



Algebra I

Skills Practice

TEACHER EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

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Algebra I

Skills Practice Guide

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Skills Practice Overview

Deliberate practice is essential for all students to build fluency in mathematics. The goal of Skills Practice is to ensure that students receive continual reinforcement that intentionally connects concepts within the TEKS and provides an additional opportunity for review. This ongoing practice increases the likelihood that students will remember the new information rather than just committing it to rote memory. Students benefit the most from practice when teachers provide them with timely and descriptive feedback. Therefore, teachers should use this resource intentionally.



Skills Practice is used on Learning Individually days, which are scheduled within a topic by the teacher at their discretion. There are suggestions for when to schedule these days in the Topic Overviews. Generally, the Learning Individually days are scheduled after every 2–4 days of Learning Together instruction. The goal is to target skills that students still need to practice and develop to achieve proficiency following a lesson, or set of lessons. This allows teachers to provide just-in-time learning and intervention.

Each section of Skills Practice includes Topic Practice, Extension, and Spaced Practice. The Topic Practice problem sets can contain topic-aligned and/or interleaved practice sets that corresponds to the lesson content. While most problem sets meet the TEKS on grade level, similar to the corresponding lesson, some problem sets incorporate prerequisite skills, while others provide opportunities for extension. For this reason, not all problems need to be assigned to all students. The teacher should intentionally assign specific problem sets to the class, small groups of students, and/or individual students based on data. Activities within the lesson correspond to Skills Practice problem sets and are identified in Skills Practice Alignment notes for teachers at point of use. Use the identified activities to determine how you will gather data and interpret student's mathematical strengths, weaknesses and/or gaps, and common misconceptions. Formative data collection may include the lesson's Talk the Talk activity, an exit ticket, student assignment performance, observation of class or small group discussions, and/or one-on-one conversations with students. The provided outline within this Skills Practice Guide will help teachers in strategically selecting which problem sets and problems to assign their students. This allows teachers to personalize learning to meet student needs.

Structure and Alignment

Skills Practice

TOPIC 1 Linear functions

Name _____ Date _____

Skills Practice is organized into sections and problem sets.

① The section number is aligned to the student lesson number of the topic.

② The section title is named with the title of the corresponding student lesson.

③ Each section consists of one or more problem set(s) that are aligned to the TEKS within the lesson and/or topic. The problem sets are lettered and directions are provided.

④ Each section includes an Extension for students who have demonstrated proficiency in grade-level content and skills and ready for a challenge.

⑤ Each section includes Spaced Practice for retrieving and reviewing skills. These problem sets can also be used as a diagnostic tool throughout the course to identify each student's proficiency level with the TEKS to make data-driven instructional decisions.

① I. Least Square Regressions ②

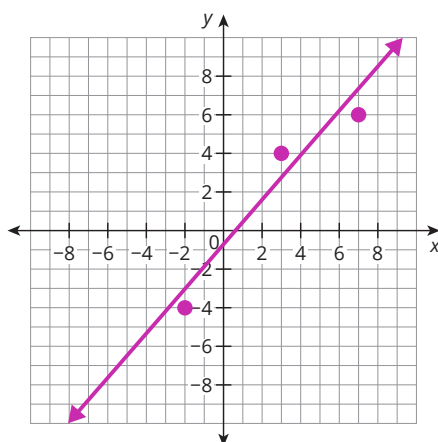
Topic Practice

- ③ A. Graph the estimated line of best fit for each set of points. Determine the estimated linear regression function for the line.

TEKS A.3C (Topic-Aligned), TEKS A.4C (Topic-Aligned)

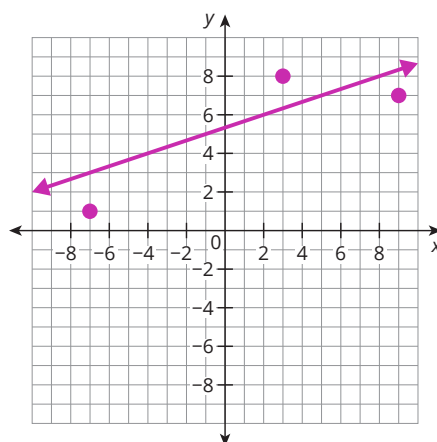
1. (3, 4), (7, 6), and (-2, -4)

Sample answer:



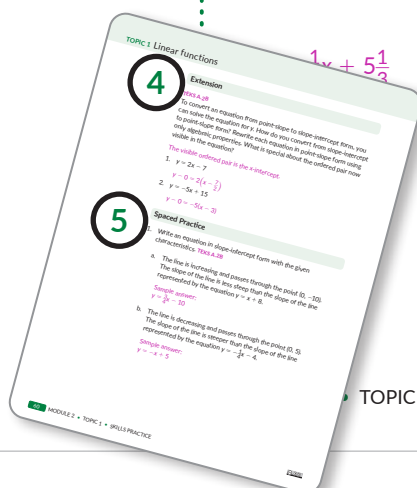
2. (-7, 1), (3, 8), and (9, 7)

Sample answer:



$$y = \frac{8}{7}x - \frac{5}{7}$$

$$y = \frac{1}{3}x + 5\frac{1}{3}$$



The TEKS-aligned Skills Practice is organized by topic for each course to align with the student lessons. Key features of the student and teacher's editions of the Skills Practice are noted.

The teacher's edition of Skills Practice has the same structure as the student edition of Skills Practice with some additional features.

6 Topic-Aligned

Each problem set is aligned to the TEKS within the lesson and/or topic and is identified in the teacher's edition of Skills Practice. This section is labeled as Topic-Aligned. Topic-Aligned indicates that students are practicing with skills to build proficiency in the aligned TEKS.

7 Answer Key

In each problem set, the answer key is provided in-line with the problems. When answers vary, a sample answer is provided.

Name _____ Date _____

I. Least Square Regressions

Topic Practice

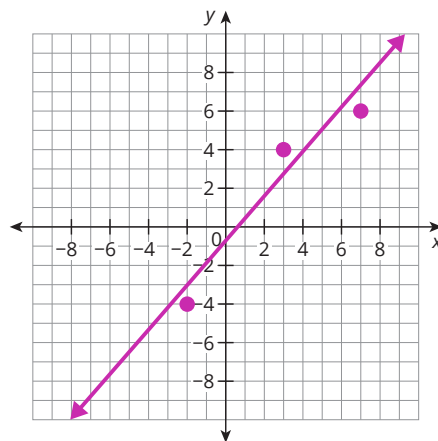
- A. Graph the estimated line of best fit for each set of points.
Determine the estimated linear regression function for the line.

TEKS A.3C (Topic-Aligned), TEKS A.4C (Topic-Aligned)

6

1. (3, 4), (7, 6), and (-2, -4)

Sample answer:

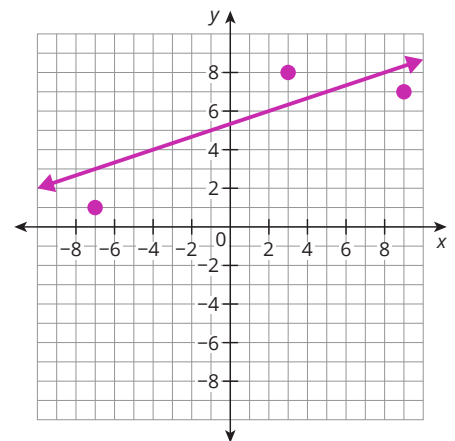


$$y = \frac{8}{7}x - \frac{5}{7}$$

2. (-7, 1), (3, 8), and (9, 7)

Sample answer:

7



$$y = \frac{1}{3}x + 5\frac{1}{3}$$

TOPIC 1 Quantities and Relationships

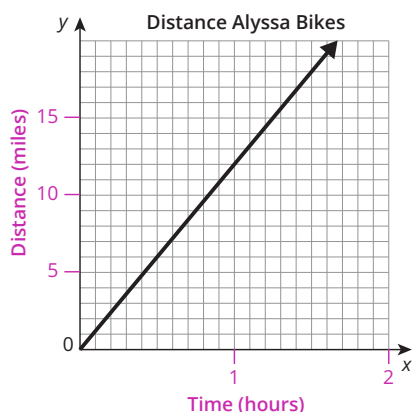
- B. Label the axes of each graph with the independent and dependent quantities. Include appropriate intervals and units of measure.**

TEKS A.3C (Interleaved Practice), TEKS A.7A (Interleaved Practice), TEKS A.9D (Interleaved Practice)

8

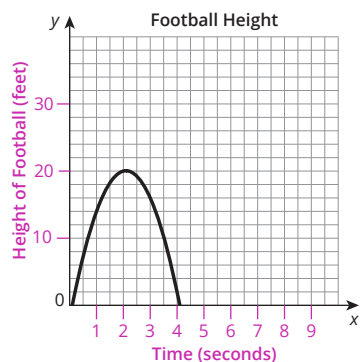
1. Alyssa enjoys bicycling for exercise. Each Saturday she bikes a course she has mapped out around her town. She averages a speed of 12 miles per hour on her journey.

Sample answer:



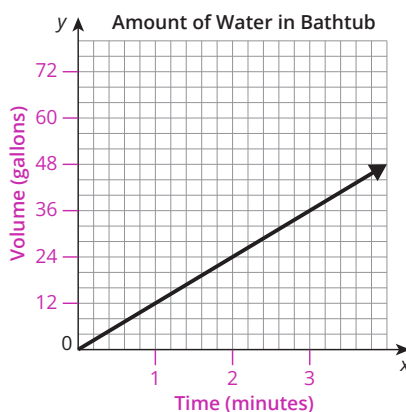
3. Mario throws a football straight up into the air. After the football reaches its maximum height of 20 feet, it descends back to the ground.

Sample answer:



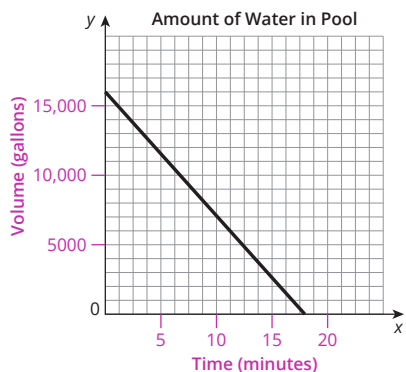
2. Natalia is filling the bathtub with water in order to give her dog Buster a bath. The faucet fills the tub at an average rate of 12 gallons per minute.

Sample answer:



4. Kaya is using a pump to drain her backyard pool to get ready for winter. The pump removes the water at an average rate of 15 gallons per minute.

Sample answer:



8 Interleaved Practice

You will see TEKS within a Topic Practice section labeled as Interleaved Practice. Interleaved Practice indicates that students are practicing multiple skills together. Interleaved Practice requires students to examine the problem to choose a strategy. Interleaved Practice improves flexibility in applying learned skills by interleaving different types of problems.



9 “Prerequisite for TEKS” is the label for problem sets in which the necessary skills to achieve proficiency are foundational, aligned to the content of the corresponding student lesson, and scaffolded to the skills aligned to the grade-level TEKS. These problem sets should only be assigned when a student needs practice in a prerequisite skill prior to or concurrently with the TEKS it supports.

10 “Extension of TEKS” is the label for problem sets in which there are also skills that extend beyond the grade-level TEKS and are considered optional. These skills are included in the Skills Practice because they align with the extension portion of the corresponding student lesson and provide opportunities for enhanced learning.

Name _____ Date _____

I. Understanding Quantities and Their Relationships

Topic Practice

- A. Determine the independent and dependent quantities in each scenario. Be sure to include the appropriate units of measure for each quantity. **Prerequisite for TEKS A.2A**

- 9** Ashley is driving to visit her grandmother who lives 325 miles away from Ashley's home. She travels an average of 60 miles per hour.

Independent quantity: time (hours)

Dependent quantity: distance (miles)
2. Jamal works at a printing company. He is making T-shirts for a high school volleyball team. The press he runs can print three T-shirts per minute with the school's mascot.

Independent quantity: time (minutes)

Dependent quantity: number of T-shirts printed
3. On her way to work each morning, Tiara purchases a small cup of coffee for \$4.25 from the coffee shop.

Independent quantity: number of cups

Dependent quantity: cost (dollars)
4. Matthew enjoys rock climbing on the weekends. At some of the less challenging locations, he can climb upwards of 12 feet per minute.

Independent quantity: time (minutes)

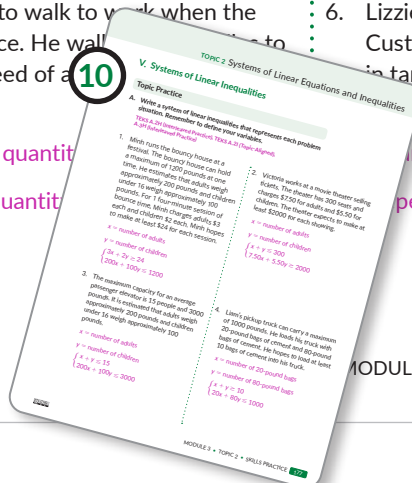
Dependent quantity: distance climbed (feet)
5. Josh prefers to walk to work when the weather is nice. He walks at a speed of 3 miles per hour.

Independent quantity: time (hours)

Dependent quantity: distance (miles)
6. Lizzie works for a skydiving company. Customers pay \$200 per jump to skydive in tandem skydives with Lizzie.

Independent quantity: number of jumps

Dependent quantity: cost (dollars)



Best Practices for Implementation of Skills Practice on Learning Individually Days

- Be intentional about grouping students to work on Skills Practice during Learning Individually days. Student groups need to be productive and not dependent on the teacher to make progress. Consider using small groups in a stations format. The teacher can be positioned at a specific station to facilitate small group instruction, or they can intentionally move around the room to observe student progress and assist small groups as needed.
- Encourage students to show their work and answer questions with complete sentences when appropriate. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context. Having students show their work gives the teacher more information about what a student does and does not know. This can help teachers give specific feedback to students beyond telling them if their answer is correct or incorrect.
- As students work, actively monitor their progress. Provide additional support to those struggling with a problem set. Use questions to help students engage in productive struggle. To help prepare questions ahead of time, consider the Questions to Support Discourse from the Teacher's Implementation Guide for the corresponding lesson.
- Problems within a set are repetitive by design. Students do not necessarily need to complete all problems within a set. Consider assigning half the problems and then have students check their work. The teacher can give feedback and decide whether further practice is needed or whether students can move on to practicing another problem set.
- When all students have achieved proficiency of the content for a particular lesson, consider not scheduling a Learning Individually day. When some students have achieved proficiency of the skills for a particular lesson, or have already completed the Skills Practice assigned to them, consider using those students as leaders of a specific group and/or station to help support their peers. This strategy helps students who need extra support while also developing the capacity of the students who provide assistance.

Preparing for Learning Individually with Skills Practice

Plan a Learning Individually day just as you would plan for a Learning Together day. Use student data to strategically schedule the Learning Individually days. Keep in mind, the topic pacing guide reflects suggestions for the placement of Learning Individually days. Ultimately, the teacher decides when it makes the most sense to schedule these days based on student need.

Tips for Planning a Learning Individually Day

- Read through the Skills Practice Outline for the specific Topic Skills Practice you are planning.
- Determine which problem sets align with the activities in the corresponding student lesson.
- Using student data, select which problems to assign the class, small groups, and/or individuals so that all students are practicing targeted skills to develop proficiency.
- Begin class with a routine, and have a warm-up or other activity to engage students in the learning.
- Set clear expectations for the structure of the Learning Individually day to maximize class time. Set up the classroom environment for stations or small groups, providing space for the teacher or student leaders to circulate around the room to support students.
- Pre-plan student groupings and determine which group(s) of students need the most targeted support.
- After students have been placed in small groups to begin working, determine the teacher's role for supporting students during this time. Some options are:
 - Circulate the room, monitoring small groups and stepping in to provide support as needed.
 - Lead a specific station to provide small group instruction.
- To keep from giving students answers, have questions prepared to help them when they get stuck. To help prepare questions, consider the Questions to Support Discourse from the Teacher's Implementation Guide for the corresponding lesson.
- When there are problem sets that the majority of students were assigned, take time to discuss these problems as a class. Select students to share their work and explain their thinking. Allow time for questions and discussion.
- Identify or create a closing activity, such as an exit ticket or a writing activity, to reflect on progress.

You Might Be Wondering . . .

Can we use Skills Practice in place of the Learning Together lesson?

No, Skills Practice should not be used in place of the Learning Together lesson. The core instructional resource for learning new concepts and skills is the Learning Together component. Research from books such as *How People Learn: Brain, Mind, Experience, and School: Expanded Edition* demonstrate and explain how learning is enhanced through socially supported interactions (National Research Council, 2000). Skills Practice provides students the opportunity to engage with problems that target each lesson's skills, concepts, and applications for development and proficiency. Individual practice is necessary for students to become fluent and build automaticity in a skill. A balance of these two components provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers while demonstrating their understanding individually.

Do my students complete all Skills Practice problems?

No, students do not need to complete every problem or set in Skills Practice. The problem sets within a section of Skills Practice scaffold the concepts and skills from the corresponding lesson. While most problem sets meet the TEKS on grade level, similar to the corresponding lesson, some problem sets incorporate prerequisite skills, while others provide opportunities for extension. Student data may indicate students have already demonstrated proficiency of a particular skill. For these reasons, not all sections, problem sets, or individual problems need to be assigned to all students. The teacher should use student data to strategically assign specific problem sets to the class, small groups, and/or individual students.

Skills Practice Outline

1 Searching for Patterns

Topic 1: Quantities and Relationships

Section	Problem Set	TEKS
I. Understanding Quantities and Their Relationships	A. Students determine the independent and dependent quantities in scenarios.	A.2A (Prerequisite)
	B. Students label the axes graphs with the independent and dependent quantities.	A.3C (Interleaved Practice) A.7A (Interleaved Practice) A.9D (Interleaved Practice)
	Extension	(E) A.2A
	Spaced Practice	(P) A.5A
II. Analyzing and Sorting Graphs	A. Students sort a variety of graphs into groups by identifying whether they are functions, whether they have a maximum or minimum, and whether they are increasing only or decreasing only.	A.3C (Interleaved Practice) A.7A (Interleaved Practice) A.9D (Interleaved Practice) A.12A (Topic-Aligned)
	Extension	(E) A.3C
	Spaced Practice	A.3C , (P) A.5A
III. Recognizing Functions and Function Families	A. Students determine whether tables, graphs, and mappings represent functions.	A.12A (Topic-Aligned)
	B. Students write the domain and the range of graphed functions in words and using inequalities.	A.2A (Interleaved Practice) A.6A (Interleaved Practice) A.9A (Interleaved Practice)
	C. Students write the domain and the range of problem situations in words and using inequalities.	A.2A (Interleaved Practice) A.6A (Interleaved Practice) A.9A (Interleaved Practice)
	Extension	2A.2A, (E) A.7A
	Spaced Practice	A.2A , (P) A.5A , A.7A

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

IV. Recognizing Functions by Characteristics	A. Students determine whether graphs represent a linear, quadratic, or an exponential function.	A.3C (Interleaved Practice) A.7A (Interleaved Practice) A.9D (Interleaved Practice)
	B. Students create and sketch graphs with given sets of characteristics.	A.3C (Interleaved Practice) A.7A (Interleaved Practice) A.9D (Interleaved Practice)
	Extension	2A.2A
	Spaced Practice	A.7A, (P)A.5A, A.9D

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

Topic 2: Sequences		
Section	Problem Set	TEKS
I. Recognizing Patterns and Sequences	A. Students describe patterns and draw figures to extend patterns.	(P)A.12D
	B. Students represent patterns and situations with numeric sequences.	(P)A.12D
	Extension	(E)A.12D
	Spaced Practice	A.9A, A.9D
II. Arithmetic and Geometric Sequences	A. Students determine common differences and predict terms in arithmetic sequences.	(P)A.12C (P)A.12D
	B. Students determine common ratios and predict terms in geometric sequences.	(P)A.12C (P)A.12D
	C. Students determine whether sequences are arithmetic or geometric, and they also predict terms.	A.12C (Topic-Aligned) (P)A.12D
	Extension	(E)A.12D
	Spaced Practice	A.3C, A.7A , A.9D, A.12D
III. Determining Recursive and Explicit Expressions from Context	A. Students determine whether sequences are arithmetic or geometric, and they represent sequences recursively and explicitly.	A.12D (Topic-Aligned)
	B. Students determine specified terms of sequences.	A.12C (Interleaved Practice)
	C. Students represent scenarios using explicit and recursive formulas.	A.12D (Topic-Aligned)
	D. Students write explicit formulas to represent terms of sequences and graph the sequences.	A.12D (Topic-Aligned)
	Extension	(E)A.12C
	Spaced Practice	A.2A, A.7A, A.9D , A.12C

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2 Exploring Constant Change

Topic 1: Linear Functions

Section	Problem Set	TEKS
I. Least Squares Regression	A. Students graph sets of points and an estimated line of best fit. Then, they determine a linear regression model for the line.	A.3C (Topic-Aligned) A.4C (Topic-Aligned)
	B. Students use linear regression to analyze scenarios.	A.4C (Topic-Aligned)
	Extension	(E)A.4C
	Spaced Practice	A.3C , A.12D
II. Correlation	A. Students examine scatterplots to determine whether they have a positive correlation, negative correlation, or no correlation. They also estimate r-values.	A.4A (Topic-Aligned)
	B. Students write linear regression models for data sets presented as tables.	A.4A (Interleaved Practice) A.4B (Interleaved Practice) A.4C (Interleaved Practice)
	C. Students examine situations to decide whether a correlation implies causation.	A.4B (Topic-Aligned)
	D. Students use data in a table to create a scatterplot, use technology to create a line of best fit, determine the correlation coefficient, describe any confounding variables, and determine if the correlation implies causation.	A.4A (Interleaved Practice) A.4B (Interleaved Practice)
	Extension	(E)A.4C
	Spaced Practice	A.2A , A.3C , A.4C, A.12C
III. Making Connections Between Arithmetic Sequences and Linear Functions	A. Students determine slope given a graph.	A.3B (Topic-Aligned)
	B. Students write an explicit formula to represent arithmetic sequences presented as graphs, lists, and situations.	A.2C (Topic-Aligned) A.12D (Interleaved Practice)
	Extension	(E) A.3B
	Spaced Practice	A.4A, A.4C, A.12B
IV. Point-Slope Form of a Line	A. Students write equations in point-slope form and slope-intercept form given a point and a slope.	A.2B (Topic-Aligned)
	B. Students write equations in point-slope form and slope-intercept form given two points.	A.2B (Topic-Aligned) A.3A (Topic-Aligned)
	C. Students write equations in point-slope form and slope-intercept form given graphs and tables.	A.2B (Interleaved Practice) A.2C (Topic-Aligned) A.3A (Topic-Aligned)
	Extension	(E)A.2B
	Spaced Practice	7.4D , 8.4A, A.2B, A.3C

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

V. Using Linear Equations	A. Students graph equations given in slope-intercept form.	A.3C (Topic-Aligned)
	B. Students graph equations given in point-slope form.	A.3C (Topic-Aligned)
	C. Students graph equations given in standard form.	A.3C (Topic-Aligned)
	D. Students write equations, graph lines, and identify the slope and y-intercept for problem situations.	A.2C (Interleaved Practice) A.3C (Topic-Aligned)
	Extension	(E)A.2B
	Spaced Practice	8.5F, A.2B, A.5B
VI. Making Sense of Different Representations of a Linear Function	A. Students solve equations involving direct variation.	A.2D (Topic-Aligned)
	B. Students determine input and output values using functions and graphs.	A.2D (Interleaved Practice) A.12B (Topic-Aligned)
	C. Students examine graphs of lines and determine the x- and y-intercepts, slope, equation in slope-intercept form, and equation in factored form.	A.3A (Topic-Aligned) A.3C (Topic-Aligned)
	Extension	(E) A.2C , (E)A.10A
	Spaced Practice	A.2C , A.4B, A.5A

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

Topic 2: Transforming and Comparing Linear Functions		
Section	Problem Set	TEKS
I. Transforming Linear Functions	A. Students write the equation of a line that is parallel to a given line and passes through a given point.	A.2E (Topic-Aligned)
	B. Students graph transformed linear functions and describe the transformations.	A.3E (Topic-Aligned)
	C. Students write equations for transformed functions.	TEKS A.2E (Interleaved Practice) TEKS A.2G (Interleaved Practice) TEKS A.3E (Interleaved Practice)
	Extension	(E) A.2C , (E)A.3E
	Spaced Practice	A.2C , A.4A, A.4B, A.4C, A.12A,
II. Vertical and Horizontal Transformations of Linear Functions	A. Students graph transformed linear functions and describe the transformations.	A.3E (Topic-Aligned)
	B. Students match transformed functions to their graphs.	A.3E (Topic-Aligned)
	Extension	(E)A.3E
	Spaced Practice	A.2A , A.3C

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

III. Determining Slopes of Perpendicular Lines	A. Students write the equations of vertical and horizontal lines.	A.2F (Interleaved Practice) A.2G (Topic-Aligned)
	B. Students write the equation of a line that is perpendicular to a given line and passes through a given point.	A.2F (Topic-Aligned)
	Extension	(E)A.2F
	Spaced Practice	A.2E, A.4B, A.4C, A.12D
IV. Comparing Linear Functions in Different Forms	A. Students determine and compare rates of change and y-intercepts for functions represented in tables and graphs. Then, they determine and represent the domain and range of each function with inequalities.	A.2A (Topic-Aligned) A.3B (Interleaved Practice) A.3C (Topic-Aligned)
	Extension	(E) A.3C
	Spaced Practice	A.2C , A.2F, A.3C , A.3E, A.4B, A.12D

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

3 Modeling Linear Equations and Inequalities

Topic 1: Linear Equations and Inequalities

Section	Problem Set	TEKS
I. Solving Linear Equations	A. Students solve linear equations with variables on both sides, and they justify their steps.	A.5A (Topic-Aligned)
	B. Students determine whether equations have one solution, no solutions, or infinitely many solutions.	A.5A (Topic-Aligned)
	C. Students write linear equations to model problem situations, and they use those equations to solve problems.	A.5A (Topic-Aligned)
	Extension	(E) A.5A , (E)A.5B
	Spaced Practice	A.2B, A.3C , A.12A, A.12B
II. Literal Equations	A. Students convert linear equations from standard form to slope-intercept form and determine the slope.	A.3A (Topic-Aligned) A.12E (Topic-Aligned)
	B. Students convert linear equations from point-slope form to slope-intercept form.	A.12E (Topic-Aligned)
	C. Students convert linear equations from standard form to slope-intercept form.	A.12E (Topic-Aligned)
	D. Students rewrite formulas to solve for a given variable.	A.12E (Topic-Aligned)
	Extension	(E)A.12E
	Spaced Practice	A.2C , A.3C , A.3E, A.5A

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

III. Modeling Linear Inequalities	A. Students analyze a single problem scenario by writing and solving linear equations and inequalities that model different outcomes.	A.2C (Interleaved Practice) A.5A (Topic-Aligned) A.5B (Topic-Aligned)
	B. Students analyze a single problem scenario by writing and solving linear equations and inequalities that model different outcomes.	A.5A (Interleaved Practice) A.5B (Interleaved Practice)
	C. Students write and solve linear inequalities that model problem situations.	A.5B (Topic-Aligned)
	D. Students solve linear inequalities and graph the solution set on a number line.	A.5B (Topic-Aligned)
	Extension	(E)A.5B
	Spaced Practice	A.2C, A.3B, A.12A, A.12E

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

Topic 2: Systems of Linear Equations and Inequalities		
Section	Problem Set	TEKS
I. Using Graphing to Solve Systems of Equations	A. Students write systems of linear equations to model problem situations, and they solve the systems by graphing.	A.2I (Topic-Aligned) A.3F (Interleaved Practice) A.3G (Topic-Aligned) A.5C (Interleaved Practice)
	B. Students solve systems of linear equations by graphing.	A.3F (Topic-Aligned) A.3G (Interleaved Practice) A.5C (Topic-Aligned)
	Extension	(E) A.5C
	Spaced Practice	A.2C, A.3A, A.4A, A.4C, A.5A
II. Using Substitution to Solve Linear Systems	A. Students write systems of linear equations to model problem situations, and they solve the systems using the substitution method.	A.2I (Topic-Aligned) A.5C (Topic-Aligned)
	B. Students write a system of equations for given graphs and tables.	A.2I (Interleaved Practice)
	C. Students solve systems of linear equations using the substitution method.	A.5C (Topic-Aligned)
	Extension	(E)A.3F, (E) A.5C
	Spaced Practice	A.3F, A.3G, A.5A, A.5B, A.5C

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III. Using Linear Combinations to Solve a System of Linear Equations	A. Students solve systems of linear equations using the linear combinations method.	A.5C (Topic-Aligned)
	B. Students write systems of linear equations to model problem situations, and they solve the systems using the linear combinations method.	A.2I (Topic-Aligned) A.5C (Topic-Aligned)
	Extension	2A.3B, (E) A.5C
	Spaced Practice	A.2I , A.3F, A.3G, A.4C, A.5C
IV. Graphing Inequalities in Two Variables	A. Students determine whether a solid or dashed boundary line is appropriate for graphing two-variable linear inequalities.	(P) A.3D
	B. Students test the point (0, 0) to determine which half-plane should be shaded when graphing two-variable linear inequalities.	(P) A.3D
	C. Students graph two-variable linear inequalities.	A.3D (Topic-Aligned)
	D. Students write two-variable linear inequalities to model problem situations, and they graph the inequalities.	A.2H (Topic-Aligned) A.3D (Interleaved Practice)
	Extension	(E) A.3D
	Spaced Practice	A.2B, A.5C
V. Systems of Linear Inequalities	A. Students write systems of two-variable linear inequalities to model problem situations.	A.2H (Interleaved Practice) A.2I (Topic-Aligned) A.3H (Interleaved Practice)
	B. Students test given ordered pairs to determine if they are solutions to systems of linear inequalities.	A.3H (Topic-Aligned)
	C. Students graph systems of linear inequalities.	A.3H (Topic-Aligned)
	Extension	2A.3E, (E)A.2I
	Spaced Practice	A.2H, A.3D , A.4C, A.5A
VI. Solving Systems of Equations and Inequalities	A. Students write systems of linear equations to analyze problem situations and draw conclusions.	A.2I (Topic-Aligned) A.5C (Topic-Aligned)
	Extension	2A.3E, 2A.3F
	Spaced Practice	A.2B, A.2G, A.3A, A.3C , A.3D , A.12E

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

4 Investigating Growth and Decay

Topic 1: Introduction to Exponential Functions

Section	Problem Set	TEKS
I. Properties of Powers with Integer Exponents	A. Students rewrite simple expressions using the power to a power rule.	A.11B (Topic-Aligned)
	B. Students rewrite expressions using the product of powers and quotient of powers rules.	A.11B (Topic-Aligned)
	C. Students rewrite more complex expressions using the power to a power rule.	A.11B (Topic-Aligned)
	Extension	(E) A.11B
	Spaced Practice	A.2D, (P)A.11A
II. Analyzing Properties of Powers	A. Students rewrite expressions with negative and zero exponents.	A.11B (Topic-Aligned)
	B. Students justify steps in sample student work using the exponent rules.	A.11B (Interleaved Practice)
	C. Students identify mistakes in examples where expressions have been rewritten incorrectly, and they correct the mistakes.	A.11B (Interleaved Practice)
	Extension	(E) A.11B
	Spaced Practice	A.3A, A.11B
III. Geometric Sequences and Exponential Functions	A. Students write an explicit geometric formula to model a sequence, and they rewrite the formula in the form $a \cdot b^x$.	A.9C (Topic-Aligned) A.9D (Topic-Aligned) A.12D (Topic-Aligned)
	B. Students write exponential functions to model tables and graphs.	A.9C (Topic-Aligned) A.9D (Topic-Aligned)
	Extension	(E) A.9D
	Spaced Practice	A.2H, A.5C
IV. Rewriting Square roots	A. Students rewrite radical expressions by extracting perfect squares.	A.11A (Topic-Aligned)
	B. Students use the quotient property of radicals to rewrite radical expressions in simplest radical form.	A.11A (Topic-Aligned)
	Extension	(E)A.11A
	Spaced Practice	A.12C

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

V. Rational Exponents and Graphs of Exponential Functions	A. Students complete a table and a graph given an exponential function. They also identify key features of the function.	A.9A (Interleaved Practice) A.9D (Topic-Aligned) A.12B (Topic-Aligned)
	B. Students rewrite rational exponents using radicals.	A.11B (Topic-Aligned)
	C. Students rewrite radical expressions using rational exponents.	A.11B (Topic-Aligned)
	D. Students rewrite expressions with rational exponents using the properties of powers.	A.11B (Topic-Aligned)
	Extension	(E) A.11B
	Spaced Practice	A.2H, A.9B, A.9D, A.11B

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

Topic 2: Using Exponential Equations		
Section	Problem Set	TEKS
I. Exponential Equations for Growth and Decay	A. Students determine whether problem situations can be represented by simple interest or compound interest.	(P)A.9B
	B. Students write exponential functions based on tables of values.	A.9C (Topic-Aligned)
	C. Students write exponential functions to model compound interest problem situations.	A.9C (Topic-Aligned)
	Extension	2A.5D, (E)A.5D
	Spaced Practice	A.9C
II. Interpreting Parameters in Context	A. Students analyze a single population problem situation. They graph the function that models the situation, and they interpret key features of the function.	A.9B (Topic Aligned) A.9D (Topic-Aligned)
	B. Students write exponential functions to model population problem situations.	A.9C (Topic-Aligned)
	C. Students analyze a single compound interest problem situation. They write an exponential function to model the situation, graph the function, and interpret the meaning of several output values when the function is evaluated.	A.9C (Topic-Aligned) A.9D (Topic-Aligned) A.12B (Interleaved Practice)
	D. Students interpret the a - and b -values of exponential functions that model problem situations.	A.9B (Topic-Aligned)
	Extension	(A)A.9B, (E) A.9C , (E) A.11B
	Spaced Practice	A.3C, A.9C, A.9D, A.11B

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

III. Modeling Using Exponential Functions	A. Students determine exponential regression models and their related r-values given tables of data.	A.9E (Topic-Aligned)
	B. Students determine the domain and range of exponential graphs using both words and inequalities.	A.9A (Interleaved Practice)
	C. Students determine exponential regression models given tables of data and problem situations, and they also make predictions using the regression models.	A.9E (Topic-Aligned)
	D. Students analyze a single exponential problem situation which is presented as a scenario, equation, and scatterplot. They interpret all key features of the function in terms of the problem situation, and they use the function to make predictions.	A.9A (Topic-Aligned), A.9B (Topic-Aligned) A.9D (Topic-Aligned)
	Extension	(E)A.4C, (E)A.9E
	Spaced Practice	A.9C

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

5 Maximizing and Minimizing

Topic 1: Introduction to Quadratic Functions

Section	Problem Set	TEKS
I. Exploring Quadratic Functions	A. Students write quadratic functions to model problem situations involving area.	A.7A (Prerequisite) A.10A (Topic-Aligned)
	B. Students use technology to identify the maximum or minimum of a quadratic function, and they interpret it in the context of a problem situation.	A.7A (Topic-Aligned)
	C. Students use technology to graph vertical motion problem situations. They identify the maximum, zeros, domain, and range in each situation.	A.6A (Topic-Aligned) A.7A (Interleaved Practice)
	D. Students use technology to identify the vertex and equation of symmetry for vertical motion models.	A.7A (Topic-Aligned)
	Extension	(E) A.7A
	Spaced Practice	A.3E, A.12A

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

II. Key Characteristics of Quadratic Functions	A. Students calculate the first and second differences for tables of values.	A.7A (Prerequisite)
	B. Students determine the x-intercepts and the axis of symmetry given a quadratic function in factored form.	A.7A (Topic-Aligned) A.7B (Interleaved Practice)
	C. Students determine the vertex of a parabola given the function in standard form and either the axis of symmetry or two symmetric points.	A.7A (Topic-Aligned)
	D. Students determine the domain and range of a quadratic function given a graph of a parabola and then they write the equation for the function in factored form.	A.6A (Interleaved Practice), A.6C (Topic-Aligned)
	E. Students use technology to determine the x-intercepts of quadratic functions given in standard form.	A.6C (Interleaved Practice) A.7A (Topic-Aligned)
	F. Students determine a symmetric point on a parabola given one point and either the axis of symmetry or the vertex.	A.7A (Topic-Aligned)
	Extension	(E) A.7C
	Spaced Practice	A.7A , A.11B , A.12E
III. Quadratic Function Transformations	A. Students describe the transformations necessary to create given functions that fit the form $af(x - c) + d$, where $f(x) = x^2$.	A.7C (Topic-Aligned)
	B. Students write and graph functions given verbal descriptions of transformations that fit the form $af(x - c) + d$, where $f(x) = x^2$.	A.7A (Topic-Aligned) A.7C (Interleaved Practice)
	C. Students verbally describe transformations given graphs of the transformed functions of the form $af(x - c) + d$, where $f(x) = x^2$.	A.7C (Topic-Aligned)
	Extension	(E) A.7C
	Spaced Practice	A.2B , A.2H , A.3H , A.5C , A.7C

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

IV. Transformations of Quadratic Functions	A. Students describe the transformations necessary to create given functions that fit the form $af(x - c) + d$, where $f(x) = x^2$.	A.7C (Topic-Aligned)
	B. Students write and graph functions given descriptions of transformations that fit the form $af(x - c) + d$, where $f(x) = x^2$.	A.7C (Topic-Aligned)
	C. Students use technology to determine the vertex of quadratic functions given in standard form, and they rewrite the functions in vertex form.	A.6B (Interleaved Practice) A.7A (Topic-Aligned)
	D. Students determine the vertex of quadratic functions given in vertex form.	A.6B (Interleaved Practice) A.7A (Topic-Aligned)
	E. Students identify the form given quadratic functions and identify key features of each function.	A.6B (Prerequisite) A.6C (Prerequisite) A.7A (Topic-Aligned)
	F. Students write equations for quadratic functions given a vertex and one other point or two zeros and another point.	A.6B (Topic-Aligned) A.6C (Prerequisite)
	Extension	(E) A.7C
	Spaced Practice	A.7A

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

Topic 2: Polynomial Operations		
Section	Problem Set	TEKS
I. Adding and Subtracting Polynomials	A. Students rewrite polynomials in standard form.	A.10A (Prerequisite) A.10B (Prerequisite)
	B. Students rewrite polynomials with the fewest number of terms.	A.10A (Topic-Aligned)
	C. Students add and subtract polynomials.	A.10A (Interleaved Practice)
	Extension	(E)A.10A
	Spaced Practice	A.6A, A.7A, A.7C
II. Multiplying Polynomials	A. Students multiply polynomials using multiplication tables.	A.10B (Topic-Aligned)
	B. Students multiply polynomials using the distributive property.	A.10B (Topic-Aligned) A.10D (Topic-Aligned)
	C. Students multiply binomials using special products.	A.10B (Topic-Aligned)
	Extension	(E)A.10B
	Spaced Practice	A.7C, A.9D , (P)A.10A, A.10A

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

III. Polynomial Division	A. Students use the Factor Theorem to determine if a given binomial is a factor of a quadratic trinomial.	A.7B (Topic-Aligned)
	B. Students divide polynomials.	A.10C (Topic-Aligned)
	Extension	(E)A.10C
	Spaced Practice	A.10A, A.10B

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

Topic 3: Solving Quadratic Equations		
Section	Problem Set	TEKS
I. Representing Solutions to Quadratic Equations	A. Students solve quadratic equations using square roots.	A.8A (Topic-Aligned) A.11A (Interleaved Practice)
	B. Students solve algebraically for the roots of quadratic equations or the zeros of quadratic functions.	A.8A (Topic-Aligned)
	Extension	(E) A.7A , (E) A.8A
	Spaced Practice	A.7A , A.7C , A.10B
II. Solutions to Quadratic Equations in Vertex Form	A. Students determine the number of real zeros for quadratic functions given graphs.	A.7A (Topic-Aligned) A.8A (Prerequisite)
	B. Students solve quadratic equations using square roots.	A.8A (Topic-Aligned)
	C. Students graph quadratic functions and determine their real zeros.	A.7A (Topic-Aligned) A.8A (Interleaved Practice)
	Extension	(E)A.6C, (E)A.10B
	Spaced Practice	A.7A , A.7C , A.10F, A.11A
III. Factoring and Completing the Square	A. Students factor polynomials using the GCF, if possible.	A.10D (Topic-Aligned)
	B. Students factor trinomials.	A.10E (Topic-Aligned) A.10F (Topic-Aligned)
	C. Students solve quadratic equations by factoring.	A.8A (Topic-Aligned)
	D. Students determine the zeros of quadratic functions algebraically or by graphing.	A.7A (Interleaved Practice) A.8A (Interleaved Practice)
	E. Students complete the square given a binomial.	A.8A (Prerequisite)
	F. Students fill in a missing term to create a perfect square trinomial.	A.8A (Prerequisite)
	G. Students solve quadratic equations by completing the square.	A.8A (Topic-Aligned) A.10E (Interleaved Practice)
	Extension	(E) A.7C , (E)A.10B, (E)A.12B
	Spaced Practice	A.7A , A.7C

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

IV. The Quadratic Formula	A. Students write and solve quadratic equations presented within problem situations.	A.8A (Topic-Aligned)
	B. Students use the quadratic formula to determine the axis of symmetry, vertex, and roots of quadratic functions given in standard form.	A.7A (Topic-Aligned) A.8A (Topic-Aligned)
	C. Students determine the zeros or roots of functions or equations.	A.7A (Topic-Aligned) A.8A (Topic-Aligned)
	Extension	(E) A.8A
	Spaced Practice	A.7A, A.8A
V. Using Quadratic Functions to Model Data	A. Students use technology to create quadratic regression models that model tables of data.	A.8B (Topic-Aligned)
	B. Students use technology to write quadratic regression models that model scenarios presented with data sets. They determine the y-intercept, x-intercepts, domain, range, and vertex of the functions, and they interpret them in terms of the scenarios.	A.6A (Topic-Aligned) A.7A (Interleaved Practice) A.8B (Interleaved Practice)
	Extension	(E)A.4C, (E)A.8B
	Spaced Practice	A.7A, A.7C, A.8A

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

Searching for Patterns

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TOPIC 1: Quantities and Relationships

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III. Determining Recursive and Explicit Expressions from Contexts	34



Name _____ Date _____

I. Understanding Quantities and Their Relationships

Topic Practice

A. Determine the independent and dependent quantities in each scenario. Be sure to include the appropriate units of measure for each quantity. **Prerequisite for TEKS A.2A**

1. Ashley is driving to visit her grandmother who lives 325 miles away from Ashley's home. She travels an average of 60 miles per hour. Independent quantity: time (hours) Dependent quantity: distance (miles)	2. Jamal works at a printing company. He is making T-shirts for a high school volleyball team. The press he runs can print three T-shirts per minute with the school's mascot. Independent quantity: time (minutes) Dependent quantity: number of T-shirts printed
3. On her way to work each morning, Tiara purchases a small cup of coffee for \$4.25 from the coffee shop. Independent quantity: number of cups Dependent quantity: cost (dollars)	4. Matthew enjoys rock climbing on the weekends. At some of the less challenging locations, he can climb upwards of 12 feet per minute. Independent quantity: time (minutes) Dependent quantity: distance climbed (feet)
5. Josh prefers to walk to work when the weather is nice. He walks the 1.5 miles to work at a speed of about 3 miles per hour. Independent quantity: time (hours) Dependent quantity: distance (miles)	6. Lizzie works for a skydiving company. Customers pay \$200 per jump to skydive in tandem skydives with Lizzie. Independent quantity: number of jumps Dependent quantity: cost (dollars)



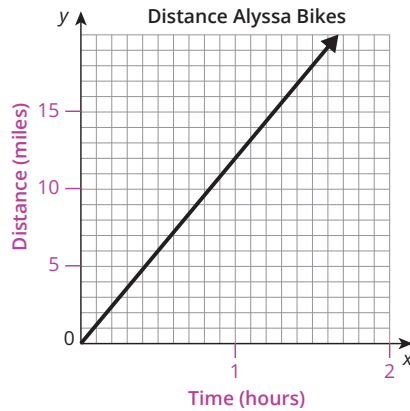
TOPIC 1 Quantities and Relationships

B. Label the axes of each graph with the independent and dependent quantities. Include appropriate intervals and units of measure.

TEKS A.3C (Interleaved Practice), TEKS A.7A (Interleaved Practice),
TEKS A.9D (Interleaved Practice)

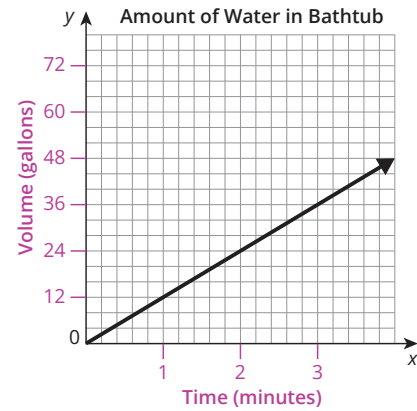
1. Alyssa enjoys bicycling for exercise. Each Saturday she bikes a course she has mapped out around her town. She averages a speed of 12 miles per hour on her journey.

Sample answer:



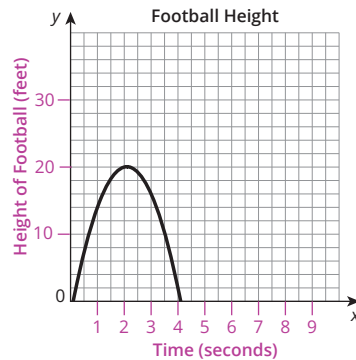
2. Natalia is filling the bathtub with water in order to give her dog Buster a bath. The faucet fills the tub at an average rate of 12 gallons per minute.

Sample answer:



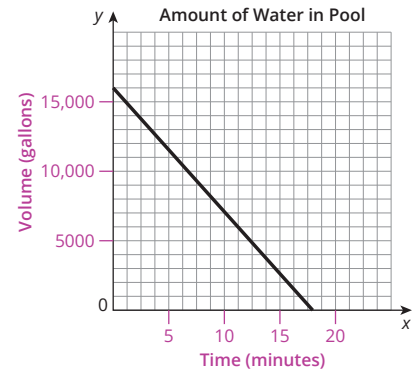
3. Mario throws a football straight up into the air. After the football reaches its maximum height of 20 feet, it descends back to the ground.

Sample answer:



4. Kaya is using a pump to drain her backyard pool to get ready for winter. The pump removes the water at an average rate of 15 gallons per minute.

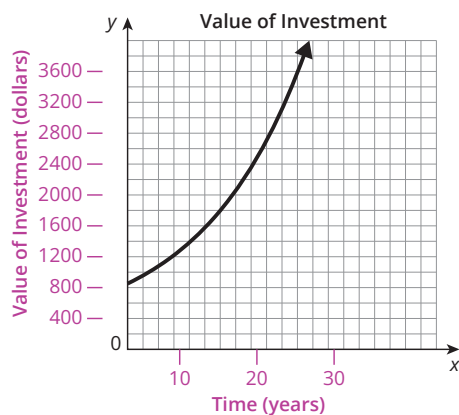
Sample answer:



TOPIC 1 Quantities and Relationships

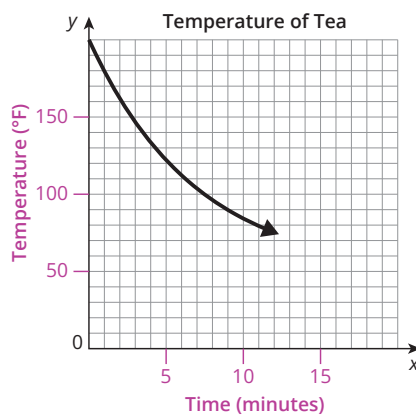
5. Andrew is saving money to purchase a used car. He places \$850 dollars in a savings account that earns 1.65% interest annually.

Sample answer:



6. A cup of hot tea is placed on a counter and begins to cool. The initial temperature of the tea was 200°F , and it cooled to room temperature.

Sample answer:



TOPIC 1 Quantities and Relationships

Extension

Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. **Extension of TEKS A.2A**

1. A student performs several experiments in which he swings a pendulum for a 20-second duration. He uses a string that is 27 cm long, and he tests pendulum masses of different sizes, varying from 2 to 12 grams. He records the number of swings each pendulum makes in 20 seconds.

Independent quantity: mass (grams)

Dependent quantity: number of swings

2. The student then decides to make a second graph showing the string length (in cm) as the independent quantity. What changes must the student make to his experiment?

The time will stay the same, and he will need to use one mass so that he can test several different string lengths.

Spaced Practice

1. Solve the equation $-2x + 8 = -3x + 14$. **Prerequisite for A.5A**

$$x = 6$$

2. Solve the equation $-3x - 6 = -5x + 8$. **Prerequisite for A.5A**

$$x = 14$$

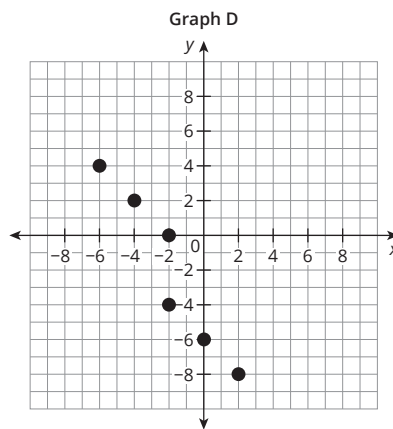
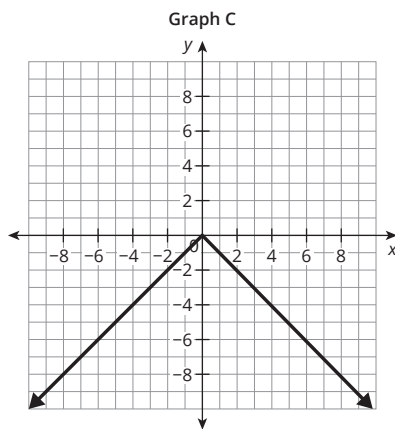
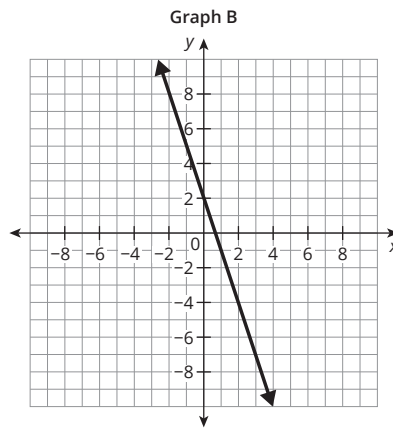
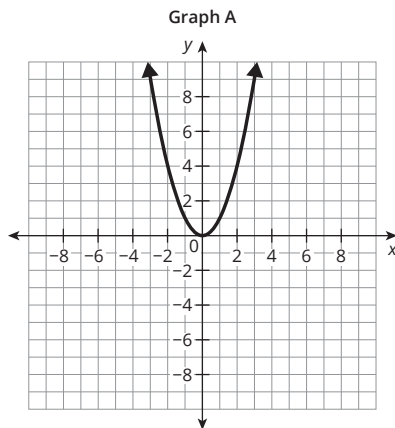
II. Analyzing and Sorting Graphs

Topic Practice

- A. Sort the graphs into groups. Write the letter of the graph in the group box. (Some graphs belong to more than one group.)

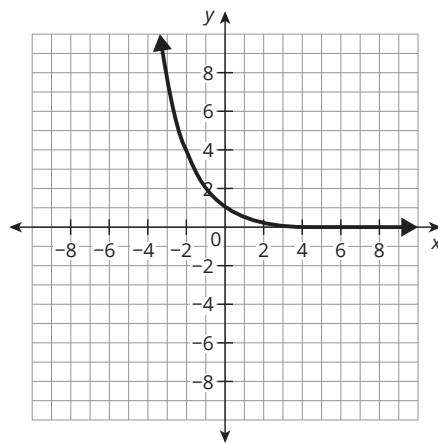
TEKS A.3C (Interleaved Practice), TEKS A.7A (Interleaved Practice),
TEKS A.9D (Interleaved Practice), TEKS A.12A (Topic-Aligned)

Function	Maximum or Minimum	Increasing Only	Decreasing Only
A, B, C, E, F, G, H, I, K	A, C, D, G, I, J	F, H, I	B, D, E

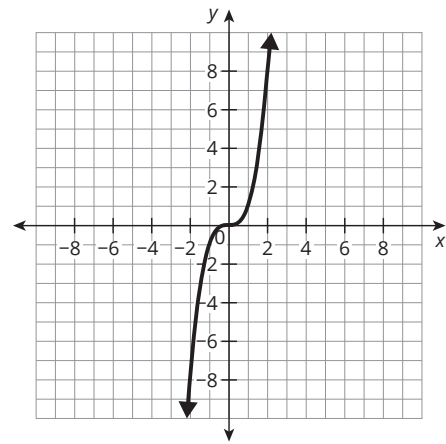


TOPIC 1 Quantities and Relationships

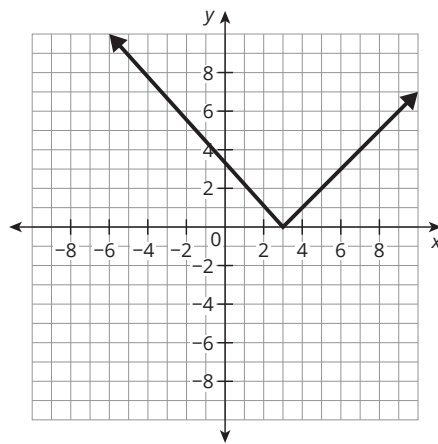
Graph E



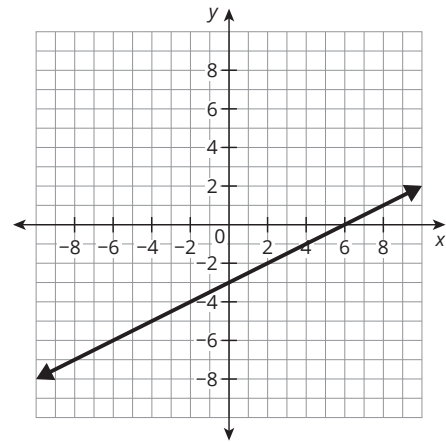
Graph F



Graph G

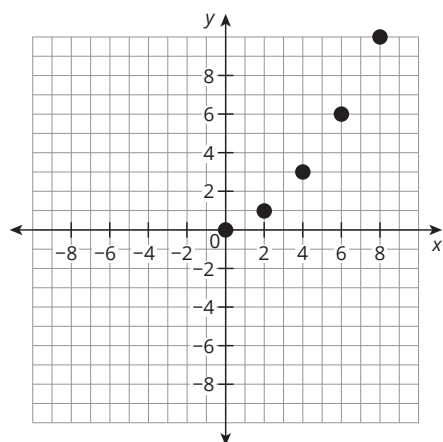


Graph H

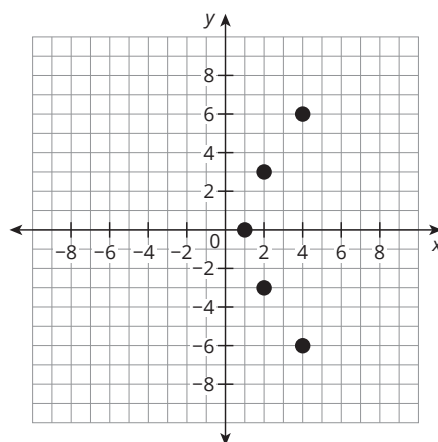


TOPIC 1 Quantities and Relationships

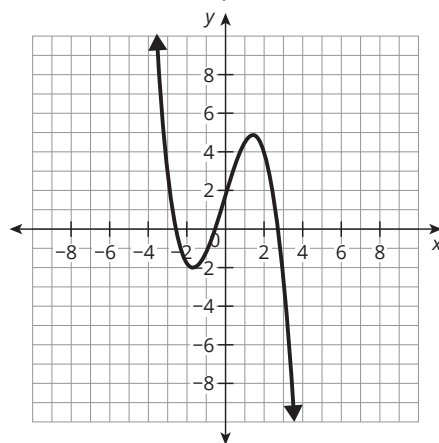
Graph I



Graph J



Graph K



TOPIC 1 Quantities and Relationships

Extension

Describe characteristics of each graph, including whether or not it has a vertical or horizontal axis of symmetry and the number of quadrants it passes through. **Extension of TEKS A.3C**

1. Diagonal line through the origin that increases from left to right

Passes through 2 quadrants

2. Diagonal line through the origin that decreases from left to right

Passes through 2 quadrants

3. Diagonal line that does not pass through the origin

Passes through 3 quadrants

4. Horizontal line below the origin

Vertical axis of symmetry, passes through 2 quadrants

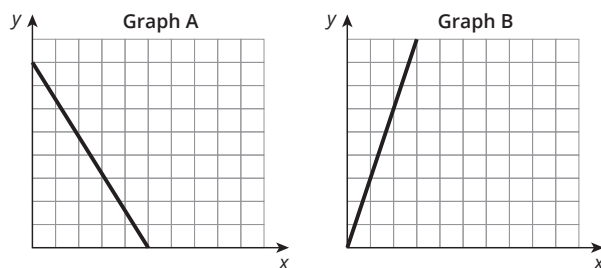
5. Vertical line to the right of the origin

Horizontal axis of symmetry, passes through 2 quadrants

Spaced Practice

1. Read the scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. Then determine which graph models the scenario.

Henry is cooking a turkey for his family. His recipe says to cook the turkey for 15 minutes per pound. **TEKS A.3C**



Independent quantity: Weight of Turkey (pounds)

Dependent quantity: Time (minutes)

Graph B

2. Solve the equation $8y + 13 = 29 - 3y$. Prerequisite for TEKS A.5A

$$y = \frac{16}{11}$$

3. Rewrite the expression $6z + 5(-2z - 7)$. Prerequisite for TEKS A.5A

$$-4z - 35$$

III. Recognizing Functions and Function Families

Topic Practice

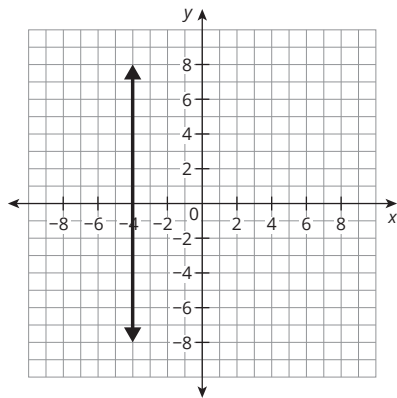
- A. Determine which relations represent functions. If the relation is not a function, state why not. TEKS A.12A (Interleaved Practice)

1.

Domain	Range
-5	8
-2	10
0	8
6	15

The relation is a function.

2.



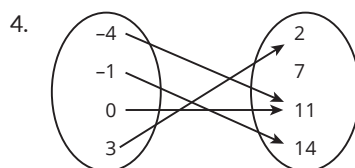
The relation is not a function, because it fails the vertical line test since there is more than one output, or y-value, for every input, or x-value.



TOPIC 1 Quantities and Relationships

3. $y = x^2 - 4$

The relation is a function.

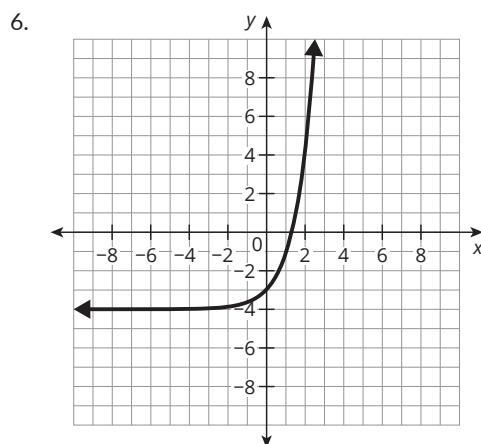


The relation is a function.

5.

Domain	Range
2	4
7	3
2	0
3	7

The relation is not a function, because the value of 2 in the domain is assigned to more than one element in the range.

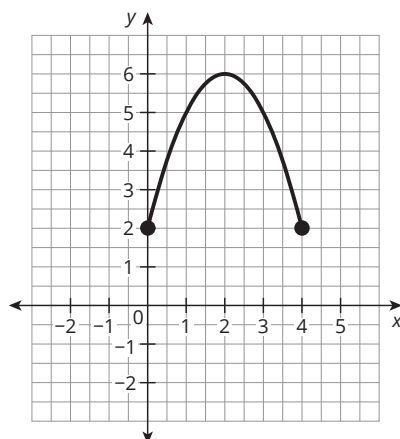


The relation is a function.

TOPIC 1 Quantities and Relationships

B. Write the domain and the range of each function in words and using inequalities notation. **TEKS A.2A (Interleaved Practice), TEKS A.6A (Interleaved Practice), TEKS A.9A (Interleaved Practice)**

1.



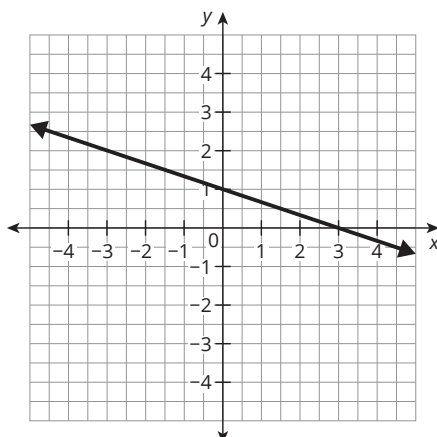
Domain in words: The domain is all real numbers greater than or equal to zero and less than or equal to 4.

Domain using inequalities: $0 \leq x \leq 4$

Range in words: The range is all real numbers greater than or equal to 2 and less than or equal to 6.

Range using inequalities: $2 \leq y \leq 6$

2.



Domain in words: The domain is all real numbers.

Domain using inequalities: $-\infty < x < \infty$

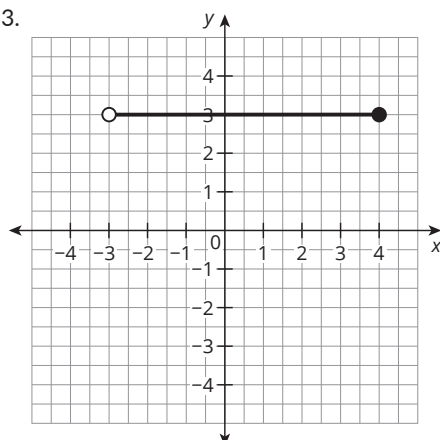
Range in words: The range is all real numbers.

Range using inequalities: $-\infty < y < \infty$



TOPIC 1 Quantities and Relationships

3.



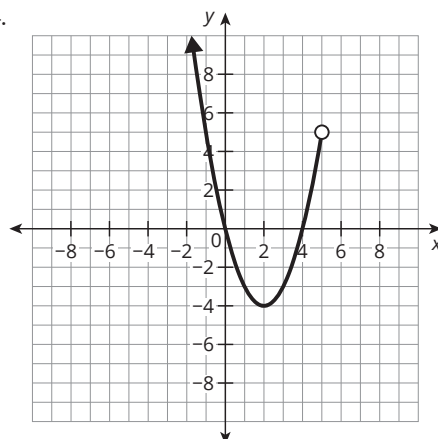
Domain in words: The domain is all real numbers greater than -3 and less than or equal to 4 .

Domain using inequalities: $-3 < x \leq 4$

Range in words: The range is three.

Range using inequalities: $x = 3$

4.



Domain in words: The domain is all real numbers less than 5 .

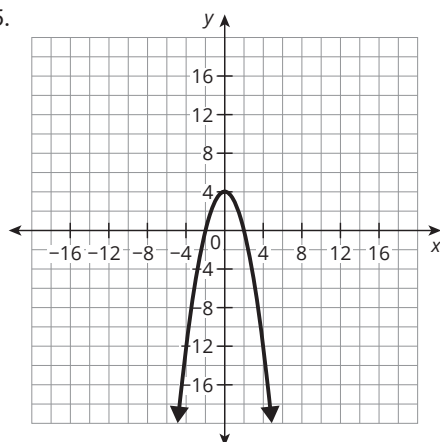
Domain using inequalities: $x < 5$

Range in words: The range is all real numbers greater than or equal to -4 .

Range using inequalities: $y \geq -4$

TOPIC 1 Quantities and Relationships

5.



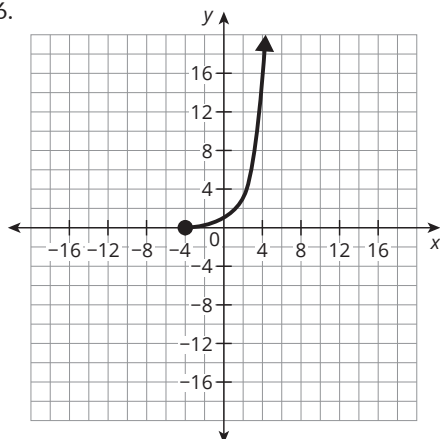
Domain in words: The domain is all real numbers.

Domain using inequalities: $-\infty < x < \infty$

Range in words: The range is all real numbers less than or equal to 4.

Range using inequalities: $y \leq 4$

6.



Domain in words: The domain is all real numbers greater than or equal to -4 .

Domain using inequalities: $y \geq -4$

Range in words: The range is all real numbers greater than or equal to 0.

Range using inequalities: $y \geq 0$



TOPIC 1 Quantities and Relationships

C. Identify the domain and range of each situation using words and inequalities. TEKS A.2A (Interleaved Practice), TEKS A.6A (Interleaved Practice), TEKS A.9A (Interleaved Practice)

1. Andrew is at an amusement park. While waiting in line, he reads the statistics on the roller coaster he is about to board. The coaster reaches a maximum speed of 75 miles per hour, and the ride lasts three minutes.
The domain is the time of the ride in minutes; $0 \leq x \leq 3$.
The range is the speed of the coaster in mph; $0 \leq y \leq 75$.
2. Alejandro uses a hot water bottle on an injury to his back he incurred playing basketball. He fills the bottle with water that is a temperature of 100°F . After 25 minutes, Alejandro finds that the bottle has cooled and he stops using it.
The domain is the time that Alejandro uses the water bottle in minutes; $0 \leq x \leq 25$.
Sample answer:
The range is the temperature of the water in $^{\circ}\text{F}$; $75 \leq y \leq 100$.
3. A township buys a new asphalt road paver. The paver costs \$50,500. The value of the paver decreases each year after purchase. Eventually, it decreases so much in value that it can only be sold for \$2,500 for parts.
The domain is the number of years that the paver is owned; $x \geq 0$.
The range is the dollar value of the asphalt road paver; $2,500 \leq y \leq 50,500$.
4. Trung and his friends are going to a local peach festival. They plan to go on rides all day. The tickets for the rides are \$0.50 each.
The domain is the number of tickets Trung and his friends buy; $x \geq 0$.
The range is cost of the tickets; $y = \{0.50, 1.00, 1.50, 2.00, \dots\}$.
5. Samantha is kayaking on a river. The cost to rent the kayak is \$25 per hour or \$200 for the entire day, from 8 a.m. to 8 p.m.
The domain is the amount of time the kayak is rented, in hours; $0 \leq x \leq 12$.
The range is the cost to rent the kayak in dollars; $0 \leq y \leq 200$.
6. Eduardo has a thermos that he takes to volleyball practice. The thermos can hold up to 18 ounces of water. The thermos develops a leak and starts losing water at a rate of 1 ounce every 2 minutes.
The domain is the amount of time water is leaking in minutes; $0 \leq x \leq 36$.
The range is the amount of water remaining in the thermos in ounces; $0 \leq y \leq 18$.

TOPIC 1 Quantities and Relationships

Extension

Graph both functions on the same screen using graphing technology. Use reasoning to classify the second function as a new family. Then describe the similarities and differences between the shapes of the graphs in terms of intervals of increase and decrease, maximums or minimums, and whether they are curves or lines. **TEKS 2A.2A; Extension of A.7A**

$$h(x) = x^2 + 9x + 14 \quad p(x) = |x^2 + 9x| + 14$$

Similarities and Differences:

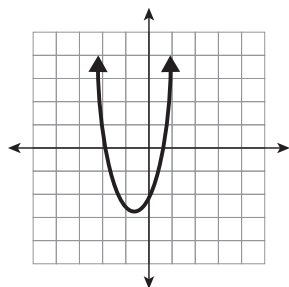
Sample answer:

The graphs are very similar in shape. Both decrease at the same rate on the left and increase at the same rate on the right. In the quadratic absolute value function, the parabola shape flips vertically at approximately $y = 14$.

Spaced Practice

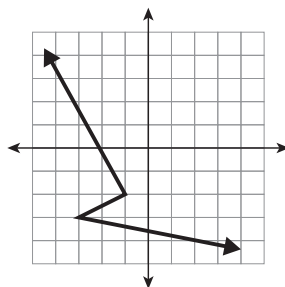
1. Identify the axis of symmetry each graph has, if any, and identify the number of quadrants it passes through. **TEKS A.7A, A.2A**

a.



Vertical axis of symmetry, passes through all 4 quadrants

b.



No axis of symmetry, passes through 3 quadrants

2. Solve the equation $18n + 40 = 14n + 16$. **Prerequisite for TEKS A.5A**

$$n = -6$$

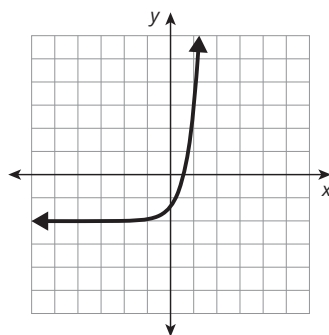


IV. Recognizing Functions by Characteristics

Topic Practice

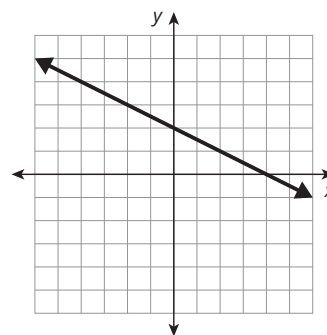
- A. Determine whether each graph represents a linear function, a quadratic function, or an exponential function. Then, identify if the function is increasing, decreasing, or both, and if the graph is continuous or discrete. **TEKS A.3C (Interleaved Practice), TEKS A.7A (Interleaved Practice), TEKS A.9D (Interleaved Practice)**

1.



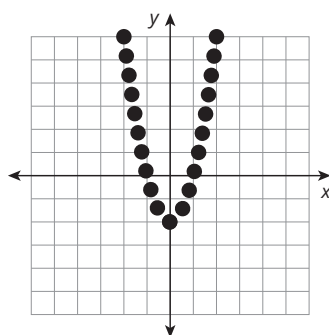
The graph represents an exponential function that is increasing and continuous.

2.



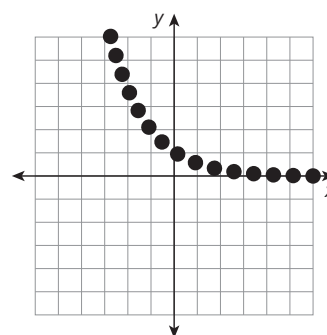
The graph represents a linear function that is decreasing and continuous.

3.



The graph represents a quadratic function that is both decreasing and increasing, and discrete.

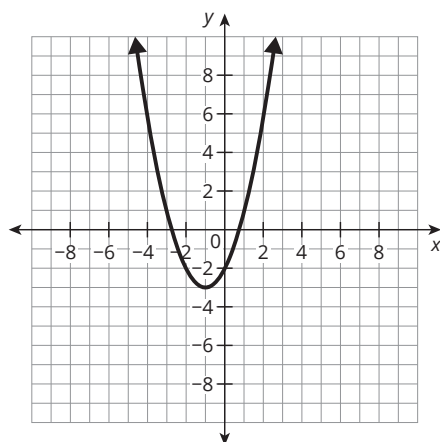
4.



The graph represents an exponential function that is decreasing and discrete.

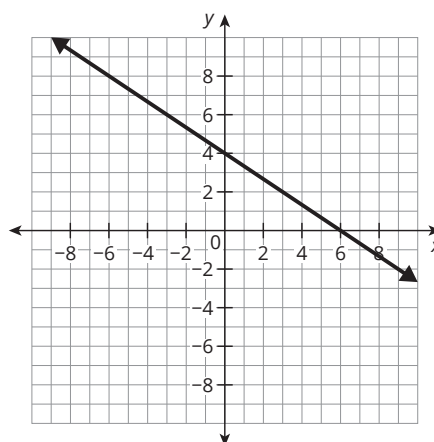
TOPIC 1 Quantities and Relationships

5.



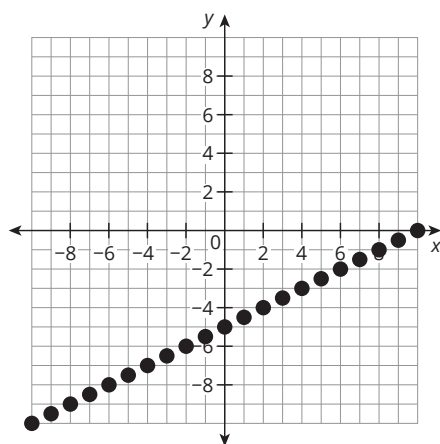
The graph represents a quadratic function that is both decreasing and increasing, and continuous.

6.



The graph represents a linear function that is decreasing and continuous.

7.



The graph represents a linear function that is increasing and discrete.



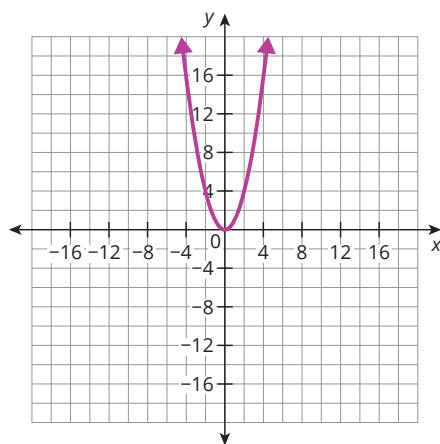
TOPIC 1 Quantities and Relationships

B. Create an equation and sketch a graph for a function with each set of given characteristics. Use values that are any real numbers between -10 and 10 . TEKS A.3C (Interleaved Practice), TEKS A.7A (Interleaved Practice), TEKS A.9D (Interleaved Practice)

1. Create an equation and sketch a graph that:
- is a smooth curve,
 - is continuous,
 - has a minimum, and
 - is quadratic.

Sample answer:

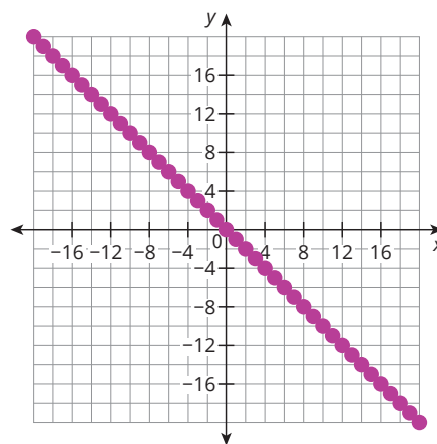
$$y = x^2$$



2. Create an equation and sketch a graph that is:
- linear,
 - discrete, and
 - decreasing across the entire domain.

Sample answer:

$$f(x) = -x, \text{ where } x \text{ is an integer}$$

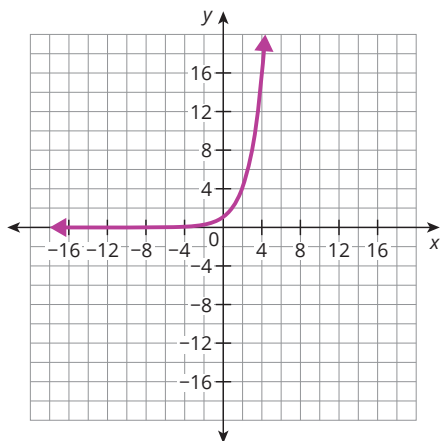


TOPIC 1 Quantities and Relationships

3. Create an equation and sketch a graph that is:
- a smooth curve,
 - increasing across the entire domain,
 - continuous, and
 - exponential.

Sample answer:

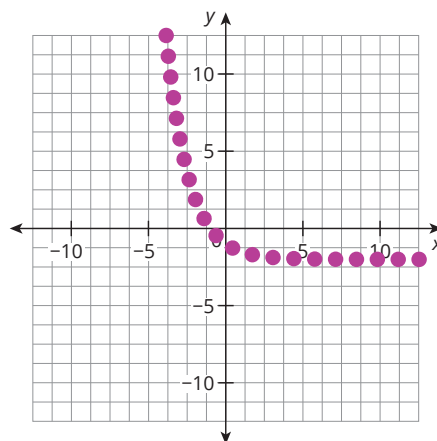
$$f(x) = 2^x$$



4. Create an equation and sketch a graph that:
- is exponential,
 - is discrete,
 - is decreasing across the entire domain, and
 - does not cross the line $y = -2$

Sample answer:

$$f(x) = 0.5^x - 2$$



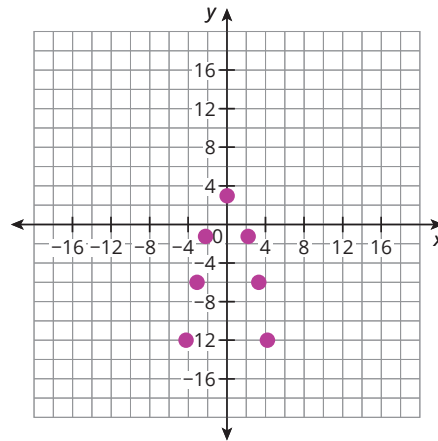
TOPIC 1 Quantities and Relationships

5. Create an equation and sketch a graph that:

- is discrete,
- has a maximum,
- does not pass through the origin, and
- is quadratic.

Sample answer:

$$f(x) = -x^2 + 3, \text{ where } x \text{ is an integer}$$



Extension

Write an equation and sketch a graph that has a minimum in Quadrant IV, is continuous, and is a linear absolute value function.

TEKS 2A.2A; Extension of A.7A

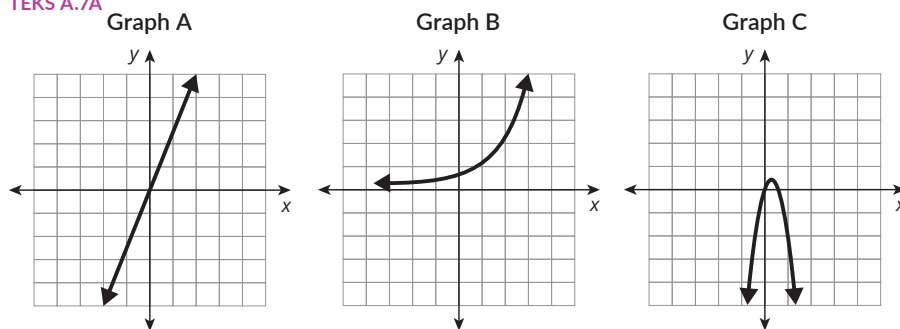
Sample answer:

$$f(x) = |x - 4| - 3$$

Spaced Practice

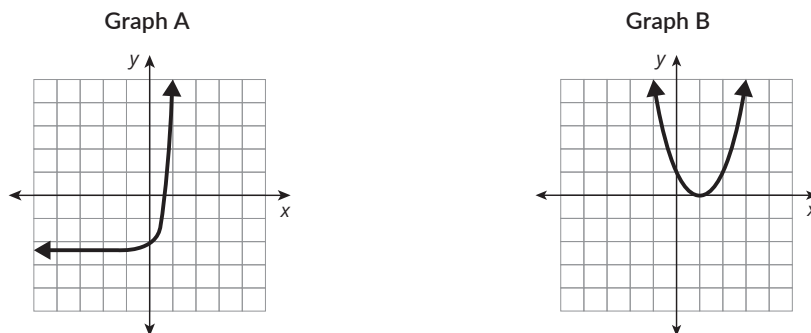
1. Choose the graph that represents the function $f(x) = -x^2 + 3x$.

TEKS A.7A



Graph C

2. Which graph represents an exponential function? TEKS A.9D



Graph A

3. Solve the equation $68 = -7 - 15b$. Prerequisite for A.5A

$$b = -5$$



Name _____ Date _____

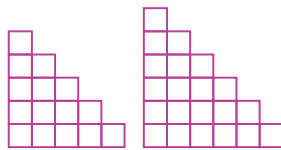
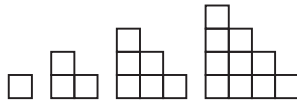
I. Recognizing Patterns and Sequences

Topic Practice

A. Describe each pattern. Draw the next two figures in the pattern.

Prerequisite for TEKS A.12D

1.



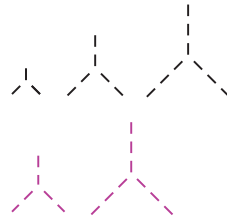
Sample answer:
The second figure has 2 more squares than the first, the third figure has 3 more squares than the second, and the fourth figure has 4 more squares than the third.

3.



Sample answer:
Each figure has 2 more circles than the previous figure.

2.



Sample answer:
Each figure has 3 more line segments than the previous figure.

4.



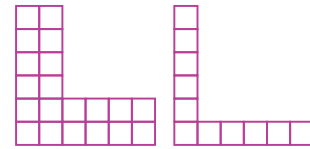
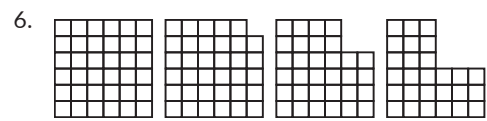
Sample answer:
The second figure has 9 blocks less than the first figure and the third figure has 7 blocks less than the second figure. The number of blocks being removed from one figure to the next decreases by 2 each time.



TOPIC 2 Sequences



Sample answer:
Each figure has twice as many triangles as the previous figure.



Sample answer:
The second figure has 1 less square than the first, the third figure has 3 less squares than the second, and the fourth figure has 5 less squares than the third. The number of squares being removed from one figure to the next starts at 1 and then increases to the next larger odd integer each time.

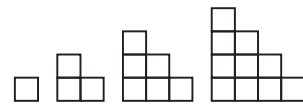
B. Represent each pattern or situation with a numeric sequence.

Prerequisite for TEKS A.12D

1. The school cafeteria begins the day with a supply of 1000 chicken nuggets. Each student that passes through the lunch line is given 5 chicken nuggets. Represent the total number of chicken nuggets remaining in the cafeteria's supply after each of the first 6 students pass through the line with a numeric sequence. Include the number of chicken nuggets the cafeteria started with.

1000, 995, 990, 985, 980, 975, 970

2. Represent the number of squares in each of the first 7 figures of the pattern with a numeric sequence.



1, 3, 6, 10, 15, 21, 28

3. Alyssa starts a job at a restaurant. She deposits \$40 from each paycheck into her savings account. There was no money in the account prior to her first deposit. Represent the amount of money in the savings account after Alyssa receives each of her first 6 paychecks with a numeric sequence.

\$40, \$80, \$120, \$160, \$200, \$240

5. Matthew is collecting canned goods for a food drive. On the first day he collects 1 can. On the second day he collects 2 cans. On the third day he collects 4 cans. On each successive day, he collects twice as many cans as he collected the previous day. Represent the total number of cans Matthew has collected by the end of each of the first 7 days of the food drive with a numeric sequence.

1, 3, 7, 15, 31, 63, 127

4. Represent the number of blocks in each of the first 5 figures of the pattern with a numeric sequence.



25, 16, 9, 4, 1

6. Represent the number of line segments in each of the first 7 figures of the pattern with a numeric sequence.



3, 6, 9, 12, 15, 18, 21

TOPIC 2 Sequences

Extension

Kaya's Kitchen

Kaya is opening a restaurant and tells her staff they have to go above and beyond to please their customers, especially on opening day. She reasons that if one customer is pleased with the restaurant, that person is likely to tell 4 people about it. Then each of those people is likely to tell 4 people about it, and so on.

- Describe the pattern for the number of customers Kaya's Kitchen will reach with each telling.
- Determine how many customers are reached after the 5th, 6th, and 7th tellings.
- Represent the number of customers reached with each telling as a numeric sequence. Then represent the sequence using a table of values.
- Identify the appropriate function family for the function. Then describe whether the function is continuous or discrete.

Extension of TEKS A.12D

Term Number	Term
1	4
2	16
3	64
4	256
5	1024
6	4096
7	16,384

Each telling reaches 4 times the customers than the previous telling.

After the 5th telling, 1024 customers will be reached.
After the 6th telling, 4096 customers will be reached.
After the 7th telling, 16,384 customers will be reached.

4, 16, 64, 256, 1024, 4096, 16,384

$$f(x) = 4^x$$

Exponential function

Discrete

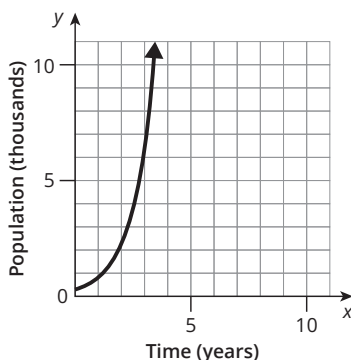
Spaced Practice

- For the scenario and graph:
 - Identify the appropriate function family.
 - Describe the domain based on the problem situation.
 - Identify the graphical behavior of the function as increasing, decreasing, or a combination.

A city discovers that its population has been tripling every year. The function graphed models the population (in thousands) after x years.

TEKS A.9A, A.9D

Exponential function; Real numbers greater than or equal to zero; Increasing



II. Arithmetic and Geometric Sequences

Topic Practice

- A. Determine the common difference and the next 3 terms in each arithmetic sequence. **Prerequisite for TEKS A.12C,**
Prerequisite for TEKS A.12D

1. 8, 14, 20, 26, _____, _____, _____, ...

Common difference: 6

Next 3 terms: 32; 38; 44

3. $\frac{3}{5}, \frac{4}{5}, 1, \frac{6}{5}, \text{---}, \text{---}, \text{---}, \dots$

Common difference: $\frac{1}{5}$

Next 3 terms: $\frac{7}{5}, \frac{8}{5}, \frac{9}{5}$

5. $-101, -112, -123, -134, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \dots$

Common difference: -11

Next 3 terms: -145 ; -156 ; -167

2. $-24, -14, -4, 6, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \dots$

Common difference: 10

Next 3 terms: 16; 26; 36

4. 12, 16.5, 21, 25.5, _____, _____, _____, ...

Common difference: 4.5

Next 3 terms: 30; 34.5; 39

6. 3.8, 5.1, 6.4, 7.7, —, —, —, ...

Common difference: 1.3

Next 3 terms: 9.0; 10.3; 11.6



TOPIC 2 Sequences

B. Determine the common ratio and the next 3 terms in each geometric sequence.

Prerequisite for TEKS A.12C, Prerequisite for TEKS A.12D

1. 3, 9, 27, 81, _____, _____, _____, ...

Common ratio: 3

Next 3 terms: 243; 729; 2187

3. 5, -10, 20, -40, _____, _____, _____, ...

Common ratio: -2

Next 3 terms: 80; -160; 320

5. 2, -2, 2, -2, _____, _____, _____, ...

Common ratio: -1

Next 3 terms: 2; -2; 2

2. 512, 256, 128, 64, _____, _____, _____, ...

Common ratio: $\frac{1}{2}$

Next 3 terms: 32; 16; 8

4. 3000, 300, 30, 3, _____, _____, _____, ...

Common ratio: 0.1

Next 3 terms: 0.3; 0.03; 0.003

6. 0.2, 1.2, 7.2, 43.2, _____, _____, _____, ...

Common ratio: 6

Next 3 terms: 259.2; 1555.2; 9331.2

C. Determine whether each given sequence is arithmetic or geometric. Write the next 3 terms of each sequence. TEKS A.12C (Topic-Aligned), Prerequisite for TEKS A.12D

1. $a_n = a_{n-1} + 4$ where $a_1 = 4$ and n is a whole number greater than 1.

The sequence is arithmetic. The next 3 terms are $a_2 = 8$, $a_3 = 12$, and $a_4 = 16$.

3. $p_n = 4 \cdot p_{n-1}$, where $p_1 = 3$ and n is a whole number greater than 1.

The sequence is geometric. The next 3 terms are $p_2 = 12$, $p_3 = 48$, and $p_4 = 192$.

5. $f(n) = \frac{1}{2}f(n-1)$, where $f(1) = 1.5$ and n is a whole number greater than 1.

The sequence is geometric. The next 3 terms are $f(2) = 0.75$, $f(3) = 0.375$, and $f(4) = 0.1875$.

2. $m_n = -0.8 + m_{n-1}$, where $m_1 = -7.2$ and n is a whole number greater than 1.

The sequence is arithmetic. The next 3 terms are $m_2 = -8$, $m_3 = -8.8$, and $m_4 = -9.6$.

4. $k_n = -2 \cdot k_{n-1}$, where $k_1 = 9$ and n is a whole number greater than 1.

The sequence is geometric. The next 3 terms are $k_2 = -18$, $k_3 = 36$, and $k_4 = -72$.

6. $h(n) = 3 + h(n-1)$, where $h(1) = 15$ and n is a whole number greater than 1.

The sequence is arithmetic. The next 3 terms are $h(2) = 18$, $h(3) = 21$, and $h(4) = 24$.

Extension

Consider the first 2 terms of the sequence $-6, 18, \dots$

1. Determine the next 5 terms in the sequence if the sequence is arithmetic. Then, write a function to represent the arithmetic sequence. **Extension of TEKS A.12D**

42, 66, 90, 114, 138

$$f(x) = 24x - 30$$

2. Determine the next 5 terms in the sequence if the sequence is geometric. Then, write a function to represent the geometric sequence. **Extension of TEKS A.12D**

$-54, 162, -486, 1458, -4374$

$$f(x) = 2 \cdot (-3)^x$$

Spaced Practice

1. Josh updates his blog regularly with trivia questions for readers to answer. The month he started this, there were 8 trivia questions on his blog. The next month, there were 19 trivia questions on his blog. The month after that, there were 30 trivia questions on his blog.

TEKS A.12D

- a. Think about the number of trivia questions on Josh's blog each month. Describe the pattern.

Each month, Josh's blog has 11 more trivia questions than the month before.

- b. Determine how many trivia questions will be on Josh's blog during months 4, 5, and 6.

Month 4: 41

Month 5: 52

Month 6: 63



TOPIC 2 Sequences

- c. Represent the number of trivia questions on Josh's blog for the first 6 months as a numeric sequence. Then, represent the sequence using a table of values.

8, 19, 30, 41, 52, 63

Term Number	Term
1	8
2	19
3	30
4	41
5	52
6	63

2. Contestants on a popular game show have an opportunity to randomly select a cash prize in 6 hidden containers. The highest possible cash prize is \$25,000. The next highest prize is \$5000, and the one after that is \$1000. **TEKS A.12D**
- a. Think about how the value of the prize changes from one container to the next. Describe the pattern.

Each container has $\frac{1}{5}$ the value of the previous container.

- b. Determine the prize values in the remaining containers.

Container 4: \$200

Container 5: \$40

Container 6: \$8

- c. Represent the prize values in all six containers as a numeric sequence. Then, represent the sequence using a table of values.

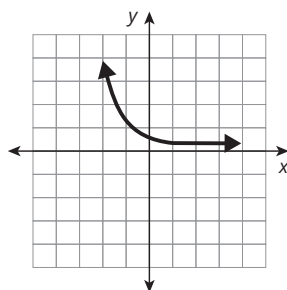
25,000, 5000, 1000, 200, 40, 8

Term Number	Term
1	25,000
2	5000
3	1000
4	200
5	40
6	8

3. Enter each function into your graphing calculator to determine the shape of its graph. Then, complete the table based on the characteristics of the function family. **TEKS A.3C, TEKS A.7A**

Function	Function Family	Increasing/Decreasing	Absolute Maximum/Minimum	Curve/Line
$h(x) = 5x^2 - 2.8x + 40$	Quadratic	Increasing and Decreasing	Absolute minimum	Smooth curve
$g(x) = 30x - 550$	Linear	Increasing	No maximum or minimum	Straight line

4. Identify the function family. **TEKS A.9D**



Exponential



III. Determining Recursive and Explicit Expressions from Contexts

Topic Practice

- A. Determine whether each sequence is arithmetic or geometric. Then, write a recursive and the simplified explicit formula for the sequence.

TEKS A.12D (Topic-Aligned)

1. 4, 8, 16, 32, ...

The sequence is geometric.

Recursive formula:

$$a_n = a_{n-1} \cdot 2$$

Explicit formula:

$$a_n = 4 \cdot 2^{n-1}$$

3. 2, -6, 18, -54, ...

The sequence is geometric.

Recursive formula:

$$a_n = a_{n-1} \cdot (-3)$$

Explicit formula:

$$a_n = 2 \cdot (-3)^{n-1}$$

5. 320, 410, 500, ...

The sequence is arithmetic.

Recursive formula:

$$a_n = a_{n-1} + 90$$

Explicit formula:

$$a_n = 320 + 90(n - 1)$$

$$a_n = 90n + 230$$

2. $a_1 = 16, a_2 = 30, a_3 = 44, a_4 = 58, \dots$

The sequence is arithmetic.

Recursive formula:

$$a_n = a_{n-1} + 14$$

Explicit formula:

$$a_n = 16 + 14(n - 1)$$

$$a_n = 14n + 2$$

4. $a_3 = 7.3, a_4 = 9.4, a_5 = 11.5,$
 $a_6 = 13.6, a_7 = 15.7, \dots$

The sequence is arithmetic.

Recursive formula:

$$a_n = a_{n-1} + 2.1$$

Explicit formula:

$$a_n = 3.1 + 2.1(n - 1)$$

$$a_n = 2.1n + 1$$

6. $a_3 = 63, a_4 = 189, a_5 = 567, a_6 = 1701, \dots$

The sequence is geometric.

Recursive formula:

$$a_n = a_{n-1} \cdot 3$$

Explicit formula:

$$a_n = 7 \cdot 3^{n-1}$$

B. Determine the specified terms of each sequence.

TEKS A.12C (Interleaved Practice)

1. What is the 10th term of the sequence

$$a_n = 3 \cdot 2^{n-1}?$$

$$a_{10} = 3 \cdot 2^{10-1}$$

$$a_{10} = 1536$$

The 10th term of the sequence is 1536.

2. What is the 50th term of the sequence

$$a_n = 100 + (-8)(n - 1)?$$

$$a_{50} = 100 + (-8)(50 - 1)$$

$$a_{50} = -292$$

The 50th term of the sequence is -292.

3. What is the 6th term of the sequence

$$a_n = a_{n-1} \cdot (-3), \text{ if } a_5 = 162?$$

$$a_6 = 162 \cdot (-3)$$

$$a_6 = -486$$

The 6th term of the sequence is -486.

4. What is the 12th term of the sequence

$$a_n = a_{n-1} + \frac{1}{3}, \text{ if the 11th term is } \frac{17}{3}?$$

$$a_{12} = \frac{17}{3} + \frac{1}{3}$$

$$a_{12} = 6$$

The 12th term of the sequence is 6.

5. What is the 23rd term of the sequence

$$a_n = a_{n-1} + 2.3, \text{ if the 21st term is 95.8?}$$

$$a_{22} = 95.8 + 2.3$$

$$a_{22} = 98.1$$

$$a_{23} = 98.1 + 2.3$$

$$a_{23} = 100.4$$

The 23rd term of the sequence is 100.4

6. What is the 19th term of the sequence

$$a_n = a_{n-1} \cdot \left(-\frac{1}{2}\right), \text{ if } a_{17} = 162?$$

$$a_{18} = 162 \cdot \left(-\frac{1}{2}\right)$$

$$a_{18} = -81$$

$$a_{19} = -81 \cdot \left(-\frac{1}{2}\right)$$

$$a_{19} = 40.5$$

The 19th term of the sequence is 40.5.



TOPIC 2 Sequences

C. Write a recursive and an explicit formula to represent the sequence that models each scenario. TEKS A.12D (Topic-Aligned)

1. The population of a city started at 378,000. Every year thereafter it decreased by 23,000 individuals.

Explicit formula: $a_n = 378,000 + (-23,000)(n - 1)$

$$a_n = -23,000n + 401,000$$

$$\text{Recursive formula: } a_n = a_{n-1} + (-23,000)$$

3. A new pet bakery sold 20 dog cakes during Week 1 of operation, 27 dog cakes during Week 2, and 34 dog cakes during Week 3.

Explicit formula: $a_n = 20 + 7(n - 1)$

$$a_n = 7n + 13$$

$$\text{Recursive formula: } a_n = a_{n-1} + 7$$

5. A petri dish starts with a sample of 3 bacteria. The number of bacteria triples every minute.

Explicit formula: $a_n = 3 \cdot 3^{(n-1)}$

$$\text{Recursive formula: } a_n = a_{n-1} \cdot 3$$

2. A test showed that after the first hour of receiving a medication, 100 milligrams remained in the body. Continued tests showed that the dose found in the body halved every hour after the first hour.

Explicit formula: $a_n = 100 \cdot \frac{1}{2}^{(n-1)}$

$$\text{Recursive formula: } a_n = a_{n-1} \cdot \frac{1}{2}$$

4. A barrel starts with 6 cups of water in it. During a heavy rainstorm, the amount of water in the barrel doubles every minute.

Explicit formula: $a_n = 6 \cdot 2^{(n-1)}$

$$\text{Recursive formula: } a_n = a_{n-1} \cdot 2$$

6. Natalia is selling cupcakes at a bake sale. She starts with 100 cupcakes and sells 10 per hour.

Explicit formula:

$$a_n = 100 + (-10)(n - 1)$$

$$a_n = -10n + 110$$

$$\text{Recursive formula: } a_n = a_{n-1} + (-10)$$

- D. Write the explicit formula you can use to represent the terms of the sequence, then represent the sequence on the coordinate plane.

TEKS A.12D (Topic-Aligned)

1. $a_4 = \frac{3}{4}, a_5 = 0, a_6 = -\frac{3}{4}, a_7 = -1\frac{1}{2}$

$$a_n = a_1 + d(n - 1)$$

$$a_4 = a_1 + 3(4 - 1)$$

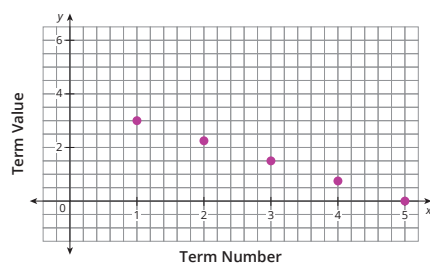
$$\frac{3}{4} = a_1 + 3 \cdot -\frac{1}{4}$$

$$\frac{3}{4} = a_1 - \frac{3}{4}$$

$$1\frac{1}{2} = a_1$$

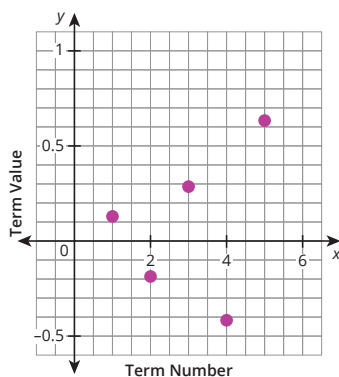
$$a_n = 1\frac{1}{2} - \frac{1}{4}(n - 1)$$

$$a_n = -\frac{1}{4}n + 1\frac{3}{4}$$



3. The first term of the sequence is $m_1 = \frac{1}{8}$ and the common ratio is -1.5 .

$$m_n = \frac{1}{8} \cdot (-1.5)^{n-1}$$



2. $k_8 = 15, k_9 = 50, k_{10} = 85, k_{11} = 120$

$$k_n = k_1 + d(n - 1)$$

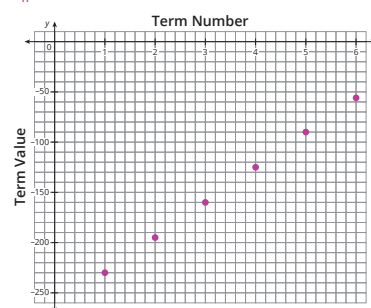
$$k_8 = k_1 + 35(8 - 1)$$

$$15 = k_1 + (35)(7)$$

$$k_1 = -230$$

$$k_n = -230 + 35(n - 1)$$

$$k_n = 35n - 265$$



4. $p_2 = 0.6, p_3 = 3.6, p_4 = 21.6, p_5 = 129.6$

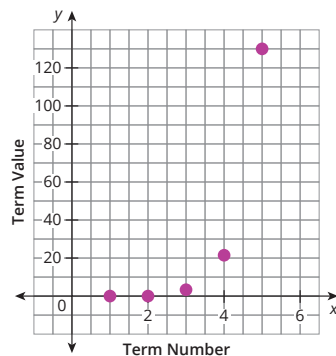
$$p_n = p_1 \cdot r^{n-1}$$

$$p_2 = p_1 \cdot 6^1$$

$$0.6 = p_1 \cdot 6$$

$$p_1 = 0.1$$

$$p_n = 0.1 \cdot 6^{n-1}$$

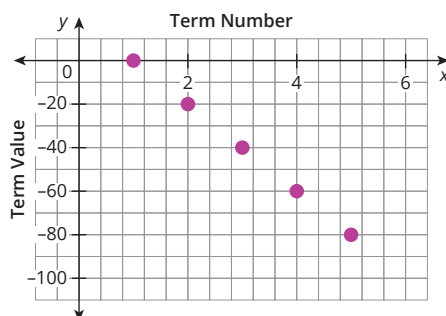


TOPIC 2 Sequences

5. The first term of the sequence is $m_1 = \frac{1}{2}$ and the common difference is -20 .

$$m_n = \frac{1}{2} - 20(n - 1)$$

$$m_n = -20n + 20\frac{1}{2}$$



6. $p_4 = -12, p_5 = -4, p_6 = -\frac{4}{3}, p_7 = -\frac{4}{9}$

$$p_n = p_1 \cdot r^{n-1}$$

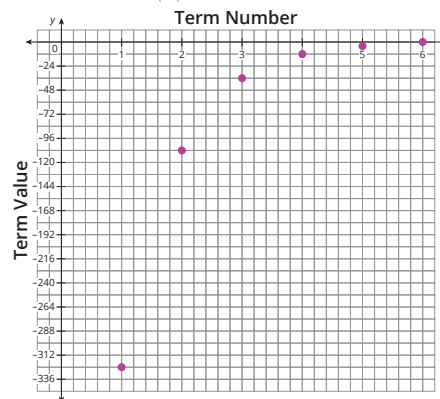
$$p_4 = p_1 \cdot r^3$$

$$-12 = p_1 \cdot \left(\frac{1}{3}\right)^3$$

$$-12 = p_1 \cdot \frac{1}{27}$$

$$p_1 = -324$$

$$p_n = -324 \cdot \left(\frac{1}{3}\right)^{n-1}$$



Extension

Consider the first two terms of this sequence $\frac{1}{16}, -\frac{3}{16}, \dots$

- Determine the 63rd term if this is an arithmetic sequence. Write your answer as an improper fraction in lowest terms.

Extension of TEKS A.12C

$$\frac{-247}{16}$$

- Determine the 63rd term if this is a geometric sequence. Write your answer in scientific notation. Extension of TEKS A.12C

$$\approx 2.38 \times 10^{28}$$

Spaced Practice

1. Determine whether each given sequence is arithmetic or geometric. Then write the next 3 terms of the sequence. **TEKS A.12C**
 - a. 3, -12, 48, -192, ...
Geometric; 768, -3072, 12288
 - b. 2.45, 3.86, 5.27, 6.68, ...
Arithmetic; 8.09, 9.5, 10.91
2. Determine the independent and dependent quantities in each scenario. Include units when possible. **TEKS A.2A**
 - a. A lamp manufacturing company produces 750 lamps per shift.
Independent quantity: number of shifts
Dependent quantity: number of lamps produced
 - b. A grocery store sells pears by the pound. A customer purchases 3 pounds for \$5.07.
Independent quantity: weight of pears (pounds)
Dependent quantity: price (dollars)
3. Determine the function family for each equation.
 - a. $g(x) = -15x^2 + 60x + 370$ **TEKS A.7A**
Quadratic function family
 - b. $h(x) = 3 \cdot (-5)^x$ **TEKS A.9D**
Exponential function family



Exploring Constant Change

TOPIC 1: Linear Functions

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TOPIC 2: Transforming and Comparing Linear Functions

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Name _____ Date _____

I. Least Square Regressions

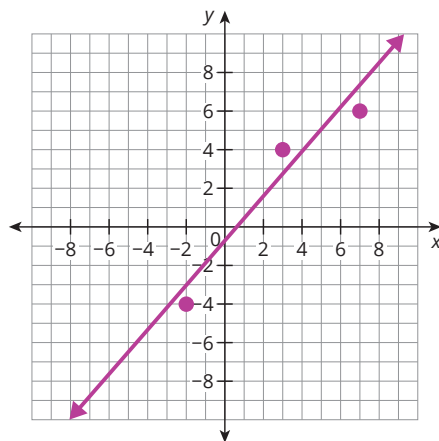
Topic Practice

- A. Graph the estimated line of best fit for each set of points.
Determine the estimated linear regression function for the line.

TEKS A.3C (Topic-Aligned), TEKS A.4C (Topic-Aligned)

1. (3, 4), (7, 6), and (-2, -4)

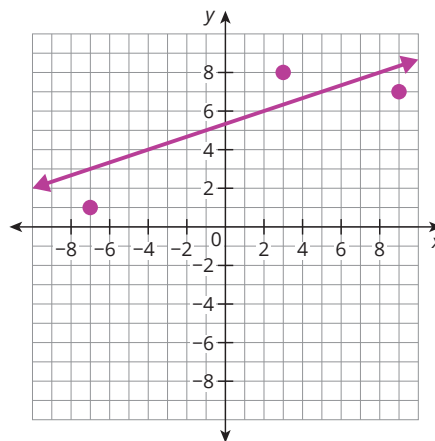
Sample answer:



$$y = \frac{8}{7}x - \frac{5}{7}$$

2. (-7, 1), (3, 8), and (9, 7)

Sample answer:

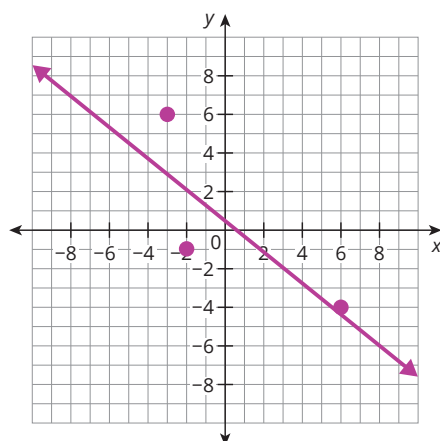


$$y = \frac{1}{3}x + 5\frac{1}{3}$$

TOPIC 1 Linear functions

3. $(-3, 6)$, $(-2, -1)$, and $(6, -4)$

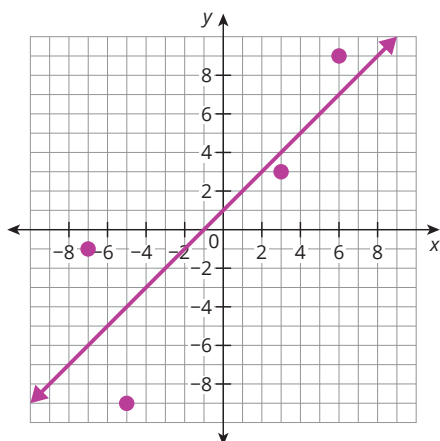
Sample answer:



$$y = -\frac{4}{5}x + \frac{1}{2}$$

5. $(-7, -1)$, $(-5, -9)$, $(3, 3)$, and $(6, 9)$

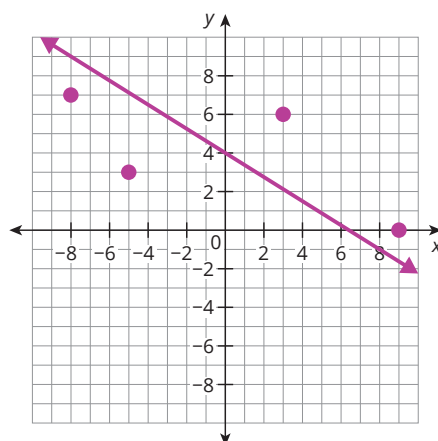
Sample answer:



$$y = x + 1$$

4. $(-8, 7)$, $(-5, 3)$, $(3, 6)$, and $(9, 0)$

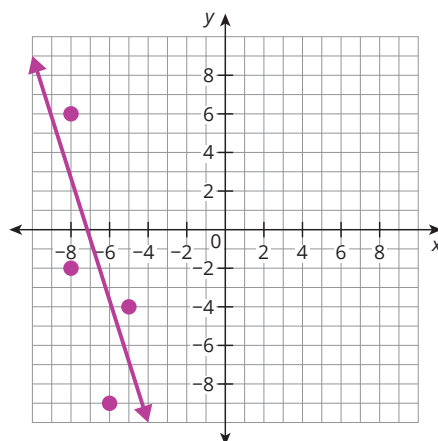
Sample answer:



$$y = -\frac{5}{8}x + 4$$

6. $(-8, 6)$, $(-8, -2)$, $(-6, -9)$, and $(-5, -4)$

Sample answer:



$$y = -\frac{13}{4}x - 23\frac{1}{4}$$

B. Use the linear regression function to analyze each scenario.

TEKS A.4C (Topic-Aligned)

1. While in high school, Xavier started his own T-shirt printing business. The table shows the number of T-shirts Xavier has sold each year since starting his business in 2010.

Years	2010	2011	2012	2013	2014	2015	2016
Numbers of T-shirts	50	75	175	125	250	350	375

The linear regression function representing the data shown in the table is $y = 57.14x + 28.57$, where x represents the number of years since 2010 and y represents the number of T-shirts sold.

- a. Complete the table for the linear regression function, which represents the data from Xavier's T-shirt printing business.

		What it Means	
Expression	Unit	Contextual Meaning	Mathematical Meaning
y	Number of T-shirts	The predicted number of T-shirts sold	Output value
57.14	Rise in number of T-shirts sold per year	The predicted change in the number of T-shirts sold per year	Slope
x	Number of years since 2010	The number of years since 2010	Input value
28.57	Number of T-shirts	The predicted number of T-shirts sold in year 2010	y-intercept



TOPIC 1 Linear functions

Use the linear regression function to predict the number of T-shirts Xavier sold during each given year. Then, compare the prediction to the actual number of T-shirts to determine whether the prediction is reasonable based on the problem situation.

b. 2012

$$\text{For 2012, } x = 2$$

$$y = 57.41x + 28.57$$

$$y = 57.14(2) + 28.57$$

$$y = 142.85$$

The predicted total number of T-shirts sold in 2012 is about 143. The actual number of T-shirts sold was 175, so the predicted value is fairly close to the actual value.

d. 2016

$$\text{For 2016, } x = 6.$$

$$y = 57.14x + 28.57$$

$$y = 57.14(6) + 28.57$$

$$y = 371.41$$

The predicted number of T-shirts sold in 2016 is about 371. The actual number of T-shirts sold was 375, so the predicted value is fairly close to the actual value.

f. 2024

$$\text{For 2024, } x = 14.$$

$$y = 57.14x + 28.57$$

$$y = 57.14(14) + 28.57$$

$$y = 828.53$$

The predicted number of T-shirts sold in 2024 is about 829. The prediction is reasonable.

c. 2014

$$\text{For 2014, } x = 4.$$

$$y = 57.14x + 28.57$$

$$y = 57.14(4) + 28.57$$

$$y = 257.13$$

The predicted number of T-shirts sold in 2014 is about 257. The actual number of T-shirts sold was 250, so the predicted value is fairly close to the actual value.

e. 2018

$$\text{For 2018, } x = 8.$$

$$y = 57.14x + 28.57$$

$$y = 57.14(8) + 28.57$$

$$y = 485.69$$

The predicted number of T-shirts sold in 2018 is about 486. The prediction is reasonable.

2. The number of students in a school chorus has increased since the school first opened 6 years ago. The table of values represents the change of students in the school chorus, where x represents the year and y represents the number of students.

Year	0	1	2	3	4	5
Number of Students in the Choir	22	36	40	59	78	83

- a. Determine the linear regression function based on the given table of values. Round the values for slope and y -intercept to the nearest tenth.

$$y = 12.9x + 20.9$$

Use your linear regression function to predict the total number students in the choir in each given year. Then, compare the prediction to the actual number of students to determine whether the prediction is reasonable based on the problem situation.

- b. The year the school opened

$$x = 0$$

$$y = 12.9x + 20.9$$

$$y = 12.9(0) + 20.9$$

$$y = 20.9$$

The predicted total number of choir students the year the school opened is about 21. The actual number of students in the choir when the school opened was 22. So, the predicted value is fairly close to the actual value.

- c. Year 2

$$x = 2$$

$$y = 12.9(2) + 20.9$$

$$y = 46.7$$

The predicted total number of choir students in year 2 is about 47. The actual number of students in the choir when the school opened was 40. So, the predicted value is fairly close to the actual value.



TOPIC 1 Linear functions

d. Year 4

$$x = 4$$

$$y = 12.9(4) + 20.9$$

$$y = 72.5$$

The predicted total number of choir students in year 4 is about 73. The actual number of students in the choir when the school opened was 78. So, the predicted value is fairly close to the actual value.

e. Year 6

$$x = 6$$

$$y = 12.9(6) + 20.9$$

$$y = 98.3$$

The predicted total number of choir students in year 6 is about 98. The prediction is reasonable.

f. Year 8

$$x = 8$$

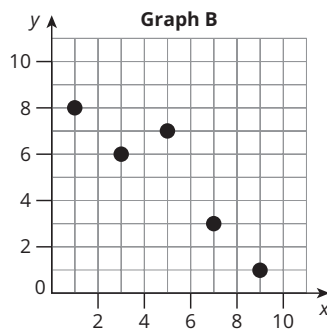
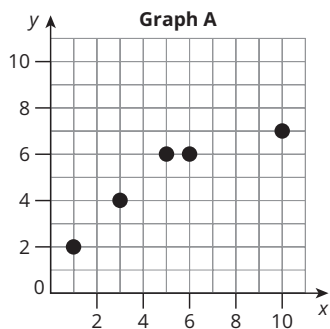
$$y = 12.9(8) + 20.9$$

$$y = 124.1$$

The predicted total number of choir students in year 8 is about 124. The prediction is reasonable.

Extension

Consider the two sets of data shown in the graphs.



- Calculate the mean of the x -values, $\bar{x}$, and the mean of the y -values, $\bar{y}$, for each graph. **Extension of TEKS A.4C**

Graph A: $\bar{x} = 5$, $\bar{y} = 5$;

Graph B: $\bar{x} = 5$, $\bar{y} = 5$

- Complete the tables for each graph. **Extension of TEKS A.4C**

Graph A				
x	$x - \bar{x}$	y	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
1	-4	2	-3	12
3	-2	4	-1	2
5	0	6	1	0
6	1	6	1	1
10	5	7	2	10
				SUM = 25

Graph B				
x	$x - \bar{x}$	y	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$
1	-4	8	3	-12
3	-2	6	1	-2
5	0	7	2	0
7	2	3	-2	-4
9	4	1	-4	-16
				SUM = -34

- Compare the two sums in the last column of each table. Determine if there seems to be a connection between the sums and the graphs of the data set. **Extension of TEKS A.4C**

Sample answer:

A positive sum corresponds to an increasing function, and a negative sum corresponds to a decreasing function.



TOPIC 1 Linear functions

Spaced Practice

1. A maintenance worker in a factory notices that a water tank is leaking. She records the amount of water in the tank each day in a table.

Day	Volume of Water (L)
1	16,000
2	12,000
3	9000
4	6750

TEKS A.12D

- a. Write a recursive formula to represent the pattern shown in the table. What predictions does this formula make for the amount of water in the tank on the 5th day?

The recursive pattern can be expressed as $V_n = \frac{3}{4}V_{n-1}$ for $n \geq 1$, assuming n is a positive integer. The amount of water left on the 5th day is 5062.5 liters.

- b. Write an explicit formula to represent the pattern shown in the table. What predictions does this formula make for the amount of water in the tank on the 10th day?

The pattern can be expressed by using the explicit formula $V_n = 16,000\left(\frac{3}{4}\right)^{n-1}$ for $n \geq 1$, assuming n is a positive integer. The amount of water left on the 10th day is 1201.4 liters.

2. The graph represents a linear relationship between x and y . TEKS A.3C

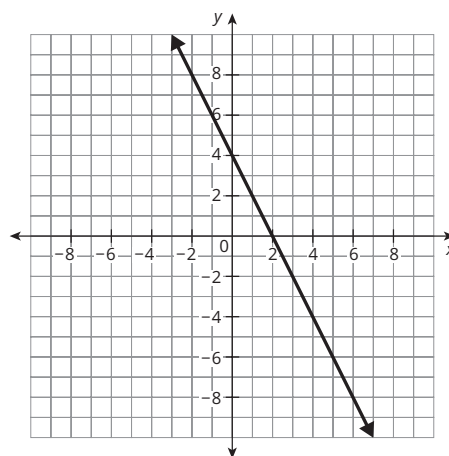
- a. Describe whether the graph is increasing or decreasing. Justify your reasoning.

The graph is decreasing because as the value of x increases, the value of y decreases.

- b. Determine the x - and y -intercept.

x -intercept: (2, 0)

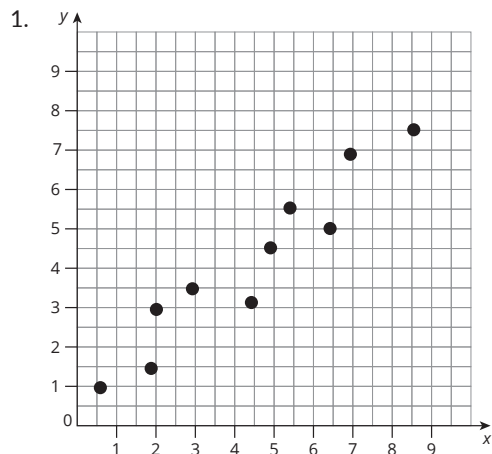
y -intercept: (0, 4)



II. Correlation

Topic Practice

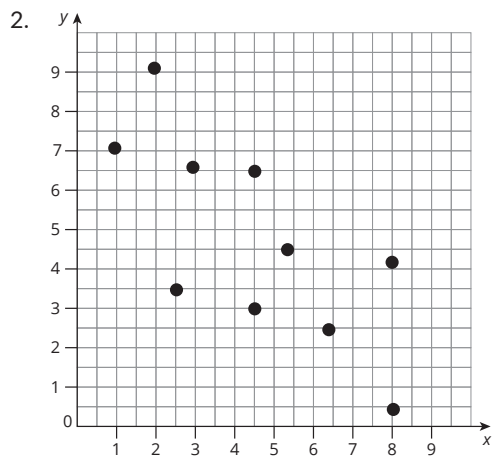
- A. Determine whether the points in each scatterplot have a positive correlation, a negative correlation, or no correlation. Then, determine which r -value is most accurate. **TEKS A.4A (Topic-Aligned)**



- A. $r = 0.097$
- B. $r = -0.57$
- C. $r = 0.83$
- D. $r = 1$

These data have a positive correlation.

(C) $r = 0.83$ would be the most accurate r -value.



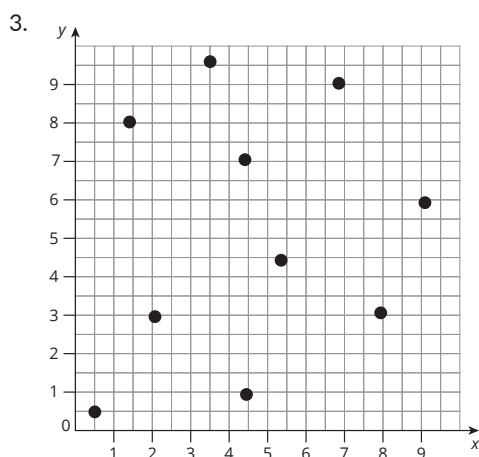
- A. $r = 0.9$
- B. $r = -0.6$
- C. $r = 0.02$
- D. $r = -0.006$

These data have a negative correlation.

(B) $r = -0.6$ would be the most accurate r -value.



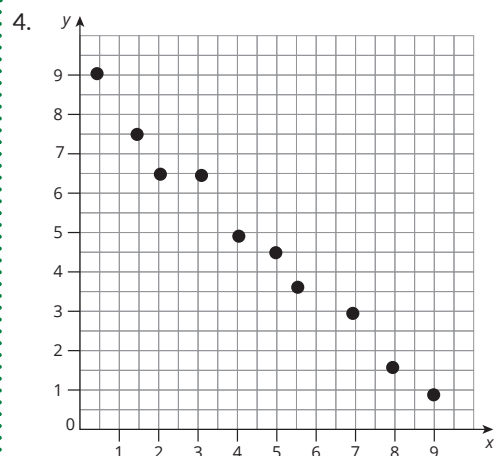
TOPIC 1 Linear functions



- A. $r = 0.01$
- B. $r = 0.8$
- C. $r = -0.5$
- D. $r = 0.08$

These data have no correlation.

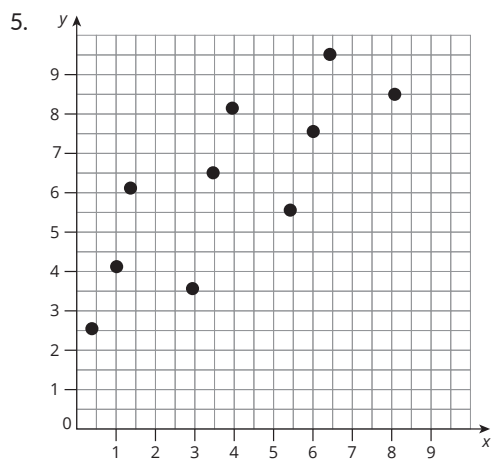
(A) $r = 0.01$ would be the most accurate r -value.



- A. $r = -0.009$
- B. $r = 0.8$
- C. $r = -0.09$
- D. $r = 0.2$

These data have a negative correlation.

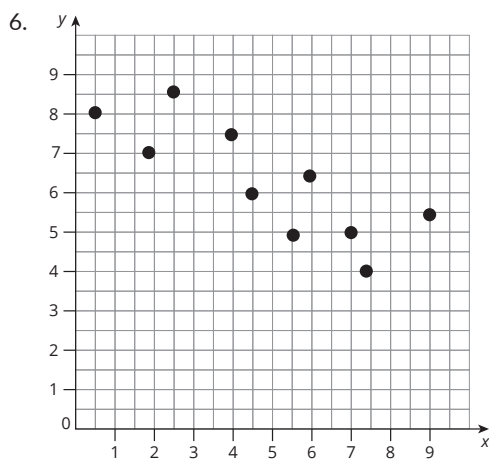
(C) $r = -0.9$ would be the most accurate r -value.



- A. $r = -0.003$
- B. $r = -0.6$
- C. $r = 0.004$
- D. $r = 0.7$

These data have a positive correlation.

(D) $r = 0.7$ would be the most accurate r -value.



- A. $r = 0.01$
- B. $r = -0.8$
- C. $r = -0.01$
- D. $r = 0.9$

These data have a negative correlation.

(B) $r = -0.8$ would be the most accurate r -value.



TOPIC 1 Linear functions

- B. Determine the linear regression function and correlation coefficient for each data set. Round your correlation coefficient to the nearest ten-thousandth. State whether the correlation is strong or weak and positive or negative. Explain your reasoning.

TEKS A.4A (Interleaved Practice), TEKS A.4B (Interleaved Practice),
TEKS A.4C (Interleaved Practice)

1.

Years since 2011	0	1	2	3	4	5
Profit (dollars)	50,000	75,000	150,000	125,000	195,000	225,000

$$y = 34,571.4x + 50,238.1$$

$$r = 0.9571$$

Because the r -value is close to 1, the correlation is strong and positive.

2.

Month	1	2	3	4	5	6
Profit (dollars)	100,000	85,000	91,000	82,000	79,500	74,000

$$y = -4442.86x + 100,800$$

$$r = -0.9060$$

Because the r -value is close to -1 , the correlation is strong and negative.

3.

Time (seconds)	0	1	2	3	4	5
Height (feet)	5	21	34	31	18	3

$$y = -0.63x + 20.24$$

$$r = -0.0915$$

Because the r -value is close to 0, the correlation is weak and negative.



4.

Month	1	2	3	4	5	6
Sales (dollars)	1480	14,105	8925	18,750	5250	2650

$$y = -311.143x + 9615.67$$

$$r = -0.0857$$

Because the r -value is close to 0, the correlation is weak and negative.

5.

Years since 2014	0	1	2	3	4	5
Units Sold	5245	7840	7075	9130	10,620	12,635

$$y = 1352.7x + 5375.7$$

$$r = 0.9613$$

Because the r -value is close to 1, the correlation is strong and positive.

6.

Time (hours)	0	1	2	3	4	5
Height (centimeters)	63	56	42	36	28	12

$$y = -9.86x + 64.14$$

$$r = -0.9910$$

Because the r -value is close to -1 , the correlation is strong and negative.



TOPIC 1 Linear functions

C. For each situation, decide whether the correlation implies causation. List reasons why or why not. **TEKS A.4B (Topic-Aligned)**

1. Since 2005, the average salary of an NBA basketball player has increased every year. The average height of an NBA player has also increased yearly since 2005.

Sample answer:
The correlation does not imply causation.
The increase in salary does not cause a player to grow taller.

3. The number of smartphones in the United States has increased yearly since 2005. The number of paper calendars sold yearly has decreased during the same period.

Sample answer:
The correlation does imply causation. It is reasonable to conclude that owning a cell phone would cause you to not purchase a paper calendar. All smartphones have calendar apps that are more convenient to use than a paper calendar.

5. The number of smartphones in the United States has increased yearly since 2005. The average weight of an adult in the United States has also increased during this time.

Sample answer:
The correlation does not imply causation.
Owning a cell phone does not cause you to gain weight.

2. Scientists have found that the number of people wearing sunscreen has increased. The number of people getting severe sunburns has decreased.

Sample answer:
The correlation does imply causation. Scientists have proved that sunscreen can prevent sunburns. So it is reasonable that when the number of people wearing sunscreen increases, the number of people getting severe sunburns would decrease. However, not all people who wear sunscreen do not get sunburns and not all people who don't wear sunscreen get sunburns.

4. The number of shark attacks on a certain beach increases on days when more ice cream is purchased.

Sample answer:
The correlation does not imply causation. Eating ice cream does not cause shark attacks. It is likely that more ice cream is purchased on days when more people are at the beach. The increase in the number of people at the beach increases the opportunity for shark attacks.

6. A scientific study found a strong correlation between the amount someone exercises and their chance of getting skin cancer.

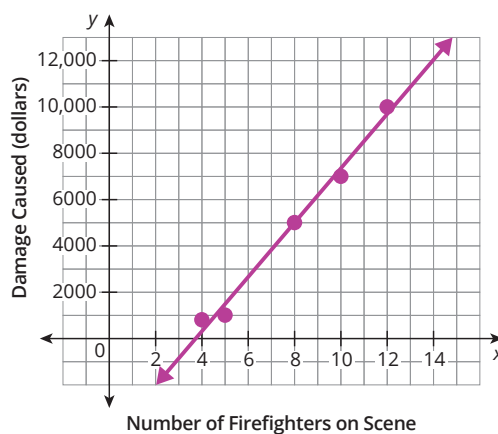
Sample answer:
The correlation does not imply causation. Exercise does not cause skin cancer. People who exercise are more likely to be outside and are exposed to the sun for greater lengths of time.

D. Use the data in the table to answer each question.

A city records the number of firefighters that arrive on a scene and the amount of monetary damage caused by the fire. The data for five fires that firefighters responded to are shown in the table.

TEKS A.4A (Interleaved Practice), TEKS A.4B (Interleaved Practice)

Number of Firefighters	Damage Caused (dollars)
5	1000
8	5000
12	10,000
10	7000
4	800



1. Create a scatterplot of the data.
2. Describe the relationship between the number of firefighters that arrive on scene and the amount of monetary damage caused by the fire.

There is a positive linear association. As the number of firefighters on scene increases, the amount of monetary damage caused by the fire increases.

3. Use technology to determine the equation for the line of best fit. Sketch the line on your scatterplot.

$$y = 1173.21x - 4391.07$$

4. Determine the correlation coefficient for the line of best fit.

$$r = 0.99$$

5. Describe any confounding variables.

Sample answer:

The amount of monetary damage is determined by the severity of the fire.



TOPIC 1 Linear functions

6. Does the correlation imply causation? Explain your reasoning.

Sample answer:

No, having more firefighters on scene does not cause the fire to do more damage. There are likely more firefighters that respond to a scene when the fire is more severe. A more severe fire will cause more damage.

Extension

Consider the points: (1, 2), (2, 3), (3, 2), (4, 5), (5, 2.5), (6, 6), (7, 3), (8, 7). The line of best fit for the graph of the points is $y = 0.5x + 1.4$.

1. Complete the table to determine the predicted values of y for each value of x using the line of best fit, and the values of the differences between the observed y -values from the points and the predicted y -values from the line of best fit. **Extension of TEKS A.4C**

x	Observed y -Value	Predicted y -Value	Observed y -Value – Predicted y -Value
1	2	1.9	0.1
2	3	2.4	0.6
3	2	2.9	–0.9
4	5	3.4	1.6
5	2.5	3.9	–1.4
6	6	4.4	1.6
7	3	4.9	–1.9
8	7	5.4	1.6

2. Determine whether there is a pattern in the differences between the y -values from the completed table. Explain what this might indicate about using the line of best fit to make predictions.

Extension of TEKS A.4C

It does not make sense to use a line of best fit because the differences between the y -values is increasing rather than remaining constant.

Spaced Practice

- The table shows the highest maximum temperature for the month of October in Philadelphia, Pennsylvania, over ten years.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Highest Maximum Temperature (°F)	64.9	53.1	61	54	63	68	61	57.9	64.9	66.9

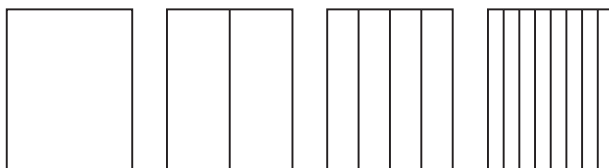
- Identify the independent and dependent quantities and their units of measure. **TEKS A.2A**

The independent quantity is the year, and the dependent quantity is the highest maximum temperature in degrees Fahrenheit.

- Use the data table and graphing technology to generate a linear regression function. What is the slope and y-intercept of the line and what do they represent? **TEKS A.4C**

$y = 0.67x + 53.05$, where x represents the number of years since 2000, and y represents the highest maximum temperature in degrees Fahrenheit. The slope is 0.67, which means that the maximum high temperature increases by 0.67 degrees Fahrenheit each year. The y-intercept is 53.05, which represents the maximum high temperature in the year 2000.

- Miguel draws a rectangle, and then in each successive figure he splits the rectangles into two rectangles as shown. **TEKS A.12C**



TOPIC 1 Linear functions

- a. Analyze the number of rectangles in each figure. Describe the pattern.

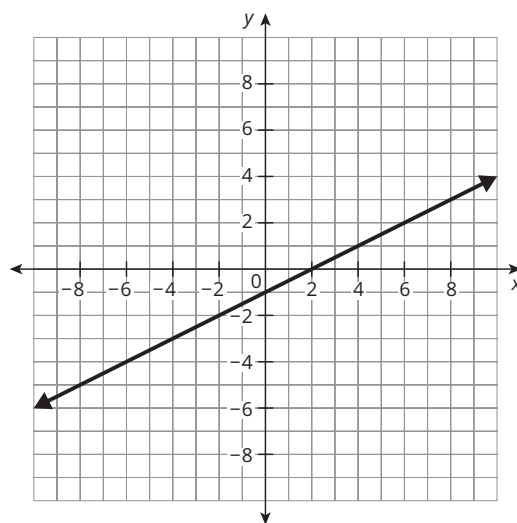
There is one rectangle in the first figure, two in the second, four in the third, and eight in the fourth. For each successive figure, the number of rectangles in each figure doubles.

- b. Write the number of rectangles in each of the first six figures as a numeric sequence.

1, 2, 4, 8, 16, 32

3. Determine the slope, x-intercept, and y-intercept of the graph.

TEKS A.3C



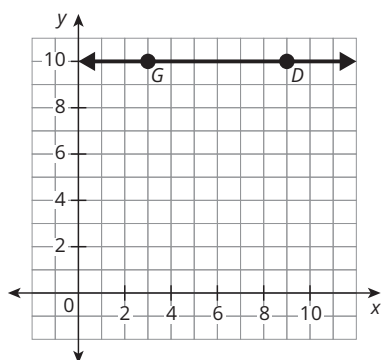
The slope is $\frac{1}{2}$, the x-intercept is (2, 0), and the y-intercept is (0, -1).

III. Making Connections Between Arithmetic Sequences and Linear Functions

Topic Practice

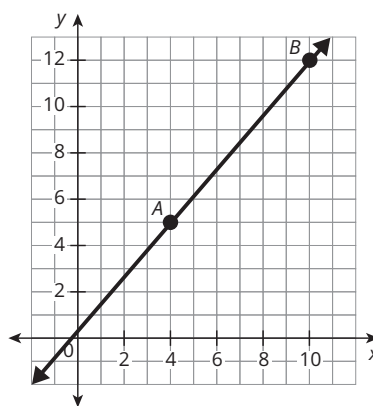
- A. Use the graph or the slope formula to determine the rate of change of each linear function. **TEKS A.3B (Topic-Aligned)**

1.



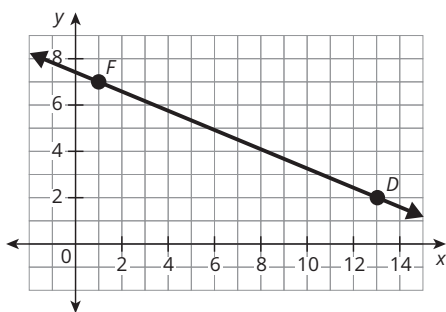
0

2.



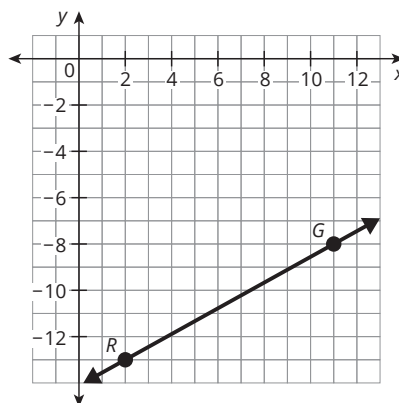
$\frac{7}{6}$

3.



$-\frac{5}{12}$

4.

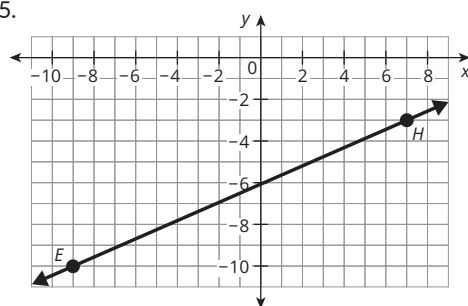


$\frac{5}{9}$



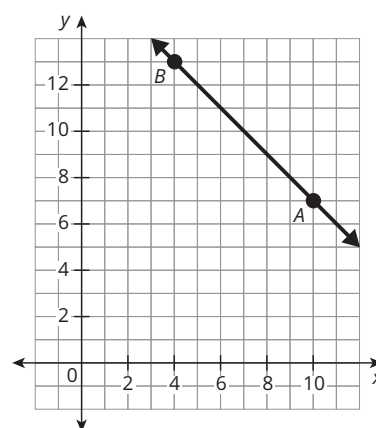
TOPIC 1 Linear functions

5.



$$\frac{7}{16}$$

6.

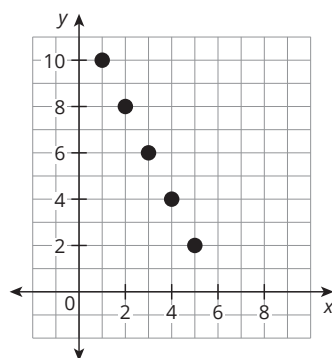


$$-1$$

B. Write an equation using the explicit formula to represent each graph or scenario. Then, rewrite the formula in function form.

TEKS A.2C (Topic-Aligned), TEKS A.12D (Interleaved Practice)

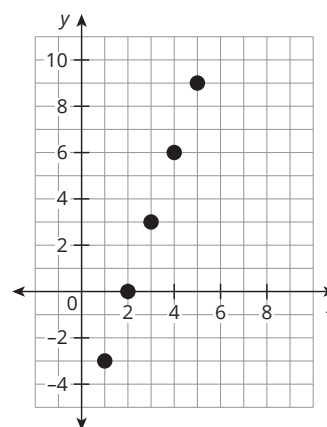
1.



$$a_n = 10 - 2(n - 1)$$

$$f(n) = 12 - 2n$$

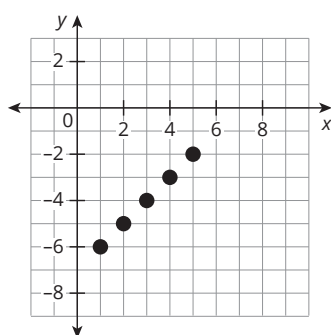
2.



$$a_n = -3 + 3(n - 1)$$

$$f(n) = -6 + 3n$$

3.



$$a_n = -6 + (n - 1)$$

$$f(n) = -7 + n$$

5. A scooter rental company charges \$3.25 for the first minute of a rental. Thereafter, the company charges an additional \$0.25/minute.

$$a_n = 3.25 + 0.25(n - 1)$$

$$f(n) = 0.25n + 3$$

4. Sequence: 19, 19.5, 20, 20.5, ...

$$a_n = 19 + 0.5(n - 1)$$

$$f(n) = 0.5n + 18.5$$

6. Joey uses 4 cups of flour in each cake he bakes. On day 1, he starts the day with 60 cups of flour.

$$a_n = 60 - 4(n - 1)$$

$$f(n) = 64 - 4n$$

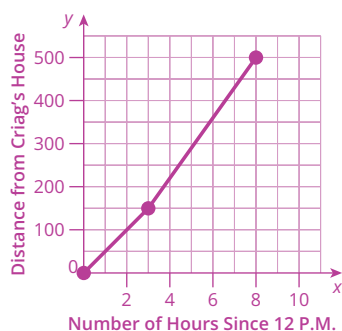


TOPIC 1 Linear functions

Extension

Nicky left his house at noon and drove 50 miles per hour until 3 P.M. Then, he drove the next 5 hours at 70 miles per hour. Graph Nicky's driving trip and calculate the average rate of change for the entire trip.

Extension of TEKS A.3B



Using $(0, 0)$ and $(8, 500)$,

$$\begin{aligned}\frac{f(t) - f(s)}{t - s} &= \frac{500 - 0}{8 - 0} \\ &= \frac{500}{8} \\ &= 62.5 \text{ mph}\end{aligned}$$

The average rate of change is neither of the rates that Nicky drove. It is the rate if Nicky drove at a constant rate for the entire trip. It is the weighted average of the rates, $\frac{(3)(50) + (5)(70)}{8}$, taking into account that Nicky drove the faster rate for a longer period of time.

Spaced Practice

Evaluate each function for the given values. TEKS A.12B

1. $f(x) = 3x - 10$	2. $f(x) = 6$	3. $f(x) = 9x + 7 - 3x$
a. $f(0)$	a. $f(0)$	a. $f(0)$
$f(0) = -10$	$f(0) = 6$	$f(0) = 7$
b. $f(5)$	b. $f(-2)$	b. $f(0.5)$
$f(5) = 5$	$f(-2) = 6$	$f(0.5) = 10$



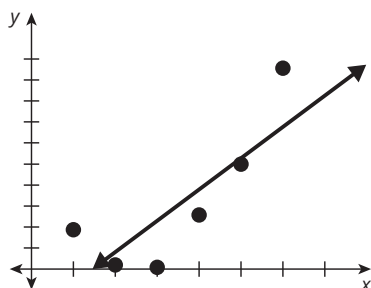
4. The linear regression function for the given data is $y = -x + 19.7$. Complete the table for the linear regression function, rounding your answers to the nearest tenth. **TEKS A.4C**

x	y	Predicted Value
2	17	17.7
4	16	15.7
6	15	13.7
8	12	11.7
10	9	9.7
12	8	7.7

5. The linear regression function for the given data is $y = 3.93x - 11.33$, $r = 0.8241$. Consider the scatterplot and the correlation coefficient. State whether a linear model is appropriate for the data. **TEKS A.4A**

x	2	4	6	8	10	12
y	9	2	1	12	25	48

Scatterplot and Line of Best Fit



Based on the shape of the scatterplot and the correlation coefficient, a linear regression model does not appear to be appropriate for the data.



IV. Point-Slope Form of a Line

Topic Practice

- A. Use point-slope form to write an equation for a line with the given information. Then, rewrite the equation in slope-intercept form.

TEKS A.2B (Topic-Aligned)

1. The slope is 3. The point (1, 2) lies on the line.
 Point-slope form: $y - 2 = 3(x - 1)$
 Slope-intercept form: $y = 3x - 1$
2. The slope is 4. The point (0, -6) lies on the line.
 Point-slope form: $y + 6 = 4x$
 Slope-intercept form: $y = 4x - 6$
3. The slope is $\frac{3}{4}$. The point (-4, -5) lies on the line.
 Point-slope form: $y + 5 = \frac{3}{4}(x + 4)$
 Slope-intercept form: $y = \frac{3}{4}x - 2$
4. The slope is $-\frac{1}{5}$. The point (-10, 3) lies on the line.
 Point-slope form:
 $y - 3 = -\frac{1}{5}(x + 10)$
 Slope-intercept form: $y = -\frac{1}{5}x + 1$
5. The slope is 5. The point (3, 4) lies on the line.
 Point-slope form: $y - 4 = 5(x - 3)$
 Slope-intercept form: $y = 5x - 11$
6. The slope is $-\frac{1}{3}$. The point (6, 0) lies on the line.
 Point-slope form: $y = -\frac{1}{3}(x - 6)$
 Slope-intercept form: $y = 4x - 6$

B. Use point-slope form to write an equation for the line that passes through each pair of points. Then, rewrite the equation in slope-intercept form. TEKS A.2B (Topic-Aligned), TEKS A.3A (Topic-Aligned)

1. Line passing through $(-4, -3)$ and $(0, -15)$

$$m = \frac{-3 - (-15)}{-4 - 0}$$

$$m = -3$$

$$\text{point-slope form: } y - (-3) = -3(x - (-4))$$

$$y + 3 = -3(x + 4)$$

$$\text{slope-intercept form: } y = -3x - 15$$

3. Line passing through $(2, -7)$ and $(-3, -9)$

$$m = \frac{-7 - (-9)}{2 - (-3)}$$

$$m = \frac{2}{5}$$

$$\text{point-slope form: } y - (-7) = \frac{2}{5}(x - 2)$$

$$y + 7 = \frac{2}{5}(x - 2)$$

$$\text{slope-intercept form: } y = \frac{2}{5}x - \frac{39}{5}$$

5. Line passing through $(0, -8)$ and $(2, 6)$

$$m = \frac{-8 - 6}{0 - 2}$$

$$m = \frac{-14}{-2}$$

$$m = 7$$

$$\text{point-slope form: } y + 8 = 7(x + 0)$$

$$y + 8 = 7x$$

$$\text{slope-intercept form: } y = 7x + (-8)$$

$$y = 7x - 8$$

2. Line passing through $(10, 5)$ and $(4, 0)$

$$m = \frac{5 - 0}{10 - 4}$$

$$m = \frac{5}{6}$$

$$\text{point-slope form: } y = \frac{5}{6}(x - 4)$$

$$\text{slope-intercept form: } y = \frac{5}{6}x - \frac{10}{3}$$

4. Line passing through $(0, 5)$ and $(3, -3)$

$$m = \frac{5 - (-3)}{0 - 3}$$

$$m = -\frac{8}{3}$$

$$\text{point-slope form: } y - 5 = -\frac{8}{3}x$$

$$\text{slope-intercept form: } y = -\frac{8}{3}x + 5$$

6. Line passing through $(-1, 1)$ and $(-3, -5)$

$$m = \frac{1 - (-5)}{-1 - (-3)}$$

$$m = \frac{6}{2}$$

$$m = 3$$

$$\text{point-slope form: } y - 1 = 3(x - (-1))$$

$$y - 1 = 3(x + 1)$$

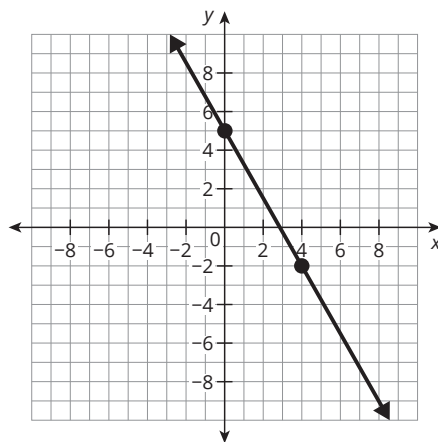
$$\text{slope-intercept form: } y = 3x + 4$$



TOPIC 1 Linear functions

C. Write a linear equation in point-slope and slope-intercept form to represent each line. TEKS A.2B (Interleaved Practice), TEKS A.2C (Topic-Aligned), TEKS A.3A (Topic-Aligned)

1. Line whose graph is shown



The line passes through (0, 5) and (4, -2).

$$m = \frac{5 - (-2)}{0 - 4}$$

$$= -\frac{7}{4}$$

slope-intercept form: $y = -\frac{7}{4}x + 5$

Sample answer:

point-slope form: $y - 5 = -\frac{7}{4}x$

2. Line represented by the table of values

x	y
3	-8
6	-7
9	-6
12	-5

$$m = \frac{-8 - (-7)}{3 - 6}$$

$$= \frac{1}{3}$$

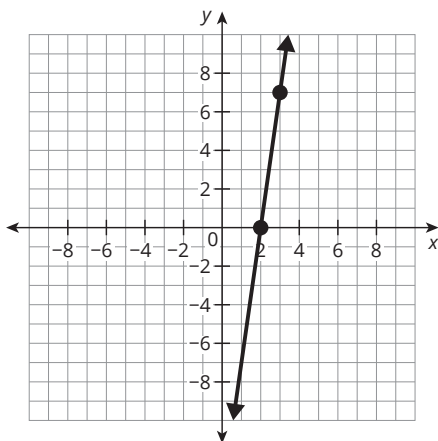
Sample answer:

point-slope form: $y - (-8) = \frac{1}{3}(x - 3)$

$$y + 8 = \frac{1}{3}(x - 3)$$

slope-intercept form: $y = \frac{1}{3}x - 9$

3. Line whose graph is shown



The line passes through (2, 0) and (3, 7).

$$\begin{aligned} m &= \frac{0 - 7}{2 - 3} \\ &= \frac{-7}{-1} \\ &= 7 \end{aligned}$$

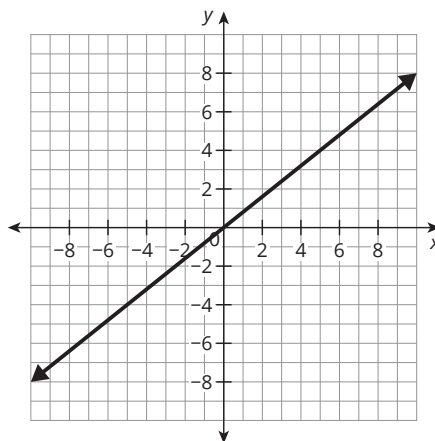
Sample answer:

point-slope form: $y - 0 = 7(x - 2)$

$$y = 7(x - 2)$$

slope-intercept form: $y = 7x - 14$

4. Line whose graph is shown



The line passes through (5, 4) and (0, 0).

$$\begin{aligned} m &= \frac{4 - 0}{5 - 0} \\ &= \frac{4}{5} \end{aligned}$$

slope-intercept form: $y = \frac{4}{5}x$

Sample answer:

point-slope form: $y - 0 = \frac{4}{5}(x - 0)$

$$y = \frac{4}{5}x$$



TOPIC 1 Linear functions

5. Line represented by the table of values

x	y
-1	42
0	30
1	18
2	6

$$\begin{aligned}m &= \frac{18 - 6}{1 - 2} \\&= \frac{12}{-1} \\&= -12\end{aligned}$$

slope-intercept form: $y = -12x + 30$

Sample answer:

point-slope form: $y - 6 = -12(x - 2)$

6. Line represented by the table of values

x	y
-8	-4
-16	-5
-24	-6
-32	-7

$$\begin{aligned}m &= \frac{-4 - (-5)}{-8 - (-16)} \\&= \frac{1}{8}\end{aligned}$$

Sample answer:

point-slope form: $y - (-4) = \frac{1}{8}(x - (-8))$

$$y + 4 = \frac{1}{8}(x + 8)$$

slope-intercept form: $y = \frac{1}{8}x - 3$



Extension

To convert an equation from point-slope to slope-intercept form, you can solve the equation for y . How do you convert from slope-intercept to point-slope form? Rewrite each equation in point-slope form using only algebraic properties. What is special about the ordered pair now visible in the equation?

The visible ordered pair is the x -intercept.

1. $y = 2x - 7$ Extension of TEKS A.2B

$$y - 0 = 2\left(x - \frac{7}{2}\right)$$

2. $y = -5x + 15$ Extension of TEKS A.2B

$$y - 0 = -5(x - 3)$$

Spaced Practice

1. Write an equation in slope-intercept form with the given characteristics. TEKS A.2B
 - a. The line is increasing and passes through the point $(0, -10)$. The slope of the line is less steep than the slope of the line represented by the equation $y = x + 8$.

Sample answer:

$$y = \frac{3}{4}x - 10$$

- b. The line is decreasing and passes through the point $(0, 5)$. The slope of the line is steeper than the slope of the line represented by the equation $y = -\frac{1}{4}x - 4$.

Sample answer:

$$y = -x + 5$$



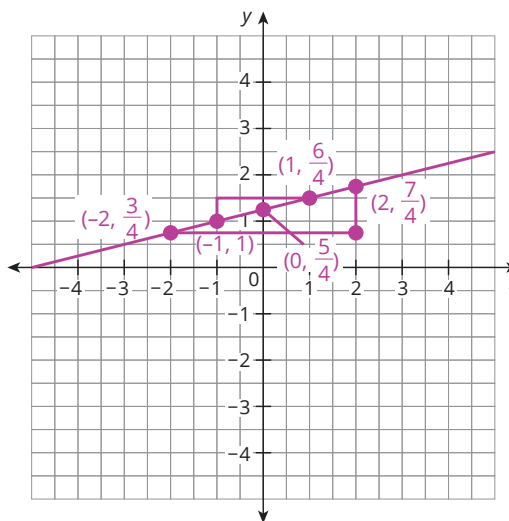
TOPIC 1 Linear functions

2. For the linear equation $x = 4y - 5$, complete each task.

a. Use a table of values to graph the linear equation. **TEKS A.3C**

x	y
-2	$\frac{3}{4}$
-1	$\frac{4}{4}$
0	$\frac{5}{4}$
1	$\frac{6}{4}$
2	$\frac{7}{4}$

b. Use the points on the graph to sketch similar triangles that may be used to show that the slope of a non-vertical line is the same between any two points on the line. **TEKS 8.4A**



- c. Verify that the slopes are the same. **TEKS 8.4A**

Sample answers:

$$m = \frac{\frac{7}{4} - \frac{6}{4}}{2 - 1} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$$

$$m = \frac{\frac{5}{4} - \frac{3}{4}}{0 - (-2)} = \frac{\frac{2}{4}}{2} = \frac{2}{4} \cdot \frac{1}{2} = \frac{1}{4}$$

3. Answer each question. **TEKS 7.4D**

- a. What is a 15% tip for a restaurant bill of \$24?

\$3.60

- b. A \$50 item was marked up 20%. What is the total increased cost of the item?

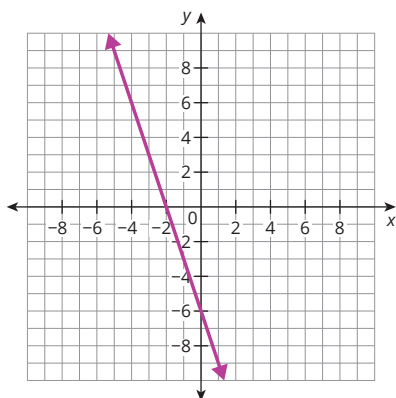
\$60.00

V. Using Linear Equations

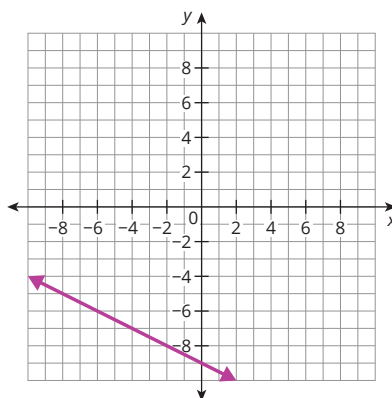
Topic Practice

- A. Graph each equation. **TEKS A.3C (Topic-Aligned)**

1. Graph the equation $y = -3x - 6$.

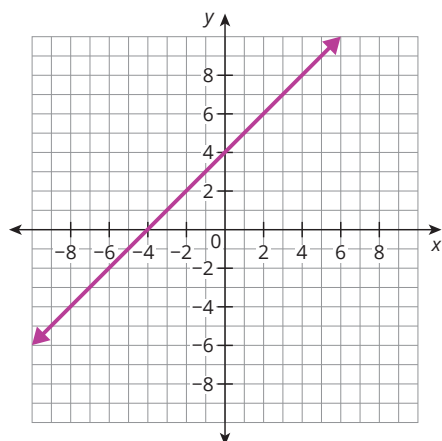


2. Graph the equation $y = -\frac{1}{2}x - 9$.

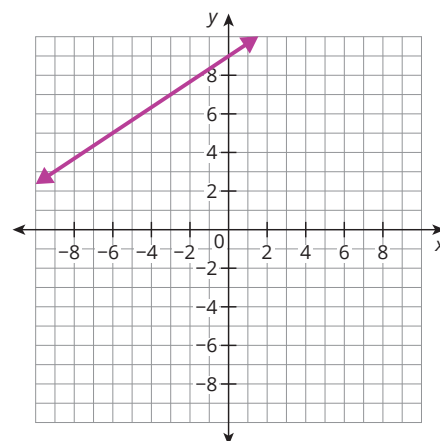


TOPIC 1 Linear functions

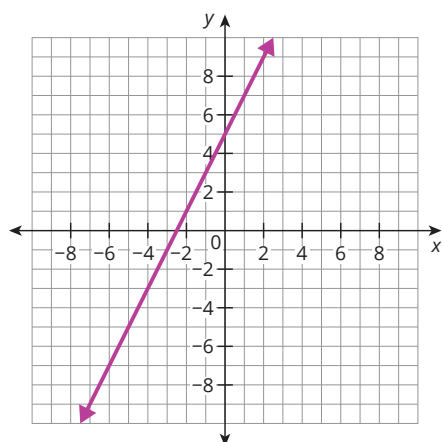
3. Graph the equation $y = x + 4$.



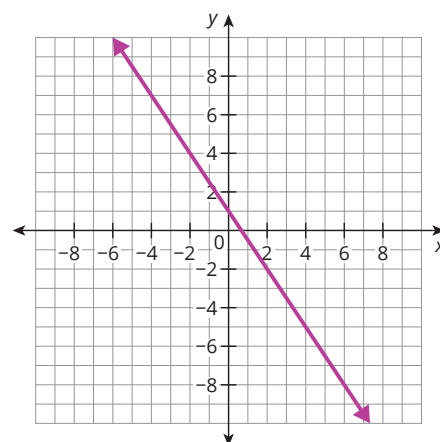
4. Graph the equation $y = \frac{2}{3}x + 9$.



5. Graph the equation $y = 2x + 5$.

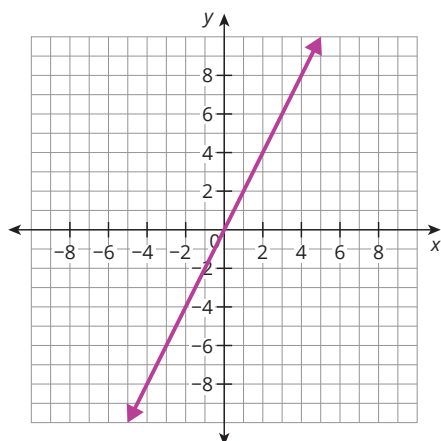


6. Graph the equation $y = -1.5x + 1$.

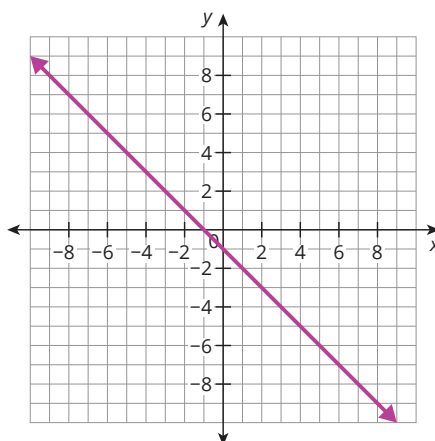


B. Graph each equation. TEKS A.3C (Topic-Aligned)

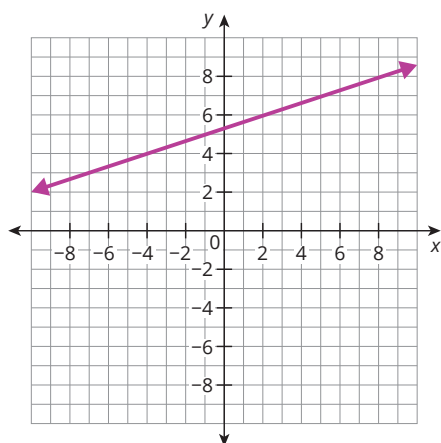
1. Graph the equation $y - 2 = 2(x - 1)$.



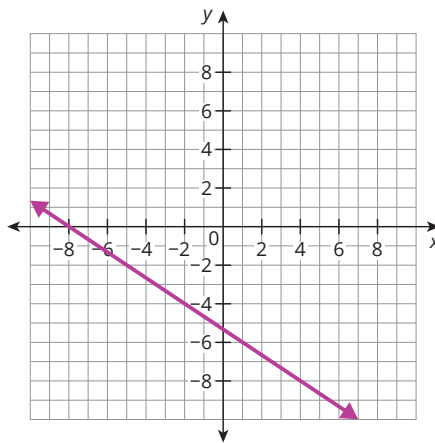
2. Graph the equation $y + 3 = -1(x - 2)$.



3. Graph the equation $y - 5 = \frac{1}{3}(x + 1)$.

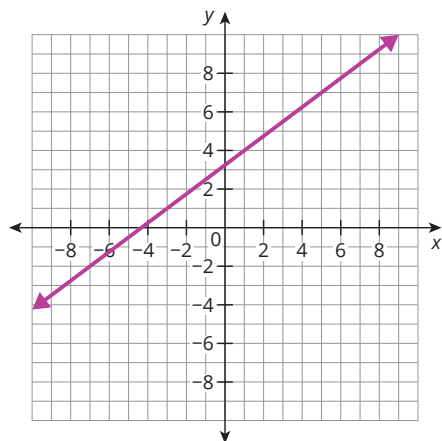


4. Graph the equation $y + 4 = -\frac{2}{3}(x + 2)$.

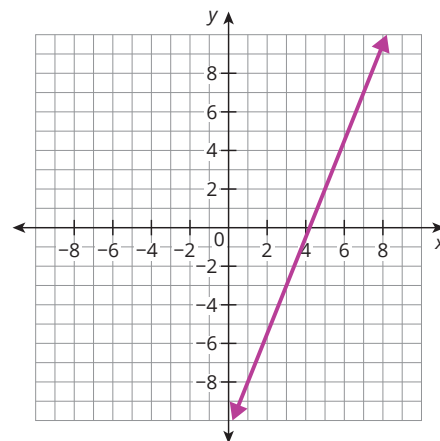


TOPIC 1 Linear functions

5. Graph the equation $y - 4 = 0.75(x - 1)$.

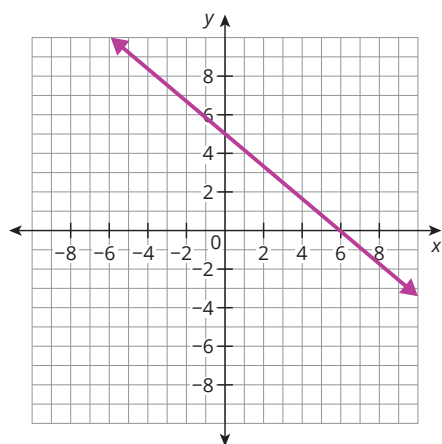


6. Graph the equation $y + 3 = 2.5(x - 3)$.

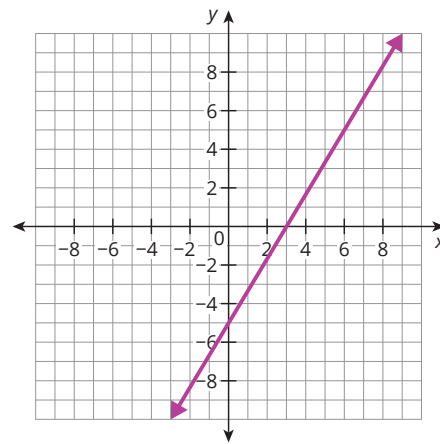


C. Graph each equation. TEKS A.3C (Topic-Aligned)

1. Graph the equation $5x + 6y = 30$.

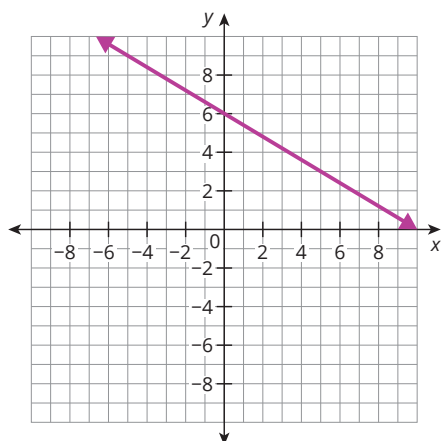


2. Graph the equation $-5x + 3y = -15$.

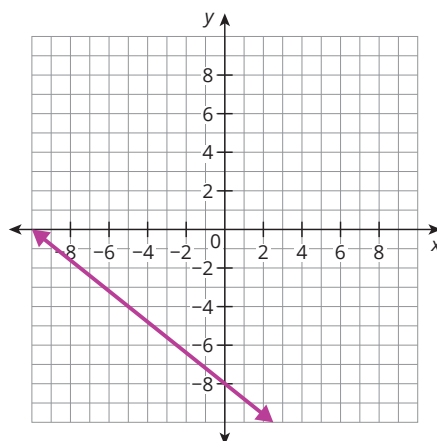


TOPIC 1 Linear functions

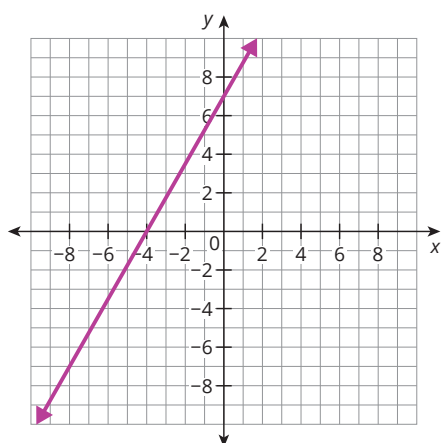
3. Graph the equation $3x + 5y = 30$.



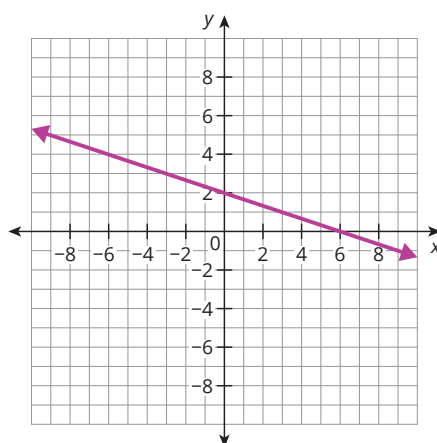
4. Graph the equation $-4x - 5y = 40$.



5. Graph the equation $7x - 4y = -28$.



6. Graph the equation $-3x - 9y = -18$.



TOPIC 1 Linear functions

D. Write an equation and create a graph for each situation. Identify the slope and y-intercept of each graph.

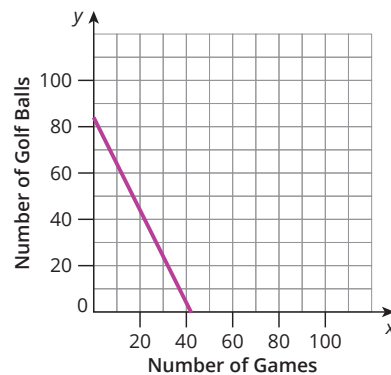
TEKS A.2C (Interleaved Practice), TEKS A.3C (Topic-Aligned)

1. At the beginning of the golf season, Jackson buys 84 golf balls. He loses 2 golf balls each time he plays a game.

- a. Write an equation that represents the number of golf balls Jackson has left given a number of times he plays a game.

Sample answer:
 $y = 84 - 2x$

- b. Graph the equation.



- c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

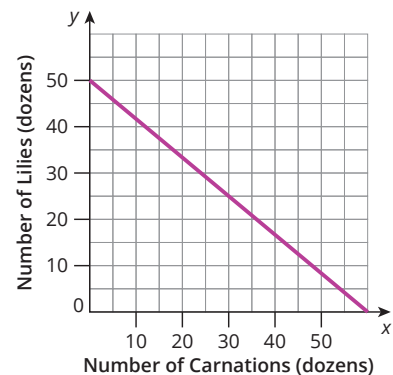
The slope is -2 and the y-intercept is 84 . The y-intercept means that Jackson begins with 84 golf balls. The slope means that he loses 2 golf balls per game played.

2. A florist sells carnations for \$11 a dozen and lilies for \$13 a dozen. During a weekend sale, the florist's goal is to earn \$650.

- a. Write an equation that represents the total amount the florist would like to earn selling carnations and lilies during the weekend sale.

Sample answer:
 $11x + 13y = 650$

- b. Graph the equation.



- c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

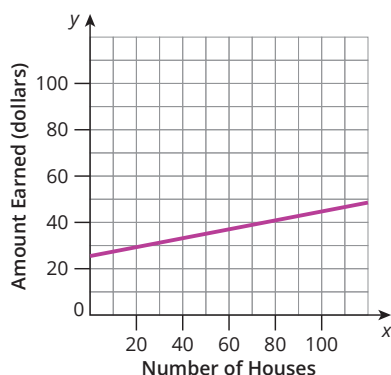
The slope is $-\frac{11}{13}$ and the y-intercept is $(0, 50)$. The y-intercept means the florist could sell 50 dozen carnations and meet her goal. The slope represents the fact that the number of lilies sold decreases by 11 for every 13 carnations sold.

3. Nia has a job delivering newspapers. Each Sunday, she earns \$24.75 plus \$0.20 for each house she delivers to.

- a. Write an equation that represents the amount Nia earns on Sunday given a certain number of houses she delivers to.

Sample answer:
 $y = 24.75 + 0.2x$

- b. Graph the equation.



- c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

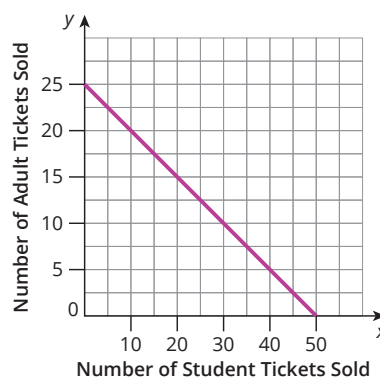
The slope is 0.2 and the y-intercept is 24.75. The y-intercept means that Jordan begins by making \$24.75 before delivering to any houses. The slope means that her pay increases by \$0.20 for every house she delivers to.

4. The high school soccer booster club sells tickets to the varsity matches for \$4 for students and \$8 for adults. The booster club hopes to earn \$200 at each match.

- a. Write an equation that represents the total amount the booster club would like to earn from ticket sales at each match.

Sample answer:
 $4x + 8y = 200$

- b. Graph the equation.



- c. Identify the slope, x-intercept, and y-intercept of the graph. Explain what each means in terms of the situation.

The slope is $-\frac{1}{2}$, the x-intercept is (50, 0), and the y-intercept is (0, 25). The y-intercept means the booster club could earn \$200 by selling 25 adult tickets and 0 student tickets. The x-intercept means the booster could also earn \$200 by selling 0 adult tickets and 50 student tickets. The slope represents the fact that the number of adult tickets sold decreases by 1 for every 2 student tickets sold.



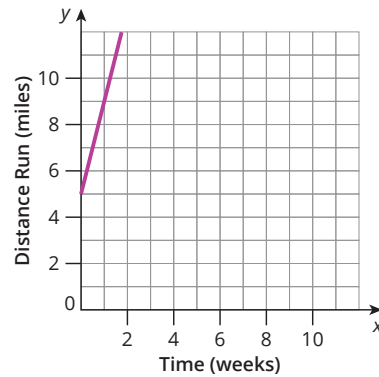
TOPIC 1 Linear functions

5. You are planning a training program to run a marathon. You add 3.9 more miles to your program each week. In week 4 you ran 20.6 miles.

- a. Write an equation that represents the distance you run in miles given a certain number of weeks.

Sample answer:
 $y - 20.6 = 3.9(x - 4)$

- b. Graph the equation.



- c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

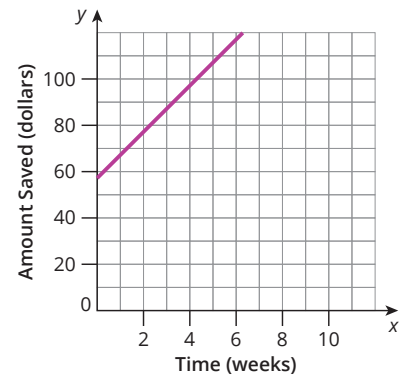
The slope is 3.9 and the y-intercept is 5. The y-intercept means that in week 0 you only run 5 miles. The slope means that for each additional week you run, the distance you run increases by 3.9 miles.

6. Ricardo is saving his money to purchase a smartphone. He already has some money saved, and he saves \$10 a week from his part-time job walking neighborhood dogs. After 3 weeks he has \$87 to put toward the purchase the phone.

- a. Write an equation that represents the total amount saved given a certain number of weeks.

Sample answer:
 $y - 87 = 10(x - 3)$

- b. Graph the equation.



- c. Identify the slope and y-intercept of the graph. Explain what each means in terms of the situation.

The slope is 10 and the y-intercept is 57. The y-intercept means that Ricardo has already saved \$57. The slope means that for each week that passes, Ricardo saves an additional \$10.

Extension

You learned how to generalize the x - and y -intercepts and the slope from the standard form of an equation. Write the point-slope and slope-intercept form of a linear equation in terms of the constants from the standard form. **Extension of TEKS A.2B**

$$y = -\frac{A}{B}x + \frac{C}{B}$$

$$y - 0 = -\frac{A}{B}(x - \frac{C}{A})$$

Spaced Practice

- Write an equation in point-slope form for each problem. **TEKS A.2B**

- $m = -8$ and passes through the point $(3, 12)$

$$y - 12 = -8(x - 3)$$

- passes through the points $(9, -18)$ and $(-3, -26)$

$$y + 18 = \frac{2}{3}(x - 9) \text{ or}$$

$$y + 26 = \frac{2}{3}(x + 3)$$

- Determine whether each table represents a proportional relationship. **TEKS 8.5F**

a.

x	y
-1	-24
2	48
4	90
8	192

No. The table does not represent a proportional relationship because the rate of change is not constant.



TOPIC 1 Linear functions

b.

x	y
2	13.5
5	33.75
10	67.5
15	101.25

Yes. The table represents a proportional relationship because there is a constant rate of change and the y-intercept of the graph is (0, 0). Continuing the pattern in this table results in an output value of 0 for an input of 0.

3. Solve each inequality. **TEKS A.5B**

a. $10 + 5x \geq -25$

$$x \geq -7$$

b. $-4x + 26 < 14$

$$x > 3$$

VI. Making Sense of Different Representations of a Linear Function

Topic Practice

A. Solve each equation involving direct variation.

TEKS A.2D (Topic-Aligned)

1. The value of y is directly proportional to the value of x . When $x = 3$, $y = 12$. What is the value of y when $x = 13$?

 $y = 52$
2. The value of y varies directly with the value of x . When $x = 7$, $y = 17.5$. What is the value of x when $y = 30$?

 $x = 12$
3. Linh mows lawns in his neighborhood to earn money. The amount of money he earns varies directly with the number of lawns he mows. When he mows 10 lawns, Linh makes \$100. How much money would make Linh make if he mowed 6 lawns?

Linh will make \$60 if he mows 6 lawns.
4. James is driving to visit a college campus. The distance he drives is directly proportional to the amount of time he has traveled. When James has driven 2 hours he has traveled 130 miles. How long has he been driving if he traveled 195 miles?

James has been driving for 3 hours if he has traveled 195 miles.
5. Catalina is selling coupon books to raise money for her school. The amount of money she raises is directly proportional to the number of books she sells. If Catalina sells 5 books, she raises 162.50. How much money would Catalina raise for her school if she sells 17 books?

Catalina would raise \$552.50 for her school if she sells 17 books.
6. Juliana is shopping for earrings. The amount of money she spends varies directly with the number of pairs of earrings she buys. When Juliana buys 2 pairs of earrings, she will spend \$45.20. When Juliana spent \$113 on earrings, how many pairs did she buy?

When Juliana spent \$113, she bought 5 pairs of earrings.



TOPIC 1 Linear functions

B. Use each equation or graph to determine input and output values.

TEKS A.2D (Interleaved Practice), TEKS A.12B (Topic-Aligned)

1. For $p(x) = -2x + 5$, determine:

a. $p(5)$

-5

b. $p(0)$

5

c. $p\left(-\frac{1}{2}\right)$

6

3. For $g(x) = 0.75x - 1.2$, determine:

a. $g(8)$

4.8

b. $g(-2)$

-2.7

c. $g(0)$

-1.2

2. For $a(x) = 5$, determine:

a. $a(-3)$

5

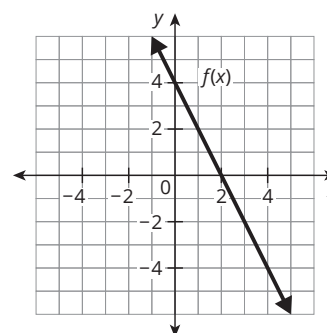
b. $a\left(\frac{1}{4}\right)$

5

c. $a(5)$

5

4. Use the graph to determine when $f(x) = 5$ and the value of $f(5)$.



$f(x) = 5$ when $x = -1$

$f(5) = -7$

5. For $t(x) = \frac{2}{3}x$, determine:

a. $t(9)$

6

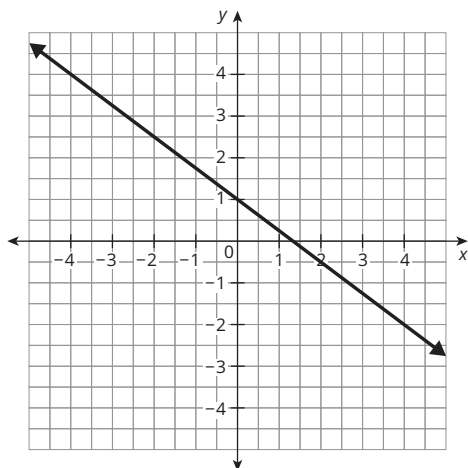
b. $t(-7)$

$-\frac{7}{3}$

c. $t(\frac{3}{5})$

$\frac{2}{5}$

7. Use the graph to determine when $f(x) = 4$ and the value of $f(4)$.



$f(x) = 4$ when $x = -4$

$f(4) = -2$

6. For $b(x) = -1.8x$, determine:

a. $b(-6.3)$

11.34

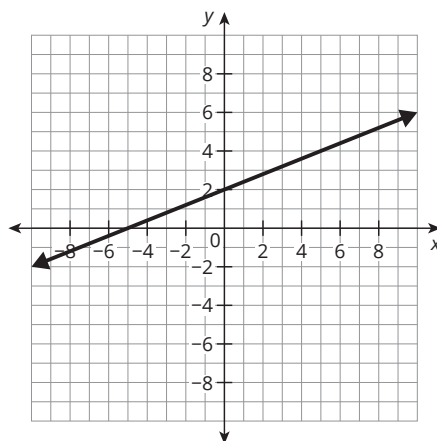
b. $b(12)$

-21.6

c. $b(-\frac{1}{3})$

$-\frac{3}{5}$

8. Use the graph to determine when $f(x) = 0$ and the value of $f(0)$.



$f(x) = 0$ when $x = -5$

$f(0) = 2$



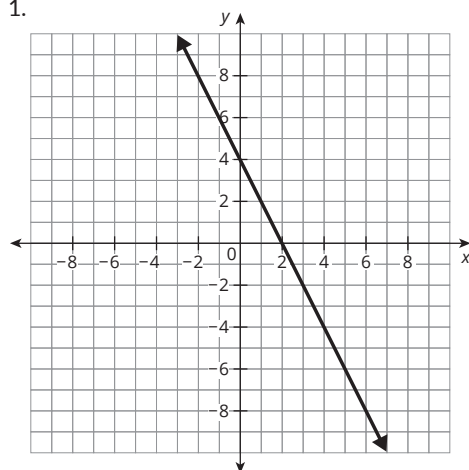
TOPIC 1 Linear functions

C. For the graph shown in each problem, determine the following:

- y-intercept
- x-intercept
- slope
- equation in slope-intercept form
- equation in factored form

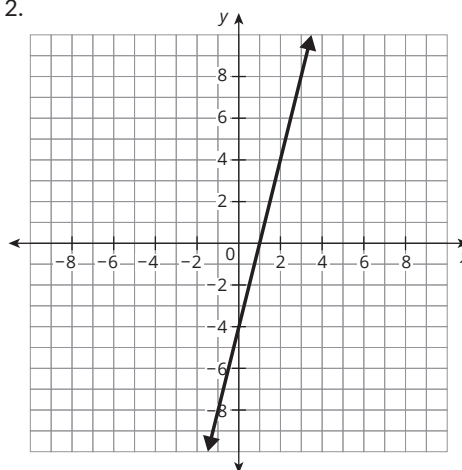
TEKS A.3A (Interleaved Practice), TEKS A.3C (Topic-Aligned)

1.



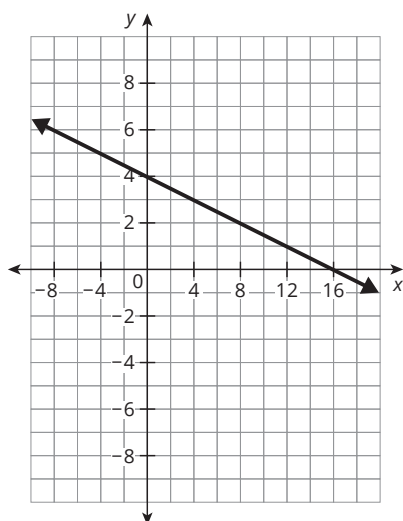
- 4
- 2
- 2
- $y = -2x + 4$
- $y = -2(x - 2)$

2.



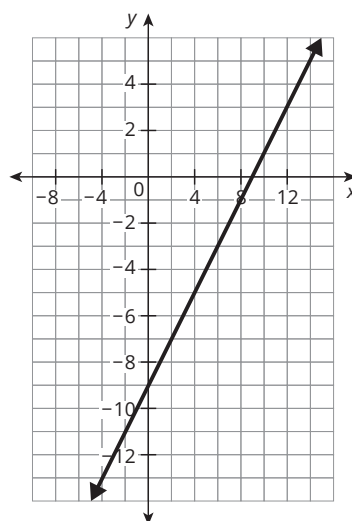
- 4
- 1
- 4
- $y = 4x - 4$
- $y = 4(x - 1)$

3.



- a. 4
- b. 16
- c. $-\frac{1}{4}$
- d. $y = -\frac{1}{4}x + 4$
- e. $y = -\frac{1}{4}(x - 16)$

4.

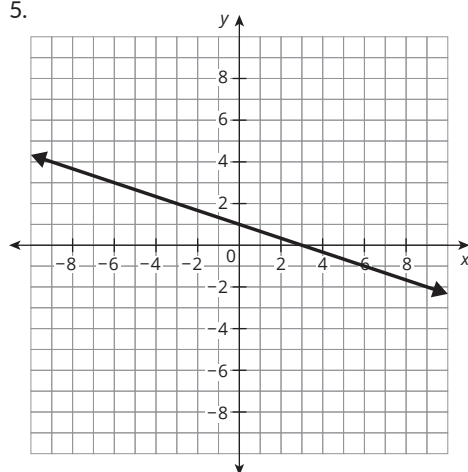


- a. -9
- b. 9
- c. 1
- d. $y = x - 9$
- e. $y = 1(x - 9)$



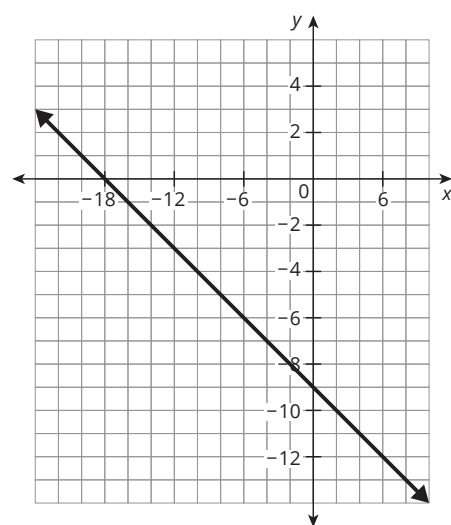
TOPIC 1 Linear functions

5.



- a. 1
- b. 3
- c. $-\frac{1}{3}$
- d. $y = -\frac{1}{3}x + 1$
- e. $y = -\frac{1}{3}(x - 3)$

6.



- a. -9
- b. -18
- c. $-\frac{1}{2}$
- d. $y = -\frac{1}{2}x - 9$
- e. $y = -\frac{1}{2}(x + 18)$

Extension

A pretzel manufacturer has two production lines. Line A produces a variety of pretzel that is sold for \$2.40 per bag. Line A typically produces 3 bags per day that do not meet company standards and cannot be sold. Line B produces a variety of pretzel that is sold for \$3.60 per bag. Line B typically produces 4 bags per day that do not meet company standards and cannot be sold. Line A produces 3 times as many bags as Line B each day.

Write a linear function that represents revenue as a function of the total number of bags the lines can produce combined.

Extension of TEKS A.2C and TEKS A.10A

$$\text{Line A: } \frac{3}{4}x$$

$$\text{Line B: } \frac{1}{4}x$$

$$a(x) = 2.4\left(\frac{3}{4}x - 3\right)$$

$$b(x) = 3.6\left(\frac{1}{4}x - 4\right)$$

$$c(x) = a(x) + b(x)$$

$$= 2.4\left(\frac{3}{4}x - 3\right) + 3.6\left(\frac{1}{4}x - 4\right)$$

$$= 1.8x - 7.2 + 0.9x - 14.4$$

$$= 2.7x - 21.6$$

The linear function $c(x) = 2.7x - 21.6$ represents the revenue as a function of the total number of bags that Line A and Line B can produce combined.



TOPIC 1 Linear functions

Spaced Practice

1. Determine whether each relationship shows a constant difference. If so, write the linear function that represents the relationship. **TEKS A.2C**

a.

x	y
2	9
3	11
4	13
5	15

Yes; $y = 2x + 5$

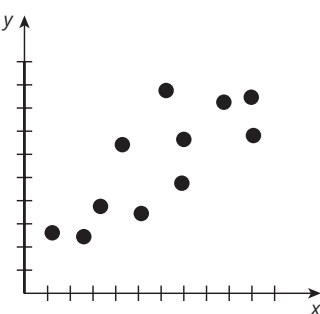
b.

x	y
1	2
2	1
3	$\frac{1}{2}$
4	$\frac{7}{2}$

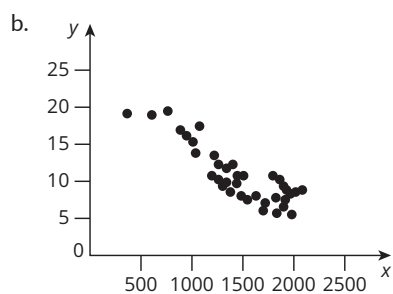
No constant difference

2. Determine whether the points in each scatterplot have a positive association, a negative association, or no association. Explain your reasoning. **TEKS A.4B**

a.



positive association



negative association

3. Solve each equation. **Prerequisite for TEKS A.5A**

a. $\frac{1}{3}x + 2 = 11$

27

b. $-5p - 12 = 19$

$-\frac{31}{5}$



Name _____ Date _____

I. Transforming Linear Functions

Topic Practice

- A. Determine an equation that represents each parallel line. Write your answer in point-slope form, slope-intercept form, and standard form. **TEKS A.2E (Topic-Aligned)**

1. What is the equation of a line parallel to $y = \frac{4}{5}x + 2$ that passes through (1, 2)?

Point-Slope Form	Slope-Intercept Form	Standard Form
$(y - 2) = \frac{4}{5}(x - 1)$	$y - 2 = \frac{4}{5}x - \frac{4}{5}$	$5(y) = 5(\frac{4}{5}x + \frac{6}{5})$
	$y = \frac{4}{5}x - \frac{4}{5} + 2$	$5y = 4x + 6$
	$y = \frac{4}{5}x + \frac{6}{5}$	$4x - 5y = -6$

2. What is the equation of a line parallel to $5x + y = 3$ that passes through (3, 1)?

Point-Slope Form	Slope-Intercept Form	Standard Form
$y - 1 = -5(x - 3)$	$y - 1 = -5x + 15$	$y + 5x = -5x + 5x + 16$
	$y = -5x + 15 + 1$	$5x + y = 16$
	$y = -5x + 16$	

3. What is the equation of a line parallel to $y = 7x - 8$ that passes through (5, -2)?

Point-Slope Form	Slope-Intercept Form	Standard Form
$y + 2 = 7(x - 5)$	$y + 2 = 7x - 35$	$y - y + 37 = 7x - 37 + 37 - y$
	$y = 7x - 35 - 2$	$7x - y = 37$
	$y = 7x - 37$	



TOPIC 2 Transforming and Comparing Linear Functions

4. What is the equation of a line parallel to $y - 7 = -\frac{1}{2}(x + 2)$ that passes through $(-4, 1)$?

Point-Slope Form

$$(y - 1) = -\frac{1}{2}(x + 4)$$

Slope-Intercept Form

$$y - 1 = -\frac{1}{2}x - \frac{4}{2}$$

$$y - 1 = -\frac{1}{2}x - 2$$

$$y = -\frac{1}{2}x - 2 + 1$$

$$y = -\frac{1}{2}x - 1$$

Standard Form

$$2(y) = 2(-\frac{1}{2}x - 1)$$

$$2y = -x - 2$$

$$x + 2y = -2$$

5. What is the equation of a line parallel to $y = \frac{1}{3}x - 4$ that passes through $(9, 8)$?

Point-Slope Form

$$y - 8 = \frac{1}{3}(x - 9)$$

Slope-Intercept Form

$$y - 8 = \frac{1}{3}x - 3$$

$$y = \frac{1}{3}x - 3 + 8$$

$$y = \frac{1}{3}x + 5$$

Standard Form

$$3(y) = 3(\frac{1}{3}x + 5)$$

$$3y = x + 15$$

$$x - 3y = -15$$

6. What is the equation of a line parallel to $4x + y = -7$ that passes through $(2, -9)$?

Point-Slope Form

$$(y + 9) = -4(x - 2)$$

Slope-Intercept Form

$$y + 9 = -4x + 8$$

$$y = -4x + 8 - 9$$

$$y = -4x - 1$$

Standard Form

$$y = -4x - 1$$

$$4x + y = -1$$

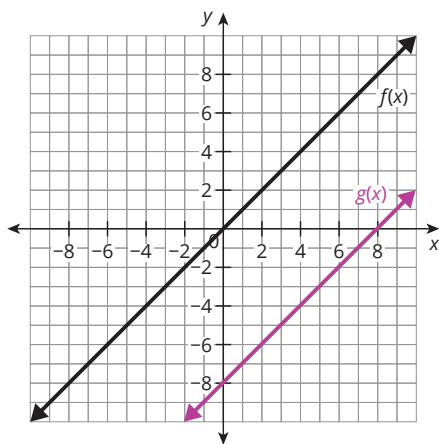


TOPIC 2 Transforming and Comparing Linear Functions

- B. The equation and graph of the parent linear function $f(x) = x$ are given. The equation of a transformed function $g(x)$ is also given. Graph $g(x)$ and describe the transformation(s) performed on $f(x)$ to produce $g(x)$. **TEKS A.3E (Topic-Aligned)**

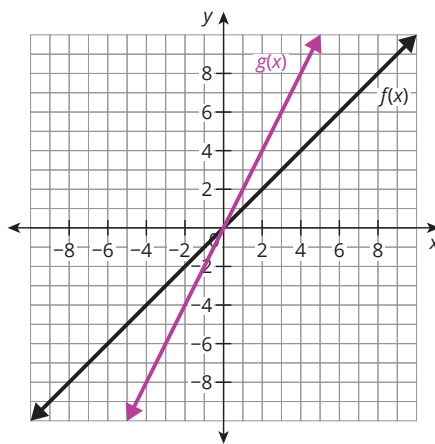
1. $f(x) = x$
 $g(x) = f(x) - 8$

The graph of the function $f(x)$ is translated down 8 units to produce $g(x)$.



2. $f(x) = x$
 $g(x) = 2f(x)$

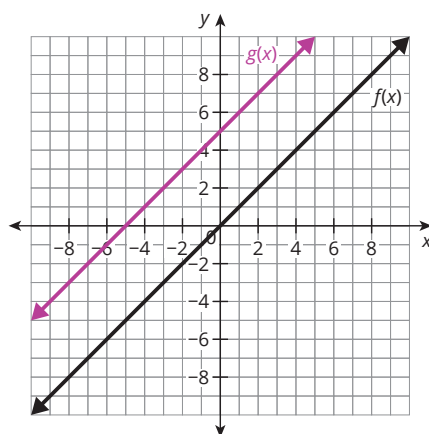
The graph of the function $f(x)$ is stretched vertically by a factor of 2 to produce $g(x)$, a steeper line.



TOPIC 2 Transforming and Comparing Linear Functions

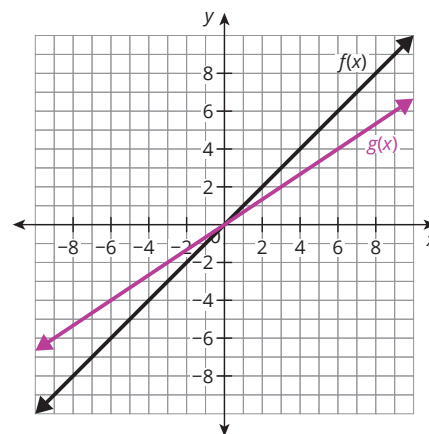
3. $f(x) = x$
 $g(x) = f(x) + 5$

The graph of the function $f(x)$ is translated up 5 units to produce $g(x)$.



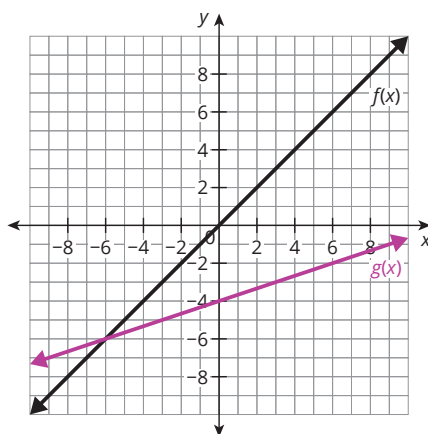
4. $f(x) = x$
 $g(x) = \frac{2}{3}f(x)$

The graph of the function $f(x)$ is compressed vertically by a factor of $\frac{2}{3}$ to produce $g(x)$, a less steep line.



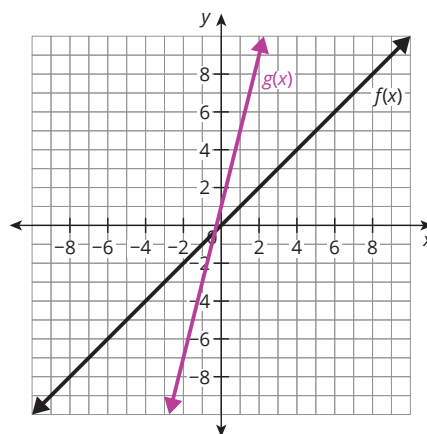
5. $f(x) = x$
 $g(x) = \frac{1}{3}f(x) - 4$

The graph of the function $f(x)$ is dilated vertically by a factor of $\frac{1}{3}$ and translated down 4 units to produce $g(x)$, a less steep line.



6. $f(x) = x$
 $g(x) = 4f(x) + 1$

The graph of the function $f(x)$ is stretched vertically by a factor of 4 and translated up 1 unit to produce $g(x)$, a steeper line.



TOPIC 2 Transforming and Comparing Linear Functions

- C. Write an equation for each transformed function $g(x)$ in terms of $f(x)$ and then simplify the equation, if necessary.

TEKS A.2E (Interleaved Practice), TEKS A.2G (Interleaved Practice),
TEKS A.3E (Interleaved Practice)

1. $f(x) = 3x + 1$ is translated 9 units up.

$$\begin{aligned} g(x) &= f(x) + 9 \\ &= 3x + 1 + 9 \\ &= 3x + 10 \end{aligned}$$

3. $g(x)$ is parallel to $f(x) = -6$ and goes through the point $(5, 8)$.

$$g(x) = 8$$

5. $f(x) = -7 - 4x$ is dilated vertically by a factor of $\frac{1}{2}$.

$$\begin{aligned} g(x) &= \frac{1}{2}f(x) \\ &= \frac{1}{2}(-7 - 4x) \\ &= -2x - \frac{7}{2} \end{aligned}$$

7. $f(x) = 3 - 5x$ is reflected across the x -axis and translated 16 units up.

$$\begin{aligned} g(x) &= -f(x) + 16 \\ &= -(3 + 5x) + 16 \\ &= -3 + 5x + 16 \\ &= 5x + 13 \end{aligned}$$

2. $g(x)$ is parallel to $f(x) = 6x - 10$ and goes through the point $(-2, -9)$.

$$\begin{aligned} g(x) &= 6x + b \\ -9 &= 6(-2) + b \\ -9 &= -12 + b \\ 3 &= b \\ g(x) &= 6x + 3 \end{aligned}$$

4. $g(x)$ is parallel to $f(x) = -2x - 7$ and goes through the point $(\frac{3}{8}, 4\frac{1}{4})$.

$$\begin{aligned} g(x) &= -2x + b \\ 4\frac{1}{4} &= -2(\frac{3}{8}) + b \\ 4\frac{1}{4} &= -\frac{3}{4} + b \\ 5 &= b \\ g(x) &= -2x + 5 \end{aligned}$$

6. $f(x) = \frac{1}{2}x + 2$ is dilated vertically by a factor of 2 and translated 4 units down.

$$\begin{aligned} g(x) &= 2f(x) - 4 \\ &= 2(\frac{1}{2}x + 2) \\ &= x + 4 - 4 \\ &= x \end{aligned}$$

8. $g(x)$ is parallel to $f(x) = 3.5$ and goes through the point $(-3, -7.25)$.

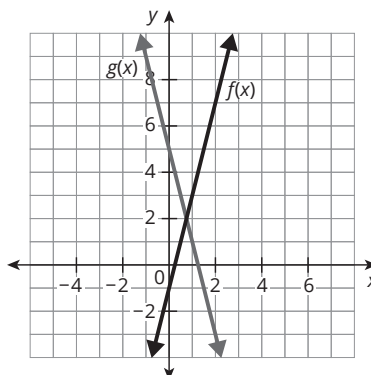
$$g(x) = -7.25.$$



TOPIC 2 Transforming and Comparing Linear Functions

Extension

The functions $f(x)$ and $g(x)$ are shown on the graph. Write an equation for each function in slope-intercept form. Then, write an equation for $g(x)$ in terms of $f(x)$. **Extension of A.2C and A.3E**



$$f(x) = 4x - 1; g(x) = -4x + 5; g(x) = -f(x) + 4$$

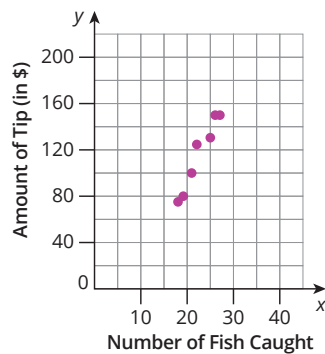
Spaced Practice

1. James works as a fly-fishing guide. The table indicates the number of fish caught on each expedition he guided in a week and the amount of the tip he received for each expedition.

Number of Fish Caught	Amount of Tip (\$)
22	125
19	80
25	130
26	150
21	100
18	75
27	150

TOPIC 2 Transforming and Comparing Linear Functions

- a. Construct a scatterplot of the data. **TEKS A.4B**



- b. Based on the shape of the scatterplot is a linear regression appropriate? What type of correlation appears to be present?

TEKS A.4B

Yes. A linear regression is appropriate. A strong positive correlation appears to be present.

- c. Use technology to write a function to represent the line of best fit. **TEKS A.4C**

$$t(x) = 8.6x - 79$$

- d. Compute and interpret the correlation coefficient. **TEKS A.4A**

$r \approx 0.97$; This value represents a strong positive linear association between the number of fish caught per expedition and the amount of the tip received for each expedition.



TOPIC 2 Transforming and Comparing Linear Functions

2. Determine whether each table of values represents a linear function. If so, write the function. If not, explain why.

a. **TEKS A.12A, A.2C**

x	y
2	3
4	4
6	5
8	6

$$y = \frac{1}{2}x + 2$$

b. **TEKS A.12A**

x	y
-4	-17
-2	-9
2	7
4	17

Not a linear function. The slope is not constant between all given points.

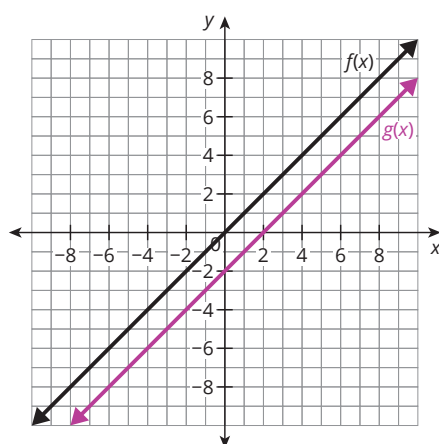
II. Vertical and Horizontal Transformations of Linear Functions

Topic Practice

- A. The equation and graph of the parent linear function $f(x) = x$ are given. The equation of a transformed function $g(x)$ is also given. Graph $g(x)$ and describe the transformation(s) performed on $f(x)$ to produce $g(x)$. TEKS A.3E (Topic-Aligned)

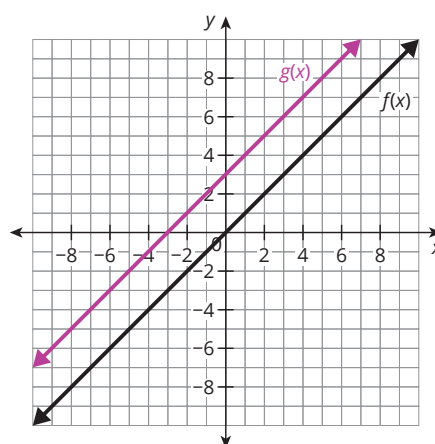
1. $f(x) = x$
 $g(x) = f(x - 2)$

The graph of the function $f(x)$ is translated right 2 units to produce $g(x)$.



2. $f(x) = x$
 $g(x) = f(x + 3)$

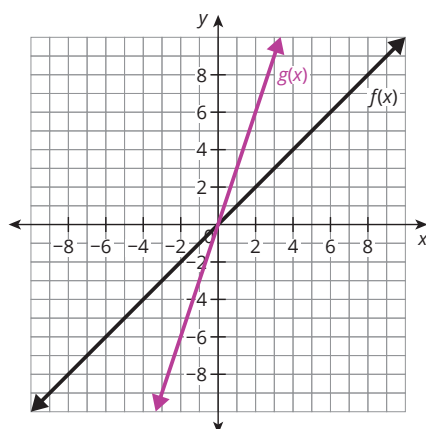
The graph of the function $f(x)$ is translated left 3 units to produce $g(x)$.



TOPIC 2 Transforming and Comparing Linear Functions

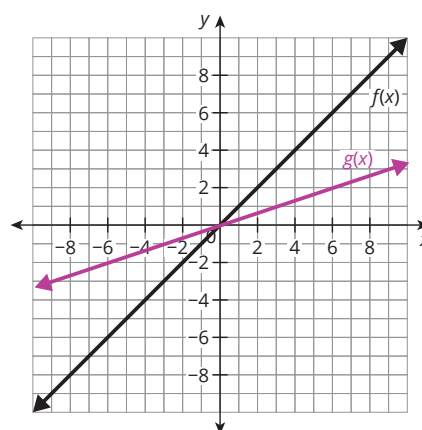
3. $f(x) = x$
 $g(x) = f(3x)$

The graph of the function $f(x)$ is horizontally compressed by a factor of 3 to produce $g(x)$, a steeper line.



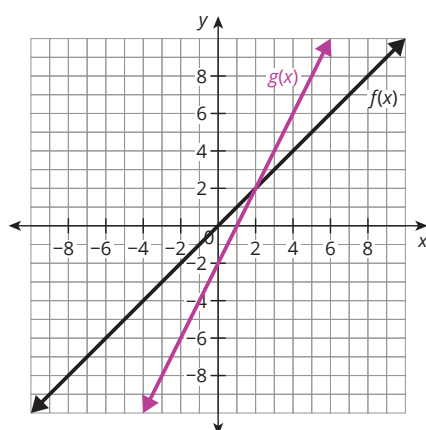
4. $f(x) = x$
 $g(x) = f\left(\frac{1}{3}x\right)$

The graph of the function $f(x)$ is horizontally stretched by a factor of 3 to produce $g(x)$, a less steep line.



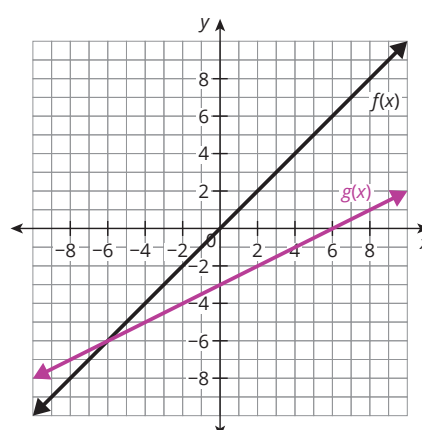
5. $f(x) = x$
 $g(x) = 2(f(x - 1))$

The graph of the function $f(x)$ is vertically stretched by a factor of 2 and translated right 1 unit to produce $g(x)$, a steeper line.



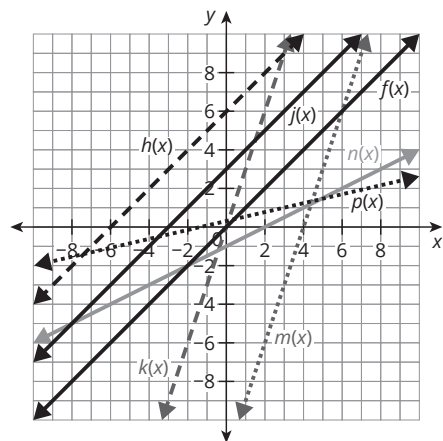
6. $f(x) = x$
 $g(x) = f\left(\frac{1}{2}x\right) - 3$

The graph of the function $f(x)$ is horizontally stretched by a factor of 2 and translated down 3 units to produce $g(x)$, a less steep line.



TOPIC 2 Transforming and Comparing Linear Functions

B. The graph shows the linear function $f(x) = x$ and six transformations of $f(x)$. TEKS A.3E (Topic-Aligned)



Equations
$\frac{1}{4} \cdot f(x)$
$3 \cdot f(x)$
$f(x + 6)$
$f(x) + 3$
$f\left(\frac{1}{2}x\right) - 1$
$3 \cdot f(x - 4)$

Match each transformed graph to the equation in the table, written in terms of $f(x)$.

1. $p(x) = \frac{1}{4} \cdot f(x)$

2. $h(x) = f(x + 6)$

3. $j(x) = f(x) + 3$

4. $m(x) = 3 \cdot f(x - 4)$

5. $k(x) = 3 \cdot f(x)$

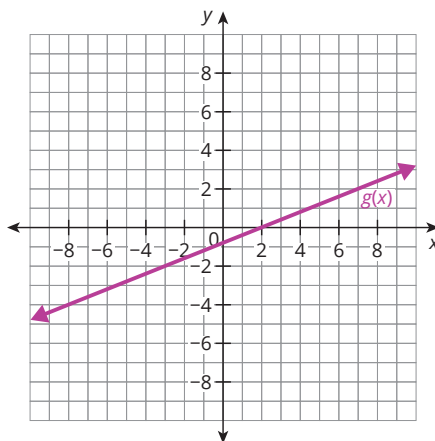
6. $n(x) = f\left(\frac{1}{2}x\right) - 1$



TOPIC 2 Transforming and Comparing Linear Functions

Extension

The function $g(x)$ is a transformation of $f(x) = x$. Write the function $g(x)$ in terms of $f(x)$. **Extension of TEKS A.3E**



$$g(x) = \frac{2}{5}f(x - 2)$$

TOPIC 2 Transforming and Comparing Linear Functions

Spaced Practice

Describe the characteristics of each function. **TEKS A.2A, A.3C**

1. $m(x) = 5x - 2$

Domain: All Real Numbers

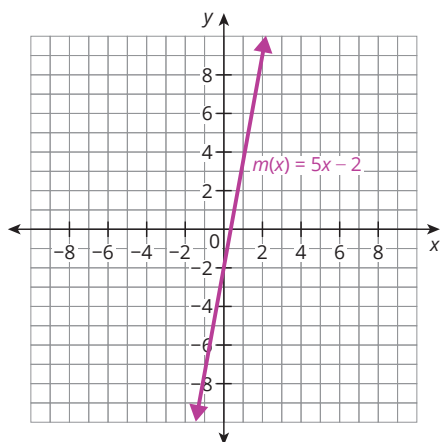
Range: All Real Numbers

x-intercept: $(\frac{2}{5}, 0)$

y-intercept: $(0, -2)$

Slope: 5

Increasing/Decreasing: Increasing over the entire domain



2. $g(x) = -\frac{1}{2}x + 6$

Domain: All Real Numbers

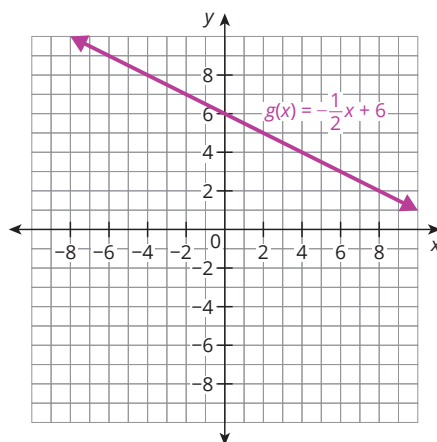
Range: All Real Numbers

x-intercept: $(12, 0)$

y-intercept: $(0, 6)$

Slope: $-\frac{1}{2}$

Increasing/Decreasing: Decreasing over the entire domain



TOPIC 2 Transforming and Comparing Linear Functions

3. $j(x) = 4 + \frac{3}{4}x$

Domain: All Real Numbers

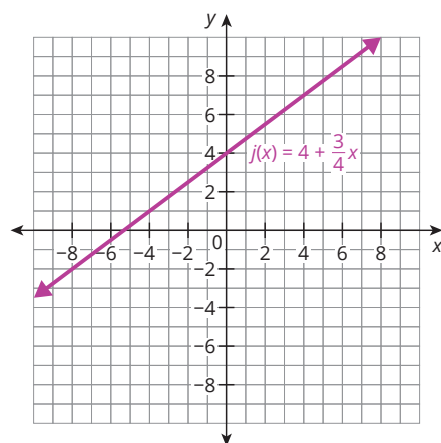
Range: All Real Numbers

x-intercept: $(-\frac{16}{3}, 0)$

y-intercept: $(0, 4)$

Slope: $\frac{3}{4}$

Increasing/Decreasing: Increasing over the entire domain



III. Determining Slopes of Perpendicular Lines

Topic Practice

A. Use the given information to write the equation and identify the slope of each line. **TEKS A.2F (Interleaved Practice), TEKS A.2G (Topic-Aligned)**

1. Write an equation for a horizontal line and an equation for a vertical line that passes through the point $(3, -3)$. Identify the slope of each line.

The equation of the horizontal line is $y = -3$ and the slope is 0. The equation of the vertical line is $x = 3$ and the slope is undefined.

3. Write an equation for a horizontal line and an equation for a vertical line that passes through the point $(0.5, 1.25)$. Identify the slope of each line.

The equation of the horizontal line is $y = 1.25$ and the slope is 0. The equation of the vertical line is $x = 0.5$ and the slope is undefined.

5. Write an equation for a line that is perpendicular to the line $x = -2$ and passes through the point $(1, 5)$. Identify the slope of the line.

The equation of the line is $y = 5$ and the slope is 0.

2. Write an equation for a horizontal line and an equation for a vertical line that passes through the point $(-1, 4)$. Identify the slope of each line.

The equation of the horizontal line is $y = 4$ and the slope is 0. The equation of the vertical line is $x = -1$ and the slope is undefined.

4. Write an equation for a line that is parallel to the line $x = 5$ and passes through the point $(-2, 3)$. Identify the slope of the line.

The equation of the line is $x = -2$ and the slope is undefined.

6. Write an equation for a line that is perpendicular to the line $y = 3$ and passes through the point $(5, 7)$. Identify the slope of the line.

The equation of the line is $x = 5$ and the slope is undefined.



TOPIC 2 Transforming and Comparing Linear Functions

B. Determine an equation that represents each perpendicular line. Write your answer in point-slope form, slope-intercept form and standard form. TEKS A.2F (Topic-Aligned)

- What is the equation of a line perpendicular to $y = 2x - 6$ that passes through $(5, 4)$?

The slope of the new line must be $-\frac{1}{2}$.

Point-Slope Form: Slope-Intercept Form: Standard Form:

$$\begin{array}{lll} y - 4 = -\frac{1}{2}(x - 5) & y - 4 = -\frac{1}{2}x + \frac{5}{2} & 2(y) = 2\left(-\frac{1}{2}x + \frac{5}{2}\right) \\ & y = -\frac{1}{2}x + \frac{5}{2} + 4 & 2y = -x + 13 \\ & y = -\frac{1}{2}x + \frac{13}{2} & x + 2y = 13 \end{array}$$

- What is the equation of a line perpendicular to $3x + y = 4$ that passes through $(-1, 6)$?

The slope of the new line must be $\frac{1}{3}$.

Point-Slope Form: Slope-Intercept Form: Standard Form:

$$\begin{array}{lll} y - 6 = \frac{1}{3}(x + 1) & y - 6 = \frac{1}{3}x + \frac{1}{3} & 3(y) = 3\left(\frac{1}{3}x + \frac{1}{3}\right) \\ & y = \frac{1}{3}x + \frac{1}{3} + 6 & 3y = x + 19 \\ & y = \frac{1}{3}x + \frac{19}{3} & x - 3y = -19 \end{array}$$

- What is the equation of a line perpendicular to $y = -\frac{2}{5}x - 1$ that passes through $(2, -8)$?

The slope of the new line must be $\frac{5}{2}$.

Point-Slope Form: Slope-Intercept Form: Standard Form:

$$\begin{array}{lll} y + 8 = \frac{5}{2}(x - 2) & y + 8 = \frac{5}{2}x - 5 & 2(y) = 2\left(\frac{5}{2}x - 13\right) \\ & y = \frac{5}{2}x - 5 - 8 & 2y = 5x - 26 \\ & y = \frac{5}{2}x - 13 & 5x - 2y = 26 \end{array}$$

TOPIC 2 Transforming and Comparing Linear Functions

4. What is the equation of a line perpendicular to $3x - 4y = -48$ that passes through $(12, 3)$?

The slope of the new line must be $-\frac{4}{3}$.

Point-Slope Form:

Slope-Intercept Form:

Standard Form:

$$y - 3 = -\frac{4}{3}(x - 12)$$

$$y - 3 = -\frac{4}{3}x + 16$$

$$3(y) = 3\left(-\frac{4}{3}x + 19\right)$$

$$y = -\frac{4}{3}x + 16 + 3$$

$$3y = -4x + 57$$

$$y = -\frac{4}{3}x + 19$$

$$4x + 3y = 57$$

5. What is the equation of a line perpendicular to $y = 6x - 5$ that passes through $(6, -3)$?

The slope of the new line must be $-\frac{1}{6}$.

Point-Slope Form:

Slope-Intercept Form:

Standard Form:

$$y + 3 = -\frac{1}{6}(x - 6)$$

$$y + 3 = -\frac{1}{6}x + 1$$

$$6(y) = 6\left(-\frac{1}{6}x - 2\right)$$

$$y = -\frac{1}{6}x + 1 - 3$$

$$6y = -x - 12$$

$$y = -\frac{1}{6}x - 2$$

$$x + 6y = -12$$

6. What is the equation of a line perpendicular to $y - 4 = \frac{5}{2}(x - 2)$ that passes through $(-1, -4)$?

The slope of the new line must be $-\frac{2}{5}$.

Point-Slope Form:

Slope-Intercept Form:

Standard Form:

$$y + 4 = -\frac{2}{5}(x + 1)$$

$$y + 4 = -\frac{2}{5}x - \frac{2}{5}$$

$$5(y) = 5\left(-\frac{2}{5}x - \frac{22}{5}\right)$$

$$y = -\frac{2}{5}x - \frac{2}{5} - 4$$

$$5y = -2x - 22$$

$$y = -\frac{2}{5}x - \frac{22}{5}$$

$$2x + 5y = -22$$

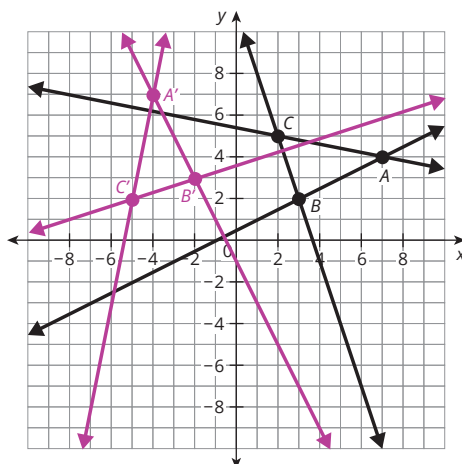


TOPIC 2 Transforming and Comparing Linear Functions

Extension

Triangle ABC is located on three lines such that the vertices occur at the points of intersection of pairs of the lines, as shown on the graph.

If $\triangle ABC$ is rotated 90° counterclockwise around the origin to form $\triangle A'B'C'$, determine the equations of the lines that would contain $\triangle A'B'C'$. Explain your reasoning. Then, draw the three lines that contain $\triangle A'B'C'$ and label $\triangle A'B'C'$. **Extension of TEKS A.2F**



If $\triangle ABC$ is rotated 90° counterclockwise, then the three lines of $\triangle A'B'C'$ will be perpendicular to the lines of $\triangle ABC$, so the slopes of those lines are negative reciprocals. Point A' is $(-4, 7)$, Point B' is $(-2, 3)$, and Point C' is $(-5, 2)$.

Line $A'C'$:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= 5(x - (-4)) \\ y &= 5x + 27 \end{aligned}$$

Line $A'B'$:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= -2(x - (-2)) \\ y &= -2x - 1 \end{aligned}$$

Line $B'C'$:

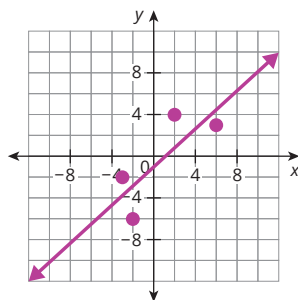
$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= \frac{1}{3}(x - (-5)) \\ y &= \frac{1}{3}x + \frac{11}{3} \end{aligned}$$

TOPIC 2 Transforming and Comparing Linear Functions

Spaced Practice

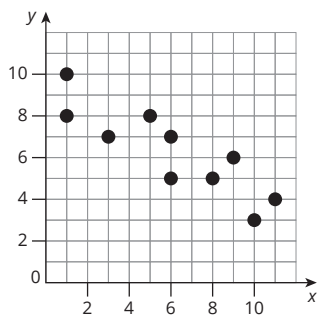
- Graph the estimated line of best fit for the set of points: $(-2, -6)$, $(-3, -2)$, $(2, 4)$, $(6, 3)$. Determine the estimated linear regression model for the line. **TEKS A.4C**

Sample answer:



$$y = \frac{11}{12}x - 1$$

- Determine whether the points on the scatterplot have a positive correlation, a negative correlation, or no correlation. Then, determine which r -value is most accurate. **TEKS A.4B**



- $r = 0.005$
- $r = -0.865$
- $r = -0.045$
- $r = 0.905$

The data has a negative correlation.

Choice b, $r = -0.865$, is the most accurate r -value.



TOPIC 2 Transforming and Comparing Linear Functions

3. Write the equation of a line parallel to the line $2x - 3y = 6$ that passes through the given points. **TEKS A.2E**

a. $(0, 3)$

$$y = \frac{2}{3}x + 3$$

b. $(-4, -1)$

$$y = \frac{2}{3}x + \frac{5}{3}$$

4. Write a recursive and explicit formula for the arithmetic sequence shown. **TEKS A.12D**
 $4, 8, 12, 16, 20, \dots$

Recursive:

$$a_n = a_{n-1} + 4;$$

$$\text{Explicit: } a_n = 4 + 4(n - 1)$$

5. Write a recursive and explicit formula for the geometric sequence shown. **TEKS A.12D**
 $9, -3, 1, -\frac{1}{3}, \frac{1}{9}, \dots$

Recursive:

$$a_n = a_{n-1} \cdot \left(-\frac{1}{3}\right);$$

Explicit:

$$a_n = 9 \cdot \left(-\frac{1}{3}\right)^{n-1}$$

6. Write the equation of a line that is perpendicular to $y = -\frac{2}{3}x + 7$ and goes through the point $(27, -3)$. **TEKS A.2F**

$$y = \frac{3}{2}x - 30$$

7. Write the equation of a line that is perpendicular to $y = 1\frac{4}{5}x - 12$ and goes through the point $(-15, -9\frac{1}{2})$. **TEKS A.2F**

$$y = \frac{5}{9}x + 5\frac{1}{2}$$



IV. Comparing Linear Functions in Different Forms

Topic Practice

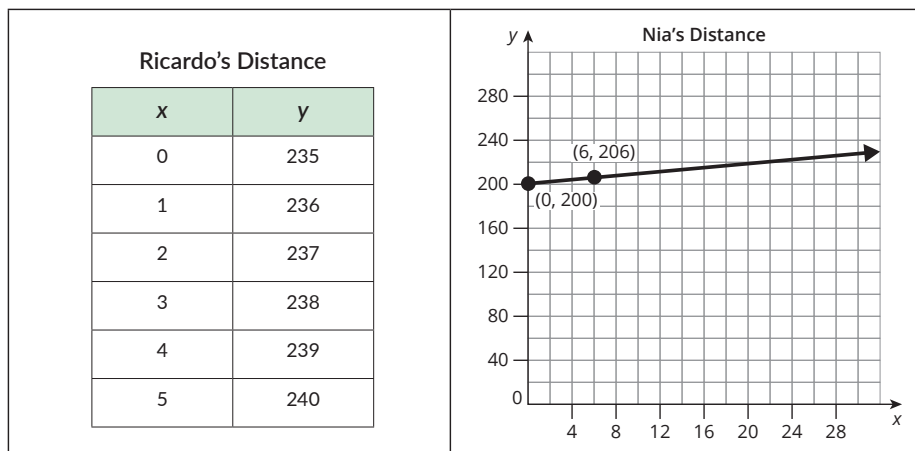
- A. For each scenario, complete the following:
- Determine and then compare the rates of change for each function in terms of the quantities represented.
 - Determine and then compare the y -intercepts of each function in terms of the quantities represented.
 - Determine the domain and range for each problem situation using words and inequalities.

TEKS A.2A (Topic-Aligned), TEKS A.3B (Interleaved Practice), TEKS A.3C (Topic-Aligned)



TOPIC 2 Transforming and Comparing Linear Functions

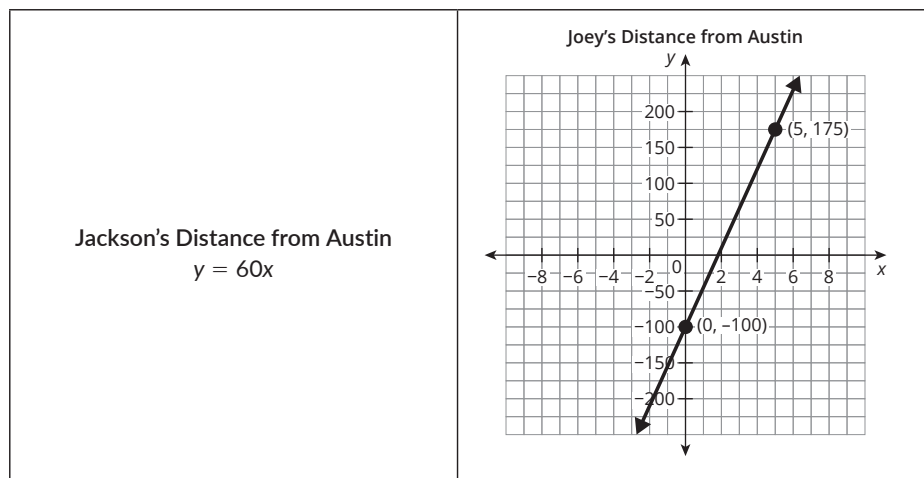
1. On the track team, two long jumpers, Ricardo and Nia, are steadily improving their maximum distances during the course of the season. These functions represent the distance jumped in inches, y , as a function of the number of months, x , that have passed during the season.



- a. The rate of change for Ricardo's distance is 1. This means he jumps 1 more inch each month. The rate of change for Nia's distance is 1. This means she jumps 1 more inch each month. They are both increasing their distances at the same rate.
- b. The y -intercept for Ricardo's distances is 235. This means that he jumped 23 inches at the beginning of the track season. The y -intercept for Nia's distances is 200. This means that she jumped 200 inches at the start of the season. Ricardo jumped further at the beginning of the track season than Nia.
- c. Ricardo's Distance:
Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
Range: The range is all real numbers greater than or equal to 235. $y \geq 235$
- Nia's Distance:
Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
Range: The range is all real numbers greater than or equal to 200. $y \geq 200$

TOPIC 2 Transforming and Comparing Linear Functions

2. Jackson and Joey take separate journeys driving north on Interstate 35 towards Dallas. Jackson begins his journey in Austin, while Joey begins his journey further south of Austin. These functions represent each person's distance in miles from Austin, y , as a function of time in hours, x , since each person began their journey.

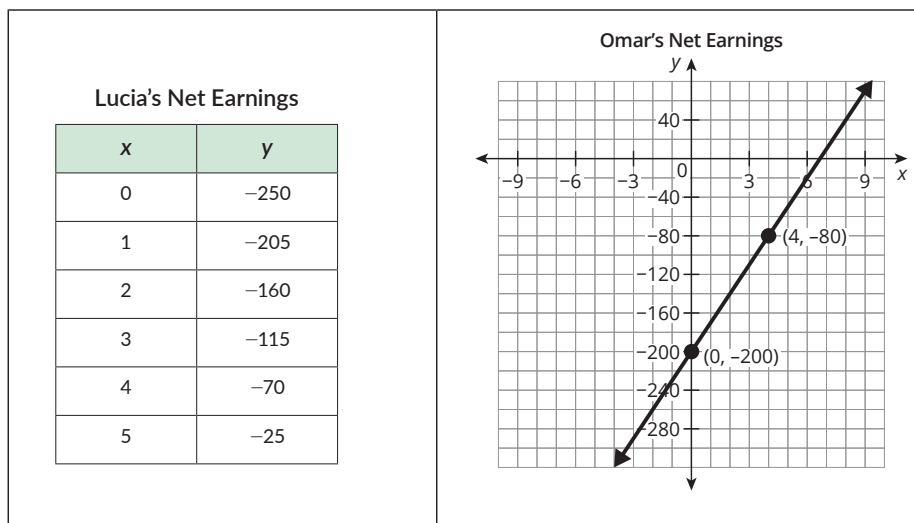


- a. The rate of change for Jackson's distance is 60. This means he is traveling 60 miles per hour. The rate of change for Joey's speed is 55. This means he is traveling 55 miles per hour. Jackson is driving at a faster rate.
- b. The y -intercept for Jackson's distance is 0. This means that he starts the trip in Austin. The y -intercept for Joey's distance is -100 . This means that he was 100 miles away from Austin before he started his trip to Dallas. Joey started farther away from Dallas than Tremain did.
- c. Jackson's Distance:
Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
Range: The range is all real numbers greater than or equal to 0. $y \geq 0$
- Joey's Distance:
Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
Range: The range is all real numbers greater than or equal to -100 . $y \geq -100$



TOPIC 2 Transforming and Comparing Linear Functions

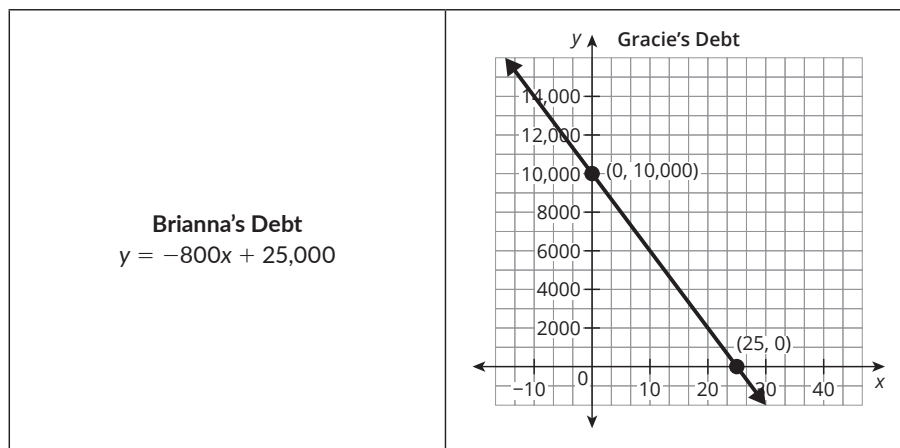
3. These functions represent the net earnings in dollars, y , as a function of the number of lawns mowed, x , for Lucia and Omar's lawn mowing businesses, including the start-up cost of buying a lawn mower.



- a. The rate of change for Lucia's net earnings is 45. This means she earns \$45 for each lawn she mows. The rate of change for Omar's net earnings is 30. This means he earns \$30 for each lawn he mows. Lucia earns more per lawn than Omar.
- b. The y-intercept for Lucia's net earnings is -250 . This means the lawn mower cost her \$250. The y-intercept for Omar's net earnings is -200 . This means the lawn mower cost him \$200. Lucia paid more for her lawnmower compared to Omar.
- c. Lucia's Net Earnings:
Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
Range: The range is all real numbers greater than or equal to -250 . $y \geq -250$
- Omar's Net Earnings:
Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
Range: The range is all real numbers greater than or equal to -200 . $y \geq -200$

TOPIC 2 Transforming and Comparing Linear Functions

4. A generous grandmother loaned money for college to two of her grandchildren, Brianna and Gracie, letting them pay it back without interest. These functions represent the amount in dollars, y , that Brianna and Gracie owe to their grandmother over the course of time in months, x .



- a. The rate of change for Brianna's debt is -800 . This means she is paying her grandmother back \$800 a month. The rate of change for Gracie's debt is -400 . This means she is paying her grandmother back \$400 a month. Brianna is paying more per month than Gracie.
- b. The y -intercept for Brianna's debt is 25,000. This means she initially borrowed \$25,000 from her grandmother. The y -intercept for Gracie's debt is 10,000. This means she initially borrowed \$10,000 from her grandmother. Brianna borrowed more money than Gracie did.
- c. Brianna's Debt:
- Domain: The domain is all real numbers greater than or equal to 0 and less than or equal to 32. $0 \leq x \leq 32$
- Range: The range is all real numbers greater than or equal to 0 and less than or equal to 25,000. $0 \leq y \leq 25,000$

Gracie's Debt:

Domain: The domain is all real numbers greater than or equal to 0 and less than or equal to 25. $0 \leq x \leq 25$

Range: The range is all real numbers greater than or equal to 0 and less than or equal to 10,000. $0 \leq y \leq 10,000$



TOPIC 2 Transforming and Comparing Linear Functions

5. Xavier and Nicky are both saving money. These functions represent the amount in dollars, y , in their savings accounts over the course of time in months, x .

Xavier's Account Balance		Nicky's Account Balance $y - (-275) = 100(x - (-5))$
x	y	
0	300	
1	425	
2	550	
3	675	
4	800	

- a. The rate of change for Xavier's account balance is 125. This means he is saving \$125 each month. The rate of change for Nicky's account balance is 100. This means she is saving \$100 each month. Marcus is saving more per month than Nicky.
- b. The y -intercept for Xavier's account balance is 300. This means he started with \$300 in his account. The y -intercept for Nicky's account balance is 225. This means she started with \$225 in her account. Xavier started with more money in his account compared to Nicky.
- c. Xavier's Account Balance :
- Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
- Range: The range is all real numbers greater than or equal to 300. $y \geq 300$
- Nicky's Account Balance:
- Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
- Range: The range is all real numbers greater than or equal to 225. $y \geq 225$

TOPIC 2 Transforming and Comparing Linear Functions

6. James and Ricardo leave Harrisburg at different times and drive towards Philadelphia. These functions represent their distances in miles, y , and time in hours since James started driving, x .

James's Distance from Harrisburg		Ricardo's Distance from Harrisburg $y = 65x + 150$
x	y	
0	0	
1	50	
2	100	
3	150	
4	200	
5	250	

- a. The rate of change for James's distance is 50. This means he is driving 50 miles per hour. The rate of change for Ricardo's distance is 65. This means he is driving 65 miles per hour. Ricardo is driving at a faster rate than James.
- b. The y -intercept for James's distance is 0. This means he started in Harrisburg. The y -intercept for Ricardo's distance is 150. This means he was 150 miles away from Harrisburg when James left Harrisburg. Louis was farther away from Harrisburg than James.
- c. James's Distance:
Domain: The domain is all real numbers greater than or equal to 0. $x \geq 0$
Range: The range is all real numbers greater than or equal to 0. $y \geq 0$
- Ricardo's Distance:
Domain: The domain is all real numbers greater than or equal to $-\frac{30}{13}$. $x \geq -\frac{30}{13}$
Range: The range is all real numbers greater than or equal to 0. $y \geq 0$



TOPIC 2 Transforming and Comparing Linear Functions

Extension

1. Omar and Antonio are twins. Their parents track their height every year between the ages of 5 and 15. Omar's height is given by the equation, and Antonio's height is shown in the graph.

Extension of TEKS A.3C

- a. Label the x- and y-axis, the origin, and the intervals on both axes. Explain your reasoning.

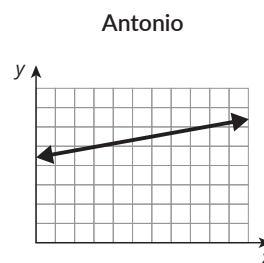
The x-axis should start with 5, be scaled by 1s, and labeled Number of Years from Age 5. The y-axis should be scaled by 10s, and labeled Height (in inches).

- b. Which twin is growing faster? Justify your answer.

Omar is growing faster. Using (8, 50) and (14, 60), Antonio's rate of growth is 1.67 inches per year, while according to the equation, Omar's rate of growth is 3.1 inches per year.

- c. At what age does one twin surpass the other in height? Explain your reasoning.

Omar
 $y = 3.1x + 40.6$



Between ages 8 and 9, Omar's height surpasses Antonio's height. At age 8, Omar is 49.9 inches tall, $3.1(3) + 40.6 = 49.9$, and Antonio is 50 inches tall. At age 9, Omar is 53 inches tall, $3.1(4) + 40.6 = 53$, and Antonio is 51.67 inches tall, $1.67(9) + 36.6$.

TOPIC 2 Transforming and Comparing Linear Functions

Spaced Practice

1. For each situation decide whether the correlation implies causation. List reasons why or why not. **TEKS A.4B**

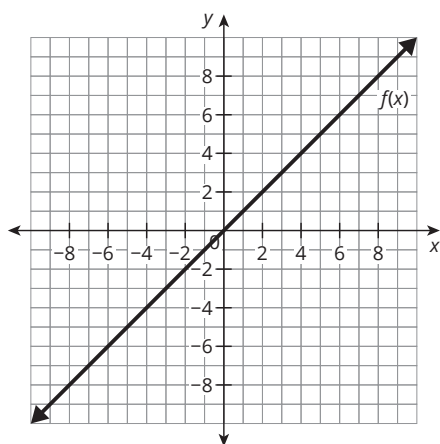
- a. The number of winter coats sold at department stores is highly correlated to average low temperatures in the area.

The correlation does imply causation. People who live in areas with colder temperatures need winter coats to function well within their environment, so colder temperatures do cause more people to buy winter coats.

- b. The number of concessions sold at a concert is highly correlated to the number of people in attendance at the concert.

The correlation does not imply causation. Although there is a correlation between the number of people attending a concert and the number of concessions sold, attending a concert does not cause people to buy the concessions.

2. The graph represents the parent function $f(x) = x$. The equation for the transformed function $g(x)$ is $g(x) = \frac{2}{3} \cdot f(x) - 2$.



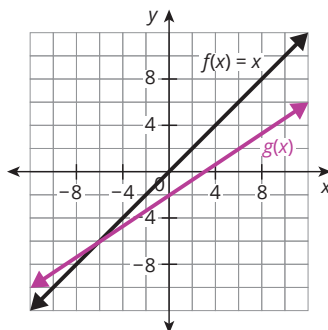
TOPIC 2 Transforming and Comparing Linear Functions

- a. Describe the transformations performed on $f(x)$ to produce $g(x)$.

TEKS A.3E

The function is vertically dilated by a factor of $\frac{2}{3}$ and is translated down 2 units.

- b. Graph $g(x)$. TEKS A.3E



- c. Write the equation of $g(x)$ in slope-intercept form and identify the slope and y-intercept. TEKS A.2C, A.3C

$g(x) = \frac{2}{3}x - 2$; The slope is $\frac{2}{3}$ and the y-intercept is -2 .

3. Write a recursive formula for each sequence. TEKS A.12D

- a. 3, -6 , 12, -24 , 48, ...

$$g_n = g_{n-1} \cdot (-2)$$

- b. 180, 160, 140, 120, 100, ...

$$a_n = a_{n-1} - 20$$

4. Write the equation of a line that is perpendicular to $y = -\frac{2}{3}x + 7$ and goes through the point $(27, -3)$. TEKS A.2F

$$y = \frac{3}{2}x - 30$$

5. Write the equation of a line that is perpendicular to $y = 1\frac{4}{5}x - 12$ and goes through the point $(-15, -9\frac{1}{2})$. TEKS A.2F

$$y = \frac{5}{9}x + 5\frac{1}{2}$$

Modeling Linear Equations and Inequalities

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Name _____ Date _____

I. Solving Linear Equations

Topic Practice

A. Solve each equation. Write the properties that justify each step.

TEKS A.5A (Topic-Aligned)

1. $-3(x - 4) = -9(x - 1)$

Sample answer:

$$-3(x - 4) = -9(x - 1)$$

$$-3x + 12 = -9x + 9$$

$$6x + 12 = 9$$

$$6x = -3$$

$$x = -\frac{1}{2}$$

distributive property

addition property of equality

subtraction property of equality

division property of equality

2. $8x - 2(x + 3) = 4x + 2$

Sample answer:

$$8x - 2(x + 3) = 4x + 2$$

$$8x - 2x - 6 = 4x + 2$$

$$6x - 6 = 4x + 2$$

$$2x - 6 = 2$$

$$2x = 8$$

$$x = 4$$

distributive property

associative property

subtraction property of equality

addition property of equality

division property of equality



TOPIC 1 Linear Equations and Inequalities

3. $\frac{-2x+1}{2} + 6 = \frac{3x}{2} - 10$

Sample answer:

$$\frac{-2x+1}{2} + 6 = \frac{3x}{2} - 10$$

$$-2x + 1 + 12 = 3x - 20$$

$$-2x + 13 = 3x - 20$$

$$-5x + 13 = -20$$

$$-5x = -33$$

$$x = \frac{33}{5}$$

multiplication property of equality

associative property

subtraction property of equality

subtraction property of equality

division property of equality

4. $12x - 4\left(\frac{1}{2}x - 5\right) = \frac{1}{3}(6x - 15)$

Sample answer:

$$12x - 4\left(\frac{1}{2}x - 5\right) = \frac{1}{3}(6x - 15)$$

$$12x - 2x + 20 = 2x - 5$$

$$10x + 20 = 2x - 5$$

$$8x + 20 = -5$$

$$8x = -25$$

$$x = -\frac{25}{8}$$

distributive property of equality

associative property

subtraction property of equality

subtraction property of equality

division property of equality

5. $\frac{7(x-1)}{4} - \frac{3}{4} = -8x + \frac{3}{4}$

Sample answer:

$$\frac{7(x-1)}{4} - \frac{3}{4} = -8x + \frac{3}{4}$$

$$7(x-1) - 3 = -32x + 3$$

$$7x - 7 - 3 = -32x + 3$$

$$7x - 10 = -32x + 3$$

$$39x - 10 = 3$$

$$39x = 13$$

$$x = \frac{1}{3}$$

multiplication property of equality

distributive property

associative property

addition property of equality

addition property of equality

division property of equality

TOPIC 1 Linear Equations and Inequalities

6. $-4(2x - 9) + 6(-x + 1) = -8x - 5\left(3x - \frac{5}{6}\right)$

Sample answer:

$$-4(2x - 9) + 6(-x + 1) = -8x - 5\left(3x - \frac{6}{5}\right)$$

$$-8x + 36 - 6x + 6 = -8x - 15x + 6$$

distributive property

$$-14x + 42 = -23x + 6$$

associative property

$$9x + 42 = 6$$

addition property of equality

$$9x = -36$$

subtraction property of equality

$$x = -4$$

division property of equality

B. Determine if the equation has one solution, no solution, or infinite solutions. Show your work. TEKS A.5A (Topic-Aligned)

1. $3(x + 2) = 3x + 6$

$$3(x + 2) = 3x + 6$$

$$3x + 6 = 3x + 6$$

$$6 = 6$$

Infinite solutions

3. $20x - 2(x + 10) = -(5 - 2x)$

$$20x - 2(x + 10) = -(5 - 2x)$$

$$20x - 2x - 20 = -5 + 2x$$

$$18x - 20 = -5 + 2x$$

$$16x = 15$$

$$x = \frac{15}{16}$$

One solution

2. $5x + 4 = 5x - 3$

$$5x + 4 = 5x - 3$$

$$4 \neq -3$$

No solution

4. $\frac{3}{5}(x - 12) = -4(x + 9) + 1$

$$\frac{3}{5}(x - 12) = -4(x + 9) + 1$$

$$\frac{3}{5}x - \frac{36}{5} = -4x - 36 + 1$$

$$\frac{3}{5}x - \frac{36}{5} = -4x - 35$$

$$\frac{23}{5}x - \frac{36}{5} = -35$$

$$\frac{23}{5}x = -\frac{139}{5}$$

$$x = -\frac{139}{23}$$

One solution



TOPIC 1 Linear Equations and Inequalities

5. $-7(x - 1) = -15x + 8(x + 2)$

$$-7(x - 1) = -15x + 8(x + 2)$$

$$-7x + 7 = -15x + 8x + 16$$

$$-7x + 7 = -7x + 16$$

$$7 \neq 16$$

No solution

6. $\frac{8(x-3)}{2} + 5x = 9(x-1) - 3$

$$\frac{8(x-3)}{2} + 5x = 9(x-1) - 3$$

$$8(x-3) + 10x = 18(x-1) - 6$$

$$8x - 24 + 10x = 18x - 18 - 6$$

$$18x - 24 = 18x - 24$$

$$-24 = -24$$

Infinite solutions

C. For each question, write and solve an equation to determine the unknown value. **TEKS A.5A (Topic-Aligned)**

1. Triangle A has a base of $y + 4$ inches and height of 10 inches. Triangle B has a height of 14 inches and a base of $2y + 1$. Determine the value of y when both triangles have the same area.

$$\frac{1}{2} \cdot 10(y + 4) = \frac{1}{2} \cdot 14(2y + 1)$$

$$5(y + 4) = 7(2y + 1)$$

$$5y + 20 = 14y + 7$$

$$13 = 9y$$

$$\frac{13}{9} = 1.\bar{4} = y$$

2. In $\triangle DEF$, $\angle D$ and $\angle F$ are congruent base angles of an isosceles triangle. Determine the value of g when $m\angle D = 3(g - 2)^\circ$ and $m\angle F = (g + 20)^\circ$.

$$3(g - 2) = g + 20$$

$$3g - 6 = g + 20$$

$$2g = 26$$

$$g = \frac{26}{2} = 13$$

TOPIC 1 Linear Equations and Inequalities

3. A rectangle has a width of 6 inches and a length of $3x + 8$ inches. A square has an area of $20x - 4$ square inches. Determine the value of x when the area of the square is equal to the area of the rectangle.

$$6(3x + 8) = 20x - 4$$

$$18x + 48 = 20x - 4$$

$$52 = 2x$$

$$26 = x$$

4. Congruent angles have the same measure. Determine the value of x , when $m\angle J = \frac{1}{2}(4x + 16)$ and $m\angle K = 4(2x - 1)$ and $\angle J$ is congruent to $\angle K$.

$$\frac{1}{2}(4x + 16) = 4(2x - 1)$$

$$2x + 8 = 8x - 4$$

$$-6x = -12$$

$$x = 2$$

5. A parallelogram has a base of 9 cm and a height of $\frac{1}{3}c + 2$. A rectangle has a base of $2c + 4$ and a height of 2 cm. Determine the value of c when the area of the parallelogram is the same as the area of the rectangle.

$$9\left(\frac{1}{3}c + 2\right) = 2(2c + 4)$$

$$3c + 18 = 4c + 8$$

$$10 = c$$

6. The side length of a square is $3x + 7$. A rectangle has side lengths of 5 and $x + 10$. Determine the value of x when the perimeter of the square is equal to the perimeter of the rectangle.

$$4(3x + 7) = 2\{(x + 10) + 5\}$$

$$12x + 28 = 2(x + 15)$$

$$12x + 28 = 2x + 30$$

$$10x = 2$$

$$x = \frac{2}{10} = 0.2$$



TOPIC 1 Linear Equations and Inequalities

Extension

1. Consider the equation $2x - 5(x - 1) = 50$.
 - a. Solve the equation for x .
Extension of TEKS A.5A
 $x = -15$
 - b. Emma was asked to solve the inequality $2x - 5(x - 1) < 50$. She gave an answer of $x < -15$. Substitute any value for x less than (-15) to determine if Emma is correct. If not, determine the correct solution. **Extension of TEKS A.5B**
Sample answer:
 $x = -100, 305 \leq 50$
No. Emma is not correct.
 $x > -15$

Spaced Practice

1. Determine whether the table of values represents a linear function. If it does represent a linear function, write the function. If it does not represent a linear function, explain why. **TEKS A.12A**

No. It does not represent a linear function.
The differences between the output values for consecutive input values is not the same.

x	$f(x)$
-2	4
-1	1
0	0
1	1

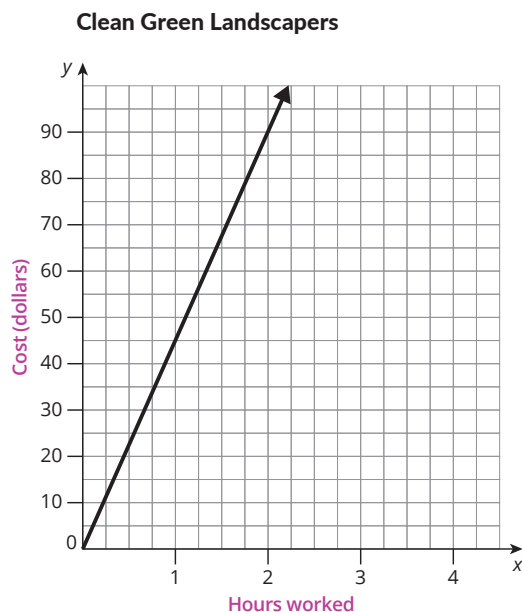
2. Ethan grows tomatoes and sells them at a nearby farmer's roadside stand. He sells them for \$2.50 each. The farmer charges him \$15 per day to use the stand. Write a linear function in point-slope form and slope-intercept form that represents the amount of money, M , Ethan will make from selling x tomatoes. **TEKS A.2B**

$$M(x) = 2.50x - 15.00; M(x) = 2.50(x - 6)$$



TOPIC 1 Linear Equations and Inequalities

3. Clean Green Landscapers uses a graph to show what they charge, and Sunshine Landscaper lists what they charge in a table. **TEKS A.3C**



Sunshine Landscaper

Hours worked	Cost (dollars)
0.5	\$25
1	\$50
1.5	\$75
2	\$100
2.5	\$125

- a. Each representation shows a functional relationship between quantities. Label the quantities and their units in the table and on the graph.

Independent value: hours worked

Dependent value: cost (dollars)

- b. Let $C(x)$ represent the function for Clean Green Landscapers, and let $S(x)$ represent the function for Sunshine Landscapers. Which function has a steeper slope? Explain how you know.

The slope of function $S(x)$ is greater than the slope of function $C(x)$.



TOPIC 1 Linear Equations and Inequalities

4. Evaluate the function $f(x) = 0.4x^2 - 3x - 8$ for the value $x = -2$.

TEKS A.12B

$$f(-2) = -0.4$$

5. The cost to install x number of central air conditioning units for a company is given by the function $C(x) = \frac{4000x + 1300}{3}$. Use the function to determine the cost to install 45 air conditioning units. TEKS A.12B

$$C(45) = 60,433.33$$

II. Literal Equations

Topic Practice

- A. Convert each equation from standard form to slope-intercept form. Determine the slope. TEKS A.12E (Interleaved Practice), TEKS A.3A (Topic-Aligned)

1. $4x + 6y = 48$

$$y = -\frac{2}{3}x + 8$$

$$m = -\frac{2}{3}$$

3. $-4x + 9y = 45$

$$y = \frac{4}{9}x + 5$$

$$m = \frac{4}{9}$$

5. $-x - 8y = 96$

$$y = -\frac{1}{8}x - 12$$

$$m = -\frac{1}{8}$$

2. $3x - 5y = 25$

$$y = \frac{3}{5}x - 5$$

$$m = \frac{3}{5}$$

4. $6x - 2y = -52$

$$y = 3x + 26$$

$$m = 3$$

6. $12x + 28y = -84$

$$y = -\frac{3}{7}x - 3$$

$$m = -\frac{3}{7}$$



TOPIC 1 Linear Equations and Inequalities

B. Convert each equation from point-slope form to the indicated form. TEKS A.12E (Topic-Aligned)

1. $y - 2 = 3(x - 1)$ to slope-intercept form

Slope-intercept form:

$$y - 2 = 3(x - 1)$$

$$y - 2 = 3x - 3$$

$$y - 2 + 2 = 3x - 3 + 2$$

$$y = 3x - 1$$

3. $y + 5 = \frac{3}{4}(x + 4)$ to standard form

Standard form:

$$y + 5 = \frac{3}{4}(x + 4)$$

$$y + 5 = \frac{3}{4}x + 3$$

$$y + 5 - 5 = \frac{3}{4}x + 3 - 5$$

$$y = \frac{3}{4}x - 2$$

$$y - \frac{3}{4}x = \frac{3}{4}x - \frac{3}{4}x - 2$$

$$4(-\frac{3}{4}x + y) = -2(4)$$

$$-3x + 4y = -8$$

5. $y - 4 = 5(x - 3)$ to standard form

Standard form:

$$y - 4 = 5(x - 3)$$

$$y - 4 = 5x - 15$$

$$y - 4 + 4 = 5x - 15 + 4$$

$$y - 5x = 5x - 5x - 11$$

$$-5x + y = -11$$

2. $y - (-6) = 4x$ to point-slope form

Standard form:

$$y - (-6) = 4x$$

$$y + 6 = 4x$$

$$y + 6 - 6 = 4x - 6$$

$$y - 4x = 4x - 4x - 6$$

$$-4x + y = -6$$

4. $y - 3 = -\frac{1}{5}(x + 10)$ to slope-intercept form

Slope-intercept form:

$$y - 3 = -\frac{1}{5}(x + 10)$$

$$y - 3 = -\frac{1}{5}x - 2$$

$$y - 3 + 3 = -\frac{1}{5}x - 2 + 3$$

$$y = -\frac{1}{5}x + 1$$

6. $y = -\frac{1}{3}(x - 6)$ to slope-intercept form

Slope-intercept form:

$$y = -\frac{1}{3}(x - 6)$$

$$y = -\frac{1}{3}x + 2$$



TOPIC 1 Linear Equations and Inequalities

C. Convert each equation from standard form to slope-intercept form.

TEKS A.12E (Topic Aligned)

1. $4x + 2y = 10$

$$\begin{aligned}4x - 4x + 2y &= 10 - 4x \\ \frac{2y}{2} &= \frac{10 - 4x}{2} \\ y &= -2x + 5\end{aligned}$$

3. $-8x + y = -24$

$$\begin{aligned}-8x + 8x + y &= -24 + 8x \\ y &= 8x - 24\end{aligned}$$

5. $x - 3y = 18$

$$\begin{aligned}x - x - 3y &= 18 - x \\ \frac{-3y}{-3} &= \frac{18 - x}{-3} \\ y &= \frac{1}{3}x - 6\end{aligned}$$

2. $3x - 4y = 12$

$$\begin{aligned}3x - 3x - 4y &= 12 - 3x \\ \frac{-4y}{-4} &= \frac{12 - 3x}{-4} \\ y &= \frac{3}{4}x - 3\end{aligned}$$

4. $12x + 8y = 16$

$$\begin{aligned}12x - 12x + 8y &= 16 - 12x \\ \frac{8y}{8} &= \frac{16 - 12x}{8} \\ y &= -1.5x + 2\end{aligned}$$

6. $9x - y = 27$

$$\begin{aligned}9x - 9x - y &= 27 - 9x \\ \frac{-y}{-1} &= \frac{27 - 9x}{-1} \\ y &= 9x - 27\end{aligned}$$



TOPIC 1 Linear Equations and Inequalities

D. Solve each equation for the variable indicated. **TEKS A.12E (Topic-Aligned)**

1. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Solve the equation for h .

$$\frac{2A}{b} = h$$

3. The formula for the volume of a cylinder is $V = \pi r^2 h$. Solve the equation for h .

$$\frac{V}{\pi r^2} = h$$

5. The Ideal Gas Law is $pV = nRT$. Solve the equation for T .

$$\frac{pV}{nR} = T$$

7. Solve the literal equation $\frac{a_1}{a_0} = \frac{b_1}{b_0}$ for b_0 .

$$b_0 = -\frac{a_0 b_1}{a_1}$$

2. The formula for the area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$. Solve the equation for b_1 .

$$\frac{2A}{h} - b_2 = b_1$$

4. The formula for the volume of a pyramid is $V = \frac{1}{3}lwh$. Solve the equation for w .

$$\frac{3V}{lh} = w$$

6. Solve the literal equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ for R_1 .

$$R_1 = \frac{RR_2}{R_2 - R}$$

8. Solve the literal equation $Z = \frac{4x}{Y^2} + 3W$ for x .

$$x = \frac{Y^2(Z - 3W)}{4}$$



TOPIC 1 Linear Equations and Inequalities

Extension

A simple pendulum is made of a long string and a small metal sphere.

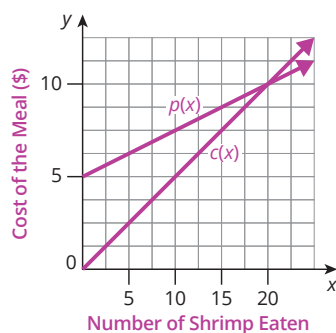
The period of oscillation can be found by the formula $T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$, where g is the acceleration due to gravity, and L is the length of the

string. Solve the formula for g , the acceleration due to gravity. **TEKS A.12E**

$$g = \frac{4L\pi^2}{T^2}$$

Spaced Practice

1. The Peters Creek restaurant has an all-you-can eat shrimp deal. Currently, the cost of the deal is 50 cents per shrimp, with free soft drinks. The cost for the shrimp deal is modeled by the function $c(x) = 0.50x$, where x represents the number of shrimp eaten. A new manager decides to change the cost of the deal to 25 cents per shrimp, but a \$5.00 charge for soft drinks. Let $p(x)$ be the function that represents the new cost for the all-you-can-eat shrimp deal.
 - a. Sketch the graph of $c(x)$ and $p(x)$ on the same coordinate plane. **TEKS A.3C**



TOPIC 1 Linear Equations and Inequalities

- b. Complete the table of corresponding points on $p(x)$. **TEKS A.3C**

Transformed Graph	
x	$p(x)$
0	5
10	7.50
40	15
60	20
70	22.50
90	27.50

- c. Write an equation for $p(x)$ in terms of $c(x)$. Describe the transformation performed on $c(x)$ to produce $p(x)$. **TEKS A.3E**

$p(x) = 0.5 \cdot c(x) + 5$. The graph of $c(x)$ is vertically compressed by a factor of 0.5 and shifted up 5 units to produce $p(x)$.

- d. Write the equation for the function $p(x)$ in slope-intercept form. **TEKS A.2C**

$$p(x) = 0.25x + 5$$

2. Solve the equation. Write the properties that justify each step.

TEKS A.5A

$$-\frac{2}{3}(-9x + 24) = 2x - 4$$

$$-\frac{2}{3}(-9x + 24) = 2x - 4$$

$$6x - 16 = 2x - 4$$

distributive property

$$4x - 16 = -4$$

subtraction property of equality

$$4x = 12$$

addition property of equality

$$x = 3$$

division property of equality

3. Determine whether the equation has one solution, no solution, or infinite solutions. **TEKS A.5A**

$$3\left(2 + \frac{2}{3}x\right) = 5 + 2(x + 1)$$

No solution

4. The table shows the relationship between y and x . Write an equation that represents the relationship between the variables. **TEKS A.2C**

$$y = 2x + 8$$

x	y
23	2
21	6
1	10
3	14
6	20



TOPIC 1 Linear Equations and Inequalities

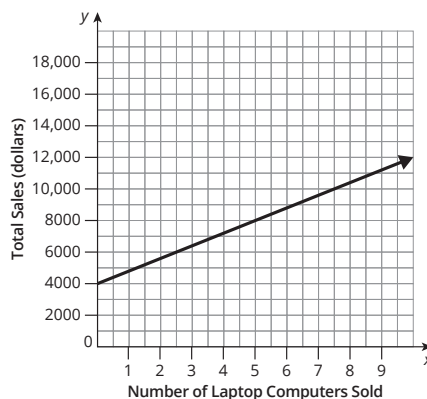
5. A clothing store decides to give out bonus points that can be used for future purchases. If a customer applies for a bonus card, they are automatically given 50 bonus points. After that, they get 25 bonus points for every \$1.00 that they spend. Write an equation that shows the number of bonus points, b , that a customer will earn for x dollars that they spend. **TEKS A.2C**

$$b = 25x + 50$$

III. Modeling Linear Inequalities

Topic Practice

- A. Samuel works at an electronics store selling computer equipment. He can earn a bonus if he sells \$10,000 worth of computer equipment this month. So far this month, he has sold \$4000 worth of computer equipment. He hopes to sell additional laptop computers for \$800 each to reach his goal. The function $f(x) = 800x + 4000$ represents Samuel's total sales as a function of the number of laptop computers he sells.



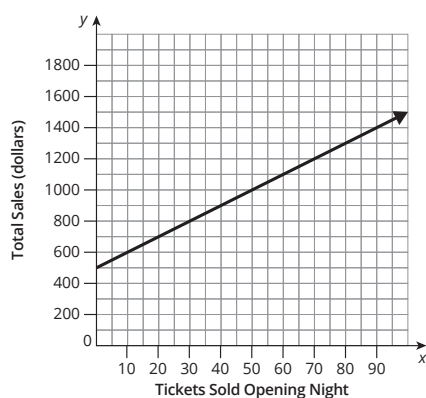
Use the graph to write an equation or inequality to determine the number of laptop computers Samuel would need to sell to earn each amount. **TEKS A.2C (Interleaved Practice), TEKS A.5A (Topic-Aligned), TEKS A.5B (Topic-Aligned)**



TOPIC 1 Linear Equations and Inequalities

1. At least \$10,000 $800x + 4000 \geq 10,000; x \geq 8$	2. Less than \$7000 $800x + 4000 < 7000; x < 4$
3. Less than \$6000 $800x + 4000 < 6000; x < 3$	4. At least \$9000 $800x + 4000 \geq 9000; x \geq 7$
5. More than \$12,000 $800x + 4000 > 12,000; x > 10$	6. Exactly \$8000 $800x + 4000 = 8000; x = 5$

- B. Isabella works at the ticket booth of a local playhouse. On the opening night of the play, tickets are \$10 each. The playhouse has already sold \$500 worth of tickets during a presale. The function $f(x) = 10x + 500$ represents the total sales as a function of tickets sold on opening night.



Use the graph of the function to answer each question. Graph each solution on the number line. **TEKS A.5A (Interleaved Practice),** **TEKS A.5B (Interleaved Practice)**

1. How many tickets must Isabella sell in order to make at least \$1000?

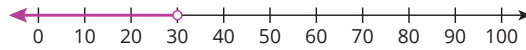
Isabella must sell at least 50 tickets. $x \geq 50$



TOPIC 1 Linear Equations and Inequalities

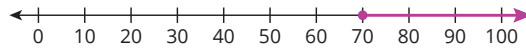
2. How many tickets must Isabella sell in order to make less than \$800?

Isabella must sell fewer than 30 tickets. $x < 30$



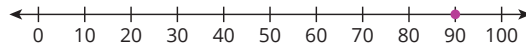
3. How many tickets must Isabella sell in order to make at least \$1200?

Isabella must sell at least 70 tickets. $x \geq 70$



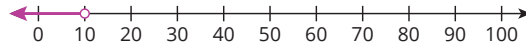
4. How many tickets must Isabella sell in order to make exactly \$1400?

Isabella must sell exactly 90 tickets. $x = 90$



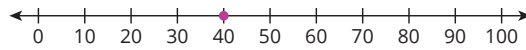
5. How many tickets must Isabella sell in order to make less than \$600?

Isabella must sell fewer than 10 tickets. $x < 10$



6. How many tickets must Isabella sell in order to make exactly \$900?

Isabella must sell exactly 40 tickets. $x = 40$



C. Write and solve a linear inequality to answer each question.

TEKS A.5B (Topic-Aligned)

1. A hot air balloon at 4000 feet begins its descent. It descends at a rate of 200 feet per minute. The function $f(x) = -200x + 4000$ represents the height of the balloon as it descends. How many minutes have passed if the balloon is below 3000 feet?

More than 5 minutes have passed if the balloon is below 3000 feet. $x > 5$

2. A bathtub filled with 55 gallons of water is drained. The water drains at a rate of 5 gallons per minute. The function $f(x) = -5x + 55$ represents the volume of water in the tub as it drains. How many minutes have passed if the tub still has more than 20 gallons of water remaining in it?

Less than 7 minutes have passed if the tub has more than 20 gallons of water remaining in it. $x < 7$



TOPIC 1 Linear Equations and Inequalities

3. Harper is walking to school at a rate of 250 feet per minute. Her school is 5000 feet from her home. The function $f(x) = 250x$ represents the distance Harper walks. How many minutes have passed if Harper still has more than 2000 feet to walk?
4. Diego is riding his bike to school at a rate of 600 feet per minute. His school is 9000 feet from his home. The function $f(x) = 600x$ represents the distance Diego rides. How many minutes have passed if Diego has less than 3000 feet left to ride?

Less than 12 minutes have passed if Harper still has more than 2000 feet to walk. $x < 12$

More than 10 minutes have passed if Diego has less than 3000 feet to ride. $x > 10$

5. A submarine begins to dive from its current depth at a rate of 20 feet per minute. The function $f(x) = -20x - 30$ represents the depth of the submarine as it dives. How many minutes have passed if the submarine is at least 160 feet below the surface?
6. Daniel plays on the varsity basketball team. So far this season, he has scored a total of 52 points. He scores an average of 13 points per game. The function $f(x) = 13x + 52$ represents the total number of points Daniel will score this season. How many more games must Daniel play in order to score at least 100 points?

At least 9.5 minutes have passed if the submarine is at least 160 feet below the surface. $x \geq 9.5$

Daniel must play in 4 or more games to score at least 100 points. $x \geq 4$

D. Solve each inequality and then graph the solution set on the number line. TEKS A.5B (Topic-Aligned)

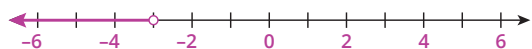
1. $4x + 3 \leq 3x - 5$

$x \leq -8$



2. $-2x > 6$

$x < -3$



TOPIC 1 Linear Equations and Inequalities

3. $\frac{1}{8}(3x - 16) < 4$

$x < 16$



4. $\frac{x-3}{2} \geq -5$

$x \geq -7$



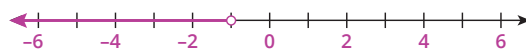
5. $-4(2 - x) \leq 6(x + 2)$

$x \geq -10$



6. $-\frac{1}{2}(4x + 20) < -7\left(x + \frac{15}{7}\right)$

$x < -1$



Extension

A company orders its nut mixes every month from a distributor. The distributor charges \$4.50 per pound of nut mix. There is a handling fee of \$8.50 for every order. There is free shipping on any order between \$100 and \$400. Write a compound inequality to represent the number of pounds of nuts the company can order and get free shipping. Solve the inequality and graph the solution on one number line.

Extension of TEKS A.5B

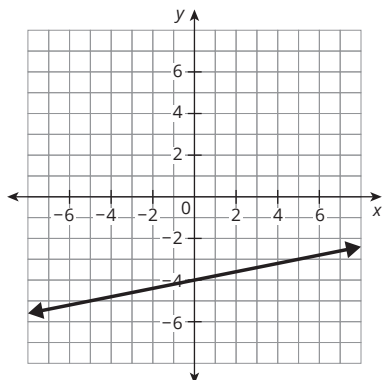
$100 < 4.50x + 8.50 \leq 400$

$20\frac{1}{3} < x \leq 87$



Spaced Practice

1. Calculate the average rate of change for the linear function using the rate of change formula. Show your work. **TEKS A.3B**



$$\frac{f(t) - f(s)}{t - s} = \frac{-4 - (-5)}{0 - (-4)} = \frac{1}{4}$$

The rate of change is $\frac{1}{4}$.

2. Determine whether the table of values represents a linear function. If so, write the function. **TEKS A.12A, A.2C**

Linear function

$$f(x) = -\frac{1}{2}x + \frac{7}{2}$$

x	y
-2	$4\frac{1}{2}$
0	$3\frac{1}{2}$
3	2
6	$\frac{1}{2}$

3. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is its height and b_1 and b_2 are the lengths of each base. **TEKS A.12E**

- a. Determine the area of a trapezoid if its height is 10 cm and the lengths of its bases are 22 cm and 18 cm.

$$200 \text{ cm}^2$$

- b. Rewrite the equation to solve for b_1 .

$$b_1 = \frac{2A - hb_2}{h}$$



TOPIC 1 Linear Equations and Inequalities

- c. Determine the length of the other base of a trapezoid if one base measures 10 meters, the height is 20 meters, and the area of the trapezoid is 600 square meters.

$$b_1 = 50 \text{ m}$$

4. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Convert the equation to solve for b . **TEKS A.12E**

$$b = \frac{2A}{h}$$



Name _____ Date _____

I. Using Graphing to Solve Systems of Equations

Topic Practice

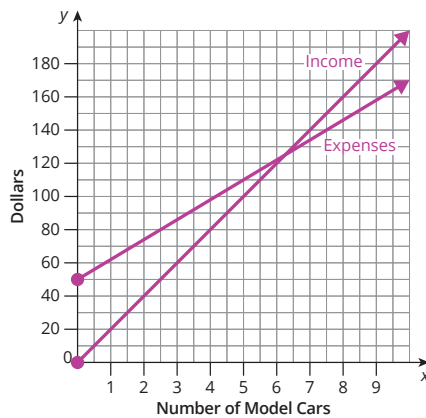
- A. Write a system of linear equations to represent each problem situation. Define each variable. Then, graph the system of equations and estimate the point of intersection. Explain what the point represents with respect to the given problem situation.

TEKS A.2I, (Topic-Aligned), TEKS A.3F (Interleaved Practice),
TEKS A.3G (Topic-Aligned), TEKS A.5C (Interleaved Practice)

- Mateo sells model cars from a booth at a local flea market. He purchases each model car from a distributor for \$12, and the flea market charges him a booth fee of \$50. Mateo sells each model car for \$20.

Mateo's income can be modeled by the equation $y = 20x$, where y represents the income (in dollars) and x represents the number of model cars he sells. Mateo's expenses can be modeled by the equation $y = 12x + 50$, where y represents the expenses (in dollars) and x represents the number of model cars he purchases from the distributor.

$$\begin{cases} y = 20x \\ y = 12x + 50 \end{cases}$$



The break-even point is between 6 and 7 model cars. Mateo must sell more than 6 model cars to make a profit.

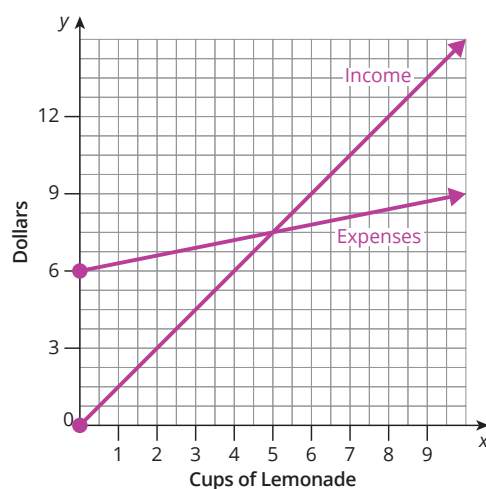


TOPIC 2 Systems of Linear Equations and Inequalities

2. Minh sets up a lemonade stand in front of her house. Each cup of lemonade costs Minh \$0.30 to make, and she spends \$6 on the advertising signs she puts up around her neighborhood. She sells each cup of lemonade for \$1.50.

Minh's income can be modeled by the equation $y = 1.50x$, where y represents the income (in dollars) and x represents the number of cups of lemonade she sells. Minh's expenses can be modeled by the equation $y = 0.30x + 6$, where y represents the expenses (in dollars) and x represents the number of cups of lemonade she makes.

$$\begin{cases} y = 1.50x \\ y = 0.30x + 6 \end{cases}$$



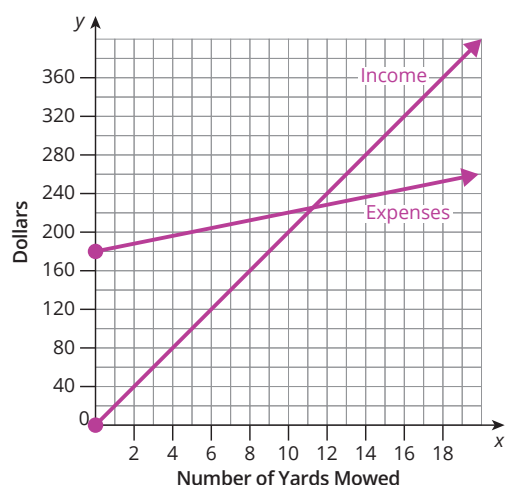
The break-even point is 5 cups of lemonade. Minh must sell more than 5 cups of lemonade to make a profit.

TOPIC 2 Systems of Linear Equations and Inequalities

3. Liam starts his own lawn mowing business. He initially spends \$180 on a new lawnmower. For each yard he mows, he receives \$20 and spends \$4 on gas.

Liam's income can be modeled by the equation $y = 20x$, where y represents the income (in dollars) and x represents the number of yards he mows. Liam's expenses can be modeled by the equation $y = 4x + 180$, where y represents the expenses (in dollars) and x represents the number of yards he mows.

$$\begin{cases} y = 20x \\ y = 4x + 180 \end{cases}$$



The break-even point is between 11 and 12 yards mowed. Liam must mow more than 11 yards to make a profit.

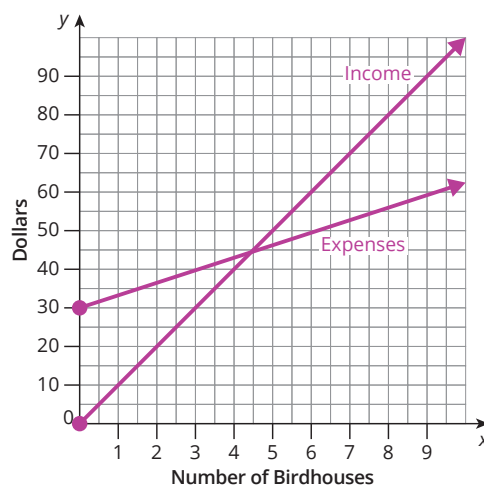


TOPIC 2 Systems of Linear Equations and Inequalities

4. Victoria is building birdhouses to raise money for a trip to Hawaii. She spends a total of \$30 on the tools needed to build the houses. The material to build each birdhouse costs \$3.25. Victoria sells each birdhouse for \$10.

Victoria's income can be modeled by the equation $y = 10x$, where y represents the income (in dollars) and x represents the number of birdhouses she sells. Victoria's expenses can be modeled by the equation $y = 3.25x + 30$, where y represents the expenses (in dollars) and x represents the number of birdhouses she builds.

$$\begin{cases} y = 10x \\ y = 3.25x + 30 \end{cases}$$



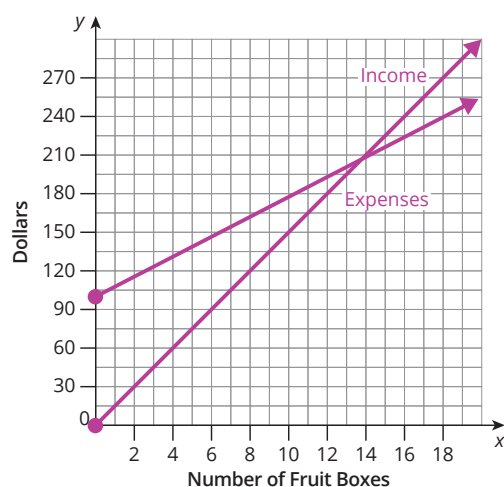
The break-even point is between 4 and 5 birdhouses. Victoria must sell more than 4 birdhouses to make a profit.

TOPIC 2 Systems of Linear Equations and Inequalities

5. The Spanish Club is selling boxes of fruit as a fundraiser. The fruit company charges the Spanish Club \$7.50 for each box of fruit and a shipping and handling fee of \$100 for the entire order. The Spanish Club sells each box of fruit for \$15.

The Spanish Club's income can be modeled by the equation $y = 15x$, where y represents the income (in dollars) and x represents the number of fruit boxes sold. The Spanish Club's expenses can be modeled by the equation $y = 7.50x + 100$, where y represents the expenses (in dollars) and x represents the number of fruit boxes ordered.

$$\begin{cases} y = 15x \\ y = 7.50x + 100 \end{cases}$$



The break-even point is between 13 and 14 boxes of fruit. The Spanish Club must sell more than 13 boxes of fruit to make a profit.

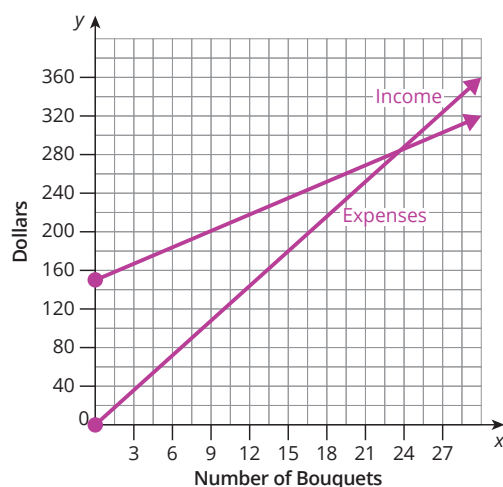


TOPIC 2 Systems of Linear Equations and Inequalities

6. Elijah sells flowers online for \$12 per bouquet. Each bouquet costs him \$5.70 to make. Jerome also paid a one-time fee of \$150 to advertise his company.

Elijah's income can be modeled by the equation $y = 12x$, where y represents the income (in dollars) and x represents the number of bouquets he sells. Elijah's expenses can be modeled by the equation $y = 5.70x + 150$, where y represents the expenses (in dollars) and x represents the number of bouquets he makes.

$$\begin{cases} y = 12x \\ y = 5.70x + 150 \end{cases}$$



The break-even point is between 23 and 24 bouquets. Elijah must sell more than 23 bouquets to make a profit.

TOPIC 2 Systems of Linear Equations and Inequalities

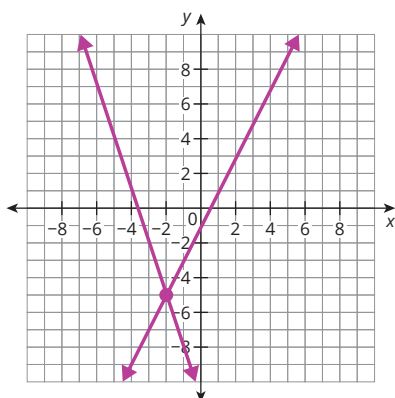
- B. Graph the equations in each system. Tell whether the system has one solution, no solution, or infinite solutions. When the system has one solution, write the values of the variables that make the equations true. Determine whether the system is consistent or inconsistent.

TEKS A.3F (Topic-Aligned), TEKS A.3G (Interleaved Practice),
TEKS A.5C (Topic-Aligned)

1.
$$\begin{cases} y = 2x - 1 \\ y = -3x - 11 \end{cases}$$

One solution; $(-2, -5)$

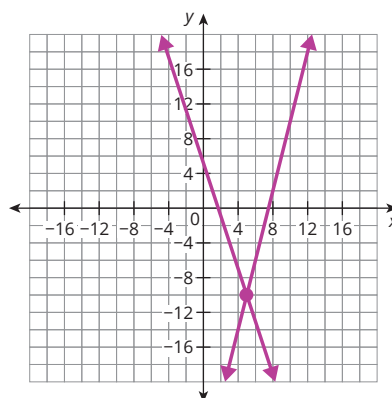
The system is consistent.



2.
$$\begin{cases} y = 4x - 30 \\ y = -3x + 5 \end{cases}$$

One solution; $(5, -10)$

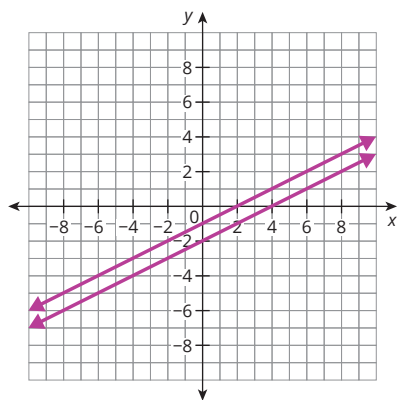
The system is consistent.



3.
$$\begin{cases} -5x + 10y = -10 \\ y = \frac{1}{2}x - 2 \end{cases}$$

No solution

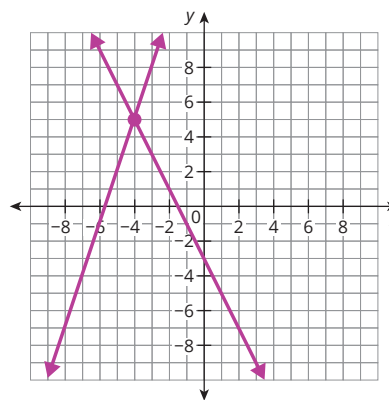
The system is inconsistent.



4.
$$\begin{cases} y = 3x + 17 \\ 10x + 5y = -15 \end{cases}$$

One solution; $(-4, 5)$

The system is consistent.

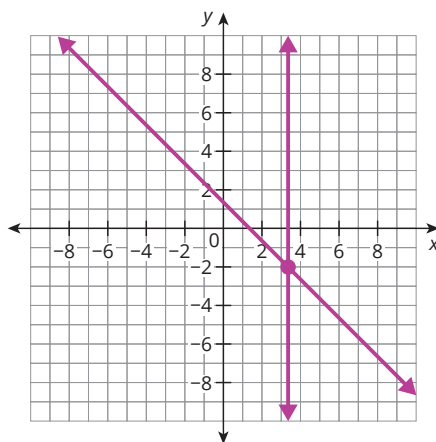


TOPIC 2 Systems of Linear Equations and Inequalities

5.
$$\begin{cases} -8x - 8y = -10 \\ x = 3.25 \end{cases}$$

One solution: $(3.25, -2)$

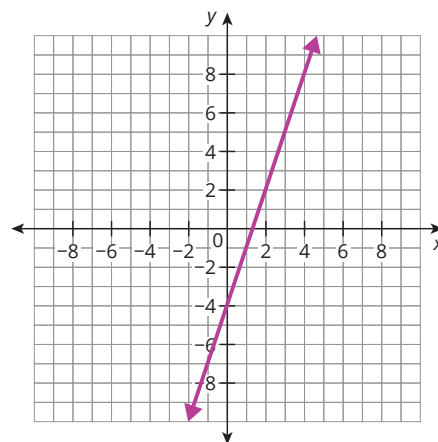
The system is inconsistent.



6.
$$\begin{cases} -1.5x + 0.5y = -2 \\ y + 7 = 3(x + 1) \end{cases}$$

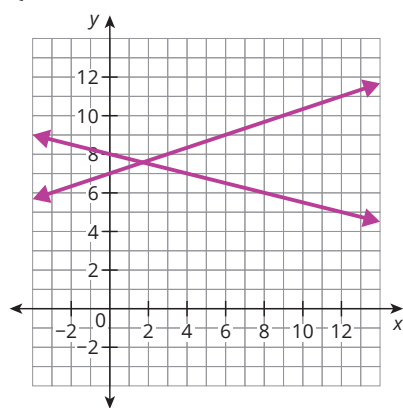
Infinite solutions

The system is inconsistent.



7. Graph the system of equations and estimate the solution of the system.

$$\begin{cases} y = \frac{1}{3}x + 7 \\ y = -\frac{1}{4}x + 8 \end{cases}$$



Sample answer: $(1.7, 7.6)$

TOPIC 2 Systems of Linear Equations and Inequalities

Extension

Solve the system of equations shown. Explain your reasoning.

Extension of TEKS A.5C

$$\begin{cases} 3x + 5y = 18 \\ y = |x - 4| \end{cases}$$

I can solve the system by graphing the two equations and determining their point of intersection. There are two solutions: (1, 3) and (4.75, 0.75).

Spaced Practice

1. Solve the equation and check your solution. TEKS A.5A

$$\frac{3}{4}x - 11 = 4 + \left(-\frac{3}{4}x + 3\right)$$

$$x = 12$$

Check:

$$\begin{aligned} \frac{3}{4}(12) - 11 &= 4 + \left[-\frac{3}{4}(12) + 3\right] \\ -2 &= -2 \end{aligned}$$

2. Consider the equation $\frac{2}{5}x - 2y = 14$. Write the equation in slope-intercept form and identify the slope and y-intercept. TEKS A.2C, TEKS A.3A

$$y = \frac{1}{5}x - 7; m = \frac{1}{5}; b = -7$$

3. Determine the linear regression model for each data set. Which regression model is the better fit? Explain your reasoning.

TEKS A.4A, TEKS A.4C

Set A

x	y
1	12
2	11
5	30
7	39
10	50

Set A:

$$y = 4.574x + 5.53$$

$$r = 0.9898$$

Set B

x	y
12	3
10	9
8	11
5	14
0	0

Set B:

$$y = 0.284x + 5.411$$

$$r = 0.23091$$

The regression model for Set A appears to be the better fit because its correlation coefficient is closer to 1 than the correlation coefficient of the regression model for Set B.



II. Using Substitution to Solve Linear Systems

Topic Practice

- A. Write a system of equations to represent each problem situation. Solve the system of equations using any method. Then, answer any associated questions. **TEKS A.2I (Topic-Aligned), TEKS A.5C (Topic-Aligned)**
1. Aaliyah needs to print some of her digital photos. She is trying to choose between Lightning Fast Foto and Snappy Shots. Lightning Fast Foto charges a base fee of \$5 plus an additional \$0.20 per photo. Snappy Shots charges a base fee of \$7 plus an additional \$0.10 per photo. Determine the number of photos for which both stores will charge the same amount. Explain which store Aaliyah should choose depending on the number of photos she needs to print.

Let x represent the number of photos printed. Let y represent the total cost (in dollars) to print x photos.

$$\begin{cases} y = 0.20x + 5 & \text{Lightning Fast Foto} \\ y = 0.10x + 7 & \text{Snappy Shots} \end{cases}$$

The solution is (20, 9). Both stores charge \$9.00 for printing 20 photos. If Aaliyah wants to print fewer than 20 photos, then she should choose Lightning Fast Foto. If Aaliyah wants to print more than 20 photos, then she should choose Snappy Shots.

TOPIC 2 Systems of Linear Equations and Inequalities

2. Kai is trying to decide which ice cream shop is the better buy. Cold & Creamy Sundaes charges \$2.50 per sundae plus an additional \$0.25 for each topping. Colder & Creamier Sundaes charges \$1.50 per sundae plus an additional \$0.50 for each topping. Determine the number of toppings for which both vendors charge the same amount. Explain which vendor is the better buy depending on the number of toppings Kai chooses.

Let x represent the number of toppings on a sundae. Let y represent the cost (in dollars) for a sundae with x toppings.

$$\begin{cases} y = 0.25x + 2.50 & \text{Cold and Creamy Sundaes} \\ y = 0.50x + 1.50 & \text{Colder and Creamier Sundaes} \end{cases}$$

The solution is (4, 3.50). Both vendors charge \$3.50 for a sundae with 4 toppings. If Kai wants fewer than 4 toppings, then Colder & Creamier Sundaes is the better buy. If Kai wants more than 4 toppings, Cold & Creamy Sundaes is the better buy.

3. Ava is selling T-shirts at the State Fair. She brings 200 shirts to sell. She has long-sleeved and short-sleeved T-shirts for sale. On the first day of the fair, she sells $\frac{1}{2}$ of her long-sleeved T-shirts and $\frac{1}{3}$ of her short-sleeved T-shirts for a total of 80 T-shirts sold. How many of each type of T-shirt did Ava bring to the fair?

Let x represent the number of long-sleeved T-shirts Ava brings to the fair. Let y represent the number of short-sleeved T-shirts Ava brings to the fair.

$$\begin{cases} x + y = 200 \\ \frac{1}{2}x + \frac{1}{3}y = 80 \end{cases}$$

The solution is (80, 120). Ava brought 80 long-sleeved T-shirts and 120 short-sleeved T-shirts to the fair.



TOPIC 2 Systems of Linear Equations and Inequalities

4. Ava has a booth at the flea market where she sells purses and wallets. All of her purses are the same price, and all of her wallets are the same price. The first hour of the day, she sells 10 purses and 6 wallets for a total of \$193. The second hour, she sells 2 purses and 1 wallet for a total of \$37.50. How much does Alicia charge for each purse and each wallet?

Let x represent the charge for each purse. Let y represent the charge for each wallet.

$$\begin{cases} 10x + 6y = 193 \\ 2x + y = 37.5 \end{cases}$$

The solution is (16, 5.5). Ava charges \$16 for each purse and \$5.50 for each wallet.

5. Noah has some nickels and dimes in his pocket, and the change is worth \$0.75. He has twice as many dimes as nickels. How many of each type of coin does Johnny have?

Let x represent the number of nickels Noah has in his pocket. Let y represent the number of dimes Noah has in his pocket.

$$\begin{cases} 0.05x + 0.10y = 0.75 \\ y = 2x \end{cases}$$

The solution is (3, 6). Noah has 3 nickels and 6 dimes in his pocket.

6. Ms. Garcia woke up one morning to find that her water heater had sprung a leak. She called two different plumbers to get their rates. The first plumber charges \$64 just to walk in the door plus \$24 per hour. The second plumber charges a flat \$56 per hour. After how many hours will the cost for both plumbers be the same? Explain which plumber Ms. Garcia should use based on the number of hours she expects the repair to take.

Let x represent the number of hours. Let y represent the total cost of the plumber working for x hours.

$$\begin{cases} y = 64 + 24x & \text{The first plumber} \\ y = 56x & \text{The second plumber} \end{cases}$$

The solution is (2, 112). Both plumbers charge \$112 for 2 hours of work. Ms. Garcia should use the first plumber if she expects the work will take longer than 2 hours. If Ms. Garcia expects the work will take less than two hours, she should use the second plumber.

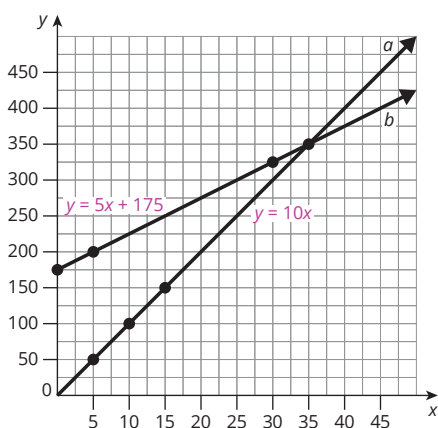


TOPIC 2 Systems of Linear Equations and Inequalities

B. Write a system of equations to represent each table or graph.

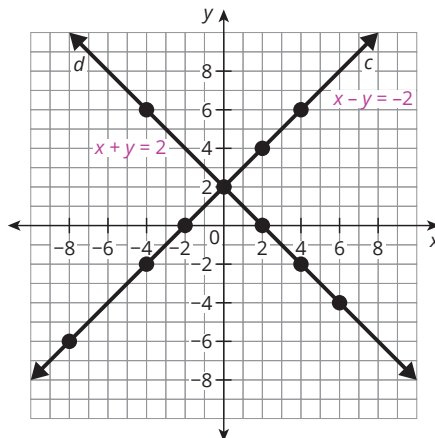
TEKS A.2I (Interleaved Practice)

1. Write a system of equations in slope-intercept form that represents lines a and b .



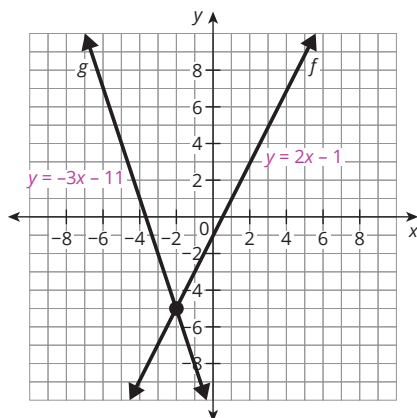
$$\begin{cases} y = 10x \\ y = 5x + 175 \end{cases}$$

2. Write a system of equations in standard form that represents lines c and d .



$$\begin{cases} x - y = -2 \\ x + y = 2 \end{cases}$$

3. Write a system of equations in slope-intercept form that represents lines f and g .



$$\begin{cases} y = 2x - 1 \\ y = -3x - 11 \end{cases}$$



TOPIC 2 Systems of Linear Equations and Inequalities

4. Write a system of equations in slope-intercept form that represents lines r and s .

Line r

x	y
-2	-6
0	-5
3	-3.5
7	-1.5

Line s

x	y
-5	-16
-1	-8
4	2
8	10

$$\begin{cases} y = \frac{1}{2}x - 5 \\ y = 2x - 6 \end{cases}$$

5. Write a system of equations in standard form that represents lines h and k .

Line h

x	y
-5	-2
-1	2
2	5
6	9

Line k

x	y
-2	20
0	12
2	4
3	0

$$\begin{cases} x - y = -3 \\ 4x + y = 12 \end{cases}$$

TOPIC 2 Systems of Linear Equations and Inequalities

6. Write a system of equations in slope-intercept form that represents lines m and n .

Line m

x	y
-4	30
-1	21
3	9
8	-6

Line n

x	y
-5	18
-2	12
1	6
4	0

$$\begin{cases} y = -3x + 18 \\ y = -2x + 8 \end{cases}$$

- C. Solve each system of equations by substitution. Determine whether the system is consistent or inconsistent.

TEKS A.5C (Topic-Aligned)

1.
$$\begin{cases} y = 2x - 3 \\ x = 4 \end{cases}$$

The solution is (4, 5).

The system is consistent.

2.
$$\begin{cases} 2x + y = 9 \\ y = 5x + 2 \end{cases}$$

The solution is (1, 7).

The system is consistent.



TOPIC 2 Systems of Linear Equations and Inequalities

3.
$$\begin{cases} y = 3x - 2 \\ y - 3x = 4 \end{cases}$$

There is no solution.

The system is inconsistent.

5.
$$\begin{cases} 0.8x - 0.2y = 1.5 \\ 0.1x + 1.2y = 0.8 \end{cases}$$

The solution is (2, 0.5).

The system is consistent.

4.
$$\begin{cases} \frac{1}{2}x + \frac{3}{2}y = -7 \\ \frac{1}{3}y = 2x - 10 \end{cases}$$

The solution is (4, -6).

The system is consistent.

6.
$$\begin{cases} 0.3y = 0.6x + 0.3 \\ 1.2x + 0.6 = 0.6y \end{cases}$$

The system has an infinite number of solutions.

The system is consistent.



TOPIC 2 Systems of Linear Equations and Inequalities

Extension

Create a system of linear equations with solution (2, 5). Solve the system using substitution to verify your system has the given solution.

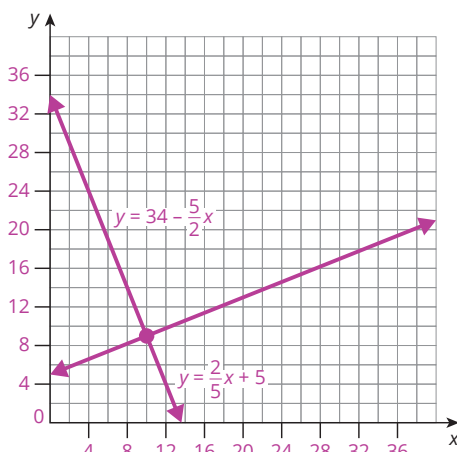
Extension of TEKS A.3F, Extension of TEKS A.5C

Answers will vary.

Spaced Practice

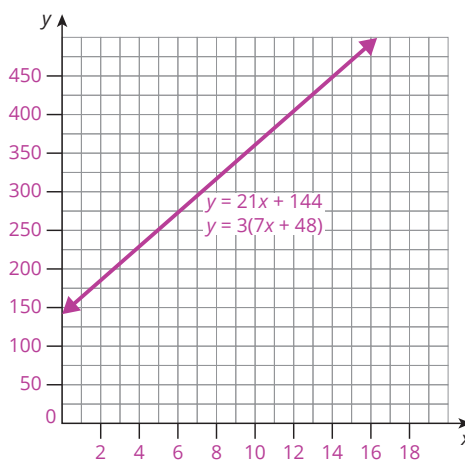
- Graph each system of linear equations to determine the solution to the system. **TEKS A.3F**

a. $y = 34 - \frac{5}{2}x$ and $y = \frac{2}{5}x + 5$



The solution is (10, 9).

b. $y = 21x + 144$ and $y = 3(7x + 48)$



There are infinitely many solutions.
The lines are the same.



TOPIC 2 Systems of Linear Equations and Inequalities

2. The population growth (in thousands) for a small town near Bay City can be represented by the expression $x + \frac{4}{5}(x + 315)$, where x represents the number of years since 2005. The population growth (in thousands) for a neighboring town can be represented by the expression $2x - \frac{1}{5}(x - 630)$, where x represents the number of years since 2005. When will the populations of the two towns be the same? **TEKS A.5C**

The two populations will never be equal because the equation formed from setting the two expressions equal to each other does not have a solution.

3. Two neighboring towns are not having population growth. In fact, they both have been losing population since 1995. The population decline for one of the towns (in thousands) can be represented by the expression $-\frac{2}{5}(x - 500)$, where x represents the number of years since 1995. The population decline for the other town (in thousands) can be represented by the expression $-\frac{1}{2}x + \frac{1}{10}(x + 2000)$, where x represents the number of years since 1995. When will the populations of the two towns be the same? **TEKS A.5C**

The populations of both towns will always be the same because the equation formed from setting the two expressions equal to each other is true for all values of x .

4. Solve each equation or inequality.

a. $8(2m + 7) = 10(m + 11)$ **TEKS A.5A**

$$m = 9$$

b. $-3(y + 20) < -9y$ **TEKS A.5B**

$$y < 10$$



III. Using Linear Combinations to Solve a System of Linear Equations

Topic Practice

A. Solve each system of equations using the linear combinations method. **TEKS A.5C (Topic-Aligned)**

1.
$$\begin{cases} 3x + 5y = 8 \\ 2x - 5y = 22 \end{cases}$$

The solution is $(6, -2)$.

2.
$$\begin{cases} 4x - y = 2 \\ 2x + 2y = 26 \end{cases}$$

The solution is $(3, 10)$.

3.
$$\begin{cases} 10x - 6y = 26 \\ 5x - 5y = 5 \end{cases}$$

The solution is $(5, 4)$.

4.
$$\begin{cases} 2x - 4y = 4 \\ -3x + 10y = 14 \end{cases}$$

The solution is $(12, 5)$.



TOPIC 2 Systems of Linear Equations and Inequalities

5.
$$\begin{cases} 3x + 2y = 14 \\ 4x + 5y = 35 \end{cases}$$

The solution is (0, 7).

7.
$$\begin{cases} 1.5x + 1.2y = 0.6 \\ 0.8x - 0.2y = 2 \end{cases}$$

The solution is (2, -2).

6.
$$\begin{cases} x + 6y = 11 \\ 2x - 12y = 10 \end{cases}$$

The solution is (8, 0.5).

8.
$$\begin{cases} \frac{3}{4}x + \frac{1}{2}y = -\frac{3}{4} \\ \frac{2}{3}x + \frac{2}{3}y = \frac{2}{3} \end{cases}$$

The solution is (-5, 6).



TOPIC 2 Systems of Linear Equations and Inequalities

B. Write a system of equations to represent each problem situation. Solve the system of equations using the linear combinations method. TEKS A.2I, (Topic-Aligned), TEKS A.5C (Topic-Aligned)

- The high school marching band is selling fruit baskets as a fundraiser. They sell a large basket containing 10 apples and 15 oranges for \$20. They sell a small basket containing 5 apples and 6 oranges for \$8.50. How much is the marching band charging for each apple and each orange?

Let x represent the amount charged for each apple. Let y represent the amount charged for each orange.

$$\begin{array}{rcl} \begin{cases} 10x + 15y = 20 \\ 5x + 6y = 8.50 \end{cases} & \begin{array}{r} 10x + 15y = 20 \\ -2(5x + 6y = 8.50) \end{array} & \\ & \begin{array}{r} 10x + 15y = 20 \\ -10x - 12y = -17 \\ \hline 3y = 3 \\ y = 1 \end{array} & \begin{array}{r} 10x + 15(1) = 20 \\ 10x + 15 = 20 \\ 10x = 5 \\ x = 0.5 \end{array} \end{array}$$

The solution is (0.5, 1). The band charges \$0.50 for each apple and \$1.00 for each orange.

- Emma works on a shipping dock at a tire manufacturing plant. She loads a pallet with 4 Mudslinger tires and 6 Roadripper tires. The tires on the pallet weigh 212 pounds. She loads a second pallet with 7 Mudslinger tires and 2 Roadripper tires. The tires on the second pallet weigh 184 pounds. How much does each Mudslinger tire and each Roadripper tire weigh?

Let x represent the weight of a Mudslinger tire. Let y represent the weight of a Roadripper tire.

$$\begin{array}{rcl} \begin{cases} 4x + 6y = 212 \\ 7x + 2y = 184 \end{cases} & \begin{array}{r} 4x + 6y = 212 \\ -3(7x + 2y = 184) \end{array} & \\ & \begin{array}{r} 4x + 6y = 212 \\ -21x - 6y = -552 \\ \hline 17x = -340 \\ x = 20 \end{array} & \begin{array}{r} 4(20) + 6y = 212 \\ 80 + 6y = 212 \\ 6y = 132 \\ y = 22 \end{array} \end{array}$$

The solution is (20, 22). Each Mudslinger tire weighs 20 pounds, and each Roadripper tire weighs 22 pounds.



TOPIC 2 Systems of Linear Equations and Inequalities

3. The Pizza Barn sells one customer 3 large pepperoni pizzas and 2 orders of breadsticks for \$30. They sell another customer 4 large pepperoni pizzas and 3 orders of breadsticks for \$41. How much does the Pizza Barn charge for each pepperoni pizza and each order of breadsticks?

Let x represent the charge for each pepperoni pizza. Let y represent the charge for each order of breadsticks.

$$\begin{cases} 3x + 2y = 30 \\ 4x + 3y = 41 \end{cases} \quad \begin{array}{r} 3(3x + 2y = 30) \\ -2(4x + 3y = 41) \\ \hline 9x + 6y = 90 \\ -8x - 6y = -82 \\ \hline x = 8 \end{array} \quad \begin{array}{r} 3(8) + 2y = 30 \\ 24 + 2y = 30 \\ 2y = 6 \\ y = 3 \end{array}$$

The solution is (8, 3). The Pizza Barn sells each pepperoni pizza for \$8 and each order of breadsticks for \$3.

4. Isabella and Ethan are making large pots of chicken noodle soup. Isabella opens 4 large cans and 6 small cans of soup and pours them into her pot. Her pot contains 115 fluid ounces of soup. Ethan opens 3 large cans and 5 small cans of soup. His pot contains 91 fluid ounces of soup. How many fluid ounces of soup does each large can and each small can contain?

Let x represent the number of fluid ounces in a large can of soup. Let y represent the number of fluid ounces in a small can of soup.

$$\begin{cases} 4x + 6y = 115 \\ 3x + 5y = 91 \end{cases} \quad \begin{array}{r} 3(4x + 6y = 115) \\ -4(3x + 5y = 91) \\ \hline 12x + 18y = 345 \\ -12x - 20y = -364 \\ \hline -2y = -19 \\ y = 9.5 \end{array} \quad \begin{array}{r} 3x + 5(9.5) = 91 \\ 3x + 47.5 = 91 \\ 3x = 43.5 \\ x = 14.5 \end{array}$$

The solution is (14.5, 9.5). Each large can contains 14.5 fluid ounces of soup, and each small can contains 9.5 fluid ounces of soup.



TOPIC 2 Systems of Linear Equations and Inequalities

5. Samuel and Harper are making block towers out of large and small blocks. They are stacking the blocks on top of each other in a single column. Samuel uses 4 large blocks and 2 small blocks to make a tower 63.8 inches tall. Harper uses 9 large blocks and 4 small blocks to make a tower 139.8 inches tall. How tall is each large block and each small block?

Let x represent the height (in inches) of each large block. Let y represent the height (in inches) of each small block.

$$\begin{array}{rcl} \begin{cases} 4x + 2y = 63.8 \\ 9x + 4y = 139.8 \end{cases} & \begin{array}{l} -2(4x + 2y = 63.8) \\ 9x + 4y = 139.8 \end{array} & \\ & \begin{array}{r} -8x - 4y = -127.6 \\ 9x + 4y = 139.8 \\ \hline x = 12.2 \end{array} & \begin{array}{l} 4(12.2) + 5y = 63.8 \\ 48.8 + 2y = 63.8 \\ \hline 2y = 15 \\ y = 7.5 \end{array} \end{array}$$

The solution is (12.2, 7.5). Each large block is 12.2 inches tall, and each small block is 7.5 inches tall.

6. Diego has 2 buckets that he uses to fill the water troughs on his horse farm. He wants to determine how many ounces each bucket holds. On Tuesday, he fills an empty 2000-fluid-ounce water trough with 7 large buckets and 5 small buckets of water. On Thursday, he fills the same empty water trough with 4 large buckets and 10 small buckets of water. How many fluid ounces does each bucket hold?

Let x represent the number of ounces the large bucket holds. Let y represent the number of fluid ounces the small bucket holds.

$$\begin{array}{rcl} \begin{cases} 7x + 5y = 2000 \\ 4x + 10y = 2000 \end{cases} & \begin{array}{l} -2(7x + 5y = 2000) \\ 4x + 10y = 2000 \end{array} & \\ & \begin{array}{r} -14x - 10y = -4000 \\ 4x + 10y = 2000 \\ \hline -10x = -2000 \\ x = 200 \end{array} & \begin{array}{l} 7(200) + 5y = 2000 \\ 1400 + 5y = 2000 \\ \hline 5y = 600 \\ y = 120 \end{array} \end{array}$$

The solution is (200, 120). The large bucket holds 200 fluid ounces. The small bucket holds 120 fluid ounces.



TOPIC 2 Systems of Linear Equations and Inequalities

Extension

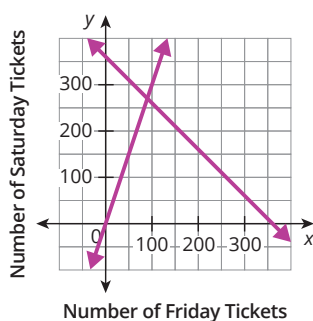
Use linear combinations to solve the given system of three equations in three variables. Show your work. **2A.3B; Extension of TEKS A.5C**

$$\begin{cases} 3x + y + 3z = -2 \\ 6x + 2y + 9z = 5 \\ -2x - y - z = 3 \end{cases}$$

$x = -5; y = 4; z = 3$

Spaced Practice

- The drama department sold a total of 360 tickets to their Friday and Saturday night shows. They sold three times as many tickets for Saturday's show than for Friday's show.
 - Write a system of equations to represent this scenario. **TEKS A.2I**
 $f + s = 360; s = 3f$
 - Graph the system of equations on a coordinate plane. **TEKS A.3F**



- How many tickets were sold for Friday? Saturday? Is there more than one solution? **TEKS A.3G**

Friday: 90 tickets

Saturday: 270 tickets

There is only one solution that satisfies both equations and conditions of the problem.

TOPIC 2 Systems of Linear Equations and Inequalities

- d. Algebraically justify that your solution is correct. **TEKS A.5C**

$$f + s = 360$$

$$s = 3f$$

$$f + (3f) = 360$$

$$4f = 360$$

$$f = 90$$

$$s = 3f$$

$$s = 270$$

Friday: 90; Saturday: 270 (90, 270)

2. Analyze the data in the table.

- a. Write the equation of the regression line for the data. **TEKS A.4C**

$$y = 76.0625x + 154.3125$$

- b. Predict the population after 20 years. Round your answer to the nearest whole number. **TEKS A.4C**

$y = 1675.5625$, The population after 20 years is predicted to be 1676.

Number of years	Population
1	240
2	360
3	280
5	500
6	625
7	830
8	720
9	813
10	900



IV. Graphing Inequalities in Two Variables

Topic Practice

A. Tell whether the graph of each linear inequality will have a dashed line or a solid line. Explain your reasoning. *Prerequisite of TEKS A.3D*

1. $x - 3y \leq 32$

The line will be solid because the symbol is $\leq$.

3. $y < 14x + 9$

The line will be dashed because the symbol is $<$.

5. $\frac{2}{3}x + \frac{4}{9}y \geq 3$

The line will be solid because the symbol is $\geq$.

7. $185x + 274y \geq 65$

The line will be solid because the symbol is $\geq$.

2. $8y + 7x > 15$

The line will be dashed because the symbol is $>$.

4. $-5.2y - 8.3x \leq -28.6$

The line will be solid because the symbol is $\leq$.

6. $y - 17 > x + 8$

The line will be dashed because the symbol is $>$.

8. $36 < 9y - 2x$

The line will be dashed because the symbol is $<$.

B. For each inequality, use the test point $(0, 0)$ to determine which half-plane should be shaded. *Prerequisite of TEKS A.3D*

1. $5x + 7y > -13$

The half-plane that includes $(0, 0)$ should be shaded because the inequality is true for that point.

3. $-8y > 6x + 12$

The half-plane that does not include $(0, 0)$ should be shaded because the inequality is false for that point.

5. $31.9x + 63.7y < -44.5$

The half-plane that does not include $(0, 0)$ should be shaded because the inequality is false for that point.

2. $y - 30 \leq 9x$

The half-plane that includes $(0, 0)$ should be shaded because the inequality is true for that point.

4. $46 \geq -5y + 10x$

The half-plane that includes $(0, 0)$ should be shaded because the inequality is true for that point.

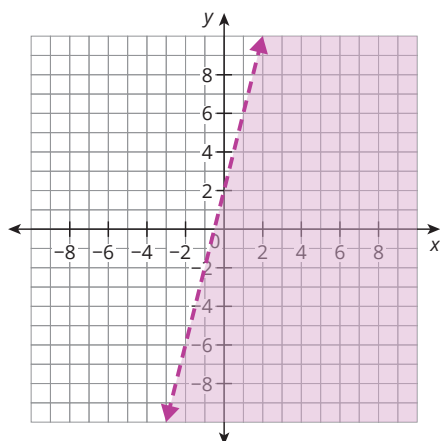
6. $y - \frac{5}{6} > \frac{1}{2}x + \frac{1}{3}$

The half-plane that does not include $(0, 0)$ should be shaded because the inequality is false for that point.

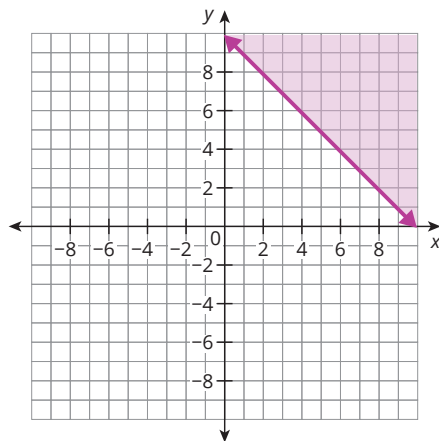
TOPIC 2 Systems of Linear Equations and Inequalities

C. Graph each linear inequality. TEKS A.3D (Topic-Aligned)

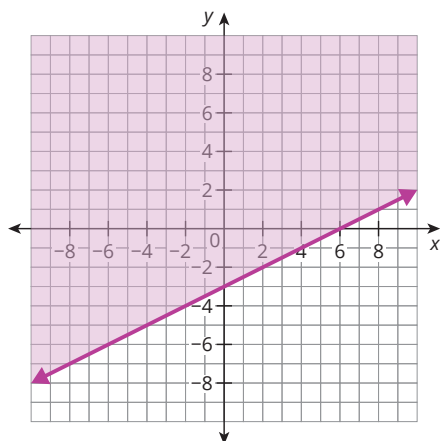
1. $y < 4x + 2$



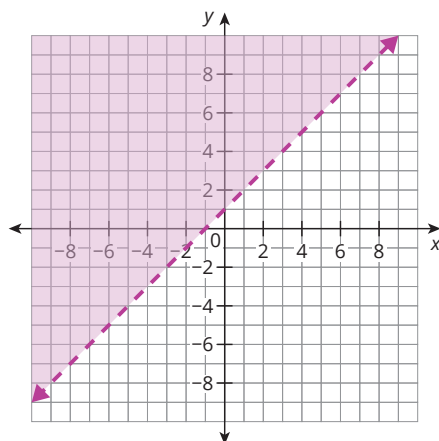
2. $y \geq 10 - x$



3. $y \geq \frac{1}{2}x - 3$

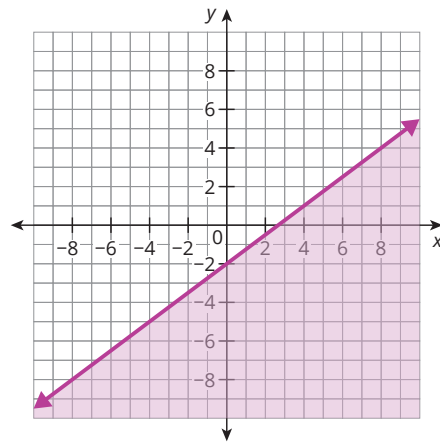


4. $-x + y > 1$

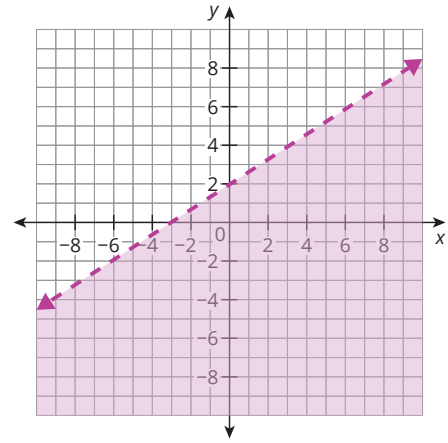


TOPIC 2 Systems of Linear Equations and Inequalities

5. $3x - 4y \geq 8$



6. $\frac{3}{8}y - \frac{1}{4}x < \frac{3}{4}$



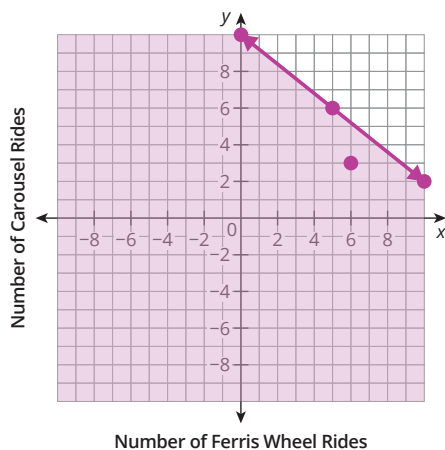
TOPIC 2 Systems of Linear Equations and Inequalities

D. Write and graph an inequality for each problem situation. Then, determine whether the ordered pair is a solution for the problem situation. **TEKS A.2H (Topic-Aligned), TEKS A.3D (Interleaved Practice)**

- Daniel has 50 tokens to spend at the school carnival. He wants to spend his tokens riding the Ferris wheel and the carousel. The table of values shows the combination of rides that expend his tokens. Write and graph an inequality that represents the possible ways Daniel could use his tokens on the two rides. Is the ordered pair (6, 3) a solution for the problem situation?

Ferris Wheel Rides	Carousel Rides
0	10
5	6
10	2

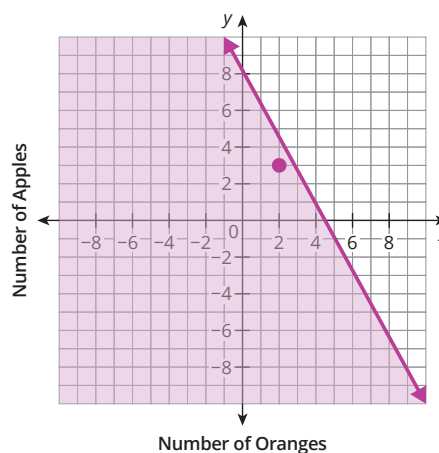
$$4x + 5y \leq 50$$



Yes. The ordered pair (6, 3) is a solution to the inequality. It is in the shaded half-plane.

- Chloe has \$2 to buy oranges and apples. Oranges cost \$0.45 each and apples cost \$0.25 each. Write and graph an inequality that represents the possible ways Chloe could spend her \$2. Is the ordered pair (2, 3) a solution for the problem situation?

$$0.45x + 0.25y \leq 2$$



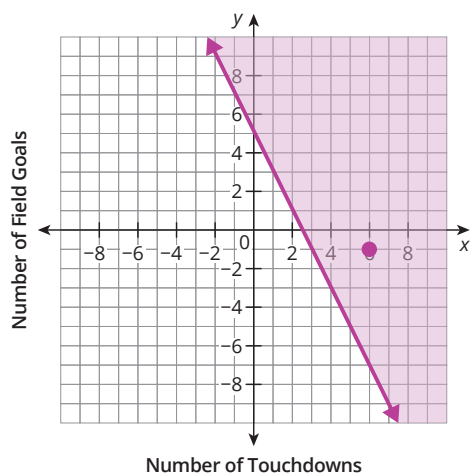
Yes. The ordered pair (2, 3) is a solution to the inequality. It is in the shaded half-plane.



TOPIC 2 Systems of Linear Equations and Inequalities

3. Juan plays football. His team's goal is to score at least 15 points per game. A touchdown is worth 6 points, and a field goal is worth 3 points. Juan's league does not allow teams to try for the extra point after a touchdown. Write and graph an inequality that represents the possible ways Juan's team could score points to reach their goal. Is the ordered pair $(6, -1)$ a solution for the problem situation?

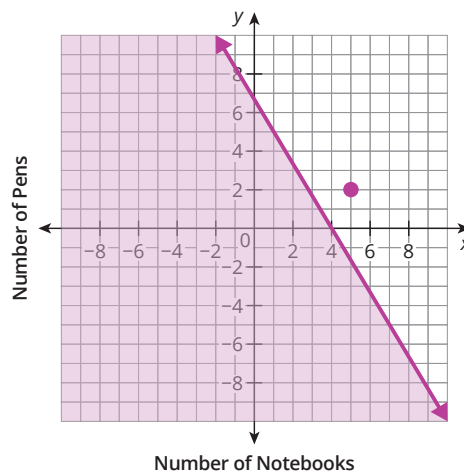
$$6x + 3y \geq 15$$



No. The ordered pair $(6, -1)$ is not a solution for the problem situation. It is in the correct shaded half-plane, but it is not a reasonable answer because Noah's team cannot score a negative number of field goals.

4. Mia has \$5 to buy notebooks and pens. Notebooks cost \$1.25 each, and pens cost \$0.75 each. Write and graph an inequality to represent the possible ways Mia could spend her \$5. Is the ordered pair $(5, 2)$ a solution for the problem situation?

$$1.25x + 0.75y \leq 5$$

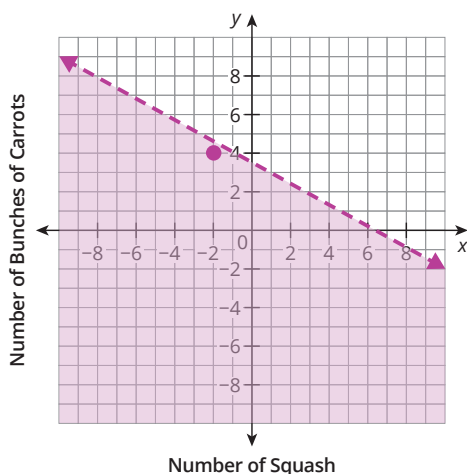


No. The ordered pair $(5, 2)$ is not a solution to the inequality. It is not in the shaded half-plane.

TOPIC 2 Systems of Linear Equations and Inequalities

5. Jayden has \$10 to buy squash and carrots. Squash costs \$1.50 each, and carrots costs \$2.75 per bunch. Write and graph an inequality that represents the possible ways Jayden could spend less than \$10. Is the ordered pair $(-2, 4)$ a solution for the problem situation?

$$1.50x + 2.75y < 10$$

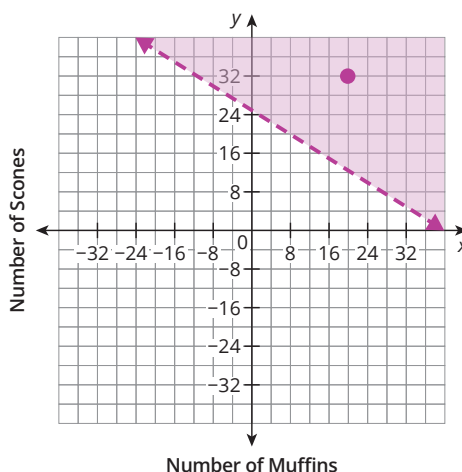


No. The ordered pair $(-2, 4)$ is not a solution for the problem situation. It is in the correct shaded half-plane, but it is not a reasonable answer because Jayden cannot purchase a negative number of squash.

6. Mateo makes and sells muffins and scones at a school bake sale. He hopes to raise more than \$20. The table of values shows the combination of sales that will raise exactly \$20. Write and graph an inequality that represents the possible ways Mateo could reach his goal. Is the ordered pair $(20, 32)$ a solution for the problem situation?

Muffins	Scones
0	25
8	20
16	15
24	10

$$0.50x + 0.80y > 20$$



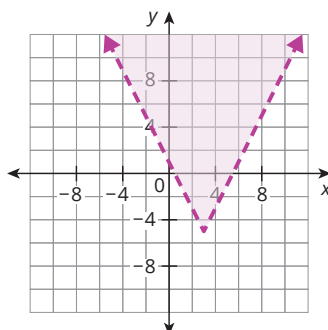
Yes. The ordered pair $(20, 32)$ is a solution to the inequality. It is in the shaded half-plane.



TOPIC 2 Systems of Linear Equations and Inequalities

Extension

Use what you know about absolute value functions to graph the inequality $y > 2|x - 3| - 5$. **Extension of TEKS A.3D**



Spaced Practice

1. Solve each system using the Linear Combinations Method. **TEKS A.5C**

a.
$$\begin{cases} 8x - 6y = -20 \\ -16x + 7y = 30 \end{cases}$$

 $(-1, 2)$

b.
$$\begin{cases} x + 3 = -7y + 3 \\ 2x - 8y = 22 \end{cases}$$

 $(7, -1)$

2. Write equations in both slope-intercept form and standard form of the line that has the given slope and passes through the point given. **TEKS A.2B**

a. $m = \frac{2}{3}; (2, -4)$
Slope-intercept form: $y = \frac{2}{3}x - \frac{16}{3}$
Standard form: $2x - 3y = 16$

b. $m = -4; (0.5, 7)$
Slope-intercept form: $y = -4x + 9$
Standard form: $2x - 3y = 16$

V. Systems of Linear Inequalities

Topic Practice

- A. Write a system of linear inequalities that represents each problem situation. Remember to define your variables.

TEKS A.2H (Interleaved Practice), TEKS A.2I (Topic-Aligned),
A.3H (Interleaved Practice)

1. Minh runs the bouncy house at a festival. The bouncy house can hold a maximum of 1200 pounds at one time. He estimates that adults weigh approximately 200 pounds and children under 16 weigh approximately 100 pounds. For 1 four-minute session of bounce time, Minh charges adults \$3 each and children \$2 each. Minh hopes to make at least \$24 for each session.

x = number of adults

y = number of children

$$\begin{cases} 3x + 2y \geq 24 \\ 200x + 100y \leq 1200 \end{cases}$$

3. The maximum capacity for an average passenger elevator is 15 people and 3000 pounds. It is estimated that adults weigh approximately 200 pounds and children under 16 weigh approximately 100 pounds.

x = number of adults

y = number of children

$$\begin{cases} x + y \leq 15 \\ 200x + 100y \leq 3000 \end{cases}$$

2. Victoria works at a movie theater selling tickets. The theater has 300 seats and charges \$7.50 for adults and \$5.50 for children. The theater expects to make at least \$2000 for each showing.

x = number of adults

y = number of children

$$\begin{cases} x + y \leq 300 \\ 7.50x + 5.50y \geq 2000 \end{cases}$$

4. Liam's pickup truck can carry a maximum of 1000 pounds. He loads his truck with 20-pound bags of cement and 80-pound bags of cement. He hopes to load at least 10 bags of cement into his truck.

x = number of 20-pound bags

y = number of 80-pound bags

$$\begin{cases} x + y \geq 10 \\ 20x + 80y \leq 1000 \end{cases}$$



TOPIC 2 Systems of Linear Equations and Inequalities

5. Elijah is drawing caricatures at a fair for 8 hours. He can complete a small drawing in 15 minutes and charges \$10 for the drawing. He can complete a larger drawing in 45 minutes and charges \$25 for the drawing. Elijah hopes to make at least \$200 at the fair.

x = number of small drawings

y = number of large drawings

8 hours = 480 minutes

$$\begin{cases} 10x + 25y \geq 200 \\ 15x + 45y \leq 480 \end{cases}$$

6. Elijah is making flower arrangements to sell in his shop. He can complete a small arrangement in 30 minutes that sells for \$20. He can complete a larger arrangement in 1 hour that sells for \$50. Elijah hopes to make at least \$350 during her 8-hour workday.

x = number of small arrangements

y = number of large arrangements

8 hours = 480 minutes

$$\begin{cases} 20x + 50y \geq 350 \\ 30x + 60y \leq 480 \end{cases}$$

B. Determine whether each given point is a solution to the system of linear inequalities. TEKS A.3H (Topic-Aligned)

1.
$$\begin{cases} 2x - y > 4 \\ -x + y \leq 7 \end{cases}$$

Point: $(-2, -10)$

Yes. The point $(-2, -10)$ is a solution to the system of inequalities.

3.
$$\begin{cases} 4x + y < 21 \\ \frac{1}{2}x \leq 36 - 5y \end{cases}$$

Point: $(3, 7)$

No. The point $(3, 7)$ is not a solution to the system of inequalities.

5.
$$\begin{cases} 15x + 25y \geq 300 \\ 20x + 30y \leq 480 \end{cases}$$

Point: $(14, 8)$

No. The point $(14, 8)$ is not a solution to the system of inequalities.

2.
$$\begin{cases} x + 5y < -1 \\ 2y \geq -3x - 2 \end{cases}$$

Point: $(0, -1)$

Yes. The point $(0, -1)$ is a solution to the system of inequalities.

4.
$$\begin{cases} 5x + 3y > 6 \\ -2x + 2y < 20 \end{cases}$$

Point: $(-2, 6)$

Yes. The point $(-2, 6)$ is a solution to the system of inequalities.

6.
$$\begin{cases} -2.1x + 7y \geq -49.5 \\ -y \leq -6.3x + 78 \end{cases}$$

Point: $(10, -8)$

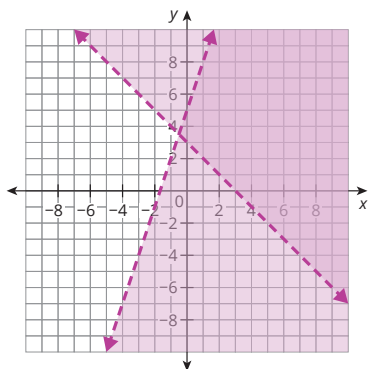
No. The point $(10, -8)$ is not a solution to the system of inequalities.



TOPIC 2 Systems of Linear Equations and Inequalities

C. Graph each system of linear inequalities and identify two solutions. **TEKS A.3H (Topic-Aligned)**

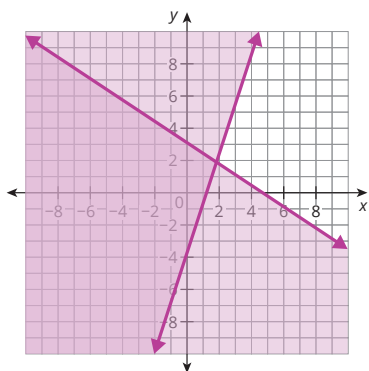
1.
$$\begin{cases} 3x - y > -5 \\ y + x > 3 \end{cases}$$



Answers will vary.

(2, 3) and (6, 0)

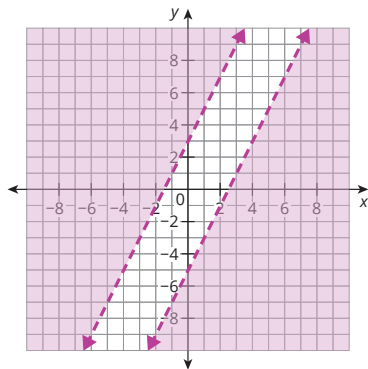
3.
$$\begin{cases} y \leq -\frac{2}{3}x + 3 \\ y \geq 3x - 4 \end{cases}$$



Answers will vary.

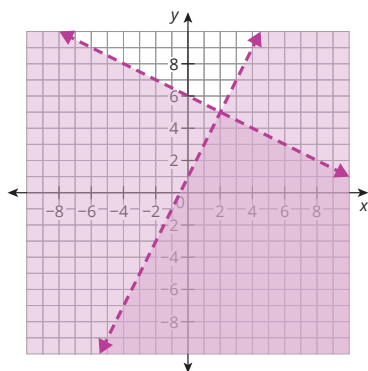
(1, 2) and (-2, 2)

2.
$$\begin{cases} y > 2x + 3 \\ y < 2x - 5 \end{cases}$$



No solution

4.
$$\begin{cases} y - 2 < -\frac{1}{2}(x - 8) \\ y < 2x + 1 \end{cases}$$



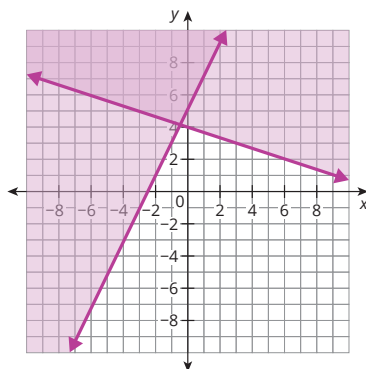
Answers will vary.

(3, 1) and (2, -2)



TOPIC 2 Systems of Linear Equations and Inequalities

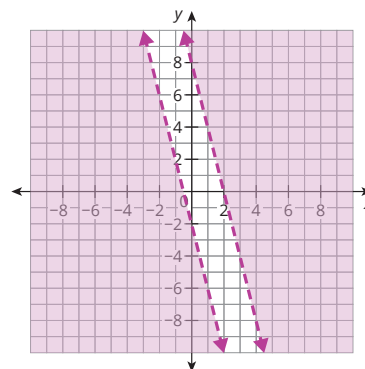
5.
$$\begin{cases} y \geq -\frac{1}{3}x + 4 \\ 2x - y \leq -5 \end{cases}$$



Answers will vary.

$(-1, 6)$ and $(1, 10)$

6.
$$\begin{cases} y > -4x + 8 \\ 4x + y < -2 \end{cases}$$



No solution

Extension

- Is it possible to create a system of inequalities that has no solutions? If so, create one and explain how the graph would show no solutions. If not, explain why. **2A.3E; Extension of TEKS A.2I**

Yes. It is possible. There would not be any region where the shading from the two inequalities overlaps.

$$\begin{cases} y \geq -2x + 13 \\ 2x + y \leq 6 \end{cases}$$

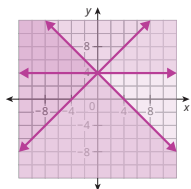
- Is it possible to create a system of two inequalities that has only one solution? If so, create one. If not, explain why. **Extension of TEKS A.2I, 2A.3E**

No. It is not possible. All systems of linear inequalities that have a solution, have infinite solutions. There is no way to have the half-planes intersect at only one point. If the inequalities are not linear, then it is possible.

TOPIC 2 Systems of Linear Equations and Inequalities

3. Is it possible to create a system of three inequalities that has only one solution? If so, sketch a graph to show the solution. If not, explain why. **2A.3E; Extension of TEKS A.2I**

Yes. It is possible. Graphs will vary.



Spaced Practice

1. Determine whether each equation has one solution, no solution, or infinite solutions. **TEKS A.5A**

a. $24x - 22 = -3(1 - 8x)$

No solution

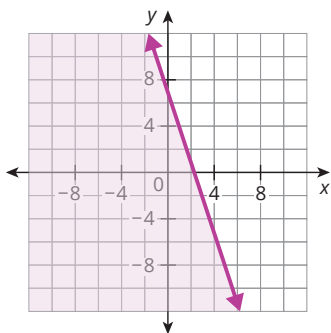
b. $-3(4a + 3) + 2(12a + 2) = 43$

One solution

c. $4(x + 1) = 6x + 4 - 2x$

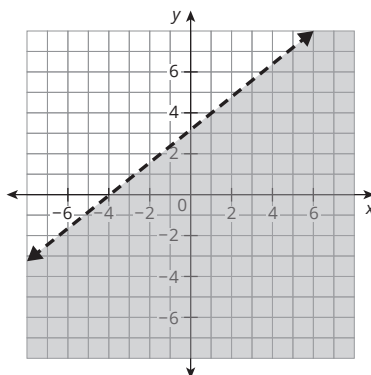
Infinite solutions

2. Graph $3x + y \leq 7$ on a coordinate plane. **TEKS A.3D**



TOPIC 2 Systems of Linear Equations and Inequalities

3. Write a linear inequality for the graph. **TEKS A.2H**



$$y < 0.8x + 3$$

4. A new workout gym opens up down the street from your house. Below are their total membership numbers for the first months of business. **TEKS A.4C**

Month	January	February	March	April	May
Number of Members	120	190	290	370	450

- a. Write the equation of the regression line for the data.

$$y \approx 84x + 32 \text{ (This assumes that January} = 1, \text{ February} = 2, \text{ etc.)}$$

- b. Use the equation to predict the gym's total membership at the end of the year.

1040 memberships



VI. Solving Systems of Equations and Inequalities

Topic Practice

- A. Write a system of equations or inequalities to represent each problem situation. Solve the system using your preferred method and answer any associated questions. **TEKS A.2I (Topic-Aligned), TEKS A.5C (Topic-Aligned)**
- Kai received two different job offers to become a real estate sales agent. Dream Homes offered Kai a base salary of \$20,000 per year plus a 2% commission on all real estate sold. Amazing Homes offered Kai a base salary of \$25,000 per year plus a 1% commission on all real estate sold. Determine the amount of real estate sales, in dollars, for which both real estate companies will pay Kai the same amount. Explain which offer Kai should accept based on the amount of real estate sales he expects to have.

Sample answer:

Let x represent the amount of Kai's real estate sales in dollars.
Let y represent the yearly income when Kai has x dollars in real estate sales.

Substitution Method:

$$0.02x + 20,000 = 0.01x + 25,000$$

$$0.01x + 20,000 = 25,000$$

$$0.01x = 5000$$

$$x = 500,000$$

$$y = 0.02(500,000) + 20,000$$

$$y = 10,000 + 20,000$$

$$y = 30,000$$

The solution is (500,000, 30,000). Both real estate companies will pay Kai \$30,000 per year for \$500,000 in real estate sales. If Kai expects to sell less than \$500,000 of real estate per year, then he should accept the offer from Amazing Homes. If Kai expects to sell more than \$500,000 of real estate per year, then he should accept the offer from Dream Homes.



TOPIC 2 Systems of Linear Equations and Inequalities

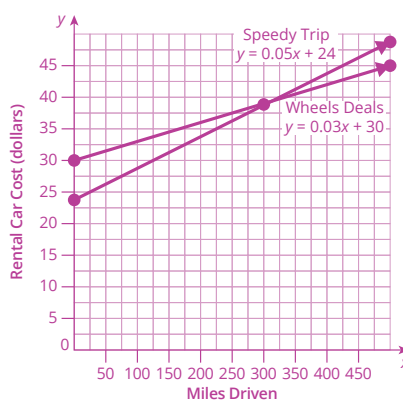
2. Ava is trying to choose between two rental car companies. Speedy Trip Rental Cars charges a base fee of \$24 plus an additional fee of \$0.05 per mile. Wheels Deals Rental Cars charges a base fee of \$30 plus an additional fee of \$0.03 per mile. Determine the amount of miles driven for which both rental car companies charge the same amount. Explain which company Ava should use based on the number of miles she expects to drive.

Sample answer:

Let x represent the number of miles driven. Let y represent the total cost of a rental car when driven x miles.

$$\begin{cases} y = 0.05x + 24 \\ y = 0.03x + 30 \end{cases}$$

Graphing Method:



The solution is (300, 39). Both rental car companies charge \$39 for a car driven 300 miles. If Ava expects to drive fewer than 300 miles, then she should use Speedy Trip Rental Cars. If Ava expects to drive more than 300 miles, then she should use Wheels Deals Rental Cars.



TOPIC 2 Systems of Linear Equations and Inequalities

3. Emma has two job offers to be a door-to-door food processor salesperson. Pro Process Processors offers her a base salary of \$15,000 per year plus an additional \$25 for each processor she sells. Puree Processors offers her a base salary of \$18,000 per year plus an additional \$21 for each processor she sells. Determine the number of food processors Emma would have to sell for both companies to pay her the same amount. Explain which job offer Emma should accept based on the number of food processors she expects to sell.

Sample answer:

Let x represent the number of food processors sold. Let y represent Emma's yearly income when she sells x food processors.

$$\begin{cases} y = 25x + 15,000 & \text{Pro Process Processors} \\ y = 21x + 18,000 & \text{Puree Processors} \end{cases}$$

Substitution Method:

$$\begin{aligned} 25x + 15,000 &= 21x + 18,000 & y &= 25(750) + 15,000 \\ 4x + 15,000 &= 18,000 & y &= 18,750 + 15,000 \\ 4x &= 3,000 & y &= 33,750 \\ x &= 750 \end{aligned}$$

The solution is (750, 33,750). Both companies will pay Emma \$33,750 for selling 750 food processors. If Emma expects to sell fewer than 750 food processors in one year, then she should accept the offer from Puree Processors. If Emma expects to sell more than 750 food processors in one year, then she should accept the offer from Pro Process Processors.



TOPIC 2 Systems of Linear Equations and Inequalities

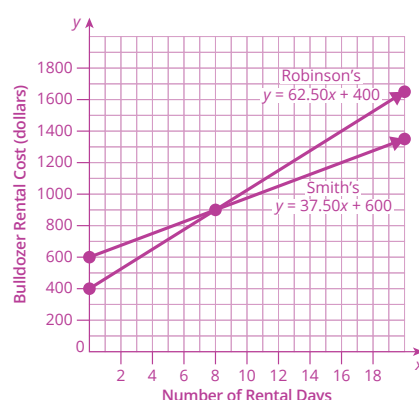
4. Noah needs to rent a bulldozer. Smith's Equipment Rentals rents bulldozers for a delivery fee of \$600 plus an additional \$37.50 per day. Robinson's Equipment Rentals rents bulldozers for a delivery fee of \$400 plus an additional \$62.50 per day. Determine the number of rental days for which both rental companies charge the same amount. Explain which company Noah should choose based on the number of days he expects to rent a bulldozer.

Sample answer:

Let x represent the number of days Noah rents a bulldozer. Let y represent the cost to rent a bulldozer for x days.

$$\begin{cases} y = 37.50x + 600 & \text{Smith's Equipment Rentals} \\ y = 62.50x + 400 & \text{Robinson's Equipment Rentals} \end{cases}$$

Graphing Method:



The solution is (8, 900). Both rental companies charge \$900 to rent a bulldozer for 8 days. If Noah expects to use the bulldozer for fewer than 8 days, then he should choose Robinson's Equipment Rental. If Noah expects to use the bulldozer for more than 8 days, then he should choose Smith's Equipment Rental.

TOPIC 2 Systems of Linear Equations and Inequalities

5. The school volleyball team is selling T-shirts and baseball hats as a fundraiser for their program. The T-shirts are selling for \$15 each, and the baseball hats are selling for \$12 each. If the school volleyball team sold a total of 84 items for a total of \$1146, determine how many of each item they sold.

Sample answer:

Let x represent the number of T-shirts sold. Let y represent the number of baseball hats sold.

$$\begin{cases} x + y = 84 \\ 15x + 12y = 1146 \end{cases}$$

Linear Combinations Method:

$$\begin{aligned} -15(x + y) &= -15(84) \\ 15x + 12y &= 1146 \end{aligned}$$

$$\begin{array}{r} -15x - 15y = -1260 \\ 15x + 12y = 1146 \\ \hline -3y = -114 \\ y = 38 \end{array}$$

$$x + 38 = 84$$

$$x = 46$$

The solution is (46, 38). The volleyball team sold 46 T-shirts and 38 baseball hats.



TOPIC 2 Systems of Linear Equations and Inequalities

6. At a diner, the total bill for one table of diners included 4 veggie burger combos and 2 beef burger combos for \$97.94. At another table, the total bill came to \$78.95 for 2 veggie burger combos and 3 beef burger combos. Determine the cost of a veggie burger combo and the cost of a beef burger combo.

Sample answer:

Let x represent the cost of a veggie burger combo. Let y represent the cost of a beef burger combo.

$$\begin{cases} 4x + 2y = 97.94 \\ 2x + 3y = 78.95 \end{cases}$$

Linear Combinations Method:

$$\begin{array}{r} 4x + 2y = 97.94 \\ -2(2x + 3y) = -2(78.95) \end{array}$$

$$\begin{array}{r} 4x + 2y = 97.94 \\ -4x - 6y = -157.90 \\ \hline -4y = -59.96 \\ y = 14.99 \end{array}$$

$$\begin{array}{r} 4x + 2(14.99) = 97.94 \\ 4x + 29.98 = 97.94 \\ 4x = 67.96 \\ x = 16.99 \end{array}$$

The solution is (16.99, 14.99). A veggie burger combo costs \$16.99 and a beef burger combo cost \$14.99.



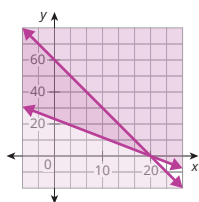
TOPIC 2 Systems of Linear Equations and Inequalities

Extension

Isla sells baked goods from her home kitchen. She offers decorated cookies for \$15 per dozen and cupcakes for \$13 per dozen. It takes her an hour to decorate a dozen cookies but only 20 minutes to decorate a dozen cupcakes. She would like to make at least \$300 per week and not put in more than 20 hours of work per week.

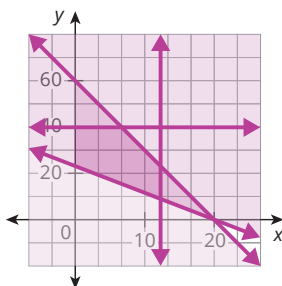
1. Create a system of linear inequalities that fits the situation and graph them. **2A.3E, 2A.3F; Extension of TEKS A.2I, A.3D**

$$\begin{cases} 15x + 13y \geq 300 \\ x + \frac{1}{3}y \leq 20 \end{cases}$$



2. Isla just discovered that she is running out of cake mix for the cupcakes and royal icing for the cookies. She can make a maximum of 40 dozen cupcakes and 12 dozen cookies. What are the new inequalities you need to add to your problem? Add them to your graph. **2A.3E, 2A.3F; Extension of TEKS A.2I, A.3DF**

$$\begin{cases} 15x + 13y \geq 300 \\ x + \frac{1}{3}y \leq 20 \\ y \leq 40 \\ x \leq 12 \end{cases}$$



TOPIC 2 Systems of Linear Equations and Inequalities

3. What is the maximum amount of baked goods that she could make? How much will she earn? How long will it take her?

2A.3F, 2A.3G; Extension of TEKS A.3D

The maximum amount of baked goods she can make is 40 dozen cupcakes and 6 dozen cookies. She will earn \$610, and it will take her 19 hours and 20 minutes.

4. What is the least amount of time she could work and still earn \$300? What baked goods would she make?

2A.3F, 2A.3G; Extension of TEKS A.3D

She could make 24 dozen cupcakes, which would only take her 8 hours, and she would make \$312.

Spaced Practice

1. Consider the equation $6x - 2y = -12$. TEKS A.3A

- a. What is the slope of the equation?

3

- b. What are the intercepts of the equation?

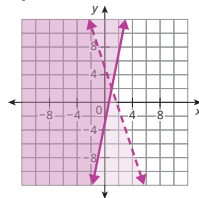
(0, 6) and (-2, 0)

2. The equation to calculate the area of a trapezoid is $A = \frac{1}{2}(a+b)h$. Rewrite the equation to solve for a . TEKS A.12F

$$a = \frac{2A}{h} - b$$

3. Graph each system of inequalities. Then, identify two points that are solutions of the system. TEKS A.3D

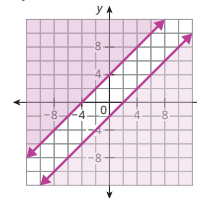
a.
$$\begin{cases} y \geq 5x - 3 \\ y < -3x + 5 \end{cases}$$



Answers will vary.

(0, 0) and (-2, 1)

b.
$$\begin{cases} y \geq x + 4 \\ x - y \geq 2 \end{cases}$$



No solution

TOPIC 2 Systems of Linear Equations and Inequalities

4. What is the equation for the line that has a slope of 0 and passes through the point (3, 7)? **TEKS A.2G**

$$y = 7$$

5. What is the equation for the line that has a slope of $\frac{1}{5}$ and passes through the point $(-\frac{2}{3}, \frac{1}{2})$? Write an equation in slope-intercept form and an equation in standard form. **TEKS A.2B**

Slope-intercept form: $y = \frac{1}{5}x + \frac{19}{30}$

Standard form: $6x - 30y = -19$



Investigating Growth and Decay

.....

TOPIC 1: Introduction to Exponential Functions

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Name _____ Date _____

I. Properties of Powers with Integer Exponents

Topic Practice

A. Write an equivalent expression in simplest form. TEKS A.11B (Topic-Aligned)

1. $(m^6)^4$ m^{24}	2. $(n^8)^{\frac{1}{2}}$ n^4	3. $(d^5)^5$ d^{25}
4. $(m^3)^6$ m^{18}	5. $(z^5)^2$ z^{10}	6. $(x^2)^9$ x^{18}

B. Write an equivalent expression in simplest form. TEKS A.11B (Topic-Aligned)

1. $2n^4m^3 \cdot 19n^2m^9$ $38n^6m^{12}$	2. $-\frac{17k^5x^3}{19k^2x}$ $-\frac{17x^2}{19k^4}$	3. $3n^5z^9(-5n^3z^6)$ $-15n^8z^{15}$
4. $\frac{15wa^5}{-6w^5a^8}$ $-\frac{5}{2w^4a^3}$	5. $-\frac{5m^9d^4}{-11m^3d}$ $\frac{5m^6d^3}{11}$	6. $(-3b^{-5}k)(9b^5k^3)$ $-27k^4$
7. $-\frac{8c^8k^5}{2c^9k^2}$ $-\frac{4k^3}{c}$	8. $(-3k^5w^3)(-13k^7w^9)$ $39k^{12}w^{12}$	9. $\frac{9z^5a^6}{16z^2a^{-7}}$ $\frac{9a^{13}}{16z}$
10. $(-5n^7j^6)(16n^7j^6)$ $-80n^{14}j^{12}$	11. $\frac{12n^5m^{-3}}{-12n^9m^5}$ $-\frac{1}{n^4m^8}$	12. $\frac{8w^9m^5}{20wm^9}$ $\frac{2w^8}{5m^4}$



TOPIC 1 Introduction to Exponential Functions

13. $(-8w^5d^6)(11w^4d^3)$

$-88w^9d^9$

16. $-y^2x^6 \cdot 8y^8x^6$

$-8y^{10}x^{12}$

14. $-\frac{3bx^8}{12b^4x^6}$

$-\frac{1}{4b^3x^2}$

17. $\frac{-20c^7b^3}{-10c^2b^4}$

$\frac{2c^5}{b}$

15. $3k^4m^6 \cdot 18k^9m^4$

$54k^{13}m^{10}$

18. $(-7a^9z^8)(7a^4z^7)$

$-49a^{13}z^{15}$

C. Write an equivalent expression in simplest form. Assume that variables are not zero. **TEKS A.11B (Topic-Aligned)**

1. $(3d^7x^3)^3$

$27d^{21}x^9$

4. $-(-4c^6d^7a^4)^3$

$64c^{18}d^{21}a^{12}$

7. $(-4a^5y^2k^5)^3$

$-64a^{15}y^6k^{15}$

10. $-(5ac^2)^2$

$-25a^2c^4$

13. $(a^8x^7b^3)^2$

$a^{16}x^{14}b^6$

16. $-(-cb^7)^3$

c^3b^{21}

2. $\left(\frac{x^3}{a^5}\right)^4$

$\frac{x^{12}}{a^{20}}$

5. $\left(\frac{x^3}{c^3}\right)^7$

$\frac{x^{21}}{c^{21}}$

8. $\left(\frac{n^9}{m^3}\right)^2$

$\frac{n^{18}}{m^6}$

11. $\left(\frac{a}{d^2}\right)^9$

$\frac{a^9}{d^{18}}$

14. $\left(\frac{b^2}{j^3}\right)^4$

$\frac{b^8}{j^{12}}$

17. $\left(\frac{n^6}{b^7}\right)^2$

$\frac{n^{12}}{b^{14}}$

3. $(-k^4w^9x)^2$

$k^8w^{18}x^2$

6. $\left(\frac{c}{b}\right)^5$

$\frac{c^5}{b^5}$

9. $(6y^2z^9)^2$

$36y^4z^{18}$

12. $-(-2w^2k^3x^4)^3$

$8w^6k^9x^{12}$

15. $\left(\frac{k^5}{c}\right)^3$

$\frac{k^{15}}{c^3}$

18. $(-6n^9y)^2$

$36n^{18}y^2$

TOPIC 1 Introduction to Exponential Functions

Extension

1. Exponents can be stacked as high as you like. Some mathematicians have used double arrows to represent repeated exponents. For example, $3 \uparrow \uparrow 3$ represents 3^{3^3} or 3^{27} .

Write different numbers using double-arrow notation. How can you write 10 billion using this notation? **Extension of TEKS A.11B**

The number 10 billion, written in double-arrow notation, is $10 \uparrow \uparrow 2$.

Spaced Practice

1. Write a direct variation equation for the table of values. **TEKS A.2D**

x	y
3	4
6	8
9	12
12	16
15	20

$$y = \frac{4}{3}x$$

2. Use what you know about approximating square roots to answer each question. **Prerequisite for TEKS A.11A**
 - a. Explain how you know that the value of $\sqrt{12}$ is between 3 and 4.
 - b. Shade between two values to show where $\sqrt{12}$ lies on the number line.



Sample answer:

$$\sqrt{(9)} = 3 \text{ and } \sqrt{16} = 4.$$

Since 12 is between 9 and 16, $\sqrt{12}$ is between 3 and 4.

Shade between 3.4 and 3.5.



TOPIC 1 Introduction to Exponential Functions

II. Analyzing Properties of Powers

Topic Practice

A. Write an equivalent expression in simplest form. **TEKS A.11B (Topic-Aligned)**

1. $-11n^{-4}$
 $-\frac{11}{n^4}$

2. $\frac{11}{17n^{-4}}$
 $\frac{11n^4}{17}$

3. $\frac{-10}{-2y^{-4}}$
 $5y^4$

4. $14c^0$
 14

5. $17z^{-7}$
 $\frac{17}{z^7}$

6. $\frac{7}{-18y^{-8}}$
 $-\frac{7y^8}{18}$

7. $-\frac{16}{19t^0}$
 $-\frac{16}{19}$

8. $6n^{-9}$
 $\frac{6}{n^9}$

9. $3x^{-4}$
 $\frac{3}{x^4}$

10. $-\frac{5}{(a^{-9})}$
 $-5a^9$

11. $-\frac{6}{5d^{-2}}$
 $-\frac{6d^2}{5}$

12. $-\frac{3}{16s^0}$
 $-\frac{3}{16}$

13. $-20x^0$
 -20

14. $-5z^{-3}$
 $-\frac{5}{z^3}$

15. $\frac{15}{-10n^0}$
 $-\frac{3}{2}$

16. $-11w^{-6}$
 $-\frac{11}{w^6}$

17. $\frac{14}{-7b^{-1}}$
 $-2b$

18. $5a^0$
 5

TOPIC 1 Introduction to Exponential Functions

B. Justify each step to rewrite the expression. Choose the properties from the box. **TEKS A.11B (Interleaved Practice)**

Product of powers rule	Power of a power rule	Negative exponent rule	Quotient of powers rule
Zero power rule	Simplify powers	Identity property of multiplication	Commutative property of multiplication

1. $4x^5 \cdot 6x^2y^6 \cdot xy$

$$\begin{aligned} 4x^5 \cdot 6x^2y^6 \cdot xy &= (4 \cdot 6)(x^5x^2x)(y^6y) \\ &= (24)(x^{5+2+1})(y^{6+1}) \\ &= 24x^8y^7 \end{aligned}$$

Commutative property of multiplication

Product of powers rule

Simplify powers

2. $3a^2b^3 \cdot 7ab^5 \cdot b^2$

$$\begin{aligned} 3a^2b^3 \cdot 7ab^5 \cdot b^2 &= (3 \cdot 7)(a^2a)(b^3b^5b^2) \\ &= (21)(a^{2+1})(b^{3+5+2}) \\ &= 21a^3b^{10} \end{aligned}$$

Commutative property of multiplication

Product of powers rule

Simplify powers

3. $(4m^2n^5)^3$

$$\begin{aligned} (4m^2n^5)^3 &= 4^3(m^2 \cdot 3)(n^{5 \cdot 3}) \\ &= 64m^6n^{15} \end{aligned}$$

Power of a power rule

Simplify powers

4. $(-3x^7y^3)^5$

$$\begin{aligned} (-3x^7y^3)^5 &= (-3)^5(x^7 \cdot 5)(y^3 \cdot 5) \\ &= -243x^{35}y^{15} \end{aligned}$$

Power of a power rule

Simplify powers

5. $\frac{27y^8z^5}{-3y^4z^2}$

$$\begin{aligned} \frac{27y^8z^5}{-3y^4z^2} &= \left(\frac{27}{-3}\right)\left(\frac{y^8}{y^4}\right)\left(\frac{z^5}{z^2}\right) \\ &= -9(y^{8-4})(z^{5-2}) \\ &= -9y^4z^3 \end{aligned}$$

Commutative property of multiplication

Quotient of powers rule

Simplify powers



TOPIC 1 Introduction to Exponential Functions

$$\begin{aligned}
 6. \quad & \frac{-96m^9n^2}{8m^2n^6} \\
 & \frac{-96m^9n^2}{8m^2n^6} = \left(\frac{-96}{8}\right)\left(\frac{m^9}{m^2}\right)\left(\frac{n^2}{n^6}\right) \\
 & = -12(m^{9-2})(n^{2-6}) \\
 & = -12m^7n^{-4} \\
 & = \frac{-12m^7}{n^4}
 \end{aligned}$$

Commutative property of multiplication

Quotient of power rules

Simplify powers

Negative exponent rule

$$\begin{aligned}
 7. \quad & -2x^5y^3 \cdot 8x^2y^{-5} \cdot x^{-9}y^2 \\
 & -2x^5y^3 \cdot 8x^2y^{-5} \cdot x^{-9}y^2 = (-2 \cdot 8)(x^5x^2x^{-9}) \\
 & \quad (y^3y^{-5}y^2) \\
 & = (-16)(x^{5+2+(-9)}) \\
 & \quad (y^{3+(-5)+2}) \\
 & = -16x^{-2}y^0 \\
 & = -16x^{-2}(1) \\
 & = \frac{-16}{x^2}
 \end{aligned}$$

Commutative property of multiplication

Product of powers rule

Simplify powers

Zero power rule

Negative exponent rule

$$\begin{aligned}
 8. \quad & \frac{42m^5n^3 \cdot m^4n^2}{6m^6n^5} \\
 & \frac{42m^5n^3 \cdot m^4n^2}{6m^6n^5} = \left(\frac{42}{6}\right)\left(\frac{m^5m^4}{m^6}\right)\left(\frac{n^3n^2}{n^5}\right) \\
 & = (7)\left(\frac{m^{5+4}}{m^6}\right)\left(\frac{n^{3+2}}{n^5}\right) \\
 & = (7)(m^{5+4-6})(n^{3+2-5}) \\
 & = 7m^3n^0 \\
 & = 7m^3(1) \\
 & = 7m^3
 \end{aligned}$$

Commutative property of multiplication

Product of powers rule

Quotient of powers rule

Simplify powers

Zero power rule

Identity property of multiplication

C. Each expression has been rewritten incorrectly. Explain the mistake that occurred and then make the correction. **TEKS A.11B (Interleaved Practice)**

$$1. \quad \left(\frac{f^3}{n^4}\right)^4 = \frac{f^{12}}{n^4}$$

The power of a power rule was used incorrectly. The exponent outside the fraction should have been applied to both the numerator and the denominator of the fraction.

$$\left(\frac{f^3}{n^4}\right)^4 = \frac{f^{12}}{n^{16}}$$



TOPIC 1 Introduction to Exponential Functions

2. $-15x^0 = -15(0) = 0$

$x^0 \neq 0$. The zero power rule says that the zero power of any number except 0 is 1.

$$-15x^0 = -15(1) = -15$$

3. $(3x^2y^4)^3 = 27x^5y^7$

The power of a power rule was used incorrectly. The exponents should be multiplied rather than added. $(3x^2y^4)^3 = 27x^6y^{12}$

4. $p^3 \cdot p^6 = p^{18}$

The product of powers rule was used incorrectly. The exponents should have been added rather than multiplied. $p^3 \cdot p^6 = p^9$

5. $\frac{m^8n^3}{m^6n^7} = m^2n^4$

The quotient of powers was used incorrectly. The power of n should be negative and then the negative exponent rule should be applied. $\frac{m^8n^3}{m^6n^7} = m^2n^{-4} = \frac{m^2}{n^4}$

6. $(-d^3h^2g^4)^7 = d^{21}h^{14}g^{28}$

The power of a power rule was used incorrectly. The negative sign on the first factor should be treated as a coefficient of -1 . $(-1)^7 = -1$ so $(-d^3h^2g^4)^7 = -d^{21}h^{14}g^{28}$



TOPIC 1 Introduction to Exponential Functions

Extension

1. Three positive integers have a sum of 10. How can you place the integers in the square brackets to form the greatest number possible? **Extension for TEKS A.11B**

$$([\quad]) \cdot ([\quad])^{\square}$$

$$[1] \cdot [4]^{[5]}$$

Spaced Practice

1. Use the properties of powers to rewrite the expressions as fractions and decimals without exponents. **TEKS A.11B**

a. $10^4 \cdot 10^{-5}$

$$\frac{1}{10} = 0.1$$

b. $2^{-6} \div 2^{-2}$

$$\frac{1}{16} = 0.0625$$

2. Determine the slope of the line given the points on the line. **TEKS A.3A**

a. A (0, 1), B (0, 7)

$$m = \frac{(7-1)}{(0-0)} = \frac{7}{0} = \text{undefined}$$

b. E (1, 4), F (1, 0)

$$m = \frac{(0-4)}{(1-1)} = \frac{7}{0} = \text{undefined}$$

III. Geometric Sequences and Exponential Functions

Topic Practice

- A. Write an explicit geometric formula for each sequence. Then, rewrite each formula as an exponential function of the form $f(x) = ab^x$. Identify the constant ratio and y-intercept of each exponential function. **TEKS A.9C (Topic-Aligned), TEKS A.9D (Topic-Aligned), TEKS A.12D (Topic-Aligned)**

1. 3, 9, 27, 81, ...

Explicit formula: $g_n = 3 \cdot 3^{n-1}$

Exponential function: $f(x) = 3^x$

Constant ratio: 3

y-intercept: (0, 1)

2. 512, 256, 128, 64, ...

Explicit formula: $g_n = 512\left(\frac{1}{2}\right)^{n-1}$

Exponential function: $f(x) = 1024\left(\frac{1}{2}\right)^x$

Constant ratio: $\frac{1}{2}$

y-intercept: (0, 1024)



TOPIC 1 Introduction to Exponential Functions

3. 0.1, 0.4, 1.6, 6.4, ...

Explicit formula: $g_n = 0.1 \cdot 4^{n-1}$

Exponential function: $f(x) = 0.025 \cdot 4^x$

Constant ratio: 4

y-intercept: (0, 0.025)

5. 45, 15, 5, $\frac{5}{3}$, ...

Explicit formula: $g_n = 45\left(\frac{1}{3}\right)^{n-1}$

Exponential function: $f(x) = 135\left(\frac{1}{3}\right)^x$

Constant ratio: $\frac{1}{3}$

y-intercept: (0, 135)

4. 3000, 300, 30, 3, ...

Explicit formula: $g_n = 3000\left(\frac{1}{10}\right)^{n-1}$

Exponential function: $f(x) = 30,000\left(\frac{1}{10}\right)^x$

Constant ratio: $\frac{1}{10}$

y-intercept: (0, 30,000)

6. -4.8, -9.6, -19.2, -38.4, ...

Explicit formula: $g_n = -4.8(2)^{n-1}$

Exponential function: $f(x) = -2.4(2)^x$

Constant ratio: 2

y-intercept: (0, -2.4)

- B. Write an exponential function for each table or graph. Then, identify the constant ratio and y-intercept of each function.

TEKS A.9C (Topic-Aligned), TEKS A.9D (Topic-Aligned)

- 1.

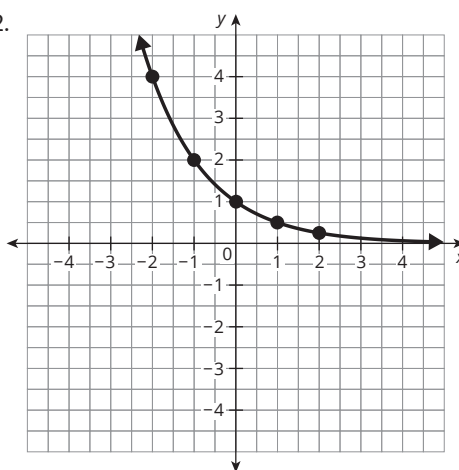
x	f(x)
-2	18
-1	6
0	2
1	$\frac{2}{3}$
2	$\frac{2}{9}$

$$f(x) = 2\left(\frac{1}{3}\right)^x$$

Constant ratio: $\frac{1}{3}$

y-intercept: (0, 2)

- 2.



$$f(x) = 1\left(\frac{1}{2}\right)^x$$

Constant ratio: $\frac{1}{2}$

y-intercept: (0, 1)



TOPIC 1 Introduction to Exponential Functions

3.

x	$f(x)$
-2	$-\frac{1}{3}$
-1	-1
0	-3
1	-9
2	-27

$$f(x) = -3(3)^x$$

Constant ratio: 3

y-intercept: (0, -3)

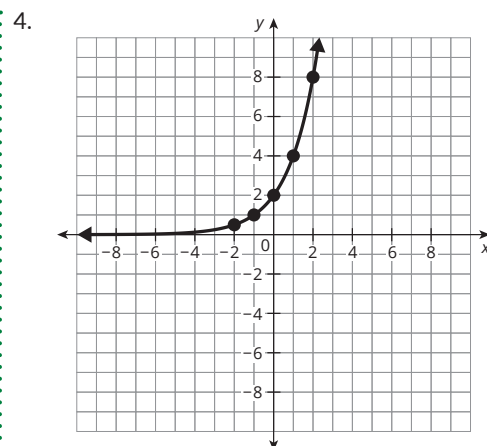
5.

x	$f(x)$
-2	-48
-1	-12
0	-3
1	$-\frac{3}{4}$
2	$-\frac{3}{16}$

$$f(x) = -3\left(\frac{1}{4}\right)^x$$

Constant ratio: $\frac{1}{4}$

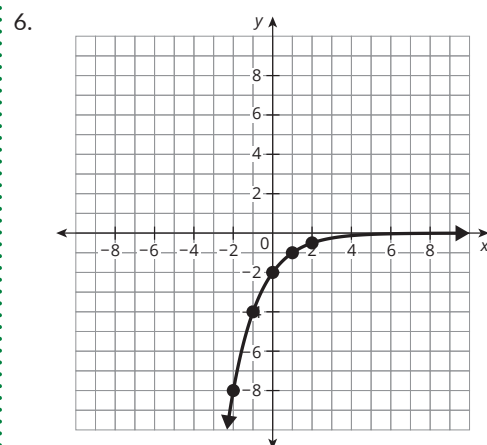
y-intercept: (0, -3)



$$f(x) = 2(2)^x$$

Constant ratio: 2

y-intercept: (0, 2)



$$f(x) = -2\left(\frac{1}{2}\right)^x$$

Constant ratio: $\frac{1}{2}$

y-intercept: (0, -2)

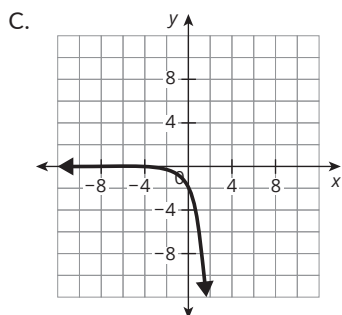
TOPIC 1 Introduction to Exponential Functions

Extension

1. Which of the functions does not fit with the others? Explain your answer. **Extension of TEKS A.9D**

A. The exponential function that goes through $(0, -3)$ and $(5, -96)$

B. $f(x) = -1 \cdot 6^x$



D.

x	y
1	$\frac{2}{3}$
2	$\frac{2}{9}$
3	$\frac{2}{27}$

Sample answer:

Functions A, B, and C are negative exponential functions, with graphs below the x-axis. Function D is a positive exponential function, $f(x) = 2\left(\frac{1}{3}\right)^x$.

Spaced Practice

1. Solve each system of linear equations. **TEKS A.5C**

a. $\begin{cases} y = -5x - 21 \\ -2x + 5y = -24 \end{cases}$

$(-3, -6)$

b. $\begin{cases} 8x - 3y = 4 \\ 7x - 10y = -26 \end{cases}$

$(2, 4)$

2. Jasmine needs to earn a score of at least 80 on her Algebra exam. The table shows the number of true or false and multiple-choice questions she needs to get correct to earn a score of 80. Write a linear inequality in two variables to represent this situation. **TEKS A.2H**

$2x + 5y \geq 80$

True or False	Multiple-Choice
0	16
5	14
20	8



IV. Rewriting Square Roots

A. Write each radical expression by extracting perfect squares.

TEKS A.11A (Topic-Aligned)

1. $\sqrt{18}$
 $3\sqrt{2}$

3. $4\sqrt{200}$
 $40\sqrt{2}$

5. $6\sqrt{15} \cdot \sqrt{3}$
 $18\sqrt{5}$

2. $\sqrt{40}$
 $2\sqrt{10}$

4. $\sqrt{12} \cdot \sqrt{8}$
 $4\sqrt{6}$

6. $3\sqrt{27} \cdot 5\sqrt{30}$
 $135\sqrt{10}$

B. Use the quotient property of radicals to determine each quotient in simplest radical form.

TEKS A.11A (Topic-Aligned)

1. $\frac{\sqrt{72}}{\sqrt{3}}$
 $2\sqrt{6}$

3. $\frac{\sqrt{98}}{\sqrt{2}}$
 7

5. $\frac{\sqrt{128}}{\sqrt{2}}$
 8

7. $\frac{\sqrt{27}}{\sqrt{9}}$
 3

2. $\frac{\sqrt{50}}{\sqrt{5}}$
 $\sqrt{10}$

4. $\frac{\sqrt{45}}{\sqrt{5}}$
 3

6. $\frac{\sqrt{200}}{\sqrt{5}}$
 $2\sqrt{10}$

8. $\frac{\sqrt{75}}{\sqrt{3}}$
 5

TOPIC 1 Introduction to Exponential Functions

Extension

Use mental math to evaluate each expression. **Extension for TEKS A.11A**

1. $\sqrt{5} + \sqrt{2} \cdot \sqrt{8} - \sqrt{5}$
4
2. $\sqrt{6} \cdot \sqrt{6} + 2\sqrt{3} \cdot 2\sqrt{3}$
18

Spaced Practice

Determine the first four terms of each sequence when n is a whole number greater than 1. **TEKS A.12C**

1. $f(n) = 5f(n - 1)$ and $f(1) = 3$
3, 15, 75, 375
2. $h(n) = \frac{1}{4}f(n - 1)$ and $f(1) = 80$
80, 20, 5, $\frac{5}{4}$

V. Rational Exponents and Graphs of Exponential Functions

Topic Practice

- A. Complete each table and graph the function. Identify the constant ratio and key characteristics.

TEKS A.9A (Interleaved Practice), TEKS A.9D (Topic-Aligned), TEKS A.12B (Topic-Aligned)

1. $f(x) = 2^x$

x	$f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

Constant ratio: 2

y-intercept: (0, 1)

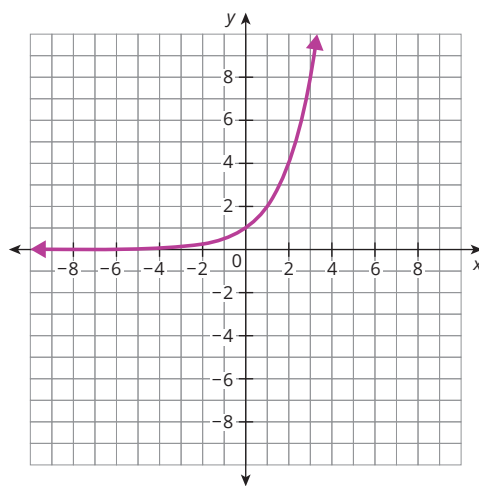
x-intercept: None

Equation of the asymptote: $y = 0$

Increasing/decreasing: Increasing

Domain: $-\infty < x < \infty$

Range: $0 < y < \infty$



TOPIC 1 Introduction to Exponential Functions

2. $f(x) = 4^x$

x	$f(x)$
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
1	4
2	16

Constant ratio: 4

y-intercept: (0, 1)

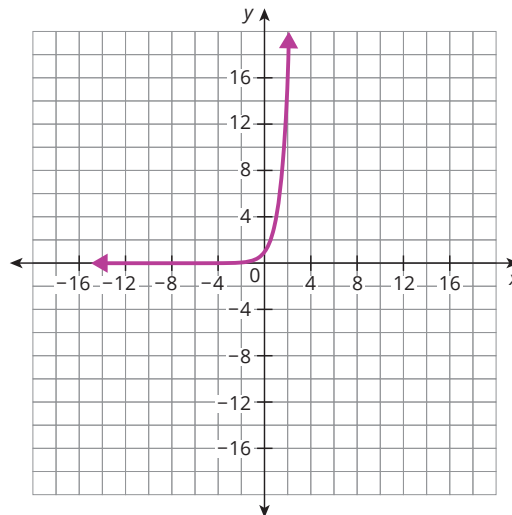
x-intercept: None

Equation of the asymptote: $y = 0$

Increasing/decreasing: Increasing

Domain: $-\infty < x < \infty$

Range: $0 < y < \infty$



3. $f(x) = \left(\frac{1}{3}\right)^x$

x	$f(x)$
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$

Constant ratio: $\frac{1}{3}$

y-intercept: (0, 1)

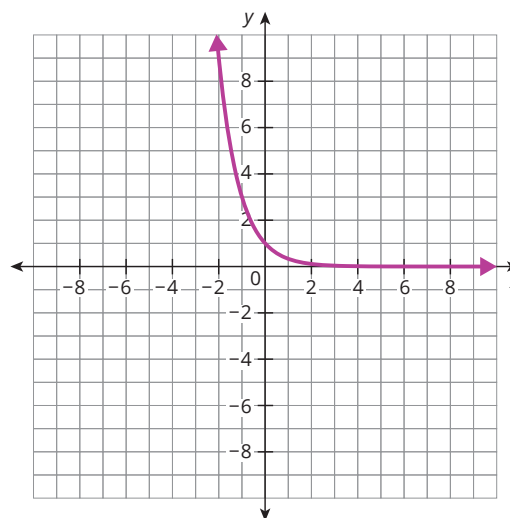
x-intercept: None

Equation of the asymptote: $y = 0$

Increasing/decreasing: Decreasing

Domain: $-\infty < x < \infty$

Range: $\infty > y > 0$



TOPIC 1 Introduction to Exponential Functions

4. $f(x) = \left(\frac{1}{4}\right)^x$

x	$f(x)$
-2	16
-1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$

Constant ratio: $\frac{1}{4}$

y-intercept: (0, 1)

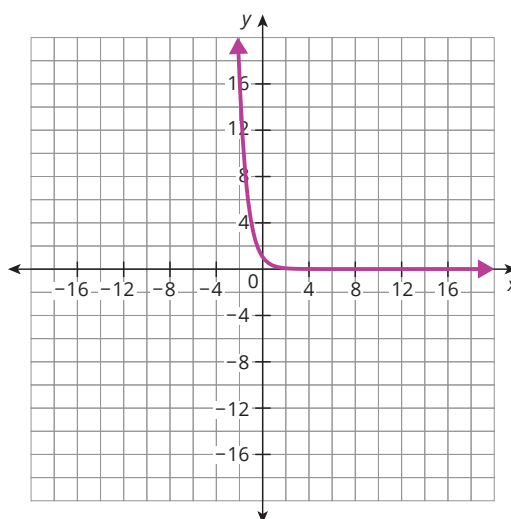
x-intercept: None

Equation of the asymptote: $y = 0$

Increasing/decreasing: Decreasing

Domain: $-\infty < x < \infty$

Range: $0 < y < \infty$



5. $f(x) = -2 \cdot 2^x$

x	$f(x)$
-2	$-\frac{1}{2}$
-1	-1
0	-2
1	-4
2	-8

Constant ratio: 2

y-intercept: (0, -2)

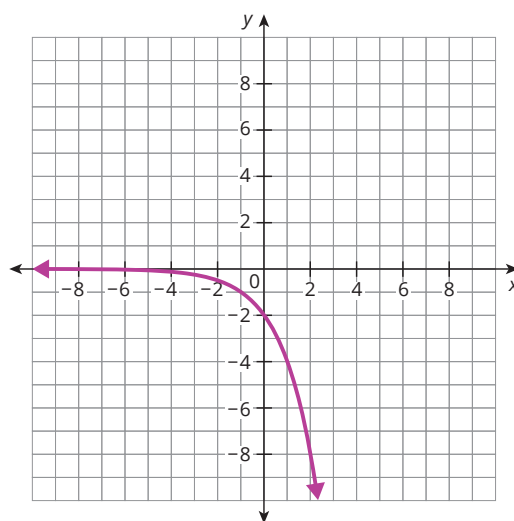
x-intercept: None

Equation of the asymptote: $y = 0$

Increasing/decreasing: Decreasing

Domain: $-\infty < x < \infty$

Range: $0 < y < \infty$



TOPIC 1 Introduction to Exponential Functions

6. $f(x) = -4^x$

x	$f(x)$
-2	$-\frac{1}{16}$
-1	$-\frac{1}{4}$
0	-1
1	-4
2	-16

Constant ratio: 4

y-intercept: $(0, -1)$

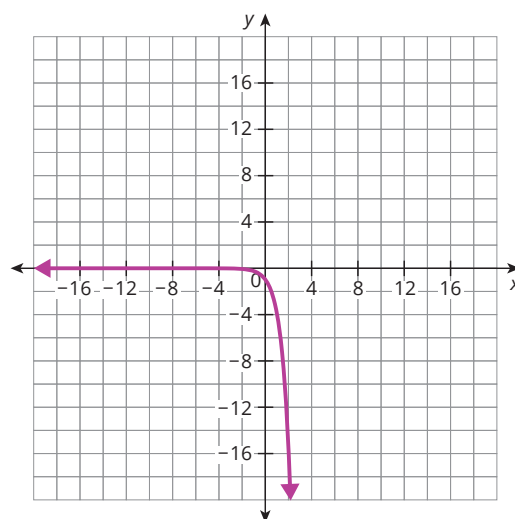
x-intercept: None

Equation of the asymptote: $y = 0$

Increasing/decreasing: Decreasing

Domain: $-\infty < x < \infty$

Range: $\infty < y < 0$



B. Write each power in radical form. TEKS A.11B (Topic-Aligned)

1. $12^{\frac{1}{3}}$
 $\sqrt[3]{12}$

2. $7^{\frac{1}{5}}$
 $\sqrt[5]{7}$

3. $18^{\frac{1}{4}}$
 $\sqrt[4]{18}$

4. $a^{\frac{1}{2}}$
 $\sqrt{a}$

5. $d^{\frac{1}{5}}$
 $\sqrt[5]{d}$

6. $c^{\frac{1}{6}}$
 $\sqrt[6]{c}$

TOPIC 1 Introduction to Exponential Functions

7. $5^{\frac{2}{3}}$
 $\sqrt[3]{5^2}$

9. $18^{\frac{3}{4}}$
 $\sqrt[4]{18^3}$

11. $y^{\frac{4}{3}}$
 $\sqrt[3]{y^4}$

8. $8^{\frac{2}{5}}$
 $\sqrt[5]{8^2}$

10. $x^{\frac{3}{5}}$
 $\sqrt[5]{x^3}$

12. $m^{\frac{5}{2}}$
 $\sqrt{m^5}$

C. Write each expression using a rational exponent. **TEKS A.11B (Topic-Aligned)**

1. $\sqrt[4]{6^3}$
 $6^{\frac{3}{4}}$

3. $\sqrt[3]{12^2}$
 $12^{\frac{2}{3}}$

5. $\sqrt[4]{p^7}$
 $p^{\frac{7}{4}}$

2. $\sqrt[5]{8^4}$
 $8^{\frac{4}{5}}$

4. $\sqrt{n^5}$
 $n^{\frac{5}{2}}$

6. $\sqrt[5]{m^3}$
 $m^{\frac{3}{5}}$



TOPIC 1 Introduction to Exponential Functions

D. Use the properties of powers to write an equivalent expression in simplest form.

TEKS A.11B (Topic-Aligned)

$$1. (y^{\frac{4}{5}})^3$$

$$y^{\frac{12}{5}}$$

$$2. (225s^8t^2)^{\frac{1}{2}}$$

$$225^{\frac{1}{2}}s^{\frac{8}{2}}t^{\frac{2}{2}}$$

$$= \sqrt{225}s^4t^1$$

$$= 15s^4t$$

$$3. \frac{\sqrt[3]{8x^6y^9}}{\sqrt{9x^{-4}y^8}}$$

$$\frac{\sqrt[3]{8x^6y^9}}{\sqrt{9x^{-4}y^8}} = \frac{(8x^6y^9)^{\frac{1}{3}}}{(9x^{-4}y^8)^{\frac{1}{2}}}$$

$$= \frac{2x^2y^3}{3x^{-2}y^4}$$

$$= \frac{2x^4}{3y}$$

$$4. (2a^{\frac{1}{2}}b^{\frac{3}{4}})^2(16a^{-\frac{5}{6}}b^{\frac{2}{3}})^0$$

$$(2a^{\frac{1}{2}}b^{\frac{3}{4}})^2(16a^{-\frac{5}{6}}b^{\frac{2}{3}})^0 = (2a^{\frac{1}{2}}b^{\frac{3}{4}})^2(1)$$

$$= 4ab^{\frac{3}{2}}$$

$$= 4ab\sqrt{b}$$

$$5. \left(\frac{\sqrt[4]{16a^3b}}{\sqrt{36a^{-2}b^5}}\right)^0$$

$$\left(\frac{\sqrt[4]{16a^3b}}{\sqrt{36a^{-2}b^5}}\right)^0 = 1$$

$$6. (3z^4)^2(z^9)^{\frac{1}{3}}$$

$$(3^2z^4 \cdot 2)(z^9 \cdot \frac{1}{3})$$

$$= (9z^8)(z^3)$$

$$= 9z^{8+3}$$

$$= 9z^{11}$$

TOPIC 1 Introduction to Exponential Functions

Extension

- How do rational exponents help you multiply or divide two radicals with different indices ($\sqrt[m]{a} \cdot \sqrt[n]{a}$ or $\frac{\sqrt[m]{a}}{\sqrt[n]{a}}$, when $m \neq n$)? Include two examples to support your answer. **Extension for TEKS A.11B**

Sample answer:

When I rewrite each radical as rational exponent, I can then add or subtract the fractional exponents using a common denominator, depending on whether I am multiplying or dividing.

$$\sqrt[m]{a} \cdot \sqrt[n]{a} = a^{\frac{1}{m}} \cdot a^{\frac{1}{n}}$$

$$= a^{\frac{1}{m} + \frac{1}{n}} = a^{\frac{n+m}{mn}}$$

$$\frac{\sqrt[m]{a}}{\sqrt[n]{a}} = \frac{a^{\frac{1}{m}}}{a^{\frac{1}{n}}} = a^{\frac{1}{m} - \frac{1}{n}} = a^{\frac{n-m}{mn}}$$

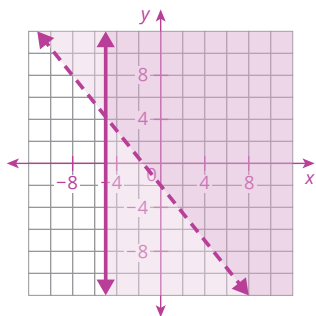
Spaced Practice

- Complete the table. **TEKS A.9B, TEKS A.9D, TEKS A.11B**

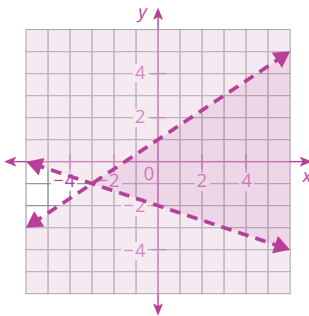
Explicit Formula	Exponential Function	Constant Ratio	y-Intercept
$840 \cdot 3^{x-1}$	$280 \cdot 3^x$	3	280
$-3\left(\frac{1}{5}\right)^{x-1}$	$-15\left(\frac{1}{5}\right)^x$	$\frac{1}{5}$	-15

- Solve each system of linear inequalities. **TEKS A.3H**

a. $\begin{cases} y > -\frac{5}{4}x - 2 \\ x \geq -5 \end{cases}$



b. $\begin{cases} 2x - 3y > -3 \\ x + 3y > -6 \end{cases}$



3. High school students in a local district are required to volunteer for at least 50 hours by the time they graduate. Carlos volunteers as the coach of a soccer team for five-year-olds. The table shows the number of practices and games Carlos must lead to earn the exact number of hours required for graduation. Define variables for this context and write a linear inequality that describes the situation. **TEKS A.2H**

Practices	Games
30	35
40	30
80	10

Let x represent the number of practices and let y represent the number of games.

$$0.5x + y \geq 50$$

Name _____ Date _____

I. Exponential Equations for Growth and Decay

Topic Practice

A. Determine whether each type of account describes simple interest or compound interest based on the scenario given. Explain your reasoning. *Prerequisite for TEKS A.9B*

1. Isaiah deposits \$300 into an account that earns 2% interest each year. After the first year, Isaiah has \$306 in the account. After the second year, Isaiah has \$312 in the account. After the third year, Isaiah has \$318 in the account.

This is a simple interest account because the interest earned at the end of each year is a percent of the original deposit amount.
2. Jasmine deposits \$600 in an account that earns 1.5% interest each year. After the first year, Jasmine has \$609 in the account. After the second year, Jasmine has \$618.14 in the account. After the third year, Jasmine has \$627.41 in the account.

This is a compound interest account because the interest earned at the end of each year is a percent of the account balance at the beginning of the year.
3. Luna deposits \$500 into an account that earns 2.5% interest each year. After the first year, Luna has \$512.50 in the account. After the second year, Luna has \$525.31 in the account. After the third year, Luna has \$538.44 in the account.

This is a compound interest account because the interest earned at the end of each year is a percent of the account balance at the beginning of the year.
4. Lucas deposits \$4000 into an account that earns 4.25% interest each year. After the first year, Lucas has \$4170 in the account. After the second year, Lucas has \$4340 in the account. After the third year, Lucas has \$4510 in the account.

This is a simple interest account because the interest earned at the end of each year is a percent of the original deposit amount.



TOPIC 2 Using Exponential Equations

5. Nahimana deposits \$725 in an account that earns 3% interest each year. After the first year, Nahimana has \$746.75 in the account. After the second year, Nahimana has \$768.50 in the account. After the third year, Nahimana has \$790.25 in the account.

This is a simple interest account because the interest earned at the end of each year is a percent of the original deposit amount.

6. Mason deposits \$3500 in an account that earns 3.75% interest each year. After the first year, Mason has \$3631.25 in the account. After the second year, Mason has \$3767.42 in the account. After the third year, Mason has \$3908.70 in the account.

This is a compound interest account because the interest earned at the end of each year is a percent of the account balance at the beginning of the year.

B. Write the exponential function represented by the table of values. TEKS A.9C (Topic-Aligned)

1.

x	f(x)
0	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{4}$

$$f(x) = 2\left(\frac{1}{2}\right)^x$$

3.

x	f(x)
0	1
1	$\frac{3}{4}$
2	$\frac{9}{16}$
3	$\frac{27}{64}$

$$f(x) = \left(\frac{3}{4}\right)^x$$

2.

x	f(x)
0	1
2	25
4	625
6	15,625

$$f(x) = 5^x$$

4.

x	f(x)
0	-1
2	-4
4	-16
6	-64

$$f(x) = -(2)^x$$

TOPIC 2 Using Exponential Equations

5.

x	f(x)
0	3
3	$\frac{1}{9}$
6	$\frac{1}{243}$
9	$\frac{1}{6561}$

$$f(x) = 3\left(\frac{1}{3}\right)^x$$

6.

x	f(x)
0	-2
1	$-\frac{1}{2}$
2	$-\frac{1}{8}$
3	$-\frac{1}{32}$

$$f(x) = -2\left(\frac{1}{4}\right)^x$$

C. Write a function to represent each problem situation. TEKS A.9C (Topic-Aligned)

1. Sebastian deposits \$500 into a compound interest account for a period of t years. The interest rate for the account is 4%.

$$P(t) = 500 \cdot 1.04^t$$

2. Kayla deposits \$250 into a compound interest account for a period of t years. The interest rate for the account is 6%.

$$P(t) = 250 \cdot 1.06^t$$

3. Adriana deposits \$1200 into a compound interest account for a period of t years. The interest rate for the account is 3.5%.

$$P(t) = 1200 \cdot 1.035^t$$

4. Angelina deposits \$2700 into a compound interest account for a period of t years. The interest rate for the account is 4.25%.

$$P(t) = 2700 \cdot 1.0425^t$$

5. Mei deposits \$300 into a compound interest account for a period of t years. After one year, Mei's balance is \$305.25.

$$P(t) = 300 \cdot 1.0175^t$$

6. Gabriel deposits \$450 into a compound interest account for a period of t years. After one year, Gabriel's balance is \$474.75.

$$P(t) = 450 \cdot 1.055^t$$



TOPIC 2 Using Exponential Equations

Extension

1. Consider a piece of paper that is 0.1 mm thick. How many times must it be folded so that it reaches the top of the Eiffel Tower? Assume the paper is as large as needed and it is possible to fold it as many times as required. **TEKS 2A.5D; Extension for A.9C**

$$f(x) = 0.1 \cdot 2^x$$

The Eiffel Tower is approximately 330 m, or 330,000 mm, tall at its tip.

$$0.1 \cdot 2^x \geq 330,000$$

$$x \geq 21.654$$

The paper must be folded 22 times to reach the top of the Eiffel Tower.

Spaced Practice

1. Diego and Valentina open a pet store and start with 5 hamsters for sale. Hamster populations usually triple every cycle. One cycle is equal to 4 months. Write an equation in function notation to represent the change in the number of hamsters as a function of the cycle number, c . Explain how you determined your equation.

TEKS A.9C

$$f(c) = 5 \cdot 3^c$$

2. Write an exponential function to model this table of values. **TEKS A.9C**

x	$g(x)$
1	0.6
2	0.06
3	0.006
4	0.0006

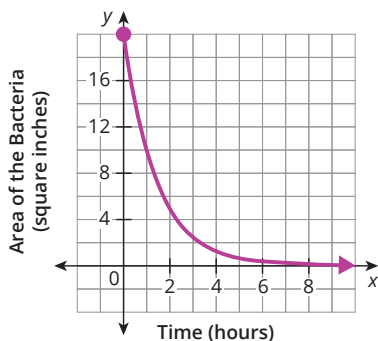
$$f(x) = 6 \cdot 0.1^x$$

II. Interpreting Parameters in Context

Topic Practice

- A. A scientist is studying the growth rate of a certain bacteria when starved of nutrients. She fills a petri dish with bacteria and measures the area covered by the bacteria every hour. The equation $f(x) = 20 \cdot 0.5^x$ represents the area, in square inches, of the bacteria after x hours. TEKS A.9B (Topic-Aligned), TEKS A.9D (Topic-Aligned)

1. Graph $f(x)$ on the coordinate plane.



3. Is $f(x)$ increasing or decreasing? What does this mean in terms of the problem situation?

$f(x)$ is decreasing. This means that the number of square inches of the petri dish covered by the bacteria decreases every hour.

2. Identify the y-intercept and interpret its meaning in terms of the problem situation.

The y-intercept is 20. This means that the bacteria covered 20 square inches of the petri dish when the study started.

4. Write an equation for the asymptote. Does the asymptote make sense in this problem situation? Explain your reasoning.

The equation of the asymptote is $y = 0$. The asymptote does not make sense in terms of the problem situation. This would mean that the area covered by the bacteria in the petri dish would approach but never reach zero. However, in reality, all the bacteria will eventually run out of nutrients and the area covered would become zero.



TOPIC 2 Using Exponential Equations

5. What is the area of the bacteria in the petri dish after three hours. Explain how you determined your answer.

After three hours, the area of bacteria in the petri dish is 2.5 square inches. I used the equation to evaluate $f(3)$.

6. When will the area of the bacteria be 14 square inches? Explain how you determined your answer.

The area of the bacteria will be 14 square inches approximately $\frac{1}{2}$ hour after the study starts. I used the graph to estimate my answer.

B. Write an exponential function that represents each population as a function of time. TEKS A.9C (Topic-Aligned)

1. A city has a population of 15,000 people. Its population is decreasing at a rate of 1.5% each year.

$$P(t) = 15,000 \cdot 0.985^t$$

2. A village has a population of 20,750 people. Its population is increasing at a rate of 2.7% each year.

$$P(t) = 20,750 \cdot 1.027^t$$

3. A village has a population of 6075 people. Its population is decreasing at a rate of 0.5% each year.

$$P(t) = 6075 \cdot 0.995^t$$

4. A city has a population of 95,000 people. Its population is decreasing at a rate of 1.375% each year.

$$P(t) = 95,000 \cdot 0.98625^t$$

5. A village's population increases from 985 to 991 from one year to the next.

$$P(t) = 985 \cdot 1.0061^t$$

6. A city's population increased from 40,500 in 2020 to 42,039 in 2021.

$$P(t) = 40,500 \cdot 1.038^t$$



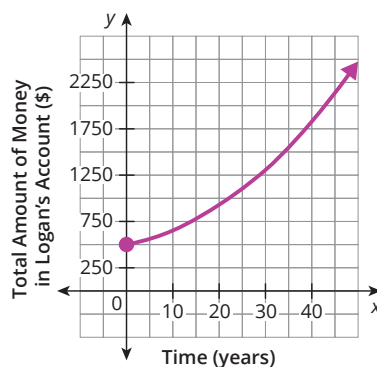
TOPIC 2 Using Exponential Equations

- C. Logan deposits \$500 into a compound interest account. The interest rate for the account is 3.25%. TEKS A.9C (Topic-Aligned), TEKS A.9D (Topic-Aligned), TEKS A.12B (Interleaved Practice)

1. Write an exponential function, $P(t)$, to represent the amount of money Logan has in his account as a function of time, t .

$$P(t) = 500 \cdot 1.0325^t$$

2. Graph $P(t)$ on the coordinate plane.



3. Evaluate $P(0)$. Explain what the value means in terms of the scenario.

In 0 years, the account balance will be \$500.

4. Evaluate $P(15)$. Explain what the value means in terms of the scenario.

In 15 years, the account balance will be \$807.83.

5. Evaluate $P(20)$. Explain what the value means in terms of the scenario.

In 20 years, the account balance will be \$947.92.

6. Evaluate $P(50)$. Explain what the value means in terms of the scenario.

In 50 years, the account balance will be \$2474.42.



TOPIC 2 Using Exponential Equations

D. For each function, interpret the a - and b -values of the given function in terms of the problem situation. Then, determine if the function is increasing or decreasing. **TEKS A.9B (Topic-Aligned)**

1. The function $P(t) = 4000 \cdot 1.03^t$ represents the population of a village as a function of time.

The a -value is 4000. This means that the current population of the village is 4000 people. The b -value is 1.03. This means the population is growing at a rate of 3%. The function is increasing.

3. The function $M(t) = 975 \cdot 0.92^t$ represents the number of milligrams of a medication in a patient's body as a function of time.

The a -value is 975. This means that the current amount of medication in the patient's body is 975 milligrams. The b -value is 0.92. This means that the amount of medication in the patient's body is decreasing at a rate of 8%. The function is decreasing.

5. The function $B(t) = 500 \cdot 1.004^t$ represents the number of bacteria in a petri dish as a function of time.

The a -value is 500. This means that the current number of bacteria is 500. The b -value is 1.004. This means that the number of bacteria is increasing at a rate of 0.4%. The function is increasing.

2. The function $B(t) = 8000 \cdot 0.98^t$ represents the number of bacteria in a petri dish as a function of time.

The a -value is 8000. This means that the current number of bacteria is 8000. The b -value is 0.98. This means the number of bacteria is decreasing at a rate of 2%. The function is decreasing.

4. The function $M(t) = 7210 \cdot 1.015^t$ represents the amount of money in a savings account as a function of time.

The a -value is 7210. This means that the current balance of the savings account is \$7210. The b -value is 1.015. This means that the amount of money in the savings account is growing at a rate of 1.5%. The function is increasing.

6. The function $V(t) = 32,000 \cdot 0.96^t$ represents the value of a new car as a function of time.

The a -value is 32,000. This means that the current value of the car is \$32,000. The b -value is 0.96. This means the value of the car is decreasing at a rate of 4%. The function is decreasing.



TOPIC 2 Using Exponential Equations

Extension

Emily and Sarah developed a new art app for smart phones. The table shows the number of customers who downloaded the app by month.

- Emily thinks that the equation that represents the data in the table is $y = 4(2)^x$. Determine if Emily is correct. Explain your reasoning. **Extension of A.9B**

Yes. The data represents $y = 4(2)^x$. The initial value is 4, and the amount is doubled each month.

- Determine a different exponential equation that represents the data in the table. Use the equation $y = a \cdot b^{f(x)}$, where $f(x)$ is a function of x and $a = 2$.

Extension of A.9C, Extension of A.11B

$$y = 2(2)^{x+1}$$

Month	Number of Downloads
0	4
1	8
2	16
3	32
4	64
5	128

Spaced Practice

- Rewrite each expression in rational exponent form in simplest terms. **TEKS A.11B**

a. $\sqrt[3]{6^4}$
 $6^{\frac{4}{3}}$

b. $(\sqrt[8]{8})^{12}$
 $8^{\frac{3}{2}}$

c. $(\sqrt[7]{x})^3$
 $x^{\frac{3}{7}}$

d. $\sqrt[10]{y^5}$
 $y^{\frac{1}{2}}$

- Camilla receives \$1500 for her birthday. She is going to spend \$500 and wants to put the rest into an account that will earn interest. She is considering two different accounts. Account A earns 6.5% annual simple interest. Account B earns 4.5% annual compound interest.
 - Write a function for each account that can be used to determine the balance in the account based on the year, t . **TEKS A.9C**

Account A: $f(t) = 65t + 1000$,

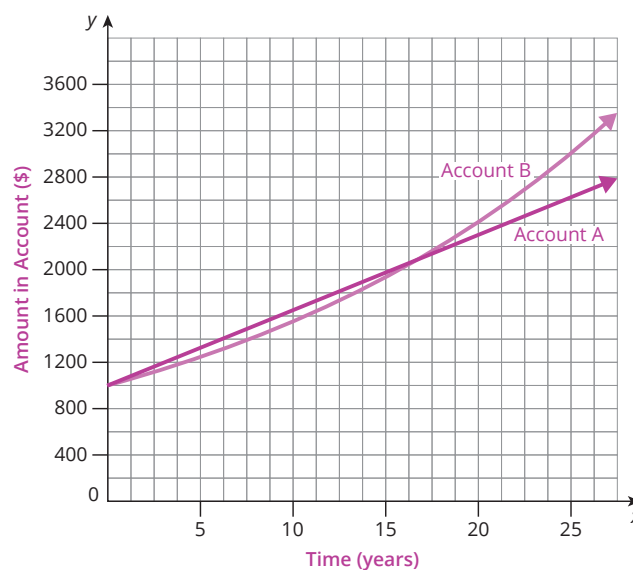
Account B: $f(t) = 1000(1.045)^t$



TOPIC 2 Using Exponential Equations

- b. Graph the functions for Accounts A and B using technology. Then, graph the functions. Be sure to label your graph.

TEKS A.3C, TEKS A.9D



- c. If Camilla plans on leaving the money in the account for 12 years, which account should she use to deposit her money? Explain your reasoning. TEKS A.9D

She should deposit her money into Account A because there will be more money in it at that time, according to the graph.

- d. If Camilla plans on leaving the money in the account for 25 years, which account should she use to deposit her money? Explain your reasoning. TEKS A.9D

She should deposit her money into Account B because there will be more money in it at that time, according to the graph.

III. Modeling Using Exponential Functions

Topic Practice

- A. For each given data set, use graphing technology to determine the exponential regression function and the value of the correlation coefficient, r . Round all values to the hundredths place.

TEKS A.9E (Topic-Aligned)

1.

x	$f(x)$
10	5
20	6
30	8
40	15
50	32
60	70
70	150

$$f(x) = 1.88(1.06)^x$$

$$r = 0.98$$

2.

x	$f(x)$
0	6000
1	2100
2	750
3	275
4	95
5	40
6	15
7	6
8	4

$$f(x) = 5004.08(0.39)^x$$

$$r = -1.00$$

3.

x	5	10	15	20	25	30	35	40
$f(x)$	12	10	25	21	45	35	80	120

$$f(x) = 6.91(1.07)^x$$

$$r = 0.95$$

4.

x	100	200	300	400	500	600	700
$f(x)$	25.4	10.5	4.5	2.1	0.8	0.3	0.4

$$f(x) = 45.86(0.99)^x$$

$$r = -0.98$$



TOPIC 2 Using Exponential Equations

5.

x	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	1200	585	272	126	42	40	14	12

$$f(x) = 2101.58(0.25)^x$$

$$r = -0.99$$

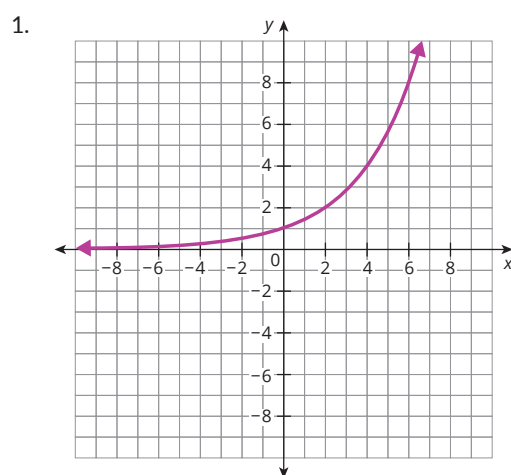
6.

x	0	100	200	300	400	500	600
$f(x)$	10	50	110	160	220	290	350

$$f(x) = 22.37(1.01)^x$$

$$r = 0.92$$

B. Consider the graph of each exponential function or partial exponential function. Write the domain and the range of each in words and using inequalities. **TEKS A.9A (Interleaved Practice)**



Domain in words:

All real numbers

Domain using inequalities:

$$-\infty < x < \infty$$

Range in words:

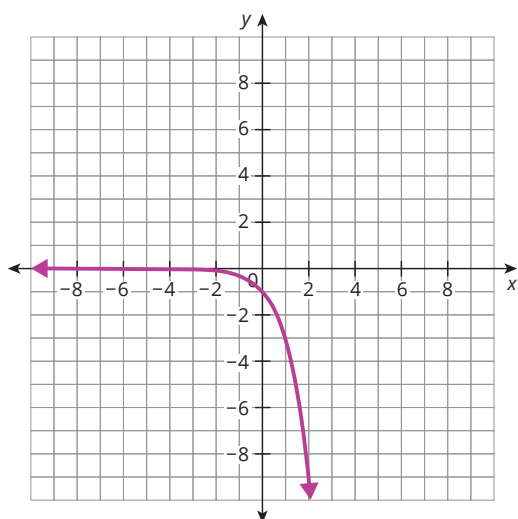
All real numbers greater than 0

Range using inequalities:

$$y > 0$$

TOPIC 2 Using Exponential Equations

2.



Domain in words:

All real numbers

Domain using inequalities:

$-\infty < x < \infty$

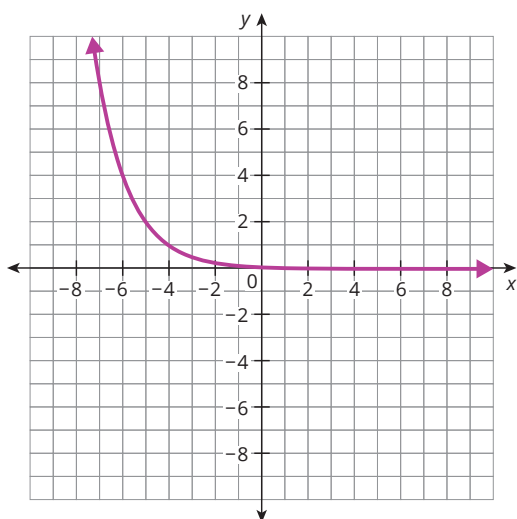
Range in words:

All real numbers less than 0

Range using inequalities:

$y < 0$

3.



Domain in words:

All real numbers

Domain using inequalities:

$-\infty < x < \infty$

Range in words:

All real numbers greater than 0

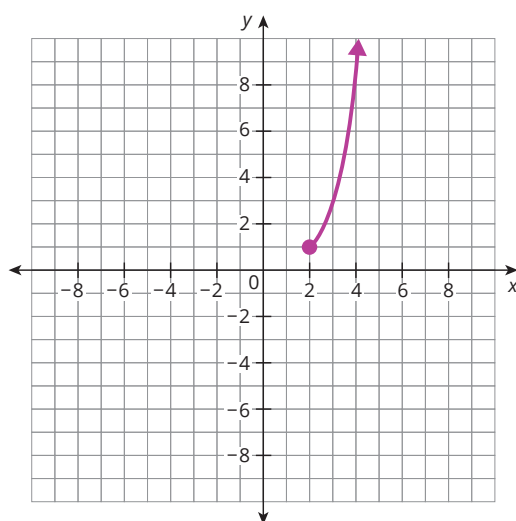
Range using inequalities:

$y > 0$



TOPIC 2 Using Exponential Equations

4.



Domain in words:

All real numbers greater than or equal to 2

Domain using inequalities:

$$x \geq 2$$

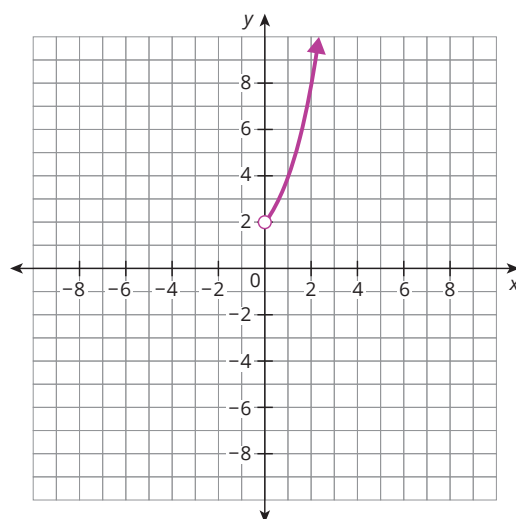
Range in words:

All real numbers greater than or equal to 1

Range using inequalities:

$$y \geq 1$$

5.



Domain in words:

All real numbers greater than 0

Domain using inequalities:

$$x > 0$$

Range in words:

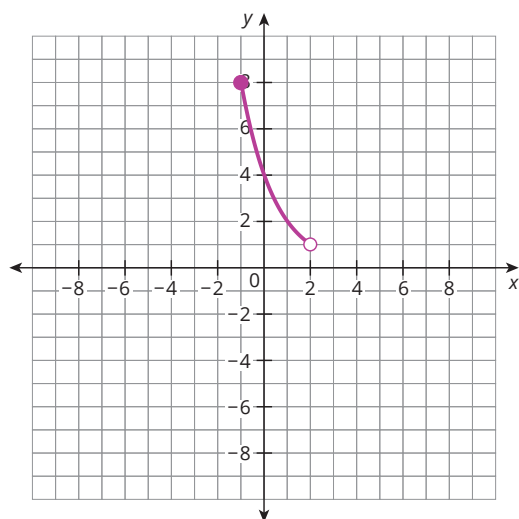
All real numbers greater than 2

Range using inequalities:

$$y > 2$$

TOPIC 2 Using Exponential Equations

6.



Domain in words:

All real numbers greater than or equal to -1 and less than 2

Domain using inequalities:

$$-1 \leq x < 2$$

Range in words:

All real numbers greater than 1 and less than or equal to 8

Range using inequalities:

$$1 < y \leq 8$$

- C. Determine the exponential regression function that models each situation. Use the function to make the associated prediction. Round all values to the hundredths place.

TEKS A.9E (Topic-Aligned)

1. Avery deposited \$500 into a savings account in 1975. The table shows the value of Avery's savings account from 1975 to 2015. Predict the account's value in 2025.

Time Since 1975 (years)	0	5	10	15	20	25	30	35	40
Account Value (dollars)	500	650	900	1150	1600	2100	2750	3850	4800

$f(x) = 497.63(1.06)^x$; The account's value will be approximately \$9166.42 in 2025.

2. Parker deposited \$1000 into a savings account in 1980. The table shows the value of Parker's savings account from 1980 to 2010. Predict when the account's value will be \$5000.

Time Since 1980 (years)	0	5	10	15	20	25	30
Account Value (dollars)	1000	1200	1480	1800	2200	2720	3250

$f(x) = 993.33(1.04)^x$; The account's value will be \$5000 around the year 2021.



TOPIC 2 Using Exponential Equations

3. A marine biologist monitors the population of sunfish in a small lake. He records 800 sunfish in his first year, 600 sunfish in his fourth year, 450 sunfish in his sixth year, and 350 sunfish in his tenth year. Predict the population of sunfish in the lake in his sixteenth year.
4. A marine biologist monitors the population of catfish in a small lake. He records 50 catfish in his first year, 170 catfish in his fourth year, 380 catfish in his sixth year, and 1900 catfish in his tenth year. Predict when the population of catfish in the lake will be 6000.

$$f(x) = 856.83(0.91)^x$$

There will be approximately 189 sunfish in the lake in his sixteenth year.

$$f(x) = 33.57(1.50)^x$$

The catfish population will be 6000 between the 12th and 13th year.

5. Every hour, a scientist records the number of cells in a colony of bacteria growing in her lab. The sample begins with 15 cells. Predict the number of cells in the colony after 7 hours.
6. Every hour, a scientist records the number of cells in a colony of bacteria growing in her lab. The sample begins with 50 cells. Predict how long it will take the sample to grow to 2000 cells.

Hour	Number of Cells
0	15
1	40
2	110
3	300
4	850

$$f(x) = 14.75(2.74)^x$$

There will be approximately 17,102 bacteria cells in the colony after 7 hours.

Hour	Number of Cells
0	50
1	90
2	160
3	290
4	530

$$f(x) = 49.78(1.80)^x$$

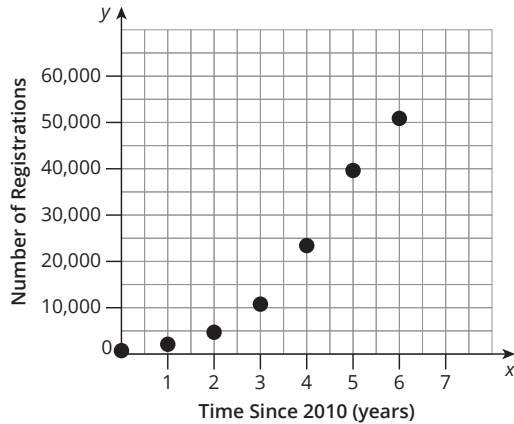
It will take approximately 6.28 hours for the sample to grow to 2000 cells.



TOPIC 2 Using Exponential Equations

- D. The scatterplot shows the registrations of plug-in electric vehicles in Norway over a period of time. The exponential regression function that best fits the data is, $p(x) = 1018.99(2.05)^x$, where $p(x)$ represents the number of registrations and x represents the number of years since 2010. The function is graphed on the grid.

Analyze this information to answer each question. **TEKS A.9A** (Topic-Aligned), **TEKS A.9B** (Topic-Aligned), **TEKS A.9D** (Topic-Aligned)



1. Identify and interpret the a - and b -values of the function in terms of the problem situation.
2. Discuss the domain and range of the function as they relate to the problem situation.

The a -value is 1018.99. This means there were approximately 1019 plug-in electric vehicles in Norway in 2010. The b -value is 2.05. This means that the number of plug-in electric cars is more than doubling every year during the period of time.

The domain is all real numbers greater than or equal to 0 because the function models the number of registrations since 2010 when x is 0. The range is all real numbers greater than or equal to approximately 1019 because the function estimates about 1019 registrations of plug-in vehicles in 2010 and the function increases as the x values increase.



TOPIC 2 Using Exponential Equations

3. Discuss the intervals of increase and decrease as they relate to the problem situation.

The function increases over the entire domain from the year 2010 to infinity.

5. Predict the number of registrations of plug-in vehicles in Norway in 2017.

When $x = 7$, $f(x) \approx 155,041$, so about 155,041 electric plug-in vehicles will be registered in 2017.

4. Discuss the x- and y-intercepts of the function as they relate to the problem situation.

The function has no x-intercepts because it does not cross the x-axis. The function has a y-intercept of approximately 1019, which represents the estimated number of registrations in 2010.

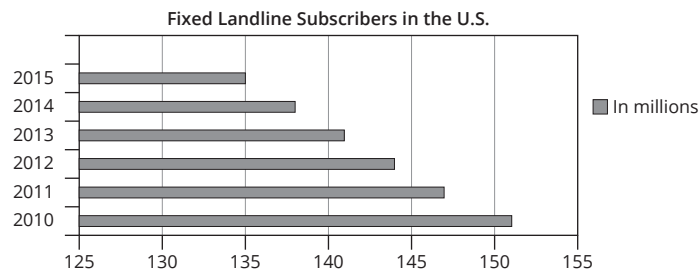
6. Why might the number of registrations of electric plug-in vehicles be increasing exponentially?

Sample answer:

The number of registrations of electric plug-in vehicles in Norway may be increasing exponentially because the citizens are given incentives to drive the vehicles to reduce the carbon footprint of the country on the environment.

Extension

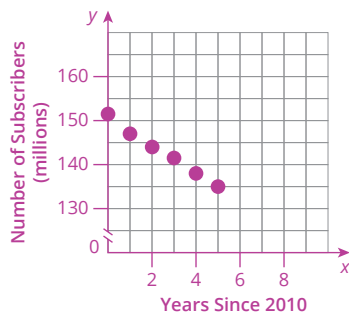
1. The number of fixed landline phone subscribers in the U.S. has been declining. The bar graph shows the decrease in the number of subscribers from 2010 to 2015. **Extension of TEKS A.4C, A.9E**



TOPIC 2 Using Exponential Equations

- a. To estimate the number of subscribers per year, create a scatterplot of the ordered pairs, with x representing the number of years since 2010 and y representing the number of subscribers in millions.

Sample answer:



- b. Determine both an exponential and a linear regression function to model the situation.

Exponential: $f(x) = 150.9(0.98)^x$

Linear: $f(x) = -3.2x + 150.8$

- c. Which model would you use from part (b)? Explain your reasoning.

Sample answer:

Both correlation coefficients round to -0.997 . According to the context, I think the number of subscribers will start to decrease more drastically, so I think the graph will be exponential.

Spaced Review

1. An experiment begins with 400 bacteria. The bacteria population doubles each day. Write an equation in function notation to represent the number of bacteria as a function of the day number, x . Explain how you determined the equation. **TEKS A.9C**

$$f(x) = 400 \cdot 2^x$$

Sample answer:

The number of bacteria can be written as a product of 400 and a power of 2. The exponent x is equal to the day number. So, to get the number of bacteria, I have to multiply 400 times 2 raised to a power equal to the day number.



Maximizing and Minimizing

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TOPIC 1: Introduction to Quadratic Functions

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Name _____ Date _____

I. Exploring Quadratic Functions

Topic Practice

A. Write a quadratic function in standard form that represents each area as a function of the width. Remember to define your variables. **TEKS A.10A (Topic-Aligned), Prerequisite for TEKS A.7A**

1. A builder is designing a rectangular parking lot. She has 300 feet of fencing to enclose the parking lot around three sides.
 Let x = the width of the parking lot.
 The length of the parking lot = $300 - 2x$.
 Let A = the area of the parking lot.
 $A(x) = -2x^2 + 300x$
2. Olivia is enclosing a new rectangular flower garden with a rabbit garden fence. She has 40 meters of fencing.
 Let x = the width of the garden.
 The length of the garden = $20 - x$.
 Let A = the area of the garden.
 $A(x) = -x^2 + 20x$
3. Michael is building a rectangular sandbox for the community park. The materials available limit the perimeter of the sandbox to at most 100 feet.
 Let x = the width of the sandbox.
 The length of the sandbox = $50 - x$.
 Let A = the area of the sandbox.
 $A(x) = -x^2 + 50x$
4. Daniela is designing a rectangular quilt. She has 16 feet of piping to finish the quilt around three sides.
 Let x = the width of the quilt.
 The length of the quilt = $16 - 2x$.
 Let A = the area of the quilt.
 $A(x) = -2x^2 + 16x$
5. Gabriela is making a rectangular vegetable garden alongside her home. She has 24 meters of fencing to enclose the garden around the three open sides.
 Let x = the width of the garden.
 The length of the garden = $24 - 2x$.
 Let A = the area of the garden.
 $A(x) = -2x^2 + 24x$
6. Javier is building a rectangular ice rink for the community park. The materials available limit the perimeter of the ice rink to at most 250 meters.
 Let x = the width of the ice rink.
 The length of the ice rink = $125 - x$.
 Let A = the area of the ice rink.
 $A(x) = -x^2 + 125x$



TOPIC 1 Introduction to Quadratic Functions

- B. Use technology to determine the absolute maximum or absolute minimum of each function. Describe what the x - and y -coordinates of this point represent in terms of the problem situation.**

TEKS A.7A (Topic-Aligned)

1. A builder is designing a rectangular parking lot. He has 400 meters of fencing to enclose the parking lot around three sides. Let x = the width of the parking lot. Let A = the area of the parking lot. The function $A(x) = -2x^2 + 400x$ represents the area of the parking lot as a function of the width.
2. Hannah is enclosing a portion of her yard to make a pen for her ferrets. She has 20 feet of fencing. Let x = the width of the pen. Let A = the area of the pen. The function $A(x) = -x^2 + 10x$ represents the area of the pen as a function of the width.

The absolute maximum of the function is at (100, 20,000). The x -coordinate of 100 represents the width in meters that produces the maximum area. The y -coordinate of 20,000 represents the maximum area in square meters of the parking lot.

The absolute maximum of the function is at (5, 25). The x -coordinate of 5 represents the width in feet that produces the maximum area. The y -coordinate of 25 represents the maximum area in square feet of the pen.

3. A baseball is thrown upward from a height of 5 feet with an initial velocity of 42 feet per second. Let t = the time in seconds after the baseball is thrown. Let h = the height of the baseball. The quadratic function $h(t) = -16t^2 + 42t + 5$ represents the height of the baseball as a function of time.
4. Fernando is standing on top of a playground set at a park. He throws a water balloon upward from a height of 12 feet with an initial velocity of 25 feet per second.

Let t = the time in seconds after the balloon is thrown. Let h = the height of the balloon. The quadratic function $h(t) = -16t^2 + 25t + 12$ represents the height of the balloon as a function of time.

The absolute maximum of the function is at about (1.31, 32.56). The x -coordinate of 1.31 represents the time in seconds after the baseball is thrown that produces the maximum height. The y -coordinate of 32.56 represents the maximum height in feet of the baseball.

The absolute maximum of the function is at about (0.78, 21.77). The x -coordinate of 0.78 represents the time in seconds after the balloon is thrown that produces the maximum height. The y -coordinate of 21.77 represents the maximum height in feet of the balloon.



TOPIC 1 Introduction to Quadratic Functions

5. Alexander is building a rectangular roller-skating rink at the community park. The materials available limit the perimeter of the skating rink to at most 180 meters. Let x = the width of the skating rink. Let A = the area of the skating rink. The function $A(x) = -x^2 + 90x$ represents the area of the skating rink as a function of the width.

The absolute maximum of the function is at (45, 2025). The x -coordinate of 45 represents the width in meters that produces the maximum area. The y -coordinate of 2025 represents the maximum area in square meters of the skating rink.

7. A bungee jumper is jumping from a height of 150 meters. Let t = the time since the start of the jump. Let h = the height of the jumper from the ground. The quadratic function $h(t) = 2t^2 - 30t + 150$ represents the height of the jumper as a function of time.

The absolute minimum of the function is about (7.5, 37.5). The x -coordinate of 7.5 represents the time in seconds when the jumper reaches the minimum height. The y -coordinate of 37.5 represents the minimum height in meters of the jumper.

6. A football is thrown upward from a height of 6 feet with an initial velocity of 65 feet per second. Let t = the time in seconds after the football is thrown. Let h = the height of the football. The quadratic function $h(t) = -16t^2 + 65t + 6$ represents the height of the football as a function of time.

The absolute maximum of the function is at about (2.03, 72.02). The x -coordinate of 2.03 represents the time in seconds after the football is thrown that produces the maximum height. The y -coordinate of 72.02 represents the maximum height in feet of the football.

8. A skateboard company has to decide how many skateboards to produce each day. The company knows that the costs to produce the skateboards go down the more they make. However, the overall cost to the company increases if they make too many skateboards due to the cost of storing overstock. Let s = the number of skateboards produced each day. Let c = the total cost to produce s skateboards. The quadratic function $c(s) = 0.04s^2 - 16s + 15,000$ represents the total cost as a function of the number of skateboards produced.

The absolute minimum of the function is (200, 13,400). The x -coordinate of 200 represents the number of skateboards produced when the cost is minimized. The y -coordinate of 13,400 represents the minimum cost of producing the skateboards.

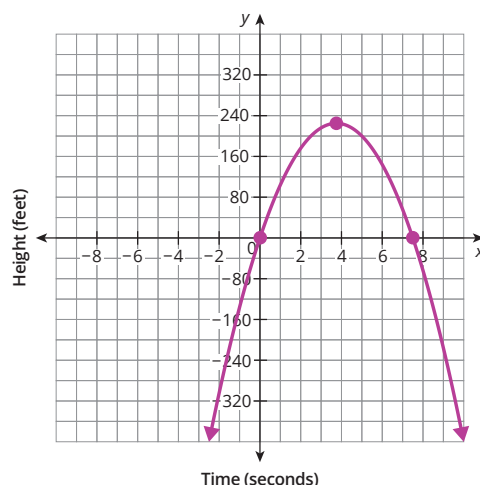


TOPIC 1 Introduction to Quadratic Functions

- C. Graph the function that represents each problem situation. Identify the absolute maximum and zeros. Then, identify the domain and range of both the function $g(t)$ and the problem situation. Round your answers to the nearest hundredth, if necessary.

TEKS A.6A (Topic-Aligned), TEKS A.7A (Interleaved Practice)

1. A model rocket is launched from the ground with an initial velocity of 120 feet per second. The function $g(t) = -16t^2 + 120t$ represents the height of the rocket, $g(t)$, t seconds after it was launched.



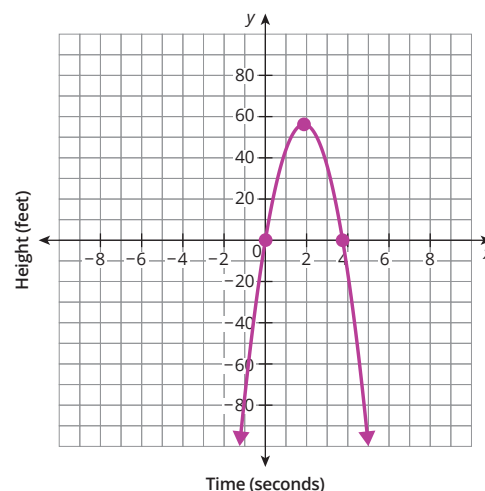
Absolute maximum: (3.75, 225)

Zeros: (0, 0), (7.5, 0)

The domain of $g(t)$ is all real numbers. The domain of the problem situation is all real numbers greater than or equal to 0 and less than or equal to 7.5.

The range of $g(t)$ is all real numbers less than or equal to 225. The range of the problem situation is all real numbers less than or equal to 225 and greater than or equal to 0.

2. A model rocket is launched from the ground with an initial velocity of 60 feet per second. The function $g(t) = -16t^2 + 60t$ represents the height of the rocket, $g(t)$, t seconds after it was launched.



Absolute maximum: (1.875, 56.25)

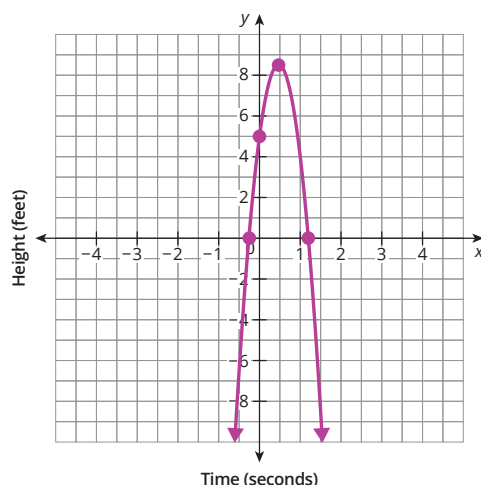
Zeros: (0, 0), (3.75, 0)

The domain of $g(t)$ is all real numbers. The domain of the problem situation is all real numbers greater than or equal to 0 and less than or equal to 3.75.

The range of $g(t)$ is all real numbers less than or equal to 56.25. The range of the problem situation is all real numbers less than or equal to 56.25 and greater than or equal to 0.

TOPIC 1 Introduction to Quadratic Functions

3. A baseball is thrown in the air from a height of 5 feet with an initial vertical velocity of 15 feet per second. The function $g(t) = -16t^2 + 15t + 5$ represents the height of the baseball, $g(t)$, t seconds after it was launched.



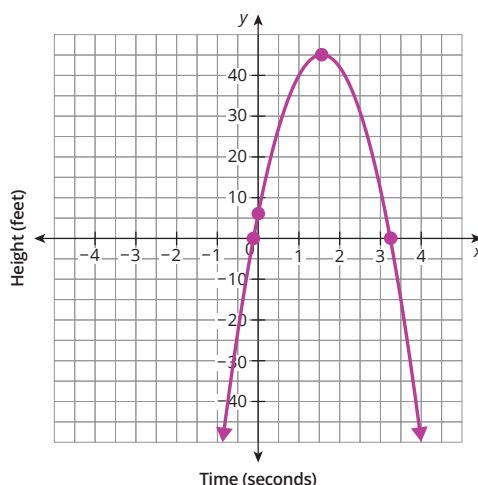
Absolute maximum: (0.47, 8.52)

Zeros: $(-0.26, 0)$, $(1.20, 0)$

The domain of $g(t)$ is all real numbers. The domain of the problem situation is all real numbers greater than or equal to 0 and less than or equal to 1.20.

The range of $g(t)$ is all real numbers less than or equal to 8.52. The range of the problem situation is all real numbers less than or equal to 8.52 and greater than or equal to 0.

4. A football is thrown in the air from a height of 6 feet with an initial vertical velocity of 50 feet per second. The function $g(t) = -16t^2 + 50t + 6$ represents the height of the football, $g(t)$, t seconds after it was launched.



Absolute maximum: (1.56, 45.06)

Zeros: $(-0.12, 0)$, $(3.24, 0)$

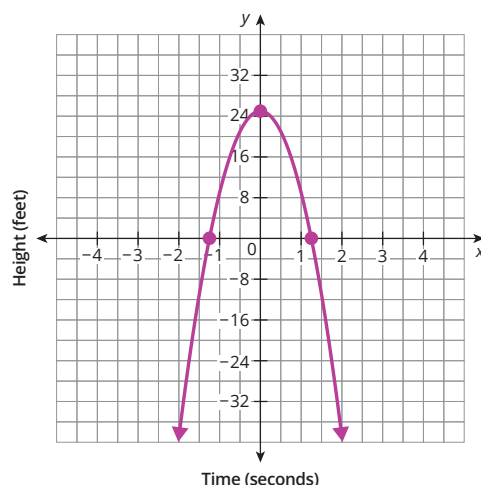
The domain of $g(t)$ is all real numbers. The domain of the problem situation is all real numbers greater than or equal to 0 and less than or equal to 3.24.

The range of $g(t)$ is all real numbers less than or equal to 45.06. The range of the problem situation is all real numbers less than or equal to 45.06 and greater than or equal to 0.



TOPIC 1 Introduction to Quadratic Functions

5. A tennis ball is dropped from a height of 25 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t) = -16t^2 + 25$ represents the height of the tennis ball, $g(t)$, t seconds after it was dropped.



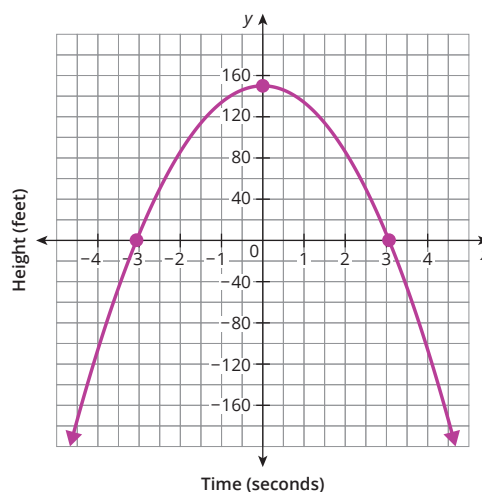
Absolute maximum: (0, 25)

Zeros: $(-1.25, 0)$, $(1.25, 0)$

The domain of $g(t)$ is all real numbers. The domain of the problem situation is all real numbers greater than or equal to 0 and less than or equal to 1.25.

The range of $g(t)$ is all real numbers less than or equal to 25. The range of the problem situation is all real numbers less than or equal to 25 and greater than or equal to 0.

6. A tennis ball is dropped from a height of 150 feet. The initial velocity of an object that is dropped is 0 feet per second. The function $g(t) = -16t^2 + 150$ represents the height of the tennis ball, $g(t)$, t seconds after it was dropped.



Absolute maximum: (0, 150)

Zeros: $(-3.06, 0)$, $(3.06, 0)$

The domain of $g(t)$ is all real numbers. The domain of the problem situation is all real numbers greater than or equal to 0 and less than or equal to 3.06.

The range of $g(t)$ is all real numbers less than or equal to 150. The range of the problem situation is all real numbers less than or equal to 150 and greater than or equal to 0.

TOPIC 1 Introduction to Quadratic Functions

D. Use technology to identify the vertex and the equation of the axis of symmetry for each vertical motion model. **TEKS A.7A (Topic-Aligned)**

1. A catapult hurls a grapefruit from a height of 24 feet at an initial velocity of 80 feet per second. The function $h(t) = -16t^2 + 80t + 24$ represents the height of the grapefruit, $h(t)$, in terms of time, t .

The vertex of the graph is (2.5, 124). The axis of symmetry is $x = 2.5$.
2. A catapult hurls a pumpkin from a height of 32 feet at an initial velocity of 96 feet per second. The function $h(t) = -16t^2 + 96t + 32$ represents the height of the pumpkin, $h(t)$, in terms of time, t .

The vertex of the graph is (3, 176). The axis of symmetry is $x = 3$.
3. A catapult hurls a watermelon from a height of 40 feet at an initial velocity of 64 feet per second. The function $h(t) = -16t^2 + 64t + 40$ represents the height of the watermelon, $h(t)$, in terms of time, t .

The vertex of the graph is (2, 104). The axis of symmetry is $x = 2$.
4. A baseball is thrown from a height of 6 feet at an initial velocity of 32 feet per second. The function $h(t) = -16t^2 + 32t + 6$ represents the height of the baseball, $h(t)$, in terms of time, t .

The vertex of the graph is (1, 22). The axis of symmetry is $x = 1$.
5. A softball is thrown from a height of 20 feet at an initial velocity of 48 feet per second. The function $h(t) = -16t^2 + 48t + 20$ represents the height of the softball, $h(t)$, in terms of time, t .

The vertex of the graph is (1.5, 56). The axis of symmetry is $x = 1.5$.
6. A rocket is launched from the ground at an initial velocity of 112 feet per second. The function $h(t) = -16t^2 + 112t$ represents the height of the rocket, $h(t)$, in terms of time, t .

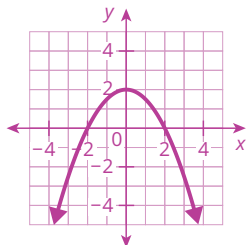
The vertex of the graph is (3.5, 196). The axis of symmetry is $x = 3.5$.



TOPIC 1 Introduction to Quadratic Functions

Extension

1. Sketch a graph of a quadratic function that has a maximum value of $(0, 2)$ and x -intercepts when $x = \pm 2$. **Extension of TEKS A.7A**



2. What is the quadratic function of your graph? Explain your reasoning. **Extension of TEKS A.7A**

$$f(x) = -\frac{1}{2}x^2 + 2$$

Sample answer:

Because the x -intercepts are at 2 and -2 , the equation must be $f(x) = a(x - 2)(x + 2)$.

Using the y -intercept of $(0, 2)$, I can substitute: $2 = a(0 - 2)(0 + 2)$.

Therefore, $a = -\frac{1}{2}$. So, the equation is $f(x) = (-\frac{1}{2})(x - 2)(x + 2)$, or $f(x) = (-\frac{1}{2})x^2 + 2$.

Spaced Practice

1. If the parent function $f(x) = x$ is translated 3 units to the right and 4 units up, what is the transformed equation? **TEKS A.3E**
2. Alexander is buying bagels. Each bagel costs \$2. Let b represent the number of bagels and c represent the total cost. Does this situation represent a function? Explain your reasoning. **TEKS A.12A**

$$g(x) = (x - 3) + 4 \text{ or } g(x) = x + 1$$

Yes, the situation represents a function. The number of bagels Alexander buys has only one cost. So, each input has only one output.

II. Key Characteristics of Quadratic Functions

Topic Practice

- A. Calculate the first and second differences for each table of values. Describe the type of function represented by the table.

Prerequisite for TEKS A.7A

1.

x	y	First Differences	Second Differences
-2	-6		
-1	-3	3	
0	0	3	0
1	3	3	0
2	6	3	0

The function represented by the table is a linear function.

2.

x	y	First Differences	Second Differences
-2	12		
-1	3	-9	
0	0	-3	6
1	3	3	6
2	12	9	6

The function represented by the table is a quadratic function.

3.

x	y	First Differences	Second Differences
-3	3		
-2	4	1	
-1	5	1	0
0	6	1	0
1	7	1	0

The function represented by the table is a linear function.

4.

x	y	First Differences	Second Differences
-1	1		
0	0	-1	
1	3	3	4
2	10	7	4
3	21	11	4

The function represented by the table is a quadratic function.



TOPIC 1 Introduction to Quadratic Functions

5.

x	y	First Differences	Second Differences
-4	-48	21	-6
-3	-27	15	-6
-2	-12	9	-6
-1	-3	3	-6
0	0		

The function represented by the table is a quadratic function.

6.

x	y	First Differences	Second Differences
-1	10	-2	0
0	8	-2	0
1	6	-2	0
2	4	-2	0
3	2		

The function represented by the table is a linear function.

B. Determine the x-intercepts and axis of symmetry of each quadratic function in factored form. TEKS A.7A (Topic-Aligned), TEKS A.7B (Interleaved Practice)

1. $f(x) = (x - 2)(x - 8)$

The x-intercepts are (2, 0) and (8, 0).
The axis of symmetry is located at $x = 5$.

2. $f(x) = (x + 1)(x - 6)$

The x-intercepts are (-1, 0) and (6, 0).
The axis of symmetry is located at $x = 2.5$.

3. $f(x) = 3(x + 4)(x - 2)$

The x-intercepts are (-4, 0) and (2, 0).
The axis of symmetry is located at $x = -1$.

4. $f(x) = 0.25(x - 1)(x - 12)$

The x-intercepts are (1, 0) and (12, 0).
The axis of symmetry is located at $x = 6.5$.

5. $f(x) = 0.5(x + 15)(x + 5)$

The x-intercepts are (-15, 0) and (-5, 0).
The axis of symmetry is located at $x = -10$.

6. $f(x) = 4(x - 1)(x - 9)$

The x-intercepts are (1, 0) and (9, 0).
The axis of symmetry is located at $x = 5$.

TOPIC 1 Introduction to Quadratic Functions

C. Determine the vertex of each parabola. TEKS A.7A (Topic-Aligned)

1. $f(x) = x^2 + 2x - 15$

Axis of symmetry: $x = -1$

The vertex is $(-1, -16)$.

3. $f(x) = x^2 + 4x - 12$

x-intercepts: $(2, 0)$ and $(-6, 0)$

The vertex is $(-2, -16)$.

5. $f(x) = -x^2 + 8x + 20$

Two symmetric points on the parabola:
 $(-1, 11)$ and $(9, 11)$

The vertex is $(4, 36)$.

2. $f(x) = x^2 - 8x + 7$

Axis of symmetry: $x = 4$

The vertex is $(4, -9)$.

4. $f(x) = -x^2 - 14x - 45$

x-intercepts: $(-9, 0)$ and $(-5, 0)$

The vertex is $(-7, 4)$.

6. $f(x) = -x^2 + 16$

Two symmetric points on the parabola:
 $(-3, 7)$ and $(3, 7)$

The vertex is $(0, 16)$.

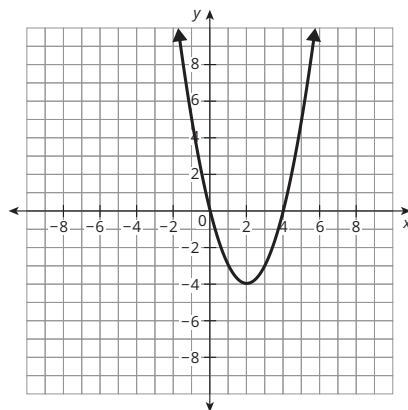


TOPIC 1 Introduction to Quadratic Functions

D. For each graph, identify the domain and range. Then, write an equation for the function in factored form.

TEKS A.6A (Interleaved Practice), TEKS A.6C (Topic-Aligned)

1.



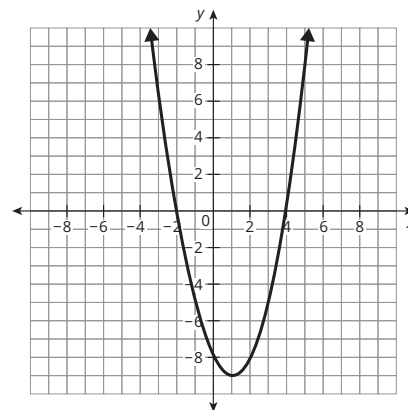
Domain: all real numbers

Range: $f(x) \geq -4$

Equation in factored form:

$$f(x) = x(x - 4)$$

2.



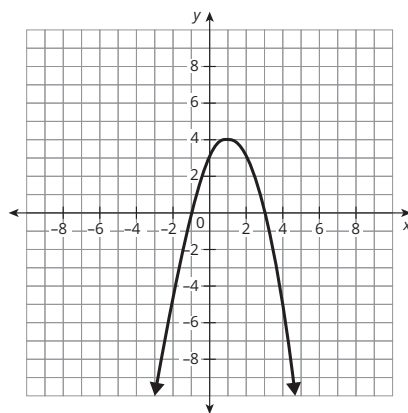
Domain: all real numbers

Range: $f(x) \geq -9$

Equation in factored form:

$$f(x) = (x + 2)(x - 4)$$

3.



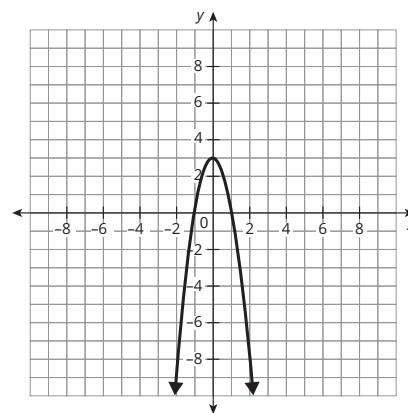
Domain: all real numbers

Range: $f(x) \leq 4$

Equation in factored form:

$$f(x) = -(x + 1)(x - 3)$$

4.



Domain: all real numbers

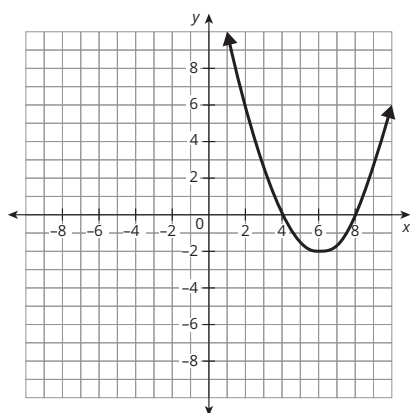
Range: $f(x) \leq 3$

Equation in factored form:

$$f(x) = -3(x - 1)(x + 1)$$

TOPIC 1 Introduction to Quadratic Functions

5.



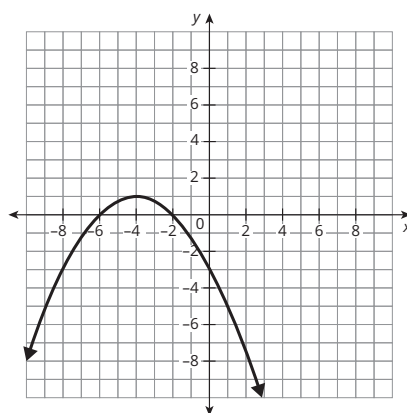
Domain: all real numbers

Range: $f(x) \geq -2$

Equation in factored form:

$$f(x) = \frac{1}{2}(x - 4)(x - 8)$$

6.



Domain: all real numbers

Range: $f(x) \leq 1$

Equation in factored form:

$$f(x) = -\frac{1}{4}(x + 6)(x + 2)$$

E. Determine the x-intercepts for each function using technology.

Write the function in factored form. A.7A (Topic-Aligned),

TEKS A.6C (Interleaved Practice)

1. $f(x) = x^2 - 8x + 7$

x-intercepts: (1, 0) and (7, 0)

Factored form: $f(x) = (x - 1)(x - 7)$

2. $f(x) = 2x^2 - 10x - 48$

x-intercepts: (-3, 0) and (8, 0)

Factored form: $f(x) = 2(x + 3)(x - 8)$

3. $f(x) = -x^2 - 20x - 75$

x-intercepts: (-5, 0) and (-15, 0)

Factored form: $f(x) = -(x + 5)(x + 15)$

4. $f(x) = x^2 + 8x + 12$

x-intercepts: (-2, 0) and (-6, 0)

Factored form: $f(x) = (x + 2)(x + 6)$

5. $f(x) = -3x^2 - 9x + 12$

x-intercepts: (-4, 0) and (1, 0)

Factored form: $f(x) = -3(x + 4)(x - 1)$

6. $f(x) = x^2 - 6x$

x-intercepts: (6, 0) and (0, 0)

Factored form: $f(x) = x(x - 6)$



TOPIC 1 Introduction to Quadratic Functions

F. Determine another point on each parabola. TEKS A.7A (Topic-Aligned)

1. The axis of symmetry is $x = 3$. A point on the parabola is $(1, 4)$.

Another point on the parabola is $(5, 4)$.

3. The axis of symmetry is $x = 1$. A point on the parabola is $(-3, 2)$.

Another point on the parabola is $(5, 2)$.

5. The vertex is $(-1, 6)$. A point on the parabola is $(2, 3)$.

Another point on the parabola is $(-4, 3)$.

2. The axis of symmetry is $x = -4$. A point on the parabola is $(0, 6)$.

Another point on the parabola is $(-8, 6)$.

4. The vertex is $(5, 2)$. A point on the parabola is $(3, -1)$.

Another point on the parabola is $(7, -1)$.

6. The vertex is $(3, -1)$. A point on the parabola is $(4, 1)$.

Another point on the parabola is $(2, 1)$.



TOPIC 1 Introduction to Quadratic Functions

Extension

1. Sketch the graph $f(x) = -3x^2 - 4$. How could you change the quadratic function to make the graph open upward? Show the change on the graph. **Extension of TEKS A.7C**

I would change the a -value to a positive number. Check students' graphs.

2. How could you change the quadratic function $f(x) = -3x^2 - 4$ to shift the graph up or down? Show on the graph. **Extension of TEKS A.7C**

I could increase the d -value, -4 , to shift it up or decrease it to shift it down. Check students' graphs.

3. How could you change the quadratic function $f(x) = -3x^2 - 4$ to shift the graph right or left? Show the change on the graph.

Extension of TEKS A.7C

I could change the c -value, 0 , to shift it right or left.

Spaced Practice

1. A camp wants to create a larger space for their albino rabbit, Clover. They want to reuse the materials from Clover's current enclosure in the construction of a new rectangular enclosure. The perimeter of Clover's current space is 6 feet. The perimeter of his new enclosure will be 3 times larger than his former enclosure.

- a. What is the area of the new enclosure $A(w)$ in terms of width, w ? **TEKS A.12E**

$$A(w) = -w^2 + 9w$$

- b. What is the maximum area of the new enclosure? What are the dimensions? **TEKS A.7A**

The maximum area is 20.25 square feet. The length and width are both 4.5 feet.

2. Is $7x^{2t} \cdot 5x^{2t}$ equivalent to $35x^{2t}$? Justify your answer. **TEKS A.11B**

No. The exponents need to be added together.

3. Is $(16^{3z})^{6y}$ equivalent to 16^{18yz} ? Justify your answer. **TEKS A.11B**

Yes. The exponents have been properly multiplied together.



TOPIC 1 Introduction to Quadratic Functions

III. Quadratic Function Transformations

Topic Practice

- A. Describe the transformation(s) necessary to transform the graph of the function $f(x) = x^2$ into the graph of each function $g(x)$.

TEKS A.7C (Topic-Aligned)

1. $g(x) = x^2 - 5$

The graph of $f(x)$ is translated down 5 units.

3. $g(x) = x^2 + 2$

The graph of $f(x)$ is translated vertically up 2 units.

5. $g(x) = 3x^2$

The graph of $f(x)$ is dilated/stretched vertically by a factor of 3.

7. $g(x) = \frac{1}{2}f(x) - 1$

The graph of $f(x)$ dilated/compressed vertically by a factor of $\frac{1}{2}$ and translated down 1 unit.

9. $g(x) = f(x + 2) - 3$

The graph of $f(x)$ is translated left 2 units and down 3 units.

11. $g(x) = f(x + 1)$

The graph of $f(x)$ is translated left 1 unit.

2. $g(x) = -x^2$

The graph of $f(x)$ has been reflected over the x-axis.

4. $g(x) = (x + 4)^2$

The graph of $f(x)$ has been translated horizontally to the left 4 units.

6. $g(x) = 2x^2 - 5$

The graph of $f(x)$ has been dilated/stretched vertically by a factor of 2 and translated vertically down 5 units.

8. $g(x) = f(x - 8)$

The graph of $f(x)$ is translated right 8 units.

10. $g(x) = -(f(x) + 2)$

The graph of $f(x)$ is translated up two units and then reflected across the x-axis.

12. $g(x) = -f(x) - 5$

The graph of $f(x)$ is reflected over the x-axis and then translated down 5 units.



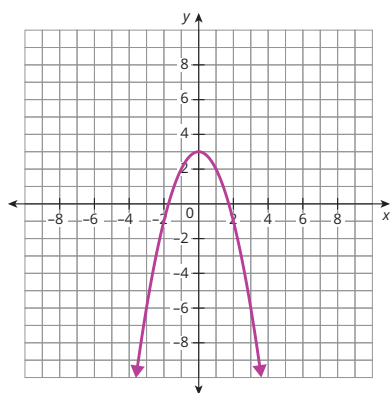
TOPIC 1 Introduction to Quadratic Functions

B. Write an equation and sketch a graph of a function $g(x)$ for each transformation of $f(x) = x^2$. **TEKS A.7A (Topic-Aligned),**

TEKS A.7C (Interleaved Practice)

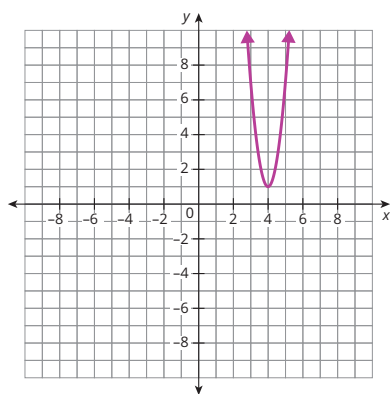
1. The graph of $g(x)$ can be created by reflecting $f(x)$ across the x -axis and then translating it 3 units up.

$$g(x) = -x^2 + 3$$



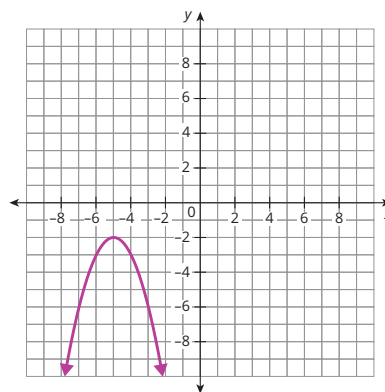
3. The graph of $g(x)$ can be created by translating $f(x)$ to the right 4 units, stretching it vertically by a factor of 6, and then translating it 1 unit up.

$$g(x) = 6(x - 4)^2 + 1$$



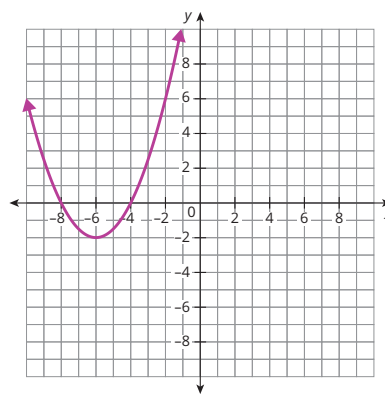
2. The graph of $g(x)$ can be created by translating $f(x)$ to the left 5 units, reflecting it across the x -axis, and then translating it 2 units down.

$$g(x) = -(x + 5)^2 - 2$$



4. The graph of $g(x)$ can be created by translating $f(x)$ to the left 6 units, compressing it vertically by a factor of 0.5, and then translating it 2 units down.

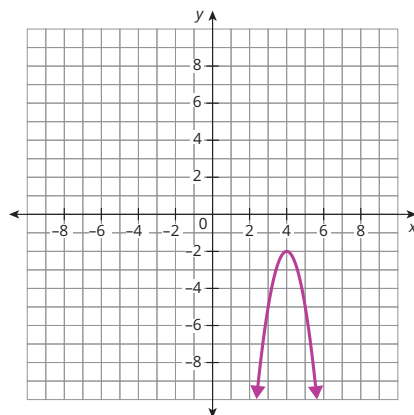
$$g(x) = \frac{1}{2}(x + 6)^2 - 2$$



TOPIC 1 Introduction to Quadratic Functions

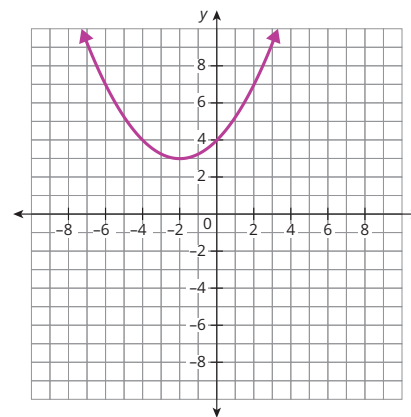
5. The graph of $g(x)$ can be created by translating $f(x)$ to the right 4 units, reflecting it across the x -axis, dilating it vertically by a factor of 3, and then translating it 2 units down.

$$g(x) = -3(x - 4)^2 - 2$$



6. The graph of $g(x)$ can be created by translating $f(x)$ to the left 2 units, dilating it vertically by a factor of $\frac{1}{2}$, and then translating it 3 units up.

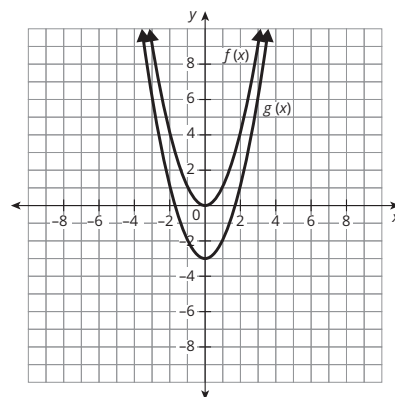
$$g(x) = \frac{1}{4}(x + 2)^2 + 3$$



- C. Describe the transformation(s) that would be required to transform $f(x)$ into $g(x)$. Then, write a function for $g(x)$ in terms of $f(x)$.

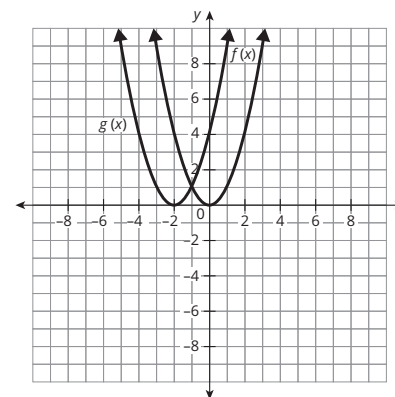
TEKS A.7C (Topic-Aligned)

1.



The graph of $f(x)$ is translated vertically down 3 units to create $g(x)$; $g(x) = f(x) - 3$.

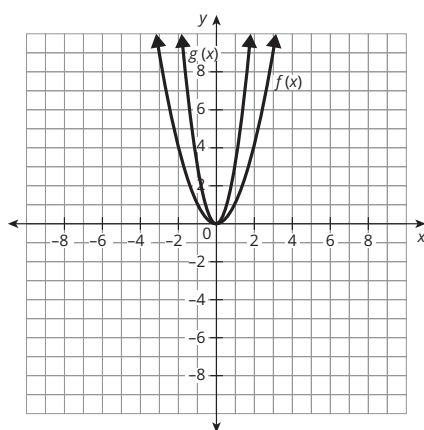
2.



The graph of $f(x)$ is translated horizontally to the left 2 units to create $g(x)$; $g(x) = f(x + 2)$.

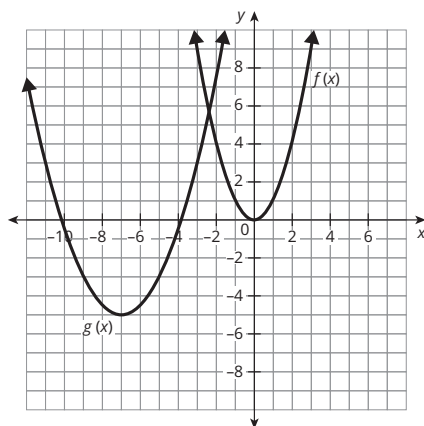
TOPIC 1 Introduction to Quadratic Functions

3.



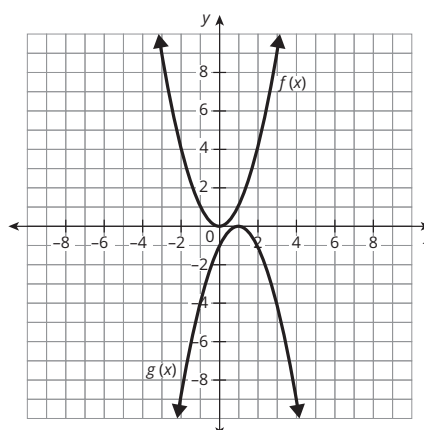
The graph of $f(x)$ is dilated/stretched vertically by a factor of 3 to create $g(x)$, which is narrower than $f(x)$.
 $g(x) = 3f(x)$

5.



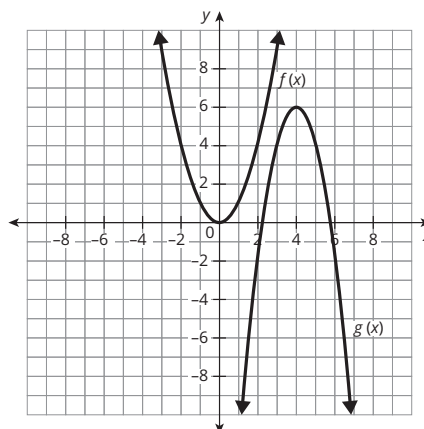
The graph of $f(x)$ is translated vertically down 5 units, translated horizontally left 7 units, and vertically dilated/compressed by a factor of $\frac{1}{2}$ to become $g(x)$, which is wider than $f(x)$.
 $g(x) = \frac{1}{2}f(x + 7) - 5$

4.



The graph of $f(x)$ is reflected across the x-axis and translated horizontally to the right 1 unit to create $g(x)$.
 $g(x) = -f(x - 1)$

6.



The graph of $f(x)$ is reflected across the x-axis, dilated/stretched vertically by a factor of 2, translated to the right 4 units, and translated up 6 units to create $g(x)$, which is narrower than $f(x)$.
 $g(x) = -2f(x - 4) + 6$



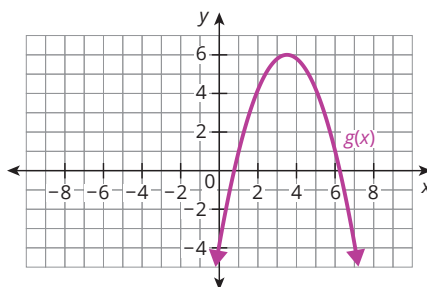
TOPIC 1 Introduction to Quadratic Functions

Extension

The function $g(x)$ is a transformation of $f(x) = x^2$.

Write the function $g(x)$. **Extension of TEKS A.7C**

$$g(x) = -\frac{4}{5}\left(x - \frac{7}{2}\right)^2 + 6$$



Spaced Practice

1. A construction company has plans for two models of the homes they build, Model A and Model B. The Model A home requires 18 single windows and 3 double windows. The Model B home requires 20 single windows and 5 double windows. A total of 1800 single windows and 375 double windows have been ordered for the developments. **TEKS A.5C**

- a. Write and solve a system of equations to represent this situation. Define your variables.

Let x be the number of Model A homes and y be the number of Model B homes.

$$\begin{cases} 18x + 20y = 1,800 \\ 3x + 5y = 375 \end{cases}$$

- b. Interpret the solution of the linear system in terms of the problem situation.

The solution: (50, 45) means that each development will have 50 Model A homes and 45 Model B homes.



TOPIC 1 Introduction to Quadratic Functions

2. A company produces two types of TV stands. Type I has 6 drawers. It requires 3 single drawer pulls and 3 double drawer pulls. The company needs 75 hours of labor to produce the Type I TV stand. Type II has 3 drawers. It requires 6 single drawer pulls. The company needs 50 hours of labor to produce the Type II TV stand. The company only has 600 labor hours available each week and a total of 60 single drawer pulls available in a week. For each Type I stand produced and sold, the company makes \$200 in profit. For each Type II stand produced and sold, the company makes \$150 in profit.

- a. Identify the constraints as a system of linear inequalities. Let x represent the number of 6 drawer TV stands produced, and let y represent the number of 3 drawer TV stands produced. **TEKS A.2H**

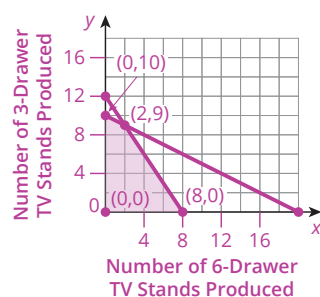
$$3x + 6y \leq 60$$

$$75x + 50y \leq 600$$

$$x \geq 0$$

$$y \geq 0$$

- b. Graph the solution set for the system of linear inequalities. Label all points of the intersection of the boundary lines. **TEKS A.3H**



- c. Write an equation in standard form for the profit, P , that the company can make. **TEKS A.2B**

$$P = 200x + 150y$$

- d. How many of each type of stand should the company make if they want to maximize their profit? What is the maximum profit? **TEKS A.3H**

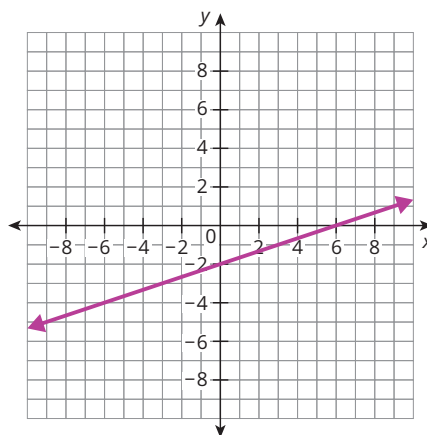
The company should produce 2 six-drawer TV stands and 9 three-drawer TV stands. The maximum profit is \$1750.



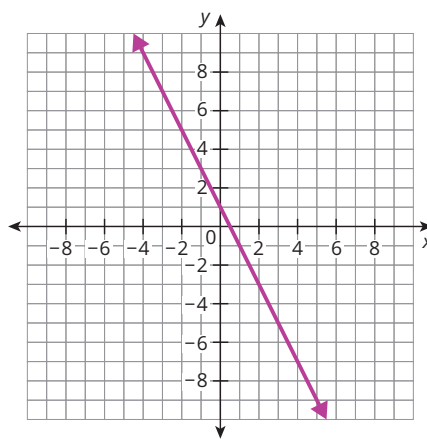
TOPIC 1 Introduction to Quadratic Functions

3. Each function is a transformation of the linear parent function $f(x) = x$. Graph each transformation. **TEKS A.7C**

a. $g(x) = \frac{1}{3}x - 2$



b. $h(x) = -2x + 1$



IV. Transformations of Quadratic Functions

Topic Practice

- A. Describe the transformation(s) necessary to transform the graph of the function $f(x) = x^2$ into the graph of each function $g(x)$.

TEKS A.7C (Topic-Aligned)

1. $g(x) = (x + 2)^2 - 7$

The graph of $f(x)$ is translated left 2 units and down 7 units.

3. $g(x) = (x + 3)^2 + 8$

The graph of $f(x)$ is translated left 3 units and up 8 units.

5. $g(x) = (x - 5)^2 - 6$

The graph of $f(x)$ is translated right 5 units and down 6 units.

7. $g(x) = f(2(x - 1)) + 3$

The graph of $f(x)$ is horizontally stretched by a factor of 2, translated right 1 unit, and translated up 3 units.

9. $g(x) = -\frac{1}{2}f(x - 8) - 4$

The graph of $f(x)$ is reflected across the x-axis, vertically dilated/compressed by a factor of $\frac{1}{2}$, translated right 8 units, and translated down 4 units.

11. $g(x) = -\frac{3}{5}f(x) - 10$

The graph of $f(x)$ is reflected across x-axis, vertically dilated/compressed by a factor of $\frac{3}{5}$, and translated up 6 units.

2. $g(x) = -(x - 2)^2 + 5$

The graph of $f(x)$ is translated to the right 2 units, reflected across the x-axis, and then translated up 5 units.

4. $g(x) = \frac{2}{3}(-x)^2 + 4$

The graph of $f(x)$ is dilated/compressed vertically by a factor of $\frac{2}{3}$, reflected across the y-axis, and translated up 4 units.

6. $g(x) = -(5x^2 - 7)$

The graph of $f(x)$ is dilated/stretched vertically by a factor of 5, translated to the left 7 units, and reflected across the x-axis.

8. $g(x) = f(-3(x + 4)) - 6$

The graph of $f(x)$ is reflected across the y-axis, horizontally compressed by a factor of 3, translated left 4 units, and translated down 6 units.

10. $g(x) = f(-x - 5) + 1$

The graph of $f(x)$ is reflected over the y-axis, translated left 5 units, and translated up 1 unit.

12. $g(x) = f\left(\frac{1}{4}x + \frac{3}{4}\right) + 9$

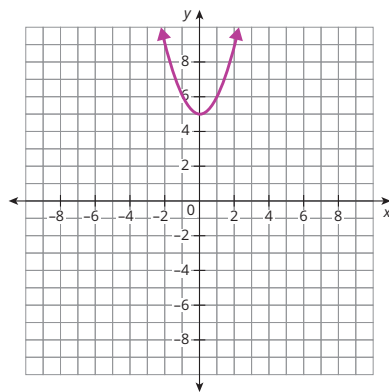
The graph of $f(x)$ is horizontally dilated/stretched by a factor of 4, translated left 3 units, and translated up 9 units.



TOPIC 1 Introduction to Quadratic Functions

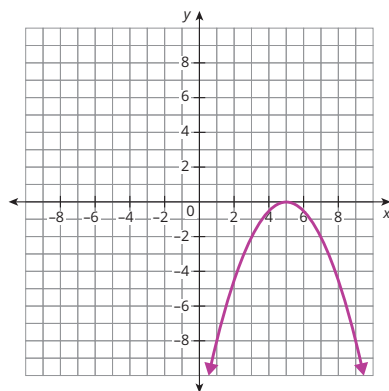
B. Write an equation and sketch a graph of a function $g(x)$ for each transformation of $f(x) = x^2$. TEKS A.7C (Topic-Aligned)

1. You can create the graph of $g(x)$ by reflecting $f(x)$ over the y -axis and then translating it 5 units up.



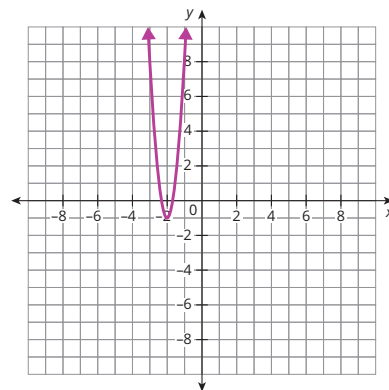
$$g(x) = (-x)^2 + 5$$

3. You can create the graph of $g(x)$ by vertically dilating $f(x)$ by a factor of 0.5, translating it right 5 units, and reflecting it over the x -axis.



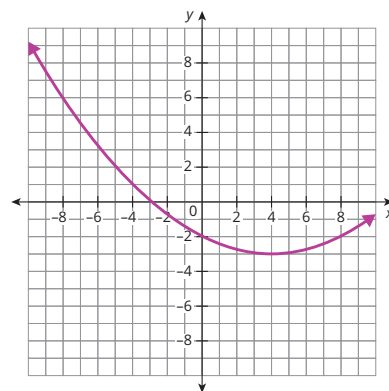
$$g(x) = -0.5(x - 5)^2$$

2. You can create the graph of $g(x)$ by horizontally compressing $f(x)$ by a factor of $\frac{1}{3}$, translating it 2 units to the left, and translating it 1 unit down.



$$g(x) = (3(x + 2))^2 - 1$$

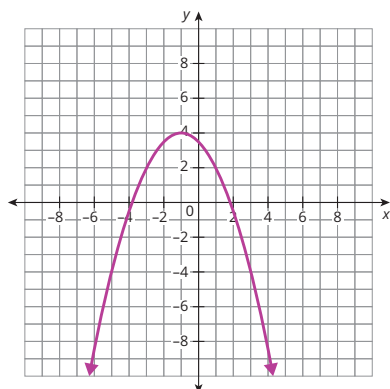
4. You can create the graph of $g(x)$ by reflecting $f(x)$ across the y -axis, stretching it horizontally by a factor of 4, translating it to the right 4 units, and translating it down 3 units.



$$g(x) = \left(-\frac{1}{4}\right)(x - 4)^2 - 3$$

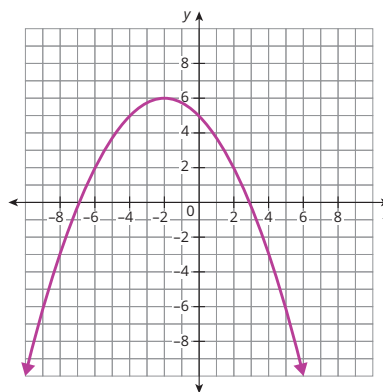
TOPIC 1 Introduction to Quadratic Functions

5. You can create the graph of $g(x)$ by reflecting $f(x)$ across the x -axis, compressing it vertically by a factor of $\frac{1}{2}$, and translating it 4 units up and 1 unit left.



$$g(x) = -\frac{1}{2}(x + 1)^2 + 4$$

6. You can create the graph of $g(x)$ by dilating $f(x)$ across the x - and y -axis, translating it up 6 units, dilating it horizontally by a factor of 2, and translating it left 2 units.



$$g(x) = -(-0.5(x + 2))^2 + 6$$

- C. Use technology to determine the vertex of each quadratic function given in standard form. Rewrite the function in vertex form.

TEKS A.6B (Interleaved Practice), TEKS A.7A (Topic-Aligned)

1. $f(x) = x^2 - 6x - 27$

The vertex is $(3, -36)$. The function in vertex form is $f(x) = (x - 3)^2 - 36$.

3. $f(x) = 2x^2 - 4x - 6$

The vertex is $(1, -8)$. The function in vertex form is $f(x) = 2(x - 1)^2 - 8$.

5. $f(x) = -x^2 + 15x - 54$

The vertex is $(7.5, 2.25)$. The function in vertex form is $f(x) = -(x - 7.5)^2 + 2.25$.

2. $f(x) = -x^2 - 2x + 15$

The vertex is $(-1, 16)$. The function in vertex form is $f(x) = -(x + 1)^2 + 16$.

4. $f(x) = x^2 - 10x + 24$

The vertex is $(5, -1)$. The function in vertex form is $f(x) = (x - 5)^2 - 1$.

6. $f(x) = -2x^2 - 14x - 12$

The vertex is $(-3.5, 12.5)$. The function in vertex form is $f(x) = -2(x + 3.5)^2 + 12.5$.



TOPIC 1 Introduction to Quadratic Functions

D. Determine the vertex of each quadratic function given in vertex form.

TEKS A.7A (Topic-Aligned), Prerequisite for TEKS A.6B

1. $f(x) = (x - 3)^2 + 8$

The vertex is (3, 8).

3. $f(x) = -2(x - 1)^2 - 8$

The vertex is (1, -8).

5. $f(x) = -(x + 9)^2 - 1$

The vertex is (-9, -1).

2. $f(x) = (x + 4)^2 + 2$

The vertex is (-4, 2).

4. $f(x) = \frac{1}{2}(x - 2)^2 + 6$

The vertex is (2, 6).

6. $f(x) = (x - 5)^2$

The vertex is (5, 0).

E. Identify the form of each quadratic function as either standard form, factored form, or vertex form. Then, state all you know about the quadratic function's key characteristics, based only on the given equation of the function. TEKS A.7A (Topic-Aligned),

Prerequisite for TEKS A.6B and A.6C

1. $f(x) = 5(x - 3)^2 + 12$

The function is in vertex form. The parabola opens upward, which means the vertex is a minimum. The vertex is (3, 12), and the axis of symmetry is $x = 3$. There are no zeros/x-intercepts.

3. $f(x) = -3x^2 + 5x$

The function is in standard form. The parabola opens downward, which means the vertex is a maximum. The y-intercept is (0, 0).

5. $f(x) = -(x + 2)^2 - 7$

The function is in vertex form. The parabola opens downward, which means the vertex is a maximum. The vertex is (-2, -7), and the axis of symmetry is $x = -2$. There are no zeros/x-intercepts.

2. $f(x) = -(x - 8)(x - 4)$

The function is in factored form. The parabola opens downward, which means the vertex is a maximum. The x-intercepts are (8, 0) and (4, 0). The axis of symmetry is $x = 6$.

4. $f(x) = \frac{2}{3}(x + 6)(x - 1)$

The function is in factored form. The parabola opens upward, which means the vertex is a minimum. The x-intercepts are (-6, 0) and (1, 0). The axis of symmetry is $x = -2.5$.

6. $f(x) = 2x^2 - 1$

The function is in standard form and vertex form. The parabola opens upward, which means the vertex is a minimum. The vertex is (0, -1), and the axis of symmetry is $x = 0$, or the y-axis. The y-intercept is (0, -1), and there are two zeros/x-intercepts.

TOPIC 1 Introduction to Quadratic Functions

F. Write an equation for a quadratic function that satisfies each set of given characteristics. TEKS A.6B (Topic-Aligned),

Prerequisite for TEKS A.6C

1. The vertex is $(-2, -3)$, and the parabola passes through the point $(1, 6)$.

$$f(x) = (x + 2)^2 - 3$$

3. The function has zeros $(6, 0)$ and $(-4, 0)$, and the parabola passes through the point $(0, 8)$.

$$f(x) = -\frac{1}{3}(x - 6)(x + 4)$$

5. The vertex is $(0, -8)$, and the parabola passes through the point $(4, 0)$.

$$f(x) = \frac{1}{2}x^2 - 8$$

2. The vertex is $(0, 0)$, and the parabola passes through the point $(-2, -8)$.

$$f(x) = -2x^2$$

4. The vertex is $(-4, 0)$, and the parabola passes through the point $(-6, 12)$.

$$f(x) = 3(x + 4)^2$$

6. The function has zeros $(5, 0)$ and $(-1, 0)$, and the parabola passes through the point $(1, -8)$.

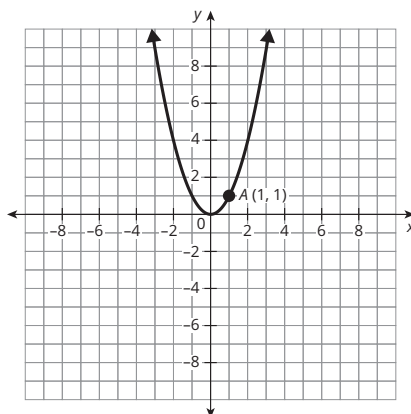
$$f(x) = (x - 5)(x + 1)$$



TOPIC 1 Introduction to Quadratic Functions

Extension

Given $f(x) = x^2$. Sketch each function. Identify point A' for each transformation. **Extension of TEKS A.7C**



1. $m(x) = f(-x + 3)$

$A'(2, 1)$

2. $n(x) = f(-(x + 3))$

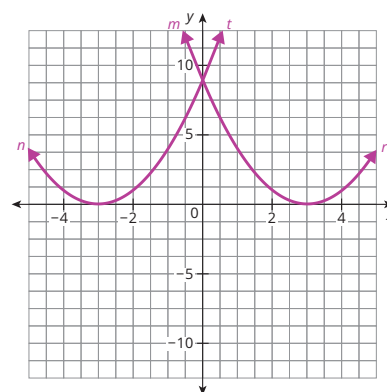
$A'(-4, 1)$

3. $r(x) = f(-(x - 3))$

$A'(2, 1)$

4. $t(x) = f(-x - 3)$

$A'(-4, 1)$



TOPIC 1 Introduction to Quadratic Functions

Spaced Practice

1. Use the equation $f(x) = \frac{1}{3}(x - 5)(x - 3)$ to determine each characteristic. **TEKS A.7A**
 - a. Axis of symmetry
 $x = 4$
 - b. x-intercepts
 $(5, 0)$ and $(3, 0)$
 - c. Will the graph open upward or downward?
Upward
2. Use the equation $f(x) = 4x^2 - 10$ to determine each characteristic. **TEKS A.7A**
 - a. Axis of symmetry
 $x = 0$
 - b. y-intercept
 $(0, -10)$
 - c. Maximum or minimum
Minimum at $(0, -10)$



TOPIC 1 Introduction to Quadratic Functions

3. Determine the exponential regression model that models each situation. Use the equation to make the associated prediction. Round all values to the hundredths place. **TEKS A.9E**

- a. Mr. Patel deposited \$1200 into a savings account in 1985. The table shows the value of Mr. Patel's savings account from 1985 to 2020. Predict the account's value in 2030.

Time Since 1985 (years)	0	5	10	15	20	25	30	35
Account Value (dollars)	1200	1450	1800	2175	2600	3015	3468	3905

$f(x) = 1252.6(1.03)^x$; The account's value will be approximately \$4736.83 in 2030.

- b. Tiara deposited \$725 into a savings account in 1985. The table shows the value of Tiara's savings account from 1985 to 2020. Predict the account's value in 2030.

Time Since 1985 (years)	0	5	10	15	20	25	30	35
Account Value (dollars)	725	850	1040	1270	1410	1830	2007	2780

$f(x) = 712.94(1.04)^x$; The account's value will be approximately \$4164.41 in 2030.

4. Determine the x-intercepts and axis of symmetry of each quadratic function in factored form. **TEKS A.7B**

a. $f(x) = (x - 7)(x - 12)$

The x-intercepts are (7, 0) and (12, 0).

The axis of symmetry is located at $x = 9.5$.

b. $f(x) = 3(x - 2)(x + 10)$

The x-intercepts are (2, 0) and (−10, 0).

The axis of symmetry is located at $x = -4$.



Name _____ Date _____

I. Adding and Subtracting Polynomials

Topic Practice

- A. Write each polynomial in standard form. Classify the polynomial by its number of terms and by its degree.

TEKS A.10A (Prerequisite), TEKS A.10B (Prerequisite)

1. $2x + 6x^2$

$6x^2 + 2x$; the polynomial is a binomial with a degree of 2.

3. $10 - 5x$

$-5x + 10$; the polynomial is a binomial with a degree of 1.

5. $15 + 4w - w^3$

$-w^3 + 4w + 15$; the polynomial is a trinomial with a degree of 3.

7. $-1 - p^4$

$-p^4 - 1$; the polynomial is a binomial with a degree of 4.

9. $-18a^3 + 54a - 22a^2$

$-18a^3 - 22a^2 + 54a$; the polynomial is a trinomial with a degree of 3.

2. $-9m^2 + 4m^3$

$4m^3 - 9m^2$; the polynomial is a binomial with a degree of 3.

4. $7x - 3 + 12x^2$

$12x^2 + 7x - 3$; the polynomial is a trinomial with a degree of 2.

6. $5x^2 - 15 + 20x$

$5x^2 + 20x - 15$; the polynomial is a trinomial with a degree of 2.

8. $-6t^2 + 4t + 3t^3$

$3t^3 - 6t^2 + 4t$; the polynomial is a trinomial with a degree of 3.

10. $x^3 - x^2 - x^5$

$-x^5 + x^3 - x^2$; the polynomial is a trinomial with a degree of 5.



TOPIC 2 Polynomial Operations

B. Rewrite each expression with the fewest number of terms.

TEKS A.10A (Topic-Aligned)

1. $(5x - 8) + (7x + 10)$

$12x + 2$

3. $(-12 + 5x - x^2) + (2x^2 - 6)$

$x^2 + 5x - 18$

5. $-5w^2 + (3w - 8) + 11 - (4w + 15w^2)$

$10w^2 - w + 3$

7. $(-x^2 + 4x) - (3x + 1)$

$-x^2 + x - 1$

9. $(3x^4 + 3x^2 - 3) - (6x^5 - 9x^3 + 2)$

$-6x^5 + 3x^4 + 9x^3 + 3x^2 - 5$

2. $(4m^2 + 9m) - (6 + 2m^2)$

$2m^2 + 9m - 6$

4. $\left(\frac{1}{2}s^2 + \frac{9}{2}\right) + \frac{1}{4} + \left(3s^2 - \frac{2}{3}s\right)$

$\frac{7}{2}s^2 - \frac{2}{3}s + \frac{19}{4}$

6. $-a^2 + (2a - 8) + 15 + (-9a + 2a^2)$

$a^2 - 7a + 7$

8. $(8x^2 - 4x + 3) - (5x + 7)$

$8x^2 - 9x - 4$

10. $(-7m^3 - m^2 - m) - (-1 - m - 10m^3)$

$3m^3 - m^2 + 1$

C. Determine the sum or difference of the polynomials.

TEKS A.10A (Interleaved Practice)

1. $(9x + 5) - (2x - 2)$

$7x + 7$

3. $\left(\frac{1}{2}pq - p - q\right) + \left(\frac{3}{4}p - pq\right)$

$-\frac{1}{2}pq - \frac{1}{4}p - q$

5. $(4x^2 + 7x - 12) - (-6x^2 + 8x - 7)$

$10x^2 - x - 5$

2. $\left(x^2 - \frac{1}{2}x + \frac{1}{8}\right) + (1 + x^2)$

$2x^2 - \frac{1}{2}x + \frac{9}{8}$

4. $(2x - 7) + (11 - 2x)$

4

6. $(-2a + 7b - 9) - (17a - 5c + 7)$

$-19a + 7b + 5c - 16$



Extension

Consider the binomials $(x + 3)$, $(2x + 1)$, and $(x - 4)$. **Extension of TEKS A.10A**

- Without adding, make a conjecture about the degree of the sum of these binomials. Explain how you determined your answer.

The degree of the sum is 1. The term x has the highest degree.

- Without adding, make a conjecture about the number of terms in the sum of these binomials. Explain your reasoning.

There will be one term, an x term.

- Determine the sum of the three binomials.

The sum is $4x$.

Spaced Practice

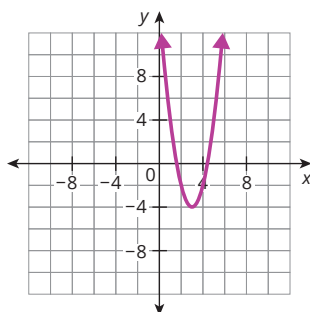
- Analyze the given representations. Then, answer each question and justify your reasoning. **TEKS A.7A**

- Which function's axis of symmetry has a greater x -value?

Function A

$$f(x) = x^2 - 4x + 9$$

Function C



Function C

Functions A and B have an axis of symmetry at $x = 2$.

Function D's axis of symmetry is at $x = 1$.

Function C's axis of symmetry is at $x = 3$.

- Which function has the greatest absolute minimum?

Function B

$$f(x) = 3(x - 2)^2 - 6$$

Function D

x	y
-1	7
0	-2
1	-5
2	-2
3	7

Function A

Function A has the greatest absolute minimum.

Function A has an absolute minimum of $y = 5$.

Function B has an absolute minimum of $y = -6$.

Function C has an absolute minimum of $y = -4$.

Function D has an absolute minimum of $y = -5$.



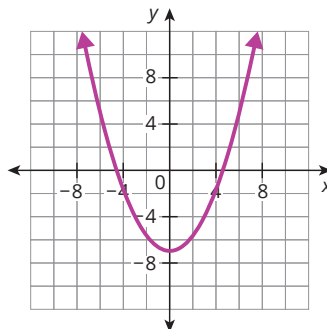
TOPIC 2 Polynomial Operations

2. Write the equation of the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2$, by reflecting it across the x -axis, vertically stretching it by a factor of 2, and translating it up 5 units.

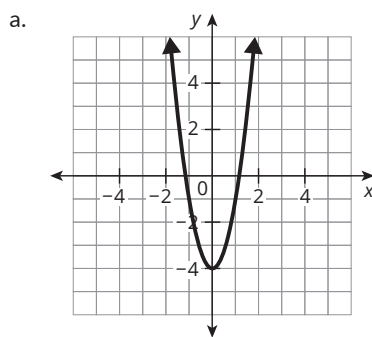
TEKS A.7C

$$g(x) = -2x^2 + 5$$

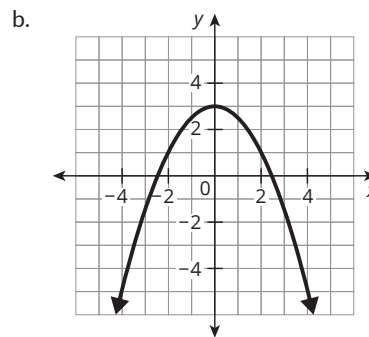
3. Graph the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2$, by vertically compressing it by a factor of $\frac{1}{3}$ and translating it down 7 units. TEKS A.7C



4. Identify the domain and range of each function. TEKS A.6A



Domain: all real numbers
Range: $y \geq -4$



Domain: all real numbers
Range: $y \leq 3$

II. Multiplying Polynomials

Topic Practice

A. Determine the product of the binomials using area models.

TEKS A.10B (Topic-Aligned)

1. $3x + 4$ and $2x + 2$

.	$2x$	2
$3x$	$6x^2$	$6x$
4	$8x$	8

$$6x^2 + 14x + 8$$

3. $6t + 5$ and $7t - 5$

.	$7t$	-5
$6t$	$42t^2$	$-30t$
5	$35t$	-25

$$42t^2 + 5t - 25$$

5. $10w - 1$ and $-9w + 8$

.	$-9w$	8
$10w$	$-90w^2$	$80w$
-1	$9w$	-8

$$-90w^2 + 89w - 8$$

2. $-5m + 3$ and $4m + 6$

.	$4m$	6
$-5m$	$-20m^2$	$-30m$
3	$12m$	18

$$-20m^2 - 18m + 18$$

4. $\frac{1}{4}x + 2$ and $\frac{1}{4}x - 2$

.	$\frac{1}{4}x$	-2
$\frac{1}{4}x$	$\frac{1}{16}x^2$	$-\frac{1}{2}x$
2	$\frac{1}{2}x$	-4

$$\frac{1}{16}x^2 - 4$$

6. $\frac{1}{2}y + 1$ and $3y + \frac{2}{3}$

.	$3y$	$\frac{2}{3}$
$\frac{1}{2}y$	$\frac{3}{2}y^2$	$\frac{1}{3}y$
1	$3y$	$\frac{2}{3}$

$$\frac{3}{2}y^2 + \frac{10}{3}y + \frac{2}{3}$$



TOPIC 2 Polynomial Operations

7. $n^2 + 3n - 2$ and $-3n + 4$

.	$-3n$	4
n^2	$-3n^3$	$4n^2$
$3n$	$-9n^2$	$12n$
-2	$6n$	-8

$$-3n^3 - 5n^2 + 18n - 8$$

8. $-2x^2 - x + 4$ and $3x - 5$

.	$3x$	-5
$-2x^2$	$-6x^3$	$10x^2$
$-x$	$-3x^2$	$5x$
4	$12x$	-20

$$-6x^3 + 7x^2 + 17x - 20$$

B. Determine the product of the polynomials using the distributive property.

TEKS A.10B (Topic-Aligned), TEKS A.10D (Topic-Aligned)

1. $3(9 - x)$

$$27 - 3x$$

3. $2x(x + 6)$

$$2x^2 + 12x$$

5. $-7x(x - 5)$

$$-7x^2 + 35x$$

7. $(-x + 3)(x^2 - 1)$

$$-x^3 + 3x^2 + x - 3$$

9. $\frac{1}{3}x(x^2 + 6x - 1)$

$$\frac{1}{3}x^3 + 2x^2 - \frac{1}{3}x$$

11. $(x + 2)(x^2 + 6x - 1)$

$$x^3 + 8x^2 + 11x - 2$$

2. $-(3x - 6)$

$$-3x + 6$$

4. $\frac{1}{4}x^2\left(x + \frac{1}{2}\right)$

$$\frac{1}{4}x^3 + \frac{1}{8}x^2$$

6. $(2x + 1)(x + 8)$

$$2x^2 + 17x + 8$$

8. $(4x + 4)(5x - 5)$

$$20x^2 - 20$$

10. $-9x(3x^2 - 4x + 2)$

$$-27x^3 + 36x^2 - 18x$$

12. $(x_2 - 4)(x^2 + 2x - 3)$

$$x^4 + 2x^3 - 7x^2 - 8x + 12$$

TOPIC 2 Polynomial Operations

C. Use special products to determine each product.

TEKS A.10B (Topic-Aligned)

1. $(x - 7)(x - 7)$

$$x^2 - 14x + 49$$

2. $(x + 4)(x - 4)$

$$x^2 - 16$$

3. $(x + 9)^2$

$$x^2 + 18x + 81$$

4. $(4x + 3)^2$

$$16x^2 + 24x + 9$$

5. $(6x - 5)(6x - 5)$

$$36x^2 - 60x + 25$$

6. $(2x - 8)(2x + 8)$

$$4x^2 - 64$$

Extension

A difference of squares can be useful when you want to remove radicals from the denominator of a fraction.

1. Show how you can use a difference of squares to rewrite the fraction $\frac{5}{\sqrt{3} + 4}$ so that the radical is in the numerator, not the denominator. **Extension of TEKS A.10B**

$$\begin{aligned} & \frac{5}{\sqrt{3} + 4} \cdot \frac{\sqrt{3} - 4}{\sqrt{3} - 4} \\ &= \frac{5(\sqrt{3} - 4)}{(\sqrt{3} + 4)(\sqrt{3} - 4)} \\ &= \frac{5(\sqrt{3} - 4)}{(\sqrt{3})^2 - 4^2} \\ &= \frac{5(\sqrt{3} - 4)}{3 - 16} \\ &= \frac{-5(\sqrt{3} - 4)}{13} \end{aligned}$$



TOPIC 2 Polynomial Operations

Spaced Practice

1. Write each polynomial in standard form. Then, classify it by its number of terms and its degree. If the expression is not a polynomial, explain why not. **Prerequisite for TEKS A.10A**

a. $8 + 9x$

$9x + 8$
Degree 1
Binomial

b. 42

42
Degree 0
Monomial

c. $|x + 9|$

Not a polynomial;
contains an
absolute value

d. $6 + 2x + 3x^2$

$3x^2 + 2x + 6$
Degree 2
Trinomial

2. Simplify the expression by determining the sum or difference. Show your work. **TEKS A.10A**

$$(3x^2 - 2x + 14) - (6x^2 + 3x - 9)$$

$$3x^2 - 2x + 14 - 6x^2 - 3x + 9$$

$$(3x^2 - 6x^2) + (-2x - 3x) + (14 + 9)$$

$$-3x^2 - 5x + 23$$

3. Identify each of the following for the function $f(x) = 3 \cdot 4^x$. **TEKS A.9D**

a. x-intercept(s)

None

b. y-intercept

$(0, 3)$

c. Asymptote

$$f(x) = 0$$

d. Domain

All real numbers

e. Range

$$f(x) > 0$$

f. Interval(s) of increase/decrease

Increasing over
the entire domain



TOPIC 2 Polynomial Operations

4. Write the equation of each function after the translation described. **TEKS A.7C**

- a. $f(x) = -10x$ after a translation 5 units to the right.

$$m(x) = -10(x - 5)$$

- b. $g(x) = 3^x$ after a translation 4 units up.

$$m(x) = 3^x + 4$$

- c. $h(x) = 2x^2$ after a translation 2 units left and 1 unit down.

$$m(x) = 2(x + 2)^2 - 1$$

5. Write an equation for a quadratic function that satisfies each set of given characteristics. **TEKS A.6B**

- a. The vertex is $(-6, 3)$ and the parabola passes through the point $(-5, 0)$.

$$f(x) = -3(x + 6)^2 + 3$$

- b. The vertex is $(2, 5)$ and the parabola passes through the point $(-3, -6)$.

$$f(x) = -0.44(x - 2)^2 + 5$$

6. Determine the axis of symmetry for each quadratic function. **TEKS A.7A**

a. $f(x) = \frac{2}{3}(x + 3)(x - 7)$

The axis of symmetry is located at $x = -2$.

b. $f(x) = -0.2(x - 16)(x - 5)$

The axis of symmetry is located at $x = -10.5$.



TOPIC 2 Polynomial Operations

III. Polynomial Division

Topic Practice

- A. Use the factor theorem to determine whether the given expression is a factor of each polynomial. Explain your reasoning.

TEKS A.7B (Topic-Aligned)

1. Is $(x - 4)$ a factor of $0.5x^2 - 20$? $p(4) = -12$ No, $(x - 4)$ is not a factor of $0.5x^2 - 20$ because 4 is not a zero.	2. Is $(x - 3)$ a factor of $-14x^2 + 52x - 30$? $p(3) = 0$ Yes, $(x - 3)$ is a factor of $-14x^2 + 52x - 30$ because 3 is a zero.
3. Is $(x + \frac{1}{2})$ a factor of $-6x^2 - 22x - 12$? $p(-\frac{1}{2}) = -2.5$ No, $(x + \frac{1}{2})$ is not a factor of $-6x^2 - 22x - 12$ because $-\frac{1}{2}$ is not a zero.	4. Is $(x - 2)$ a factor of $9x^2 + 15x - 6$? $p(2) = 60$ No, $(x - 2)$ is not a factor of $9x^2 + 15x - 6$ because 2 is not a zero.
5. Is $(x - 1)$ a factor of $24x^2 - 66x + 42$? $p(1) = 0$ Yes, $(x - 1)$ is a factor of $24x^2 - 66x + 42$ because 1 is a zero.	6. Is $(x + \frac{4}{3})$ a factor of $21x^2 + 16x - 16$? $p(-\frac{4}{3}) = 0$ Yes, $(x + \frac{4}{3})$ is a factor of $21x^2 + 16x - 16$ because $-\frac{4}{3}$ is a zero.



TOPIC 2 Polynomial Operations

B. Determine the quotient for each problem. Show all of your work.

TEKS A.10C (Topic-Aligned)

1. $(8x - 16) \div (x - 2)$

$$\begin{array}{r} 8 \\ x-2 \overline{)8x-16} \\ \underline{-(8x-16)} \\ 0 \end{array}$$

3. $(x^2 + 2x - 15) \div (x + 5)$

$$\begin{array}{r} x-3 \\ x+5 \overline{)x^2+2x-15} \\ \underline{-(x^2+5x)} \\ -3x-15 \\ \underline{-(-3x-15)} \\ 0 \end{array}$$

5. $(x^2 + 6x - 10) \div (x - 5)$

$$\begin{array}{r} x+11 \text{ R45} \\ x-5 \overline{)x^2+6x-10} \\ \underline{-(x^2-5x)} \\ 11x-10 \\ \underline{-(11x-55)} \\ 45 \end{array}$$

7. $(16x^2 - 12x + 10) \div (4x - 4)$

$$\begin{array}{r} 4x+1 \text{ R14} \\ 4x-4 \overline{)16x^2-12x+10} \\ \underline{-(16x^2-16x)} \\ 4x+10 \\ \underline{-(4x-4)} \\ 14 \end{array}$$

2. $(5x + 21) \div (x + 4)$

$$\begin{array}{r} 5 \text{ R1} \\ x+4 \overline{)5x+21} \\ \underline{-(5x+20)} \\ 1 \end{array}$$

4. $(x^2 - 7x - 30) \div (x + 3)$

$$\begin{array}{r} x-10 \\ x+3 \overline{)x^2-7x-30} \\ \underline{-(x^2+3x)} \\ -10x-30 \\ \underline{-(-10x-30)} \\ 0 \end{array}$$

6. $(9x^2 - 20x - 21) \div (x - 3)$

$$\begin{array}{r} 9x+7 \\ x-3 \overline{)9x^2-20x-21} \\ \underline{-(9x^2-27x)} \\ 7x-21 \\ \underline{-(7x-21)} \\ 0 \end{array}$$

8. $(24x^2 - 83x + 63) \div (8x^2 - 9)$

$$\begin{array}{r} 3 \text{ R } (-83x + 90) \\ 8x^2-9 \overline{)24x^2-83x+63} \\ \underline{-(24x^2 \quad \quad -27)} \\ -83x+90 \end{array}$$



Extension

Determine the quotient of $\frac{4x^4 - 19x^2 - 8}{x - 2}$. Show all your work.

Extension of TEKS A.10C

$$\begin{array}{r}
 4x^3 + 8x^2 - 3x - 6 \\
 x - 2 \overline{) 4x^4 + 0x^3 - 19x^2 + 0x - 8} \\
 \underline{-(4x^4 - 8x^3)} \\
 8x^3 - 19x^2 \\
 \underline{-(8x^3 - 16x^2)} \\
 -3x^2 + 0x \\
 \underline{-(-3x^2 + 6x)} \\
 -6x - 8 \\
 \underline{-(-6x + 12)} \\
 -20
 \end{array}$$

The quotient is

$$4x^3 + 8x^2 - 3x - 6 \text{ R } (-20)$$

or

$$4x^3 + 8x^2 - 3x - 6 - \frac{20}{x - 2}$$

Spaced Practice

1. Perform each operation.

a. $(8m^2 - 7m + 3) - (3m^2 + 2m - 3)$ **TEKS A.10A**

$$5m^2 - 9m + 6$$

b. $(2x - 5)(3x^2 - x + 4)$ **TEKS A.10B**

$$6x^3 - 17x^2 + 13x - 20$$

c. $7a(a + 3) + (a - 4)(2a - 7) - (4a^2 - 11a + 3)$ **TEKS A.10A**

$$5a^2 + 17a + 25$$

d. $(5x^2 + 2x - 1)(3x^2 - 3x + 5)$ **TEKS A.10B**

$$15x^4 - 9x^3 + 16x^2 + 13x - 5$$

2. Use the distributive property to determine the product of the polynomials. **TEKS A.10D**

a. $3x(3x^2 + 2x - 7)$

$$9x^3 + 6x^2 - 21x$$

b. $(x - 6)(4x + 2)$

$$4x^2 - 22x - 12$$



Name _____ Date _____

I. Representing Solutions to Quadratic Equations

Topic Practice

- A. Solve each quadratic equation. If necessary, leave solutions in simplest radical form. **TEKS A.8A (Topic-Aligned),**
TEKS A.11A (Interleaved Practice)

1. $x^2 = 48$

$x = \pm 4\sqrt{3}$

3. $x^2 = 27$

$x = \pm 3\sqrt{3}$

5. $x^2 = 40$

$x = \pm 2\sqrt{10}$

2. $x^2 = 52$

$x = \pm 2\sqrt{13}$

4. $x^2 = 175$

$x = \pm 5\sqrt{7}$

6. $x^2 = 147$

$x = \pm 7\sqrt{3}$

- B. Solve algebraically for the roots of each quadratic equation or the zeros of each quadratic function. **TEKS A.8A (Topic-Aligned)**

1. $x^2 - 100 = 0$

The roots are -10 and 10 .

3. $f(x) = x^2 - 225$

The zeros are -15 and 15 .

5. $f(x) = 8x^2 - 50$

The zeros are $-\frac{5}{2}$ and $\frac{5}{2}$.

2. $4x^2 - 9 = 0$

The roots are $-\frac{3}{2}$ and $\frac{3}{2}$.

4. $f(x) = 9x^2 - 1$

The zeros are $-\frac{1}{3}$ and $\frac{1}{3}$.

6. $16x^2 - 25 = 0$

The roots are $-\frac{5}{4}$ and $\frac{5}{4}$.



TOPIC 3 Solving Quadratic Equations

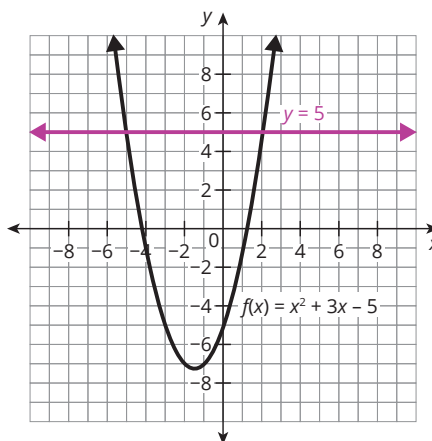
Extension

1. Consider the graph of the function $f(x) = x^2 + 3x - 5$.

Extension of TEKS A.7A, A.8A

- a. Determine the solutions for the equation $x^2 + 3x - 5 = 5$. Identify the solutions on the graph.

Solutions are $x = -5$ and $x = 2$.



- b. Rewrite the equation from part (a) so that the right side of the equation is 0. What do the solutions from part (a) represent in this new equation?

$$x^2 + 3x - 10 = 0$$

The solutions from part (a) are the roots of this equation, or the values of x for which the value of the function is 0.

- c. Use your solutions from part (a) to write a product of two binomials, $(x - a)(x - b)$, where a and b are the solutions from part (a). What is the relationship between this and the left side of the equation in part (b)?

$$(x + 5)(x - 2)$$

When the two binomials are multiplied together, the result is the same as the left side of the equation from part (b).

TOPIC 3 Solving Quadratic Equations

Spaced Practice

1. Identify the axis of symmetry of the graph of $f(x) = -5(x - 3)(x + 12)$.

TEKS A.7A

$$x = \frac{3 + (-12)}{2} = -4.5$$

2. Write a quadratic function in factored form to represent a parabola that opens downward and has zeros at $(-6, 0)$ and $(-2, 0)$. TEKS A.7A

Answers will vary. However, answers should have a negative leading coefficient.

Sample answer:

$$f(x) = -3(x + 6)(x + 2).$$

3. Determine each product. Show your work. TEKS A.10B

a. $(2x - 3)(4x + 7)$

$$(2x - 3)(4x + 7)$$

$$= 8x^2 + 14x - 12x - 21$$

$$= 8x^2 + 2x - 21$$

⋮

b. $(3x + 5)\left(-\frac{1}{2}x + 16\right)$.

$$(3x + 5)\left(-\frac{1}{2}x + 16\right)$$

$$= -\frac{3}{2}x^2 + 48x +$$

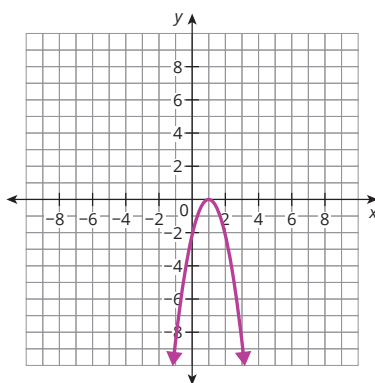
$$\left(-\frac{5}{2}x\right) + 80$$

$$= -\frac{3}{2}x^2 + \frac{91}{2}x + 80$$

4. Write the equation of the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2 + 1$ by reflecting it across the x -axis, shifting it up 6 units, and shifting it to the left 4 units. TEKS A.7C

$$g(x) = -(x + 4)^2 + 5$$

5. Graph the function, $g(x)$, in which its graph transforms the graph $f(x) = (x - 4)^2$ by vertically stretching it by a factor of 2, reflecting it across the x -axis, and moving it to the left 3 units. TEKS A.7C



TOPIC 3 Solving Quadratic Equations

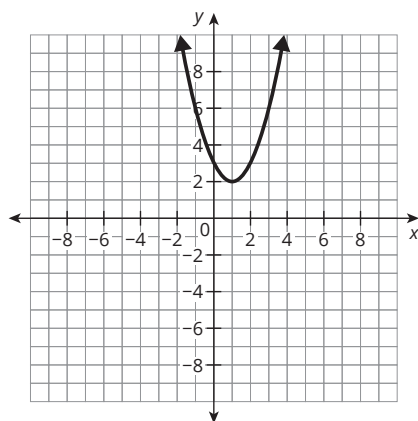
II. Solutions to Quadratic Equations in Vertex Form

Topic Practice

- A. For each given graph, determine the number of real zeros for the quadratic function. **TEKS A.7A (Topic-Aligned),**

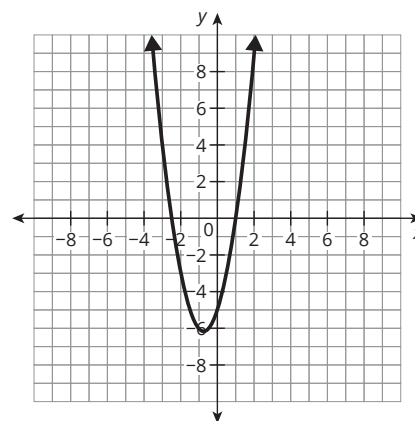
Prerequisite of TEKS A.8A

1.



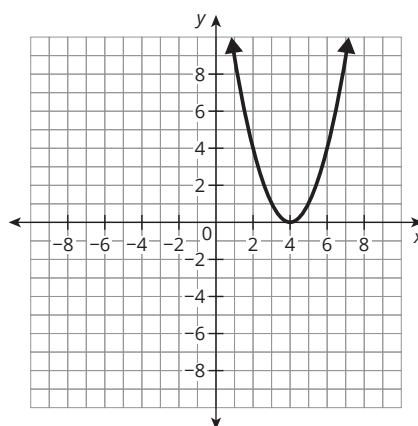
The function has no real roots. The graph of $f(x)$ does not cross the x-axis.

2.



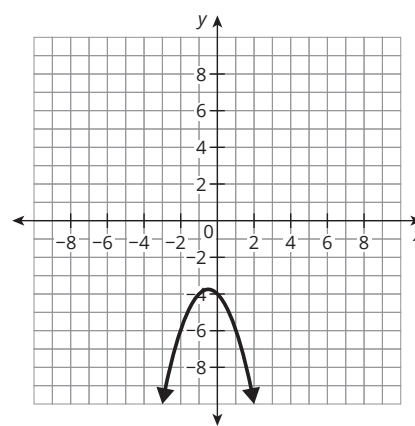
The function has two real zeros.

3.



The function has one real zero.

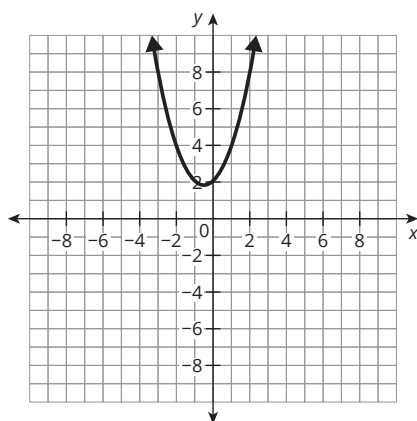
4.



The function has no real zeros. The graph of $f(x)$ does not cross the x-axis.

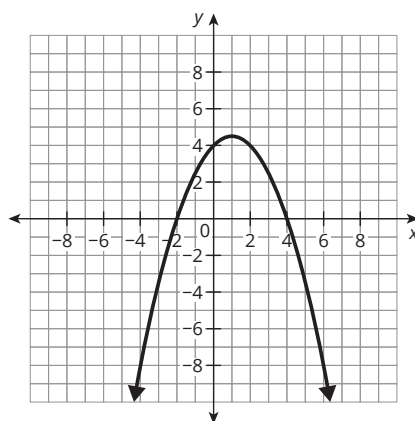
TOPIC 3 Solving Quadratic Equations

5.



The function has no real zeros. The graph of $f(x)$ does not cross the x -axis.

6.



The function has two real zeros.

B. Solve each quadratic equation. Give both exact and approximate solutions rounded to nearest hundredth. TEKS A.8A (Topic-Aligned)

1. $(x - 2)^2 = 16$

$$x = 2 \pm 4$$

$$x = 6 \text{ or } x = -2$$

3. $-6(x + 3)^2 = -30$

$$x = -3 \pm \sqrt{5}$$

$$x \approx -0.76 \text{ or } x \approx -5.24$$

5. $-\frac{1}{3}(x - 4)^2 = -12$

$$x = 4 \pm 6$$

$$x = 10 \text{ or } x = -2$$

2. $3(x + 1)^2 = 6$

$$x = -1 \pm \sqrt{2}$$

$$x \approx 0.41 \text{ or } x \approx -2.41$$

4. $7(x - 9)^2 = 36$

$$x = 9 \pm \sqrt{\frac{36}{7}}$$

$$x \approx 11.27 \text{ or } x \approx 6.73$$

6. $3(10 - x)^2 = 27$

$$x = 10 \pm 3$$

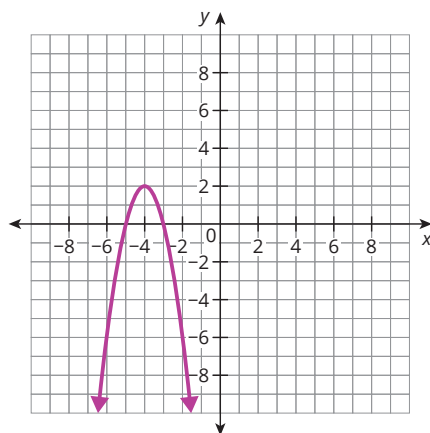
$$x = 7 \text{ or } x = 13$$



TOPIC 3 Solving Quadratic Equations

- C. Sketch a graph of each quadratic function. Describe the number of zeros of each function and solve for the zeros algebraically. Give both exact and approximate solutions rounded to the nearest hundredth. **TEKS A.7A (Topic-Aligned),**
TEKS A.8A (Interleaved Practice)

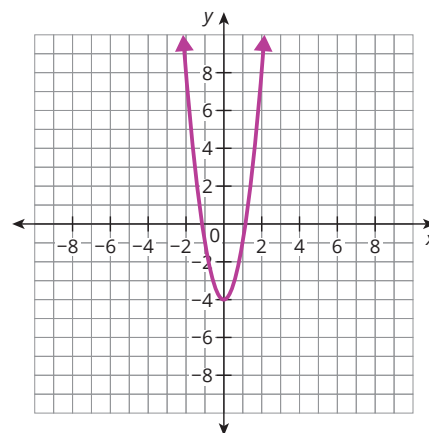
1. $f(x) = -2(x + 4)^2 + 2$



There are two real zeros.

$$x = -3 \text{ and } x = -5$$

2. $f(x) = 3x^2 - 4$



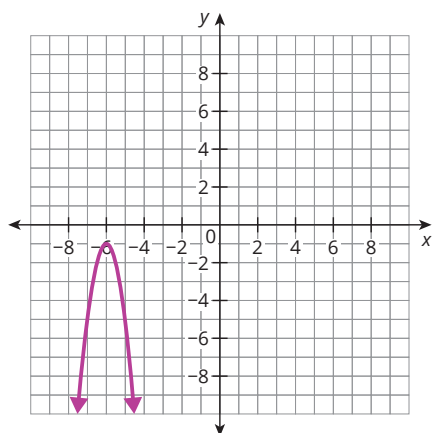
There are two real zeros.

$$x = \pm \sqrt{\frac{4}{3}}$$

$$x \approx 1.15 \text{ and } x \approx -1.15$$

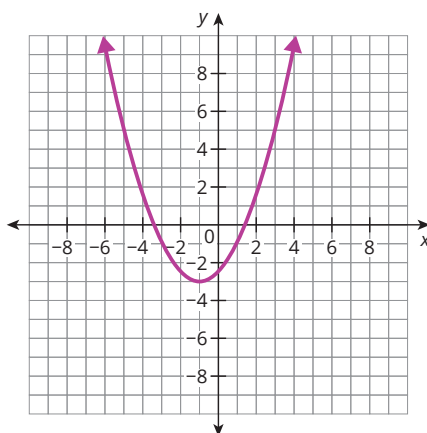
TOPIC 3 Solving Quadratic Equations

3. $f(x) = -4(x + 6)^2 - 1$



There are no real zeros. The graph of $f(x)$ does not cross the x -axis.

4. $f(x) = 0.5(x + 1)^2 - 3$

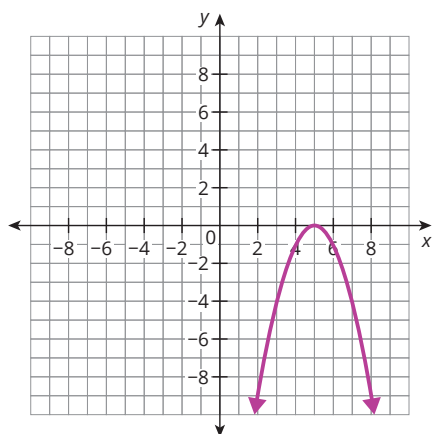


There are two real zeros.

$$x = -1 \pm \sqrt{6}$$

$$x \approx 1.45 \text{ and } x \approx -3.45$$

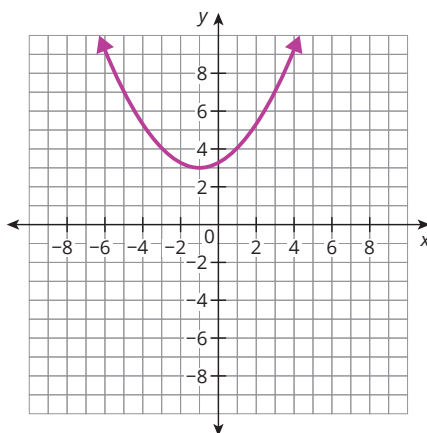
5. $f(x) = -(x - 5)^2$



There is one real zero.

$$x = 5$$

6. $f(x) = \frac{1}{4}(x + 1)^2 + 3$



There are no real zeros. The graph of $f(x)$ does not cross the x -axis.



TOPIC 3 Solving Quadratic Equations

Extension

A quadratic function has zeros at $x = -2 \pm \sqrt{15}$. Write the function in standard form. Show your work. **Extension of TEKS A.6C, A.10B**

Sample answer:

$$f(x) = (x - (-2 + \sqrt{15}))(x - (-2 - \sqrt{15}))$$

$$f(x) = (x + 2 - \sqrt{15})(x + 2 + \sqrt{15})$$

$$f(x) = (x^2 + 2x + \sqrt{15}x + 2x + 4 + 2\sqrt{15} - \sqrt{15}x - 2\sqrt{15} - (\sqrt{15})^2)$$

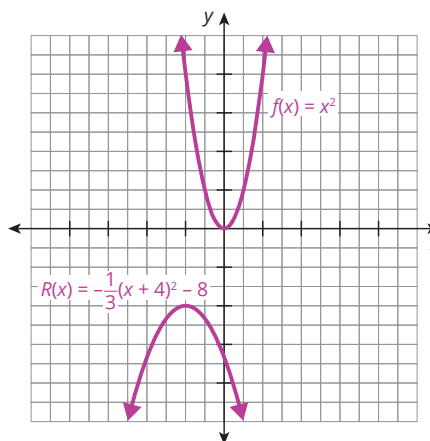
$$f(x) = x^2 + 4x + 4 - 15$$

$$f(x) = x^2 + 4x - 11$$

Spaced Practice

1. Use the given characteristics to write a function $R(x)$ in vertex form. Then, sketch the graph of $R(x)$ and the parent function $f(x) = x^2$ on a coordinate plane. **TEKS A.7C**
 - a. The function has an absolute maximum, is vertically dilated by a factor of $\frac{1}{3}$, and is translated 8 units down and 4 units to the left.

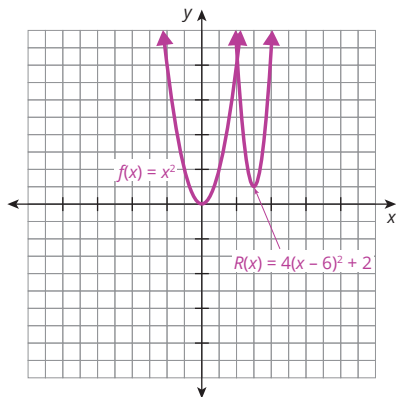
$$R(x) = -\frac{1}{3}(x + 4)^2 - 8$$



TOPIC 3 Solving Quadratic Equations

- b. The function has an absolute minimum, is vertically dilated by a factor of 4, and is translated 2 units up and 6 units to the right.

$$R(x) = 4(x - 6)^2 + 2$$



2. Estimate the value of the radical expression $\sqrt{54}$. Then, rewrite the radical by extracting all perfect squares, if possible. **TEKS A.11A**

$$\sqrt{54} \approx 7.3; \sqrt{54} = 3\sqrt{6}$$

3. Rewrite the quadratic function, $f(x) = 16x^2 - 3$, as the product of linear factors. **TEKS A.10F**

$$f(x) = (4x - \sqrt{3})(4x + \sqrt{3})$$

4. Identify the form of each quadratic equation. Then, identify what characteristic of the function can be determined by the structure of the equation. **TEKS A.7A**

a. $y = (x - 7)(x + 5)$

Factored form

The roots are 7 and (-5) and the axis of symmetry is

$$x = \frac{7 + (-5)}{2} = 1. \text{ The parabola opens up.}$$

b. $y = -3(x + 1)^2 - 4$

Vertex form

The vertex is at $(-1, -4)$ and the axis of symmetry is $x = -1$.

The parabola opens down.



TOPIC 3 Solving Quadratic Equations

III. Factoring and Completing the Square

Topic Practice

A. Factor out the greatest common factor of each polynomial, if possible. **TEKS A.10D (Topic-Aligned)**

1. $x^2 + 9x$

$x(x + 9)$

2. $m^2 - 4m$

$m(m - 4)$

3. $5x^2 + 20x - 15$

$5(x^2 + 4x - 3)$

4. $-24w^2 + 16$

$-8(3w^2 - 2)$

5. $3w + 10$

$3w + 10$ (the greatest common factor is 1).

6. $2x^2 + 10x$

$2x(x + 5)$

7. $\frac{1}{4}x + \frac{3}{4}$

$\frac{1}{4}(x + 3)$

8. $\frac{4}{5}x + \frac{1}{5}x^2$

$\frac{1}{5}x(4 + x)$

9. $20x^3 + 16x^2 + 8x$

$4x(5x^2 + 4x + 2)$

10. $15x^3 + 4$

$15x^3 + 4$ (the greatest common factor is 1).



TOPIC 3 Solving Quadratic Equations

B. Factor each trinomial. TEKS A.10E (Topic-Aligned),

TEKS A.10F (Topic-Aligned)

1. $x^2 - 2x - 8$
 $(x - 4)(x + 2)$

3. $m^2 + 6m - 7$
 $(m + 7)(m - 1)$

5. $b^2 - 16$
 $(b + 4)(b - 4)$

7. $3m^3 + 36m^2 + 60m$
 $3m(m + 10)(m + 2)$

9. $x^2 + 11x + 10$
 $(x + 1)(x + 10)$

11. $m^2 + 2m - 35$
 $(m + 7)(m - 5)$

13. $6x^2 + 9x - 15$
 $3(2x + 5)(x - 1)$

2. $y^2 + 13y + 42$
 $(y + 6)(y + 7)$

4. $x^2 - 9x + 18$
 $(x - 3)(x - 6)$

6. $2t^3 - 14t^2 + 24t$
 $2t(t - 3)(t - 4)$

8. $3x^2 + 14x - 5$
 $(3x - 1)(x + 5)$

10. $w^2 + 10w + 25$
 $(w + 5)(w + 5)$

12. $9x^2 - 100$
 $(3x - 10)(3x + 10)$

14. $2x^2 + 22x + 60$
 $2(x + 5)(x + 6)$



TOPIC 3 Solving Quadratic Equations

C. Determine the solutions of each quadratic equation using the factoring method. **TEKS A.8A (Topic-Aligned)**

1. $x^2 + 5x + 6 = 0$

$x = -3$ and $x = -2$

2. $x^2 - 3x - 4 = 0$

$x = 4$ and $x = -1$

3. $m^2 + 2m - 35 = 0$

$m = -7$ and $m = 5$

4. $-x^2 - 4x + 12 = 0$

$x = 2$ and $x = -6$

5. $x^2 + 8x = 0$

$x = 0$ and $x = -8$

6. $w^2 + 50 = -15w$

$w = -10$ and $w = -5$

7. $-t^2 + 12t = 32$

$t = 4$ and $t = 8$

8. $x^2 + 2x + 2 = 0$

The equation has no real roots.



TOPIC 3 Solving Quadratic Equations

9. $2t^2 + t - 3 = 0$

$t = -\frac{3}{2}$ and $t = 1$

10. $w^2 + 5w - 32 = 2w - 4$

$w = -7$ and $w = 4$

11. $-14x^2 + 28x + 42 = 0$

$x = -1$ and $x = 3$

12. $-12x^2 + 38x = 6$

$x = 3$ and $x = \frac{1}{6}$

D. Algebraically or graphically determine the zeros of each quadratic function, if possible. TEKS A.7A (Interleaved Practice),

TEKS A.8A (Interleaved Practice)

1. $f(x) = x^2 - 5x$

The zeros are 0 and 5.

2. $f(x) = 3x^2 + 6x$

The zeros are 0 and -2 .

3. $f(x) = x^2 + 11x + 30$

The zeros are -6 and -5 .

4. $f(x) = x^2 - 9x - 36$

The zeros are 12 and -3 .



TOPIC 3 Solving Quadratic Equations

5. $f(x) = 2x^2 + 9x + 10$

The zeros are $-\frac{5}{2}$ and -2 .

6. $f(x) = x^2 + 5x + 14$

The function has no real zeros.

7. $f(x) = 3x^2 + 3x - 6$

The zeros are -2 and 1 .

8. $f(x) = \frac{1}{2}x^2 - \frac{3}{4}x$

The zeros are 0 and $\frac{3}{2}$.

E. Use an area model to complete the square for each expression. Factor the resulting trinomial and write the original expression in vertex form. Prerequisite of TEKS A.8A, TEKS A.10E (Interleaved Practice)

1. $x^2 + 2x$

1	x	1
x	x^2	x
	x	1

$$x^2 + 2x + 1 = (x + 1)^2$$

$$x^2 + 2x = (x + 1)^2 - 1$$

2. $x^2 + 4x$

2	2x	4
x	x^2	2x
	x	2

$$x^2 + 4x + 4 = (x + 2)^2$$

$$x^2 + 4x = (x + 2)^2 - 4$$

3. $x^2 + 12x$

6	6x	36
x	x^2	6x
	x	6

$$x^2 + 12x + 36 = (x + 6)^2$$

$$x^2 + 12x = (x + 6)^2 - 36$$

4. $x^2 + 9x$

$\frac{9}{2}$	$\frac{9}{2}x$	$\frac{81}{4}$
x	x^2	$\frac{9}{2}x$
	x	$\frac{9}{2}$

$$x^2 + 9x + \frac{81}{4} = \left(x + \frac{9}{2}\right)^2$$

$$x^2 + 9x = \left(x + \frac{9}{2}\right)^2 - \frac{81}{4}$$

TOPIC 3 Solving Quadratic Equations

5. $x^2 + 11x$

$\frac{11}{2}$	$\frac{11}{2}x$	$\frac{121}{4}$
x	x^2	$\frac{11}{2}x$

$$x \qquad \frac{11}{2}$$

$$x^2 + 11x + \frac{121}{4} = \left(x + \frac{11}{2}\right)^2$$

$$x^2 + 11x = \left(x + \frac{11}{2}\right)^2 - \frac{121}{4}$$

6. $x^2 + 28x$

14	14x	196
x	x^2	14x

$$x \qquad 14$$

$$x^2 + 28x + 196 = (x + 14)^2$$

$$x^2 + 28x = (x + 14)^2 - 196$$

F. Determine the unknown value that would make each trinomial a perfect square. Prerequisite of TEKS A.8A

1. $x^2 - 10x + \underline{\hspace{2cm}}$
25

2. $x^2 + 14x + \underline{\hspace{2cm}}$
49

3. $x^2 + \underline{\hspace{2cm}}x + 9$
6

4. $x^2 - \underline{\hspace{2cm}}x + 81$
18

5. $x^2 + 7x + \underline{\hspace{2cm}}$
 $\frac{49}{4}$

6. $x^2 - 15x + \underline{\hspace{2cm}}$
 $\frac{225}{4}$

7. $x^2 - \underline{\hspace{2cm}}x + 169$
26

8. $x^2 + \underline{\hspace{2cm}}x + \frac{9}{4}$
3



TOPIC 3 Solving Quadratic Equations

G. Determine the solutions of each quadratic equation by completing the square. Round your answer to the nearest hundredth. Check your answer algebraically. TEKS A.8A (Topic-Aligned), TEKS A.10E (Topic-Aligned)

1. $x^2 + 4x - 6 = 0$

$x = 1.16$ and $x = -5.16$

Check:

$(1.16)^2 + 4(1.16) - 6 \approx 0$

$(-5.16)^2 + 4(-5.16) - 6 \approx 0$

2. $x^2 - 2x - 4 = 0$

$x = 3.24$ and $x = -1.24$

Check:

$(3.24)^2 - 2(3.24) - 4 \approx 0$

$(-1.24)^2 - 2(-1.24) - 4 \approx 0$

3. $x^2 + 10x + 2 = 0$

$x = -0.20$ and $x = -9.80$

Check:

$(-0.20)^2 + 10(-0.20) + 2 \approx 0$

$(-9.80)^2 + 10(-9.80) + 2 \approx 0$

4. $x^2 - 12x + 25 = 0$

$x = 9.32$ and $x = 2.68$

Check:

$(9.32)^2 - 12(9.32) + 25 \approx 0$

$(2.68)^2 - 12(2.68) + 25 \approx 0$

5. $x^2 + 3x - 1 = 0$

$x = 0.30$ and $x = -3.30$

Check:

$(0.30)^2 + 3(0.30) - 1 \approx 0$

$(-3.30)^2 + 3(-3.30) - 1 \approx 0$

6. $x^2 + x - 10 = 0$

$x = 2.70$ and $x = -3.70$

Check:

$(2.70)^2 + (2.70) - 10 \approx 0$

$(-3.70)^2 + (-3.70) - 10 \approx 0$



TOPIC 3 Solving Quadratic Equations

Extension

The function g is defined by $g(x) = x^2 - 3x - 10$. If $g(x + 3) = x^2 + bx - c$, what are the values of b and c ? Show your work and justify your answer.

Extension of TEKS A.7C, A.10B, A.12B

$$\begin{aligned} g(x + 3) &= (x + 3)^2 - 3(x + 3) - 10 \\ &= x^2 + 6x + 9 - 3x - 9 - 10 \\ &= x^2 + 3x - 10 \end{aligned}$$

So, $b = 3$ and $c = 10$.

Justifications will vary.

Spaced Practice

- For each quadratic function, determine if it has an absolute minimum or absolute maximum and if the graph opens upward or downward. Identify the y-intercept. **TEKS A.7A**

a. $f(x) = 3x^2 + 6x - 72$

absolute minimum; opens upward; y-intercept = $(0, -72)$

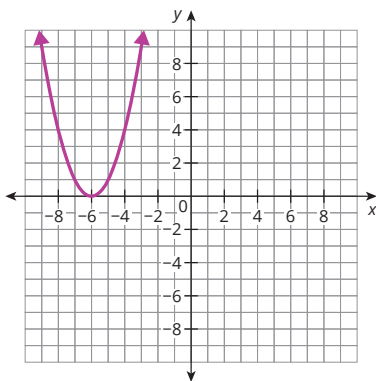
b. $f(x) = -\frac{1}{2}(x - 2)(x + 5)$

absolute maximum; opens downward; y-intercept = $(0, 5)$

- Sketch a graph of each quadratic function. Determine the zeros of each function. **TEKS A.7A**

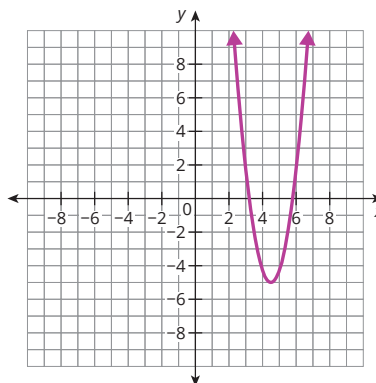
a. $f(x) = (x + 6)^2$

$x = -6$; double root



b. $f(x) = \left(3x - \frac{9}{2}\right)^2 - 5$

$x = \frac{9}{2} \pm \sqrt{\frac{5}{3}}$

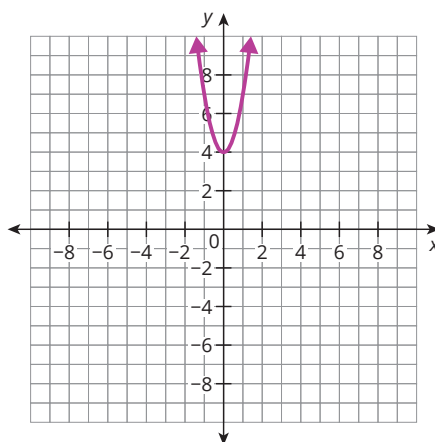


TOPIC 3 Solving Quadratic Equations

3. Write the equation of the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2$ by reflecting it across the x -axis, vertically compressing it by a factor of $\frac{1}{2}$, and moving it down 3 units. **TEKS A.7C**

$$f(x) = -\frac{1}{2}x^2 - 3$$

4. Graph the function, $g(x)$, in which its graph transforms the graph $f(x) = x^2$ by vertically stretching it by a factor of 3 and moving it up 4 units. **TEKS A.7C**



Determine the quotient for each problem. **TEKS A.10C**

5. $(6x^2 - 54) \div (2x - 6)$

$$3x + 9$$

6. $(6x^2 - 16x - 32) \div (3x + 4)$

$$2x - 8$$



IV. The Quadratic Formula

Topic Practice

A. Solve each quadratic equation. TEKS A.8A (Topic-Aligned)

1. A water balloon is thrown upward from a height of 5 feet with an initial velocity of 35 feet per second. The quadratic function $h(t) = -16t^2 + 35t + 5$ represents the height of the balloon, h , in feet t seconds after it is thrown. How long does it take for the balloon to reach the ground? Round your answer to the nearest tenth.

$$0 = -16t^2 + 35t + 5$$

$$a = -16, b = 35, c = 5$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(35) \pm \sqrt{(35)^2 - 4(-16)(5)}}{2(-16)}$$

$$t = \frac{-35 \pm \sqrt{1225 + 320}}{-32}$$

$$t = \frac{\sqrt{-35 \pm 1545}}{-32}$$

$$t = \frac{-35 + 39.306}{-32}$$

$$t \approx -0.1346$$

or

$$t = \frac{-35 - 39.306}{-32}$$

$$t \approx 2.322$$

It will take just over 2.3 seconds for the balloon to reach the ground.

2. Gabriela has saved up some money and decides to take a risk and invest in some stocks. She invests her money in Doogle, a popular computer company. Unfortunately, she lost it all over a matter of months. The change in her money during this investment can be represented by the function $v(x) = 75 + 72x - 3x^2$, where v is the value of her investment and x is the time in months. Determine when her portfolio reached a value of \$360.

$$360 = 75 + 72x - 3x^2$$

$$0 = -285 + 72x - 3x^2$$

$$0 = -3(95 - 24x + x^2)$$

$$0 = 95 - 24x + x^2$$

$$0 = (x - 5)(x - 19)$$

$$x = 5 \text{ and } x = 19$$

The portfolio reached a value of \$360 when it had been invested for 5 months and when it had been invested for 19 months.



TOPIC 3 Solving Quadratic Equations

3. Alexander invested some of his money in Home-mart, a large home improvement store in his town. A few years after investing, the company went out of business and Alexander lost all his money. The growth and decline of his money over this time can be represented by the function $v(x) = -2x^2 + 98x + 100$, where v is the value of his investment and x is the time in months. When did Alexander have \$1288.00 in his account?

$$1288 = -2x^2 + 98x + 100$$

$$0 = -1188 - 2x^2 + 98x$$

$$0 = -2(594 + x^2 - 49x)$$

$$0 = 594 + x^2 - 49x$$

$$0 = (x - 22)(x - 27)$$

$$x = 22 \text{ and } x = 27$$

Alexander had \$1288.00 in his account at months 22 and 27.

4. Jacob throws a softball up in the air. The height of the ball in meters can be determined by the function $h(t) = -4.9(t - 3)^2 + 60$, where t is the time it is in the air in seconds. When will the ball reach a height of 20 feet? Round your answer to the nearest tenth.

$$20 = -4.9(t - 3)^2 + 60$$

$$-40 = -4.9(t - 3)^2$$

$$0.0978 \approx (t - 3)^2$$

$$\pm 0.3127 \approx t - 3$$

$$-0.3127 \approx t - 3 \qquad 0.3127 \approx t - 3$$

$$t \approx 2.6873 \quad \text{and} \quad t \approx 3.3127$$

The ball will reach a height of 20 feet approximately 2.7 and 3.3 seconds after it is thrown.



TOPIC 3 Solving Quadratic Equations

5. A company knows that the more it advertises the more product it will sell. However, advertising more will also cost more money, which then takes away some of the profit. The profit will follow the path of a parabola because it will increase from more advertising but eventually decrease if too much money is spent on advertising. The profit (in thousands of dollars) can be represented by the function $P(x) = -2x^2 + 14x + 60$, where x represents the amount of money spent (in thousands of dollars). How much money should the company spend on advertising if they want their profit to be \$24,000?

$$24 = -2x^2 + 14x + 60$$

$$0 = -2x^2 + 14x + 36$$

$$0 = -2(x^2 - 7x - 18)$$

$$0 = x^2 - 7x - 18$$

$$0 = (x - 9)(x + 2)$$

$$x = 9 \quad \text{and} \quad x = -2$$

$x = -2$ doesn't make sense in the problem situation.

The company should spend \$9000 on advertising to make a profit of \$24,000.

6. The owners of a botanical park would like to create a walkway around one of their premier gardens. The garden is 100 feet long and 100 feet wide. The drawing below shows the layout of the garden and walkway. The function $A(x) = 4(x + 50)^2$ represents the total area of the garden and walkway when x represents the width of the walkway. What should the width of the walkway be if the owners want the total area to be 12,100 square feet?

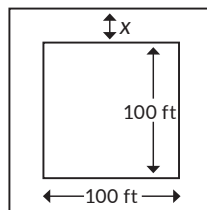
$$12,100 = 4(x + 50)^2$$

$$3025 = (x + 50)^2$$

$$\pm 55 = x + 50$$

$$-55 = x + 50 \qquad 55 = x + 50$$

$$x = -5 \quad \text{and} \quad x = 5$$



$x = -5$ does not make sense in the problem situation.

The width of the walkway should be 5 feet if the owners want the total area to be 12,100 square feet.



TOPIC 3 Solving Quadratic Equations

B. Use the quadratic formula to determine the axis of symmetry, the vertex, and the roots of each function. **TEKS A.7A (Topic-Aligned),**
TEKS A.8A (Topic-Aligned)

1. $f(x) = x^2 + 2x - 2$

Axis of Symmetry: $x = -1$

Vertex: $(-1, -3)$

Roots: $x = -1 \pm \sqrt{3}$

3. $f(x) = 2x^2 - 12x + 13$

Axis of Symmetry: $x = 3$

Vertex: $(3, -5)$

Roots: $x = 3 \pm \frac{\sqrt{10}}{2}$

5. $f(x) = -3x^2 - 18x - 25$

Axis of Symmetry: $x = -3$

Vertex: $(-3, 2)$

Roots: $x = -3 \pm \frac{\sqrt{6}}{3}$

2. $f(x) = -x^2 + 6x - 7$

Axis of Symmetry: $x = 3$

Vertex: $(3, 2)$

Roots: $x = 3 \pm \sqrt{2}$

4. $f(x) = -x^2 - 2x + 4$

Axis of Symmetry: $x = -1$

Vertex: $(-1, 5)$

Roots: $x = -1 \pm \sqrt{5}$

6. $f(x) = -x^2 + 10x - 26$

Axis of Symmetry: $x = 5$

Vertex: $(5, -1)$

Roots: no real roots



TOPIC 3 Solving Quadratic Equations

C. Determine the zeros or roots of each function or equation.

TEKS A.7A (Topic-Aligned), TEKS A.8A (Topic-Aligned)

1. $f(x) = -2x^2 + 5x - 1$

$$x = \frac{5 - \sqrt{17}}{4} \text{ or } x = \frac{5 + \sqrt{17}}{4}$$

3. $f(x) = 9x^2 + 5x + 1$

The function has no real zeros.

5. $f(x) = 5x^2 + 10x + 5$

$$x = -1$$

2. $-3x^2 + 8x - 2 = -6$

$$x = \frac{4 + 2\sqrt{7}}{3} \text{ or } x = \frac{4 - 2\sqrt{7}}{3}$$

4. $6x^2 + 3x - 5 = 2$

$$x = \frac{-3 + \sqrt{177}}{12} \text{ or } x = \frac{-3 - \sqrt{177}}{12}$$

6. $f(x) = 7x^2 + 9x + 5$

The function has no real zeros.



TOPIC 3 Solving Quadratic Equations

Extension

Consider the function $f(x) = -2x^2 + bx - 5$. Determine the b -value(s) that would ensure the function has two real roots. Explain your reasoning. **Extension of TEKS A.8A**

$$b > \sqrt{40} \text{ or } b < -\sqrt{40}$$

To have two real roots, the discriminant $b^2 - 4ac$ must be positive.

$$b^2 - 4ac > 0$$

$$b^2 - 4(-2)(-5) > 0$$

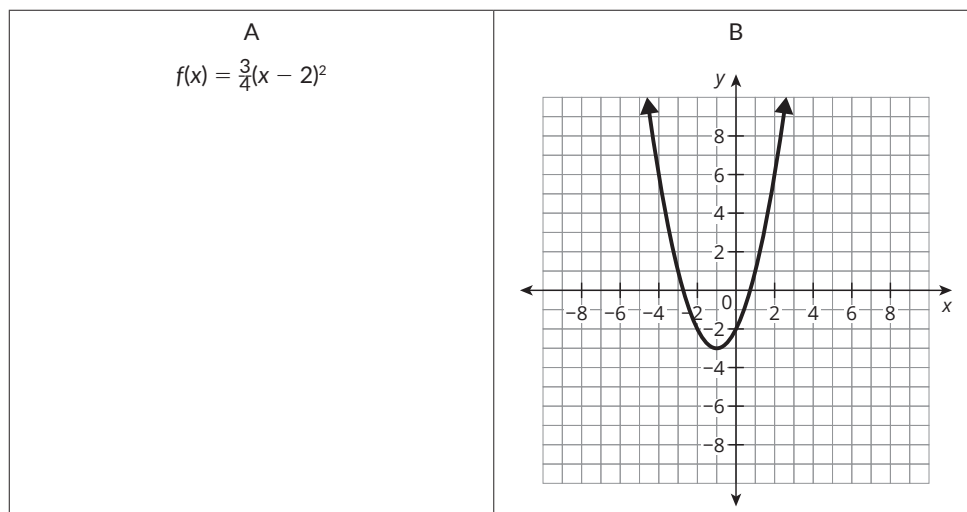
$$b^2 - 40 > 0$$

$$b^2 > 40$$

$$b > \sqrt{40} \text{ or } b < -\sqrt{40}$$

Spaced Practice

1. Analyze each pair of representations. Then, answer each question and justify your reasoning. **TEKS A.7A**
 - a. Which function has a greater y-intercept?



Function A

The y-intercept for Function A is (0, 3). The y-intercept for Function B is (0, -2).

TOPIC 3 Solving Quadratic Equations

- b. Which function has a greater absolute maximum?

A		B
		$f(x) = -2x^2 + 15x - 4$
x	y	
0	24	
3	25	
6	24	

Function B

The y-value for the absolute maximum of Function A is 25. The y-value for the absolute maximum of Function B is 24.125.

2. Complete the square to determine the roots of each equation.
Show your work. **TEKS A.8A**

a. $y = 2x^2 + 5x - 14$

Check students' work.

$x \approx -4.176$ and $x \approx 1.676$

b. $y = -3x^2 - 6x + 10$

Check students' work.

$x \approx 1.082$ and $x \approx -3.082$

3. Consider the function $f(x) = \left(x + \frac{1}{2}\right)\left(x - \frac{3}{4}\right)$. **TEKS A.7A**

- a. Identify the form of the function as factored, standard, or vertex.

factored form

- b. Identify the zeros and axis of symmetry of the function.

The roots are $-\frac{1}{2}$ and $\frac{3}{4}$.

The axis of symmetry is $x = \frac{1}{8}$.



TOPIC 3 Solving Quadratic Equations

V. Using Quadratic Functions to Model Data

Topic Practice

- A. Use technology to determine the quadratic function that models each data set. Round decimals to the nearest thousandth.

TEKS A.8B (Topic-Aligned)

1.

x	y
1	22
6	6
20	20
42	64
80	99

$$y = 0.002x^2 + x + 9.259$$

2.

x	y
12	38
24	12
30	3
40	16
54	54

$$y = 0.090x^2 - 5.527x + 91.384$$

3.

x	y
25	60
50	80
75	140
100	210
125	250
150	322
175	400

$$y = 0.006x^2 + 1.201x + 19.714$$

4.

x	y
10	14
20	36
30	70
40	120
50	180
60	240
70	350

$$y = 0.070x^2 - 0.169x + 10.571$$

TOPIC 3 Solving Quadratic Equations

5.

x	y
0.1	8
0.2	12
0.3	14
0.4	16
0.5	18
0.6	10
0.7	2

$$y = -130.952x^2 + 98.333x - 1.714$$

6.

x	y
0.1	2.2
0.2	3.1
0.3	4
0.4	4.8
0.5	5.4
0.6	6
0.7	4.2

$$y = -18.333x^2 + 19.381x + 0.157$$



TOPIC 3 Solving Quadratic Equations

- B. Use technology to write a quadratic function that can be used to model each scenario. Then, determine the y-intercept, x-intercepts, domain, range, and vertex of the graph. Interpret these characteristics and explain if they make sense in terms of the scenario.

TEKS A.6A (Topic-Aligned), TEKS A.7A (Interleaved Practice), TEKS A.8B (Interleaved Practice)

1. The data table represents the speed of a car, s , in miles per hour and the car's average fuel efficiency, g , in miles per gallon at that speed.

s (mph)	g (mpg)
10	8.3
11	9.8
12	11.4
13	12.7
14	14.7
15	15.6
16	17.8
17	18.7
18	19.9
19	21.2

The quadratic regression model that fits the data for the scenario is $g(s) = -0.024s^2 + 2.15s - 10.934$.

The y-intercept is the point $(0, -10.9)$. The y-intercept represents the fuel efficiency of the car when it is traveling at 0 miles per hour (at a stop). Because the fuel efficiency is a negative number, this value does not make sense in terms of the scenario.

The x-intercepts are the points $(5.4, 0)$ and $(83.5, 0)$. The x-intercepts represent the speed of the car when the fuel efficiency is zero. The fuel efficiency would be zero when the car is not moving, not when the car is traveling 5.4 miles per hour or 83.5 miles per hour. So, these values do not make sense in terms of the scenario.

The domain of the function is all real numbers. However, in the problem scenario, s must be greater than or equal to zero. It does not make sense for speed to be negative. The range of the function is $y \leq 36.9$. However, fuel efficiency cannot be less than 0 so the range of the problem scenario is $0 \leq y \leq 36.9$.

The vertex is the point $(44.4, 36.9)$. The vertex represents the speed at which the car is getting the highest fuel efficiency. So, when the car is traveling at 44.4 miles per hour, it gets its highest fuel efficiency, which is 36.9 miles per gallon. This value makes sense in terms of the scenario.



TOPIC 3 Solving Quadratic Equations

2. An athlete throws a disc upward at an angle. The data table represents height in feet, h , of the disc and the distance in feet, d , that the disc has traveled horizontally.

d (ft)	h (ft)
1	6.0
2	6.4
3	6.8
6	8.2
7	8.7
8	9.1
10	9.9
11	10.4
13	11.2
14	11.6

The quadratic regression model that fits the data for the scenario is $h(d) = -0.002d^2 + 0.466d + 5.487$.

The y -intercept is the point $(0, 5.5)$. The y -intercept represents the height of the disc when it has traveled 0 feet horizontally. In other words, this is the starting point. This value makes sense because the athlete is holding the disc at the starting point, and a height of 5.5 feet seems reasonable.

The x -intercepts are the points $(-11.2, 0)$ and $(235.6, 0)$. The x -intercepts represent the distance the disc has traveled when it is at a height of 0 feet, or on the ground. The point $(-11.2, 0)$ does not make sense because it represents a negative distance. The point $(235.6, 0)$ makes sense because it means that the disc had traveled a horizontal distance of 235.6 feet when it hit the ground.

The domain of the function is all real numbers. However, in the problem scenario, d must be greater than zero. It does not make sense for the distance to be negative. The range of the function is $y \leq 31.7$. However, it does not make sense for the height to be less than 0. The range of the problem scenario is $0 \leq y \leq 31.7$.

The vertex is the point $(112.2, 31.7)$. The vertex represents the horizontal distance the disc has traveled when it is at its highest point vertically. So, when the disc travels 110 feet, it is at its highest point, which is 31.7 feet. This value makes sense in terms of the scenario.



TOPIC 3 Solving Quadratic Equations

3. Amir and his friend are playing catch with a baseball. Amir tosses the ball to his friend, but he overthrows it, and it hits the ground. The data table represents the height in feet, h , of the baseball and the distance in feet, x , that the baseball has traveled horizontally.

x (ft)	h (ft)
1	5.8
2	6.7
3	7.1
4	8.2
5	9.0
8	10.6
9	11.6
12	13.4
13	13.4
14	14.3

The quadratic regression model that fits the data for the scenario is $h(x) = -0.013x^2 + 0.835x + 4.97$.

The y -intercept is the point $(0, 4.97)$. The y -intercept represents the height of the baseball when it has traveled 0 feet horizontally. In other words, this is the starting point. This value makes sense because Amir is holding the ball at the starting point, and a height of 4.97 feet for Amir seems reasonable.

The x -intercepts are the points $(-5.5, 0)$ and $(71.8, 0)$. The x -intercepts represent the distance the ball has traveled when it is at a height of 0 feet, or on the ground. The point $(-5.5, 0)$ does not make sense because it represents a negative distance. The point $(71.8, 0)$ makes sense because it means that the ball traveled a horizontal distance of 71.8 feet after Amir threw it before it hit the ground.

The domain of the function is all real numbers. However, in the problem scenario, x must be greater than zero. It does not make sense for the distance to be negative. The range of

the function is $y \leq 18.8$. However, it does not make sense for the height to be less than 0. The range of the problem scenario is $0 \leq y \leq 18.8$.

The vertex is the point $(33.2, 18.8)$. The vertex represents the horizontal distance the ball has traveled when it is at its highest point vertically. So, when the ball travels 33.2 feet, it is at its highest point, which is 18.8 feet. This value makes sense in terms of the scenario.



4. The data table represents a company's profit in dollars, p , and the number of years, x , since the company was started.

x (years)	p (dollars)
3	379.45
4	3686.16
5	6732.61
6	9630.78
7	12102.85
8	14006.43
9	16830.09
10	17817.00
11	19758.89
12	21312.82

The quadratic regression model that fits the data for the scenario is $p(x) = -118.658x^2 + 4081.88x + 10735$.

The y-intercept is the point $(0, -10734.99)$. The y-intercept represents the company's profit after they have been in business for 0 years. In other words, this is the starting point. This value makes sense because a negative number represents a loss, which could be attributed to start-up costs.

The x-intercepts are the points $(2.869, 0)$ and $(31.531, 0)$. The x-intercepts represent the number of years since the company was started when the profit was 0 dollars, or they broke even. Both points make sense. The points mean that the company did not make a profit until about 3 years after they started, and they broke even about 31 years after they started.

The domain of the function is all real numbers. However, in the problem scenario, x must be greater than or equal to 0. It does not make sense to talk about profit before the

company was started. The range of the function is $y \leq 24,368.71$. This makes sense in terms of the problem scenario because the company can have a negative profit if it spends more than it makes.

The vertex is the point $(17.2, 24369.71)$. The vertex represents the number of years that the company has been in business and the company's profit at that time. When the company has been in business for about 17 years, they reach their highest profit of approximately \$24,369.71. This value makes sense in terms of the problem situation.



TOPIC 3 Solving Quadratic Equations

5. The data table represents temperatures, t , in degrees Fahrenheit recorded during a 10-hour winter snowstorm and the number of hours, h , the storm has lasted.

h (hours)	t (°F)
2	21.8
3	16.8
4	14.1
5	12.8
6	13.4
7	14.9
8	18.0

The quadratic regression model that fits the data for the scenario is $t(h) = 0.777h^2 - 8.34h + 35.136$.

The y-intercept is the point (0, 35.14). The y-intercept represents the temperature at the beginning of the storm. This is the starting point. This value makes sense because the temperature is cold enough for a winter storm.

There are no x-intercepts for this graph. This means that the temperature never reached 0°. This makes sense because it would be rare for the temperature to drop from 35 to 0 degrees in a short amount of time.

The domain of the function is all real numbers. However, in the problem scenario, h must be greater than or equal to zero. The model measured the temperature from the time

the storm started so it would not be an appropriate measure of the temperature before the storm. It also would not be accurate to use the model to predict the temperature after the storm. Therefore, the domain of the problem situation is $0 \leq x \leq 10$. The range of the function is $y \geq 12.76$. This the problem scenario, the temperature is limited to the 10 hours during which the storm occurred. The temperature was the highest when the storm began. The range of the problem situation is $12.76 \leq y \leq 35.14$.

The vertex is the point (5.4, 12.76). The vertex represents the lowest temperature during the storm. This makes sense because after about 5.5 hours, the temperature dropped to about 13°F.



TOPIC 3 Solving Quadratic Equations

6. The data table represents a company's daily profit, p , from selling graphing calculators and the price of the graphing calculator, x .

x (dollars)	p (dollars)
45	4996.9
46	5614.2
47	6167.7
48	6936.6
49	7471.3
71	18563.4
72	17368.1
82	19063
96	17926.1
110	13231.1

The quadratic regression model that fits the data for the scenario is $p(x) = -9.369x^2 + 1579.85x - 47275.42$.

The y-intercept is the point $(0, -47275.42)$. The y-intercept represents the company's daily profit if they sell the calculators for \$0 that day. This makes sense because the negative number represents a loss, which could be attributed to the cost of making the calculator.

The x-intercepts are the points $(38.896, 0)$ and $(129.726, 0)$. The x-intercepts represent the selling price of the calculators when the profit is \$0. This makes sense. If they sell the calculator for too little, they will not make a profit because the cost of making the calculator will be greater than the amount it sold for. If they sell the calculator for too much, they will not make a profit because no one will buy the calculator.

The domain of the function is all real numbers. However, in the problem scenario, x must be greater than zero. It does not make sense to sell the calculators for less than \$0. The range of the function is $y \leq 19,323.98$. The company would not lose

any more than the amount if they sold the calculators for \$0. The range of the problem scenario is $-47,275.42 \leq y \leq 19,323.98$.

The vertex is the point $(84.31, 19323.98)$. The vertex represents the selling price that results in the largest daily profit. This point makes sense because a selling price of \$84.31 is reasonable and could result in a profit of \$19,323.98.



TOPIC 3 Solving Quadratic Equations

Extension

Use your graphing technology to determine the quadratic regression model and coefficient of determination for the line of best fit of the given data set. Determine if the model is a good fit for the data. Would a linear function be a better model? Round your answers to the nearest hundredth. **Extension of TEKS A.4C, A.8B**

x	y
-5	-0.5
-4	5
-1	7
0	3
0.5	-1
1	3

$$y = -0.73x^2 - 2.93x + 3.84$$

r^2 is approximately 0.55.

Because the r^2 -value is not close to 1, the quadratic regression model is not a good fit for the data.

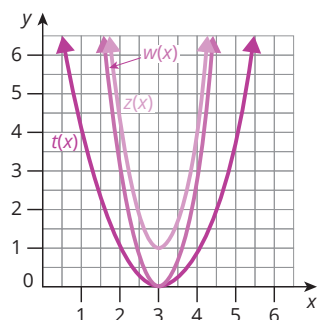
No. The linear regression model has a correlation coefficient of approximately 0.04.



Spaced Practice

1. Consider each function shown. **TEKS A.7C**

- a. Graph each function on the same coordinate plane.



$$t(x) = (x - 3)^2$$

$$w(x) = 3(x - 3)^2$$

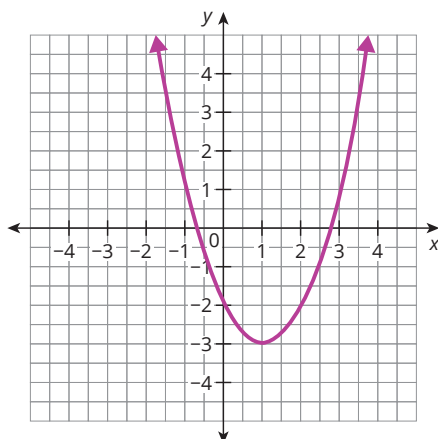
$$z(x) = 3(x - 3)^2 + 1$$

- b. Describe how functions w and z have been transformed from function t .

$w(x)$ is dilated vertically by the factor 3. $z(x)$ is also dilated vertically by the factor 3 and translated up 1 unit.

2. Consider the function $f(x) = x^2 - 2x - 2$. **TEKS A.7A**

- a. Graph the function.



TOPIC 3 Solving Quadratic Equations

- b. Describe the key characteristics of the graph.

decreasing: $-\infty < x \leq 1$

increasing: $1 \leq x < \infty$

minimum value = $(1, -3)$

x-intercepts = $(-0.73, 0)$ and $(2.73, 0)$

y-intercept = $(0, -2)$

3. The cost of producing chapter books for a company is $C(x) = 6x + 81$. The company's revenue for every chapter book sold is $R(x) = 36x - x^2$. **TEKS A.8A**

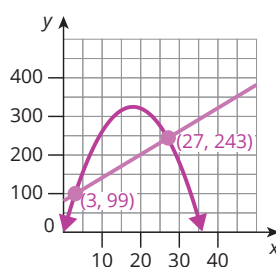
- a. What is the company's break-even point for the production and sales of chapter books?

$x = 3$ and $x = 27$

- b. What does the solution mean?

$(3, 99)$ and $(27, 243)$; If the company makes 3 books a month, then the cost and revenue will be \$99. If the company makes 27 books a month, then the cost and revenue will be \$243.

- c. Show the solution graphically.



4. Factor each trinomial. **TEKS A.10E**

a. $x^2 + 4x - 21$

$(x + 7)(x - 3)$

b. $5x^2 - 47x + 18$

$(5x - 2)(x - 9)$

TOPIC 3 Solving Quadratic Equations

5. Use technology to determine the quadratic function that models each data set. Round decimals to the nearest hundredth. **TEKS A.8B**

a.

x	y
5	18
7	13
10	9
13	7
17	5

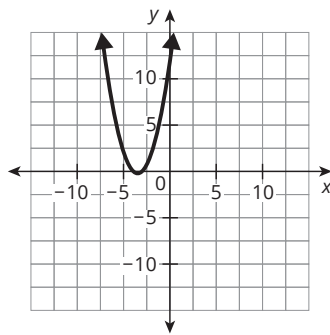
$$y = 0.09x^2 - 3.05x + 30.52$$

b.

x	y
-6	9
-4	3
2	8
5	9
12	18

$$y = 0.06x^2 + 0.25x + 6.09$$

6. Write a function to represent the quadratic function shown on the grid. **TEKS A.6C**



Sample answer:

$$f(x) = x^2 + 7x + 12$$



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