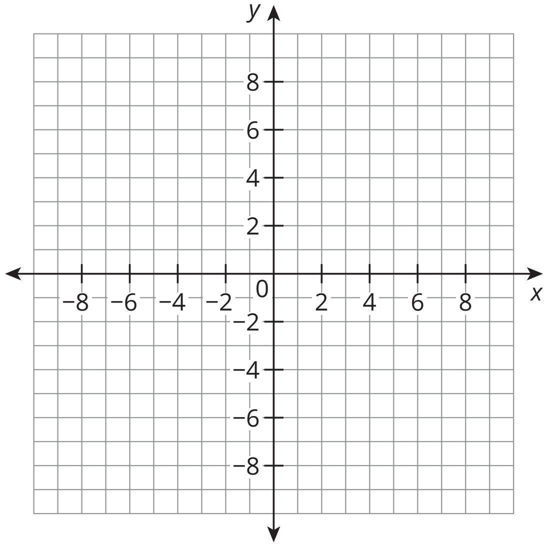
b. A $50 item was marked up 20%. What is the total increased cost of the item?

V. Using Linear Equations

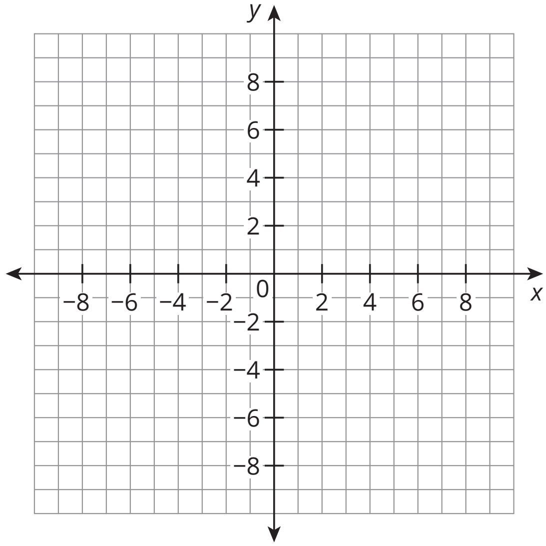
Topic Practice

A. Graph each equation.

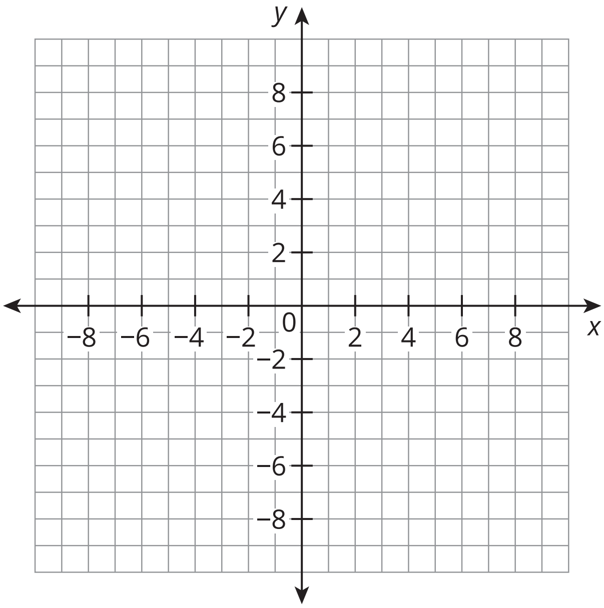
1. Graph the equation $y = -3x - 6$.



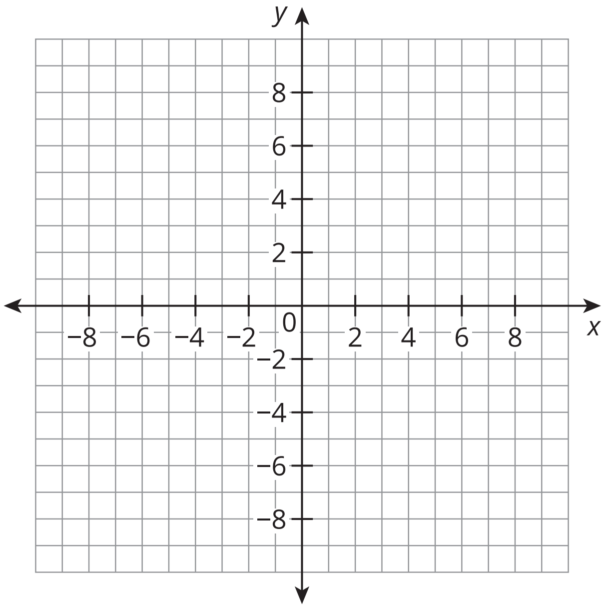
2. Graph the equation $y = -\frac{1}{2}x - 9$.



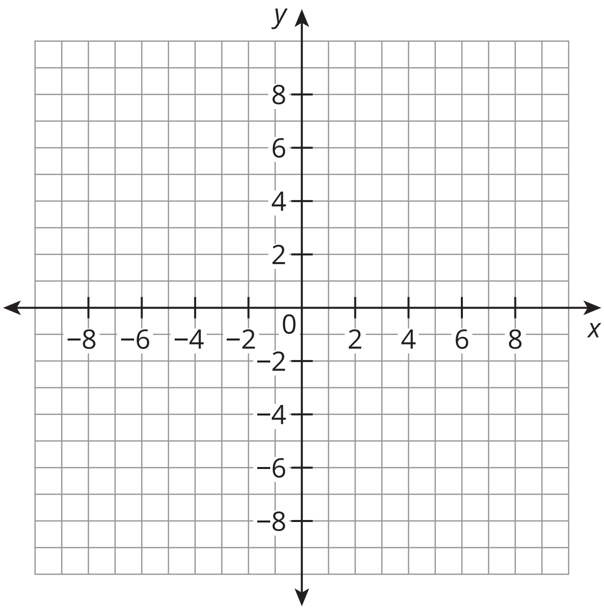
3. Graph the equation $y = x + 4$.



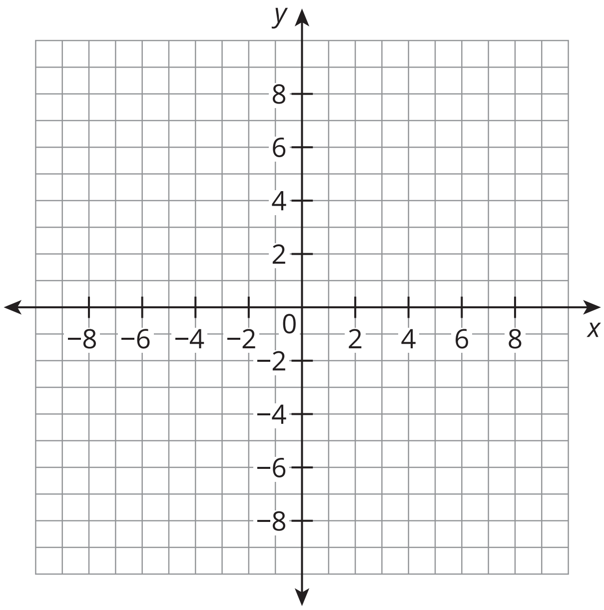
4. Graph the equation $y = \frac{2}{3}x + 9$.



5. Graph the equation $y = 2x + 5$.

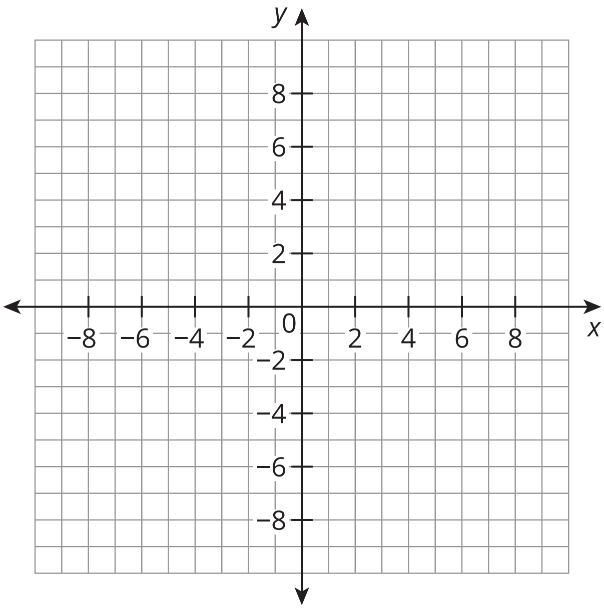


6. Graph the equation $y = -1.5x + 1$.

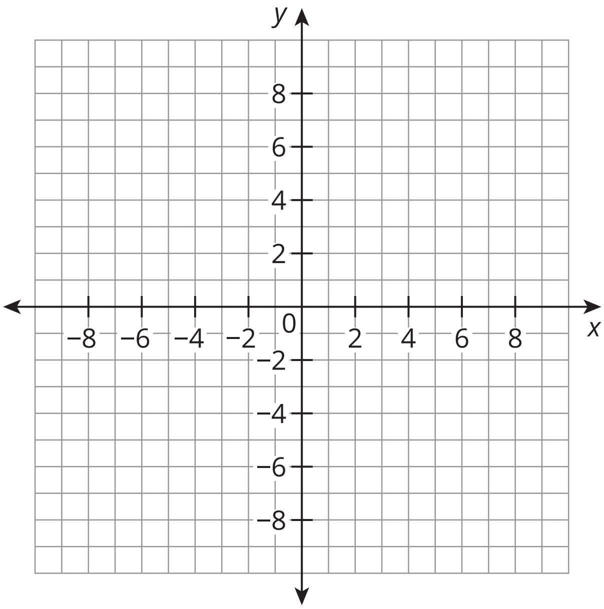


B. Graph each equation.

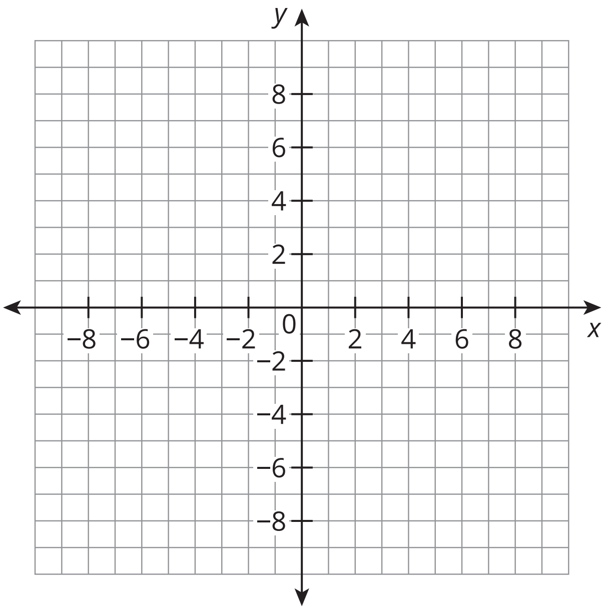
1. Graph the equation $y - 2 = 2(x - 1)$.



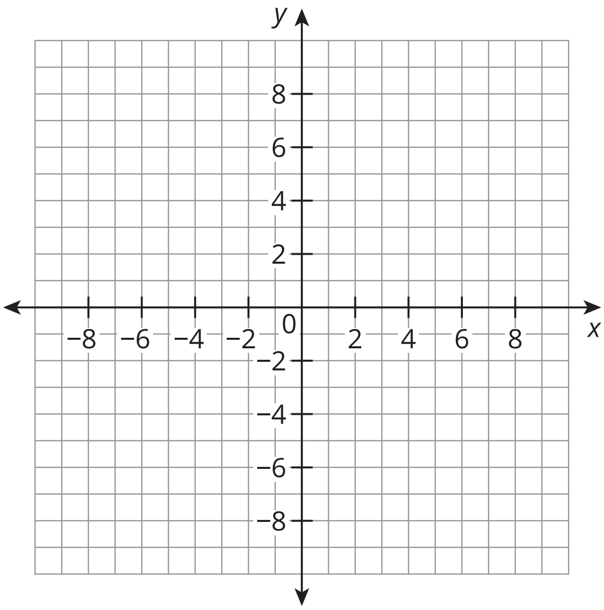
2. Graph the equation $y + 3 = -1(x - 2)$.



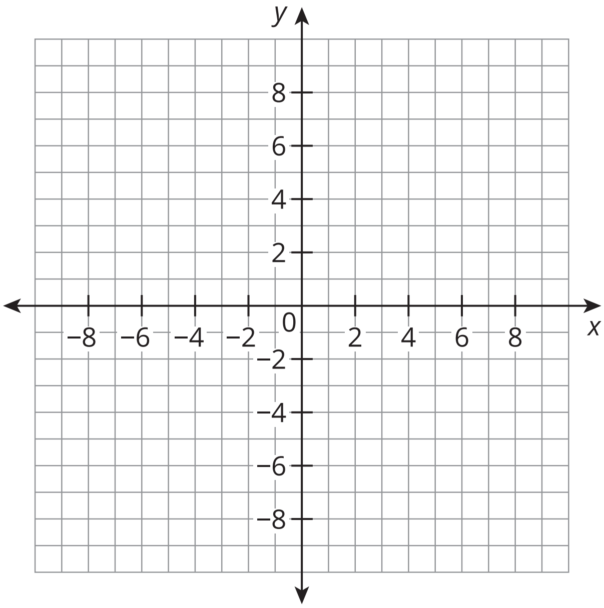
3. Graph the equation $y - 5 = \frac{1}{3}(x + 1)$.



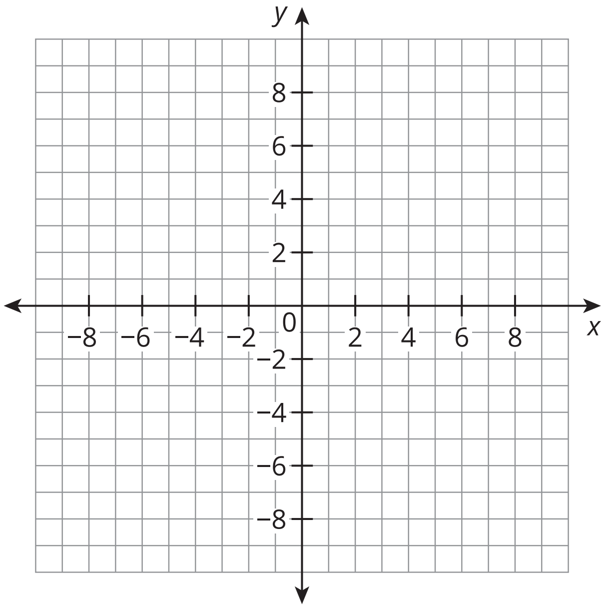
4. Graph the equation $y + 4 = -\frac{2}{3}(x + 2)$.



5. Graph the equation $y - 4 = 0.75(x - 1)$.

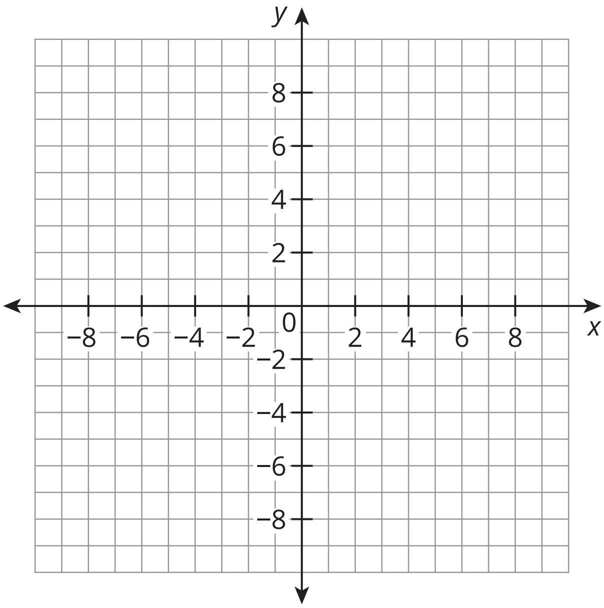


6. Graph the equation $y + 3 = 2.5(x - 3)$.

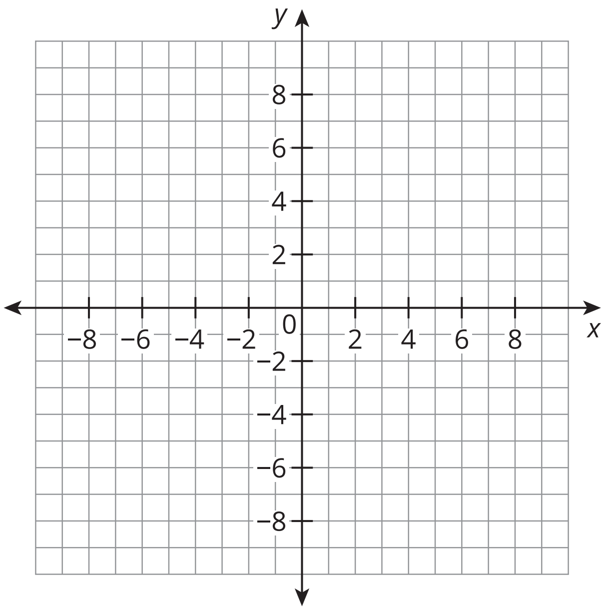


C. Graph each equation.

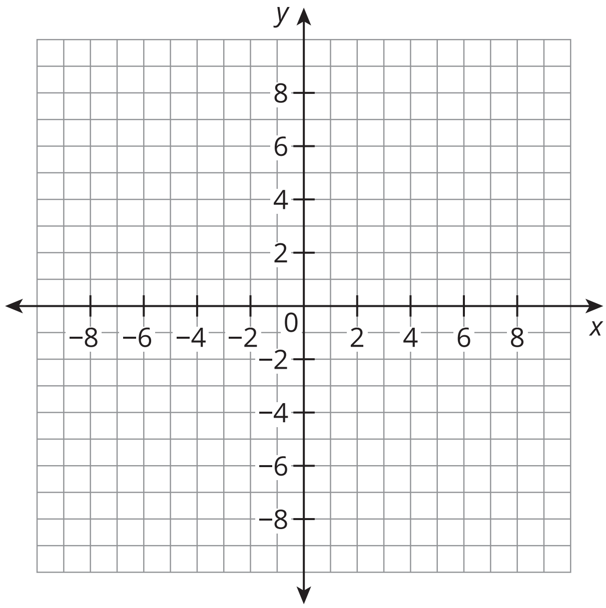
1. Graph the equation $5x + 6y = 30$.



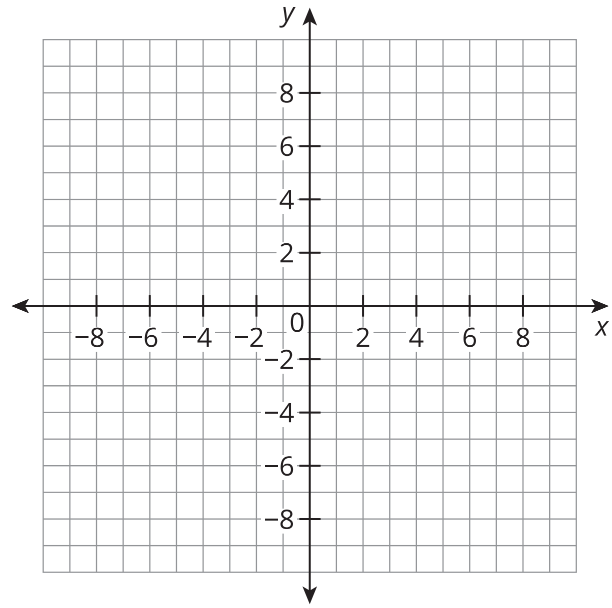
2. Graph the equation $-5x + 3y = -15$.



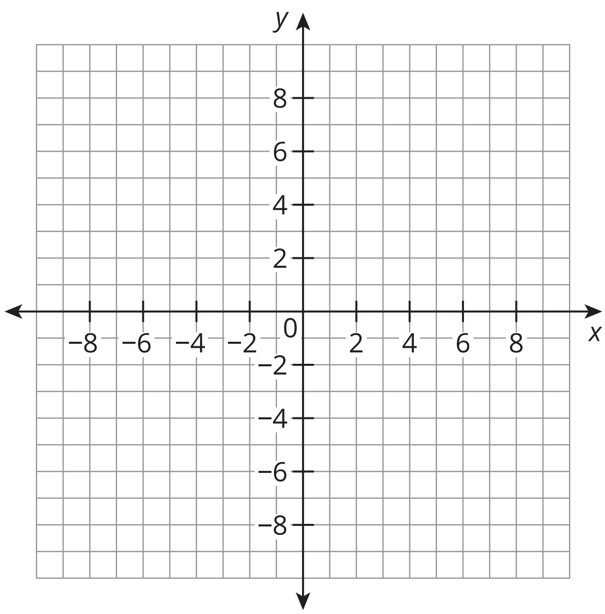
3. Graph the equation $3x + 5y = 30$.



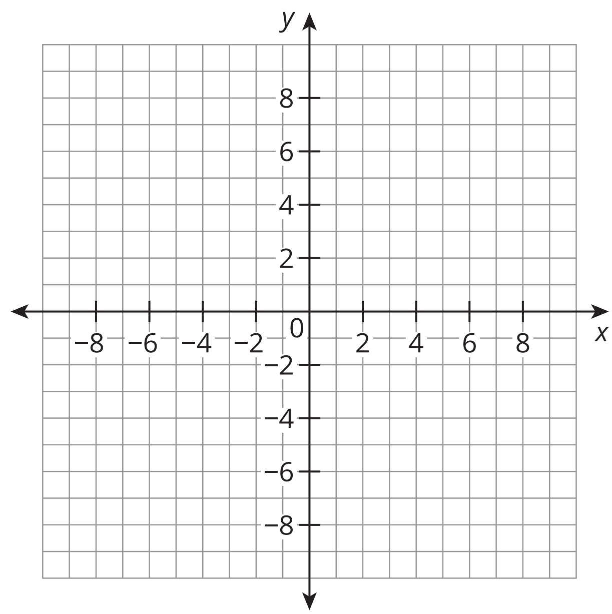
4. Graph the equation $-4x - 5y = 40$.



5. Graph the equation $7x - 4y = -28$.



6. Graph the equation $-3x - 9y = -18$.

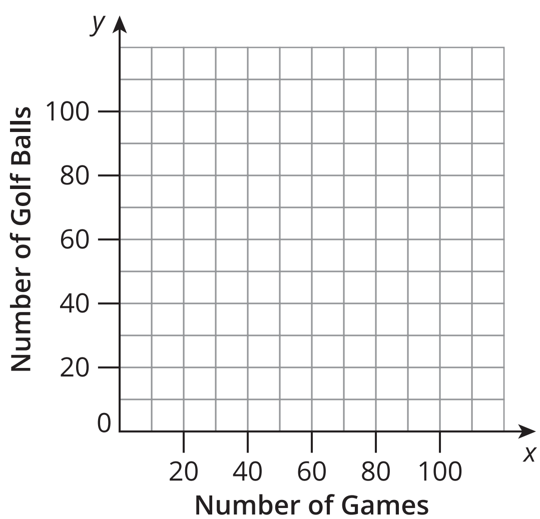


D. Write an equation and create a graph for each situation. Identify the slope and *y*-intercept of each graph.

1. At the beginning of the golf season, Jackson buys 84 golf balls. He loses 2 golf balls each time he plays a game.

   a. Write an equation that represents the number of golf balls Jackson has left given a number of times he plays a game.

   b. Graph the equation.

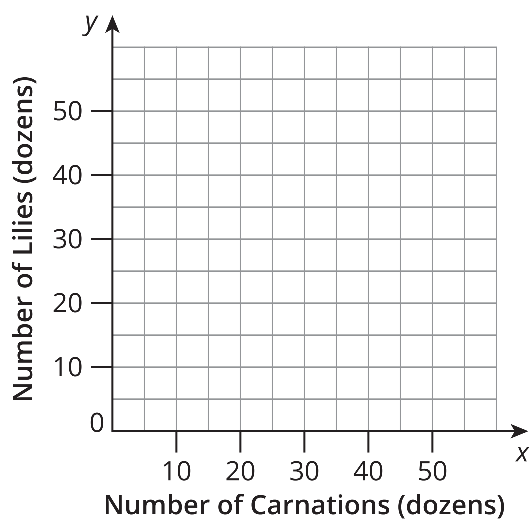


   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

2. A florist sells carnations for $11 a dozen and lilies for $13 a dozen. During a weekend sale, the florist's goal is to earn $650.

   a. Write an equation that represents the total amount the florist would like to earn selling carnations and lilies during the weekend sale.

   b. Graph the equation.

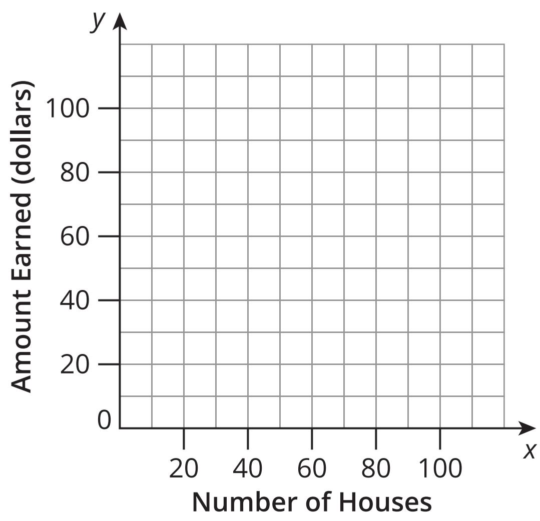


   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

3. Nia has a job delivering newspapers. Each Sunday, she earns \$24.75 plus \$0.20 for each house she delivers to.

   a. Write an equation that represents the amount Nia earns on Sunday given a certain number of houses she delivers to.

   b. Graph the equation.

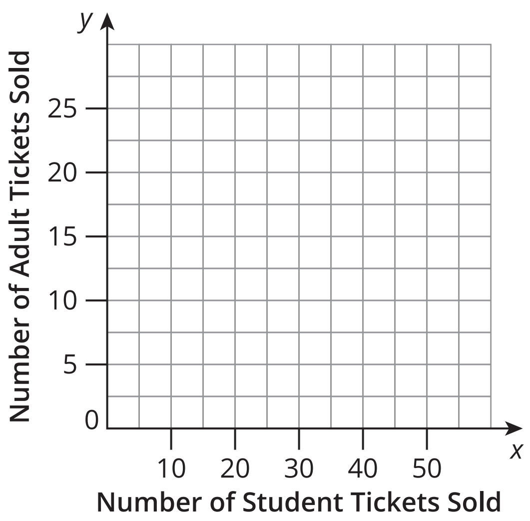


   c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

4. The high school soccer booster club sells tickets to the varsity matches for \$4 for students and \$8 for adults. The booster club hopes to earn \$200 at each match.

   a. Write an equation that represents the total amount the booster club would like to earn from ticket sales at each match.

   b. Graph the equation.

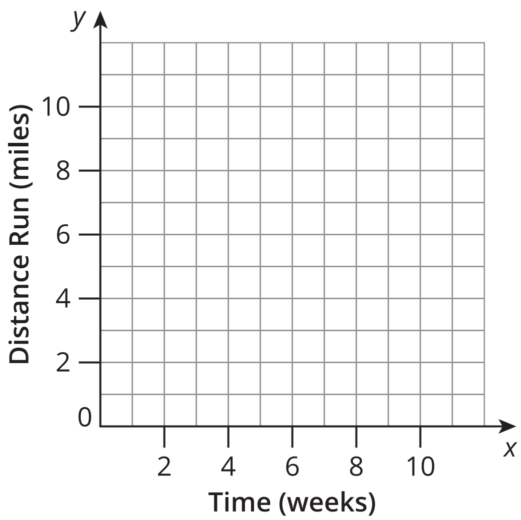


   c. Identify the slope, x-intercept, and y-intercept of the graph. Explain what each means in terms of the situation.

5. You are planning a training program to run a marathon. You add 3.9 more miles to your program each week. In week 4 you ran 20.6 miles.

   a. Write an equation that represents the distance you run in miles given a certain number of weeks.

   b. Graph the equation.

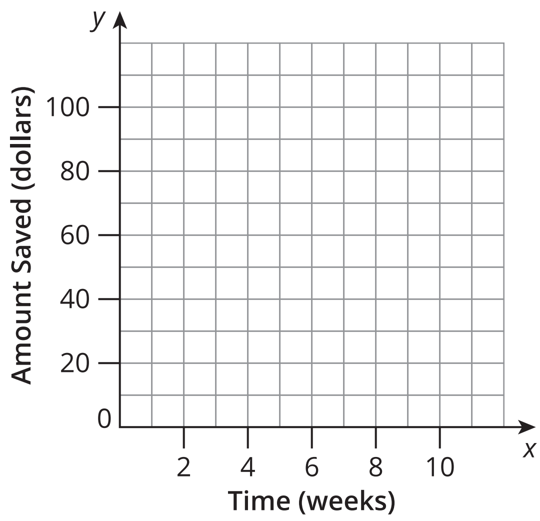


   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

6. Ricardo is saving his money to purchase a smartphone. He already has some money saved, and he saves $10 a week from his part-time job walking neighborhood dogs. After 3 weeks he has $87 to put toward the purchase the phone.

   a. Write an equation that represents the total amount saved given a certain number of weeks.

   b. Graph the equation.



   c. Identify the slope and *y*-intercept of the graph. Explain what each means in terms of the situation.

Extension

You learned how to generalize the x- and y-intercepts and the slope from the standard form of an equation. Write the point-slope and slope-intercept form of a linear equation in terms of the constants from the standard form.

Spaced Practice

1. Write an equation in point-slope form for each problem.

   a. $m = -8$ and passes through the point (3, 12)

   b. passes through the points (9, −18) and (−3, −26)

2. Determine whether each table represents a proportional relationship.

   a.

x	y
−1	−24
2	48
4	90
8	192

b.

x	y
2	13.5
5	33.75
10	67.5
15	101.25

3. Solve each inequality.

   a. $10 + 5x \geq -25$

   b. $-4x + 26 < 14$

VI. Making Sense of Different Representations of a Linear Function

Topic Practice

A. Solve each equation involving direct variation.

1. The value of y is directly proportional to the value of x. When $x = 3$, $y = 12$. What is the value of y when $x = 13$?

2. The value of y varies directly with the value of x. When $x = 7$, $y = 17.5$. What is the value of x when $y = 30$?

3. Linh mows lawns in his neighborhood to earn money. The amount of money he earns varies directly with the number of lawns he mows. When he mows 10 lawns, Linh makes $100. How much money would make Linh make if he mowed 6 lawns?

4. James is driving to visit a college campus. The distance he drives is directly proportional to the amount of time he has traveled. When James has driven 2 hours he has traveled 130 miles. How long has he been driving if he traveled 195 miles?

5. Catalina is selling coupon books to raise money for her school. The amount of money she raises is directly proportional to the number of books she sells. If Catalina sells 5 books, she raises 162.50. How much money would Catalina raise for her school if she sells 17 books?

6. Juliana is shopping for earrings. The amount of money she spends varies directly with the number of pairs of earrings she buys. When Juliana buys 2 pairs of earrings, she will spend $45.20. When Juliana spent $113 on earrings, how many pairs did she buy?

B. Use each equation or graph to determine input and output values.

1. For $p(x) = -2x + 5$, determine:

   a. $p(5)$

   b. $p(0)$

   c. $p\left(-\frac{1}{2}\right)$

2. For $a(x) = 5$, determine:

   a. $a(-3)$

   b. $a\left(\frac{1}{4}\right)$

   c. $a(5)$

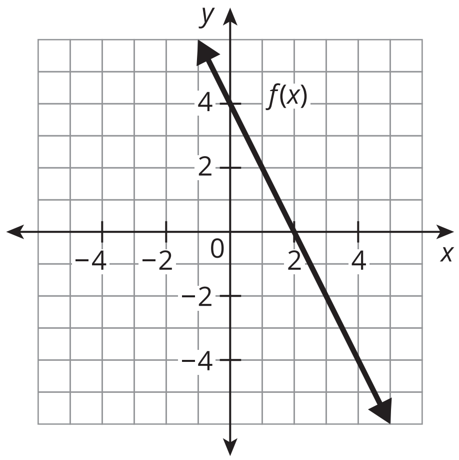
3. For $g(x) = 0.75x - 1.2$, determine:

   a. $g(8)$

   b. $g(-2)$

   c. $g(0)$

4. Use the graph to determine when $f(x) = 5$ and the value of $f(5)$.



5. For $t(x) = \frac{2}{3}x$, determine:

   a. $t(9)$

   b. $t(-7)$

   c. $t\left(\frac{3}{5}\right)$

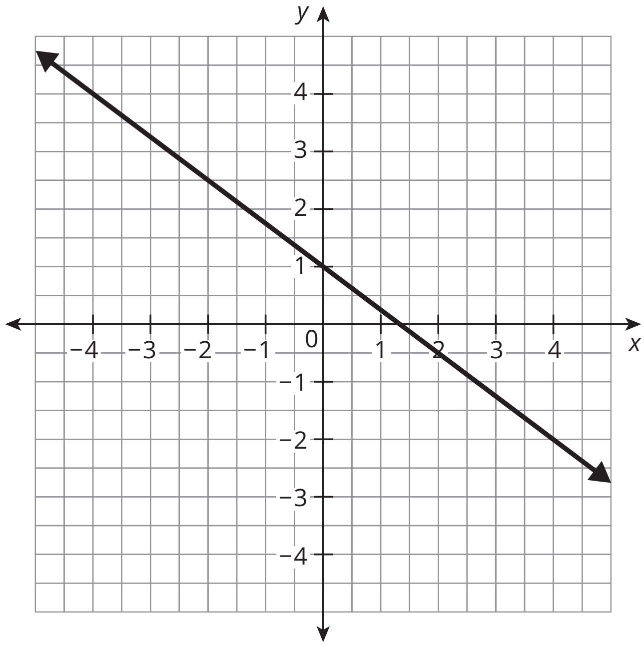
6. For $b(x) = -1.8x$, determine:

   a. $b(-6.3)$

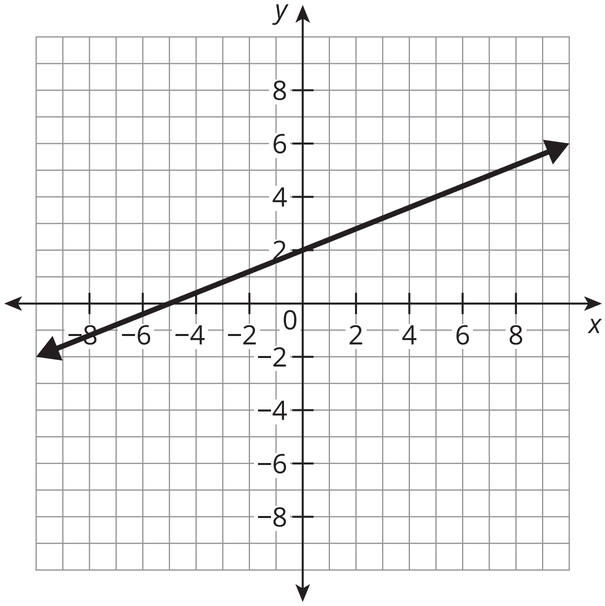
   b. $b(12)$

   c. $b\left(-\frac{1}{3}\right)$

7. Use the graph to determine when $f(x) = 4$ and the value of $f(4)$.



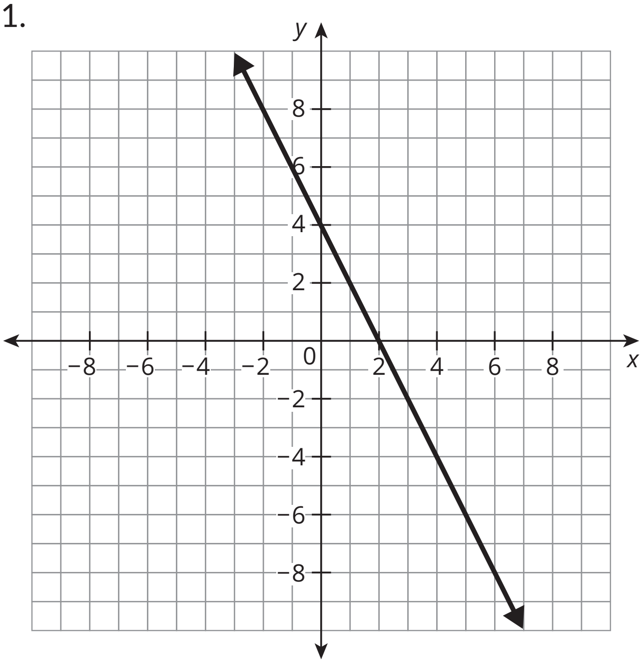
8. Use the graph to determine when $f(x) = 0$ and the value of $f(0)$.



C. For the graph shown in each problem, determine the following:

a. *y*-intercept
b. *x*-intercept
c. slope
d. equation in slope-intercept form
e. equation in factored form

1.



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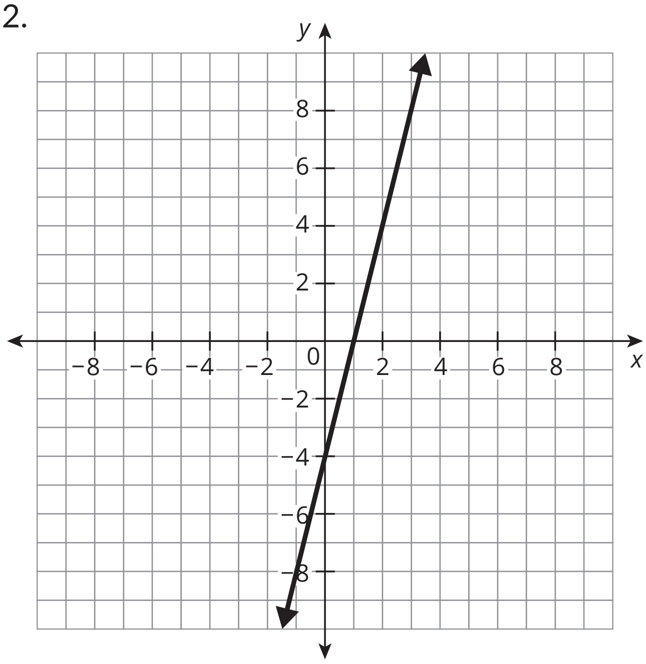

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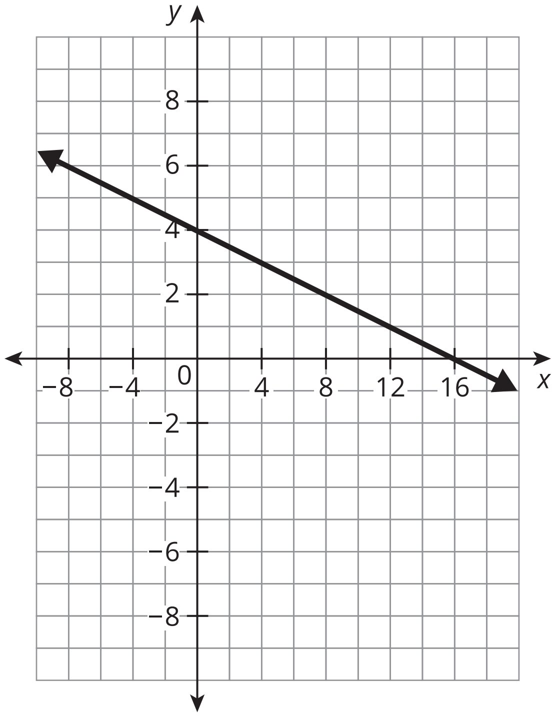

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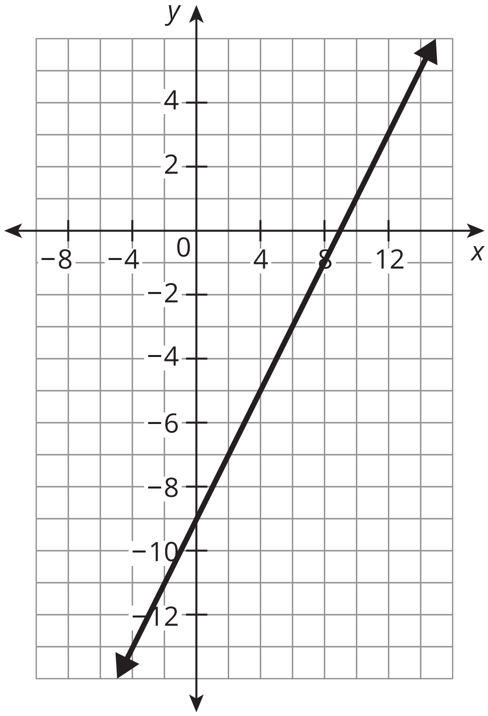

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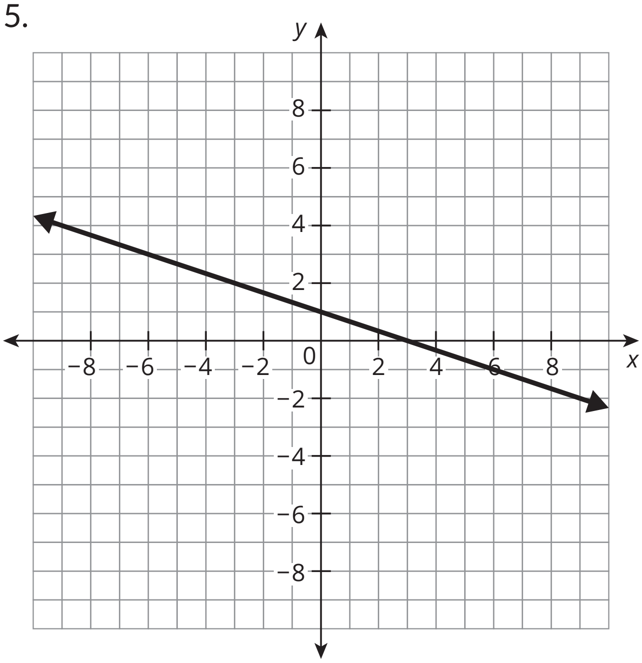

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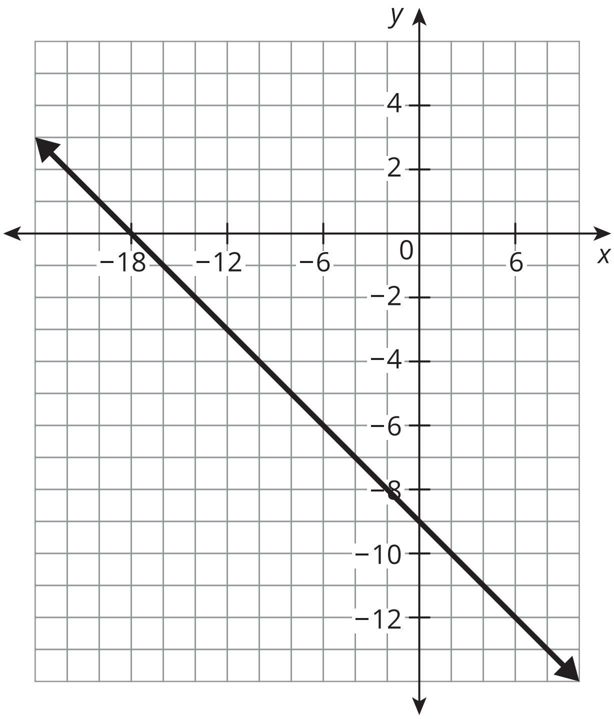


Extension

A pretzel manufacturer has two production lines. Line A produces a variety of pretzel that is sold for $2.40 per bag. Line A typically produces 3 bags per day that do not meet company standards and cannot be sold. Line B produces a variety of pretzel that is sold for $3.60 per bag. Line B typically produces 4 bags per day that do not meet company standards and cannot be sold. Line A produces 3 times as many bags as Line B each day.

Write a linear function that represents revenue as a function of the total number of bags the lines can produce combined.

Spaced Practice

1. Determine whether each relationship shows a constant difference. If so, write the linear function that represents the relationship.

a.

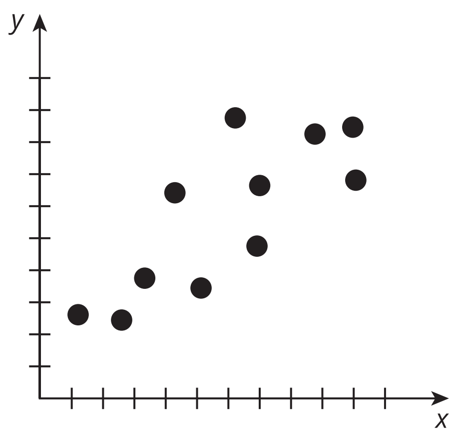
x	y
2	9
3	11
4	13
5	15

b.

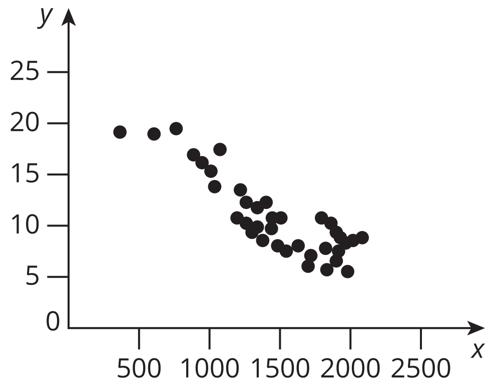
x	y
1	2
2	1
3	$\frac{1}{2}$
4	$\frac{7}{2}$

2. Determine whether the points in each scatterplot have a positive association, a negative association, or no association. Explain your reasoning.

a.



b.



3. Solve each equation.

a. $\frac{1}{3}x + 2 = 11$

b. $-5p - 12 = 19$

Name ______________________________ Date ______________

I. Transforming Linear Functions

Topic Practice

A. Determine an equation that represents each parallel line. Write your answer in point-slope form, slope-intercept form, and standard form.

1. What is the equation of a line parallel to $y = \frac{4}{5}x + 2$ that passes through (1, 2)?

2. What is the equation of a line parallel to $5x + y = 3$ that passes through (3, 1)?

3. What is the equation of a line parallel to $y = 7x - 8$ that passes through (5, -2)?

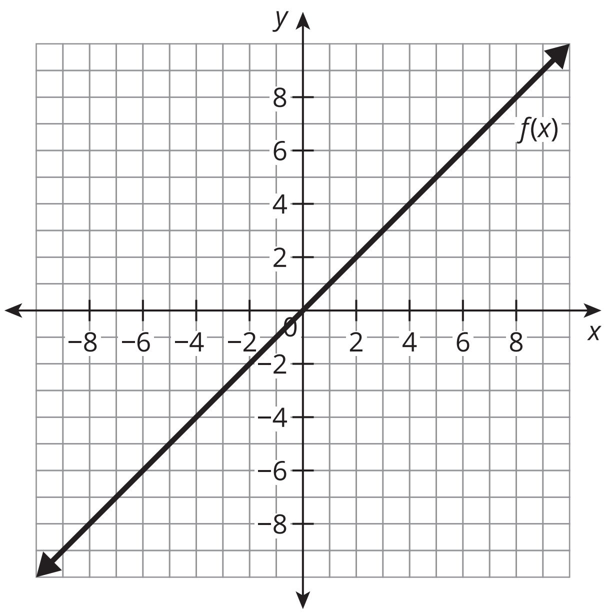
4. What is the equation of a line parallel to $y - 7 = -\frac{1}{2}(x + 2)$ that passes through $(-4, 1)$?

5. What is the equation of a line parallel to $y = \frac{1}{3}x - 4$ that passes through $(9, 8)$?

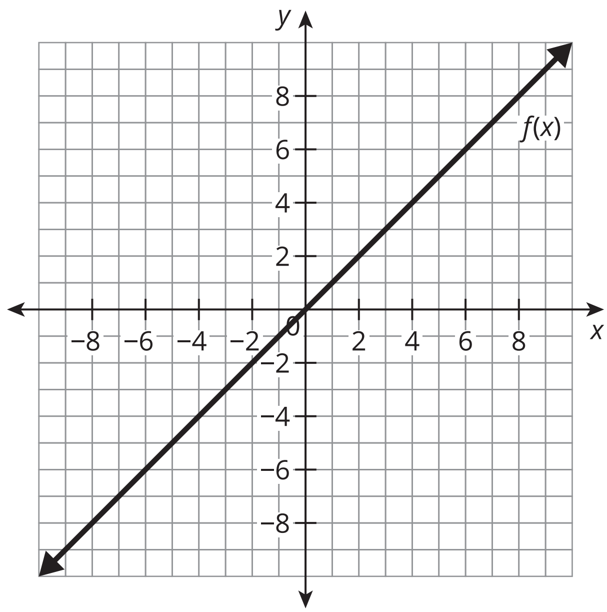
6. What is the equation of a line parallel to $4x + y = -7$ that passes through $(2, -9)$?

B. The equation and graph of the parent linear function $f(x) = x$ are given. The equation of a transformed function $g(x)$ is also given. Graph $g(x)$ and describe the transformation(s) performed on $f(x)$ to produce $g(x)$.

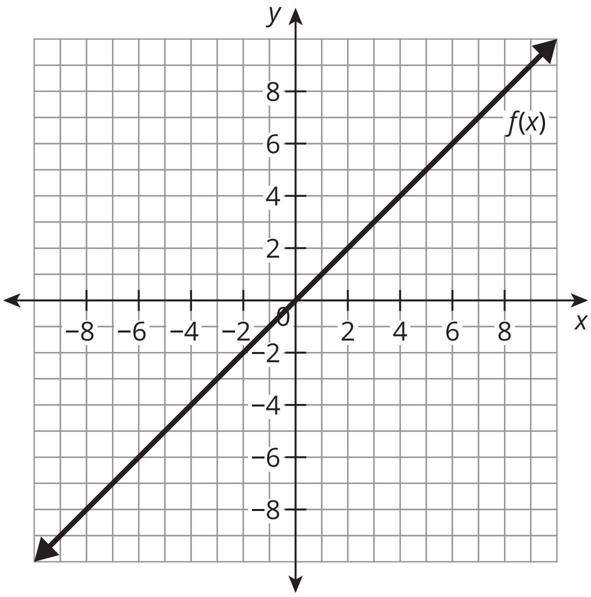
1. $f(x) = x$
   $g(x) = f(x) - 8$



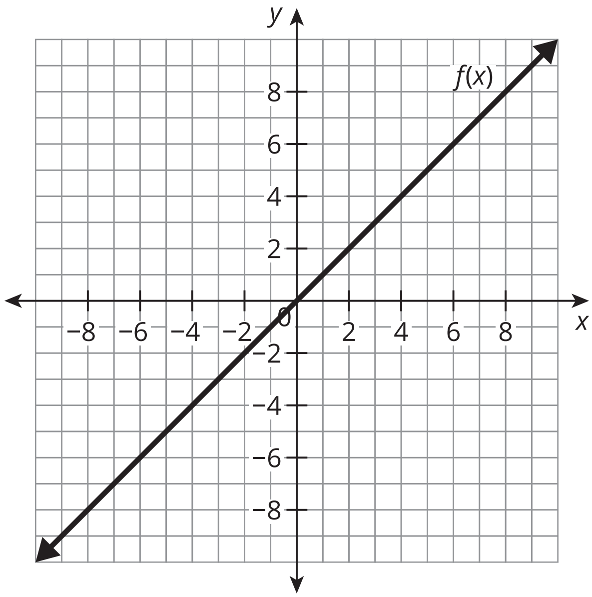
2. $f(x) = x$
   $g(x) = 2f(x)$



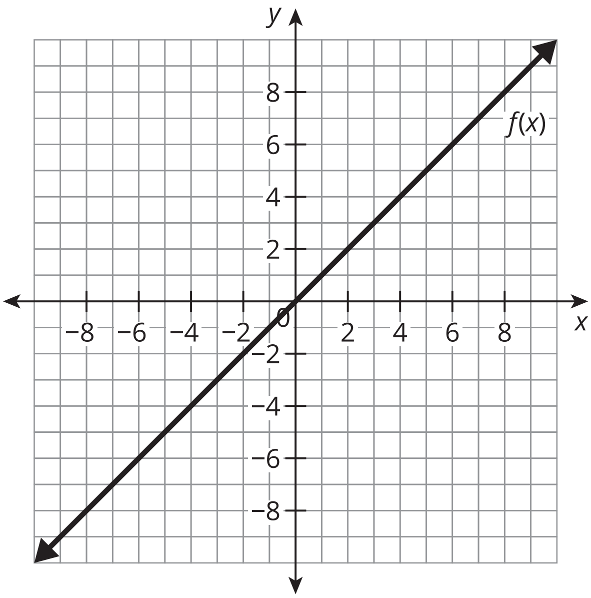
3. $f(x) = x$
$g(x) = f(x) + 5$



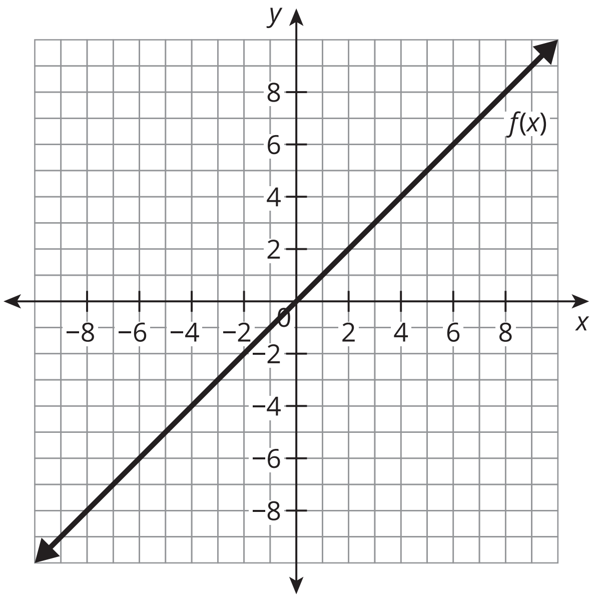
4. $f(x) = x$
$g(x) = \frac{2}{3}f(x)$



5. $f(x) = x$
$g(x) = \frac{1}{3}f(x) - 4$



6. $f(x) = x$
$g(x) = 4f(x) + 1$



C. Write an equation for each transformed function $g(x)$ in terms of $f(x)$ and then simplify the equation, if necessary.

1. $f(x) = 3x + 1$ is translated 9 units up.

2. $g(x)$ is parallel to $f(x) = 6x - 10$ and goes through the point $(-2, -9)$.

3. $g(x)$ is parallel to $f(x) = -6$ and goes through the point $(5, 8)$.

4. $g(x)$ is parallel to $f(x) = -2x - 7$ and goes through the point $\left(\frac{3}{8}, 4\frac{1}{4}\right)$.

5. $f(x) = -7 - 4x$ is dilated vertically by a factor of $\frac{1}{2}$.

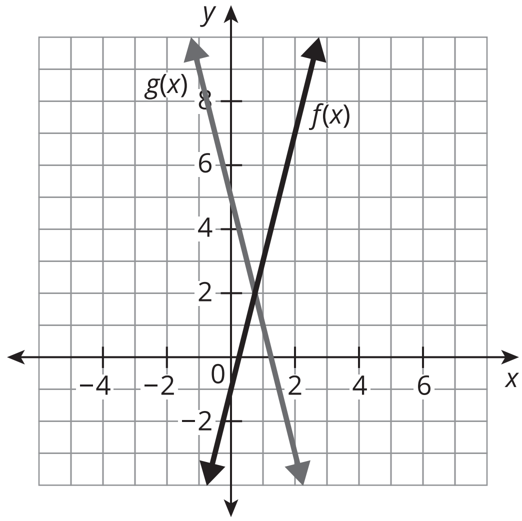
6. $f(x) = \frac{1}{2}x + 2$ is dilated vertically by a factor of 2 and translated 4 units down.

7. $f(x) = 3 - 5x$ is reflected across the x-axis and translated 16 units up.

8. $g(x)$ is parallel to $f(x) = 3.5$ and goes through the point $(-3, -7.25)$.

Extension

The functions $f(x)$ and $g(x)$ are shown on the graph. Write an equation for each function in slope-intercept form. Then, write an equation for $g(x)$ in terms of $f(x)$.

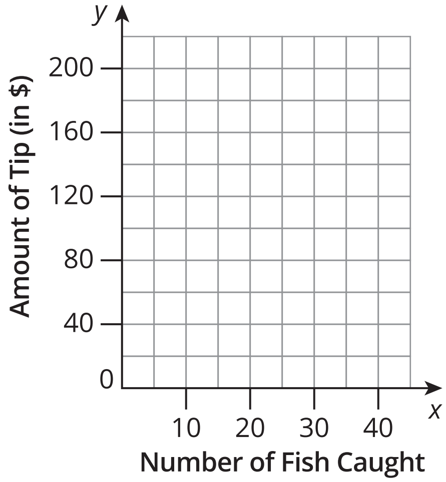


Spaced Practice

1. James works as a fly-fishing guide. The table indicates the number of fish caught on each expedition he guided in a week and the amount of the tip he received for each expedition.

Number of Fish Caught	Amount of Tip ($)
22	125
19	80
25	130
26	150
21	100
18	75
27	150

a. Construct a scatterplot of the data.



b. Based on the shape of the scatterplot is a linear regression appropriate? What type of correlation appears to be present?

c. Use technology to write a function to represent the line of best fit.

d. Compute and interpret the correlation coefficient.

2. Determine whether each table of values represents a linear function. If so, write the function. If not, explain why.

a.

x	y
2	3
4	4
6	5
8	6

b.

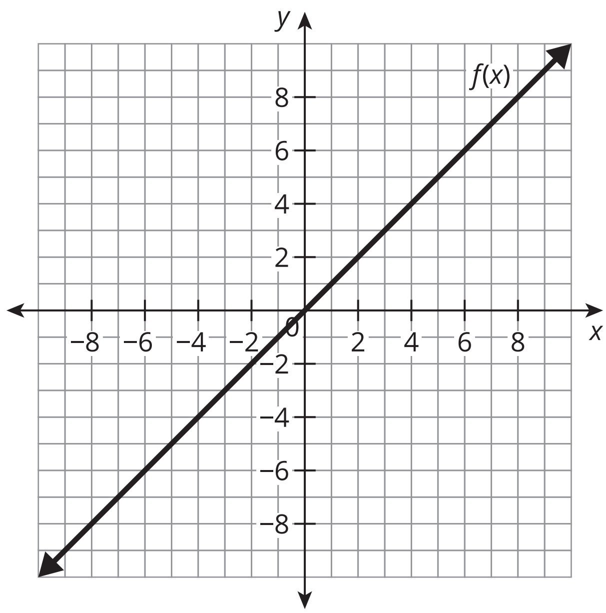
x	y
−4	−17
−2	−9
2	7
4	17

II. Vertical and Horizontal Transformations of Linear Functions

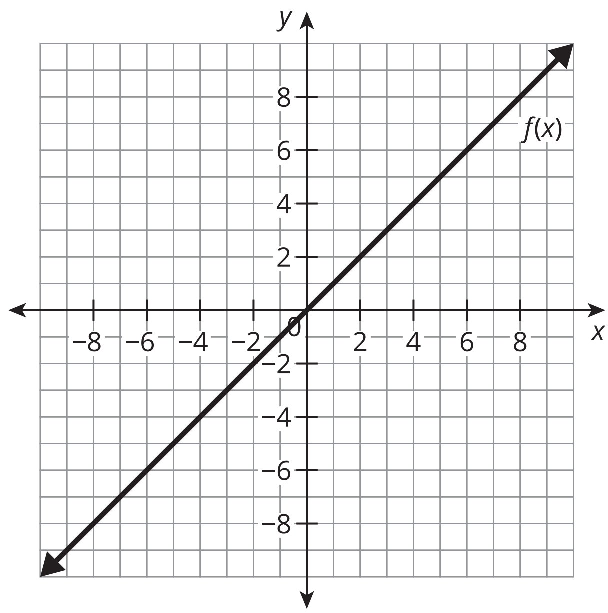
Topic Practice

A. The equation and graph of the parent linear function $f(x) = x$ are given. The equation of a transformed function $g(x)$ is also given. Graph $g(x)$ and describe the transformation(s) performed on $f(x)$ to produce $g(x)$.

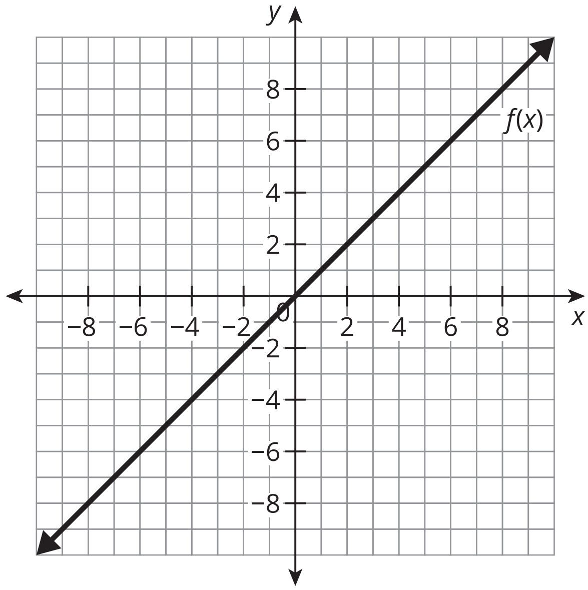
1. $f(x) = x$
   $g(x) = f(x - 2)$



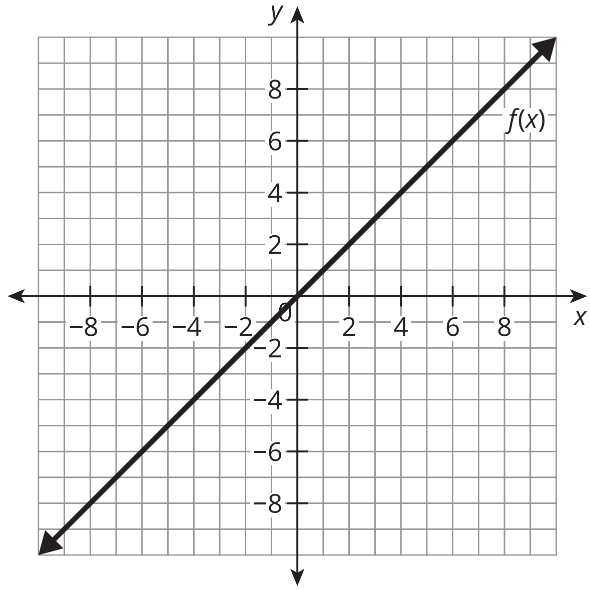
2. $f(x) = x$
   $g(x) = f(x + 3)$



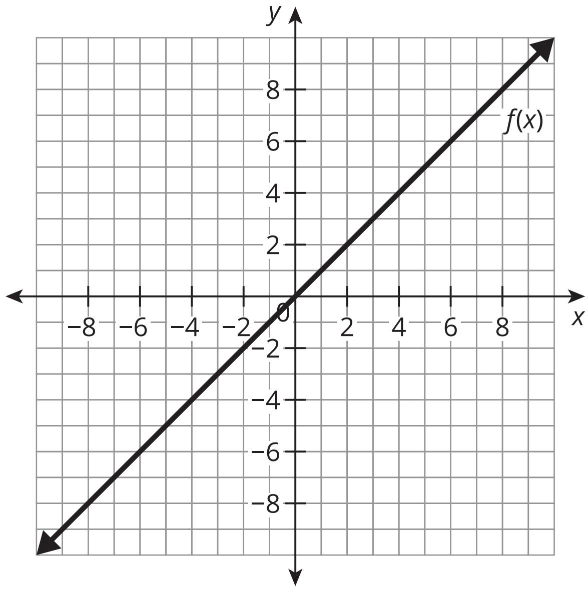
3. $f(x) = x$
$g(x) = f(3x)$



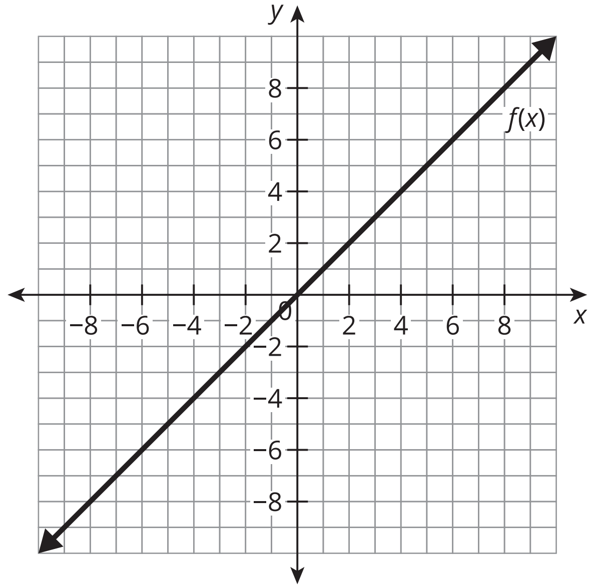
4. $f(x) = x$
$g(x) = f\left(\frac{1}{3}x\right)$



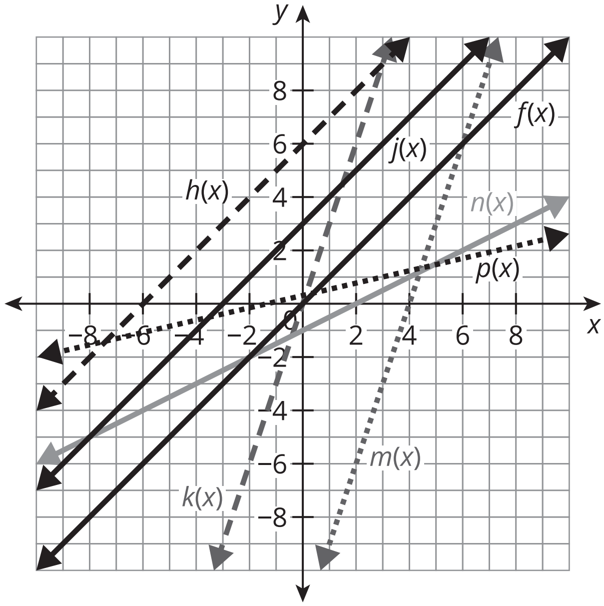
5. $f(x) = x$
$g(x) = 2(f(x - 1))$



6. $f(x) = x$
$g(x) = f\left(\frac{1}{2}x\right) - 3$



B. The graph shows the linear function $f(x) = x$ and six transformations of $f(x)$.



Equations
$\frac{1}{4} \cdot f(x)$
$3 \cdot f(x)$
$f(x + 6)$
$f(x) + 3$
$f\left(\frac{1}{2}x\right) - 1$
$3 \cdot f(x - 4)$

Match each transformed graph to the equation in the table, written in terms of $f(x)$.

1. $p(x) =$ ___________

2. $h(x) =$ ___________

3. $j(x) =$ ___________

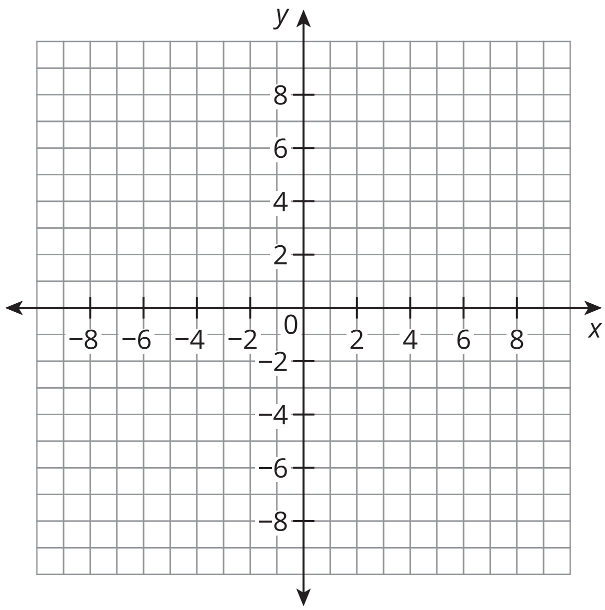
4. $m(x) =$ ___________

5. $k(x) =$ ___________

6. $n(x) =$ ___________

Extension

The function $g(x)$ is a transformation of $f(x) = x$. Write the function $g(x)$ in terms of $f(x)$.



Spaced Practice

Describe the characteristics of each function.

1. $m(x) = 5x - 2$

Domain: ________________

Range: ________________

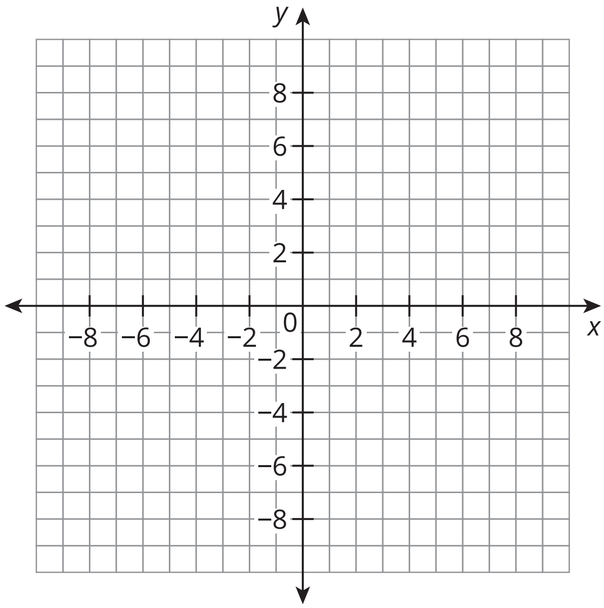
x-intercept: ______

y-intercept: ______

Slope: _____

Increasing/Decreasing: ________________

____



2. $g(x) = -\frac{1}{2}x + 6$

Domain: ________________

Range: ________________

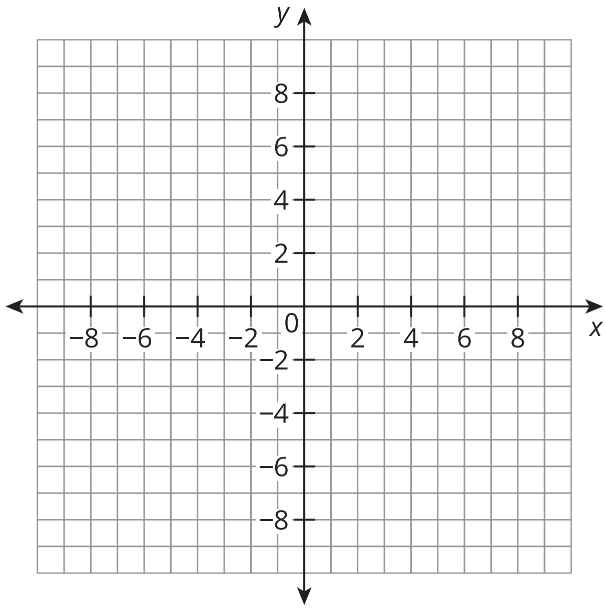
x-intercept: _______

y-intercept: ______

Slope: ______

Increasing/Decreasing: ________________

____



3. $j(x) = 4 + \frac{3}{4}x$

Domain: ___________________

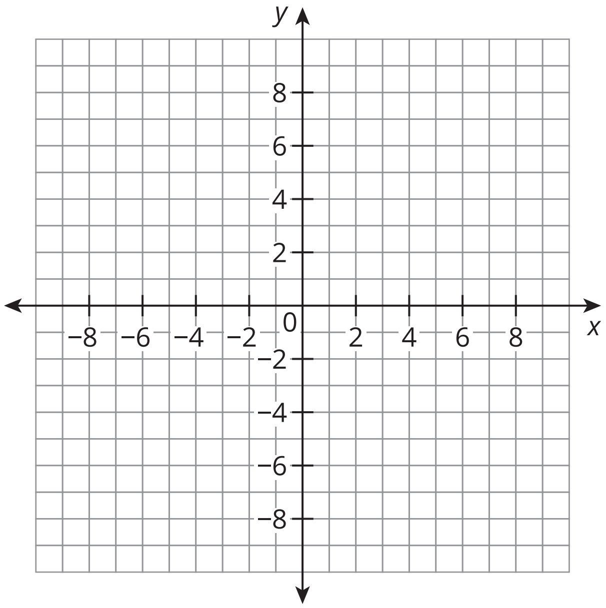
Range: ___________________

x-intercept: _________

y-intercept: _______

Slope: _______

Increasing/Decreasing: ________________________________



III. Determining Slopes of Perpendicular Lines

Topic Practice

A. Use the given information to write the equation and identify the slope of each line.

1. Write an equation for a horizontal line and an equation for a vertical line that passes through the point $(3, -3)$. Identify the slope of each line.

2. Write an equation for a horizontal line and an equation for a vertical line that passes through the point $(-1, 4)$. Identify the slope of each line.

3. Write an equation for a horizontal line and an equation for a vertical line that passes through the point $(0.5, 1.25)$. Identify the slope of each line.

4. Write an equation for a line that is parallel to the line $x = 5$ and passes through the point $(-2, 3)$. Identify the slope of the line.

5. Write an equation for a line that is perpendicular to the line $x = -2$ and passes through the point $(1, 5)$. Identify the slope of the line.

6. Write an equation for a line that is perpendicular to the line $y = 3$ and passes through the point $(5, 7)$. Identify the slope of the line.

B. Determine an equation that represents each perpendicular line. Write your answer in point-slope form, slope-intercept form and standard form.

1. What is the equation of a line perpendicular to $y = 2x - 6$ that passes through (5, 4)?

2. What is the equation of a line perpendicular to $3x + y = 4$ that passes through $(-1, 6)$?

3. What is the equation of a line perpendicular to $y = -\frac{2}{5}x - 1$ that passes through $(2, -8)$?

4. What is the equation of a line perpendicular to $3x - 4y = -48$ that passes through (12, 3)?

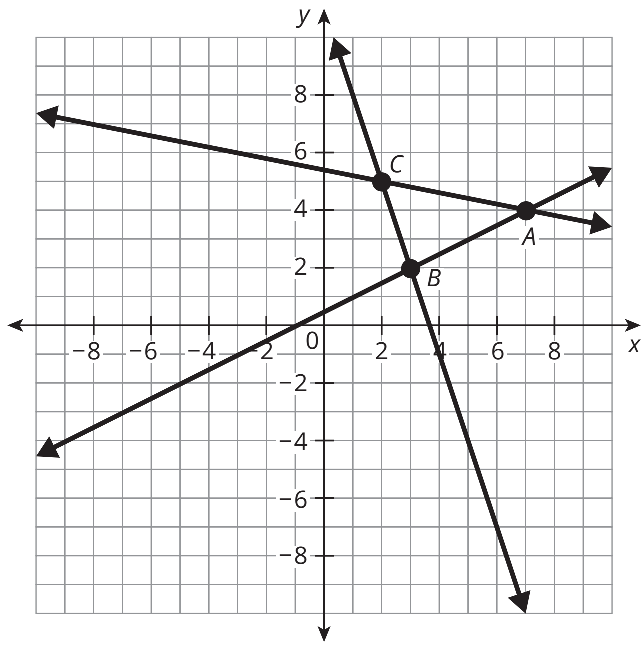
5. What is the equation of a line perpendicular to $y = 6x - 5$ that passes through $(6, -3)$?

6. What is the equation of a line perpendicular to $y - 4 = \frac{5}{2}(x - 2)$ that passes through $(-1, -4)$?

Extension

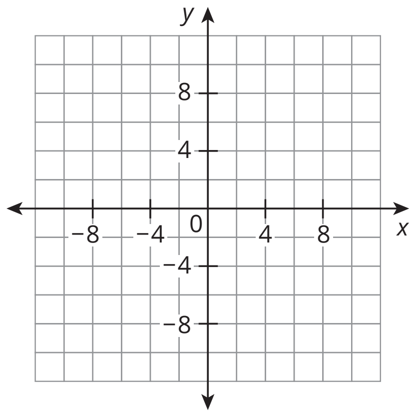
Triangle *ABC* is located on three lines such that the vertices occur at the points of intersection of pairs of the lines, as shown on the graph.

If △*ABC* is rotated 90° counterclockwise around the origin to form △*A′B′C′*, determine the equations of the lines that would contain △*A′B′C′*. Explain your reasoning. Then, draw the three lines that contain △*A′B′C′* and label △*A′B′C′*.

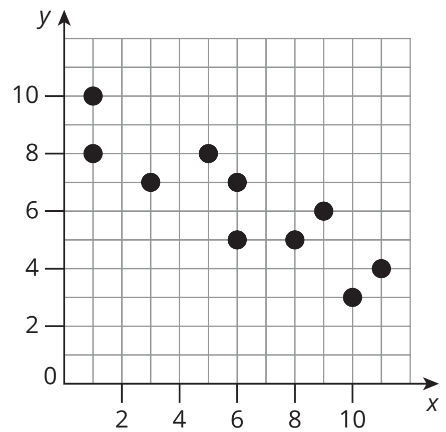


Spaced Practice

1. Graph the estimated line of best fit for the set of points: (−2, −6), (−3, −2), (2, 4), (6, 3). Determine the estimated linear regression model for the line.



2. Determine whether the points on the scatterplot have a positive correlation, a negative correlation, or no correlation. Then, determine which *r*-value is most accurate.



a. $r = 0.005$

b. $r = -0.865$

c. $r = -0.045$

d. $r = 0.905$

3. Write the equation of a line parallel to the line $2x - 3y = 6$ that passes through the given points.

   a. (0, 3)

   b. (−4, −1)

4. Write a recursive and explicit formula for the arithmetic sequence shown.
   4, 8, 12, 16, 20, . . .

5. Write a recursive and explicit formula for the geometric sequence shown.
   $9, -3, 1, -\frac{1}{3}, \frac{1}{9}, \ldots$

6. Write the equation of a line that is perpendicular to $y = -\frac{2}{3}x + 7$ and goes through the point (27, −3).

7. Write the equation of a line that is perpendicular to $y = 1\frac{4}{5}x - 12$ and goes through the point $(-15, -9\frac{1}{2})$.

IV. Comparing Linear Functions in Different Forms

Topic Practice

A. For each scenario, complete the following:

a. Determine and then compare the rates of change for each function in terms of the quantities represented.

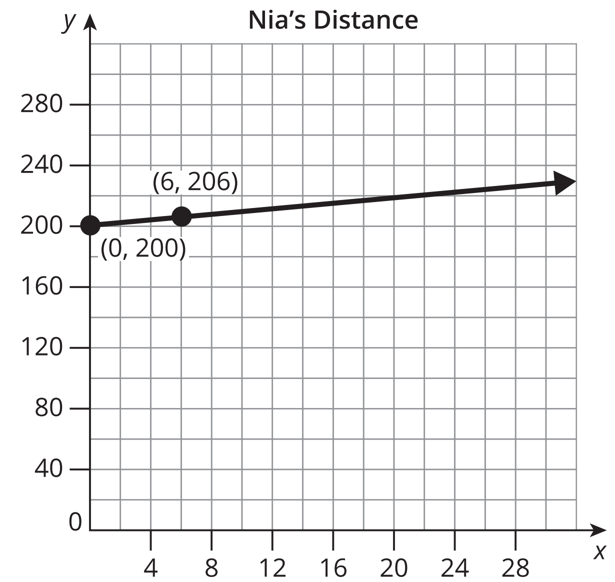
b. Determine and then compare the *y*-intercepts of each function in terms of thequantities represented.

c. Determine the domain and range for each problem situation using words and inequalities.

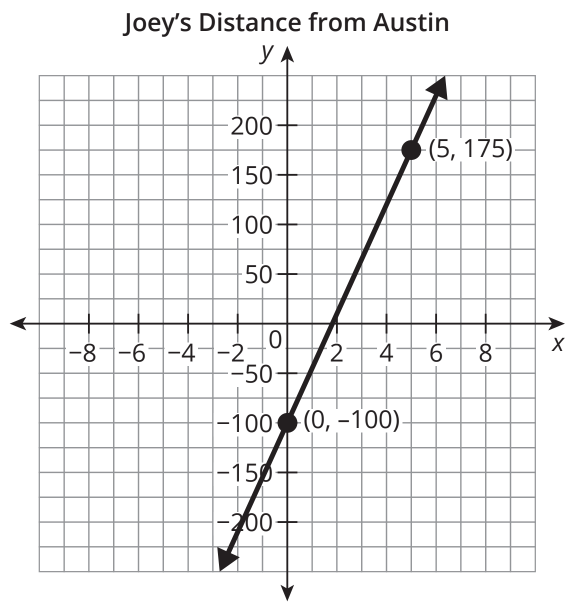
1. On the track team, two long jumpers, Ricardo and Nia, are steadily improving their maximum distances during the course of the season. These functions represent the distance jumped in inches, y, as a function of the number of months, x, that have passed during the season.

Ricardo's Distance

x	y
0	235
1	236
2	237
3	238
4	239
5	240



2. Jackson and Joey take separate journeys driving north on Interstate 35 towards Dallas. Jackson begins his journey in Austin, while Joey begins his journey further south of Austin. These functions represent each person's distance in miles from Austin, y, as a function of time in hours, x, since each person began their journey.

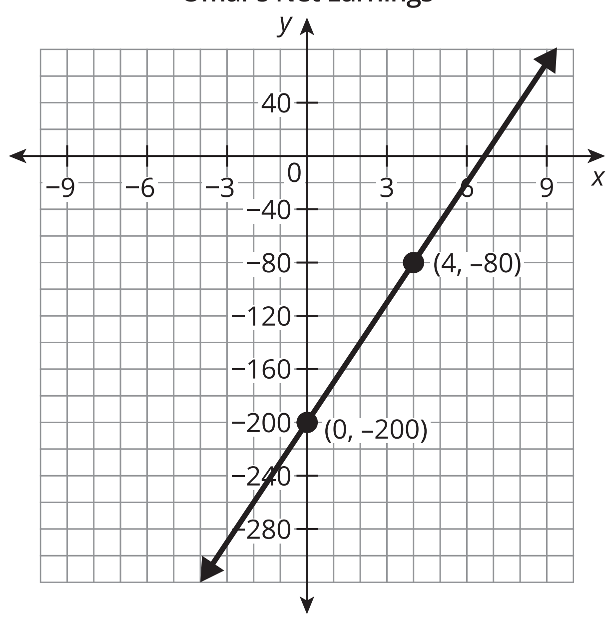
Jackson's Distance from Austin $y = 60x$	

3. These functions represent the net earnings in dollars, *y*, as a function of the number of lawns mowed, *x*, for Lucia and Omar's lawn mowing businesses, including the start-up cost of buying a lawn mower.

Lucia's Net Earnings

x	*y*
0	−250
1	−205
2	−160
3	−115
4	−70
5	−25

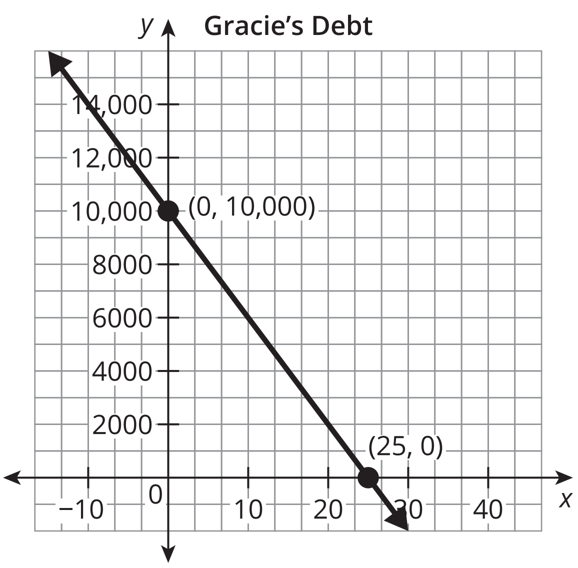
Omar's Net Earnings



4. A generous grandmother loaned money for college to two of her grandchildren, Brianna and Gracie, letting them pay it back without interest. These functions represent the amount in dollars, y, that Brianna and Gracie owe to their grandmother over the course of time in months, x.

Brianna's Debt

$y = -800x + 25{,}000$



5. Xavier and Nicky are both saving money. These functions represent the amount in dollars, y, in their savings accounts over the course of time in months, x.

Xavier's Account Balance

x	y
0	300
1	425
2	550
3	675
4	800

Nicky's Account Balance

$y - (-275) = 100(x - (-5))$

6. James and Ricardo leave Harrisburg at different times and drive towards Philadelphia. These functions represent their distances in miles, y, and time in hours since James started driving, x.

James's Distance from Harrisburg

x	y
0	0
1	50
2	100
3	150
4	200
5	250

Ricardo's Distance from Harrisburg

$y = 65x + 150$

Extension

1. Omar and Antonio are twins. Their parents track their height every year between the ages of 5 and 15. Omar's height is given by the equation, and Antonio's height is shown in the graph.

   a. Label the *x*- and *y*-axis, the origin, and the intervals on both axes. Explain your reasoning.

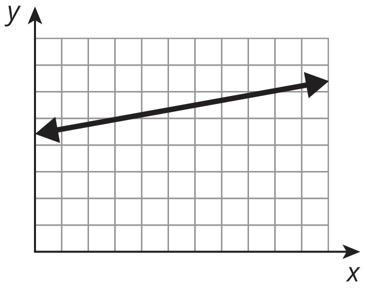
   b. Which twin is growing faster? Justify your answer.

   c. At what age does one twin surpass the other in height? Explain your reasoning.

Omar

$y = 3.1x + 40.6$

Antonio



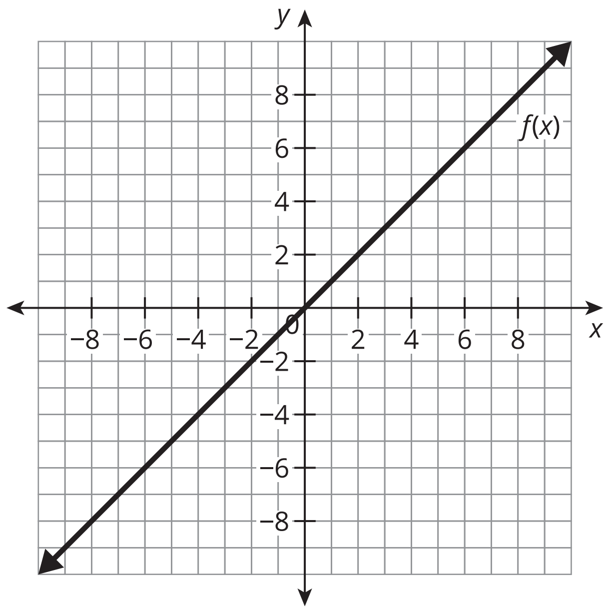
Spaced Practice

1. For each situation decide whether the correlation implies causation. List reasons why or why not.

   a. The number of winter coats sold at department stores is highly correlated to average low temperatures in the area.

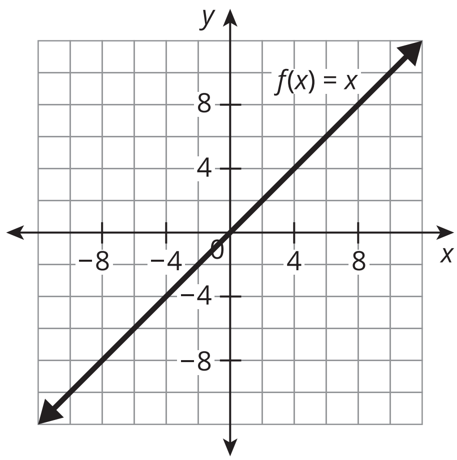
   b. The number of concessions sold at a concert is highly correlated to the number of people in attendance at the concert.

2. The graph represents the parent function $f(x) = x$. The equation for the transformed function $g(x)$ is $g(x) = \frac{2}{3} \cdot f(x) - 2$.



a. Describe the transformations performed on $f(x)$ to produce $g(x)$.

b. Graph $g(x)$.



c. Write the equation of $g(x)$ in slope-intercept form and identify the slope and y-intercept.

3. Write a recursive formula for each sequence.

a. 3, −6, 12, −24, 48, . . .

b. 180, 160, 140, 120, 100, . . .

4. Write the equation of a line that is perpendicular to $y = -\frac{2}{3}x + 7$ and goes through the point (27, −3).

5. Write the equation of a line that is perpendicular to $y = 1\frac{4}{5}x - 12$ and goes through the point $(-15, -9\frac{1}{2})$.

MODULE
3

Modeling Linear Equations and Inequalities

TOPIC 1: Linear Equations and Inequalities

TOPIC 2: Systems of Linear Equations and Inequalities

Name ________________________ Date ____________

I. Solving Linear Equations

Topic Practice

A. Solve each equation. Write the properties that justify each step.

1. $-3(x - 4) = -9(x - 1)$

2. $8x - 2(x + 3) = 4x + 2$

3. $\frac{-2x + 1}{2} + 6 = \frac{3x}{2} - 10$

4. $12x - 4\left(\frac{1}{2}x - 5\right) = \frac{1}{3}(6x - 15)$

5. $\frac{7(x - 1)}{4} - \frac{3}{4} = -8x + \frac{3}{4}$

6. $-4(2x - 9) + 6(-x + 1) = -8x - 5\left(3x - \frac{5}{6}\right)$

B. Determine if the equation has one solution, no solution, or infinite solutions. Show your work.

1. $3(x + 2) = 3x + 6$

2. $5x + 4 = 5x - 3$

3. $20x - 2(x + 10) = -(5 - 2x)$

4. $\frac{3}{5}(x - 12) = -4(x + 9) + 1$

5. $-7(x - 1) = -15x + 8(x + 2)$

6. $\frac{8(x - 3)}{2} + 5x = 9(x - 1) - 3$

C. For each question, write and solve an equation to determine the unknown value.

1. Triangle *A* has a base of $y + 4$ inches and height of 10 inches. Triangle *B* has a height of 14 inches and a base of $2y + 1$. Determine the value of *y* when both triangles have the same area.

2. In ΔDEF, $\angle D$ and $\angle F$ are congruent base angles of an isosceles triangle. Determine the value of *g* when $m\angle \text{D} = 3(g - 2)^\circ$ and $m\angle \text{F} = (g + 20)^\circ$.

3. A rectangle has a width of 6 inches and a length of $3x + 8$ inches. A square has an area of $20x - 4$ square inches. Determine the value of x when the area of the square is equal to the area of the rectangle.

4. Congruent angles have the same measure. Determine the value of x, when $m\angle J = \frac{1}{2}(4x + 16)$ and $m\angle K = 4\{2x - 1)$ and $\angle J$ is congruent to $\angle K$.

5. A parallelogram has a base of 9 cm and a height of $\frac{1}{3}c + 2$. A rectangle has a base of $2c + 4$ and a height of 2 cm. Determine the value of c when the area of the parallelogram is the same as the area of the rectangle.

6. The side length of a square is $3x + 7$. A rectangle has side lengths of 5 and $x + 10$. Determine the value of x when the perimeter of the square is equal to the perimeter of the rectangle.

Extension

1. Consider the equation $2x - 5(x - 1) = 50$.

   a. Solve the equation for x.

   b. Emma was asked to solve the inequality $2x - 5(x - 1) < 50$. She gave an answer of $x < -15$. Substitute any value for x less than (-15) to determine if Emma is correct. If not, determine the correct solution.

Spaced Practice

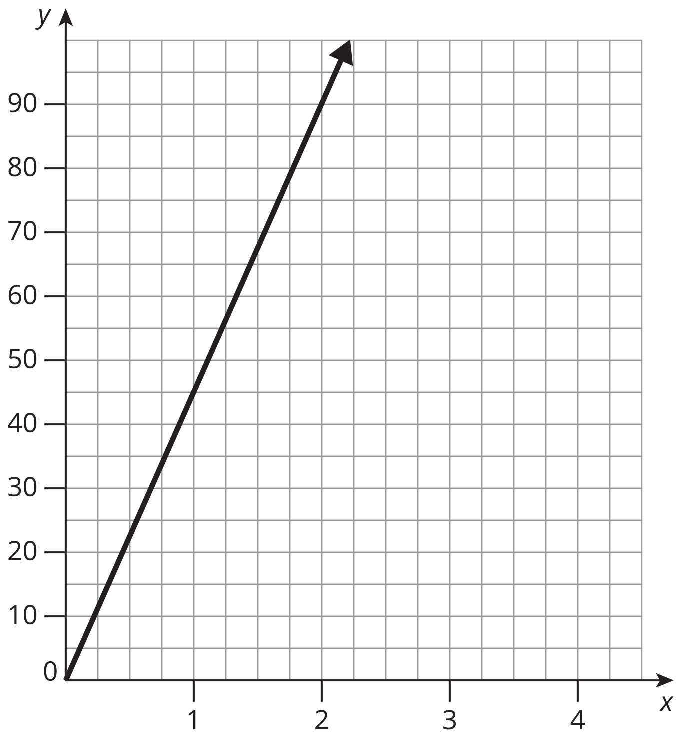
1. Determine whether the table of values represents a linear function. If it does represent a linear function, write the function. If it does not represent a linear function, explain why.

x	$f(x)$
−2	4
−1	1
0	0
1	1

2. Ethan grows tomatoes and sells them at a nearby farmer's roadside stand. He sells them for $2.50 each. The farmer charges him $15 per day to use the stand. Write a linear function in point-slope form and slope-intercept form that represents the amount of money, M, Ethan will make from selling x tomatoes.

3. Clean Green Landscapers uses a graph to show what they charge, and Sunshine Landscaper lists what they charge in a table.

Clean Green Landscapers



Sunshine Landscaper

0.5	$25
1	$50
1.5	$75
2	$100
2.5	$125

a. Each representation shows a functional relationship between quantities. Label the quantities and their units in the table and on the graph.

b. Let *C*(*x*) represent the function for Clean Green Landscapers, and let *S*(*x*) represent the function for Sunshine Landscapers. Which function has a steeper slope? Explain how you know.

4. Evaluate the function $f(x) = 0.4x^2 - 3x - 8$ for the value $x = -2$.

5. The cost to install x number of central air conditioning units for a company is given by the function $C(x) = \frac{4000x + 1300}{3}$. Use the function to determine the cost to install 45 air conditioning units.

II. Literal Equations

Topic Practice

A. Convert each equation from standard form to slope-intercept form. Determine the slope.

1. $4x + 6y = 48$

2. $3x - 5y = 25$

3. $-4x + 9y = 45$

4. $6x - 2y = -52$

5. $-x - 8y = 96$

6. $12x + 28y = -84$

B. Convert each equation from point-slope form to the indicated form.

1. $y - 2 = 3(x - 1)$ to slope-intercept form

2. $y - (-6) = 4x$ to point-slope form

3. $y + 5 = \frac{3}{4}(x + 4)$ to standard form

4. $y - 3 = -\frac{1}{5}(x + 10)$ to slope-intercept form

5. $y - 4 = 5(x - 3)$ to standard form

6. $y = -\frac{1}{3}(x - 6)$ to slope-intercept form

C. Convert each equation from standard form to slope-intercept form.

1. $4x + 2y = 10$

2. $3x - 4y = 12$

3. $-8x + y = -24$

4. $12x + 8y = 16$

5. $x - 3y = 18$

6. $9x - y = 27$

D. Solve each equation for the variable indicated.

1. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Solve the equation for h.

2. The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$. Solve the equation for b_1.

3. The formula for the volume of a cylinder is $V = \pi r^2 h$. Solve the equation for h.

4. The formula for the volume of a pyramid is $V = \frac{1}{3}lwh$. Solve the equation for w.

5. The Ideal Gas Law is $pV = nRT$. Solve the equation for T.

6. Solve the literal equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_1.

7. Solve the literal equation $\frac{a_1}{a_0} = \frac{b_1}{b_0}$ for b_0.

8. Solve the literal equation $Z = \frac{4x}{Y^2} + 3W$ for x.

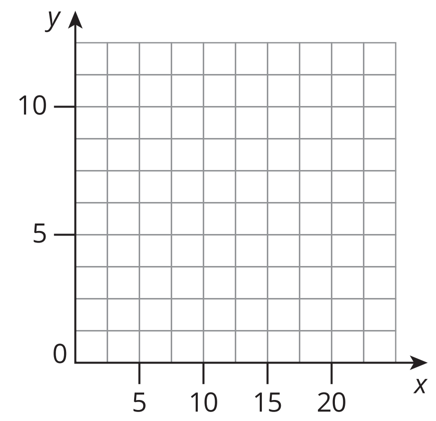
Extension

A simple pendulum is made of a long string and a small metal sphere. The period of oscillation can be found by the formula $T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$, where g is the acceleration due to gravity, and L is the length of the string. Solve the formula for g, the acceleration due to gravity.

Spaced Practice

1. The Peters Creek restaurant has an all-you-can eat shrimp deal. Currently, the cost of the deal is 50 cents per shrimp, with free soft drinks. The cost for the shrimp deal is modeled by the function $c(x) = 0.50x$, where x represents the number of shrimp eaten. A new manager decides to change the cost of the deal to 25 cents per shrimp, but a $5.00 charge for soft drinks. Let $p(x)$ be the function that represents the new cost for the all-you-can-eat shrimp deal.

   a. Sketch the graph of $c(x)$ and $p(x)$ on the same coordinate plane.



b. Complete the table of corresponding points on $p(x)$.

Transformed Graph	
x	$p(x)$
0	
10	
40	
60	
70	
90	

c. Write an equation for $p(x)$ in terms of $c(x)$. Describe the transformation performed on $c(x)$ to produce $p(x)$.

d. Write the equation for the function $p(x)$ in slope-intercept form.

2. Solve the equation. Write the properties that justify each step.

$$-\frac{2}{3}(-9x + 24) = 2x - 4$$

3. Determine whether the equation has one solution, no solution, or infinite solutions.

$$3\left(2 + \frac{2}{3}x\right) = 5 + 2(x + 1)$$

4. The table shows the relationship between y and x. Write an equation that represents the relationship between the variables.

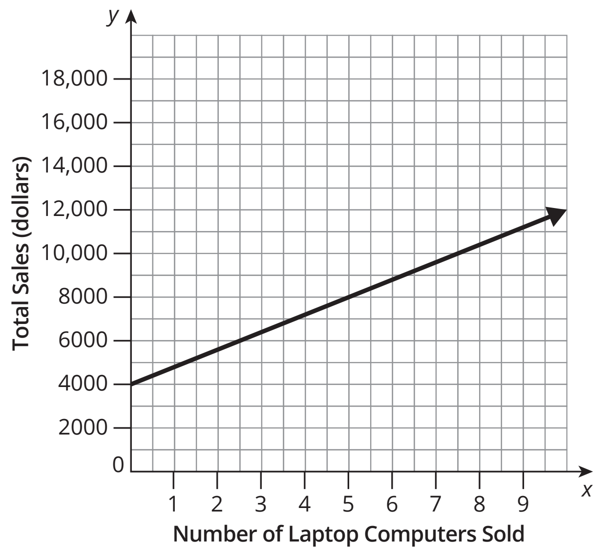
x	y
23	2
21	6
1	10
3	14
6	20

5. A clothing store decides to give out bonus points that can be used for future purchases. If a customer applies for a bonus card, they are automatically given 50 bonus points. After that, they get 25 bonus points for every $1.00 that they spend. Write an equation that shows the number of bonus points, *b*, that a customer will earn for *x* dollars that they spend.

III. Modeling Linear Inequalities

Topic Practice

A. Samuel works at an electronics store selling computer equipment. He can earn a bonus if he sells $10,000 worth of computer equipment this month. So far this month, he has sold $4000 worth of computer equipment. He hopes to sell additional laptop computers for $800 each to reach his goal. The function *f* (*x*) = 800*x* + 4000 represents Samuel's total sales as a function of the number of laptop computers he sells.



Use the graph to write an equation or inequality to determine the number of laptop computers Samuel would need to sell to earn each amount.

1. At least $10,000

2. Less than $7000

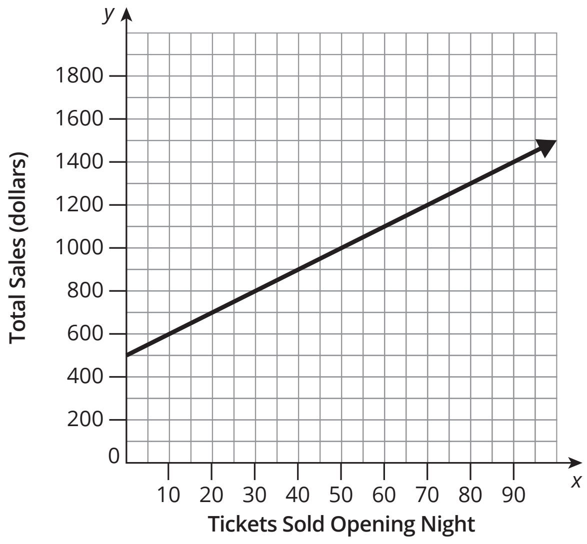
3. Less than $6000

4. At least $9000

5. More than $12,000

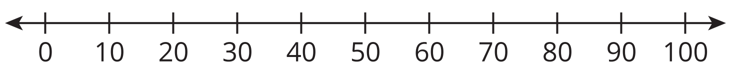
6. Exactly $8000

B. Isabella works at the ticket booth of a local playhouse. On the opening night of the play, tickets are $10 each. The playhouse has already sold $500 worth of tickets during a presale. The function $f(x) = 10x + 500$ represents the total sales as a function of tickets sold on opening night.

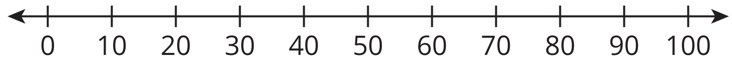


Use the graph of the function to answer each question. Graph each solution on the number line.

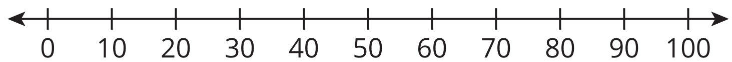
1. How many tickets must Isabella sell in order to make at least $1000?



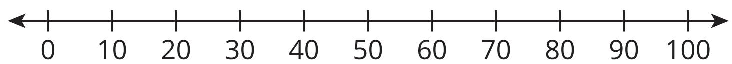
2. How many tickets must Isabella sell in order to make less than $800?



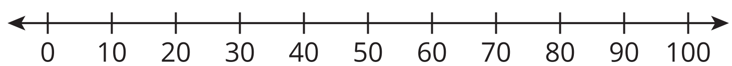
3. How many tickets must Isabella sell in order to make at least $1200?



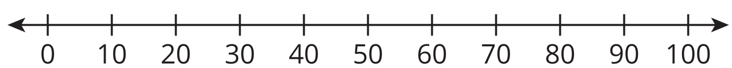
4. How many tickets must Isabella sell in order to make exactly $1400?



5. How many tickets must Isabella sell in order to make less than $600?



6. How many tickets must Isabella sell in order to make exactly $900?



C. Write and solve a linear inequality to answer each question.

1. A hot air balloon at 4000 feet begins its descent. It descends at a rate of 200 feet per minute. The function $f(x) = -200x + 4000$ represents the height of the balloon as it descends. How many minutes have passed if the balloon is below 3000 feet?

2. A bathtub filled with 55 gallons of water is drained. The water drains at a rate of 5 gallons per minute. The function $f(x) = -5x + 55$ represents the volume of water in the tub as it drains. How many minutes have passed if the tub still has more than 20 gallons of water remaining in it?

3. Harper is walking to school at a rate of 250 feet per minute. Her school is 5000 feet from her home. The function $f(x) = 250x$ represents the distance Harper walks. How many minutes have passed if Harper still has more than 2000 feet to walk?

4. Diego is riding his bike to school at a rate of 600 feet per minute. His school is 9000 feet from his home. The function $f(x) = 600x$ represents the distance Diego rides. How many minutes have passed if Diego has less than 3000 feet left to ride?

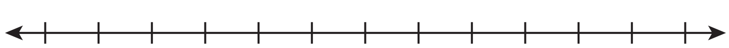
5. A submarine begins to dive from its current depth at a rate of 20 feet per minute. The function $f(x) = -20x - 30$ represents the depth of the submarine as it dives. How many minutes have passed if the submarine is at least 160 feet below the surface?

6. Daniel plays on the varsity basketball team. So far this season, he has scored a total of 52 points. He scores an average of 13 points per game. The function $f(x) = 13x + 52$ represents the total number of points Daniel will score this season. How many more games must Daniel play in order to score at least 100 points?

D. Solve each inequality and then graph the solution set on the number line.

1. $4x + 3 \leq 3x - 5$

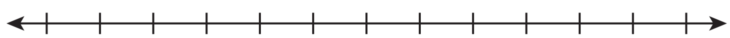
2. $-2x > 6$



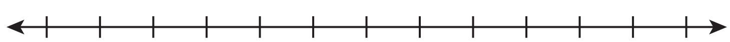
3. $\frac{1}{8}(3x - 16) < 4$

4. $\frac{x - 3}{2} \geq -5$

5. $-4(2 - x) \leq 6(x + 2)$

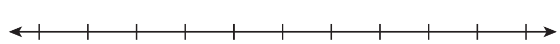


6. $-\frac{1}{2}(4x + 20) < -7\left(x + \frac{15}{7}\right)$



Extension

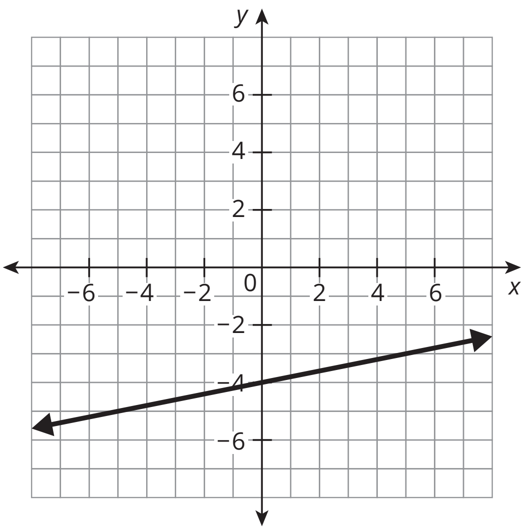
A company orders its nut mixes every month from a distributor. The distributor charges $4.50 per pound of nut mix. There is a handling fee of $8.50 for every order. There is free shipping on any order between $100 and $400. Write a compound inequality to represent the number of pounds of nuts the company can order and get free shipping. Solve the inequality and graph the solution on one number line.



Nuts (pounds)

Spaced Practice

1. Calculate the average rate of change for the linear function using the rate of change formula. Show your work.



2. Determine whether the table of values represents a linear function. If so, write the function.

x	y
−2	$4\frac{1}{2}$
0	$3\frac{1}{2}$
3	2
6	$\frac{1}{2}$

3. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is its height and b_1 and b_2 are the lengths of each base.

   a. Determine the area of a trapezoid if its height is 10 cm and the lengths of its bases are 22 cm and 18 cm.

   b. Rewrite the equation to solve for b_1.

c. Determine the length of the other base of a trapezoid if one base measures 10 meters, the height is 20 meters, and the area of the trapezoid is 600 square meters.

4. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Convert the equation to solve for *b*.

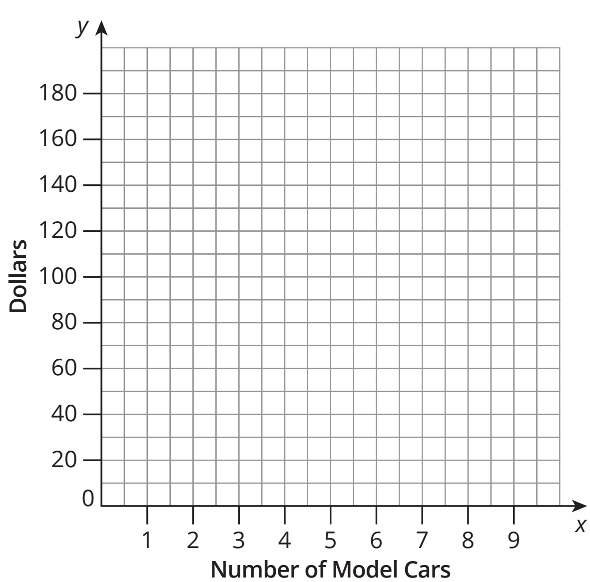
Name ______________________ Date ____________

I. Using Graphing to Solve Systems of Equations

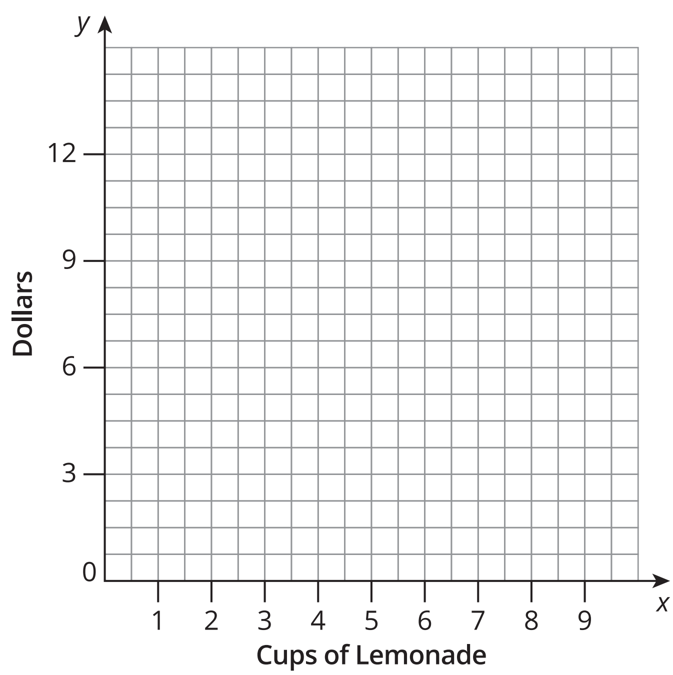
Topic Practice

A. Write a system of linear equations to represent each problem situation. Define each variable. Then, graph the system of equations and estimate the point of intersection. Explain what the point represents with respect to the given problem situation.

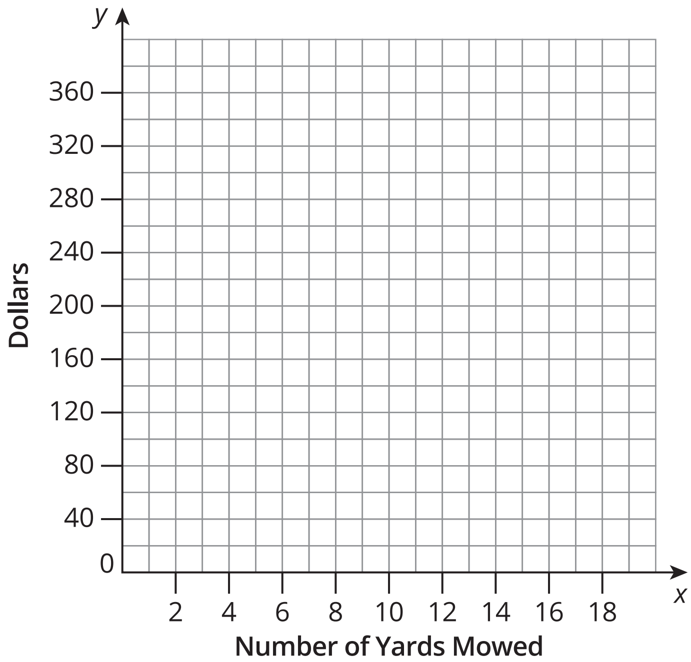
1. Mateo sells model cars from a booth at a local flea market. He purchases each model car from a distributor for $12, and the flea market charges him a booth fee of $50. Mateo sells each model car for $20.



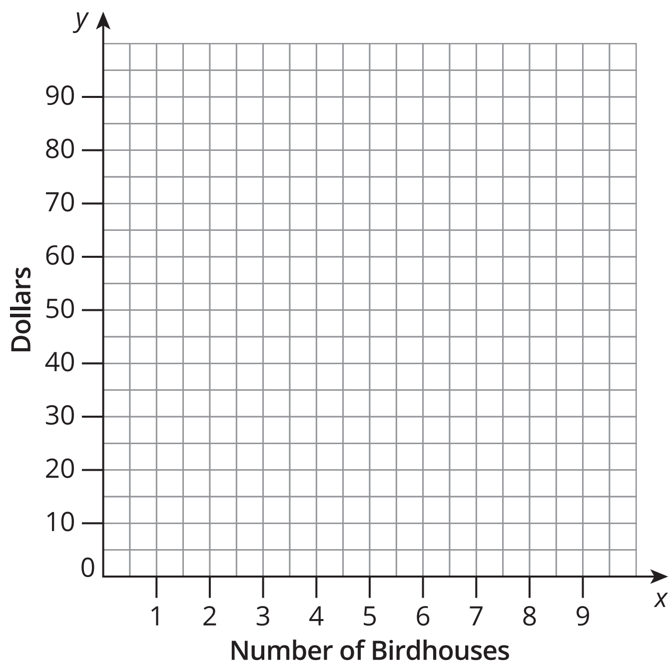
2. Minh sets up a lemonade stand in front of her house. Each cup of lemonade costs Minh $0.30 to make, and she spends $6 on the advertising signs she puts up around her neighborhood. She sells each cup of lemonade for $1.50.



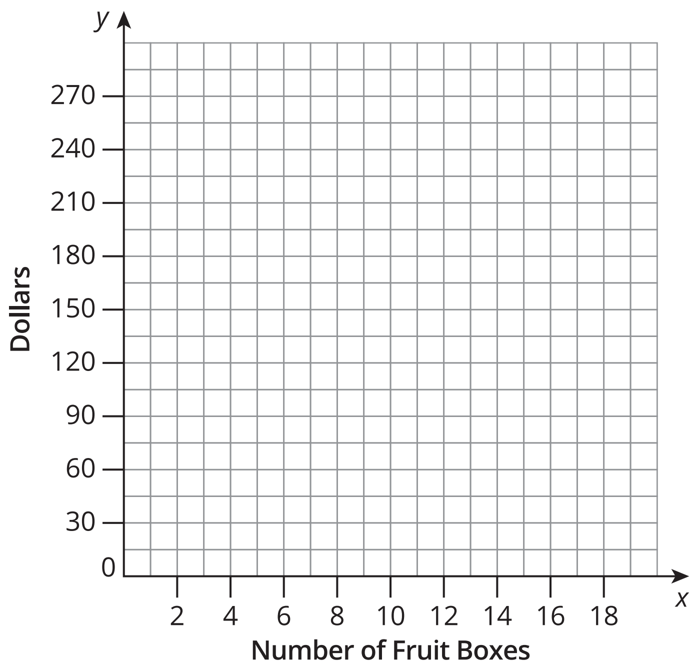
3. Liam starts his own lawn mowing business. He initially spends $180 on a new lawnmower. For each yard he mows, he receives $20 and spends $4 on gas.



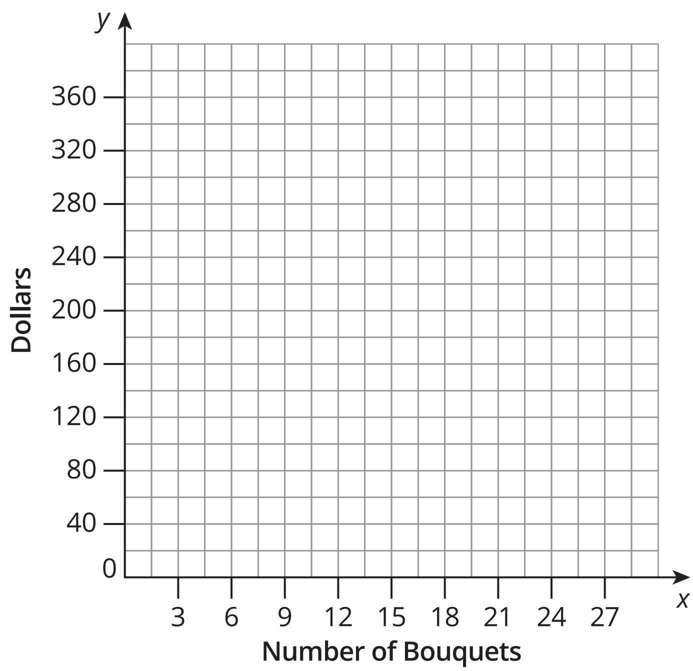
4. Victoria is building birdhouses to raise money for a trip to Hawaii. She spends a total of $30 on the tools needed to build the houses. The material to build each birdhouse costs $3.25. Victoria sells each birdhouse for $10.



5. The Spanish Club is selling boxes of fruit as a fundraiser. The fruit company charges the Spanish Club $7.50 for each box of fruit and a shipping and handling fee of $100 for the entire order. The Spanish Club sells each box of fruit for $15.

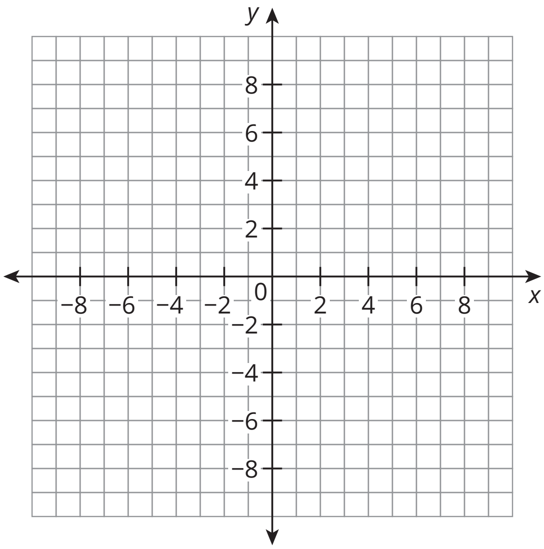


6. Elijah sells flowers online for $12 per bouquet. Each bouquet costs him $5.70 to make. Jerome also paid a one-time fee of $150 to advertise his company.

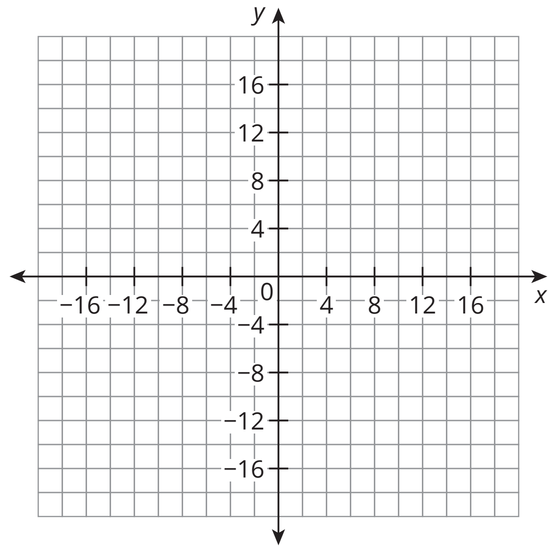


B. Graph the equations in each system. Tell whether the system has one solution, no solution, or infinite solutions. When the system has one solution, write the values of the variables that make the equations true. Determine whether the system is consistent or inconsistent.

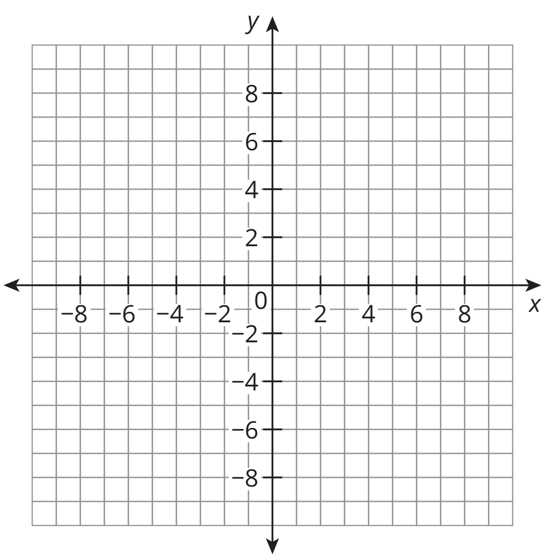
1. $\begin{cases} y = 2x - 1 \\ y = -3x - 11 \end{cases}$



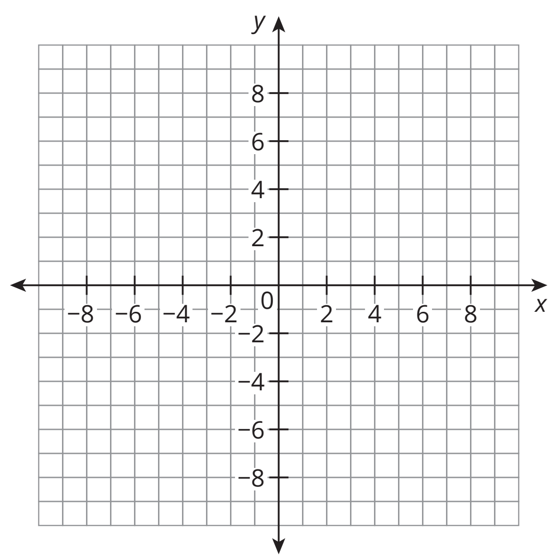
2. $\begin{cases} y = 4x - 30 \\ y = -3x + 5 \end{cases}$



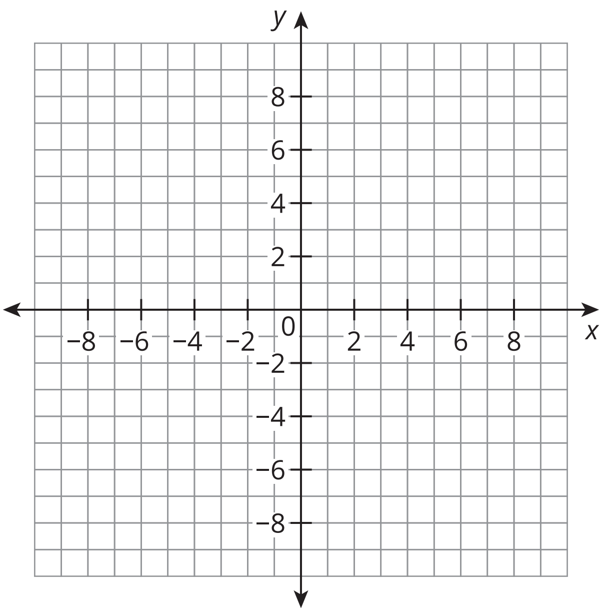
3. $\begin{cases} -5x + 10y = -10 \\ y = \frac{1}{2}x - 2 \end{cases}$



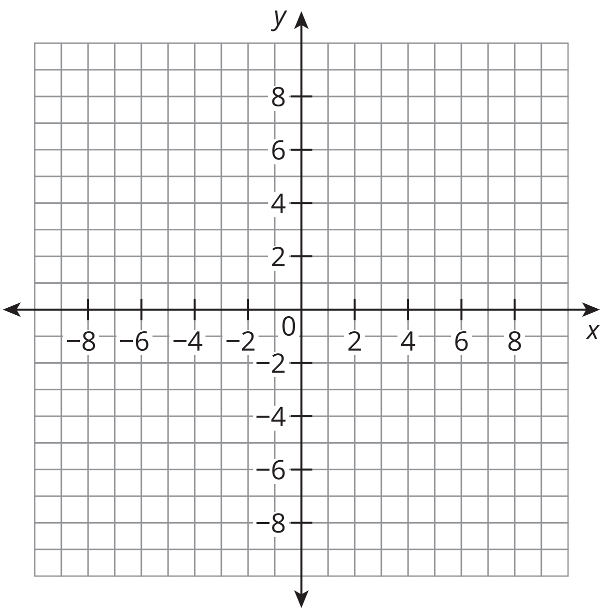
4. $\begin{cases} y = 3x + 17 \\ 10x + 5y = -15 \end{cases}$



5. $\begin{cases} -8x - 8y = -10 \\ x = 3.25 \end{cases}$

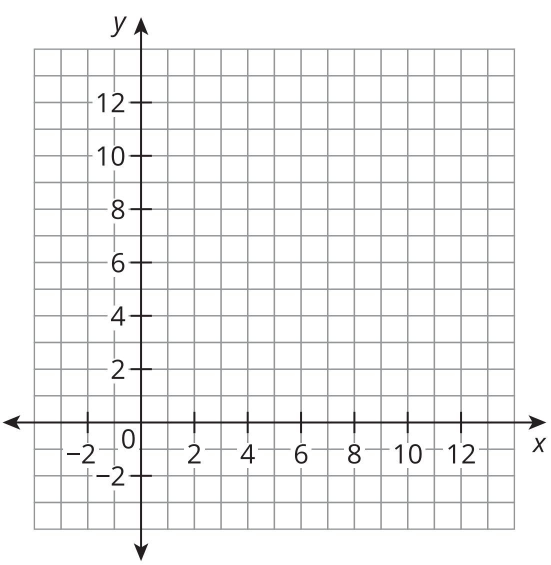


6. $\begin{cases} -1.5x + 0.5y = -2 \\ y + 7 = 3(x + 1) \end{cases}$



7. Graph the system of equations and estimate the solution of the system.

$\begin{cases} y = \frac{1}{3}x + 7 \\ y = -\frac{1}{4} + 8 \end{cases}$



Extension

Solve the system of equations shown. Explain your reasoning.

$$\begin{cases} 3x + 5y = 18 \\ y = |x - 4| \end{cases}$$

Spaced Practice

1. Solve the equation and check your solution.
$\frac{3}{4}x - 11 = 4 + \left(-\frac{3}{4}x + 3\right)$

2. Consider the equation $\frac{2}{5}x - 2y = 14$. Write the equation in slope-intercept form and identify the slope and y-intercept.

3. Determine the linear regression model for each data set. Which regression model is the better fit? Explain your reasoning.

Set A

x	y
1	12
2	11
5	30
7	39
10	50

Set B

x	y
12	3
10	9
8	11
5	14
0	0

II. Using Substitution to Solve Linear Systems

Topic Practice

A. Write a system of equations to represent each problem situation. Solve the system of equations using any method. Then, answer any associated questions.

1. Aaliyah needs to print some of her digital photos. She is trying to choose between Lightning Fast Foto and Snappy Shots. Lightning Fast Foto charges a base fee of $5 plus an additional $0.20 per photo. Snappy Shots charges a base fee of $7 plus an additional $0.10 per photo. Determine the number of photos for which both stores will charge the same amount. Explain which store Aaliyah should choose depending on the number of photos she needs to print.

2. Kai is trying to decide which ice cream shop is the better buy. Cold & Creamy Sundaes charges \$2.50 per sundae plus an additional \$0.25 for each topping. Colder & Creamier Sundaes charges \$1.50 per sundae plus an additional \$0.50 for each topping. Determine the number of toppings for which both vendors charge the same amount. Explain which vendor is the better buy depending on the number of toppings Kai chooses.

3. Ava is selling T-shirts at the State Fair. She brings 200 shirts to sell. She has long-sleeved and short-sleeved T-shirts for sale. On the first day of the fair, she sells $\frac{1}{2}$ of her long-sleeved T-shirts and $\frac{1}{3}$ of her short-sleeved T-shirts for a total of 80 T-shirts sold. How many of each type of T-shirt did Ava bring to the fair?

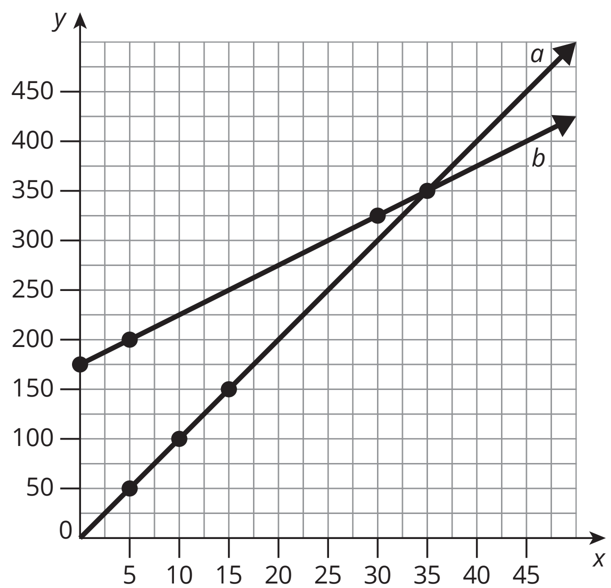
4. Ava has a booth at the flea market where she sells purses and wallets. All of her purses are the same price, and all of her wallets are the same price. The first hour of the day, she sells 10 purses and 6 wallets for a total of $193. The second hour, she sells 2 purses and 1 wallet for a total of $37.50. How much does Alicia charge for each purse and each wallet?

5. Noah has some nickels and dimes in his pocket, and the change is worth $0.75. He has twice as many dimes as nickels. How many of each type of coin does Johnny have?

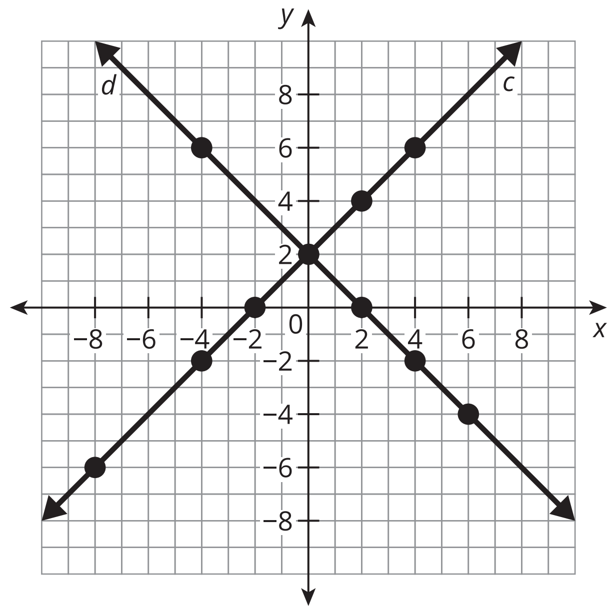
6. Ms. Garcia woke up one morning to find that her water heater had sprung a leak. She called two different plumbers to get their rates. The first plumber charges $64 just to walk in the door plus $24 per hour. The second plumber charges a flat $56 per hour. After how many hours will the cost for both plumbers be the same? Explain which plumber Ms. Garcia should use based on the number of hours she expects the repair to take.

B. Write a system of equations to represent each table or graph.

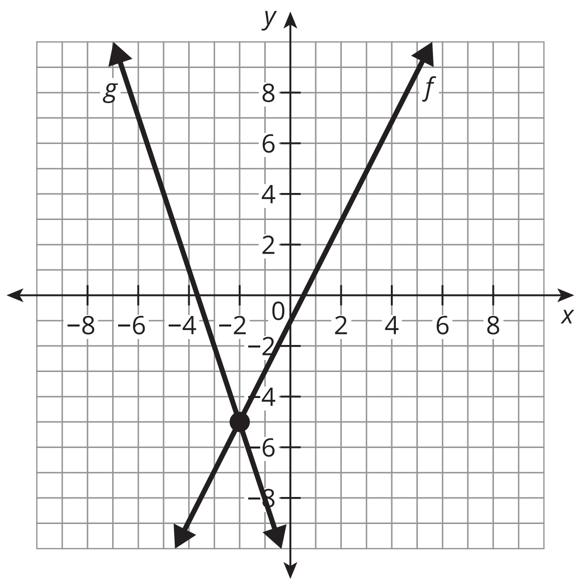
1. Write a system of equations in slope-intercept form that represents lines *a* and *b*.



2. Write a system of equations in standard form that represents lines *c* and *d*.



3. Write a system of equations in slope-intercept form that represents lines *f* and *g*.



4. Write a system of equations in slope-intercept form that represents lines *r* and *s*.

Line *r*

x	y
−2	−6
0	−5
3	−3.5
7	−1.5

Line *s*

x	y
−5	−16
−1	−8
4	2
8	10

5. Write a system of equations in standard form that represents lines *h* and *k*.

Line *h*

x	y
−5	−2
−1	2
2	5
6	9

Line *k*

x	y
−2	20
0	12
2	4
3	0

6. Write a system of equations in slope-intercept form that represents lines *m* and *n*.

Line *m*

x	y
-4	30
-1	21
3	9
8	-6

Line *n*

x	y
-5	18
-2	12
1	6
4	0

C. Solve each system of equations by substitution. Determine whether the system is consistent or inconsistent.

1. $\begin{cases} y = 2x - 3 \\ x = 4 \end{cases}$

2. $\begin{cases} 2x + y = 9 \\ y = 5x + 2 \end{cases}$

3. $\begin{cases} y = 3x - 2 \\ y - 3x = 4 \end{cases}$

4. $\begin{cases} \frac{1}{2}x + \frac{3}{2}y = -7 \\ \frac{1}{3}y = 2x - 10 \end{cases}$

5. $\begin{cases} 0.8x - 0.2y = 1.5 \\ 0.1x + 1.2y = 0.8 \end{cases}$

6. $\begin{cases} 0.3y = 0.6x + 0.3 \\ 1.2x + 0.6 = 0.6y \end{cases}$

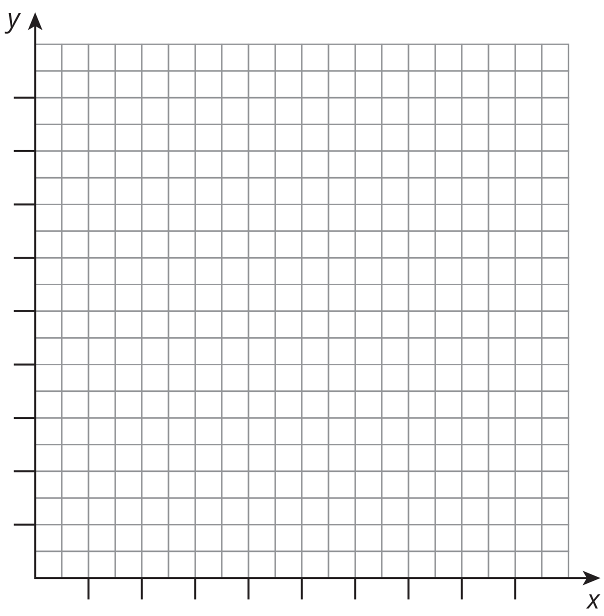
Extension

Create a system of linear equations with solution (2, 5). Solve the system using substitution to verify your system has the given solution.

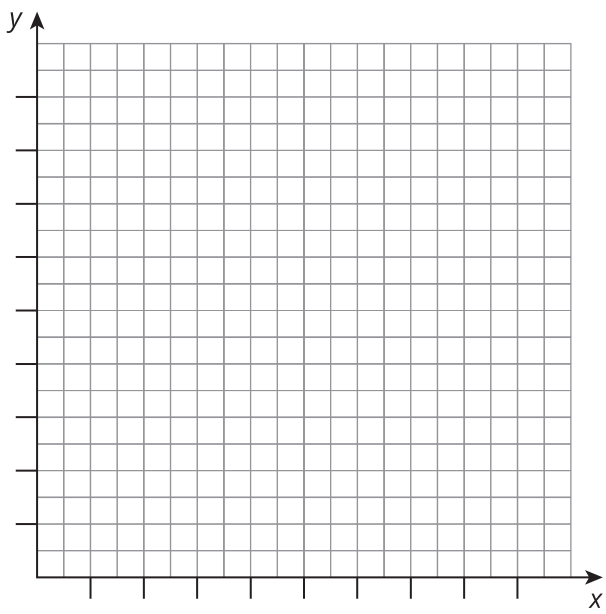
Spaced Practice

1. Graph each system of linear equations to determine the solution to the system.

a. $y = 34 - \frac{5}{2}x$ and $y = \frac{2}{5}x + 5$



b. $y = 21x + 144$ and $y = 3(7x + 48)$



2. The population growth (in thousands) for a small town near Bay City can be represented by the expression $x + \frac{4}{5}(x + 315)$, where x represents the number of years since 2005. The population growth (in thousands) for a neighboring town can be represented by the expression $2x - \frac{1}{5}(x - 630)$, where x represents the number of years since 2005. When will the populations of the two towns be the same?

3. Two neighboring towns are not having population growth. In fact, they both have been losing population since 1995. The population decline for one of the towns (in thousands) can be represented by the expression $-\frac{2}{5}(x - 500)$, where x represents the number of years since 1995. The population decline for the other town (in thousands) can be represented by the expression $-\frac{1}{2}x + \frac{1}{10}(x + 2000)$, where x represents the number of years since 1995. When will the populations of the two towns be the same?

4. Solve each equation or inequality.

   a. $8(2m + 7) = 10(m + 11)$

   b. $-3(y + 20) < -9y$

III. Using Linear Combinations to Solve a System of Linear Equations

Topic Practice

A. Solve each system of equations using the linear combinations method.

1. $\begin{cases} 3x + 5y = 8 \\ 2x - 5y = 22 \end{cases}$

2. $\begin{cases} 4x - y = 2 \\ 2x + 2y = 26 \end{cases}$

3. $\begin{cases} 10x - 6y = 26 \\ 5x - 5y = 5 \end{cases}$

4. $\begin{cases} 2x - 4y = 4 \\ -3x + 10y = 14 \end{cases}$

5. $\begin{cases} 3x + 2y = 14 \\ 4x + 5y = 35 \end{cases}$

6. $\begin{cases} x + 6y = 11 \\ 2x - 12y = 10 \end{cases}$

7. $\begin{cases} 1.5x + 1.2y = 0.6 \\ 0.8x - 0.2y = 2 \end{cases}$

8. $\begin{cases} \frac{3}{4}x + \frac{1}{2}y = -\frac{3}{4} \\ \frac{2}{3}x + \frac{2}{3}y = \frac{2}{3} \end{cases}$

B. Write a system of equations to represent each problem situation. Solve the system of equations using the linear combinations method.

1. The high school marching band is selling fruit baskets as a fundraiser. They sell a large basket containing 10 apples and 15 oranges for $20. They sell a small basket containing 5 apples and 6 oranges for $8.50. How much is the marching band charging for each apple and each orange?

2. Emma works on a shipping dock at a tire manufacturing plant. She loads a pallet with 4 Mudslinger tires and 6 Roadripper tires. The tires on the pallet weigh 212 pounds. She loads a second pallet with 7 Mudslinger tires and 2 Roadripper tires. The tires on the second pallet weigh 184 pounds. How much does each Mudslinger tire and each Roadripper tire weigh?

3. The Pizza Barn sells one customer 3 large pepperoni pizzas and 2 orders of breadsticks for $30. They sell another customer 4 large pepperoni pizzas and 3 orders of breadsticks for $41. How much does the Pizza Barn charge for each pepperoni pizza and each order of breadsticks?

4. Isabella and Ethan are making large pots of chicken noodle soup. Isabella opens 4 large cans and 6 small cans of soup and pours them into her pot. Her pot contains 115 fluid ounces of soup. Ethan opens 3 large cans and 5 small cans of soup. His pot contains 91 fluid ounces of soup. How many fluid ounces of soup does each large can and each small can contain?

5. Samuel and Harper are making block towers out of large and small blocks. They are stacking the blocks on top of each other in a single column. Samuel uses 4 large blocks and 2 small blocks to make a tower 63.8 inches tall. Harper uses 9 large blocks and 4 small blocks to make a tower 139.8 inches tall. How tall is each large block and each small block?

6. Diego has 2 buckets that he uses to fill the water troughs on his horse farm. He wants to determine how many ounces each bucket holds. On Tuesday, he fills an empty 2000-fluid-ounce water trough with 7 large buckets and 5 small buckets of water. On Thursday, he fills the same empty water trough with 4 large buckets and 10 small buckets of water. How many fluid ounces does each bucket hold?

Extension

Use linear combinations to solve the given system of three equations in three variables. Show your work.

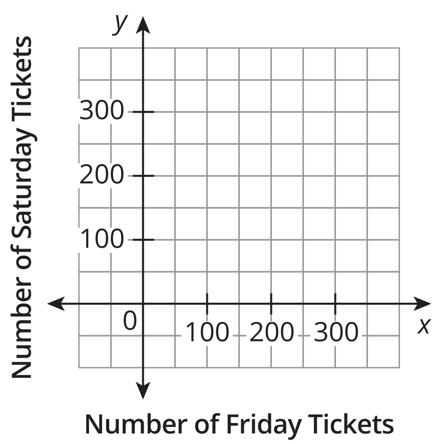
$$\begin{cases} 3x + y + 3z = -2 \\ 6x + 2y + 9z = 5 \\ -2x - y - z = 3 \end{cases}$$

Spaced Practice

1. The drama department sold a total of 360 tickets to their Friday and Saturday night shows. They sold three times as many tickets for Saturday's show than for Friday's show.

   a. Write a system of equations to represent this scenario.

   b. Graph the system of equations on a coordinate plane.



   c. How many tickets were sold for Friday? Saturday? Is there more than one solution?

d. Algebraically justify that your solution is correct.

2. Analyze the data in the table.

a. Write the equation of the regression line for the data.

b. Predict the population after 20 years. Round your answer to the nearest whole number.

Number of years	Population
1	240
2	360
3	280
5	500
6	625
7	830
8	720
9	813
10	900

IV. Graphing Inequalities in Two Variables

Topic Practice

A. Tell whether the graph of each linear inequality will have a dashed line or a solid line. Explain your reasoning.

1. $x - 3y \leq 32$

2. $8y + 7x > 15$

3. $y < 14x + 9$

4. $-5.2y - 8.3x \leq -28.6$

5. $\frac{2}{3}x + \frac{4}{9}y \geq 3$

6. $y - 17 > x + 8$

7. $185x + 274y \geq 65$

8. $36 < 9y - 2x$

B. For each inequality, use the test point (0, 0) to determine which half-plane should be shaded.

1. $5x + 7y > -13$

2. $y - 30 \leq 9x$

3. $-8y > 6x + 12$

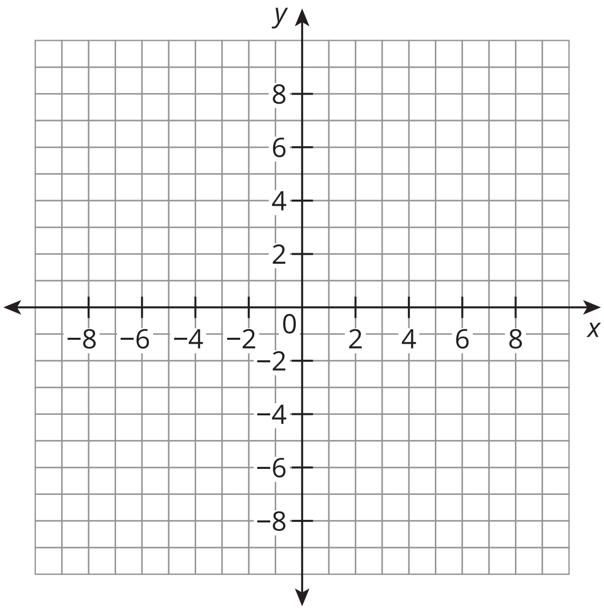
4. $46 \geq -5y + 10x$

5. $31.9x + 63.7y < -44.5$

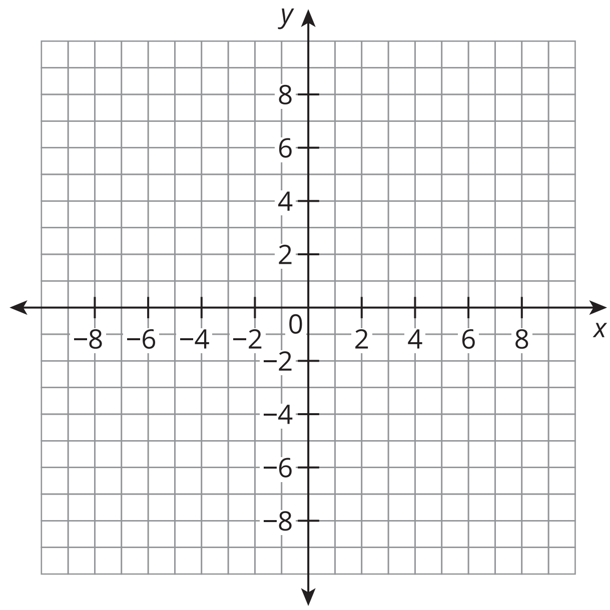
6. $y - \frac{5}{6} > \frac{1}{2}x + \frac{1}{3}$

C. Graph each linear inequality.

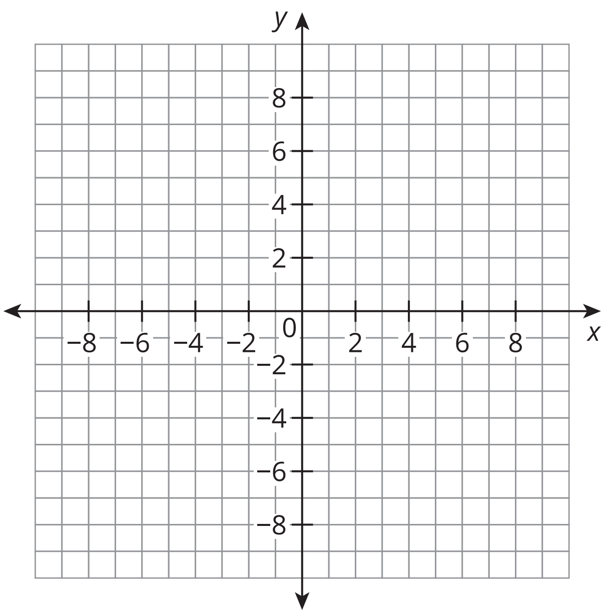
1. $y < 4x + 2$



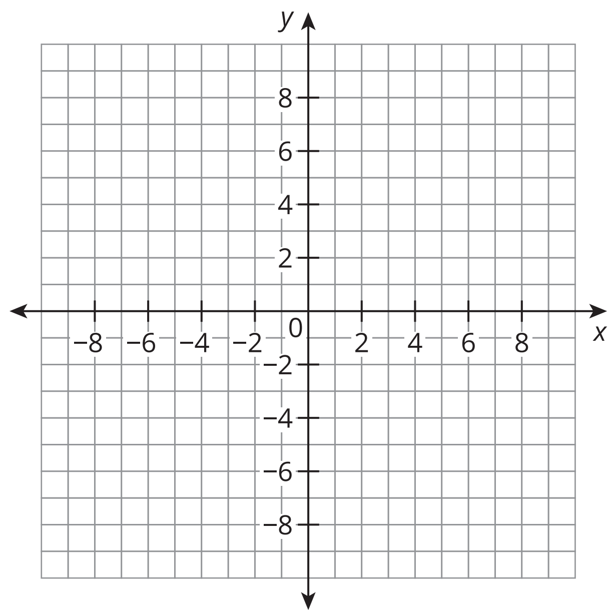
2. $y \geq 10 - x$



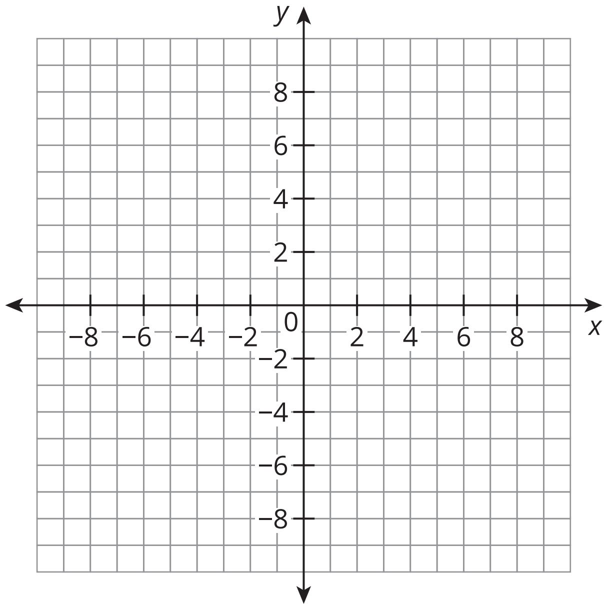
3. $y \geq \frac{1}{2}x - 3$



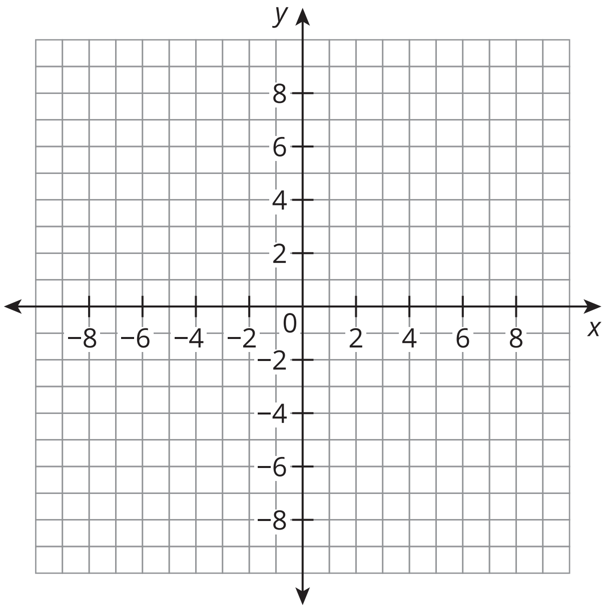
4. $-x + y > 1$



5. $3x - 4y \geq 8$



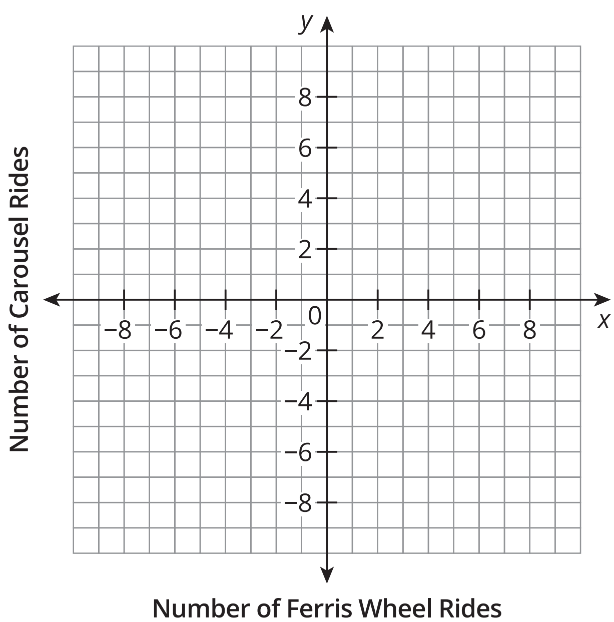
6. $\frac{3}{8}y - \frac{1}{4}x < \frac{3}{4}$



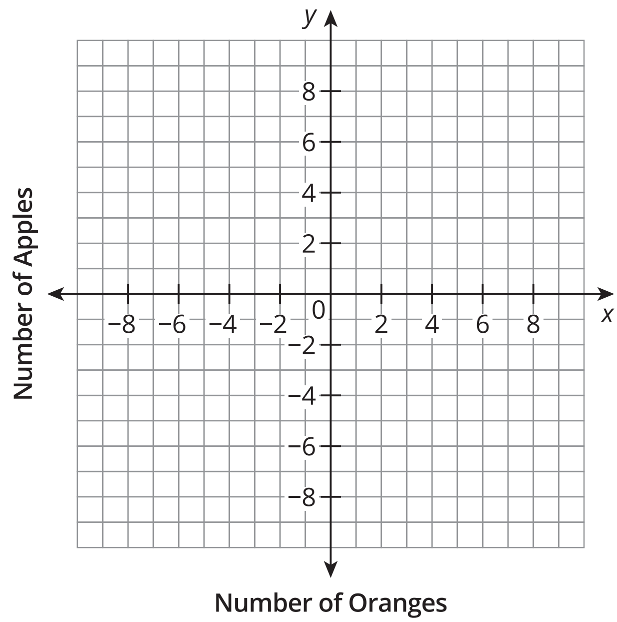
D. Write and graph an inequality for each problem situation. Then, determine whether the ordered pair is a solution for the problem situation.

1. Daniel has 50 tokens to spend at the school carnival. He wants to spend his tokens riding the Ferris wheel and the carousel. The table of values shows the combination of rides that expend his tokens. Write and graph an inequality that represents the possible ways Daniel could use his tokens on the two rides. Is the ordered pair (6, 3) a solution for the problem situation?

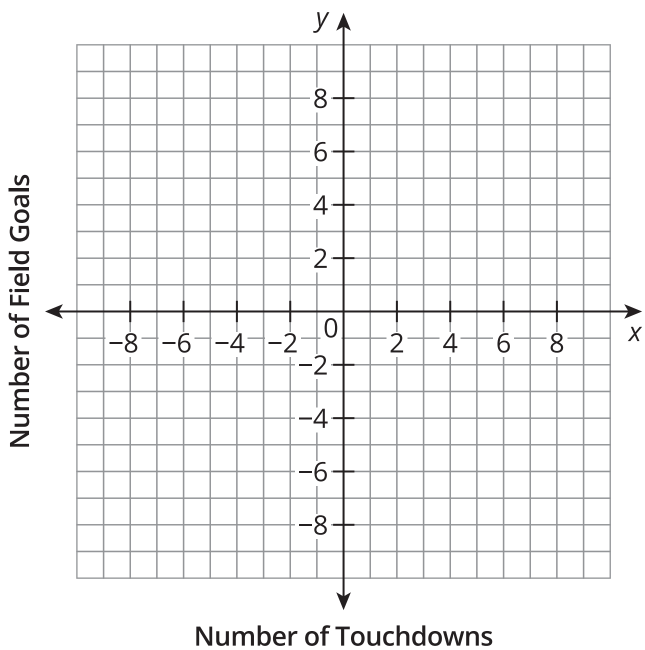
Ferris Wheel Rides	Carousel Rides
0	10
5	6
10	2



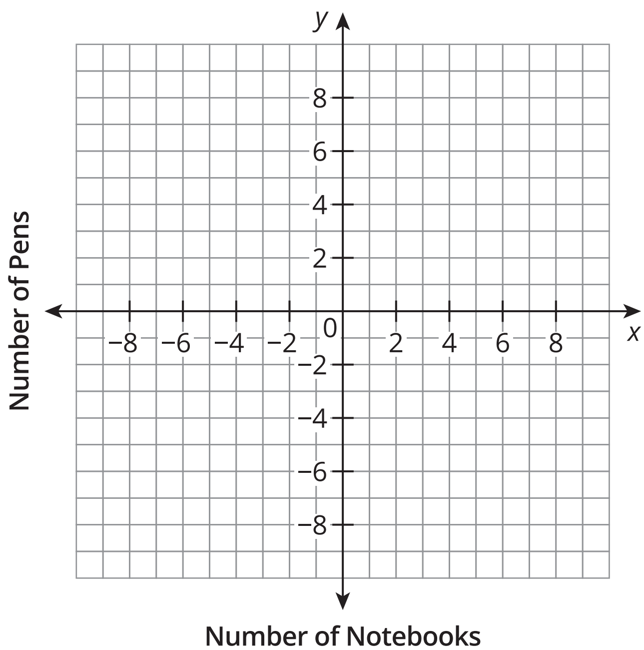
2. Chloe has $2 to buy oranges and apples. Oranges cost $0.45 each and apples cost $0.25 each. Write and graph an inequality that represents the possible ways Chloe could spend her $2. Is the ordered pair (2, 3) a solution for the problem situation?



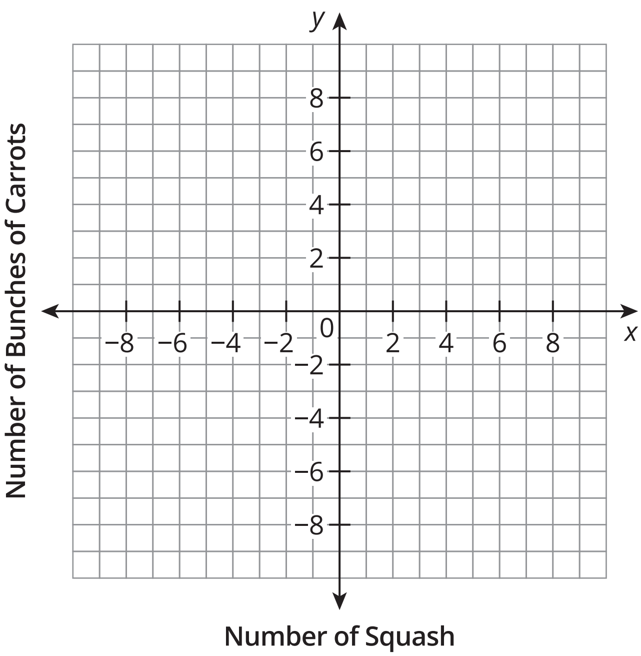
3. Juan plays football. His team's goal is to score at least 15 points per game. A touchdown is worth 6 points, and a field goal is worth 3 points. Juan's league does not allow teams to try for the extra point after a touchdown. Write and graph an inequality that represents the possible ways Juan's team could score points to reach their goal. Is the ordered pair (6, −1) a solution for the problem situation?



4. Mia has $5 to buy notebooks and pens. Notebooks cost $1.25 each, and pens cost $0.75 each. Write and graph an inequality to represent the possible ways Mia could spend her $5. Is the ordered pair (5, 2) a solution for the problem situation?

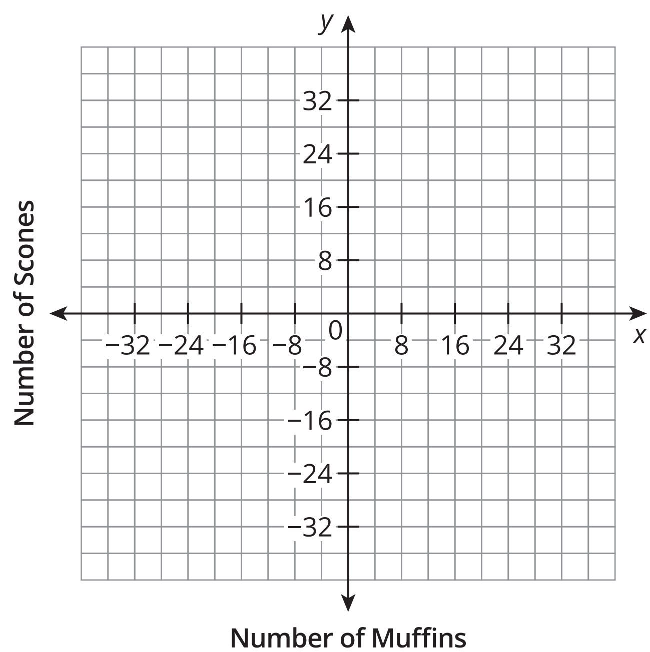


5. Jayden has \$10 to buy squash and carrots. Squash costs \$1.50 each, and carrots costs \$2.75 per bunch. Write and graph an inequality that represents the possible ways Jayden could spend less than \$10. Is the ordered pair (−2, 4) a solution for the problem situation?



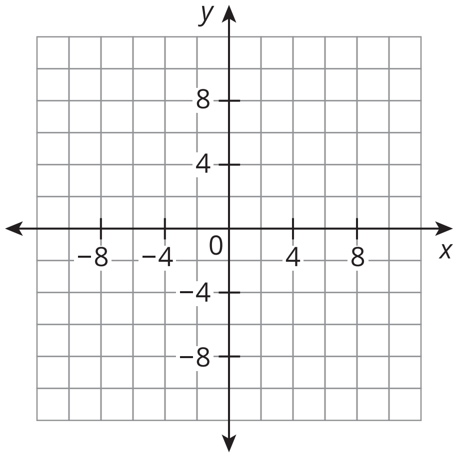
6. Mateo makes and sells muffins and scones at a school bake sale. He hopes to raise more than \$20. The table of values shows the combination of sales that will raise exactly \$20. Write and graph an inequality that represents the possible ways Mateo could reach his goal. Is the ordered pair (20, 32) a solution for the problem situation?

Muffins	Scones
0	25
8	20
16	15
24	10



Extension

Use what you know about absolute value functions to graph the inequality $y > 2|x - 3| - 5$.



Spaced Practice

1. Solve each system using the Linear Combinations Method.

   a. $\begin{cases} 8x - 6y = -20 \\ -16x + 7y = 30 \end{cases}$

   b. $\begin{cases} x + 3 = -7y + 3 \\ 2x - 8y = 22 \end{cases}$

2. Write equations in both slope-intercept form and standard form of the line that has the given slope and passes through the point given.

   a. $m = \frac{2}{3}$; $(2, -4)$

   b. $m = -4$; $(0.5, 7)$

V. Systems of Linear Inequalities

Topic Practice

A. Write a system of linear inequalities that represents each problem situation. Remember to define your variables.

1. Minh runs the bouncy house at a festival. The bouncy house can hold a maximum of 1200 pounds at one time. He estimates that adults weigh approximately 200 pounds and children under 16 weigh approximately 100 pounds. For 1 four-minute session of bounce time, Minh charges adults $3 each and children $2 each. Minh hopes to make at least $24 for each session.

2. Victoria works at a movie theater selling tickets. The theater has 300 seats and charges $7.50 for adults and $5.50 for children. The theater expects to make at least $2000 for each showing.

3. The maximum capacity for an average passenger elevator is 15 people and 3000 pounds. It is estimated that adults weigh approximately 200 pounds and children under 16 weigh approximately 100 pounds.

4. Liam's pickup truck can carry a maximum of 1000 pounds. He loads his truck with 20-pound bags of cement and 80-pound bags of cement. He hopes to load at least 10 bags of cement into his truck.

5. Elijah is drawing caricatures at a fair for 8 hours. He can complete a small drawing in 15 minutes and charges $10 for the drawing. He can complete a larger drawing in 45 minutes and charges $25 for the drawing. Elijah hopes to make at least $200 at the fair.

6. Elijah is making flower arrangements to sell in his shop. He can complete a small arrangement in 30 minutes that sells for $20. He can complete a larger arrangement in 1 hour that sells for $50. Elijah hopes to make at least $350 during her 8-hour workday.

B. Determine whether each given point is a solution to the system of linear inequalities.

1. $\begin{cases} 2x - y > 4 \\ -x + y \leq 7 \end{cases}$

   Point: $(-2, -10)$

2. $\begin{cases} x + 5y < -1 \\ 2y \geq -3x - 2 \end{cases}$

   Point: $(0, -1)$

3. $\begin{cases} 4x + y < 21 \\ \frac{1}{2}x \leq 36 - 5y \end{cases}$

   Point: $(3, 7)$

4. $\begin{cases} 5x + 3y > 6 \\ -2x + 2y < 20 \end{cases}$

   Point: $(-2, 6)$

5. $\begin{cases} 15x + 25y \geq 300 \\ 20x + 30y \leq 480 \end{cases}$

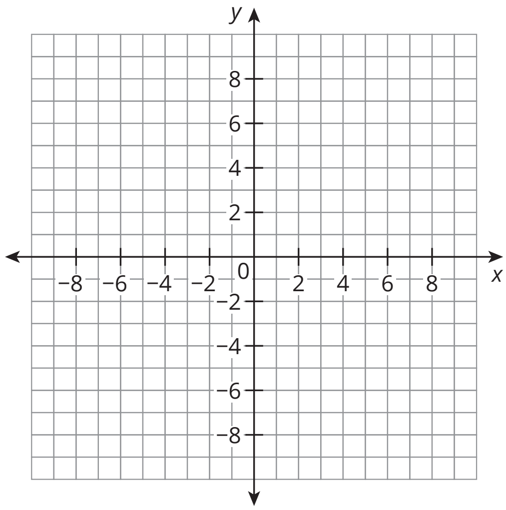
   Point: $(14, 8)$

6. $\begin{cases} -2.1x + 7y \geq -49.5 \\ -y \leq -6.3x + 78 \end{cases}$

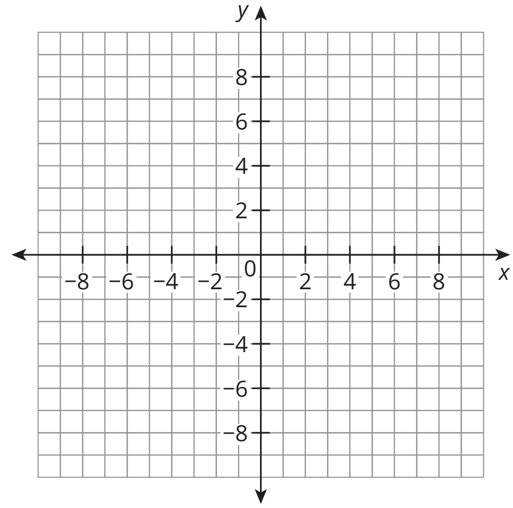
   Point: $(10, -8)$

C. Graph each system of linear inequalities and identify two solutions.

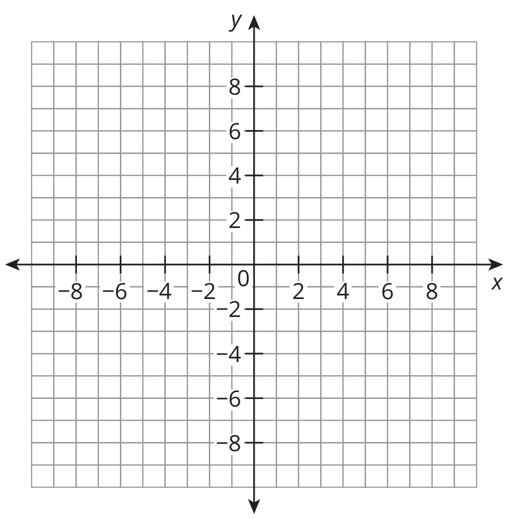
1. $\begin{cases} 3x - y > -5 \\ y + x > 3 \end{cases}$



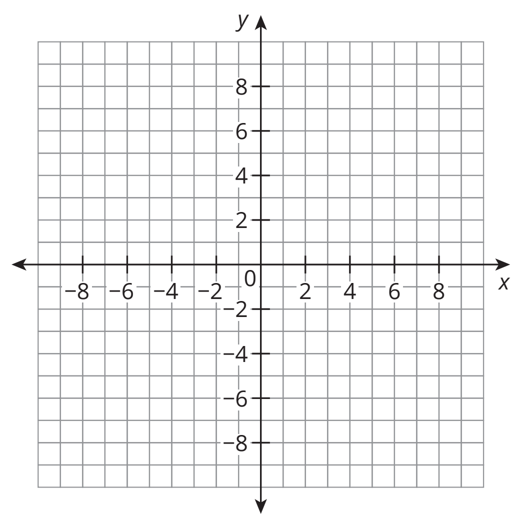
2. $\begin{cases} y > 2x + 3 \\ y < 2x - 5 \end{cases}$



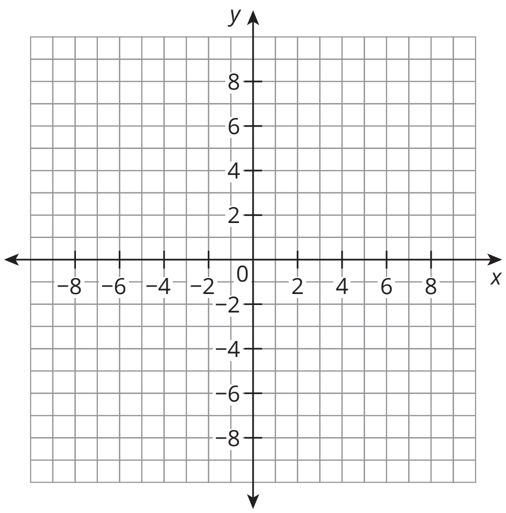
3. $\begin{cases} y \leq -\frac{2}{3}x + 3 \\ y \geq 3x - 4 \end{cases}$



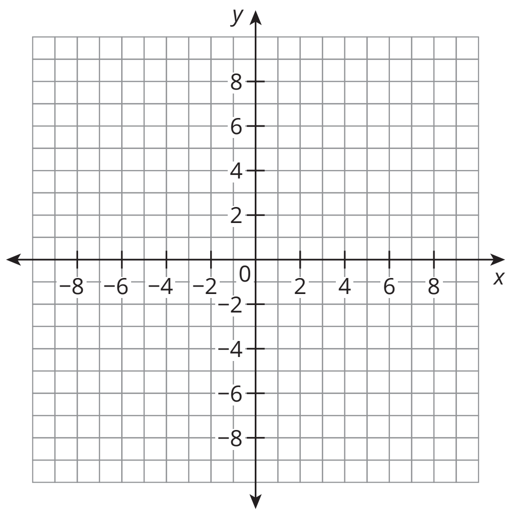
4. $\begin{cases} y - 2 < -\frac{1}{2}(x - 8) \\ y < 2x + 1 \end{cases}$



5. $\begin{cases} y \geq -\frac{1}{3}x + 4 \\ 2x - y \leq -5 \end{cases}$



6. $\begin{cases} y > -4x + 8 \\ 4x + y < -2 \end{cases}$

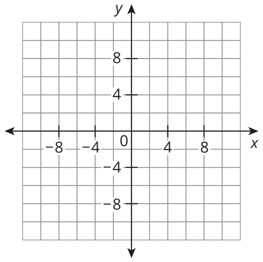


Extension

1. Is it possible to create a system of inequalities that has no solutions? If so, create one and explain how the graph would show no solutions. If not, explain why.

2. Is it possible to create a system of two inequalities that has only one solution? If so, create one. If not, explain why.

3. Is it possible to create a system of three inequalities that has only one solution? If so, sketch a graph to show the solution. If not, explain why.



Spaced Practice

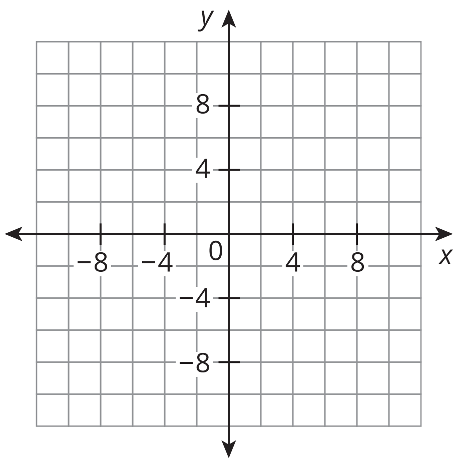
1. Determine whether each equation has one solution, no solution, or infinite solutions.

   a. $24x - 22 = -3(1 - 8x)$

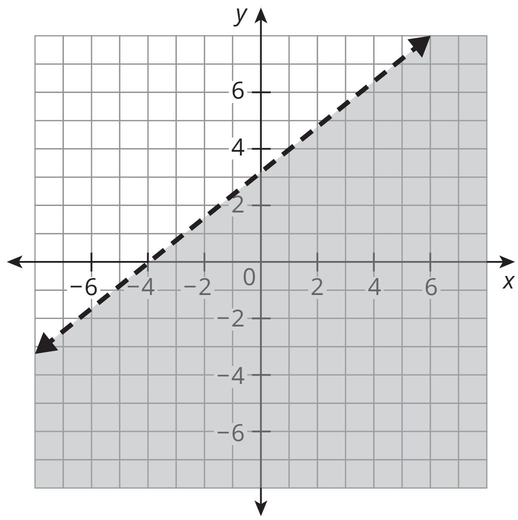
   b. $-3(4a + 3) + 2(12a + 2) = 43$

   c. $4(x + 1) = 6x + 4 - 2x$

2. Graph $3x + y \leq 7$ on a coordinate plane.



3. Write a linear inequality for the graph.



4. A new workout gym opens up down the street from your house. Below are their total membership numbers for the first months of business.

Month	January	February	March	April	May
Number of Members	120	190	290	370	450

a. Write the equation of the regression line for the data.

b. Use the equation to predict the gym's total membership at the end of the year.

VI. Solving Systems of Equations and Inequalities

Topic Practice

A. Write a system of equations or inequalities to represent each problem situation. Solve the system using your preferred method and answer any associated questions.

1. Kai received two different job offers to become a real estate sales agent. Dream Homes offered Kai a base salary of $20,000 per year plus a 2% commission on all real estate sold. Amazing Homes offered Kai a base salary of $25,000 per year plus a 1% commission on all real estate sold. Determine the amount of real estate sales, in dollars, for which both real estate companies will pay Kai the same amount. Explain which offer Kai should accept based on the amount of real estate sales he expects to have.

2. Ava is trying to choose between two rental car companies. Speedy Trip Rental Cars charges a base fee of $24 plus an additional fee of $0.05 per mile. Wheels Deals Rental Cars charges a base fee of $30 plus an additional fee of $0.03 per mile. Determine the amount of miles driven for which both rental car companies charge the same amount. Explain which company Ava should use based on the number of miles she expects to drive.

3. Emma has two job offers to be a door-to-door food processor salesperson. Pro Process Processors offers her a base salary of $15,000 per year plus an additional $25 for each processor she sells. Puree Processors offers her a base salary of $18,000 per year plus an additional $21 for each processor she sells. Determine the number of food processors Emma would have to sell for both companies to pay her the same amount. Explain which job offer Emma should accept based on the number of food processors she expects to sell.

4. Noah needs to rent a bulldozer. Smith's Equipment Rentals rents bulldozers for a delivery fee of $600 plus an additional $37.50 per day. Robinson's Equipment Rentals rents bulldozers for a delivery fee of $400 plus an additional $62.50 per day. Determine the number of rental days for which both rental companies charge the same amount. Explain which company Noah should choose based on the number of days he expects to rent a bulldozer.

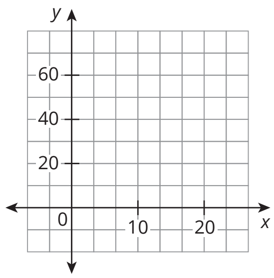
5. The school volleyball team is selling T-shirts and baseball hats as a fundraiser for their program. The T-shirts are selling for $15 each, and the baseball hats are selling for $12 each. If the school volleyball team sold a total of 84 items for a total of $1146, determine how many of each item they sold.

6. At a diner, the total bill for one table of diners included 4 veggie burger combos and 2 beef burger combos for $97.94. At another table, the total bill came to $78.95 for 2 veggie burger combos and 3 beef burger combos. Determine the cost of a veggie burger combo and the cost of a beef burger combo.

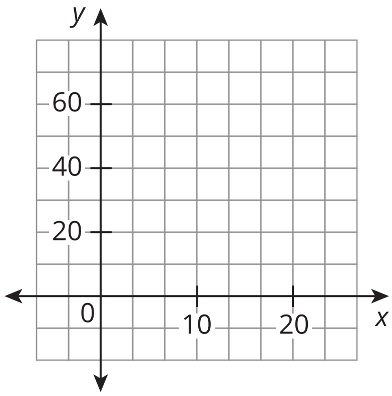
Extension

Isla sells baked goods from her home kitchen. She offers decorated cookies for $15 per dozen and cupcakes for $13 per dozen. It takes her an hour to decorate a dozen cookies but only 20 minutes to decorate a dozen cupcakes. She would like to make at least $300 per week and not put in more than 20 hours of work per week.

1. Create a system of linear inequalities that fits the situation and graph them.



2. Isla just discovered that she is running out of cake mix for the cupcakes and royal icing for the cookies. She can make a maximum of 40 dozen cupcakes and 12 dozen cookies. What are the new inequalities you need to add to your problem? Add them to your graph.



3. What is the maximum amount of baked goods that she could make? How much will she earn? How long will it take her?

4. What is the least amount of time she could work and still earn \$300? What baked goods would she make?

Spaced Practice

1. Consider the equation $6x - 2y = -12$.

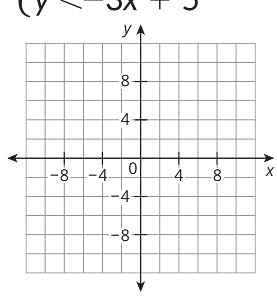
   a. What is the slope of the equation?

   b. What are the intercepts of the equation?

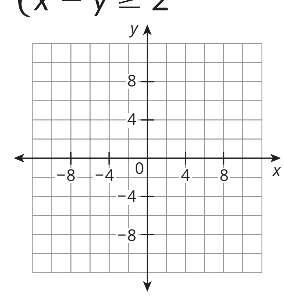
2. The equation to calculate the area of a trapezoid is $A = \frac{1}{2}(a+b)h$. Rewrite the equation to solve for a.

3. Graph each system of inequalities. Then, identify two points that are solutions of the system.

   a. $\begin{cases} y \geq 5x - 3 \\ y < -3x + 5 \end{cases}$



   b. $\begin{cases} y \geq x + 4 \\ x - y \geq 2 \end{cases}$



4. What is the equation for the line that has a slope of 0 and passes through the point (3, 7)?

5. What is the equation for the line that has a slope of $\frac{1}{5}$ and passes through the point $\left(-\frac{2}{3}, \frac{1}{2}\right)$? Write an equation in slope-intercept form and an equation in standard form.

MODULE
4

Investigating Growth and Decay

Skills Practice

Name ______________________________ Date ____________

I. Properties of Powers with Integer Exponents

Topic Practice

A. Write an equivalent expression in simplest form.

1. $(m^6)^4$

2. $(n^8)^{\frac{1}{2}}$

3. $(d^5)^5$

4. $(m^3)^6$

5. $(z^5)^2$

6. $(x^2)^9$

B. Write an equivalent expression in simplest form.

1. $2n^4m^3 \cdot 19n^2m^9$

2. $-\frac{17k^5x^3}{19k^9x}$

3. $3n^5z^9(-5n^3z^6)$

4. $\frac{15wa^5}{-6w^5a^8}$

5. $\frac{-5m^9d^4}{-11m^3d}$

6. $(-3b^{-5}k)(9b^5k^3)$

7. $-\frac{8c^8k^5}{2c^9k^2}$

8. $(-3k^5w^3)(-13k^7w^9)$

9. $\frac{9z^5a^6}{16z^6a^{-7}}$

10. $(-5n^7j^6)(16n^7j^6)$

11. $\frac{12n^5m^{-3}}{-12n^9m^5}$

12. $\frac{8w^9m^5}{20wm^9}$

13. $(-8w^5d^6)(11w^4d^3)$

14. $-\frac{3bx^{-8}}{12b^4x^{-6}}$

15. $3k^4m^6 \cdot 18k^9m^4$

16. $-y^2x^6 \cdot 8y^8x^6$

17. $\frac{-20c^7b^3}{-10c^2b^4}$

18. $(-7a^9z^8)(7a^4z^7)$

C. Write an equivalent expression in simplest form. Assume that variables are not zero.

1. $(3d^7x^3)^3$

2. $\left(\frac{x^3}{a^5}\right)^4$

3. $(-k^4w^9x)^2$

4. $-(-4c^6d^7a^4)^3$

5. $\left(\frac{x^3}{c^3}\right)^7$

6. $\left(\frac{c}{b}\right)^5$

7. $(-4a^5y^2k^5)^3$

8. $\left(\frac{n^9}{m^3}\right)^2$

9. $(6y^2z^9)^2$

10. $-(5ac^2)^2$

11. $\left(\frac{a}{d^2}\right)^9$

12. $-(-2w^2k^3x^4)^3$

13. $(a^8x^7b^3)^2$

14. $\left(\frac{b^2}{j^3}\right)^4$

15. $\left(\frac{k^5}{c}\right)^3$

16. $-(-cb^7)^3$

17. $\left(\frac{n^6}{b^7}\right)^2$

18. $(-6n^9yj)^2$

Extension

1. Exponents can be stacked as high as you like. Some mathematicians have used double arrows to represent repeated exponents. For example, $3 \uparrow\uparrow 3$ represents 3^{3^3} or 3^{27}.

   Write different numbers using double-arrow notation. How can you write 10 billion using this notation?

Spaced Practice

1. Write a direct variation equation for the table of values.

x	*y*
3	4
6	8
9	12
12	16
15	20

2. Use what you know about approximating square roots to answer each question.

   a. Explain how you know that the value of $\sqrt{12}$ is between 3 and 4.

   b. Shade between two values to show where $\sqrt{12}$ lies on the number line.

II. Analyzing Properties of Powers

Topic Practice

A. Write an equivalent expression in simplest form.

1. $-11n^{-4}$

2. $\frac{11}{17n^{-4}}$

3. $\frac{-10}{-2y^{-4}}$

4. $14c^{0}$

5. $17z^{-7}$

6. $\frac{7}{-18y^{-8}}$

7. $-\frac{16}{19t^{0}}$

8. $6n^{-9}$

9. $3x^{-4}$

10. $-\frac{5}{(a^{-9})}$

11. $-\frac{6}{5d^{-2}}$

12. $-\frac{3}{16s^{0}}$

13. $-20x^{0}$

14. $-5z^{-3}$

15. $\frac{15}{-10n^{0}}$

16. $-11w^{-6}$

17. $\frac{14}{-7b^{-1}}$

18. $5a^{0}$

B. Justify each step to rewrite the expression. Choose the properties from the box.

Product of powers rule	Power of a power rule	Negative exponent rule	Quotient of powers rule
Zero power rule	Simplify powers	Identity property of multiplication	Commutative property of multiplication

1. $4x^5 \cdot 6x^2y^6 \cdot xy$

$4x^5 \cdot 6x^2y^6 \cdot xy = (4 \cdot 6)(x^5x^2x)(y^6y)$ ____________________

$= (24)(x^{5+2+1})(y^{6+1})$ ____________________

$= 24x^8y^7$ ____________________

2. $3a^2b^3 \cdot 7ab^5 \cdot b^2$

$3a^2b^3 \cdot 7ab^5 \cdot b^2 = (3 \cdot 7)(a^2a)(b^3b^5b^2)$ ____________________

$= (21)(a^{2+1})(b^{3+5+2})$ ____________________

$= 21a^3b^{10}$ ____________________

3. $(4m^2n^5)^3$

$(4m^2n^5)^3 = 4^3(m^{2 \cdot 3})(n^{5 \cdot 3})$ ____________________

$= 64m^6n^{15}$ ____________________

4. $(-3x^7y^3)^5$

$(-3x^7y^3)^5 = (-3)^5(x^{7 \cdot 5})(y^{3 \cdot 5})$ ____________________

$= -243x^{35}y^{15}$ ____________________

5. $\dfrac{27y^8z^5}{-3y^4z^2}$

$\dfrac{27y^8z^5}{-3y^4z^2} = \left(\dfrac{27}{-3}\right)\left(\dfrac{y^8}{y^4}\right)\left(\dfrac{z^5}{z^2}\right)$ ____________________

$= -9(y^{8-4})(z^{5-2})$ ____________________

$= -9y^4z^3$ ____________________

6. $\frac{-96m^9n^2}{8m^2n^6}$

$$\frac{-96m^9n^2}{8m^2n^6} = \left(\frac{-96}{8}\right)\left(\frac{m^9}{m^2}\right)\left(\frac{n^2}{n^6}\right)$$
$$= -12(m^{9-2})(n^{2-6})$$
$$= -12m^7n^{-4}$$
$$= \frac{-12m^7}{n^4}$$

7. $-2x^5y^3 \cdot 8x^2y^{-5} \cdot x^{-9}y^2$

$$-2x^5y^3 \cdot 8x^2y^{-5} \cdot x^{-9}y^2 = (-2 \cdot 8)(x^5x^2x^{-9})(y^3y^{-5}y^2)$$
$$= (-16)(x^{5+2+(-9)})(y^{3+(-5)+2})$$
$$= -16x^{-2}y^0$$
$$= -16x^{-2}(1)$$
$$= \frac{-16}{x^2}$$

8. $\frac{42m^5n^3 \cdot m^4n^2}{6m^6n^5}$

$$\frac{42m^5n^3 \cdot m^4n^2}{6m^6n^5} = \left(\frac{42}{6}\right)\left(\frac{m^5m^4}{m^6}\right)\left(\frac{n^3n^2}{n^5}\right)$$
$$= (7)\left(\frac{m^{5+4}}{m^6}\right)\left(\frac{n^{3+2}}{n^5}\right)$$
$$= (7)(m^{5+4-6})(n^{3+2-5})$$
$$= 7m^3n^0$$
$$= 7m^3(1)$$
$$= 7m^3$$

C. Each expression has been rewritten incorrectly. Explain the mistake that occurred and then make the correction.

1. $\left(\frac{f^3}{n^4}\right)^4 = \frac{f^{12}}{n^4}$

2. $-15x^0 = -15(0) = 0$

3. $(3x^2y^4)^3 = 27x^5y^7$

4. $p^3 \cdot p^6 = p^{18}$

5. $\frac{m^8n^3}{m^6n^7} = m^2n^4$

6. $(-d^3h^2g^4)^7 = d^{21}h^{14}g^{28}$

Extension

1. Three positive integers have a sum of 10. How can you place the integers in the square brackets to form the greatest number possible?

   ([   ]) · ([   ])^[ ]

Spaced Practice

1. Use the properties of powers to rewrite the expressions as fractions and decimals without exponents.

   a. $10^4 \cdot 10^{-5}$

   b. $2^{-6} \div 2^{-2}$

2. Determine the slope of the line given the points on the line.

   a. A (0, 1), B (0, 7)

   b. E (1, 4), F (1, 0)

III. Geometric Sequences and Exponential Functions

Topic Practice

A. Write an explicit geometric formula for each sequence. Then, rewrite each formula as an exponential function of the form $f(x) = ab^x$. Identify the constant ratio and y-intercept of each exponential function.

1. 3, 9, 27, 81, …

   Explicit formula:

   Exponential function:

   Constant ratio:

   y-intercept:

2. 512, 256, 128, 64, …

   Explicit formula:

   Exponential function:

   Constant ratio:

   y-intercept:

3. 0.1, 0.4, 1.6, 6.4, ...

Explicit formula:

Exponential function:

Constant ratio:

y-intercept:

4. 3000, 300, 30, 3, ...

Explicit formula:

Exponential function:

Constant ratio:

y-intercept:

5. 45, 15, 5, $\frac{5}{3}$, ...

Explicit formula:

Exponential function:

Constant ratio:

y-intercept:

6. −4.8, −9.6, −19.2, −38.4, ...

Explicit formula:

Exponential function:

Constant ratio:

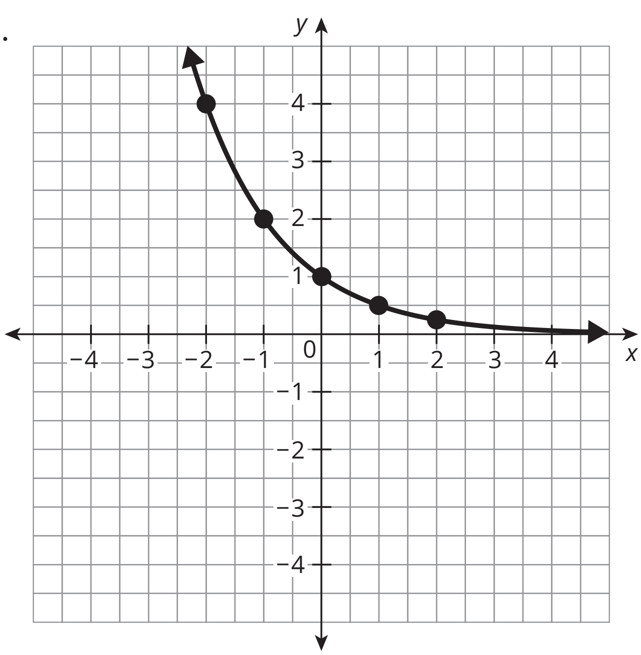
y-intercept:

B. Write an exponential function for each table or graph. Then, identify the constant ratio and *y*-intercept of each function.

1.

x	f(x)
−2	18
−1	6
0	2
1	$\frac{2}{3}$
2	$\frac{2}{9}$

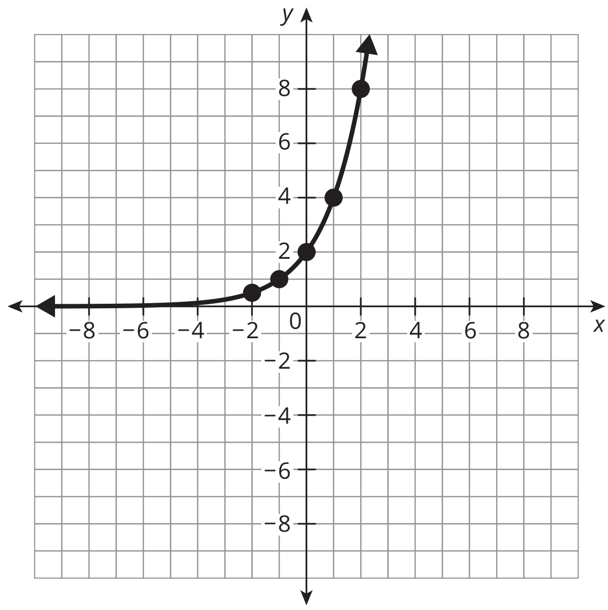
2.



3.

x	$f(x)$
-2	$-\frac{1}{3}$
-1	-1
0	-3
1	-9
2	-27

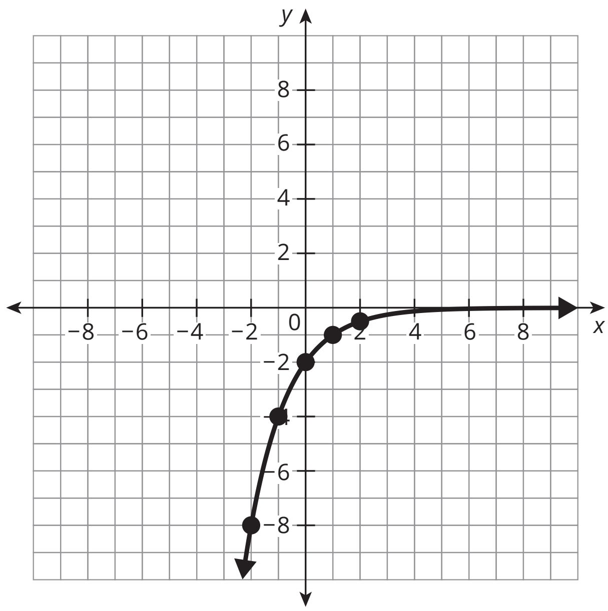
4.



5.

x	$f(x)$
-2	-48
-1	-12
0	-3
1	$-\frac{3}{4}$
2	$-\frac{3}{16}$

6.



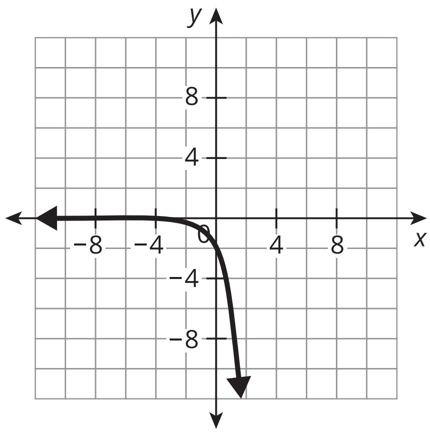
Extension

1. Which of the functions does not fit with the others? Explain your answer.

A. The exponential function that goes through (0, −3) and (5, −96)

B. $f(x) = -1 \cdot 6^x$

C.



D.

x	y
1	$\frac{2}{3}$
2	$\frac{2}{9}$
3	$\frac{2}{27}$

Spaced Practice

1. Solve each system of linear equations.

a. $\begin{cases} y = -5x - 21 \\ -2x + 5y = -24 \end{cases}$

b. $\begin{cases} 8x - 3y = 4 \\ 7x - 10y = -26 \end{cases}$

2. Jasmine needs to earn a score of at least 80 on her Algebra exam. The table shows the number of true or false and multiple-choice questions she needs to get correct to earn a score of 80. Write a linear inequality in two variables to represent this situation.

True or False	Multiple-Choice
0	16
5	14
20	8

IV. Rewriting Square Roots

A. Write each radical expression by extracting perfect squares.

1. $\sqrt{18}$

2. $\sqrt{40}$

3. $4\sqrt{200}$

4. $\sqrt{12} \cdot \sqrt{8}$

5. $6\sqrt{15} \cdot \sqrt{3}$

6. $3\sqrt{27} \cdot 5\sqrt{30}$

B. Use the quotient property of radicals to determine each quotient in simplest radical form.

1. $\frac{\sqrt{72}}{\sqrt{3}}$

2. $\frac{\sqrt{50}}{\sqrt{5}}$

3. $\frac{\sqrt{98}}{\sqrt{2}}$

4. $\frac{\sqrt{45}}{\sqrt{5}}$

5. $\frac{\sqrt{128}}{\sqrt{2}}$

6. $\frac{\sqrt{200}}{\sqrt{5}}$

7. $\frac{\sqrt{27}}{\sqrt{9}}$

8. $\frac{\sqrt{75}}{\sqrt{3}}$

Extension

Use mental math to evaluate each expression.

1. $\sqrt{5} + \sqrt{2} \cdot \sqrt{8} - \sqrt{5}$

2. $\sqrt{6} \cdot \sqrt{6} + 2\sqrt{3} \cdot 2\sqrt{3}$

Spaced Practice

Determine the first four terms of each sequence when n is a whole number greater than 1.

1. $f(n) = 5f(n - 1)$ and $f(1) = 3$

2. $h(n) = \frac{1}{4}f(n - 1)$ and $f(1) = 80$

V. Rational Exponents and Graphs of Exponential Functions

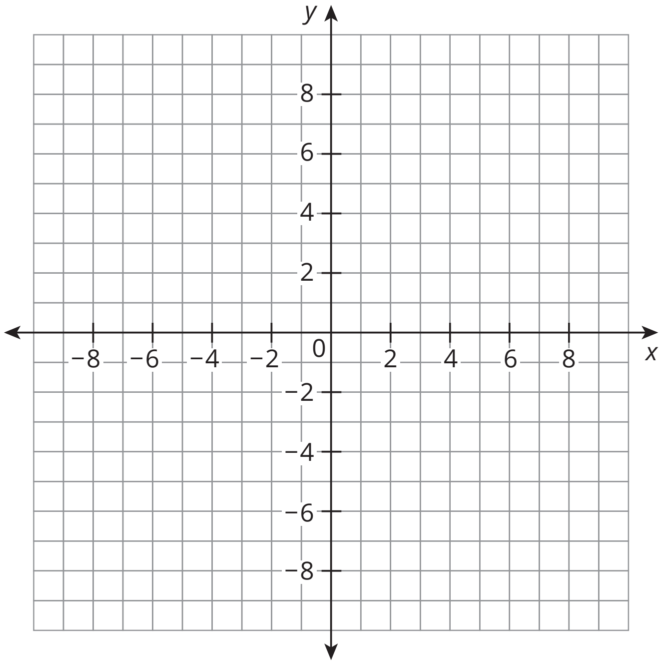
Topic Practice

A. Complete each table and graph the function. Identify the constant ratio and key characteristics.

1. $f(x) = 2^x$

x	$f(x)$
-2	
-1	
0	
1	
2	

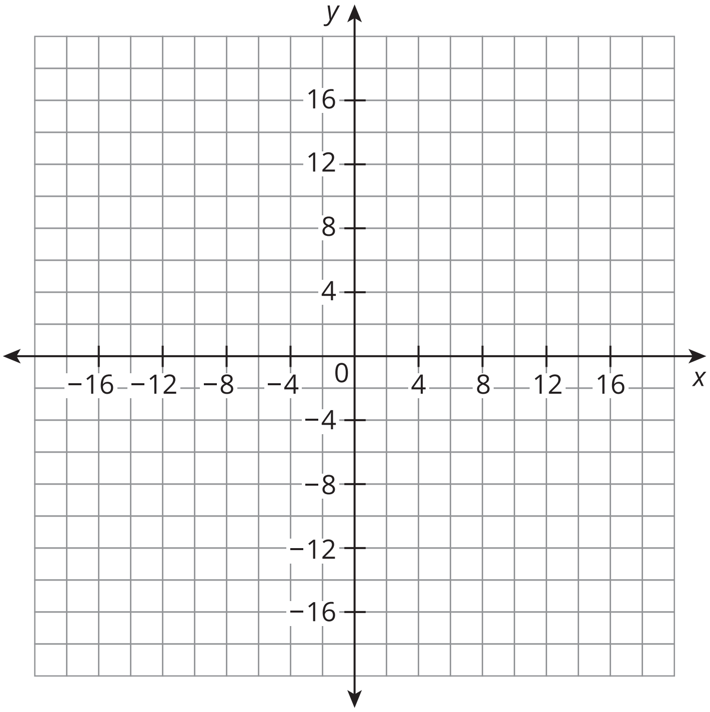
Constant ratio:
y-intercept:
x-intercept:
Equation of the asymptote:
Increasing/decreasing:
Domain:
Range:



2. $f(x) = 4^x$

x	$f(x)$
-2	
-1	
0	
1	
2	

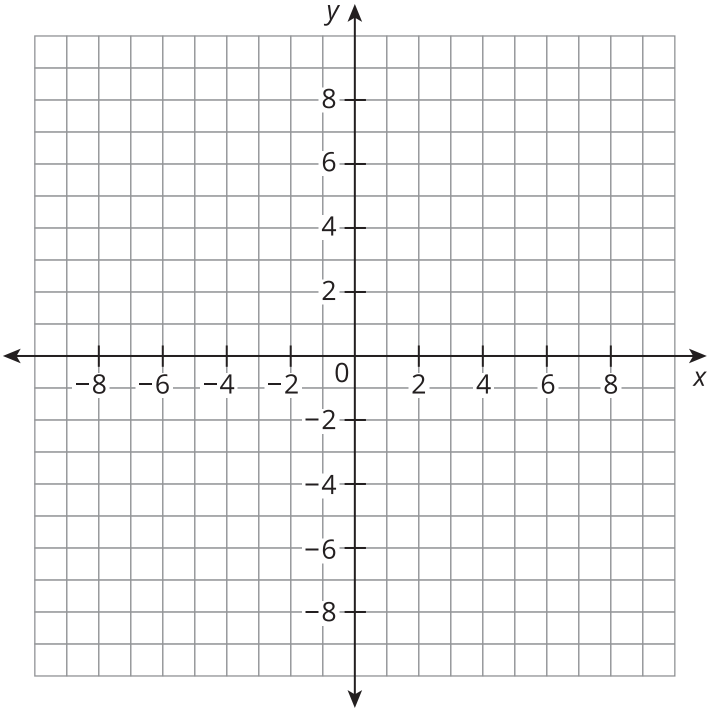
Constant ratio:
y-intercept:
x-intercept:
Equation of the asymptote:
Increasing/decreasing:
Domain:
Range:



3. $f(x) = \left(\frac{1}{3}\right)^x$

x	$f(x)$
-2	
-1	
0	
1	
2	

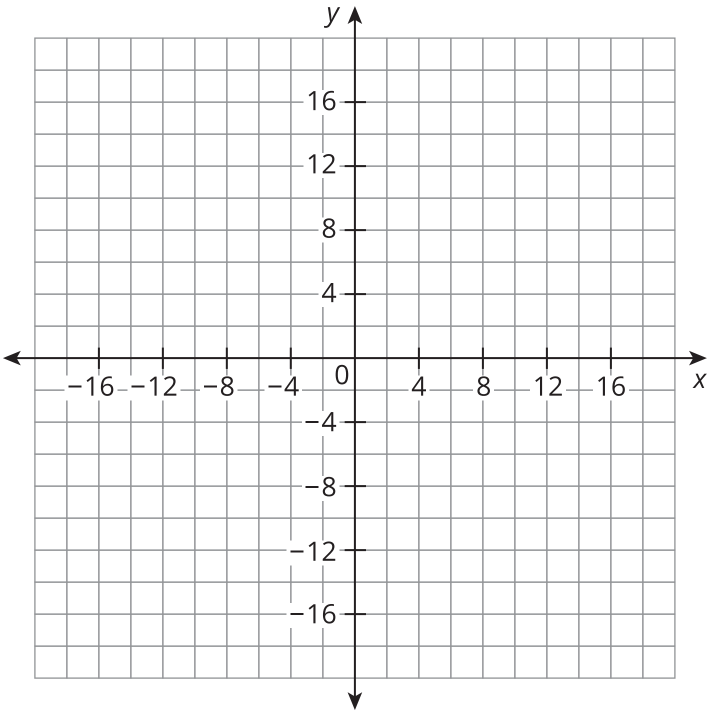
Constant ratio:
y-intercept:
x-intercept:
Equation of the asymptote:
Increasing/decreasing:
Domain:
Range:



4. $f(x) = \left(\frac{1}{4}\right)^x$

x	$f(x)$
−2	
−1	
0	
1	
2	

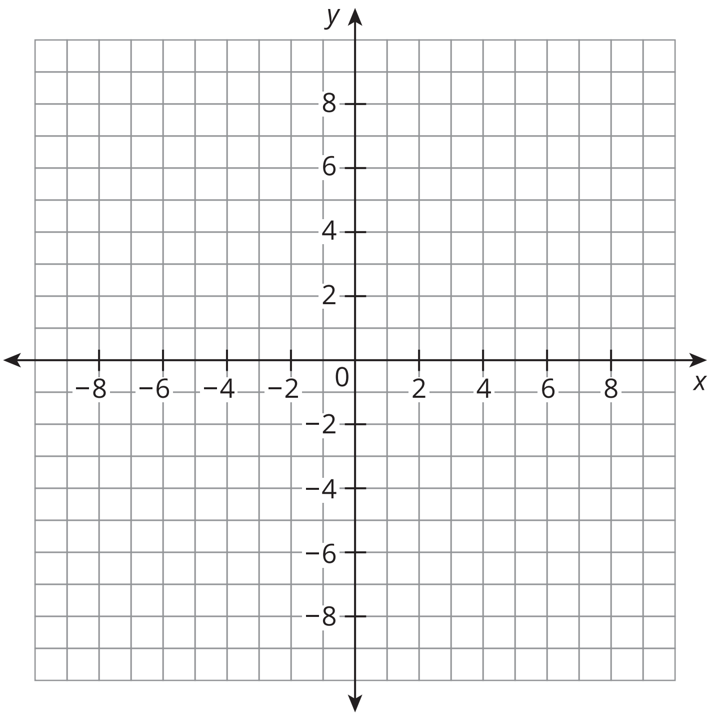
Constant ratio:
y-intercept:
x-intercept:
Equation of the asymptote:
Increasing/decreasing:
Domain:
Range:



5. $f(x) = -2 \cdot 2^x$

x	$f(x)$
−2	
−1	
0	
1	
2	

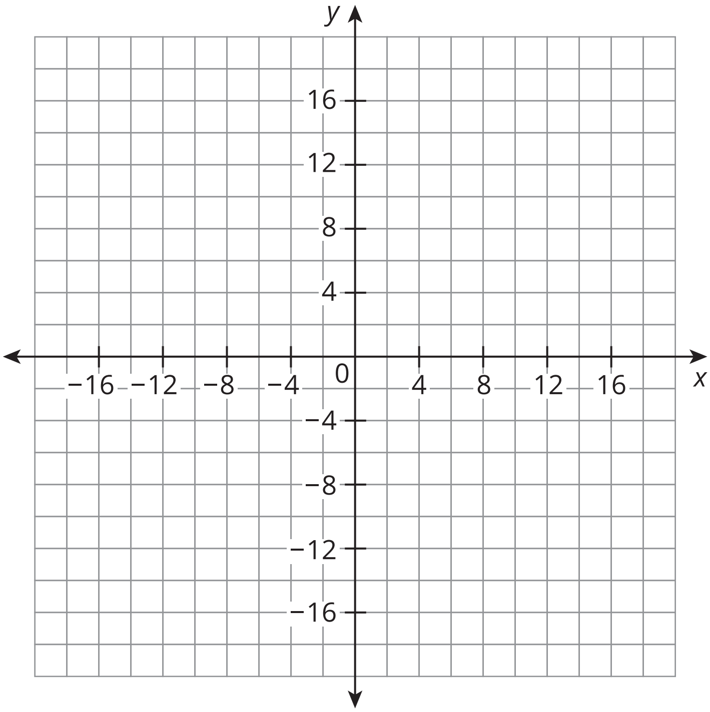
Constant ratio:
y-intercept:
x-intercept:
Equation of the asymptote:
Increasing/decreasing:
Domain:
Range:



6. $f(x) = -4^x$

x	$f(x)$
-2	
-1	
0	
1	
2	

Constant ratio:
y-intercept:
x-intercept:
Equation of the asymptote:
Increasing/decreasing:
Domain:
Range:



B. Write each power in radical form.

1. $12^{\frac{1}{3}}$

2. $7^{\frac{1}{5}}$

3. $18^{\frac{1}{4}}$

4. $a^{\frac{1}{2}}$

5. $d^{\frac{1}{5}}$

6. $c^{\frac{1}{6}}$

7. $5^{\frac{2}{3}}$

8. $8^{\frac{2}{5}}$

9. $18^{\frac{3}{4}}$

10. $x^{\frac{3}{5}}$

11. $y^{\frac{4}{3}}$

12. $m^{\frac{5}{2}}$

C. Write each expression using a rational exponent.

1. $\sqrt[4]{6^3}$

2. $\sqrt[5]{8^4}$

3. $\sqrt[3]{12^2}$

4. $\sqrt{n^5}$

5. $\sqrt[4]{p^7}$

6. $\sqrt[5]{m^3}$

D. Use the properties of powers to write an equivalent expression in simplest form.

1. $(y^{\frac{4}{5}})^3$

2. $(225s^8t^2)^{\frac{1}{2}}$

3. $\dfrac{\sqrt[3]{8x^6y^9}}{\sqrt{9x^{-4}y^8}}$

4. $\left(2a^{\frac{1}{2}}b^{\frac{3}{4}}\right)^2\left(16a^{-\frac{5}{6}}b^{\frac{2}{3}}\right)^0$

5. $\left(\dfrac{\sqrt[4]{16a^3b}}{\sqrt{36a^{-2}b^5}}\right)^0$

6. $(3z^4)^2(z^9)^{\frac{1}{3}}$

Extension

1. How do rational exponents help you multiply or divide two radicals with different indices ($\sqrt[m]{a} \cdot \sqrt[n]{a}$ or $\frac{\sqrt[m]{a}}{\sqrt[n]{a}}$, when $m \neq n$)? Include two examples to support your answer.

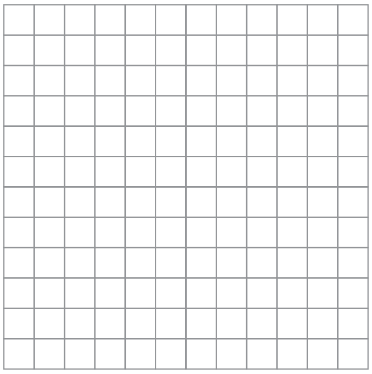
Spaced Practice

1. Complete the table.

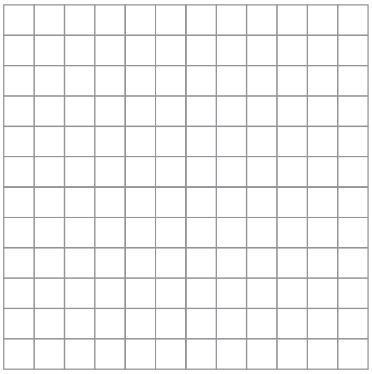
Explicit Formula	Exponential Function	Constant Ratio	y-Intercept
$840 \cdot 3^{x-1}$			
$-3\left(\frac{1}{5}\right)^{x-1}$			

2. Solve each system of linear inequalities.

a. $\begin{cases} y > -\frac{5}{4}x - 2 \\ x \geq -5 \end{cases}$



b. $\begin{cases} 2x - 3y > -3 \\ x + 3y > -6 \end{cases}$



3. High school students in a local district are required to volunteer for at least 50 hours by the time they graduate. Carlos volunteers as the coach of a soccer team for five-year-olds. The table shows the number of practices and games Carlos must lead to earn the exact number of hours required for graduation. Define variables for this context and write a linear inequality that describes the situation.

Practices	Games
30	35
40	30
80	10

Name ______________________________ Date ______________

I. Exponential Equations for Growth and Decay

Topic Practice

A. Determine whether each type of account describes simple interest or compound interest based on the scenario given. Explain your reasoning.

1. Isaiah deposits \$300 into an account that earns 2% interest each year. After the first year, Isaiah has \$306 in the account. After the second year, Isaiah has \$312 in the account. After the third year, Isaiah has \$318 in the account.

2. Jasmine deposits \$600 in an account that earns 1.5% interest each year. After the first year, Jasmine has \$609 in the account. After the second year, Jasmine has \$618.14 in the account. After the third year, Jasmine has \$627.41 in the account.

3. Luna deposits \$500 into an account that earns 2.5% interest each year. After the first year, Luna has \$512.50 in the account. After the second year, Luna has \$525.31 in the account. After the third year, Luna has \$538.44 in the account.

4. Lucas deposits \$4000 into an account that earns 4.25% interest each year. After the first year, Lucas has \$4170 in the account. After the second year, Lucas has \$4340 in the account. After the third year, Lucas has \$4510 in the account.

5. Nahimana deposits \$725 in an account that earns 3% interest each year. After the first year, Nahimana has \$746.75 in the account. After the second year, Nahimana has \$768.50 in the account. After the third year, Nahimana has \$790.25 in the account.

6. Mason deposits \$3500 in an account that earns 3.75% interest each year. After the first year, Mason has \$3631.25 in the account. After the second year, Mason has \$3767.42 in the account. After the third year, Mason has \$3908.70 in the account.

B. Write the exponential function represented by the table of values.

1.

x	$f(x)$
0	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{4}$

2.

x	$f(x)$
0	1
2	25
4	625
6	15,625

3.

x	$f(x)$
0	1
1	$\frac{3}{4}$
2	$\frac{9}{16}$
3	$\frac{27}{64}$

4.

x	$f(x)$
0	-1
2	-4
4	-16
6	-64

5.

x	$f(x)$
0	3
3	$\frac{1}{9}$
6	$\frac{1}{243}$
9	$\frac{1}{6561}$

6.

x	$f(x)$
0	-2
1	$-\frac{1}{2}$
2	$-\frac{1}{8}$
3	$-\frac{1}{32}$

C. Write a function to represent each problem situation.

1. Sebastian deposits \$500 into a compound interest account for a period of t years. The interest rate for the account is 4%.

2. Kayla deposits \$250 into a compound interest account for a period of t years. The interest rate for the account is 6%.

3. Adriana deposits \$1200 into a compound interest account for a period of t years. The interest rate for the account is 3.5%.

4. Angelina deposits \$2700 into a compound interest account for a period of t years. The interest rate for the account is 4.25%.

5. Mei deposits \$300 into a compound interest account for a period of t years. After one year, Mei's balance is \$305.25.

6. Gabriel deposits \$450 into a compound interest account for a period of t years. After one year, Gabriel's balance is \$474.75.

Extension

1. Consider a piece of paper that is 0.1 mm thick. How many times must it be folded so that it reaches the top of the Eiffel Tower? Assume the paper is as large as needed and it is possible to fold it as many times as required.

Spaced Practice

1. Diego and Valentina open a pet store and start with 5 hamsters for sale. Hamster populations usually triple every cycle. One cycle is equal to 4 months. Write an equation in function notation to represent the change in the number of hamsters as a function of the cycle number, *c*. Explain how you determined your equation.

2. Write an exponential function to model this table of values.

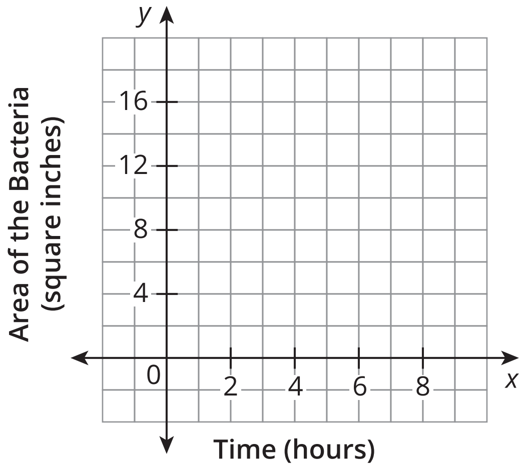
x	$g(x)$
1	0.6
2	0.06
3	0.006
4	0.0006

II. Interpreting Parameters in Context

Topic Practice

A. A scientist is studying the growth rate of a certain bacteria when starved of nutrients. She fills a petri dish with bacteria and measures the area covered by the bacteria every hour. The equation $f(x) = 20 \cdot 0.5^x$ represents the area, in square inches, of the bacteria after x hours.

1. Graph $f(x)$ on the coordinate plane.



2. Identify the y-intercept and interpret its meaning in terms of the problem situation.

3. Is $f(x)$ increasing or decreasing? What does this mean in terms of the problem situation?

4. Write an equation for the asymptote. Does the asymptote make sense in this problem situation? Explain your reasoning.

5. What is the area of the bacteria in the petri dish after three hours. Explain how you determined your answer.

6. When will the area of the bacteria be 14 square inches? Explain how you determined your answer.

B. Write an exponential function that represents each population as a function of time.

1. A city has a population of 15,000 people. Its population is decreasing at a rate of 1.5% each year.

2. A village has a population of 20,750 people. Its population is increasing at a rate of 2.7% each year.

3. A village has a population of 6075 people. Its population is decreasing at a rate of 0.5% each year.

4. A city has a population of 95,000 people. Its population is decreasing at a rate of 1.375% each year.

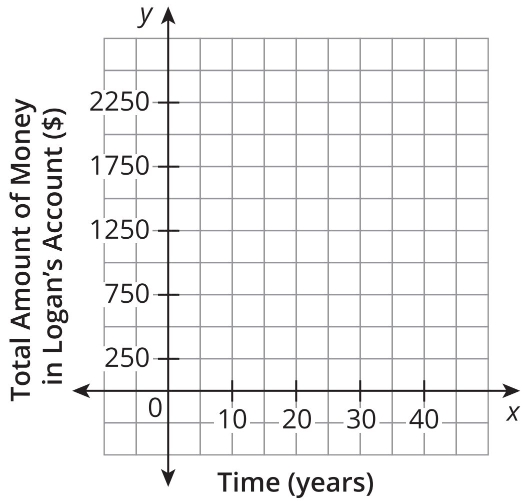
5. A village's population increases from 985 to 991 from one year to the next.

6. A city's population increased from 40,500 in 2020 to 42,039 in 2021.

C. Logan deposits $500 into a compound interest account. The interest rate for the account is 3.25%.

1. Write an exponential function, $P(t)$, to represent the amount of money Logan has in his account as a function of time, t.

2. Graph $P(t)$ on the coordinate plane.



3. Evaluate $P(0)$. Explain what the value means in terms of the scenario.

4. Evaluate $P(15)$. Explain what the value means in terms of the scenario.

5. Evaluate $P(20)$. Explain what the value means in terms of the scenario.

6. Evaluate $P(50)$. Explain what the value means in terms of the scenario.

D. For each function, interpret the *a*- and *b*-values of the given function in terms of the problem situation. Then, determine if the function is increasing or decreasing.

1. The function $P(t) = 4000 \cdot 1.03^t$ represents the population of a village as a function of time.

2. The function $B(t) = 8000 \cdot 0.98^t$ represents the number of bacteria in a petri dish as a function of time.

3. The function $M(t) = 975 \cdot 0.92^t$ represents the number of milligrams of a medication in a patient's body as a function of time.

4. The function $M(t) = 7210 \cdot 1.015^t$ represents the amount of money in a savings account as a function of time.

5. The function $B(t) = 500 \cdot 1.004^t$ represents the number of bacteria in a petri dish as a function of time.

6. The function $V(t) = 32{,}000 \cdot 0.96^t$ represents the value of a new car as a function of time.

Extension

Emily and Sarah developed a new art app for smart phones. The table shows the number of customers who downloaded the app by month.

1. Emily thinks that the equation that represents the data in the table is $y = 4(2)^x$. Determine if Emily is correct. Explain your reasoning.

Month	Number of Downloads
0	4
1	8
2	16
3	32
4	64
5	128

2. Determine a different exponential equation that represents the data in the table. Use the equation $y = a \cdot b^{f(x)}$, where $f(x)$ is a function of x and $a = 2$.

Spaced Practice

1. Rewrite each expression in rational exponent form in simplest terms.

   a. $\sqrt[3]{6^4}$

   b. $(\sqrt[8]{8})^{12}$

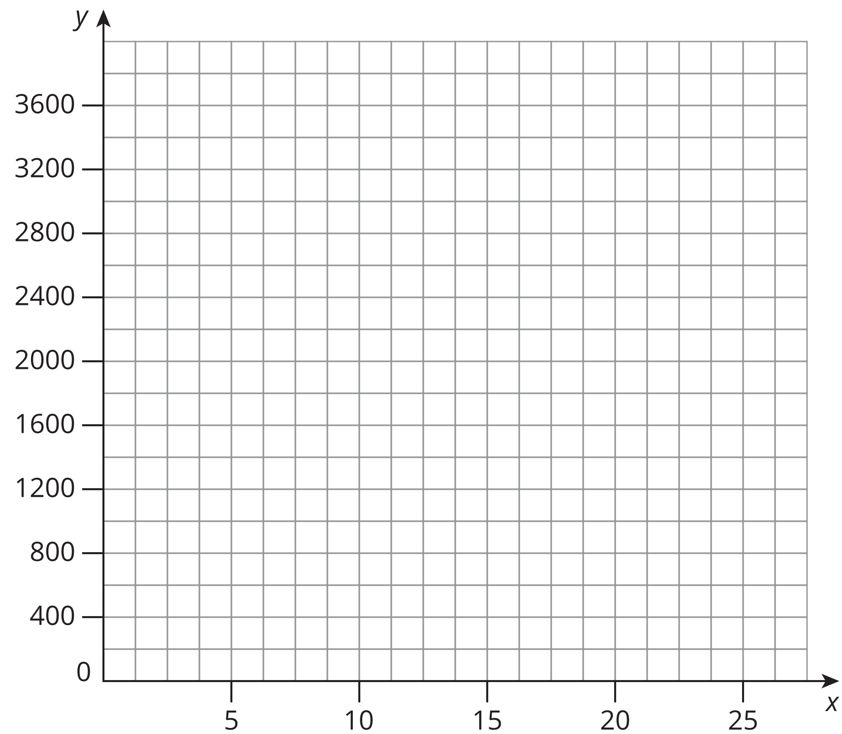
   c. $(\sqrt[7]{x})^3$

   d. $\sqrt[10]{y^5}$

2. Camilla receives \$1500 for her birthday. She is going to spend \$500 and wants to put the rest into an account that will earn interest. She is considering two different accounts. Account A earns 6.5% annual simple interest. Account B earns 4.5% annual compound interest.

   a. Write a function for each account that can be used to determine the balance in the account based on the year, t.

b. Graph the functions for Accounts A and B using technology. Then, graph the functions. Be sure to label your graph.



c. If Camilla plans on leaving the money in the account for 12 years, which account should she use to deposit her money? Explain your reasoning.

d. If Camilla plans on leaving the money in the account for 25 years, which account should she use to deposit her money? Explain your reasoning.

III. Modeling Using Exponential Functions

Topic Practice

A. For each given data set, use graphing technology to determine the exponential regression function and the value of the correlation coefficient, *r*. Round all values to the hundredths place.

1.

x	*f*(*x*)
10	5
20	6
30	8
40	15
50	32
60	70
70	150

2.

x	*f*(*x*)
0	6000
1	2100
2	750
3	275
4	95
5	40
6	15
7	6
8	4

3.

x	5	10	15	20	25	30	35	40
f(*x*)	12	10	25	21	45	35	80	120

4.

x	100	200	300	400	500	600	700
f(*x*)	25.4	10.5	4.5	2.1	0.8	0.3	0.4

5.

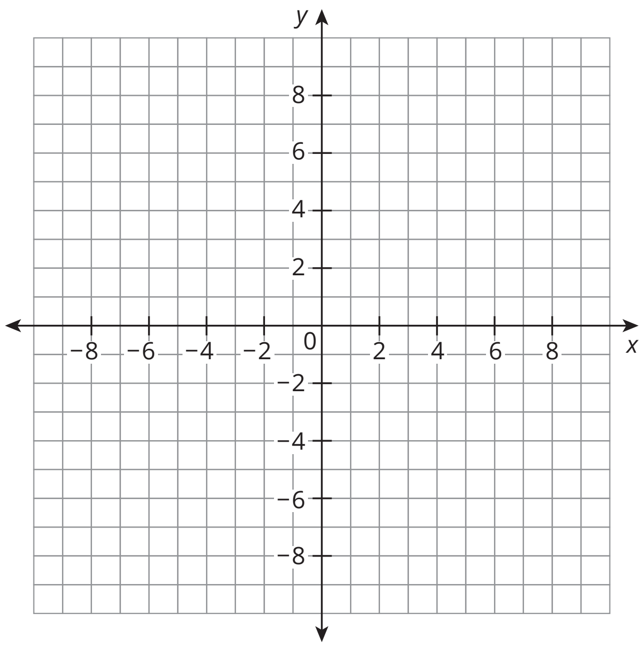
x	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	1200	585	272	126	42	40	14	12

6.

x	0	100	200	300	400	500	600
$f(x)$	10	50	110	160	220	290	350

B. Consider the graph of each exponential function or partial exponential function. Write the domain and the range of each in words and using inequalities.

1.



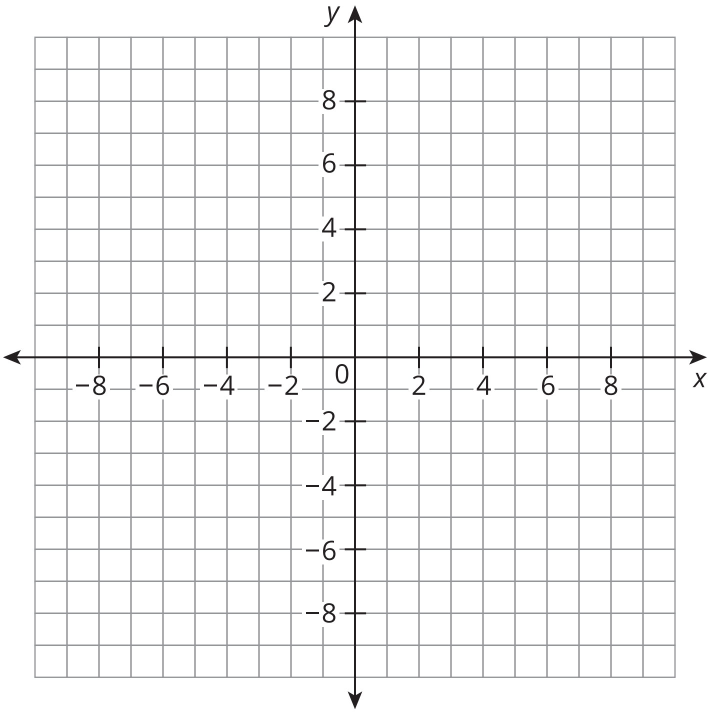
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

2.



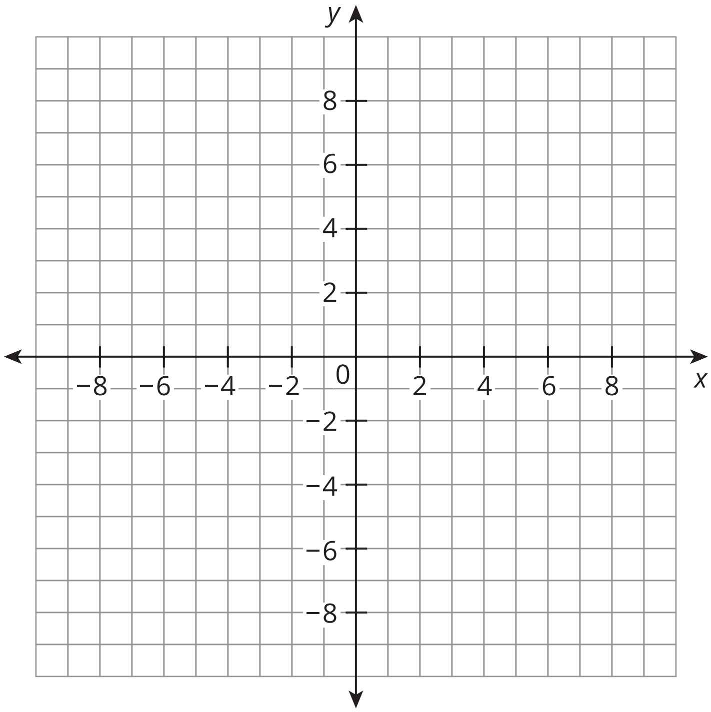
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

3.



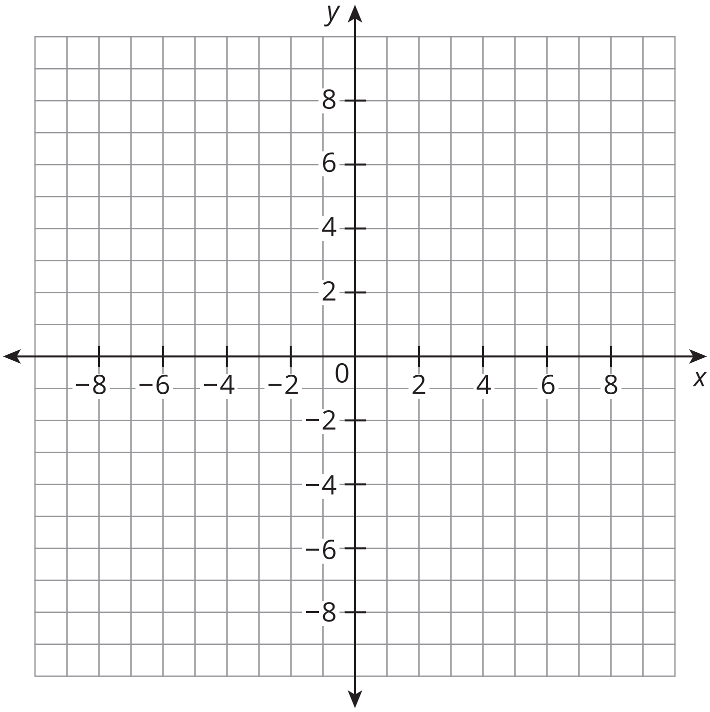
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

4.



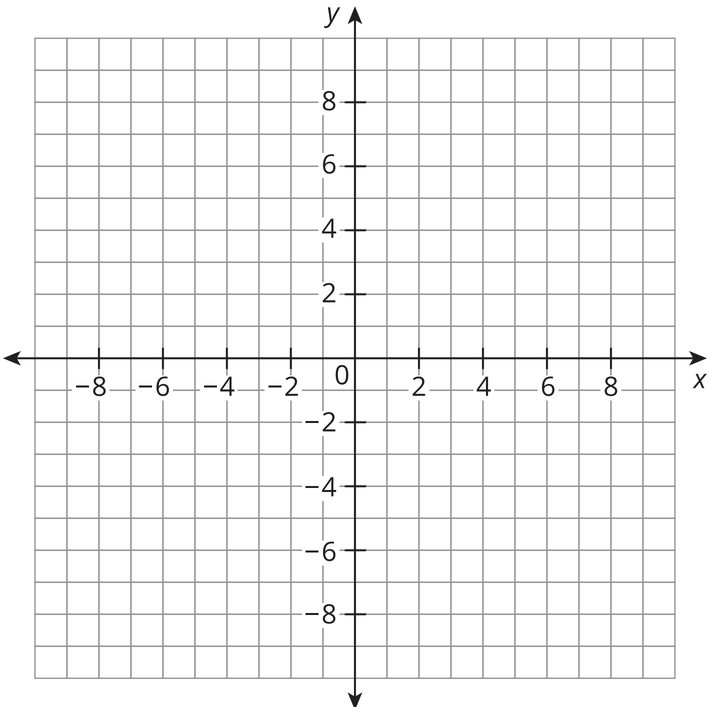
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

5.



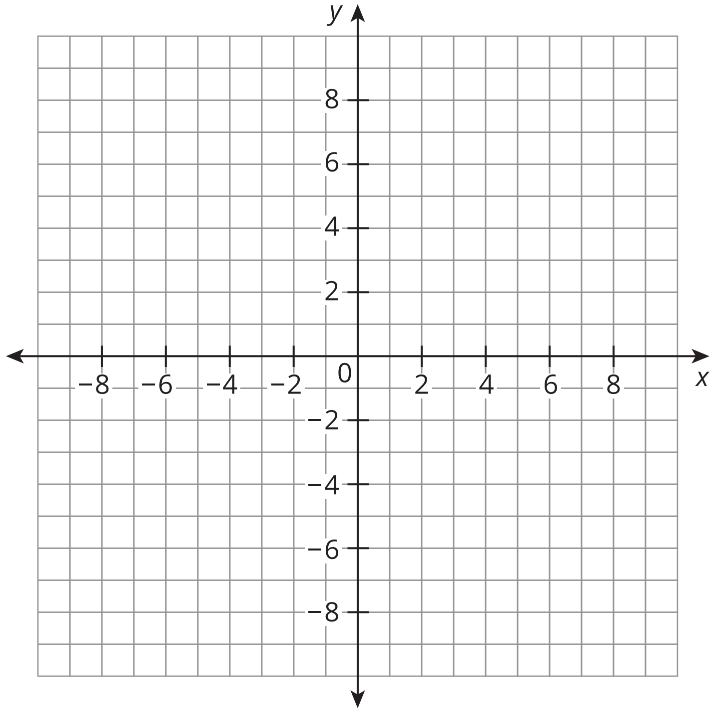
Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

6.



Domain in words:

Domain using inequalities:

Range in words:

Range using inequalities:

C. Determine the exponential regression function that models each situation. Use the function to make the associated prediction. Round all values to the hundredths place.

1. Avery deposited $500 into a savings account in 1975. The table shows the value of Avery's savings account from 1975 to 2015. Predict the account's value in 2025.

Time Since 1975 (years)	0	5	10	15	20	25	30	35	40
Account Value (dollars)	500	650	900	1150	1600	2100	2750	3850	4800

2. Parker deposited $1000 into a savings account in 1980. The table shows the value of Parker's savings account from 1980 to 2010. Predict when the account's value will be $5000.

Time Since 1980 (years)	0	5	10	15	20	25	30
Account Value (dollars)	1000	1200	1480	1800	2200	2720	3250

3. A marine biologist monitors the population of sunfish in a small lake. He records 800 sunfish in his first year, 600 sunfish in his fourth year, 450 sunfish in his sixth year, and 350 sunfish in his tenth year. Predict the population of sunfish in the lake in his sixteenth year.

4. A marine biologist monitors the population of catfish in a small lake. He records 50 catfish in his first year, 170 catfish in his fourth year, 380 catfish in his sixth year, and 1900 catfish in his tenth year. Predict when the population of catfish in the lake will be 6000.

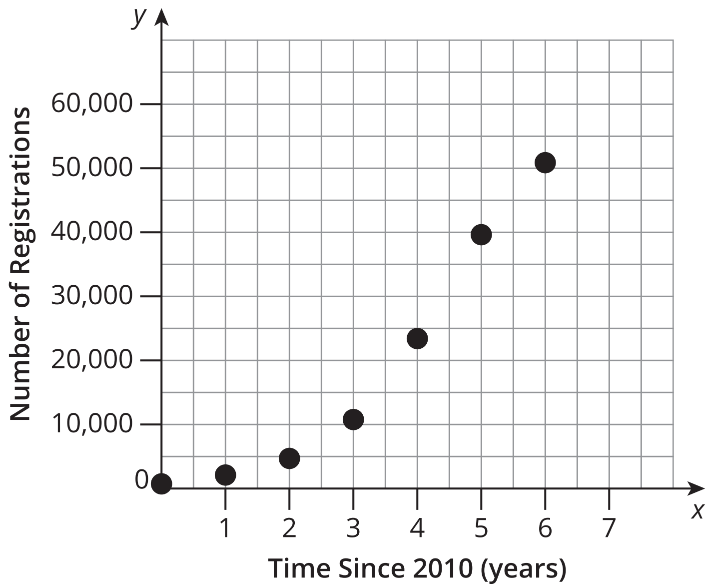
5. Every hour, a scientist records the number of cells in a colony of bacteria growing in her lab. The sample begins with 15 cells. Predict the number of cells in the colony after 7 hours.

Hour	Number of Cells
0	15
1	40
2	110
3	300
4	850

6. Every hour, a scientist records the number of cells in a colony of bacteria growing in her lab. The sample begins with 50 cells. Predict how long it will take the sample to grow to 2000 cells.

Hour	Number of Cells
0	50
1	90
2	160
3	290
4	530

D. The scatterplot shows the registrations of plug-in electric vehicles in Norway over a period of time. The exponential regression function that best fits the data is, $p(x) = 1018.99(2.05)^x$, where $p(x)$ represents the number of registrations and x represents the number of years since 2010. The function is graphed on the grid. Analyze this information to answer each question.



1. Identify and interpret the *a*- and *b*-values of the function in terms of the problem situation.

2. Discuss the domain and range of the function as they relate to the problem situation.

3. Discuss the intervals of increase and decrease as they relate to the problem situation.

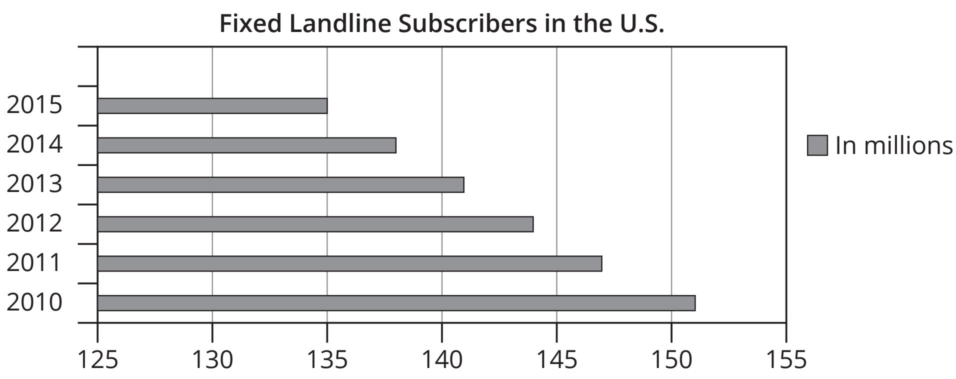
4. Discuss the *x*- and *y*-intercepts of the function as they relate to the problem situation.

5. Predict the number of registrations of plug-in vehicles in Norway in 2017.

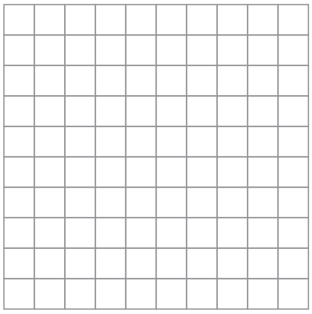
6. Why might the number of registrations of electric plug-in vehicles be increasing exponentially?

Extension

1. The number of fixed landline phone subscribers in the U.S. has been declining. The bar graph shows the decrease in the number of subscribers from 2010 to 2015.



a. To estimate the number of subscribers per year, create a scatterplot of the ordered pairs, with *x* representing the number of years since 2010 and *y* representing the number of subscribers in millions.



b. Determine both an exponential and a linear regression function to model the situation.

c. Which model would you use from part (b)? Explain your reasoning.

Spaced Review

1. An experiment begins with 400 bacteria. The bacteria population doubles each day. Write an equation in function notation to represent the number of bacteria as a function of the day number, *x*. Explain how you determined the equation.

MODULE
5

Maximizing and Minimizing

TOPIC 1: Introduction to Quadratic Functions

TOPIC 2: Polynomial Operations

TOPIC 3: Solving Quadratic Equations

Name ______________________________ Date ______________

I. Exploring Quadratic Functions

Topic Practice

A. Write a quadratic function in standard form that represents each area as a function of the width. Remember to define your variables.

1. A builder is designing a rectangular parking lot. She has 300 feet of fencing to enclose the parking lot around three sides.

2. Olivia is enclosing a new rectangular flower garden with a rabbit garden fence. She has 40 meters of fencing.

3. Michael is building a rectangular sandbox for the community park. The materials available limit the perimeter of the sandbox to at most 100 feet.

4. Daniela is designing a rectangular quilt. She has 16 feet of piping to finish the quilt around three sides.

5. Gabriela is making a rectangular vegetable garden alongside her home. She has 24 meters of fencing to enclose the garden around the three open sides.

6. Javier is building a rectangular ice rink for the community park. The materials available limit the perimeter of the ice rink to at most 250 meters.

B. Use technology to determine the absolute maximum or absolute minimum of each function. Describe what the *x*- and *y*-coordinates of this point represent in terms of the problem situation.

1. A builder is designing a rectangular parking lot. He has 400 meters of fencing to enclose the parking lot around three sides. Let x = the width of the parking lot. Let A = the area of the parking lot. The function $A(x) = -2x^2 + 400x$ represents the area of the parking lot as a function of the width.

2. Hannah is enclosing a portion of her yard to make a pen for her ferrets. She has 20 feet of fencing. Let x = the width of the pen. Let A = the area of the pen. The function $A(x) = -x^2 + 10x$ represents the area of the pen as a function of the width.

3. A baseball is thrown upward from a height of 5 feet with an initial velocity of 42 feet per second. Let t = the time in seconds after the baseball is thrown. Let h = the height of the baseball. The quadratic function $h(t) = -16t^2 + 42t + 5$ represents the height of the baseball as a function of time.

4. Fernando is standing on top of a playground set at a park. He throws a water balloon upward from a height of 12 feet with an initial velocity of 25 feet per second.
Let t = the time in seconds after the balloon is thrown. Let h = the height of the balloon. The quadratic function $h(t) = -16t^2 + 25t + 12$ represents the height of the balloon as a function of time.

5. Alexander is building a rectangular roller-skating rink at the community park. The materials available limit the perimeter of the skating rink to at most 180 meters. Let x = the width of the skating rink. Let A = the area of the skating rink. The function $A(x) = -x^2 + 90x$ represents the area of the skating rink as a function of the width.

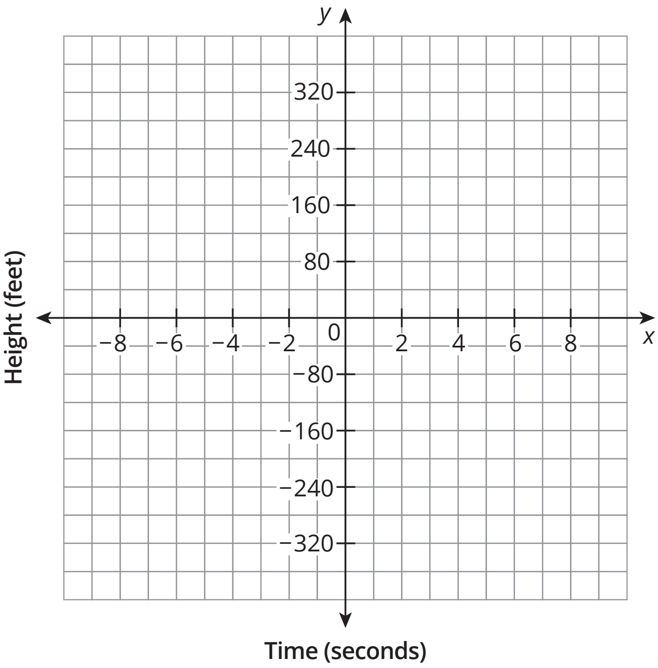
6. A football is thrown upward from a height of 6 feet with an initial velocity of 65 feet per second. Let t = the time in seconds after the football is thrown. Let h = the height of the football. The quadratic function $h(t) = -16t^2 + 65t + 6$ represents the height of the football as a function of time.

7. A bungee jumper is jumping from a height of 150 meters. Let t = the time since the start of the jump. Let h = the height of the jumper from the ground. The quadratic function $h(t) = 2t^2 - 30t + 150$ represents the height of the jumper as a function of time.

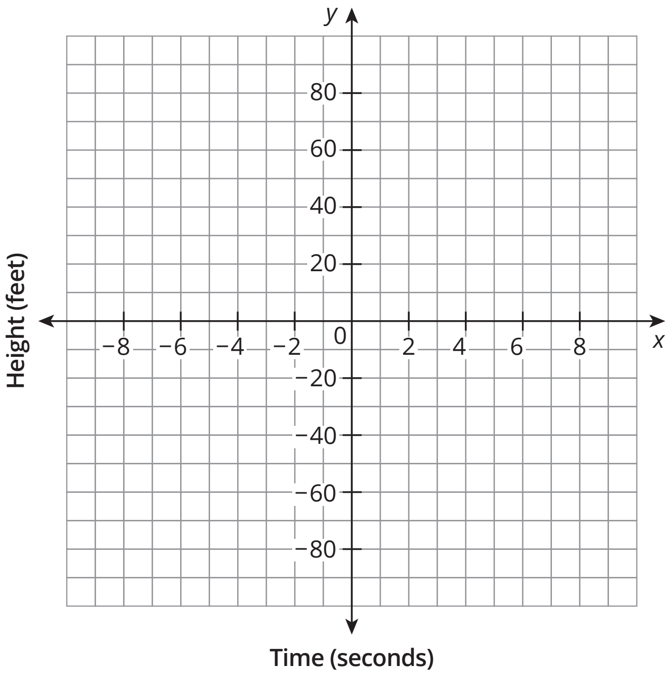
8. A skateboard company has to decide how many skateboards to produce each day. The company knows that the costs to produce the skateboards go down the more they make. However, the overall cost to the company increases if they make too many skateboards due to the cost of storing overstock. Let s = the number of skateboards produced each day. Let c = the total cost to produce s skateboards. The quadratic function $c(s) = 0.04s^2 - 16s + 15{,}000$ represents the total cost as a function of the number of skateboards produced.

C. Graph the function that represents each problem situation. Identify the absolute maximum and zeros. Then, identify the domain and range of both the function *g*(*t*) and the problem situation. Round your answers to the nearest hundredth, if necessary.

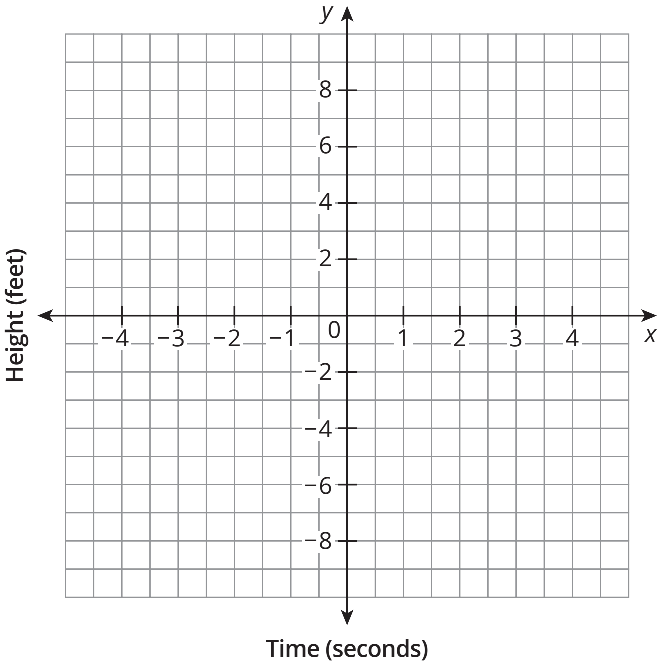
1. A model rocket is launched from the ground with an initial velocity of 120 feet per second. The function $g(t) = -16t^2 + 120t$ represents the height of the rocket, $g(t)$, t seconds after it was launched.



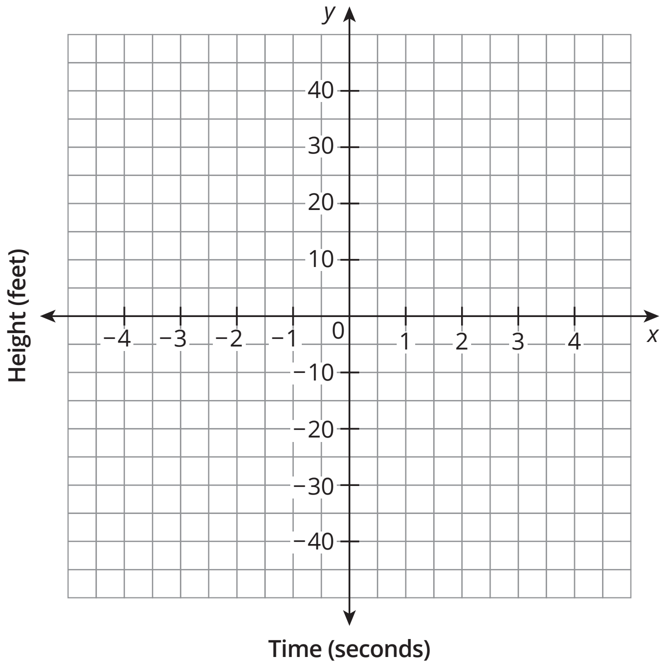
2. A model rocket is launched from the ground with an initial velocity of 60 feet per second. The function $g(t) = -16t^2 + 60t$ represents the height of the rocket, $g(t)$, t seconds after it was launched.



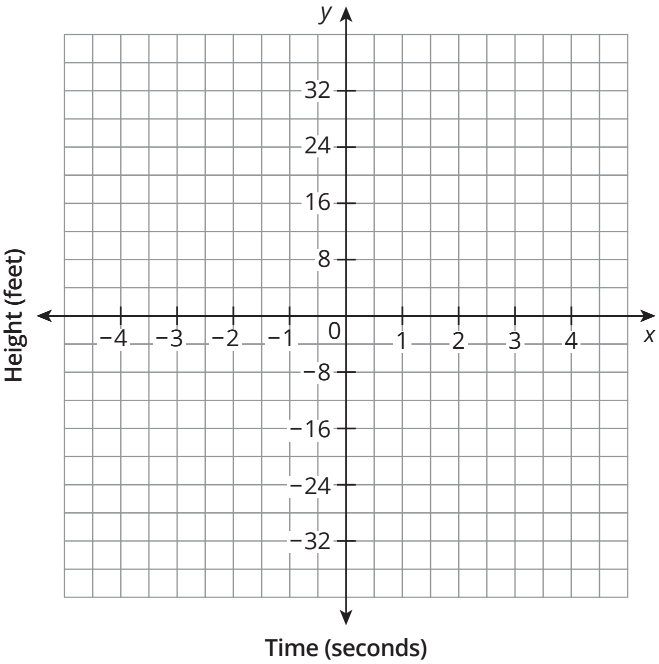
3. A baseball is thrown in the air from a height of 5 feet with an initial vertical velocity of 15 feet per second. The function $g(t) = -16t^2 + 15t + 5$ represents the height of the baseball, $g(t)$, t seconds after it was launched.



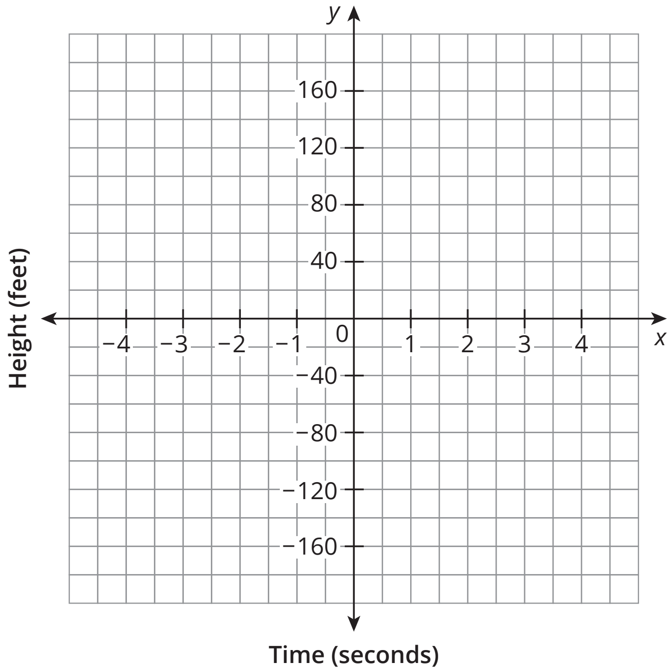
4. A football is thrown in the air from a height of 6 feet with an initial vertical velocity of 50 feet per second. The function $g(t) = -16t^2 + 50t + 6$ represents the height of the football, $g(t)$, t seconds after it was launched.



5. A tennis ball is dropped from a height of 25 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t) = -16t^2 + 25$ represents the height of the tennis ball, $g(t)$, t seconds after it was dropped.



6. A tennis ball is dropped from a height of 150 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t) = -16t^2 + 150$ represents the height of the tennis ball, $g(t)$, t seconds after it was dropped.



D. Use technology to identify the vertex and the equation of the axis of symmetry for each vertical motion model.

1. A catapult hurls a grapefruit from a height of 24 feet at an initial velocity of 80 feet per second. The function $h(t) = -16t^2 + 80t + 24$ represents the height of the grapefruit, $h(t)$, in terms of time, t.

2. A catapult hurls a pumpkin from a height of 32 feet at an initial velocity of 96 feet per second. The function $h(t) = -16t^2 + 96t + 32$ represents the height of the pumpkin, $h(t)$, in terms of time, t.

3. A catapult hurls a watermelon from a height of 40 feet at an initial velocity of 64 feet per second. The function $h(t) = -16t^2 + 64t + 40$ represents the height of the watermelon, $h(t)$, in terms of time, t.

4. A baseball is thrown from a height of 6 feet at an initial velocity of 32 feet per second. The function $h(t) = -16t^2 + 32t + 6$ represents the height of the baseball, $h(t)$, in terms of time, t.

5. A softball is thrown from a height of 20 feet at an initial velocity of 48 feet per second. The function $h(t) = -16t^2 + 48t + 20$ represents the height of the softball, $h(t)$, in terms of time, t.

6. A rocket is launched from the ground at an initial velocity of 112 feet per second. The function $h(t) = -16t^2 + 112t$ represents the height of the rocket, $h(t)$, in terms of time, t.

Extension

1. Sketch a graph of a quadratic function that has a maximum value of (0, 2) and x-intercepts when $x = \pm 2$.

2. What is the quadratic function of your graph? Explain your reasoning.

Spaced Practice

1. If the parent function $f(x) = x$ is translated 3 units to the right and 4 units up, what is the transformed equation?

2. Alexander is buying bagels. Each bagel costs $2. Let b represent the number of bagels and c represent the total cost. Does this situation represent a function? Explain your reasoning.

II. Key Characteristics of Quadratic Functions

Topic Practice

A. Calculate the first and second differences for each table of values. Describe the type of function represented by the table.

1.

x	y
−2	−6
−1	−3
0	0
1	3
2	6

First Differences ____ ____ ____ ____

Second Differences ____ ____ ____

2.

x	y
−2	12
−1	3
0	0
1	3
2	12

First Differences ____ ____ ____ ____

Second Differences ____ ____ ____

3.

x	y
−3	3
−2	4
−1	5
0	6
1	7

First Differences ____ ____ ____ ____

Second Differences ____ ____ ____

4.

x	y
−1	1
0	0
1	3
2	10
3	21

First Differences ____ ____ ____ ____

Second Differences ____ ____ ____

5.

x	y
−4	−48
−3	−27
−2	−12
−1	−3
0	0

First Differences

_____

_____

_____

_____

Second Differences

_____

_____

_____

6.

x	y
−1	10
0	8
1	6
2	4
3	2

First Differences

_____

_____

_____

_____

Second Differences

_____

_____

_____

B. Determine the *x*-intercepts and axis of symmetry of each quadratic function in factored form.

1. $f(x) = (x - 2)(x - 8)$

2. $f(x) = (x + 1)(x - 6)$

3. $f(x) = 3(x + 4)(x - 2)$

4. $f(x) = 0.25(x - 1)(x - 12)$

5. $f(x) = 0.5(x + 15)(x + 5)$

6. $f(x) = 4(x - 1)(x - 9)$

C. Determine the vertex of each parabola.

1. $f(x) = x^2 + 2x - 15$

   Axis of symmetry: $x = -1$

2. $f(x) = x^2 - 8x + 7$

   Axis of symmetry: $x = 4$

3. $f(x) = x^2 + 4x - 12$

   x-intercepts: (2, 0) and (−6, 0)

4. $f(x) = -x^2 - 14x - 45$

   x-intercepts: (−9, 0) and (−5, 0)

5. $f(x) = -x^2 + 8x + 20$

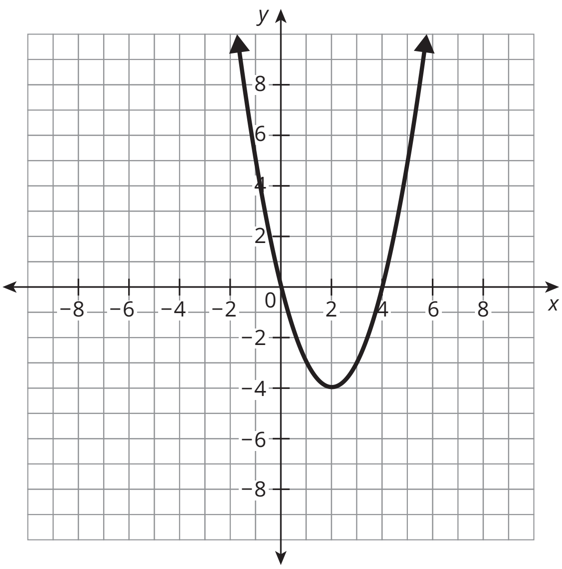
   Two symmetric points on the parabola: (−1, 11) and (9, 11)

6. $f(x) = -x^2 + 16$

   Two symmetric points on the parabola: (−3, 7) and (3, 7)

D. For each graph, identify the domain and range. Then, write an equation for the function in factored form.

1.

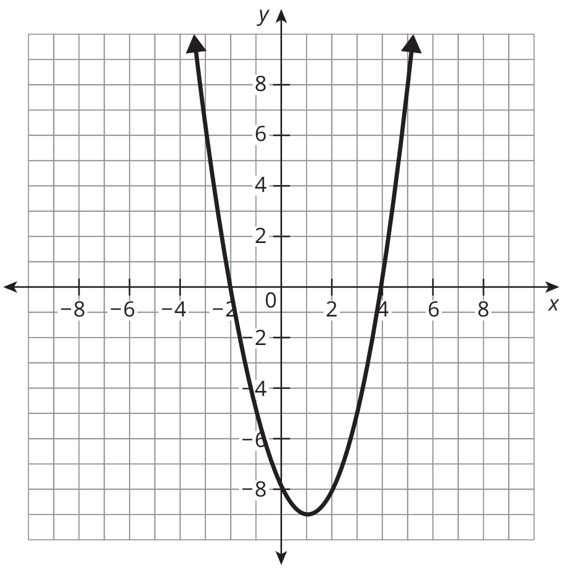


Domain:

Range:

Equation in factored form:

2.

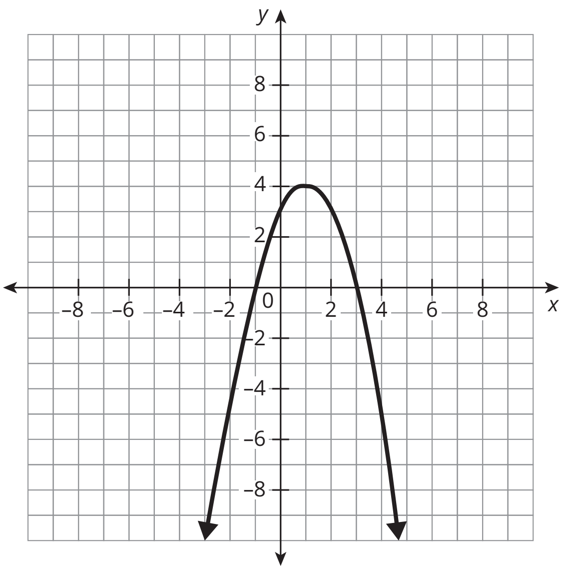


Domain:

Range:

Equation in factored form:

3.

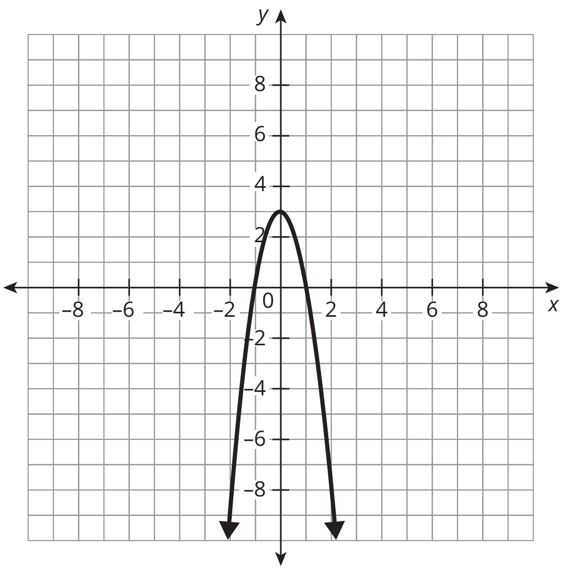


Domain:

Range:

Equation in factored form:

4.

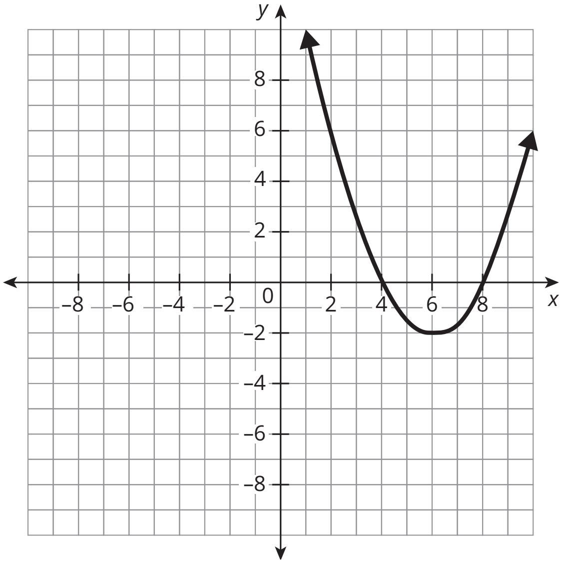


Domain:

Range:

Equation in factored form:

5.

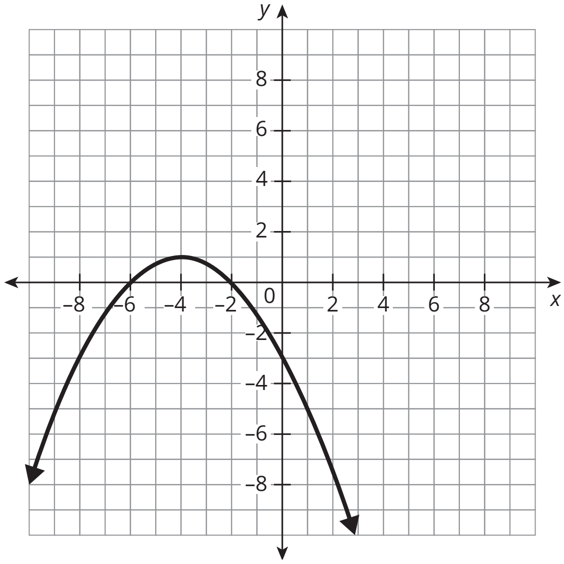


Domain:

Range:

Equation in factored form:

6.



Domain:

Range:

Equation in factored form:

E. Determine the *x*-intercepts for each function using technology. Write the function in factored form.

1. $f(x) = x^2 - 8x + 7$

2. $f(x) = 2x^2 - 10x - 48$

3. $f(x) = -x^2 - 20x - 75$

4. $f(x) = x^2 + 8x + 12$

5. $f(x) = -3x^2 - 9x + 12$

6. $f(x) = x^2 - 6x$

F. Determine another point on each parabola.

1. The axis of symmetry is $x = 3$. A point on the parabola is (1, 4).

2. The axis of symmetry is $x = -4$. A point on the parabola is (0, 6).

3. The axis of symmetry is $x = 1$. A point on the parabola is (−3, 2).

4. The vertex is (5, 2). A point on the parabola is (3, −1).

5. The vertex is (−1, 6). A point on the parabola is (2, 3).

6. The vertex is (3, −1). A point on the parabola is (4, 1).

Extension

1. Sketch the graph $f(x) = -3x^2 - 4$. How could you change the quadratic function to make the graph open upward? Show the change on the graph.

2. How could you change the quadratic function $f(x) = -3x^2 - 4$ to shift the graph up or down? Show on the graph.

3. How could you change the quadratic function $f(x) = -3x^2 - 4$ to shift the graph right or left? Show the change on the graph.

Spaced Practice

1. A camp wants to create a larger space for their albino rabbit, Clover. They want to reuse the materials from Clover's current enclosure in the construction of a new rectangular enclosure. The perimeter of Clover's current space is 6 feet. The perimeter of his new enclosure will be 3 times larger than his former enclosure.

   a. What is the area of the new enclosure $A(w)$ in terms of width, w?

   b. What is the maximum area of the new enclosure? What are the dimensions?

2. Is $7x^{2t} \cdot 5x^{2t}$ equivalent to $35x^{2t}$? Justify your answer.

3. Is $(16^{3z})^{6y}$ equivalent to 16^{18yz}? Justify your answer.

III. Quadratic Function Transformations

Topic Practice

A. Describe the transformation(s) necessary to transform the graph of the function $f(x) = x^2$ into the graph of each function $g(x)$.

1. $g(x) = x^2 - 5$

2. $g(x) = -x^2$

3. $g(x) = x^2 + 2$

4. $g(x) = (x + 4)^2$

5. $g(x) = 3x^2$

6. $g(x) = 2x^2 - 5$

7. $g(x) = \frac{1}{2}f(x) - 1$

8. $g(x) = f(x - 8)$

9. $g(x) = f(x + 2) - 3$

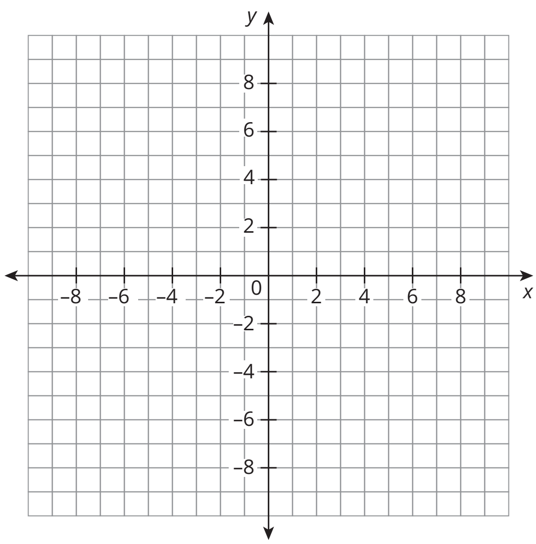
10. $g(x) = -(f(x) + 2)$

11. $g(x) = f(x + 1)$

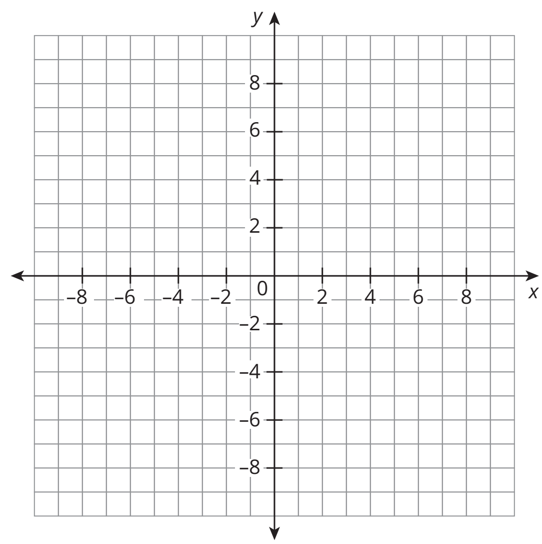
12. $g(x) = -f(x) - 5$

B. Write an equation and sketch a graph of a function $g(x)$ for each transformation of $f(x) = x^2$.

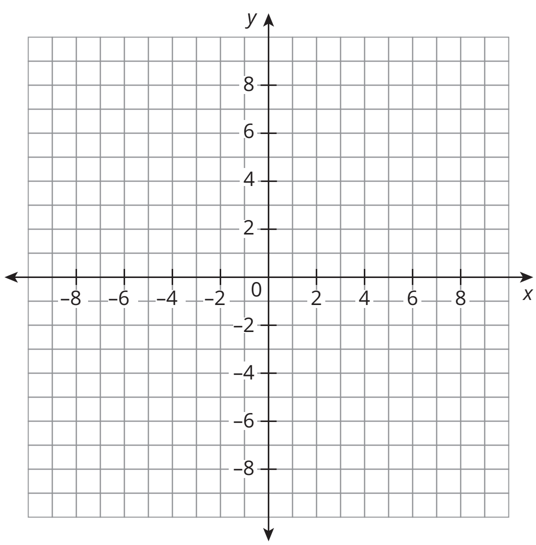
1. The graph of $g(x)$ can be created by reflecting $f(x)$ across the x-axis and then translating it 3 units up.



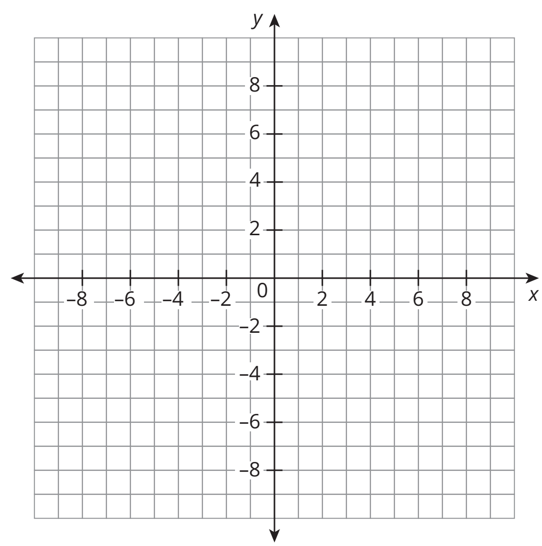
2. The graph of $g(x)$ can be created by translating $f(x)$ to the left 5 units, reflecting it across the x-axis, and then translating it 2 units down.



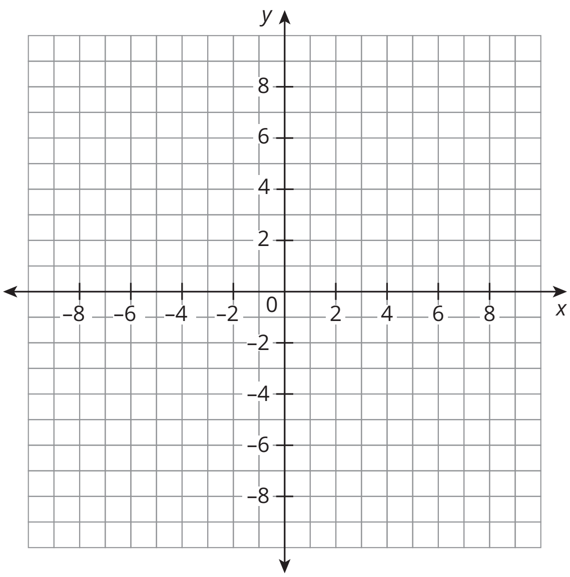
3. The graph of $g(x)$ can be created by translating $f(x)$ to the right 4 units, stretching it vertically by a factor of 6, and then translating it 1 unit up.



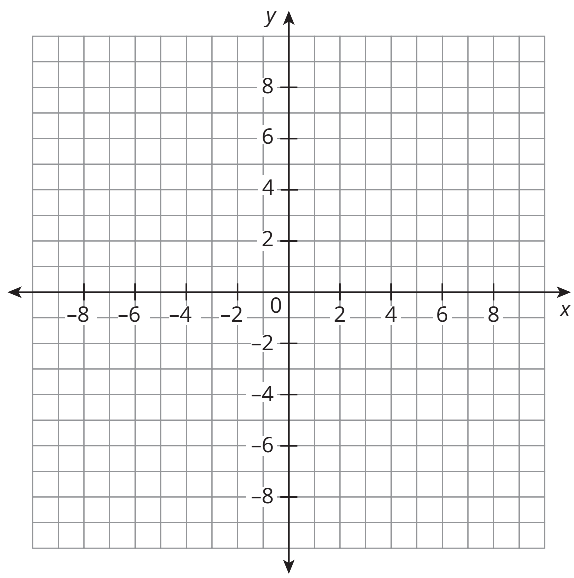
4. The graph of $g(x)$ can be created by translating $f(x)$ to the left 6 units, compressing it vertically by a factor of 0.5, and then translating it 2 units down.



5. The graph of $g(x)$ can be created by translating $f(x)$ to the right 4 units, reflecting it across the x-axis, dilating it vertically by a factor of 3, and then translating it 2 units down.

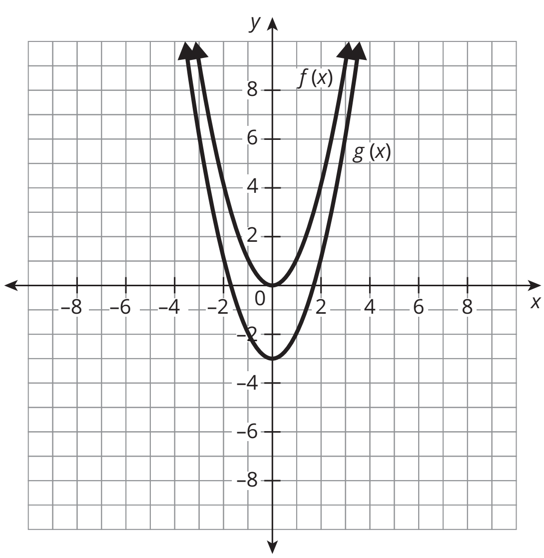


6. The graph of $g(x)$ can be created by translating $f(x)$ to the left 2 units, dilating it vertically by a factor of $\frac{1}{2}$, and then translating it 3 units up.

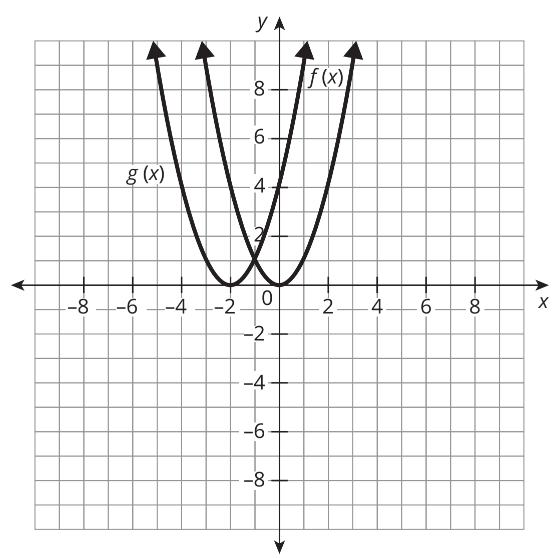


C. Describe the transformation(s) that would be required to transform $f(x)$ into $g(x)$. Then, write a function for $g(x)$ in terms of $f(x)$.

1.



2.



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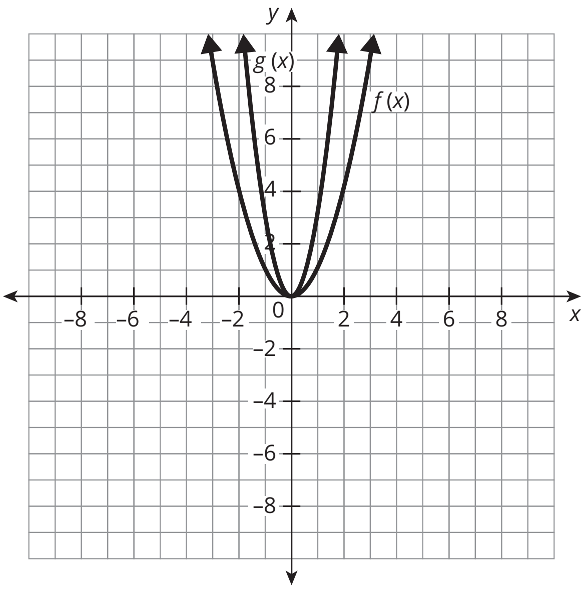
y
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g (x)
f (x)
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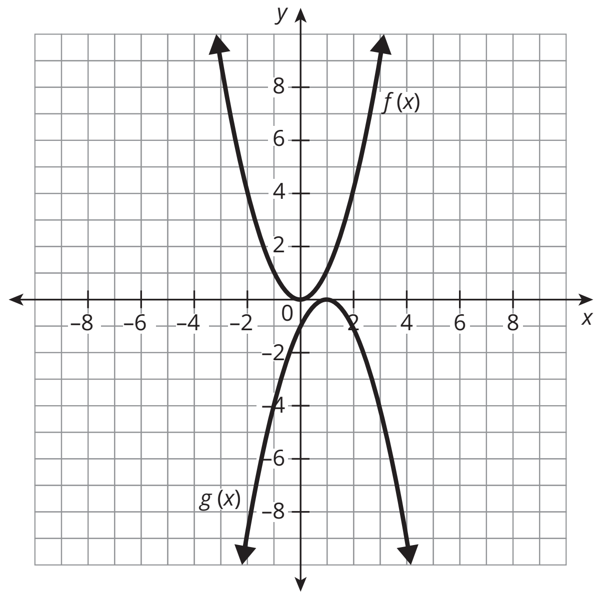
8

4.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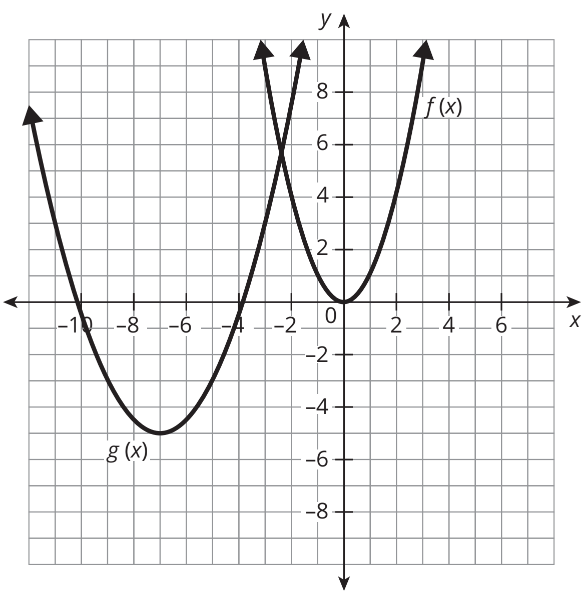
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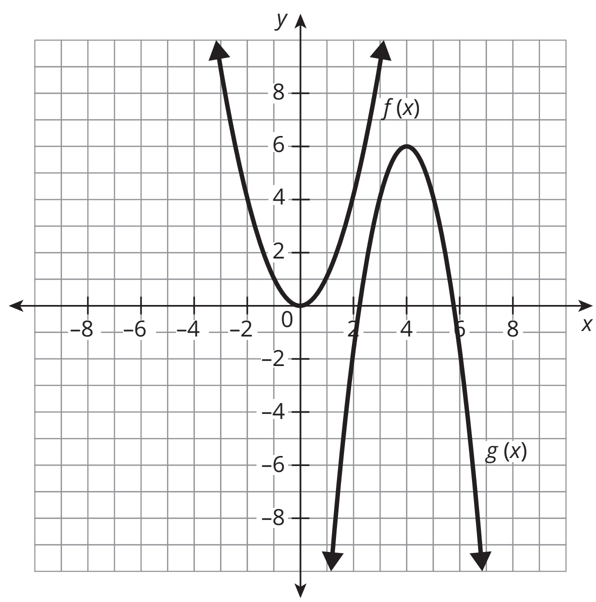
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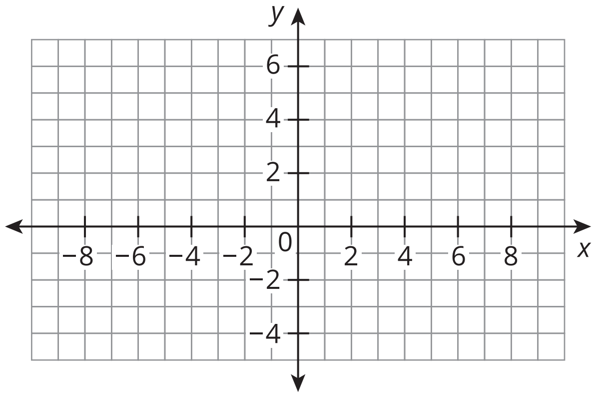
8

Extension

The function $g(x)$ is a transformation of $f(x) = x^2$.

Write the function $g(x)$.



Spaced Practice

1. A construction company has plans for two models of the homes they build, Model A and Model B. The Model A home requires 18 single windows and 3 double windows. The Model B home requires 20 single windows and 5 double windows. A total of 1800 single windows and 375 double windows have been ordered for the developments.

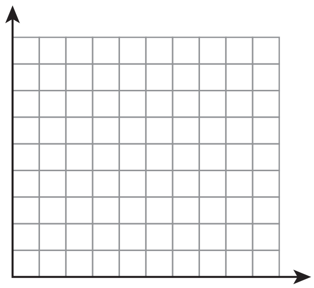
   a. Write and solve a system of equations to represent this situation. Define your variables.

   b. Interpret the solution of the linear system in terms of the problem situation.

2. A company produces two types of TV stands. Type I has 6 drawers. It requires 3 single drawer pulls and 3 double drawer pulls. The company needs 75 hours of labor to produce the Type I TV stand. Type II has 3 drawers. It requires 6 single drawer pulls. The company needs 50 hours of labor to produce the Type II TV stand. The company only has 600 labor hours available each week and a total of 60 single drawer pulls available in a week. For each Type I stand produced and sold, the company makes $200 in profit. For each Type II stand produced and sold, the company makes $150 in profit.

   a. Identify the constraints as a system of linear inequalities. Let x represent the number of 6 drawer TV stands produced, and let y represent the number of 3 drawer TV stands produced.

   b. Graph the solution set for the system of linear inequalities. Label all points of the intersection of the boundary lines.

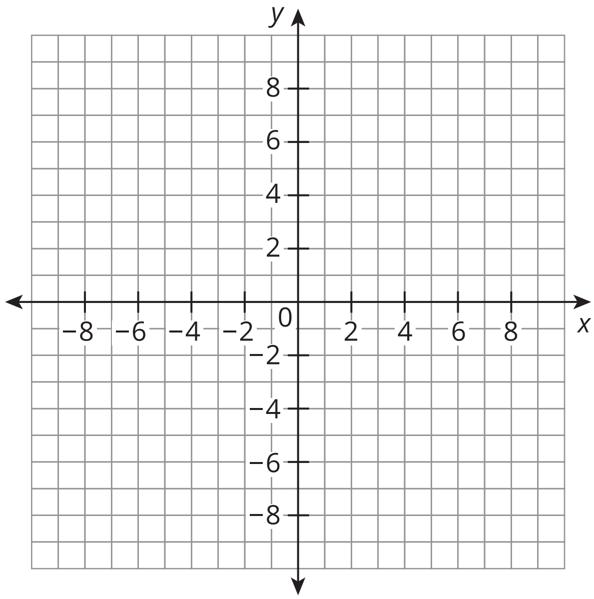


   c. Write an equation in standard form for the profit, P, that the company can make.

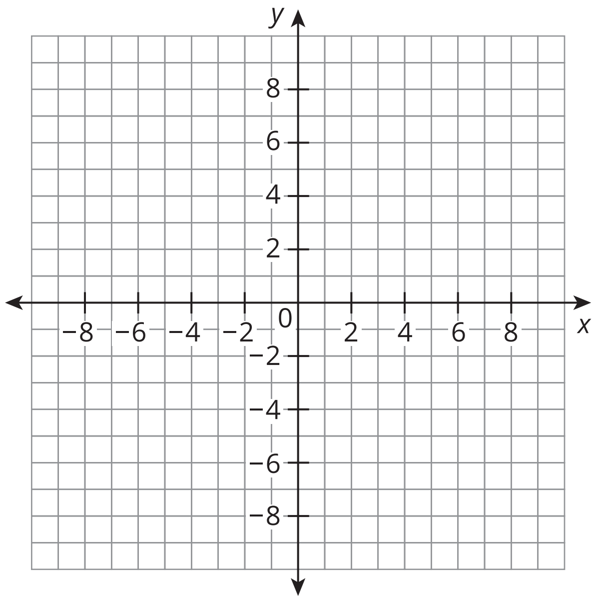
   d. How many of each type of stand should the company make if they want to maximize their profit? What is the maximum profit?

3. Each function is a transformation of the linear parent function $f(x) = x$. Graph each transformation.

a. $g(x) = \frac{1}{3}x - 2$



b. $h(x) = -2x + 1$



IV. Transformations of Quadratic Functions

Topic Practice

A. Describe the transformation(s) necessary to transform the graph of the function $f(x) = x^2$ into the graph of each function $g(x)$.

1. $g(x) = (x + 2)^2 - 7$

2. $g(x) = -(x - 2)^2 + 5$

3. $g(x) = (x + 3)^2 + 8$

4. $g(x) = \frac{2}{3}(-x)^2 + 4$

5. $g(x) = (x - 5)^2 - 6$

6. $g(x) = -(5x^2 - 7)$

7. $g(x) = f(2(x - 1)) + 3$

8. $g(x) = f(-3(x + 4)) - 6$

9. $g(x) = -\frac{1}{2}f(x - 8) - 4$

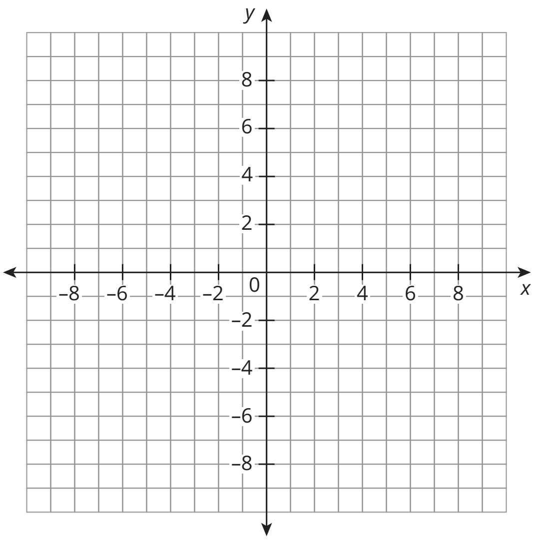
10. $g(x) = f(-x - 5) + 1$

11. $g(x) = -\frac{3}{5}(f(x) - 10)$

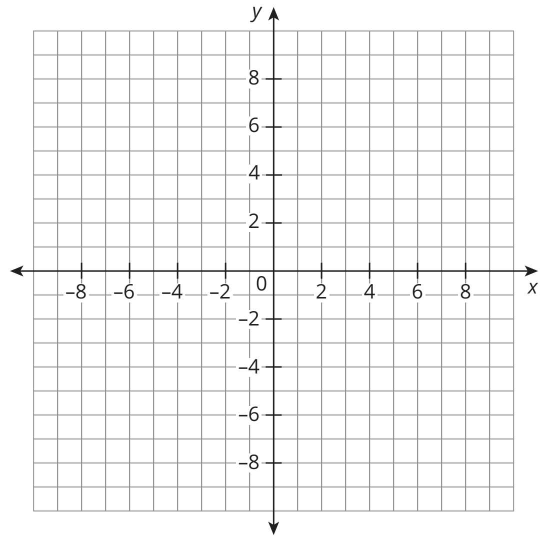
12. $g(x) = f\left(\frac{1}{4}x + \frac{3}{4}\right) + 9$

B. Write an equation and sketch a graph of a function $g(x)$ for each transformation of $f(x) = x^2$.

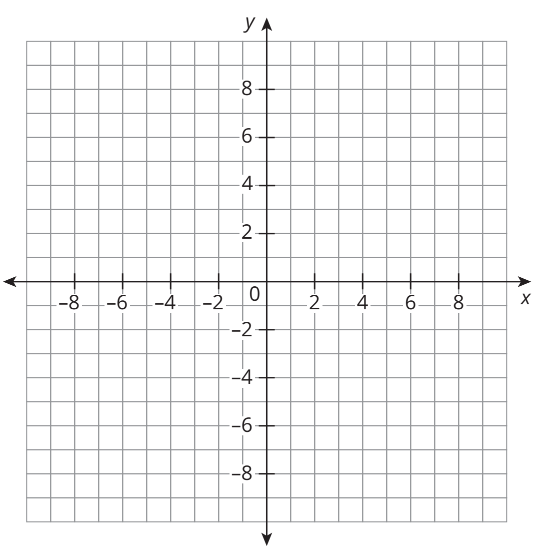
1. You can create the graph of $g(x)$ by reflecting $f(x)$ over the y-axis and then translating it 5 units up.



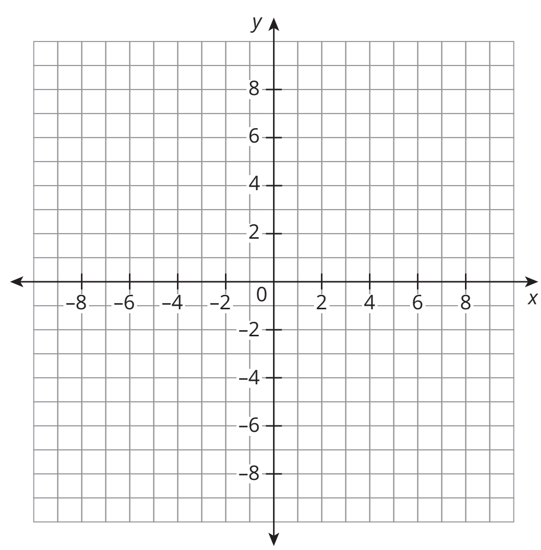
2. You can create the graph of $g(x)$ by horizontally compressing $f(x)$ by a factor of $\frac{1}{3}$, translating it 2 units to the left, and translating it 1 unit down.



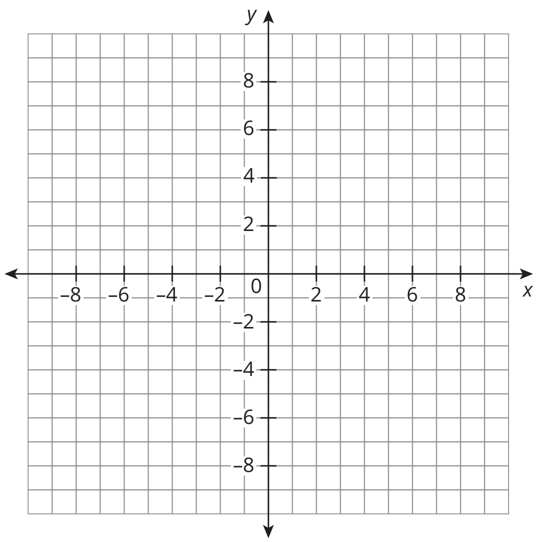
3. You can create the graph of $g(x)$ by vertically dilating $f(x)$ by a factor of 0.5, translating it right 5 units, and reflecting it over the x-axis.



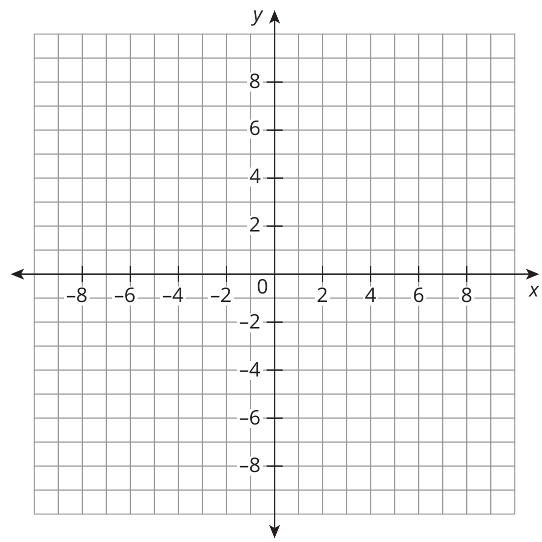
4. You can create the graph of $g(x)$ by reflecting $f(x)$ across the y-axis, stretching it horizontally by a factor of 4, translating it to the right 4 units, and translating it down 3 units.



5. You can create the graph of $g(x)$ by reflecting $f(x)$ across the x-axis, compressing it vertically by a factor of $\frac{1}{2}$, and translating it 4 units up and 1 unit left.



6. You can create the graph of $g(x)$ by dilating $f(x)$ across the x- and y-axis, translating it up 6 units, dilating it horizontally by a factor of 2, and translating it left 2 units.



C. Use technology to determine the vertex of each quadratic function given in standard form. Rewrite the function in vertex form.

1. $f(x) = x^2 - 6x - 27$

2. $f(x) = -x^2 - 2x + 15$

3. $f(x) = 2x^2 - 4x - 6$

4. $f(x) = x^2 - 10x + 24$

5. $f(x) = -x^2 + 15x - 54$

6. $f(x) = -2x^2 - 14x - 12$

D. Determine the vertex of each quadratic function given in vertex form.

1. $f(x) = (x - 3)^2 + 8$

2. $f(x) = (x + 4)^2 + 2$

3. $f(x) = -2(x - 1)^2 - 8$

4. $f(x) = \frac{1}{2}(x - 2)^2 + 6$

5. $f(x) = -(x + 9)^2 - 1$

6. $f(x) = (x - 5)^2$

E. Identify the form of each quadratic function as either standard form, factored form, or vertex form. Then, state all you know about the quadratic function's key characteristics, based only on the given equation of the function.

1. $f(x) = 5(x - 3)^2 + 12$

2. $f(x) = -(x - 8)(x - 4)$

3. $f(x) = -3x^2 + 5x$

4. $f(x) = \frac{2}{3}(x + 6)(x - 1)$

5. $f(x) = -(x + 2)^2 - 7$

6. $f(x) = 2x^2 - 1$

F. Write an equation for a quadratic function that satisfies each set of given characteristics.

1. The vertex is (−2, −3), and the parabola passes through the point (1, 6).

2. The vertex is (0, 0), and the parabola passes through the point (−2, −8).

3. The function has zeros (6, 0) and (−4, 0), and the parabola passes through the point (0, 8).

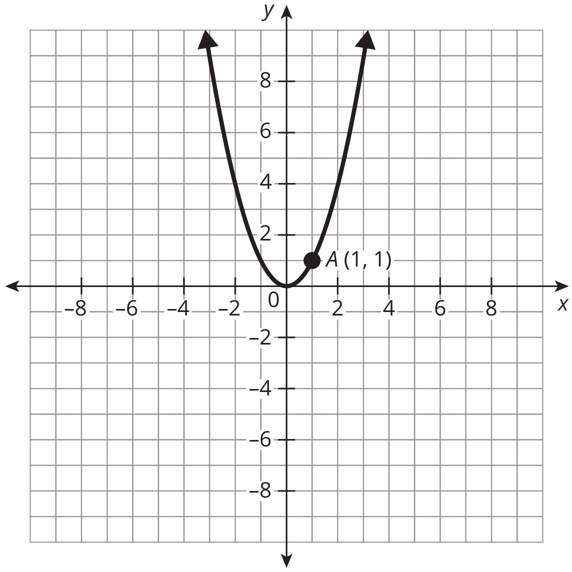
4. The vertex is (−4, 0), and the parabola passes through the point (−6, 12).

5. The vertex is (0, −8), and the parabola passes through the point (4, 0).

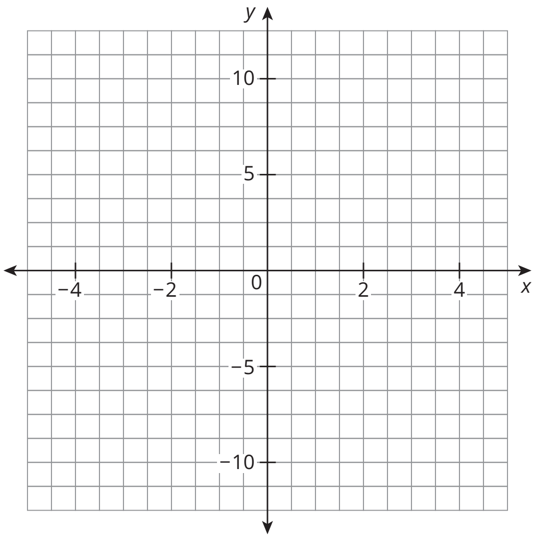
6. The function has zeros (5, 0) and (−1, 0), and the parabola passes through the point (1, −8).

Extension

Given $f(x) = x^2$. Sketch each function. Identify point A' for each transformation.



1. $m(x) = f(-x + 3)$



2. $n(x) = f(-(x + 3))$

3. $r(x) = f(-(x - 3))$

4. $t(x) = f(-x - 3)$

Spaced Practice

1. Use the equation $f(x) = \frac{1}{3}(x - 5)(x - 3)$ to determine each characteristic.

   a. Axis of symmetry

   b. x-intercepts

   c. Will the graph open upward or downward?

2. Use the equation $f(x) = 4x^2 - 10$ to determine each characteristic.

   a. Axis of symmetry

   b. y-intercept

   c. Maximum or minimum

3. Determine the exponential regression model that models each situation. Use the equation to make the associated prediction. Round all values to the hundredths place.

   a. Mr. Patel deposited $1200 into a savings account in 1985. The table shows the value of Mr. Patel's savings account from 1985 to 2020. Predict the account's value in 2030.

Time Since 1985 (years)	0	5	10	15	20	25	30	35
Account Value (dollars)	1200	1450	1800	2175	2600	3015	3468	3905

   b. Tiara deposited $725 into a savings account in 1985. The table shows the value of Tiara's savings account from 1985 to 2020. Predict the account's value in 2030.

Time Since 1985 (years)	0	5	10	15	20	25	30	35
Account Value (dollars)	725	850	1040	1270	1410	1830	2007	2780

4. Determine the x-intercepts and axis of symmetry of each quadratic function in factored form.

   a. $f(x) = (x - 7)(x - 12)$

   b. $f(x) = 3(x - 2)(x + 10)$

Name ____________________ Date ____________

I. Adding and Subtracting Polynomials

Topic Practice

A. Write each polynomial in standard form. Classify the polynomial by its number of terms and by its degree.

1. $2x + 6x^2$

2. $-9m^2 + 4m^3$

3. $10 - 5x$

4. $7x - 3 + 12x^2$

5. $15 + 4w - w^3$

6. $5x^2 - 15 + 20x$

7. $-1 - p^4$

8. $-6t^2 + 4t + 3t^3$

9. $-18a^3 + 54a - 22a^2$

10. $x^3 - x^2 - x^5$

B. Rewrite each expression with the fewest number of terms.

1. $(5x - 8) + (7x + 10)$

2. $(4m^2 + 9m) - (6 + 2m^2)$

3. $(-12 + 5x - x^2) + (2x^2 - 6)$

4. $\left(\frac{1}{2}s^2 + \frac{9}{2}\right) + \frac{1}{4} + \left(3s^2 - \frac{2}{3}s\right)$

5. $-5w^2 + (3w - 8) + 11 - (4w + 15w^2)$

6. $-a^2 + (2a - 8) + 15 + (-9a + 2a^2)$

7. $(-x^2 + 4x) - (3x + 1)$

8. $(8x^2 - 4x + 3) - (5x + 7)$

9. $(3x^4 + 3x^2 - 3) - (6x^5 - 9x^3 + 2)$

10. $(-7m^3 - m^2 - m) - (-1 - m - 10m^3)$

C. Determine the sum or difference of the polynomials.

1. $(9x + 5) - (2x - 2)$

2. $\left(x^2 - \frac{1}{2}x + \frac{1}{8}\right) + (1 + x^2)$

3. $\left(\frac{1}{2}pq - p - q\right) + \left(\frac{3}{4}p - pq\right)$

4. $(2x - 7) + (11 - 2x)$

5. $(4x^2 + 7x - 12) - (-6x^2 + 8x - 7)$

6. $(-2a + 7b - 9) - (17a - 5c + 7)$

Extension

Consider the binomials $(x + 3)$, $(2x + 1)$, and $(x - 4)$.

1. Without adding, make a conjecture about the degree of the sum of these binomials. Explain how you determined your answer.

2. Without adding, make a conjecture about the number of terms in the sum of these binomials. Explain your reasoning.

3. Determine the sum of the three binomials.

Spaced Practice

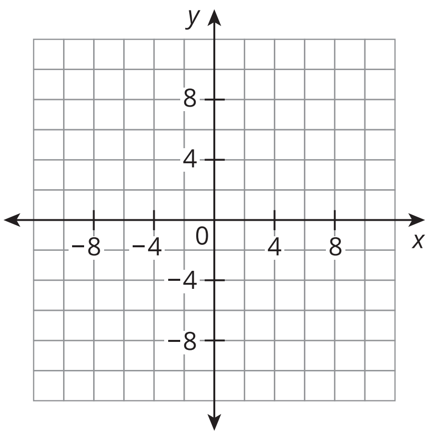
1. Analyze the given representations. Then, answer each question and justify your reasoning.

a. Which function's axis of symmetry has a greater x-value?

Function A

$f(x) = x^2 - 4x + 9$

Function C



b. Which function has the greatest absolute minimum?

Function B

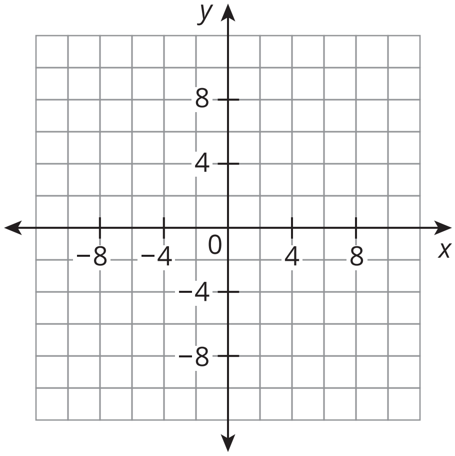
$f(x) = 3(x - 2)^2 - 6$

Function D

x	y
−1	7
0	−2
1	−5
2	−2
3	7

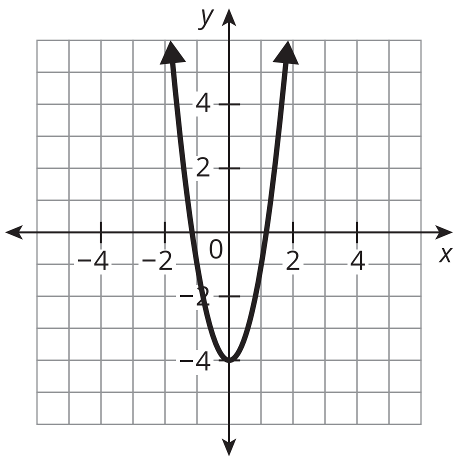
2. Write the equation of the function, g(x), in which its graph transforms the graph $f(x) = x^2$, by reflecting it across the x-axis, vertically stretching it by a factor of 2, and translating it up 5 units.

3. Graph the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2$, by vertically compressing it by a factor of $\frac{1}{3}$ and translating it down 7 units.

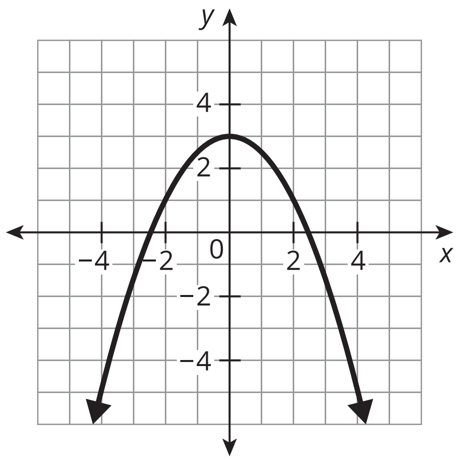


4. Identify the domain and range of each function.

a.



b.



II. Multiplying Polynomials

Topic Practice

A. Determine the product of the binomials using area models.

1. $3x + 4$ and $2x + 2$

2. $-5m + 3$ and $4m + 6$

3. $6t + 5$ and $7t - 5$

4. $\frac{1}{4}x + 2$ and $\frac{1}{4}x - 2$

5. $10w - 1$ and $-9w + 8$

6. $\frac{1}{2}y + 1$ and $3y + \frac{2}{3}$

7. $n^2 + 3n - 2$ and $-3n + 4$

8. $-2x^2 - x + 4$ and $3x - 5$

B. Determine the product of the polynomials using the distributive property.

1. $3(9 - x)$

2. $-(3x - 6)$

3. $2x(x + 6)$

4. $\frac{1}{4}x^2\left(x + \frac{1}{2}\right)$

5. $-7x(x - 5)$

6. $(2x + 1)(x + 8)$

7. $(-x + 3)(x^2 - 1)$

8. $(4x + 4)(5x - 5)$

9. $\frac{1}{3}x(x^2 + 6x - 1)$

10. $-9x(3x^2 - 4x + 2)$

11. $(x + 2)(x^2 + 6x - 1)$

12. $(x_2 - 4)(x^2 + 2x - 3)$

C. Use special products to determine each product.

1. $(x - 7)(x - 7)$

2. $(x + 4)(x - 4)$

3. $(x + 9)^2$

4. $(4x + 3)^2$

5. $(6x - 5)(6x - 5)$

6. $(2x - 8)(2x + 8)$

Extension

A difference of squares can be useful when you want to remove radicals from the denominator of a fraction.

1. Show how you can use a difference of squares to rewrite the fraction $\frac{5}{\sqrt{3} + 4}$ so that the radical is in the numerator, not the denominator.

Spaced Practice

1. Write each polynomial in standard form. Then, classify it by its number of terms and its degree. If the expression is not a polynomial, explain why not.

   a. $8 + 9x$

   b. 42

   c. $|x + 9|$

   d. $6 + 2x + 3x^2$

2. Simplify the expression by determining the sum or difference. Show your work.

   $(3x^2 - 2x + 14) - (6x^2 + 3x - 9)$

3. Identify each of the following for the function $f(x) = 3 \cdot 4^x$.

   a. x-intercept(s)

   b. y-intercept

   c. Asymptote

   d. Domain

   e. Range

   f. Interval(s) of increase/decrease

4. Write the equation of each function after the translation described.

   a. $f(x) = -10x$ after a translation 5 units to the right.

   b. $g(x) = 3^x$ after a translation 4 units up.

   c. $h(x) = 2x^2$ after a translation 2 units left and 1 unit down.

5. Write an equation for a quadratic function that satisfies each set of given characteristics.

   a. The vertex is $(-6, 3)$ and the parabola passes through the point $(-5, 0)$.

   b. The vertex is $(2, 5)$ and the parabola passes through the point $(-3, -6)$.

6. Determine the axis of symmetry for each quadratic function.

   a. $f(x) = \frac{2}{3}(x + 3)(x - 7)$

   b. $f(x) = -0.2(x - 16)(x - 5)$

III. Polynomial Division

Topic Practice

A. Use the factor theorem to determine whether the given expression is a factor of each polynomial. Explain your reasoning.

1. Is $(x - 4)$ a factor of $0.5x^2 - 20$?

2. Is $(x - 3)$ a factor of $-14x^2 + 52x - 30$?

3. Is $\left(x + \frac{1}{2}\right)$ a factor of $-6x^2 - 22x - 12$?

4. Is $(x - 2)$ a factor of $9x^2 + 15x - 6$?

5. Is $(x - 1)$ a factor of $24x^2 - 66x + 42$?

6. Is $\left(x + \frac{4}{3}\right)$ a factor of $21x^2 + 16x - 16$?

B. Determine the quotient for each problem. Show all of your work.

1. $(8x - 16) \div (x - 2)$

2. $(5x + 21) \div (x + 4)$

3. $(x^2 + 2x - 15) \div (x + 5)$

4. $(x^2 - 7x - 30) \div (x + 3)$

5. $(x^2 + 6x - 10) \div (x - 5)$

6. $(9x^2 - 20x - 21) \div (x - 3)$

7. $(16x^2 - 12x + 10) \div (4x - 4)$

8. $(24x^2 - 83x + 63) \div (8x^2 - 9)$

Extension

Determine the quotient of $\frac{4x^4 - 19x^2 - 8}{x - 2}$. Show all your work.

Spaced Practice

1. Perform each operation.

   a. $(8m^2 - 7m + 3) - (3m^2 + 2m - 3)$

   b. $(2x - 5)(3x^2 - x + 4)$

   c. $7a(a + 3) + (a - 4)(2a - 7) - (4a^2 - 11a + 3)$

   d. $(5x^2 + 2x - 1)(3x^2 - 3x + 5)$

2. Use the distributive property to determine the product of the polynomials.

   a. $3x(3x^2 + 2x - 7)$

   b. $(x - 6)(4x + 2)$

Name ________________________________ Date ______________

I. Representing Solutions to Quadratic Equations

Topic Practice

A. Solve each quadratic equation. If necessary, leave solutions in simplest radical form.

1. $x^2 = 48$

2. $x^2 = 52$

3. $x^2 = 27$

4. $x^2 = 175$

5. $x^2 = 40$

6. $x^2 = 147$

B. Solve algebraically for the roots of each quadratic equation or the zeros of each quadratic function.

1. $x^2 - 100 = 0$

2. $4x^2 - 9 = 0$

3. $f(x) = x^2 - 225$

4. $f(x) = 9x^2 - 1$

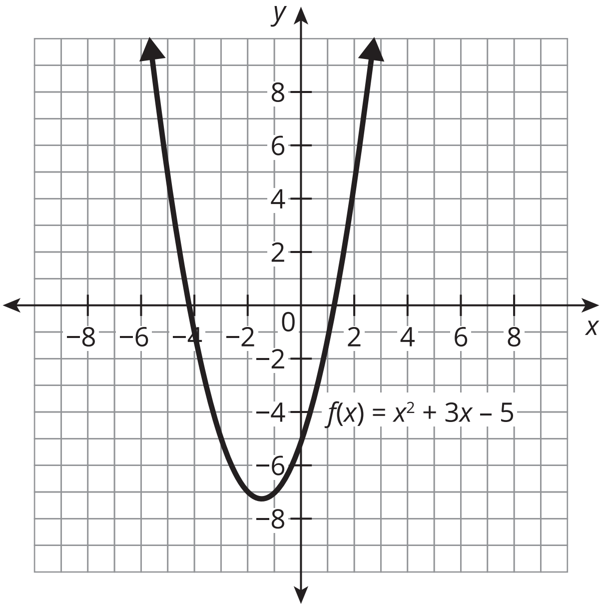
5. $f(x) = 8x^2 - 50$

6. $16x^2 - 25 = 0$

Extension

1. Consider the graph of the function $f(x) = x^2 + 3x - 5$.

   a. Determine the solutions for the equation $x^2 + 3x - 5 = 5$. Identify the solutions on the graph.



   b. Rewrite the equation from part (a) so that the right side of the equation is 0. What do the solutions from part (a) represent in this new equation?

   c. Use your solutions from part (a) to write a product of two binomials, $(x - a)(x - b)$, where a and b are the solutions from part (a). What is the relationship between this and the left side of the equation in part (b)?

Spaced Practice

1. Identify the axis of symmetry of the graph of $f(x) = -5(x - 3)(x + 12)$.

2. Write a quadratic function in factored form to represent a parabola that opens downward and has zeros at $(-6, 0)$ and $(-2, 0)$.

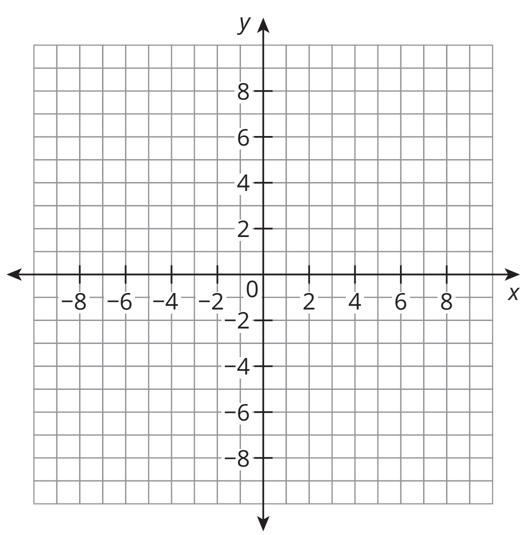
3. Determine each product. Show your work.

   a. $(2x - 3)(4x + 7)$

   b. $(3x + 5)\left(-\frac{1}{2}x + 16\right)$.

4. Write the equation of the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2 + 1$ by reflecting it across the x-axis, shifting it up 6 units, and shifting it to the left 4 units.

5. Graph the function, $g(x)$, in which its graph transforms the graph $f(x) = (x - 4)^2$ by vertically stretching it by a factor of 2, reflecting it across the x-axis, and moving it to the left 3 units.

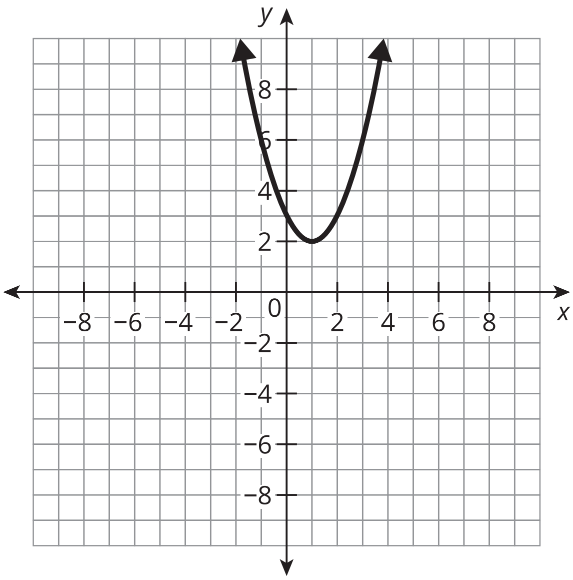


II. Solutions to Quadratic Equations in Vertex Form

Topic Practice

A. For each given graph, determine the number of real zeros for the quadratic function.

1.



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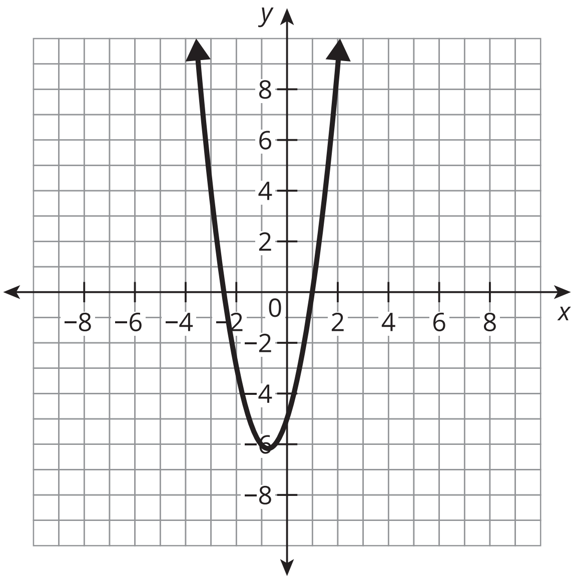

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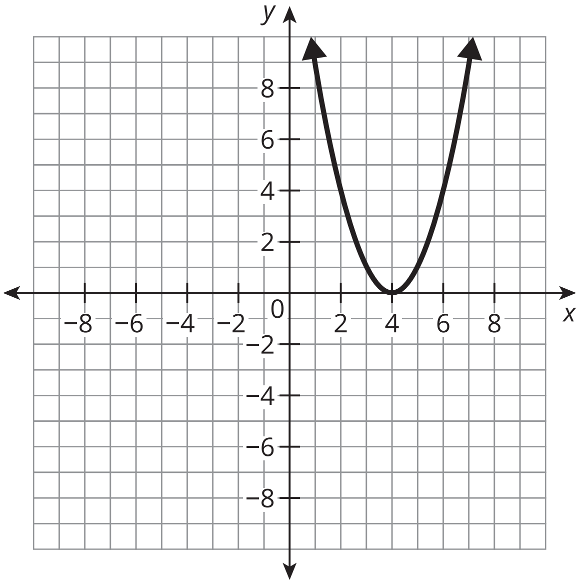

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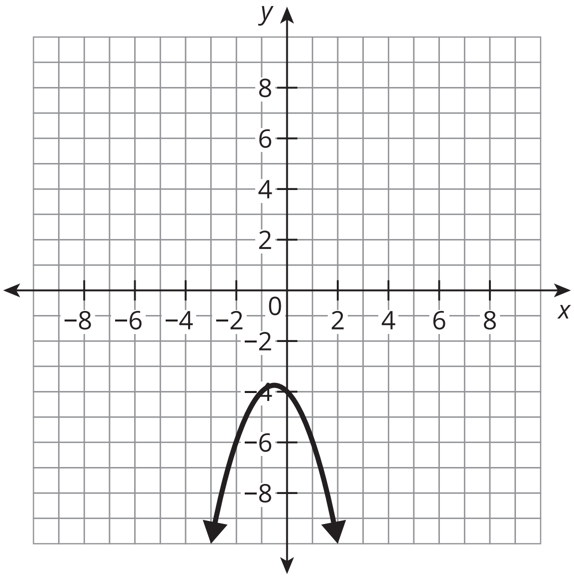

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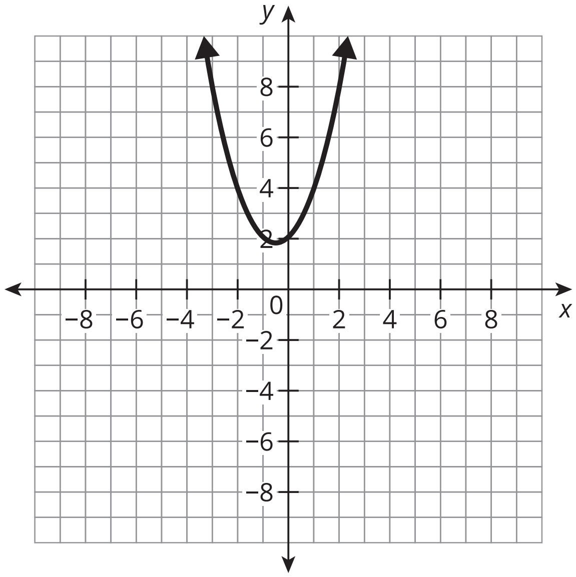

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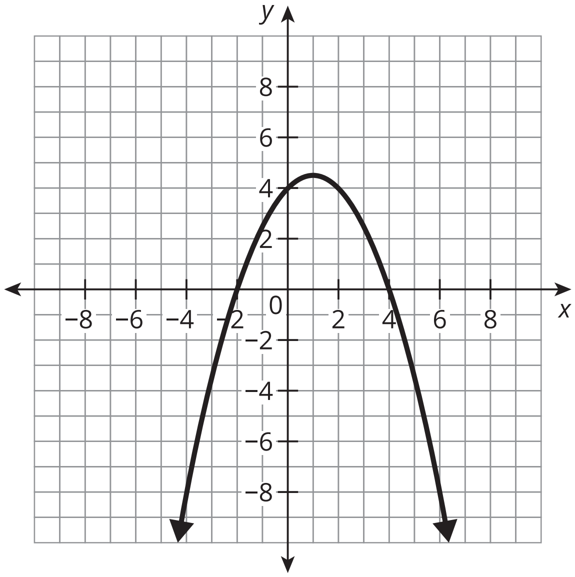


B. Solve each quadratic equation. Give both exact and approximate solutions rounded to nearest hundredth.

1. $(x - 2)^2 = 16$

2. $3(x + 1)^2 = 6$

3. $-6(x + 3)^2 = -30$

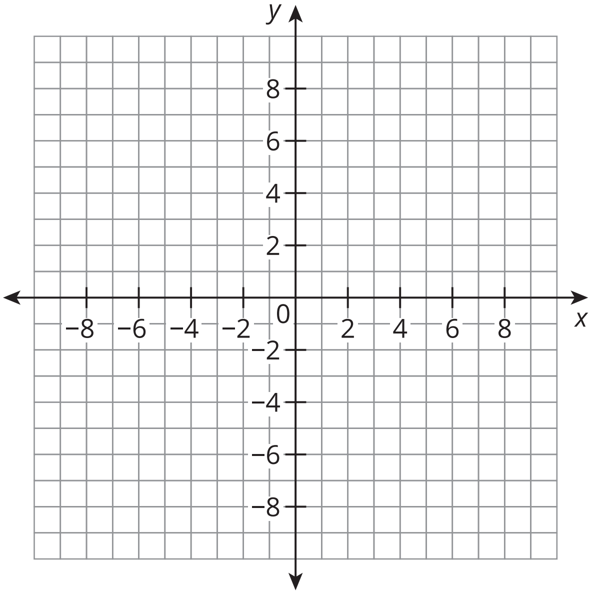
4. $7(x - 9)^2 = 36$

5. $-\frac{1}{3}(x - 4)^2 = -12$

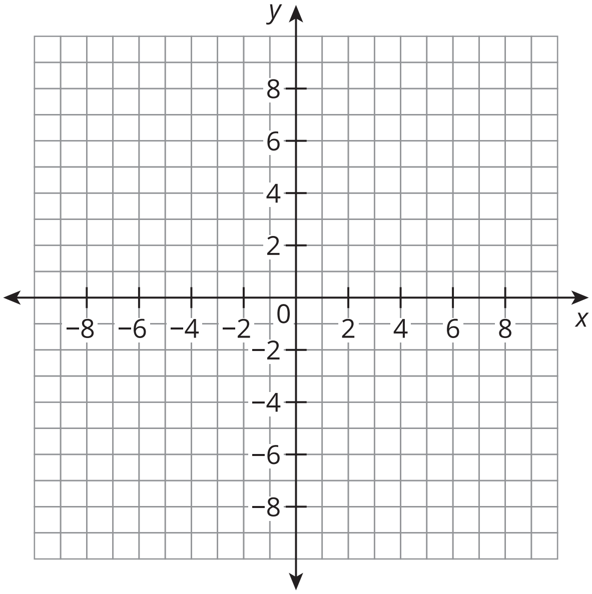
6. $3(10 - x)^2 = 27$

C. Sketch a graph of each quadratic function. Describe the number of zeros of each function and solve for the zeros algebraically. Give both exact and approximate solutions rounded to the nearest hundredth.

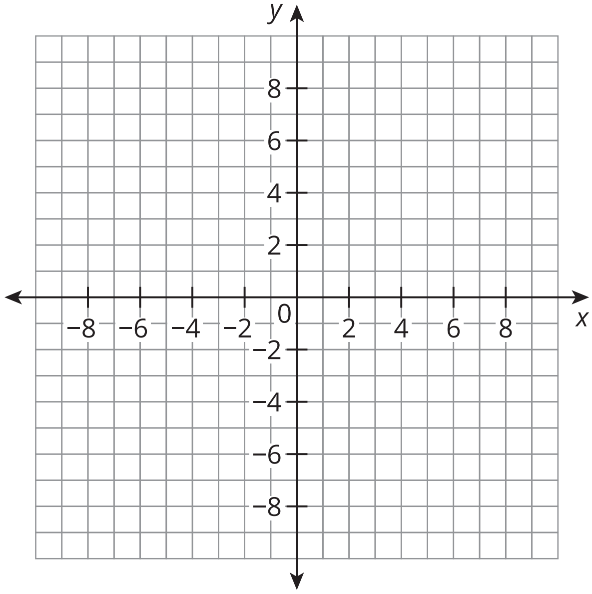
1. $f(x) = -2(x + 4)^2 + 2$



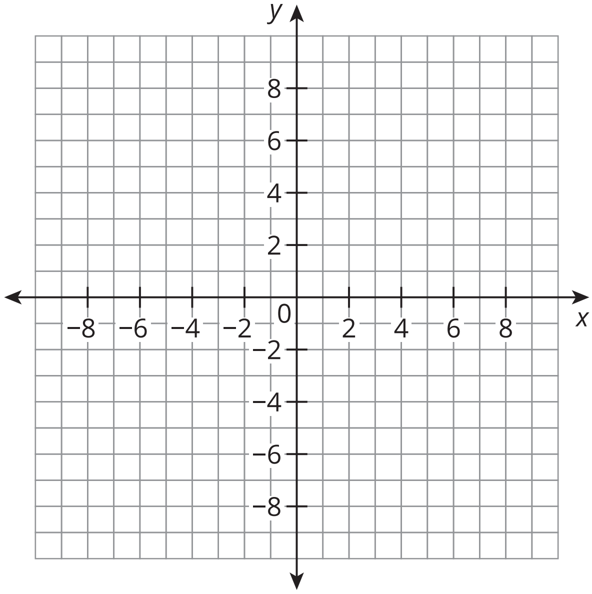
2. $f(x) = 3x^2 - 4$



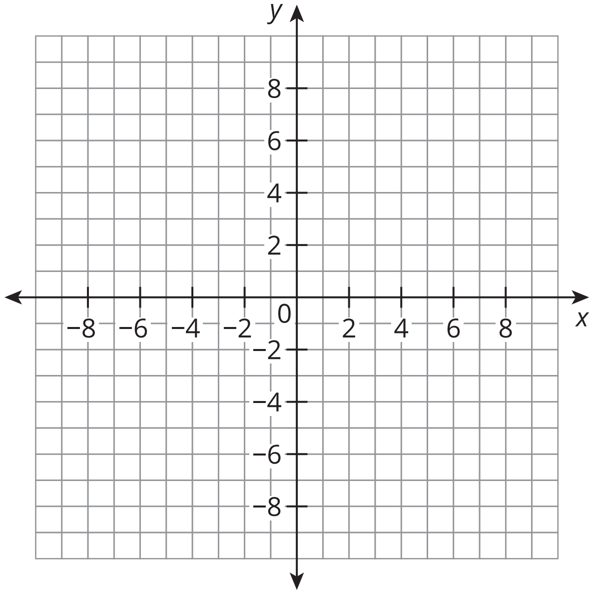
3. $f(x) = -4(x + 6)^2 - 1$



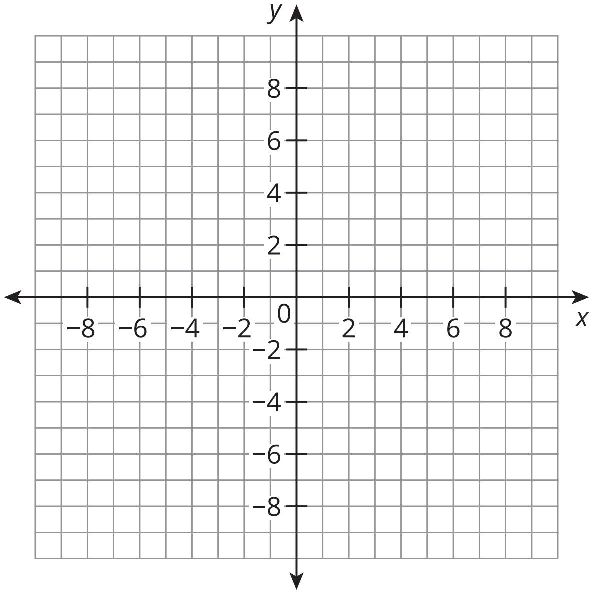
4. $f(x) = 0.5(x + 1)^2 - 3$



5. $f(x) = -(x - 5)^2$



6. $f(x) = \frac{1}{4}(x + 1)^2 + 3$



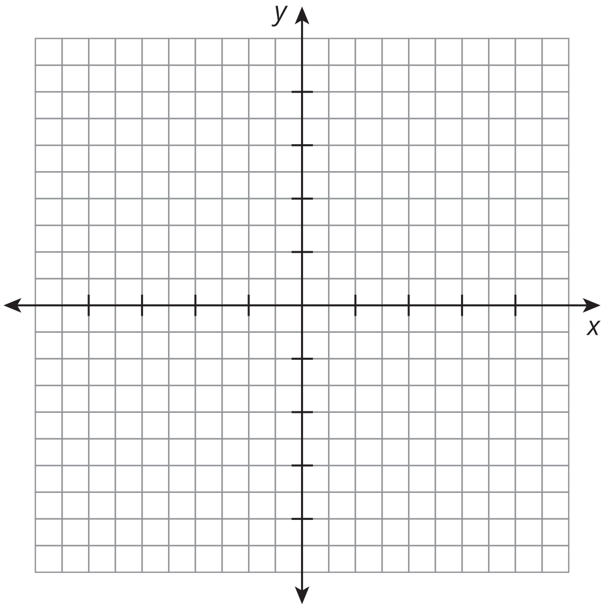
Extension

A quadratic function has zeros at $x = -2 \pm \sqrt{15}$. Write the function in standard form. Show your work.

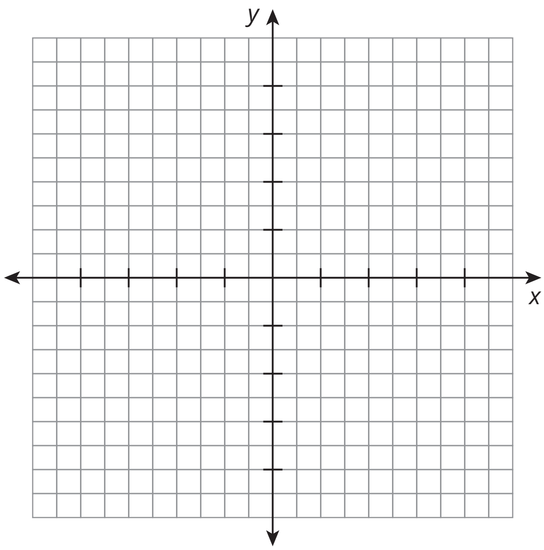
Spaced Practice

1. Use the given characteristics to write a function $R(x)$ in vertex form. Then, sketch the graph of $R(x)$ and the parent function $f(x) = x^2$ on a coordinate plane.

   a. The function has an absolute maximum, is vertically dilated by a factor of $\frac{1}{3}$, and is translated 8 units down and 4 units to the left.



b. The function has an absolute minimum, is vertically dilated by a factor of 4, and is translated 2 units up and 6 units to the right.



2. Estimate the value of the radical expression $\sqrt{54}$. Then, rewrite the radical by extracting all perfect squares, if possible.

3. Rewrite the quadratic function, $f(x) = 16x^2 - 3$, as the product of linear factors.

4. Identify the form of each quadratic equation. Then, identify what characteristic of the function can be determined by the structure of the equation.

a. $y = (x - 7)(x + 5)$

b. $y = -3(x + 1)^2 - 4$

III. Factoring and Completing the Square

Topic Practice

A. Factor out the greatest common factor of each polynomial, if possible.

1. $x^2 + 9x$

2. $m^2 - 4m$

3. $5x^2 + 20x - 15$

4. $-24w^2 + 16$

5. $3w + 10$

6. $2x^2 + 10x$

7. $\frac{1}{4}x + \frac{3}{4}$

8. $\frac{4}{5}x + \frac{1}{5}x^2$

9. $20x^3 + 16x^2 + 8x$

10. $15x^3 + 4$

B. Factor each trinomial.

1. $x^2 - 2x - 8$

2. $y^2 + 13y + 42$

3. $m^2 + 6m - 7$

4. $x^2 - 9x + 18$

5. $b^2 - 16$

6. $2t^3 - 14t^2 + 24t$

7. $3m^3 + 36m^2 + 60m$

8. $3x^2 + 14x - 5$

9. $x^2 + 11x + 10$

10. $w^2 + 10w + 25$

11. $m^2 + 2m - 35$

12. $9x^2 - 100$

13. $6x^2 + 9x - 15$

14. $2x^2 + 22x + 60$

C. Determine the solutions of each quadratic equation using the factoring method.

1. $x^2 + 5x + 6 = 0$

2. $x^2 - 3x - 4 = 0$

3. $m^2 + 2m - 35 = 0$

4. $-x^2 - 4x + 12 = 0$

5. $x^2 + 8x = 0$

6. $w^2 + 50 = -15w$

7. $-t^2 + 12t = 32$

8. $x^2 + 2x + 2 = 0$

9. $2t^2 + t - 3 = 0$

10. $w^2 + 5w - 32 = 2w - 4$

11. $-14x^2 + 28x + 42 = 0$

12. $-12x^2 + 38x = 6$

D. Algebraically or graphically determine the zeros of each quadratic function, if possible.

1. $f(x) = x^2 - 5x$

2. $f(x) = 3x^2 + 6x$

3. $f(x) = x^2 + 11x + 30$

4. $f(x) = x^2 - 9x - 36$

5. $f(x) = 2x^2 + 9x + 10$

6. $f(x) = x^2 + 5x + 14$

7. $f(x) = 3x^2 + 3x - 6$

8. $f(x) = \frac{1}{2}x^2 - \frac{3}{4}x$

E. Use an area model to complete the square for each expression. Factor the resulting trinomial and write the original expression in vertex form.

1. $x^2 + 2x$

2. $x^2 + 4x$

3. $x^2 + 12x$

4. $x^2 + 9x$

5. $x^2 + 11x$

6. $x^2 + 28x$

F. Determine the unknown value that would make each trinomial a perfect square.

1. $x^2 - 10x +$ ________

2. $x^2 + 14x +$ ________

3. $x^2 +$ ________ $x + 9$

4. $x^2 -$ ________ $x + 81$

5. $x^2 + 7x +$ ________

6. $x^2 - 15x +$ ________

7. $x^2 -$ ________ $x + 169$

8. $x^2 +$ ________ $x + \frac{9}{4}$

G. Determine the solutions of each quadratic equation by completing the square. Round your answer to the nearest hundredth. Check your answer algebraically.

1. $x^2 + 4x - 6 = 0$

2. $x^2 - 2x - 4 = 0$

3. $x^2 + 10x + 2 = 0$

4. $x^2 - 12x + 25 = 0$

5. $x^2 + 3x - 1 = 0$

6. $x^2 + x - 10 = 0$

Extension

The function g is defined by $g(x) = x^2 - 3x - 10$. If $g(x + 3) = x^2 + bx - c$, what are the values of b and c? Show your work and justify your answer.

Spaced Practice

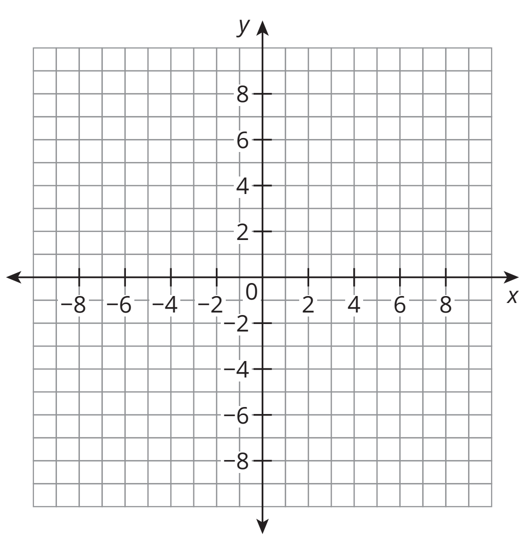
1. For each quadratic function, determine if it has an absolute minimum or absolute maximum and if the graph opens upward or downward. Identify the y-intercept.

   a. $f(x) = 3x^2 + 6x - 72$

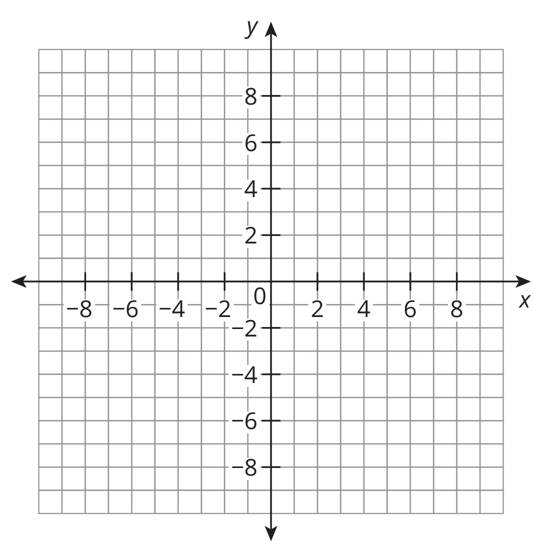
   b. $f(x) = -\frac{1}{2}(x - 2)(x + 5)$

2. Sketch a graph of each quadratic function. Determine the zeros of each function.

   a. $f(x) = (x + 6)^2$

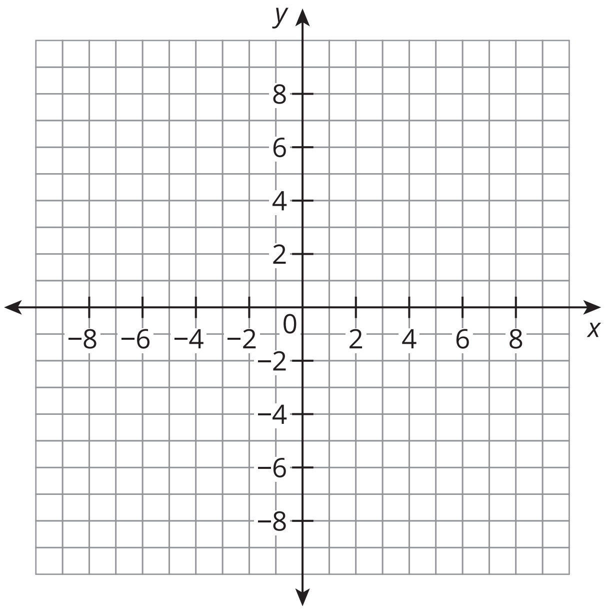


   b. $f(x) = \left(3x - \frac{9}{2}\right)^2 - 5$



3. Write the equation of the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2$ by reflecting it across the x-axis, vertically compressing it by a factor of $\frac{1}{2}$, and moving it down 3 units.

4. Graph the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2$ by vertically stretching it by a factor of 3 and moving it up 4 units.



Determine the quotient for each problem.

5. $(6x^2 - 54) \div (2x - 6)$

6. $(6x^2 - 16x - 32) \div (3x + 4)$

IV. The Quadratic Formula

Topic Practice

A. Solve each quadratic equation.

1. A water balloon is thrown upward from a height of 5 feet with an initial velocity of 35 feet per second. The quadratic function $h(t) = -16t^2 + 35t + 5$ represents the height of the balloon, h, in feet t seconds after it is thrown. How long does it take for the balloon to reach the ground? Round your answer to the nearest tenth.

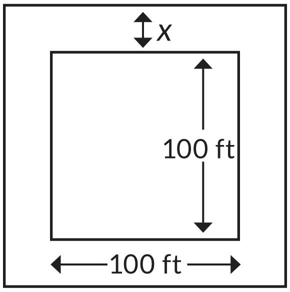
2. Gabriela has saved up some money and decides to take a risk and invest in some stocks. She invests her money in Doogle, a popular computer company. Unfortunately, she lost it all over a matter of months. The change in her money during this investment can be represented by the function $v(x) = 75 + 72x - 3x^2$, where v is the value of her investment and x is the time in months. Determine when her portfolio reached a value of $360.

3. Alexander invested some of his money in Home-mart, a large home improvement store in his town. A few years after investing, the company went out of business and Alexander lost all his money. The growth and decline of his money over this time can be represented by the function $v(x) = -2x^2 + 98x + 100$, where v is the value of his investment and x is the time in months. When did Alexander have $1288.00 in his account?

4. Jacob throws a softball up in the air. The height of the ball in meters can be determined by the function $h(t) = -4.9(t - 3)^2 + 60$, where t is the time it is in the air in seconds. When will the ball reach a height of 20 feet? Round your answer to the nearest tenth.

5. A company knows that the more it advertises the more product it will sell. However, advertising more will also cost more money, which then takes away some of the profit. The profit will follow the path of a parabola because it will increase from more advertising but eventually decrease if too much money is spent on advertising. The profit (in thousands of dollars) can be represented by the function $P(x) = -2x^2 + 14x + 60$, where x represents the amount of money spent (in thousands of dollars). How much money should the company spend on advertising if they want their profit to be $24,000?

6. The owners of a botanical park would like to create a walkway around one of their premier gardens. The garden is 100 feet long and 100 feet wide. The drawing below shows the layout of the garden and walkway. The function $A(x) = 4(x + 50)^2$ represents the total area of the garden and walkway when x represents the width of the walkway. What should the width of the walkway be if the owners want the total area to be 12,100 square feet?



B. Use the quadratic formula to determine the axis of symmetry, the vertex, and the roots of each function.

1. $f(x) = x^2 + 2x - 2$

2. $f(x) = -x^2 + 6x - 7$

3. $f(x) = 2x^2 - 12x + 13$

4. $f(x) = -x^2 - 2x + 4$

5. $f(x) = -3x^2 - 18x - 25$

6. $f(x) = -x^2 + 10x - 26$

C. Determine the zeros or roots of each function or equation.

1. $f(x) = -2x^2 + 5x - 1$

2. $-3x^2 + 8x - 2 = -6$

3. $f(x) = 9x^2 + 5x + 1$

4. $6x^2 + 3x - 5 = 2$

5. $f(x) = 5x^2 + 10x + 5$

6. $f(x) = 7x^2 + 9x + 5$

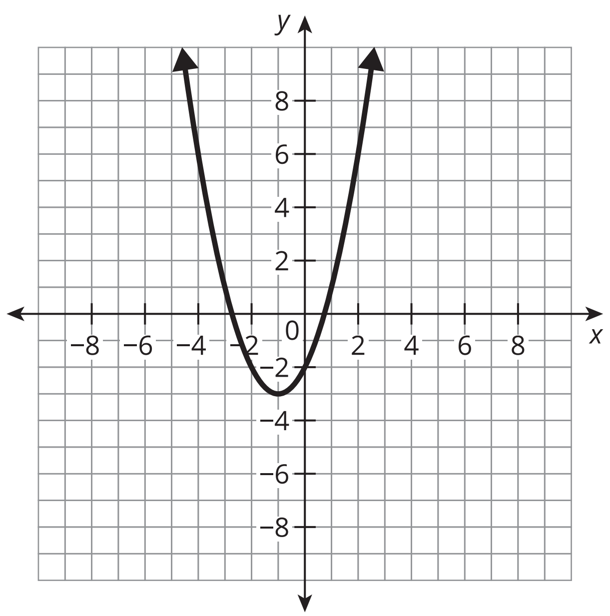
Extension

Consider the function $f(x) = -2x^2 + bx - 5$. Determine the b-value(s) that would ensure the function has two real roots. Explain your reasoning.

Spaced Practice

1. Analyze each pair of representations. Then, answer each question and justify your reasoning.

   a. Which function has a greater y-intercept?

A	B
$f(x) = \frac{3}{4}(x - 2)^2$	

b. Which function has a greater absolute maximum?

A	B
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>24</td></tr><tr><td>3</td><td>25</td></tr><tr><td>6</td><td>24</td></tr></table>	$f(x) = -2x^2 + 15x - 4$

2. Complete the square to determine the roots of each equation. Show your work.

a. $y = 2x^2 + 5x - 14$

b. $y = -3x^2 - 6x + 10$

3. Consider the function $f(x) = \left(x + \frac{1}{2}\right)\left(x - \frac{3}{4}\right)$.

a. Identify the form of the function as factored, standard, or vertex.

b. Identify the zeros and axis of symmetry of the function.

V. Using Quadratic Functions to Model Data

Topic Practice

A. Use technology to determine the quadratic function that models each data set. Round decimals to the nearest thousandth.

1.

x	*y*
1	22
6	6
20	20
42	64
80	99

2.

x	*y*
12	38
24	12
30	3
40	16
54	54

3.

x	*y*
25	60
50	80
75	140
100	210
125	250
150	322
175	400

4.

x	*y*
10	14
20	36
30	70
40	120
50	180
60	240
70	350

5.

x	y
0.1	8
0.2	12
0.3	14
0.4	16
0.5	18
0.6	10
0.7	2

6.

x	y
0.1	2.2
0.2	3.1
0.3	4
0.4	4.8
0.5	5.4
0.6	6
0.7	4.2

B. Use technology to write a quadratic function that can be used to model each scenario. Then, determine the *y*-intercept, *x*-intercepts, domain, range, and vertex of the graph. Interpret these characteristics and explain if they make sense in terms of the scenario.

1. The data table represents the speed of a car, *s*, in miles per hour and the car's average fuel efficiency, *g*, in miles per gallon at that speed.

s (mph)	*g* (mpg)
10	8.3
11	9.8
12	11.4
13	12.7
14	14.7
15	15.6
16	17.8
17	18.7
18	19.9
19	21.2

2. An athlete throws a disc upward at an angle. The data table represents height in feet, *h*, of the disc and the distance in feet, *d*, that the disc has traveled horizontally.

d (ft)	*h* (ft)
1	6.0
2	6.4
3	6.8
6	8.2
7	8.7
8	9.1
10	9.9
11	10.4
13	11.2
14	11.6

3. Amir and his friend are playing catch with a baseball. Amir tosses the ball to his friend, but he overthrows it, and it hits the ground. The data table represents the height in feet, h, of the baseball and the distance in feet, x, that the baseball has traveled horizontally.

x (ft)	h (ft)
1	5.8
2	6.7
3	7.1
4	8.2
5	9.0
8	10.6
9	11.6
12	13.4
13	13.4
14	14.3

4. The data table represents a company's profit in dollars, p, and the number of years, x, since the company was started.

x (years)	p (dollars)
3	379.45
4	3686.16
5	6732.61
6	9630.78
7	12102.85
8	14006.43
9	16830.09
10	17817.00
11	19758.89
12	21312.82

5. The data table represents temperatures, t, in degrees Fahrenheit recorded during a 10-hour winter snowstorm and the number of hours, h, the storm has lasted.

h (hours)	t (°F)
2	21.8
3	16.8
4	14.1
5	12.8
6	13.4
7	14.9
8	18.0

6. The data table represents a company's daily profit, *p*, from selling graphing calculators and the price of the graphing calculator, *x*.

x (dollars)	*p* (dollars)
45	4996.9
46	5614.2
47	6167.7
48	6936.6
49	7471.3
71	18563.4
72	17368.1
82	19063
96	17926.1
110	13231.1

Extension

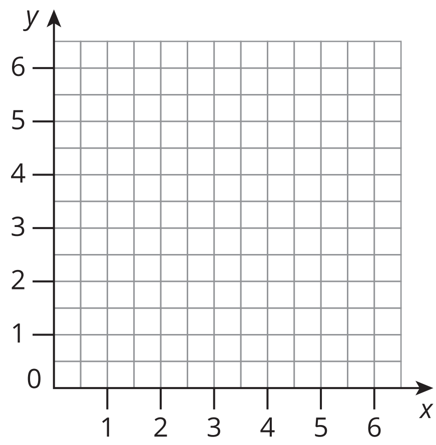
Use your graphing technology to determine the quadratic regression model and coefficient of determination for the line of best fit of the given data set. Determine if the model is a good fit for the data. Would a linear function be a better model? Round your answers to the nearest hundredth.

x	y
−5	−0.5
−4	5
−1	7
0	3
0.5	−1
1	3

Spaced Practice

1. Consider each function shown.

   a. Graph each function on the same coordinate plane.



$t(x) = (x - 3)^2$

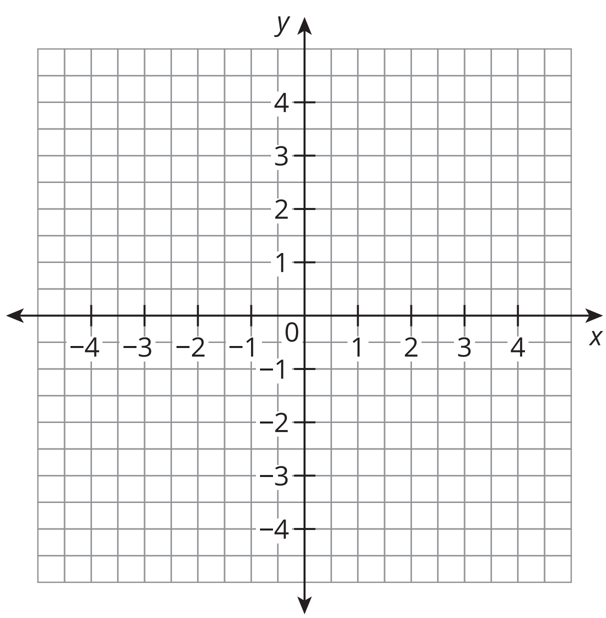
$w(x) = 3(x - 3)^2$

$z(x) = 3(x - 3)^2 + 1$

   b. Describe how functions w and z have been transformed from function t.

2. Consider the function $f(x) = x^2 - 2x - 2$.

   a. Graph the function.



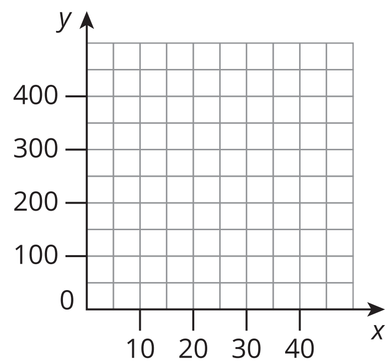
b. Describe the key characteristics of the graph.

3. The cost of producing chapter books for a company is $C(x) = 6x + 81$. The company's revenue for every chapter book sold is $R(x) = 36x - x^2$.

   a. What is the company's break-even point for the production and sales of chapter books?

   b. What does the solution mean?

   c. Show the solution graphically.



4. Factor each trinomial.

   a. $x^2 + 4x - 21$

   b. $5x^2 - 47x + 18$

5. Use technology to determine the quadratic function that models each data set. Round decimals to the nearest hundredth.

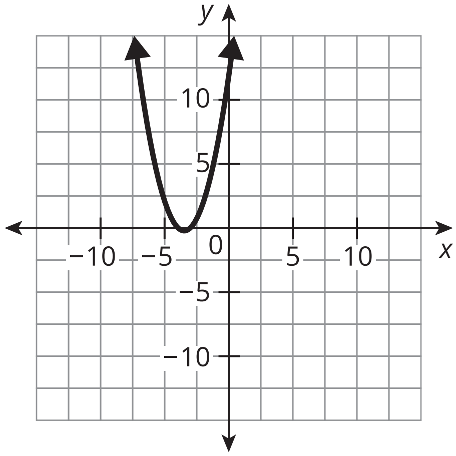
a.

x	y
5	18
7	13
10	9
13	7
17	5

b.

x	y
−6	9
−4	3
2	8
5	9
12	18

6. Write a function to represent the quadratic function shown on the grid.



ISBN: 198-9-79127-636-9

Printed in the USA

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