OER Secondary Mathematics
EDITION 1

Algebra I

Volume 1

TEACHER EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

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Secondary Mathematics

EDITION 1

Algebra I

Course and Implementation Guide

Welcome to the Course Implementation Guide for Secondary Mathematics, Algebra I

The Instructional Approach

These instructional materials follow the path that has been proven most effective by research and classroom experience. The instructional approach and design are informed by books like *Adding It Up: Helping Children Learn Mathematics* (National Research Council, 2001), *How People Learn: Brain, Mind, Experience, and School: Expanded Edition* (National Research Council, 2000), *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics, 2014).

The instructional approach utilized is a culmination of the collective knowledge of researchers, instructional designers, cognitive learning scientists, and master practitioners. The approach is based on a scientific understanding of how people learn as well as an understanding of how to apply the science to the classroom. At its core, the instructional approach is based on three key components:

ENGAGE

Getting Started

- Building Off Intuition
- Accessing Prior Mathematical Knowledge
- Establishing a Scenario

DEVELOP

Activities

- Problem Solving
- Classification
- Investigation
- Explicit Instruction
    - Worked Example
    - Peer Work Analysis

DEMONSTRATE

Talk the Talk

- Graphic Organizer
- Presentation
- Generalization
- Writing Task
- Procedure
- Application

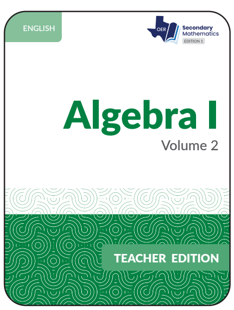
The Getting Started for each lesson activates student thinking by tapping into prior knowledge and real-world experiences. The Activities in each lesson vary in complexity and build a deep understanding of the mathematics. The Talk the Talk for each lesson is an opportunity to engage students in reflection and assess their learning.

Instructional Design

To provide both a toolkit of instructional strategies and opportunities for targeted individualized instruction, the course consists of both a Learning Together and a Learning Individually component. These two components work together to engage students with the various learning experiences they need to understand the mathematics in this course.

Learning Together

On **Learning Together** days, you spend time facilitating active learning so that students build their mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the student-facing materials for each lesson.



STUDENT EDITION

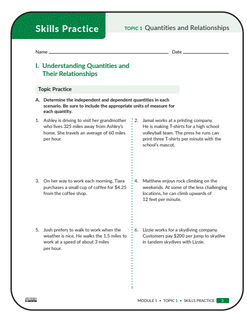
I am a record of student thinking, reasoning, and problem solving.

My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On **Learning Individually** days, you spend time on targeted instruction to meet the needs of each student. Skills Practice offers students the opportunity to engage with problems aligned to each lesson's essential ideas. It also provides opportunities for interleaved practice, which encourages students to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning.

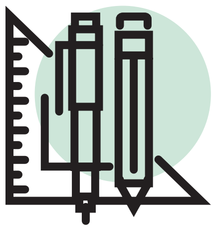


SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide additional problem sets for teachers to assign as needed for differentiated instruction, enrichment, and extension.

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure students thoroughly understand the mathematical concepts. The instructional materials guide students to connect related ideas holistically, supporting the integration of their evolving mathematical understanding and developing proficiency with mathematical processes.



Intentional Mathematics Design

Mathematical Coherence: The arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

TEKS Mathematical Process Standards: The instructional materials embody the TEKS mathematical process standards as they encourage experimentation, creativity, and various solution strategies. These mathematical processes empower students to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work (Chi et. al, 2008). The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture student interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for students to build their new understanding on the firm foundation of what they already know. Students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem Solving: Solving problems is an essential life skill that students need to develop. The problem-solving model provides a structure to support students as they analyze and solve problems. It is a strategy they can continue to use as they solve problems in everyday life.

Seeing Connections: Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. Activities focus on relevant real-world situations to demonstrate the usefulness of mathematics.

Exploring Structure: Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships.

Reflecting and Communicating: Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. During collaborative activities and peer work analysis, students give and receive feedback, which leads to verifying, clarifying, and/or improving the strategy. Using the Topic Self-Reflection and Assessment Reflection tools, students reflect on their own learning. This allows students to communicate where they need support, adjust strategies, and set learning goals.

Additional information on the principles that guide the design and organization of the materials is included in the Program and Implementation Guide.

Connecting Learning Experiences

The instructional development aids students in the effective transition from their intuitive understanding of the world to the abstract language of mathematics.

Once students have ample opportunities to build understanding, procedural problems and exercises are presented to increase computational fluency.

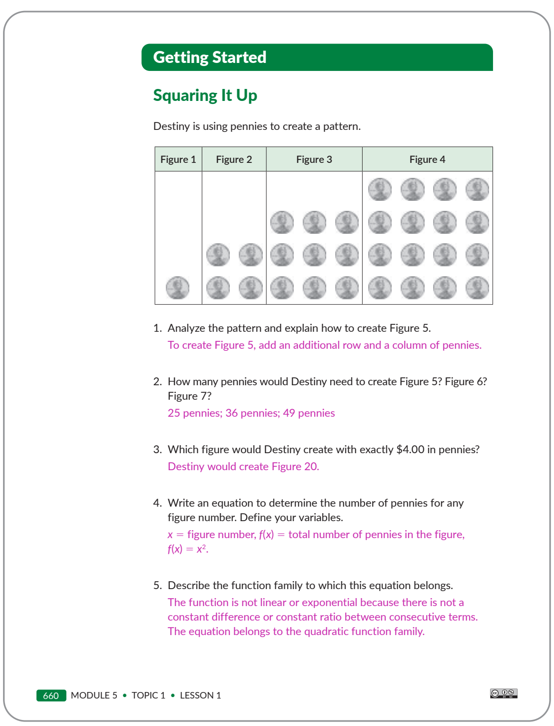
A thoughtful progression from the use of manipulatives and visual aids to representations and drawings that bridge to more abstract understanding benefits all students.

By linking these learning experiences, students can make meaningful connections. These progressions can occur across topics, across lessons, and within lessons.

One example of this sequence occurs in the topic *Introduction to Quadratic Functions*, In this topic, students determine the domain and range of quadratic functions and represent the domain and range using inequalities (**TEKS A.6A**). They graph quadratic functions on the coordinate plane and use the graph to identify key attributes, including x-intercepts, y-intercepts, zeros, maximum values, minimum values, vertex, and the equation of the axis of symmetry (**TEKS A.7A**).

Building Understanding

In Lesson 1, *Exploring Quadratic Functions*, students explore a real-world scenario to develop their intuitive understanding of quadratic functions. They recognize they can write the basic quadratic function, x^2, to represent the pattern.



Getting Started

Squaring It Up

Destiny is using pennies to create a pattern.

Figure 1	Figure 2	Figure 3	Figure 4

1. Analyze the pattern and explain how to create Figure 5.
   To create Figure 5, add an additional row and a column of pennies.
2. How many pennies would Destiny need to create Figure 5? Figure 6? Figure 7?
   25 pennies; 36 pennies; 49 pennies
3. Which figure would Destiny create with exactly $4.00 in pennies?
   Destiny would create Figure 20.
4. Write an equation to determine the number of pennies for any figure number. Define your variables.
   x = figure number, $f(x)$ = total number of pennies in the figure, $f(x) = x^2$.
5. Describe the function family to which this equation belongs.
   The function is not linear or exponential because there is not a constant difference or constant ratio between consecutive terms. The equation belongs to the quadratic function family.

660 MODULE 5 • TOPIC 1 • LESSON 1

Think About . . .
Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks.

Connecting Learning Experiences

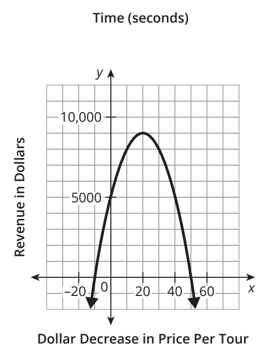
Representing the Concept

In the next lesson, *Key Characteristics of Quadratic Functions* students learn that different forms of quadratic functions reveal specific key characteristics of the graph.

Alamo Tours

$r(x) = -10(x + 10)(x - 50)$

$= -10x^2 + 400x + 5000$



x	$r(x)$
0	5000
1	5390
2	5760
3	6110
4	6440

Moving to Symbols

In Activity 2.1, students formalize their ability to recognize quadratic relationships represented by tables. They determine that a table represents a quadratic function when the second differences are constant.

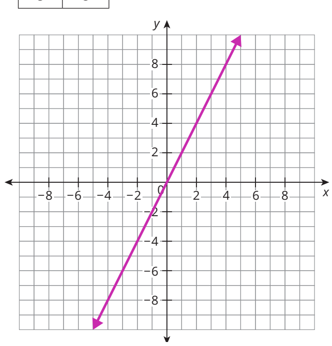
5. Identify each equation as linear or quadratic. Complete the table to calculate the first and second differences. Then, sketch the graph.

a. $y = 2x$ Linear

x	y
−3	−6
−2	−4
−1	−2
0	0
1	2
2	4
3	6

First Differences: 2, 2, 2, 2, 2, 2

Second Differences: 0, 0, 0, 0, 0

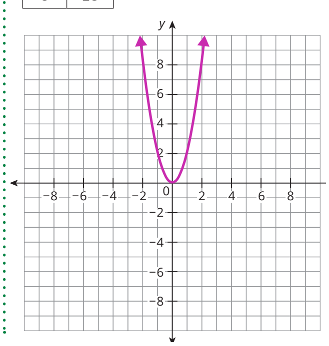


b. $y = 2x^2$ Quadratic

x	y
−3	18
−2	8
−1	2
0	0
1	2
2	8
3	18

First Differences: −10, −6, −2, 2, 6, 10

Second Differences: 4, 4, 4, 4, 4

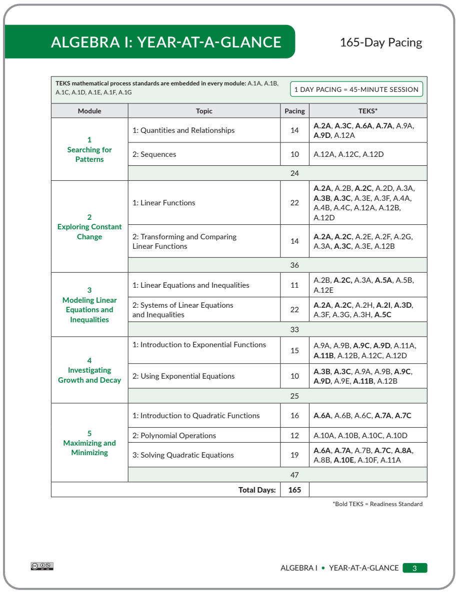


Look for similar sequences throughout the materials.

Course-Level Documents

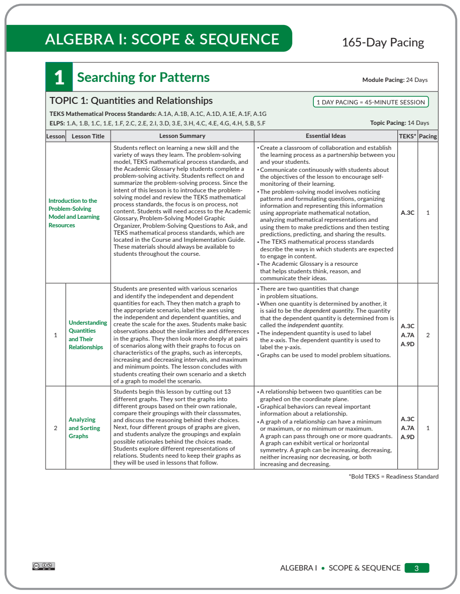
Year-at-a-Glance

The Year-at-a-Glance highlights the sequence of topics and the number of instructional days allocated for the course.



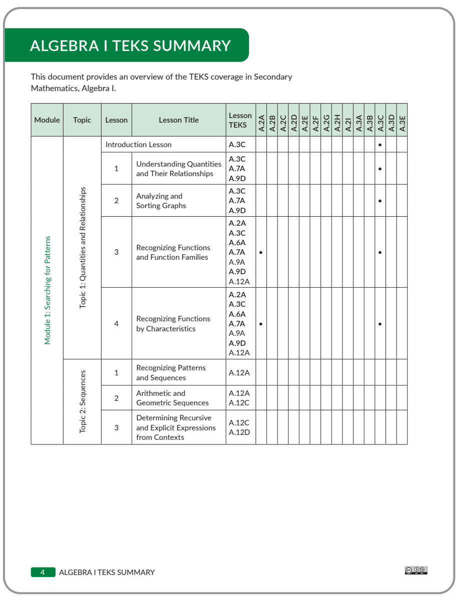
Scope and Sequence

The Scope and Sequence provides lesson highlights, pacing suggestions, and essential ideas for each lesson.



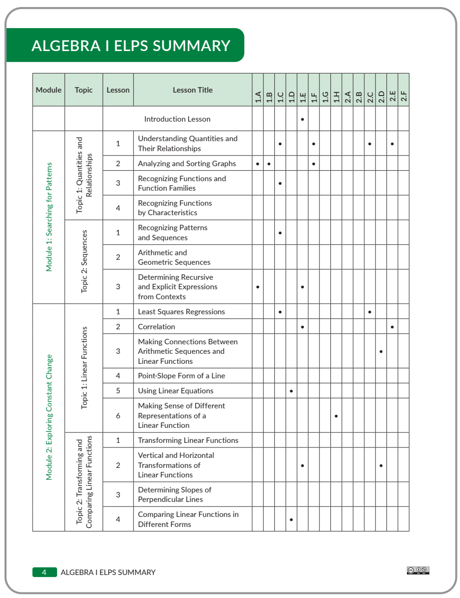
TEKS Summary

The TEKS Summary visually represents where each standard is addressed within the course.



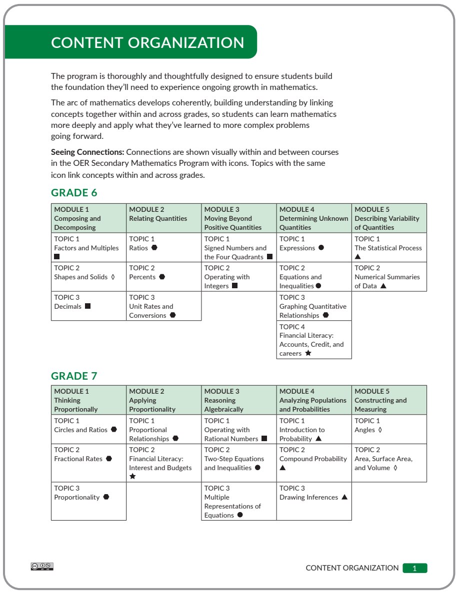
ELPS Summary

The ELPS Summary visually represents where each standard is addressed within the course.



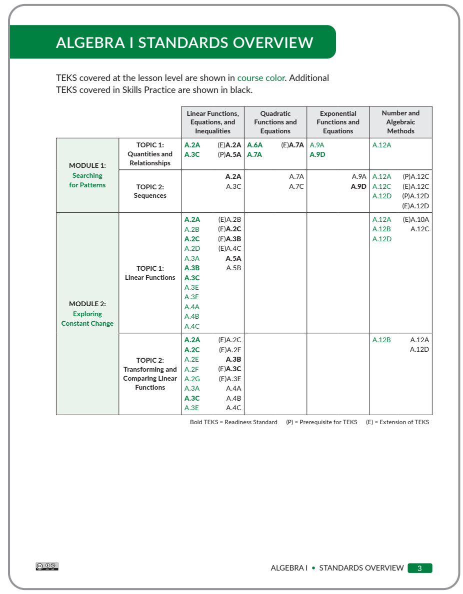
Content Organization

The Content Organization document provides insights into how the content is organized. It also identifies content connections within and across different courses.



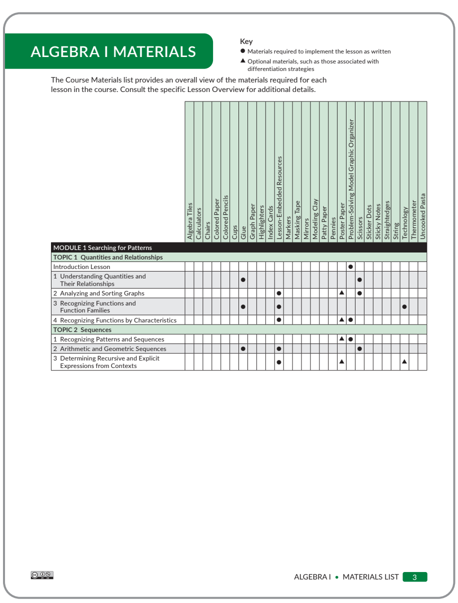
Standards Overview

The Standards Overview lists the standards addressed in each topic. The Standards Overview shows the lesson and skills practice standards.



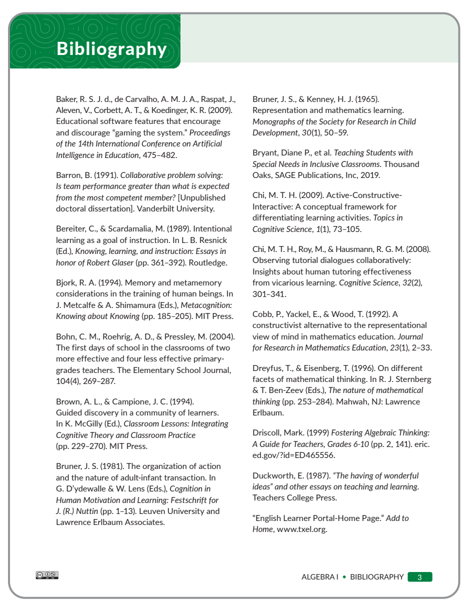
Materials List

The Materials List highlights the materials needed for the course.



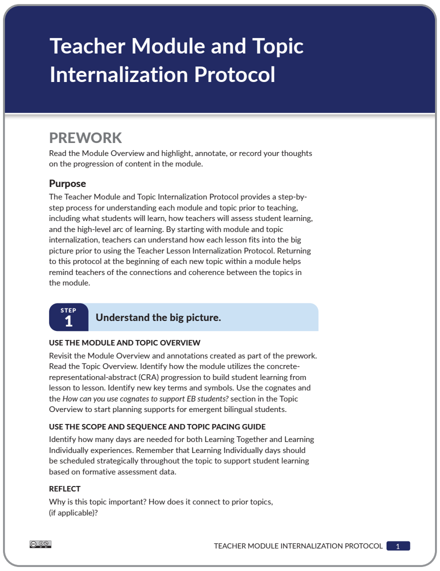
Bibliography

The Bibliography contains a list of peer-reviewed articles and research used to influence the content of the instructional materials.



Module and Topic Internalization Protocol

The Module and Topic Internalization Protocol supports understanding of what students will learn, how students will be assessed, and the high-level arc of learning over the course of the module and topic. There is a Teacher and Coach version of the Module and Topic Internalization Protocol.



Teacher Module and Topic Internalization Protocol

PREWORK

Read the Module Overview and highlight, annotate, or record your thoughts on the progression of content in the module.

Purpose

The Teacher Module and Topic Internalization Protocol provides a step-by-step process for understanding each module and topic prior to teaching, including what students will learn, how teachers will assess student learning, and the high-level arc of learning. By starting with module and topic internalization, teachers can understand how each lesson fits into the big picture prior to using the Teacher Lesson Internalization Protocol. Returning to this protocol at the beginning of each new topic within a module helps remind teachers of the connections and coherence between the topics in the module.

STEP 1 Understand the big picture.

USE THE MODULE AND TOPIC OVERVIEW

Revisit the Module Overview and annotations created as part of the prework. Read the Topic Overview. Identify how the module utilizes the concrete-representational-abstract (CRA) progression to build student learning from lesson to lesson. Identify new key terms and symbols. Use the cognates and the *How can you use cognates to support EB students?* section in the Topic Overview to start planning supports for emergent bilingual students.

USE THE SCOPE AND SEQUENCE AND TOPIC PACING GUIDE

Identify how many days are needed for both Learning Together and Learning Individually experiences. Remember that Learning Individually days should be scheduled strategically throughout the topic to support student learning based on formative assessment data.

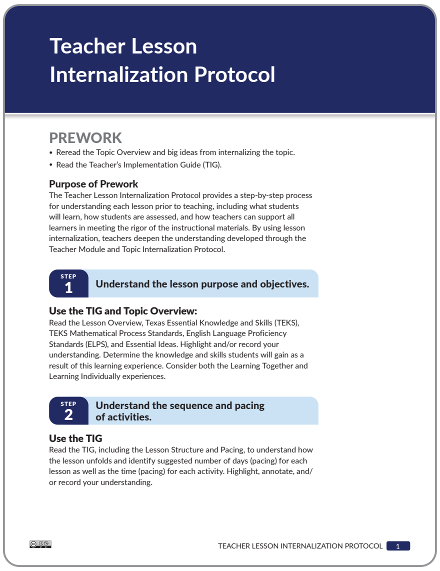
REFLECT

Why is this topic important? How does it connect to prior topics, (if applicable)?

TEACHER MODULE INTERNALIZATION PROTOCOL 1

Lesson Internalization Protocol

The Lesson Internalization Protocol focuses on what students will learn in a specific lesson, how students will be assessed, and helps determine how to make decisions about teaching the lesson to support all students' success. There is a Teacher and Coach version of the Lesson Internalization Protocol.



Teacher Lesson Internalization Protocol

PREWORK

- Reread the Topic Overview and big ideas from internalizing the topic.
- Read the Teacher's Implementation Guide (TIG).

Purpose of Prework

The Teacher Lesson Internalization Protocol provides a step-by-step process for understanding each lesson prior to teaching, including what students will learn, how students are assessed, and how teachers can support all learners in meeting the rigor of the instructional materials. By using lesson internalization, teachers deepen the understanding developed through the Teacher Module and Topic Internalization Protocol.

STEP 1 Understand the lesson purpose and objectives.

Use the TIG and Topic Overview:

Read the Lesson Overview, Texas Essential Knowledge and Skills (TEKS), TEKS Mathematical Process Standards, English Language Proficiency Standards (ELPS), and Essential Ideas. Highlight and/or record your understanding. Determine the knowledge and skills students will gain as a result of this learning experience. Consider both the Learning Together and Learning Individually experiences.

STEP 2 Understand the sequence and pacing of activities.

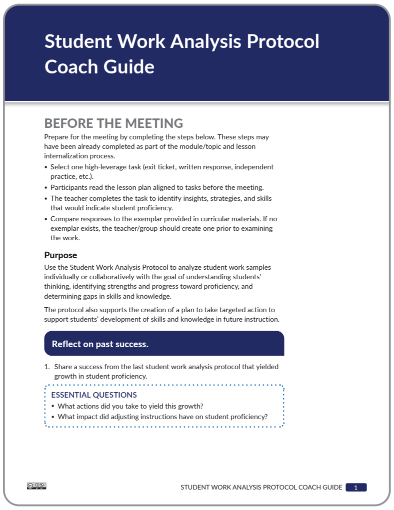
Use the TIG

Read the TIG, including the Lesson Structure and Pacing, to understand how the lesson unfolds and identify suggested number of days (pacing) for each lesson as well as the time (pacing) for each activity. Highlight, annotate, and/or record your understanding.

TEACHER LESSON INTERNALIZATION PROTOCOL 1

Student Work Analysis Protocol

The Student Work Analysis Protocol is a tool for analyzing student work samples individually or collaboratively to understand students' thinking, identify strengths and progress toward proficiency, and determine gaps in skills and knowledge. There is a Teacher and Coach version of the Student Work Analysis Protocol.



Student Work Analysis Protocol Coach Guide

BEFORE THE MEETING

Prepare for the meeting by completing the steps below. These steps may have been already completed as part of the module/topic and lesson internalization process.

- Select one high-leverage task (exit ticket, written response, independent practice, etc.).
- Participants read the lesson plan aligned to tasks before the meeting.
- The teacher completes the task to identify insights, strategies, and skills that would indicate student proficiency.
- Compare responses to the exemplar provided in curricular materials. If no exemplar exists, the teacher/group should create one prior to examining the work.

Purpose

Use the Student Work Analysis Protocol to analyze student work samples individually or collaboratively with the goal of understanding students' thinking, identifying strengths and progress toward proficiency, and determining gaps in skills and knowledge.

The protocol also supports the creation of a plan to take targeted action to support students' development of skills and knowledge in future instruction.

Reflect on past success.

1. Share a success from the last student work analysis protocol that yielded growth in student proficiency.

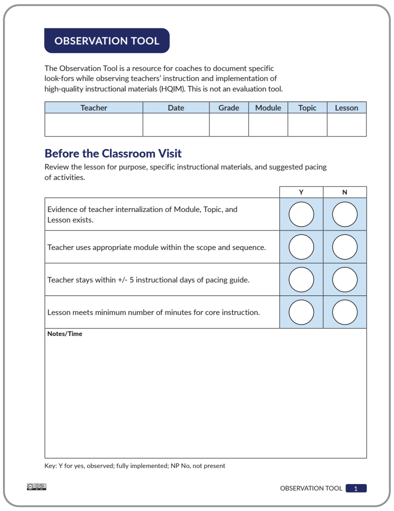
ESSENTIAL QUESTIONS

- What actions did you take to yield this growth?
- What impact did adjusting instructions have on student proficiency?

STUDENT WORK ANALYSIS PROTOCOL COACH GUIDE 1

Observation Tool

The Observation Tool is a resource to document specific look-fors while observing instruction and implementation of High-Quality Instructional Materials (HQIM). Teachers and coaches can use this document to reflect on instruction and implementation.



OBSERVATION TOOL

The Observation Tool is a resource for coaches to document specific look-fors while observing teachers' instruction and implementation of high-quality instructional materials (HQIM). This is not an evaluation tool.

Teacher	Date	Grade	Module	Topic	Lesson

Before the Classroom Visit

Review the lesson for purpose, specific instructional materials, and suggested pacing of activities.

	Y	N
Evidence of teacher internalization of Module, Topic, and Lesson exists.		
Teacher uses appropriate module within the scope and sequence.		
Teacher stays within +/- 5 instructional days of pacing guide.		
Lesson meets minimum number of minutes for core instruction.		

Notes/Time

Key: Y for yes, observed; fully implemented; NP No, not present

OBSERVATION TOOL 1

Connecting Content and Practice

LESSON STRUCTURE
Each lesson of the course has the same structure. This consistency allows both you and your students to internalize the lesson progression. Key features of each lesson are noted.

1 Objectives
Objectives are stated for each lesson to help you take ownership of the objectives.

2 Essential Question
Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3 New Key Terms
The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

4 Point-Slope Form of a Line

1 **OBJECTIVES**

- Use the slope formula to derive the point-slope form of a linear equation.
- Construct an equation in point-slope form to model a linear relationship between two quantities.
- Write equations for vertical and horizontal lines.

3 **NEW KEY TERM**

- point-slope form

2 You have used the slope-intercept form to represent linear relationships. Are there other forms of a linear equation that you can use? How do you write equations for horizontal and vertical lines?

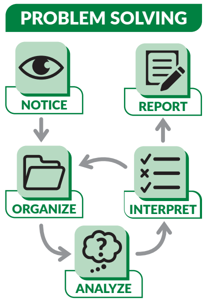
Engage

Establishing Mathematical Goals to Focus Learning

Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the objectives of the lesson to encourage self-monitoring of their learning.

4 Getting Started



Draining the Pool

Miguel and Nia are pool cleaners who have been hired to drain the community diving pools at the end of the summer. They are comparing the rate at which the three pools drain.

1. For each pool, write an equation in slope-intercept form to represent the linear relationship.

   a. Pool *A* is at a water level of 14 feet and drains at a rate of 3 feet per hour.

   b. Pool *B* is at a water level of 10 feet after draining for 3 hours and drains at a rate of 2 feet per hour.

   c. Pool *C* is at a water level of 15 feet after draining for 2 hours and at 12 feet after draining for 4 hours.

I wonder whether there is a way to make writing the equation of a line more efficient.

2. Compare your process for writing each equation. How are the processes different?

4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning. Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

5 Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

5 ACTIVITY 4.1

Writing Equations in Point-Slope Form

In the previous lesson, you used the slope, the y-intercept, and the slope formula to write a linear equation. You can also determine the equation of a line without knowing the y-intercept.

WORKED EXAMPLE

To write an equation of a line from a table of values, you can use the slope formula.

x	y
2	6
4	5
6	4

- First, calculate the slope. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-6}{4-2} = \frac{-1}{2} = -\frac{1}{2}$
- Next, choose any point from the table. (2, 6)
- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, (2, 6), and the unknown point (x, y). $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad -\frac{1}{2} = \frac{y-6}{x-2}$
- Finally, rewrite the equation with no variables in a denominator. $-\frac{1}{2} = \frac{y-6}{x-2}$ $\quad -\frac{1}{2}(x-2) = y - 6$

The equation is $y - 6 = -\frac{1}{2}(x - 2)$.

This linear equation in the Worked Example is written in *point-slope form*. The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is any point on the line.

1. Solve the equation in the Worked Example for y so that the linear equation is in slope-intercept form. What unique information does each form of the linear equation provide? How are they similar?

Develop

Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

6 Talk the Talk

Say What?

You have learned about two forms of a linear equation: the slope-intercept form, $y = mx + b$, and the point-slope form, $y - y_1 = m(x - x_1)$.

1. What information can you determine about each line by looking at the structure of the equation?

   a. $y = \frac{3}{5}x - 4$

   b. $y - 6 = 2(x + 1)$

   c. $y + 4 = 2(x - 0)$

   d. $y = -\frac{2}{7}x$

   e. $y + 5 = -(x - 4)$

   f. $y = 19$

2. Create a context that represents a linear relationship that passes through the point (2, 56) and has an increasing slope. Then, write the equation of the line in point-slope form and slope-intercept form.

6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Demonstrate

Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress toward demonstrating proficiency of the objectives.

Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

ASSIGNMENT
An intentionally designed assignment follows each lesson. The assignment is an additional component that is useful for Tier 1 instruction.

7 Write
Reflect on your work and clarify your thinking.

8 Remember
Take note of the key concepts from the lesson.

9 Practice
Use the concepts learned in the lesson to solve problems.

There is one assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the assignment students can complete based on that day's progress.

Use the assignment as homework throughout or after a lesson, as additional practice before students leave class, or as a review tool before assessments.

Lesson 4 Assignment

7 Write

Compare the slope-intercept and point-slope forms of a linear equation.

Remember 8

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

9 Practice

Write an equation in point-slope form.

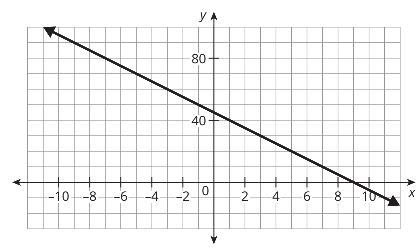
1. $m = 2$; (5, 6)
2. $m = -9.2$; (−17, 10)
3. (−2, −3) and (8, −8)
4. (79, 52) and (−87, 550)
5. A photography studio charges $50 for a sitting fee and 6 prints. Luigi increased his order to 11 prints and paid $65.
6. Lucia is taking the stairs in her building from her floor to the top of the building. After 2 minutes, she was 100 steps from the bottom floor. After 5 minutes, she was 196 steps from the bottom floor.

Write an equation in any form.

7. A newspaper charges a flat fee plus a charge per day to place a classified ad.

Number of Days	Total Charge ($)
2	8.00
4	13.00
6	18.00

8.

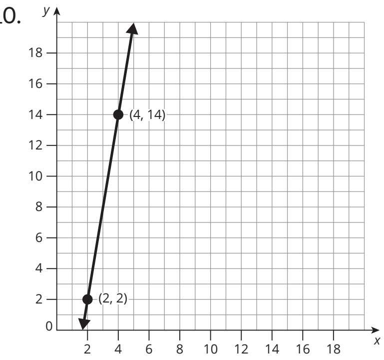


Lesson 4 Assignment

9.

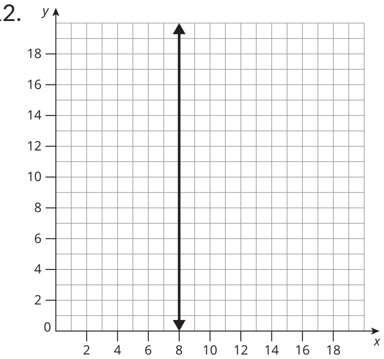
x	y
−10	50
−2	10
4	−20
14	−70

10.



11. Xavier is traveling on a toll road. He plans to exit the road 5 miles ahead and pay $1.75. He changes his plans and travels 9 miles and pays $2.75.

12.



10 Prepare

Solve each equation for *y*.

1. $-2y = -x + 7$

2. $\frac{3}{4}y = x - 6$

3. $2x + 3y = 6$

4. $\frac{1}{2}x - 4y = 8$

ASSIGNMENT

10 Prepare

Get ready for the next lesson.

The **Prepare** section provides spaced retrieval of concepts related to previous learning and fluency skills important for the course.

Note

Look at each lesson and assignment in the context of the over-arching story of the mathematics for the course. One lesson or activity may not fully cover a particular TEKS. However, in these cases, the lesson or activity is a building block to another lesson or topic where the TEKS is fully covered.

Research-Based Strategies

Internal researchers collaborated with various independent research organizations, tirelessly working to understand more about how people learn and how to best facilitate learning. This information was supplemented with feedback and data from educators, students, and the community to continuously evaluate and elevate the instructional approach and delivery.

The embedded strategies, tools, and guidance provided in these instructional resources are informed by books like *Adding it Up*, *How People Learn*, and *Principles to Actions*.

Worked Examples

- Worked Examples provide a means for students to view each step taken to solve the example problem.
- The questions that follow serve as a model for self-questioning and self-explanations. They represent and mimic an internal dialog about the mathematics and the strategies.
- This approach doesn't allow students to skip over the example without interacting with it, thinking about it, and responding to the questions.
- This approach will help students recognize the value in identifying steps, organizing their thinking, and codifying processes for problem solving.

WORKED EXAMPLE

Consider the sequence generated using $a_n = a_{n-1} + (-2)$, where $a_1 = 11$ and n is a whole number greater than 1.

a_1 represents the first term of the sequence and a_n represents the n^{th} term of the sequence.

Since I know the first term of the sequence, to determine the second term a_2, I add -2 to 11.

$a_2 = a_{2-1} + -2 = a_1 + -2 = 11 + -2 = 9$

I can determine the 3rd and 4th term of the sequence by continuing the pattern.

$a_3 = a_{3-1} + -2 = a_2 + -2 = 9 + -2 = 7$

$a_4 = a_{4-1} + -2 = a_3 + -2 = 7 + -2 = 5$

The sequence is 11, 9, 7, 5, ...

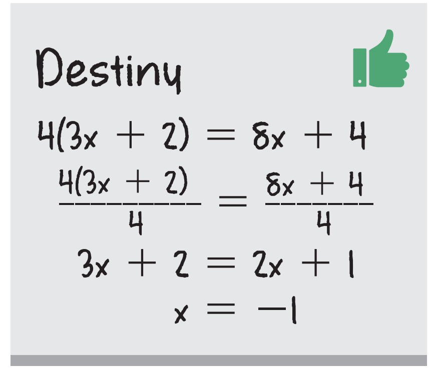
The pattern is to add the same negative number, -2, to each term to determine the next term.

Sequence: $\underline{11} \xrightarrow{\text{add } -2} \underline{9} \xrightarrow{\text{add } -2} \underline{7} \xrightarrow{\text{add } -2} \underline{5}, \ldots$

This sequence is arithmetic and the common difference d is -2.

Thumbs Up/Thumbs Down

- Thumbs Up problems allow students the opportunity to analyze viable methods and problem-solving strategies. We present questions to help students think more in-depth about the various strategies and analyze correct responses.
- Research shows that only providing positive examples does not eliminate some of the common misconceptions students may have. Negative examples provide a way to directly address common misconceptions.
- Thumbs Down problems, showing incorrect responses, allow students to identify and explain errors and make corrections.



Who's Correct?

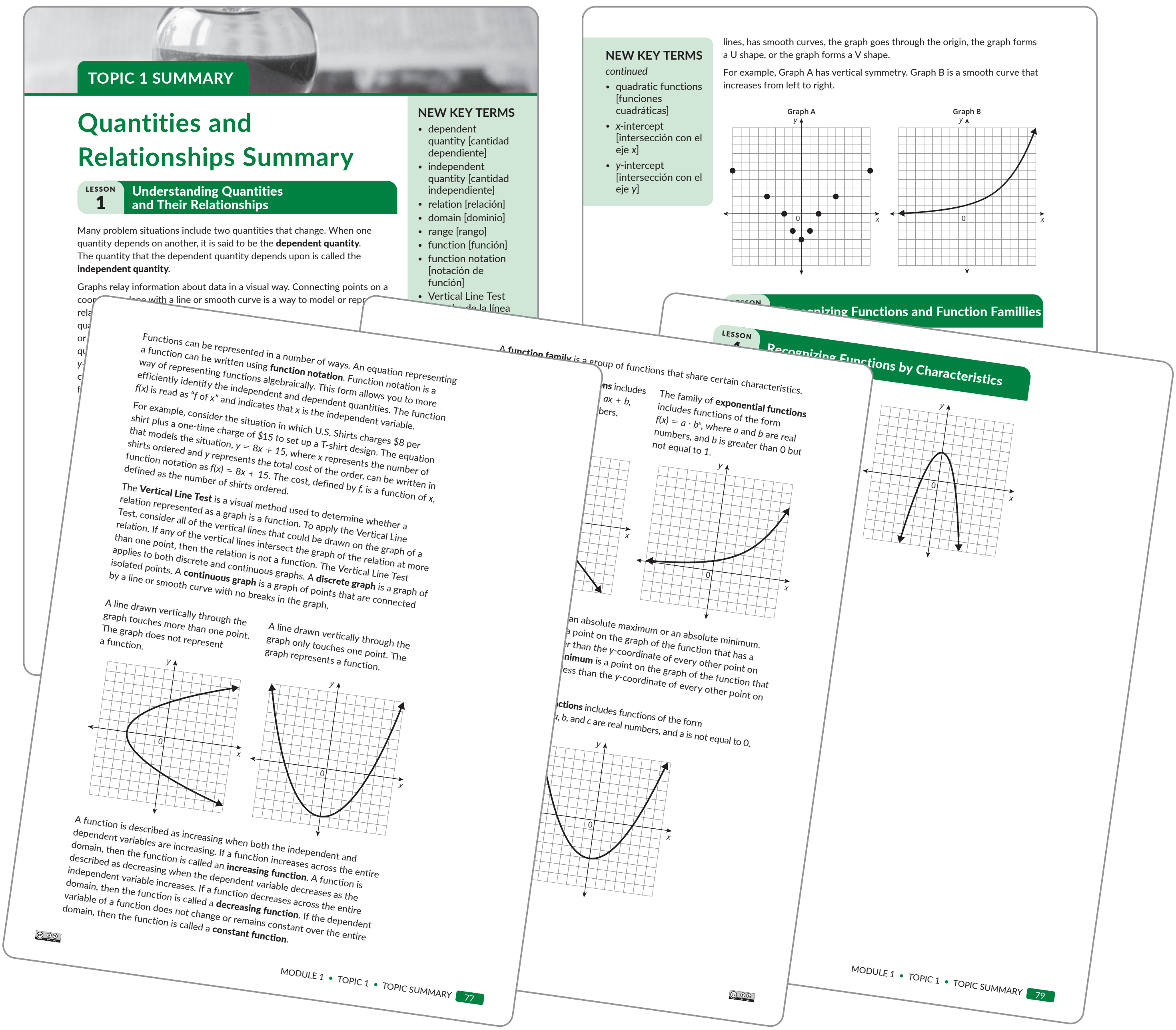
Who's Correct? problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean and whether each of the solutions makes sense.

5. Consider a sequence in which the first term is 64 and each term after that is calculated by dividing the previous term by 4. Hannah says that this sequence ends at 1 because there are no whole numbers that come after 1. Jasmine disagrees and says that the sequence continues beyond 1. Who is correct? When Hannah is correct, explain why. When Jasmine is correct, predict the next two terms of the sequence.

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all new key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or Worked Examples.

Consider using the Topic Summary to help students review content when they were absent or are preparing for a test. The Topic Summary can also provide insight into the key ideas and examples from each lesson.



Topic Self-Reflection

The Topic Self-Reflection allows students to reflect on their understanding of the concepts and skills they learn in the topic. The Topic Self-Reflection is designed for students to reflect on their understanding of concepts at the beginning, middle, and end of the topic, so they can track and monitor their own progress and growth.

You can use this information to guide conversations with students about their strengths, areas for improvement, and strategies for growth. Additionally, the Topic Self-Reflection can highlight areas for additional practice or review.

TOPIC 1 SELF-REFLECTION Name: ____________________

Quantities and Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Quantities and Relationships* topic by:

TOPIC 1: *Quantities and Relationships*	Beginning of Topic	Middle of Topic	End of Topic
choosing appropriate scale and origin for graphs.	○	○	○
identifying the appropriate unit of measure for each variable or quantity.	○	○	○
analyzing a graph and stating the key characteristics of the graph.	○	○	○
using a problem situation to explain what the key features of a graph mean in real-world context.	○	○	○
deciding whether relations represented verbally, tabularly, graphically, and symbolically define a function.	○	○	○
recognizing a linear, exponential, or quadratic function by its equation or graph.	○	○	○
evaluating functions, expressed in function notation, given one or more elements in their domain.	○	○	○
determining the domain and range and the independent and dependent quantities in a relationship.	○	○	○

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Quantities and Relationships* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

Teacher's Edition

The Teacher's Edition is designed to fully support a wide range of teachers implementing our materials: from new teachers to experienced teachers.

One goal in developing the Teacher's Edition was to make the instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

What makes this Teacher's Edition useful?

Effective Lesson Design

Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing

Each course is designed to be taught in a 165-day school year. Pacing suggestions are provided for each lesson. Each day in the Pacing Guide is equivalent to about a 45-minute instructional session.

Instructional Supports

Guiding questions are provided for teachers to use as they're circulating the room as well as differentiation strategies, common student misconceptions, and student look-fors.

Clearly Defined Mathematics

The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The Teacher's Edition is critical to understanding how the mathematics that students encounter should be realized in the classroom. The Teacher's Edition describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students.

Module and Topic Overviews

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics and how topics build to achieve understanding throughout the course.

> "Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)."
>
> *Drake & Sherin,* 2009 | Page 325

Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, and the connections to future learning.

Topic Overview

The Topic Overview describes how the topic is organized from the entry point for students to how students will demonstrate understanding. The Topic Overview explains why the mathematics of the topic is important and how the activities within the topic promote expertise in the TEKS mathematical process standards. A materials list, visual representations or strategies used, and suggested pacing information is also provided.

MODULE 1 OVERVIEW

TEKS Addressed: **A.2A**, **A.3C**, **A.6A**, **A.7A**, A.9A, **A.9D**, A.12A, A.12C, A.12D

*Bold TEKS = Readiness Standard

Searching for Patterns

Sessions: **24**

Why is this module named *Searching for Patterns*?

Students have searched for patterns in previous courses. They have recognized patterns in lists, learned to extend a pattern beyond a given list, and applied a rule to determine an arbitrary value well beyond the given list. This module extends students' understanding of functions to explore specific function families, including linear, exponential, and quadratic.

Throughout the module, students are searching for, recognizing, and defining patterns in relationships between quantities.

In the first topic, students explore various functions, presented as graphs and equations, and investigate their differentiating characteristics. Once they recognize patterns in the graphs and equations, they sort the functions into their corresponding function families.

In the second topic, students search for patterns in sequences of numbers. They recognize that while all sequences are functions, arithmetic sequences are linear functions, and some geometric sequences are exponential functions.

The Research Shows . . .

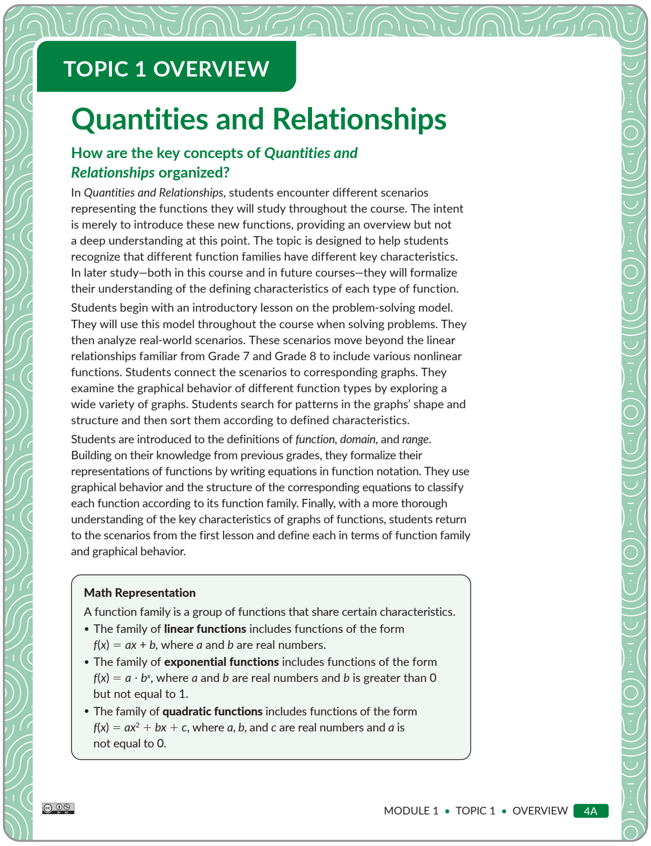
"Critical to algebraic thinking is the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined functional rules."

Fostering Algebraic Thinking: A Guide For Teacher Grades 6–10, | Page 2

What is the mathematics of *Searching for Patterns*?

Searching for Patterns contains two topics: *Quantities and Relationships* and *Sequences*. Students recognize and identify the key characteristics of different function families. They write recursive and explicit formulas for arithmetic and geometric sequences. Students will revisit all of the function families, key characteristics and types of sequences again in later modules.

MODULE 1 • OVERVIEW 2A



TOPIC 1 OVERVIEW

Quantities and Relationships

How are the key concepts of *Quantities and Relationships* organized?

In *Quantities and Relationships*, students encounter different scenarios representing the functions they will study throughout the course. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study—both in this course and in future courses—they will formalize their understanding of the defining characteristics of each type of function.

Students begin with an introductory lesson on the problem-solving model. They will use this model throughout the course when solving problems. They then analyze real-world scenarios. These scenarios move beyond the linear relationships familiar from Grade 7 and Grade 8 to include various nonlinear functions. Students connect the scenarios to corresponding graphs. They examine the graphical behavior of different function types by exploring a wide variety of graphs. Students search for patterns in the graphs' shape and structure and then sort them according to defined characteristics.

Students are introduced to the definitions of *function*, *domain*, and *range*. Building on their knowledge from previous grades, they formalize their representations of functions by writing equations in function notation. They use graphical behavior and the structure of the corresponding equations to classify each function according to its function family. Finally, with a more thorough understanding of the key characteristics of graphs of functions, students return to the scenarios from the first lesson and define each in terms of function family and graphical behavior.

Math Representation

A function family is a group of functions that share certain characteristics.

- The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.
- The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers and b is greater than 0 but not equal to 1.
- The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and a is not equal to 0.

MODULE 1 • TOPIC 1 • OVERVIEW 4A

1 **Materials**

Materials required for the lesson are identified.

2 **Lesson Overview**

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

3 **TEKS Addressed**

The highlighted TEKS mathematical process standards and TEKS content standards for each lesson are listed. A visual display accompanies the content standards to show coverage of the TEKS throughout the course. When the circle is completely shaded in course color, that is the last lesson associated with that particular TEKS. A star inside the circle indicates a readiness standard.

4 **ELPS Addressed**

Highlighted English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

Facilitation Notes support your planning process. Point-of-use strategies are your resource for planning, guiding, and facilitating student learning.

1 Properties of Powers with Integer Exponents

1 MATERIALS

Calculator

2 LESSON OVERVIEW

The terms *power*, *base of a power*, and *exponent of a power* are defined. Students write and evaluate expressions with positive integer exponents. They begin with a context using the power with a base of 2. Students then investigate positive and negative integer bases, where the negative sign may or may not be raised to a power depending on the placement of parentheses. Some expressions also contain variables.

3 ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Number and Algebraic Methods

(11) The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms.

The student is expected to:

A.11B simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

4 ELPS

(1) Learning Strategies

The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(3) Speaking

The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

5 ESSENTIAL IDEAS

- Large numbers that have factors that are repeated can be written as a product of powers.
- Placement of parentheses in an expression with an exponent determines what portion of the expression is raised to the exponent.
- When a negative value is raised to an exponent that is an even integer, the simplified expression is a positive value.
- When a negative value is raised to an exponent that is an odd integer, the simplified expression is a negative value.

5 **Essential Ideas**

These statements are derived from the standards and state the concepts students will develop.

LESSON STRUCTURE AND PACING: 3 DAYS 6

DAY 1

7 **ENGAGE**

Getting Started: Three Generations 5–10 minutes

ESTABLISH A SITUATION

A chart that represents a puppy's lineage is given. Students analyze a pattern between each generation of the puppy's lineage. This activity is designed to engage students in thinking about repeated multiplication patterns, which will be formalized in the lesson.

DEVELOP

Activity 1.1: Review of Powers and Exponents 15–20 minutes

WORKED EXAMPLE, REAL-WORLD PROBLEM SOLVING

Students complete a table detailing the puppy's lineage back seven generations, continuing work from the previous activity. They express the number of sires and dams for each generation in expanded notation and power notation and then answer related questions.

Activity 1.2: Practice with Powers 10–15 minutes

MATHEMATICAL PROBLEM SOLVING

Students investigate the role of parentheses in expressions containing exponents, including negative integers raised to an even or odd power. They evaluate expressions containing exponents and reverse the process to write expressions containing exponents that represent the product of factors.

DAY 2

Activity 1.3: Multiplying and Dividing Powers 20–25 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

A medium sized eBook contains about one megabyte (MB) of information. One gigabyte (GB) is 1024 megabytes, one megabyte (MB) is 1024 kilobytes, and one kilobyte is 1024 bytes. Students calculate the storage capacity of eBooks and jump drives. This activity provides a context that creates the opportunity for students to perform multiplication and division on expressions with exponents.

Activity 1.4: Product of Powers 15–20 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

Students use expanded notation of expressions to develop the product of powers rule. The product of powers rule states that to multiply powers with the same base, keep the base the same and add the exponents.

6 **Pacing**

Lessons often span more than one 45-minute instructional session. Suggested pacing is provided for each lesson so that the entire course can be completed in a 165-Day Instructional Calendar.

7 **Lesson Structure**

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity is included.

8 Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, common misconceptions, differentiation strategies, student look-fors, and an activity summary.

9 Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the objectives.

10 Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners. For example, to extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

8 Getting Started

ENGAGE

Three Generations

9 Facilitation Notes

In this lesson, students are given a scenario represented in chart form. Students analyze a pattern between each generation of the puppy's lineage. This activity is designed to engage students in thinking about repeated multiplication patterns, which will be formalized in the lesson.

Have a student read the introduction. As a class, discuss the lineage provided in the chart. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

10 DIFFERENTIATION STRATEGY

Access for All

If students look at the sequence of numbers without the context, they might recognize the pattern of repeated addition of the previous number. Acknowledge that this is a correct pattern, but it is not as useful as the pattern of multiplication by 2; it is easier to write and use a pattern where the number stays constant instead of being based upon a previous term.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Express the number of dogs in each generation as a sequence. • Why does this pattern make sense for this context?

STAMP THE LEARNING

Summary

Charts can be used to organize data and look for patterns.

ACTIVITY 1.1 Review of Powers and Exponents

DEVELOP

Facilitation Notes

In this activity, students complete a table detailing the puppy's lineage back seven generations, continuing work from the previous activity. They express the number of sires and dams for each generation in expanded notation and power notation and answer related questions.

Have students work with a partner or in a group to complete Question 1. Note that students are only completing the second column of the table. Have students share their strategies for completing the number of sires and dams in Rickson's lineage. This question does not require students to use powers.

11 AS STUDENTS WORK, LOOK FOR

The use of powers. Although it is not expected here, some students may have recognized the pattern and started using exponential notation.

12 QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do the values 2, 4, and 8 in the table represent in the context?
Probing	• How did you complete the table without extending the diagram? • Why do the values in this problem increase by a factor of two for each generation?

Ask a student to read the information and definitions following the table aloud. Analyze the Worked Example as a class. Then, have students work with a partner or in a group to complete Questions 2 and 3. Have students share their completed tables with another partner or group. Share responses as a class.

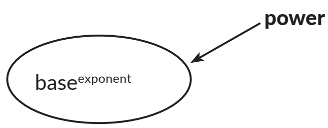
COMMON MISCONCEPTION

Students may inaccurately refer to the exponent as the power. This misconception is strengthened by the fact that 2^7 may be read as "2 to the 7th power." Remind students that a power is comprised of a base and an exponent.

DIFFERENTIATION STRATEGY

Access for All

Have students take notes on the definitions and Worked Example.



- Include a diagram.
- Have students write "= 128" in the Worked Example and then write an additional example to demonstrate that placement of the base and exponent makes a difference: $7^2 = (7)(7) = 49$.
- Have students write examples that include variables, such as 3^x and x^3. After they complete Questions 2 and 3, have them write these powers in expanded notation, $3^x = (3)(3)(3)\ldots x$ times and $x^3 = (x)(x)(x)$. Students cannot solve for a single value because they do not know the value of x.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are the different ways to read each power of 2 in the table?
Probing	• How is the generation number related to the exponent of the power representing the generation?
Seeing structure	• What is the difference between a power and an exponent?

11 As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

12 Questions to Support Discourse

As you facilitate the lesson, use the Questions to Support Discourse to assess students' sense-making and reasoning, to gauge what they know, and generate evidence of student learning.

13 Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Research supports that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

13 **Have students answer Questions 4 and 5 in partners or in a group. Share responses as a class.**

AS STUDENTS WORK, LOOK FOR

- Use of expanded notation by typing twelve 2s into the calculator.
- Knowledge and use of the ^ key or a^b key to enter the exponent into a calculator.

COMMON MISCONCEPTION

Students may attempt to answer Question 5, asking for a total, by using exponents exclusively. Discuss the fact that addition is still required to solve this problem.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you calculate 2^{12} using technology? What is another way? • How did you calculate the total number of sires and dams in all three generations?

STAMP THE LEARNING

Summary

A power is an expression used to represent the product of a repeated multiplication. The base of a power is the expression that is used as a factor in the repeated multiplication, and the exponent of a power is the number of times that the base is used as a factor in the repeated multiplication.

ACTIVITY 1.2 Practice with Powers

Facilitation Notes

In this activity, students investigate the role of parentheses in expressions containing exponents, including negative integers raised to an even or odd power. They evaluate expressions containing exponents and reverse the process to write expressions containing exponents that represent the product of factors.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class. Then, answer Questions 3 and 4 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know how many times to write the negative sign when writing an exponent as a product? • How many negative signs are in the expanded expression for -1^5? $(-1)^5$? • What is the sign of a positive number raised to an odd power? An even power?

Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How is this situation's structure the same as and different than the Grains of Rice problem?
Probing	• How did you identify the common ratio from the situation? • Why is 3 the y-intercept? • Explain your substitution step to construct the function. • Why do you need to check the common ratio for every pair of consecutive values?

Complete Question 2, parts (c) and (d), as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

- Suggest that students create a new table for function form, where the input values have their own column.

Inputs Time	Outputs Number of New Participants
x	ab^x
$x + 1$	ab^{x+1}
$x + 2$	ab^{x+2}
$x + 3$	ab^{x+3}
$x + 4$	ab^{x+4}

14 COMMON MISCONCEPTION

Students are accustomed to $f(x)$ or $f(\text{constant})$, but they may have a difficult time interpreting $f(x + \text{constant})$ as the function of a single value. In that case, consider practice outside of the format of the table to help students make sense of the notation.

$f(x) = 3(4)^x$ $f(2) = 3(4)^2$ $f(y) = 3(4)^y$ $f(x + 2) = 3(4)^{x+2}$
$f(x) = ab^x$ $f(3) = ab^3$ $f(y) = ab^y$ $f(x + 2) = ab^{x+2}$

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you answer this question using the table? The formula? • Explain how you completed the Function Form column. • How do you calculate the constant ratio with algebraic expressions? • How can you apply the properties of powers to rewrite the ratio? • Is the result what you expected? Why or why not?
Gathering	• What does $f(x + 2)$ mean?

14 **Common Misconceptions**

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

15 **White Space**

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

16 **Summary**

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

DEMONSTRATE

Talk the Talk

A WRITER AND A MATHEMATICIAN

Facilitation Notes

In this activity, students create a scenario based upon a possible trip to school. They then sketch a graph to model their scenario. They share their work with classmates and note similarities and differences.

15

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you label the axes? • What was the distance of the trip? • How did you inform the reader about the pace? • Did you embellish your scenario so that the graph wasn't a straight line or smooth curve? If so, explain how. • Could more than one possible graph model your scenario? Explain. • How did you use the axes' labels and scales to support your response?
Seeing structure	• Is your graph increasing or decreasing? Why? • What type of scenario would create a curved graph?

DIFFERENTIATION STRATEGIES

Access for All

- When setting up their graph, ask students questions about the significance of points on the *x*-axis, on the *y*-axis, and at the origin.

Challenge Opportunity

- Have students create a scenario and trade papers with a partner. Then, have the partner draw a graph for the scenario. Have partners discuss if the graph drawn was the intent of the writer of the scenario.
- Have students create two graphs, one with the *y*-axis labeled as the distance from home and the other with the *y*-axis labeled distance from school. Then, have them compare the characteristics of the graphs.

Have students read and answer the Essential Question on the lesson opener page.

Summary

16

A graph is an efficient way to model and interpret a scenario.

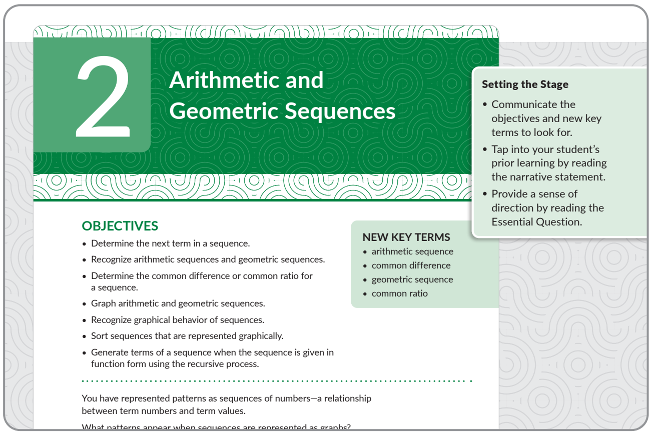
STAMP THE LEARNING

Embedded Teacher Supports

Within the lesson, additional teacher embedded notes, when applicable, are provided at point of use within a lesson.

Setting the Stage

Each Lesson Overview provides guidance to help students anticipate how the new information will connect to prior learning.



2 Arithmetic and Geometric Sequences

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Determine the next term in a sequence.
- Recognize arithmetic sequences and geometric sequences.
- Determine the common difference or common ratio for a sequence.
- Graph arithmetic and geometric sequences.
- Recognize graphical behavior of sequences.
- Sort sequences that are represented graphically.
- Generate terms of a sequence when the sequence is given in function form using the recursive process.

NEW KEY TERMS

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

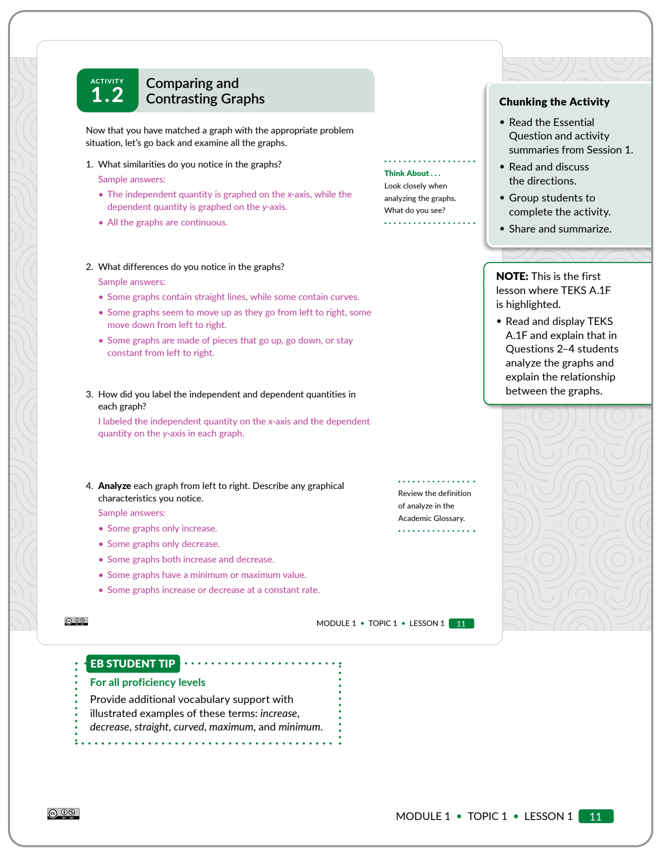
You have represented patterns as sequences of numbers—a relationship between term numbers and term values.

What patterns appear when sequences are represented as graphs?

Chunking the Activity This section provides suggestions for separating the activity into smaller meaningful parts or chunks to help students process the information. This is the same activity chunking provided within the facilitation notes.

TEKS Mathematical Process Standards Notes

Each note references a particular TEKS mathematical process standard. The first instance of a TEKS mathematical process standard is highlighted in a lesson and encourages you to introduce the standard to your students. After the first time a process standard is highlighted, additional notes help you assess whether students are demonstrating proficiency with the process standards.



ACTIVITY 1.2 Comparing and Contrasting Graphs

Now that you have matched a graph with the appropriate problem situation, let's go back and examine all the graphs.

1. What similarities do you notice in the graphs?
   Sample answers:
   - The independent quantity is graphed on the x-axis, while the dependent quantity is graphed on the y-axis.
   - All the graphs are continuous.

Think About . . .
Look closely when analyzing the graphs. What do you see?

2. What differences do you notice in the graphs?
   Sample answers:
   - Some graphs contain straight lines, while some contain curves.
   - Some graphs seem to move up as they go from left to right, some move down from left to right.
   - Some graphs are made of pieces that go up, go down, or stay constant from left to right.

3. How did you label the independent and dependent quantities in each graph?
   I labeled the independent quantity on the x-axis and the dependent quantity on the y-axis in each graph.

4. **Analyze** each graph from left to right. Describe any graphical characteristics you notice.
   Sample answers:
   - Some graphs only increase.
   - Some graphs only decrease.
   - Some graphs both increase and decrease.
   - Some graphs have a minimum or maximum value.
   - Some graphs increase or decrease at a constant rate.

Review the definition of analyze in the Academic Glossary.

MODULE 1 • TOPIC 1 • LESSON 1 11

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

NOTE: This is the first lesson where TEKS A.1F is highlighted.

- Read and display TEKS A.1F and explain that in Questions 2–4 students analyze the graphs and explain the relationship between the graphs.

EB STUDENT TIP
For all proficiency levels
Provide additional vocabulary support with illustrated examples of these terms: *increase*, *decrease*, *straight*, *curved*, *maximum*, and *minimum*.

MODULE 1 • TOPIC 1 • LESSON 1 11

Stamp the Learning The Stamp the Learning icon provides prompts and guided instructions to support you in communicating, explaining, and modeling the concepts directly and explicitly.

STAMP THE LEARNING

The definition and representations provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB Student Tip EB Student Tips provide instructional strategies to support students with varying levels of English language proficiency.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Allow students time to identify vocabulary and create basic descriptions for each group of graphs to prepare for classroom discussions. As you ask groups members for their grouping decisions, encourage students to orally contribute with the help of their prepared descriptions.

Optimizing Learning These notes indicate opportunities for purposeful learning. These strategies provide access to the course content for all learners.

Optimizing Learning

This common misconception provides options that support students by using multiple tools for construction and composition.

Skills Practice Alignment Notes These notes indicate the section(s) of Skills Practice that align to activities within the lessons.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice sorting a variety of graphs into groups based on characteristics, assign Skills Practice Set A for this lesson.

Modeling Moment These notes provide instructional guidance surrounding when and how to utilize the Problem-Solving Model Graphic Organizer.

Self-Monitoring Strategies Some lessons include self-monitoring strategy notes at point of use.

> **SELF-MONITORING STRATEGY**
> Look for students using self-motivation and self-discipline to persevere in solving problems. Refer to the Course and Implementation Guide for further details on these look fors.

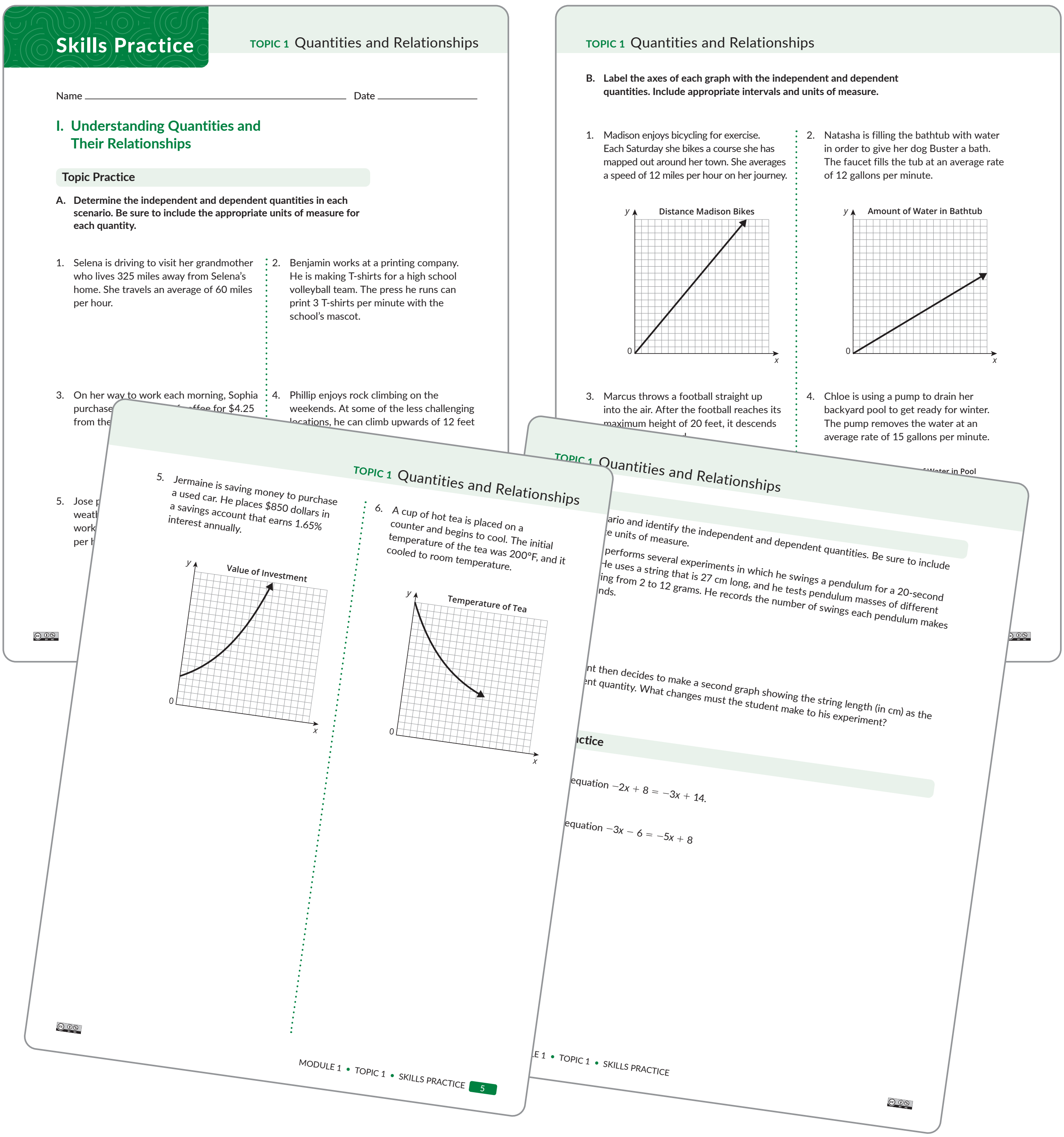
Providing students with purposeful guidance as they develop self-monitoring skills is critical to their growth as holistic learners. Throughout the text, there are callouts placed strategically where there are opportunities for students to practice self-monitoring. Support students by acknowledging those demonstrating the actions outlined by the callout, and encouraging other students to make an intentional attempt at implementing the strategies. Over time, these actions will become habits for students that they can apply throughout all their academic courses and their lives more broadly. As the Self-Monitoring Strategies indicate, look for students:

- Making a reasoned judgment.
- Using self-motivation and self-discipline to persevere in problem solving.
- Engaging in productive struggle as they use their reasoning to solve problems.
- Recognizing the importance of developing critical thinking skills.
- Showing a sense of confidence and optimism as they approach the problem.
- Accurately perceiving their strengths and limitations.
- Using organizational strategies while solving problems.
- Monitoring their understanding by referring back to concrete examples.
- Demonstrating self-awareness by perseverance through problem solving.
- Reviewing and reflecting on their goals.

> **Modeling Moment**
> - Provide students with the Problem-Solving Model Graphic Organizer.
> - For Question 1, have students work in pairs, ask themselves the questions from the first step of the model, and share their reasoning.
> - Complete the remaining steps in the graphic organizer together as a class.
> - Have students work in pairs and use the problem-solving model to complete Questions 2–4.

Skills Practice

Each topic includes a set of Skills Practice problems that focus on skills and concepts students learn in the topic. Use Skills Practice during Learning Individually days to support students in developing skill proficiency. You can use formative and summative assessment data to determine when to schedule Learning Individually days. Skills Practice provides opportunity for targeted instruction as you determine which problem sets to assign to individual students.



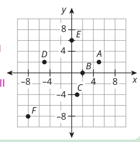
Formative Assessments

To prepare for the upcoming lesson, use the Prepare section of the assignment from the previous lesson as a diagnostic tool, either as a warm-up or an exit ticket, to assess whether your students are ready for new learning.

Prepare

1. Write the coordinates of each point and name the quadrant or axis where the point is located.

   Point A: (5, 2) Quadrant I
   Point B: (2, 0) on the x-axis
   Point C: (1, −4) Quadrant IV
   Point D: (−5, 2) Quadrant II
   Point E: (0, 6) on the y-axis
   Point F: (−8, −8) Quadrant III



To set the stage for new learning, Lesson Overviews provide Objectives and New Key Terms. Most importantly, you will see a statement that connects to prior knowledge and Essential Questions that anticipates new learning. At the end of the lesson, students return to and answer the Essential Question to demonstrate their learning. Use student responses to the Essential Question as data to drive your instructional practice and decision making.

1 Understanding Quantities and Their Relationships

Setting the Stage

- Communicate the learning goals and new key terms to look for
- Tap into your student's prior learning by reading the narrative statement
- Provide a sense of direction by reading the Essential Question

OBJECTIVES

- Understand quantities and their relationships with each other.
- Identify the independent and dependent quantities for a scenario.
- Match a graph with an appropriate scenario.
- Use a reasonable scale for a graph modeling a scenario.
- Identify key characteristics of graphs.
- Describe similarities and differences between pairs of graphs and scenarios.

NEW KEY TERMS

- dependent quantity
- independent quantity

You have analyzed graphs of relationships and identified important features, such as intercepts and slopes. How can the key characteristics of a graph tell a story?

Sample answer:
When one quantity depends on another, it is a dependent quantity. The quantity it depends upon is the independent quantity. The independent quantity is represented on the x-axis, and the dependent quantity is represented on the y-axis.

MODULE 1 • TOPIC 1 • LESSON 1 5

MODULE 1 • TOPIC 1 • LESSON 1 5

As you facilitate lessons, use the Questions to Support Discourse to assess students' sense-making and reasoning, to gauge what they know, and to generate evidence of student learning.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Which strategy do you prefer? Why? • What is an example of an irrational number? A rational number?
Probing	• Why is 15 · 5 rewritten as 3 · 5 · 5? • What is the difference between the two strategies? • Why is it inefficient to multiply the values under the radical symbols before extracting roots?
Seeing structure	• Do the same rules apply for integer and rational exponents? Explain.
Reflecting and justifying	• What expression(s) in Question 7 support your response?

As you facilitate the learning, use the Stamp the learning icon to identify opportunities to provide direct and explicit instruction on key concepts within the lesson. The Stamp the Learning icon highlights these opportunities. To assess students' understanding of these key concepts, have them restate the information in their own words.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

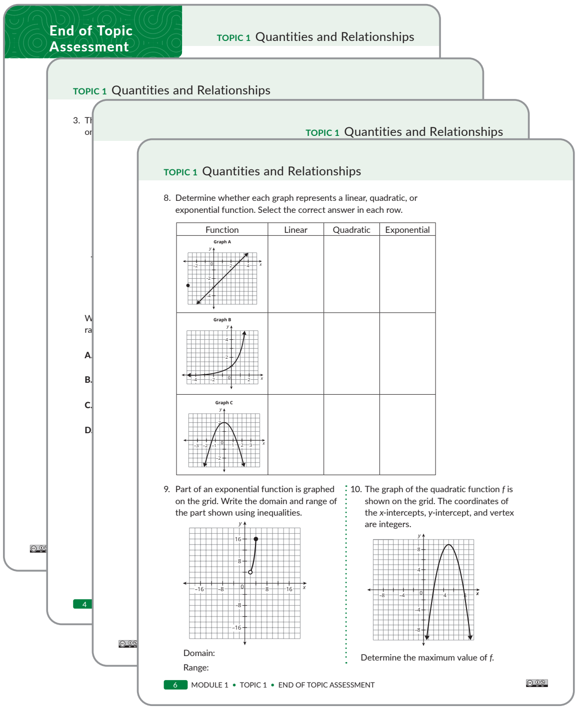
As students self-reflect during the Talk the Talk, you can interpret how well they have demonstrated the learning outcomes and prepare for what's next.

Talk the Talk

Interception!

Recall that the ***x*-intercept** is the point where a graph crosses the x-axis. The ***y*-intercept** is the point where a graph crosses the y-axis.

1. The graphs shown represent relations with just the x- and y-intercepts plotted. If possible, draw a function that has the given intercepts. If it is not possible, explain why not.

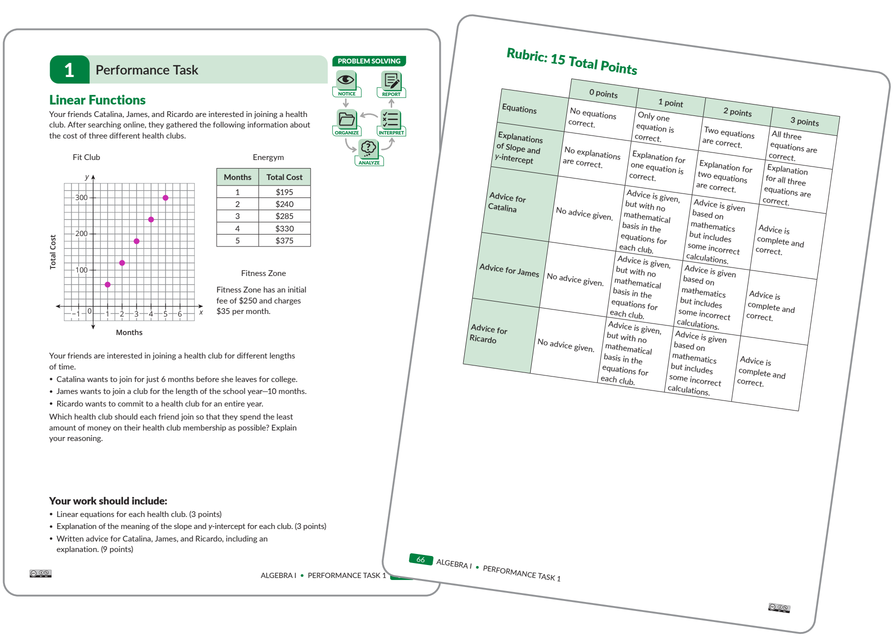


Summative Assessments

End of Topic Assessments are provided to measure student performance on a clearly denoted set of standards. There are three problem types students will encounter on the assessment: multiple-choice, multiselect, and open-response questions. These questions are thoughtfully designed to prepare students for digitally enhanced standardized tests.

The answer keys provide teachers with sample answers for open-response questions as well as for the multiple-choice and multiselect questions, when applicable.

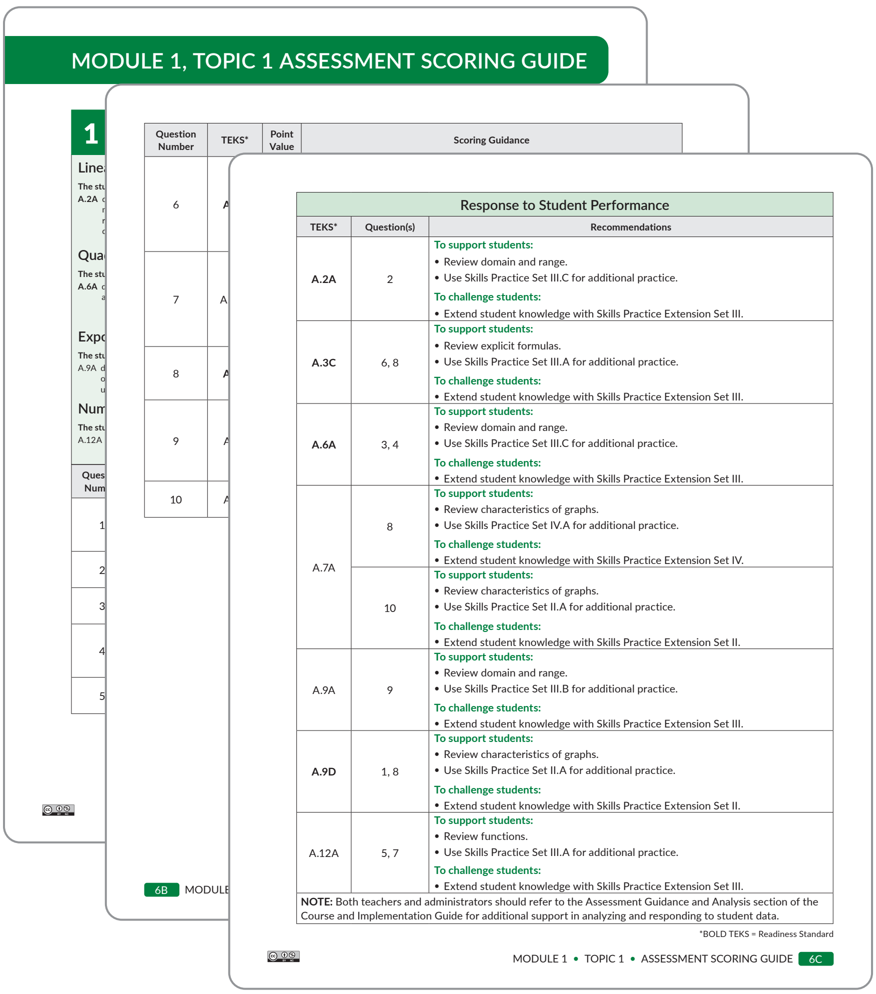
Question & Test Interoperability (QTI) files are provided for the implementation of digital assessments aligned to each End of Topic Assessment. There are many problem types that students will encounter on digital assessments: multiple-choice, multiselect, text entry/equation editor, graphing, inline choice, hot spot, drag and drop, and match table grid. It is recommended to administer either the print or the digital assessment method of the End of Topic Assessment. Students should not take both assessment types at the end of a topic.



There are a set of optional Performance Tasks students can complete after certain modules/topics. These tasks cover selected priority TEKS content from the course. You can use the performance task as either a formative or summative assessment. These tasks include a rubric that you can utilize to assess individual or class depth of understanding as aligned to the TEKS.

Assessment Guidance and Analysis

An Assessment Scoring Guide is provided for each End of Topic Assessment. The scoring guide highlights the TEKS aligned to each question on the assessment. It identifies possible point values for each question, along with a rubric for how to score each question. Each Assessment Scoring Guide also provides recommendations for how to respond to students' performance on the assessment.



MODULE 1, TOPIC 1 ASSESSMENT SCORING GUIDE

Response to Student Performance

TEKS*	Question(s)	Recommendations
A.2A	2	To support students: • Review domain and range. • Use Skills Practice Set III.C for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set III.
A.3C	6, 8	To support students: • Review explicit formulas. • Use Skills Practice Set III.A for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set III.
A.6A	3, 4	To support students: • Review domain and range. • Use Skills Practice Set III.C for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set III.
A.7A	8	To support students: • Review characteristics of graphs. • Use Skills Practice Set IV.A for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set IV.
	10	To support students: • Review characteristics of graphs. • Use Skills Practice Set II.A for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set II.
A.9A	9	To support students: • Review domain and range. • Use Skills Practice Set III.B for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set III.
A.9D	1, 8	To support students: • Review characteristics of graphs. • Use Skills Practice Set II.A for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set II.
A.12A	5, 7	To support students: • Review functions. • Use Skills Practice Set III.A for additional practice. To challenge students: • Extend student knowledge with Skills Practice Extension Set III.

NOTE: Both teachers and administrators should refer to the Assessment Guidance and Analysis section of the Course and Implementation Guide for additional support in analyzing and responding to student data.

*BOLD TEKS = Readiness Standard

MODULE 1 • TOPIC 1 • ASSESSMENT SCORING GUIDE 6C

Assessments are designed to cover the focus TEKS of the topic at the depth and rigor of the standard. When a TEKS addressed in a topic is not assessed on a topic assessment it will be assessed to its full depth in another topic. The only TEKS that are not assessed in the course are the unassessed standards in the course.

The Assessment Scoring Guide's Response to Student Performance section is still applicable when digital assessments are utilized.

Promoting Self-Reflection

The Crew

Characters are embedded throughout the course to remind students to stop and think in order to promote productive reflection. The characters are used in a variety of ways: they may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

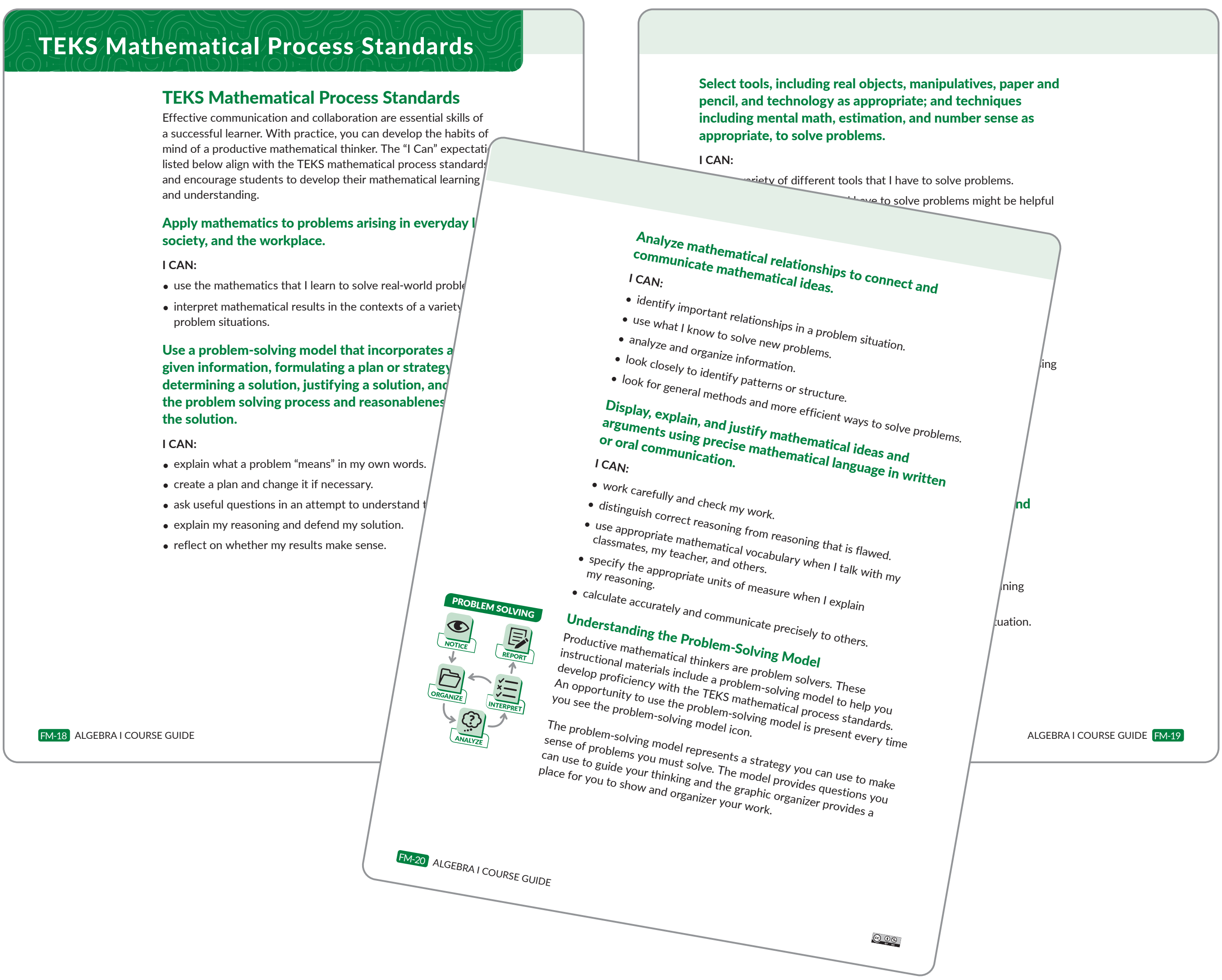
ALGEBRA I COURSE GUIDE FM-17

TEKS Mathematical Process Standards

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

Consider having students create "I can" statements to promote their self-reflection.


TEKS Mathematical Process Standards

TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I Can" expectati... listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday l... society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world probl...
- interpret mathematical results in the contexts of a variety... problem situations.

Use a problem-solving model that incorporates a... given information, formulating a plan or strategy... determining a solution, justifying a solution, and... the problem solving process and reasonablenes... the solution.

I CAN:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand t...
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

FM-18 ALGEBRA I COURSE GUIDE

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I CAN:

ALGEBRA I COURSE GUIDE FM-19

Analyze mathematical relationships to connect and communicate mathematical ideas.

I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Understanding the Problem-Solving Model

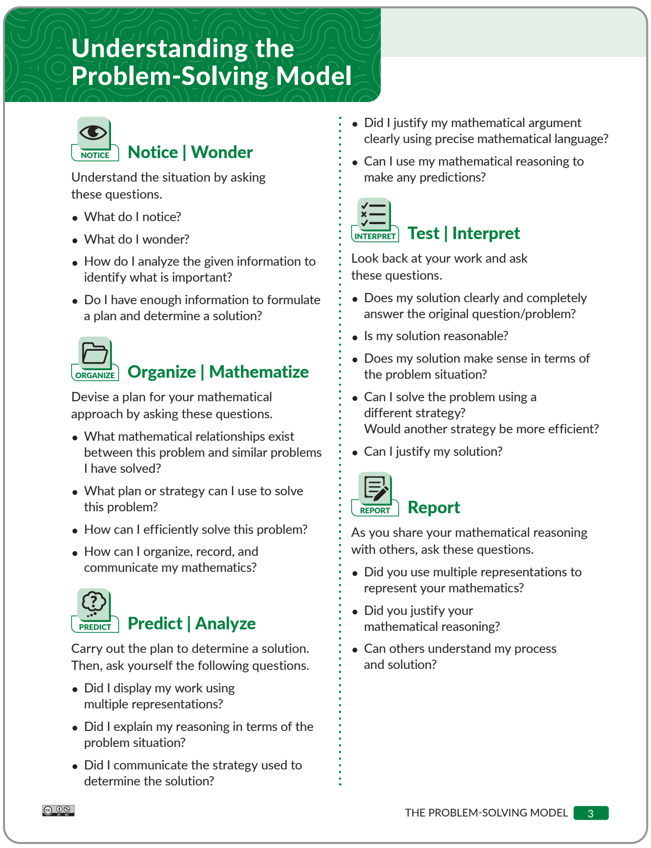
Productive mathematical thinkers are problem solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

FM-20 ALGEBRA I COURSE GUIDE

Understanding the Problem-Solving Model

Productive mathematical thinkers are problem solvers. These instructional materials include a problem-solving model to help students develop proficiency with the TEKS mathematical process standards and to make sense of the problems they must solve. As students engage with the problem-solving model, have them use the provided questions to guide their thinking. As students collaborate, suggest they use the provided questions to spark discussion. When appropriate, provide students with the Problem-Solving Model Graphic Organizer to complete as they solve problems.



Understanding the Problem-Solving Model

NOTICE **Notice | Wonder**

Understand the situation by asking these questions.

- What do I notice?
- What do I wonder?
- How do I analyze the given information to identify what is important?
- Do I have enough information to formulate a plan and determine a solution?

ORGANIZE **Organize | Mathematize**

Devise a plan for your mathematical approach by asking these questions.

- What mathematical relationships exist between this problem and similar problems I have solved?
- What plan or strategy can I use to solve this problem?
- How can I efficiently solve this problem?
- How can I organize, record, and communicate my mathematics?

PREDICT **Predict | Analyze**

Carry out the plan to determine a solution. Then, ask yourself the following questions.

- Did I display my work using multiple representations?
- Did I explain my reasoning in terms of the problem situation?
- Did I communicate the strategy used to determine the solution?
- Did I justify my mathematical argument clearly using precise mathematical language?
- Can I use my mathematical reasoning to make any predictions?

INTERPRET **Test | Interpret**

Look back at your work and ask these questions.

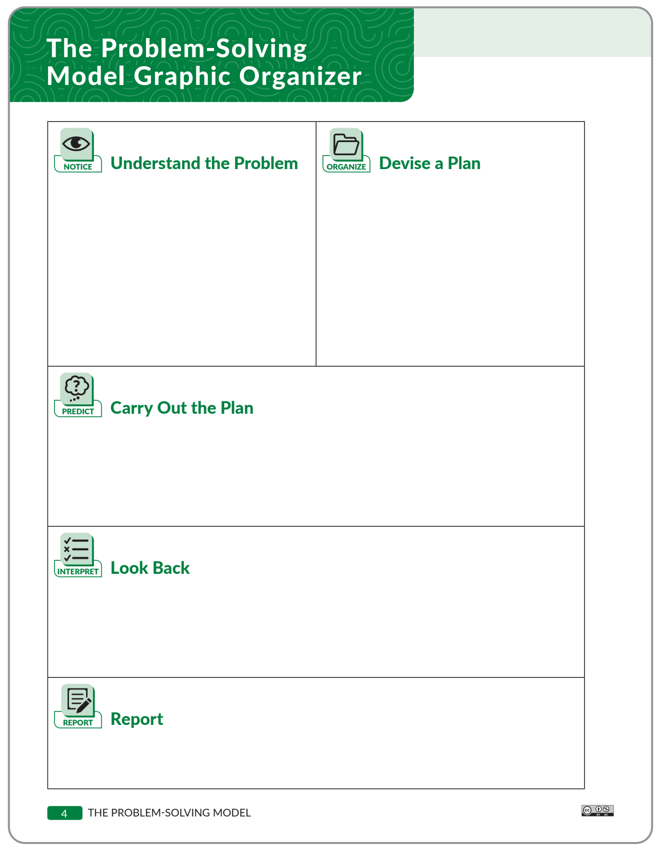
- Does my solution clearly and completely answer the original question/problem?
- Is my solution reasonable?
- Does my solution make sense in terms of the problem situation?
- Can I solve the problem using a different strategy? Would another strategy be more efficient?
- Can I justify my solution?

REPORT **Report**

As you share your mathematical reasoning with others, ask these questions.

- Did you use multiple representations to represent your mathematics?
- Did you justify your mathematical reasoning?
- Can others understand my process and solution?

THE PROBLEM-SOLVING MODEL 3



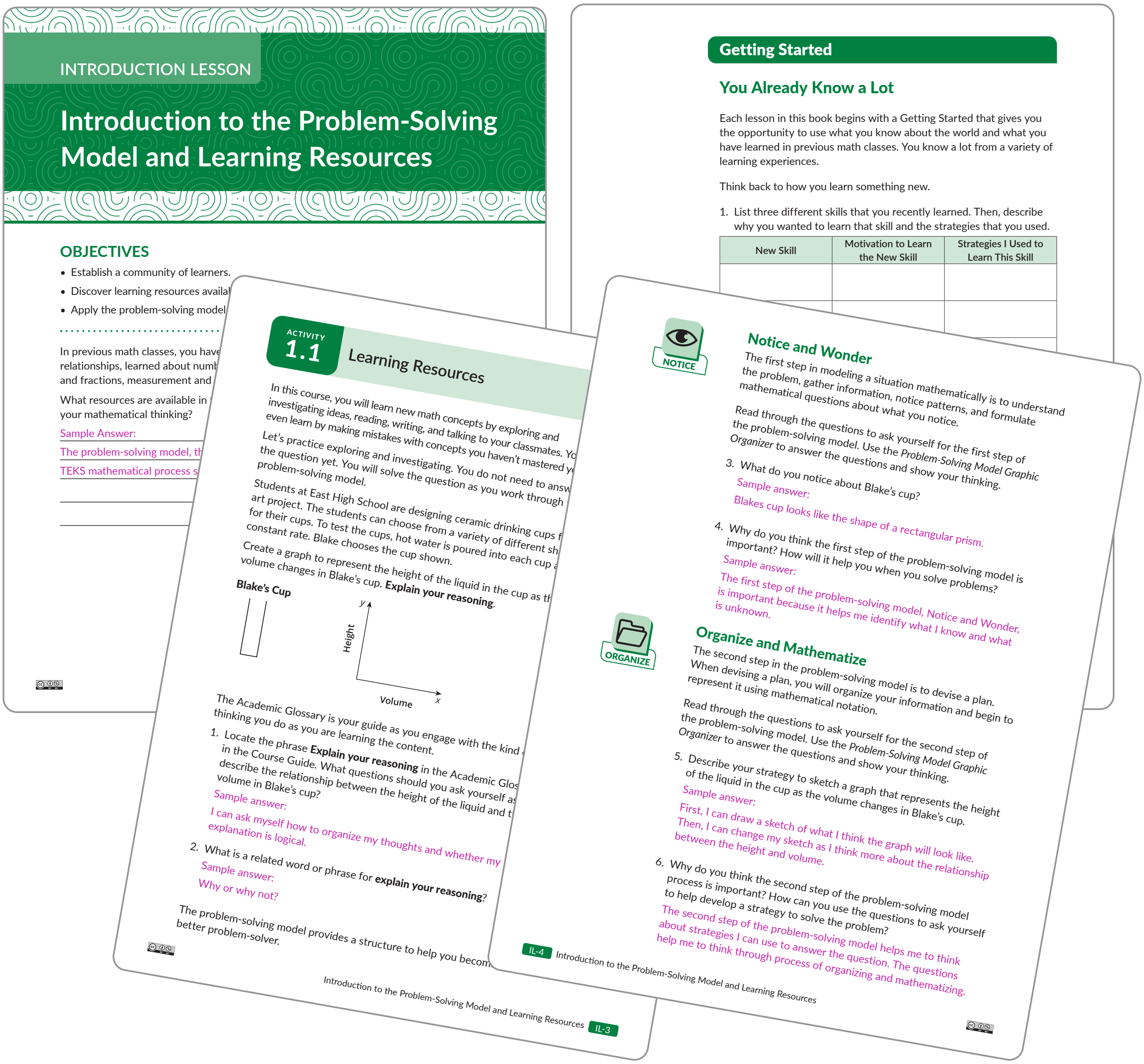
The Problem-Solving Model Graphic Organizer

NOTICE **Understand the Problem**	ORGANIZE **Devise a Plan**
PREDICT **Carry Out the Plan**	
INTERPRET **Look Back**	
REPORT **Report**	

4 THE PROBLEM-SOLVING MODEL

Introduction Lesson

The introduction lesson is designed to establish a community of learners. This lesson introduces students to the learning process and the learning resources they will utilize throughout the course. The lesson introduces or reviews the TEKS mathematical process standards, the Academic Glossary, the Math Glossary, and the Problem-Solving Model Graphic Organizer. Students will access these tools throughout the entire course. The Problem-Solving Model Graphic Organizer helps students organize and communicate their mathematics. The introduction lesson empowers students with tools to persevere during challenging mathematical tasks.



Academic Glossary

Language Expectations

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

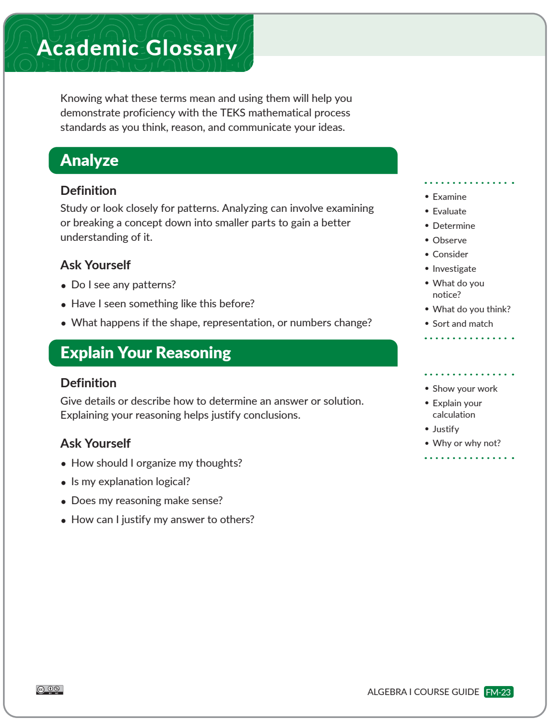
Be explicit about your expectations of language use and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different from quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.



Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition
Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself
- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

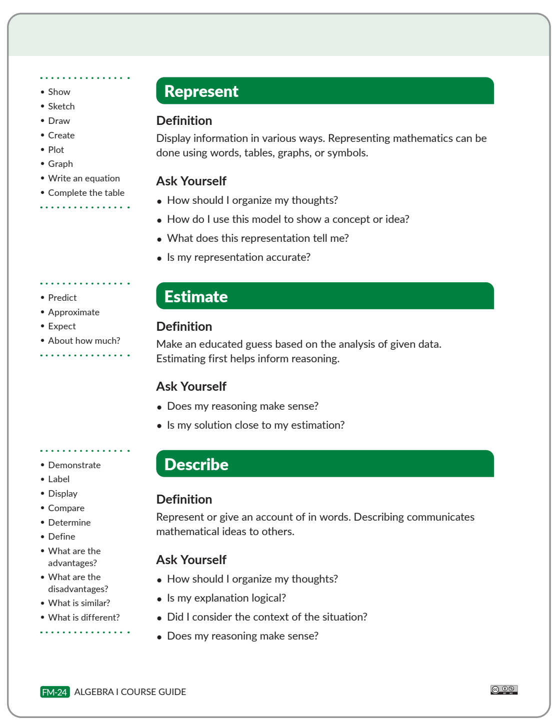
Explain Your Reasoning

Definition
Give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself
- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

ALGEBRA I COURSE GUIDE FM-23



Represent

Definition
Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself
- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Estimate

Definition
Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself
- Does my reasoning make sense?
- Is my solution close to my estimation?

- Predict
- Approximate
- Expect
- About how much?

Describe

Definition
Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself
- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

FM-24 ALGEBRA I COURSE GUIDE

Math Glossary

A course-specific math glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.

Math Glossary

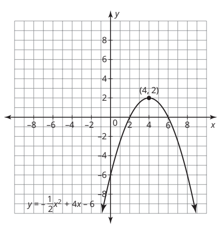
A

absolute maximum

A function has an absolute maximum if there is a point that has a y-coordinate that is greater than the y-coordinates of every other point on the graph.

Example

The ordered pair (4, 2) is the absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$.

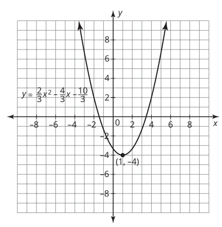


absolute minimum

A function has an absolute minimum if there is a point that has a y-coordinate that is less than the y-coordinates of every other point on the graph.

Example

The ordered pair (1, −4) is the absolute minimum of the graph of the function $y = \frac{2}{3}x^2 - \frac{4}{3}x - \frac{10}{3}$.



argument of a function

The argument of a function is the variable on which the function operates.

Example

In the function $f(x + 5) = 32$, the argument is $x + 5$.

A Collaborative Classroom

Early research on teaching and learning shows that what happens in the classroom in the first three days determines the environment for the entire year. Effective implementation is most likely to occur in the collaborative classroom, a classroom, where knowledge is shared.

The instructional materials are grounded in the belief that students develop understanding and skills by actively participating in their environment. Furthermore, effective communication and collaboration are essential skills for the successful learner. Through dialogue and discussion of different strategies and perspectives, students become knowledgeable independent learners.

Defining A Collaborative Classroom

A collaborative classroom is an environment where teachers and students share knowledge and authority. In a collaborative classroom, teachers are facilitators and students are active participants. All students, not segregated by ability level, linguistic proficiency, interest, or achievement, benefit from the environment created in the collaborative classroom.

Teachers in the collaborative classroom combine their extensive knowledge about teaching and learning, content, and skills with their students' informal and formal knowledge, strategies, and individual experiences. The collaborative classroom differs from the traditional classroom, where the teacher is seen as an information giver.

Characteristics Of The Collaborative Classroom

The collaborative classroom is identified by discussion, with in-depth, accountable talk and two-way interactions, whether among members of the whole class or small groups. It is a well-structured environment that values questioning and dialogue and sets appropriate parameters for active learning. Given the importance of dialogue, the teacher needs to understand the linguistic proficiencies of your emergent bilingual students to foster a safe environment for students to speak freely. Careful planning by the teacher ensures students can work together to attain individual and collective goals and develop learning strategies.

In the collaborative classroom, students are encouraged to take responsibility for their learning through monitoring and reflective self-evaluation. The collaborative classroom is where teachers spend more time in true academic interactions, guiding students to search for information and share what they know. As facilitators, teachers support each student's needs by providing appropriate hints, probing questions, feedback, linguistic support, and help clarifying, thinking, or using a particular strategy.

Learning In The Collaborative Classroom

Critical to teaching and learning in the collaborative environment is the ability to define the teacher's and students' responsibilities. For effective collaboration and teamwork, teachers and students must agree to specific responsibilities that support the learning process. The table reflects the parallel responsibilities of teachers and students.

Teacher Responsibilities	Student Responsibilities
Monitor student behavior.	Develop the skills to work cooperatively.
Provide assistance when needed.	Learn to talk and discuss problems with each other to accomplish the group goal.
Answer questions only when they are group questions.	Ask for help only after each group member has considered the problem and the group has a question for the teacher.
Interrupt the process to reinforce cooperative skills or to provide direct instruction to all students.	Believe that all members of the group work together toward a common goal. Understand all members share the success or failure of the group.
Provide closure for the lesson.	Reflect on the work of the group.
Evaluate the group process by discussing the actions of the group members.	Appreciate that working together is a process and encourage group members to interact with and relate to the rest of the group members.
Help students to become individually accountable for learning and reinforce this understanding regularly.	Realize that each member must contribute as much as possible to the group goal. Understand that the success of the group depends on each member's individual work and that students are accountable for their own learning.

Facilitating Productive Struggle

As students learn mathematics, it is critical that they develop characteristics, such as creative thinking, resilience, and belief in their own ability as problem solvers. *Productive struggle* is a process that supports the development of those characteristics. Engaging in productive struggle motivates students to exert effort, develop persistence, and employ effective problem-solving strategies, leading to deeper learning and improved academic performance. Research also suggests that experiencing productive struggle can increase students' self-efficacy, resilience, and perseverance through problem solving, ultimately preparing them to tackle future challenges with confidence and adaptability.

Consider teaching practices when creating opportunities for students to engage in productive struggle.

- Select non-routine tasks that allow all students to engage in productive struggle.
- Provide time for students to try different strategies and ask questions.
- Support students as they develop resilience by having them share their thought process instead of their solutions or conclusions.

What is Productive Struggle?

Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process *productive struggle*. Productive struggle is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

ALGEBRA I COURSE GUIDE FM-25

Supporting All Learners

The course materials are designed to support all learners. The course supports provide adaptations to meet the needs of emergent bilingual students, students of special populations, and gifted and talented students.

Supporting Students of Special Populations

This course is specifically designed to engage and support all learners. Supports included within this course provide ways to adapt and differentiate instruction.

To support students of special populations, differentiate instruction by:

- using embedded Differentiation Strategies.
- providing options for answering questions, such as drawing a picture or answering aloud.
- shortening assignments on a one-by-one basis.
- using the Math Glossary as a pre-teaching tool.
- utilizing the Skills Practice to target specific skills and complete prerequisite or space practice sets as needed.
- using the Prepare section of the lesson assignment to preview the material in the upcoming lesson.
- using alternative grouping strategies.
- having students interact with Worked Examples by highlighting key steps or information.

DIFFERENTIATION STRATEGY

Just in Time Support

- Encourage students to extend the graph to cover the coordinate plane when it includes an arrow.
- To determine the domain, suggest students repeat the process. This time, move their pencil from the bottom to the top of the graph and record the value of the graph's first and last points.
- To determine the range, suggest students use their pencil as if they were applying the vertical line test, but this time record the value of the graph's lowest and highest points.

Supporting Gifted and Talented Students

To support gifted and talented students or any student who is showing proficiency in a standard and is ready for a challenge and/or extension differentiate instruction by:

- using embedded Differentiation Strategies labeled as Challenge Opportunities.
- utilizing the Extension section of the Skills Practice.
- scaffolding up the academic glossary by encouraging students to apply the terminology across disciplines and in real-world applications.
- using alternative grouping strategies.

DIFFERENTIATION STRATEGIES

Access for All

- Provide students the opportunity to use function notation.
  - Ask students to rewrite $y = 3x + 2$ in function notation.
  - Show their response, $f(x) = 3x + 2$.
  - Ask students to solve $f(5)$.
  - Show substitution, $f(5) = 3(5) + 2$.
  - Show solution $f(5) = 15 + 2 = 17$.
  - Ask students to calculate the value of x when the function equals 32.
  - Since $f(x) = 32$, write the problem as $32 = 3x + 2$.
  - Solve the equation, resulting in $x = 10$.

Challenge Opportunity

- Provide an equation, such as $y = 3x + 2$. Ask students to rewrite the statement using function notation.
  - What is the value of the function when $x = 5$?
  - What is the value of x when the function equals 10?

Emergent Bilingual Students

Some emergent bilingual students may face challenges in the mathematics classroom beyond language development skills, including, peer-to-peer understanding, and building solid conceptual proficiency. The instructional materials seek to support emergent bilingual students as they develop skills in both mathematics and language.

The Topic Overview includes cognates for new key terms, when applicable. It also includes guidance on how to use cognates to support emergent bilingual students.

Throughout instruction, EB Student Tips are placed for teachers at point-of-use on the mini-lesson page in the Teacher's Edition.
They provide additional scaffolds to support this population.

These tips:

- Inform teachers of potential learning barriers specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Quantities and Relationships* when they can:

- Choose appropriate scale and origin for graphs.
- Identify the appropriate unit of measure for each variable or quantity.
- Analyze a graph and state the key characteristics of the graph.
- Use a problem situation to explain what the key features of a graph mean in real-world context.
- Decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.
- Recognize a linear, exponential, or quadratic function by its equation or graph.
- Evaluate functions, expressed in function notation, given one or more elements in their domain.
- Determine the domain and range and the independent and dependent quantities in a relationship.

How do the activities in *Quantities and Relationships* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Throughout *Quantities and Relationships*, applying mathematics to everyday life (A.1A), using the problem-solving model (A.1B), communicating mathematical ideas through multiple representations (A1.D), and connecting mathematical ideas (A.1F) are highlighted. Students search for patterns in tables, equations, and scenarios. They examine the structure of these function representations to identify common characteristics of function types. They should notice that the equations of graphs in the same family all take the same general form.

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Strategically encourage students to keep a bilingual math journal, recording reflections and background knowledge on new topics, in either written or verbal format, with added visuals for clarity. Incorporate journal excerpts into a shared word wall or digital bilingual glossary, with a focus on highlighting cognates.

NEW KEY TERMS

- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- relation [relación]
- domain [dominio]
- range [rango]
- function [función]
- function notation [notación de función]
- Vertical Line Test [Prueba de la línea vertical]
- discrete graph [gráfica discreta/discontinua]
- continuous graph [gráfica continua]
- increasing function [función creciente]
- decreasing function [función decreciente]
- constant function [función constante]
- function family [familia de funciones]
- linear functions [funciones lineales]
- exponential functions [funciones exponenciales]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]
- quadratic functions [funciones cuadráticas]
- *x*-intercept [intersección con el eje *x*]
- *y*-intercept [intersección con el eje *y*]

NEW SYMBOL

Symbol	Description
$f(x)$	Function notation

9. Determine which relations represent functions. If the relation is not a function, state why not.

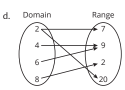
a. $y = 3^x$
The relation is a function.

b. For every house, there is one and only one street address.
The relation is a function.

c.

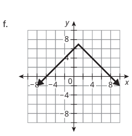
Domain	Range
−1	4
0	0
3	−2
0	4

The relation is not a function; the domain value of 0 has more than one range value.

d.


The relation is not a function; the domain value of 2 has more than one range value.

e. {(−7, 5), (−5, 5), (2, −2), (3, 5)}
The relation is a function.

f.


The relation is a function.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Ask students to identify what the prefix *non-* means in *non-function*. Follow up with additional examples of words with the prefix *non-*, including nonsmoking, nonstop, and nonfat. Define these words and highlight the connection between the prefix *non-* and the words *not* and *no*. Encourage students to remember this connection to assist them in comprehension when they come across a word with this prefix.

Preparing to Facilitate Learning

The classroom teachers make the material come alive for students, transforming the way math is taught. Implementation requires integrating Learning Together and Learning Individually. It is important to balance the two and set a structure that students can follow.

- Use the structure of the lesson to guide teaching on Learning Together days.
  - Begin by establishing mathematical objectives to focus and activate student learning.
  - Align teaching to learning by facilitating collaborative discourse and providing time for students to engage in productive struggle during learning activities.
  - Have students reflect and demonstrate their learning and provide feedback.
- Set clear expectations for the structure of the Learning Individually day to maximize class time. Set up the classroom environment for stations or small groups, providing space for the teacher or student leaders to circulate around the room to support students.

Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Develop and understanding of how each lesson fits into the big picture by using the Teacher Module and Topic Internalization Protocol.
- Understand the lesson prior to teaching by using the Teacher Lesson Internalization Protocol.
- Prepare team-building activities to intentionally create a student-centered environment.

Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of working individually and why practice is important.

- Read through Skills practice for the topic.
- Read through skills practice alignment notes at point of use in the topic lessons.
- Determine how you will interpret student's strengths, weaknesses and/or gaps, and common misconceptions from the identified activities within lessons that correspond to Skills Practice problem sets.

Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of new key terms.

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
    - Their written work is a record of their learning and is to be used as a reference for any assignments or assessments you give.
    - They will be doing the thinking, talking, and writing in your classroom.
    - They will be working and sharing their strategies and reasoning with their peers.
    - Mistakes and struggles are normal and necessary.

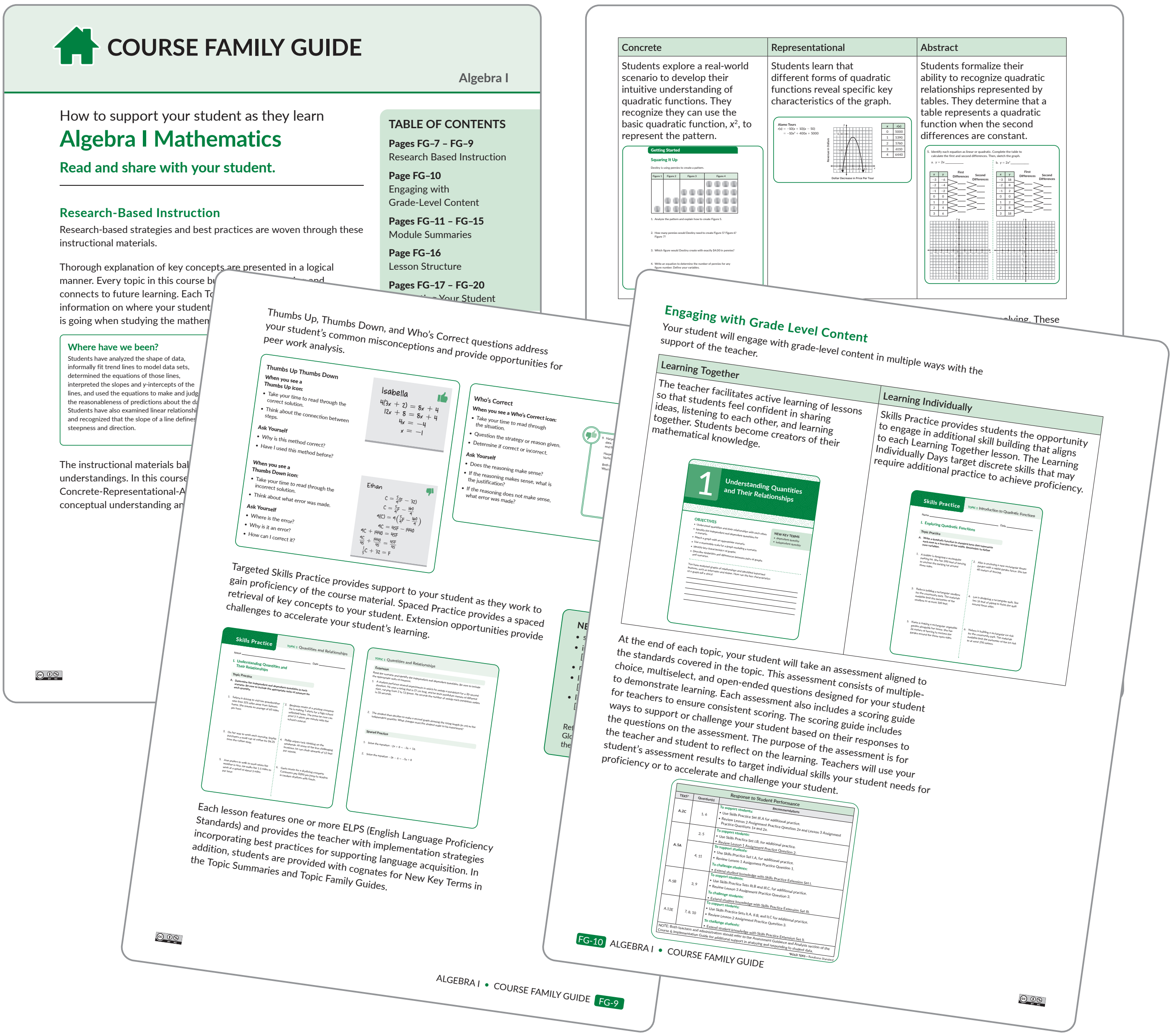
Prepare the Support

- Send home the Course Family Letter and the Course Family Guide on the first day.
- Encourage families to read the Student Course Guide.
- Ensure that families receive the Topic Family Guide at the start of each module.
- Consider a Family Math Night some time within the first few weeks of the school year.

Course Family Guide

The Course Family Guide, available in both English and Spanish, provides families with strategies they can use to support their students. The guide explains the research-based instructional approach of the course. It details the course structure as well as an overview of the content in each module of the course. The guide details the resources available to support learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

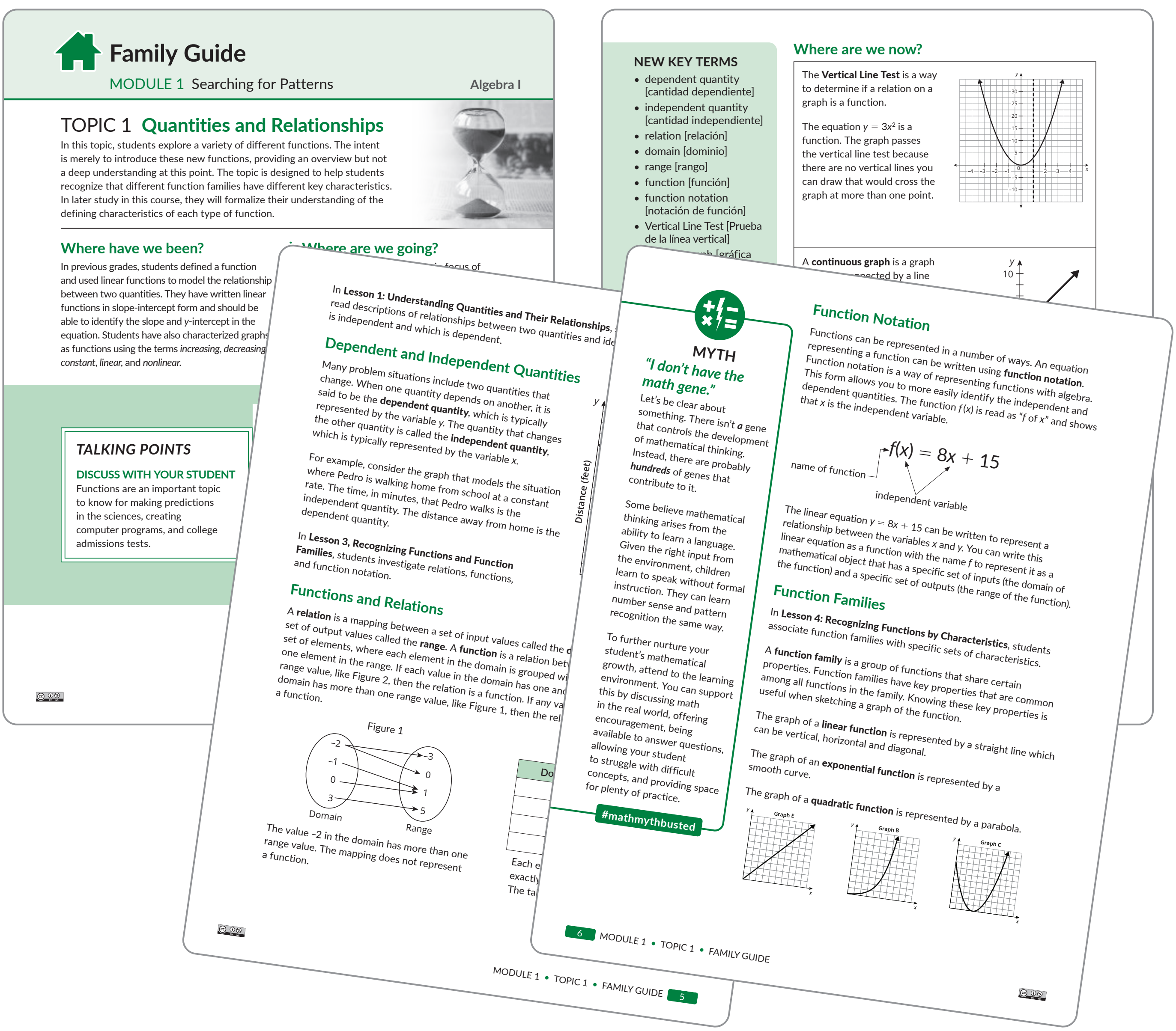
The purpose of the Course Family Guide is to bridge student learning in the classroom to student learning at home. The goal is to empower families to understand the concepts and skills learned in the classroom so that families can review, discuss, and solidify the understanding of these key concepts together.



Topic Family Guides

Each topic contains a Topic Family Guide, which is available in both English and Spanish. The Topic Family Guide provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how they will use that knowledge in future learning. It provides families an example of a math model or strategy their student is learning in the topic, busting of a math myth, questions to ask their student to support their learning, and the new key terms their student will learn.

Learning outside of the classroom is crucial to students' success at school. While families are not expected to be math teachers, the Topic Family Guides are designed to assist families as they talk to their students about what they are learning. The hope is that both the students and their families will read and benefit from the guides.



Technology Use

Technology is a useful tool in the classroom. Graphing technology can aid students in their mathematical concept development by exploring patterns, such as characteristics of graphs. In addition, there are places where additional online tools or resources are suggested in the materials for a lesson to enhance student learning. When using any website or online tool in the classroom, it is important to preview and vet the tool prior to classroom use.

Course Table of Contents

MODULE 1 Searching For Patterns

MODULE 2 Exploring Constant Change

MODULE 3 Modeling Linear Equations And Inequalities

MODULE 4 Investigating Growth And Decay

MODULE 5 Maximizing And Minimizing

MODULE
1

Searching for Patterns

MODULE 1 OVERVIEW

TEKS Addressed:
A.2A, **A.3C**, **A.6A**, **A.7A**, A.9A, **A.9D**, A.12A, A.12C, A.12D

*Bold TEKS = Readiness Standard

Searching for Patterns

Sessions: **24**

Why is this module named *Searching for Patterns*?

Students have searched for patterns in previous courses. They have recognized patterns in lists, learned to extend a pattern beyond a given list, and applied a rule to determine an arbitrary value well beyond the given list. This module extends students' understanding of functions to explore specific function families, including linear, exponential, and quadratic.

Throughout the module, students are searching for, recognizing, and defining patterns in relationships between quantities.

In the first topic, students explore various functions, presented as graphs and equations, and investigate their differentiating characteristics. Once they recognize patterns in the graphs and equations, they sort the functions into their corresponding function families.

In the second topic, students search for patterns in sequences of numbers. They recognize that while all sequences are functions, arithmetic sequences are linear functions, and some geometric sequences are exponential functions.

The Research Shows . . .

"Critical to algebraic thinking is the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined functional rules."

Fostering Algebraic Thinking: A Guide For Teacher Grades 6–10, | Page 2

What is the mathematics of *Searching for Patterns*?

Searching for Patterns contains two topics: *Quantities and Relationships* and *Sequences*. Students recognize and identify the key characteristics of different function families. They write recursive and explicit formulas for arithmetic and geometric sequences. Students will revisit all of the function families, key characteristics and types of sequences again in later modules.

1 DAY PACING = 45-MINUTE SESSION

14 SESSIONS
13 LEARNING • 1 ASSESSMENT

TOPIC 1 *Quantities and Relationships*

Learning Together: 9 Sessions

TEKS: **A.2A**, **A.3C**, **A.6A**, **A.7A**, A.9A, **A.9D**, A.12A

Students analyze scenarios and graphs representing the functions they will study throughout the course.

- Students learn to write equations for functions in function notation.
- Students recognize that different function families have different key characteristics.
- Students use graphical behavior to classify functions according to their function families.

Learning Individually: 4 Sessions

Targeted Skills Practice for *Quantities and Relationships*

- Students identify independent and dependent quantities in situations.
- Students label the axes of graphs with the independent and dependent quantities.
- Students identify domain and range, sort graphs of functions by their characteristics, and determine whether a relation is a function.
- Students classify graphs by function type.
- Students create an equation and sketch a graph for a function.

10 SESSIONS
9 LEARNING • 1 ASSESSMENT

TOPIC 2 *Sequences*

Learning Together: 6 Sessions

TEKS: A.12A, A.12C, A.12D

Students explore sequences represented as lists of numbers, tables of values, equations, and graphs modeled on the coordinate plane.

- Students recognize that all sequences are functions.
- Students recognize the characteristics of arithmetic and geometric sequences and write recursive and explicit formulas for both.

Learning Individually: 3 Sessions

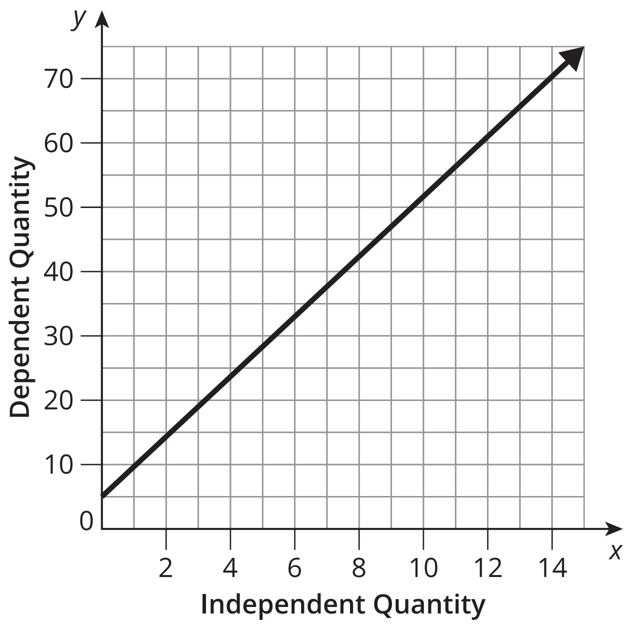
Targeted Skills Practice for *Sequences*

- Students describe patterns in sequences.
- Students determine the common difference in arithmetic sequences and the common ratio in geometric sequences and extend each type of sequence.
- Students identify sequences as arithmetic or geometric and use recursive and explicit formulas to determine unknown terms.

How is *Searching for Patterns* connected to prior learning?

Students have been reasoning with quantities in previous courses and are familiar with independent and dependent quantities. They have analyzed and interpreted linear relationships.

Independent Quantity	Dependent Quantity
0	5
1	10
2	15
3	20



You can represent this relationship with the equation $y = 5x + 5$.

When will students use knowledge from *Searching for Patterns* in future learning?

The concept of a function is the underpinning for the study of algebra. Students will explore three of the function families introduced in *Searching for Patterns*—linear, exponential, and quadratic—in more detail throughout the remainder of the course. They will use their understanding of arithmetic sequences to launch their study of linear functions.

Arithmetic Sequence	Linear Function	Mathematical Meaning
$a_n = a_1 + d(n - 1)$	$f(x) = ax + b$	
a_n	$f(x)$	output value
d	a	slope
n	x	input value
$a_1 - d$	b	y-intercept

1 Searching for Patterns

Module 1 Assessment Summary

Topic	Topic Title	Name	Administered	TEKS*
1	Quantities and Relationships	End of Topic Assessment	After Topic 1	**A.2A** **A.3C** **A.6A** **A.7A** A.9A **A.9D** A.12A
2	Sequences	End of Topic Assessment	After Topic 2	A.12A A.12C A.12D

*Bold TEKS = Readiness Standard

ALGEBRA I

TOPIC 1

The amount of sand in the lower bulb of an hourglass is directly proportional to the time since the glass was turned over.

Quantities and Relationships

TOPIC 1 OVERVIEW

Quantities and Relationships

How are the key concepts of *Quantities and Relationships* organized?

In *Quantities and Relationships*, students encounter different scenarios representing the functions they will study throughout the course. The intent is merely to introduce these new functions, providing an overview but not a deep understanding at this point. The topic is designed to help students recognize that different function families have different key characteristics. In later study—both in this course and in future courses—they will formalize their understanding of the defining characteristics of each type of function.

Students begin with an introductory lesson on the problem-solving model. They will use this model throughout the course when solving problems. They then analyze real-world scenarios. These scenarios move beyond the linear relationships familiar from Grade 7 and Grade 8 to include various nonlinear functions. Students connect the scenarios to corresponding graphs. They examine the graphical behavior of different function types by exploring a wide variety of graphs. Students search for patterns in the graphs' shape and structure and then sort them according to defined characteristics.

Students are introduced to the definitions of *function*, *domain*, and *range*. Building on their knowledge from previous grades, they formalize their representations of functions by writing equations in function notation. They use graphical behavior and the structure of the corresponding equations to classify each function according to its function family. Finally, with a more thorough understanding of the key characteristics of graphs of functions, students return to the scenarios from the first lesson and define each in terms of function family and graphical behavior.

Math Representation

A function family is a group of functions that share certain characteristics.

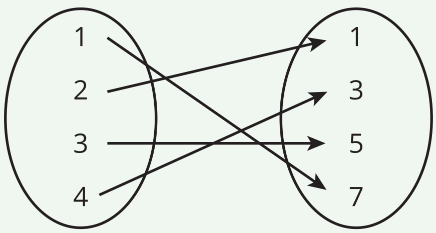
- The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.
- The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers and b is greater than 0 but not equal to 1.
- The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and a is not equal to 0.

What is the entry point for students?

Students have defined independent and dependent variables and have used them to write equations and create tables for various relationships. They have defined a function and used linear functions to model the relationship between two quantities.

Math Representation

In each mapping shown, the domain is {1, 2, 3, 4}.



The range is {1, 3, 5, 7}. The range is {1, 3, 7}.

Each mapping represents a function because no input, or domain value, is mapped to more than one output, or range value.

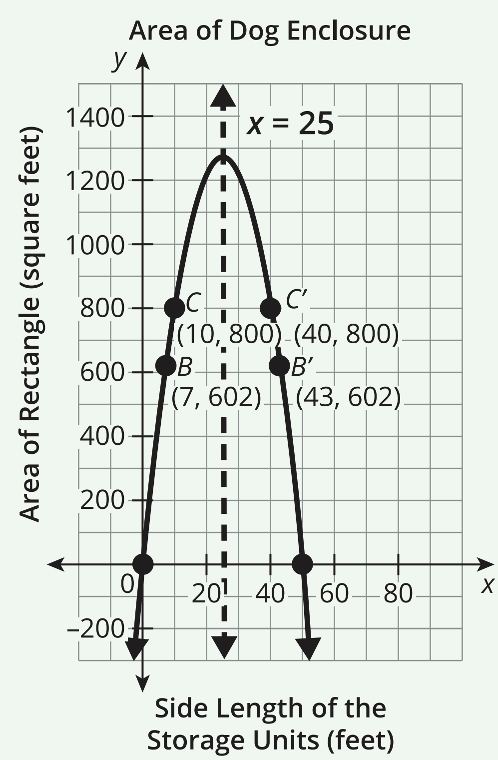
Why is *Quantities and Relationships* important?

Recognizing patterns and structure in multiple function representations allows students to generalize patterns across function families.

Math Representation

The axis of symmetry is the vertical line that passes through the vertex and divides the parabola into two mirror images.

By analyzing the symmetric point, you can see that the y-coordinates of symmetric points are the same and the horizontal distance of each symmetric point from the axis of symmetry is the same. In this situation, the axis of symmetry of $x = 25$ indicates that a side length of 25 feet for the storage area will yield the maximum area for the dog pen.



How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Quantities and Relationships* when they can:

- Choose appropriate scale and origin for graphs.
- Identify the appropriate unit of measure for each variable or quantity.
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- function [función]
- function notation [notación de función]
- Vertical Line Test [Prueba de la línea vertical]
- discrete graph [gráfica discreta/ discontinua]
- continuous graph [gráfica continua]
- increasing function [función creciente]
- decreasing function [función decreciente]
- constant function [función constante]
- function family [familia de funciones]
- linear functions [funciones lineales]
- exponential functions [funciones exponenciales]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]
- quadratic functions [funciones cuadráticas]
- *x*-intercept [intersección con el eje *x*]
- *y*-intercept [intersección con el eje *y*]

NEW SYMBOL

Symbol	Description
$f(x)$	Function notation

1 Searching for Patterns

TOPIC 1: Quantities and Relationships

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.A, 1.B, 1.C, 1.E, 1.F, 2.C, 2.E, 2.I, 3.D, 3.E, 3.H, 4.C, 4.E, 4.G, 4.H, 5.B, 5.F

Topic Pacing: 14 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
	Introduction to the Problem-Solving Model and Learning Resources	Students reflect on learning a new skill and the variety of ways they learn. The problem-solving model, TEKS mathematical process standards, and the Academic Glossary help students complete a problem-solving activity. Students reflect on and summarize the problem-solving process. Since the intent of this lesson is to introduce the problem-solving model and review the TEKS mathematical process standards, the focus is on process, not content. Students will need access to the Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Model Questions to Ask, and TEKS mathematical process standards which are located in the Course Guide. These materials should always be available to students throughout the course. **Materials Needed:** (located in the Course Guide) Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Model Questions to Ask, TEKS Mathematical Process Standards	**A.3C**	1
1	Understanding Quantities and Their Relationship	Students are presented with various scenarios and identify the independent and dependent quantities for each. They then match a graph to the appropriate scenario, label the axes using the independent and dependent quantities, and create the scale for the axes. Students make basic observations about the similarities and differences in the graphs. They then look more deeply at pairs of scenarios along with their graphs to focus on characteristics of the graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points. The lesson concludes with students creating their own scenario and a sketch of a graph to model the scenario. **Materials Needed:** Glue Sticks, Scissors	**A.3C** **A.7A** **A.9D**	2
2	Analyzing and Sorting Graphs	Students begin this lesson by cutting out 13 different graphs. They sort the graphs into different groups based on their own rationale, compare their groupings with their classmates, and discuss the reasoning behind their choices. Next, four different groups of graphs are given, and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations. Students need to keep their graphs as they will be used in lessons that follow. **Materials Needed:** Scissors, Graph Cards (located at the end of the lesson)	**A.3C** **A.7A** **A.9D**	1

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	**Recognizing Functions and Function Families**	The definitions *function* and *function notation* are introduced in this lesson. For the remainder of the lesson, students use graphing technology to connect equations written in function forms to their graphs and then identify the function family to which they belong. The terms *increasing function*, *decreasing function*, and *constant function* are defined, and students sort the graphs from the previous lesson into these groups and a group labeled for functions that include a combination of increasing, decreasing, and constant intervals. The terms *function family*, *linear function*, and *exponential function* are then defined, and students sort the increasing constant and decreasing functions into one of these families. Next, the terms *absolute minimum* and *absolute maximum* are defined as well as the term *quadratic function*. Students sort the functions with an absolute minimum or absolute maximum. Students then complete a graphic organizer for each function family that describes the graphical behavior and displays graphical examples. In the final activity, students use their knowledge of the function families to demonstrate how the families differ with respect to their *x*- and *y*-intercepts. Graphing technology is necessary to help students connect some equations and their graphs. **Materials Needed:** Graphs from *Analyzing and Sorting Graphs*, Graphing Technology, Glue Sticks	**A.2A** **A.3C** **A.6A** **A.7A** A.9A **A.9D** A.12A	3
4	**Recognizing Functions by Characteristics**	Given characteristics describing the graphical behavior of specific functions, students name the possible function family/families that fit each description. Students revisit the scenarios and graphs from the first lesson, name the function family associated with each scenario, identify the domain, and describe the graph. Students then write equations and sketch graphs to satisfy a list of characteristics. They conclude by determining that a function or equation, not just a list of characteristics, is required to generate a unique graph. **Materials Needed:** Graphs from *Analyzing and Sorting Graphs*, Problem-Solving Model Graphic Organizer	**A.2A** **A.3C** **A.6A** **A.7A** A.9A **A.9D** A.12A	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				4

*Bold TEKS = Readiness Standard

MODULE 1, TOPIC 1 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: **A.3C** Introduction to the Problem-Solving Model and Learning Resources GETTING STARTED ACTIVITY 1 TALK THE TALK	TEKS: **A.3C, A.7A, A.9D** **LESSON 1** Understanding Quantities and Their Relationships GETTING STARTED ACTIVITY 1	**LESSON 1** continued ACTIVITY 2 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.3C, A.7A, A.9D** **LESSON 2** Analyzing and Sorting Graphs GETTING STARTED ACTIVITY 1 TALK THE TALK
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LEARNING INDIVIDUALLY Skills Practice *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.2A, A.3C, A.6A, A.7A,** A.9A, **A.9D,** A.12A **LESSON 3** Recognizing Functions and Function Families GETTING STARTED ACTIVITY 1	**LESSON 3** continued ACTIVITY 2 ACTIVITY 3	**LESSON 3** continued ACTIVITY 4 ACTIVITY 5 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice *This is a suggested placement. Move based on student data and individual needs.*
Day 11	**Day 12**	**Day 13**	**Day 14**	
TEKS: **A.2A, A.3C, A.6A, A.7A,** A.9A, **A.9D,** A.12A **LESSON 4** Recognizing Functions by Characteristics GETTING STARTED ACTIVITY 1	**LESSON 4** continued ACTIVITY 2 TALK THE TALK	**LEARNING INDIVIDUALLY** Skills Practice *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**	

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are four Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice set of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources

LESSON OVERVIEW

Students reflect on learning a new skill and the variety of ways they learn. The problem-solving model, TEKS mathematical process standards, and the Academic Glossary help students complete a problem-solving activity. Students reflect on and summarize the problem-solving model. Since the intent of this lesson is to introduce the problem-solving model and review the TEKS mathematical process standards, the focus is on process not content. Students will need access to the Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Questions to Ask, and TEKS mathematical process standards which are located in the Course Guide. These materials should always be available to students throughout the course.

MATERIALS

(located in the Course Guide)

Academic Glossary

Problem-Solving Model Graphic Organizer

Problem-Solving Model Questions to Ask

TEKS Mathematical Process Standards

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1E create and use representations to organize, record, and communicate mathematical ideas.

(TEKS continued on next page)

ELPS

(1) Learning Strategies

The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(2) Listening

The student is expected to:

(I) demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.

(4) Reading

The student is expected to:

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

(H) read silently with increasing ease and comprehension for longer periods.

ALGEBRA I TEKS *(TEKS continued from previous page)*

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

ESSENTIAL IDEAS

- Create a classroom of collaboration and establish the learning process as a partnership between you and your students.
- Communicate continuously with students about the objectives of the lesson to encourage self-monitoring of their learning.
- The problem-solving model involves noticing patterns and formulating questions, organizing information and representing this information using appropriate mathematical notation, analyzing mathematical representations and using them to make predictions, and then testing predictions, predicting, and sharing the results.
- The TEKS mathematical process standards describe the ways in which students are expected to engage in content.
- The Academic Glossary is a resource that helps students think, reason, and communicate their ideas.

LESSON STRUCTURE AND PACING: 1 DAY

ENGAGE

Getting Started: You Already Know A Lot! 10–15 minutes

BUILDING OFF INTUITION

Students describe different strategies they use to learn new skills. They identify their motivation to learn new skills. Students learn that collaboration is important to learning new skills.

DEVELOP

Activity 1: Learning Resources 30 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

Students use the problem-solving model, TEKS mathematical process standards, and Academic Glossary to complete a problem-solving task.

DEMONSTRATE

Talk the Talk: The Problem-Solving Model 5 minutes

GENERALIZATION

Students reflect on the modeling process and summarize what is involved in each phase.

Getting Started

ENGAGE

You Already Know A Lot!

Facilitation Notes

In this activity, students describe different strategies they use to learn new skills. They identify their motivation to learn new skills. Students learn that collaboration is important to learning new skills.

Read and discuss the introduction. Have students work individually to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Think of a time when you helped someone learn how to do something. What did you do?
Reflecting and justifying	• How do the strategies you use to learn new skills help you persevere when solving problems?

COMMON MISCONCEPTION

Students often believe that reading is not a skill regularly used in math class. They need support and opportunities to build their capacity and stamina for reading in math. When planning, think about how you will scaffold the text for students. Think about breaking the text into smaller pieces to analyze, reading the text aloud to the whole class, or other strategies to support students as they read the text.

Optimizing Learning

This differentiation strategy provides options for fostering collaboration and community.

DIFFERENTIATION STRATEGY

Access for All

To encourage collaboration, select team-building activities to do with your class at the end of this activity.

Read and discuss the directions. Have students work with a partner or in a group to complete Question 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did your list change when you started talking to other students? • Do you learn things the same way? • How long does it take you to learn something new?
Reflecting and justifying	• At what point are you confident in your knowledge?

STAMP THE LEARNING

Summary

Learning happens in different ways, using different strategies. Math is not different. This course uses a variety of strategies to support learning.

ACTIVITY 1.1 Learning Resources

DEVELOP

Facilitation Notes

In this activity, students use the problem-solving model, TEKS mathematical process standards, and Academic Glossary to complete a problem-solving task.

Read the introduction together as a class. Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are some situations where the Academic Glossary will be helpful?
Probing	• Which part(s) of the Academic Glossary are most helpful for you?
Reflecting and justifying	• How is the Academic Glossary different from the Math Glossary

DIFFERENTIATION STRATEGY

Access for All

Use random groups for this activity at the start of the school year as you learn more about your students. As the year progresses, use what you know about students, along with data from Skills Practice and other assessments, to strategize how to group your students.

Read the information following Question 2 together as a class. Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why would Mario need to test his cup?
Reflecting and justifying	• How is this problem similar to other problems you have solved?

Read the information following Question 4 as a class. Have students work with a partner or in a group to complete Questions 5 and 6.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Describe your strategy. • How can you document your steps so that you can remember them? • What mathematical processes are a part of your strategy? • Did you and your partner approach the problem the same way?
Reflecting and justifying	• Did you and your partner approach the problem the same way? • What did you learn from listening to and sharing with your partner?

Read the information following Question 6 as a class. Have students work with a partner or in a group to complete Questions 7 and 8.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• Did you use your initial strategy? Explain your reasoning. • Is there another strategy you could use? Explain your reasoning. • Did you communicate your mathematics in an organized way that is easy to understand?

Read the information following Question 8 as a class. Have students work with a partner or in a group to complete Questions 9 and 10.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How does your strategy change when the shape of the cup changes?
Reflecting and justifying	• How do you know when you have determined all the different ways you can organize the cards?

Read the information following Question 10 as a class. Share student solutions as a class. Have students individually answer Question 11.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• How does your strategy compare to other students' strategies? • How can you give constructive feedback to another student? • What norms will help our class collaborate and share ideas?

Review the TEKS mathematical process standards as a class. Have students work with a partner or in a group to complete Question 12. Share responses as a class.

Reflecting and justifying	• Which TEKS mathematical process standard do you feel the most familiar with? • Why do you think the TEKS mathematical process standards are important?

Summary

STAMP THE LEARNING

The problem-solving model, TEKS mathematical process standards, and the Academic Glossary are resources available to support your learning in this course.

DEMONSTRATE

Talk the Talk

THE PROBLEM-SOLVING MODEL

Facilitation Notes

In this activity, students reflect on the modeling process: Notice and Wonder, Organize and Mathematize, Predict and Analyze, Test and Interpret, and Report.

Have students work with a partner or in a group to complete the activity. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Describe how you engaged in each phase as you solved the problem. • Why is it essential that Notice and Wonder is the first phase? • How is the Organize and Mathematize phase related to efficiency? • What should you do when your results don't match your prediction in the Test and Interpret phase?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

The mathematical modeling process includes the basic steps: (1) Notice and Wonder, (2) Organize and Mathematize, (3) Predict and Analyze, (4) Test and Interpret, and (5) Report.

INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Establish a community of learners.
- Discover learning resources available for this course.
- Apply the problem-solving model to a real-life situation.

In previous math classes, you have analyzed patterns and relationships, learned about numbers and operations in base ten and fractions, measurement and data, and geometry.

What resources are available in this course to help you extend your mathematical thinking?

Sample Answer:

The problem-solving model, the Academic Glossary, and the TEKS mathematical process standards

MODULE 1 • INTRODUCTION TO THE PROBLEM-SOLVING MODEL AND LEARNING RESOURCES IL-1

Chunking the Activity

- Read and discuss the directions.
- Have students work individually to complete Question 1.
- Read and discuss the directions.
- Group students to complete Question 2.
- Share and summarize.

Getting Started

You Already Know A Lot

Each lesson in this book begins with a Getting Started that gives you the opportunity to use what you know about the world and what you have learned in previous math classes. You know a lot from a variety of learning experiences.

Think back to how you learn something new.

1. List three different skills that you recently learned. Then, describe why you wanted to learn that skill and the strategies that you used.

New Skill	Motivation to Learn the New Skill	Strategies I Used to Learn This Skill

Ask Yourself . . .
How do your strategies change based on what you are learning and what you already know about it?

One learning strategy is to talk with your peers. In this course, you will work with your classmates to solve problems, discuss strategies, and learn together.

Compare and discuss your list with a classmate.

2. Which strategies do you have in common? Which strategies does your classmate have that you did not think of on your own?

Think About . . .
Listening well, cooperating with others, and appreciating different perspectives are essential life skills.

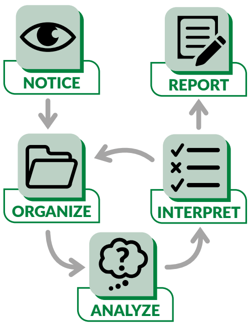
Be prepared to share your list of learning strategies with the class.

IL-2 MODULE 1 • INTRODUCTION TO THE PROBLEM-SOLVING MODEL AND LEARNING RESOURCES

ACTIVITY 1.1

Learning Resources

PROBLEM SOLVING



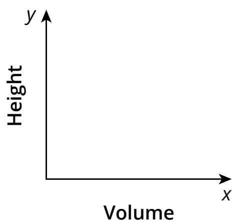
In this course, you will learn new math concepts by exploring and investigating ideas, reading, writing, and talking to your classmates. You will even learn by making mistakes with concepts you haven't mastered yet.

Let's practice exploring and investigating. You do not need to answer the question yet. You will solve the question as you work through the problem-solving model.

Students at East High School are designing ceramic drinking cups for an art project. The students can choose from a variety of different shapes for their cups. To test the cups, hot water is poured into each cup at a constant rate. Mario chooses the cup shown.

Create a graph to represent the height of the liquid in the cup as the volume changes in Mario's cup. **Explain your reasoning**.

Mario's Cup



The Academic Glossary provides definitions of terms you will see throughout the course as you think, reason, and communicate your ideas. The Math Glossary provides the definitions of new key terms in each lesson.

The Academic Glossary is your guide as you engage with the kind of thinking you do as you are learning the content.

1. Locate the phrase **Explain your reasoning** in the Academic Glossary in the Course Guide. What questions should you ask yourself as you describe the relationship between the height of the liquid and the volume in Mario's cup?

   Sample answer:

   I can ask myself how to organize my thoughts and whether my explanation is logical.

2. What is a related word or phrase for **explain your reasoning**?

   Sample answer:

   Why or why not?

The problem-solving model provides a structure to help you become a better problem-solver.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Group students to complete Questions 3 and 4.
- Check in and share.
- Group students to complete Questions 5 and 6.
- Check in and share.
- Group students to complete Questions 7 and 8.
- Check in and share.
- Group students to complete Questions 9 and 10.
- Have students individually answer Question 11.
- Group students to complete Question 12.
- Share and summarize.

EB STUDENT TIP

For all proficiency levels

While students have encountered icons for apps, games, or programs, ensure that they understand *icon* refers to the symbols used to represent features in the text, such as the icon for each step of the problem-solving model.

Notice and Wonder

The first step in modeling a situation mathematically is to understand the problem, gather information, notice patterns, and formulate mathematical questions about what you notice.

Read through the questions to ask yourself for the first step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

3. What do you notice about Mario's cup?

   Sample answer:

   Mario's cup looks like the shape of a rectangular prism.

4. Why do you think the first step of the problem-solving model is important? How will it help you when you solve problems?

   Sample answer:

   The first step of the problem-solving model, Notice and Wonder, is important because it helps me identify what I know and what is unknown.

Organize and Mathematize

The second step in the problem-solving model is to devise a plan. When devising a plan, you will organize your information and begin to represent it using mathematical notation.

Read through the questions to ask yourself for the second step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

5. Describe your strategy to sketch a graph that represents the height of the liquid in the cup as the volume changes in Mario's cup.

   Sample answer:

   First, I can draw a sketch of what I think the graph will look like. Then, I can change my sketch as I think more about the relationship between the height and volume.

6. Why do you think the second step of the problem-solving model process is important? How can you use the questions to ask yourself to help develop a strategy to solve the problem?

   The second step of the problem-solving model helps me to think about strategies I can use to answer the question. The questions help me to think through process of organizing and mathematizing.

IL-4 MODULE 1 • INTRODUCTION TO THE PROBLEM-SOLVING MODEL AND LEARNING RESOURCES

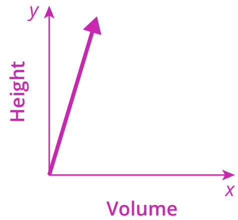
Predict and Analyze

PREDICT

The third step in the modeling process involves carrying out your plan. As you carry out the plan you will complete operations, analyze mathematical results, make predictions, and extend patterns.

Read through the questions to ask yourself for the third step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

7. Use your strategy to create a graph that represents the height of the liquid in the cup as the volume changes in Blake's cup.



NOTE: Allow students an opportunity to experience productive struggle. See the Facilitating Productive Struggle section in the Course and Implementation Guide for additional guidance on supporting students through problem-solving activities.

8. How can the questions to ask for the third step help you communicate your mathematical thinking?

   Sample answer:

   The questions to ask remind me to show my work and explain my solution.

Test and Interpret

INTERPRET

The fourth step in the modeling process is to look back, interpret your results, and test your mathematical predictions in the real world. When your predictions are incorrect, or your results are not reasonable, you can revisit your mathematical work and make adjustments—or start all over!

Read through the questions to ask yourself for the fourth step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

NOTE: Remind students to turn to the Academic Glossary and read about the term **Explain Your Reasoning** before answering Question 9.

9. Alyssa choses a different design. Can you apply the same reasoning to create a graph to represent the height of the liquid in the cup as the volume changes in Jacquelyn's cup.

   Sample answer:

   Yes, the graph will be different, but I can use the same process.

MODULE 1 • INTRODUCTION TO THE PROBLEM-SOLVING MODEL AND LEARNING RESOURCES IL-5

10. How do the questions for the fourth step of the problem-solving model help you evaluate the reasonableness of your solution?

    Sample answer:

    The questions remind me to check the reasonableness of my answer and to check that I answered the question completely.

Report

The final step in the modeling process is to share your results.

Optimizing Learning

This activity provides options that support students to develop self-assessment and reflection.

11. How does listening to others help you learn mathematics?

    Sample answer:

    When I listen to others, I learn other ways of thinking about the question.

Locate the TEKS mathematical process standards in the Course Guide.

12. Which TEKS mathematical process standard(s) did you use to create a graph that represents the height of the liquid in the cup as the volume changes in Mario's cup?

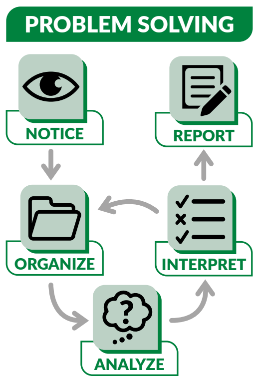
    Sample answer:

    I used all the TEKS mathematical process standards. The problem is set in a real-world situation. I used a problem-solving model to answer the question. I selected to use paper and pencil and graph to solve the problem. I communicated my mathematical ideas using diagrams and language. I recorded my mathematical ideas. I analyzed the height of the liquid in the cup as the volume changed. I explained my answer using precise mathematical language when I shared my solution.

IL-6 MODULE 1 • INTRODUCTION TO THE PROBLEM-SOLVING MODEL AND LEARNING RESOURCES

Talk the Talk

The Problem-Solving Model



In this lesson, you used a problem-solving model to solve a real-world problem. The basic steps of the problem-solving model are summarized in the diagram.

Summarize what is involved in each phase of this problem-solving model.

Sample answers:

Notice and Wonder

Gather information, notice patterns, and formulate mathematical questions about what you notice.

Organize and Mathematize

Organize your information and represent it using mathematical notation.

Predict and Analyze

Extend the patterns created, complete operations, make predictions, and analyze the mathematical results.

Test and Interpret

Interpret your results and test your mathematical predictions in the real world. Make adjustments when necessary.

Report

Report your results to a partner, small group, or the class.

You will see this symbol throughout the course to remind you to use the problem-solving model.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

MODULE 1 • INTRODUCTION TO THE PROBLEM-SOLVING MODEL AND LEARNING RESOURCES IL-7

1 Understanding Quantities and Their Relationships

MATERIALS

Glue Sticks

Scissors

LESSON OVERVIEW

Students are presented with various scenarios and identify the independent and dependent quantities for each. They then match a graph to the appropriate scenario, label the axes using the independent and dependent quantities, and create the scale for the axes. Students make basic observations about the similarities and differences in the graphs. They then look more deeply at pairs of scenarios along with their graphs to focus on characteristics of the graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points. The lesson concludes with students creating their own scenario and a sketch of a graph to model the scenario.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
The student is expected to:

A.3C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

(TEKS continued on next page)

ELPS

(1) Learning Strategies
The student is expected to:

(C) use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary.

(F) use accessible language and learn new and essential language in the process.

(2) Listening
The student is expected to:

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

(E) use visual, contextual, and linguistic support to enhance and confirm understanding of increasingly complex and elaborate spoken language.

(4) Reading
The student is expected to:

(E) read linguistically accommodated content area material with a decreasing need for linguistic accommodations as more English is learned.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Quadratic Functions and Equations

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations.

The student is expected to:

A.7A graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data.

The student is expected to:

A.9D graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

ESSENTIAL IDEAS

- There are two quantities that change in problem situations.
- When one quantity is determined by another, it is said to be the dependent quantity. The quantity that the dependent quantity is determined from is called the independent quantity.
- The independent quantity is used to label the x-axis. The dependent quantity is used to label the y-axis.
- Graphs can be used to model problem situations.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: What Comes First? 5–10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students read descriptions of relationships between two quantities and identify which is independent and which is dependent.

DEVELOP

Activity 1.1: Connecting Scenarios and Their Graphs 30–35 minutes

REAL-WORLD PROBLEM SOLVING

Students are presented with six different scenarios. For each scenario, they identify the independent and dependent quantities and match a graph. Students then scale the axes and determine the domain and range for each scenario.

DAY 2

Activity 1.2: Comparing and Contrasting Graphs 30 minutes

MATHEMATICAL PROBLEM SOLVING

Students make basic observations about the similarities and differences in the graphs from the previous activity. They then look more deeply at pairs of scenarios along with their graphs to focus on key characteristics, such as intercepts, increasing and decreasing intervals, and maximum and minimum points.

DEMONSTRATE

Talk the Talk: A Writer and a Mathematician 10–15 minutes

EXIT TICKET APPLICATION

Students create a scenario based upon a possible trip to school. They then sketch a graph to model their scenario. Students share their work with classmates and note similarities and differences.

Getting Started

ENGAGE

What Comes First?

Facilitation Notes

In this activity, students read descriptions of relationships between two quantities and identify which is the independent and which is the dependent.

Ask a student to read the introduction before Question 1 aloud. Review the definitions of dependent quantity and independent quantity as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Strategies and phrases they use to determine which quantity depends on the other.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why did you decide that quantity is the independent quantity? Dependent quantity? • What is another way to explain how to determine the independent and dependent quantities?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Provide opposing scenarios to make independence and dependence more explicit. For example:
  "I have 6 eggs in my refrigerator, so I will make 3 cakes."
  "I am going to make 3 cakes, so I need 6 eggs."
  Because knowing the number of cakes first makes more sense, the independent quantity is the number of cakes.
- Suggest students ask themselves similar questions to identify the independent and dependent quantities in the remaining questions.

COMMON MISCONCEPTION

Students may confuse the independent variable with the dependent variable. For example, they could think the number of movie tickets is determined by the total cost of the tickets (if the cost of three tickets is $22.50, then each ticket must have been $7.50). Just because the value of one variable can be determined using the value of a second variable, this does not signify dependence or independence.

Summary

STAMP THE LEARNING

There are two quantities that change in problem situations. When one quantity is determined by another, it is said to be the dependent quantity. The quantity that the dependent quantity is determined from is called the independent quantity.

ACTIVITY 1.1 Connecting Scenarios and Their Graphs

DEVELOP

Facilitation Notes

In this activity, students are presented with six different scenarios. For each scenario, they identify the independent and dependent quantities and match a graph. Students then scale the axes and determine the domain and range for each scenario.

Ask a student to read the introduction before Question 1 aloud. As a class, discuss the directions to this task because it has several parts and includes cutting out and gluing graphs next to their scenario descriptions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Characteristics of the graphs that students use to connect them to the scenarios.
- Clues students use in the scenarios to determine the scale.

DIFFERENTIATION STRATEGY

Access for All

As an alternative grouping method, use the jigsaw strategy for scaling the axes for each scenario. This strategy is meant to save time while providing a brief recall of scaling, but the sharing part is necessary so that students can use the information to determine the domain and range for each problem.

QUESTIONS TO SUPPORT DISCOURSE FOR *MUSIC CLUB*

Probing	• How is the number of songs measured? • How is the cost measured? • What is the cost for zero songs? One song? Five songs? • Does the number of songs determine the cost, or does the cost determine the number of songs? • Can this scenario be described using a rate of change? • Is this an increasing or decreasing function? • What is the meaning of the point located on the x-axis?

QUESTIONS TO SUPPORT DISCOURSE FOR *SOMETHING'S FISHY*

Probing	• Did you select a graph that was increasing or decreasing? Why? • How did you determine the units and intervals for the axes? • What is the meaning of the y-intercept? • Is the dependent quantity the amount of water draining or the remaining water in the aquarium? Explain your thinking.
Seeing structure	• How did you know the graph is a straight line?

QUESTIONS TO SUPPORT DISCOURSE FOR *SMART PHONE, BUT IS IT A SMART DEAL?*

Probing	• Did you select a graph that was increasing or decreasing? Why? • How did you determine the units and intervals for the axes? • What is the meaning of the y-intercept? • Explain how the graph shows the interest for the first several weeks.
Seeing structure	• Why did you use a graph with a curve to represent this situation?

QUESTIONS TO SUPPORT DISCOURSE FOR *IT'S MAGIC*

Probing	• Did you select a graph that was increasing or decreasing? Why? • How did you determine the units and intervals for the axes? • What does the y-intercept represent?
Seeing structure	• How do this scenario and graph relate to the Smart Phone scenario and its graph?

QUESTIONS TO SUPPORT DISCOURSE FOR *BATON TWIRLING*

Probing	• Did you select a graph that was increasing or decreasing? Why? • How did you determine the units and intervals for the axes? • What does each point on the graph represent? • What characteristic does this graph have that the others do not?
Seeing structure	• When the baton goes straight up and comes straight back down, why isn't the graph a vertical segment?

QUESTIONS TO SUPPORT DISCOURSE FOR *SKATEBOARDING*

Probing	• How did you determine the units and intervals for the axes? • What is the meaning of the *x*-intercept?
Seeing structure	• What is the relationship between the symmetry of this graph and the scenario?

To close the Day 1 session, have the students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

Graphs can be used to model scenarios. Knowing the independent and dependent variables, as well as the domain and range, is helpful in making connections between the scenario and its graph.

ACTIVITY 1.2 Comparing and Contrasting Graphs

Facilitation Notes

In this activity, students make basic observations about the similarities and differences in the graphs from the previous activity. They then look more deeply at pairs of scenarios along with their graphs to focus on key characteristics, such as intercepts, increasing and decreasing intervals, and maximum and minimum points.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Mathematical terms used to describe similarities and differences in the graphs.
- Instances where students would benefit from an increased mathematical vocabulary to describe graphical characteristics.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which features are always the same when you set up a graph? • How can you tell whether a graph is increasing or decreasing? • Do all graphs start at the origin? Explain why or why not.
Seeing structure	• What does a graph with a constant rate look like? • What are different ways a graph that models a non-constant rate might appear?
Probing	• Why are both graphs increasing in Question 5 part (a)?
Seeing structure	• Why is one graph in Question 5 part (a) composed of line segments while the other is a curve? • Explain why one graph in Question 5 part (b) is a line and the other is a curve. • Why do both graphs in Question 5 part (c) have symmetry? • Which type of equation represents each graph in part (c)?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Provide strategies to determine whether a graph is increasing or decreasing.
- Read the graph from left to right, just as you read text.
- Use the sentence structure, “As the *x*-value increases, the *y*-value increases/decreases.”

STAMP THE LEARNING

Summary

Key characteristics of graphs, such as intercepts, increasing and decreasing intervals, and maximum and minimum points are used to interpret scenarios and differentiate graphs.

DEMONSTRATE

Talk the Talk

A WRITER AND A MATHEMATICIAN

Facilitation Notes

In this activity, students create a scenario based upon a possible trip to school. They then sketch a graph to model their scenario. They share their work with classmates and note similarities and differences.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you label the axes? • What was the distance of the trip? • How did you inform the reader about the pace? • Did you embellish your scenario so that the graph wasn't a straight line or smooth curve? If so, explain how. • Could more than one possible graph model your scenario? Explain. • How did you use the axes' labels and scales to support your response?
Seeing structure	• Is your graph increasing or decreasing? Why? • What type of scenario would create a curved graph?

DIFFERENTIATION STRATEGIES

Access for All

- When setting up their graph, ask students questions about the significance of points on the *x*-axis, on the *y*-axis, and at the origin.

Challenge Opportunity

- Have students create a scenario and trade papers with a partner. Then, have the partner draw a graph for the scenario. Have partners discuss if the graph drawn was the intent of the writer of the scenario.
- Have students create two graphs, one with the *y*-axis labeled as the distance from home and the other with the *y*-axis labeled distance from school. Then, have them compare the characteristics of the graphs.

Have students read and answer the Essential Question on the lesson opener page.

Summary

A graph is an efficient way to model and interpret a scenario.

STAMP THE LEARNING

1 Understanding Quantities and Their Relationships

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Understand quantities and their relationships with each other.
- Identify the independent and dependent quantities for a scenario.
- Match a graph with an appropriate scenario.
- Use a reasonable scale for a graph modeling a scenario.
- Identify key characteristics of graphs.
- Describe similarities and differences between pairs of graphs and scenarios.

NEW KEY TERMS

- dependent quantity
- independent quantity

You have analyzed graphs of relationships and identified important features, such as intercepts and slopes. How can the key characteristics of a graph tell a story?

Sample answer:

When one quantity depends on another, it is a dependent quantity. The quantity it depends upon is the independent quantity. The independent quantity is represented on the *x*-axis, and the dependent quantity is represented on the *y*-axis.

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MODULE 1 • TOPIC 1 • LESSON 1 5

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice identifying independent and dependent quantities, assign Skills Practice Set A for this lesson.

Getting Started

What Comes First?

Have you ever planned a birthday party? You may have purchased ice, gone grocery shopping, selected music, made food, or even cleaned in preparation. Many times, these tasks depend on another task being done first. For instance, you wouldn't make food before grocery shopping, now would you?

Consider the two quantities that are changing in each relationship.

- The number of movie tickets purchased and the total cost
- The number of eggs used and the number of cakes baked
- The number of students in attendance at school and the number of lunches served
- The number of hours driven and the number of miles to a vacation destination
- The number of minutes a swimming pool is filled with water and the number of gallons of water in the swimming pool

When one quantity is determined by another in a problem situation, it is called the **dependent quantity**. The quantity it is determined from is called the **independent quantity**.

1. Circle the independent quantity and underline the dependent quantity in each relationship.

2. Describe how you can determine which quantity is independent and which quantity is dependent in any problem situation.

   The independent quantity is the one that is necessary to know first. It affects the dependent quantity.

EB STUDENT TIP

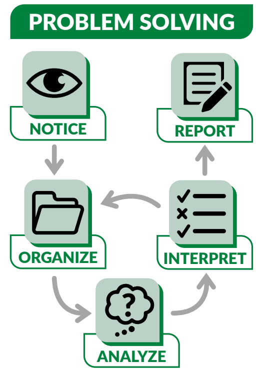
For "Intermediate" and higher proficiency levels

The terms *independent* and *dependent* are cognates in many languages and may be easily identified by emergent bilingual students. Review the verb *depend* with students and discuss how it is related to *independent* and *dependent*. Give them the following sentence frame to practice sentence structure and independent/dependent identification.

"__________ depend(s) on __________ to __________.
__________ is dependent, and __________ is independent."

ACTIVITY **1.1**

Connecting Scenarios and Their Graphs



While a person can describe the monthly cost to operate a business or talk about a marathon pace a runner ran to break a world record, graphs on a coordinate plane enable people to see the data. Graphs relay information about data in a visual way.

You can use lines or smooth curves to represent relationships between points on a graph. In some problem situations, all the points on the line will make sense. In other problem situations, not all the points will make sense. So, when you model a relationship with a line or a curve, it is up to you to consider the situation and interpret the meaning of the data values shown.

This activity includes six scenarios and six graphs that are located at the end of the lesson.

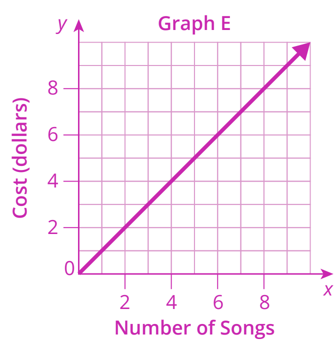
Ask Yourself . . .
How can graphs be used to tell a story in everyday life?

1. Read each scenario. Determine the independent and dependent quantities. Then, match each scenario to its corresponding graph. Glue the graph next to the scenario. For each graph, label the *x*- and *y*-axes with the appropriate quantity and a reasonable scale and then interpret the meaning of the origin.

Be sure to include the appropriate units of measure for each quantity.

Music Club
Natalia loves music. She can lip sync almost any song at a moment's notice. She joined Songs When I Want Them, an online music store. By becoming a member, Natalia can purchase just about any song she wants. Natalia pays $1 per song.

- Independent quantity:
  Number of songs; *x*-axis: interval of 1
- Dependent quantity:
  Cost (dollars); *y*-axis: interval of 1
  Origin: (0 songs, 0 dollar cost)



Chunking the Activity
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Optimizing Learning
This activity highlights patterns, critical features, big ideas and relationships.

NOTE: This is the first lesson where TEKS A.1A is highlighted.
- Read and display TEKS A.1A and explain that this activity is an example of applying math to everyday life.

EB STUDENT TIP
For all proficiency levels
Students who understand the concept of independent and dependent quantities may still struggle to express this understanding if they lack familiarity with a given cultural context or its related vocabulary in English for a given scenario. Ensure students have a clear understanding of all variables before they begin.

MODULE 1 • TOPIC 1 • LESSON 1 7

NOTE: This is the first lesson where TEKS A.1C is highlighted.

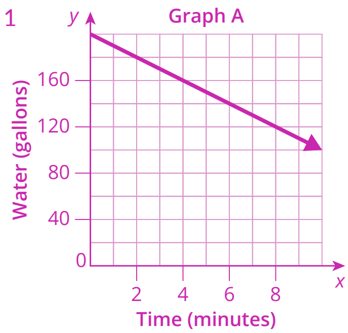
- Read and display TEKS A.1C and explain that students can select tools including paper and pencil or technology to determine what a graph that models each scenario looks like.

Ask Yourself . . .
What strategies will you use to match each graph with one of the six scenarios?

Something's Fishy

Kaya is the building manager for an office building. One of her responsibilities is cleaning the office building's 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.

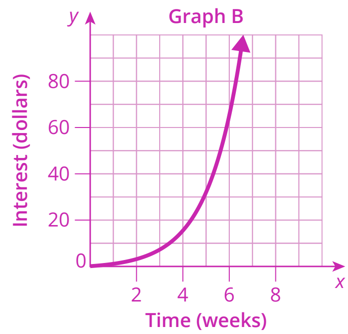
- Independent quantity:
  Time (minutes); *x*-axis: interval of 1
- Dependent quantity:
  Water (gallons); *y*-axis: interval of 20
  Origin: (0 minutes, 0 gallons of water)



Smart Phone, but Is It a Smart Deal?

You have your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him back with interest. He tells you that you only need to pay $1 in interest initially and then the interest will double each week after that. You consider his offer and wonder if this *really* is a good deal.

- Independent quantity:
  Time (weeks); *x*-axis: interval of 1
- Dependent quantity:
  Interest (dollars); *y*-axis: interval of 10
  Origin: (0 weeks, 0 dollars of interest)



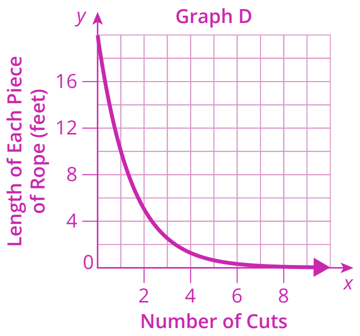
8 MODULE 1 • TOPIC 1 • LESSON 1

It's Magic

The Amazing Alejandro is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it.

- Independent quantity:

  Number of cuts; *x*-axis: interval of 1



- Dependent quantity:

  Length of each piece of rope (feet); *y*-axis: interval of 2

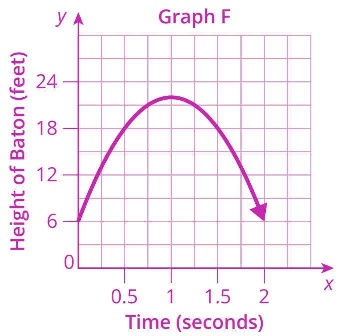
  Origin: (0 cuts, 0 feet of rope)

Baton Twirling

Samantha is a drum major for the high school marching band. For the finale of the halftime performance, Samantha tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

- Independent quantity:

  Time (seconds); *x*-axis: interval of 0.25



- Dependent quantity:

  Height of baton (feet); *y*-axis: interval of 3

  Origin: (0 seconds, height of 0 feet)

MODULE 1 • TOPIC 1 • LESSON 1 9

Activity 1.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice labeling the axes of graphs with independent and dependent quantities, assign Skills Practice Set B for this lesson.

Skateboarding

Andrew is skateboarding on a 4-foot-tall half-pipe at the local skate park. He wants to know how far his skateboard's wheels are from the ground as he skates from the top of one side of the half-pipe to the top of the other side.

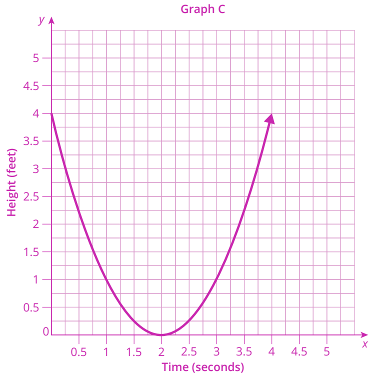
- Independent quantity:

  Time (seconds); *x*-axis: interval of 0.5

- Dependent quantity:

  Height of skateboard's wheels from the ground (feet); *y*-axis: interval of 0.5

  Origin: (0 seconds, height of 0)



SELF-MONITORING STRATEGY

Look for students using self-motivation and self-discipline to persevere in solving problems. Refer to the Course and Implementation Guide for further details on these look-fors.

ACTIVITY **1.2**

Comparing and Contrasting Graphs

Now that you have matched a graph with the appropriate problem situation, let's go back and examine all the graphs.

1. What similarities do you notice in the graphs?

   Sample answers:

   - The independent quantity is graphed on the *x*-axis, while the dependent quantity is graphed on the *y*-axis.
   - All the graphs are continuous.

Think About . . .

Look closely when analyzing the graphs. What do you see?

2. What differences do you notice in the graphs?

   Sample answers:

   - Some graphs contain straight lines, while some contain curves.
   - Some graphs seem to move up as they go from left to right, some move down from left to right.
   - Some graphs are made of pieces that go up, go down, or stay constant from left to right.

3. How did you label the independent and dependent quantities in each graph?

   I labeled the independent quantity on the *x*-axis and the dependent quantity on the *y*-axis in each graph.

4. **Analyze** each graph from left to right. Describe any graphical characteristics you notice.

   Sample answers:

   - Some graphs only increase.
   - Some graphs only decrease.
   - Some graphs both increase and decrease.
   - Some graphs have a minimum or maximum value.
   - Some graphs increase or decrease at a constant rate.

Review the definition of analyze in the Academic Glossary.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

NOTE: This is the first lesson where TEKS A.1F is highlighted.

- Read and display TEKS A.1F and explain that in Questions 2–4 students analyze the graphs and explain the relationship between the graphs.

EB STUDENT TIP

For all proficiency levels

Provide additional vocabulary support with illustrated examples of these terms: *increase*, *decrease*, *straight*, *curved*, *maximum*, and *minimum*.

Think About . . .
What do the points on each graph represent?

5. Compare the graphs for each pair of scenarios given and describe any similarities and differences you notice.

   a. *Smart Phone, but Is It a Smart Deal?* and *Music Club*

      Sample answers:

      - Both graphs increase from left to right.
      - The graph of the *Smart Phone, but is it a Smart Deal?* situation is a smooth curve, but the graph of the *Music Club* situation is a straight line.

   b. *Something's Fishy* and *It's Magic*

      Sample answers:

      - Both graphs decrease from left to right.
      - The graph of the *Something's Fishy* situation is a straight line, but the graph of the *It's Magic* situation is a smooth curve.

   c. *Baton Twirling* and *Skateboarding*

      Sample answers:

      - The graphs have either a minimum or a maximum value.
      - Both graphs increase and decrease.
      - The graph of the *Baton Twirling* situation is a smooth curve with a maximum value, but the graph of the *Skateboarding* situation is a smooth curve with a minimum value.

12 MODULE 1 • TOPIC 1 • LESSON 1

Talk the Talk

A Writer and a Mathematician

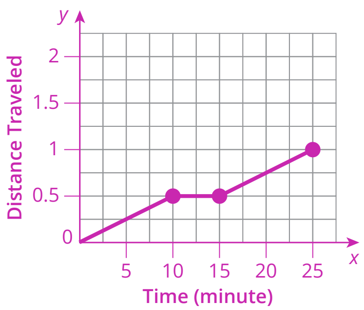
1. Write a scenario and sketch a graph to describe a possible trip to school.

Scenario

Sample answer:

I walk half a mile to school in 10 minutes. Then, I stop to talk to a friend and tie my shoes for 5 minutes. I walk the remaining half-mile to school in 10 minutes.

Graph



2. Describe the meaning of the points, or smooth curve, represented by your graph.

Sample answer:

Each point on the graph represents possible times and the corresponding distances.

3. Compare your scenario and sketch with your classmates' scenarios and sketches. What similarities do you notice? What differences do you notice?

Answers will vary based on each classroom.

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

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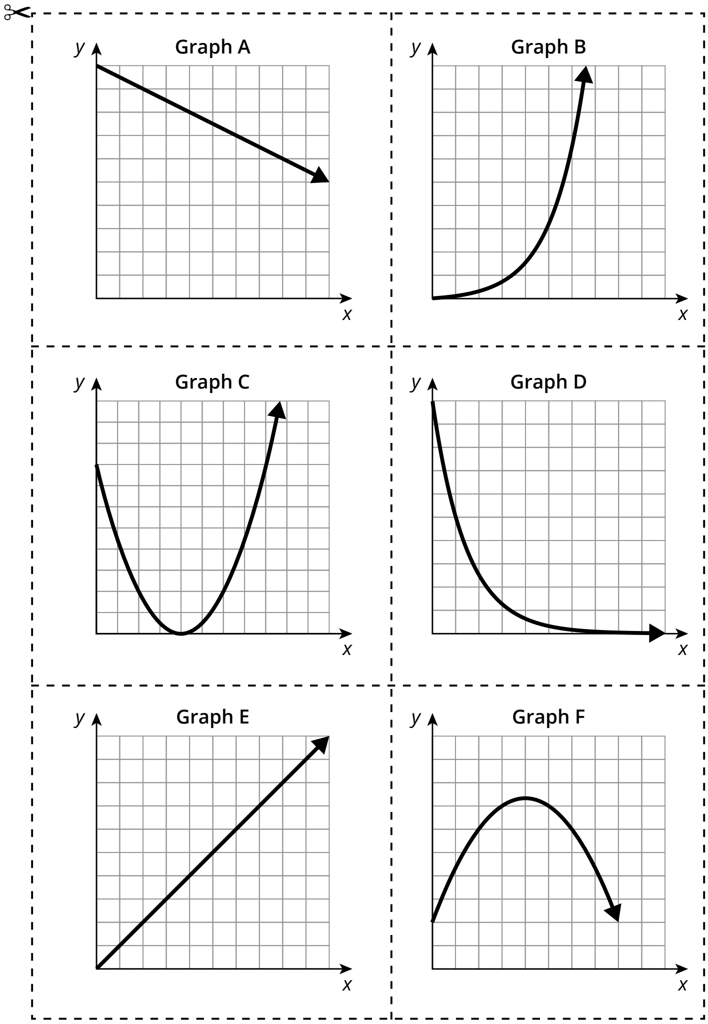
MODULE 1 • TOPIC 1 • LESSON 1 13

Graph Cutouts



MODULE 1 • TOPIC 1 • LESSON 1 15

Why is this page blank?

So you can cut out the graphs on the other side

16 MODULE 1 • TOPIC 1 • LESSON 1

Lesson 1 Assignment

Write

Describe how you can distinguish between an independent quantity and a dependent quantity. Use an example in your description.

Remember

When one quantity is determined by another in a problem situation, it is called the *dependent quantity*. The quantity it is determined from is called the *independent quantity*. The independent quantity is represented on the *x*-axis and the dependent quantity is represented on the *y*-axis.

Practice

1. Read each scenario and identify the independent and dependent quantities. Be sure to include the appropriate units of measure. Then, analyze each graph and determine which of the provided scenarios it models. For each graph, label the *x*- and *y*-axes with the appropriate quantity and unit of measure.

   a. **Endangered Species**

   A local animal conservation organization is working with various reptile species to increase their populations. The initial population of 450 endangered turtles tripled each year for the past five years.

   IQ: time (years)
   DQ: number of turtles
   Graph C
   x-axis: Time (years)
   y-axis: Number of Turtles

   b. **Video Games**

   Trung is playing video games at an arcade. Trung starts with $40 and is playing games that cost 50 cents per game.

   IQ: number of games played
   DQ: money (dollars)
   Graph B
   x-axis: Number of Games Played
   y-axis: Money (dollars)

   c. **Sales Commission**

   Eduardo works as a salesman. He receives a monthly salary of $3000 as well as a 10% commission on the amount of sales.

   IQ: monthly sales (dollars)
   DQ: monthly earnings (dollars)
   Graph A
   x-axis: Monthly Sales (dollars)
   y-axis: Monthly Earnings (dollars)

   d. **Cooling Tea**

   A freshly made cup of tea is served at a temperature of about 180°F. The tea cools rapidly at first and then slows down gradually as it approaches room temperature.

   IQ: time (minutes)
   DQ: temperature (degrees F)
   Graph D
   x-axis: Time (minutes)
   y-axis: Temperature (degrees F)

Write

The independent quantity can be any value that you choose. The dependent quantity depends on the value of the other quantity. For example, I have a number of nickels, which is the independent quantity. The value of the nickels depends on the number of nickels, so it is the dependent quantity.

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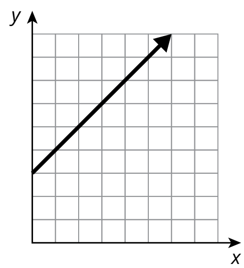
MODULE 1 • TOPIC 1 • LESSON 1 ASSIGNMENT 17

Lesson 1 Assignment

A.


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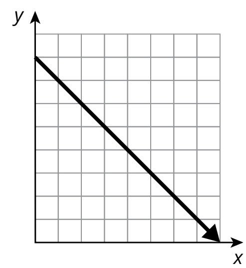

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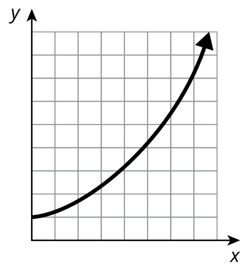

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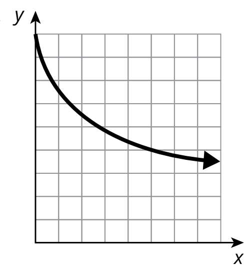
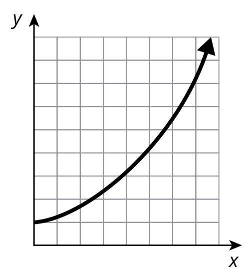
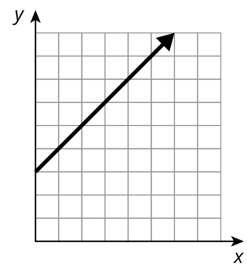


2. Compare the pair of graphs and describe any similarities and differences you notice.





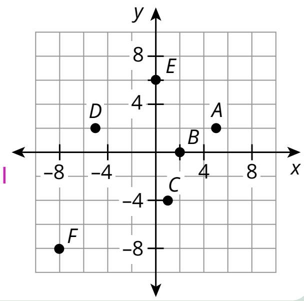
Sample answer:
Both graphs are increasing. One graph is a curve; the other is a line.

18 MODULE 1 • TOPIC 1 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

Prepare

1. Write the coordinates of each point and name the quadrant or axis where the point is located.



Point *A*: (5, 2) Quadrant I

Point *B*: (2, 0) on the *x*-axis

Point *C*: (1, −4) Quadrant IV

Point *D*: (−5, 2) Quadrant II

Point *E*: (0, 6) on the *y*-axis

Point *F*: (−8, −8) Quadrant III

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MODULE 1 • TOPIC 1 • LESSON 1 ASSIGNMENT 19

2 Analyzing and Sorting Graphs

MATERIALS

Scissors

Graph Cards (located at the end of the lesson)

LESSON OVERVIEW

Students begin this lesson by cutting out 13 different graphs. They sort the graphs into different groups based on their own rationale, compare their groupings with their classmates, and discuss the reasoning behind their choices. Next, four different groups of graphs are given, and students analyze the groupings and explain possible rationales behind the choices made. Students explore different representations of relations. Students need to keep their graphs as they will be used in lessons that follow.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1E create and use representations to organize, record, and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
The student is expected to:

A.3C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

(TEKS continued on next page)

ELPS

(1) Learning Strategies
The student is expected to:

(A) use prior knowledge and experiences to understand meanings in English.

(B) monitor oral and written language production and employ self-corrective techniques or other resources.

(F) use accessible language and learn new and essential language in the process.

(3) Speaking
The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

(E) share information in cooperative learning interactions.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Quadratic Functions and Equations

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations.

The student is expected to:

A.7A graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data.

The student is expected to:

A.9D graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

ESSENTIAL IDEAS

- A relationship between two quantities can be graphed on the coordinate plane.
- Graphical behaviors can reveal important information about a relationship.
- A graph of a relationship can have a minimum or maximum, or no minimum or maximum. A graph can pass through one or more quadrants. A graph can exhibit vertical or horizontal symmetry. A graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing.

LESSON STRUCTURE AND PACING: 1 DAY

ENGAGE

Getting Started: Let's Sort Some Graphs 15–20 minutes

ESTABLISH A SITUATION

Students cut out 17 graphs and sort the graphs into different categories based on their own rationale. They then compare their categorizations with their classmates' choices and explain their reasoning. The emphasis is on the variety of ways to correctly categorize these graphs.

DEVELOP

Activity 2.1: Identifying Graphical Behaviors 10–15 minutes

CLASSIFICATION, PEER WORK ANALYSIS

Four different scenarios that show groups of graphs are given, and students explain the rationale behind the groups and the errors in the reasoning behind a grouping. Rationales for groups include graphs being discrete, having vertical symmetry, existing in only two quadrants, and not being a function.

DEMONSTRATE

Talk the Talk: Compare and Contrast 10 minutes

GENERALIZATION

Students use the graphs they cut out and sorted at the beginning of the lesson to create a list of all the different types graphical behaviors.

Getting Started

ENGAGE

Let's Sort Some Graphs

Facilitation Notes

In this activity, students cut out 13 graphs and sort the graphs into different categories based on their own rationale. They then compare their categorizations with their classmates' choices and explain their reasoning. The emphasis is on the variety of ways to correctly categorize these graphs.

Ask a student to read the introduction aloud and discuss the activity as a class. Provide scissors and the time necessary to cut out each of the 13 graph cards.

Have students work with a partner or in a group to complete the activity. Student responses will be shared in the Talk the Talk at the end of the lesson.

AS STUDENTS WORK, LOOK FOR

- Conflicts and reasoning about the best way to group the graphs.
- Creative strategies, such as Venn diagrams, to deal with more than one graphical characteristic at a time.

DIFFERENTIATION STRATEGY

Just in Time Support

Reduce the number of graphs that students must sort.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you decide which groups to make? • Did you and your group members disagree about any particular groupings? If so, explain your viewpoints. • Did you encounter any graphs that belonged to more than one group? How did you address that situation? • Did you create any subgroups? If so, explain your thinking. • Do any of your groups contain a single graph? • What title did you give that group?

Optimizing Learning

This common misconception provides options that support students by using multiple tools for construction and composition.

COMMON MISCONCEPTION

Students may not realize the significance of the arrowheads included on various graphs in terms of continuation. In these situations, you could suggest that they visualize these graphs beyond the viewable window.

STAMP THE LEARNING

Summary

Graphs of relationships have a variety of characteristics.

ACTIVITY 2.1

Identifying Graphical Behaviors

DEVELOP

Facilitation Notes

In this activity, four different scenarios that show groups of graphs are given, and students explain the rationale behind the groups and the errors in the reasoning behind a grouping. Rationales for categorizations include graphs being discrete, having vertical symmetry, existing in only two quadrants, and not being a function.

At this point, students are not required to use the terms *discrete* and *continuous*. These terms will be defined in the next lesson.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Did you create a group that is the same as Chris's group?
Probing	• How could Chris separate this group into subgroups?
Seeing structure	• Create another graph that would belong in this group.
Probing	• How could Paola separate this group into subgroups?
Seeing structure	• In Graphs *G*, *E*, and *H*, describe the relationship between the point on each side of the line of symmetry. • Are Jorge's graphs only what they appear to be, or is only part of each graph visible? Explain.
Probing	• For each graph, estimate the ordered pairs when $x = 2$.
Seeing structure	• Is there any relationship between these graphs and a line of symmetry? Explain your thinking. • Create another graph that would belong in this group.

COMMON MISCONCEPTION

Students may think the term *axis of symmetry* implies that the axis of symmetry must be the *x*-axis or *y*-axis. Use graph *L* to disprove this claim. If helpful, use the term *line of symmetry* rather than the term *axis of symmetry*.

DIFFERENTIATION STRATEGIES

Access for All

- Have students write the word example above the Thumbs Up strategy as an indicator that it is a correct solution to reference later. As students read through the strategy and think about the connections, suggest they ask themselves:
  - Why is this method correct?
  - Have I used this method before?

Just in Time Support

- For Question 4, remind students of the Vertical Line Test and have them label points that have the same x-value but different y-values. Revisit graphs of functions to emphasize the difference.

Challenge Opportunity

- To extend the activity, ask students to sketch other graphs that could belong in the groups.

STAMP THE LEARNING

Summary

Graphs of relationships can exhibit symmetry and can represent functions and non-functions.

DEMONSTRATE

Talk the Talk

COMPARE AND CONTRAST

Facilitation Notes

In this activity, students use the graphs they cut out and sorted at the beginning of the lesson to create a list of different graphical behaviors. Remind students to keep their graph cutouts. They will need them for the next two lessons.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What mathematical term describes that graphical behavior?
Probing	• How did your groupings compare to your classmates' groupings? • Would you prefer to change your groupings after seeing other groupings? If so, why? • Did anyone use subgroups or Venn diagrams? What additional information do those grouping methods provide?

continued

continued

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Poster Paper

Do a gallery walk as a way for students to share their results.

- Have each group create a poster with their results from Activity 1. They should glue one set of their graphs into groups and name each group. (They need these same graphs for the next lesson; however, they can share graphs as they work together.)
- Have the groups display their posters around the room, with each group standing next to their poster.
- Then, have the groups circulate in a clockwise direction, timing them for 90 seconds to view each poster and document any differences among the graph groups.
- Once each group returns to their poster, follow-up with a discussion to answer Question 1.

Have students read and answer the Essential Question on the lesson opener page.

Summary

Graphs of relationships have a variety of characteristics.

STAMP THE LEARNING

2 Analyzing and Sorting Graphs

Setting the Stage

- Communicate the objectives.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Review and analyze graphs and graphical behavior.
- Determine similarities and differences among various graphs.
- Sort graphs and give reasons for the similarities and differences between the groups of graphs.

You have used graphs to analyze the relationship between independent and dependent quantities. Do the graphs of certain types of relationships share any characteristics?

Sample answer:

Graphs of relationships between quantities have characteristics that can give you essential information about the relationship. For example, a graph can increase, decrease, neither increase nor decrease, or both increase and decrease. A graph can have straight lines or smooth curves, a maximum or a minimum, or no maximum or minimum.

MODULE 1 • TOPIC 1 • LESSON 2 21

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

NOTE: This is the first lesson where TEKS A.1E is highlighted.

- Read and display TEKS A.1E and explain that this activity is an example of using mathematical representations to communicate mathematical ideas.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice sorting a variety of graphs into groups based on characteristics, assign Skills Practice Set A for this lesson.

Getting Started

Let's Sort Some Graphs

Keep your graphs. You will need them in the next lesson.

Mathematics is the science of patterns and relationships. Looking for patterns and sorting patterns into different groups based on similarities and differences can provide valuable insights. In this lesson, you will analyze many different graphs and sort them into various groups.

1. Cut out the 13 graphs at the end of the lesson. Then, analyze and sort the graphs into at least two different groups. You may group the graphs in any way you feel is appropriate.

   Record the following information for each of your groups.

   - Name each group of graphs.
   - List the letters of the graphs in each group.
   - Provide a rationale for why you created each group.
     Answers will vary based on each classroom.

EB STUDENT TIP

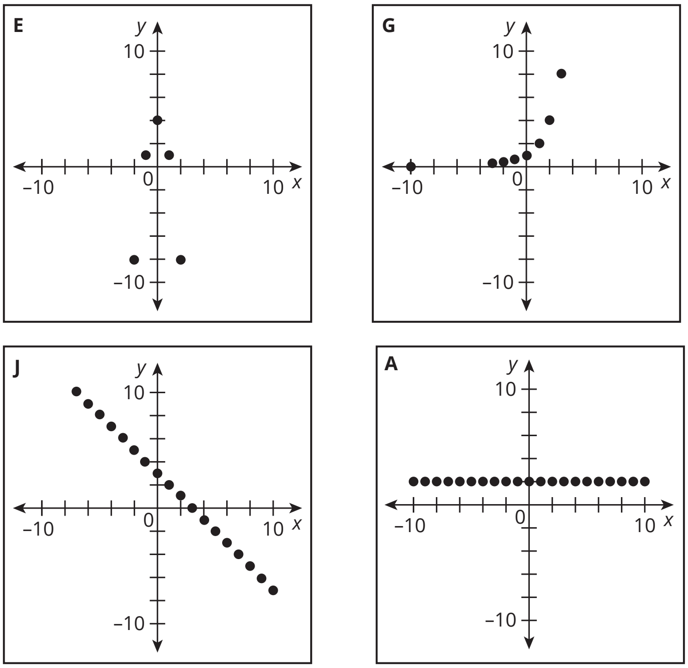
For "Intermediate" and higher proficiency levels

Allow students time to identify vocabulary and create basic descriptions for each group of graphs to prepare for classroom discussions. As you ask groups members for their grouping decisions, encourage students to orally contribute with the help of their prepared descriptions.

ACTIVITY 2.1 Identifying Graphical Behaviors

In this activity, consider the different ways the graphs are grouped.

1. Chris grouped these graphs together. Why do you think Chris put these graphs in the same group?



Sample answer:
The graphs have points that are not connected.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

NOTE: This is the first lesson where TEKS A.1G is highlighted.

- Read and display TEKS A.1G and explain that this activity is an example of explaining and justifying mathematical ideas using precise mathematical language.

EB STUDENT TIP

For all proficiency levels

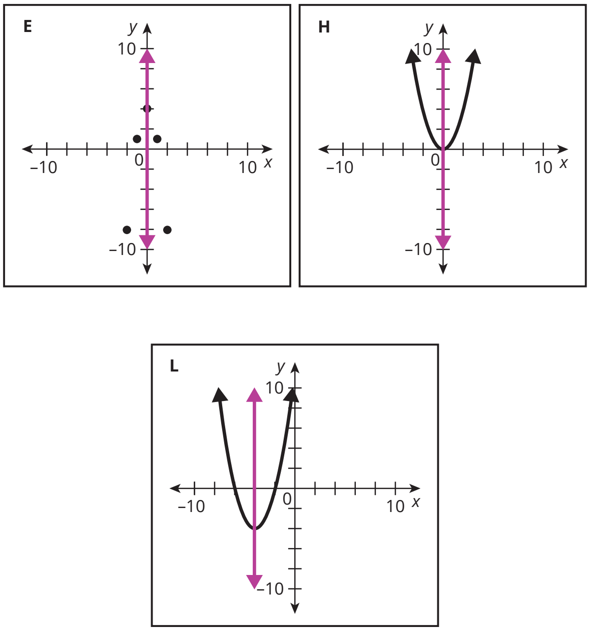
Have students define each term as needed with an illustration: *symmetry, horizontal, vertical, linear.* Provide connections with familiar terms, such as *horizon/horizontal* and *line/linear.*

MODULE 1 • TOPIC 1 • LESSON 2 23

2. Consider Paola's correct grouping.

Paola

I grouped these graphs together because they all have a vertical axis of symmetry. If I draw a vertical line through the middle of the graph, the image is the same on both sides.



a. Show why Paola's reasoning is correct.

Each of the graphs can be divided in half by drawing a vertical line.

b. When possible, identify other graphs in this activity that have a vertical axis of symmetry.

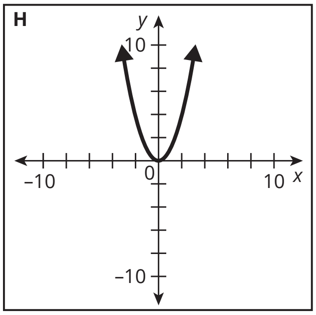
Graphs A, B, D

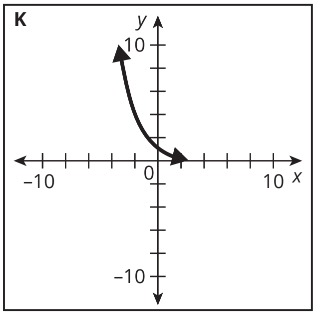
24 MODULE 1 • TOPIC 1 • LESSON 2

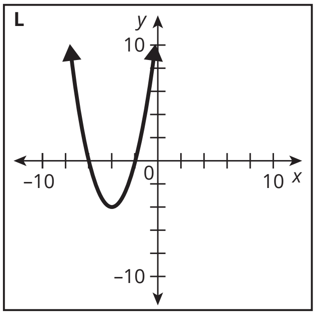
3. Consider Jorge's incorrect grouping.

Jorge

I grouped these graphs together because each graph goes through only two quadrants.







a. Explain why Jorge's reasoning is not correct.

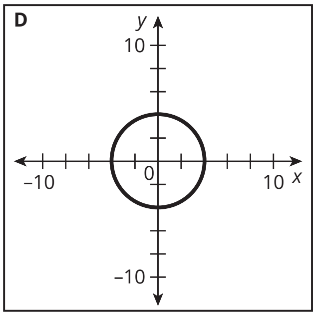
Even though it is not visible, Graph L continues into the first quadrant. Therefore, the graph goes through three quadrants. Each of the other graphs H and K satisfy Jorge's reasoning.

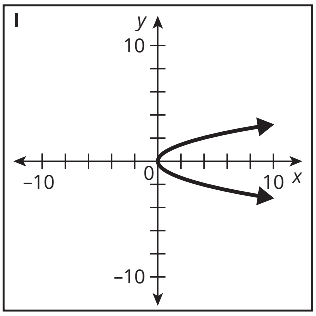
b. When possible, identify other graphs in this activity that go through only two quadrants.

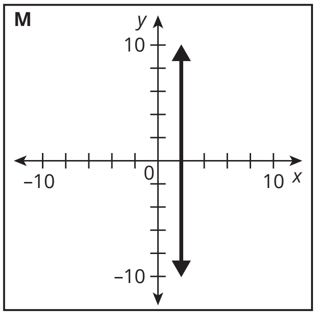
Graphs A, F, G, I, M

MODULE 1 • TOPIC 1 • LESSON 2 25

4. Elena grouped these graphs together but did not provide any rationale.







a. What do you notice about the graphs?

Sample answer:

In each graph, for at least one value of *x*, there is more than one value of *y*. Each of these graphs has a horizontal axis of symmetry.

b. What rationale could Elena have provided?

Sample answer:

The graphs are not functions.

26 MODULE 1 • TOPIC 1 • LESSON 2

Talk the Talk

Compare and Contrast

1. Compare your groups with your classmates' groups. Create a list of the different graphical behaviors you noticed.

   Sample answers:

   Possible graphical behaviors:

   - Always increasing from left to right
   - Always decreasing from left to right
   - The graph both increases and decreases
   - Straight lines
   - Smooth curves
   - Discrete data values
   - The graph has a maximum value
   - The graph has a minimum value
   - The graph is a function
   - The graph is not a function
   - The graph goes through the origin
   - The graph forms a U shape

Ask Yourself . . .
Are any of the graphical behaviors shared among your groups? Or are they unique to each group?

Chunking the Activity

- Read and discuss the directions.
- Group the students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

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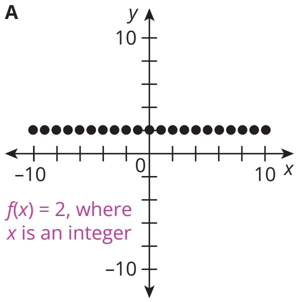
MODULE 1 • TOPIC 1 • LESSON 2 27

Graph Cards

A



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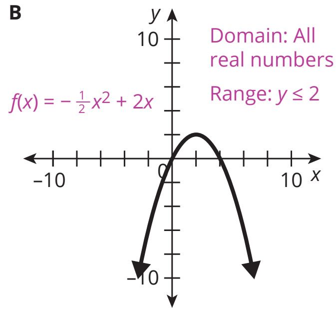

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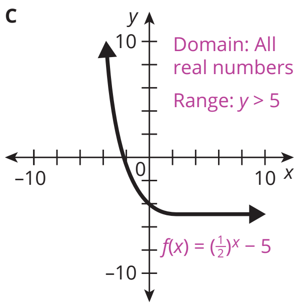

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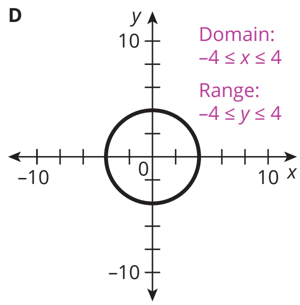

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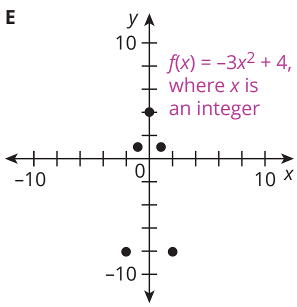
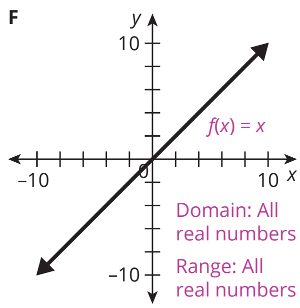


F



NOTE: Answers for Lesson 3 are included on the Graph Cards.

MODULE 1 • TOPIC 1 • LESSON 2 29

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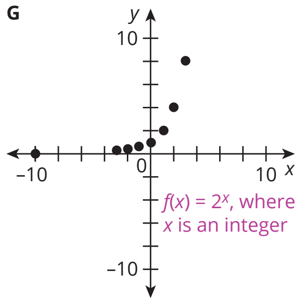
So you can cut out the Graph Cards on the other side

30 MODULE 1 • TOPIC 1 • LESSON 2

G



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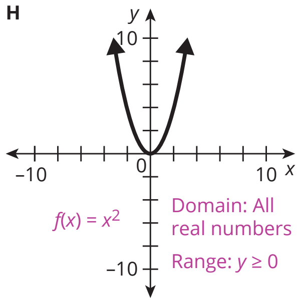

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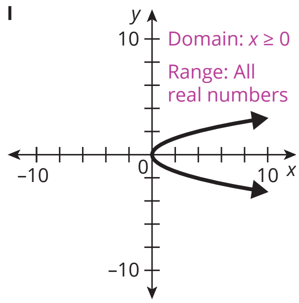
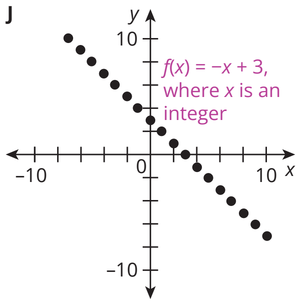


J



NOTE: Answers for Lesson 3 are included on the Graph Cards.

MODULE 1 • TOPIC 1 • LESSON 2 31

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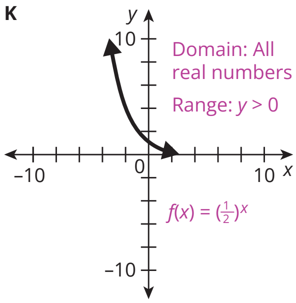
32 MODULE 1 • TOPIC 1 • LESSON 2

NOTE: Answers for Lesson 3 are included on the Graph Cards.

K



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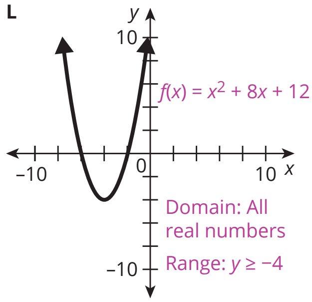
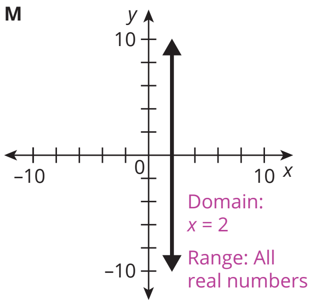


M



MODULE 1 • TOPIC 1 • LESSON 2 33

Why is this page blank?

So you can cut out the Graph Cards on the other side

34 MODULE 1 • TOPIC 1 • LESSON 2

Lesson 2 Assignment

Write

Describe the importance of graphical representations.

Remember

Graphs of relationships between quantities have characteristics that can give you important information about the relationship. For example, a graph can be increasing, decreasing, neither increasing nor decreasing, or both increasing and decreasing. A graph can have straight lines or smooth curves, a maximum or a minimum, or no maximum or minimum, and so on.

Write

Sample answer: Graphs provide a visual representation of a function. They provide a model to see and interpret key features.

Practice

1. Record the letter of each graph with the given characteristic.

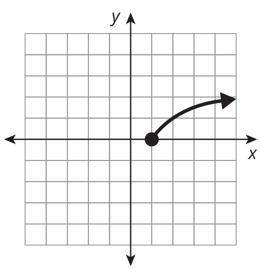
   a. Has a vertical axis of symmetry
   B, C

   b. Passes through exactly 1 quadrant
   A, D

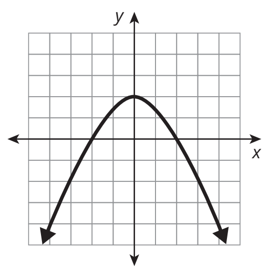
   c. Has a horizontal axis of symmetry
   C, D

   d. Passes through all 4 quadrants
   B, E

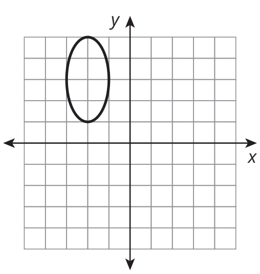
A.



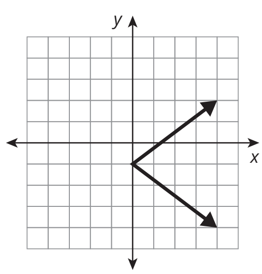
B.



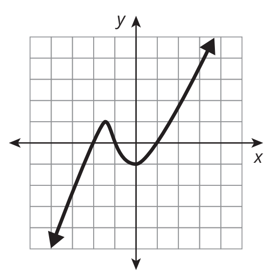
C.



D.



E.



MODULE 1 • TOPIC 1 • LESSON 2 ASSIGNMENT 35

Lesson 2 Assignment

Prepare

Create a list of the *x*- and *y*-values of the relation described by each set of ordered pairs. Write an equation using the variables *x* and *y* that could map the *x*-value to the *y*-value.

1. $\{(-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2)\}$

   x-values: $-3, -2, -1, 0, 1$

   y-values: $-6, -4, -2, 0, 2$

   $y = -2x$

2. $\{(-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

   x-values: $-3, -2, -1, 0, 1, 2$

   y-values: $9, 4, 1, 0$

   $y = x^2$

3. $\{(-3, 3), (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

   x-values: $-3, -2, -1, 0, 1, 2$

   y-values: $3, 2, 1, 0, -1, -2$

   $y = -x$

36 MODULE 1 • TOPIC 1 • LESSON 2 ASSIGNMENT

3 Recognizing Functions and Function Families

MATERIALS

Graphs from *Analyzing and Sorting Graphs*
Graphing Technology
Glue Sticks

LESSON OVERVIEW

The definition of *relation*, *function*, *function notation*, *domain*, and *range* are introduced in this lesson. For the remainder of the lesson, students use graphing technology to connect equations written in function form to their graph and then identify the function family to which they belong. The terms *Vertical Line Test*, *continuous graph*, and *discrete graph* are defined, and students sort the graphs from the previous lesson into functions and non-functions. Then, the terms *Vertical Line Test*, *increasing function*, *decreasing function*, *constant function*, *discrete function*, and *continuous function* are defined, and students sort the graphs from the previous lesson into these groups and a group labeled for functions that include a combination of increasing and decreasing intervals. The terms *function family*, *linear function*, and *exponential function* are then defined, and students sort the increasing, constant, and decreasing functions into one of these families. Next, the terms *absolute minimum* and *absolute maximum* are defined as well as the term *quadratic function*. Finally, students recall the definition of *x*-intercept and *y*-intercept. Students then complete a graphic organizer for each function family that describes the graphical behavior and displays graphical examples. In the final activity, students use their knowledge of the function families to demonstrate how the families differ with respect to their *x*- and *y*-intercepts. Graphing technology is necessary to help students connect some equations and their graphs.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(TEKS continued on next page)

ELPS

(1) Learning Strategies
The student is expected to:

(C) use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary.

(3) Speaking
The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

(H) narrate, describe, and explain with increasing specificity and detail as more English is acquired.

(ELPS continued on next page)

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
The student is expected to:

A.3C graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

Quadratic Functions and Equations

(6) The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations.
The student is expected to:

A.6A determine the domain and range of quadratic functions and represent the domain and range inequalities.

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations.
The student is expected to:

A.7A graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

(TEKS continued on next page)

ELPS *(ELPS continued from previous page)*

(4) Reading
The student is expected to:

(C) develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data.

The student is expected to:

A.9A determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

A.9D graph exponential functions that model growth and decay and identify key features, including y-intercept and asymptote, in mathematical and real-world problems.

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

The student is expected to:

A.12A decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

ESSENTIAL IDEAS

- A function is a relation that assigns to each element of the domain exactly one element of the range.
- The family of linear functions includes functions of the form $f(x) = ax + b$, where a and b are real numbers.
- The family of exponential functions includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but is not equal to 1.
- The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Odd One Out 5 minutes

BUILD OFF INTUITION

Students are presented with four numberless graphs of relations and decide which of the graphs does not belong with the others. This activity solicits students' prior knowledge related to the characteristics of graphs, which they explored in the previous lesson. This activity has no correct answer. There are a variety of reasons related to graphical behavior to explain why each of the four graphs does not belong with the others.

DEVELOP

Activity 3.1: Functions and Non-Functions 35–40 minutes

CLASSIFICATION

Students are provided the definitions of the terms *relation*, *domain*, *range*, *function*, and *function notation*. Multiple representations of relations are shown and then analyzed to determine which relations are functions. Students are given an equation written in function notation that models a scenario. They identify which expression in the function equation represents the domain and range and then determine the possible domain and range of the function in the context of the scenario. The *Vertical Line Test* is reviewed as a visual method used to determine whether a relation represented as a graph is a function, and the terms *discrete graph* and *continuous graph* are defined. Additional relations are analyzed to determine which are functions. Students then sort the graphs from the previous lesson into function and non-function groups.

DAY 2

Activity 3.2: Domain and Range of a Function 15–20 minutes

WORKED EXAMPLE

Students revisit the graphs from the previous lesson and use different notations, including words and inequalities, to describe the domain and range of each graph.

Activity 3.3: Linear, Constant, and Exponential Functions 20–25 minutes

CLASSIFICATION, MATHEMATICAL PROBLEM SOLVING

Students use a sorting activity and the graphs from the previous lesson to distinguish among *increasing*, *decreasing*, and *constant functions*, and functions that show a combination of increasing and decreasing behaviors. Next, focusing only on the graphs of increasing, decreasing, and constant functions, they match each graph with the appropriate equation written in function notation. They sort these graphs again into two groups based on the equation of each function. The terms *function family*, *linear functions*, and *exponential functions* are described, and students identify which group is best represented using these terms.

DAY 3

Activity 3.4: Quadratic Functions 15–20 minutes

CLASSIFICATION, MATHEMATICAL PROBLEM SOLVING

Students sort the graphs that are both increasing and decreasing into two groups: having the characteristics of *absolute minimum* and having the characteristics of *absolute maximum*. Focusing only on the graphs containing absolute minimums or absolute maximums, they match each graph with the appropriate equation written in function notation. The term *quadratic functions* is defined.

Activity 3.5: Function Families 15–20 minutes

CLASSIFICATION

If they have not done so already, students paste their equations and linear, exponential, and quadratic graphs into appropriate graphic organizers. Students then describe the graphical behavior of each function.

DEMONSTRATE

Talk the Talk: Interception! 5 minutes

EXIT TICKET APPLICATION

Students recall the definitions of *x-intercept* and *y-intercept*. They then use their knowledge about functions and function families to draw functions on numberless graphs, given only the *x*- and *y*-intercepts of the functions. This activity is designed to solicit students' reasoning about the possibilities for the graphs of functions.

Getting Started

ENGAGE

Odd One Out

Facilitation Notes

In this activity, students are presented with four numberless graphs of relations and decide which of the graphs does not belong with the others. This activity solicits students' prior knowledge related to the characteristics of graphs, which they explored in the previous lesson. This activity has no correct answer. There are a variety of reasons related to graphical behavior to explain why each of the four graphs does not belong with the others.

Have students complete this activity independently or with a partner. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Identification of different graphs as not belonging with the others.
- Use of math terminology related to the characteristics of the graphs.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Did anyone choose the same graph for a different reason? Explain the differences in your reasoning. • Did anyone select a different graph? If so, why? • Which characteristic do Graphs *A* and *B* have in common that is not a characteristic for Graphs *C* and *D*? • Which characteristic do Graphs *A* and *C* have in common that is not a characteristic for Graphs *B* and *D*? • What is a characteristic all of the graphs have in common?

STAMP THE LEARNING

Summary

Graphical behaviors can help distinguish one type of relation from another.

ACTIVITY 3.1

Functions and Non-Functions

DEVELOP

Facilitation Notes

In this activity, the terms *relation*, *domain*, *range*, *function* and *function notation* are defined. Multiple representations of relations are shown and then analyzed to determine which relations are functions. Students are then given an equation written in function notation that models a scenario. They identify which expression in the function equation represents the domain and range and then determine the possible domain and range in the context of the scenario. The *Vertical Line Test* is reviewed as a visual method used to determine whether a relation represented as a graph is a function, and the terms *discrete graph* and *continuous graph* are defined. Additional relations are analyzed to determine which are functions. Students then sort the graphs from the previous lesson into function and non-function groups.

Have a student read the definition of *relation* aloud. As a class, discuss the six different representations of a relation provided.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are the input values for this relation? • What are the output values for this relation?
Probing	• Which relations have a limited number of values? An unlimited number of values?

Ask a student to read the definition of *function*.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain what a function is in your own words. • What is the difference between a relation and a function?

Have students work independently or with a partner to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can the table represent a function when both 5 and 6 have the same y-value?
Seeing structure	• How can you modify the table so that it does not represent a function?
Gathering	• What would this mapping look like as a table of values?
Probing	• Is this relation still a function when two students have the same birthday? Explain your thinking.

DIFFERENTIATION STRATEGY

Challenge Opportunity

Have students create tables, mappings, and verbal representations that are functions. Then, ask students to modify each function to create a non-function.

COMMON MISCONCEPTION

Students may connect the math term *relation* to its common usage as *relationship*; however, the math term *function* is more restrictive than its common usage. As an alternative method of understanding, address the fact that technology can only perform operations involving functions; each input can only have one output.

Have a student read the information and definition following Question 3 aloud. Complete Questions 4 and 5 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What do you think $C(10)$ means? • What do you think $C(s) = 55$ means?
Probing	• Identify the domain and range for when you can order up to 5 shirts.

DIFFERENTIATION STRATEGIES

Access for All

- Provide students the opportunity to use function notation.
  - Ask students to rewrite $y = 3x + 2$ in function notation.
  - Show their response, $f(x) = 3x + 2$.
  - Ask students to solve $f(5)$.
  - Show substitution, $f(5) = 3(5) + 2$.
  - Show solution $f(5) = 15 + 2 = 17$.
  - Ask students to calculate the value of x when the function equals 32.
  - Since $f(x) = 32$, write the problem as $32 = 3x + 2$.
  - Solve the equation, resulting in $x = 10$.

Challenge Opportunity

- Provide an equation, such as $y = 3x + 2$. Ask students to rewrite the statement using function notation.
  - What is the value of the function when $x = 5$?
  - What is the value of x when the function equals 10?

Discuss the Worked Example as a class. Have students work with a partner or in a group to complete Question 6.

Have a student read the definitions of *Vertical Line Test*, *discrete graph*, and *continuous graph* aloud. Discuss as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why does the Vertical Line Test work?
Seeing structure	• Why do you use a vertical line rather than a horizontal line?
Reflecting and justifying	• Explain how the Vertical Line Test relates to the definition of a *function*.

DIFFERENTIATION STRATEGIES

Access for All

- Place a non-function on a coordinate plane. Have students identify coordinate pairs that demonstrate that it does not fit the definition of a function.

Just in Time Support

- Suggest students lay their pencil on top of the arrow and move it horizontally across the graph to apply the vertical line test.

Have students work with a partner or in a group to complete Questions 7 through 11. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What's the difference between a relation and a function?
Probing	• How do you test whether an equation is a function? • Is the table not a function due to the repeated 0s or 4s? Explain your thinking. • Is the mapping not a function because of two arrows from the 2 or two arrows to the 9? Explain your thinking.
Seeing structure	• How could you add a point to the graph so that it is no longer a function? • Use the definition of a function to explain why with all the repeated 5s, this set is still a function.
Probing	• Would a horizontal line also not be a function? Explain. • Are slanted lines functions? Why or why not?
Reflecting and justifying	• Provide a pair of points to support your conclusion.

DIFFERENTIATION STRATEGY

Access for All

To clarify that every equation is not a function, provide a non-example.

- Consider $y = \sqrt{x}$. Ask students to solve the equation when $x = 9$. Students should see that $y = 3$ and -3.
- Discuss why this equation is not a function.
- Note that technology will provide only the primary root because technology provides only one output per input.

To close the Day 1 session, have the students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

A function is a relation that assigns to each element of the domain exactly one element of the range.

ACTIVITY 3.2 Domain and Range of a Function

Facilitation Notes

In this activity, students revisit their graphs from the previous lesson and use different notations, including words and inequalities, to describe the domain and range of each graph.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work independently to complete Question 1. Share responses as a class. Discuss the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How do the domain and range relate to the dependent and independent quantities? • The x-value and y-values? • How do you know whether to use $<$ or $\leq$? • How do you express the set of real numbers using notation?
Probing	• Why is the domain of the second graph all real numbers? • Why don't you need to write the range of the second graph as a compound inequality? • How could you express the range of the second graph as a compound inequality?
Seeing structure	• Why is -2 not included in the domain of $g(x)$? • Why is -1 included in the range of $g(x)$?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Encourage students to extend the graph to cover the coordinate plane when it includes an arrow.
- To determine the domain, suggest students repeat the process. This time, move their pencil from the bottom to the top of the graph and record the value of the graph's first and last points.
- To determine the range, suggest students use their pencil as if they were applying the vertical line test, but this time record the value of the graph's lowest and highest points.

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Students using the correct inequality symbols when writing the domain and range in interval notation.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What is the difference between the domain and range of an exponential function?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Introduce students to interval notation. An interval is defined as the set of real numbers between two given numbers. To describe an interval, use this notation:

- A closed interval [*a*, *b*] describes the set of all numbers between *a* and *b*, including the endpoints *a* and *b*. Graphically, a closed interval is indicated by closed-circle endpoints.
- An open interval (*a*, *b*) describes the set of all numbers between *a* and *b*, not including the endpoints *a* and *b*. Graphically, an open interval is indicated by open-circle endpoints.
- A half-closed or half-open interval (*a*, *b*] describes the set of all numbers between *a* and *b*, including *b* but not including *a*. Or, [*a*, *b*) describes the set of all numbers between *a* and *b*, including *a* but not including *b*.

Summary

The domain and range of a function can be represented in words or using inequalities.

STAMP THE LEARNING

ACTIVITY 3.3

Linear, Constant, and Exponential Functions

Facilitation Notes

In this activity, students use a sorting activity and the graphs from the previous lesson to distinguish among increasing and decreasing behaviors and functions that show a combination of increasing and decreasing behaviors. Next, focusing only on the graphs of increasing, decreasing, and constant functions, they match each graph with the appropriate equation written in function notation. They sort these graphs again into two groups based on the equation of each function. The terms *function family*, *linear functions*, and *exponential functions* are described, and students identify which group is best represented using these terms.

Ask a student to read the introduction and definitions aloud. Discuss the behaviors of increasing functions, decreasing functions, and constant functions as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Note: Vertical transformations of exponential functions are used in this lesson only to help students visualize and understand key features using different functions. Students are not expected to know or become proficient with transformations of exponential functions in Algebra I.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Explain why a horizontal line is a constant function but a vertical line is not.
Probing	• Why do some of the functions include x as an integer?
Seeing structure	• What is the difference between the graphs of $f(x) = x$ and $f(x) = -x + 3$? • What is the difference between the graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = \left(\frac{1}{2}\right)^x - 5$? • What is the difference between the graphs of $f(x) = \left(\frac{1}{2}\right)^x$ and $f(x) = 2^x$? • Why did you include Graph *A* with Graphs *F* and *J*?

Ask a student to read the definitions following Question 3 aloud. Discuss the definitions as a class.

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Select a function from each group and explain how its equation fits the function form for its family. • Are constant functions linear functions? Why or why not? • Are horizontal lines linear functions? Why or why not? • Are vertical lines linear functions? Why or why not?

To close the Day 2 session, have the students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

The family of linear functions includes functions of the form $f(x) = ax + b$, where a and b are real numbers. The family of exponential functions includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but is not equal to 1.

ACTIVITY 3.4 Quadratic Functions

Facilitation Notes

In this activity, students sort the graphs that are both increasing and decreasing into two groups based on the characteristics of absolute minimum and absolute maximum. They match each graph with the appropriate equation written in function notation. The term quadratic functions is defined.

To begin the Day 3 session, have a student read the Essential Question aloud.

Ask a student to read the definitions aloud. Discuss as a class.

DIFFERENTIATION STRATEGY

Access for All

Compare an absolute maximum and a relative maximum. For an example that includes both maximums, have students use technology to graph $f(x) = -0.75(x - 3)(x + 1)(x - 1)(x + 2)$.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What is different among the graphs with an absolute minimum? Absolute maximum? • Describe the relationship between the graph shapes and their equations. • What do the equations that represent graphs with an absolute maximum have in common?

Ask a student to read the definitions following Question 3 aloud, then complete Question 4 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How would you describe the graph of a quadratic function? • How can you tell which function family Graph *E* belongs to from its graph rather than its equation?

STAMP THE LEARNING

Summary

The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

ACTIVITY 3.5

Function Families

Facilitation Notes

In this activity, students paste their equations and linear, exponential, and quadratic graphs into graphic organizers. They then describe the graphical behavior of each function.

Have students work with a partner or in a group to complete this activity.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Which families of functions contain straight lines? • Does a linear function contain an absolute minimum or an absolute maximum? • Does an exponential function contain an absolute minimum or an absolute maximum?

Have students read the closing paragraph.

Note: The intent of this topic is to introduce these new functions, providing an overview but not a deep understanding. Students will formalize their understanding of the defining characteristics of each type of function later in this course.

Summary

Functions can be classified as linear, exponential, and quadratic.

STAMP THE LEARNING

Talk the Talk

DEMONSTRATE

INTERCEPTION!

Facilitation Notes

In this activity, students review the definitions of *x-intercept* and *y-intercept*. They then use their knowledge about function families to draw functions on numberless graphs given only the *x*- and *y*-intercepts of the functions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Is there more than one possible graph? Why or why not? Why can you graph a linear or exponential function through the two points? • Why can you plot a quadratic function through the three points? • Why can't you draw a slanted line through this single point? • Why can't you draw a function through the two points?
Seeing structure	• How many *x*-intercepts can a function have? • Why can't a function have more than one *y*-intercept?

Have students read and answer the Essential Question on the lesson opener page.

Summary

Different function families can have different numbers of *x*-intercepts, and at most, one *y*-intercept.

STAMP THE LEARNING

3 Recognizing Functions and Function Families

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Define a *function* as a relation that assigns each element of the domain to exactly one element of the range.
- Write equations using function notation.
- Recognize multiple representations of functions.
- Determine and recognize characteristics of functions.
- Determine and recognize characteristics of function families.

NEW KEY TERMS

- relation
- domain
- range
- function
- function notation
- Vertical Line Test
- discrete graph
- continuous graph
- increasing function
- decreasing function
- constant function
- function family
- linear functions
- exponential functions
- absolute minimum
- absolute maximum
- quadratic functions
- *x*-intercept
- *y*-intercept

You have sorted graphs by their graphical behaviors. How can you describe the common characteristics of the graphs of the functions?

Sample answer:

A function is a relation that assigns exactly one element of the range to each element of the domain.

Some different function families include linear functions, exponential functions, and quadratic functions.

MODULE 1 • TOPIC 1 • LESSON 3 37

Chunking the Activity

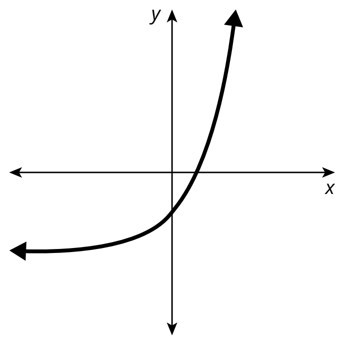
- Group students to complete the activity.
- Share and summarize.

Getting Started

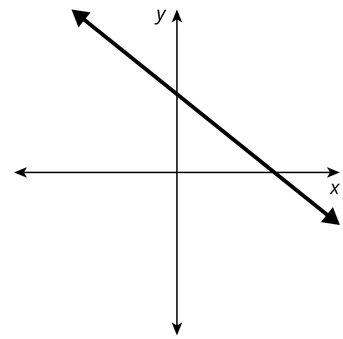
Odd One Out

1. Which of the graphs does not belong with the others? Explain your reasoning.

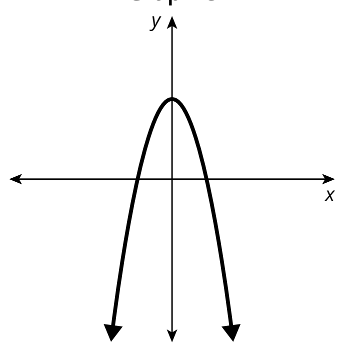
Graph A



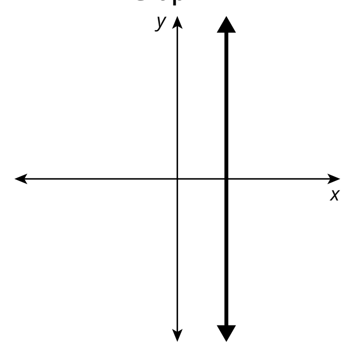
Graph B



Graph C



Graph D



Sample answer:

Graph *D*. It is neither increasing or decreasing or both, like the others.

SELF-MONITORING STRATEGIES

Look for students using self-awareness to persevere in solving problems.

Refer to the Course and Implementation Guide for further details on these look-fors.

38 MODULE 1 • TOPIC 1 • LESSON 3

ACTIVITY 3.1
Functions and Non-Functions

A relation can be represented in the following ways.

Ordered Pairs

{(−2, 2), (0, 2), (3, −4), (3, 5)}

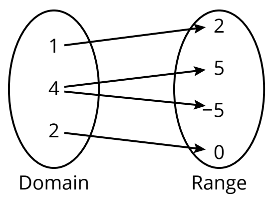
Equation

$y = \frac{2}{3}x - 1$

Verbal

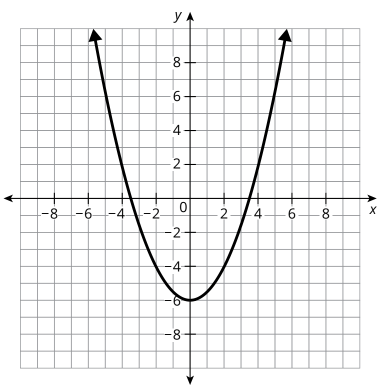
The relation between students in your school and each student's birthday

Mapping



A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

Graph



Table

Domain	Range
−1	1
2	0
5	−5
6	−5
7	−8

Chunking the Activity

- Read and discuss the definitions and the six representations.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the information and definition.
- Group students to complete Questions 4 and 5.
- Check in and share.
- Read and discuss the Worked Example.
- Group students to complete Question 6.
- Check in and share.
- Read and discuss the definitions.
- Group students to complete Questions 7–11.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definition and representations provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

NOTE: This is the first lesson where TEKS A.1D is highlighted.

- Read and display TEKS A.1D and explain that this activity is an example of communicating mathematical ideas through multiple representations.

MODULE 1 • TOPIC 1 • LESSON 3 39

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Think About . . .

So, all functions are relations, but only some relations are functions.

A **function** is a relation that assigns to each element of the domain exactly one element of the range. Functions can be represented in a number of ways.

1. Analyze the relation represented as a table. Is the relation a function? Explain your reasoning.

   Yes. Each element in the domain has exactly one element in the range.

2. Analyze the relation represented as a mapping. Is the relation a function? Explain your reasoning.

   No. An element in the domain maps to more than one element in the range.

3. Analyze the relation represented verbally. Is the relation a function? Explain your reasoning.

   Yes. Each student has one and only one birthday.

40 MODULE 1 • TOPIC 1 • LESSON 3

You can write an equation representing a function using *function notation*. Let's look at the relationship between an equation and function notation.

Consider this scenario. A shirt company charges $8 per shirt plus a one-time charge of $15 to set up a T-shirt design.

The equation $y = 8x + 15$ can be written to model this situation. The independent variable x represents the number of shirts ordered, and the dependent variable y represents the total cost of the order, in dollars.

This is a function because for each number of shirts ordered (independent value) there is exactly one total cost (dependent value) associated with it. Because this relationship is a function, you can write $y = 8x + 15$ in function notation.

$$f(x) = 8x + 15$$

name of function

independent variable

The cost, defined by f, is a function of x, defined as the number of shirts ordered.

You can write a function in a number of different ways. You could write the T-shirt cost function as $C(s) = 8s + 15$, where the cost, defined as C, is a function of s, the number of shirts ordered.

4. Consider the function, $C(s) = 8s + 15$. What expression in the function equation represents:

   a. the domain of the function?

      The expression s represents the domain of the function.

   b. the range of the function?

      The expressions $8s + 15$ and $C(s)$ each represent the range of the function.

5. Describe the possible domain and range for this situation.

   The domain is the set of whole numbers. The range is the corresponding set of whole numbers that result from substituting values into the expression $8s + 15$. The range can be interpreted to either include the value at $C = 15$ or not depending on whether the design was set, but no T-shirts were ordered yet.

Function notation is a way of representing functions algebraically. The function notation $f(x)$ is read as "f of x" and indicates that x is the independent variable.

When f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x.

Optimizing Learning

This activity supports development of vocabulary and symbols.

STAMP THE LEARNING

The definition and paragraphs provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 1 • TOPIC 1 • LESSON 3 41

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

WORKED EXAMPLE

You can write domain and range using set notation.

A teacher is restocking his pencil supply for this semester's tutoring group. All new students to the tutoring group receive 12 pencils. The teacher expects to have no more than 6 new students attending his group.

The domain for this situation is {0, 1, 2, 3, 4, 5, 6} for the possibility of having 0 to 6 new students.

The range for this situation is {0, 12, 24, 36, 48, 60, 72} for the possibility of needing between 0 and 72 pencils.

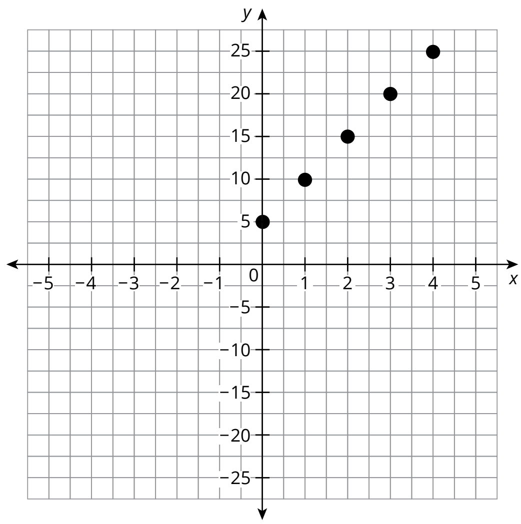
6. Write the domain and range using set notation.

   a. A neighborhood bakery makes and sells bagels every morning. The bagels are sold in sets of 6 and only the first 9 customers can purchase them.

   Domain {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

   Range {0, 6, 12, 18, 24, 30, 36, 42, 48, 54}

   b.

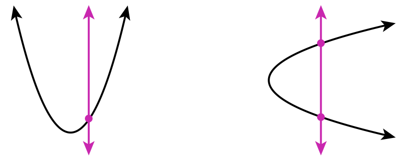


   Domain {0, 1, 2, 3, 4}

   Range {5, 10, 15, 20, 25}

42 MODULE 1 • TOPIC 1 • LESSON 3

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.



Function   Not a function

The Vertical Line Test applies for both *discrete* and *continuous graphs*.

A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

7. How can you determine if a relation represented as a set of ordered pairs is a function? Explain your reasoning.

   A set of ordered pairs represents a function when each x-value in the set maps to exactly one y-value.

8. How can you determine if a relation represented as an equation is a function? Explain your reasoning.

   An equation represents a function when I can solve for y and get only one y-value for any given x-value.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 7 and 8 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice identifying which relations represent functions, assign Skills Practice Set A for this lesson.

EB STUDENT TIP

For all proficiency levels

Review with students the difference between *continuous* and *discrete* functions.

Beginning: Use visuals to show how continuous functions can be represented by a smooth line or curve, while discrete functions can be shown as distinct, separate points. Use familiar contexts, such as a water level rising in a tank (continuous) versus the number of students in a classroom each day (discrete). Students can identify whether real-world examples are continuous or discrete by pointing to corresponding visuals on an anchor chart.

Intermediate: Provide sentence frames like: "When we represent ________, this would likely be a ________ graph because ________."

Advanced/Advanced High: Students can describe in their own words the difference between continuous and discrete functions. They should provide their own real-world examples, explaining why a particular function type is appropriate for each scenario.

9. Determine which relations represent functions. If the relation is not a function, state why not.

a. $y = 3^x$

The relation is a function.

b. For every house, there is one and only one street address.

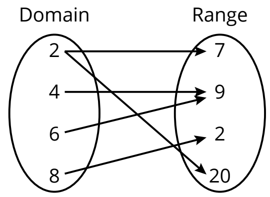
The relation is a function.

c.

Domain	Range
-1	4
0	0
3	-2
0	4

The relation is not a function; the domain value of 0 has more than one range value.

d.

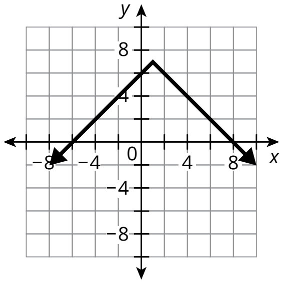


The relation is not a function; the domain value of 2 has more than one range value.

e. $\{(-7, 5), (-5, 5), (2, -2), (3, 5)\}$

The relation is a function.

f.



The relation is a function.

EB STUDENT TIP

For “Intermediate” and higher proficiency levels

Ask students to identify what the prefix *non-* means in *non-function*. Follow up with additional examples of words with the prefix *non-*, including nonsmoking, nonstop, and nonfat. Define these words and highlight the connection between the prefix *non-* and the words *not* and *no*. Encourage students to remember this connection to assist them in comprehension when they come across a word with this prefix.

10. Analyze the three graphs Elena grouped together in the previous lesson, graphs *D*, *I*, and *M*. Are the graphs she grouped functions? Explain your conclusion.

No. The graphs do not pass the Vertical Line Test.

11. Use the Vertical Line Test to sort the graphs in the previous lesson into two groups: functions and non-functions. Record your results by writing the letter of each graph in the appropriate column in the table shown.

Functions	Non-Function
A, B, C, E, F, G, H, J, K, L	D, I, M

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MODULE 1 • TOPIC 1 • LESSON 3 45

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Have students complete Question 1 independently.
- Check in and share.
- Read and discuss the Worked Example.
- Group students to complete Question 2.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice identifying and writing the domain and range of functions, assign Skills Practice Sets B and C for this lesson.

ACTIVITY **3.2**

Domain and Range of a Function

You have identified the domain and range of a function given its equation.

Ask Yourself . . .
How does an open or closed circle on a graph reflect the domain? The range?

1. Explain how you can identify the domain and range of a function given:

   a. a verbal statement.

   Domain: The possible values that make sense as the independent quantities

   Range: The possible values that make sense as the dependent quantities

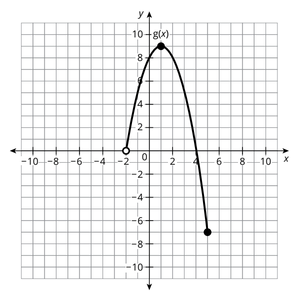
   b. a graph.

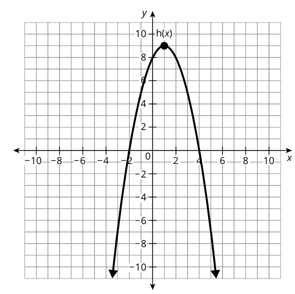
   Domain: The set of *x*-values represented by the graph

   Range: The set of *y*-values represented by the graph

WORKED EXAMPLE

There are different ways to write the domain and range of a function given its graph.





	Domain		Range	
	$g(x)$	$h(x)$	$g(x)$	$h(x)$
In Words	The domain is all real numbers greater than -2 and less than or equal to 5.	The domain is the set of all real numbers.	The range is all real numbers greater than or equal to -8 and less than or equal to 8.	The range is all real numbers less than or equal to 8.
Using Notation	$-2 < x \le 5$	$-\infty < x < \infty$	$-8 \le x \le 8$	$y \le 8$

2. Label each of the Graph Cards from the previous activity with the appropriate domain and range.

   See answers on Graph Cards in Lesson 2: *Analyzing and Sorting Graphs*

46 MODULE 1 • TOPIC 1 • LESSON 3

ACTIVITY 3.3 Linear, Constant, and Exponential Functions

Gather all of the graphs that you identified as functions.

A function is described as increasing when the value of the dependent variable increases as the value of the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

A function is described as decreasing when the value of the dependent variable decreases as the value of the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the value of the dependent variable of a function remains constant over the entire domain, then the function is called a **constant function**.

When determining whether a graph is increasing or decreasing, read the graph from left to right.

1. Analyze each graph from left to right. Sort all the graphs into one of the four groups listed.
   - Increasing function
   - Decreasing function
   - Constant function
   - A combination of increasing and decreasing

Record the function letter in the appropriate column of the table shown.

Increasing Function	Decreasing Function	Constant Function	Combination of Increasing and Decreasing
F, G	C, J, K	A	B, E, H, L

Chunking the Activity

- Read and discuss the introduction and definitions.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the definitions.
- Group students to complete Questions 4 and 5.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definitions are an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 1 • TOPIC 1 • LESSON 3 47

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Use graphing technology to determine the shape of its graph. Then, match the function to its corresponding graph by writing the function directly on the graph that it represents.

Think About . . .
Be sure to correctly interpret the domain of each function. Also, remember to use parentheses when entering fractions into your graphing technology.

- $f(x) = x$
  Graph F
- $f(x) = \left(\frac{1}{2}\right)^x$
  Graph C
- $f(x) = \left(\frac{1}{2}\right)^x - 5$, where x is an integer
  Graph G
- $f(x) = 2$, where x is an integer
  Graph J
- $f(x) = 2^x$, where x is an integer
  Graph K
- $f(x) = -x + 3$, where x is an integer
  Graph A

3. Consider the six graphs and functions that are increasing functions, decreasing functions, or constant functions.

   a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2
A, F, J Linear/Constant	C, G, K Exponential

   b. What is the same about all the functions in each group?
   Sample answer:
   Group 1 represents lines. Group 2 represents smooth curves that are either increasing or decreasing.

48 MODULE 1 • TOPIC 1 • LESSON 3

You have just sorted the graphs into their own *function families*. A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but not equal to 1.

Ask Yourself . . .
What other variables have you used to represent a linear function?

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. When $f(x) = ax + b$ represents a linear function, describe the a and b values that produce a constant function.
   When $a = 0$ and b is any real number, the result will be a constant function.

Place these two groups of graphs off to the side. You will need them again.

EB STUDENT TIP

For “Intermediate” and higher proficiency levels

Ensure that students understand the term *function family*. Because much of the focus of this lesson is on increasing, decreasing, and constant functions, students may think that all functions that are decreasing are part of the same function family. Display a graph of a decreasing linear function and a graph of a decreasing exponential function. Ask whether the functions belong to the same function family because they are both decreasing, and have students explain their answers.

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the definitions.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the definitions.
- Complete Question 4 as a class.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 3.4 Quadratic Functions

A function has an **absolute minimum** if there is a point on the graph of the function that has a *y*-coordinate that is less than the *y*-coordinate of every other point on the graph. A function has an **absolute maximum** if there is a point on the graph of the function that has a *y*-coordinate that is greater than the *y*-coordinate of every other point on the graph.

1. Sort the graphs from the combination of increasing and decreasing category in the previous activity into one of the two groups listed.
   - Those that have an absolute minimum value
   - Those that have an absolute maximum value

   Then, record the function letter in the appropriate column of the table shown.

Absolute Minimum	Absolute Maximum
H, L	B, E

50 MODULE 1 • TOPIC 1 • LESSON 3

2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Use graphing technology to determine the shape of its graph. Then, match the function to its corresponding graph by writing the function directly on the graph that it represents. Identify the absolute maximum or minimum of each graph.

- $f(x) = x^2 + 8x + 12$
  Graph L
  Absolute minimum: $(-4, -4)$
- $f(x) = -3x^2 + 4$, where x is integer
  Graph E
  Absolute maximum: $(0, 4)$
- $f(x) = x^2$
  Graph H
  Absolute minimum: $(0, 0)$

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For all proficiency levels

The visual nature of this activity provides a good opportunity to reinforce the vocabulary needed to describe functions. Write the words *increasing*, *decreasing*, *maximum*, *minimum*, *curve*, and *line* on the board. Use the words as you describe the graphs and have students point to that feature of the graph to show they understand what you are saying.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Optimizing Learning

This activity optimizes transfer and generalization.

ACTIVITY 3.5

Function Families

You have now sorted each of the graphs and equations representing functions into one of three function families: linear, exponential, and quadratic.

1. Glue your sorted graphs and functions to the appropriate function family graphic organizer located at the end of the lesson. Write a description of the graphical behavior for each function family.

   Check students' graphic organizers.

Hang on to your graphic organizers. They will be a great resource moving forward!

You've done a lot of work up to this point! You've been introduced to linear, exponential, and quadratic functions. Don't worry—you don't need to know everything there is to know about these function families right now. As you progress through this course, you will learn more about each function family.

52 MODULE 1 • TOPIC 1 • LESSON 3

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Talk the Talk

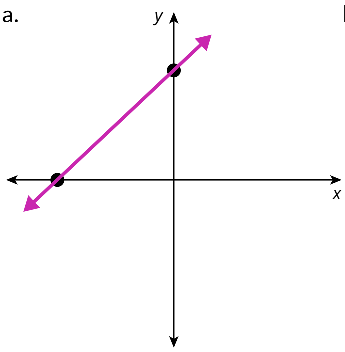
Interception!

Recall that the **x-intercept** is the point where a graph crosses the x-axis. The **y-intercept** is the point where a graph crosses the y-axis.

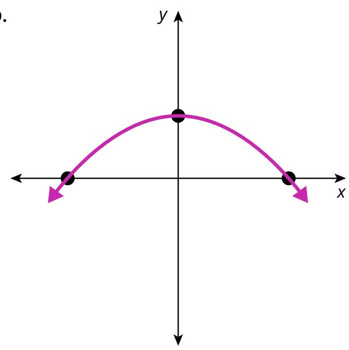
1. The graphs shown represent relations with just the x- and y-intercepts plotted. If possible, draw a function that has the given intercepts. If it is not possible, explain why not.

Sample answers:

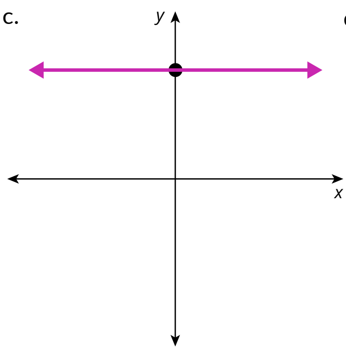
a.



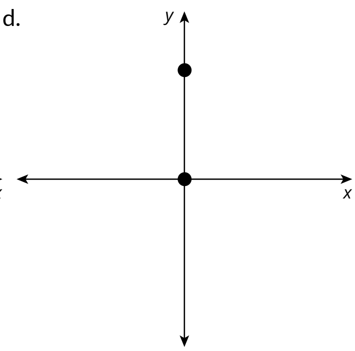
b.



c.



d.



d. It is not possible. A function can have only one y-value for an x-value, but according to the graphed points, when $x = 0$ there are two different y-values.

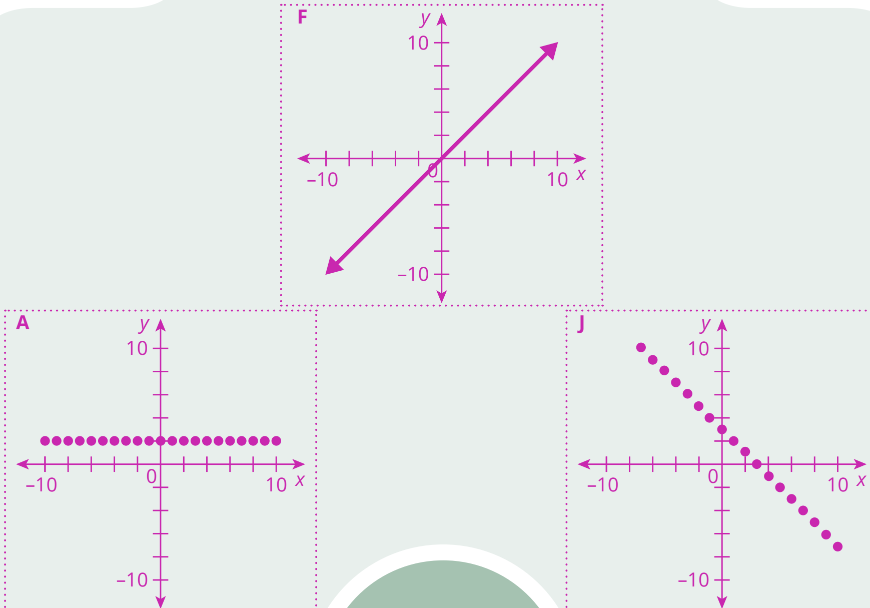
Chunking the Activity

- Read and discuss the definitions.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

MODULE 1 • TOPIC 1 • LESSON 3 53

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

Graphs



Linear Functions

Increasing/Decreasing/Constant:
The graph either increases, decreases, or is constant.

Domain and Range:
Domain: All real numbers
Range: All real numbers

Maximum/Minimum:
None

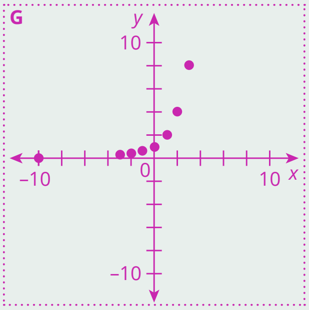
Curve/Line:
Line

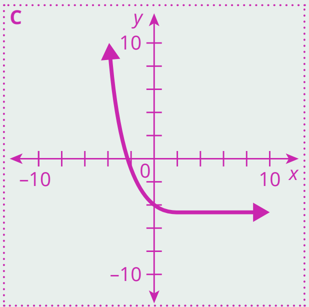
Graphical Behaviors

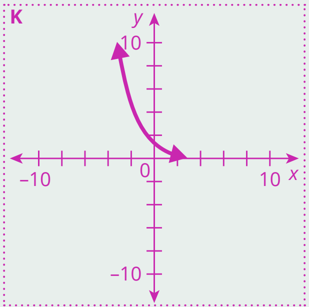
54 MODULE 1 • TOPIC 1 • LESSON 3

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but not equal to 1.

Graphs







Exponential Functions

Increasing/Decreasing/Constant:

The graph either increases or decreases.

Domain and Range:

Domain: All real numbers
Range: A subset of real numbers

Maximum/Minimum:

None

Curve/Line:

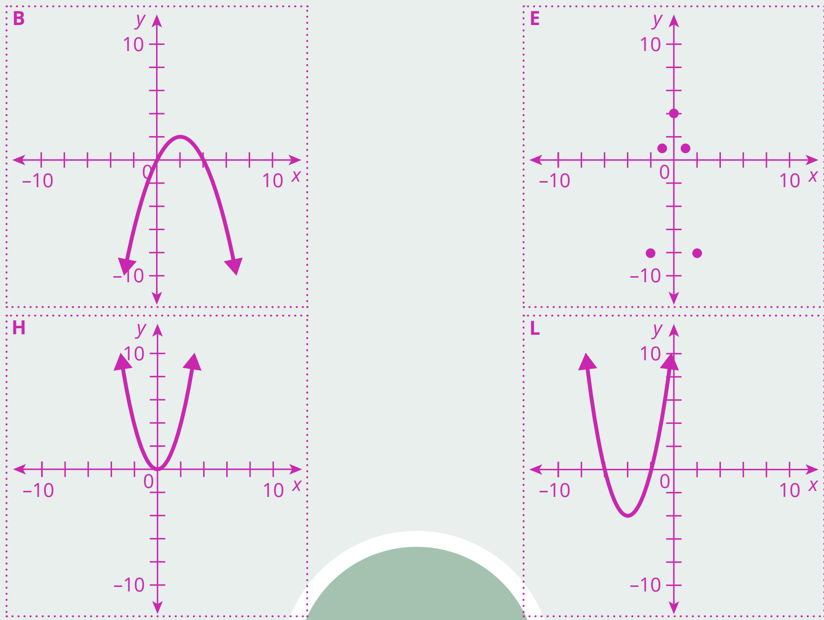
Curve

Graphical Behaviors

MODULE 1 • TOPIC 1 • LESSON 3 55

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

Graphs



Quadratic Functions

Increasing/Decreasing/Constant:
The graph either increases and then decreases or vise versa.

Domain and Range:
Domain: All real numbers
Range: A subset of real numbers

Maximum/Minimum:
Contains either a maximum or a minimum

Curve/Line:
Curve

Graphical Behaviors

56 MODULE 1 • TOPIC 1 • LESSON 3

Lesson 3 Assignment

Write

function notation	increasing function	constant function
absolute maximum	decreasing function	absolute minimum

Choose the term that best completes each statement.

1. A way to represent equations algebraically that makes it more efficient to recognize the independent and dependent variables is called __________________.
2. When both the independent and dependent variables of a function increase across the entire domain, the function is called a(n) __________________.
3. A function has a(n) __________________ if there is a point on its graph that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph.
4. When the dependent variable of a function decreases as the independent variable increases across the entire domain, the function is called a(n) __________________.
5. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a(n) __________________.
6. A function has a(n) __________________ if there is a point on its graph that has a *y*-coordinate that is less than the *y*-coordinate of every other point on the graph.

Remember

A function is a relation that assigns to each element of the domain exactly one element of the range. The different function families include linear functions, exponential functions, and quadratic functions.

Write

1. function notation
2. increasing function
3. absolute maximum
4. decreasing function
5. constant function
6. absolute minimum

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MODULE 1 • TOPIC 1 • LESSON 3 ASSIGNMENT 57

Lesson 3 Assignment

Practice

For each scenario, use graphing technology to determine the shape of its graph. Then, identify the function family, whether it is increasing, decreasing, or a combination of both, and whether it is a smooth curve or straight line. Finally, identify whether the function has an absolute minimum or absolute maximum, if there is one.

1. A fitness company is selling DVDs for one of its new cardio routines. Each DVD will sell for \$15. Due to fixed and variable costs, the profit that the company will see after selling x DVDs can be represented by the function $P(x) = 11.5x - 0.1x^2 - 150$.
   Quadratic function; increasing, then decreasing; absolute maximum; smooth curve

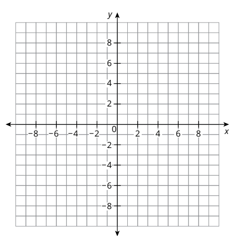
2. Mariana is going to put \$500 into an account with her bank. The bank is offering a 3% interest rate compounded annually. The amount of money that Mariana will have after x years can be represented by the function $A(x) = 500(1.03)^x$.
   Exponential function; increasing; no maximum/minimum; curve

3. A calendar company is going to buy a new 3D printer for \$20,000. In order to plan for the future, the owners are interested in the salvage value of the printer each year. The salvage value after x years can be represented by the function $S(x) = 20{,}000 - 2000x$.
   Linear function; decreasing; absolute minimum; line

Prepare

1. Sketch a graph and write an equation for each function.

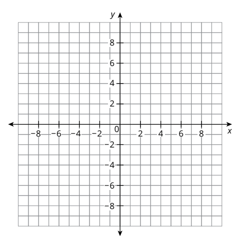
   a. Decreasing linear function

   Sample answer:

   $f(x) = -2x + 1$

   b. Increasing exponential function

   Sample answer:

   $f(x) = 3^x$

58 MODULE 1 • TOPIC 1 • LESSON 3 ASSIGNMENT

4 Recognizing Functions by Characteristics

LESSON OVERVIEW

Given characteristics describing graphical behavior, students name the possible function family or families that fit each description. Using the scenarios and their graphs from the first lesson of the topic, they complete a table by naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant, or both increasing and decreasing. Students then work with a partner and write equations and sketch graphs to satisfy different lists of characteristics. They conclude the lesson by creating their own list of characteristics, providing two graphs that include those characteristics, and determining that an equation, not just a list of characteristics, is required to generate a unique graph.

MATERIALS

Graphs from *Analyzing and Sorting Graphs*
Problem-Solving Model
Graphic Organizer

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

A.1E create and use representations to organize, record, and communicate mathematical ideas.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

(TEKS continued on next page)

ELPS

(2) Listening
The student is expected to:

(I) demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.

(3) Speaking
The student is expected to:

(H) narrate, describe, and explain with increasing specificity and detail as more English is acquired.

(5) Writing
The student is expected to:

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary.

(F) write using a variety of grade-appropriate sentence lengths, patterns, and connecting words to combine phrases, clauses, and sentences in increasingly accurate ways as more English is acquired.

ALGEBRA I TEKS *(TEKS continued from previous page)*

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
The student is expected to:

A.3C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

Quadratic Functions and Equations

(6) The student applies the mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations.
The student is expected to:

A.6A determine the domain and range of quadratic functions and represent the domain and range using inequalities.

(7) The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations.
The student is expected to:

A.7A graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including *x*-intercept, *y*-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.

Exponential Functions and Equations

(9) The student applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions. The student formulates statistical relationships and evaluates their reasonableness based on real-world data.
The student is expected to:

A.9A determine the domain and range of exponential functions of the form $f(x) = ab^x$ and represent the domain and range using inequalities.

A.9D graph exponential functions that model growth and decay and identify key features, including *y*-intercept and asymptote, in mathematical and real-world problems.

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.
The student is expected to:

A.12A decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

ESSENTIAL IDEAS

- The graph of an exponential or quadratic function is a curve.
- The graph of a linear function is a line.
- The graph of a linear or exponential function is either increasing or decreasing.
- The graph of a quadratic function has intervals where it is increasing and intervals where it is decreasing. Quadratic functions also have an absolute maximum or an absolute minimum.
- Key characteristics of graphs help to determine the function family to which it belongs.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Name That Function! 10–15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students are given one or two characteristics of a graph and determine whether the function could be a member of the linear, exponential, and/or quadratic function family.

DEVELOP

Activity 4.1: Categorizing Scenarios into Their Function Families 25–30 minutes

CLASSIFICATION, MATHEMATICAL PROBLEM SOLVING

Students revisit the scenarios and their graphs from the first lesson of the topic to complete a table naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant, or both increasing and decreasing.

DAY 2

Activity 4.2: Building Graphs from Characteristics 20–25 minutes

MATHEMATICAL PROBLEM SOLVING

Students write equations and sketch graphs to satisfy different lists of characteristics. They then create their own function and describe characteristics of the function so that another person can sketch the graph. They exchange descriptions with a partner and sketch their partner's function.

DEMONSTRATE

Talk the Talk: Trying to Be Unique 15–20 minutes

EXIT TICKET APPLICATION

Students create their own list of characteristics, provide two graphs that include those characteristics, and determine that an equation, not just a list of characteristics, is required to generate a unique graph.

Getting Started

ENGAGE

Name That Function!

Facilitation Notes

In this activity, students are given one or two characteristics of a graph and determine whether the function could be a member of the linear, exponential, and/or quadratic function family.

NOTE: Previously in this topic, students were introduced to these functions. The focus of this lesson is for students to analyze general characteristics of functions. Students will formalize their understanding of the defining characteristics of each type of function later in this course.

DIFFERENTIATION STRATEGY

Just in Time Support

Prior to beginning the lesson, review the three function families and their graphs as a class: linear, exponential, and quadratic. This will support students with the descriptions that are used in this activity.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What does a smooth curve look like?
Probing	• What is another function that has this characteristic?
Gathering	• Explain what *symmetry* means.
Probing	• Provide an example to support each response. • Why don't any of the five functions apply to the criteria in part (a)?
Seeing structure	• Can you draw a quadratic function that doesn't have symmetry? Explain your thinking.

STAMP THE LEARNING

Summary

Graphs described as straight lines may be associated with a linear function, while those described as smooth curves may be associated with an exponential or quadratic function. The graph of a linear or exponential function is either increasing or decreasing, while the graph of a quadratic function has an interval where it is increasing and an interval where it is decreasing.

ACTIVITY 4.1

Categorizing Scenarios into Their Function Families

DEVELOP

Facilitation Notes

In this activity, students revisit the scenarios and their graphs from the first lesson of the topic to complete a table naming the function family associated with each scenario, identifying the domain, and describing the graphical behavior as increasing, decreasing, constant, or both increasing and decreasing.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you tell from a scenario whether it represents a function? • How can you tell from the scenario whether the domain is continuous or discrete?

COMMON MISCONCEPTION

Students may think the graphs are incorrect because they all are continuous, while some of the scenarios have domains that are discrete. Discuss the fact that the graphs relate to functions that are mathematical models of the scenarios; the scenarios require an interpretation of the necessary components of the mathematical model.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

A scenario and its graph provide the necessary characteristics to determine the function family to which it belongs.

ACTIVITY 4.2

Building Graphs from Characteristics

Facilitation Notes

In this activity, students write equations and sketch graphs to satisfy different lists of characteristics. They then create their own function and describe characteristics of the function so that another person can sketch the graph. They exchange descriptions with a partner and sketch their partner's function.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

To scaffold support, have students complete Question 1 with a partner. Decide who begins with part (a) and who begins with part (b), then create an equation based on the given criteria. Have students swap equations with their partner, check each other's equations, and together correct any errors. Then, have students graph the equation written by their partner, then swap graphs and check for correctness again. Repeat this process with parts (c) and (d). Both partners should write an equation for part (e), and graph their partner's equation.

AS STUDENTS WORK, LOOK FOR

- Students who efficiently create a correct equation and graph on their first attempt.
- Students who must self-correct as they attempt to create an equation and graph.
- Students who reorder the characteristics prior to creating a graph.
- Students who attempt to graph first and then write the equation from the graph.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is another possible equation?
Seeing structure	• Explain how you constructed an exponential equation that was decreasing. • How did the value of *c* affect your graph?
Probing	• How did you identify that the function is discrete? How did you represent this on the graph?
Seeing structure	• How would you modify your equation to have a maximum? • Which value in your equation identifies the line as increasing? • What effect does the *b*-value have on your graph?
Probing	• How did you identify that the function is discrete? How did you represent this on the graph?
Seeing structure	• How would you modify your equation to have a maximum? • Which value in your equation identifies the line as increasing? • What effect does the *b*-value have on your graph?

Probing *(continued)*	• Describe how your partner's graph includes the characteristics you listed.
Seeing structure	• Were your partner's graph and equation the same as those you used to create their problem? Why or why not?

Summary

A list of characteristics can be used to write an equation and the equation can then be used to generate a graph.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

TRYING TO BE UNIQUE

Facilitation Notes

In this activity, students create their own list of characteristics, provide two graphs that include those characteristics, and determine that an equation, not just a list of characteristics, is required to generate a unique graph.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Did you include the characteristic function or non-function? • Did you include the characteristic continuous or discrete? • Did you include the characteristic smooth curve or straight line? • Did you include the characteristic absolute minimum or absolute maximum?

Have students read and answer the Essential Question on the lesson opener page.

Summary

An equation, not just a list of characteristics, is required to generate a unique graph.

STAMP THE LEARNING

4 Recognizing Functions by Characteristics

Setting the Stage

- Communicate the objectives.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Recognize similar characteristics among function families.
- Recognize different characteristics among function families.
- Determine function types given certain characteristics.

You have identified key characteristics of graphs.

How can the key characteristics help you sketch the graph of a function?

Sample answer:

Function families have key characteristics that are common among all functions in the family.

Knowing these key characteristics is useful when sketching a graph of the function.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Materials Needed: Poster Paper

Create an anchor chart to identify the various characteristics of functions. Ask students what they think of when they hear the word *characteristics*. Compare and contrast *characteristics* of a person to *characteristics* of a function. Ask students to sketch examples of graphs that represent different types of functions and label the graphs with the *characteristics* of each function.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The paragraph provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Getting Started

Name That Function!

Function Families
linear
exponential
quadratic

You have sorted graphs according to their function family. Now, consider which function families have the given characteristics.

1. Which function families can be described by the characteristic provided? Choose from the given list.

   a. The graph is a smooth curve.
   Exponential function or quadratic function

   b. The graph is made up of a straight line.
   Linear function

   c. The graph increases or decreases over the entire domain.
   Linear function or exponential function

   d. The graph has an absolute maximum or minimum.
   Quadratic function

2. One or more characteristics have been added to the graphical description of each function. Name the possible function families.

   a. The graph has an absolute minimum or absolute maximum and is a smooth curve.
   Quadratic function

   b. The graph either increases or decreases over the entire domain and is a straight line.
   Linear function

   c. The graph is a smooth curve, and either increases or decreases over the entire domain.
   Exponential function

Each function family has certain graphical behaviors, with some behaviors common among different function families. Notice, the more specific characteristics that are given, the more specifically you can name that function!

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Review the difference between behaviors of functions and characteristics of functions. Discuss as a class the characteristics of a person compared to the behaviors of a person. Help students make the connection that characteristics are usually nouns and behaviors are usually verbs. Create a list of words that are characteristics of a function and a list of words that can be described as behaviors of a function. Have students add to the list as they come across the different functions

ACTIVITY 4.1 Categorizing Scenarios into Their Function Families

You have been introduced to several function families: linear, exponential, and quadratic. Let's revisit the first lesson: *Understanding Quantities and Their Relationships*. Each of the scenarios in that lesson represents one of these function families.

Remember . . .
Each of the graphs representing the scenarios was drawn with either a continuous line or a continuous smooth curve to model the problem situation.

1. Describe how each scenario represents a function.
   Each scenario describes a function because there is one unique output value for each input value.

2. Complete the table on the following pages to describe each scenario. See answers in the table.

   a. Identify the appropriate function family under the scenario name.

   b. Based on the context, identify the domain as continuous or discrete.

   c. Describe the graphical behavior as increasing, decreasing, constant, or a combination.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days.
To provide additional practice determining what types of functions graphs represent, assign Skills Practice Set A for this lesson.

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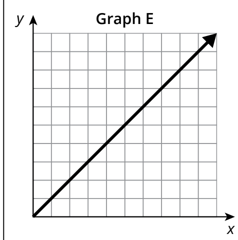
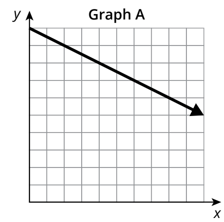
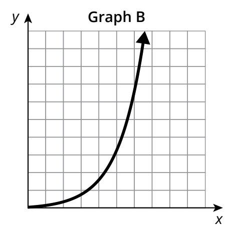
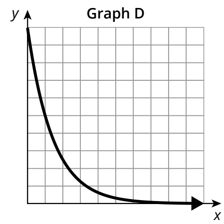
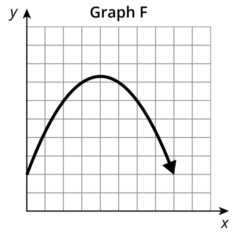
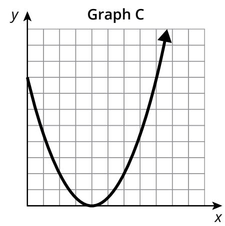
MODULE 1 • TOPIC 1 • LESSON 4 61

Scenario	Domain of the Real-World Situation	Graph of the Mathematical Model	Graphical Behavior
Music Club Linear	Discrete		Increasing
Something's Fishy Linear	Continuous		Decreasing
Smart Phone, but Is It a Smart Deal? Exponential	Discrete		Increasing

62 MODULE 1 • TOPIC 1 • LESSON 4

Scenario	Domain of the Real-World Situation	Graph of the Mathematical Model	Graphical Behavior
It's Magic Exponential	Discrete		Decreasing
Baton Twirling Quadratic	Continuous		Increasing and Decreasing
Skateboarding Quadratic	Continuous		Decreasing and Increasing

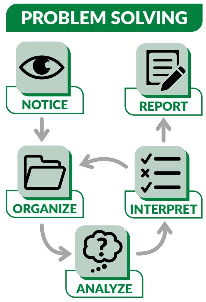
MODULE 1 • TOPIC 1 • LESSON 4 63

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Refer students to the problem-solving model in the Course and Implementation Guide.
- Have students read the questions to ask aloud for each step.
- Work through Question 1 part (a) together as a class to complete the graphic organizer.
- Have students work in pairs and use the problem-solving model to complete Questions 1 and 2.



ACTIVITY 4.2 Building Graphs from Characteristics

In this activity, you will write equations and sketch a graph based on given characteristics for the three main function families we will study this year in Algebra I: Linear Functions, Exponential Functions, and Quadratic Functions.

Use the problem-solving model whenever you see this icon.

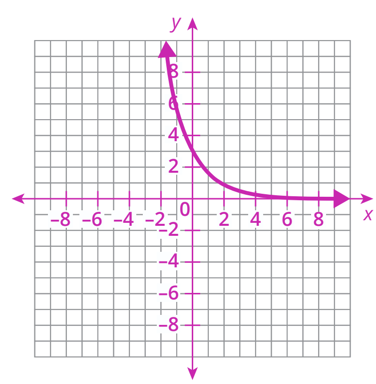
1. Use the given characteristics to create an equation and sketch a graph. Use the equations given in the box as a guide. When creating your equation, use a, b, and c values that are any real numbers between -3 and 3. Do not use any functions that were used previously in this topic.

> **Linear Function**
> $f(x) = ax + b$
> **Exponential Function**
> $f(x) = a \cdot b^x$
> **Quadratic Function**
> $f(x) = ax^2 + bx + c$

   a. Create an equation and sketch a graph that is:
      - a function,
      - exponential,
      - continuous, and
      - decreasing.

      Sample answers:

      Equation: $f(x) = 3\left(\frac{1}{2}\right)^x$



Think About . . .
Don't forget about the function family graphic organizers you created if you need some help.

NOTE: This is the first lesson where TEKS A.1B is highlighted.

- To encourage persevering through productive struggle, have students ask themselves the questions from the model to persevere in solving problems.

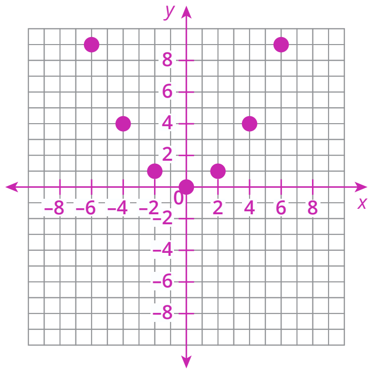
64 MODULE 1 • TOPIC 1 • LESSON 4

b. Create an equation and sketch a graph that:

- has a minimum,
- is discrete, and
- is a quadratic function.

Sample answers:

Equation: $f(x) = \frac{1}{4}x^2$



Domain is the set of integers.

Ask Yourself . . .
Is the domain the same or different for each function?

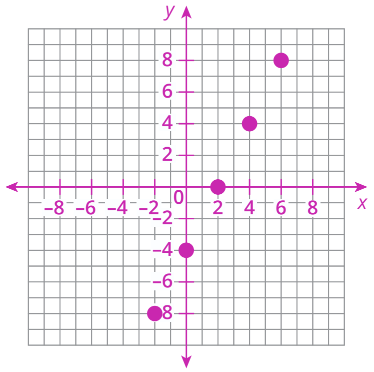
Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice creating equations and graphs with given sets of characteristics, assign Skills Practice Set B for this lesson.

c. Create an equation and sketch a graph that is:

- linear,
- discrete,
- increasing, and
- a function.

Sample answers:

Equation: $f(x) = 2x - 1$



Domain is the set of integers.

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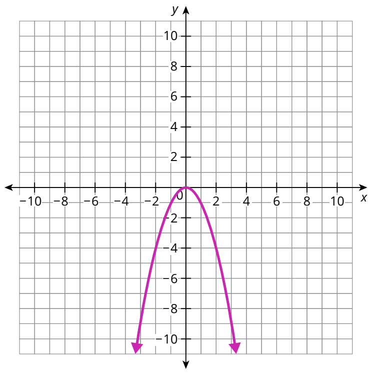
MODULE 1 • TOPIC 1 • LESSON 4 65

d. Create an equation and sketch a graph that:

- is continuous,
- has a maximum,
- is a function, and
- is quadratic.

Sample answers:

Equation: $f(x) = -x^2$

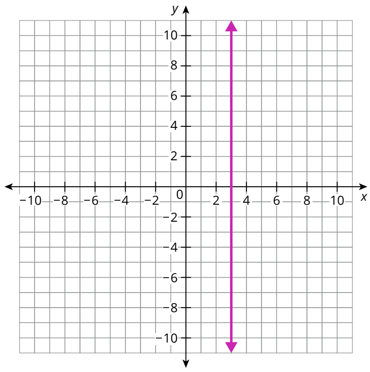


e. Create an equation and sketch a graph that is:

- not a function,
- continuous, and
- a straight line.

Sample answers:

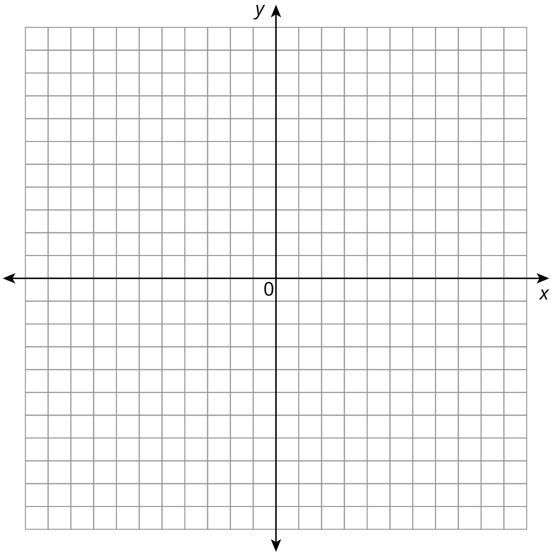
Equation: $x = 3$



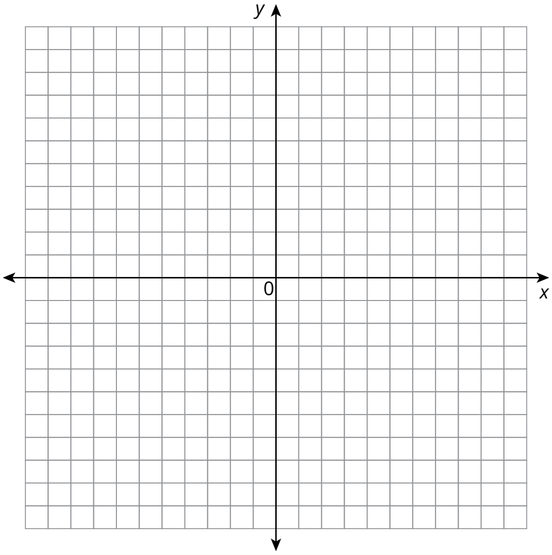
66 MODULE 1 • TOPIC 1 • LESSON 4

2. Create your own function. Describe certain characteristics of the function and see if your partner can sketch it. Then, sketch your partner's function based on characteristics provided. Answers will vary.

Your Function:



Your Partner's Function:



MODULE 1 • TOPIC 1 • LESSON 4 67

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Trying to Be Unique

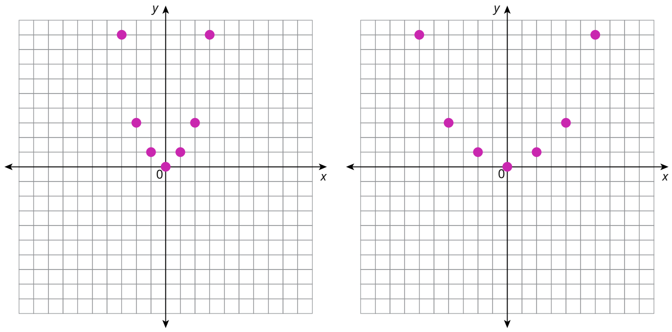
Throughout this lesson, you used characteristics to describe graphs.

Sample answer:

1. Write a list of four characteristics to describe a graph.
   - a function
   - quadratic
   - discrete
   - has a minimum

2. Sketch two possible graphs based on your characteristics.

   Sample answers:



3. How could you modify your list of characteristics to describe a unique graph?

   Change the directions to write an equation, or provide a specific number of points that lie on the graph.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Ask students what they think of when they hear the word *unique*. Have them define the term with examples of how it would be used in their culture. Discuss examples of ways to make objects *unique* and then connect the concept of *uniqueness* to graphs. Ask for volunteers to discuss how to make a particular type of graph *unique*. For example, show a graph of two linear functions, one with a negative slope and one with a positive slope. Ask students, "Although both of these graphs represent linear functions, how is each one *unique*?"

Lesson 4 Assignment

Write

Identify the function family or families that are described by the given characteristic(s). Choose from linear, exponential, and quadratic functions.

1. The graph of this function family has an absolute minimum.
2. The graph of this function family is decreasing over the entire domain.
3. The graph of this function family has an increasing interval and a decreasing interval and forms a U shape.
4. The graph of this function family does not have an absolute maximum or absolute minimum and is a smooth curve.
5. The graph of this function family contains straight lines and does not have an absolute maximum or absolute minimum.

Remember

Function families have key characteristics that are common among all functions in the family. Knowing these key characteristics is useful when sketching a graph of the function.

Write

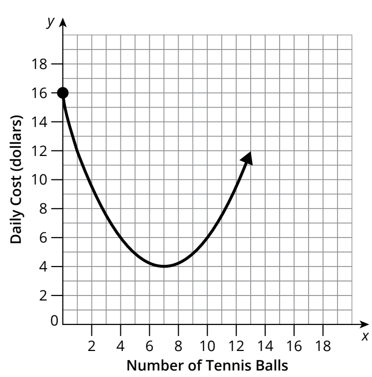
1. Quadratic functions
2. Linear functions or exponential functions
3. Quadratic functions
4. Exponential functions
5. Linear functions

Practice

For each scenario and its graph, identify the appropriate function family. Then, based on the problem situation, identify whether the data values represented in the graph are discrete or continuous. Finally, identify the graphical behavior of the function that models the scenario based on the characteristics of its function family.

1. A manufacturing company finds that the daily costs associated with making tennis balls is high if they don't make enough balls and then becomes high again if they make too many balls. The function graphed models the daily costs of making x tennis balls.

   Quadratic
   Discrete
   Decreasing and increasing
   Has a minimum



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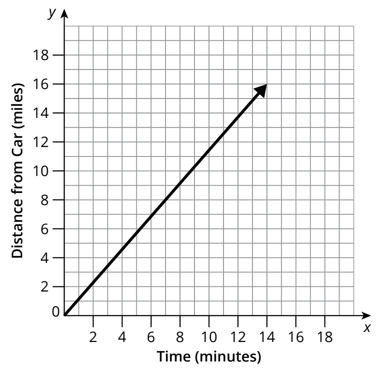
MODULE 1 • TOPIC 1 • LESSON 4 ASSIGNMENT 69

Lesson 4 Assignment

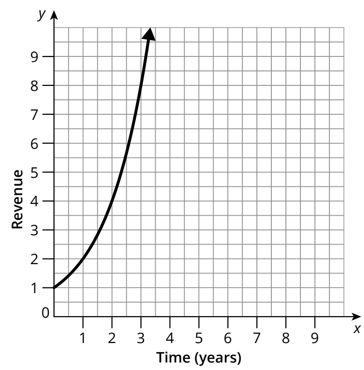
2. Jamal is training for a mountain bike race. He bikes 14 miles. If he bikes at a constant rate, the function graphed models the distance he bikes after *x* minutes.

   Linear
   Continuous
   Increasing



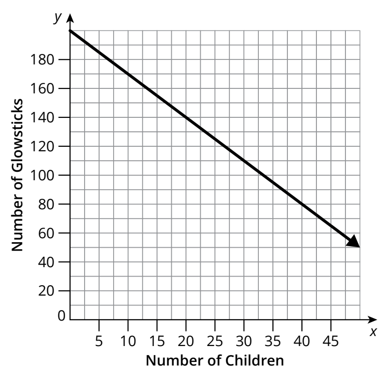
3. A local television company determines that the revenue it gets from running ads doubles each year. The function graphed models the revenue from advertising after *x* years.

   Exponential
   Continuous
   Increasing



4. Mr. Patel's neighborhood is hosting a summer nighttime party in the park. They are handing out glow sticks to all the children who attend. They start with 200 glow sticks and each child receives 3 glow sticks. The function graphed models the number of glow sticks they have left after *x* children have entered.

   Linear
   Discrete
   Decreasing



70 MODULE 1 • TOPIC 1 • LESSON 4 ASSIGNMENT

Lesson 4 Assignment

Prepare

Write the next three terms in each pattern and explain how you generated each term.

1. J, F, M, A, M, J, J, A, S, ...
   O, N, D; They are the first letter of each month.

2. S, M, T, W, ...
   T, F, S; They are the first letter of each day of the week.

3. 5, 10, 15, 20, ...
   25, 30, 35; They are all increasing by 5.

4. 100, 81, 64, 49, ...
   36, 25, 16; They are all decreasing perfect square numbers beginning with 10.

MODULE 1 • TOPIC 1 • LESSON 4 ASSIGNMENT 71

TOPIC 1 SELF-REFLECTION

Name: ______________________

Notes

Quantities and Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Quantities and Relationships* topic by:

TOPIC 1: *Quantities and Relationships*	Beginning of Topic	Middle of Topic	End of Topic
choosing appropriate scale and origin for graphs.	○	○	○
identifying the appropriate unit of measure for each variable or quantity.	○	○	○
analyzing a graph and stating the key characteristics of the graph.	○	○	○
using a problem situation to explain what the key features of a graph mean in real-world context.	○	○	○
deciding whether relations represented verbally, tabularly, graphically, and symbolically define a function.	○	○	○
recognizing a linear, exponential, or quadratic function by its equation or graph.	○	○	○
evaluating functions, expressed in function notation, given one or more elements in their domain.	○	○	○
determining the domain and range and the independent and dependent quantities in a relationship.	○	○	○

continued on the next page

MODULE 1 • TOPIC 1 • SELF-REFLECTION 73

Notes

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Quantities and Relationships* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

74 MODULE 1 • TOPIC 1 • SELF-REFLECTION

TOPIC 1 SUMMARY

Notes

Quantities and Relationships Summary

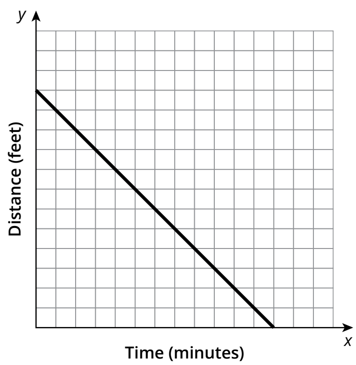
LESSON 1 Understanding Quantities and Their Relationships

Many problem situations include two quantities that change. When one quantity depends on another, it is said to be the **dependent quantity**. The quantity that the dependent quantity depends upon is called the **independent quantity**.

Graphs relay information about data in a visual way. Connecting points on a coordinate plane with a line or smooth curve is a way to model or represent relationships. The independent quantity is graphed on the horizontal, or *x*-axis, while the dependent quantity is graphed on the vertical, or *y*-axis. Graphs can be straight lines or curves and can increase or decrease from left to right.

For example, consider the graph that models the situation where Pedro is walking home from school.

Time, in minutes, is the independent quantity and the distance Pedro is from home is the dependent quantity.



LESSON 2 Analyzing and Sorting Graphs

Looking for patterns can help when sorting and comparing graphs. Some graphs show vertical symmetry (when a vertical line is drawn through the middle of the graph, the image is the same on both sides). Other possible patterns to look for include: only goes through two quadrants, always increases from left to right, always decreases from left to right, has straight

NEW KEY TERMS

- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- relation [relación]
- domain [dominio]
- range [rango]
- function [función]
- function notation [notación de función]
- Vertical Line Test [Prueba de la línea vertical]
- discrete graph [gráfica discreta/ discontinua]
- continuous graph [gráfica continua]
- increasing function [función creciente]
- decreasing function [función decreciente]
- constant function [función constante]
- function family [familia de funciones]
- linear functions [funciones lineales]
- exponential functions [funciones exponenciales]
- absolute maximum [máximo absoluto]
- absolute minimum [mínimo absoluto]

MODULE 1 • TOPIC 1 • TOPIC SUMMARY 75

Notes

NEW KEY TERMS

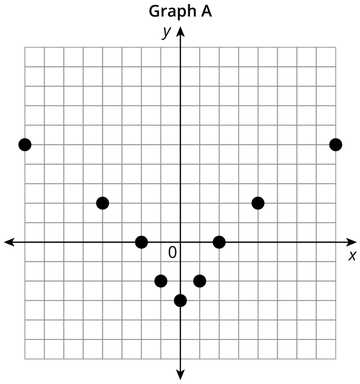
continued

- quadratic functions [funciones cuadráticas]
- *x*-intercept [intersección con el eje *x*]
- *y*-intercept [intersección con el eje *y*]

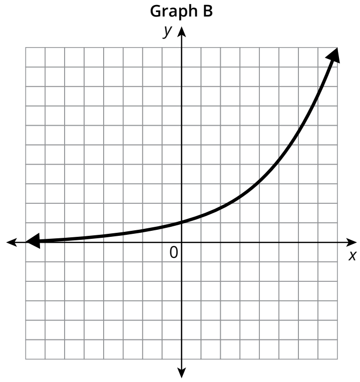
lines, has smooth curves, the graph goes through the origin, the graph forms a U shape, or the graph forms a V shape.

For example, Graph A has vertical symmetry. Graph B is a smooth curve that increases from left to right.

Graph A



Graph B

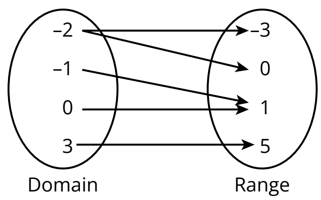


LESSON 3 Recognizing Functions and Function Families

A **relation** is the mapping between a set of input values called the **domain** and a set of output values called the **range**.

A **function** is a relation between a given set of elements such that for each element in the domain, there exists exactly one element in the range. If each value in the domain has one and only one range value, then the relation is a function. If any value in the domain has more than one range value, then the relation is not a function.

The value −2 in the domain has more than one range value. The mapping does not represent a function.



Each element in the domain has exactly one element in the range. The table represents a function.

Domain	Range
2	1
6	3
10	5
14	7

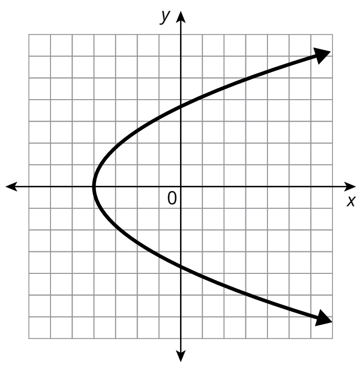
76 MODULE 1 • TOPIC 1 • TOPIC SUMMARY

Functions can be represented in a number of ways. An equation representing a function can be written using **function notation**. Function notation is a way of representing functions algebraically. This form allows you to more efficiently identify the independent and dependent quantities. The function $f(x)$ is read as "f of x" and indicates that x is the independent variable.

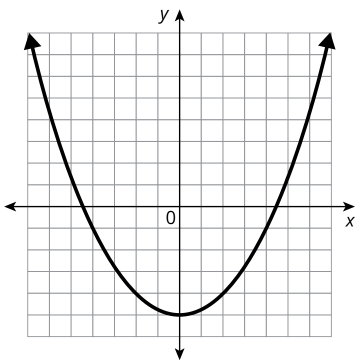
For example, consider the situation in which U.S. Shirts charges $8 per shirt plus a one-time charge of $15 to set up a T-shirt design. The equation that models the situation, $y = 8x + 15$, where x represents the number of shirts ordered and y represents the total cost of the order, can be written in function notation as $f(x) = 8x + 15$. The cost, defined by f, is a function of x, defined as the number of shirts ordered.

The **Vertical Line Test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the Vertical Line Test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function. The Vertical Line Test applies to both discrete and continuous graphs. A **discrete graph** is a graph of isolated points. A **continuous graph** is a graph of points that are connected by a line or smooth curve with no breaks in the graph.

A line drawn vertically through the graph touches more than one point. The graph does not represent a function.



A line drawn vertically through the graph only touches one point. The graph represents a function.



A function is described as increasing when both the independent and dependent variables are increasing. If a function increases across the entire domain, then the function is called an **increasing function**. A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**. If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a **constant function**.

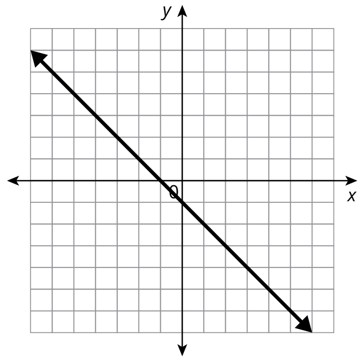
Notes

MODULE 1 • TOPIC 1 • TOPIC SUMMARY 77

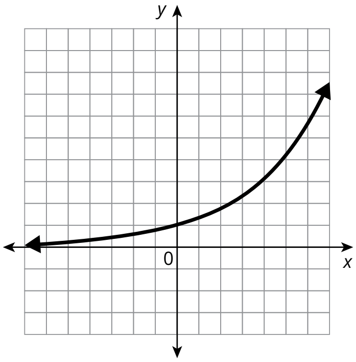
Notes

A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

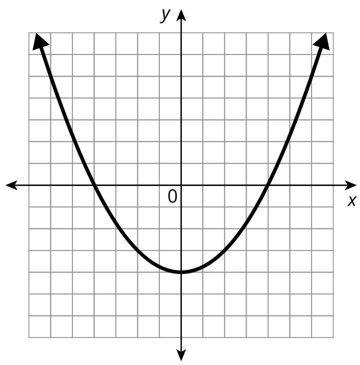


The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but not equal to 1.



Quadratic functions have an absolute maximum or an absolute minimum. An **absolute maximum** is a point on the graph of the function that has a y-coordinate that is greater than the y-coordinate of every other point on the graph. An **absolute minimum** is a point on the graph of the function that has a y-coordinate that is less than the y-coordinate of every other point on the graph.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

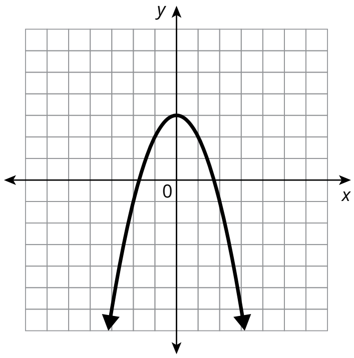


78 MODULE 1 • TOPIC 1 • TOPIC SUMMARY

LESSON 4

Recognizing Functions by Characteristics

Function families have key characteristics that are common among all functions in the family. Knowing these key characteristics is useful when sketching a graph of the function. For example, to sketch a graph of a continuous quadratic function with a maximum, the graph should be a smooth, U-shaped curve that increases to point and then decreases again.



Notes

(CC BY NC)

MODULE 1 • TOPIC 1 • TOPIC SUMMARY 79

ALGEBRA I

TOPIC 2

Everyday sequences might include sizes or ages. What other sequences have you noticed today?

Sequences

Sequences

How are the key concepts of *Sequences* organized?

In *Sequences*, students progress from recognizing patterns of numbers, letters, and shapes to identifying arithmetic and geometric sequences. They explore sequences represented as lists of numbers, in tables of values, by equations, and as graphs on the coordinate plane. The intent of this topic is to move students from their intuitive understanding of patterns to a more formal approach of representing sequences as functions. In later modules, they will use the connection between arithmetic sequences and linear functions and between some geometric sequences and exponential functions to examine the structure of each function family.

As in *Quantities and Relationships*, students begin *Sequences* by analyzing sequences presented in scenarios. They infer a rule for each sequence, identify additional terms, and represent the sequence as a table. They explain why all sequences are functions and differentiate between a finite sequence and an *infinite sequence*. After articulating the differences between different sequences, students define *arithmetic sequences* as those with a common difference and *geometric sequences* as those with a common ratio. They then match sequences to corresponding graphs.

Once familiar with the structure of sequences, students write recursive and explicit formulas for arithmetic and geometric sequences. Students return to the scenarios from the first lesson and write an arithmetic or geometric formula for each.

Math Representation

Recursive and explicit formulas provide ways to determine unknown terms of a sequence.

	A recursive formula expresses each new term of a sequence based on the preceding term in the sequence.	An explicit formula of a sequence is a formula to calculate the nth term of a sequence using the term's position in the sequence.
Arithmetic Sequence	$\overset{\text{nth term}}{a_n} = \underbrace{a_{n-1}}_{\text{previous term}} + \overset{\text{common difference}}{d}$	$\overset{\text{nth term}}{g_n} = \underbrace{g_{n-1}}_{\text{previous term}} \cdot \overset{\text{common ratio}}{r}$
Geometric Sequence	$\overset{\text{nth term}}{a_n} = \underset{\text{1st term}}{a_1} + \overset{\text{common difference}}{d}\underbrace{(n-1)}_{\text{previous term number}}$	$\overset{\text{nth term}}{g_n} = \underset{\text{1st term}}{g_1} \cdot \underset{\text{common ratio}}{r}^{\overbrace{n-1}^{\text{previous term number}}}$

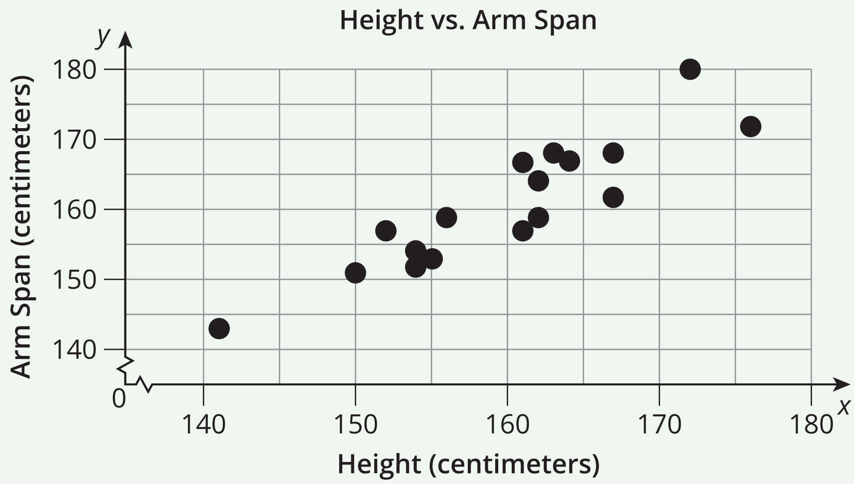
What is the entry point for students?

Students have been analyzing and extending numeric patterns since elementary school. They have used data to create tables of values and graph scatterplots. They have then used scatterplots to determine whether data represents a linear or nonlinear relationship.

Math Representation

The scatterplot represents the relationship between a person's height and their arm span.

As height increases, so does the arm span. This relationship is linear and increasing.



Why is *Sequences* important?

As students deepen their understanding of functions throughout this course and beyond, recognizing that all sequences are functions is an important building block. A rich understanding of arithmetic sequences, including their graphical and algebraic representations, is the foundation for linear functions. Likewise, students need to recognize that some geometric sequences represent exponential functions. By recognizing structure in sequences of numbers, they become aware of the possible functions that can model a scenario, allowing them to solve more complicated problems.

Math Representation

You can write the explicit formula for geometric sequences as an exponential function.

Represent $a_n = 45 \cdot 2^{n-1}$ as a function in the form $f(x) = ab^x$.

$a_n = 45 \cdot 2^{n-1}$

$f(n) = 45 \cdot 2^{n-1}$

Next, rewrite the expression $45 \cdot 2^{n-1}$.

$f(n) = 45 \cdot 2^n \cdot 2^{-1}$	product rule of powers
$f(n) = 45 \cdot 2^{-1} \cdot 2^n$	commutative property
$f(n) = 45 \cdot \frac{1}{2} \cdot 2^n$	definition of negative exponent
$f(n) = \frac{45}{2} \cdot 2^n$	multiply

So, $a_n = 45 \cdot 2^{n-1}$ written in function notation is $f(n) = \left(\frac{45}{2}\right)2^n$, or $f(n) = (22.5)2^n$.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Sequences* when they can:

- Understand that a sequence represents a relationship between term numbers (inputs) and term values (outputs).

- State the appropriate domain for a sequence.
- Distinguish between arithmetic and geometric sequences.
- Recognize that an arithmetic sequence has a common difference between terms and a geometric sequence has a common ratio between terms.
- Determine the common difference between two terms in an arithmetic sequence and the common ratio between two terms in a geometric sequence represented in tables and graphs.
- Describe the graph of an arithmetic and geometric sequence.
- Explain that a recursive formula tells you how to determine the next value of a sequence from the previous value.
- Explain that an explicit formula tells you how to determine any value given the term number.
- Distinguish between explicit and recursive formulas.
- Write recursive and explicit formulas for any sequence, including those presented as real-world scenarios.
- Rewrite explicit formulas of arithmetic sequences using the distributive property.
- Translate between explicit and recursive formulas.
- Decide when real-world problems model an arithmetic or geometric sequence.

NEW KEY TERMS

- sequence [secuencia/ sucesión]
- term of a sequence [término de una secuencia]
- infinite sequence [secuencia infinita]
- finite sequence [secuencia finita]
- arithmetic sequence [secuencia aritmética]
- common difference [diferencia común]
- geometric sequence [secuencia geométrica]
- common ratio [razón común]
- recursive formula [fórmula recursiva]
- explicit formula [fórmula explícita]
- mathematical modeling [modelado matemático]

How do the activities in *Sequences* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Students use tools—tables, graphs, and equations—to model situations as arithmetic and geometric sequences (A.1C, A.1D, A.1E). They examine the structure of these sequences to recognize key and defining characteristics (A.1F). Finally, students use the modeling process to solve a real-world sequence problem (A.1A).

NEW SYMBOL

Symbol	Description
. . .	Ellipsis, which means “and so on”

How can you use cognates to support EB students?

Cognates are provided for new key terms, when applicable. Strategically presenting information and modeling your thought process in both languages highlights the connections between cognates in each language and the linguistic knowledge and assets students already possess. Anchor such discussions within real-world contexts that are likely familiar to students wherever possible.

MODULE 1, TOPIC 2 PACING GUIDE

165-Day Pacing

1 Searching for Patterns

TOPIC 2: Sequences

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.A, 1.C, 1.E, 3.D, 3.E, 3.J, 4.B, 5.A, 5.B, 5.F

Topic Pacing: 10 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Recognizing Patterns and Sequences	Students begin by exploring various patterns in Pascal's triangle. *Sequence* and *term of a sequence* are defined. Given four geometric patterns or contexts, students write a numeric sequence to represent each problem. They are guided to represent each sequence as a table of values and conclude that all sequences are functions. Students then organize the sequences in a table, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences have a domain that includes only positive integers. *Infinite sequence* and *finite sequence* are defined and included as another characteristic for students to consider as they write sequences. **Materials Needed:** Problem-Solving Model Graphic Organizer	A.12A	1
2	Arithmetic and Geometric Sequences	Given eight numeric sequences, students generate several additional terms for each sequence and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale. The terms *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio* are then defined, examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers, identifying four different representations for each arithmetic and geometric sequence. In the first activity, they glue each arithmetic and geometric sequence to a separate graphic organizer and label them, and in the second activity, the corresponding graph is added. The remaining representations are completed in the following lessons. This lesson concludes with students writing sequences given a first term and a common difference or common ratio and identifying whether the sequences are arithmetic or geometric. **Materials Needed:** Scissors, Glue, Sequence and Graph Cards (located at the end of the lesson)	A.12A A.12C	1

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	Determining Recursive and Explicit Expressions from Contexts	Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of terms in each sequence. The term *recursive formula* is defined and used to generate term values. As the term number increases, it becomes more time consuming to generate the term value. This sets the stage for *explicit formulas* to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences. Students write recursive and explicit formulas for sequences and represent sequences as graphs. **Materials Needed:** Graphic Organizers from Lesson 2: *Arithmetic and Geometric Sequences*	A.12C A.12D	4
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				3

*Bold TEKS = Readiness Standard

MODULE 1, TOPIC 2 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: A.12A **LESSON 1** Recognizing Patterns and Sequences **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.12A, A.12C **LESSON 2** Arithmetic and Geometric Sequences **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.12C, A.12D **LESSON 3** Determining Recursive and Explicit Expressions from Contexts **GETTING STARTED** **ACTIVITY 1**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 3 continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 3** continued **ACTIVITY 4**	**LESSON 3** continued **ACTIVITY 5** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are three Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice problem sets for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether your students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Recognizing Patterns and Sequences

MATERIALS

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students begin by exploring various patterns in Pascal's triangle. *Sequence* and *term of a sequence* are defined. Given four geometric patterns or contexts, students write a numeric sequence to represent each problem. They are guided to represent each sequence as a table of values and conclude that all sequences are functions. Students then organize the sequences in a table, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. They determine that all sequences have a domain that includes only positive integers. *Infinite sequence* and *finite sequence* are defined and included as another characteristic for students to consider as they write sequences.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

(TEKS continued on next page)

ELPS

(1) Learning Strategies

The student is expected to:

(C) use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary.

(3) Speaking

The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

(J) respond orally to information presented in a wide variety of print, electronic, audio, and visual media to build and reinforce concept and language attainment.

(5) Writing

The student is expected to:

(A) learn relationships between sounds and letters of the English language to represent sounds when writing in English.

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

The student is expected to:

A.12A decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

ESSENTIAL IDEAS

- A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.
- A term of a sequence is an individual number, figure, or letter in the sequence.
- A sequence can be written as a function. The domain includes only positive integers.
- An infinite sequence is a sequence that continues forever, or never ends.
- A finite sequence is a sequence that terminates, or has an end term.

LESSON STRUCTURE AND PACING: 1 DAY

ENGAGE

Getting Started: A Pyramid of Patterns 10 minutes

BUILD OFF INTUITION

Students explore various patterns in Pascal's triangle. They first explore patterns on their own and then are guided to recognize and explain specific patterns.

DEVELOP

Activity 1.1: Patterns to Sequences to Tables 15 minutes

INVESTIGATION

Students are given the definitions of *sequence* and *term of a sequence*. Given four geometric patterns or contexts, students describe each pattern, determine the next few figures or numbers in the patterns, write a numeric sequence for each pattern, and represent each sequence using a table of values.

Activity 1.2: Looking at Sequences More Closely 15 minutes

INVESTIGATION, PEER WORK ANALYSIS

Students organize sequences in a table, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. *Infinite sequence* and *finite sequence* are defined, and examples are provided.

DEMONSTRATE

Talk the Talk: Searching for a Sequence 5–10 minutes

GENERALIZATION

Students are provided characteristics, including the newly defined terms *infinite* and *finite*, to build sequences.

Getting Started

ENGAGE

A Pyramid of Patterns

Facilitation Notes

In this activity, students identify patterns using the first 7 rows of Pascal's Triangle.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Describe the patterns in the diagonals. • Describe the pattern in terms of mathematical operations. • Does your pattern include addition or subtraction? • What is the pattern within the values added or subtracted?
Seeing structure	• Why does this number pattern create symmetry? • How can you express this doubling pattern using exponents?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Have students research other patterns in Pascal's Triangle, such as the hockey stick identity, the lazy caterer's sequence, and the Fibonacci numbers. Encourage students to look for these and other mathematical patterns in the world around them.

STAMP THE LEARNING

Summary

Pascal's Triangle is a famous geometric and numeric figure that generates many patterns.

ACTIVITY 1.1

Patterns to Sequences to Tables

DEVELOP

Facilitation Notes

In this activity, students are given the definitions of *sequence* and *term of a sequence*. Given four geometric patterns or contexts, students describe each pattern, determine the next few figures or numbers in the patterns, write a numeric sequence for each pattern, and represent each sequence using a table of values.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Arithmetic errors that may prevent them from recognizing patterns.
- Language that demonstrates a generalization of patterns.

QUESTIONS TO SUPPORT DISCOURSE FOR *POSITIVE THINKING*

Probing	• How does the sequence relate to the diagram?
Seeing structure	• Why can you represent all sequences by a table of values?

QUESTIONS TO SUPPORT DISCOURSE FOR *FAMILY TREE*

Probing	• How does this sequence relate to the family tree? • Which mathematical operation did you use to generate the terms? What is another way?
Seeing structure	• How is this pattern different than the one in the Positive Thinking problem?

QUESTIONS TO SUPPORT DISCOURSE FOR *PAOLA'S PATTERN*

Gathering	• Is this sequence increasing or decreasing? Explain.
Probing	• Generate a new sequence for the sum of the flowers up to the third, sixth, ninth, etc. rows. Explain why this new sequence makes sense mathematically.

QUESTIONS TO SUPPORT DISCOURSE FOR *GAMER GURU*

Probing	• How did you determine the first term of the sequence?
Seeing structure	• Which other sequence in this activity has a growth pattern similar to this one? Explain your thinking.

DIFFERENTIATION STRATEGIES

Access for All

Materials Needed: Poster Paper, Chart Paper, or Butcher Paper

- Create groups of three or four students. Ensure there are at least four groups. Assign each group a different problem from the activity. Have them create a poster showing the pattern and answering the questions for each situation. Leave time for students to do a gallery walk, where they visit each poster, recording the sequence in the table in Activity 2, Question 1.

Just in Time Support

- To scaffold support, demonstrate how any sequence can be converted into a table of values. Allow tables to be set up horizontally or vertically.

Term number	1	2	3	4
Term value	25	21	17	13

STAMP THE LEARNING

Summary

All numeric sequences can be represented as a function. The independent variable is the term number beginning with 1, and the dependent variable is the term of the sequence.

ACTIVITY 1.2 Looking at Sequences More Closely

Facilitation Notes

In this activity, students organize sequences in a table, state whether each sequence is increasing or decreasing, and describe the sequence using a starting value and operation. *Infinite sequence* and *finite sequence* are defined, and examples are provided.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which sequences are more challenging to describe than others? Why is that the case?
Seeing structure	• Do you think a description of a starting number and operation with a value will create a unique sequence? Explain your thinking.
Gathering	• How would you describe that similarity using mathematical terms?
Seeing structure	• What is a function?
Gathering	• With which value does the domain of a sequence start? • Which type of numbers are the domain of a sequence?
Seeing structure	• Why do all sequences have the same domain? • Are there any restrictions placed on the range of a sequence? Explain your thinking.

DIFFERENTIATION STRATEGY

Access for All

Encourage students to take time to read through each strategy, thinking about and annotating the connections, to determine whether each strategy is correct or not. Suggest students ask themselves:

- Does the reasoning make sense?
- When the reasoning makes sense, what is the justification?
- When the reasoning does not make sense, what error was made?

Ask a student to read the definitions and example following Question 5 aloud, then complete Questions 6 through 8 in groups or pairs.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the difference between an infinite sequence and a finite sequence? • How can you determine whether a sequence is infinite or finite?
Probing	• Are all sequences either infinite or finite? Why or why not? • Is it possible to visually represent zero blocks?
Seeing structure	• What is another example of an infinite sequence? A finite sequence?

DIFFERENTIATION STRATEGY

Just in Time Support

For Question 8, each group or pair can be assigned one of the sequence scenarios to determine whether the sequence is a function.

STAMP THE LEARNING

Summary

The domain of a sequence is the set of term numbers, and the range of a sequence is the set of term values. A sequence that continues on forever is called an *infinite sequence*, and a sequence that terminates is called a *finite sequence*.

DEMONSTRATE

Talk the Talk

SEARCHING FOR A SEQUENCE

Facilitation Notes

In this activity, students build sequences to fit given criteria.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How can you show that your sequence is finite? Infinite?
Probing	• Why doesn't each description describe a unique sequence? • Which type of number must you use as a multiplier to create a sequence that is decreasing by multiplication? • What is a sequence that increases by the addition of a constant? • What is a sequence that increases by the addition of non-constant values that also includes a pattern? Describe the pattern.

DIFFERENTIATION STRATEGY

Challenge Opportunity

To extend the activity, have students create their own sequences. Have the class categorize them by increasing or decreasing, type of operation used, and infinite or finite.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Sequences can be built from a list of characteristics. Characteristics may include a starting value, whether the sequence is increasing or decreasing, operations used between consecutive terms, and whether the sequence is finite or infinite.

1 Recognizing Patterns and Sequences

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Recognize and describe patterns.
- Represent patterns as sequences.
- Predict the next term in a sequence.
- Represent a sequence as a table of values.

NEW KEY TERMS

- sequence
- term of a sequence
- infinite sequence
- finite sequence

Since early elementary school, you have been recognizing and writing patterns involving shapes, colors, letters, and numbers.

How are patterns related to sequences and how can sequences be represented using a table of values?

Sample answer:

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. I can represent all numeric sequences as functions.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Getting Started

A Pyramid of Patterns

Pascal's Triangle is a famous pattern named after the French mathematician and philosopher Blaise Pascal. A portion of the pattern is shown.

								1								
							1		1							
						1		2		1						
					1		3		3		1					
				1		4		6		4		1				
			1		5		10		10		5		1			
		1		6		15		20		15		6		1		
	1		7		21		35		35		21		7		1	
1		8		28		56		70		56		28		8		1

1. List at least two patterns that you notice.

   Answers will vary based on each classroom.

2. Describe the pattern for the number of terms in each row.

   In each row, the number of terms is 1 more than the row above it.

3. Describe the pattern within each row.

   Sample answer:
   Each row is symmetrical.

4. Describe the pattern that results from determining the sum of each row.

   The sum of each row generates the pattern 1, 2, 4, 8, 16, 32, 64.

5. Determine the next two rows in Pascal's Triangle. See diagram above. Explain your reasoning.

   Each row begins and ends with 1. All other terms are the sum of the numbers to the immediate left and right of it in the row above.

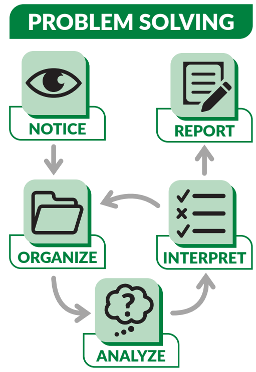
EB STUDENT TIP

For all proficiency levels

Explore the word *pattern* and its various applications in both everyday and mathematical contexts, using the Spanish cognate *patrón* to aid comprehension for emergent bilingual students. Facilitate a discussion on the diverse types of patterns, such as those found in nature, like leaf arrangements and wave formations, as well as those in human-made objects, including tablecloth designs and tile floor layouts. Transition into mathematical examples with Pascal's Triangle, inviting students to discover and discuss the various patterns it embodies. Following this exploration, guide students in creating visual word walls with labels for the various kinds of mathematical and non-mathematical patterns.

ACTIVITY **1.1**

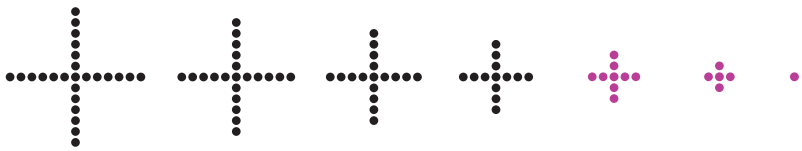
Patterns to Sequences to Tables



A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term of a sequence** is an individual number, figure, or letter in the sequence.

Four examples of sequences are given in this activity. For each sequence, describe the pattern, draw or describe the next terms, and represent each sequence numerically.

1. **Positive Thinking**



a. Analyze the number of dots. Describe the pattern.

Each figure has 4 fewer dots than the figure before it.

b. Draw the next three figures of the pattern.

See sequence.

c. Represent the number of dots in each of the seven figures as a numeric sequence.

25, 21, 17, 13, 9, 5, 1

d. Represent the number of dots in each of the first seven figures as a function using a table of values.

Term Number	1	2	3	4	5	6	7
Term Value	25	21	17	13	9	5	1

All numeric sequences can be represented as functions. The independent variable is the term number beginning with 1, and the dependent variable is the term value of the sequence.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Chunking the Activity

- Read and discuss the definitions.
- Group students to complete the activity.
- Share and summarize.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice describing patterns and drawing figures to extend patterns, assign Skills Practice Set A.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work in pairs, ask themselves the questions from the first step of the model, and share their reasoning.
- Complete the remaining steps in the graphic organizer together as a class.
- Have students work in pairs and use the problem-solving model to complete Questions 2–4.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Review the term *sequence*. Discuss the relationship between patterns and sequences. Before beginning the exercises in the activity, ask students to create their own pattern of dots, similar to the example given in Activity 1.1. Ask for volunteers to explain how their pattern can be represented as a numeric sequence.

MODULE 1 • TOPIC 2 • LESSON 1 85

Questions 2–4 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with representing patterns and situations as numeric sequences, assign Skills Practice Set B for this lesson.

2. **Family Tree**
Trung is investigating her family tree by researching each generation, or set, of parents. She learns all she can about the first four generations, which include her two parents, her grandparents, her great-grandparents, and her great-great-grandparents.

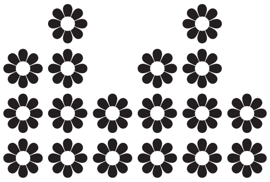
   a. Think about the number of parents. Describe the pattern.
   Each generation has 2 times the number of parents as the generation after it.

   b. Determine the number of parents in the fifth and sixth generations.
   The fifth generation has $2^5 = 32$ parents, and the sixth generation has $2^6 = 64$ parents.

   c. Represent the number of parents in each of the 6 generations as a numeric sequence. Then, represent the sequence using a table of values.

Term Number	1	2	3	4	5	6
Term Value	2	4	8	16	32	64

3. **Paola's Pattern**
Paola is decorating the top border of her bedroom walls with a flower pattern. She is applying decals with each column having a specific number of flowers.



   a. Think about the number of flowers in each column. Describe the pattern.
   The number of flowers repeat in the pattern 3, 4, 2.

EB STUDENT TIP

For all proficiency levels

Some non-mathematical terms that appear in this lesson are *generation*, *decals*, and *auditorium*. Create a vocabulary chart that shows each term followed by a picture and synonyms that describe each term in students' native language.

86 MODULE 1 • TOPIC 2 • LESSON 1

b. Determine the number of flowers in each of the next two columns.

The seventh column has 3 flowers, and the eighth column has 4 flowers.

c. Represent the number of flowers in each of the first 8 columns as a numeric sequence. Then, represent the sequence using a table of values.

Term Number	1	2	3	4	5	6	7	8
Term Value	3	4	2	3	4	2	3	4

4. **Gamer Guru**

Eduardo is trying to beat his high score on his favorite video game. He unlocks some special mini-games where he earns points for each one he completes. Before he begins playing the mini-games, Eduardo has 500 points. After completing 1 mini-game, he has a total of 550 points; after completing 2 mini-games, he has a total of 600 points; and after completing 3 mini-games, he has a total of 650 points.

a. Think about the total number of points Eduardo gains from mini-games. Describe the pattern.

Eduardo gains 50 points for each mini-game he plays.

Ask Yourself . . .
How else can you represent this information?

b. Determine Eduardo's total points after he plays the next two mini-games.

After playing 4 mini-games, Eduardo has 700 points. After playing 5 mini-games, Eduardo has 750 points.

c. Represent Eduardo's total points after completing each of the first 5 mini-games as a numeric sequence. Be sure to include the number of points he started with. Then, represent the sequence using a table of values.

Term Number	1	2	3	4	5	6
Term Value	500	550	600	650	700	750

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MODULE 1 • TOPIC 2 • LESSON 1 87

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–5.
- Check in and share.
- Read and discuss the definitions and example.
- Group students to complete Questions 6–8.
- Share and summarize.

STAMP THE LEARNING

The paragraph provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 1.2 Looking at Sequences More Closely

There are many different patterns that can generate a sequence of numbers. For example, you may have noticed that some of the sequences in the previous activity were generated by performing the same operation using a constant number. In other sequences, you may have noticed a different pattern.

The next term in a sequence is calculated by determining the pattern of the sequence and then using that pattern on the last known term of the sequence.

1. For each sequence in the previous activity, write the numeric sequence record whether the sequence increases or decreases, and describe the sequence by stating the first term and the operation(s) used to create the sequence. The first one has been completed for you.

Problem Name	Numeric Sequence	Increases or Decreases	Sequence Description
Positive Thinking	25, 21, 17, 13, 9, 5, 1	Decreases	Begin at 25. Subtract 4 from each term.
Family Tree	2, 4, 8, 16, 32, 64	Increasing	Begin at 2. Multiply each term by 2.
Paola's Pattern	3, 4, 2, 3, 4, 2, 3, 4	Increases and decreases	Begin at 3. Continue with 4 and 2. Repeat this 3, 4, 2 pattern.
Gamer Guru	500, 550, 600, 650, 700, 750	Increases	Begin at 500. Add 50 to each term.

2. Which sequences are similar? Explain your reasoning.

   Sample answer:

   The sequences Positive Thinking and Gamer Guru are similar. They increase/decrease by adding or subtracting.

88 MODULE 1 • TOPIC 2 • LESSON 1
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3. What do all sequences have in common?

   All of the sequences are functions.

4. Consider a sequence in which the first term is 64 and each term after that is calculated by dividing the previous term by 4. Elena says that this sequence ends at 1 because there are no whole numbers that come after 1. Chris disagrees and says that the sequence continues beyond 1. Who is correct? If Elena is correct, explain why. If Chris is correct, predict the next two terms of the sequence.

   Chris is correct. Even though the sequence begins with whole numbers, this does not mean that it must contain only whole numbers. After 1, the next two terms of the sequence are $1 \div 4 = \frac{1}{4}$ and $\frac{1}{4} \div 4 = \frac{1}{16}$.

5. What is the domain of a sequence? What is the range?

   The domain of a sequence is the term numbers, being all integers beginning with 1; the range of a sequence is the term values and will vary depending upon the function.

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MODULE 1 • TOPIC 2 • LESSON 1 89

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

An ellipsis is three periods, which means "and so on." An infinite sequence can be represented using an ellipsis.

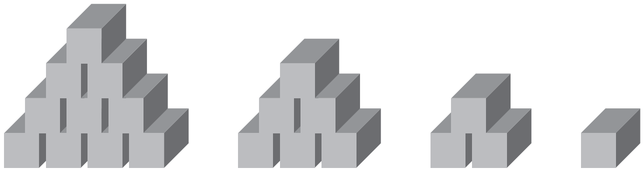
When a sequence continues on forever, it is called an **infinite sequence**. If a sequence terminates, it is called a **finite sequence**.

For example, consider an auditorium where the seats are arranged according to a specific pattern. There are 22 seats in the first row, 26 seats in the second row, 30 seats in the third row, and so on. Numerically, the sequence is 22, 26, 30, ... , which continues infinitely. However, in the context of the problem, it does not make sense for the number of seats in each row to increase infinitely. Eventually, the auditorium would run out of space! Suppose that this auditorium can hold a total of 10 rows of seats. The correct sequence for this problem situation is:

22, 26, 30, 34, 38, 42, 46, 50, 54, 58.

Therefore, because of the problem situation, the sequence is a finite sequence.

6. Does the pattern shown represent an infinite or finite sequence? Explain your reasoning.



This is a finite sequence. Each stage results by taking away the bottom row of the pyramid until the pyramid no longer exists.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

The terms *infinite* and *finite* are cognates in many languages and may be easily identified by students. Discuss how the word *finite* is related to the words final or finish. Also discuss how *infinite* is an antonym of *finite*. Model the use and meanings of the terms with an example, such as, "There are a *finite* number of positive factors for the number 100, but there are an *infinite* number of multiples of the number 100." Ask students to create their own sentences showing the contrast between a *finite* amount and an *infinite* amount of something.

7. For each sequence in the previous activity, write the domain and range using set notation.

   a. Positive Thinking
      Domain: {1, 2, 3, 4, 5, 6, 7}; Range: {25, 21, 17, 13, 9, 5, 1}

   b. Family Tree (6 generations):
      Domain: {1, 2, 3, 4, 5, 6}; Range: {2, 4, 8, 16, 32, 64}

   c. Paola's Pattern (8 columns)
      Domain: {1, 2, 3, 4, 5, 6, 7, 8}; Range: {2, 3, 4}

   d. Gamer Guru (completes 5 mini games)
      Domain: {1, 2, 3, 4, 5, 6}; Range: {500, 550, 600, 650, 700, 750}

8. For each sequence in the previous activity, determine whether the sequence is a function.

   a. Positive Thinking
      Each input has only one output, so this is a function.

   b. Family Tree
      Each input has only one output, so this is a function.

   c. Paola's Pattern
      Each input has only one output, so this is a function.

   d. Gamer Guru
      Each input has only one output, so this is a function.

MODULE 1 • TOPIC 2 • LESSON 1 91

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Searching for a Sequence

In this lesson, you have seen that many different patterns can generate a sequence of numbers.

1. Explain why the definition of a function applies to all sequences.

   In a function, each independent value has one corresponding dependent value. In a sequence, each term number (independent value) has one term (dependent value) that corresponds to it.

2. Create a sequence to fit the given criteria. Describe your sequence using figures, words, or numbers. Provide the first four terms of the sequence. Explain how you know that it is a sequence.

   a. Create a sequence that begins with a positive integer, is decreasing by multiplication, and is finite.

   Sample answer:

   $27, 9, 3, 1, \frac{1}{3}$

   b. Create a sequence that begins with a negative rational number, is increasing by addition, and is infinite.

   Sample answer:

   $-2.5, -1.5, -0.5, 0.5, \ldots$

92 MODULE 1 • TOPIC 2 • LESSON 1

Lesson 1 Assignment

Write

Explain why all sequences can be described as functions.

A sequence is made up of term numbers and term values. Each term number (input) has exactly one term value (output).

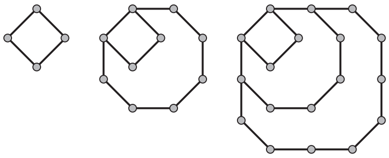
Remember

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. All numeric sequences can be represented as functions.

Practice

Consider the three sequences given. For each sequence, describe the pattern. Then, represent the sequence as a numeric sequence and as a table of values, including the first six terms.

1. **Stellar Segments**



The second figure has 7 more segments than the first, the third figure has 9 more segments than the second, and the fourth figure has 11 more segments than the third.

4, 11, 20, 31, 44, 59

Term Number	1	2	3	4	5	6
Term Value	4	11	20	31	44	59

2. **Welcome Home**

A local construction company specializes in building and selling homes. When the company first started, they sold 1 home the first month, 3 homes the second month, 9 homes the third month, and 27 homes the fourth month.

Each month the company sold 3 times the number of homes as the month before.

1, 3, 9, 27, 81, 243

Term Number	1	2	3	4	5	6
Term Value	1	3	9	27	81	243

MODULE 1 • TOPIC 2 • LESSON 1 ASSIGNMENT 93

Lesson 1 Assignment

3. **Samantha's Streaming**
Samantha is a yoga instructor who regularly streams exercise videos on a website for her clients. One week after launching the website, she had posted a total of 6 videos. At the end of week 2, she had a total of 10 videos. At the end of week 3, she had a total of 14 videos. At the end of week 4, she had a total of 18 videos.

Each week Samantha's website has 4 more videos than the week before.

6, 10, 14, 18, 22, 26

Term Number	1	2	3	4	5	6
Term Value	6	10	14	18	22	26

Prepare

Write the next three terms in each sequence. Explain how you determined each term and whether it is a function.

1. −2, 4, −8, 16, ...
   −32, 64, −128; multiply the previous term by −2; function

2. 60, 53, 46, 39, 32, ...
   25, 18, 11; subtract 7 from the previous term; function

3. 1, 5, 17, 53, 161, 485, ...
   1457, 4373, 13,121; multiply the previous term by 3, then add 2; function

4. 4, 10, 16, 22, ...
   28, 34, 40; add 6 to the previous term; function

94 MODULE 1 • TOPIC 2 • LESSON 1 ASSIGNMENT

2 Arithmetic and Geometric Sequences

MATERIALS

Scissors

Glue

Sequence and Graph Cards (located at the end of the lesson)

LESSON OVERVIEW

Given eight numeric sequences, students generate several additional terms for each sequence and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale. The terms *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio* are then defined, examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers, identifying four different representations for each arithmetic and geometric sequence. In the first activity, they glue each arithmetic and geometric sequence to a separate graphic organizer and label them, and in the second activity, the corresponding graph is added. The remaining representations are completed in the following lesson. This lesson concludes with students writing sequences given a first term and a common difference or common ratio and identifying whether the sequences are arithmetic or geometric.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1E create and use representations to organize, record, and communicate mathematical ideas.

(TEKS continued on next page)

ELPS

(3) Speaking
The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

(E) share information in cooperative learning interactions.

(4) Reading
The student is expected to:

(B) recognize directionality of English reading such as left to right and top to bottom.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

The student is expected to:

A.12A decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

A.12C identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

ESSENTIAL IDEAS

- An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a positive or negative constant. This constant is called the *common difference* and is represented by the variable *d*.
- A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant. This constant is called the *common ratio* and is represented by the variable *r*.
- The graph of a sequence is a set of discrete points.
- The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing.
- The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.

LESSON STRUCTURE AND PACING: 1 DAY

ENGAGE

Getting Started: What Comes Next, and How Do You Know? 10–15 minutes

ESTABLISH A SITUATION

Students generate several additional terms for 8 different numeric sequences and describe the rule they used for each sequence. They sort the sequences into groups based upon common characteristics of their choosing and explain their rationale.

DEVELOP

Activity 2.1: Defining Arithmetic and Geometric Sequences 20 minutes

WORKED EXAMPLE, PEER WORK ANALYSIS, CLASSIFICATION

Students are provided the definitions of *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio*. Examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers, identifying four different representations for each sequence. In this activity, students glue each arithmetic and geometric sequence to a separate graphic organizer.

Activity 2.2: Matching Graphs and Sequences 10–15 minutes

CLASSIFICATION

Students match graphs to their corresponding numeric sequence and then add the graphs to each graphic organizer.

DEMONSTRATE

Talk the Talk: Name That Sequence! 5 minutes

EXIT TICKET PROCEDURES

Students are given a first term and a common difference or common ratio, and they must identify the unique sequence it describes and state whether the sequence is arithmetic or geometric.

Getting Started

ENGAGE

What Comes Next, and How Do You Know?

Facilitation Notes

In this activity, students cut out sequence cards, generate additional terms for 8 different numeric sequences, and then describe the rule they used for each sequence. A sort activity is used to categorize the sequences based upon common characteristics.

Have students work with a partner or in a group to complete Questions 1 through 3. Make sure that students understand that they are just describing a pattern; they do not have to write a rule. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Support students by reducing the number of sequences they are working with. Ensure to maintain variety so that students see patterns that represent arithmetic sequences, geometric sequences, or neither.

AS STUDENTS WORK, LOOK FOR

Strategies and phrases they use to determine the next terms of the sequences.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What strategies did you use to determine the patterns in the sequences with fractions? • How can you use mathematical operations to describe the pattern in Sequence *C*? • How can you identify the pattern in Sequence *G* using division? Multiplication? • How can you use addition to describe the pattern in Sequence *B*? • What is another way to sort the sequences?
Seeing structure	• Which sequences involve addition by numbers in a pattern each time? Explain your thinking.

STAMP THE LEARNING

Summary

Different operations can be used to generate sequences.

ACTIVITY **2.1** Defining Arithmetic and Geometric Sequences

DEVELOP

Facilitation Notes

In this activity, students are provided the definitions of *arithmetic sequence*, *common difference*, *geometric sequence*, and *common ratio*. Examples are provided, and students respond to clarifying questions. They then categorize the sequences from the beginning of the lesson as arithmetic, geometric, or neither and identify the common difference or common ratio where appropriate. Students begin to create graphic organizers, identifying four different representations for each sequence. In this activity, students glue each arithmetic and geometric sequence to a separate graphic organizer.

Ask a student to read the introduction and definitions aloud. Review the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• When you subtract the same number each time to determine the next value in a sequence, is the sequence arithmetic? Explain. • Why does the term common difference make sense?
Probing	• Explain how to adjust your thinking to identify the common difference when you used subtraction to get the next term. • How would you determine the common difference when it is not apparent, such as in the sequence 11, 9.31, 7.62, ...?

DIFFERENTIATION STRATEGIES

Access for All

- Encourage students to take time to read through the Worked Example and annotate key ideas or steps in the process. As they think about the connections, have students ask themselves:
  - Why is this method correct?
  - Have I used this method before?
- Substitute 2, 3, and 4 in place of *n* in the function form as a class. Relate both notations to the sequence.

Have students work individually or with a partner to complete Question 1 and discuss as a class. Then, have students work with a partner or in a group to complete Question 2. Share responses as a class

COMMON MISCONCEPTION

Students may confuse the term *arithmetic* (noun) with the term *arithmetic* (adjective). Emphasize how to pronounce *arithmetic* when it is an adjective rather than a noun.

Ask a student read the definitions following Question 2 aloud. Review the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the difference between an arithmetic sequence and a geometric sequence? • When you divide by the same number each time to determine the next value in a sequence, is the sequence arithmetic? Explain.
Probing	• Show how to use ratios to determine the common ratio is 2. • What is the relationship between the procedures to calculate the common difference and common ratio? • How do you apply a common ratio to generate the term values in a sequence?

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

COMMON MISCONCEPTION

Students already have an understanding of the terms *arithmetic* and *geometry*. Address how previous use of these terms is the same and different as how they are used with sequences.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How can you tell by the common ratio whether the geometric sequence will increase or decrease?
Probing	• Why do the values alternate between positive and negative numbers? • How would the sequence change when the common ratio is -1?

Have students work with a partner or in a group to complete Questions 6 through 11. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain to Jaylen how to adjust his thinking to identify the common ratio when he uses division to get the next term. • In which order did you write term values in a ratio to get the common ratio?
Reflecting and justifying	• How can you check that a common ratio is correct?
Seeing structure	• Create a similar problem where you can use the same thinking to develop two correct sequences.
Probing	• What is the common difference when it is an arithmetic sequence? • What is the common ratio when it is a geometric sequence?

DIFFERENTIATION STRATEGY

Just in Time Support

While students may intuitively see the pattern as dividing by 3 and agree with Jaylen, ensure they relate the common ratio to a multiplier. Suggest they explore what would happen to a sequence when multiplying by a common ratio of 3 versus what happens when they multiply by $\frac{1}{3}$.

Summary

STAMP THE LEARNING

An arithmetic sequence is a sequence of numbers in which a positive or negative constant, called the *constant difference*, is added to each term to produce the next term. A geometric sequence is a sequence of numbers in which you multiply each term by a constant, called the *common ratio*, to determine the next term.

ACTIVITY 2.2

Matching Graphs and Sequences

Facilitation Notes

In this activity, students cut out and match several graphs to the appropriate numeric sequence and then attach the graphs to each graphic organizer.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why are all the graphs discrete? • Why don't any of the graphs have a *y*-intercept?
Probing	• Did you use the point coordinates or scale to match any graph to its sequence? When so, explain your thinking.
Seeing structure	• What relationship do you notice between arithmetic sequences and the shape of their graphs? Why does this relationship make sense? • Why do you think the graph of a geometric sequence is a curve rather than a line? • What connection did you notice between whether a graph is increasing or decreasing and its matching sequence?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Suggest students sort the graphs into two piles: ones that form lines and ones that do not form lines. Have them connect each pile to a type of sequence. They can then focus on whether the graph is increasing, decreasing, or both increasing and decreasing to connect the graph to its sequence.

Challenge Opportunity

- Challenge students to use the graphs to create a table and write an equation for the sequence.

STAMP THE LEARNING

Summary

All sequences are functions. The graph of a sequence is a set of discrete points. The points of an arithmetic sequence lie on a line. When the common difference is a positive, the graph is increasing, and when the common difference is a negative, the graph is decreasing. The points of a geometric sequence do not lie on a line. When the common ratio is greater than 1, the graph is increasing; when the common ratio is between 0 and 1, the graph is decreasing; and when the common ratio is less than 0, the graph alternates between increasing and decreasing between consecutive points.

Talk the Talk

NAME THAT SEQUENCE!

Facilitation Notes

In this activity, students are given a first term and a common difference or common ratio. Using those criteria, they write the first five terms of a unique sequence and state whether the sequence is arithmetic or geometric.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Challenge Opportunity

Have students design their own problems.

- Ask students to write a first term and either common difference or common ratio. Give the information to their partner and ask them to generate the first few terms in the sequence.
- Ask students to create a sequence using their own rule, then ask their partner to identify the rule.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you apply a common difference to create a sequence?
Seeing structure	• Is there more than one possible sequence for this criteria? Explain your thinking.
Gathering	• How do you know this sequence is geometric?
Probing	• How do you apply a common ratio to create a sequence?

Have students read and answer the Essential Question on the lesson opener page.

Summary

A unique sequence can be described by a first term and common difference or common ratio.

STAMP THE LEARNING

2 Arithmetic and Geometric Sequences

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Determine the next term in a sequence.
- Recognize arithmetic sequences and geometric sequences.
- Determine the common difference or common ratio for a sequence.
- Graph arithmetic and geometric sequences.
- Recognize graphical behavior of sequences.
- Sort sequences that are represented graphically.
- Generate terms of a sequence when the sequence is given in function form using the recursive process.

NEW KEY TERMS

- arithmetic sequence
- common difference
- geometric sequence
- common ratio

You have represented patterns as sequences of numbers—a relationship between term numbers and term values.

What patterns appear when sequences are represented as graphs?

Sample answer:

All sequences appear discrete on graphs; however, arithmetic sequences increase or decrease by their common difference, while geometric sequences increase or decrease by their common ratios.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

Getting Started

What Comes Next, and How Do You Know?

Review the definition of **describe** in the Academic Glossary.

Cut out Sequences *A* through *H* located at the end of the lesson.

1. Determine the unknown terms of each sequence. Then, describe the pattern under each sequence.
   See answers on sequence cards in Lesson 2.

2. Sort the sequences into groups based on common characteristics. In the space provided, record the following information for each of your groups.
   - List the letters of the sequences in each group.
   - Provide a rationale as to why you created each group.
     Sample answer:

     A, *E*, and *G*: Sequences that change by multiplying or dividing by the same number each time

     B, *D*, and *H*: Sequences that change by adding or subtracting by the same number each time

     C and *F*: Sequences that change in some other way

3. What mathematical operation(s) did you perform in order to determine the next terms of each sequence?
   Some sequences required addition, subtraction, multiplication, or division by the same number each time. Some sequences involved operations such as operations with consecutive numbers or switching signs each time.

EB STUDENT TIP

For all proficiency levels

Review the term *rationale* and create a list of synonyms for the term. Ask students for examples of when *rationale* is used in different contexts.

ACTIVITY **2.1**

Defining Arithmetic and Geometric Sequences

For some sequences, you can describe the pattern as adding a constant to each term to determine the next term. For other sequences, you can describe the pattern as multiplying each term by a constant to determine the next term. Still other sequences cannot be described either way.

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which a constant is added to each term to produce the next term. This constant is called the **common difference**. The common difference is typically represented by the variable d.

The common difference of a sequence is positive when the same positive number is added to each term to produce the next term. The common difference of a sequence is negative when the same negative number is added to each term to produce the next term.

Remember:
When you add a negative number, it is the same as subtracting a positive number.

WORKED EXAMPLE

Consider the sequence generated using $a_n = a_{n-1} + (-2)$, where $a_1 = 11$ and n is a whole number greater than 1.

a_1 represents the first term of the sequence and a_n represents the n^{th} term of the sequence.

Since I know the first term of the sequence, to determine the second term a_2, I add -2 to 11.

$a_2 = a_{2-1} + -2 = a_1 + -2 = 11 + -2 = 9$

I can determine the 3rd and 4th term of the sequence by continuing the pattern.

$a_3 = a_{3-1} + -2 = a_2 + -2 = 9 + -2 = 7$

$a_4 = a_{4-1} + -2 = a_3 + -2 = 7 + -2 = 5$

The sequence is 11, 9, 7, 5, ...

The pattern is to add the same negative number, -2, to each term to determine the next term.

Sequence: $11 \xrightarrow{\text{add } -2} 9 \xrightarrow{\text{add } -2} 7 \xrightarrow{\text{add } -2} 5$, ...

This sequence is arithmetic and the common difference d is -2.

a_{n-1} is the term before a_n. So, when $n = 2$, a_{n-1} is a_1.

Chunking the Activity

- Read and discuss the introduction, definitions, and Worked Example.
- Group students to complete Question 1.
- Check in and share.
- Group students to complete Question 2.
- Check in and share.
- Read and discuss the definitions and Worked Example.
- Group students to complete Questions 3–5.
- Check in and share.
- Group students to complete Questions 6–11.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The Worked Examples provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For all proficiency levels

Ask students what is meant by *consecutive terms*. Discuss how the word *consecutive* is used in mathematical and non-mathematical situations.

Beginning: Use everyday examples with visual representations, like days of the week or numbers on a number line. Then, relate this to familiar mathematical sequences, like counting numbers, where students can point out consecutive (next to each other) and non-consecutive (with gaps) terms.

Intermediate: Offer the sentence frame: "In mathematics, the terms ____________ and ____________ are consecutive because ____________."

Advanced/Advanced High: Have students use their own words to write out definitions for the terms consecutive and non-consecutive, providing clear examples in both everyday real-world contexts as well as within the context of mathematics.

Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining the common difference and extending arithmetic sequences, assign Skills Practice Set A.

Ask Yourself . . .
How does the function notation relate to the notation in the Worked Example?

You can also write the sequence from the Worked Example in function form. Given that $f(1) = 11$ and n is a whole number greater than 1, the function $f(n) = f(n-1) + -2$ represents the sequence.

1. Suppose a sequence has the same starting number as the sequence in the Worked Example, but is generated by $a_n = a_{n-1} + 4$.

   a. Write the first four terms of the sequence.

   $a_1 = 11$
   $a_2 = a_1 + 4 = 11 + 4 = 15$
   $a_3 = a_2 + 4 = 15 + 4 = 19$
   $a_4 = a_3 + 4 = 19 + 4 = 23$

   b. How does the pattern change?

   The sequence would increase by 4 instead of decreasing by 2.

   c. Is the sequence still arithmetic? Why or why not?

   Yes. The sequence is still arithmetic because the difference between each consecutive term is constant. The common difference is 4.

2. Analyze the sequences you cut out in the Getting Started.

   a. List the sequences that are arithmetic.

   Sequences *B*, *D*, and *H*

   b. Write the common difference of each arithmetic sequence you identified.

   Sequence *B*: $d = -\frac{9}{4}$

   Sequence *D*: $d = 4$

   Sequence *H*: $d = -20.5$

98 MODULE 1 • TOPIC 2 • LESSON 2

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. In other words, it is a sequence of numbers in which you multiply each term by a constant to determine the next term. This integer or fraction constant is called the *common ratio*. The **common ratio** is represented by the variable *r*.

WORKED EXAMPLE

Consider the sequence generated by the function $g_n = g_{n-1} \cdot 2$ where $g_1 = 1$ and *n* is a whole number greater than 1.

Since I know the first term of the sequence, to determine the second term g_2, I multiply 1 by 2.

$g_2 = g_1 \cdot 2 = 1 \cdot 2 = 1$
$g_3 = g_2 \cdot 2 = 2 \cdot 2 = 4$
$g_4 = g_3 \cdot 2 = 4 \cdot 2 = 8$

The sequence is 1, 2, 4, 8, ...

The pattern is to multiply each term by the same number, 2, to determine the next term.

$$\text{Sequence: } \underline{1} \xrightarrow{\text{multiply by 2}} \underline{2} \xrightarrow{\text{multiply by 2}} \underline{4} \xrightarrow{\text{multiply by 2}} \underline{8}, \ldots$$

This sequence is geometric and the common ratio *r* is 2.

You can also write the sequence from the Worked Example in function form. Given that $f(1) = 1$ and *n* is a whole number greater than 1, the function $f(n) = 2f(n-1)$ describes the sequence.

3. Suppose a sequence has the same starting number as the sequence in the Worked Example, but is generated by $a_n = a_{n-1} \cdot 3$.

   a. Write the first four terms of the sequence.

   $a_1 = 1$
   $a_2 = a_1 \cdot 3 = 1 \cdot 3 = 3$
   $a_3 = a_2 \cdot 3 = 3 \cdot 3 = 9$
   $a_4 = a_3 \cdot 3 = 9 \cdot 3 = 27$

   b. How does the pattern change?

   The sequence would still increase, but the terms would be different. The sequence would increase more rapidly.

Ask Yourself . . .
How does the function notation relate to the notation in the Worked Example?

STAMP THE LEARNING

The definitions and Worked Example an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Question 3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining the common ratio and extending geometric sequences, assign Skills Practice Set B.

MODULE 1 • TOPIC 2 • LESSON 2 99

c. Is the sequence still geometric? Explain your reasoning.

Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

4. Suppose a sequence has the same starting number as the sequence in the Worked Example, but its common ratio is $\frac{1}{3}$.

a. How would the pattern change?

The sequence would decrease.

b. Is the sequence still geometric? Why or why not?

Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

c. Write the first four terms of the new sequence.

$a_1 = 1$

$a_2 = \frac{1}{3}$

$a_3 = \frac{1}{9}$

$a_4 = \frac{1}{27}$

5. Suppose a sequence has the same starting number as the sequence in the Worked Example, but it is generated by $g_n = 1 \cdot (-2)^{n-1}$.

a. How would the pattern change?

The sequence would decrease and increase and contain alternating positive and negative integers.

b. Is the sequence still geometric? Explain your reasoning.

Yes. The sequence is still geometric because the ratio between any two consecutive terms is constant.

c. Write the first four terms of the new sequence.

$g_1 = 1$

$g_2 = -2$

$g_3 = 4$

$g_4 = -8$

100 MODULE 1 • TOPIC 2 • LESSON 2

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6. Consider the sequence shown.

$$270, 90, 30, 10, \ldots$$

Jorge says that he can determine each term of this sequence by multiplying each term by $\frac{1}{3}$, so the common ratio is $\frac{1}{3}$. Jaylen says that he can determine each term of this sequence by dividing each term by 3, so the common ratio is 3. Who is correct? Explain your reasoning.

Jorge is correct. The next term in the sequence can be determined by multiplying the previous term by $\frac{1}{3}$. Jaylen is correct in that he can determine the sequence by dividing each term by 3, but the common ratio represents the number by which each term is multiplied. Each term in this sequence is not multiplied by 3, it is multiplied by $\frac{1}{3}$.

7. Consider the sequences you cut out in the Getting Started. List the sequences that are geometric. Then, write the common ratio on each Sequence Card.

Sequence *A*: $r = 3$

Sequence *E*: $r = \frac{1}{2}$

Sequence *G*: $r = -\frac{1}{4}$

8. Consider the sequences that are neither arithmetic nor geometric. List these sequences. Explain why these sequences are neither arithmetic nor geometric.

Sequences *C* and *F* are neither arithmetic nor geometric because there is no common difference or common ratio for either of these sequences.

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MODULE 1 • TOPIC 2 • LESSON 2 101

Question 9 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining whether sequences are arithmetic or geometric, assign Skills Practice Set C.

Optimizing Learning

This activity optimizes transfer and generalization.

9. Consider the first two terms of the sequence 3, 6, ...

Mariana says, "This is how I wrote the sequence for the given terms."

3, 6, 9, 12, ...

Ashley says, "This is the sequence I wrote."

3, 6, 12, 24, ...

Who is correct? Explain your reasoning.

Both are correct. From the first two terms, Mariana or Ashley did not know whether the sequence was arithmetic or geometric. Mariana assumed it was arithmetic with a common difference of 3. Ashley assumed it was geometric with a common ratio of 2.

10. Consider the sequence 2, 2, 2, 2, 2 ... Identify the type of sequence it is and describe the pattern.

Sample answers:

This sequence could be arithmetic in that I could add 0 to each term.

This sequence could be geometric in that I could multiply each term by 1.

This sequence could be neither arithmetic nor geometric in that the term 2 could just be repeating.

11. Begin to complete the graphic organizers located at the end of the lesson to identify arithmetic and geometric sequences. Glue each arithmetic sequence and each geometric sequence to a separate graphic organizer according to its type. Discard all other sequences.

See answers on sequence cards in Lesson 2.

102 MODULE 1 • TOPIC 2 • LESSON 2

ACTIVITY 2.2

Matching Graphs and Sequences

As you have already discovered when studying functions, graphs can help you see trends of a sequence—and at times can help you predict the next term in a sequence.

1. The graphs representing the arithmetic and geometric sequences from the previous activity are located at the end of this lesson. Cut out these graphs. Match each graph to its appropriate sequence and glue it into the Graph section of its graphic organizer.

   Sequence *A*, Graph 2

   Sequence *B*, Graph 1

   Sequence *D*, Graph 3

   Sequence *E*, Graph 5

   Sequence *G*, Graph 6

   Sequence *H*, Graph 4

2. What strategies did you use to match the graphs to their corresponding sequences?

   Answers will vary based on each classroom.

3. How can you use the graphs to verify that all sequences are functions?

   Sample answer:

   The graphs all pass the vertical line test.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Group students to complete the activity.
- Share and summarize.

MODULE 1 • TOPIC 2 • LESSON 2 103

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Name That Sequence!

Write the first five terms of each sequence described and identify the sequence as arithmetic or geometric.

1. The first term of the sequence is 0 and the common difference is -6.

   $0, -6, -12, -18, -24$; arithmetic

2. The first term of the sequence is -3 and the common ratio is $-\frac{1}{4}$.

   $-3, \frac{3}{4}, -\frac{3}{16}, \frac{3}{64}, -\frac{3}{256}$; geometric

104 MODULE 1 • TOPIC 2 • LESSON 2

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Sequence Cards

A $-2, -6, -18, -54,$ $\underline{-162}, \underline{-486}, \ldots$ Multiply by 3. Geometric $r = 3$	**B** $4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}, \underline{-5}, \underline{-\frac{29}{4}}, \ldots$ Subtract $-\frac{9}{4}$. Arithmetic $d = -\frac{9}{4}$
C $1, -2, 3, -4, \underline{5}, \underline{-6}, \ldots$ Consecutive numbers, every other number negative Neither	**D** $-20, -16, -12, -8, -4,$ $\underline{0}, \underline{4}, \ldots$ Add 4. Arithmetic $d = 4$
E $-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8},$ $\underline{-\frac{5}{16}}, \underline{-\frac{5}{32}}, \ldots$ Multiply by $\frac{1}{2}$. Geometric $r = \frac{1}{2}$	**F** $86, 85, 83, 80, 76, \underline{71}, \underline{65}, \ldots$ Subtract 1, then 2, then 3, ... Neither
G $-16, 4, -1, \frac{1}{4}, \underline{-\frac{1}{16}}, \underline{-\frac{1}{64}}, \ldots$ Divide by -4. Geometric $r = -\frac{1}{4}$	**H** $1473.2, 1452.7, 1432.2, 1411.7,$ $\underline{1391.2}, \underline{1370.7}, \ldots$ Subtract 20.5. Arithmetic $d = -20.5$

MODULE 1 • TOPIC 2 • LESSON 2 105

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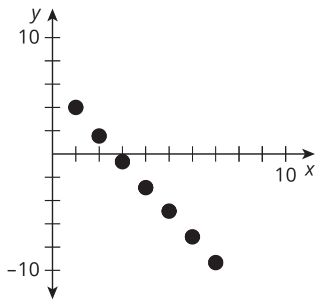
So you can cut out the Sequence Cards on the other side

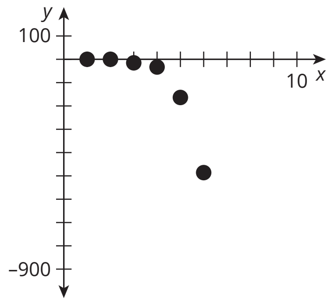
106 MODULE 1 • TOPIC 2 • LESSON 2

Graph Cards

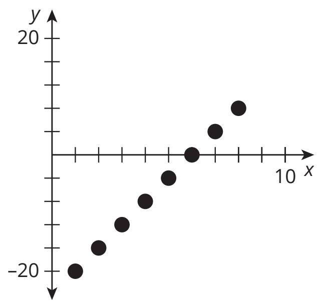
Graph 1
Sequence *B*



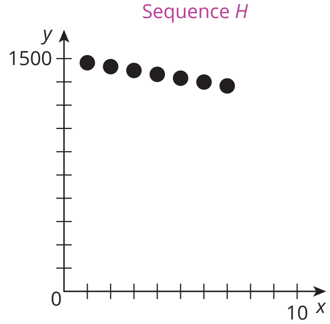
Graph 2
Sequence *A*



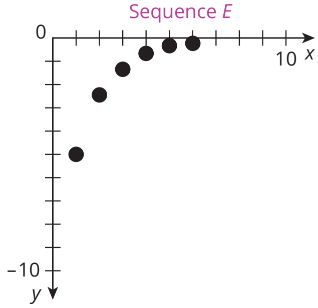
Graph 3
Sequence *D*



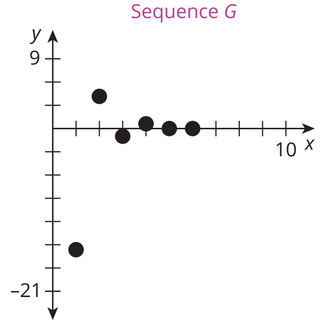
Graph 4
Sequence *H*



Graph 5
Sequence *E*



Graph 6
Sequence *G*



MODULE 1 • TOPIC 2 • LESSON 2 107

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So you can cut out the Graph Cards on the other side

108 MODULE 1 • TOPIC 2 • LESSON 2

Sequence

B

$4, \frac{7}{4}, -\frac{1}{2}, -\frac{11}{4}, -5 - \frac{29}{4}, \ldots$

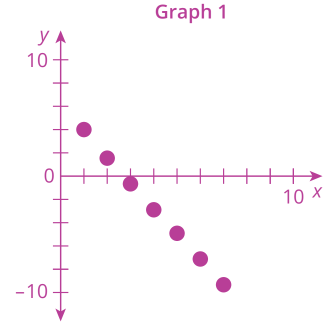
Subtract $-\frac{9}{4}$.

Arithmetic

$d = -\frac{9}{4}$

Graph

Graph 1



Arithmetic Sequence

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = a_{n-1} - \frac{9}{4}$

Recursive Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = 4 - \frac{9}{4}(n - 1)$

$a_n = -\frac{9}{4}n + 6.25$

Explicit Formula

MODULE 1 • TOPIC 2 • LESSON 2 109

Sequence

D

$-20, -16, -12, -8, -4, 0, 4, \ldots$

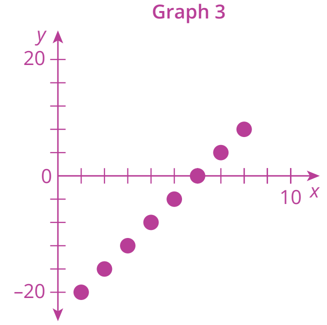
Add 4.

Arithmetic

$d = 4$

Graph

Graph 3



Arithmetic Sequence

Recursive Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = a_{n-1} + 4$

Explicit Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = -20 + 4(n - 1)$

$a_n = 4n - 24$

110 MODULE 1 • TOPIC 2 • LESSON 2

Sequence

H

1473.2, 1452.7, 1432.2, 1411.7, 1391.2, 1370.7, 1350.2, …

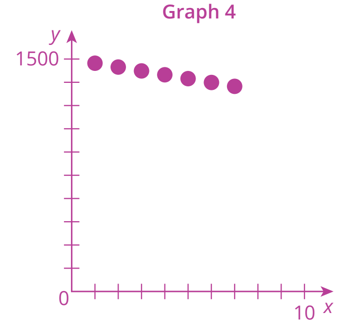
Subtract 20.5.

Arithmetic

$d = -20.5$

Graph

Graph 4



Arithmetic Sequence

Recursive Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = a_{n-1} - 20.5$

Explicit Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = 1473.2 - 20.5(n - 1)$

$a_n = -20.5n + 1493.7$

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MODULE 1 • TOPIC 2 • LESSON 2 111

Sequence

A

−2, −6, −18, −54, −162, −486, ...

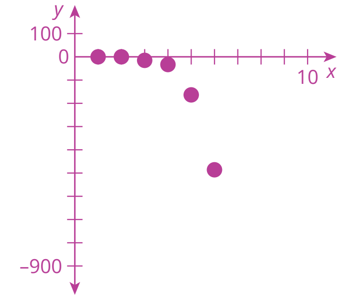
Multiply by 3.

Geometric

$r = 3$

Graph

Graph 2



Geometric Sequence

Student add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = a_{n-1} \cdot 3$

Recursive Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = -2 \cdot 3^{n-1}$

Explicit Formula

112 MODULE 1 • TOPIC 2 • LESSON 2

Sequence

E

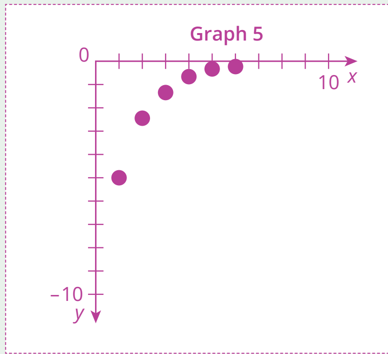
$-5, -\frac{5}{2}, -\frac{5}{4}, -\frac{5}{8}, -\frac{5}{16}, -\frac{5}{32}, \ldots$

Multiply by $\frac{1}{2}$.

Geometric

$r = \frac{1}{2}$

Graph



Geometric Sequence

Student add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = a_{n-1} \cdot \frac{1}{2}$

Recursive Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = (-5) \cdot \left(\frac{1}{2}\right)^{n-1}$

Explicit Formula

MODULE 1 • TOPIC 2 • LESSON 2 113

Sequence

G

$-16, 4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}, \ldots$

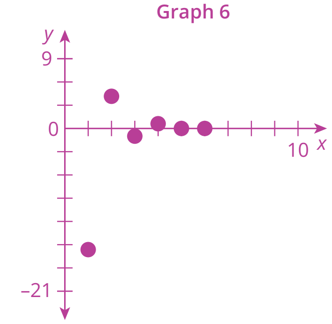
Multiply by $-\frac{1}{4}$.

Geometric

$r = -\frac{1}{4}$

Graph

Graph 6



Geometric Sequence

Student add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = a_{n-1} \cdot -\frac{1}{4}$

Recursive Formula

Students add this answer in Lesson 3 *Determining Recursive and Explicit Expressions from Contexts.*

$a_n = -16 \cdot \left(-\frac{1}{4}\right)^{n-1}$

Explicit Formula

114 MODULE 1 • TOPIC 2 • LESSON 2

Lesson 2 Assignment

Write

Complete each sentence.

1. A sequence which terminates is called a(n) ______________ .
2. A(n) ______________ is an individual number, figure, or letter in a sequence.
3. A(n) ______________ is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.
4. A sequence which continues forever is called a(n) ______________ .

Remember

An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a constant.

A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant.

Practice

Consider the first two terms of the sequence 28, 14, ...

1. Determine whether the sequence is arithmetic or geometric. Explain your reasoning.

   Sample answer:

   This sequence could be either arithmetic or geometric.

   When I add −14 to 28, the result is 14. This would indicate that the sequence is arithmetic.

   When I multiply 28 by $\frac{1}{2}$, the result is 14. This would indicate that the sequence is geometric.

2. Suppose the sequence 28, 14, . . . is arithmetic.

   a. Determine the common difference.
   −14

   b. List the next 3 terms in the sequence. Explain your reasoning.
   0, −14, −28

   c. Determine whether the sequence is finite or infinite. Explain your reasoning.

   The sequence is infinite.

   Sample explanation: The sequence will continue forever as the value of each new term approaches negative infinity.

Write

1. finite sequence
2. term
3. sequence
4. finite sequence

Lesson 2 Assignment

3. Suppose the sequence 28, 14, ... is geometric.

   a. Determine the common ratio.

   $\frac{1}{2}$

   b. List the next 3 terms in the sequence. Explain your reasoning.

   $7, \frac{7}{2}, \frac{7}{4}$

   c. Determine whether the sequence is finite or infinite. Explain your reasoning.

   The sequence is infinite.

   Sample explanation: The sequence will continue forever as the value of each new term approaches zero.

4. Using the first two terms 28 and 14, write the next three terms of a sequence that is neither arithmetic nor geometric.

   Sample answer:

   28, 14, 28

5. A sequence is generated by the function $a_n = a_{(n-1)} + 5$ where $a_1 = 4\frac{1}{2}$ and n is a whole number greater than 1.

   a. Determine the first four terms of the sequence.

   $a_1 = 4\frac{1}{2}$ $a_2 = 4\frac{1}{2} + 5 = 9\frac{1}{2}$ $a_3 = 9\frac{1}{2} + 5 = 14\frac{1}{2}$

   $a_4 = 14\frac{1}{2} + 5 = 19\frac{1}{2}$

   b. Identify the sequence type and the common difference or common ratio.

   Arithmetic sequence

   Common difference: $d = 5$

116 MODULE 1 • TOPIC 2 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

6. A sequence is generated by the function $a_n = a_{n-1} \cdot \frac{1}{4}$ where $a_1 = 12$ and n is a whole number greater than 1.

   a. Determine the first four terms of the sequence.

   $a_1 = 12$ $\quad a_2 = 12 \cdot \frac{1}{4} = 3$ $\quad a_3 = 3 \cdot \frac{1}{4} = \frac{3}{4}$

   $a_4 = \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16}$

   b. Identify the sequence type and the common difference or common ratio.

   Geometric sequence

   Common ratio: $r = \frac{1}{4}$

Prepare

The local bank has agreed to donate $250 to the annual turkey fund to help feed families in need. In addition, for every bank customer that donates $50, the bank will donate $25.

1. A sequence describes the relationship between the number of $50 donations and the amount of the bank's donation. Is the sequence arithmetic or geometric?

   The sequence is arithmetic because the common difference is 25.

2. How can you calculate the 10th term based on the 9th term?

   Add 25 to the ninth term.

3. What is the 20th term?

   The 20th term is $725.

CC BY NC

MODULE 1 • TOPIC 2 • LESSON 2 ASSIGNMENT 117

3 Determining Recursive and Explicit Expressions from Contexts

MATERIALS

Graphic Organizers from Lesson 2: *Arithmetic and Geometric Sequences*

LESSON OVERVIEW

Scenarios are presented that can be represented by arithmetic and geometric sequences. Students determine the value of terms in each sequence. The term *recursive formula* is defined and used to generate term values. As the term number increases, it becomes more time consuming to generate the term value. This sets the stage for *explicit formulas* to be defined and used. Students practice using these formulas to determine the values of terms in both arithmetic and geometric sequences. Students write recursive and explicit formulas for sequences and represent sequences as graphs.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.
The student is expected to:

A.12C identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes.

A.12D write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms.

ELPS

(1) Learning Strategies
The student is expected to:

(A) use prior knowledge and experiences to understand meanings in English.

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(5) Writing
The student is expected to:

(F) write using a variety of grade-appropriate sentence lengths, patterns, and connecting words to combine phrases, clauses, and sentences in increasingly accurate ways as more English is acquired.

ESSENTIAL IDEAS

- A recursive formula expresses each new term of a sequence based on a preceding term of the sequence.
- An explicit formula for a sequence is a formula for calculating each term of the sequence using the term's position in the sequence.
- The explicit formula for determining the *n*th term of an arithmetic sequence is $a_n = a_1 + d(n - 1)$, where n is the term number, a_1 is the first term in the sequence, a_n is the *n*th term in the sequence, and d is the common difference.
- The explicit formula for determining the nth term of a geometric sequence is $g_n = g_1 \cdot r^{(n-1)}$, where n is the term number, g_1 is the first term in the sequence, g_n is the *n*th term in the sequence, and r is the common ratio.

LESSON STRUCTURE AND PACING: 4 DAYS

DAY 1

ENGAGE

Getting Started: Can I Get a Formula? 10–15 minutes

ESTABLISH A SITUATION

A scenario is given that can be represented by an arithmetic sequence. Students complete a table of values listing each term number and the value of the first ten terms. This is an introduction to the problem situation presented in Activity 3.1.

DEVELOP

Activity 3.1: Writing Formulas for Arithmetic Sequences 25–30 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

Students use two Worked Examples to understand recursive and explicit formulas for arithmetic sequences. They use this understanding to write recursive and explicit formulas for the sequence described by the problem situation from the Getting Started. The problem situation is then changed, and students answer questions about the new problem situation by rewriting the explicit formula. Finally, students rewrite arithmetic sequences using the distributive property.

DAY 2

Activity 3.2: Writing Formulas for Geometric Sequences 20–25 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

Students are given a new problem situation and determine that the situation can be represented by a geometric sequence. They analyze two Worked Examples to understand recursive and explicit formulas for geometric sequences. Students then use this understanding to write recursive and explicit formulas for the sequence described by the problem situation. The problem situation is then changed, and they answer questions about the new problem situation by rewriting the explicit formula.

Activity 3.3: Writing Recursive and Explicit Formulas 20–25 minutes

MATHEMATICAL PROBLEM SOLVING

Students use what they now know about recursive and explicit formulas for arithmetic and geometric sequences to write both types of formula for each of the sequences they studied in the previous lesson.

DAY 3

Activity 3.4: Arithmetic and Geometric Sequences 40–45 minutes

MATHEMATICAL PROBLEM SOLVING

Students write recursive and explicit formulas to represent sequences represented in different ways. Then, students use given information to determine the specific term of a sequence.

DAY 4

Activity 3.5: Graphing Sequences 30–35 minutes

MATHEMATICAL PROBLEM SOLVING

In this activity, students write an explicit formula to represent terms of a sequence and represent sequences on the coordinate plane.

DEMONSTRATE

Talk the Talk: Pros and Cons 10–15 minutes

GENERALIZATION

Students write paragraphs to describe the advantages and disadvantages of using recursive and explicit formulas to determine term values of arithmetic and geometric sequences.

Getting Started

ENGAGE

Can I Get a Formula?

Facilitation Notes

In this activity, a scenario is given that can be represented by an arithmetic sequence. Students complete a table of values listing each term number and the value of the first ten terms. This is an introduction to the problem situation presented in Activity 3.1.

Have students work with a partner or in a group to complete the table of values and answer Questions 1 through 4. Share responses as class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you tell the sequence is arithmetic from the situation? Term values? • How can you identify the common difference from the situation? Term values?
Gathering	• Why do the term numbers start with 1 rather than 0?
Probing	• What is the relationship among the numbers 0, 1, and 125 in the first row?
Seeing structure	• How many 18s did you add to 125 to get to the value of 287? • What is the relationship between the number of 18s and the number of home runs? The term number?

Summary

STAMP THE LEARNING

An arithmetic sequence can be used to model a situation by creating additional term values using the common difference. The term numbers and term values can be organized in a table.

ACTIVITY 3.1

Writing Formulas for Arithmetic Sequences

DEVELOP

Facilitation Notes

In this activity, students analyze two Worked Examples to understand recursive and explicit formulas for arithmetic sequences. They use this understanding to write recursive and explicit formulas for the sequence described by Jamal's donations to the baseball team. The problem situation is then changed, and students answer questions about the new problem situation by rewriting the explicit formula.

Ask a student to read the definition aloud. Review the Worked Example as a class.

Have students work with a partner or in a group to answer Questions 1 and 2. Share responses as class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do these steps relate to how you completed the table in the Getting Started? • Why does your procedure include multiplication? • When it is the 20th term, why do you need nineteen 18s rather than twenty 18s?

Ask a student to read the information and definition following Question 2 aloud. Review the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain what this formula means in your own words.
Probing	• What is the advantage of using an explicit formula? • How is this formula the same and different from the recursive formula?
Seeing structure	• Why does this formula include multiplication? • Why do you multiply the common difference by $(n - 1)$ instead of n?
Probing	• Explain the substitutions in the formula.

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as class.

DIFFERENTIATION STRATEGY

Access for All

Help students connect the new terminology to words they already know. Recursive has the prefix *re-* and means *repeating something*, in this case, repeating the same operation to get the next term. *Explicit* means *clear*, such as giving explicit directions; in this case, the explicit formula is more clear or direct.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how you used the formula to solve each problem. • What is the mathematical term for what \$75 represents? • Where will you substitute \$500 in the formula? Why? • How do you know what value to use for *n*?
Reflecting and justifying	• Explain why your calculations make sense.

Have a student read the Worked Example aloud. Have students work with a partner or in a group to complete Question 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you determine the first term of a sequence when it has been rewritten using the distributive property?

To close the Day 1 session, have the students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

An arithmetic sequence can be represented using a recursive formula or an explicit formula. The explicit formula is more efficient in determining any term value without having to calculate all the terms before it.

ACTIVITY 3.2 Writing Formulas for Geometric Sequences

Facilitation Notes

In this activity, students are given a problem situation that can be represented by a geometric sequence. They analyze two Worked Examples to understand recursive and explicit formulas for geometric sequences. They then write and use recursive and explicit formulas for the sequence described by the problem situation.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction aloud.

Have students work with a partner or in a group to complete Question 1. Share responses as class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you identify the common ratio? What is another way?
Seeing structure	• How many 2s did you multiply 1 by to get to the value of 512? • Use the definition of a function to explain why with all the repeated 5s, this set is still a function. • What is the relationship between the number of 2s and the number of cell divisions? The term number?

Ask a student to read the description of the recursive formula associated with a geometric sequence aloud. Review the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain what the formula means in your own words.
Probing	• Explain the substitutions in the formula.

Have students work with a partner or in a group to complete Question 2. Share responses as class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do these steps relate to how you completed the table in Question 1?

Ask a student to read the description of the explicit formula associated with a geometric sequence aloud. Review the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain what this formula means in your own words. • How is this formula the same and different from the recursive formula?
Seeing structure	• Why does this formula include an exponent? • Why does the common ratio have an exponent of $(n - 1)$ instead of n?
Probing	• Explain the substitutions in the formula. • When it is the 20th term, why does the conclusion say there were 19 cell divisions?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Challenge students to use the properties of exponents to rewrite the explicit formula with an exponent of n instead of $n - 1$.

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how you used the formula to solve the problem. • What is the mathematical term for what the 3 represents? • Where will you substitute 5 in the formula? Why? • How do you know which value to use for *n*? • Explain why your calculations make sense.

COMMON MISCONCEPTION

Students sometimes misunderstand the meaning of the first term value of a sequence. Sequences always start with term number 1. Based upon the phrasing of the scenario, the first term number usually represents a starting value, and the 2nd term represents the first time the operation is performed. For example, the first term is the number of cells after 0 divisions, not after one division, so the 100th term represents the number of cells after 99 divisions, not after 100 divisions. Sometimes, students get this concept, but go in the reverse direction. As students solve these problems, have them explain the value they substitute in the formula and the meaning of the result. The clarification now will help later when students connect sequences and functions and realize that the first term of a sequence is not the same as the *y*-intercept.

Summary

STAMP THE LEARNING

A geometric sequence can be represented using a recursive formula or an explicit formula. The explicit formula is more efficient to determine any term value without having to calculate all the terms before it.

ACTIVITY 3.3 Writing Recursive and Explicit Formulas

Facilitation Notes

In this activity, students use what they know about recursive and explicit formulas for arithmetic and geometric sequences to write both types of formulas for each of the sequences they studied in the previous lesson.

Have students work with a partner or in a group to complete this activity. Share responses with the class.

AS STUDENTS WORK, LOOK FOR

- Arithmetic sequences written two different ways when *d* is a negative value.
- Proper use of parentheses when *r* is a negative value.

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials Needed: Graphing Technology

To extend the activity, show students how to use graphing technology to identify a specified term.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the formulas for the arithmetic sequences? The geometric sequences?

To close the Day 2 session, have the students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Recursive and explicit formulas can be used to generate arithmetic and geometric sequences.

ACTIVITY 3.4 Arithmetic and Geometric Sequences

Facilitation Notes

In this activity, students write recursive and explicit formulas to represent sequences represented in different ways. Then, students use given information to determine the specific term of a sequence.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Question 1.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the difference between an arithmetic and geometric sequence? • What information do you need to write the recursive formula of an arithmetic sequence? • What information do you need to write the explicit formula of an arithmetic sequence? • What information do you need to write the recursive formula of a geometric sequence? • What information do you need to write the explicit formula of a geometric sequence?
Seeing structure	• Describe the graph of an arithmetic sequence when the common difference is a positive rational number. • Describe the graph of an arithmetic sequence when the common difference is a negative rational number. • Describe the graph of a geometric sequence when the common difference is a positive rational number. • Describe the graph of a geometric sequence when the common difference is a negative rational number.
Reflecting and justifying	• How did you determine the common difference or common ratio?
Probing	• How can you determine the terms of a sequence from a graph? • What do you do when the first term of the sequence is not given to you? • Did you use the recursive formula or the explicit formula to determine the first term of the sequence? Why?

Have students work with a partner or in a group to complete Questions 2 through 5.

Reflecting and justifying	• What type of sequence represents the number of eggs left when the chef makes omelets? Explain your reasoning. • What type of sequence represents the size of the slice compared to the whole pizza? Explain your reasoning.
Gathering	• Which term of the sequence represents the number of eggs left when the chef makes 20 omelets? Explain your reasoning. • Which term of the sequence represents the size of the slice after 10 cuts? Explain your reasoning.
Reflecting and justifying	• Did you use the recursive or the explicit formula to determine the specific term of the sequence? Why?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Challenge students to graph the sequences representing the scenarios in Questions 2 and 3. Discuss how students labeled the axis and what points students plotted on the graph.

To close the Day 3 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

You can write a recursive or explicit formula to represent an arithmetic or geometric sequence from different representations of the sequence. To determine a specific term of a sequence you can use a recursive or explicit formula. You must determine the first term of a sequence before writing the explicit formula.

ACTIVITY 3.5 Graphing Sequences

Facilitation Notes

In this activity, students write an explicit formula to represent terms of a sequence and represent sequences on the coordinate plane.

To begin the Day 4 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 4.

DIFFERENTIATION STRATEGY

Access for All

Have students create a table of values before graphing the sequences.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the scale for your graph? • Why does the sequence in Question 1 alternate between positive and negative term values?
Reflecting and justifying	• For Question 2, did everyone write the explicit formula the same way? Why or why not? • Why don't you see the first term in the explicit formula of the arithmetic sequence when you rewrite it using the distributive property?
Probing	• In Question 2, is the graph increasing or decreasing? • When does the graph of an arithmetic sequence decrease? • How does the common difference of an arithmetic sequence impact the steepness of the graph? • How did you determine the scale for your graph?
Reflecting and justifying	• Why does the graph of the sequence decrease? • Did you use the recursive or explicit formula to determine the first term of the sequence? Why?

Summary

STAMP THE LEARNING

You can write an explicit formula for terms of a sequence and represent an arithmetic or a geometric sequence on a graph. The graph of an arithmetic sequence increases when the common difference is a positive rational number and decreases when the common difference is negative. The graph of a geometric sequence may increase, decrease, or both increase and decrease.

Talk the Talk

PROS AND CONS

Facilitation Notes

In this activity, students write paragraphs describing the advantages and disadvantages of using recursive and explicit formulas to determine term values of arithmetic and geometric sequences.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How do the definitions of arithmetic and geometric sequences relate to their recursive formulas? • How is the repeated addition using an arithmetic sequence's recursive formula addressed more efficiently in its explicit formula? • How is the repeated multiplication using a geometric sequence's recursive formula addressed more efficiently in its explicit formula?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Provide students with the sequences from the previous lesson that were neither arithmetic nor geometric. Ask students to try to write recursive or explicit formulas for those sequences.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

There are advantages and disadvantages to using either an explicit or recursive formula to represent an arithmetic or geometric sequence.

3 Determining Recursive and Explicit Expressions from Contexts

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your student's prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Write recursive formulas for arithmetic and geometric sequences from contexts.
- Write explicit expressions for arithmetic and geometric sequences from contexts.
- Use formulas to determine unknown terms of a sequence.

NEW KEY TERMS

- recursive formula
- explicit formula

You have learned that arithmetic and geometric sequences always describe functions. How can you write equations to represent these functions?

Sample answer:

All sequences describe functions. The explicit formula for an arithmetic sequence is $a_n = a_1 + d(n - 1)$, where n is the term number, a_1 is the first term in the sequence, n is the nth term in the sequence, and d is the common difference. The explicit formula for a geometric sequence is $a_n = a_1 \cdot r^{(n-1)}$, where n is the term number, a_1 is the first term in the sequence, a_n is the nth term in the sequence, and r is the common ratio.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Assess students' prior knowledge of the word *donate*. Create a list of synonyms for the word and discuss the distinction between donating money and giving money to a friend, for example. Ask for volunteers to share examples of scenarios of money *donations*.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Can I Get a Formula?

Notice that the first term in this sequence is the amount Jamal donates if the team hits 0 home runs.

While a common ratio or a common difference can help you determine the next term in a sequence, how can they help you determine the thousandth term of a sequence? The ten-thousandth term of a sequence?

Consider the sequence represented in this situation.

Jamal owns a sporting goods store. He has agreed to donate $125 to his old high school's baseball team for their equipment fund. In addition, he will donate $18 for every home run the team hits during the season. The sequence shown represents the possible dollar amounts that Jamal could donate for the season.

125, 143, 161, 179, ...

Number of Home Runs	Term Number (n)	Donation Amount (dollars)
0	1	125
1	2	143
2	3	161
3	4	179
4	5	197
5	6	215
6	7	233
7	8	251
8	9	269
9	10	287

1. Identify the sequence type. Describe how you know.
   The sequence is arithmetic. It is arithmetic because a constant is added to each term to produce the next term.

2. Determine the common difference or common ratio for the sequence.
   The common difference is 18.

3. Complete the table.
   See answers in the table.

4. Explain how you can calculate the tenth term based on the ninth term.
   To calculate the tenth term, add 18 to the ninth term.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Materials Needed: Poster Paper

Review the terms *common difference* and *common ratio*. Create an anchor chart with two columns using the terms as the headers for each column. Discuss the similarities and differences between the terms and fill in the anchor chart with key ideas about each term. Ask students to give examples of sequences that have a *common difference* as well as sequences that have a *common ratio*. Ensure students' understanding of which term applies to an arithmetic sequence and which term applies to a geometric sequence.

ACTIVITY **3.1**

Writing Formulas for Arithmetic Sequences

In the previous lesson you used the **recursive formula** to determine term values in a sequence. The recursive formula expresses each new term of a sequence based on the preceding term in the sequence. The recursive formula to determine the *n*th term of an arithmetic sequence is:

$$n\text{th term} \longrightarrow a_n = \underbrace{a_{n-1}}_{\text{previous term}} + d \longleftarrow \text{common difference}$$

WORKED EXAMPLE

Consider the sequence −2, −9, −16, −23, ...
You can use the recursive formula to determine the 5th term.

$$a_n = a_{n-1} + d$$
$$a_5 = a_{5-1} + -7$$

The expression a_5 represents the 5th term. The previous term is −23, and the common difference is −7.

$$a_5 = a_4 + -7$$
$$a_5 = -23 + -7$$
$$a_5 = -30$$

The 5th term of the sequence is −30.

You only need to know the previous term and the common difference to use the recursive formula.

Ask Yourself . . .
What observations can you make?

Consider the sequence showing Jamal's contribution to his old high school's baseball team in terms of the number of home runs hit.

1. Use a recursive formula to determine the 11th term in the sequence. Explain what this value means in terms of this problem situation.

   $a_{11} = a_{10} + 18;$
   $a_{11} = 287 + 18;$
   $a_{11} = 305;$

   Jamal will donate a total of $305 when 10 home runs are hit.

2. Is there a way to calculate the 20th term without first calculating the 19th term? If so, describe the strategy.

   Answers will vary based on each classroom.

Ask Yourself . . .
Did you justify your mathematical reasoning?

Chunking the Activity

- Read and discuss the definition and Worked Example.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Read and discuss the information, definition, and Worked Example.
- Group students to complete Questions 3–5.
- Share and summarize.
- Read and discuss the Worked Example.
- Group students to complete Question 6.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Ask students to identify what the prefix *pre-* means in the word *preceding*. Follow up with additional examples of words with the prefix *pre-*, including pretest, preview, and precooked. Define these words and then ask students to explain why preceding means "the term before" in the context of "the preceding term in the sequence." Create a list of words beginning with the prefix *pre-* and have students add to it as they encounter additional words with this prefix in the lesson.

STAMP THE LEARNING

The definition and Worked Example provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Remember . . .

The first term in this sequence is the amount Jamal donates when the team hits 0 home runs. So, the 93rd term represents the amount Jamal donates if the team hits 92 home runs.

You can determine the 93rd term of the sequence by calculating each term before it and then adding 18 to the 92nd term, but this will probably take a while! A more efficient way to calculate any term of a sequence is to use an *explicit formula*.

An **explicit formula** of a sequence is a formula to calculate the *n*th term of a sequence using the term's position in the sequence. The explicit formula for determining the *n*th term of an arithmetic sequence is:

*n*th term   common difference

$$a_n = a_1 + d\underbrace{(n - 1)}_{\text{previous term number}}$$

1st term

WORKED EXAMPLE

You can use the explicit formula to determine the 93rd term in this problem situation.

$$a_n = a_1 + d(n - 1)$$
$$a_{93} = 125 + 18(93 - 1)$$

The expression a_{93} represents the 93rd term. The first term is 125, and the common difference is 18.

$$a_{93} = 125 + 18(92)$$
$$a_{93} = 125 + 1656$$
$$a_{93} = 1781$$

The 93rd term of the sequence is 1781.

This means Jamal will contribute a total of $1781 when his old high school's baseball team hits 92 home runs.

122 MODULE 1 • TOPIC 2 • LESSON 3

3. Use the explicit formula to determine the amount of money Jamal will contribute for each number of home runs hit.

   a. 35 home runs

   $a_n = a_1 + d(n - 1)$

   $a_{36} = 125 + 18(36 - 1)$

   $755

   b. 48 home runs

   $a_n = a_1 + d(n - 1)$

   $a_{49} = 125 + 18(49 - 1)$

   $989

   c. 86 home runs

   $a_n = a_1 + d(n - 1)$

   $a_{87} = 125 + 18(87 - 1)$

   $1673

   d. 214 home runs

   $a_n = a_1 + d(n - 1)$

   $a_{215} = 125 + 18(215 - 1)$

   $3977

Jamal decides to increase his initial contribution and amount donated per home run hit. He decides to contribute $500 and will donate $75 for every home run the team hits.

4. Write the first five terms of the sequence representing the new contribution Jamal will donate to the the baseball team.

   500, 575, 650, 725, 800

5. Determine Jamal's contribution for each number of home runs hit.

   a. 39 home runs

   $a_n = a_1 + d(n - 1)$

   $a_{40} = 500 + 75(40 - 1)$

   $3425

   b. 50 home runs

   $a_n = a_1 + d(n - 1)$

   $a_{51} = 500 + 75(51 - 1)$

   $4250

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MODULE 1 • TOPIC 2 • LESSON 3 123

WORKED EXAMPLE

Use the distributive property to rewrite the explicit formula for the problem situation.

The explicit formula for the problem situation is $a_n = 125 + 18(n + 1)$.

Step 1: Use the distributive property.

$a_n = 125 + 18n - 18$.

Step 2: Combine like terms.

$a_n = 18n + 107$

6. For each arithmetic sequence, identify the first term and the common difference. Then, use the distributive property to rewrite the explicit formula.

a. $p_n = 16 - \frac{1}{2}(n - 1)$

The common difference is $d = -\frac{1}{2}$.

The first term is $p_1 = 16$

$p_n = -\frac{1}{2}n + 16\frac{1}{2}$

b. $k_n = 0.6 + 0.2(n - 1)$

The common difference is $d = 0.2$

The first term is $k_1 = 0.6$

$k_n = 0.2n + 0.4$

c. $r_n = -7(n - 1) - 2$

The common difference is $d = -7$.

The first term is $r_1 = -2$

$r_n = -7n + 5$

124 MODULE 1 • TOPIC 2 • LESSON 3

ACTIVITY 3.2

Writing Formulas for Geometric Sequences

When it comes to bugs, bats, spiders, and—ugh, any other creepy crawlers—finding one in your house is finding one too many! Then again, when it comes to cells, the more the better. Animals, plants, fungi, slime, molds, and other living creatures are composed of eukaryotic cells. During growth, generally there is a cell called a "mother cell" that divides itself into two "daughter cells." Each of those daughter cells then divides into two more daughter cells, and so on.

Notice that the 1st term in this sequence is the total number of cells after 0 divisions (that is, the mother cell).

1. The sequence shown represents the growth of eukaryotic cells.

   1, 2, 4, 8, 16, ...

Number of Cell Divisions	Term Number (n)	Total Number of Cells
0	1	1
1	2	2
2	3	4
3	4	8
4	5	16
5	6	32
6	7	64
7	8	128
8	9	256
9	10	512

   a. Describe why this sequence is geometric and identify the common ratio.

   This sequence is geometric because each term is multiplied by a constant to produce the next term. The common ratio is 2.

   b. Complete the table of values. Use the number of cell divisions to identify the term number and the total number of cells after each division.

   See answers in the table.

   c. Explain how you can calculate the tenth term based on the ninth term.

   Multiply the ninth term by 2.

In the previous lesson, you used the recursive formula for a geometric sequence. The recursive formula to determine the nth term of a geometric sequence is:

$$\underset{n\text{th term}}{a_n} = \underbrace{a_{n-1}}_{\text{previous term}} \cdot \underset{\text{common ratio}}{r}$$

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete Question 1.
- Check in and share.
- Read and discuss the formula and the Worked Example.
- Group students to complete Question 2.
- Check in and share.
- Read and discuss the formula and the Worked Example.
- Group students to complete Questions 3–5.
- Share and summarize.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining whether sequences are arithmetic or geometric then representing them recursively and explicitly, assign Skills Practice Set A.

EB STUDENT TIP

For "Advanced" and "Advanced High" proficiency levels

Review the scientific terms given in the example for the activity. Ask students to make a list of terms, such as *cells*, *mother cells*, *daughter cells*, *petri dish*, and *hypothesis*. Discuss how the terms are used in the activity and ask students to create a sentence using each term to demonstrate their understanding. Also ask students to create a list of synonyms for *hypothesis*.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 2–5 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining specific terms of sequences, assign Skills Practice Set B.

WORKED EXAMPLE

Consider the sequence shown.

4, 12, 36, 108, ...

You can use the recursive formula to determine the 5th term.

$$b_n = b_{n-1} \cdot r$$
$$b_5 = b_{5-1} \cdot (3)$$

The expression a_5 represents the 5th term. The previous term is 108, and the common ratio is 3.

$$b_5 = b_4 \cdot (3)$$
$$b_5 = 108 \cdot (3)$$
$$b_5 = 324$$

The 5th term of the sequence is 324.

Consider the sequence of cell divisions and the total number of resulting cells.

2. Write a recursive formula for the sequence and use the formula to determine the 12th term in the sequence. Explain what your result means in terms of this problem situation.

   $a_{12} = 1024 \cdot 2$; $a_{12} = 2048$;

   There are a total of 2048 cells after 11 divisions.

The explicit formula to determine the nth term of a geometric sequence is:

$$\underset{\text{nth term}}{a_n} = \underset{\text{1st term}}{a_1} \cdot \underset{\text{common ratio}}{r}^{\overbrace{n-1}^{\text{previous term number}}}$$

126 MODULE 1 • TOPIC 2 • LESSON 3

WORKED EXAMPLE

You can use the explicit formula to determine the 20th term in this problem situation.

$$a_n = a_1 \cdot r^{n-1}$$
$$a_{20} = 1 \cdot 2^{20-1}$$

The expression a_{20} represents the 20th term. The first term is 1, and the common ratio is 2.

$$a_{20} = 1 \cdot 2^{19}$$
$$a_{20} = 1 \cdot 524{,}288$$
$$a_{20} = 524{,}288$$

The 20th term of the sequence is 524,288.

This means that after 19 cell divisions, there are a total of 524,288 cells.

Remember . . .
The first term in this sequence is the total number of cells after 0 divisions. So, the 20th term represents the total number of cells after 19 divisions.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

3. Use the explicit formula to determine the total number of cells for each number of divisions.

a. 11 divisions

$a_n = a_1 \cdot r^{n-1}$
$a_{12} = 1 \cdot 2^{12-1}$
2048

b. 14 divisions

$a_n = a_1 \cdot r^{n-1}$
$a_{15} = 1 \cdot 2^{15-1}$
16,384

c. 18 divisions

$a_n = a_1 \cdot r^{n-1}$
$a_{19} = 1 \cdot 2^{19-1}$
262,144

d. 22 divisions

$a_n = a_1 \cdot r^{n-1}$
$a_{23} = 1 \cdot 2^{23-1}$
4,194,304

Remember . . .
You can use parentheses or a · to represent multiplication.

Suppose that a scientist has 5 eukaryotic cells in a petri dish. She wonders how the growth pattern would change when each mother cell divides into 3 daughter cells.

4. Write the first five terms of the sequence for the scientist's hypothesis.

5, 15, 45, 135, 405

5. Determine the total number of cells in the petri dish for each number of divisions.

a. 13 divisions

$a_n = a_1 r^{n-1}$
$a_{14} = (5)3^{14-1}$
7,971,615

b. 16 divisions

$a_n = a_1 r^{n-1}$
$a_{17} = (5)3^{17-1}$
215,233,605

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MODULE 1 • TOPIC 2 • LESSON 3 127

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Optimizing Learning

This activity optimizes transfer and generalization.

ACTIVITY 3.3 Writing Recursive and Explicit Formulas

In the previous lesson, you identified sequences as either arithmetic or geometric and then matched a corresponding graph.

1. Go back to the graphic organizers from the previous lesson. Write the recursive and explicit formulas for each sequence.

128 MODULE 1 • TOPIC 2 • LESSON 3

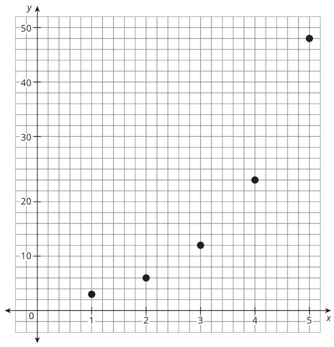
ACTIVITY 3.4 Arithmetic and Geometric Sequences

1. For each sequence, identify the sequence type and the common difference or ratio. Write an explicit formula and a recursive formula to represent the sequence.

   a. $a_1 = 2.3, a_2 = 0.3, a_3 = -1.7, a_4 = -3.7$

   Arithmetic sequence common difference: $d = -2$
   Recursive formula: $a_n = a_{n-1} - 6$
   Explicit formula: $a_n = 2.3 - 6(n - 1)$
   $a_n = 8.3 - 6n$

   b.



   Geometric sequence
   Common ratio: $r = 2$
   Recursive formula: $b_n = b_{n-1} \cdot 2$
   Explicit formula: $b_n = 3 \cdot b^{n-1}$

   c.

n	c_n
4	−567
5	189
6	−63
7	21
8	−7

   Geometric sequence Common ratio: $r = -\frac{1}{3}$
   Recursive formula: $c_n = -\frac{1}{3} \cdot c_{n-1}$
   Explicit formula:
   $-567 = c_1 \cdot \left(-\frac{1}{3}\right)^{4-1}$
   $-567 = c_1 \cdot \left(-\frac{1}{3}\right)^{3}$
   $-567 = c_1 \cdot -\frac{1}{27}$
   $c_1 = 15{,}309$
   $c_n = 15{,}309 \left(-\frac{1}{3}\right)^{n-1}$

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Group students to complete Question 1.
- Check in and share.
- Group students to complete Questions 2–5.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Activity 3.4 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice representing scenarios using explicit and recursive formulas, assign Skills Practice Set C.

MODULE 1 • TOPIC 2 • LESSON 3 129

d. $a_5 = -5.9,\ a_6 = -5.3,\ a_7 = -4.7,\ a_8 = -4.1$

Arithmetic sequence

Common difference: $d = 0.6$

Recursive formula: $a_n = a_{n-1} + 0.6$

Explicit formula:

$-5.9 = a_1 + 0.6(5 - 1)$

$-5.9 = a_1 + 0.6(4)$

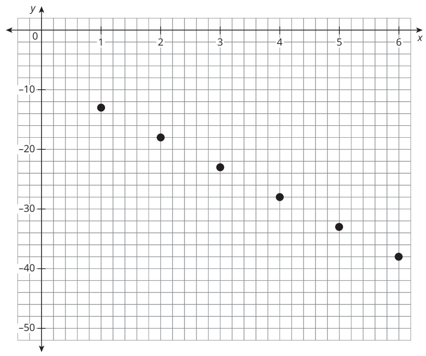
$-5.9 = a_1 + 2.4$

$-8.3 = a_1$

$a_n = -8.3 + 0.6(n - 1)$

$a_n = -8.9 + 0.6n$

e.



Arithmetic sequence

Common difference: $d = -5$

Recursive formula: $e_n = e_{n-1} - 5$

Explicit formula: $e_n = -13 - 5(n - 1)$

$e_n = -8 - 5n$

2. The local diner makes the best omelets in town! The chef begins each day with 150 eggs to make his famous omelets. After making 1 omelet, he has 144 eggs. After making 2 omelets, he has 138 eggs left. After making 3 omelets, he has 132 eggs left. Write a recursive and explicit formula to represent the sequence. Then, determine how many eggs the chef will have left after he makes 20 omelets.

Recursive formula: $a_n = a_{n-1} - 6$

Explicit formula: $a_n = 150 - 6(n - 1)$

The 21st term represents the number of eggs left after the chef makes 20 omelets.

$a_{21} = 150 - 6(21 - 1)$

$a_{21} = 150 - 120 = 30$

The chef has 30 eggs left after he makes 20 omelets.

130 MODULE 1 • TOPIC 2 • LESSON 3

3. Tiara is participating in a pizza-making contest. Each contestant has to bake the largest and most delicious pizza they can. Tiara's pizza has a 6-foot diameter! After the contest, she plans to cut the pizza so that she can pass the slices out to share. She begins with 1 whole pizza. Then, she cuts it in half. After that, she cuts each of those slices in half. Then she cuts each of those slices in half and so on. Write a recursive and an explicit formula for the sequence. Then, determine the size of the slice compared to the whole pizza after 10 cuts.

   Recursive formula: $k_n = \frac{1}{2} \cdot k_{n-1}$

   Explicit formula: $k_n = 1 \cdot \left(\frac{1}{2}\right)^{n-1}$

   The 11th term of the sequence represents the size of the slice compared to the whole pizza after 10 cuts.

   $a_{11} = 1 \cdot \left(\frac{1}{2}\right)^{11-1}$

   $a_{11} = 1 \cdot \left(\frac{1}{2}\right)^{10}$

   $a_{11} = \frac{1}{1024}$

   After 10 cuts the size of the slice is $\frac{1}{1024}$ the size of the whole pizza.

4. Determine the 9th term of the sequence $a_n = a_{n-1} \cdot 8$ and $a_3 = 2$.

   $a_4 = 16$
   $a_5 = 128$
   $a_6 = 1024$
   $a_7 = 8192$
   $a_8 = 65{,}536$
   $a_9 = 524{,}288$

5. Determine the 12th term of the sequence $a_n = a_{n-1} + \frac{3}{4}$ and $a_9 = 6$.

   $a_{10} = 6\frac{3}{4}$

   $a_{11} = 7\frac{1}{2}$

   $a_{12} = 8\frac{1}{4}$

MODULE 1 • TOPIC 2 • LESSON 3 131

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1–3.
- Group students to complete the activity.
- Share and summarize.

Activity 3.5 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional writing an explicit formula to represent terms of a sequence and representing a sequence with a graph assign Skills Practice Set D.

ACTIVITY 3.5 Graphing Sequences

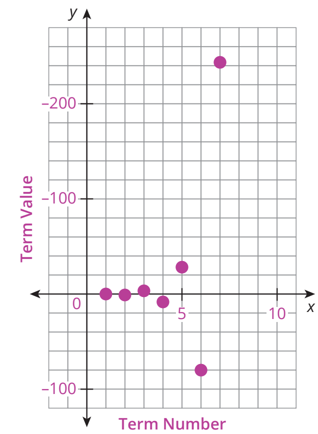
Identify the sequence type and the common difference or ratio for each sequence. Write the explicit formula you can use to represent the terms of the sequence, then represent the sequence on the coordinate plane.

1. $a_1 = \frac{1}{3}, a_2 = -1, a_3 = 3, a_4 = -9, a_5 = 27$

   Geometric sequence

   Common ratio: $r = -3$

   $a_n = \frac{1}{3}(-3)^{n-1}$

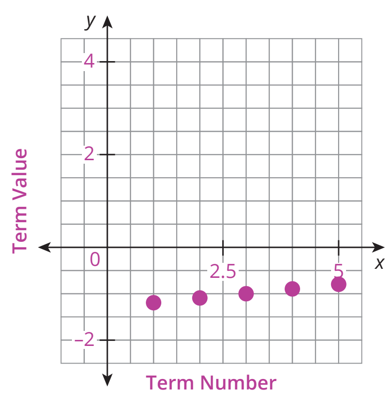


2. $h_1 = -1.2, h_2 = -1.1, h_3 = -1.0, h_4 = -0.9, h_5 = -0.8$

   Arithmetic sequence

   Common difference: $d = 0.1$

   $h_n = -1.3 + 0.1n$



132 MODULE 1 • TOPIC 2 • LESSON 3

3. $a_3 = -12, a_4 = -14, a_5 = -16, a_6 = -18$

Arithmetic sequence

Common difference: $d = -2$

$a_n = a_1 + d(n - 1)$

$a_3 = a_1 + (-2)(3 - 1)$

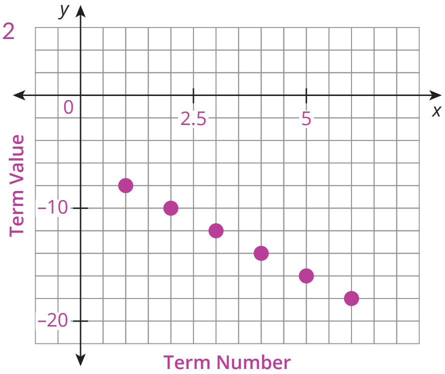
$-12 = a_1 + (-2)(2)$

$-12 = a_1 + (-4)$

$a_1 = -8$

$a_n = -8 - 2(n - 1)$

$a_n = -6 - 2n$



4. $k_3 = \frac{1}{8}, k_4 = \frac{1}{32}, k_5 = \frac{1}{128}, k_6 = \frac{1}{512}$

Geometric sequence

Common ratio: $r = \frac{1}{4}$

$k_n = k_1 \cdot \left(\frac{1}{4}\right)^{n-1}$

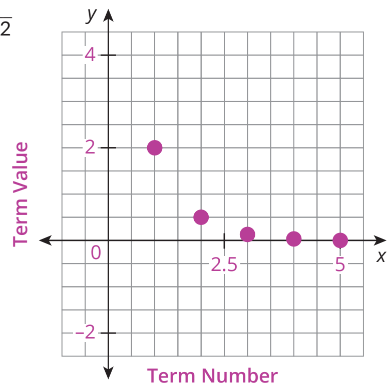
$k_3 = k_1 \cdot \left(\frac{1}{4}\right)^{3-1}$

$\frac{1}{8} = k_1 \cdot \left(\frac{1}{4}\right)^{2}$

$\frac{1}{8} = k_1 \cdot \frac{1}{16}$

$2 = k_1$

$k_n = 2 \cdot \left(\frac{1}{4}\right)^{n-1}$



MODULE 1 • TOPIC 2 • LESSON 3 133

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener.

Talk the Talk

Pros and Cons

1. Explain the advantages and disadvantages of using a recursive formula.

   Sample answer:

   Advantage: It enables you to make sense of the growth pattern of the sequence.

   Disadvantage: It is not an efficient method when determining the term value for a large term number.

2. Explain the advantages and disadvantages of using an explicit formula.

   Sample answer:

   Advantage: It is an efficient method when determining the term value for a large term number.

   Disadvantage: It takes a little more effort to determine an explicit formula than it does to determine a recursive formula.

SELF-MONITORING STRATEGY

Look for students using self-motivation and self-discipline to persevere in solving problems. Refer to the Course and Implementation Guide for further details on these look fors.

134 MODULE 1 • TOPIC 2 • LESSON 3

Lesson 3 Assignment

Write

Explain the difference between a recursive formula and an explicit formula in your own words.

Remember

All sequences describe functions.

The explicit formula for an arithmetic sequence is $a_n = a_1 + d(n - 1)$, where n is the term number, a_1 is the first term in the sequence, a_n is the nth term in the sequence, and d is the common difference.

The explicit formula for a geometric sequence is $a_n = a_1 \cdot r^{(n-1)}$, where n is the term number, a_1 is the first term in the sequence, a_n is the nth term in the sequence, and r is the common ratio.

Practice

1. Lizzie must volunteer 225 hours for a community service project. She plans to volunteer for 6 hours each week. The sequence shown represents the number of volunteer hours she has left after three weeks have passed.

   225, 219, 213, 207, ...

   a. Describe this sequence.

   This arithmetic sequence starts with 225 and has a common difference of −6.

   b. Write and simplify the explicit formula for the problem situation.

   $a_n = 225 + (-6)(n - 1)$

   $a_n = 225 - 6n - 6$

   $a_n = 231 - 6n$

Write

Sample answer:

To calculate the term value that corresponds to a term number using a recursive formula, I must generate all the terms that come before it in the sequence. When using an explicit formula, I can calculate the term value without having to calculate the value of all previous terms.

MODULE 1 • TOPIC 2 • LESSON 3 ASSIGNMENT 135

Lesson 3 Assignment

c. Use your explicit formula from part (b) to determine how many volunteer hours Lizzie has left to fulfill her requirement after 33 weeks have passed. Show your work.

$a_n = 231 - 6n$

$a_{34} = 231 - 6(34)$

$a_{34} = 231 - 204$

$a_{34} = 27$

27 hours

d. Which formula should you use to determine how many volunteer hours Lizzie has left to fulfill her requirement after 40 weeks have passed? Explain your reasoning.

I should use the explicit formula because I do not know how many volunteer hours Lizzie has left after 39 weeks have passed to fulfill her requirement.

e. Calculate the number of volunteer hours Lizzie has left to fulfill her requirement after 40 weeks have passed. Explain what your answer means in terms of the problem situation.

$a_n = 225 + (-6)(n - 1)$

$a_n = 231 - 6n$

$a_{41} = 231 - 6(41)$

$a_{41} = 231 - 246$

$a_{41} = -15$

My answer of −15 hours means that Lizzie will complete her community service before 40 weeks.

2. The half-life of a substance is defined as the period of time it takes for the amount of the substance to decay by half. The sequence below shows the amount of a substance that will be left after a certain number of half-lives have elapsed.

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

a. Describe this sequence.

This geometric sequence starts with 1 and has a common ratio of $\frac{1}{2}$.

136 MODULE 1 • TOPIC 2 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

b. Calculate how much of the substance will be left after 21 half-lives have elapsed. Show your work. Does your answer make sense in this problem context? Why or why not?

$a_n = 1 \cdot \left(\frac{1}{2}\right)^{n-1}$

$a_{22} = 1 \cdot \left(\frac{1}{2}\right)^{21}$

$a_{22} = \left(\frac{1}{2}\right)^{21}$ amount is $\frac{1}{2{,}097{,}152}$

Yes, the amount of the substance will continue to get smaller and smaller as it decays.

Prepare

Consider the first two terms of this sequence $\frac{1}{16}, -\frac{3}{16}, \ldots$

- Determine the 63rd term if this is an arithmetic sequence. Write your answer as an improper fraction in lowest terms.

  $\frac{-247}{16}$

- Determine the 63rd term if this is a geometric sequence. Write your answer in scientific notation.

  $\approx 2.38 \times 10^{28}$

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MODULE 1 • TOPIC 2 • LESSON 3 ASSIGNMENT 137

TOPIC 2 SELF-REFLECTION

Name: ____________________

Sequences

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill.**

I can demonstrate an understanding of the standards in the *Sequences* topic by:

TOPIC 2: *Sequences*	Beginning of Topic	Middle of Topic	End of Topic
understanding that a sequence represents a relationship between term numbers (inputs) and term values (outputs).	◯	◯	◯
stating the appropriate domain for a sequence.	◯	◯	◯
distinguishing between arithmetic and geometric sequences.	◯	◯	◯
recognizing that an arithmetic sequence has a common difference between terms and a geometric sequence has a common ratio between terms.	◯	◯	◯
determining the common difference between two terms in an arithmetic sequence and the common ratio between two terms in a geometric sequence represented in tables and graphs.	◯	◯	◯
describing the graph of an arithmetic and geometric sequence.	◯	◯	◯
explaining that a recursive formula tells you how to determine the next value of a sequence from the previous value.	◯	◯	◯
explaining that an explicit formula tells you how to determine any value given the term number.	◯	◯	◯

continued on the next page

Notes

Notes

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Sequences*	Beginning of Topic	Middle of Topic	End of Topic
distinguishing between explicit and recursive formulas.			
writing recursive and explicit formulas for any sequence, including those presented as real-world scenarios.			
rewriting explicit formulas of arithmetic sequences using the distributive property.			
translating between explicit and recursive formulas.			
deciding when real-world problems model an arithmetic or geometric sequence.			

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Sequences* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

140 MODULE 1 • TOPIC 2 • SELF-REFLECTION

TOPIC 2 SUMMARY

Sequences Summary

LESSON 1 Recognizing Patterns and Sequences

A **sequence** is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects. A **term in a sequence** is an individual number, figure, or letter in the sequence. Many different patterns can generate a sequence of numbers.

A sequence that continues on forever is called an **infinite sequence**. A sequence that terminates is called a **finite sequence**.

For example, consider the situation in which an album that can hold 275 baseball cards is filled with 15 baseball cards at the end of each week. A sequence to represent how many baseball cards can fit into the album after 6 weeks is 275 cards, 260 cards, 245 cards, 230 cards, 215 cards, and 200 cards. This sequence begins at 275 and decreases by 15 with each term. The pattern cannot continue forever since you cannot have a negative number of cards, so this is a finite sequence.

LESSON 2 Arithmetic and Geometric Sequences

An **arithmetic sequence** is a sequence of numbers in which the difference between any two consecutive terms is a constant. This constant is called the **common difference** and is typically represented by the variable *d*. The common difference of a sequence is positive when the same positive number is added to each term to produce the next term. The common difference of a sequence is negative when the same negative number is added to each term to produce the next term.

For example, consider the sequence 14, $16\frac{1}{2}$, 19, $21\frac{1}{2}$, ... The pattern of this sequence is to add $2\frac{1}{2}$ to each term to produce the next term. This is an arithmetic sequence, and the common difference d is $2\frac{1}{2}$.

A **geometric sequence** is a sequence of numbers in which the ratio between any two consecutive terms is a constant. The constant, which is either an integer or a fraction, is called the **common ratio** and is typically represented by the variable *r*.

NEW KEY TERMS

- sequence [secuencia/sucesión]
- term of a sequence [término de una secuencia]
- infinite sequence [secuencia infinita]
- finite sequence [secuencia finita]
- arithmetic sequence [secuencia aritmética]
- common difference [diferencia común]
- geometric sequence [secuencia geométrica]
- common ratio [razón común]
- recursive formula [fórmula recursiva]
- explicit formula [fórmula explícita]
- mathematical modeling [modelado matemático]

Notes

Notes

For example, consider the sequence 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$. The pattern is to multiply each term by the same number, $\frac{1}{3}$, to determine the next term. Therefore, this sequence is geometric and the common ratio, r, is $\frac{1}{3}$.

LESSON 3 Determining Recursive and Explicit Expressions from Context

A **recursive formula** expresses each new term of a sequence based on a preceding term of the sequence. The recursive formula to determine the nth term of an arithmetic sequence is $a_n = a_{n-1} + d$. The recursive formula to determine the nth term of a geometric sequence is $a_n = a_{n-1} \cdot r$. When using the recursive formula, it is not necessary to know the first term of the sequence.

For example, consider the geometric sequence 32, 8, 2, $\frac{1}{2}$, ... with a common ratio of $\frac{1}{4}$. The 5th term of the sequence can be determined using the recursive formula.

$$a_n = a_{n-1} \cdot r$$
$$a_5 = a_4 \cdot r$$
$$a_5 = \frac{1}{2} \cdot \frac{1}{4}$$
$$a_5 = \frac{1}{8}$$

The 5th term of the sequence is $\frac{1}{8}$.

An **explicit formula** for a sequence is a formula for calculating each term of the sequence using the index, which is a term's position in the sequence. The explicit formula to determine the nth term of an arithmetic sequence is $a_n = a_1 + d(n - 1)$. You can use the distributive property to rewrite the formula for an arithmetic sequence. The explicit formula to determine the nth term of a geometric sequence is $g_n = g_1 \cdot r^{n-1}$.

For example, consider the situation of a cactus that is currently 3 inches tall and will grow $\frac{1}{4}$ inch every month. The explicit formula for arithmetic sequences can be used to determine how tall the cactus will be in 12 months.

$$a_n = 3 + \frac{1}{4}(n - 1)$$
$$a_n = 3 + \frac{1}{4}n - \frac{1}{4}$$
$$a_n = \frac{1}{4}n + 2.75$$
$$a_{12} = \frac{1}{4}(12) + 2.75$$
$$a_{12} = 3 + 2.75$$
$$a_{12} = 5.75 \text{ or } 5\frac{3}{4}$$

In 12 months, the cactus will be $5\frac{3}{4}$ inches tall.

142 MODULE 1 • TOPIC 1 • TOPIC SUMMARY

MODULE 2

Exploring Constant Change

TEKS* Addressed:
A.2A, A.2B, **A.2C**, A.2D, A.2E, A.2F, A.2G, A.3A, **A.3B**, **A.3C**, A.3E, A.3F, A.4A, A.4B, A.4C, A.12A, A.12B, A.12D
*Bold TEKS = Readiness Standard

Exploring Constant Change

Sessions: **36**

Why is this module named *Exploring Constant Change*?

The study of algebra is the study of the relationships among points given in each of the four representations (verbal, numeric, algebraic, and graphical). The defining characteristic of a linear function is the constant rate of change between any set of points in the relationship.

For students to understand functions defined by more complicated relationships, they first need to understand constant change. The idea of constant change is not new for students; they have been exploring proportional and linear relationships since 6th grade.

Throughout this module, students examine the structure of equations and their corresponding graphs. They are introduced to transformation notation and begin thinking about functions as objects.

Even though students are exploring only functions with constant change in this module, they learn that all functions have similar behaviors when transforming their graphs or equations.

The Research Shows . . .

"A fluency in linking and translating among multiple representations seems to be critical in the development of algebraic thinking. The learner who can, for a particular mathematical problem, move fluidly among different mathematical representations has access to a perspective on the mathematics in the problem that is greater than the perspective any one representation can provide."

Fostering Algebraic Thinking: A Guide For Teacher Grades 6–10 | Page 141

What is the mathematics of *Exploring Constant Change*?

Exploring Constant Change contains two topics: *Linear Functions* and *Transforming and Comparing Linear Functions*. Students recognize and identify the key characteristics of different function families. They write recursive and explicit formulas for arithmetic and geometric sequences.

*1 DAY PACING = 45-MINUTE SESSION

22 SESSIONS
21 LEARNING • 1 ASSESSMENT

TOPIC 1 *Linear Functions*

Learning Together: 15 Sessions

TEKS: **A.2A**, A.2B, **A.2C**, A.2D, A.3A, **A.3B**, **A.3C**, A.3E, A.3F, A.4A, A.4B, A.4C, A.12A, A.12B, A.12D

Students use lines of best fit to model bivariate data and connect arithmetic sequences to linear functions.

- Students use technology to generate linear regression functions to model data.
- Students differentiate between correlation and causation.
- Students examine the structure of equations representing functions and compare the graphs to determine what their differences indicate about the functions and the scenarios they model.

Learning Individually: 6 Sessions

Targeted Skills Practice for *Linear Functions*

- Students determine linear regression functions and correlation coefficients for data sets.
- Students use linear regression functions to make predictions.
- Students write linear equations in slope-intercept, point-slope, and standard form.
- Students graph linear equations and identify key characteristics of graphs.
- Students evaluate equations in function notation.
- Students solve direct variation equations.

14 SESSIONS
13 LEARNING • 1 ASSESSMENT

TOPIC 2 *Transforming and Comparing Linear Functions*

Learning Together: 9 Sessions

TEKS: **A.2A**, **A.2C**, A.2E, A.2F, A.2G, **A.3C**, A.3A, A.3E, A.12B

Students begin thinking about functions as objects.

- Students learn transformation notation, $y = a \cdot f(b(x - c)) + d$.
- Students compare key characteristics of linear functions in different forms.

Learning Individually: 4 Sessions

Targeted Skills Practice for *Transforming and Comparing Linear Functions*

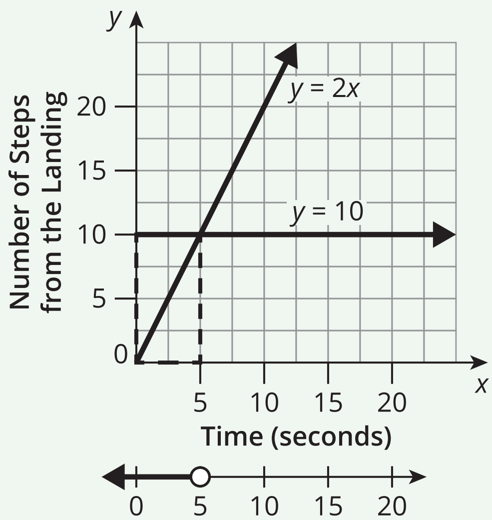
- Students graph transformed functions.
- Students write equations for linear transformations.
- Students write equations that represent parallel and perpendicular lines in point-slope, slope-intercept, and standard form.
- Students compare key features of linear functions given different representations.

How is *Linear Functions* connected to prior learning?

Students have defined the constant of proportionality and extended their understanding of proportional relationships to understand linear relationships.

Math Representation

Catalina walks upstairs at a rate of two steps per second. The graph shows the relationship between the time in seconds and the number of steps Catalina travels.



The **rectangle** shows the time when Catalina is fewer than 10 steps above the landing.

Catalina is fewer than 10 steps above the landing for times less than 5 seconds.

When will students use knowledge from *Searching for Patterns* in future learning?

Students will use what they know about constant change to compare and contrast with exponential and quadratic functions.

x	y
–3	–5
–2	0
–1	3
0	4
1	3
2	0
3	–5

First Differences

$0 - (-5) = 5$

$3 - 0 = 3$

$4 - 3 = 1$

$3 - 4 = -1$

$0 - 3 = -3$

$-5 - 0 = -5$

Second Differences

$3 - 5 = -2$

$1 - 3 = -2$

$-1 - 1 = -2$

$-3 - (-1) = -2$

$-5 - (-3) = -2$

In a linear function, the first difference is constant.

In a quadratic function, the second difference is constant.

2 Exploring Constant Change

MODULE 2: Assessment Summary

Topic	Topic Title	Name	Administered	TEKS*
1	Linear Functions	End of Topic Assessment	After Topic 1	**A.2A** A.2B **A.2C** A.3A **A.3B** **A.3C** A.4A A.4B A.4C A.12B
2	Transforming and Comparing of Linear Functions	End of Topic Assessment	After Topic 2	**A.2A** **A.2C** A.2E A.2F A.2G **A.3C** A.3E

*Bold TEKS = Readiness Standard

ALGEBRA I

TOPIC 1

A set of points in a straight line can be modeled by a linear function.

Linear Functions

Linear Functions

How are the key concepts of *Linear Functions* organized?

In *Linear Functions*, students increase their fluency in analyzing linear relationships. Students begin by focusing on the patterns that are evident in certain data sets and use linear functions to model those patterns.

First, students explore a data set, represent it with a scatterplot, and estimate lines of best fit based on observable patterns. They learn about the *Least Squares Method* and how to use technology to determine the linear regression function. Students need access to graphing technology that can render such equations. Students use the linear regression functions to make and assess predictions, and they differentiate between *extrapolation* and *interpolation*.

Next, students learn to use correlation coefficients to measure the appropriateness of a linear fit. They analyze the formula for the correlation coefficient but use graphing technology to actually calculate the value. Students differentiate between *correlation* and *causation*, recognizing that a correlation between two quantities does not necessarily mean that there is a causal relationship.

Students prove that the common difference of an arithmetic sequence and the slope of the corresponding linear function are both constant and equal. They connect the average rate of change of a linear function to the slope of a line.

Students review slope-intercept form that was developed in Grade 8, and use the slope formula to derive the point-slope form. They write equations of lines, including horizontal and vertical lines, and graph lines presented in slope-intercept, point-slope, and standard forms.

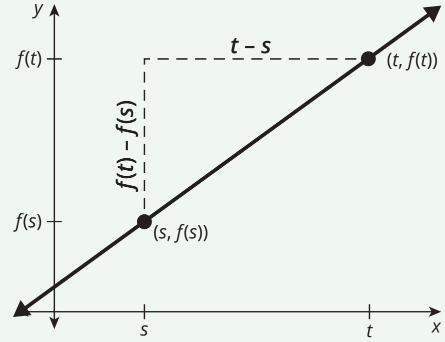
Students determine whether functions presented in different forms are linear functions. They examine the structure of tables, using the defining characteristics of linear functions to determine whether given tables represent functions. Students explore a scenario represented by equations in three forms: $f(x) = ax$, $f(x) = a(x - c)$, and $f(x) = a(x - c) + d$. They compare the graphs to determine what their differences indicate about the functions and the scenarios they model, intuitively transforming functions on the coordinate plane. While they only consider dilations and vertical translations in the abstract, this comparison gives students an intuitive understanding about the horizontal translation of a linear function. In addition to graphic transformations, students analyze the structure of each equation. Focusing on the

Math Representation

The average rate of change, $\frac{f(t) - f(s)}{t - s}$, represents the ratio of the change in the output values to the change in the corresponding input values.

The slope formula, $m = \frac{\Delta y}{\Delta x}$ or $\frac{y_2 - y_1}{x_2 - x_1}$, represents the ratio of the change in the dependent quantities to the corresponding independent quantities.

The formulas represent the same ratio.



general and factored form of the equation places linear functions within the wider framework of polynomial functions. Students relate the structure of the general form $f(x) = ax + b$ to the slope-intercept form of a line. Likewise, they interpret the meaning of the terms in $f(x) = a(x - c)$ and make connections to the slope and zeros of the function.

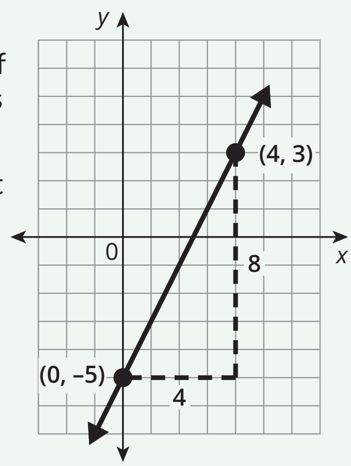
Math Representation

You can calculate the slope of a line using two ordered pairs and the formula,
$m \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$, where the first point is (x_1, y_1) and the second point is (x_2, y_2).

Consider the line with the two points $(0, -5)$ and $(4, 3)$.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3 - (-5)}{4 - 0} = \frac{8}{4} = 2$$

The slope of the line is 2.



What is the entry point for students?

Over the last few years, students have had extensive experience with linear relationships. They have represented relationships using tables, graphs, and equations. They understand slope as the steepness and direction of a graph and as a unit rate of change.

In the first topic of this course, *Quantities and Relationships*, students learned to use function notation to represent the equation of a function.

Why is *Linear Functions* important?

Functions are objects that can be represented in a multitude of ways—using scenarios, tables of values, equations, or graphs. This topic explores how a linear function is represented and interpreted in each form, making connections between each. From this topic, students should understand the key and defining characteristics of a linear function in each of these forms. Solving equations using horizontal lines on the graph lays the foundation for solving systems of linear equations as well as the more complicated nonlinear equations that they will encounter throughout future courses.

Math Representation

System of Two Linear Equations	Consistent Systems		Inconsistent Systems
Description of *y*-Intercepts	Same or different *y*-intercepts	Same *y*-intercepts	Different *y*-intercepts
Number of Solutions	One solution	Infinite solutions	No solutions
Description of Graph	Lines intersect	Lines are the same	Lines are parallel

The *x*- and *y*-values of the point of intersection of the two graphs makes both equations true.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Linear Functions* when they can:

- Determine when a data set should be modeled by a linear function.
- Use technology to determine the linear regression function and correlation coefficient for a data set.
- Make predictions using the line of best fit.
- Interpret the meaning of the slope and y-intercept of a linear regression function in terms of the problem context.
- Explain the correlation coefficient as a measure of how well a function fits a data set.
- Understand that the sign of a correlation coefficient indicates the direction of the association and that the magnitude indicates the strength of the fit.
- Analyze a function on a scatterplot and its correlation coefficient to determine whether a function is an appropriate fit for a data set.
- Recognize that correlation does not imply causation.
- Recognize that all arithmetic sequences are linear functions and rewrite an arithmetic sequence as a linear function using function notation.
- Write equations of lines in the appropriate form: slope-intercept, point-slope, or standard form.
- Graph lines from an equation written in slope-intercept, point-slope, or standard form.
- Write and solve equations involving direct variation.
- Connect average rate of change of a linear function to the slope of a line.
- Construct a linear function from a scenario, table of values, graph, or arithmetic sequence.
- Interpret the real-world meaning of the coefficients and constants in a linear function.
- Recognize a linear relationship and write an equation to represent that situation.
- Identify the variables and quantities of a linear function.
- Determine key features of graphs of linear functions including domain, range, x-intercept, y-intercept, zeros and slope.

NEW KEY TERMS

- conjecture [conjetura]
- first differences
- average rate of change
- point-slope form
- standard form [forma estándar/general]
- polynomial [polinomio]
- degree
- leading coefficient
- zero of a function [cero de una función]
- Least Squares Method
- centroid [centroide]
- linear regression function [función de regresión lineal]
- interpolation [interpolación]
- extrapolation [extrapolación]
- correlation [correlación]
- correlation coefficient [coeficiente de correlación]
- coefficient of determination [coeficiente de determinación]
- causation [causalidad]
- necessary condition [condición necesaria]
- sufficient condition [condición suficiente]
- common response [respuesta común]
- confounding variable [variable de confusión]

How do the activities in *Linear Functions* promote student expertise in the TEKS mathematical process standards?

Every topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Throughout *Linear Functions*, students model real-world situations with multiple representations (TEKS A.1A, TEKS A.1D, TEKS A.1E). They use the problem-solving model to think through and solve real-world problems (TEKS A.1B). Students are expected to decontextualize real-world situations as they create tables, equations, and graphs (TEKS A.1E), and then re-contextualize them to interpret their meanings (TEKS A.1A). They must consider the structure of the equation, quantities, units, and scales when creating graphs in an effort to create precise and useful representations of problem situations (TEKS A.1A, TEKS A.1E, TEKS A.1F, TEKS A.1G). Students must also choose which representation to use when representing a function (TEKS A.1C).

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Have students pair up and enact a role-play scenario with each student using a different language but with corresponding mathematical cognates. A potential scenario students can act out is one student pretending not to fully understand the other and using their knowledge of cognates to reconstruct and restate what the other student is saying.

2 Exploring Constant Change

TOPIC 1: Linear Functions

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.C, 1.D, 1.E, 1.H, 2.C, 2.D, 2.E, 2.G, 3.A, 3.B, 3.C, 4.A, 4.C, 4.D, 4.F, 4.K, 5.E, 5.G

Topic Pacing: 22 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Least Squares Regressions	Students informally determine a line of best fit by visual approximation of a hand-drawn line. They are then introduced to a formal method to determine the linear regression function of a data set using graphing technology; the mathematics behind the calculator function is explained using the related terms *Least Squares Method*, *linear regression function*, and *centroid*. Students then use the linear regression function to make predictions and distinguish between the terms *interpolation* and *extrapolation*. **Materials Needed:** Uncooked Spaghetti, Graphing Technology, Problem-Solving Model Graphic Organizer	**A.3C** A.4C A.12A	2
2	Correlation	This lesson provides several definitions related to correlations. The terms *correlation* and *correlation coefficient* are defined. The formula to compute the correlation coefficient is given; however, students are only required to use technology to determine the value of r or to estimate correlation coefficients from a list of choices. The distinction is then made between the meanings of r and r^2, the coefficient of determination. Students use these terms to make decisions regarding the model that best fits the data. It is suggested that students revisit the modeling process as they solve these problems in context. The terms *causation*, *necessary condition*, and *sufficient condition* are defined. Examples are provided to help students see the difference between correlation and causation. The terms *common response* and *confounding variable* are defined as relationships often mistaken for causation. **Materials Needed:** Graphing Technology, Problem-Solving Model Graphic Organizer	A.4A A.4B A.4C	2

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	**Making Connections Between Arithmetic Sequences and Linear Functions**	Students are provided two sequences. They must identify each sequence as *arithmetic* or *geometric*, write the explicit formula for the sequence, and graph the sequence. Students then list and compare characteristics of each graphical representation. The remainder of the lesson focuses on connecting arithmetic sequences to linear functions. Students match the explicit formulas for arithmetic sequences and their graphs. A worked example demonstrates how to rewrite an arithmetic sequence in explicit form as a linear function in slope-intercept form. Students then use the context of stacking chairs to make connections among the terms of the explicit formula of a sequence and the linear function that models it. Students compare the terms of each equation and recognize that the common difference and the slope are constant and equal; however, the first term of the sequence is equal to $f(1)$ rather than the y-intercept of the linear function. Using tables of values for this context, *first differences* is defined as a strategy to determine if a relationship is linear. Students move from the concrete example to generalize that the constant difference of an arithmetic sequence is equal to the slope of the corresponding linear function by completing an algebraic proof. Next, *average rate of change* is defined and presented graphically as a method to determine the unit rate using non-consecutive x-values. Students solidify these new concepts by revisiting the sequences from the start of the lesson, practicing their newly-developed skills, and verifying their conclusions. The special case of a constant function is then addressed. Finally, students complete a graphic organizer to summarize the characteristics and representations of linear functions. **Materials Needed:** None	**A.2A** A.2B **A.2C** A.3A **A.3B** A.12D	3
4	**Point-Slope Form of a Line**	Students use the slope formula to derive the point-slope form of a linear equation. They write equations in point-slope and slope-intercept form given different sets of information: a table of values, two points, a context, a slope and the y-intercept, a slope and a point, a graph with a visible y-intercept, and a graph with a non-visible y-intercept. Students explore the slopes, intercepts, and equations of horizontal and vertical lines. Finally, they match equations written in slope-intercept or point-slope form with contexts and tables. **Materials Needed:** Scissors, Problem-Solving Model Graphic Organizer, Representation Cards (located at the end of the lesson)	A.2B **A.2C** A.3A **A.3C**	2

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
5	**Using Linear Equations**	Students use three different forms of a linear equation to graph linear relationships. First, they learn how to use the slope-intercept and point-slope forms of a line to graph. Students explore the standard form of a linear equation and connect relationships among the coefficients of the standard form with the x-intercept, y-intercept, and slope of a line. They then practice writing and graphing equations in standard form. Finally, students identify the slope and intercept of linear equations in different forms and evaluate the usefulness of each form of a linear equation. **Materials Needed:** Straightedges	A.2B **A.2C** A.3A **A.3C**	3
6	**Making Sense of Different Representations of a Linear Function**	Students determine whether tables of values with non-consecutive input values represent linear functions. They evaluate functions and analyze worked examples that demonstrate how to solve equations algebraically and graphically. For the remainder of the lesson, students deal with a context, a graph, and two translations of the graph based on additions to the context. They focus on two equivalent linear functions, one written in general form, $f(x) = ax + b$, and the other written in factored form, $f(x) = a(x - c)$. Students interpret the meaning of the terms of each function and analyze their structure. The form $f(x) = ax + b$ relates to the slope-intercept form of a line, while $f(x) = a(x - c)$ connects with the slope and zero of the function. Linear functions are placed within the wider framework of polynomial functions. The terms *polynomial*, *degree*, *leading coefficient*, and *zero of a function* are defined, setting a frame of reference for future work with other functions. Students use a graphic organizer to summarize four representations—general form, factored form, graph, and table—of a linear function. **Materials Needed:** Problem-Solving Model Graphic Organizer	**A.2C** A.2D A.3A **A.3C** A.3E A.3F A.12A A.12B	3
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				6

*Bold TEKS = Readiness Standard

MODULE 2, TOPIC 1 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: **A.3C**, A.4C, A.12A **LESSON 1** Least Squares Regression **GETTING STARTED** **ACTIVITY 1**	**LESSON 1** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.4A, A.4B, A.4C **LESSON 2** Correlation **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 2** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.2A,** A.2B, **A.2C,** A.3A, **A.3B,** A.12D **LESSON 3** Making Connections Between Arithmetic Sequences and Linear Functions **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 3** continued **ACTIVITY 4** **ACTIVITY 5** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
TEKS: A.2B, **A.2C**, A.3A, **A.3C** **LESSON 4** Point-Slope Form of a Line **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 4** continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.2B, **A.2C**, A.3A, **A.3C** **LESSON 5** Using Linear Equations **GETTING STARTED** **ACTIVITY 1**	**LESSON 5** continued **ACTIVITY 2** **ACTIVITY 3**

*Bold TEKS = Readiness Standard

Day 16	Day 17	Day 18	Day 19	Day 20
LESSON 5 continued **ACTIVITY 4** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.2C**, A.2D, A.3A, **A.3C**, A.3E, A.3F, A.12A, A.12B **LESSON 6** Making Sense of Different Representations of a Linear Function **GETTING STARTED** **ACTIVITY 1**	**LESSON 6** continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 6** continued **ACTIVITY 4** **TALK THE TALK**

Day 21	Day 22
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are six Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice set of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Least Square Regressions

MATERIALS

Uncooked Spaghetti
Graphing Technology
Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students informally determine a line of best fit by visual approximation of a hand-drawn line. They are then introduced to a formal method to determine the linear regression line of a data set using graphing technology; the mathematics behind the calculator function is explained using the related terms *Least Squares Method*, *linear regression function*, and *centroid*. Students then use the line of best fit to make predictions and distinguish between the terms *interpolation* and *extrapolation*.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
The student is expected to:

A.3C graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

(4) The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data.
The student is expected to:

A.4C write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

(TEKS continued on next page)

ELPS

(1) Learning Strategies
The student is expected to:

(C) use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary.

(2) Listening
The student is expected to:

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

(4) Reading
The student is expected to:

(C) develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

The student is expected to:

A.12A decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

ESSENTIAL IDEAS

- Interpolation is the process of using a regression function to make predictions within the data set.
- Extrapolation is the process of using a regression function to make predictions beyond the data set.
- A linear regression function is the line of best fit that minimizes the squares of the distances of the points from the line.
- You can use regression methods to build linear functions to model data.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Frozen Yogurt . . . When It's Freezing? 15–20 minutes

ESTABLISH A SITUATION

Students analyze data by creating a scatterplot and using a piece of spaghetti to estimate the line of best fit. They readjust the line of best fit (piece of spaghetti) as each data point is added to the scatterplot. Students conclude that with only one data value, an infinite number of lines are possible, and with two data values, only one line is possible. Once a third non-collinear point is introduced, students must make judgments about the appropriate position for the piece of spaghetti for the line of best fit. Students then estimate a line of best fit for the entire data set and interpret its meaning in terms of the problem situation.

DEVELOP

Activity 1.1: A Line Of Best Fit 25–30 minutes

INVESTIGATION, PEER WORK ANALYSIS

Students informally determine a line of best fit by visual approximation of a hand-drawn line and use their equation to make predictions. They are introduced to a formal method to determine the linear regression function of a data set using graphing technology; the mathematics behind the calculator function is explained using the related terms *Least Squares Method*, *linear regression function*, and *centroid*. Students calculate the *linear regression function* via graphing technology and use the function to make predictions. Finally, they compare the two sets of predictions.

DAY 2

Activity 1.2: Making Predictions 15–20 minutes

REAL-WORLD PROBLEM SOLVING, MATHEMATICAL PROBLEM SOLVING

Students use graphing technology to generate a linear regression function and then interpret the contextual and mathematical meanings of each element of the equation.

Activity 1.3: Making Predictions Within And Outside a Data Set 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students analyze a data set and use technology to create a linear regression function. They then make predictions with values that lie within the parameters of the given domain and some predictions from values that lie outside the range of the given domain. The terms *interpolation* and *extrapolation* are defined.

DEMONSTRATE

Talk the Talk: Tell Me Ev-ery-thing 5–10 minutes

GENERALIZATION

Students discuss the use and accuracy of the linear regression function for making predictions for data points outside of the domain of the given data set.

Getting Started

ENGAGE

Frozen Yogurt . . . When It's Freezing?

Facilitation Notes

In this activity, students analyze data by creating a scatterplot and using a piece of spaghetti to estimate the line of best fit. They readjust the line of best fit (piece of spaghetti) as each data point is added to the scatterplot. Students conclude that with only one data value, an infinite number of lines are possible, and with two data values, only one line is possible. Once the third non-collinear point is introduced, students must make judgments about the appropriate position for the piece of spaghetti for the line of best fit. Students then estimate a line of best fit for the entire data set and interpret its meaning in terms of the problem situation.

Ask a student to read the introduction aloud. Discuss the scenario and directions as a class before distributing a piece of spaghetti to each student.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How many lines go through one point? Two points? • What is another way to explain how to determine the independent and dependent quantities?
Probing	• How did you determine your line when it wasn't possible to pass through all of the points? • How does your response compare to your classmates' responses?
Seeing structure	• Why is your line increasing for this situation?

STAMP THE LEARNING

Summary

Different relationships can exist when you only analyze parts of data sets. To understand and describe relationships in data, the entire data set must be considered.

ACTIVITY 1.1

A Line Of Best Fit

DEVELOP

Facilitation Notes

In this activity, students informally determine a line of best fit by visual approximation of a hand-drawn line and use their equation to make predictions. They are introduced to a formal method to determine the linear regression function of a data set using graphing technology; the mathematics behind the calculator function is explained using the related terms *Least Squares Method*, *linear regression function*, and *centroid*. Students calculate the linear regression function via graphing technology and use the function to make predictions. They compare the two sets of predictions.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the equation for your line? • Does your line have to pass through the origin? Any of the given points? Explain your thinking. • Do your predictions seem reasonable? Why or why not? • How can you use the graph to make predictions?

Have students work with a partner or in a group to complete Questions 4 through 7. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

- Model the graphing technology process for the class so that you can respond to technology questions as they arise and help students make sense of the process.
- In addition to using the graphing technology to determine the equation, take the time to go through the process of creating the scatterplot, graphing the equation, and accessing the table of values for the equation. This allows students to see an exact graphical answer and have access to a table of values.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you determine the independent and dependent quantities from the situation? The table? The graph? • According to your equation, how much does the temperature need to increase to get one additional customer? • How can you use benchmark fractions to approximate the slope with a fraction? Interpret the fraction's meaning.
Gathering	• How do your results compare to those in Questions 2?

COMMON MISCONCEPTION

- Students sometimes confuse the two tables available with graphing technology, one with the real data entered to create the linear regression function and one with the values generated from the linear regression function. To clear the confusion, select points from each table and have students locate them on the graph.

Ask a student to read the information and definitions following Question 7 aloud and complete Question 8 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What's the centroid for the data set in this activity?
Probing	• How can you show that the line generated by technology passes through the centroid?

Have students work with a partner or in a group to complete Questions 9 and 10. Share responses as a class.

COMMON MISCONCEPTIONS

- Students are sometimes uncertain when to use the term *line of best fit* and when to use the term *linear regression function*. *Line of best fit* is a general term that can be used whether the line was estimated by being hand-drawn or calculated using a regression process. *Linear regression function* is used only when technology determines the equation of the line.
- Students sometimes think the line of best fit must pass through points on the scatterplot, especially the points containing the smallest and largest *x*-values. To counteract this thinking, note that this is not the case with the Frozen Yogurt Problem.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why does the name Least Squares Method make sense? • Why does it make sense to use technology to use the Least Squares Method?
Probing	• Explain the difference in how Linh and James chose to place their line.

DIFFERENTIATION STRATEGY

Challenge Opportunity

- Have students create a graph modeling Linh's and James's thinking.

To close the Day 1 session, have students reread the Essential Question and reach the activity summaries to the class.

STAMP THE LEARNING

Summary

Graphing technology uses the Least Squares Method to determine the line of best fit. The line of best fit can be used to model data and predict the dependent value when given an independent value.

ACTIVITY 1.2 Making Predictions

Facilitation Notes

In this activity, students use graphing technology to generate a linear regression function and then interpret the contextual and mathematical meanings of each element of the equation.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How can you tell whether a table represents a function?
Probing	• Is it easier to recognize an association from the table or graph? Explain.

DIFFERENTIATION STRATEGIES

Access for All

- Model the steps of adding the first few data points as a class. Then, have students work with their groups to add the rest of the data for the scatterplot.
- Have students read the remember note about the types of linear association. Then, remind students that they learned about these types of associations in Grade 8.

Have students work with a partner or in a group to complete Questions 6 through 10. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you know which value to substitute into the equation? • Does your prediction make sense? Why or why not?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Create additional word boxes for students to use. Depending on students' needs, create a word box for the units or a word box for the contextual meaning.

Challenge Opportunity

- Have students use their linear regression function to predict the year since 2010 when the population was 21,075. Encourage students to think about what a negative value means in this situation and whether their conclusion makes sense.

COMMON MISCONCEPTION

- Students may assume that using the least squares method resulting in a line of best fit models the problem situation at every point of the domain rather than an appropriate subset of the domain. Be sure to always ask the students whether the prediction makes sense in the particular problem situation.

STAMP THE LEARNING

Summary

A line of best fit is a way to model linear trends in real-world data.

ACTIVITY 1.3 Making Predictions Within and Outside a Data Set

Facilitation Notes

In this activity, students analyze a data set and use technology to determine a linear regression function. They then make predictions for values that lie within the given domain and outside of the given domain. The terms *interpolation* and *extrapolation* are defined.

Have students work with a partner or in a group to complete Questions 1 through 7. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Discuss a method for entering the data for the independent variable into the graphing technology. If needed, suggest they represent the start year in the data set, 2010, as 0 on their data list.
- Have students relate the terms *interpolation* and *extrapolation* with words that have the same prefixes that they already know, such as *interior* and *exterior*. Discuss how interpolation and extrapolation relate to the interior and exterior of the data points.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How is the year 2010 represented in your data set? • Which ordered pair did you use to represent the data point?
Gathering	• Why is this question considered an interpolation question? • What value did you use to represent 2013 in your equation? • Why is this question considered an extrapolation? • What value did you use to represent 2004 in your equation?
Probing	• Does a y-value greater than 100 make sense in this situation? Explain your reasoning. • Does a negative y-value make sense in this problem situation? Explain your reasoning.
Seeing structure	• Why do you think the extrapolation results do not make sense, but the interpolation results did make sense?

Summary

Lines of best fit are most appropriately used as predictors within the bounds of the domain of the given data. Caution must be exercised when using a prediction equation to make a prediction outside the boundaries of the original data set.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

TELL ME EV-ERY-THING

Facilitation Notes

In this activity, students discuss the use and accuracy of the regression line for making predictions for data points outside of the domain of the given data set.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why is it impossible for you to complete the least squares regression model by hand?
Gathering	• What is the relationship between the interpolation values and the original data set?

Have students read and answer the Essential Question on the lesson opener page.

Summary

A linear regression function can be used to make predictions. Predictions made by extrapolation will likely be less accurate than predictions made by interpolation.

STAMP THE LEARNING

1 Least Squares Regressions

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Create a graph of data points with and without technology.
- Determine an equation for a line of best fit by visual approximation of a hand-drawn line.
- Determine a linear regression function using technology.
- Make predictions about data using a linear regression function.
- Explain the calculations involved in the Least Squares Method.
- Choose a level of accuracy appropriate when reporting quantities.

NEW KEY TERMS

- Least Squares Method
- centroid
- linear regression function
- interpolation
- extrapolation

You have searched for patterns in graphs and sequences of numbers.
How can you use what you know to identify patterns in sets of data?

Sample answer:

You can model patterns in data with lines of best fit. The Least Squares Method is one way to create a linear regression function, and it is the method that graphing technology tends to use.

MODULE 2 • TOPIC 1 • LESSON 1 147

Chunking the Activity

- Read and discuss the introduction, scenario, and directions.
- Group students to complete the activity.
- Share and summarize.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1A:

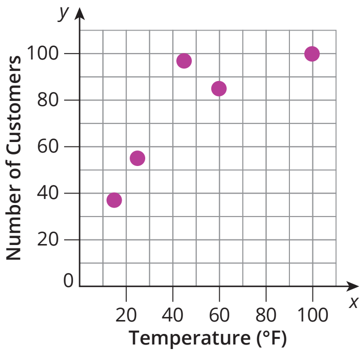
- Do students persevere in real-world problem solving?
- Do students recognize they can use linear models outside of the math classroom?

Getting Started

Frozen Yogurt . . . When It's Freezing?

A school's club for aspiring business leaders is helping a local frozen yogurt shop analyze how the business is affected by the weather. The owner is wondering whether there is a relationship between the temperature and the number of customers that buy yogurt during the 2 hours immediately after school. The school's club collected these data.

Temperature (°F)	Number of Customers
45	97
25	55
60	85
15	37
100	100



1. Construct a scatterplot of the collected data.

   a. Plot the first data point. Is there a pattern? Use a piece of spaghetti to approximate a line that models the data.

   There is no pattern with just one data point. An infinite number of lines can pass through the point.

   b. Add the second data point to the graph. Is there a pattern? Adjust the piece of spaghetti to approximate a line that models the data with the additional point.

   I can connect the data points to make a straight line.

EB STUDENT TIP

For all proficiency levels

Make sure students understand what frozen yogurt is to make sense of the relationship between temperature and number of customers. Ask students to share examples of frozen treats that they enjoy, and inquire whether anyone has any special traditions or fun routines centered around frozen treats within their family.

c. Add the third data point to the graph. Describe the pattern that you see. Approximate the line using the spaghetti.

Answers will vary based on each classroom.

d. Continue this process until all five data points are plotted and recorded in the table.

Answers will vary based on each classroom.

2. Use your linear model to describe the relationship between the temperature outside and the number of customers at the frozen yogurt shop.

There is a positive association. As the temperature increases, the number of customers increases.

MODULE 2 • TOPIC 1 • LESSON 1 149

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Questions 4–7.
- Read and discuss the information and definitions.
- Complete Question 8 as a class.
- Group students to complete Questions 9 and 10.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1C:

- Are students considering which strategies or tools to use?

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing sets of points, estimating a line of best fit, and determining a linear regression function for the line, assign Skills Practice Set A for this lesson.

ACTIVITY 1.1 A Line Of Best Fit

You have approximated the line that best represents the data with each additional data point.

1. Use the full data set and the line that you approximated to write an equation that you think best represents the data.

   Sample answer:

   $y = \frac{3}{5}x + 45$

2. Based on your equation, predict the number of customers to visit the frozen yogurt shop in the two hours after school for each given temperature.

   Sample answers based on equation in Question 1.

   a. 85°F — 96 customers

   b. 115°F — 114 customers

   c. 10°F — 51 customers

3. Compare your predictions with your classmates. Did your predictions differ from the other groups? Explain why or why not.

   The predictions varied among my classmates because we wrote different equations for the line of best fit.

You have noticed that estimating a line of best fit can give different predictions. Fortunately, with technology you can create prediction equations as well as a scatterplots from tables of data. You just need to build a data table that has an independent variable and a dependent variable.

4. Identify the independent and dependent variable. What is the significance of those designations?

   The temperature outside is the independent variable and the number of customers is the dependent variable. The independent variable represents the domain of the function, and the value of the dependent variable is generated by the value of the independent variable.

5. Use the data table and graphing technology to generate a line of best fit. What is the slope and *y*-intercept of the line and what do they represent?

   $y = 0.687x + 41.158$; the *y*-intercept means that the frozen yogurt shop could expect about 41 customers if the temperature were 0°F; the slope means that for every degree that the temperature increases, the number of customers goes up by 0.687 customers.

6. Use the new line of best fit to predict the number of customers at the frozen yogurt shop immediately after school for each given temperature.

   a. 85°F

      At 85°F, the yogurt shop should expect approximately 99 customers.

   b. 115°F

      At 115°F, the yogurt shop should expect approximately 120 customers.

   c. 10°F

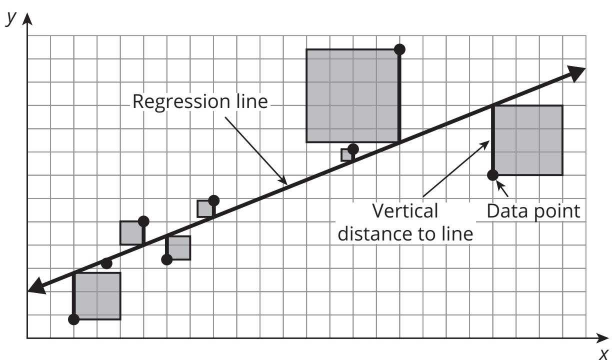
      At 10°F, the yogurt shop should expect approximately 48 customers.

7. How do your predictions compare to the predictions from the other groups?

   Our predictions are the same because we are using the same linear regression function.

The equation that your graphing technology uses to give you the line of best fit is called the **Least Squares Method**. This is a method that creates a line of best fit for a scatterplot that has two basic requirements:

- The line must contain the *centroid* of the data set. The **centroid** is a point whose *x*-value is the mean of all the *x*-values of the points on the scatterplot and its *y*-value is the mean of all the *y*-values of the points on the scatterplot.
- Even though infinitely many lines can pass through the centroid, the **linear regression function** has the smallest possible vertical distances from each given data point to the regression line. The sum of the squares of those distances are at a minimum with this line.



Optimizing Learning

This activity optimizes presenting key concepts in one form of symbolic representation.

MODULE 2 • TOPIC 1 • LESSON 1 151

8. Consider the graph of the sample linear regression function.

   a. What do the vertical lines in bold represent?

      Each vertical line represents the difference in the y-value of a data point and the y-value of the point on the linear regression function for a given x.

   b. What do the shaded squares represent? How do they relate to the Least Squares Method?

      The area of each shaded square is equal to the vertical distance squared. The line that provides the minimum sum of the area of the squares is the least squares regression line.

9. Linh and James each draw a line of best fit to model a set of data. They both record the vertical distances between each point and the line of best fit.

   Linh
   Vertical Distances: 2, 2, 2, 2, 2

   James
   Vertical Distances: 1, 1, 1, 1, 6

   Both students believe they drew the least square regression line. Who's correct? Justify your choice.

   Linh is correct. The sum of the squares of the vertical distances from her line equals 20, which is lower than the sum of the squares of James's, which is 40.

10. How does your decision in Question 9 inform you about the placement of a line of best fit using the Least Squares Method?

    A linear regression function is a line that is fairly close to all data points instead of very close to most points, but one point is very distant from the line.

152 MODULE 2 • TOPIC 1 • LESSON 1

ACTIVITY
1.2 Making Predictions

The table shows the population of a small town for each year from 2010 to 2017.

Years	2010	2011	2012	2013	2014	2015	2016	2017
Population	21,359	22,906	22,542	23,048	23,562	23,609	24,008	24,716

1. What is the range of the data set?
   24,716 − 21,359 = 3357 people

2. Determine a linear regression model for the data. Round the slope and *y*-intercept to the nearest whole number.
   $f(x) = 390x + 21{,}855$, where x represents the number of years since 2003

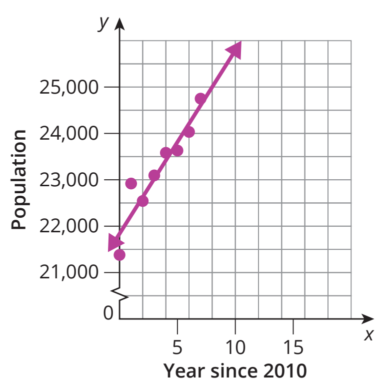
3. Identify the independent and dependent variables and their units of measure.
   The number of the years since 2010 is the independent variable and the population in number of people is the dependent variable.

4. Does the data represent a function? Does it appear that there is a specific function that could model this data set? When so, describe the function. When not, state why not.
   Each ordered pair has exactly one member of the domain paired with one member of the range, so it represents a function.

   It appears that the data is growing at a constant rate and could be modeled by a linear function.

5. Use technology to graph a scatterplot demonstrating the relationship between the year and population. What association do you notice?
   There appears to be a positive association.

6. Between which consecutive years was there a decrease in the population?
   Between 2011 and 2012 there was a decrease in the population.



Remember . . .
Data comparing two variables can show a positive association, negative association, or no association.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Group students to complete Questions 1–6.
- Check in and share.
- Group students to complete Questions 6–10.
- Share and summarize.

MODULE 2 • TOPIC 1 • LESSON 1 153

Optimizing Learning

This activity supports students in the use of graphing technology.

Activity 1.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice using linear regression to analyze scenarios, assign Skills Practice Set B for this lesson.

7. Use graphing technology to determine the linear regression function for the population. Then, sketch the data points and the line of best fit that you see.

   $f(x) = 390x + 21{,}855$, where x represents the number of years since 2010

8. What is the relationship between the equation for the line of best fit and any association you notice in the graph? Do you think that this line fits the data well?

   There appears to be a positive association, which is reflected in the positive slope of the linear regression line. The line fits the data well.

9. For each expression from your linear regression function about populations, write an appropriate unit of measure and describe the contextual meaning. Then, choose a term from the word box to describe the mathematical meaning of each part.

Word Box

- input value
- output value
- rate of change
- y-intercept

		What it Means	
Expression	**Unit**	**Contextual Meaning**	**Mathematical Meaning**
$f(x)$	Number of people	The predicted population	Output value
390	Population rise per year	The predicted change in the population per year	Rate of change
x	Number of years since 2010	Number of years since 2010	Input value
21,855	Number of people	The predicted population for the year 2010	y-intercept

10. Use your linear regression function to predict the population for the year 2024.

    2024 is 14 years after 2010, so the prediction is $f(x) = 390(14) + 21{,}855 = 27{,}315$ people

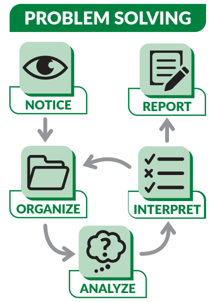
EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Students may be unfamiliar with the term *contextual meaning*. Discuss the definition of *context* and how *contextual meaning* is different from mathematical meaning. Ask students to demonstrate their understanding of the *contextual meaning* of the regression function. If students provide different wording, discuss the differences and explain how the wording that explains the *contextual meaning* can be different as long as it correctly represents the context.

ACTIVITY **1.3**

Making Predictions Within and Outside a Data Set



The music industry is constantly changing how it delivers music to its listeners. The table shows the percent of total U.S. music sales revenues from streaming.

Year	2010	2011	2012	2013	2014	2015
Percent of Total U.S. Music Sales Revenue From Streaming	7	9	15	21	27	34

1. Use graphing technology to determine the linear regression function for the data.

   Answers may vary based on rounding.

   $g(x) = 5.6x + 4.9$ where x represents the number of years since 2010, and y represents the percent of revenue from streaming.

2. Interpret the equation of the line in terms of this problem situation.

   The percentage of US music sales revenue from streaming rises about 5.6% every year since 2010, when it was about 4.9%.

When there is a linear association between the independent and dependent variables of a data set, you can use a linear regression function to make predictions within the data set. Using a linear regression function to make predictions within the data set is called **interpolation**.

3. Use your equation to predict the percent of streaming revenues in 2013. Compare the predicted value percent in 2013 with the actual value.

   21.7%; the predicted value percent is 0.7% higher than the actual value.

4. Compute the predicted value percent for 2011 and compare it with the actual value.

   10.5%; they are 1.5% apart.

Ask Yourself . . .
What is an appropriate level of accuracy needed throughout this situation?

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the problem-solving model graphic organizer.
- For Question 1, have students work in pairs, ask themselves the questions to ask from the first two steps of the model, and share their reasoning.
- Complete the remaining steps in the graphic organizer together as a class.
- Have students work in pairs and use the problem-solving model to complete Questions 2–7.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Students may not be familiar with the terms *industry*, *revenues*, and *streaming*. Discuss the meaning of these terms using vocabulary that students understand and by providing examples. Review the data table at the beginning of the problem and clarify any remaining misunderstandings about the context of the problem.

MODULE 2 • TOPIC 1 • LESSON 1 155

5. Do you think a prediction made using interpolation will always be close to the actual value? Explain your reasoning.

   Sample answer:
   Probably, as long as the line fits the data well.

To make predictions for values of x that are outside of the data set is called **extrapolation**.

6. Use the equation to predict the percent of streaming revenues:

   a. in 2040.
      $x = 30$; 172.9%

   b. in 2004.
      $x = -6$; -28.7%

7. Are these predictions reasonable? Explain your reasoning.
   No. More than 100% of music revenues from streaming isn't possible, and a negative percent from streaming isn't possible either.

EB STUDENT TIP

For all proficiency levels

Relate the terms *interpolation* and *extrapolation* to other terms students might be more familiar with, such as *interior* and *exterior.* Inform students that just as interior can refer to the space inside a boundary, interpolation can refer to predictions that are within a data set. Exterior and extrapolation, on the other hand, have outside-related meanings, with exterior referring to the space outside of a boundary and extrapolation referring to predictions outside of a data set.

The Spanish words *interior* and *exterior* are cognates and have closely related meanings and similar pronunciations to their English counterparts.

Talk the Talk

Tell Me Ev-ery-thing

You have used technology to determine linear regression functions. You have then used those linear regression functions to predict unknown values within and without a data set.

1. Why is the linear regression function generated using technology more accurate than the line of best fit that you can write using two points?

   Sample answer:
   The linear regression function generated using graphing technology uses the Least Squares Method for all possible cases and includes the centroid.

2. Why are predictions made by extrapolation more likely to be less accurate than predictions made by interpolation?

   Sample answer:
   The linear regression function best fits the data given. Data outside of the given set may vary widely due to changes in the situation, or it may not be reasonable for the data to grow the same way outside the given set.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

CC BY NC

MODULE 2 • TOPIC 1 • LESSON 1 157

158 MODULE 2 • TOPIC 1 • LESSON 1

Lesson 1 Assignment

Write

Complete each sentence with the appropriate vocabulary term.

1. A <u>data table</u> can be used to organize and display the values of two variables in a data set.

2. A <u>linear regression function</u> models the relationship between two variables in a data set by producing a line of best fit.

3. A(n) <u>linear regression function</u> is a line that best approximates the linear relationship between two variables in a data set.

4. The <u>Least Squares Method</u> is used to approximate a line of best fit by minimizing the squares of the distances of the points from the line.

5. <u>Interpolation</u> is using a linear regression function to make predictions within the data set.

6. Using a linear regression function to make predictions outside of the data set is <u>extrapolation</u>.

7. After a scatterplot is created, the <u>centroid</u> is a point with an *x*-value that is the mean of all the *x*-values of the points on the plot and a *y*-value that is the mean of all the *y*-values of the points on the plot.

Remember

Patterns in data can be modeled with lines of best fit. The Least Squares Method is one way to create a linear regression function, and it is the method that graphing technology tends to use.

MODULE 2 • TOPIC 1 • LESSON 1 ASSIGNMENT 159

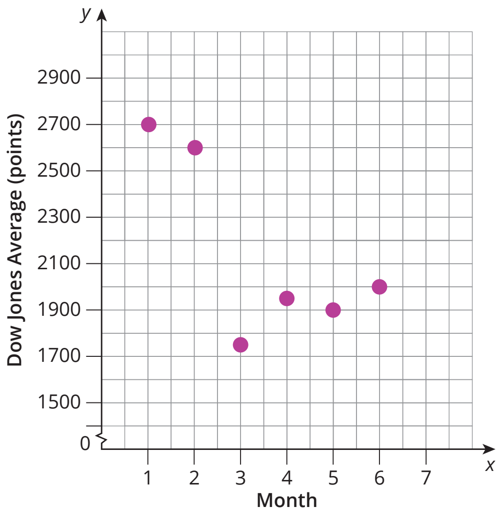
Lesson 1 Assignment

Practice

1. The table shows the Dow Jones Industrial Average on the NYC Stock Exchange over a period of six consecutive months (August through January) in 1987. On Monday, October 16, 1987, stock markets around the world crashed. This decline was the greatest one-day percentage decline in stock market history to date.

Time (months)	Dow Jones Average (points)
1	2700
2	2600
3	1750
4	1950
5	1900
6	2000

a. Create a scatterplot of the NYC Stock Exchange data. What information can you gather about the stock market from the scatterplot?



Sample answer:
The graph has a large drop in the data but then a large rise. It appears that the data will continue to increase.

b. Generate a linear regression function that would model the Dow Jones Average using the data for months 1–6.

$f(x) = -154.29x + 2690$, where x represents the time in months, and $f(x)$ represents the Dow Jones Average in points.

160 MODULE 2 • TOPIC 1 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

c. Generate a linear regression function that would model the Dow Jones Average using only the data for months 3–6.

$g(x) = 70x + 1585$, where x represents the time in months, and $g(x)$ represents the Dow Jones Average in points.

d. When will the model for the data from months 3–6 provide a better prediction?

The new model will be best for making predictions for months 3 and beyond.

e. Interpret the slope and y-intercept of the linear regression function you wrote in part (b). What do these values represent in terms of the problem situation?

The y-intercept is 1585. According to the equation, the Dow Jones Average was 1585 points in August 1987.

The slope is 70. The Down Jones Industrial Average rose by about 70 points each month.

f. Use the linear regression function from part (b) to determine the Dow Jones Average in October 1987. Is this the same as the Dow Jones Average recorded in the table? If not, explain the difference.

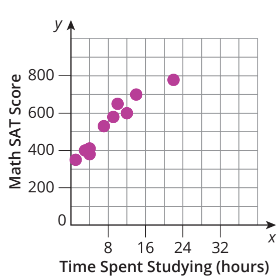
$g(2) = 15{,}725$

According to the linear regression function, the Dow Jones Average was about 15,725 points in October 1987. According to the table, the Dow Jones Average was 1750 points in October 1987. The linear regression function is the best fit of the for months 3–6, so October 1987 is outside of the data range.

2. Mr. Flores is a high school teacher and is preparing his students to take the SAT test. He collected data from 10 students who took the test last year and presented this information to the students in a table. The highest math SAT score a student can achieve is 800. Analyze the data in the table.

Time Spent Studying (hours)	Math SAT Score
1	350
22	780
12	600
14	700
4	380
10	650
9	580
3	400
7	530
4	410

a. Construct a scatterplot of the data and describe any patterns you see in the data.

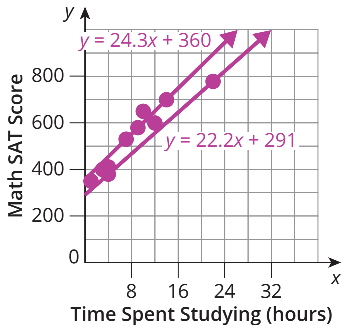


The data have a linear pattern. As the number of hours studied increases, the math SAT scores improve.

MODULE 2 • TOPIC 1 • LESSON 1 ASSIGNMENT 161

Lesson 1 Assignment

b. Determine the equation of a line passing through (14, 700) and (7, 530). Then, determine the equation of a line passing through (22, 780) and (4, 380). Graph both lines on the same graph as the scatterplot.



c. Which line seems to best fit the data? Would you use either one of these lines to make predictions about a student's math SAT score based on the amount of studying they do? Why or why not?

The first line seems to fit the data points the best because it is closer to more of the points. I would not use either line to make a prediction because neither line is the line of best fit.

d. Use graphing technology to determine the linear regression function.

$f(x) = 22.54x + 344.19$

e. Interpret the linear regression function in terms of the problem situation.

For every increase of 1 hour in studying, the math SAT score will increase by about 22.54 points. If a student studies for 0 hours, their math SAT score will be 344.19.

f. Use the linear regression function to predict the math SAT score for a student who studies for 17 hours. Did you use interpolation or extrapolation to make this prediction? Is this prediction reasonable for this problem situation? Explain your reasoning.

$y = 727.37$; interpolation

Sample explanation:
This prediction is reasonable because 17 hours of studying would probably result in a high test score.

162 MODULE 2 • TOPIC 1 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

g. Use the linear regression function to predict the math SAT score for a student who studies for 40 hours. Did you use interpolation or extrapolation to make this prediction? Is this prediction reasonable for this problem situation? Explain your reasoning.

$y = 1245.79$; extrapolation

Sample answer:
This prediction is not reasonable because the highest possible math SAT score is 800.

h. One of Mr. Flores's students comes back to him the following year and says that he studied for 15 hours for the math SAT and got a score of 610. He argues that the linear regression function predicted that he would have scored a 682. What do you think explains the discrepancy?

Sample answer:
A linear regression function model trends in data, but may not be an exact predictor for one specific case.

Prepare

Describe a possible flaw in the reasoning for each situation.

1. When I wash my hands regularly, I will not get sick.

   Sample answer:
   People who wash their hands regularly still get sick on occasion.

2. When I practice my guitar every day, I will be a rock star.

   Sample answer:
   Many people practice a guitar every day and are not rock stars.

3. When I wear my favorite football jersey to support the team, they will win the game.

   Sample answer:
   Many people wear their favorite jerseys, but the team they are supporting still loses the game.

4. When I am a good driver, I will not have an accident.

   Sample answer:
   Unfortunately, good drivers still have accidents.

MODULE 2 • TOPIC 1 • LESSON 1 ASSIGNMENT 163

164 MODULE 2 • TOPIC 1 • LESSON 1 ASSIGNMENT

2 Correlation

MATERIALS

Graphing Technology

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

This lesson provides several definitions related to correlations. The terms *correlation* and *correlation coefficient* are defined. The formula to compute the correlation coefficient is given; however, students are only required to use technology to determine the value of r or to estimate correlation coefficients from a list of choices. The distinction is then made between the meanings of r and r^2, the *coefficient of determination*. Students use these terms to make decisions regarding the model that best fits the data.

The terms *causation*, *necessary condition*, and *sufficient condition* are defined. Examples are provided to help students see the difference between correlation and causation. The terms *common response* and *confounding variable* are defined as relationships often mistaken for causation.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1E create and use representations to organize, record, and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(TEKS continued on next page)

ELPS

(1) Learning Strategies

The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(2) Listening

The student is expected to:

(E) use visual, contextual, and linguistic support to enhance and confirm understanding of increasingly complex and elaborated spoken language.

(5) Writing

The student is expected to:

(G) narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(4) The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data.

The student is expected to:

- **A.4A** calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association.
- **A.4B** compare and contrast association and causation in real-world problems.
- **A.4C** write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.

ESSENTIAL IDEAS

- A *correlation* is a measure of how well a regression model fits a data set.
- The *correlation coefficient*, *r*, is a value between −1 and 1 that indicates the type (positive or negative) of association and the strength of the relationship. Values close to 1 or −1 demonstrate a strong association, while a value of 0 signifies no association.
- *Causation* is when one event causes a second event. A correlation is a necessary condition for causation, but not a sufficient condition for causation.
- Two relationships that are often mistaken for causation are a *common response*, when some other reason may cause the same result, and a *confounding variable*, when there are other variables that are unknown or unobserved.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Associate, Formulate, Correlate! 5 minutes

CONNECT TO PRIOR KNOWLEDGE

Students describe the type of association between the independent and dependent variable of three scatterplots. They then draw a line of best fit for each graph.

DEVELOP

Activity 2.1: The Correlation Coefficient 15–20 minutes

MATHEMATICAL PROBLEM SOLVING

The term *correlation coefficient* is defined. Students are given three scatterplots and must choose the r-value that best fits each graph and describe the correlations. The formula to compute the correlation coefficient is given, but students use technology to determine the value of r.

Activity 2.2: Is it Linear? 15–20 minutes

REAL-WORLD PROBLEM SOLVING

Students analyze data, calculate linear regression functions and compute values for the correlation coefficient. They then look more closely at the data and scatterplot to decide whether the data is best described by a linear function or if another type of function may better fit the data. Students analyze data and use graphing technology to compute the r-value and r^2-value. They conclude a linear regression model is appropriate. The term *coefficient of determination* is defined. Its value is always between 0 and 1, and when expressed as a percent, it represents the percent of the data that lies close to the line of best fit.

DAY 2

Activity 2.3: Using the Correlation Coefficient to Assess a Line of Best Fit 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students analyze data and use technology to determine a linear regression function, then compute the r-value and r^2-value to determine the appropriateness of a linear regression model. In this situation, they conclude an exponential regression model may be more appropriate.

Activity 2.4: Correlation Vs. Causation 20–25 minutes

PEER WORK ANALYSIS, REAL-WORLD PROBLEM SOLVING

Students are provided contexts and make judgments about correlation vs. causation. Causation is when one event causes a second event. A correlation is a necessary condition for causation, but a correlation is not a sufficient condition for causation. While determining a correlation is straightforward, using it to establish causation is very difficult.

DEMONSTRATE

Talk the Talk: Correlations R Us 5–10 minutes

EXIT TICKET PROCEDURES

Students calculate the linear regression function for two data sets and decide which of the linear regression functions is the better fit based upon the correlation coefficient.

Getting Started

ENGAGE

Associate, Formulate, Correlate!

Facilitation Notes

In this activity the students analyze scatterplots and describe any associations between the independent and dependent variables, drawing lines of best fit when possible.

Have students work independently or with a partner to complete this activity. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

- Provide explicit directions to determine the type of association in a graph.
    - It is customary to read the graph from left to right, as in reading text. For this reason, it makes sense to say, “As the *x*-values are increasing, the *y*-values are increasing/decreasing.”
    - An increasing graph relates to a positive association, while a decreasing graph relates to a negative association.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• As the weight of a vehicle increases, how would you describe the change in the miles per gallon? • As the IQ of a person increases, how would you describe the height of a person? • As the time spent studying increases, how would you describe the grade on the Algebra test? • Does each scatterplot show a negative, positive, or no association? How do you know?

STAMP THE LEARNING

Summary

Scatterplots can show a positive, negative, or no association between the independent and dependent variables.

ACTIVITY 2.1

The Correlation Coefficient

DEVELOP

Facilitation Notes

In this activity, students are given scatterplots and must choose the *r*-value that best fits each graph and describe the correlation. The term *correlation coefficient* is defined, and the formula to calculate the correlation coefficient is given. Students are not expected to use the formula, but rather, use graphing technology to determine the value of *r*.

Ask a student to read the introduction and definitions aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

COMMON MISCONCEPTION

- Students sometimes incorrectly believe that a negative correlation means there is no correlation and is not helpful in making predictions. Emphasize that a negative correlation supplies very good information; it means that as the *x*-values increase, the *y*-values decrease. This is just as valuable as knowing that as the *x*-values increase, the *y*-values increase. What is not valuable for prediction purposes is when there is no correlation, and this occurs when the *r*-value is close to zero. Help students understand and remember this concept by connecting *no* and *zero*.

DIFFERENTIATION STRATEGY

Just in Time Support

- Provide a process to selecting the most appropriate *r*-value. First, determine whether the correlation coefficient should be positive or negative. Then, decide which value makes sense for how the points are scattered.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the relationship between the terms *association* and *correlation*?
Gathering	• How can you tell whether a correlation is positive or negative? • Once you determine the sign, how did you identify the numeric value?
Seeing structure	• Describe how scatterplots, with correlation coefficients of 1 and −1 look alike? Look different?

Ask a student to read the information following Question 1 aloud. Discuss as a class.

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials Needed: Graphing Technology

- Have students predict the correlation coefficients for data representing horizontal and vertical lines. Then, have them check their predictions using graphing technology.

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What would a scatterplot of these data look like? • Why is the *r*-value close to 1? • Why is the *r*-value positive? • Does the slope of the line of best fit need to be close to 1 for the correlation coefficient to be close to 1?

STAMP THE LEARNING

Summary

The correlation coefficient, *r*, is a value between −1 and 1, which indicates how close the data are to the graph of the regression function. The closer the correlation coefficient is to 1 or −1, the stronger the relationship is between the two variables.

ACTIVITY 2.2

Is it Linear?

Facilitation Notes

In this activity, students analyze a data set, create a scatterplot, determine the line of best fit, interpret the equation, and use the correlation coefficient to determine whether a linear function is an appropriate model for the data. The *coefficient of determination*, r^2, is defined and explained in comparison to the value of the correlation coefficient, *r*.

Use this activity as an opportunity to practice using the problem-solving model. The steps of the problem-solving model are: *Notice and Wonder*, *Organize and Mathematize*, *Predict and Analyze*, *Test and Interpret*, and *Report*. An expanded explanation of the mapping of the problem-solving model onto this activity is as follows:

- Question 1: As students look at the table and identify independent and dependent quantities, ask them to *notice* and *wonder* about the data trend (increases in both). Encourage them to wonder about the type of association this might suggest (positive).
- Question 2 part (a): As students sketch and label the scatterplot, point out that a scatterplot is one of many methods used to *organize* and *mathematize* a set of data points.
- Question 2 part (b): As students think about the behavior of this graph, and possibly associate it with a linear regression function, they are *analyzing* the scatterplot and *predicting* the algebraic model of the problem situation before verifying it with a correlation coefficient.
- Question 3: As students determine and *interpret* the linear regression function, the correlation coefficient *tests* the appropriateness of this equation representing the line of best fit for this problem situation.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

DIFFERENTIATION STRATEGIES

Challenge Opportunity

- As you are sharing responses as a class, have students explain how the questions map to the problem-solving model.
- Provide students the context and table, then ask them to predict the monthly rent given another monthly net income. Do not provide Questions 1 through 4 to scaffold their thinking. Afterwards, have students reflect upon how their thinking coincided with the problem-solving model. Did they benefit from scaffolded questions? Are they able to begin to use the problem-solving model to guide themselves through an efficient solution path?

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Does the data set appear to have a positive or negative correlation? • How does a positive correlation relate to the situation? • Can you tell from the table that a line represents this situation? Explain your thinking.
Seeing structure	• What does the slope represent in terms of the problem situation? The y-intercept? • Explain how the correlation coefficient relates to the scatterplot.
Gathering	• What different information do the correlation coefficient and the coefficient of determination provide?
Probing	• What is the range of possible values for the coefficient of determination? Why does this make sense?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

The coefficient of determination represents the percentage of variation of the observed values of the data points from their predicted values. The problem-solving model provides a structure to solve problems.

ACTIVITY 2.3 Using the Correlation Coefficient to Assess a Line of Best Fit

Facilitation Notes

In this activity, students analyze data and use technology to determine a linear regression function, then compute the r-value and r^2-value to determine the appropriateness of a linear regression model. In this situation, they conclude an exponential regression model may be more appropriate.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

DIFFERENTIATION STRATEGY

Challenge Opportunity

- As you are sharing responses as a class, have students explain how the questions map to the problem-solving process.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Use the situation and table to explain why the slope is negative. • Why is the coefficient of determination positive? Less than the correlation coefficient?
Gathering	• Are you surprised by the shape of the scatterplot? Why or why not?
Probing	• What function do you think models the relationship in the data? • Which sources of information should you use to determine whether a line of best fit is appropriate?

STAMP THE LEARNING

Summary

The r-value, r^2-value, table of values, scatterplot, and context are all helpful in determining the function that best fits a data set.

ACTIVITY 2.4

Correlation Vs. Causation

Facilitation Notes

In this activity, students are provided contexts and make judgments about correlation vs. causation. Students then learn that a correlation is a necessary condition for causation, but a correlation is not a sufficient condition for causation.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is another possible reason that both smartphones and flat-screen television sales have increased? • What is another possible reason that both NFL salaries and the weight of NFL players have increased? • Why does this conclusion make more sense than the other ones Jackson and Ricardo made?

Ask a student to read the information and definitions following Question 3. Discuss as a class.

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 4

Gathering	• What is the difference between correlation and causation? • What is meant by the phrase necessary but not sufficient? • Why is determining a correlation straightforward? • Why is it more difficult to determine causation? • What is the difference between Question 4, parts (a) and (b)?
Probing	• Do people who do not wash their hands get sick more often? • Has it been proven that washing hands prevents the spread of germs? • What extra step do you need to determine causation? • Why does it make sense to check that there is a correlation first?

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 5

Probing	• Do people who spend long periods of time outside get sun burns? • Has it been proven that sun exposure causes sun burns?

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 6

Gathering	• How would you measure absenteeism? Poor performance?
Probing	• What other factors might affect poor performance? • How can you ensure it is absenteeism rather than other factors that cause poor performance?

COMMON MISCONCEPTION

- Students may think that correlation implies causation. To help students understand this is not true, emphasize other factors that could affect each situation.

Have students work with a partner or in a group to read the definitions after Question 6 and complete Question 7. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Provide an example of a common response and a confounding variable for the previous questions in this activity.
Probing	• Does a correlation between these variables make sense? Explain your reasoning. • Why is it more difficult to determine causation rather than correlation?

DIFFERENTIATION STRATEGY

Challenge Opportunity

- Have students share examples from their own life experiences of a situation when a false conclusion was reached due to a common response or a confounding variable.

STAMP THE LEARNING

Summary

A correlation is a necessary condition for causation, but a correlation is not a sufficient condition for causation.

DEMONSTRATE

Talk the Talk

CORRELATIONS R US

Facilitation Notes

In this activity, students determine linear regression functions and their corresponding correlation coefficients and coefficients of determination for two sets of data. This information is used to decide which linear regression function has a stronger correlation and better fit.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why does one equation have a positive slope and the other have a negative slope? • Why isn't the y-intercept of the linear regression function 24 as indicated in the table for Set A? • Why does a negative y-intercept make sense for Set B? • What is the relationship between the signs of the correlation coefficients and the lines? • Use your responses to describe the scatterplots of Set A and Set B.

Have students read and answer the Essential Question on the lesson opener page.

Summary

STAMP THE LEARNING

Data can sometimes be modeled by a linear regression function. The correlation coefficient and the coefficient of determination assess how well the regression equation fits the data.

2 Correlation

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Determine the correlation coefficient using technology.
- Interpret the correlation coefficient for a set of data.
- Understand the difference between r and r^2.
- Understand the difference between correlation and causation.
- Understand necessary conditions.
- Understand sufficient conditions.
- Choose a level of accuracy appropriate when reporting quantities.

NEW KEY TERMS

- correlation
- correlation coefficient
- coefficient of determination
- causation
- necessary condition
- sufficient condition
- common response
- confounding variable

You have learned how to write a line of best fit relating two variables using the Least Squares Method.

Is there a way to measure the strength of the relationship between the variables?

Sample answer:

You can model sets of data by using a linear function called a regression function. You can also calculate a numeric summary called the correlation coefficient to determine the strength and the direction of the relationship between the two variables.

MODULE 2 • TOPIC 1 • LESSON 2 165

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

EB STUDENT TIP

For “Intermediate” and higher proficiency levels

Review the meaning of the phrase *line of best fit* by breaking it down and first discussing the meaning of the phrase *best fit*. When you feel students have a solid grasp, connect this shared understanding back to the entire phrase within the context of a linear regression function.

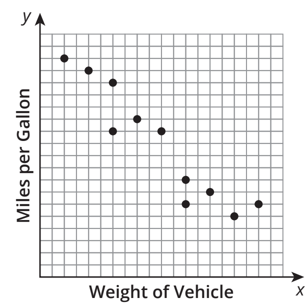
Getting Started

Associate, Formulate, Correlate!

Consider each relationship shown.

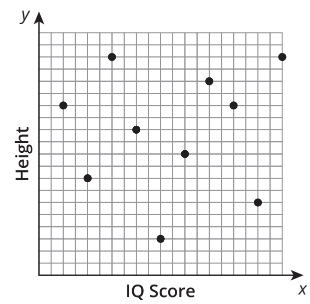
1. Describe any associations between the independent and dependent variables, and then draw a line of best fit, if possible.

a.



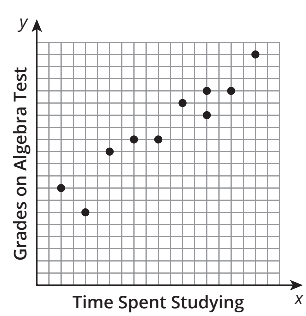
There is a negative association between weight and miles per gallon. As weight increases, miles per gallon decreases.

b.



There is no association between IQ score and height. No line of best fit would make sense.

c.



There is a positive association between time spent studying and grades on an algebra test. As time spent studying increases, grades on an algebra test increase.

EB STUDENT TIP

For all proficiency levels

Help students understand the word *correlation* by connecting it with its root word *relate* and having them practice using it in context.

Beginning: Discuss familiar examples of related things, such as height and shoe size for a *positive correlation* and screen time on an electronic device and remaining battery life for a *negative correlation.* Explain *no correlation* with unrelated pairs, such as the color of a car and its speed. Display corresponding scatterplots for these examples and consider having students place images or drawings on the plots to demonstrate their understanding. For example, students might place visual depictions of various shoe sizes or battery icons with different amounts of power remaining along a plot. Have students point when prompted or say the

(continued on next page)

166 MODULE 2 • TOPIC 1 • LESSON 2

ACTIVITY 2.1 The Correlation Coefficient

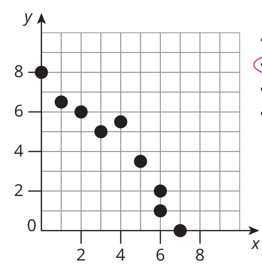
A measure of how well a regression model fits a set of data is called **correlation**. The **correlation coefficient** is a value between −1 and 1, which indicates how close the data are to the graph of the regression function. The closer the correlation coefficient is to 1 or −1, the stronger the relationship is between the two variables. The variable *r* is used to represent the correlation coefficient.

The correlation coefficient falls between −1 and 0 when the data show a negative association or between 0 and 1 when the data show a positive association.

1. Determine whether the points in each scatterplot have a positive correlation, a negative correlation, or no correlation. Four possible *r*-values are given. Circle the *r*-value you think is most appropriate. Explain your reasoning for each.

The closer the *r*-value gets to 0, the less of a linear relationship there is in the data.

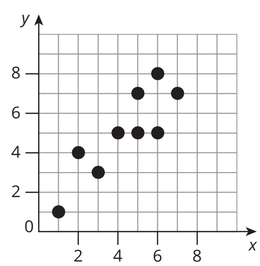
a.



- $r = 0.9$
- $r = -0.9$
- $r = 0.09$
- $r = -0.09$

$r = -0.9$; The correlation coefficient of −0.9 is reasonable because there is a negative linear relationship and the relationship between the variables is really strong so the correlation coefficient of −0.9 is closest to −1.

b.



- $r = 0.7$
- $r = -0.7$
- $r = -0.7$
- $r = -0.07$

$r = 0.7$; The correlation coefficient of 0.7 is reasonable because there is a positive linear relationship and the relationship between the variables is strong so the correlation coefficient of 0.7 is closest to 1.

Chunking the Activity

- Read and discuss the definitions.
- Group students to complete Question 1.
- Check in and share.
- Read and discuss the formula.
- Group students to complete Question 2.
- Share and summarize.

Student Look-Fors
Whether students are demonstrating proficiencies related to TEKS A.1D:

- Do students represent their mathematical reasoning with appropriate representations?
- Can students communicate how a chosen representation demonstrates their reasoning?
- Can students identify the advantages and disadvantages of different representations?

EB STUDENT TIP *(continued)*

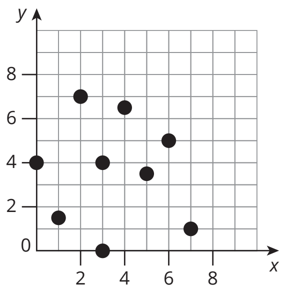
corresponding phrase of "(positive/negative/no) correlation" as they point to a scatterplot.

Intermediate: Have students discuss scatterplots based on familiar data points using a sentence frame such as: "When _________ increases and _________ also increases, this shows a positive correlation." For negative correlation: "If _________ increases but _________ decreases, this shows a negative correlation." For no correlation, use "There is no correlation between _________ and _________ because their changes do not relate."

Advanced/Advanced High: Have students develop their own examples of the different kinds of correlations, creating plausible scatterplots with written explanations for their reasoning in choosing each example, making sure to use the words *relate* and *correlate*.

MODULE 2 • TOPIC 1 • LESSON 2 167

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice examining scatterplots to determine whether they have positive, negative or no correlation, and estimating *r*-values, assign Skills Practice Set A for this lesson.

c. 

- $r = 1$
- $r = 0.5$
- $r = 0.01$
- $r = -0.1$

$r = 0.01$; The correlation coefficient of 0.01 is reasonable because the relationship between the variables is very weak, so the correlation coefficient of 0.01 is farthest from 1.

You can calculate the correlation coefficient of a data set using the formula:

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Fortunately graphing technology can do this arithmetic. Previously you used graphing technology to determine the linear regression function using the Least Squares Method. Along with calculating the equation for the line, graphing technology also calculated the value *r*, the correlation coefficient.

Let's use technology to compute the value of the correlation coefficient.

2. Consider the data set $(-3, -3)$, $(1, 2)$, and $(3, 4)$.

   a. Use technology to compute the correlation coefficient.

      $r = 0.999$

   b. Interpret the correlation coefficient of the data set.

      There is a very strong positive linear correlation.

168 MODULE 2 • TOPIC 1 • LESSON 2

ACTIVITY
2.2 Is It Linear?

A group of friends completed a survey about their monthly income and how much they pay for rent each month. The table shows the results.

Monthly Net Income (dollars)	Monthly Rent (dollars)
1400	450
1550	505
2000	545
2600	715
3000	930
3400	1000

Ask Yourself . . .
What do you notice as you read through the data?

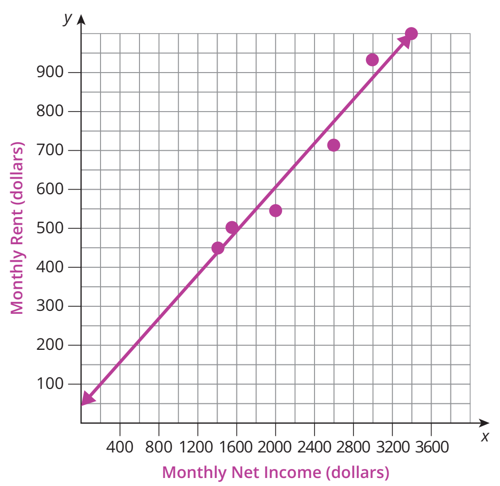
1. Identify the independent and dependent variables in this problem situation.

   Independent variable: monthly net income

   Dependent variable: monthly rent

2. Construct a scatterplot of the data using technology.

   a. Sketch and label the scatterplot.



   b. Do you think a linear regression function would best describe this situation? Explain your reasoning.

   Yes. The graph appears to be linear.

Chunking the Activity

- Read and discuss the situation.
- Group students to complete Questions 1–4.
- Check in and share.
- Read and discuss the definition.
- Return to the lesson opener and read the Essential Question.

MODULE 2 • TOPIC 1 • LESSON 2 169

Question 3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing linear regression functions for data sets presented as tables, assign Skills Practice Set B for this lesson.

Ask Yourself . . .
What is the appropriate level of accuracy needed for this linear regression function?

3. Use technology to determine whether a line of best fit is appropriate for these data.

   a. Determine and interpret the linear regression function.

   $y = 0.282x + 36.3$

   The coefficient of x is the percent of net income that goes towards rent. The y-intercept represents for a \$0 income, rent would be \$36.30. This does not make sense.

   b. Compute the correlation coefficient.

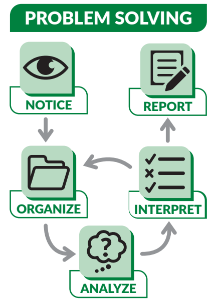
   $r = 0.9817$

4. Would a line of best fit be appropriate for this data set? Explain your reasoning.

   Yes. The r-value is very close to 1.

The correlation coefficient, r, indicates the type (positive or negative) and strength of the relationship that may exist for a given set of data points. The **coefficient of determination**, r^2, measures how well the graph of the regression fits the data. It represents the percentage of variation of the observed values of the data points from their predicted values.

ACTIVITY 2.3 Using the Correlation Coefficient to Assess a Line of Best Fit



The amount of antibiotic that remains in your body over a period of time varies from one drug to the next. The table given shows the amount of Antibiotic X that remains in your body over a period of two days.

Time (hours)	Amount of Antibiotic X in Body (mg)
0	60
6	36
12	22
18	13
24	7.8
30	4.7
36	2.8
42	1.7
48	1

1. Determine and interpret a linear regression function for this data set.

   $y = -1.1x + 42.3$

   As time increases by 1 hour, the amount of antibiotic in the body decreases by 1.1 mg.

2. Compute and interpret both the correlation coefficient and coefficient of determination of this data set.

   The value of the correlation coefficient is -0.8832. There is a negative correlation between the time that Antibiotic X is in the body and the amount of the antibiotic in the body. The value of the coefficient of determination is 0.7800. It assigns a percentage value to the fit of the observed values and their predicted values.

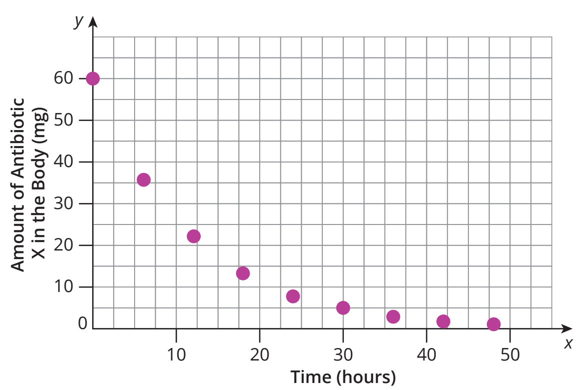
3. Does it seem appropriate to use a line of best fit? If no, explain your reasoning.

   No. The r-value of -0.8832 and r^2-value of 0.7800 are not extremely close to -1 and 1 respectively, indicating that a linear function may not be the best model for this data.

4. Sketch a scatterplot of the data.

5. Look at the graph of the data. Do you still agree with your answer to Question 3? Explain your reasoning.

   Sample answer:
   Yes. I still do not think a line of best fit is appropriate. The data appear to be exponential.



EB STUDENT TIP

For "Intermediate" and higher proficiency levels

To help students understand the meaning of the term *antibiotic*, compare it to its Spanish cognate *antibiótico* and provide examples of other words with the prefix *anti-*, such as *antifreeze*, *antisocial*, *antidote*, and *anti-aging*. Discuss the meaning of the prefix, then help students make sense of the context by discussing why an *antibiotic* is used.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the problem-solving model graphic organizer.
- For Question 1, have students work in pairs, ask themselves the questions to ask from the first three steps of the model, and share their reasoning.
- Complete the remaining steps in the graphic organizer together as a class.
- Have students work in pairs and use the problem-solving model to complete Questions 2–5.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1E:

- Do students create representations to present and organize their mathematical reasoning?

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the definitions.
- Group students to complete Questions 4–6.
- Check in and share.
- Read and discuss definitions.
- Group students to complete Question 7.
- Share and summarize.

Student Look-Fors
Whether students are demonstrating proficiencies related to TEKS A.1G:

- Do students defend their mathematical reasoning?
- Do students use precise mathematical language when communicating?

ACTIVITY
2.4 Correlation Vs. Causation

Does correlation mean causation? What do you think causation means? That is a question that statisticians are always trying to determine.

Read the three true statements that Jackson and Ricardo are given by their Algebra I teacher. She asks them to decide what conclusions they can draw from the data. Do you agree with them? Why or why not?

1. The number of smartphones sold in the United States has increased every year since 2005. The number of flat screen televisions sold in the United States has also increased during the same period of time.

   Jackson and Ricardo reached the conclusion that owning a cell phone causes a person to buy a flat screen television.

   There is no evidence that buying a smartphone causes a person to buy a flat screen television. Most people buy a flat screen because they are the only type available. Also, all technology sales are increasing.

2. Since 2004, the average salary of an NFL football player has increased every year. The average weight of an NFL player has also increased yearly since 2004.

   After much discussion, Jackson and Ricardo reached the conclusion that higher salaries cause the players to gain weight.

   The increase in pay does not cause a player to gain weight. However, it could be argued that increasing pay has given an incentive to players to become bigger and stronger.

3. Worldwide, the number of automobiles sold annually has steadily increased since 1920. Gasoline production has also steadily increased since 1920.

   Jackson and Ricardo concluded that the increase in the number of automobiles sold caused an increase in the amount of gasoline produced.

   It does seem reasonable that the increase in the number of automobiles has caused an increase in the amount of gasoline produced.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Help students make sense of the terms *causation* and *statistician* in the introduction by connecting them to words they are familiar with, such as *cause* and *statistics*, respectively.

172 MODULE 2 • TOPIC 1 • LESSON 2

Proving causation is challenging. The scenarios Jackson and Ricardo analyzed demonstrate that even though two quantities are correlated, this does not mean that one quantity caused the other. This is one of the most misunderstood and misapplied uses of statistics.

Causation is when one event effects the outcome of a second event. A correlation is a **necessary condition** for causation, but a correlation is not a **sufficient condition** for causation. While determining a correlation is straightforward, using statistics to establish causation is very difficult.

4. Many medical studies have tried to prove that not washing your hands can cause you to get sick.

   a. Is not washing your hands a necessary condition for getting sick? Why or why not?

      No. People can get sick even when they wash their hands.

   b. Is not washing your hands a sufficient condition for getting sick?

      No. Not every person doesn't wash their hands gets sick.

   c. Is there a correlation between people who don't wash their hands and people who get sick? Explain your reasoning.

      Yes. People who don't wash their hands are more likely to get sick than those who do wash their hands.

   d. Is it true that not washing your hands causes you to get sick? If so, how was it proven?

      Yes. Scientists have proven that not washing your hands leads to the spread of germs that can then make you sick. Washing your hands kills germs and bacteria that may be on them, reducing your chances of getting sick.

MODULE 2 • TOPIC 1 • LESSON 2 173

5. It is often said that being outside for an extended amount of time can cause you to get a sunburn.

   a. Is going outside for a extended amount of time a necessary condition for a sunburn? Why or why not?

      No. Some people can get a sunburn after being outside for a short amount of time.

   b. Is going outside for an extended amount of time a sufficient condition for a sunburn? Why or why not?

      No. Not everyone who is outside for an extended amount of time gets a sunburn.

   c. Is there a correlation between being outside for an extended amount of time and getting a sunburn? Explain your reasoning.

      Yes. The more time you spend outside, the more likely you are to get a sunburn.

   d. Is it true that being outside for an extended amount of time causes sunburns? Explain your reasoning.

      Yes. It has been proved that prolonged exposure to UV radiation from the sun damages the skin cells and leads to sunburn.

Does school absenteeism cause poor performance in school? A correlation between the independent variable of days absent to the dependent variable of grades makes sense. However, this alone does not prove causation.

6. To prove that the number of days that a student is absent causes the student to get poor grades, we would need to conduct more controlled experiments.

   a. List several ways that you could design experiments to attempt to prove this assertion.

      Sample answers:

      I could have the same students go to school every day for a semester and stay home 20 days for the next semester, and then compare their grades from one semester to the next.

      I could choose two students with similar grades and have one stay home more than the other, and then compare their grades after a period of time.

      I could examine the total number of days absent and the grades for a much larger number of students.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Some students may be unfamiliar with the term *assertion* in Question 6. Discuss the definition of *assert*, and then provide an example of what it means to *assert* oneself, as well as examples of making an *assertion*. Explain why the statement is labeled as an *assertion* in the question and discuss the differences between making a statement and making an *assertion*.

b. Will any of these experiments prove the assertion? Explain your reasoning.

Answers will vary based on each classroom.

There are two relationships that are often mistaken for causation. A **common response** is when some other reason may cause the same result. A **confounding variable** is when there are other variables that are unknown or unobserved.

7. Consider each relationship. List two or more common responses that could also cause this result.

a. In North Carolina, the number of shark attacks increases when the temperature increases. Therefore, a temperature increase appears to cause sharks to attack.

Sample answers:

When the temperature increases, more people go swimming, increasing the opportunity for shark attacks.

The temperature increases in the summer when more people go on vacation and are more likely to be at the beach.

When the temperature increases, the water temperature increases, and warmer water may bring sharks closer to shore.

b. A company claims that their weight loss pill caused people to lose 20 pounds when following the accompanying exercise program.

Sample answers:

People just following the exercise program could have lost the same amount of weight.

People given a placebo could have lost the same amount of weight.

People lost the weight for other reasons, such as eating less or exercising more.

Question 7 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice examining situations and deciding whether a correlation implies causation, assign Skills Practice Sets C and D for this lesson.

MODULE 2 • TOPIC 1 • LESSON 2 175

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Correlations R Us

Consider the given data sets.

Set A

x	y
0	24
2	19
5	12
10	6
20	0

Set B

x	y
8	13
10	4
14	15
15	14
19	73

1. Determine the linear regression function for each set.

   Set A: $f(x) = -1.15x + 20.71$

   Set B: $g(x) = 5.08x - 43.29$

2. Compare the correlation coefficient and the coefficient of determination of each data set. Describe which linear regression function is the better fit and why.

   Set A: $r = -0.9512$,
   $r^2 = 0.9048$

   Set B: $r = 0.7892$,
   $r^2 = 0.6228$

   Set A is a better fit because it's r-value is closer to -1 than Set B's r-value is to 1. Also, Set A's r^2-value is a higher percentage than Set B's r^2-value.

176 MODULE 2 • TOPIC 1 • LESSON 2

Lesson 2 Assignment

Write

Complete each sentence.

1. A correlation is a necessary condition for causation, but a correlation is not a sufficient condition for causation.
2. A common response is when some other reason may cause the same result.
3. Causation is when one event causes a second event.
4. A confounding variable is when there are other variables that are unknown or unobserved.
5. The correlation coefficient is a value between −1 and 1 that indicates how close the data are to forming a straight line.
6. The percentage of variation of the observed values of the data points from their predicted values is represented by the coefficient of determination.

Remember

Sets of data can frequently be modeled by using a linear function called a *linear regression function.* A value called the *correlation coefficient* can also be calculated to assist in determining how well the regression model fits the data.

Practice

1. The table shows the percent of the United States population who did not receive needed dental care services.

Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Percent	7.9	8.1	8.7	8.6	9.2	10.7	10.7	10.8	10.5	12.6	13.3

a. Do you think a linear regression function would best describe this situation? Why or why not?

Yes, a linear regression function is best because as the years increase by 1, the percent increases by a fairly constant amount.

b. Determine the linear regression function for these data. Interpret the equation in terms of this problem situation.

Let x represent the number of years since 1999.

$y = 0.51x + 7.54$

For every increase of 1 year, the percent of the population that did not receive needed dental care services increased by 0.51%. The percent for the year 1999 was 7.54%.

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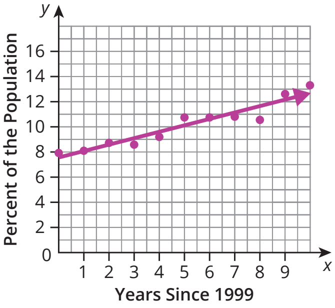
Lesson 2 Assignment

c. Compute and interpret the correlation coefficient of this data set. Does it seem appropriate to use a line of best fit? Explain your reasoning.

The value of $r \approx 0.953$. There is a positive correlation; as the number of years increases, the percent increases. A line of best fit is appropriate because the correlation 1d. coefficient is close to 1.

d. Sketch a scatterplot of the data. Then, plot the equation of the linear regression function on the same grid. Do you still think a linear regression model is appropriate? Explain your answer.



The line fits the data well, so a linear regression model is appropriate.

2. A teacher claims that students who study will receive good grades.

a. Do you think that studying is a necessary condition for a student to receive good grades?

No. It may be possible for a student to receive good grades without studying.

b. Do you think that studying is a sufficient condition for a student to receive good grades?

No. Not every student who studies receives good grades.

c. Do you think that there is a correlation between students who study and students who receive good grades?

Yes. Students who study have a much higher chance of receiving good grades than those who do not study.

178 MODULE 2 • TOPIC 1 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

d. Do you think that it is true that studying will cause a student to receive good grades?

While students who study have a much higher chance of receiving good grades, studying does not actually cause a student to receive good grades.

e. List two or more confounding variables that could have an effect on this claim.

Sample answers:
A student who has a high IQ may receive good grades. A student who pays attention in class may receive good grades.

3. For each situation, decide whether the correlation implies causation. List reasons why or why not.

a. The number of newspapers sold in a city is highly correlated to the number of runs scored by the city's professional baseball team.

The correlation does not imply causation. This may be the case for championship games, but overall, people get news other ways as well and buy newspapers at times other than baseball season.

b. The number of mouse traps found in a person's house is highly correlated to the number of mice found in their house.

The correlation does imply causation. People with a mouse problem tend to have mouse traps in their houses.

Prepare

Use what you know about arithmetic sequences to complete each task.

1. Rewrite the sequence $a_n = 10 - 3(n - 1)$ in simplest terms. Then, write the first 5 terms of the sequences generated by a_n.

$a_n = -3n + 13$

10, 7, 4, 1, -2

2. Given the function $f(x) = -3x + 10$, calculate $f(1)$, $f(2)$, $f(3)$, $f(4)$, and $f(5)$.

$f(1) = 7$, $f(2) = 4$, $f(3) = 1$, $f(4) = -2$, $f(5) = -5$

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MODULE 2 • TOPIC 1 • LESSON 2 ASSIGNMENT 179

3 Making Connections Between Arithmetic Sequences and Linear Functions

MATERIALS

None

LESSON OVERVIEW

Students are provided two sequences. They must identify each sequence as arithmetic or geometric, write the explicit formula for the sequence, and graph the sequence. Students then list and compare characteristics of each graphical representation. The remainder of the lesson focuses on connecting arithmetic sequences to linear functions. Students match the explicit formulas for arithmetic sequences and their graphs. A Worked Example demonstrates how to rewrite an arithmetic sequence in explicit form as a linear function in slope-intercept form. Students then use the context of stacking chairs to make connections among the terms of the explicit formula of a sequence and the linear function that models it. Students compare the terms of each equation and recognize that the common difference and the slope are constant and equal; however, the first term of the sequence is equal to $f(1)$ rather than the y-intercept of the linear function. Using tables of values for this context, *first differences* is defined as a strategy to determine whether a relationship is linear.

Students move from the concrete example to generalize that the constant difference of an arithmetic sequence is equal to the slope of the corresponding linear function by completing an algebraic proof. Next, *average rate of change* is defined and presented graphically as a method to determine the unit rate using non-consecutive x-values. Students solidify these new concepts by revisiting the sequences from the start of the lesson, practicing their newly-developed skills, and verifying their conclusions. The special case of a constant function is then addressed. Finally, students complete a graphic organizer to summarize the characteristics and representations of linear functions.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

(TEKS continued on next page)

ELPS

(2) Listening
The student is expected to:

(D) monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed.

(ELPS continued on next page)

ALGEBRA I TEKS *(TEKS continued from previous page)*

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.

The student is expected to:

A.2A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

A.2B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3A determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

A.3B calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems.

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

The student is expected to:

A.12D write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms.

ELPS *(continued from previous page)*

(4) Reading

The student is expected to:

(A) learn relationships between sounds and letters of the English language and decode (sound out) words using a combination of skills such as recognizing sound-letter relationships and identifying cognates, affixes, roots and base words.

(K) demonstrate English comprehension and expand reading skills by employing analytical skills such as evaluating written information and performing critical analyses commensurate with content area and grade-level needs.

ESSENTIAL IDEAS

- The explicit formula of an arithmetic sequence can be rewritten as the slope-intercept form of a linear function using algebraic properties.
- The explicit formula of an arithmetic sequence, $a_n = a_1 + d(n - 1)$, includes the first term of the sequence, $f(1)$, and the common difference. The slope-intercept form of a linear function, $f(x) = mx + b$, includes $f(0)$ and the slope.
- Both the average rate of change formula and slope formula calculate the unit rate over a given interval. The average rate of change formula refers to the dependent variable as $f(x)$, while the slope formula uses y.
- First differences is a strategy to determine whether a table of values can be modeled by a linear function. First differences are the values determined by subtracting consecutive output values when the input values have an interval of 1.
- The domain of an arithmetic sequence is consecutive integers beginning with 1, while the domain of a linear function may include all real numbers.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Line Up in Sequential Order 10–15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students are provided two sequences. They must identify each sequence as arithmetic or geometric, write the explicit formula for the sequence, and graph the sequence. They list 3 characteristics of the graphs and compare them.

DEVELOP

Activity 3.1: Connecting Forms 30–35 minutes

WORKED EXAMPLE, PEER WORK ANALYSIS

Students match the explicit formulas for arithmetic sequences and their graphs. They first conjecture and then conclude that the graphs of arithmetic sequences belong to the linear function family. A Worked Example demonstrates how to rewrite an arithmetic sequence in explicit form as a linear function in the general form $f(x) = ax + b$. Students relate the common difference of an arithmetic sequence to the slope of the line, connecting the terms of the explicit form of an arithmetic sequence to the terms of a linear function. Students notice that the first term of an arithmetic sequence is not the *y*-intercept, but can be represented by the expression $a_1 - d$.

DAY 2

Activity 3.2: Connecting Constant Difference and Slope 20–25 minutes

REAL-WORLD PROBLEM SOLVING, MATHEMATICAL PROBLEM SOLVING

Students use the context of stacking chairs to connect constant difference and slope. Moving forward, the term *constant difference*, rather than *common difference*, is used. Students expand on Hank's reasoning from the previous activity to realize that while the first term of a sequence is not equal to the *y*-intercept, the term a_0 represents the *y*-intercept. They then determine whether a table of values represents a linear function by analyzing first differences; when the first differences are constant, the table of values represents a linear relationship.

Activity 3.3: Connecting Constant Difference, Slope, and Average Rate of Change 20–25 minutes

MATHEMATICAL PROBLEM SOLVING

Students generalize the concepts of constant difference and slope by using variable expressions for consecutive terms. They use a table, a graph, and algebraic reasoning to demonstrate that the slope of a linear function is always equal to the constant difference of the corresponding arithmetic sequence. The *average rate of change formula* using function notation is introduced as another form of the slope formula.

DAY 3

Activity 3.4: Verifying that Common Differences and Slopes are the Same 15–20 minutes

MATHEMATICAL PROBLEM SOLVING

Students revisit the sequences and graphs from Activity 1 and summarize their conclusions. They identify the constant difference on the graph, write the sequences in function notation, verify the constant difference is the slope, and describe the relationship between the first term of the sequence and the *y*-intercept.

Activity 3.5: Describing Constant Sequences 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students consider a constant sequence. They write the explicit formula and the linear function to represent the sequence. Students then discover that a sequence with a common difference of 0 is the same as a constant function with a slope of 0.

DEMONSTRATE

Talk the Talk: Making It Plain and Clear 10–15 minutes

GRAPHIC ORGANIZER

Students complete a graphic organizer to summarize the characteristics and representations of linear functions.

Getting Started

ENGAGE

Line Up in Sequential Order

Facilitation Notes

In this activity, students are provided two sequences. They must identify each sequence as arithmetic or geometric, write the explicit formula for the sequence, and graph the sequence. They list 3 characteristics of the graphs and compare them.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how you wrote the explicit formula for each sequence.
Seeing structure	• How can you tell whether a sequence is geometric or arithmetic? • What part of each formula tells you how the sequence grows? • What does $(n - 1)$ represent in explicit form? • How are the common difference and common ratio visible in the graphs? • Do you think all geometric sequences grow faster than arithmetic sequences? Explain your thinking.

DIFFERENTIATION STRATEGY

Just in Time Support

- Provide students with the explicit formulas for arithmetic and geometric sequences.

  Arithmetic: $a_n = a_1 + d(n - 1)$
  Geometric: $g_v = g_1 \cdot r^{v - 1}$

Summary

STAMP THE LEARNING

An arithmetic sequence grows by addition of a common difference, and the points on its graph are collinear. A geometric sequence grows by multiplication by a common ratio, and the points on its graph may form a smooth curve.

Facilitation Notes

In this activity, students match the explicit formulas for arithmetic sequences and their graphs and conjecture that arithmetic sequences belong to the linear function family. The explicit form of a sequence is provided, along with its graph and table of values. Through the table, students relate the terms of the sequence and coordinate pairs of the line. Graphically, they build connections between arithmetic sequences and linear functions by connecting the points on the graph of the sequence to form a line and then writing the equation of the line. A Worked Example demonstrates how to use algebraic properties to rewrite an arithmetic sequence in explicit form as a linear function in the general form $f(x) = ax + b$. Students compare the terms of the explicit form of an arithmetic sequence to the terms of a linear function. They notice that the common difference of an arithmetic sequence is the same as the slope of the line; however, the first term of an arithmetic sequence is $f(1)$ and not the y-intercept of the line.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Use of the structure and term values to match the formulas and graphs
- Substitution of n-values to match the formulas and graphs

DIFFERENTIATION STRATEGY

Just in Time Support

- To scaffold support, have students create an example for reference.

$$a_n = a_1 + d(n - 1)$$
$$a_n = 2 - 4(n - 1)$$

First term

increasing or decreasing

common difference

COMMON MISCONCEPTION

- When focusing on an explicit formula with a negative d-value, students sometimes do not realize the negative sign is part of the d-term. Help students connect $a_n = a_1 + d(n - 1)$ with the substituted values. This will help build toward later cases when formulas have negative signs in front of variables.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What point on the graph can you easily identify from the formula? • How can you tell whether the graph is increasing or decreasing from the formula? • What happens when you substitute $n = 1$ into the formula? How does your answer relate to the graph?

Have a student read the information after Question 2 and the introduction to the explicit formula.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How can you tell that the graph represents the explicit formula? • Why does the term value column in the table have two columns? What is that trying to tell you?

DIFFERENTIATION STRATEGY

Just in Time Support

- Have students label the points on the graph.

Have students work with a partner or in a group to complete Questions 3 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you identify the common difference? • How did you determine the slope and y-intercept of the equation?
Seeing structure	• Is there any difference between how to identify the common difference and slope from a graph? Explain your thinking. • Do all sequences have the same domain? Why or why not?

Have students work with a partner or in a group to read the introduction and Worked Example, then complete Questions 7 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the coordinate pair for the y-intercept? The first term of an arithmetic sequence?
Seeing structure	• What value from the linear function is the same as the first term of the arithmetic sequence? • Why doesn't a sequence have a y-intercept?

COMMON MISCONCEPTION

- Students may refer to the *y*-intercept as the “starting value” due to a process they previously learned to graph lines. A line is infinite and has no starting value. Also, this language may add confusion because the starting value of a sequence is different from what they are referring to as a starting value for a line. Suggest more mathematical language, such as, “The *y*-intercept of a line is the value when $x = 0$” or “The first term of a sequence is when $n = 1$.”

DIFFERENTIATION STRATEGY

Access for All

- Throughout this lesson, have students create a table to organize their thoughts, placing common meanings in the same column. Additions will be noted throughout this lesson.

Linear relationship	• Arithmetic Sequence • $a_n = a_1 + d(n-1)$ • Linear Equation • $y = ax + b$ • Linear Function • $f(x) = ax + b$
Slope	• Common difference of a sequence, *d* • The *a*-value in $y = ax + b$
y-intercept	• $f(0)$ • $(0, y)$
$f(1)$	• a_1 of a sequence • $(1, y)$

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

All sequences are functions. All arithmetic sequences are linear functions. The explicit formula of an arithmetic sequence can be rewritten as a linear function. The common difference of an arithmetic sequence is the same as the slope of its linear function. The value of a_1 of a sequence is equal to $f(1)$ of its linear function.

ACTIVITY 3.2 Connecting Constant Difference and Slope

Facilitation Notes

In this activity, students use the context of stacking chairs to connect constant difference and slope. Students expand on Hank’s reasoning from the previous activity to realize that while the first term of a sequence is not the *y*-intercept, the term a_0 represents the *y*-intercept. They then determine whether a table of values represents a linear function by analyzing first differences; when the first differences are constant, the table of values represents a linear relationship.

Have a student read the introduction to the class. Discuss the context.

To begin the Day 2 session, have a student read the Essential Question aloud.

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Classroom Chairs or Plastic Cups

- Have students demonstrate the context by stacking classroom chairs. Replace the values in the context with the actual measurements from the demonstration. When chairs are not available, you can use plastic cups.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What are the independent and dependent quantities? • What does the product of $8(n - 1)$ represent in the context? • What does 26 represent in the context? • How can you check that your explicit formula and function are equivalent representations? • What is the domain for your explicit formula and function?

Have a student read the summary paragraph and table information. Discuss as a class.

DIFFERENTIATION STRATEGY

Access for All

- Have students enter additional terms in the table as they are addressed in the lesson and through student discussions.

Linear relationship	• $a_n = a_0 + d_n$
Slope	• Constant difference • Unit rate of change • Constant change • $\frac{y_2 - y_1}{x_2 - x_1}$ • Direction and steepness of a line • Difference in output value divided by difference in corresponding input values
y-intercept	• The b-value in $y = ax + b$ • a_0 in an arithmetic sequence
$f(1)$	• $a_0 + d$

Have a student read the paragraph following the table and the first differences strategy. Then have students work with a partner to complete Questions 4 through 6. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Guide students to take notes on the paragraph from the text for the first differences strategy.

  First Differences Strategy

  1. Check that input values have an interval of 1.
  2. Subtract consecutive output values.
  3. When the differences are constant, the relationship is linear.
- Have them use arrows for direction and write the subtraction for common differences.

n	a_n
1	34
2	42
3	50
4	58

$42 - 34 = 8$

$50 - 42 = 8$

$58 - 50 = 8$

Access for All

- Clarify the use of the terms *first differences*, *constant differences*, and *common differences*. *First differences* is a strategy to determine whether a table of values represents a linear relationship. Because *constant differences* and *common differences* have the same meaning, and common differences is primarily used for sequences, moving forward the term *constant differences* will be used.

COMMON MISCONCEPTION

- Students are sometimes confused as to whether the order of subtraction of table values matters. Clarify that when the x-values are in ascending order, subtracting $y_2 - y_1$ consistently gives you first differences for comparison purposes and tells you the correct slope. Remember, slope describes both the steepness and direction of a line. When the x-values are in ascending order, subtracting $y_1 - y_2$ consistently gives you first differences for comparison purposes, but it will not give you the correct slope. In terms of slope, the steepness value will be correct, but the direction (increasing or decreasing) will be wrong.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why does one table have an input of 0 and the other table does not?
Probing	• Does it matter in which order you subtract the values to determine whether a table represents a linear relationship? Explain your thinking.
Seeing structure	• How does calculating first differences relate to using the slope formula? • Why do the first differences represent the slope without applying the division step of the slope formula?
Probing	• In part (b), are the first differences of 4 and −4 both correct? Explain your thinking. • In part (b), how did you identify the slope and *y*-intercept? • How did you determine the linear equation for part (d)?
Seeing structure	• In part (d), what will the graph representing this table look like?

Summary

STAMP THE LEARNING

The expression $a_n = a_1 + d(n - 1)$ can be rewritten as $a_n = a_0 + dn$, where a_0 represents the *y*-intercept. First differences is a strategy to determine whether a table of values represents a linear function.

ACTIVITY 3.3 Connecting Constant Difference, Slope, and Average Rate of Change

Facilitation Notes

In this activity, students generalize the concepts of constant difference and slope by using variable expressions for the consecutive terms. They use a table, a graph, and algebraic reasoning to demonstrate that the slope of a linear function is always equal to the constant difference of the corresponding arithmetic sequence. The *average rate of change formula* using function notation is introduced as another form of the slope formula.

Have a students read the introduction. Discuss as a class.

AS STUDENTS WORK, LOOK FOR

- Confusion related to variable expressions being used as term numbers and why they are being used instead of constants.
- Sense-making that an expression with more than one term can be substituted for *x* in *f*(*x*).
- Incorrect use of the distributive property and subtraction leading to errors in determining the constant difference.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

- Have students write $y = ax + b$ above the Term Value column of the table and refer to it as they complete their substitutions.
- Use color coding to assist in substitution. For example, $y = ax + b$ and $y = a(n + 2) + b$.
- For Question 3, model the process for a set of consecutive values and have students repeat the process with a different set of consecutive values.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain the meaning of the labels on the *x*-axis and *y*-axis. • Select a point on the graph and identify its coordinates. • How did you know to make that substitution for the table values? • Explain your steps to evaluate the expression.
Reflecting and justifying	• How can you calculate the slope using non-consecutive points on the graph? • What is the purpose of calculating the slope in general terms using variables? • What does it mean that you completed this algebraic process and got *m* as a result?

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

- Provide a template with the slope formula and partially completed steps. Then, ask students to complete the remainder of the process.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How is the process for answering Question 4 part (b) different from the process to answer Question 3? • Why don't you have to repeat this process for all non-consecutive pairs in the table? • What does it mean that you completed this algebraic process and got $a = a$ as the result? • How is the conclusion to Question 4 part (b) more powerful than the conclusion to Question 3?

Have a student read the introduction to the diagram. Discuss the diagram and answer Question 5 as a class.

COMMON MISCONCEPTION

- When students hear the term *average* they sometimes want to calculate the mean. Explain the term *average* will have a different meaning later when it is used to estimate the rate of change between any two points in a nonlinear function.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the difference between the slope formula and the average rate of change formula? Why?
Seeing structure	• How does the average rate of change formula relate to the diagram? Does this formula relate to consecutive or non-consecutive terms? How can you tell?

DIFFERENTIATION STRATEGY

Access for All

- Have students enter the additional terms in the table as they are addressed in the lesson and through student discussion.

Slope	• Average rate of change • average rate of change formula, $\frac{f(t) - f(s)}{t - s}$ • Ratio of the change in the outputs and the change in the inputs

Have students work with a partner or in a group to complete Question 6.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Does it matter which sets of ordered pairs are used for determining the average rate of change?
Reflecting and justifying	• Does your strategy to determine average rate of change depending on the different situations?

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Using variables rather than numeric values in a formula generalizes the conclusions for all cases. In this case, for any linear function, the constant difference is the same for any two consecutive values, and the slope is the same for any two ordered pairs regardless of whether they are consecutive or not. The average rate of change formula $\frac{f(t) - f(s)}{t - s}$ has the same structure as the slope formula, $\frac{y_2 - y_1}{x_2 - x_1}$.

ACTIVITY 3.4

Verifying that Common Differences and Slopes are the Same

Facilitation Notes

In this activity, students revisit the sequences and graphs from Activity 1 and summarize their conclusions. They identify the constant difference on the graph, write the sequences in function notation, verify the constant difference is the slope, and describe the relationship between the first term of the sequence and the *y*-intercept.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work independently or with a partner to complete this activity. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Where is the constant difference visible in the explicit formula? • How did you label the constant difference on the graph? • How can you check that the function notation is equivalent to the explicit formula? • What calculations did you use to determine the constant difference? • What notation can you use to express how the first term and *y*-intercept are related?

DIFFERENTIATION STRATEGY

Access for All

- Have students connect the points to create lines to make the *y*-intercept visible. This will help students check the accuracy of their function notation and verify the relationship between the first term and the *y*-intercept.

STAMP THE LEARNING

Summary

All arithmetic sequences are linear functions. The explicit formula of an arithmetic sequence can be rewritten as a linear function. The common difference of an arithmetic sequence is the same as the slope of its linear function.

ACTIVITY **3.5** Describing Constant Sequences

Facilitation Notes

In this activity, students consider a constant sequence. They write the explicit formula and the linear function to represent the sequence. Students then discover that a sequence with a common difference of 0 is the same as a constant function with a slope of 0.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Can this function be written as $f(x) = 0x + 3$? • Can the explicit formula be written as $3 + 0(n - 1)$? • Are the graphs of all constant functions horizontal lines? • Why is it called a constant function?

Summary

STAMP THE LEARNING

An arithmetic sequence with a common difference of 0 can be represented as a constant linear function.

DEMONSTRATE

Talk the Talk

MAKING IT PLAIN AND CLEAR

Facilitation Notes

In this activity, students complete a graphic organizer to summarize the characteristics and representations of linear functions.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• How can you determine whether a table with consecutive values represents a linear relationship? With non-consecutive values? • How can you determine whether consecutive points on a graph represent a linear relationship? Non-consecutive points?

DIFFERENTIATION STRATEGY

Just in Time Support

- Provide a word bank of possible choices for the mathematical meaning column.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

All arithmetic sequences are linear functions. The explicit formula of an arithmetic sequence can be rewritten as a linear function. The common difference of an arithmetic sequence is the same as the slope of its linear function. The explicit form displays the value of $f(1)$ while the slope-intercept form displays the value of $f(0)$.

3 Making Connections Between Arithmetic Sequences and Linear Functions

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Use algebraic properties to prove the explicit formula for an arithmetic sequence is equivalent to the equation of a linear function.
- Relate the defining characteristics of an arithmetic sequence, the first term and common difference, and the defining characteristics of a linear function, the y-intercept and slope.
- Connect the slope of a line to the average rate of change of a function.

NEW KEY TERMS

- conjecture
- first differences
- average rate of change

You know that all sequences are functions. What type of function is an arithmetic sequence?

Sample answer:

All arithmetic sequences are linear functions. The average rate of change of any linear relationship is constant between any two points on that line.

MODULE 2 • TOPIC 1 • LESSON 3 181

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Ask Yourself . . .
What is the difference between an arithmetic and geometric sequence?

Getting Started

Line Up in Sequential Order

Lauren counts the number of new flowers that are blooming in her garden each day in the spring. Sequence A represents the number of new flowers on day 1, day 2, etc.

Sequence A: 3, 6, 12, 24, 48

She also measures the height of the first sunflower that has started blooming. Sequence B represents the height in centimeters of the sunflower on day 1, day 2, etc.

Sequence B: 3, 6, 9, 12, 15

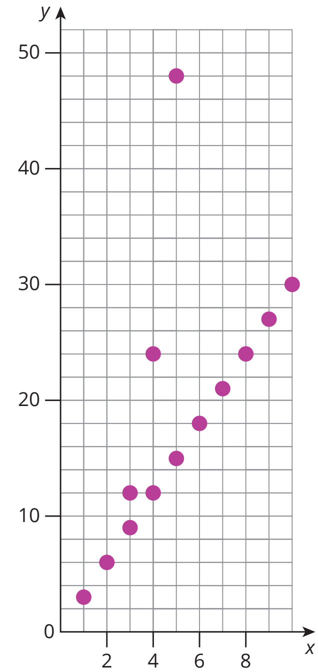
1. For each sequence, determine whether it is arithmetic or geometric and write the explicit formula that generates the sequence. Then, graph and label each on the coordinate plane.

   Sequence A:

   Sequence A: Geometric
   $g_n = 3 \cdot 2^{n-1}$

   Sequence B:

   Sequence B: Arithmetic
   $a_n = 3 + 3(n - 1)$
   $= 3n$



2. List at least three common characteristics of the graphs. How do the two sequences compare?

   Sample answer:

   Both graphs start at $x = 1$, include discrete points, and are increasing from left to right. The arithmetic sequence builds by a common difference, and the geometric sequence builds by a common ratio. Both sequences have the same first two terms.

182 MODULE 2 • TOPIC 1 • LESSON 3

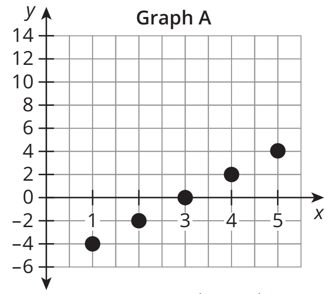
ACTIVITY 3.1

Connecting Forms

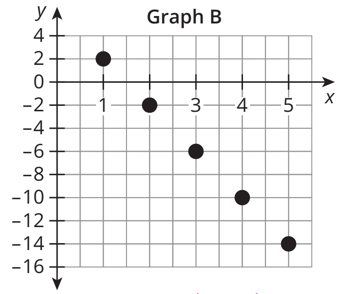
Consider the four explicit formulas, each representing a different arithmetic sequence.

- $a_n = 2 - 4(n - 1)$
- $a_n = 2 + 4(n - 1)$
- $a_n = -4 + 2(n - 1)$
- $a_n = 4 + 2(n - 1)$

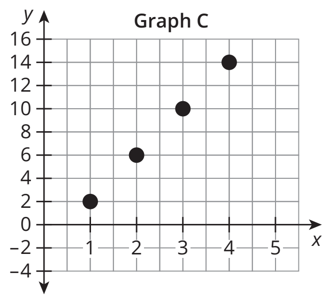
1. Match each explicit formula with its graph. Describe the strategies you used.



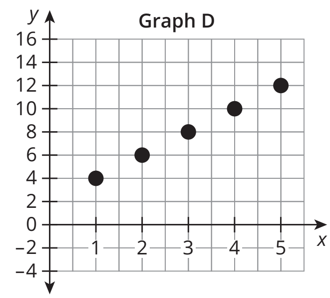
$a_n = -4 + 2(n - 1)$



$a_n = 2 - 4(n - 1)$



$a_n = 2 + 4(n - 1)$



$a_n = 4 + 2(n - 1)$

2. Consider the set of graphs and identify the function family represented. Based on these formulas and graphs, do you think that all arithmetic sequences belong to this function family? Explain your *conjecture*.

   Sample answer:

   The graphs belong to the linear function family. The graph of any arithmetic sequence is linear because both arithmetic sequences and linear functions have a constant rate of change.

Ask Yourself . . .
How do you know by the form of the explicit formula that it represents an arithmetic sequence?

A **conjecture** is a mathematical statement that appears to be true, but has not been formally proven.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Read and discuss the information.
- Group students to complete Questions 3–6.
- Check in and share.
- Read and discuss the introduction and Worked Example.
- Group students to complete Questions 7–9.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1F:

- Can students identify and explain mathematical relationships?

Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining the slope given a graph, assign Skills Practice Set A for this lesson.

EB STUDENT TIP

For all proficiency levels

Provide students a word bank of function families. Have them sketch graphs to accompany each function family. Explain that the term *family* is used instead of the term *group*. For example, all lines in the linear function *family* are related by common characteristics. Connect the root word *line* to *linear function*.

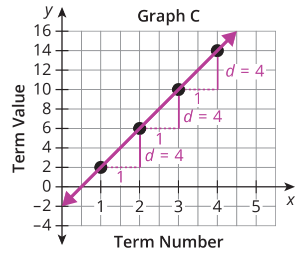
MODULE 2 • TOPIC 1 • LESSON 3 183

Remember . . .

An arithmetic sequence is a sequence of numbers in which the differences between any two consecutive terms is constant. The explicit formula is of the form $a_n = a_1 + d(n - 1)$.

Let's take a closer look at the relationship between arithmetic sequences and the family of linear functions. You know a lot about each relationship.

The explicit formula and the table of values that represents Graph C are shown.



$a_n = 2 + 4(n - 1)$

Term Number n	Term Value a_n	
1	a_1	2
2	a_2	6
3	a_3	10
4	a_4	14

$6 - 2 = 4$

$10 - 6 = 4$

$14 - 10 = 4$

3. Describe the domain of the sequence.

   The domain of the sequence is all integers starting with 1 or the natural numbers.

4. Identify the common difference in each representation.

   See table and graph.

A linear function written in general form is $f(x) = ax + b$, where a and b are real numbers. In this form, a represents the slope and b represents the y-intercept.

5. Draw a line to model the linear relationship seen in the graph. Then, write the equation to represent your line. Describe your strategy.

   $y = 4x - 2$

184 MODULE 2 • TOPIC 1 • LESSON 3

6. Describe the domain of the graph of the linear model.

   The domain of the graph of the linear equation is all real numbers.

An explicit formula is one equation that you can use to model an arithmetic sequence. You can rewrite the explicit formula for the arithmetic sequence $a_n = 2 + 4(n - 1)$ in function notation.

WORKED EXAMPLE

You can represent a_n using function notation.

$$a_n = 2 + 4(n - 1)$$
$$f(n) = 2 + 4(n - 1)$$

Next, rewrite the expression $2 + 4(n - 1)$.

$f(n) = 2 + 4n - 4$	Distributive property
$= 4n + 2 - 4$	Commutative property
$= 4n - 2$	Combine like terms

So, $a_n = 2 + 4(n - 1)$ written in function notation is $f(n) = 4n - 2$.

7. Compare the equation you wrote to model the graph of the arithmetic sequence to the explicit formula written in function form. What do you notice?

   The equation and function have the same slope and y-intercept. The only difference is that the equation has a y where the function has $f(x)$.

8. Compare the common difference with the slope. What do you notice?

   The common difference of an arithmetic sequence is the slope of a linear function. The common difference is 4 between consecutive terms. The slope is $\frac{4}{1}$ between consecutive points or equivalent to that ratio when the points are further apart.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

The Worked Example presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing an equation using the explicit formula to represent graphs and scenarios then rewriting in function form, assign Skills Practice Set B for this lesson.

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MODULE 2 • TOPIC 1 • LESSON 3 185

Chunking the Activity

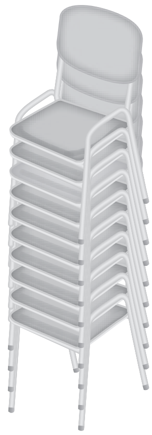
- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction and context.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the summary paragraph and table information.
- Read and discuss the first differences strategy.
- Group students to complete Questions 4–6.
- Share and summarize.

9. Explain why Nakota's reasoning is not correct.

> **Nakota**
>
> The y-intercept of a linear function is the same as the first term of an arithmetic sequence.

Nakota's reasoning is incorrect because all sequences begin with term number 1, which translates to $x = 1$. The y-intercept of a linear function is when $x = 0$.

ACTIVITY 3.2 Connecting Constant Difference and Slope



At the end of the last show, stagehands at the community theater stack the audience chairs and place them in storage. The height of one chair is 34 inches, and as each additional chair is stacked, the height increases by 8 inches.

1. Write an equation using the explicit formula to represent this scenario.

   $a_n = 34 + 8(n - 1)$

2. Rewrite the explicit formula in function form and define your variables.

   Let x represent the number of chairs and $f(x)$ represent the height, in inches, of the stack of chairs. $f(x) = 8x + 26$

3. Compare the two algebraic representations of this scenario.

   a. How is the value of d in the explicit formula related to the value of a in function form? How are d and a represented in the scenario? Be sure to include units of measure.

   The values of d and a represent the unit rate, 8 inches per chair.

   b. How is the value of a_1 from the explicit formula related to b in function form? How are a_1 and b represented in the scenario?

   The expression a_1 represents the first term in the sequence or $f(1)$ of the function. It represents the height of a stack with one chair, 34 inches.

   The expression b represents the y-intercept or $f(0)$ of the function. It represents the height of one partial chair, not counting the top 8 inches.

EB STUDENT TIP

For all proficiency levels

The word *difference* can be interpreted as the opposite of similar or as *subtraction*. Draw attention to the fact that the terms common *difference*, *constant difference*, and *first difference* all involve subtraction. Address the terms *common* and *constant*. Use everyday examples of characteristics that items have in *common* or events that occur *constantly*, so students have a better understanding that they mean the *same*.

c. If a_1 represents the first term of a sequence, what does a_0 represent? How can you rewrite the arithmetic sequence using a_0?

The variable a_0 represents the height of one partial chair, not counting the top 8 inches.

$a_n =$ $a_0 + d(n)$

In this stacked chair scenario, both *d* and *a* represent the additional 8 inches of height per one chair, or the unit rate of change. The common difference, *d*, of the explicit formula is the same as the slope, *a*, of a general linear function. They both represent constant change.

In a sequence, the common, or constant, difference is the difference in term values between consecutive terms.	In a linear function, the slope describes the direction and steepness of the line. It is the difference in output values divided by the difference in corresponding input values.

When you see a graph or rewrite an explicit formula for an arithmetic sequence, it is apparent that it represents a linear function. However, the structure of a table requires other strategies to determine whether it represents a linear function.

One strategy is to examine *first differences*. **First differences** are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear.

The tables that represent the explicit formula and function form of the stacking chairs scenario are shown.

n	a_n
1	34
2	42
3	50
4	58

x	$y = f(x)$
0	26
1	34
2	42
3	50

The expression $y = f(x)$ means that the value of *y* depends on the value of *x*. That is, for different values of *x*, there is a function *f* which determines the value of *y*.

4. Determine the first differences in each table to verify they both represent a linear relationship.

In both tables, the first differences are 8 or 8 inches per chair.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Ask students what is meant by the term *consecutive*. Discuss how the word *consecutive* is used in mathematical and non-mathematical situations. Have students provide mathematical examples of *consecutive* and *non-consecutive* terms relating to arithmetic sequences. Review the activity and ask students to explain what the terms *consecutive* values and *consecutive points* mean in the context of the problems.

5. Joey and Alejandra agree that the table shown represents a linear relationship because there is a constant difference between consecutive points. Joey claims the slope is 5 and Alejandra claims the slope is −5.

Who's correct? Explain your reasoning.

x	y
1	22
2	17
3	12
4	7

22 − 17 = 5

17 − 12 = 5

12 − 7 = 5

Alejandra is correct. The slope is −5 because as the x-values increase, the y-values decrease. The 5 describes the steepness of the line, and the negative describes the fact that it is decreasing in direction.

Ask Yourself . . .
What does slope describe?

6. Use first differences to determine whether each table represents a linear function. Then, create your own table to represent a linear function. Describe your strategy.

a.

x	y
5	12
6	15
7	21
8	30

No. The first differences are 3, 6, and 9.

b.

x	y
−2	18
−1	14
0	10
1	6

Yes. The first differences are −4.

c.

x	y
10	1
11	4
12	9
13	16

No. The first differences are 3, 5, and 7.

d.

x	y

Answers will vary based on each classroom.

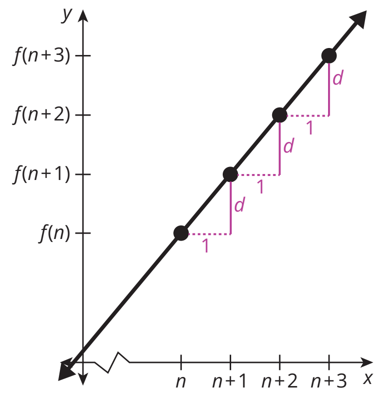
188 MODULE 2 • TOPIC 1 • LESSON 3

ACTIVITY **3.3**

Connecting Constant Difference, Slope, and Average Rate of Change

In the previous activity, you determined that the constant difference of the chair heights and the slope of the line are both equal to 8 inches per chair. Is the slope of a linear function always equal to the constant difference of the corresponding arithmetic sequence?

Consider the graph of the arithmetic sequence represented by the general linear function $f(x) = ax + b$.



1. Identify the constant difference of the sequence in the graph.

2. Complete the table for consecutive values of the input.

Term Number	Term Value	
n	$f(n)$	$a(n) + b$
$n + 1$	$f(n + 1)$	$a(n + 1) + b$
$n + 2$	$f(n + 2)$	$a(n + 2) + b$
$n + 3$	$f(n + 3)$	$a(n + 3) + b$

3. Select any two consecutive input values in the sequence. Use the expressions for the term values to determine the constant difference of the sequence.

   Sample answer:

   $= a(n + 1) + b - (a(n) + b)$

   $= an + a + b - an - b$

   $= an - an + b - b + a$

   $= a$

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Question 4.
- Read and discuss the introduction and diagram.
- Complete Question 5 as a class.
- Group students to complete Question 6.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

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MODULE 2 • TOPIC 1 • LESSON 3 189

Recall that the slope of a line is constant between any two points on the line, not just consecutive points.

Remember . . .

The slope formula is $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\Delta y}{\Delta x}$. Delta ($\Delta$) is a Greek letter used in mathematics to represent "change in". So, $\frac{\Delta y}{\Delta x}$ means "change in y" divided by "change in x".

4. Use the table and graph from Question 2 to complete each task.

   a. Identify the slope of the function on the graph.

   b. Select two non-consecutive points in the table and determine the slope between those two points.

   Sample answer:

   $$a = \frac{a(n+3) + b - (a(n) + b)}{(n+3) - n}$$

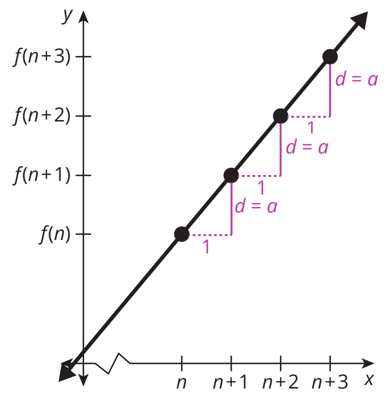
   $$a = \frac{an + 3a + b - an - b}{n + 3 - n}$$

   $$a = \frac{an - an + 3a + b - b}{n - n + 3}$$

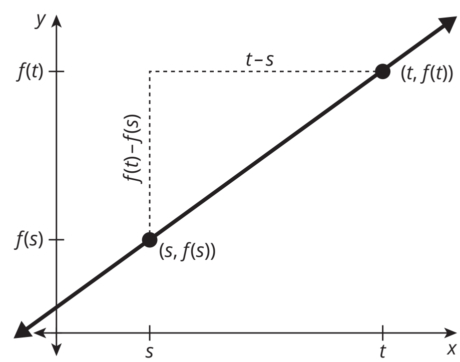
   $$a = \frac{3a}{3}$$

   $$a = a$$

   The slope is constant.



The slope, a, is equal to the constant difference. Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is $\frac{f(t)-f(s)}{t-s}$. This represents the change in the output as the input changes from s to t.



EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Reinforce and review the terms *input values* and *output values*. Some examples of synonyms for *input values* are *domain* and *independent variable*. Some examples of synonyms for *output values* are *range* and *dependent variable*. Discuss the two most common forms in which *input* and *output values* are presented, specifically in table format and equation format.

5. Show that the slope formula and the average rate of change formula represent the same ratio.

The average rate of change, $\frac{f(t) - f(s)}{t - s}$, represents the ratio of the change in the output values to the change in the corresponding input values.

The slope formula, $\frac{y_2 - y_1}{x_2 - x_1}$, represents the ratio of the change in the dependent quantities to the corresponding independent quantities. The formulas represent the same ratio.

6. Determine the average rate of change for each graph, table, or function.

a. $g(x) = 3x + 2$

3

b.

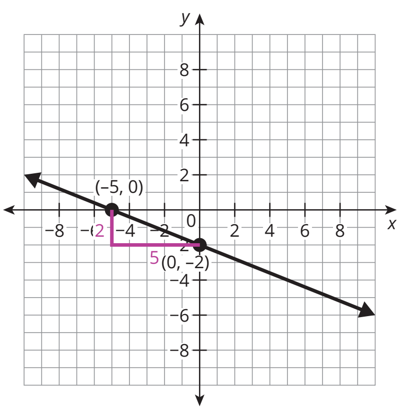
x	y
4	1
6	−1
8	−3
10	−5

$10 - 8 = 2$

$-5 - (-3) = -2$

$-\frac{2}{2} = -1$

c.



$-\frac{2}{5}$

d. The table shows the linear relationships between the distance Hailey runs and the time he spends running.

Average Distance Traveled

$12 - 8 = 4$

Time (minutes)	4	6	8	12
Distance (miles)	0.5	0.75	1	1.5

$1.5 - 1 = 0.5$

$\frac{0.5}{4} = 0.125$

0.125 mile per minute

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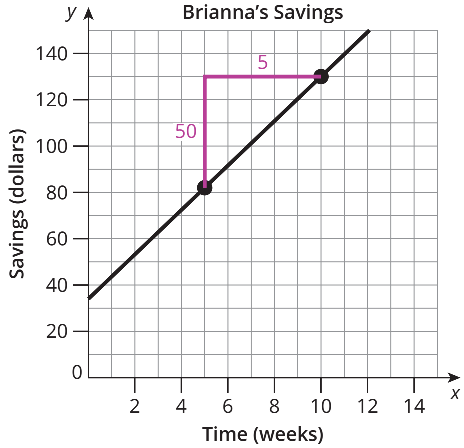
MODULE 2 • TOPIC 1 • LESSON 3 191

e. The graph shows the relationship between the balance of Brianna's bank account and the number of weeks she has been saving.

Sample answer:



$10 per week

f. The function $y = 2.5x - 15.25$ models the amount of money the student council earns from selling bagels after spending money on supplies for the sale.

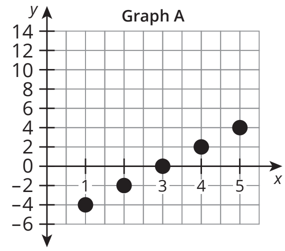
$2.50 per bagel

192 MODULE 2 • TOPIC 1 • LESSON 3

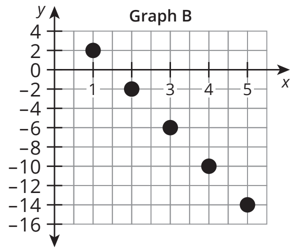
ACTIVITY 3.4

Verifying that Common Differences and Slopes are the Same

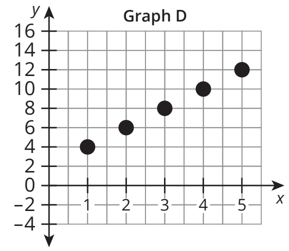
The remaining explicit formulas and graphs from Activity 1 are shown.



$a_n = -4 + 2(n - 1)$



$a_n = 2 - 4(n - 1)$



$a_n = 4 + 2(n - 1)$

1. For each graph and arithmetic sequence, complete each task.

   a. Identify the constant difference in the explicit formula and on the graph.
   Graph A: $d = 2$
   Graph B: $d = -4$
   Graph D: $d = 2$

   b. Rewrite each explicit formula in function notation.
   Graph A: $f(n) = 2n - 6$
   Graph B: $f(n) = 24n + 6$
   Graph D: $f(n) = 2n + 2$

   c. Verify that the constant difference is the same as the slope of the linear function.
   Graph A: $f(1) - f(0) = -4 - (-6) = 2$, slope is 2
   Graph B: $f(1) - f(0) = 2 - 6 = -4$, slope is -4
   Graph D: $f(1) - f(0) = 4 - 2 = 2$, slope is 2

   d. Describe how the first term in the explicit formula is related to the *y*-intercept of the function.
   $a_1 - d = f(0)$
   Graph A: $-4 - 2 = -6$
   Graph B: $2 - (-4) = 6$
   Graph D: $4 - 2 = 2$

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

MODULE 2 • TOPIC 1 • LESSON 3 193

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

ACTIVITY 3.5 Describing Constant Sequences

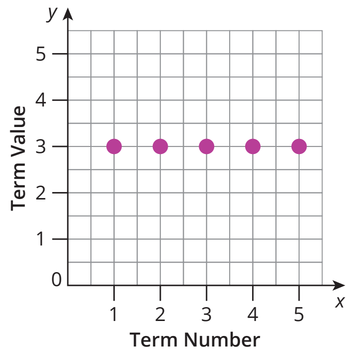
Lauren's cat loves to knock over flowerpots. Each morning, she counts the number of flowerpots her cat knocked over during the night before she uprights them again. Sequence C represents the number of flowerpots knocked over on day 1, day, 2, etc.

Sequence C: 3, 3, 3, 3, 3

1. Determine the constant difference and write the explicit formula to represent Sequence C. Then, create a table of values and graph it.

   $d = 0$; $a_n = 3$

Term Number (n)	Term Value (a_n)
1	3
2	3
3	3
4	3
5	3



2. Use function notation to write an equation representing the relationship between the number of days, x, and the number of knocked over flowerpots, C. Interpret the function in terms of the context.

   $C(x) = 3$
   This is a constant function. The slope is 0 and the y-intercept is (0, 3).

3. Prove that the slope of $C(x)$ is equal to the common difference of Sequence C.

   The ratio of the change in output values to the change in corresponding input values is zero between any two ordered pairs.

194 MODULE 2 • TOPIC 1 • LESSON 3

Consider a similar sequence.

Sequence D: $-5, -5, -5, -5, -5$

4. Write the function, $D(x)$, to model this sequence.
   $D(x) = -5$

5. Prove $D(x)$ is a constant function.
   The slope of $D(x)$ is zero. The values of the dependent variable remain constant over the entire domain.

Remember . . .
When the the values of the dependent variable of a function remain constant over the entire domain, then the function is called a constant function.

MODULE 2 • TOPIC 1 • LESSON 3 195

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Optimizing Learning

This activity uses a graphic organizer to emphasize key ideas and relationships.

Talk the Talk

Making It Plain and Clear

You have proven that all arithmetic sequences can be represented by linear functions.

1. Complete the graphic organizer to summarize the connections between arithmetic sequences and linear functions. Then, describe how you can tell a linear relationship exists given a table of values or a graph. Answers will vary based on each classroom.

Equation

Arithmetic Sequence $a_n = a_1 + d(n - 1)$	Linear Function $f(x) = ax + b$	Mathematical Meaning
a_n	$f(x)$	Output value
d	a	Slope
n	x	Input value
$a_1 - d$	b	y-intercept

Characteristics and Representations of Linear Functions

Table of Values

Graph

196 MODULE 2 • TOPIC 1 • LESSON 3

Lesson 3 Assignment

Write

Describe how the terms *constant difference*, *slope*, and *average rate of change* are related.

Remember

The explicit formula of an arithmetic sequence can be rewritten as a linear function in the general form $f(x) = ax + b$, where a and b are real numbers, using algebraic properties. The constant difference of an arithmetic sequence is always equal to the slope of the corresponding linear function.

Write

Answers will vary based on each classroom.

Practice

1. Gracie claims that the equation $f(n) = 5n - 7$ is the function notation for the sequence that is represented by the explicit formula $a_n = -2 + 5(n - 1)$. Nicky doesn't understand how this can be the case.

   a. Help Nicky by listing the steps to write the explicit formula of the given sequence in function notation. Provide a rationale for each step.

   $a_n = -2 + 5(n - 1)$

   $f(n) = -2 + 5(n - 1)$

   $f(n) = -2 + 5n - 5$

   Distributive property

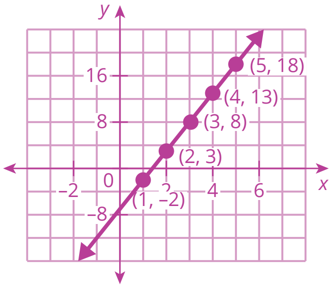
   $f(n) = 5n - 2 - 5$

   Commutative property

   $f(n) = 5n - 7$

   Combining like terms

   b. Graph the function. Label the first 5 values of the sequence on the graph.



MODULE 2 • TOPIC 1 • LESSON 3 ASSIGNMENT 197

Lesson 3 Assignment

2. Determine whether each table of values represents a linear function. For those that represent linear functions, write the function. For those that do not, explain why not.

a.

x	$f(x)$
3	14
4	18
5	23
6	29

This table does not represent a linear function because the first differences are not constant. The first differences are 4, 5, and 6.

b.

x	$g(x)$
0	2
1	−1
2	−4
3	−7

Linear function
$g(x) = -3x + 2$

c.

x	$h(x)$
1	11
2	16
3	21
4	26

Linear function
$h(x) = 5x + 6$

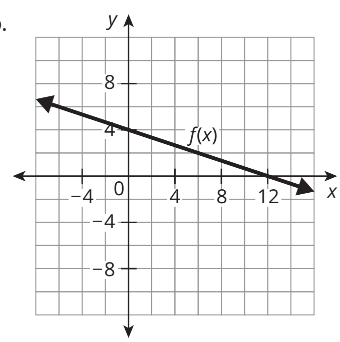
3. Calculate the average rate of change for each linear function using the formula. Show your work.

a.

x	$f(x)$
3	−4
7	4
9	8
12	14

2

b.



$-\frac{1}{3}$

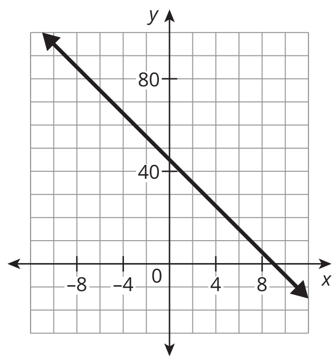
198 MODULE 2 • TOPIC 1 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

Prepare

Write an equation for each linear relationship.

1. The contestant on a game show has already won a total of $2750 from a previous episode when the game show continued today. He earns an additional $250 for each question he answers correctly today.
   $y = 2750 + 250x$

2. 

   $y = 45 - 5x$

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MODULE 2 • TOPIC 1 • LESSON 3 ASSIGNMENT 199

4 Point-Slope Form of a Line

MATERIALS

Scissors

Problem-Solving Model graphic Organizer

Representation Cards (located at the end of the lesson)

LESSON OVERVIEW

Students use the slope formula to derive the point-slope form of a linear equation. They write equations in point-slope and slope-intercept form given different sets of information: a table of values, two points, a context, a slope and the *y*-intercept, a slope and a point, a graph with a visible *y*-intercept, and a graph with a non-visible *y*-intercept. Students explore the slopes, intercepts, and equations of horizontal and vertical lines. Finally, they match equations written in slope-intercept or point-slope form with contexts and tables.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1E create and use representations to organize, record, and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

(TEKS continued on next page)

ELPS

(2) Listening
The student is expected to:

(G) understand the general meaning, main points, and important details of spoken language ranging from situations in which topics, language, and contexts are familiar to unfamiliar.

(3) Speaking
The student is expected to:

(A) practice producing sounds of newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters to pronounce English words in a manner that is increasingly comprehensible.

(5) Writing
The student is expected to:

(E) employ increasingly complex grammatical structures in content area writing commensurate with grade-level expectations such as (i) using correct verbs, tenses, and pronouns/antecedents; (ii) using possessive case (apostrophe -s) correctly; and, (iii) using negatives and contractions correctly.

ALGEBRA I TEKS *(TEKS continued from previous page)*

A.3A determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

A.3C graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

ESSENTIAL IDEAS

- The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept of the line.
- The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line.
- Horizontal lines have a slope of 0. The equation of a horizontal line that passes through $(0, b)$ is $y = b$.
- Vertical lines have an undefined slope. The equation of a vertical line that passes through $(a, 0)$ is $x = a$.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Draining the Pool 10–15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students write equations to represent the water level as three pools are drained. The rate and initial value are provided for one pool, and the water level at three distinct times are provided for the other pool. Students then compare the process of writing each equation.

DEVELOP

Activity 4.1: Writing Equations in Point-Slope Form 20–25 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

Students use the slope formula to derive the the point-slope form of a linear equation. They write equations in point-slope and slope-intercept form given different sets of information: a table of values, two points, a context, a slope and the y-intercept, a slope and a point, a graph with visible *y*-intercept, and a graph with non-visible *y*-intercept. Students explain when they prefer to use each form of a linear equation.

Activity 4.2: Horizontal and Vertical Lines 10–15 minutes

MATHEMATICAL PROBLEM SOLVING

Students analyze tables of values for horizontal and vertical lines. They determine that the slope of a horizontal line is 0 and its equation is $y = a\ constant$. They determine that the slope of a vertical line is undefined and its equation is $x = a\ constant$. In both cases, students practice writing equations and relating the equations to a graph and a table.

DAY 2

Activity 4.3: Matching Representations 30 minutes

INVESTIGATION

Students cut out and analyze graphs, tables, and contexts representing linear relationships. They use what they have learned about slopes, *y*-intercepts, and equations of a line to match either a table, graph, or scenario with an equation. Students may recognize some of the representations from previous activities.

DEMONSTRATE

Talk the Talk: Say What? 10–15 minutes

EXIT TICKET PROCEDURES

Students explain what information they know about a linear relationship by looking at its equation in a variety of forms. They also create a context given specific characteristics.

Getting Started

ENGAGE

Draining the Pool

Facilitation Notes

In this activity, students write equations to represent the water level as three pools are drained. The rate and initial value are provided for one pool, the rate and water level at one distinct time is provided for another pool, and the water level at two distinct times are provided for the third pool. Students then compare the process of writing each equation.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why is the slope negative? • Why might someone prefer to write the equation as $y = 14 - 3x$ rather than $y = 3x - 14$?
Probing	• How did you determine the initial water level at Pool *B*?
Seeing structure	• What does the 18 in your equation represent?
Gathering	• What is the general equation you used to write the equations for these lines?
Probing	• How did you identify the slope and *y*-intercept in each situation?

STAMP THE LEARNING

Summary

Real-world situations modeling linear relationships can be expressed as an equation written in slope-intercept form, $y = mx + b$, where m is the slope and b is the y-intercept.

ACTIVITY 4.1

Writing Equations in Point-Slope Form

DEVELOP

Facilitation Notes

In this activity, students use the slope formula to derive the point-slope form of a linear equation. They write equations in point-slope and slope-intercept form given different sets of information: a table of values, two points, a context, a slope and the *y*-intercept, a slope and a point, a graph with visible *y*-intercept, and a graph with non-visible *y*-intercept. Students explain when they prefer to use each form of a linear equation.

Ask a student to read the introduction and definition aloud. Discuss the Worked Example and complete Question 1 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What information was substituted into the slope formula? • Why do you think it is more efficient to substitute (x, y) for (x_2, y_2)? • What algebra steps were used to rewrite $-\frac{1}{2} = \frac{y - 6}{x - 2}$ as $-\frac{1}{2}(x - 2) = y - 6$
Seeing structure	• Why do you think this form is the point-slope form?
Probing	• If the equation was $y + 6 = -\frac{1}{2}(x + 2)$, what is the slope? The point? How can you tell?

DIFFERENTIATION STRATEGIES

Challenge Opportunity

- Compare the Worked Examples for slope-intercept form and point-slope form side-by-side so that students see that the same process was used each time. Discuss the fact that the y-intercept is just a special case of the point in point-slope form.
- Have students use their literal equation-solving techniques to derive the point-slope form of a linear equation from the slope-formula.

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Describe the graph of this line. • Can you identify the slope from your equation? • Can you identify the y-intercept from your equation? • What other information does your equation provide?

DIFFERENTIATION STRATEGY

Access for All

- Ask half the class (or one partner in a pair) to determine the equation using point-slope form, and the other half to determine the equation in slope-intercept form. Then, discuss which process was more efficient.

Have students work with a partner or in a group to complete Questions 4 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Did you write the equation in point-slope form or slope-intercept form? Why did you choose that form? • Can you visualize the graph more easily from one form than the other? Explain your thinking.
Probing	• What is the equation written in the other form?

STAMP THE LEARNING

Summary

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is any point on the line.

ACTIVITY 4.2 Horizontal and Vertical Lines

Facilitation Notes

In this activity, students analyze tables of values for horizontal and vertical lines. They determine that the slope of a horizontal line is 0, and its equation is $y = a\ constant$. They determine that the slope of a vertical line is undefined, and its equation is $x = a\ constant$. In both cases, students practice writing equations and relating the equations to a graph and a table.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

Materials Needed: Graph Paper

- Provide six graphs scaled from −12 to 12 on both axes so they can graph the points to help determine or check their answers.
- Have them rewrite equations where $y = a\ constant$ in $y = mx + b$ form; for example, $y = 3$ can be rewritten as $y = 0x + 3$.

COMMON MISCONCEPTION

- It may be counterintuitive to students that $x = a\ constant$ is parallel to the y-axis, and $y = a\ constant$ is parallel to the x-axis. Instead of focusing on what axis the equation is parallel to, have them focus on what axis the line intersects; for example, $x = -2$ intersects the x-axis at −2, and $y = -6$ intersects the y-axis at −6.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How could you rewrite this equation in $y = mx + b$ form? • How do you know that this graph has a *y*-intercept? • Does the graph of this line pass through (0, −6) or (−6, 0)? • When given ordered pairs, how do you know what variable and value to use for your equation? • Why can't you write this equation in $y = mx + b$ form? • Why does it make sense that the slope is undefined?
Gathering	• When given ordered pairs, how do you know which variable and value to use for your equation?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

The equation for a horizontal line is $y = $ *a constant* because there is no change in the y-values as the x-values change, therefore the slope is 0. The equation for a vertical line is $x = $ *a constant* because there is no change in the *x*-values as the *y*-values change, therefore the slope is undefined.

ACTIVITY 4.3 Matching Representations

Facilitation Notes

In this activity, students cut out and analyze graphs, tables, and contexts representing linear relationships. They use what they have learned about slopes, *y*-intercepts, and equations of a line to match either a table, graph, or scenario with an equation. Students may recognize some of the representations from previous activities.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Have students work with two sets of three representations and equations at a time, rather than the entire set of representations and equations at one time.

Challenge Opportunity

- To extend the activity, have students create the two representations that were not provided for each equation.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What clues did you use from the graphs, tables, and contexts to match them to the equations? • Did you start with the equations or the other representations when matching them? Explain your strategy. • Explain how you rewrote the equation for Graph *A* in slope-intercept form. • How did you determine the slope from each table? • What are two different ways you could have determined the y-intercept for Table *D*? • How could you calculate the rate of $1.20 per topping from Context *F*?

STAMP THE LEARNING

Summary

Linear relationships can be modeled using graphs, tables, contexts, and equations.

DEMONSTRATE

Talk the Talk

SAY WHAT?

Facilitation Notes

In this activity, students examine equations written in different forms and identify specific information such as slope, y-intercept, and the coordinates of a point on the line.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What equations are in the same form? How can you tell? • Is part (d) in slope-intercept form? How could you rewrite it as $y = mx + b$? • Is part (f) in slope-intercept form? How could you rewrite it as $y = mx + b$? • How could you rewrite the equation in part (c) so that all substitutions in the point-slope form are visible? Which point can you identify from this form? • Rewrite the equation in part (c) in slope-intercept form. What is the y-intercept?

continued

continued

COMMON MISCONCEPTION

- When determining a point that lies on a line from point-slope form, students often make sign errors. Work backwards from an example, and discuss how when a point is substituted into $y - y_1 = m(x - x_1)$, the signs in front of x_1 and y_1 are negative signs, taking the opposite of the substituted coordinates of the point. For example,

  Given slope $= 2$ and point $(-1, 6)$:

  By substitution in $y - y_1 = m(x - x_1)$

  $$y - 6 = m(x - (-1))$$
  $$y - 6 = m(x + 1).$$

Reading the point from the equation, the *y*-value is the opposite of what is written after *y*, and the *x*-value is the opposite of what is written after *x*. The point is $(-1, 6)$.

Have students read and answer the Essential Question on the lesson opener page.

Summary

Characteristics of linear relationships can be identified when their equations are written in slope-intercept form and point-slope form.

STAMP THE LEARNING

4 Point-Slope Form of a Line

Setting the Stage

- Communicate the objectives and new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Use the slope formula to derive the point-slope form of a linear equation.
- Construct an equation in point-slope form to model a linear relationship between two quantities.
- Write equations for vertical and horizontal lines.

NEW KEY TERM

- point-slope form

You have used the slope-intercept form to represent linear relationships. Are there other forms of a linear equation that you can use? How do you write equations for horizontal and vertical lines?

Sample answer:

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

MODULE 2 • TOPIC 1 • LESSON 4 201

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Student Look-Fors

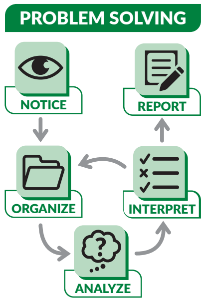
Whether students are demonstrating proficiencies related to TEKS A.1B:

- Are students referring to the problem-solving model?
- Do students evaluate the reasonableness of their solution?
- Do students adapt their plan as needed?

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work in pairs, ask themselves the questions to ask from the four steps of the model, and share their reasoning.
- Complete the remaining step in the graphic organizer together as a class.
- Have students work in pairs and use the problem-solving model to complete Question 2.

Getting Started



Draining the Pool

Miguel and Nia are pool cleaners who have been hired to drain the community diving pools at the end of the summer. They are comparing the rate at which the three pools drain.

1. For each pool, write an equation in slope-intercept form to represent the linear relationship.

   a. Pool A is at a water level of 14 feet and drains at a rate of 3 feet per hour.

   $y = 14 - 3x$

   b. Pool B is at a water level of 10 feet after draining for 3 hours and drains at a rate of 2 feet per hour.

   $y = -2x + 16$

   c. Pool C is at a water level of 15 feet after draining for 2 hours and at 12 feet after draining for 4 hours.

   $y = -\frac{3}{2}x + 18$

I wonder whether there is a way to make writing the equation of a line more efficient.

2. Compare your process for writing each equation. How are the processes different?

   Sample answer:
   The slope and y-intercept were given for Pool A. The slope was given for Pool B, but I had to calculate the y-intercept. I had to calculate the slope and y-intercept for Pool C.

EB STUDENT TIP

For all proficiency levels

Explain to students that there is more than one way to write the equation of a line. Ask students about things in their life that can be represented in multiple ways, but still mean the same thing. Explain to students in this lesson they will learn another way of writing the equation of a line. Ask them the name of the way they already know, which they learned about in the previous lesson.

ACTIVITY **4.1**

Writing Equations in Point-Slope Form

In the previous lesson, you used the slope, the *y*-intercept, and the slope formula to write a linear equation. You can also determine the equation of a line without knowing the *y*-intercept.

WORKED EXAMPLE

To write an equation of a line from a table of values, you can use the slope formula.

x	y
2	6
4	5
6	4

- First, calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-6}{4-2}$$
$$= \frac{-1}{2} = -\frac{1}{2}$$

- Next, choose any point from the table.

(2, 6)

- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, (2, 6), and the unknown point (x, y).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-\frac{1}{2} = \frac{y-6}{x-2}$$

- Finally, rewrite the equation with no variables in a denominator.

$$-\frac{1}{2} = \frac{y-6}{x-2}$$
$$-\frac{1}{2}(x-2) = y - 6$$

The equation is $y - 6 = -\frac{1}{2}(x - 2)$.

This linear equation in the Worked Example is written in *point-slope form*. The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is any point on the line.

1. Solve the equation in the Worked Example for *y* so that the linear equation is in slope-intercept form. What unique information does each form of the linear equation provide? How are they similar?

$$-\frac{1}{2}(x-2) = y - 6$$
$$-\frac{1}{2}x + 1 = y - 6$$
$$y = -\frac{1}{2}x + 7$$

Both forms display the slope of the line. The point-slope form gives a point that may not be the *y*-intercept. The slope-intercept form gives the *y*-intercept.

Chunking the Activity

- Read and discuss the Worked Example and definition.
- Complete Question 1 as a class.
- Group students to complete Questions 2 and 3.
- Check in and share.
- Group students to complete Questions 4–9.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 2 • TOPIC 1 • LESSON 4 203

Questions 2–5 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing equations in point-slope form given a point and a slope, assign Skills Practice Set A for this lesson.

Question 6 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing equations in point-slope form and slope-intercept form given two points, assign Skills Practice Set B for this lesson.

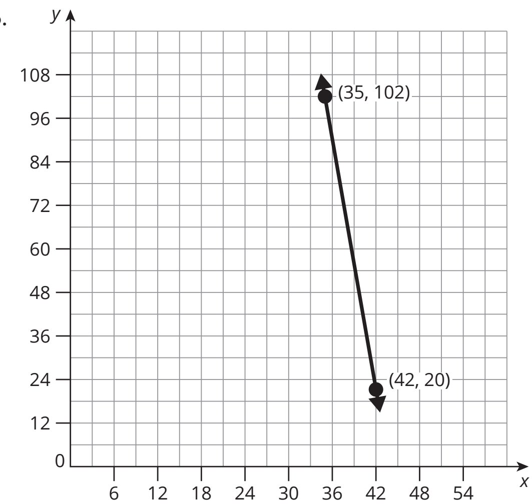
Write the equation for each linear relationship in point-slope form.

2. The slope is −8. The point (3, 12) lies on the line.
   $y - 12 = -8(x - 3)$

3. The slope is $\frac{2}{3}$. The point (−4, 5) lies on the line.
   $y - 5 = \frac{2}{3}(x + 4)$

4. It costs $1.10 per pound plus a basic shipping charge to mail a package. A 10-pound package costs $14 to mail.
   $y - 14 = 1.1(x - 10)$

5. $m = -\frac{3}{8}$ and point (50, 7)
   $y - 7 = -\frac{3}{8}(x - 50)$

6. 

   Sample answer:
   $y - 20 = -\frac{82}{7}(x - 42)$

204 MODULE 2 • TOPIC 1 • LESSON 4

7.

x	y
−5	−6
1	−6
2	−6

Sample answer:
$y + 6 = 0(x - 1)$

8. Rewrite the equation you wrote for Question 7 in slope-intercept form. What do you notice about this equation?

$$y + 6 = 0(x - 1)$$
$$y + 6 = 0x - 0$$
$$y = -6$$

Sample answer:
This equation has no x-term. It is a constant function.

9. Consider the information you may have about a linear relationship. Which form of the equation do you prefer to use in each case? Explain your reasoning.

a. Given slope and y-intercept

Reasoning may vary.
I would use the slope-intercept form.

b. Given two points

Reasoning may vary.
I would need to calculate the slope first, and then I would use the point-slope form.

c. Given slope and a point other than the y-intercept

Reasoning may vary.
I would use the point-slope form.

Questions 6 and 7 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing equations in point-slope form and slope-intercept form given graphs and tables, assign Skills Practice Set C for this lesson.

MODULE 2 • TOPIC 1 • LESSON 4 205

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 4.2 Horizontal and Vertical Lines

Horizontal and vertical lines represent linear relationships, but their equations are different from the equations of lines that are not horizontal or vertical.

1. Consider the equation, $y = -6$, that you wrote for the table shown in the previous activity.

   a. How is this equation different from the other equations? What is its slope?

   This equation does not have an x. The slope of the equation is 0.

   b. Describe the graph of the coordinate pairs in this table. Why does the value of its slope make sense?

x	y
−5	−6
1	−6
2	−6

   The graph of this linear relationship is a horizontal line passing through (0, −6).

   What is the y-intercept?

   c. Explain why the equation makes sense in terms of the graph and the table.

   The equation states that y is always equal to −6. The table shows that the y-value is always −6, and the graph always has a y-value of −6.

2. Write an equation for each linear relationship. Describe the graph of the linear relationship. State the slope and y-intercept.

   a.

x	y
−7	11
−2	11
0	11

   $y = 11$

   The graph is a horizontal line. It has a slope of 0 and a y-intercept of (0, 11).

   b. A line that passes through (−15, −3.75) and (89, −3.75)

   $y = -3.75$

   The graph is a horizontal line. It has a slope of 0 and a y-intercept of (0, −3.75).

206 MODULE 2 • TOPIC 1 • LESSON 4

3. Consider a new table of values representing a linear relationship.

x	y
−2	5
−2	14
−2	29

   a. Explain how this table is similar to and different from the tables in Questions 1 and 2.

   The table has all the same values in a single column, but this time, the values are in the x column.

   b. Write an equation for the linear relationship in the table.

   $x = -2$

   c. Describe the graph of this linear relationship.

   The graph will be a vertical line passing through (−2, 0).

   d. Use the slope formula to calculate the slope between two points in the table. What do you notice?

   $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{29 - 5}{-2 - (-2)} = \frac{24}{0}$$

   The slope is undefined.

   e. What is the y-intercept of this linear relationship? Explain why this makes sense.

   There is no y-intercept because the graph goes directly up and down and does not slope toward the y-axis.

4. Write an equation for each linear relationship. Describe the graph of the linear relationship.

   a.

x	y
$\frac{17}{2}$	−18
$\frac{17}{2}$	23
$\frac{17}{2}$	267

   $x = \frac{17}{2}$.
   The graph is a vertical line passing through the point $\left(\frac{17}{2}, 0\right)$.

   b. A line that passes through (−7, −973) and (−7, 542)

   $x = -7$. The graph is a vertical line passing through the point (−7, 0).

   c. Create an additional table of values and write the equation for a vertical line.

   Answers will vary based on each classroom.

In a horizontal line there is no change in the y-values as the x-values change. Therefore, the slope is 0. A horizontal line has zero steepness. In a vertical line there is no change in the x-values as the y-values change. Therefore, the slope is undefined. A vertical line has an undefined steepness.

STAMP THE LEARNING

The paragraph provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 2 • TOPIC 1 • LESSON 4 207

Chunking the Activity

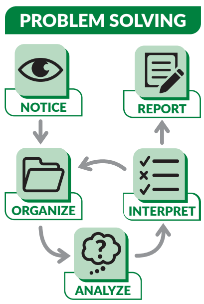
- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the problem-solving model graphic organizer.
- For Question 1, have students work in pairs to complete the graphic organizer and share their reasoning.
- Have students work in pairs and use the problem-solving model to complete Questions 2–4.

ACTIVITY 4.3 Matching Representations

Take out your scissors. It's time to cut and sort!



1. Carefully cut out the graphs, tables, contexts, and equations located the end of the lesson. Match each equation with its correct graph, table, or context. Explain how you matched the equations with the representations.

   Explanations will vary.

   A: $y - 200 = -5(x - 24)$

   B: $y = \frac{1}{2}x + 40$

   C: $y - x = 2$

   D: $y = -\frac{3}{4}x + \frac{33}{2}$

   E: $y = 8x + 120$

   F: $y = 1.2x + 9$

2. Compare the graphs.

   a. How are they different? How are these differences reflected in the slope-intercept form of their equation?

   One of the graphs goes down from left to right, and one goes up from left to right. The graph that goes down from left to right has a negative slope, and the graph that goes up from left to right has a positive slope.

   b. Identify the *y*-intercept for each graph. How can you determine this point in the slope-intercept form of the equation for each graph?

   If the equation is written in slope-intercept form, the number that is added in each equation is the *y*-coordinate of the *y*-intercept.

   c. Identify the slope for each graph. How is the slope represented in the slope-intercept form of each equation?

   The slope for Graph *A* is -5. The slope for Graph *C* is 1. The slope is the coefficient of *x* in the equation when it is written in slope-intercept form.

208 MODULE 2 • TOPIC 1 • LESSON 4

3. Analyze the equation for each table.

   a. Determine the coefficient of x for each linear relationship using the slope formula.

   Table D: $m = \frac{y_2 - y_1}{x_2 - x_1}$

   $= \frac{9 - 15}{10 - 2} = -\frac{6}{8} = -\frac{3}{4}$

   Table E: $m = \frac{y_2 - y_1}{x_2 - x_1}$

   $= \frac{264 - 216}{18 - 12} = \frac{48}{6} = \frac{8}{1}$

   b. How can the number that is added in each equation written in slope-intercept form be determined from the table?

   The number that is added in each equation is the y-value when the x-value is 0. If it is not in the table, it can be determined by using the slope formula to determine the slope, then by using the slope formula a second time substituting the value for m, a point, and the point (0, y).

Can you remember the ways to determine the rate of change from a table?

4. Analyze the equation for each context. Explain what each term of the equation means in each context.

   For Context B, the term $\frac{1}{2}x$ means that Michele is reading one page every two minutes, or one-half page per minute. The x represents the independent variable, the number of minutes. The term 40 represents the number of pages she has already read.

   For Context F, the term $1.2x$ means that there is a cost of \$1.20 per additional topping. The x represents the independent variable, the number of additional toppings. The term 9 represents the starting cost of a large pizza.

MODULE 2 • TOPIC 1 • LESSON 4 209

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Say What?

You have learned about two forms of a linear equation: the slope-intercept form, $y = mx + b$, and the point-slope form, $y - y_1 = m(x - x_1)$.

1. What information can you determine about each line by looking at the structure of the equation?

   a. $y = \frac{3}{5}x - 4$

   The slope is $\frac{3}{5}$ and the y-intercept is $(0, -4)$.

   b. $y - 6 = 2(x + 1)$

   The slope is 2 and a point on the line is $(-1, 6)$.

   c. $y + 4 = 2(x - 0)$

   The slope is 2 and a point on the line is $(0, -4)$.

   d. $y = -\frac{2}{7}x$

   The slope is $-\frac{2}{7}$ and the y-intercept is $(0, 0)$.

   e. $y + 5 = -(x - 4)$

   The slope is -1 and a point on the line is $(4, -5)$.

   f. $y = 19$

   The slope is 0 and the y-intercept is $(0, 19)$. It is a horizontal line through $(0, 19)$.

2. Create a context that represents a linear relationship that passes through the point (2, 56) and has an increasing slope. Then, write the equation of the line in point-slope form and slope-intercept form.

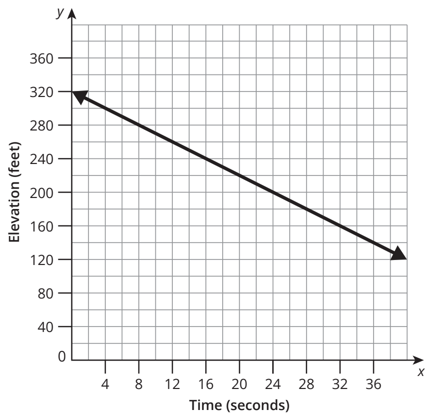
   Sample answer:

   Omar plays a game in which he earns credits to activate game enhancers. For each game he plays, he earns 4 credits. After playing 2 games today, Omar now has 56 credits.

   $y - 56 = 4(x - 2)$

210 MODULE 2 • TOPIC 1 • LESSON 4

a.

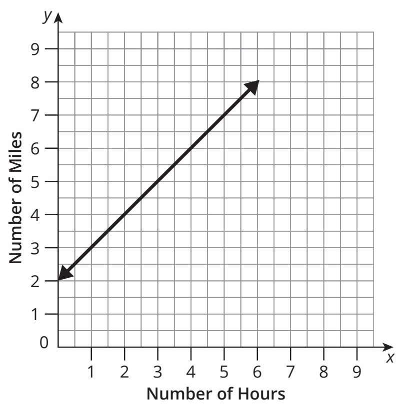


$y - 200 = -5(x - 24)$

b. Juliana read the first 40 pages of a mystery novel before she fell asleep. The next day, she read one page every two minutes until she finished the book, which was a total of 325 pages.

$y = \frac{1}{2}x + 40$

c.



$y - x = 2$

d.

Time (hours)	Water Level (feet)
x	**y**
2	15
4	13.5
8	10.5
10	9

$y = -\frac{3}{4}x + \frac{33}{2}$

e.

Number of Games Ben Won Today	Number of Credits on Ben's Player's Card
x	**y**
12	216
18	264
25	320
40	440

$y = 8x + 120$

f. A pizza shop charges \$4.50 for a small pizza, \$7.00 for a medium pizza, and \$9.00 for a large pizza. Additional toppings cost extra depending on the size of the pizza ordered. Antonio ordered a large pizza with three toppings that cost a total of \$12.60.

$y = 1.2x + 9$

MODULE 2 • TOPIC 1 • LESSON 4 211

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So you can cut out the cards on the other side.

212 MODULE 2 • TOPIC 1 • LESSON 4

$y = 1.2x + 9$	$y = -\frac{3}{4x} + \frac{33}{2}$
$y = \frac{1}{2}x + 40$	$y - x = 2$
$y - 200 = -5(x - 24)$	$y = 8x + 120$

MODULE 2 • TOPIC 1 • LESSON 4 213

Why is this page blank?

So you can cut out the cards on the other side.

214 MODULE 2 • TOPIC 1 • LESSON 4

Lesson 4 Assignment

Write

Compare the slope-intercept and point-slope forms of a linear equation.

Remember

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

Practice

Write an equation in point-slope form.

1. $m = 2$; (5, 6)
   $y - 6 = 2(x - 5)$

2. $m = -9.2$; (−17, 10)
   $y - 10 = -9.2(x + 17)$

3. (−2, −3) and (8, −8)
   $y + 8 = -\frac{1}{2}(x - 8)$ or
   $y + 3 = -\frac{1}{2}(x + 2)$

4. (79, 52) and (−87, 550)
   $y - 550 = -3(x + 87)$ or
   $y - 52 = -3(x - 79)$

5. A photography studio charges $50 for a sitting fee and 6 prints. Luigi increased his order to 11 prints and paid $65.
   $y - 50 = 3(x - 6)$ or
   $y - 65 = 3(x - 11)$

6. Lucia is taking the stairs in her building from her floor to the top of the building. After 2 minutes, she was 100 steps from the bottom floor. After 5 minutes, she was 196 steps from the bottom floor.
   $y - 100 = 32(x - 2)$ or
   $y - 196 = 32(x - 5)$

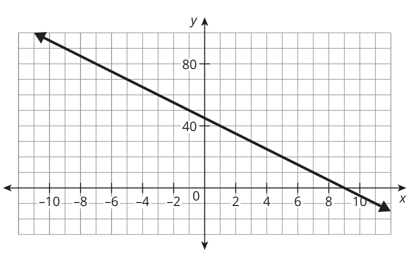
Write an equation in any form.

7. A newspaper charges a flat fee plus a charge per day to place a classified ad.

Number of Days	Total Charge ($)
2	8.00
4	13.00
6	18.00

Sample answer:
$y = 2.5x + 3$

8.



Sample answer:
$y = 45 - 5x$

Write

Answers will vary based on each classroom. Both forms display the slope of the line. Both can display the y-intercept, but the point-slope form can display a point other than the y-intercept.

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MODULE 2 • TOPIC 1 • LESSON 4 ASSIGNMENT 215

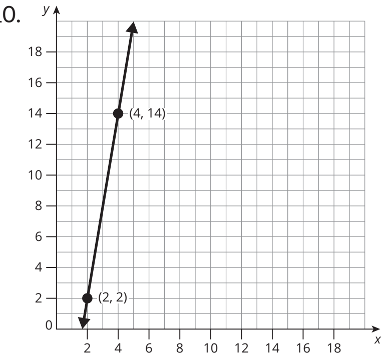
Lesson 4 Assignment

9.

x	y
−10	50
−2	10
4	−20
14	−70

Sample answer:
$y - 50 = -5(x + 10)$

10.

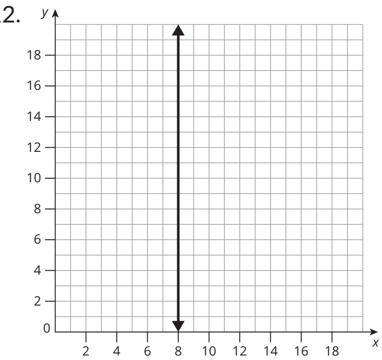


Sample answer:
$y - 2 = 6(x - 2)$

11. Xavier is traveling on a toll road. He plans to exit the road 5 miles ahead and pay $1.75. He changes his plans and travels 9 miles and pays $2.75.

Sample answer:
$y - 1.75 = 0.25(x - 5)$

12.



$x = 8$

Prepare

Solve each equation for y.

1. $-2y = -x + 7$
   $y = \frac{1}{2}x - \frac{7}{2}$

2. $\frac{3}{4}y = x - 6$
   $y = \frac{4}{3}x - 8$

3. $2x + 3y = 6$
   $y = 2 - \frac{2}{3}x$

4. $\frac{1}{2}x - 4y = 8$
   $y = -2 + \frac{1}{8}x$

216 MODULE 2 • TOPIC 1 • LESSON 4 ASSIGNMENT

5 Using Linear Equations

MATERIALS

Straightedges

LESSON OVERVIEW

Students use three different forms of a linear equation to graph linear relationships. First, they learn how to use the slope-intercept and point-slope forms of a line to graph. Students explore the standard form of a linear equation and connect relationships among the coefficients of the standard form with the x-intercept, y-intercept, and slope of a line. They then practice writing and graphing equations in standard form. Finally, students identify the slope and intercept of linear equations in different forms and evaluate the usefulness of each form of a linear equation.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.

The student is expected to:

A.2B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

(TEKS continued on next page)

ELPS

(1) Learning Strategies

The student is expected to:

(D) speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution (conveying ideas by defining or describing when exact English words are not known).

(3) Speaking

The student is expected to:

(B) expand and internalize initial English vocabulary by learning and using high-frequency English words necessary for identifying and describing people, places, and objects, by retelling simple stories and basic information represented or supported by pictures, and by learning and using routine language needed for classroom communication.

(4) Reading

The student is expected to:

(F) use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language.

ALGEBRA I TEKS *(TEKS continued from previous page)*

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3A determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$

A.3C graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems

ESSENTIAL IDEAS

- The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept of the line.
- The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line.
- The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.
- The information contained in the equation of a line can be used to graph the line.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Jump in the Line 10–15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students describe what they know about graphs of lines from given equations in different forms, and they brainstorm how they might graph the lines from their equations.

DEVELOP

Activity 5.1: Using Slope-Intercept Form to Graph a Line 25–30 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

In this activity, students learn through a Worked Example how to graph equations written in slope-intercept form. They graph lines from equations written in slope-intercept form or contexts that can be modeled using slope-intercept form without creating a table of values.

DAY 2

Activity 5.2: Using Point-Slope Form to Graph a Line 15–20 minutes

MATHEMATICAL PROBLEM SOLVING, REAL-WORLD PROBLEM SOLVING

Students learn through written steps how to graph equations written in point-slope form. They graph lines from equations written in point-slope form and from a context that can be modeled using point-slope form without creating a table of values.

Activity 5.3: Standard Form of a Linear Equation 20–25 minutes

CONNECT TO PRIOR KNOWLEDGE

Students are given a scenario which includes an equation written in standard form, and they explain the meaning of each term, define the variables, solve for the intercepts, and use the intercepts to graph the situation. Students use equations (written in standard form) to identify the intercepts and slope of the equation. Next, they match graphs of lines to their respective equations and write and graph equations from given contexts.

DAY 3

Activity 5.4: Identifying Slope and y-Intercept 20–25 minutes

MATHEMATICAL PROBLEM SOLVING

Students determine the slope and y-intercept of linear relationships represented in slope-intercept, point-slope, and standard form. Some equations require algebraic simplification, and the special cases of $y = b$ and $x = a$ are included in the questions.

DEMONSTRATE

Talk the Talk: Choose Your Medium 20–25 minutes

MATHEMATICAL PROBLEM SOLVING

Students write and graph linear equations from contexts. They choose and explain which form of the equation to use and how to graph the line. Students evaluate the usefulness of each form of a linear equation.

Getting Started

ENGAGE

Jump in the Line

Facilitation Notes

In this activity, students describe what they know about graphs of lines from given equations in different forms, and they brainstorm how they might graph the lines from their equations.

The intent of this lesson is to see what students understand and are able to do. Most students will resort to creating tables and plotting points to graph the lines. Students may struggle with how to use similar triangles or transformations to create graphs. The following activities guide students to graph from slope-intercept, point-slope, and standard forms of equations of lines.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Identify the y-intercept as a point.
Seeing structure	• What is the form of this equation? • Explain how you determined the signs of the point that lies on this line.
Probing	• What are the coordinates of several points on this line? • How could you work with this equation to determine its slope or any points that lie on it?
Seeing structure	• How would you describe the structure of this equation?

Summary

STAMP THE LEARNING

Characteristics of linear relationships can be identified when their equations are written in different forms such slope-intercept form, point-slope form, and standard form.

ACTIVITY 5.1

Using Slope-Intercept Form to Graph a Line

DEVELOP

Facilitation Notes

In this activity, students learn through a Worked Example how to graph equations written in slope-intercept form. They graph lines from equations written in slope-intercept form or contexts that can be modeled using slope-intercept form without creating a table of values.

Ask a student to read the introduction aloud. Review the Worked Example as a class.

DIFFERENTIATION STRATEGIES

Access for All

- In the Getting Started, students graphed lines from equations. You may want to see how students attempt to graph a context, too, prior to any guidance.
  - Have students read the first two paragraphs, discuss what the independent and dependent variables are, provide scaled graphs, and give students time to graph the context.
  - Have students explain how they used the context to move from the y-intercept to (1, 8). Lead students to use phrasing such as, "As the number of people who requested tickets increased by 1, the number of tickets remaining decreased by 2." or, "Every time the number of tickets remaining decreased by 2, 1 additional person requested tickets."
  - Then, reconvene using the textbook at the 3rd paragraph. As you go through the Worked Example, make connections to the graph generated from the context.

Materials Needed: Colored Pencils

- Have students interact with the Worked Example.
  - Students should highlight (0, 10) with a colored pencil and mark that point on the graph with the same color.
  - Students highlight $m = -\frac{2}{1}$ and trace the arrows on the part of the graph that applies to it using a second color.
  - Repeat the same process for $m = \frac{2}{1}$ using a third color.
- Require that students list the y-intercept and two equivalent fractions for the slope for each problem.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- The counting of spaces rather than units in Questions 3 through 5.
- Confusion in realizing an equivalent rate for $\frac{3}{2}$ is $\frac{-3}{-2}$.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which point did you plot first? Where did you locate it? • How did you apply the slope $\frac{2}{3}$ to plot an additional point?
Seeing structure	• How is graphing using the slope related to slope triangles? • In which direction do you move with a positive numerator? A negative numerator? A positive denominator? A negative denominator? • How do you know the direction of the line is correct? • Why does it help to write the number 10 as a fraction? • Why is this graph increasing? • Use the graph to show how the total savings increases by 15 for every one lawn mowed? • Why is this graph decreasing? • Use the graph to show how the amount owed decreased by 10 for every one week.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

An equation written in slope-intercept form can be graphed by first plotting the *y*-intercept and then using the slope to locate two additional points by counting from that initial point.

ACTIVITY 5.2 Using Point-Slope Form to Graph a Line

Facilitation Notes

In this activity, students learn through written steps how to graph equations written in point-slope form. They graph lines from equations written in point-slope form and from a context that can be modeled using point-slope form without creating a table of values.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- The counting of spaces rather than units in Questions 1 and 3.
- Confusion because the graph cannot be extended to the right in Question 2 part (a).

- Varying scales on the *x*-axis for Question 3, such as Time (half-hours), Time (hours), or Time (minutes).
- Graphs that are short segments connecting the few points rather than extended lines.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why did it make sense to use the point-slope form for this situation?
Probing	• What other way did you express the slope $\frac{5}{1}$ to graph points on the other side of (3, 50)?
Seeing structure	• How is this process the same and different from graphing a point in slope-intercept form?
Gathering	• What is the known point? • What are the two different ways you expressed the slope?
Reflecting and justifying	• How do you know that the direction of the line is correct?
Gathering	• How did you rewrite the given rate to substitute the value for the slope in your equation?
Reflecting and justifying	• Why is this graph increasing? • Use the graph to show how the number of bracelets increased by 14 for every 1 hour.

DIFFERENTIATION STRATEGIES

Access for All

- Compare and contrast slope-intercept method and point-slope method.
- Compare and contrast graphs that have different scales in Question 3.

STAMP THE LEARNING

Summary

An equation written in point-slope form can be graphed by first plotting the coordinates of the known point and then using the slope to locate two additional points by counting from that initial point.

ACTIVITY 5.3

Standard Form of a Linear Equation

Facilitation Notes

In this activity, students are given a scenario which includes an equation written in standard form, and they explain the meaning of each term, define the variables, solve for the intercepts, and use the intercepts to graph the situation. Students use equations (written in standard form) to identify the intercepts and slope of the equation. Next, they match graphs of lines to their respective equations and write and graph equations from given contexts.

Ask a student to read the introduction and definition aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Describe the structure of this equation. • Why can't both *A* and *B* be 0? • Why is standard form appropriate to use for this problem situation? • Why is it difficult to tell which quantity is independent and which quantity is dependent?
Gathering	• Use the graph to identify another coordinate pair that makes sense in this situation.
Probing	• How did you determine the slope? What is another way?

Have students work with a partner or in a group to complete Questions 6 and 7. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Sign errors when attempting to calculate slope mentally; the result may be due to the calculation of $\frac{y\text{-intercept value}}{x\text{-intercept value}}$ rather than $\frac{y_2 - y_1}{x_2 - x_1}$.

COMMON MISCONCEPTION

- Students sometimes confuse slope-intercept and standard form. They overgeneralize and think that the coefficient of *x* is the slope regardless of what form the equation of a line is written in. Clarify this error in thinking while sharing responses to Question 6.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how you determined the slope.
Gathering	• What are the integers *A*, *B*, and *C* in this equation?
Probing	• Label the *x*-intercept, *y*-intercept, and slope with the variables *A*, *B*, and *C*.

DIFFERENTIATION STRATEGY

Challenge Opportunity

- Have students solve literal equations to get the generalized forms for the intercepts and slope. For the *x*-intercept, substitute 0 for *y* in the standard form equation, resulting in $Ax = C$; $x = \frac{C}{A}$ and the *x*-intercept is $\left(\frac{C}{A}, 0\right)$. Similarly, the *y*-intercept is $\left(0, \frac{C}{B}\right)$. Use these points to calculate slope using the slope formula, $\frac{\frac{C}{B}}{-\frac{C}{A}} = -\frac{A}{B}$.

- After the formulas have been developed, discuss how to convert an equation from standard form to slope-intercept form; this way, students can see another way the formula $-\frac{A}{B}$ makes sense. Also, some students may prefer to use this method to calculate slope from a linear equation in standard form. Using the introductory problem:

$$\begin{aligned} 5x + 8y &= 1600 \\ -5x \qquad &\quad -5x \\ \hline \frac{8y}{8} &= \frac{1600 - 5x}{8} \\ \hline y &= 200 - \frac{5}{8}x \\ y &= -\frac{5}{8}x + 200 \end{aligned}$$

Have students work with a partner or in a group to complete Questions 8 through 10. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Rates written with different units for Question 9, $\frac{20 \text{ calories}}{5 \text{ minutes}}$ or $\frac{4 \text{ calories}}{1 \text{ minute}}$.
- Differences in the independent and dependent variables for Questions 9 and 10.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Show the steps using the formula in Question 8 to identify the slope of each line. • How did you use the slopes to identify each line?
Gathering	• Use the graph to identify another coordinate pair that makes sense in this situation.
Probing	• Is it incorrect to reverse the axes on your graph? Why not?

Analyze the Worked Example together as a class. Then, have students work with a partner or in a group to complete Question 11.

Gathering	• What is the equation for the standard form of a linear equation?
Probing	• How did you use the other forms of linear equations to write the equation in standard form? • What is the slope of the line? Why do you need to determine the slope?
Reflecting and justifying	• Which form of a linear equation do you prefer to write when given two points? Explain your reasoning.

COMMON MISCONCEPTION

- Students may write the equation in Question 4 part (c) with a fractional *A*-value. Refer students back to the definition of standard form and help students think through rewriting the equation so that *A*, *B*, and *C* are integers.

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero. An equation in standard form can be graphed by calculating and graphing the intercepts.

ACTIVITY 5.4 Identifying Slope and *y*-Intercept

Facilitation Notes

In this activity, students determine the slope and *y*-intercept of linear relationships represented in slope-intercept, point-slope, and standard form. Some equations require algebraic simplification, and the special cases of $y = b$ and $x = a$ are included in the questions.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 10. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Failure to include the sign for B when calculating slope using $-\frac{A}{B}$ for an equation written in standard form.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the slope? What is another way? • How did you determine the *y*-intercept? What is another way?
Seeing structure	• Why doesn't this line have a *y*-intercept?

Summary

STAMP THE LEARNING

The slope of an equation can be identified directly when equations are written in slope-intercept or point-slope form; it can be calculated using $-\frac{A}{B}$ in the standard form equation, $Ax + By = C$. The *y*-intercept can be identified by using substitution of $x = 0$ in any form.

DEMONSTRATE

Talk the Talk

CHOOSE YOUR MEDIUM

Facilitation Notes

In this activity, students write and graph linear equations from contexts. They choose and explain which form of the equation to use and how to graph the line. Students evaluate the usefulness of each form of a linear equation.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Does $2x + 3y = 36$ or $3x + 2y = 36$ represent this situation? • Which axis represents the number of correct multiple-choice questions, and which represents the number of correct short answer questions? • Does $y = 5x + 20$ or $y = 5x - 20$ represent this situation? • Is $y = 5x + 20$ written in slope-intercept form or point-intercept form? • Which axis represents the number of weeks, and which represents the total savings? • What are the coordinates of the intercepts in each situation?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Different forms of equations, such as standard form, slope-intercept form, and point-slope form can be used to represent linear relationships.

5 Using Linear Equations

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Construct linear equations to model relationships between two quantities.
- Graph lines using the slope-intercept form of a linear equation.
- Graph lines using the point-slope form of a linear equation.
- Graph lines using the standard form of a linear equation.
- Convert equations from point-slope or standard form to slope-intercept form.
- Discuss the advantages and disadvantages of slope-intercept, point-slope, and standard form.

NEW KEY TERMS

- standard form
- *x*-intercept

You have graphed equations using tables of values.

Is there a more efficient method to graphing a linear relationship? Can you use the equation of a linear relationship to create a graphical representation?

Sample answer:

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

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MODULE 2 • TOPIC 1 • LESSON 5 217

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Jump In the Line

Describe what you know about the graph of each relationship by analyzing each equation. Then, explain how you might graph each line given its equation.

1. $y = \frac{2}{3}x + 7$

   Sample answer:

   The slope is $\frac{2}{3}$ and the y-intercept is 7. I can start at the y-intercept (0, 7) and determine a second point (3, 9) and connect those two points to graph a line.

2. $y - 3 = 5(x + 1)$

   Sample answer:

   The slope is 5 and a point on the line is (−1, 3). I can start at the point (−1, 3) and determine a second point (0, 8) and connect those two points to graph a line.

3. $x = -4$

   Sample answer:

   This is a vertical line at $x = -4$.

4. $-3x + 8y = 10$

   Sample answer:

   This equation doesn't directly tell me the slope or y-intercept. I can create a table of values and plot the ordered pairs to create a line. Points on the line include (2, 2) and (10, 5).

218 MODULE 2 • TOPIC 1 • LESSON 5

ACTIVITY 5.1

Using Slope-Intercept Form to Graph a Line

As you learned previously, the slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept. You can use the equation to graph the relationship without first creating a table of values using the y-intercept and the slope.

Douglas is giving away tickets to a concert that he won from a radio station contest. Currently, he has 10 tickets remaining. He gives a pair of tickets to each person who asks for them.

This situation can be modeled by the equation $y = -2x + 10$, where x represents the number of people who request tickets and y represents the number of tickets available.

To graph the equation $y = -2x + 10$, you will first plot the y-intercept, $(0, 10)$, and then use the slope, -2, to plot two more points. Remember, slope describes the steepness and direction of a line. Slope is the ratio of the change in y-values to the change in x-values, commonly referred to as rise over run. In this equation, you can think of $m = -2$ as two different ratios: $\frac{-2}{1}$ or $\frac{2}{-1}$. The sign of the number tells you the direction to go to plot a new point. The ratio $\frac{-2}{1}$ has a negative rise and a positive run. It is interpreted as down 2 units and to the right 1 unit. The ratio $\frac{2}{-1}$ has a positive rise and a negative run. It is interpreted as up 2 units and to the left 1 unit.

A rule of thumb when graphing a line is to plot at least three points.

WORKED EXAMPLE

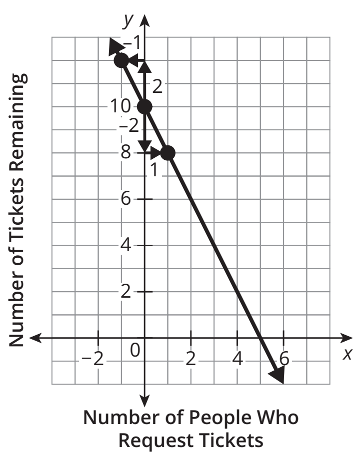
Graph $y = -2x + 10$.

Begin by plotting the y-intercept, $(0, 10)$.

Use the slope and count from the y-intercept to graph two more points on the line.

- For $m = \frac{-2}{1}$, go down 2 units and to the right 1 unit.
- For $m = \frac{2}{-1}$, go up 2 units and to the left 1 unit.

Connect the points to form a straight line.



Chunking the Activity

- Read and discuss the Worked Example.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

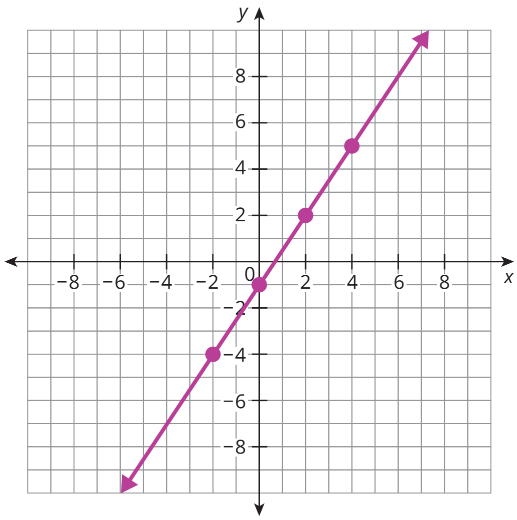
STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

MODULE 2 • TOPIC 1 • LESSON 5 219

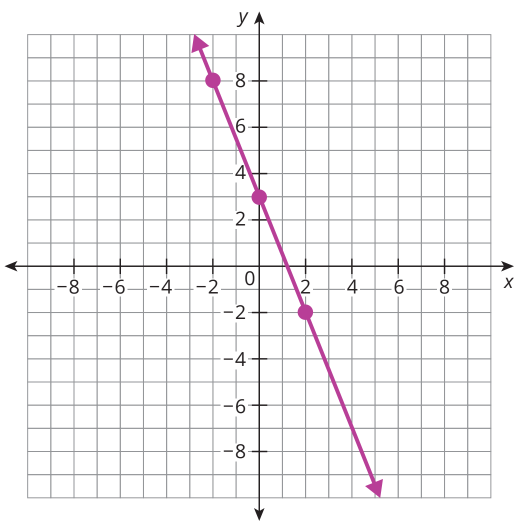
Use the equations to graph each line.

1. $y = \frac{3}{2}x - 1$



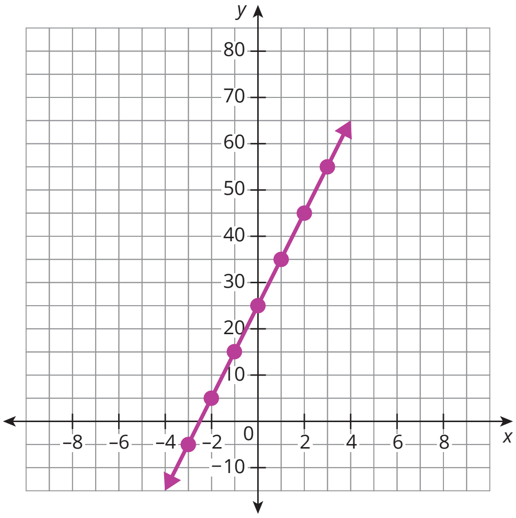
Think about the direction of the line before you start graphing.

2. $y = -\frac{5}{2}x + 3$



220 MODULE 2 • TOPIC 1 • LESSON 5

3. $y = 10x + 25$



Use a straightedge to draw your lines.

Ask Yourself . . .
Should each graph have arrows or points on each end? When using a point, how do you know whether to use an open circle or a closed circle?

Questions 1–3 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing equations given in slope-intercept form, assign Skills Practice Set A for this lesson.

Write an equation for each problem situation. Then, graph each equation.

4. Avery wants to buy a virtual reality headset. He already has $30 saved and plans to mow lawns for $15 each.

$y = 30 + 15x$

Remember to label the axes with the appropriate variable quantities.

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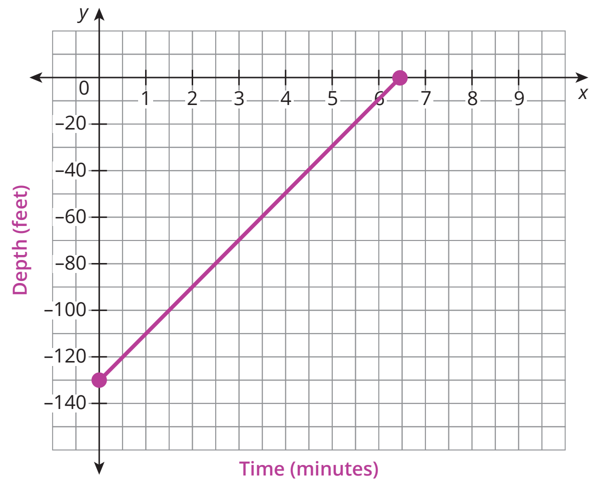
MODULE 2 • TOPIC 1 • LESSON 5 221

5. A scuba diver is 130 feet below sea level. The diver ascends at a constant rate of 20 feet per minute.

$y = -130 + 20x$



222 MODULE 2 • TOPIC 1 • LESSON 5

ACTIVITY 5.2 Using Point-Slope Form to Graph a Line

In the previous lesson, you learned that the point-slope form of a linear equation is $y - y_1 = m(x - x_1)$ where m is the slope of the line and (x_1, y_1) is a point on the line. Use this form of a linear equation for the next problem situation.

The jazz band is selling tickets to raise money for new music stands. They already had some money in their account when they started selling tickets at $5.00 each. After selling 3 tickets, the band had a total of $50 in their account.

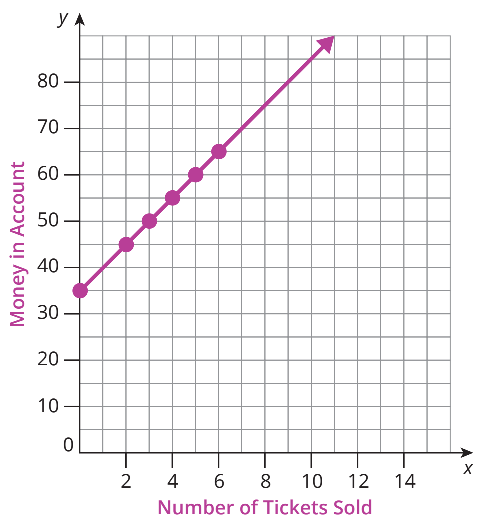
1. Let x = the number of tickets sold, and let y = the total amount of money in the jazz band's account.

   a. Write an equation in point-slope form.

   $y - 50 = 5(x - 3)$

   b. Use the point-slope form to graph the equation.

   - Write the coordinates for the known point. Plot the point on the coordinate plane.
   - Write the slope as a ratio. Then, use the slope and count from the point. To identify another point on the graph, start at the point and count either down (negative) or up (positive) for the rise. Then, count either left (negative) or right (positive) for the run.
   - Continue the counting process to plot at least two more points.
   - Connect the points to form a straight line.



   Point: (3, 50); Slope: $m = \frac{5}{1} = \frac{-5}{-1}$;
   Other points could include (2, 45), (4, 55), (5, 60), (6, 65)

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

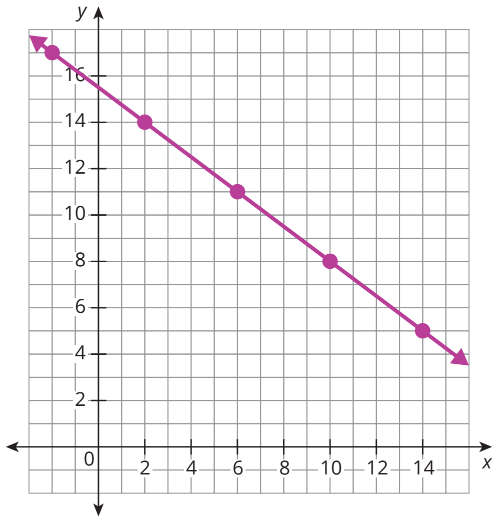
MODULE 2 • TOPIC 1 • LESSON 5 223

Question 2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing equations given in point-slope form, assign Skills Practice Set B for this lesson.

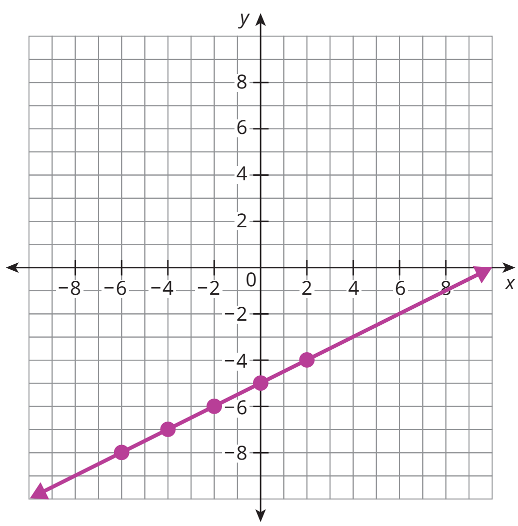
Remember, start with the given point and then use the slope to plot two more points.

2. Identify the slope and a point on the line from the given equation. Then, graph each line. Be careful to take into account the scales on the axes and the signs of the points given in the equation.

   a. $y - 5 = -\frac{3}{4}(x - 14)$



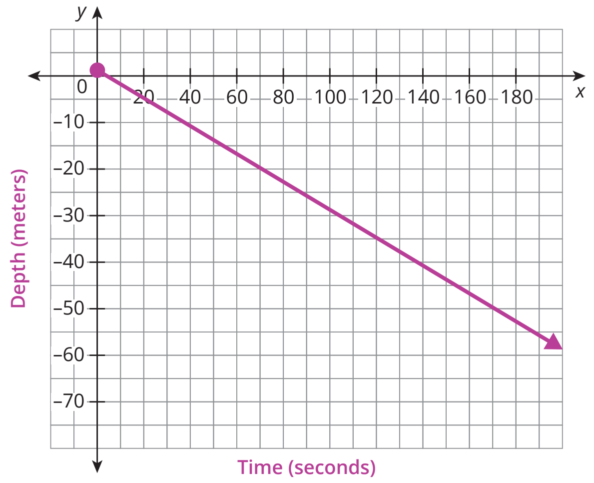
   b. $y + 8 = \frac{1}{2}(x + 6)$



224 MODULE 2 • TOPIC 1 • LESSON 5

3. An underwater remote operated vehicle (ROV) is lowered from a boat at a constant rate of 0.3 meters per second. After 90 seconds, the ROV is at a depth of −25.8 meters.

Define your variables and units. Then, write and graph an equation for the depth of the submarine over time.



x = the amount of time, in minutes, the ROV is lowered

y = the depth in meters of the ROV

$y - (-25.8) = -0.3(x - 90)$

$y + 25.8 = -0.3(x - 90)$

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MODULE 2 • TOPIC 1 • LESSON 5 225

Chunking the Activity

- Read and discuss the situation and definition.
- Group students to complete Questions 1–5.
- Check in and share.
- Group students to complete Questions 6 and 7.
- Check in and share.
- Group students to complete Questions 8–10.
- Share and summarize.
- Review the Worked Example as a class.
- Group students to complete Question 11.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

EB STUDENT TIP

For all proficiency levels

Have emergent bilingual students work in pairs to create a table comparing the three forms they have learned for the equation of a line. Use an example to write the equation in each form, and describe the advantages of each.

ACTIVITY 5.3

Standard Form of a Linear Equation

Tickets for a school play cost \$5.00 for students and \$8.00 for adults. On opening night, \$1600 is collected in ticket sales.

This situation can be modeled by the equation $5x + 8y = 1600$. You can define the variables as shown.

x = number of student tickets sold
y = number of adult tickets sold

This equation is not written in slope-intercept form or in point-slope form. It is written in *standard form*. The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

1. Explain what each term of the equation represents in the problem situation.

   a. $5x$

   5x is the cost of student tickets multiplied by the number of student tickets sold.

   b. $8y$

   8y is the cost of adult tickets multiplied by the number of adult tickets sold.

   c. 1600

   1600 is the total amount of money collected in ticket sales.

2. What is the independent variable? What is the dependent variable? Explain your reasoning.

   In this context, either variable could be the independent variable or dependent variable. The number of student tickets sold could depend upon the number of adult tickets sold to get to the \$1600 collected in ticket sales, or vice versa.

Equation of a Line			
Form	Point-slope $y - y_1 = m(x - x_1)$	Slope-intercept $y = mx + b$	Standard $Ax + By = C$
Example	$y - 5 = 2(x - 4)$	$y = 2x + 3$	$2x + 1y = 3$
Advantages	Best for identifying the slope and a point on the line	Best for identifying the slope and y-intercept	Best for identifying both intercepts

Remember, the y-intercept, $(0, b)$ is where a line crosses the y-axis, so the value of x is 0. To calculate a y-intercept, substitute 0 for x and solve the equation for b.

The **x-intercept**, $(x, 0)$, is where the line crosses the x-axis, so the value of y is 0. To calculate an x-intercept, substitute 0 for y and solve the equation for x.

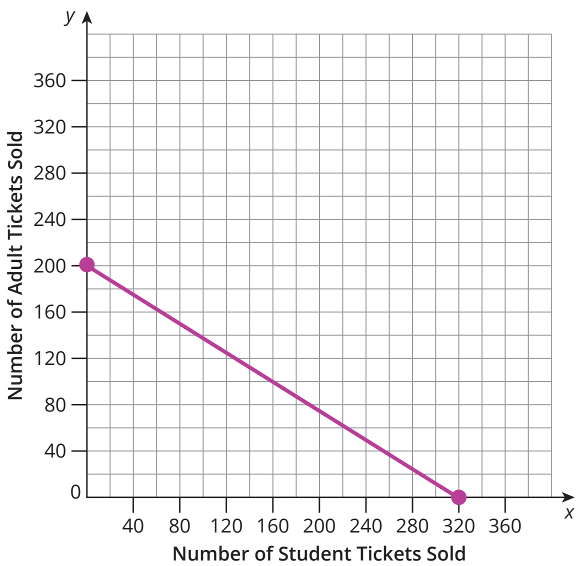
3. Calculate and interpret the meanings of the x-intercept and y-intercept for this equation.

   The x-intercept is (320, 0), and the y-intercept is (0, 200).

   The x-intercept means that if 320 student tickets are sold, then no adult tickets were sold to collect the $1600.

   The y-intercept means that if 200 adult tickets are sold, then no student tickets were sold to collect the $1600.

4. Use the x-intercept and y-intercept to graph the equation of the line.



5. Determine the slope of this line. Interpret the meaning of the slope in this problem situation.

   The slope of the line is $-\frac{5}{8}$. It represents the fact that the number of adult tickets sold decreases by 5 for every 8 student tickets sold.

MODULE 2 • TOPIC 1 • LESSON 5 227

6. Complete the table.

Standard Form	x-Intercept	y-Intercept	Slope
$5x + 2y = 6$	$\left(\frac{6}{5}, 0\right)$	(0, 3)	$m = -\frac{5}{2}$
$3x + 4y = 7$	$\left(\frac{7}{3}, 0\right)$	$\left(0, \frac{4}{3}\right)$	$m = -\frac{3}{4}$
$2x - 3y = 9$	$\left(\frac{9}{2}, 0\right)$	(0, 23)	$m = \frac{2}{3}$
$-5x + 7y = 11$	$\left(-\frac{11}{5}, 0\right)$	$\left(0, \frac{11}{7}\right)$	$m = \frac{5}{7}$

7. What do you notice about the relationship between the integers *A*, *B*, and *C* from the standard form and

   a. the *x*-intercepts?

   The *x*-intercepts are $\left(\frac{C}{A}, 0\right)$.

   b. the *y*-intercepts?

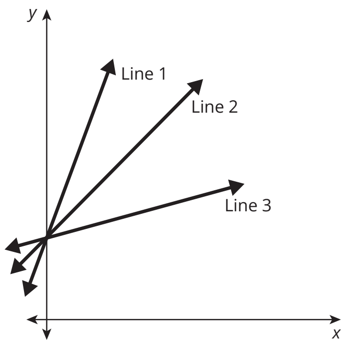
   The *y*-intercepts are $\left(0, \frac{C}{B}\right)$.

   c. the slope?

   The slope is $m = -\frac{A}{B}$.

228 MODULE 2 • TOPIC 1 • LESSON 5

8. Match each graph with the correct equation written in standard form. Explain your reasoning.



a. $3x - 12y = -60$ Line 3

b. $6x - 2y = -10$ Line 1

c. $9x - 9y = -45$ Line 2

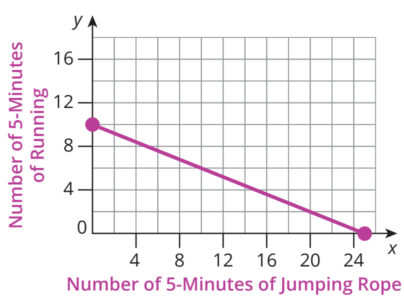
Sample explanation: Because all three lines had the same y-intercept, I need to use the slopes of the lines to distinguish among them. The slopes of the equations are (a) $m = \frac{1}{4}$, (b) $m = 3$, and (c) $m = 1$. The equation with the greatest coefficient of x matches the steepest line, so equation (b) matches Line 1. Following this reasoning, equation (c) matches Line 2, and equation (a) matches Line 3.

Notice that there are no values on the x- and y-axis. What strategies can you use to determine which graph goes with which equation?

Question 8 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing equations given in standard form, assign Skills Practice Set C for this lesson.

Define variables for each problem situation. Then, write an equation in standard form and use the intercepts to graph the linear relationship.

9. Nia burns 20 calories for every 5 minutes she jumps rope and 50 calories for every 5 minutes she runs. On Tuesday, Nia burned a total of 500 calories.

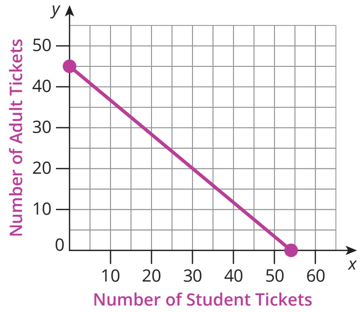


Let x = the number of 5-minute increments Nia jumps rope, and let y = the number of 5-minute increments Nia runs. $20x + 50y = 500$

Ask Yourself . . .
Why is the standard form of a linear equation appropriate to represent this situation?

MODULE 2 • TOPIC 1 • LESSON 5 229

10. For the show choir's holiday performance, they are selling tickets for \$5 per student and \$6.00 per adult. On the night of their final performance, they collect \$270 in ticket sales.



Let x = the number of student tickets, and let y = the number of adult tickets. $5x + 6y = 270$

WORKED EXAMPLE

Write a linear equation in standard form, $Ax + By = C$, that goes through the points $(-1, 3)$ and $(-4, -3)$.

Begin by determining the slope of the line.

$$m = \frac{(-5 - 3)}{(-4 - (-1))} = \frac{(-6)}{(-3)} = 2$$

Pick one of the points and write an equation in point-slope form for the line.

$y - 3 = 2(x - (-1))$

$y - 3 = 2(x + 1)$

Rewrite the equation in standard form.

$$y - 3 = 2x + 2$$
$$y = 2x + 5$$
$$-2x + y = 5$$

In standard form, A, B, and C are integers.

11. Write a linear equation in standard form given two points on the line.

a. $(0, -4)$ and $(2, 2)$

$-3x + y = -4$

b. $(1, -5)$ and $(-2, 7)$

$4x + y = -1$

c. $(12, 10)$ and $(16, 12)$

Sample answer:
$x - 2y = -8$

230 MODULE 2 • TOPIC 1 • LESSON 5

ACTIVITY 5.4 Identifying Slope and *y*-Intercept

Each equation represents a linear relationship. Examine each and determine the slope and the *y*-intercept. Write the *y*-intercept as an ordered pair.

1. $y = 11x - 9$

   Slope $= 11$;
   y-intercept $(0, -9)$

2. $4x + 6y = 270$

   Slope $= -\frac{2}{3}$;
   y-intercept $(0, 45)$

3. $y = 5(2x - 9)$

   Slope $= 10$;
   y-intercept $(0, -45)$

4. $8y = -6x + 24$

   Slope $= -\frac{3}{4}$;
   y-intercept $(0, 3)$

5. $y + 9 = 6(x - 3)$

   Slope $= 6$;
   y-intercept $(0, -27)$

6. $y = 9$

   Slope $= 0$;
   y-intercept $(0, 9)$

7. $4x - 12y = 48$

   Slope $= \frac{1}{3}$;
   y-intercept $(0, -4)$

8. $x = 10$

   Slope: undefined;
   no *y*-intercept

9. $y - 4 = 3(x + 1)$

   Slope $= 3$;
   y-intercept $(0, 7)$

10. $y = 9 - \frac{1}{2}x$

    Slope $= -\frac{1}{2}$;
    y-intercept $(0, 9)$

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

Activity 5.4 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing equations, graphing lines and identifying the slope and *y*-intercept for problem situations, assign Skills Practice Set D for this lesson.

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MODULE 2 • TOPIC 1 • LESSON 5 231

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Talk the Talk

Choose Your Medium

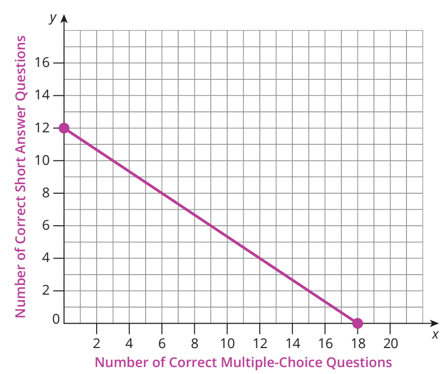
For each context, complete each task:

- Write an equation in slope-intercept, point-slope, or standard form.
- State the form of the equation you used and your reason for using it.
- Graph the line using any method.
- Explain the graphing method you used and your reason for using it.
- Match each transformed graph to the equation in the table, written in terms of $f(x)$.

1. On a math quiz, students earned 2 points for every correct multiple-choice question and 3 points for every correct short answer question. Joey earned a total of 36 points on the quiz.

   Let x = number of correct multiple-choice questions
   Let y = number of correct short answer questions
   Equation: $2x + 3y = 36$

   Reasoning: I chose standard form because I could write the equation directly from the context.



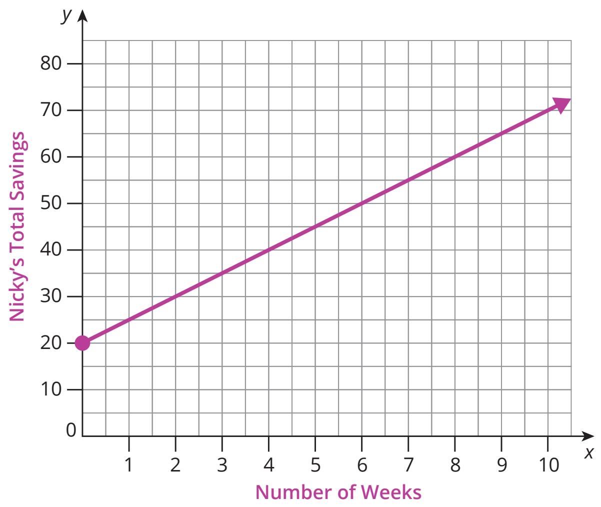
Reasoning:
Answers will vary.
I used the two intercepts because that is an advantage of having an equation in standard form.

232 MODULE 2 • TOPIC 1 • LESSON 5

2. Nicky has \$20, and he plans to save an additional \$10 every two weeks.

Let x = number of weeks
Let y = Nicky's total savings
Equation: $y = 5x + 20$

Reasoning: I chose slope-intercept form because the rate and y-intercept were directly stated in the problem.



Reasoning:
Since my equation was in slope-intercept form, I graphed the line by plotting the y-intercept and then using *rise/run* to plot additional points.

3. What are the advantages and disadvantages of using each form of a linear equation?

   a. Slope-intercept form

      Sample answer:
      One advantage of slope-intercept form is that I can write the equation for some contexts if I know an initial value and a rate of change. Also, I can graph using the intercept and slope. However, if the intercept doesn't make sense in the problem or is not in the part of the graph I need to analyze, it would not be as helpful.

MODULE 2 • TOPIC 1 • LESSON 5 233

b. Point-slope form

Sample answer:
One advantage of the point-slope form is that I can easily graph using the slope and a point. One disadvantage is that I do not know the *y*-intercept.

c. Standard form

Sample answer:
One advantage of standard form is that it lends itself toward writing an equation for some types of contexts. Another advantage of standard form is you can identify both the *x*-intercept and *y*-intercept, and then use them to graph the equation. The disadvantage is that you need to use a formula to calculate slope.

234 MODULE 2 • TOPIC 1 • LESSON 5

Lesson 5 Assignment

Write

Explain how to graph a line when the equation is written in slope-intercept form, point-slope form, or standard form.

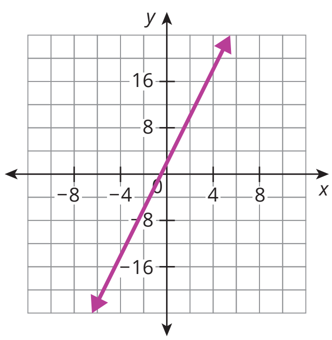
Remember

The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

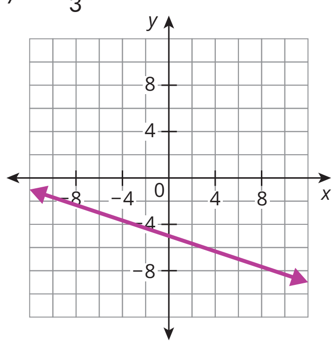
Practice

1. Graph each equation using its given form.

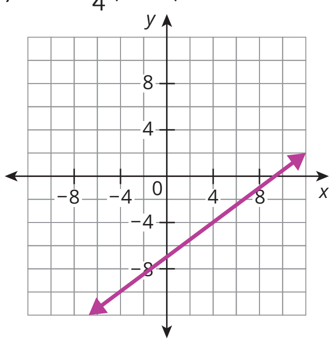
a. $y = 4x + 2$



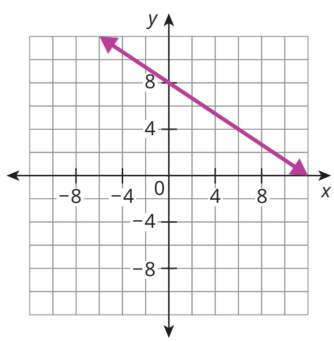
b. $y = -\frac{1}{3}x - 5$



c. $y + 1 = \frac{3}{4}(x - 8)$



d. $y - 4 = -\frac{2}{3}(x - 6)$



Write

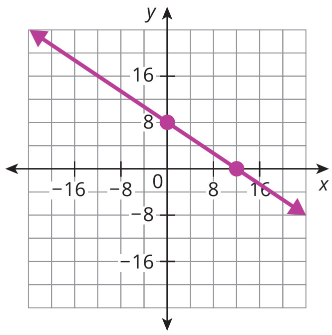
Sample answer: When the equation is in slope-intercept form, I can plot the y-intercept and use the slope to determine another point on the line. When the equation is in point-slope form, I can plot the point given in the equation and use the slope to determine another point on the line. When the equation is in standard form, I can determine the x- and y-intercepts and use them to graph the line.

MODULE 2 • TOPIC 1 • LESSON 5 ASSIGNMENT 235

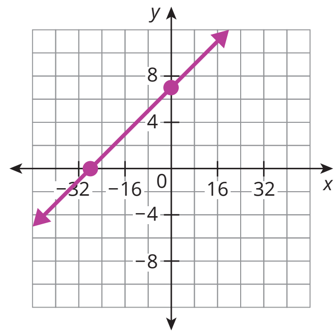
Lesson 5 Assignment

2. Graph each equation using its intercepts.

   a. $4x + 6y = 48$



   b. $-2x + 8y = 56$



3. Write a linear equation in standard form given two points on the line.

   a. (0, −9) and (−3, 6)

   $5x + y = -9$

   b. (−4, 2) and (2, 5)

   $x - 2y = -8$

4. Brianna bought magazines for \$6 each and paperback books for \$3 each for a total of \$54.

   a. Define your variables and write an equation in standard form to represent the situation.

   Let x be the number of magazines and let y be the number of paperback books. The equation is $6x + 3y = 54$.

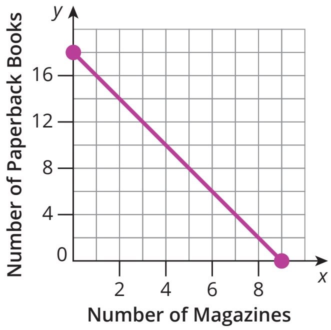
   b. Calculate and interpret the x-intercept and the y-intercept for this equation.

   The y-intercept is (0, 18). If Brianna bought only paperback books, she could buy 18 of them. The x-intercept is (9, 0). If she bought only magazines, she could buy 9 of them.

236 MODULE 2 • TOPIC 1 • LESSON 5 ASSIGNMENT

Lesson 5 Assignment

c. Graph the equation of the line using the intercepts.



d. Calculate and interpret the slope of this line.

The slope of the line is −2. This means that for each additional magazine bought, Eugenie can buy 2 fewer paperback books.

5. Each equation represents a linear relation. State the slope and y-intercept for each.

a. $-9x + 2y = -36$

$m = \frac{9}{2}$; y-intercept $= (0, -18)$

b. $y + 5 = -7(x + 3)$

$m = -7$; y-intercept $= (0, -26)$

c. $y = 2$

$m = 0$; y-intercept $= (0, 2)$

d. $y = \frac{5}{2}x - 9$

$m = \frac{5}{2}$; y-intercept $= (0, -9)$

Prepare

Determine the slope of the line between each pair of points.

1. (0, 10) and (3, 12)

$m = \frac{2}{3}$

2. (−1, 4.5) and (1, −4.5)

$m = -\frac{9}{2}$

3. (21, 0) and (0, 12)

$m = -\frac{4}{7}$

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MODULE 2 • TOPIC 1 • LESSON 5 ASSIGNMENT 237

238 MODULE 2 • TOPIC 1 • LESSON 5 ASSIGNMENT

6 Making Sense of Different Representations of a Linear Function

MATERIALS

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students determine whether tables of values with non-consecutive input values represent linear functions. They evaluate functions and analyze Worked Examples that demonstrate how to solve equations algebraically and graphically. For the remainder of the lesson, students deal with a context, a graph, and two translations of the graph based on additions to the context. They focus on two equivalent linear functions, one written in general form, $f(x) = ax + b$, and the other written in factored form, $f(x) = a(x - c)$. Students interpret the meaning of the terms of each function and analyze their structures. The form $f(x) = ax + b$ relates to the slope-intercept form of a line, while $f(x) = a(x - c)$ connects with the slope and zero of the function. Linear functions are placed within the wider framework of polynomial functions. The terms *polynomial*, *degree*, *leading coefficient*, and *zero of a function* are defined, setting a frame of reference for future work with other functions. Students use a graphic organizer to summarize four representations—general form, factored form, graph, and table—of a linear function.

ALGEBRA I TEKS

Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

(TEKS continued on next page)

ELPS

(1) Learning Strategies

The student is expected to:

(H) develop and expand repertoire of learning strategies such as reasoning inductively or deductively, looking for patterns in language, and analyzing sayings and expressions commensurate with grade-level learning expectations.

(3) Speaking

The student is expected to:

(C) speak using a variety of grammatical structures, sentence lengths, sentence types, and connecting words with increasing accuracy and ease as more English is acquired.

(4) Reading

The student is expected to:

(D) use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

A.2D write and solve equations involving direct variation.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
The student is expected to:

A.3A determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

A.3C graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

A.3E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

A.3F graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.
The student is expected to:

A.12A decide whether relations represented verbally, tabularly, graphically, and symbolically define a function.

A.12B evaluate functions, expressed in function notation, given one or more elements in their domains.

ESSENTIAL IDEAS

- If a table represents a linear function, the slope, or average rate of change, is constant between all given points.
- Using an equation to solve for the independent value given the dependent value always results in an exact answer. Using a graph or a table to determine the independent value sometimes results in an exact answer.
- The graph of an equation plotted on the coordinate plane represents the set of all its solutions.
- The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient, which describes the steepness and direction of the line. The b-value describes the y-intercept.
- The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a polynomial is in factored form, the value of x that makes each factor equal to zero is the x-intercept. This value is called the zero of the function.
- A linear function is a polynomial with a degree of one.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Well, Are Ya or Aren't Ya? 10–15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students analyze scenarios, equations, tables, and graphs and determine whether they represent linear functions.

DEVELOP

Activity 6.1: Interpreting Linear Functions 25–30 minutes

REAL-WORLD PROBLEM SOLVING

Students determine whether tables of values with non-consecutive input values represent linear functions. Given contexts and tables of values, they determine the linear function that models the context and solve for independent and dependent variables.

DAY 2

Activity 6.2: Interpreting Graphs of Linear Functions 20–25 minutes

WORKED EXAMPLE, REAL-WORLD PROBLEM SOLVING

Students begin with a graph and context. They interpret the graph with reference to the context and write a function to model the graph. Students evaluate the function at various values of the independent variable and interpret the meaning of each. They then solve for the independent variable both algebraically and graphically. A Worked Example is provided for each method, and students compare the methods.

Activity 6.3: Interpreting Changes to the Graph of a Linear Function 20–25 minutes

WORKED EXAMPLE, PEER WORK ANALYSIS

Students are given another situation related to the context from the previous activity, resulting in a graph that is a translation of the original graph; they interpret the meaning of the slope and y-intercept of each graph in terms of the context. The remainder of the activity focuses on two equivalent linear functions, one written in general form, $f(x) = ax + b$, and the other written in factored form, $f(x) = a(x - c)$. Students interpret the meaning of the terms of each function and analyze their structure. The form $f(x) = ax + b$ relates to the slope-intercept form of a line, while the form $f(x) = a(x - c)$ connects with the slope and zeros of the function. Linear functions are placed within the wider framework of polynomial functions. The terms *polynomial*, *degree*, *leading coefficient*, and *zero of a function* are defined, setting a frame of reference for future work with other functions.

DAY 3

Activity 6.4: Interpreting More Changes to the Graph of a Linear Function 15–20 minutes

REAL-WORLD PROBLEM SOLVING

Students have the opportunity to solidify their understanding of the structure of the general and factored forms of a linear function through interpreting an additional function.

DEMONSTRATE

Talk the Talk: Reading Between the Lines 20–25 minutes

GRAPHIC ORGANIZER

Students summarize the usefulness of tables, graphs, and equations in determining an approximate and exact answer when solving equations. They write functions from tables of values and use each function to solve for independent and dependent values. Students use a graphic organizer to summarize four representations—general form, factored form, graph, and table—of a linear function.

Getting Started

ENGAGE

Well, Are Ya or Aren't Ya?

Facilitation Notes

In this activity, students analyze scenarios, equations, tables, and graphs to determine whether they represent linear or nonlinear functions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Methods, vocabulary, and notation used to determine whether a function is linear.
- A particular representation that students gravitate to when determining whether a representation models a linear function. For example, students may make tables for the scenarios and equations to determine whether they model a line, instead of determining linearity directly from the scenarios and equations.

COMMON MISCONCEPTIONS

- Students may think that just because they know a formula for a relationship, such as in Scenario C, that the relationship is linear. Discuss how to use the formula to determine linearity or generate data from the formula and place it in a table.
- Misunderstandings of the scenarios; encourage students to create an additional representation of each scenario to better visualize its rate of change.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Suggest students create an additional representation of each situation to visualize its rate of change.
- Encourage students to convert other representations to one they are comfortable with to determine whether the representation models a linear or nonlinear function.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you tell whether each scenario describes a linear function? • Did you create a table for any scenario? If so, explain your results. • What is another way to describe decreases by 50% each hour? • Which feature identifies whether an equation is a linear function? • Describe why Table B is a linear function and Table A is not. • What strategy can you use to identify whether Table C represents a linear function? • What table does Graph C model? Why isn't this line a linear function?

STAMP THE LEARNING

Summary

Methods exist to determine whether a context, equation, table, or graph represents a linear function.

ACTIVITY 6.1 Interpreting Linear Functions

DEVELOP

Facilitation Notes

In this activity, students determine whether tables of values with non-consecutive input values represent linear functions. Given contexts and tables of values, they determine the linear function that models the context and solve for independent and dependent variables.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Inappropriate use of first differences in a table of non-consecutive values.
- Generation of table values between the given ones to create consecutive values.
- Confusion when phrases do not explicitly state what the *x*-values or *y*-values are.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why can't you use first differences to determine whether these tables represent linear functions? • How can you tell whether a table with non-consecutive values represents a linear function? • How did you determine the *y*-intercept for the table in part (a)? • What is the difference between *y*-values in part (b)? Then why doesn't this table represent a linear function? • How can the column titles help you relate the slope to the context? • What equation represents this table? • How do you know whether the slope represents the price of a ticket or the entrance fee? • How can you calculate the entrance fee? • How did you figure out the number of ride tickets Nakota can buy? • Why doesn't tripling the table results for 6 result in $20? • Did you use the table or equation to determine the tank's capacity? • How can you represent the empty tank mathematically? • How did you calculate when the tank will be empty?

COMMON MISCONCEPTION

- Students often attempt to use first differences even though the values in the table are not consecutive.
  - Remind students to verify the *x*-values are consecutive before determining the first difference.
  - Encourage students to generate table values between the given ones to create consecutive values.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

If a table represents a linear function, the slope, or average rate of change, is constant between all given points.

ACTIVITY **6.2**

Interpreting Graphs of Linear Functions

Facilitation Notes

In this activity, students begin with a graph and context. They interpret the graph with reference to the context and write a function to model the graph. Students evaluate the function at various values of the independent variable and interpret the meaning of each. They then solve for the independent variable both algebraically and graphically. A Worked Example is provided for each method, and students compare the methods.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

To begin the Day 2 session, have a student read the Essential Question aloud.

AS STUDENTS WORK, LOOK FOR

- Identification of the domain and range of the function rather than the domain and range of the scenario.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Have students label additional ordered pairs on the line and interpret their meaning.
- Provide a strategy to relate any function notation back to *x* and *y*.

  $(t, E(t))$ $(x, f(x))$ (x, y)

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How much money does Gracie's mom earn for each T-shirt sold?
Seeing structure	• Did you use one or two points on the graph to calculate the earnings per T-shirt sold? Explain the relationship between the two strategies.
Probing	• Why doesn't the equation have a constant value?

Ask a student to read the information following Question 5 aloud. Analyze the Worked Examples and complete Questions 6 and 7 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know whether to substitute the given value for t or $f(t)$? • How did you decide that Gracie needed to sell 7 T-shirts instead of 6 T-shirts? • How much money will Gracie earn selling 6 T-shirts? 7 T-shirts? • Why does it make sense to rely on technology when using a graph? • How could you solve this equation using a table with technology?

DIFFERENTIATION STRATEGY

Access for All

- Encourage students to take time to read through the Worked Example and annotate key ideas or steps in the process. As they think about the connections, have students ask themselves:
  - Why is this method correct?
  - Have I used this method before?

STAMP THE LEARNING

Summary

An equation or a graph can be used to determine the value of an independent variable. Sometimes a graph can only give an approximate answer; however, using an equation always results in an exact answer.

ACTIVITY 6.3

Interpreting Changes to the Graph of a Linear Function

Facilitation Notes

In this activity, students are given another situation related to the context from the previous activity, resulting in a graph that is a translation of the original graph; they interpret the meaning of the slope and y-intercept of each graph in terms of the context. The remainder of the activity focuses on two equivalent linear functions, one written in general form, $f(x) = ax + b$, and the other written in factored form, $f(x) = a(x - c)$. Students interpret the meaning of the terms of each function and analyze their structure. The form $f(x) = ax + b$ relates to the slope-intercept form of a line, while $f(x) = a(x - c)$ connects with the slope and zero of the function. Linear functions are placed within the wider framework of polynomial functions. The terms *polynomial*, *degree*, *leading coefficient*, and *zero of a function* are defined, setting a frame of reference for future work with other functions.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Statements used to interpret $y = G(0)$ and $G(t) = 0$.
- How students use the unit of each expression to guide their thinking about the contextual and mathematical meaning of the expression.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Provide a strategy to relate any function notation back to x and y for Question 1.
  $(t, E(t))$ $(x, f(x))$ (x, y)
- Provide a word bank for mathematical meanings in table for Question 4

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do the parallel lines relate to the context? • Does this notation represent the x-intercept or y-intercept? • Which property did you apply to show the equations are equivalent? • Explain how Brianna's and Hailey's perspectives are different. • How did you interpret the solution in terms of the context? • Compare this question and response to the Worked Example in Activity 2.

Ask a student to read the information and definitions following Questions 4 and 5 aloud. Review the Worked Example and complete Questions 5 through 7 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Rewrite the constant 7 with a variable and exponent. • What is the exponent of the variable in a linear function? • What is an example of a linear function with a coefficient of 1? • Write an example of a cubic function with four terms. • Explain how your label for *a* makes sense. • Where is another possible location for *a*? • Explain how you solved for *c*. • What information do general form and factored form both provide?
Seeing structure	• What information is available in general form that is not available in factored form? • What information is available in factored form that is not available in general form?

COMMON MISCONCEPTION

- Students often confuse whether to identify *x*-intercepts and zeros by a single value or an ordered pair.
    - Intercepts are actual locations on a coordinate plane described using ordered pairs, such as (−3, 0), (0, 0) or (7, 0).
    - The zeros of a function are the values when $f(x) = 0$, and you express them using values such as −3, 0, or 7.

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

The general form of a linear function and the factored form of a linear function each have a structure that identifies characteristics of the graph of the linear function it models.

ACTIVITY 6.4

Interpreting More Changes to the Graph of a Linear Function

Facilitation Notes

In this activity, students have the opportunity to solidify their understanding of the structure of the general and factored forms of a linear function through interpreting an additional function.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain what the coordinates of the new y-intercept mean in this context. • How did the \$35 fee affect how many T-shirts Gracie needs to sell to earn \$100? • How can you rewrite the equation in general form to create the equation in factored form? • How can you use what you know about factored form to determine the factor $\left(\frac{t-16}{3}\right)$?

Summary

In the general form of a linear function, $f(x) = ax + b$, a is the slope and b is the y-intercept. In the factored form of a linear function, $f(x) = a(x - c)$, a is the slope and c is the x-intercept or zero of the function.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

READING BETWEEN THE LINES

Facilitation Notes

In this activity, students summarize the usefulness of tables, graphs, and equations in determining an approximate and exact value for the independent variable. They write functions from tables of values and use each function to solve for independent and dependent values. Students use a graphic organizer to summarize four representations—general form, factored form, graph, and table—of a linear function.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Students who consider only the first two terms in the table when writing the equation.
- Students that do not notice the input values are not consecutive in Table 2.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you know that each function is linear? • How did you calculate the slope and y-intercept? • Explain how to determine the value of x for a given $f(x)$.

Summary

Both the general and factored form of a linear function relate to the graph and table of the function they model. The equation always provides exact answers, while the graph and table sometimes may only be able to provide approximate values.

STAMP THE LEARNING

6 Making Sense of Different Representations of a Linear Function

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

LEARNING OBJECTIVES

- Determine whether a scenario, equation, table, or graph represents a linear relationship.
- Calculate the average rate of change from a table.
- Write functions given a table of values.
- Interpret expressions that represent different quantities in terms of a context and a graph.
- Compare different equation representations of linear functions.

NEW KEY TERMS

- polynomial
- degree
- leading coefficient
- zero of a function

You know how to determine whether a relationship represents a linear function, and you know how to write an equation for the function. How can you use the structure of the equation to identify characteristics of the function?

Sample answer:

The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient that describes the steepness and direction of the line. The b-value describes the y-intercept. The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. In this form, the a-value is the slope and the value of x that makes the factor $(x - c)$ equal to zero is the x-intercept.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Review the term *average rate of change*. Ensure that students can connect *average rate of change* with the slope of a line. Discuss the components necessary for determining the *average rate of change*. Ask students to explain the formula and provide an example of calculating the *average rate of change* between two points.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Well, Are Ya or Aren't Ya?

1. Determine whether each representation models a linear or nonlinear function. Explain your reasoning.

Scenario A
A tree grows 3.5 inches each year.
Linear function

Scenario B
The strength of a medication decreases by 50% each hour it is in the patient's system.
Nonlinear function

Scenario C
The area of a square depends on its side length.
Nonlinear function

Equation A
$y = 14 - 9x$
Linear function

Equation B
$y = 2^x + 1$
Nonlinear function

Equation C
$y = \frac{1}{4}(x + 7) - 1$
Linear function

Table A

x	y
1	3
1	4
1	5

Not a function

Table B

x	y
3	1
4	1
5	1

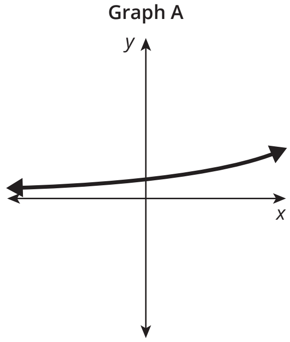
Linear function

Table C

x	y
−9	45
−8	30
−7	15

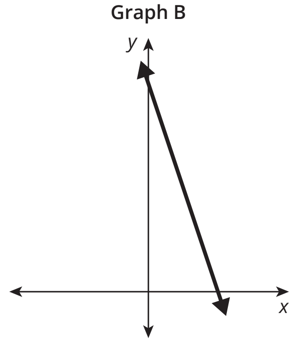
Linear function

Graph A



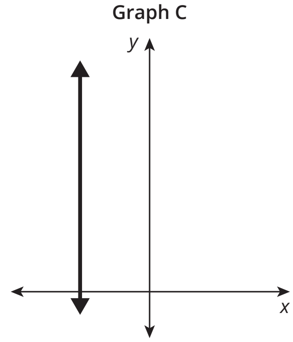
Nonlinear function

Graph B



Linear function

Graph C



Not a function

240 MODULE 2 • TOPIC 1 • LESSON 6

ACTIVITY 6.1 Interpreting Linear Functions

Each table of values in the previous activity had consecutive input values. Tables in this format allow you to use differences to determine whether the representation is linear. Often, input values are in intervals other than 1, and sometimes the input values are in random order. To determine whether these tables represent linear functions, you need to make sure the slope, or average rate of change, is constant between all given points.

Ask Yourself . . .
Is there a pattern in the input values?

1. Determine whether each table represents a linear function. If so, write the function.

a.

x	y
−2	5.5
1	4.75
4	4
7	3.25

$f(x) = -0.25x + 5$

b.

x	y
0	5
2	13
4	21
8	29

Not a linear function

Remember . . .
In a linear relationship, when the value of the dependent variable is directly proportional to the value of the independent variable, the relationship is a direct variation. Another way to say this is that in a direct variation the value of the dependent variable varies directly with the independent value. You can use these phrases interchangeably.

Analyze each situation represented as a table of values.

2. Nakota sells pretzels at festivals on weekends. The table shown is a record of past sales.

Number of Pretzels Sold	Amount of Money Earned (dollars)
15	37.5
42	105
58	145
29	72.5

a. What does this table tell you about his sales?
Nakota earned \$2.50 per pretzel

b. Determine whether the amount of money Nakota earns is directly proportional to the number of pretzels sold. Justify your reasoning.
The amount of money Nakota earns is directly proportional to the number of pretzels sold.

c. Write an equation to represent the situation.
$y = 2.5x$

d. How much money does Nakota earn when he sells 75 pretzels?
\$187.50

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

CC BY NC
MODULE 2 • TOPIC 1 • LESSON 6 241

Question 3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice solving equations involving direct variation, assign Skills Practice Set A for this lesson.

3. Alejandra and her friends recently went to the community fair. They had to pay an entrance fee and then purchase 1 ticket for each ride. Lauren is going to the fair tomorrow and wants to know the cost of each ride ticket. Alejandra and her friends help Lauren by writing down how much money they spent and the number of tickets they purchased.

Number of Ride Tickets	Amount of Money Spent (dollars)
2	7.5
4	9
6	10.5
11	14.25

a. What does this table tell you about the cost to go to the fair and ride the rides?

Ride tickets cost \$0.75 each; the entrance fee is \$6.

b. Determine whether the amount of money Alejandra and and her friends spent varies directly with the number of ride tickets they purchased. Justify your reasoning.

The amount of money Alejandra and her friends spent does not vary directly with the number of tickets purchased due to the entrance fee.

c. Write an equation to represent the situation.

$y = 0.75x + 6$

d. If Lauren has \$20 to spend, how many ride tickets can she buy?

18 tickets

4. The local pet store has a fish tank on display at the community fair. Darren is responsible for draining the tank at the end of the fair. The pet store manager provides him with this information from when they drained the tank at the end of the fair last year.

Time (hours)	Amount of Water Remaining (gallons)
$\frac{1}{4}$	169
$\frac{1}{2}$	163
$\frac{3}{4}$	157
1	151

a. Determine whether the amount of water remaining is directly proportional to the time the drain spent draining the tank. Justify your reasoning.

The amount of water remaining does not vary directly with the time spent draining the tank.

b. How many gallons did the fish tank hold?

175 gallons

c. Write an equation to represent the situation.

$y = 175 - 24x$

d. When will the tank be empty?

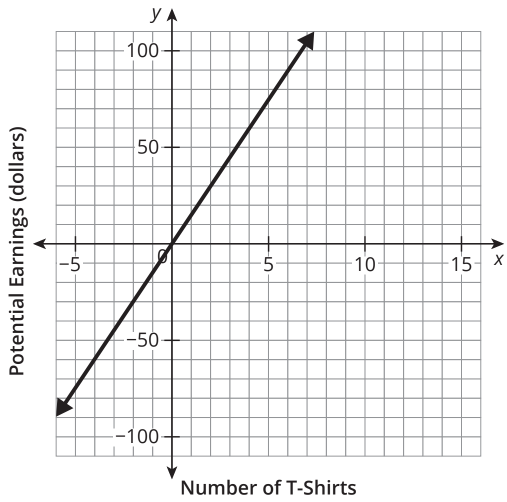
About 7.29 hours or 7 hours 17.5 minutes

242 MODULE 2 • TOPIC 1 • LESSON 6

ACTIVITY 6.2

Interpreting Graphs of Linear Functions

Gracie sells silk screened T-shirts for her mom at local festivals. After each festival, she returns whatever money she earns to her mom. The graph shown represents her potential earnings based on the number of T-shirts she sells.



Ask Yourself . . .
What is the meaning of the slope, *x*- and *y*-intercepts, domain, and range in terms of this situation?

1. Analyze and interpret the graph. List as many facts as you can about the scenario based on what you see in the graph and describe how they relate to the scenario.

   Sample answer:
   Gracie starts with \$0 for 0 T-shirts sold. If Gracie sells 4 T-shirts, she earns \$60. Gracie makes \$15 per T-shirt.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete Questions 1–5.
- Check in and share.
- Read and discuss the Worked Examples and complete Questions 6 and 7 as a class.
- Share and summarize.

MODULE 2 • TOPIC 1 • LESSON 6 243

Ask Yourself . . .
Does the function $E(t)$ represent a direct variation? Explain your reasoning.

2. Interpret the meaning of the x- and y-intercept.
   $E(0) = 0$; If no T-shirts are sold, Gracie does not earn any money.

3. Write a function, $E(t)$, to model Gracie's potential earnings given the number of T-shirts she sells.
   $E(t) = 15t$

4. What does $(t, E(t))$ represent in terms of the function and the graph?
   Function: $(t, E(t))$ t represents the number of T-shirts, and $f(t)$ represents the number of T-shirts multiplied by $15 per T-shirt.
   Graph: Each point on the graph represents the number of T-shirts and the money earned for that amount of T-shirts.

5. Evaluate each and interpret the meaning in terms of the equation, the graph, and the scenario.

   a. $E(2)$
   (2, 30) means that when 2 T-shirts are sold, Gracie earns $30.

   b. $E(5)$
   $E(5) = 75$; When 5 T-shirts are sold, Gracie earns $75.

   c. $E(2.75)$
   $E(2.75) = 41.25$; This does not make sense, Gracie cannot sell 2.75 shirts.

Gracie has a goal to earn $100 at the festival. Let's consider how to determine the number of T-shirts she needs to sell to meet her goal.

WORKED EXAMPLE

To determine the number of T-shirt sales it takes to earn $100 using the function, $E(t) = 15t$, substitute 100 for $E(t)$ and solve.

$$E(t) = 15t$$
$$100 = 15t$$
$$\frac{100}{15} = t$$
$$6.67 = t$$

6. Consider the Worked Example.

   a. Interpret the meaning of $t = 6.67$.
   To earn $100, 6.67 T-shirts must be sold. Therefore, to earn at least $100, Gracie must sell 7 T-shirts.

STAMP THE LEARNING
The Worked Examples provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

244 MODULE 2 • TOPIC 1 • LESSON 6

b. Why can you substitute 100 for $E(t)$?

$E(t)$ represents the dollars earned based upon t, the number of T-shirts sold. Because \$100 is dollar amount to be earned, it can replace the expression. $E(t)$.

You can also use the graph to determine the number of T-shirts Gracie needs to sell to earn \$100.

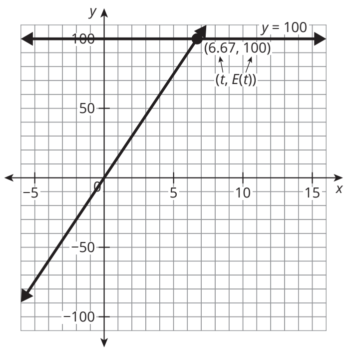
Remember . . .

The graph of an equation plotted on the coordinate plane represents the set of all its solutions.

WORKED EXAMPLE

To determine the number of T-shirts sales it takes to earn \$100 using your graph, you need to determine the intersection of the two lines represented by the equation $100 = 15t$.

First, graph the function defined by each side of the equation, and then determine the intersection point of the two graphs.



$$h(t) = 15t$$
$$100 = 15t$$
$$y = 100 \qquad y = 15x$$

Solution: (6.67, 100)

In terms of the graph, Gracie needs to sell 6.67 T-shirts to earn \$100. In terms of the context, she needs to sell 7 T-shirts.

Activity 6.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide determining input and output values using functions and graphs, assign Skills Practice Set B for this lesson.

7. Consider the equation and graphical representations. What are the limitations of using each to answer questions about the number of T-shirts sold or the amount of money earned?

a. Equation

Sample answer:
Sometimes an equation provides an answer that does not make sense in the context, and the answer must be rounded up or down to answer the actual question.

b. Graph

Sample answer:
Sometimes it is difficult to read an exact answer from a graph.

MODULE 2 • TOPIC 1 • LESSON 6 245

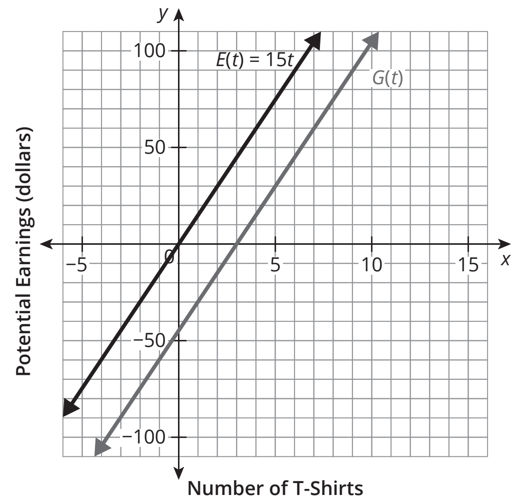
Chunking the Activity

- Read and discuss the situation.
- Group students to complete Questions 1–4.
- Check in and share.
- Read and discuss the definitions and the Worked Example.
- Group students to complete Question 5.
- Check in and share.
- Read and discuss the definition.
- Group students to complete Questions 6 and 7.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY **6.3**

Interpreting Changes to the Graph of a Linear Function

For the next festival, Gracie's mom suggests that she still sells each T-shirt for $15, but should give away 3 T-shirts in a raffle. This new relationship, $G(t)$, is shown on the graph.



1. Compare the two graphs. What do you notice?

   a. How do the graphs show the selling price per T-shirt remains the same?

   Both graphs have the same slope or average rate of change.

   b. Determine and interpret the meaning of $y = G(0)$ in terms of the graph and this scenario. Label the point on the graph.

   $y = G(0)$ is represented by the point $(0, -45)$ on the graph. It means that if no T-shirts are sold, Gracie's potential earnings are $\$-45$. This point doesn't make sense in terms of this situation.

   c. Determine and interpret the meaning of $G(t) = 0$ in terms of the graph and this scenario. Label the point on the graph.

   $G(t) = 0$ is represented by the point $(3, 0)$ on the graph. It means that once Gracie sells/gives away 3 T-shirts, her earnings are $0. She has no money to return to her mom.

Ask Yourself . . .
Does $G(t)$ vary directly with t?

246 MODULE 2 • TOPIC 1 • LESSON 6

Brianna and Hailey each wrote an equation to describe the effect of giving away three T-shirts.

Brianna

Gracie is giving away 3 T-shirts, so she has 3 fewer shirts to sell.

$G(t) = 15(t - 3)$

Hailey

The cost of giving away three shirts is $45.

$G(t) = 15t - 45$

2. Verify the two equation representations are equivalent.

   $G(t) = 15(t - 3)$ By distribution, $G(t) = 15t - 45$.

3. How many T-shirts will Gracie need to sell to earn $100? Use the graph and an equation.

   $G(t) = 100$ when $t = 9.67$; Gracie needs 10 T-shirts to earn $100 Since she gave away three, she needs to sell 7 T-shirts to earn $100.

4. Consider the expressions in the first two rows that define the quantities of the function and then the parts of each equation written by Brianna and Hailey to complete the table. First, determine the unit of measure for each expression. Then, describe the contextual meaning and the mathematical meaning of each part of the function.

		What It Means	
Expression	Unit	Contextual Meaning	Mathematical Meaning
t	T-shirts	Number of T-shirts sold	Input
$G(t)$	Dollars	Potential earnings for selling t T-shirts	Output
15	Dollars per T-shirt	Selling price of a T-shirt	Average rate of change
$(t - 3)$	T-shirts	The number of T-shirts to be sold, not counting the 3 free ones	The input minus a constant
$15t$	Dollars	The earnings for t T-shirts being sold	The rate times the input
-45	Dollars	The dollars not being earned because 3 T-shirts were given away for free	The y-intercept

MODULE 2 • TOPIC 1 • LESSON 6 247

The linear functions that Brianna and Hailey each wrote are equivalent; however, they are written in different forms. The linear function $G(t) = 15(t - 3)$ is written in factored form and $G(t) = 15t - 45$ is written in general form.

Polynomial comes from *poly-* meaning "many" and *-nomial* meaning "term," so it means "many terms."

A linear function can also be referred to as a *polynomial* function. A **polynomial** is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. The **degree** of a polynomial is the greatest variable exponent in the expression. The **leading coefficient** of a polynomial is the numeric coefficient of the term with the greatest power.

WORKED EXAMPLE

A few examples of polynomial functions.

Polynomial Functions		Degree
Constant	$P(x) = 7$	0
Linear	$P(x) = 2x - 5$	1
Quadratic	$P(x) = 3x^2 - 2x + 4$	2
Cubic	$P(x) = 4x^3 - 2$	3

When you graph a polynomial the degree tells you the maximum number of times the graph can cross the *x*-axis.

The structure of each linear function tells you important information about the graph. Let's consider the general form of a linear function, $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient, which describes the steepness and direction of the line. The b-value describes the y-intercept.

The variables used to represent any real number in the general linear form are irrelevant. Think about the position of the number as either the leading coefficient or a constant and the potential effect on the function.

You know the form $y = mx + b$ as slope-intercept form, where m represents the slope and b represents the y-intercept. Notice that the general form has the same structure. The general form shows that a linear equation is a polynomial of degree 1. You will learn more about polynomials as you progress through future mathematical courses.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

248 MODULE 2 • TOPIC 1 • LESSON 6

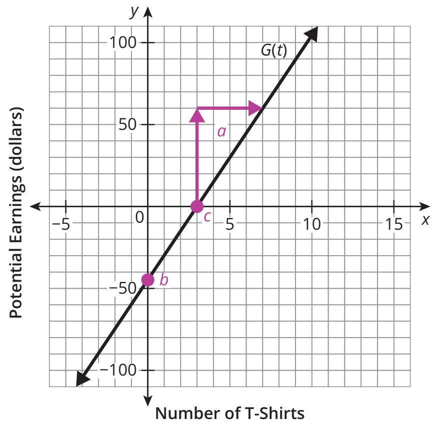
5. Consider the general form of the linear function $G(t)$.

   a. Label a on the graph.

   The a-value is a slope triangle on the graph.

   b. Label b on the graph.

   The b-value is the y-intercept, $(0, -45)$.



To provide additional practice examine graphs of lines and determine the x- and y-intercepts, slope, equation in slope-intercept form, and equation in factored form, assign Skills Practice Set C for this lesson.

Next, consider the factored form of a linear function, $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a polynomial is in factored form, the value of x that makes the factor $(x - c)$ equal to zero is the x-intercept. This value is called the *zero of the function*. A **zero of a function** is a real number that makes the value of the function equal to zero, or $f(x) = 0$.

You can set $(x - c)$ equal to zero and determine the point where the graph crosses the x-axis.

6. Consider the factored form of the linear function $G(t)$.

   a. Label a on the graph.

   The a-value is a slope triangle on the graph.

   b. Label c on the graph.

   The c-value is the x-intercept, $(3, 0)$.

7. What is the zero of $G(t)$? Explain your reasoning.

   The zero of $G(t)$ is 3. It is the value of t that makes $15(t - 3) = 0$.

MODULE 2 • TOPIC 1 • LESSON 6 249

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

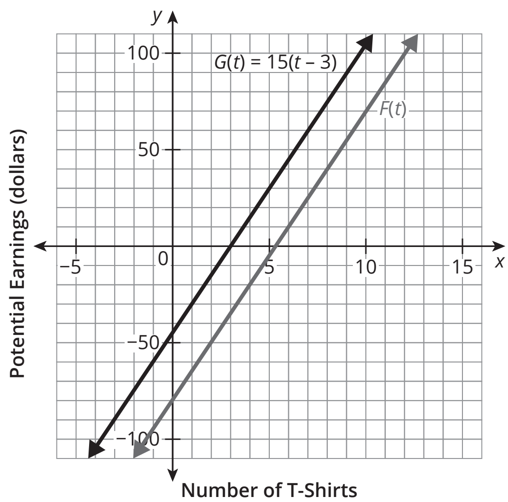
- Provide students with the problem-solving model graphic organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Question 2, have students work individually or with a partner to complete the graphic organizer.

ACTIVITY **6.4**

Interpreting More Changes to the Graph of a Linear Function

The next festival that Gracie is attending charges a \$35 fee to rent a booth. She is still selling her mom's T-shirts for \$15 each and giving 3 away in a raffle. The graph shows this new relationship, $F(t)$.

Ask Yourself . . .
Is $F(t)$ directly proportional to t?



PROBLEM SOLVING

1. Consider the relationship between graphs of $G(t)$ and $F(t)$.

   a. How do the graphs show that the selling price per T-shirt remains the same?

   Both graphs have the same slope, or average rate of change.

   b. How did the new booth fee of \$35 change the graph?

   The y-intercept shifted down from −45 to −80.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Some non-mathematical terms that appear in the activities are *festival, community fair, raffle*, and *booth*. Create a vocabulary chart that displays each term, along with cognates (e.g., *festival* and *feria de la comunidad*) and synonyms for each term. Discuss how the terms are commonly used in real-world scenarios.

c. How many T-shirts will Gracie need to sell before she will have any money to return to her mom? Explain your reasoning.

6 T-shirts; Looking at the graph, the y-values which represent potential earnings are positive when the number of T-shirts is 6 or more.

d. How many T-shirts will Gracie need to sell to earn $100?

$F(t) = 100$ when $t = 12$; Since Gracie gave away 3 shirts and paid a $35 fee to rent a booth, she needs to sell an additional 6 T-shirts to earn $100.

2. Consider the relationship between the equations of $G(t)$ and $F(t)$.

a. Write the function $F(t)$ in terms of $G(t)$.

$F(t) = G(t) - 35$

b. Rewrite $F(t)$ in general form. Then describe how the a- and b-values are represented on the graph.

$F(t) = 15t - 80$; a is evident by a slope triangle, and b is the y-intercept, $(0, -80)$.

The general form of a linear function is $f(x) = ax + b$.

The factored form of a linear function is $f(x) = a(x - c)$.

The values a, b, and c are real numbers and $a \neq 0$.

c. Rewrite $F(t)$ in factored form. Use a fraction to represent the c-value. Then describe how the a- and c-values are represented on the graph.

$F(t) = 15\left(t - \frac{16}{3}\right)$; a is evident by a slope triangle, and c is an x-intercept, $\left(5\frac{1}{3}, 0\right)$.

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MODULE 2 • TOPIC 1 • LESSON 6 251

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Reading Between the Lines

Complete each "I can" sentence using *always*, *sometimes*, or *never.*

1. Suppose you are given a dependent value and need to calculate an independent value of a linear function.

   a. I can ____always____ use a table to determine an *approximate* value.

   b. I can ____sometimes____ use a table to calculate an *exact* value.

   c. I can ____always____ use a graph to determine an *approximate* value.

   d. I can ____sometimes____ use a graph to calculate an *exact* value.

   e. I can ____always____ use an equation to determine an *approximate* value.

   f. I can ____always____ use an equation to calculate an *exact* value.

2. Write the function that models each table of values. Then, evaluate the function for each independent and dependent value.

   a.

x	1	2	3	4	5
$f(x)$	−20	5	30	55	80

   $f(x) = 25x - 45$

   $f(12) = 255$

   $f(x) = -145$ $f(-4) = -145$

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Review the terms *independent value* and *dependent value*. Ask students to define the terms *independent* and *dependent* in a non-mathematical context, and then ask them to connect the terms in the context of values in a function. Provide an example of a function in which students are asked to calculate an *independent value* given a *dependent value*. Review the problems in the activity and clarify any additional misunderstandings.

b.

x	1	3	5	7	9
$g(x)$	18	6	−6	−18	−30

$g(x) = -6x + 24$

$g(-9) = 78$

$g(x) = -54$ $g(13) = -54$

3. Complete the graphic organizer located at the end of this lesson for the linear function $f(x) = 2x - 8$.

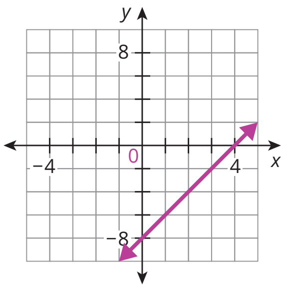
   a. Write $f(x)$ in general form. Then, describe the information given in this form.

   $f(x) = 2x - 8$; slope $= 2$, y-intercept $= -8$

   b. Write $f(x)$ in factored form. Then, describe the information given in this form.

   $f(x) = 2(x - 4)$; slope $= 2$, zero $= 4$

   c. Graph $f(x)$. Describe how you know this graph can cross the x-axis only one time.



The zeros identify where the graph crosses the x-axis. Using factored form, $f(x) = 2(x - 4)$ has exactly one zero, $x = 4$, when the factor $(x - 4) = 0$.

   d. Create a table of values for $f(x)$.

Sample answer:

x	y
−1	−10
0	−8
1	−6
2	−4

Optimizing Learning

This activity provides a graphic organizer to emphasize key ideas and relationships.

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MODULE 2 • TOPIC 1 • LESSON 6 253

Graphic Organizer

Sample answers:

General Form

$f(x) = ax + b$, where a and b are real numbers and $a \neq 0$

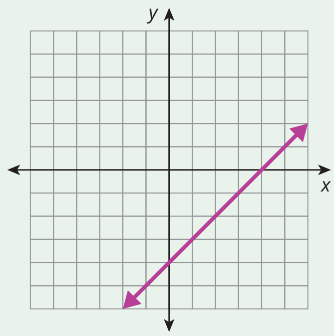
$f(x) = 2x - 8$

Factored Form

$f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$

$f(x) = 2(x - 4)$

$f(x) = 2x - 8$



Graph

x	y
0	−8
2	−4
4	0
6	4

Table

254 MODULE 2 • TOPIC 1 • LESSON 6

Lesson 6 Assignment

Write

Describe a zero of a function in your own words.

Remember

The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient which describes the steepness and direction of the line. The b-value describes the y-intercept.

The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. In this form, the a-value is the slope and the value of x that makes the factor $(x - c)$ equal to zero is the x-intercept.

Write

Sample answer: A zero of a function is a real number that makes the value of the function equal to zero.

Practice

Determine whether the table of values represents a linear function. If so, write the function.

1.

x	y
−2	$5\frac{2}{3}$
0	5
2	$4\frac{1}{3}$
4	$3\frac{2}{3}$

Yes; $y = -\frac{1}{3}x + 5$

2.

x	y
−5	−27
0	−2
5	20
10	48

Not a linear function

For each scenario, write a linear function in factored form and in general form. Then, sketch a graph and label the x- and y-intercepts. Finally, answer each question.

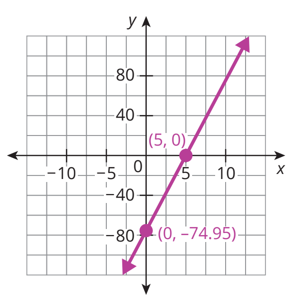
3. Omar prints and sells T-shirts for $14.99 each. Each month 5 T-shirts are misprinted and cannot be sold. How much money will he earn when he prints 22 T-shirts? How many T-shirts will he need to sell to earn $200?

   $f(x) = 14.99(x - 5)$

   $f(x) = 14.99x - 74.95$

   He will earn $254.83 if he prints 22 shirts.

   He will have to print 19 shirts to earn $200.



MODULE 2 • TOPIC 1 • LESSON 6 ASSIGNMENT 255

Lesson 6 Assignment

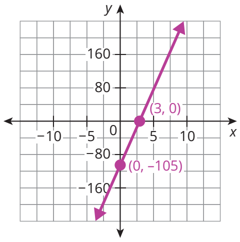
4. Juliana paints and sells ceramic vases for \$35 each. Each month, she typically breaks 3 vases in the kiln. How much money will she earn if she sells 17 ceramic vases? How many ceramic vases will she need to sell to earn \$600?

   $f(x) = 35(x - 3)$

   $f(x) = 35x - 105$

   She will earn \$490 if she makes 17 ceramic vases.

   She will have to sell 21 ceramic vases to earn \$600.



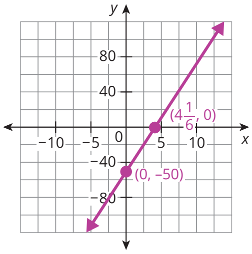
5. Antonio builds and sells homemade wooden toys for \$12 each. The festival he is attending charges \$50 to set up his booth. How much money will he earn if sells 35 wooden toys? How many wooden toys will he need to sell to earn \$250?

   $f(x) = 12x - 50$

   $f(x) = 12\left(x - \frac{25}{6}\right)$

   He will earn \$370 if he sells 35 wooden toys.

   He will have to sell 25 wooden toys to earn \$250.



Prepare

Determine each value, given the function $h(x) = -2x - 5$.

1. $h(3)$ $\quad -11$
2. $2 \cdot h(-2)$ $\quad -2$
3 $-1 \cdot h(1) + 5$ $\quad 12$
4. $5 \cdot h(0) + 5$ $\quad -20$

256 MODULE 2 • TOPIC 1 • LESSON 6 ASSIGNMENT

TOPIC 1 SELF-REFLECTION

Name: ____________________

Linear Functions

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Linear Functions* topic by:

TOPIC 1: *Linear Functions*	Beginning of Topic	Middle of Topic	End of Topic
determining when a data set should be modeled by a linear function.	○	○	○
using technology to determine the linear regression function for a data set.	○	○	○
making predictions using the linear regression function.	○	○	○
interpreting the meaning of the slope and y-intercept of a linear regression function in terms of the problem context.	○	○	○
explaining the correlation coefficient as a measure of how well a function fits a data set.	○	○	○
understanding that the sign of a correlation coefficient indicates the direction of the association and that the magnitude indicates the strength of the fit.	○	○	○
analyzing a function on a scatterplot and its correlation coefficient to determine whether a function is an appropriate fit for a data set.	○	○	○
recognizing that correlation does not imply causation.	○	○	○

continued on the next page

Notes

MODULE 2 • TOPIC 1 • SELF-REFLECTION 257

Notes

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Linear Functions*	Beginning of Topic	Middle of Topic	End of Topic
recognizing that all arithmetic sequences are linear functions and rewriting an arithmetic sequence as a linear function using function notation.	◯	◯	◯
writing equations of lines in the appropriate form: slope-intercept, point-slope, or standard form.	◯	◯	◯
graphing lines from an equation written in slope-intercept, point-slope, or standard form.	◯	◯	◯
writing and solving equations involving direct variation.	◯	◯	◯
connecting average rate of change of a linear function to the slope of a line.	◯	◯	◯
constructing a linear function from a scenario, table of values, graph, or arithmetic sequence.	◯	◯	◯
interpreting the real-world meaning of the coefficients and constants in a linear function.	◯	◯	◯
recognizing a linear relationship and writing an equation to represent that situation.	◯	◯	◯
identifying the variables and quantities of a linear function.	◯	◯	◯
determining key features of graphs of linear functions including domain, range, x-intercept, y-intercept, zeros, and slope.	◯	◯	◯

continued on the next page

258 MODULE 2 • TOPIC 1 • SELF-REFLECTION

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Linear Functions* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

Notes

Notes

260 MODULE 2 • TOPIC 1 • SELF-REFLECTION

TOPIC 1 SUMMARY

Linear Functions Summary

LESSON 1 Least Squares Regression

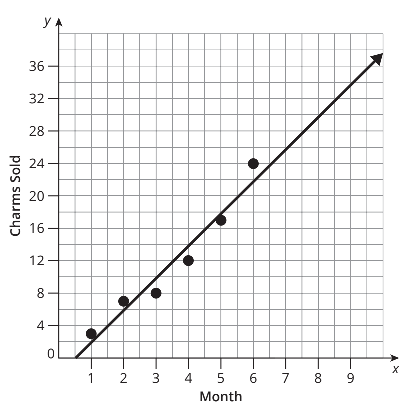
Technology calculates the line of best fit of data on a scatterplot using the **Least Squares Method**. The line includes the **centroid**, or the point whose *x* value is the mean of all *x* values and whose *y* value is the mean of all *y* values. The **linear regression function** has the smallest possible vertical distance from each given data point to the line. The sum of the squares of these distances is at a minimum with the linear regression function.

When there is a linear association between the independent and dependent variables, you can use a linear regression function to make predictions within the data set. Using a linear regression function to make predictions within the data set is called **interpolation**. To make predictions outside the data set is called **extrapolation**.

For example, consider the situation of Lucia selling charms to her classmates. The table records the sales of her charms over the months since she began selling them.

Month	Charms Sold
1	3
2	7
3	8
4	12
5	17
6	24

The linear regression function modeling the situation is graphed on the scatterplot shown.



The linear regression function is $y = 3.97x - 2.07$.

Using the equation to interpolate, Lucia should sell about 14 charms in the fourth month.

$$y = 3.97(4) - 2.07$$
$$= 13.81$$

Using the equation to extrapolate, Lucia should sell about 30 charms in the eighth month.

$$y = 3.97(8) - 2.07$$
$$= 29.69$$

NEW KEY TERMS

- Least Squares Method
- centroid [centroide]
- linear regression function [función de regresión lineal]
- interpolation [interpolación]
- extrapolation [extrapolación]
- correlation [correlación]
- correlation coefficient [coeficiente de correlación]
- coefficient of determination [coeficiente de determinación]
- causation [causalidad]
- necessary condition [condición necesaria]
- sufficient condition [condición suficiente]
- common response [respuesta común]
- confounding variable [variable de confusión]
- conjecture [conjetura]
- first differences

Notes

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MODULE 2 • TOPIC 1 • TOPIC SUMMARY 261

Notes

NEW KEY TERMS

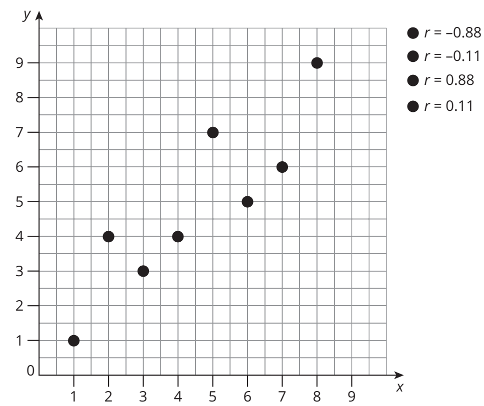
- average rate of change
- point-slope form
- standard form [forma estándar/general]
- polynomial [polinomio]
- degree
- leading coefficient
- zero of a function [cero de una función]

LESSON 2

Correlation

The measure of how well a regression fits a set of data is called **correlation**. When dealing with regression functions, the variable r is used to represent a value called the **correlation coefficient**. The correlation coefficient indicates how close the data are to the graph of the regression function. The correlation coefficient falls between -1 and 0 when the data show a negative association or between 0 and 1 when the data show a positive association. The closer the r-value is to 1 or -1, the stronger the relationship is between the two. The **coefficient of determination**, r^2, measures how well the regression line fits the data. It represents the percentage of variation of the observed values of the data points from their predicted values.

For example, consider the possible r-values for a linear regression function given for the data graphed in the scatterplot.



The data has a positive correlation. Because of this, r-value must be positive. Also, the data are fairly close to forming a straight line, so of the choices, $r = 0.88$ would be the most accurate. Technology can be used to verify the correlation coefficient. The coefficient of determination for this data set is 0.7744.

When interpreting the correlation between two variables, you are looking at the association between the variables. While an association may exist, that does not mean there is causation between the variables. **Causation** is when one event causes a second event. A correlation is a **necessary condition** for causation, but a correlation is not a **sufficient condition** for causation. Correlation may be due to a **common response**, which is when another reason may cause the same result, or a **confounding variable**, which is when other variables are either unknown or unobserved.

For example, consider an experiment conducted by a group of college students that found that more class absences correlated to rainy days. The group concluded that rain causes students to be sick. However, this correlation does not imply causation. Rain is neither a necessary condition (because students can get sick on days it does not rain) nor a sufficient condition (because not every student who is absent is necessarily sick) for students being sick.

262 MODULE 2 • TOPIC 1 • TOPIC SUMMARY

LESSON 3

Making Connections Between Arithmetic Sequences and Linear Functions

You can make a conjecture that the graphs of arithmetic sequences represent linear functions. A **conjecture** is a mathematical statement that appears to be true, but has not been formally proven. An explicit formula for an arithmetic sequence can be rewritten in function notation to prove your conjecture.

For example, to write the explicit formula $a_n = 2 + 4(n - 1)$ as a linear function in general form, first use function notation to represent a_n.

$$a_n = 2 + 4(n - 1)$$
$$f(n) = 2 + 4(n - 1)$$

Next, rewrite the expression $2 + 4(n - 1)$.

$$\begin{aligned} f(n) &= 2 + 4n - 4 && \text{distributive property} \\ &= 4n + 2 - 4 && \text{commutative property} \\ &= 4n - 2 && \text{combine like terms} \end{aligned}$$

So, $a_n = 2 + 4(n - 1)$ written in function notation is $f(n) = 4n - 2$.

One strategy to determine if a table of values represents a linear function is to examine first differences. **First differences** are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear.

x	y	
1	22	
2	17	$17 - 22 = -5$
3	12	$12 - 17 = -5$
4	7	$7 - 12 = -5$

For example, the first differences of the table shown are constant, so the relationship is linear.

The slope, a, of a linear function is equal to the constant difference of an arithmetic sequence. Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is $\frac{f(t) - f(s)}{t - s}$. This represents the change in the output as the input changes from s to t.

You can determine slope of a line when given two points using the formula $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\Delta y}{\Delta x}$. Delta ($\Delta$) is a greek letter used in mathematics to represent "change in". So $\frac{\Delta y}{\Delta x}$ means "change in y" divided by "change in x".

Notes

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MODULE 2 • TOPIC 1 • TOPIC SUMMARY 263

Notes

LESSON 4 Point-Slope Form of a Line

You can determine the equation of a line from a table of values without knowing the y-intercept using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

For example, to write an equation of a line from a table of values, you can use the slope formula.

x	y
2	6
4	5
6	4

- First, calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-6}{4-2}$$
$$= \frac{-1}{2} = -\frac{1}{2}$$

- Next, choose any point from the table. Let's use (2, 6).
- Then, substitute what you know into the slope formula: $m = -\frac{1}{2}$, (2, 6), and the unknown point (x, y).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$-\frac{1}{2} = \frac{y-6}{x-2}$$

- Finally, rewrite the equation with no variables in a denominator.

$$-\frac{1}{2} = \frac{y-6}{x-2}$$
$$-\frac{1}{2}(x-2) = y-6$$

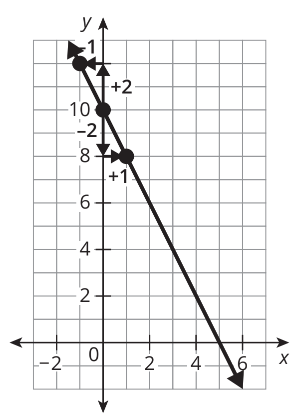
The equation is $y - 6 = -\frac{1}{2}(x - 2)$.

The linear equation you wrote is written in point-slope form. The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is any point on the line. For example, given a slope of 3 and the point (1, 4), you can write the equation for the line in point-slope form as $y - 4 = 3(x - 1)$.

The slope of a horizontal line is 0. The slope of a vertical line is undefined.

LESSON 5 Using Linear Equations

You can use an equation in slope-intercept form to graph a relationship without first creating a table of values using the y-intercept and the slope.



For example, graph the equation $y = -2x + 10$. Begin by plotting the y-intercept, (0, 10). Use the slope and count from the y-intercept to graph two more points on the line.

- For $m = \frac{-2}{1}$, go down 2 units and to the right 1 unit.
- For $m = \frac{-2}{1}$, go up 2 units and to the left 1 unit.

Connect the points to form a straight line.

264 MODULE 2 • TOPIC 1 • TOPIC SUMMARY

Another way to write a linear equation is in standard form. The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are integers, and A and B are not both zero.

When an equation is in standard form, you can calculate a y-intercept, where a line crosses the y-axis, by substituting 0 for x and solving the equation for y. To calculate an x-intercept, where the line crosses the x-axis, substitute 0 for y and solve the equation for x.

To write a linear equation in standard form when given two points, first write the equation in point-slope form. Then, rewrite the equation in standard form.

Write a linear equation in standard form, $Ax + By = C$, that goes through the points $(-1, 3)$ and $(-4, -3)$.

Begin by determining the slope of the line.

$$m = \frac{(-5 - 3)}{(-4 - (-1))} = \frac{(-\ 6)}{(-3)} = 2$$

Pick one of the points and write an equation in point-slope form for the line.

$$y - 3 = 2(x - (-1))$$
$$y - 3 = 2(x + 1)$$

Rewrite the equation in standard form.

$$\begin{aligned} y - 3 &= 2x + 2 \\ y &= 2x + 5 \\ -2x + y &= 5 \end{aligned}$$

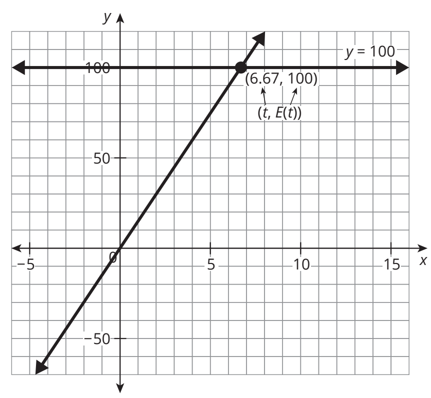
LESSON 6 Making Sense of Different Representations of a Linear Function

To determine whether a table of values represents a linear function, the slope, or average rate of change, needs to be constant between all given points.

You can use substitution or a graph to determine the output for a given input of a function.

For example, consider the function $E(t) = 15t$ which models the amount of money Gracie earns for selling t T-shirts. To determine the number of shirts she needs to sell to earn \$100, substitute \$100 for $E(t)$ and solve.

$$\begin{aligned} E(t) &= 15t \\ 100 &= 15t \\ 6.67 &= t \end{aligned}$$



Notes

MODULE 2 • TOPIC 1 • TOPIC SUMMARY 265

Notes

Or you can determine the intersection of the graphs of the two lines represented by the equation $100 = 15x$.

Gracie needs to sell 6.67 T-shirts to earn \$100. In terms of the context, she needs to sell 7 T-shirts.

A linear function can also be referred to as a polynomial function. A **polynomial** is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. The **degree** of a polynomial is the greatest variable exponent in the expression. The **leading coefficient** of a polynomial is the numeric coefficient of the term with the greatest power.

The chart shows a few examples of polynomial functions.

Polynomial Function		Degree
Constant	$P(x) = 7$	0
Linear	$P(x) = 2x - 5$	1
Quadratic	$P(x) = 3x^2 - 2x + 4$	2
Cubic	$P(x) = 4x^3 - 2$	3

The structure of each linear function provides important information about the graph. The general form of a linear function is $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$. In this form, the a-value is the leading coefficient, which describes the steepness and direction of the line. The b-value describes the y-intercept.

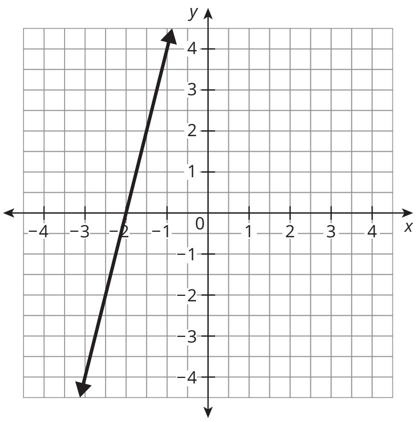
When you graph a polynomial, the degree indicates the maximum number of times the graph can cross the x-axis. A linear function has a degree of 1, so it crosses the x-axis at most one time.

The factored form of a linear function is $f(x) = a(x - c)$, where a and c are real numbers and $a \neq 0$. When a linear function is in factored form, the value of x that makes the factor $(x - c)$ equal to zero is the x-intercept. This value is called the zero of the function. A **zero of a function** is a real number that makes the value of the function equal to zero, $f(x) = 0$.

You can set the factor $(x - c)$ equal to zero to determine the point where the graph crosses the x-axis.

For example, the linear function $f(x) = 4x + 8$ in factored form is $f(x) = 4(x + 2)$. Set the factor $x + 2$ equal to zero and then solve for x to determine the zero of the function, which is the point at which the graph of the function will cross the x-axis.

$$x + 2 = 0$$
$$x = -2$$



266 MODULE 2 • TOPIC 1 • TOPIC SUMMARY

ALGEBRA I

TOPIC 2

A region can be described as above, below, to the left, or to the right of a line.

Transforming and Comparing Linear Functions

Transforming and Comparing Linear Functions

How are the key concepts of *Transforming and Comparing Linear Functions* organized?

In *Transforming and Comparing Linear Functions*, students continue to analyze linear relationships, transforming linear functions, and comparing linear functions represented in different forms.

Students are introduced to the transformation notation $y = a \cdot f(b(x - c)) + d$, which represents translations and dilations given specific values of a, b, c, and d. Students compare horizontal and vertical translations by the same value for c and d in transformation form and determine that for the parent function $f(x) = x$, a horizontal translation right n units produces the same graph as a vertical translation down n units. In addition, a horizontal translation left n units produces the same graph as a vertical translation up n units. Then, students compare vertical and horizontal dilations and investigate the relationships between the a and b values in transformation form. They determine that for the parent function $f(x) = x$, a vertical dilation by a factor of n units produces the same graph as a horizontal dilation by a factor of $\frac{1}{n}$ units. In addition, a horizontal dilation by a factor of n units produces the same graph as a vertical dilation by a factor of $\frac{1}{n}$ units.

Next, students focus on the effects of replacing a (vertical dilations and/or reflections across the x-axis) and d (vertical translations) with specific values. Students prove that a line and its translation are parallel to each other; therefore, parallel lines have the same slope. They also learn that perpendicular lines form a right angle at the point of intersection. Perpendicular lines can be thought of as a line and its rotation 90 degrees

Math Representation

Given the graph of $y = f(x)$, the graph of $y = a \cdot f(b(x - c)) + d$ represents transformations given specific values of a, b, c, and d.

Vertical dilation by a factor of a;
a reflection across the x-axis when $a < 0$.

Horizontal translation of c units,
to the right when $c > 0$ and to the left when $c < 0$.

$$a \cdot f(b(x - c)) + d$$

Horizontal dilation by a factor of $\frac{1}{|b|}$;
a reflection across the y-axis when $b < 0$.

Vertical translation of d units,
up when $d > 0$ or down when $d < 0$.

about a point, which is also the point of intersection. The product of the slopes of perpendicular lines is -1; therefore, the slopes of perpendicular lines are negative reciprocals.

Graphing linear functions by transformation is critical to students developing a unified understanding of functions. Because linear functions behave like all other function types, understanding the rules of transformations for linear functions lays the groundwork for students to transform any function type.

The topic concludes with students comparing linear functions presented in different forms, including such key characteristics as slopes, y-intercepts, independent and dependent quantities, and appropriate units of measure.

What is the entry point for students?

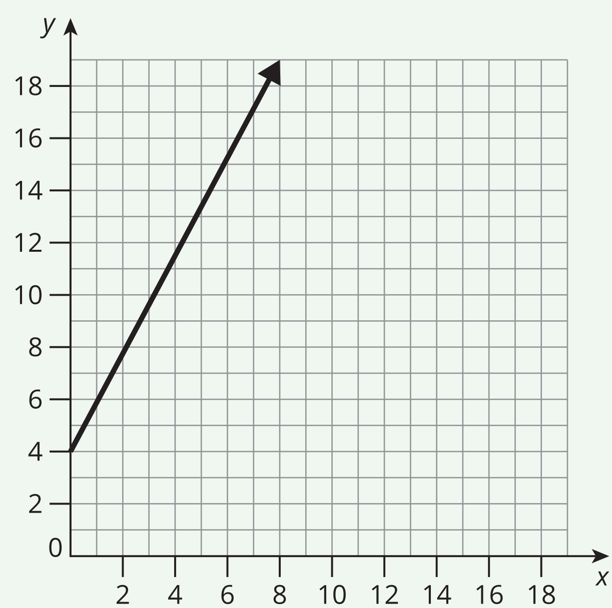
Over the last few years, students have had extensive experience with linear relationships. As students progressed from proportional relationships—represented as straight lines passing through the origin—to linear functions, students learned that vertical translations are related to the y-intercept, and that dilations are connected to the slope of a line.

In the first topic of this course, *Quantities and Relationships*, students learned to use function notation to represent the equation of a function. In this topic, students transform functions using that notation. This helps students to recognize patterns in transformations and generalize to other functions.

Math Representation

The equation $y = mx$ represents a proportional relationship. The equation represents every point (x, y) on the graph of a line with slope m that passes through the origin (0, 0).

An equation in the form $y = mx + b$, where b is not equal to 0, represents a non-proportional relationship. This equation represents every point (x, y) on the graph of a line with slope m that passes through the point $(0, b)$. For example, the graph shown represents a non-proportional relationship where $m = 2$ and $b = 4$.



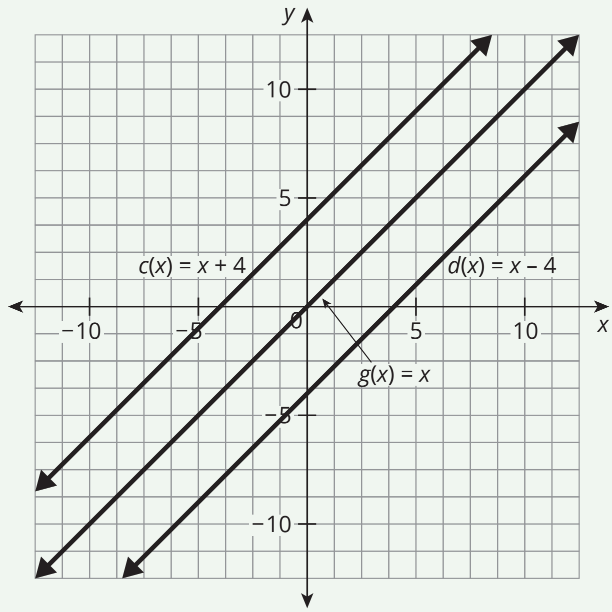
Why is *Transforming and Comparing Linear Functions* important?

Functions are objects that can be represented in a multitude of ways—using scenarios, tables of values, equations, or graphs. This topic explores how a linear function is transformed and represented in various representations.

Ultimately, linear functions are the first of many function types students will study in this course and beyond. Establishing a strong foundation in linear functions is critical in preparing students for more advanced mathematics.

Math Representation

$g(x) = x$	parent function
$c(x) = g(x) + 4$	$g(x)$ translated 4 units up, so $(x, y) \longrightarrow (x, y + 4)$
$d(x) = g(x) - 4$	$g(x)$ translated 4 units down, so (x, y) $(x, y - 4)$



How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Transforming and Comparing Linear Functions* when they can:

- Recognize that the parent function for linear functions is $f(x) = x$.
- Determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $a \cdot f(x)$, $f(x) + d$, $f(x - c)$ and $f(bx)$.
- Describe the transformations performed on a parent function to create a new function in transformation form $a \cdot f(b(x - c)) + d$.
- Write the equation of a line that contains a given point and is parallel or perpendicular to a given line.

NEW KEY TERM

- parent function

How do the activities in *Transforming and Comparing Linear Functions* promote student expertise in the TEKS mathematical process standards?

Every topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Throughout *Transforming and Comparing Linear Functions*, use structure and patterns to develop an understanding of the rules of transformation (A.1E, A.1G). Students are expected to use precision when analyzing graphs and describing transformations (A.1F, A.1G). They apply what they learn throughout the topic to solve real-world problems using the problem-solving model. (A.1A, A.1B). Students communicate their mathematical reasoning using tables, graphs, and equations (A1.D).

2 Exploring Constant Change

TOPIC 2: Transforming and Comparing Linear Functions

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1D, A.1E, A.1F, A.1G

ELPS: 1.D, 1.E, 2.D, 2.G, 2.H, 2.I, 3.C, 3.E, 3.F, 4.A, 4.F, 4.K

Topic Pacing: 14 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Transforming Linear Functions	Students identify key characteristics of several linear functions. A graph and a table of values for the parent linear function $f(x) = x$ is provided, and they investigate $f(x) + d$ and $a \cdot f(x)$. Given a function $g(x)$ in terms of $f(x)$, students graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$. They prove algebraically that a line and its translation are parallel to one another and write equations of lines parallel to a given line through a given point. Finally, students use their knowledge of linear function transformations to test a video game that uses linear functions to shoot targets. They write the function transformations several ways and identify the domains, ranges, slopes, and y-intercepts of the new functions. **Materials Needed:** Masking Tape, Markers, Problem-Solving Model Graphic Organizer	**A.2A** **A.2C** A.2E **A.3C** A.3E	3
2	Vertical and Horizontal Transformations of Linear Functions	Students identify key characteristics of several linear functions. A graph and a table of values for the parent linear function $f(x) = x$ is provided, and students will translate this function horizontally and vertically to determine which transformations affect the input and output values. They will also dilate the function $f(x) = x$ horizontally and vertically. Students will generalize about equivalent translations for the parent function $f(x) = x$, and determine that these relationships do not hold true for all linear functions. Given a function $g(x)$ in terms of $f(x)$, students will graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$. **Materials Needed:** None	**A.2A** **A.2C** **A.3C** A.3E	2
3	Determining Slopes of Perpendicular Lines	Students rotate a line segment on the coordinate plane in increments of 90° counterclockwise and recognize patterns in the slopes and coordinates of the endpoints of the images. They analyze a proof of a theorem stating that if two lines are perpendicular, the slopes of the lines are negative reciprocals. Students then explore relationships between vertical and horizontal lines. Finally, they write the equation of a line perpendicular to a given a line that passes through a given point. **Materials Needed:** Patty Paper, Straightedges, Problem-Solving Model Graphic Organizer	A.2F A.2G	2

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
4	Comparing Linear Functions in Different Forms	Students analyze functions represented as tables, graphs, equations, and verbal descriptions. They explore slope with particular attention to parallelism and perpendicularity in different representations. Students compare properties, such as slope, y-intercept, and the units for independent and dependent quantities, all in terms of the situations they represent. Students also identify the scale and origin on the graph of a function given a situation description. Finally, they generate and compare their own linear functions using tables, graphs, and equations. **Materials Needed:** Problem-Solving Model Graphic Organizer	A.3A **A.3C** A.12B	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				4

*Bold TEKS = Readiness Standard

MODULE 2, TOPIC 2 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1 TEKS: **A.2A, A.2C,** A.2E, **A.3C**, A.3E **LESSON 1** Transforming Linear Functions **GETTING STARTED** **ACTIVITY 1**	**Day 2** **LESSON 1** continued **ACTIVITY 2** **ACTIVITY 3**	**Day 3** **LESSON 1** continued **ACTIVITY 4** **ACTIVITY 5** **TALK THE TALK**	**Day 4** **LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**Day 5** TEKS: **A.2A, A.2C, A.3C,** A.3E **LESSON 2** Vertical and Horizontal Transformations of Linear Functions **GETTING STARTED** **ACTIVITY 1**
Day 6 **LESSON 2** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**Day 7** **LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**Day 8** TEKS: A.2F, A.2G **LESSON 3** Determining Slopes of Perpendicular Lines **GETTING STARTED** **ACTIVITY 1**	**Day 9** **LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**Day 10** **LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 11 TEKS: A.3A, **A.3C,** A.12B **LESSON 4** Comparing Linear Functions in Different Forms **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**Day 12** **LESSON 4** continued **ACTIVITY 3** **ACTIVITY 4** **TALK THE TALK**	**Day 13** **LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**Day 14** **END OF TOPIC ASSESSMENT**	

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are four Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Transforming Linear Functions

LESSON OVERVIEW

Students identify key characteristics of several linear functions. A graph and a table of values for the parent linear function $f(x) = x$ is provided, and they investigate $f(x) + d$ and $a \cdot f(x)$. Given a function $g(x)$ in terms of $f(x)$, students graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$. They prove algebraically that a line and its translation are parallel to one another and write equations of lines parallel to a given line through a given point. Finally, students use their knowledge of linear function transformations to test a video game that uses linear functions to shoot targets. They write the function transformations several ways and identify the domains, ranges, slopes, and y-intercepts of the new functions.

MATERIALS

Masking Tape

Markers

Problem-Solving Model Graphic Organizer

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.

The student is expected to:

A.2A determine the domain and range of a linear function in mathematical problems, determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

A.2E write the equation of a line that contains a given point and is parallel to a given line.

(TEKS continued on next page)

ELPS

(2) Listening

The student is expected to:

(H) understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations.

(3) Speaking

The student is expected to:

(F) ask and give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments.

(4) Reading

The student is expected to:

(F) use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3C graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.

A.3E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

ESSENTIAL IDEAS

- For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values of the function. For $d > 0$, the graph vertically shifts up. For $d < 0$, the graph vertically shifts down. The amount of shift is given by $|d|$.
- For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ affects the output values of the function. For $|a| > 1$, the graph stretches vertically by a factor of a units. For $0 < |a| < 1$, the graph compresses vertically by a factor of a units. For $a < 0$, the graph reflects across the x-axis.
- A line and its translation are parallel to one another.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Returning to Transformation Station 15–20 minutes

BUILD OFF INTUITION

Students revisit the idea of transforming geometric figures on the coordinate plane. They form a line segment on a floor coordinate plane, using individual students as points. The student "points" move to vertically translate and then vertically dilate the line segment. Students describe, using mathematical notation, the effect of a vertical translation on the coordinates of the points of a line segment. Students graph a translated line segment and a dilated line segment and then compare the pre-image with the image.

DEVELOP

Activity 1.1: Vertical Translations of Functions 20–25 minutes

INVESTIGATION, WORKED EXAMPLE

Students learn that functions can be translated in the same way that geometric figures can be translated on the coordinate plane. Students graph vertical translations of linear functions using tables of values. They vertically translate $f(x) = x$ and $f(x) = 2x - 1$. In each case, they identify the slope and y-intercept of the translated function and write its equation in slope-intercept form. For translations of the function $f(x) = 2x - 1$, they also rewrite the equation of the translated function in terms of the original function. Students generalize that the D-value in the function transformation form $f(x) + D$ shifts the graph of the function up or down.

DAY 2

Activity 1.2: Slopes of Parallel Lines 15–20 minutes

INVESTIGATION, PEER WORK ANALYSIS

Students relate translations to parallel lines. They use algebra to prove that a line and its translation are parallel to one another. Students analyze student work on how to write the equation of a line parallel to a given line through a given point and then practice this skill.

Activity 1.3: Vertical Dilations of Functions 20–25 minutes

INVESTIGATION

Students graph vertical dilations (stretches) of linear functions using tables of values. They then identify the slopes and y-intercepts of the two functions and write the equation for the translated function in slope-intercept form. Students generalize that the a-value in the function transformation form $a \cdot f(x)$ stretches or compresses the graph of the function. And, when a is negative, the a-value reflects the function across the x-axis.

DAY 3

Activity 1.4: Vertical Dilations and Vertical Translations of Functions 15–20 minutes

INVESTIGATION

Students graph functions of the form $a \cdot f(x) + d$, which are vertical translations and vertical dilations of the original functions. Students consider whether the order in which they apply the transformations (vertical dilation before vertical translation or vice versa) affects the end result. Students generalize that the *a*-value affects the slope of the function, while the *d*-value affects the *y*-intercept.

Activity 1.5: Applying Linear Function Transformations 15–20 minutes

REAL-WORLD PROBLEM SOLVING

Students apply what they have learned about vertical linear function transformations in the context of testing a video game that generates linear functions of the form $a \cdot f(x) + d$ as the paths of a cannon firing cannonballs at targets. Students are asked first to transform the default function $f(x) = x$ any way they like to estimate lines from the cannon to the targets. Students are also asked to consider the restricted domain of the cannon shot, since it can only shoot in one direction. Then, students are asked to write equations for transformed functions assuming (a) the cannon remains fixed at the origin, so only a change in *a*-value will generate lines that will hit the targets, or (b) the cannon's angle remains fixed on the line $f(x) = x$, so only a change in *d*-value will generate lines that will hit the targets.

DEMONSTRATE

Talk the Talk: Function Matching 5 minutes

EXIT TICKET PROCEDURE

Students are given the graphs of four function transformations. They choose from a list of four equations, written in terms of the original function, to match to each graph.

Getting Started

ENGAGE

Returning to Transformation Station

Facilitation Notes

In this activity, students revisit the idea of transforming geometric figures on the coordinate plane. They form a line segment on a floor coordinate plane, using individual students as points. The student "points" move to vertically translate and then vertically dilate the line segment. Students describe, using mathematical notation, the effect of a vertical translation on the coordinates of the points of a line segment. They graph a translated line segment and a dilated line segment and then compare the pre-image with the image.

In advance, use masking tape and a marker to create a coordinate plane on the floor of the classroom. The coordinate plane must extend from at least −16 to +16 on each axis. Use the tape to create gridlines—enough so that students can stand at the points on the given line segment.

During the lesson, ask for student volunteers to stand at specific locations on the coordinate plane. Some students can provide the coordinates while others stand at those locations. Physically perform the movement of the points through the translations and dilation.

As a class, complete Questions 1 through 6. Discuss the instructions and transformations throughout the activity as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What does *translate* mean?
Probing	• When you moved on the coordinate plane, did you change the x-value or y-value of the original coordinate pair? Explain. • Does the translated line still go through the origin? Why not?
Seeing structure	• What's the difference between the y-values on the original and translated lines for the same x-value? • What does the y in $y + 4$ represent?
Probing	• Why are the lines parallel? • Compare the y-intercepts. • When you moved on the coordinate plane, did you change the x-value or y-value of the original coordinate pair? Explain your thinking. • Does the newly graphed line still pass through the origin? Why? • Why does it make sense that the line is steeper?
Seeing structure	• How are the lines graphed by multiplying by 2 and −2 the same? Different?

AS STUDENTS WORK, LOOK FOR

- Students changing both the *x*-value and the *y*-value when performing a vertical translation or dilation.
- Students confusing the concepts of translation and dilation.

COMMON MISCONCEPTION

- Some student may compare the graphs of $\overline{AB}$ and $\overline{CD}$ and think that $\overline{CD}$ is a horizontal, rather than vertical, translation of $\overline{AB}$. Clarify this misconception using two methods:
  - Focus on individual points. Have students refer to, or redo the human graph. Because a student represents a "point," address where that specific student is mapped to after the translation.
  - Focus on the line segments they graphed. If the translation were horizontal, the endpoints would be lined up horizontally; that is not the case.

  Use this activity to help students correct their thinking. These underpinnings will be helpful because the next activity deals with transformations of lines without endpoints for reference. Horizontal translations will be addressed with exponential functions and then practiced with quadratic functions later in the course.

STAMP THE LEARNING

Summary

When a line segment is vertically translated on the coordinate plane, the *x*-value of each point remains unchanged, and the *y*-value changes according to the translation.

ACTIVITY 1.1

Vertical Translations of Functions

DEVELOP

Facilitation Notes

In this activity, students learn that functions can be translated in the same way geometric figures can be translated on the coordinate plane. Students graph vertical translations of linear functions using tables of values. They vertically translate $f(x) = x$ and $f(x) = 2x - 1$. In each case, they identify the slope and *y*-intercept of the translated function and write its equation in slope-intercept form. For translations of the function $f(x) = 2x - 1$, they also rewrite the equation of the translated function in terms of the original function. Students generalize regarding the *d*-values in the function transformation form $f(x) + d$, where the *d*-value shifts the graph of the function up or down vertically.

Ask a student to read the introduction and definition aloud. Discuss the Worked Example as a class.

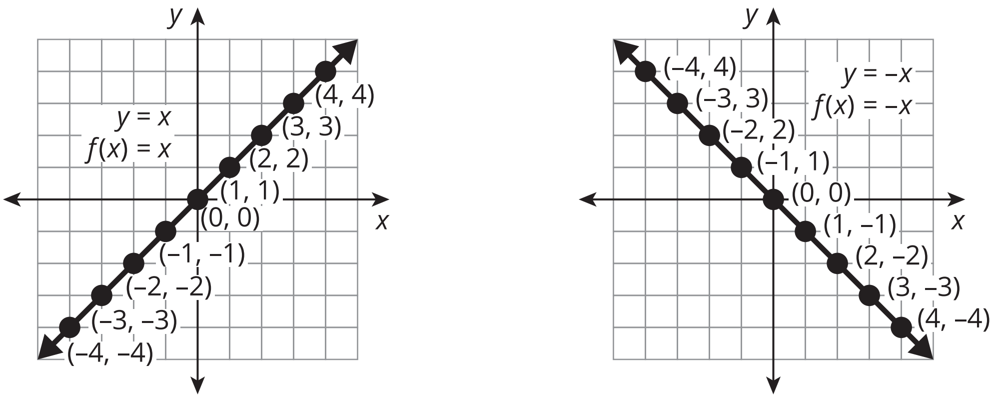
Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Poster Paper

- To visually assist with making sense of transformations, students must have a clear understanding of the parent function, $f(x) = x$, and be able to graph it with automaticity. Display a poster for a quick reference.



Optimizing Learning

This differentiation strategy provides options for activating or supplying background knowledge.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why is the y-intercept of the transformed graph (0, −5)? • Did you complete any calculations to determine the slope? Explain. • How did you determine the points that lie on $p(x)$? • Do vertical translations of $f(x)$, such as $p(x)$ and $m(x)$, affect the range? Why not?
Seeing structure	• Explain the relationship between the graph, table, and equation.

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you know which operation to include with $q(x)$? • Explain how you rewrote each function in terms of x.
Seeing structure	• How is writing the equation of a translated function the same and different when the original function is not the parent function $f(x) = x$?

DIFFERENTIATION STRATEGIES

Access for All

- Call attention to the advantages of using function notation. Function notation makes it easy to reference several graphs on the same coordinate plane and show how their equations are related to one another.

Challenge Opportunity

- Challenge students to think about their translation of $j(x)$ as adding two functions. Make it explicit to students that they can represent translating the function up 4 units using a constant function, $k(x) = 4$. So, $q(x)$ is the sum of $j(x)$ and $k(x)$. All students will learn more about this concept later in the course when they add polynomial functions.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ shows a vertical translation of the function. The slopes of vertically translated linear functions do not change, which means that the graphs of the functions are parallel lines.

ACTIVITY 1.2 Slopes of Parallel Lines

Facilitation Notes

In this activity, students relate translations to parallel lines. They use algebra to prove that a line and its translation are parallel to one another. Students analyze student work on how to write the equation of a line parallel to a given line through a given point and then practice this skill.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

- Suggest that students use the graph to make sense of the coordinates of the points.
    - Suggest students label the lines with the terms *pre-image* and *image*.
    - Have students draw vertical arrows from the pre-image to the image so that they can see that the x-coordinates stay the same, and the y-coordinates change when completing a translation.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which line is the image? Which is the pre-image?
Seeing structure	• Are the *x*-coordinates, *y*-coordinates, or both, changed when completing a translation? How do you know? • Does the order of the coordinates matter when using the slope formula? Explain.

Have students work with a partner or in a group to analyze the student work and complete Questions 3 and 4. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

- Assign students to teams of four. Have one pair of students analyze Catalina's method and the other pair of students analyze James's method. Then, have each pair teach the other pair their method.
- Stress the efficiency of James's thinking and the power of the slope formula. The slope formula isn't just for calculating the slope of a line, it can also be used to write the equation of a line. Knowing and using the slope formula to write the equation of a line, whether a point and slope or two points are provided, eliminates the need for memorizing different formulas and procedures. (Actually, students may recognize how James's work is connected to the point-slope form of a line.)

Just in Time Support

- Encourage students to graph the given information prior to analyzing the strategies to make better sense of each method.

COMMON MISCONCEPTION

- Students may have difficulty understanding Catalina's method because they are accustomed to performing a translation rather than reversing the process and using algebra to determine what translation occurred. A discussion and diagram making this explicit may be beneficial.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why did Catalina substitute the value of x into her equation? • How was the value of y when $x = 2$ helpful to Catalina? • How did Catalina know the translation was up 5 units rather than down 5 units?
Reflecting and justifying	• Why did James use both the x-value and the y-value in his substitution step, but Catalina only used the x-value in her substitution step?
Seeing structure	• How did you get the slope of your equation? • How did you use the given point to write the equation of the line?

Summary

A line and its translation are parallel to one another.

STAMP THE LEARNING

ACTIVITY 1.3

Vertical Dilations of Functions

Facilitation Notes

In this activity, students graph vertical dilations of linear functions using tables of values. They then identify the slopes and y-intercepts of the two functions and write the equation for each transformed function in slope-intercept form. Students generalize that the *a*-value in the function transformation form, $a \cdot f(x)$, stretches or compresses the graph of the function. When *a* is negative, the *a*-value reflects the function across the *x*-axis.

Discuss the difference between a translation and a dilation, then complete Question 1 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why is this transformation called a *vertical dilation*?
Probing	• How did you complete the table? • How is the graph affected by the dilation by a factor of 4? • How do you think the graph would be affected by a factor of 10? • How did you determine the slope? The *y*-intercept?
Seeing structure	• How can you tell from the equation that a dilation occurred?

Have students work with a partner or in a group to complete Questions 2 through 4. Share responses as a class.

COMMON MISCONCEPTION

- Students may look at a graph that is vertically stretched and see it as horizontally compressed. Address the fact that without more information, the graph could be described both ways; however, the function provides additional information. The *a*-value deals with vertical movement. Therefore, the *y*-values are affected by the transformation, while the *x*-values stay constant. Have students take notes with sketches for a graph that is vertically stretched and a graph that is vertically compressed. Horizontal transformations will be addressed in the next lesson and with exponential and quadratic functions.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why isn't the transformed line as steep as the original line? • What is the slope of a line less steep than $b(x)$? • What is the slope of a line that lies between $f(x)$ and $b(x)$? • Why did the y-intercept remain at the origin? • Describe two ways the graph of $c(x)$ is different from the graph of $f(x)$? • How does the factor of 4 affect the graph? • How does multiplication by a negative number affect the graph? • What does a vertical stretch look like? • Why does the name vertical compression make sense? • How can you tell that the reflection is across the x-axis rather than the y-axis?

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

For the parent function $f(x) = x$, the transformed function, $a \cdot f(x)$, is a vertical dilation of the function. When a is negative, the function is reflected across the x-axis.

ACTIVITY 1.4 Vertical Dilations and Vertical Translations of Functions

Facilitation Notes

In this activity, students graph functions of the form $a \cdot f(x) + d$, which are vertical translations and vertical dilations of the original functions. They conclude that the a-value affects the slope of the function, while the d-value affects the y-intercept.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Students who create tables and students who rename points from the original line to the transformed line mentally without creating tables. If students perform the latter method, you may want to have them label their points on the graph.

DIFFERENTIATION STRATEGY

Access for All

- To build a foundation for transformations, graph lines by transforming individual points rather than by using the slope and y-intercept. This approach emphasizes the underpinnings of transformations and helps students to avoid confusion when dealing with horizontal transformations later in the course. Future topics refer to these points as *reference points*.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you rewrite the function in general form? • How did you graph the line? Explain your process. • Is the transformed graph a vertical stretch or compression? How can you tell from the equation and graph? • Which ordered pair in the transformed function corresponds to (1, 1) in the original function? • How is the graph affected by an a-value greater than 1? An a-value between 0 and 1? A negative a-value? • How can you describe the effects using the terms *vertical stretch* and *vertical compression*? • How is the graph affected by a d-value greater than 1? A d-value less than 1?

STAMP THE LEARNING

Summary

When a function is both translated and stretched vertically, the resulting function can be written in the form $a \cdot f(x) + d$, where d represents the vertical translation of $f(x)$ and a represents the vertical dilation of $f(x)$.

ACTIVITY 1.5

Applying Linear Function Transformations

Facilitation Notes

In this activity, students apply their knowledge of transformations in the context of testing a video game. Students are asked first to transform the default function $f(x) = x$ and consider restricted domains. They write equations for transformed functions assuming (a) the cannon remains fixed at the origin, so only a change in a-value will generate lines that will hit the targets or (b) the cannon's angle remains fixed on the line $f(x) = x$, so only a change in d-value will generate lines that will hit the targets. There are no precise correct answers.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Students choosing to use all constant functions to hit the targets. Under this circumstance, the *a*-values would be 0, and the *d*-values would be the *y*-intercepts of the lines.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Have students begin with Version C and focus on the *a*-value since the cannon stays at the origin. As students move on to Version B, suggest they decide whether they want *a* to be 1 or −1 before they attempt to determine the *d*-value.

Challenge Opportunity

- Have students determine two ways to hit each target: one using a positive *a*-value and one using *a* negative a-value.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How can you write an equation to change the cannon's angle? The cannon's location on the *y*-axis?
Gathering	• What are the approximate coordinates of the target in Quadrant I?
Probing	• How could you hit the targets with horizontal lines? • For Version B, why is the *a*-value the same every time? • For Version B, how did you identify the *d*-value that would hit the target? • When using Version C, how did you know when you needed a negative *a*-value?
Seeing structure	• Which version of the game uses only a translation? • Which version of the game uses only a dilation?

Summary

STAMP THE LEARNING

Linear function transformations are used in real-life situations.

DEMONSTRATE

Talk the Talk

FUNCTION MATCHING

Facilitation Notes

In this activity, students are given the graphs of four function transformations. They choose from a list of four equations, written in terms of the original function, to match to each graph.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which characteristic of the line did you use to identify it? • Which two lines included a vertical dilation? How can you tell which *a*-value applies to each line? • Which two lines included a vertical translation? • How can you tell which *d*-value applies to each line?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

An equation can be generated given the graph of a function transformation.

1 Transforming Linear Functions

Setting the Stage

- Communicate the objectives and the new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Determine the effects on the graph of a linear function when $f(x)$ is replaced by $f(x) + d$ or $a \cdot f(x)$.
- Graph linear function transformations expressed symbolically and show intercepts.
- Identify key characteristics of the graphs of linear functions, such as slope and y-intercept, in terms of quantities from a verbal description.
- Prove that a translated line and its pre-image have the same slope and are, therefore, parallel.
- Write equations of parallel lines.

NEW KEY TERM

- parent function

You have learned about linear functions and their characteristics, including slope and y-intercept.

How can you transform a function? What effects do different transformations have on the characteristics of linear functions?

Sample answer:

For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ shows a vertical translation of the function. For $d > 0$, the resulting graph vertically shifts up. For $d < 0$, the resulting graph vertically shifts down. The parent function and the resulting graph are parallel because they have the same slope but different y-intercepts.

The transformed function $y = af(x)$ shows a vertical dilation of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph is vertically stretched or compressed and is reflected across the x-axis.

EB STUDENT TIP

For all proficiency levels

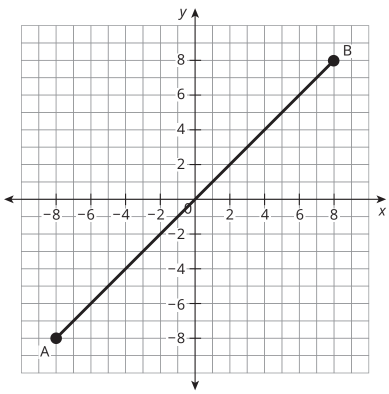
Discuss the meaning of the term *transformation* and compare it to its Spanish cognate, *transformación*. In this case, the prefix *trans-* means to *change thoroughly*. Mathematically, *transformations* are used to change the form of an equation and its graph.

Chunking the Activity

- Read and discuss the directions.
- Complete the activity as a class.
- Share and summarize.

Getting Started

Returning to Transformation Station



Consider $\overline{AB}$ with coordinates $A(-8, -8)$ and $B(8, 8)$.

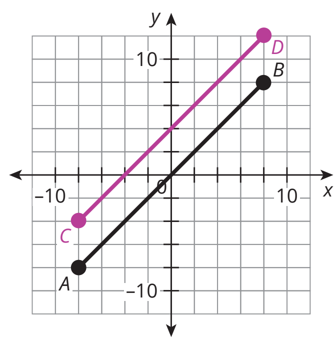
Follow your teacher's instructions to copy this line segment on a coordinate plane on the floor of your classroom, with different students standing at different points of the line segment.

1. Move on the coordinate plane to translate the entire line segment up 4 units and then down 4 units. Describe how the student "points" move.
   The points of the figure move straight up or straight down for a vertical translation. There is no movement left or right.

2. How do the translations affect the coordinates of the figure?

Original Point	4 Units Up	4 Units Down
(x, y)	$(x, y + 4)$	$(x, y - 4)$

3. Draw $\overline{CD}$ on the coordinate plane so that it is a vertical translation of $\overline{AB}$ up 4 units. Compare the two line segments. Describe how they are related.



The segments are parallel, with $\overline{CD}$ lying 4 units above $\overline{AB}$.

EB STUDENT TIP

For all proficiency levels

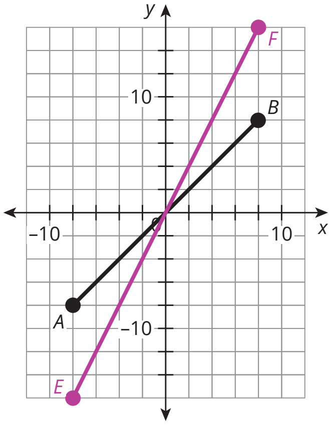
Discuss the mathematical meaning of the term *translate* and compare it to its Spanish cognate, *trasladar*. Students may be familiar with *translating* a word from one language to another, and while *trasladar* can also be used in this sense, it is very formal and many Spanish-speaking students may not be familiar with this usage. The much more common Spanish word for translating languages is *traducir*. Mathematically, *translations* involve changing the location of a shape from one location to another by a sliding motion where it retains its original orientation. Translations are a type of transformation.

270 MODULE 2 • TOPIC 2 • LESSON 1

4. Move back to where you started. Then, multiply all the *y*-coordinates by 2 and then by −2. Describe how the student “points” move.

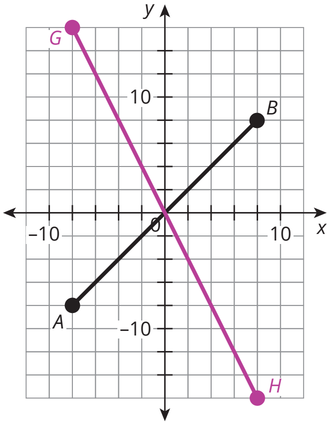
   The movement of points causes the line to stretch vertically or be steeper. When the factor is negative, the points also reflect across the *x*-axis.

5. Draw $\overline{EF}$ on the coordinate plane by multiplying all the *y*-coordinates of $\overline{AB}$ by 2. Compare the two line segments. Describe how they are related.



   The image is steeper than the original.

6. Draw $\overline{GH}$ on the coordinate plane by multiplying all the *y*-coordinates of $\overline{AB}$ by −2. Compare the two line segments. Describe how they are related.



   The image is steeper than the original. It is also reflected across the *x*-axis.

MODULE 2 • TOPIC 2 • LESSON 1 271

Chunking the Activity

- Read and discuss the introduction and Worked Example.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Question 4.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definition and Worked Example provide an opportunity for explicit instruction. Interact with this information as a class, and encourage students to restate or explain the information in their own words

Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

ACTIVITY 1.1 Vertical Translations of Functions

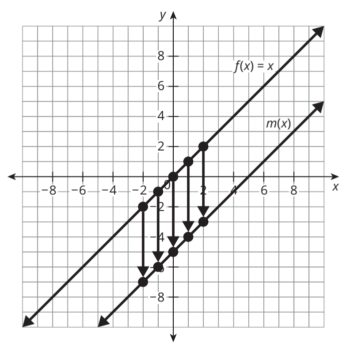
Let's determine how translations impact the graph of the linear function $f(x) = x$. The function $f(x) = x$ is the *parent function* for the linear function family. A **parent function** is the simplest function of its type.

For the family of linear functions having the general form $f(x) = ax + b$, where a and b are any real numbers, the function $f(x) = x$, where $a = 1$ and $b = 0$, is the parent function. It is the simplest linear function.

WORKED EXAMPLE

You can translate the graph of $f(x)$ down 5 units by moving each point 5 units down. The transformed graph is labeled as $m(x)$.

To translate the point $(-2, -2)$ on $f(x)$, subtract 5 units from the output value, or y-value. The input value, or x-value, remains unchanged. The coordinates of the translated point on $m(x)$ are $(-2, -7)$. The coordinates of four additional points on $f(x)$ are translated for you.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$m(x)$
−2	−7
−1	−6
0	−5
1	−4
2	−3

1. Consider the translated function, $m(x)$. Identify the slope and y-intercept of the graph of the function. Then, write the equation for the function in general form.

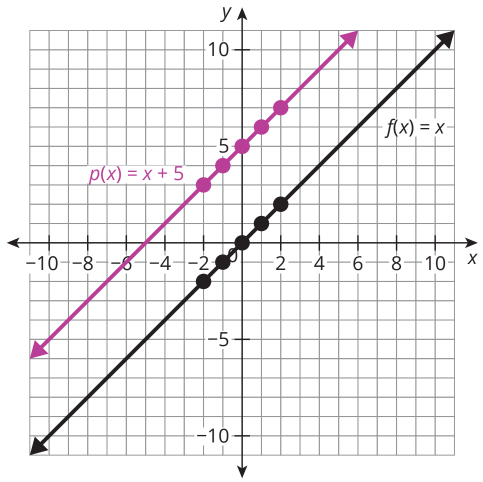
   Slope: 1, y-intercept: −5

   $m(x) = 1x + (-5)$ or $m(x) = x - 5$

272 MODULE 2 • TOPIC 2 • LESSON 1

2. Translate $f(x)$ again to create a new function, $p(x)$.

a. Translate the graph of $f(x)$ up 5 units. Label your graph as $p(x)$. Complete the table of corresponding points on $p(x)$.



Original Graph		Transformed Graph	
x	$f(x)$	x	$p(x)$
−2	−2	−2	3
−1	−1	−1	4
0	0	0	5
1	1	1	6
2	2	2	7

b. Identify the slope and y-intercept of the graph of the function. Then, write the equation for the function in general form.

Slope: 1, y-intercept: 5

$p(x) = 1x + 5$ or $p(x) = x + 5$

For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ shows a vertical translation of the function. This translation affects the output values, or y-values, of the function. For $d > 0$, the resulting graph vertically shifts up. For $d < 0$, the resulting graph vertically shifts down. The distance the graph is shifted is the absolute value of d, or $|d|$.

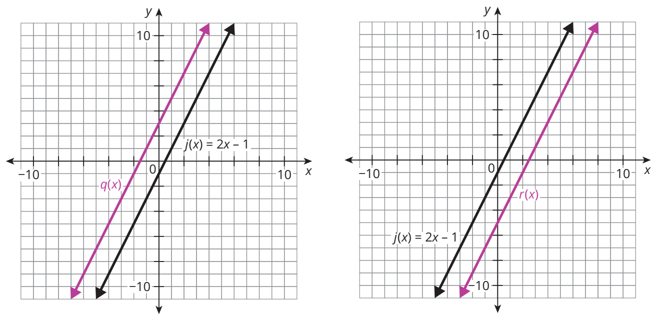
3. Compare the values of $f(2)$ and $p(2)$. How did the transformation of the function affect the value of the function at $x = 2$?

$f(2) = 2$, $p(2) = 7$

The vertical translation increased the value of the function at $x = 2$ by 5.

MODULE 2 • TOPIC 2 • LESSON 1 273

4. Consider the graph of $j(x) = 2x - 1$.



a. Translate the graph of $j(x)$ up 4 units. Label the graph as $q(x)$. Then, write an equation for $q(x)$ in terms of $j(x)$.

$q(x) = j(x) + 4$

b. Translate the graph of $j(x)$ down 4 units. Label the graph as $r(x)$. Then, write an equation for $r(x)$ in terms of $j(x)$.

$r(x) = j(x) - 4$

c. Rewrite $q(x)$ and $r(x)$ in general form.

$q(x) = 2x + 3$

$r(x) = 2x - 5$

Ask Yourself . . .
Will the graphs of $j(x)$, $q(x)$, and $r(x)$ ever intersect?

d. Compare the equations and graphs of $j(x)$, $q(x)$, and $r(x)$. What do you notice?

Sample answer:
The lines have the same slope; they are parallel.

274 MODULE 2 • TOPIC 2 • LESSON 1

ACTIVITY 1.2 Slopes of Parallel Lines

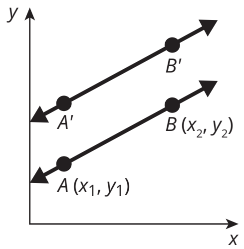
In the previous activity you translated the function $j(x) = 2x - 1$ up 4 units to create $q(x) = 2x + 3$.

1. Will the graphs of $j(x)$ and $q(x)$ ever intersect? Explain your reasoning.

   No. The graphs of the functions will never intersect because they will always remain 4 units apart.

You can algebraically prove that a line and its translation are parallel to each other.

2. Line AB was translated a units up to create line $A'B'$.



   a. Identify the x- and y-coordinates of each corresponding point on the image.

   $A' = (x_1, y_1 + a)$

   $B' = (x_2, y_2 + a)$

   b. Use the slope formula to calculate the slope of the pre-image.

   Slope of pre-image $= \dfrac{y_2 - y_1}{x_2 - x_1}$

   c. Use the slope formula to calculate the slope of the image.

   Slope of image $= \dfrac{(y_2 + a) - (y_1 + a)}{x_2 - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Group students to complete Questions 3–4.
- Share and summarize.

MODULE 2 • TOPIC 2 • LESSON 1 275

Activity 1.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing the equations of lines that are parallel to a given line that passes through a point, assign Skills Practice Set A for this lesson.

Ask Yourself . . .
The original figure is the pre-image and the transformation of the figure is the image.

d. How does the slope of the image compare to the slope of the pre-image?

The slopes of the pre-image and image are the same.

e. How would you describe the relationship between the graph of the image and the graph of the pre-image?

The graphs of the pre-image and image are parallel lines.

Catalina and James each write the equation of a line that is parallel to $y = \frac{1}{2}x + 3$ and passes through the point (2, 9).

Catalina

I know that since the lines are parallel, the new line is a translation of $y = \frac{1}{2}x + 3$. I can substitute the x-value from (2, 9) into $y = \frac{1}{2}x + 3$ to determine the corresponding point on the pre-image.

$$y = \frac{1}{2}(2) + 3 = 4$$

The point that corresponds to the point (2, 9) on $y = \frac{1}{2}x + 3$ is (2, 4). Going from (2, 4) to (2, 9) is a translation up five units.

Since

$y = \left(\frac{1}{2}x + 3\right) + 5 = \frac{1}{2}x + 8$, the equation of the parallel line is $y = \frac{1}{2}x + 8$.

James

To write the equation of the line, all I need to know is the slope formula,

$m = \frac{y_2 - y_1}{x_2 - x_1}$.

I used $m = \frac{1}{2}$ and (2, 9) for (x, y).

$\frac{1}{2} = \frac{y - 9}{x - 2}$.

Then, I rearranged the equation to write it in general form.

$$2(y - 9) = x - 2$$
$$y - 9 = \frac{1}{2}(x - 2)$$
$$y = \frac{1}{2}(x - 2) + 9$$
$$y = \frac{1}{2}x - 1 + 9$$
$$y = \frac{1}{2}x + 8$$

3. Which student's method do you prefer and why?

Answers will vary based on each classroom.

4. Write the equation of a line that is parallel to $y = -3x - 1$ and passes through the point (−1, 5).

$y = -3x + 2$

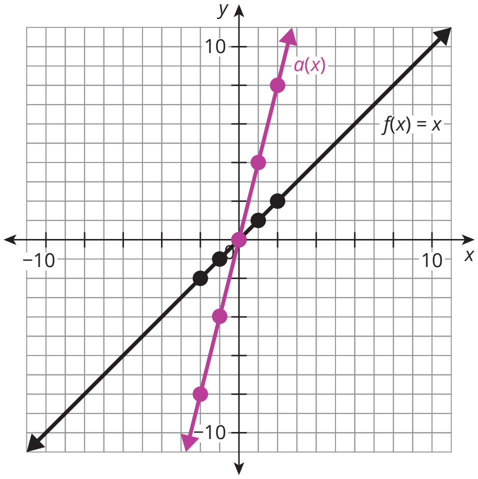
276 MODULE 2 • TOPIC 2 • LESSON 1

ACTIVITY 1.3 Vertical Dilations of Functions

In this activity, let's consider how dilations impact the graph of the linear function $f(x) = x$.

1. Suppose the output values of $f(x)$ are changed by a factor of 4 to create $a(x)$.

   a. Sketch the graph of $a(x)$ and complete the table of values.



Original Graph		Transformed Graph	
x	$f(x)$	x	$a(x)$
−2	−2	−2	−8
−1	−1	−1	−4
0	0	0	0
1	1	1	4
2	2	2	8

   b. Identify the slope and y-intercept of the function $a(x)$. Then, write the equation for the function $a(x)$ in general form.

   Slope: 4, y-intercept: 0

   $a(x) = 4x + 0$ or $a(x) = 4x$

Chunking the Activity

- Read and discuss the introduction.
- Complete Question 1 as a class.
- Group students to complete Questions 2–4.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

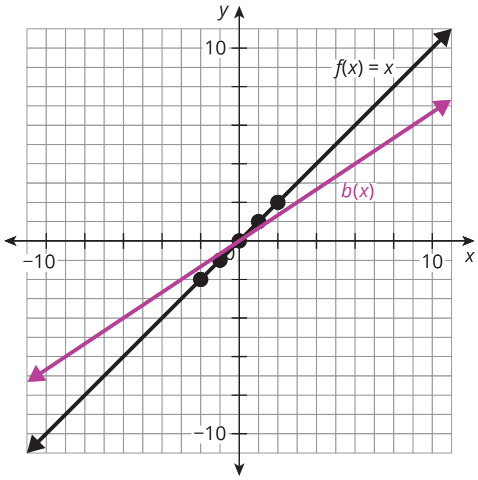
EB STUDENT TIP

For all proficiency levels

Provide the term *dilation* as another type of transformation and compare it to its Spanish cognate, *dilatar*. Students may be familiar with the non-mathematical use of *dilate*, where the pupils of their eyes become larger in the dark. Mathematically, *dilate* means to *make a figure larger or smaller while retaining its shape.*

2. Suppose the output values of $f(x)$ are changed by a factor of $\frac{2}{3}$ to create $b(x)$.

a. Sketch the graph of $b(x)$ and complete the table of values.



Original Graph		Transformed Graph	
x	$f(x)$	x	$b(x)$
−2	−2	−2	$-\frac{4}{3}$
−1	−1	−1	$-\frac{2}{3}$
0	0	0	0
1	1	1	$\frac{2}{3}$
2	2	2	$\frac{4}{3}$

b. Identify the slope and y-intercept of the function $b(x)$. Then, write the equation for the function $b(x)$ in general form.

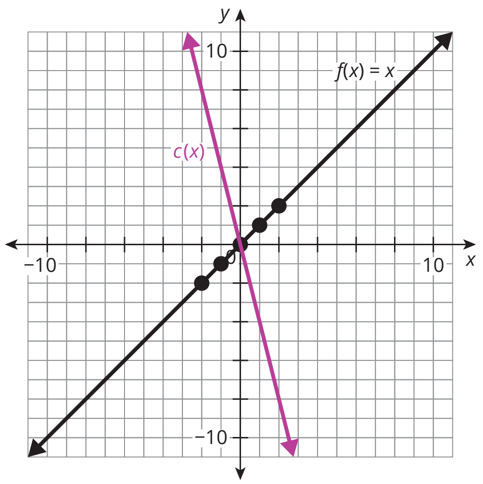
Slope: $\frac{2}{3}$, y-intercept: 0

$b(x) = \frac{2}{3}x + 0$ or $b(x) = \frac{2}{3}x$

278 MODULE 2 • TOPIC 2 • LESSON 1

3. Suppose the output values of $f(x)$ are changed by a factor of -4 to create $c(x)$.

   a. Sketch the graph of $c(x)$ and complete the table of values.



Original Graph		Transformed Graph	
x	$f(x)$	x	$c(x)$
−2	−2	−2	8
−1	−1	−1	4
0	0	0	0
1	1	1	−4
2	2	2	−8

   b. Identify the slope and y-intercept of the function $c(x)$. Then, write the equation for the function $c(x)$ in general form.

   Slope: -4, y-intercept: 0

   $c(x) = -4x + 0$ or $c(x) = -4x$

For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ shows a vertical dilation of the function. This dilation affects the output values, or y-values, of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph is vertically stretched or compressed and is reflected across the x-axis.

4. Compare the values $f(-1)$ and $c(-1)$. How did the transformation of the function affect the value of the function at $x = -1$?

   $f(-1) = -1$, $c(-1) = 4$; the vertical dilation multiplied the value of the function at $x = -1$ by -4

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MODULE 2 • TOPIC 2 • LESSON 1 279

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the directions.
- Group students to complete Questions 1 and 2.
- Share and summarize.

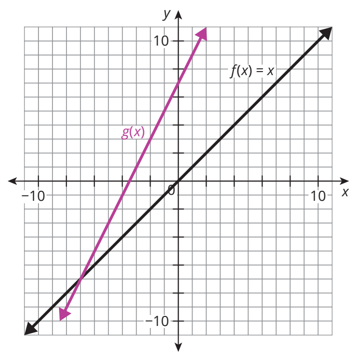
Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing transformed linear functions and describing the transformations, assign Skills Practice Set B for this lesson.

ACTIVITY 1.4 Vertical Dilations and Vertical Translations of Functions

Let's consider more transformations of the parent function $f(x) = x$.

1. Describe the transformations performed on $f(x)$ to produce $g(x)$. Then, graph $g(x)$. Write the function equation in general form.

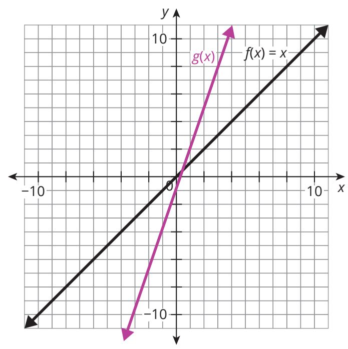
a. $g(x) = 2 \cdot f(x) + 7$



$g(x) = 2x + 7$

The function is vertically dilated by a factor of 2 and is translated up 7 units.

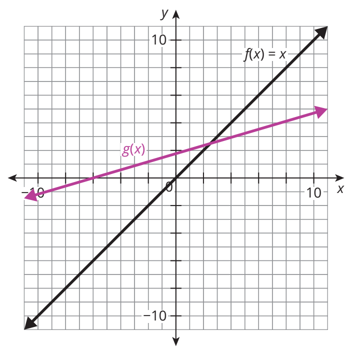
b. $g(x) = 3 \cdot f(x) - 1$



$g(x) = 3x - 1$

The function is vertically dilated by a factor of 3 and is translated down 1 unit.

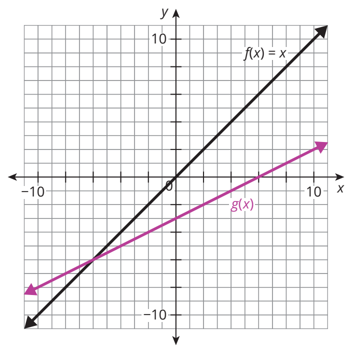
c. $g(x) = \frac{1}{3} \cdot f(x) + 2$



$g(x) = \frac{1}{3}x + 2$

The function is vertically dilated by a factor of $\frac{1}{3}$ and is translated up 2 units.

d. $g(x) = \frac{1}{2} \cdot f(x) - 3$

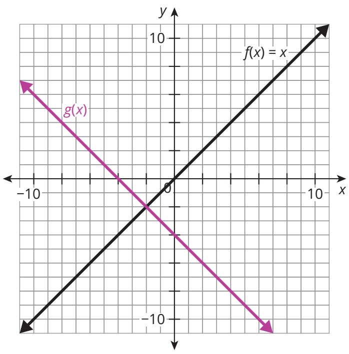


$g(x) = \frac{1}{2}x - 3$

The function is vertically dilated by a factor of $\frac{1}{2}$ and is translated down 3 units.

280 MODULE 2 • TOPIC 2 • LESSON 1

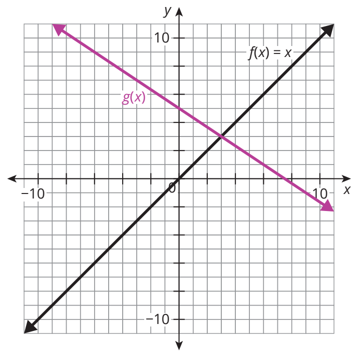
e. $g(x) = -1 \cdot f(x) - 4$



$g(x) = -1x - 4$

The function is reflected across the x-axis and is translated down 4 units.

f. $g(x) = -\frac{2}{3} \cdot f(x) + 5$



$g(x) = -\frac{2}{3}x + 5$

The function is reflected across the x-axis, dilated by a factor of $\frac{2}{3}$, and translated up 5 units.

Question 2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing equations for transformed functions, assign Skills Practice Set C for this lesson

When a function is both translated and stretched vertically, the resulting function can be written in the form $a \cdot f(x) + d$, where d represents the vertical translation of $f(x)$, and a represents the vertical dilation of $f(x)$.

2. Consider the function $g(x) = a \cdot f(x) + d$.

a. How does changing the a-value affect the slope of the function? The y-intercept of the function?

Changing the a-value does not affect the y-intercept of the function, but it changes the slope. If it is greater than 1 or less than −1, the slope increases; if it is between −1 and 1, the slope decreases. If it is exactly 1, the slope doesn't change; if it is exactly −1, the slope becomes the opposite of the original slope.

b. How does changing the d-value affect the slope of the function? The y-intercept of the function?

Changing the d-value does not affect the slope of the function, but it changes the y-intercept. If d is positive, the y-intercept increases; if d is negative, the y-intercept decreases.

MODULE 2 • TOPIC 2 • LESSON 1 281

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The situation provides an opportunity for explicit instruction. Interact with this information as a class, and encourage students to restate or explain the information in their own words.

Modeling Moment

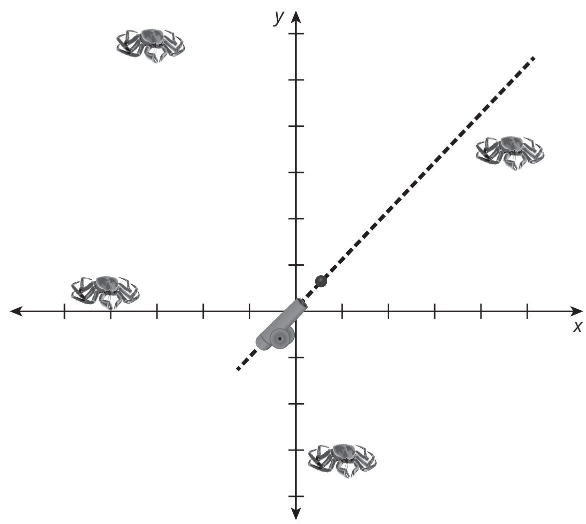
- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Questions 2 and 3, have students work individually or with a partner to complete the graphic organizer.
- See the Facilitating Productive Struggle section in the Course and Implementation Guide for additional guidance on supporting students through problem-solving activities.

PROBLEM SOLVING

ACTIVITY 1.5 Applying Linear Function Transformations

Your company is developing a new video game for kids, which involves scoring points by shooting targets with a cannon. The cannon starts at the origin of a coordinate system as shown.

By default it shoots along the line $f(x) = x$. Players can move the cannon up and down the y-axis and also change the angle of the cannon. When a player shoots the cannon, the game program determines the values of a and d for the linear function $a \cdot f(x) + d$. If the target is on that line, the program will show an animation of the cannonball hitting the target.



Ask Yourself . . .
The cannon can shoot in only one direction. Does that affect the domain or range of the linear function?

1. Test the program. Determine values for a and d for four linear functions that should hit the targets shown.

   Sample answers:

   $a = 0$ and $d = 6$; $a = 0$ and $d = 4$; $a = 0$ and $d = 1$; $a = 0$ and $d = -3$

EB STUDENT TIP

For all proficiency levels

Beginning: Review the terms *domain* and *range* along with their Spanish cognates *dominio* and *rango*. Connect these terms to the concepts of *input* and *output* by using a familiar example, such as a vending machine, where the domain is the set of buttons (*input*) and the range is the set of possible snacks (*output*) that come out. Explain that sometimes a domain is *restricted* and only specific inputs will work, just as vending machines will only accept specific inputs based on what is available in the machine.

(continued on next page)

282 MODULE 2 • TOPIC 2 • LESSON 1

2. Identify the domain on which each graph hits the target.

   For the upper left target in Quadrant II, the domain for the linear function is the set of numbers less than or equal to 0, $x \le 0$; for the lower target in Quadrant II, the domain is the set of numbers less than or equal to 0, $x \le 0$; for the target in Quadrant I, the domain is the set of numbers greater than or equal to 0; for the target in Quadrant IV, the domain is the set of numbers greater than or equal to 0.

The game developer is also testing two other versions of the game, with different abilities for the cannon.

Version B	**Version C**
The cannon can only shoot along a line parallel to $f(x) = x$ or $f(x) = -x$. It can move up and down the y-axis.	The cannon can change angles but remains fixed at the origin.

3. Determine values for a and d, along with their corresponding domains, for four linear functions that can hit the targets in each version of the game. Show your work.

   For Version B:
   Sample answers:
   (clockwise, from top left)
   a: -1, d: 2 (domain: $x \le 0$)
   or a: 1, d: 10 (domain: $x \le 0$);
   a: 1, d: -1 (domain: $x \ge 0$)
   or a: -1, d: 9 (domain: $x \ge 0$);
   a: -1, d: -2 (domain: $x \ge 0$) or a: 1, d: -4 (domain: $x \ge 0$);
   a: -1, d: -4 (domain: $x \le 0$) or a: 1, d: 4 (domain: $x \le 0$)

   For Version C:
   Sample answers:
   (clockwise, from top left)
   a: $-\frac{6}{4}$, d: 0 (domain: $x \le 0$);
   a: $\frac{4}{5}$, d: 0 (domain: $x \ge 0$);
   a: -3, d: 0 (domain: $x \ge 0$);
   a: $-\frac{1}{5}$, d: 0 (domain: $x \le 0$)

EB STUDENT TIP ***(continued)***

Intermediate: Review the terms as described for the Beginning proficiency level, but extend this by having students practice using the term *domain* with the sentence frame "The *domain*, or acceptable *inputs* I can use, are ________."

Advanced/Advanced High: Review the terms as described in the previous sections, but further challenge students to think of other examples beyond a video game where using a *restricted domain* would make sense. Have students write descriptions of these examples, using the terms *domain*, *restricted*, and *range*, optionally having them sketch graphs or visuals that correspond to their descriptions.

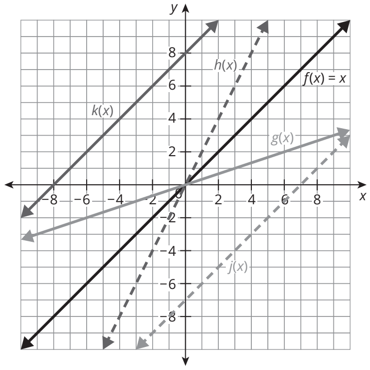
Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Function Matching

The graph shows the linear function $f(x) = x$ and four transformations of $f(x)$.



Transformations
$\frac{1}{3} \cdot f(x)$
$2 \cdot f(x)$
$f(x) - 7$
$f(x) + 8$

Match each transformed graph to one of the transformations in the table. Explain your reasoning.

Ask Yourself . . .
Did you justify your mathematical reasoning?

1. $k(x) =$ $f(x) + 8$

   Sample explanation: $k(x)$ is the graph of $f(x)$ transformed up 8 units.

2. $h(x) =$ $2 \cdot f(x)$

   Sample explanation: $h(x)$ is the graph of $f(x)$ stretched vertically by a factor of 2.

3. $g(x) =$ $\frac{1}{3} \cdot f(x)$

   Sample explanation: $g(x)$ is the graph of $f(x)$ compressed vertically by a factor of $\frac{1}{3}$.

4. $j(x) =$ $f(x) - 7$

   Sample explanation: $j(x)$ is the graph of $f(x)$ transformed down 7 units.

284 MODULE 2 • TOPIC 2 • LESSON 1

Lesson 1 Assignment

Write

Describe the term *parent function* in the context of transformations using your own words.

Remember

For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ shows a vertical translation of the function. For $d > 0$, the resulting graph vertically shifts up. For $d < 0$, the resulting graph vertically shifts down. The parent function and the resulting graph are parallel because they have the same slope but different y-intercepts.

The transformed function $y = af(x)$ shows a vertical dilation of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph is vertically stretched or compressed and is reflected across the x-axis.

Write

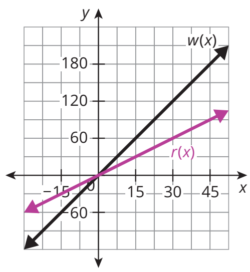
Sample answer:

A parent function is the simplest function of its type before any transformations.

Practice

1. Given $w(x) = 4x$.

   a. Graph $r(x) = \frac{1}{2} \cdot w(x)$. Then, complete the table of corresponding points on $r(x)$.



x	$w(x)$	$r(x)$
0	0	0
15	60	30
30	120	60
45	180	90

   b. Describe the transformation performed on $w(x)$ to produce $r(x)$.

   The graph of $w(x)$ is vertically compressed by a factor of $\frac{1}{2}$ to produce $r(x)$.

   c. Write the equation for the function $r(x)$ in general form.

   $r(x) = 2x$

2. Write the equation of a line parallel to the line $y = 2x$ that passes through the given points.

   a. (0, 4) $y = 2x + 4$

   b. (−2, −1) $y = 2x + 3$

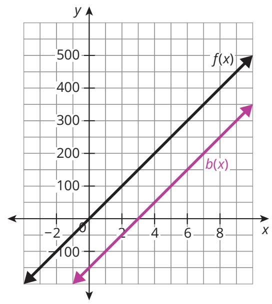
   c. (2, 0) $y = 2x - 4$

MODULE 2 • TOPIC 2 • LESSON 1 ASSIGNMENT 285

Lesson 1 Assignment

3. Given $f(x) = 50x$.

   a. Graph $b(x) = f(x) - 150$. Then, complete the table of corresponding points on $b(x)$.



x	f(x)	b(x)
2	100	−50
4	200	50
6	300	150
8	400	250

   b. Write the equation for the function $b(x)$ in general form.

   $b(x) = 50x - 150$

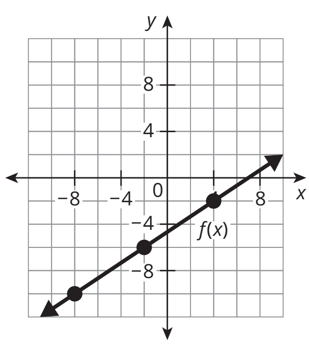
   c. Describe the transformation performed on $f(x)$ to produce $b(x)$.

   The graph of $f(x)$ is shifted down 150 units to produce $b(x)$.

Prepare

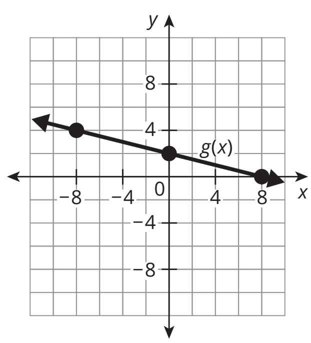
Write a linear function for each graph.

1.



$f(x) = \frac{2}{3}x - 5$

2.



$g(x) = -\frac{1}{4}x + 2$

286 MODULE 2 • TOPIC 2 • LESSON 1 ASSIGNMENT

2 Vertical and Horizontal Transformations of Linear Functions

MATERIALS

None

LESSON OVERVIEW

Students will identify key characteristics of several linear functions. A graph and a table of values for the parent linear function $f(x) = x$ is given, and students will translate this function horizontally and vertically to determine which transformations affect the input and output values. They will also dilate the function $f(x) = x$ horizontally and vertically. Students will generalize about equivalent translations for the parent function $f(x) = x$ and determine that these relationships do not hold true for all linear functions. Given a function $g(x)$ in terms of $f(x)$, students will graph $g(x)$ and describe each transformation on $f(x)$ to produce $g(x)$.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

(TEKS continued on next page)

ELPS

(1) Learning Strategies
The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(2) Listening
The student is expected to:

(D) monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed.

(3) Speaking
The student is expected to:

(C) speak using a variety of grammatical structures, sentence lengths, sentence types, and connecting words with increasing accuracy and ease as more English is acquired.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

- **A.3C** graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems.
- **A.3E** determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.

ESSENTIAL IDEAS

- For the parent function $f(x) = x$, a horizontal translation right n units is equivalent to a vertical translation down n units, and a horizontal translation left n units is equivalent to a vertical translation up n units.
- For the parent function $f(x) = x$, the transformed function $y = f(x - c)$ affects the input values of the function. For $c > 0$, the graph horizontally shifts right c units. For $c < 0$, the graph horizontally shifts left c units.
- For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values of the function. For $d > 0$, the graph vertically shifts up d units. For $d < 0$, the graph vertically shifts down c units.
- For the parent function $f(x) = x$, a horizontal dilation by a factor of n units is equivalent to a vertical dilation by a factor of $\frac{1}{n}$ units.
- For the parent function $f(x) = x$, the transformed function $y = af(x)$ affects the output values of the function. For $|a| > 1$, the graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the graph vertically compresses by a factor of a units. For $a < 0$, the graph reflects across the x-axis.
- For the parent function $f(x) = x$, the transformed function $y = f(bx)$ affects the input values of the function. For $|b| > 1$, the graph horizontally compresses by a factor of $\frac{1}{|b|}$ units. For $0 < |b| < 1$, the graph horizontally stretches by a factor of $\frac{1}{|b|}$ units. For $b < 0$, the graph reflects across the y-axis.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Hey, I Remember This! 5–10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students graph linear equations and identify key characteristics of each graph, including domain, range, *x*-intercept/zero, *y*-intercept, slope, and intervals of increase and decrease.

DEVELOP

Activity 2.1: Comparing Vertical and Horizontal Translations 30–35 minutes

WORKED EXAMPLE, INVESTIGATION

The graph of the linear function $f(x) = x$ is provided and students translate the graph horizontally and vertically. They complete a table of values listing corresponding points for each translated function. Students compare the tables of values to determine whether the input or output values of $f(x) = x$ are affected by each translation. They generalize about equivalent horizontal and vertical translations for the parent function $f(x) = x$. Students then translate the graph of the linear function $j(x) = 2x - 1$ horizontally and vertically. They write equations for each translated graph. Students compare their equations to determine whether horizontal and vertical translations are equivalent for any linear function.

DAY 2

Activity 2.2: Comparing Vertical and Horizontal Dilations 15–20 minutes

INVESTIGATION

The graph of the linear function $f(x) = x$ is provided and students dilate the graph horizontally and vertically. They complete a table of values listing corresponding points for each translated function. Students compare the tables of values to determine the factor by which the input values and output values changed given a horizontal or vertical dilation. They generalize about equivalent horizontal and vertical dilations for the parent function $f(x) = x$. Students then determine whether there is a relationship between horizontal and vertical dilations for any linear function, providing a counterexample when necessary.

Activity 2.3: Graphing Linear Transformations 15–20 minutes

MATHEMATICAL PROBLEM SOLVING

The graph of the linear function $f(x) = x$ is provided. Students will analyze the equation of $f(x)$ and $g(x)$, describe the transformations performed on $f(x)$ to produce $g(x)$, then graph $g(x)$.

DEMONSTRATE

Talk the Talk: Show Me What You've Got! 5 minutes

EXIT TICKET PROCEDURE

The graph of four transformations of the parent function $f(x)$ are provided along with the equation for each transformation in terms of $f(x)$. Students match each transformed graph to its corresponding equation.

Getting Started

ENGAGE

Hey, I Remember This!

Facilitation Notes

In this activity, students graph linear equations and identify key characteristics of each graph, including domain, range, *x*-intercept/zero, *y*-intercept, slope, and intervals of increase and decrease.

Have students complete all parts of Questions 1 and 2 with a partner. Then have students share their responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Are the domain and range of $g(x)$ the same as $h(x)$? • How do you determine the *x*-intercept/zero? • How do you determine the *y*-intercept? • How do you determine the slope? • How do you determine whether the function is increasing or decreasing?

STAMP THE LEARNING

Summary

The graph and key characteristics of a linear function can be determined from the equation.

ACTIVITY 2.1 Comparing Vertical and Horizontal Translations

DEVELOP

Facilitation Notes

In this activity, the graph of the linear function $f(x) = x$ is provided, and students translate the graph horizontally and vertically. They complete a table of values listing corresponding points for each translated function. Students compare the tables of values to determine whether the input or output values of $f(x) = x$ are affected by each translation. They generalize about equivalent horizontal and vertical translations for the parent function $f(x) = x$. Students then translate the graph of the linear function $j(x) = 2x - 1$ horizontally and vertically. They write equations for each translated graph. Students compare their equations to determine whether horizontal and vertical translations are equivalent for any linear function.

Ask a student to read the Worked Example. Discuss as a class.

Have students complete Questions 1 through 5 with a partner. Then, have students share their responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Do horizontal translations of $f(x)$, such as $m(x)$ and $t(x)$, affect the domain? • Do horizontal translations of $f(x)$ affect the input values of the translated function? • Do vertical translations of $f(x)$ affect the output values of the translated function? • For the parent function $f(x)$, is a horizontal translation right 5 units equivalent to a vertical translation down 5 units? • For the parent function $f(x)$, is a vertical translation up 3 units equivalent to a horizontal translation left 3 units?

COMMON MISCONCEPTION

Materials Needed: Graphing Technology

- Students might have difficulty realizing that a horizontal translation right is equivalent to a vertical translation down and a horizontal translation left is equivalent to a vertical translation up. Use graphing technology to have students manipulate several graphs until they see the patterns.

Ask a student to read the information. Discuss as a class.

Have students complete Questions 6 through 9 with a partner. Then, have students share their responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Is $q(x)$ written as $q(x) = j(x) + 4$ or written as $q(x) = j(x + 4)$? • Is $r(x)$ written as $r(x) = j(x) + 4$ or written as $r(x) = j(x + 4)$? • Is $b(x)$ written as $b(x) = j(x - 2)$ or written as $b(x) = j(x) - 2$? • Is $v(x)$ written as $v(x) = j(x - 2)$ or written as $v(x) = j(x) - 2$? • For any linear function, is a vertical translation up 4 units equivalent to a horizontal translation left 4 units? • For any linear function, is a horizontal translation right 2 units equivalent to a vertical translation down 2 units?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

For the parent function $f(x) = x$, a horizontal translation right n units produces the same graph as a vertical translation down n units In addition, a horizontal translation left n units produces the same graph as a vertical translation up n units.

ACTIVITY 2.2 Comparing Vertical and Horizontal Dilations

Facilitation Notes

In this activity, the graph of the linear function $f(x) = x$ is provided and students dilate the graph horizontally and vertically. They complete a table of values listing corresponding points for each translated function. Students compare the tables of values to determine the factor by which the input values and output values changed given a horizontal or vertical dilation. They generalize about equivalent horizontal and vertical dilations for the parent function $f(x) = x$. Students then determine whether there is a relationship between horizontal and vertical dilations for any linear function, providing a counterexample when necessary.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students complete all parts of Questions 1 and 2 with a partner. Then, have students share their responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you change the output values of $a(x)$ by a factor of 2? • How do you change the input values of $b(x)$ by a factor of $\frac{1}{2}$? • How do you change the output values of $c(x)$ by a factor of $\frac{4}{5}$? • How do you change the input values of $d(x)$ by a factor of $\frac{5}{4}$?

DIFFERENTIATION STRATEGY

Just in Time Support

Materials Needed: Modeling Clay or Dough

- Use modeling clay or dough to have students visualize horizontally and vertically stretching and compressing an object. Relate this experience to stretching and compressing a function.

Ask a student to read the information. Discuss as a class.

Have students complete Question 3 with a partner. Then, have students share their responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Does this relationship between horizontal and vertical dilations hold true for all linear functions with a slope of 1? • Does this relationship between horizontal and vertical dilations hold true for all linear functions with a *y*-intercept of 0? • Does your counterexample include graphs, equations, or both?

Summary

STAMP THE LEARNING

For the parent function $f(x) = x$, changing the output values by a factor of n units is equivalent to changing the input values by a factor of $\frac{1}{n}$ units. This relationship between horizontal and vertical dilations is only true for the parent function $f(x) = x$.

ACTIVITY 2.3 Graphing Linear Transformations

Facilitation Notes

In this activity, the graph of the linear function $f(x) = x$ is provided. Students will analyze the equation of $f(x)$ and $g(x)$, describe the transformations performed on $f(x)$ to produce $g(x)$, then graph $g(x)$.

Have students complete Questions 1 through 6 with a partner. Then, have students share their responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Is the transformed function $g(x)$ translated horizontally? If so, by how many units? • Is the transformed function $g(x)$ translated vertically? If so, by how many units? • Is the transformed function $g(x)$ dilated horizontally? If so, by what factor? • Is the transformed function $g(x)$ dilated vertically? If so, by what factor?

Summary

STAMP THE LEARNING

Changing the output values of the parent function $f(x) = x$, affects the graph of the function vertically. Changes to the input values of the parent function $f(x) = x$, affects the graph of the function horizontally.

DEMONSTRATE

Talk the Talk

SHOW ME WHAT YOU'VE GOT!

Facilitation Notes

The graph of four transformations of the parent function $f(x)$ are provided along with the equation for each transformation in terms of $f(x)$. Students match each transformed graph to the equation.

Have students work with a partner or in groups to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• Which function equations affect the input values? • Which function equations affect the output values? • How do horizontal transformations affect the graph of the parent function $f(x)$?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Horizontal transformations of linear functions are caused by changes to the x-values, and vertical transformations of linear functions are caused by changes to the y-values.

2 Vertical and Horizontal Transformations of Linear Functions

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Identify key characteristics of linear functions.
- Determine the effects on the graph of a linear function when $f(x)$ is replaced by $f(x) + d$, $f(x - c)$, $af(x)$, and $f(bx)$.

You have learned about the effects of vertical transformations on linear functions.

What effect do horizontal transformations have on linear functions? How do they compare to vertical transformations?

Sample answer:

When $f(x)$ is replaced by $af(b(x - c)) + d$, b- and c- affect the input values of the function where as a- and d- affect the output values of the function.

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MODULE 2 • TOPIC 2 • LESSON 2 287

Chunking the Activity

- Read and discuss the directions.
- Complete the activity as a class.
- Share and summarize.

Getting Started

Hey, I Remember This!

Graph each linear function and describe the key characteristics of the graph.

1. $g(x) = 3x - 8$

   Domain: All real numbers

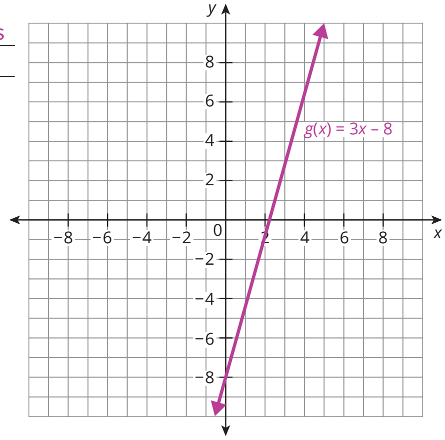
   Range: All real numbers

   x-intercept: $\left(\frac{8}{3}, 0\right)$

   y-intercept: $(0, -8)$

   Slope: 3

   Increasing/Decreasing: Increasing



2. $h(x) = -\frac{2}{3}x + 5$

   Domain: All real numbers

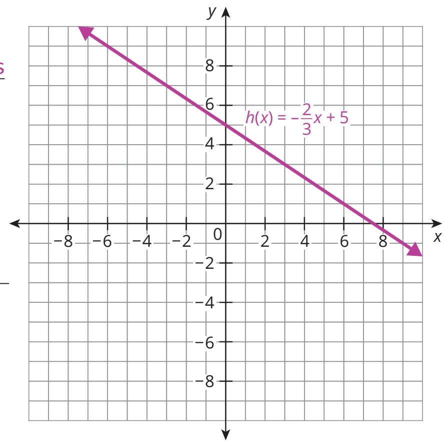
   Range: All real numbers

   x-intercept: $(7.5, 0)$

   y-intercept: $(0, 5)$

   Slope: $-\frac{2}{3}$

   Increasing/Decreasing: Decreasing



288 MODULE 2 • TOPIC 2 • LESSON 2

ACTIVITY
2.1

Comparing Vertical and Horizontal Translations

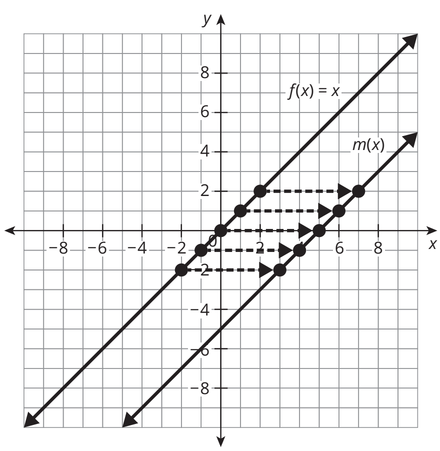
Let's determine how translations impact the graph of the linear function $f(x) = x$.

WORKED EXAMPLE

You can translate the graph of $f(x)$ to the right 5 units by moving each point 5 units to the right. The transformed graph is labeled as $m(x)$.

To translate the point $(-2, -2)$ on $f(x)$, add 5 units to the input value, or x-value. The output value, or y-value, remains unchanged. The coordinates of the translated point on $m(x)$ are $(3, -2)$.

The coordinates of four additional points on $f(x)$ are translated for you.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$m(x)$
3	−2
4	−1
5	0
6	1
7	2

Chunking the Activity

- Read and discuss the introduction and Worked Example.
- Group students to complete Questions 1–5.
- Check in and share.
- Read and discuss the information.
- Group students to complete Questions 6–9.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

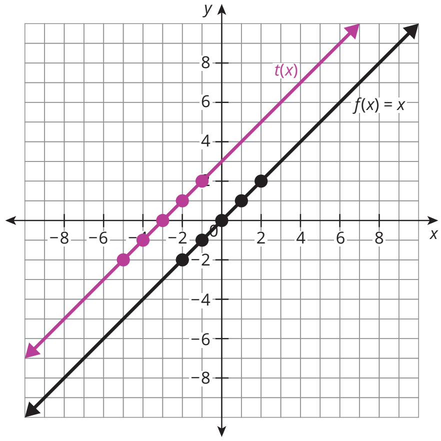
Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

MODULE 2 • TOPIC 2 • LESSON 2 289

1. Given the graph and a table of values for $f(x) = x$, complete the following transformations.

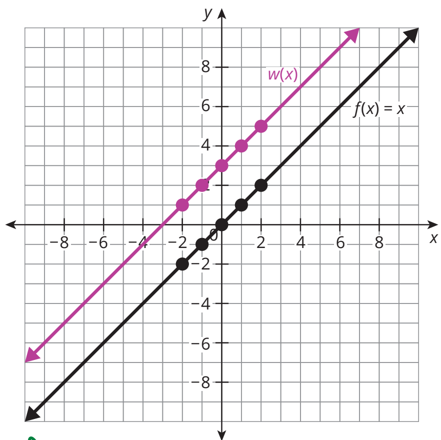
   a. Translate the graph of $f(x)$ left 3 units. Label your graph as $t(x)$. Complete the table of corresponding points on $t(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$t(x)$
−5	−2
−4	−1
−3	0
−2	1
−1	2

   b. Translate the graph of $f(x)$ up 3 units. Label your graph as $w(x)$. Complete the table of corresponding points on $w(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$t(x)$
−2	1
−1	2
0	3
1	4
2	5

Ask Yourself . . .
What observations can you make?

290 MODULE 2 • TOPIC 2 • LESSON 2

c. Compare the table of values for $f(x)$ and $t(x)$. Are the input values or output values affected by this horizontal translation?

The horizontal translation affects the input values. The output values remain the same.

d. Compare the table of values for $f(x)$ and $w(x)$. Are the input values or output values affected by this vertical translation?

The vertical translation affects the output values. The input values remain the same.

e. Compare the graphs and tables of values for $t(x)$ and $w(x)$. What do you notice?

Although the graphs of $t(x)$ and $w(x)$ are the same, their tables of values are different.

2. Which transformation(s) of the parent function $f(x) = x$ affect the input values of the corresponding points on the transformed function?

Horizontal translations of $f(x)$ affect the input values of the corresponding points on the transformed function. The output values of the corresponding points on the transformed function remain the same.

3. Which transformation(s) of the parent function $f(x) = x$ affect the output values of the corresponding points on the transformed function?

Vertical translations of $f(x)$ affect the output values of the corresponding points on the transformed function. The input values of the corresponding points on the transformed function remain the same.

4. Based on your observations, horizontally translating the parent function $f(x) = x$ right n units is equivalent to what other transformation?

For the parent function $f(x) = x$, a horizontal translation right n units is equivalent to a vertical translation down n units.

5. Based on your observations, vertically translating the parent function $f(x) = x$ up n units is equivalent to what other transformation?

For the parent function $f(x) = x$, a vertical translation up n units is equivalent to a horizontal translation left n units.

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MODULE 2 • TOPIC 2 • LESSON 2 291

STAMP THE LEARNING

The paragraphs provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

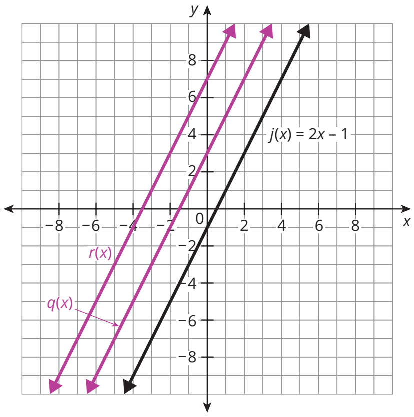
For the parent function $f(x) = x$, the transformed function $y = f(x - c)$ affects the input values, or x-values, of the function. For $c > 0$, the resulting graph horizontally shifts right c units. For $c < 0$, the resulting graph horizontally shifts left c units.

For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values, or y-values, of the function. For $d > 0$, the resulting graph vertically shifts up d units. For $d < 0$, the resulting graph vertically shifts down d units.

You have determined that for the parent function $f(x) = x$, a horizontal translation right n units produces the same graph as a vertical translation down n units. In addition, a horizontal translation left n units produces the same graph as a vertical translation up n units.

Let's see whether these translations have similar effects on the graph of any linear function.

6. Consider the graph of $j(x) = 2x - 1$.



   a. Sketch the graph of $j(x)$ shifted up 4 units. Label the graph as $q(x)$.

   b. Write an equation for $q(x)$ in terms of $j(x)$.

   $q(x) = j(x) + 4$

   c. Sketch the graph of $j(x)$ shifted left 4 units. Label the graph as $r(x)$.

   d. Write an equation for $r(x)$ in terms of $j(x)$.

   $r(x) = j(x + 4)$

EB STUDENT TIP

All Proficiency Levels

Help students connect the words *shift*, *stretch*, and *compress* to the concepts of *translation* and *dilation*. Explain that when we translate something, it moves or shifts. For example, if a friend is sitting on a bench and you want to sit by them, you might ask them to shift over to give you more space to sit down. When we speak of this geometrically, we can say, "When we translate a triangle, we shift it from one place to another without turning it." Clarify that when we talk about stretching or compressing a shape, this is another way to talk about the dilation of a shape and is similar to zooming in or out on a photo. Using these words in a mathematical context, we can say, "Dilating a circle can stretch it larger or compress it smaller, but its round shape stays the same." Have students practice using these terms both in everyday contexts and mathematical contexts.

e. Compare the graphs of $q(x)$ and $r(x)$. What do you notice?

The graphs of $q(x)$ and $r(x)$ are not the same.

f. The equation for $q(x)$ is simplified for you. Simplify the equation for $r(x)$. What do you notice?

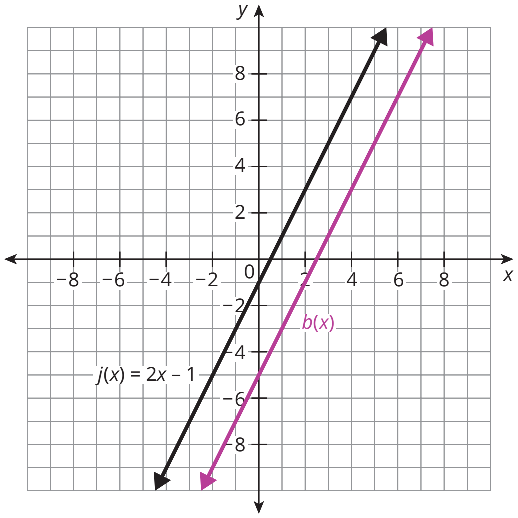
$q(x) = j(x) + 4$
$= 2x - 1 + 4$
$q(x) = 2x + 3$

$r(x) = j(x) + 4$
$r(x) = j(x + 4)$
$= 2(x + 4) - 1$
$= 2x + 8 - 1$
$r(x) = 2x + 7$

The equations for $q(x)$ and $r(x)$ are not the same.

7. Again, consider the graph of $j(x) = 2x - 1$.

a. Sketch the graph of $j(x)$ shifted right 2 units. Label the graph as $b(x)$.



b. Write an equation for $b(x)$ in terms of $j(x)$.

$b(x) = j(x - 2)$

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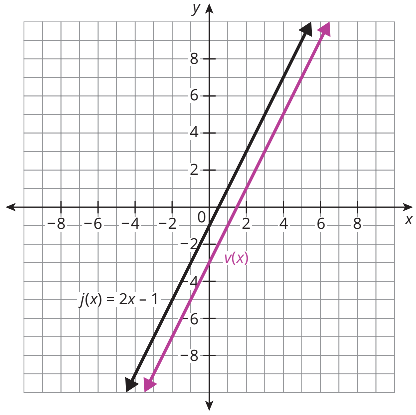
MODULE 2 • TOPIC 2 • LESSON 2 293

c. Sketch the graph of $j(x)$ shifted down 2 units. Label the graph as $v(x)$.



d. Write an equation for $v(x)$ in terms of $j(x)$.

$v(x) = j(x) - 2$

e. Compare the graphs of $b(x)$ and $v(x)$. What do you notice?

The graphs of $b(x)$ and $v(x)$ are not the same.

f. Simplify the equations for $b(x)$ and $v(x)$. What do you notice?

$b(x) =$	$v(x) =$
$b(x) = j(x - 2)$	$v(x) = j(x) - 2$
$= 2(x - 2) - 1$	$= 2x - 1 - 2$
$= 2x - 4 - 1$	$= 2x - 3$
$b(x) = 2x - 5$	

The equations for $b(x)$ and $v(x)$ are not the same.

8. For any linear function, does a horizontal translation right n units produce the same graph as a vertical translation down n units?

No. For any linear function, a horizontal translation right n units does not produce the same graph as a vertical translation down n units.

9. For any linear function, does a horizontal translation left n units produce the same graph as a vertical translation up n units?

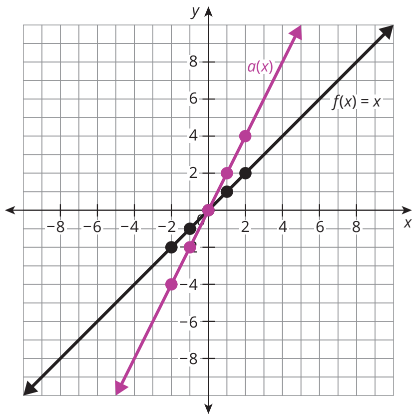
No. For any linear function, a horizontal translation left n units does not produce the same graph as a vertical translation up n units.

294 MODULE 2 • TOPIC 2 • LESSON 2

ACTIVITY 2.2 Comparing Vertical and Horizontal Dilations

1. The graph and a table of values for $f(x) = x$ are provided for you.

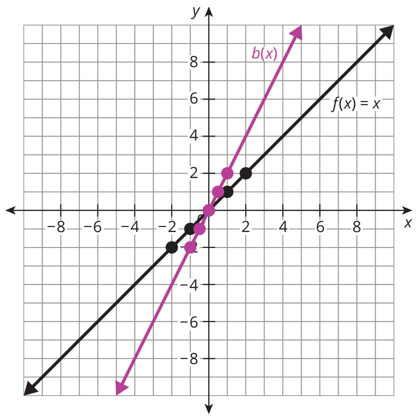
   a. Sketch the graph of $a(x)$, if the output values of $a(x)$ are changed by a factor of 2. Complete the table of corresponding points on $a(x)$.



Original Graph	
x	$f(x)$
–2	–2
–1	–1
0	0
1	1
2	2

Transformed Graph	
x	$a(x)$
–2	–4
–1	–2
0	0
1	2
2	4

   b. Sketch the graph of $b(x)$, if the input values of $b(x)$ are changed by a factor of $\frac{1}{2}$. Complete the table of corresponding points on $b(x)$.



Original Graph	
x	$f(x)$
–2	–2
–1	–1
0	0
1	1
2	2

Transformed Graph	
x	$b(x)$
–1	–2
$-\frac{1}{2}$	–1
0	0
$\frac{1}{2}$	1
1	2

   c. Compare the graphs and tables of values for $a(x)$ and $b(x)$. What do you notice?

   Although the graphs of $a(x)$ and $b(x)$ are the same, their tables of values are different.

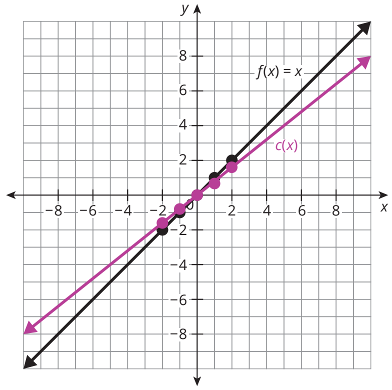
Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Read and discuss the information.
- Group students to complete Question 3.
- Share and summarize.

MODULE 2 • TOPIC 2 • LESSON 2 295

2. Again, the graph and a table of values for $f(x) = x$ are provided for you.

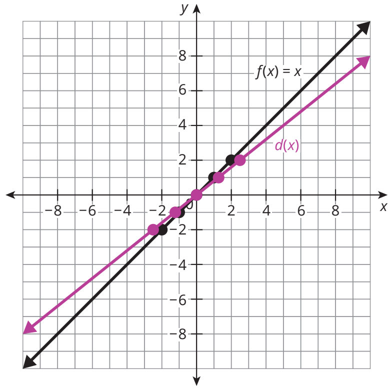
a. Sketch the graph of $c(x)$, if the output values of $c(x)$ are changed by a factor of $\frac{4}{5}$. Complete the table of corresponding points on $c(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$c(x)$
−2	$-1\frac{3}{5}$
−1	$-\frac{4}{5}$
0	0
1	$\frac{4}{5}$
2	$1\frac{3}{5}$

b. Sketch the graph of $d(x)$, if the input values of $d(x)$ are changed by a factor of $\frac{5}{4}$. Complete the table of corresponding points on $d(x)$.



Original Graph	
x	$f(x)$
−2	−2
−1	−1
0	0
1	1
2	2

Transformed Graph	
x	$d(x)$
$-2\frac{1}{2}$	−2
$-1\frac{1}{4}$	−1
0	0
$1\frac{1}{4}$	1
$2\frac{1}{2}$	2

c. Compare the graphs and tables of values for $c(x)$ and $d(x)$. What do you notice?

Although the graphs of $c(x)$ and $d(x)$ are the same, their tables of values are different.

d. Based on your observations, changing the output values of the parent function $f(x) = x$ by a factor of n units is equivalent to changing the input values by what factor?

For the parent function $f(x) = x$, changing the output values by a factor of n units is equivalent to changing the input values by a factor of $\frac{1}{n}$ units.

296 MODULE 2 • TOPIC 2 • LESSON 2

e. Based on your observations, changing the input values of the parent function $f(x) = x$ by a factor of n units is equivalent to changing the output values by what factor?

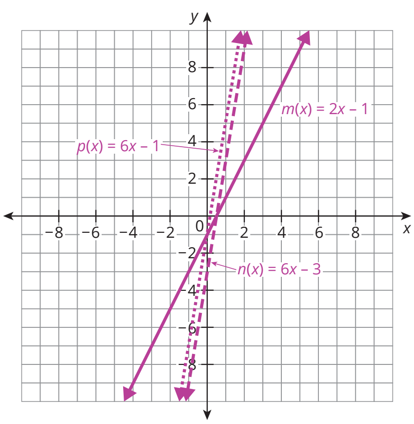
For the parent function $f(x) = x$, changing the input values by a factor of n units is equivalent to changing the output values by a factor of $\frac{1}{n}$ units.

For the parent function $f(x) = x$, the transformed function $y = af(x)$ affects the output values, or y-values, of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph reflects across the x-axis.

For the parent function $f(x) = x$, the transformed function $y = f(bx)$ affects the input values, or x-values, of the function. For $|b| > 1$, the resulting graph horizontally compresses by a factor of $\frac{1}{|b|}$ units. For $0 < |b| < 1$, the resulting graph horizontally stretches by a factor of $\frac{1}{|b|}$ units. For $b < 0$, the resulting graph reflects across the y-axis.

You have determined that for the parent function $f(x) = x$, a vertical dilation by a factor of n units produces the same graph as a horizontal dilation by a factor of $\frac{1}{n}$ units. In addition, a horizontal dilation by a factor of n units produces the same graph as a vertical dilation by a factor of $\frac{1}{n}$ units.

3. Does this relationship between horizontal and vertical dilations hold true for any linear function? Explain your reasoning. Provide a counterexample if necessary.



No. This relationship between horizontal and vertical dilations is only true for the parent function $f(x) = x$.

Counterexamples will vary.

Given the graph of $m(x) = 2x - 1$, the graph of $n(x) = 3m(x)$ is not the same as the graph of $p(x) = m(3x)$.

$n(x) = 3m(x)$

$= 3(2x - 1)$

$n(x) = 6x - 3$

$p(x) = m(3x)$

$= 2(3x) - 1$

$p(x) = 6x - 1$

STAMP THE LEARNING

The paragraphs provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

MODULE 2 • TOPIC 2 • LESSON 2 297

Chunking the Activity

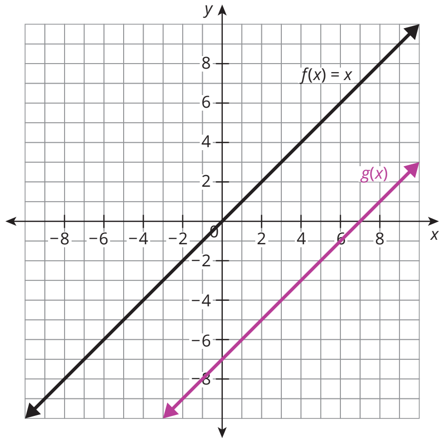
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

ACTIVITY

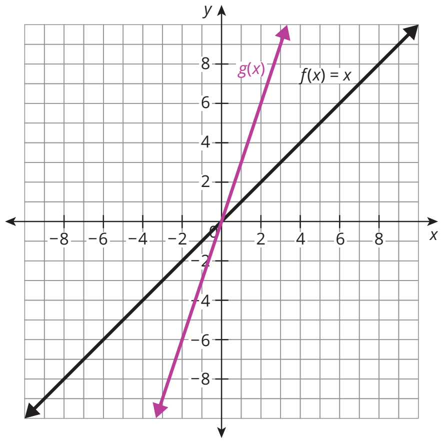
2.3 Graphing Linear Transformations

The equation of the parent linear function $f(x)$ is given. The equation for the transformed function $g(x)$ is also given. Describe the transformation(s) performed on $f(x)$ to produce $g(x)$. Then, graph $g(x)$.

1. $f(x) = x$
   $g(x) = f(x - 7)$
   The graph of the function $f(x)$ is translated right 7 units to produce $g(x)$.

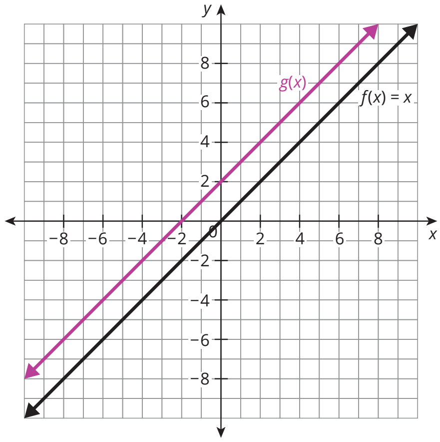


2. $f(x) = x$
   $g(x) = 3f(x)$
   The graph of the function $f(x)$ is stretched vertically by a factor of 3 to produce $g(x)$.



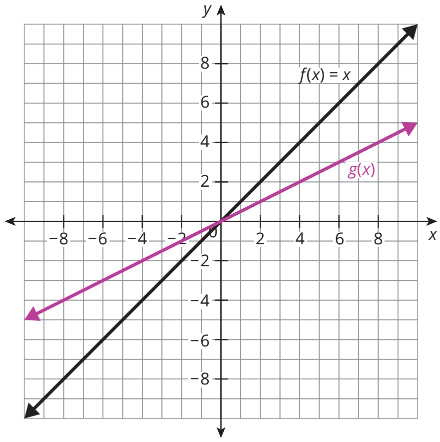
298 MODULE 2 • TOPIC 2 • LESSON 2

3. $f(x) = x$
   $g(x) = f(x) + 2$



The graph of the function $f(x)$ is translated up 2 units to produce $g(x)$.

4. $f(x) = x$
   $g(x) = f(\frac{1}{2}x)$

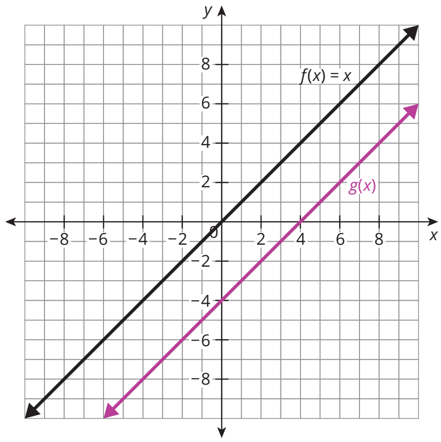


The graph of the function $f(x)$ is stretched horizontally by a factor of 2 to produce $g(x)$.

MODULE 2 • TOPIC 2 • LESSON 2 299

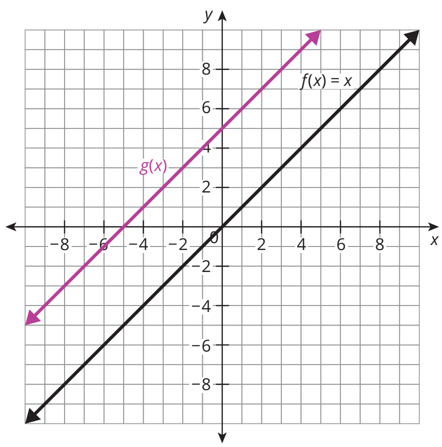
Activity 2.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing transformed linear functions and describing the transformations, assign Skills Practice Set A for this lesson.

5. $f(x) = x$
   $g(x) = f(x) - 4$



The graph of the function $f(x)$ is translated down 4 units to produce $g(x)$.

6. $f(x) = x$
   $g(x) = f(x + 5)$



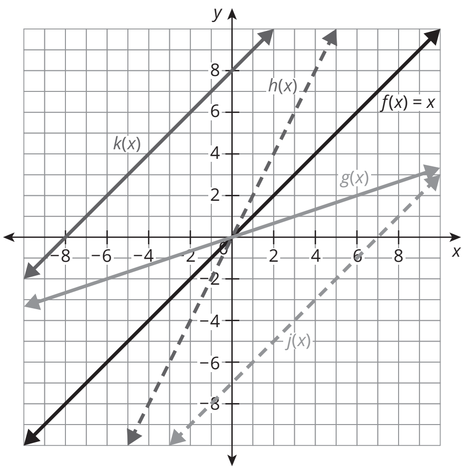
The graph of the function $f(x)$ is translated left 5 units to produce $g(x)$.

300 MODULE 2 • TOPIC 2 • LESSON 2

Talk the Talk

Show Me What You've Got!

The graph shows the linear function $f(x) = x$ and four transformations of $f(x)$.



Equation
$f(\frac{1}{3}x)$
$2f(x)$
$f(x - 7)$
$f(x) + 8$

Match each transformed graph to the equation in the table, written in terms of $f(x)$.

1. $k(x) =$ $f(x) + 8$
2. $h(x) =$ $2f(x)$
3. $g(x) =$ $f(\frac{1}{3}x)$
4. $j(x) =$ $f(x - 7)$

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

The Talk the Talk presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice matching transformed functions to their graphs, assign Skills Practice Set B for this lesson.

302 MODULE 2 • TOPIC 2 • LESSON 2

Lesson 2 Assignment

Write

For the parent function, $f(x) = x$, how do horizontal transformations compare to vertical transformations?

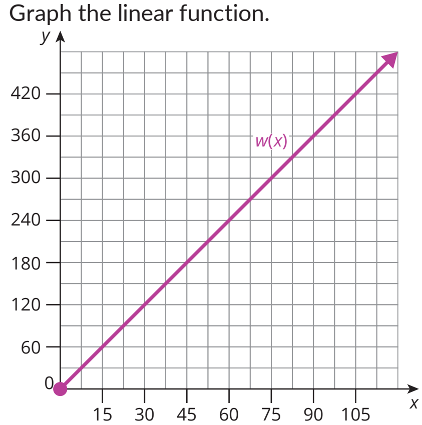
Remember

When $f(x)$ is replaced by $af(b(x - c)) + d$, b and c affect the input values of the function.

Practice

1. Ricardo started an exercise program. He researched the number of calories burned by walking and calisthenics. He learned that walking burns 4 calories per minute. The number of calories Ricardo can burn by walking is modeled by the function $w(x) = 4x$, where x represents the number of minutes he walks.

   a. Graph the linear function.



   b. Describe the key characteristics of the graph.
   Domain:
   Range:
   x-intercept/zero:
   Domain: All Real Numbers
   Range: All Real Numbers
   x-intercept/zero: (0, 0)

   c. Do the domain and range make sense in terms of the problem situation? Explain your reasoning. If the domain and range do not make sense, how could they be changed to better model the situation?
   The domain and range do not make sense in this situation because Ricardo could not walk for a negative number of minutes or burn a negative number of calories. Using a domain of $x \geq 0$ and a range of $y \geq 0$ is a more accurate representation of the problem situation.

Write

For the parent function, $f(x) = x$, a horizontal translation right n units produces the same graph as a vertical translation down n units. In addition, a horizontal translation left n units produces the same graph as a vertical translation up n units. A vertical dilation by a factor of n units produces the same graph as a horizontal dilation by a factor of $\frac{1}{n}$ units. In addition, a horizontal dilation by a factor of n units produces the same graph as a vertical dilation by a factor of $\frac{1}{n}$ units.

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MODULE 2 • TOPIC 2 • LESSON 2 ASSIGNMENT 303

Lesson 2 Assignment

d. Ricardo burned 330 calories doing calisthenics before going for a walk. Let $c(x)$ be the function that represents Ricardo's calories burned from doing calisthenics and walking. Write an equation for $c(x)$ in terms of $w(x)$.

$c(x) = w(x) + 330$

e. Simplify the equation for $c(x)$.

$c(x) = w(x) + 330$
$c(x) = 4x + 330$

f. Describe the transformation performed on $w(x)$ to produce $c(x)$.

The graph of $w(x)$ is shifted up 330 units to produce $c(x)$.

Prepare

Determine the reciprocal of each value.

1. 3

$\frac{1}{3}$

2. -10

$\frac{1}{10}$

3. $\frac{1}{5}$

5

4. $-\frac{5}{4}$

$-\frac{4}{5}$

5. $-c$

$-\frac{1}{c}$

6. $\frac{a}{b}$

$\frac{b}{a}$

304 MODULE 2 • TOPIC 2 • LESSON 2 ASSIGNMENT

3 Determining Slopes of Perpendicular Lines

MATERIALS

Patty Paper
Straightedges
Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students rotate a line segment on the coordinate plane in increments of 90° counterclockwise and recognize patterns in the slopes and coordinates of the endpoints of the images. They analyze a proof of a theorem stating that when two lines are perpendicular, the slopes of the lines are negative reciprocals. Students then explore relationships between vertical and horizontal lines. Finally, they write the equation of a line perpendicular to a given line that passes through a given point.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1E create and use representations to organize, record, and communicate mathematical ideas.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2F write the equation of a line that contains a given point and is perpendicular to a given line.

A.2G write an equation of a line that is parallel or perpendicular to the *x*- or *y*-axis and determine whether the slope of the line is zero or undefined.

ELPS

(2) Listening
The student is expected to:

(I) demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.

(3) Speaking
The student is expected to:

(E) share information in cooperative learning interactions.

(4) Reading
The student is expected to:

(K) demonstrate English comprehension and expand reading skills by employing analytical skills such as evaluating written information and performing critical analyses commensurate with content area and grade-level needs.

ESSENTIAL IDEAS

- Transformations can be used to create perpendicular lines. By rotating a line 90°, the pre-image and image form perpendicular lines.
- If two lines are perpendicular, their slopes are negative reciprocals.
- All horizontal lines have a slope of zero, are parallel to one another, and are perpendicular to vertical lines. All vertical lines have a slope that is undefined, are parallel to one another, and are perpendicular to horizontal lines.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Coordinate Rotation 15–20 minutes

CONNECT TO PRIOR KNOWLEDGE

Students determine the coordinates of the images of a line segment that has been rotated 90°, 180°, and 270° counterclockwise. They notice patterns in the coordinates of the endpoints of the images and the slopes of the images.

DEVELOP

Activity 3.1: Slopes of Perpendicular Lines 20–25 minutes

WORKED EXAMPLE

Students formalize what they recognized about the slope of perpendicular lines in the Getting Started. They rotate lines 90° counterclockwise with centers of rotation other than the origin and write the equations for each pair of lines. They then analyze a proof of the theorem that states that if two lines are perpendicular, the slopes of the lines are negative reciprocals.

DAY 2

Activity 3.2: Horizontal and Vertical Lines 15–20 minutes

INVESTIGATION, WORKED EXAMPLE

Students explore vertical and horizontal relationships between lines. They conclude that the slopes of horizontal lines are equal to zero, and the slopes of vertical lines are undefined. Students are asked to write equations for horizontal and vertical lines.

Activity 3.3: Writing Equations of Perpendicular Lines 15–20 minutes

MATHEMATICAL PROBLEM SOLVING

Students write the equation of a line perpendicular to a given line that passes through a given point. Based on the complexity of the information given, additional calculations may be required.

DEMONSTRATE

Talk the Talk: Things Aren't Always What They Seem 5 minutes

GENERALIZATION

Students analyze graphs of lines on a coordinate plane to determine whether parallel and perpendicular relationships exist.

Getting Started

ENGAGE

Coordinate Rotation

Facilitation Notes

In this activity, students determine the coordinates of the images of a line segment that has been rotated 90°, 180°, and 270° counterclockwise. They notice patterns in the coordinates of the endpoints of the images and the slopes of the images.

Have students work with a partner or in a group to complete Question 1. Share responses as a class. Make sure students have correct responses to Question 1 before moving on to Question 2.

COMMON MISCONCEPTION

Students may create images that are reflections across the *x*-axis and *y*-axis rather than 90° rotations. Common incorrect answers are (−8, 5) for 90° and (8, −5) for 270°.

Optimizing Learning

This differentiation strategy offers ways of customizing the display of information.

DIFFERENTIATION STRATEGY

Just in Time Support

Materials Needed: Patty Paper

Use patty paper to demonstrate how a 90° counterclockwise rotation maps the positive *x*-axis onto the positive *y*-axis.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know that your rotations are exactly 90°? • Were you surprised by the locations of each rotated line segment? If so, why? • Why do you think $\overline{AB}$ and $\overline{A''B''}$ connect to make a straight line?

Have students work with a partner or in a group to complete Questions 2 through 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

The use of mathematical vocabulary, such as *opposite*, *negative*, *reciprocal*, and *multiplicative inverse*.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How do the endpoints of each line segment compare after any 90° rotation? • How do the numeric components of the coordinates compare? How do the signs compare? • How do the sign changes connect with what you know about the signs used in each quadrant of the coordinate plane? • Does it make a difference in the slope when the negative sign lies in the numerator or denominator of the fraction? • Which segments have the same slope? How can you tell from the graph?
Probing	• What two statements could you use to describe the difference in slopes of consecutive line segments?

DIFFERENTIATION STRATEGIES

Challenge Opportunity

- Have students use the graph and the table to explain the relationship between a 180° rotation of a point and a reflection of a point.
- Ask students to explain why there are lines of reflection, and perpendicular bisectors in their diagrams.
- Have students experiment with the difference between reflecting and rotating a point 180°, and reflecting and rotating a line segment 180°.

Summary

A point (x, y) that is rotated 90° counterclockwise has the coordinates ($-y$, x).

STAMP THE LEARNING

ACTIVITY 3.1

Slopes of Perpendicular Lines

DEVELOP

Facilitation Notes

In this activity, student formalize what they recognized about the slope of perpendicular lines in the Getting Started. They rotate lines 90° counterclockwise with centers of rotation other than the origin and write the equations for each pair of lines. They then analyze a proof of the theorem that states that if two lines are perpendicular, the slopes of the lines are negative reciprocals.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- The equation of each line written from the graph, then a comparison made of their equations.
- The equation of the first line written from the graph and the second line written by taking the negative reciprocal of the first line for its slope.
- Failure to recognize that the slopes are negative reciprocals because the fractional slopes are not simplified.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why does this specific rotation create perpendicular lines?
Seeing structure	• How could you use patty paper to verify that the lines are perpendicular?
Probing	• What strategy did you use to write the equation of each line?
Seeing structure	• What is the relationship between the slopes? • Is there any relationship between perpendicular lines and their *y*-intercepts? • Why can't you use these equations to prove perpendicular lines have reciprocal slopes with opposite signs?

Have students work with a partner or in a group to analyze the Worked Example and complete Questions 3 and 4. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

- Analyze the proof as a class. Then have students answer Question 3 in more detail by creating a diagram for the 90° clockwise rotation and rewriting the proof to apply to their diagram.
- Encourage students to take time to read through the Worked Example and annotate key ideas or steps in the process. As they think about the connections, have students ask themselves:
    - Why is this method correct?
    - Have I used this method before?

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How does the Assumption statement relate to the If Then statement? The Conclusion statement? • What is meant by negative reciprocals? • Explain what the conclusion $m_1 = \frac{1}{m_2}$ means. • Does the negative sign mean the second line always has a negative slope? Explain your thinking.
Probing	• Explain why the coordinates of point *D* are $(-b, a)$. • Explain how the side lengths of each triangle were determined. • How can you use the labeled diagram to determine each slope? • Explain the steps to rewrite the complex fraction.
Seeing structure	• Why does the Worked Example demonstrate formal reasoning?
Probing	• How did you determine the negative reciprocal of -3?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

If two lines are perpendicular, their slopes are negative reciprocals.

STAMP THE LEARNING

ACTIVITY 3.2 Horizontal and Vertical Lines

Facilitation Notes

In this activity, students extend their understanding of perpendicular lines to include horizontal and vertical lines. They reason why the slopes of horizontal lines are zero and the slopes of vertical lines are undefined, relate their slopes to parallelism and perpendicularity, and write equations for lines perpendicular to horizontal or vertical lines through a given point.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Reasoning using the graphs of the lines, such as “Because horizontal lines are flat, their slope is zero.”
- Explanations using $m = \frac{y_2 - y_1}{x_2 - x_1}$.

DIFFERENTIATION STRATEGY

Access for All

- Suggest that students select points on the lines and substitute them in the slope formula to make sense of the slopes of horizontal and vertical lines.

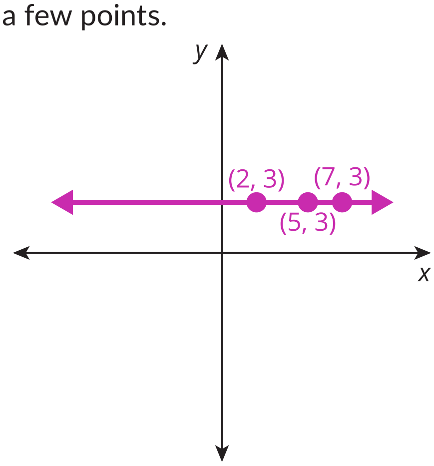
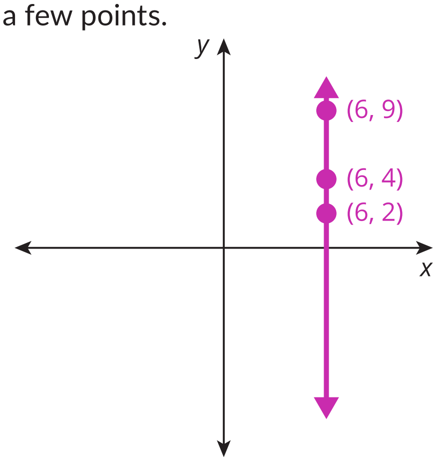
QUESTIONS TO SUPPORT DISCOURSE

Probing	• Demonstrate the slope is zero using two points from a line. • Demonstrate the slope is zero using points (x_1, y_1) and (x_2, y_2).
Seeing structure	• If one line has a slope of zero, how do you know all the other lines have a slope of zero?
Probing	• What is another way to demonstrate the slope is undefined?

Have students work with a partner or in a group to complete Questions 6 through 10. Share responses as a class.

COMMON MISCONCEPTION

- Students often confuse the fact that the equation for a horizontal line is $y = a$ *constant*, and the equation for a vertical line is $x = a$ *constant*. Suggest a method that students can use to construct the information they need rather than relying on memorization.

Sketch a horizontal line and label a few points.	Sketch a vertical line and label a few points.
	
In all cases, $y = 3$.	In all cases, $x = 6$.
Therefore, for horizontal lines, use $y = a$ *constant*.	Therefore, for vertical lines, use $x = a$ *constant*.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How do you know the form of the equation for a horizontal or vertical line? • How can you use a table of values to verify your reasoning? A graph?
Probing	• What is the slope of the original line? What is the reciprocal of that slope? • How do you know which number from the ordered pair to use in your equation?
Seeing structure	• Does the relationship that perpendicular lines have negative reciprocal slopes apply to vertical and horizontal lines? Explain your thinking.
Reflecting and justifying	• Sketch a graph to verify your thinking.

Summary

Any vertical line and any horizontal line are perpendicular to each other.

STAMP THE LEARNING

ACTIVITY 3.3

Writing Equations of Perpendicular Lines

Facilitation Notes

In this activity, students write the equation of a line perpendicular to a given line that passes through a given point. Based on the complexity of the information given, additional calculations may be required.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

- Suggest students label their steps with notations, such as "$m =$" and "$\perp m =$" for organization of their work.
- Encourage the use of sketches to make sense of Questions 3 through 5.

Just in Time Support

- Modify Questions 3 through 5 to provide the same information as Questions 1 and 2.

AS STUDENTS WORK, LOOK FOR

- Errors identifying the perpendicular slope, mainly due to forgetting to take the opposite sign of the original slope.
- Unnecessary steps when solving Questions 3 through 5, such as determining the equation for the line through the two given points, rather than just calculating the slope. When this occurs, discuss the more efficient method with students.
- Use of the slope formula to write the equation of the line.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you know what slope to use in your equation? • How did you use the point the line passes through to help write your equation? • Explain how you used the slope formula to write the equation of the line. • How can you check that your answer is correct? • How did you determine the slope to use in your equation without being given a perpendicular slope?
Seeing structure	• How was knowing the point of intersection helpful? • How was the information given in Question 5 similar to the information given in Questions 3 and 4?

Summary

STAMP THE LEARNING

To write the equation of a line perpendicular to a given line, it is necessary to use the negative reciprocal relationship between their slopes.

DEMONSTRATE

Talk the Talk

THINGS AREN'T ALWAYS WHAT THEY SEEM

Facilitation Notes

In this activity, students analyze graphs of lines on a coordinate plane to determine whether parallel and perpendicular relationships exist.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How can algebra be used to determine whether the lines are perpendicular? • How can patty paper be used to determine whether the lines are perpendicular?
Probing	• When you know that one line out of a set of parallel lines is not perpendicular to a line out of another set of parallel lines, can you conclude that none of the lines are perpendicular? Why or why not?

Have students read and answer the Essential Question on the lesson opener page.

Summary

The slopes of lines graphed on a coordinate plane must be negative reciprocals in order for the lines to be defined as perpendicular.

STAMP THE LEARNING

3 Determining Slopes of Perpendicular Lines

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Identify and write the equations of lines perpendicular to given lines.
- Identify and write the equations of horizontal and vertical lines.

You have translated the graphs of linear equations to determine the slopes of parallel lines. How can you rotate the graphs of linear equations to determine the slopes of perpendicular lines?

Sample answer:

The slopes of perpendicular lines are negative reciprocals. Any vertical line is perpendicular to any horizontal line.

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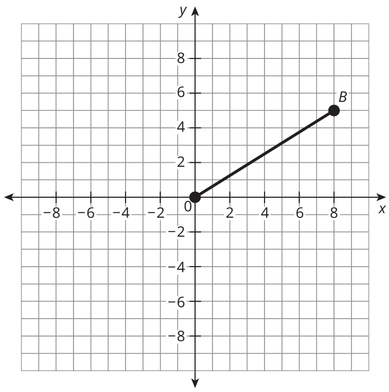
MODULE 2 • TOPIC 2 • LESSON 3 305

Chunking the Activity

- Read and discuss the directions.
- Group students to complete Question 1.
- Check in and share.
- Group students to complete Questions 2–4.
- Share and summarize.

Getting Started

Coordinate Rotation



Consider $\overline{AB}$ with coordinates A (0, 0) and B (8, 5).

1. Suppose the line segment is rotated through an angle with the origin as the center of rotation. Complete the table with the coordinates of the rotated figure.

Original Line Segment	90° Counterclockwise	180°	270° Counterclockwise
$\overline{AB}$	$\overline{A'B'}$	$\overline{A''B''}$	$\overline{A'''B'''}$
A (0, 0)	A'(0, 0)	A''(0, 0)	A'''(0, 0)
B (8, 5)	B'(−5, 8)	B''(−8, −5)	B'''(5, −8)

2. What do you notice about the coordinates of the endpoints of each rotated figure?

   Each time the endpoints are rotated another 90° counterclockwise, the coordinates change from (x, y) to (−y, x). Because one endpoint of the original line segment is on the origin, it does not change.

3. Determine the slope of each line segment.

   a. $\overline{AB}$ $m = \frac{5}{8}$

   b. $\overline{A'B'}$ $m = -\frac{8}{5}$

   c. $\overline{A''B''}$ $m = \frac{5}{8}$

   d. $\overline{A'''B'''}$ $m = -\frac{8}{5}$

Review the definition of **describe** in the Academic Glossary.

4. **Describe** any patterns you notice in the slopes of the figures.

   The slopes of the original line segment and its image rotated 180° are equal. The slopes of the images rotated 90° and 270° counterclockwise are equal. The two slopes are reciprocals of each other with opposite signs.

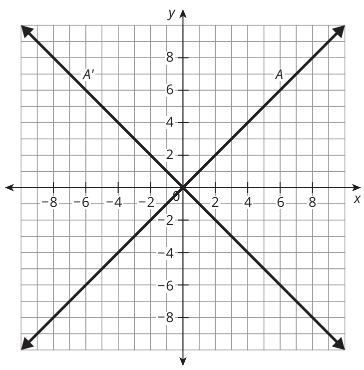
306 MODULE 2 • TOPIC 2 • LESSON 3

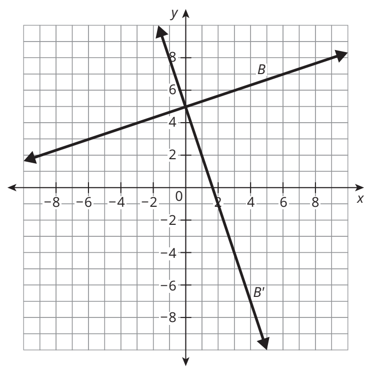
ACTIVITY 3.1

Slopes of Perpendicular Lines

Recall that perpendicular lines or line segments form a right angle at the point of intersection.

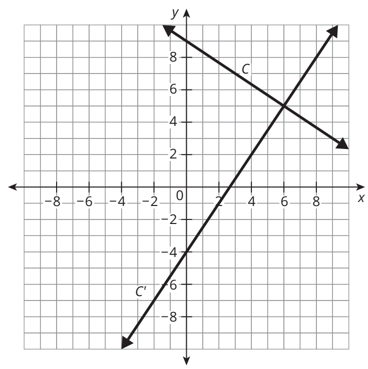
Consider the three graphs shown. Each shows a line and its rotation 90° about a point, which is also the point of intersection.





1. Are the lines in each graph perpendicular? Explain your reasoning.

   Yes. Each line is rotated 90°, so the angle formed by the intersection of the lines measures 90°, which is a right angle. When two lines form right angles, they are perpendicular.



2. Write the equation for each line and its transformation. What do you notice?

   A: $y = x$

   A': $y = -x$

   B: $y = \frac{1}{3}x + 5$

   B': $y = -3x + 5$

   C: $y = -\frac{2}{3}x + 9$

   C': $y = \frac{3}{2}x - 4$

   The slopes of each pair of perpendicular lines are reciprocals of each other with opposite signs.

Remember . . .

The reciprocal of a number $\frac{a}{b}$ is the number $\frac{b}{a}$, where a and b are nonzero numbers. Because the product of a number and its reciprocal is one, reciprocal numbers are also known as multiplicative inverses.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Read and discuss the Worked Example.
- Group students to complete Questions 3 and 4.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

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MODULE 2 • TOPIC 2 • LESSON 3 307

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Consider the theorem.

Remember . . .
The symbol $\perp$ means *is perpendicular* to.

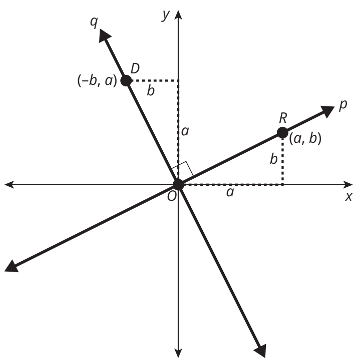
The product of the slopes of perpendicular lines is -1.

The proof shown is a paragraph proof. You will learn different formats of proof as you investigate more properties of lines and figures.

WORKED EXAMPLE

Theorem: If two lines are perpendicular, their slopes are negative reciprocals.

Use the graph and the proof shown to analyze the validity of the theorem.



Given: $p \perp q$

Let m_1 = slope of line p and let m_2 = slope of line q.

Point R lies on line p.

Prove: $m_1 = -\frac{1}{m_2}$

Rotate point R 90° counterclockwise using point O as the center of rotation. Since p and q are perpendicular, the image (point D) will lie on line q under this 90° rotation.

Since this rotation maps the positive x-axis to the positive y-axis and the positive y-axis to the negative x-axis, then the coordinates of $R(a, b)$ are transformed into the coordinates of $D(-b, a)$. Graphically, you can see the movement of lengths a and b under the rotation.

The graph shows the slope of line p, $m_1 = \frac{b}{a}$, and the slope of line q, $m_2 = \frac{a}{-b}$.

Using these slopes, you can demonstrate that $m_1 = -\frac{1}{m_2}$.

$$\frac{b}{a} = -\frac{1}{\frac{a}{-b}}$$
$$= -1 \cdot \frac{-b}{a}$$
$$= \frac{b}{a}$$

The slope of line q is the negative reciprocal of the slope of line p.

EB STUDENT TIP

For all proficiency levels

Read aloud the sentence after the theorem in the Worked Example, "Use the graph and the proof to analyze the validity of the theorem." Assess students' prior knowledge of the term *validity*. Discuss the definition of the root word *valid*, which means *having a reasonable basis in logic or fact*. Compare *valid* along with *validity* to their Spanish cognates, *válido* and *validez*. Provide examples of common uses of *validity*, such as the *validity* of a driver's license, the *validity* of testimony in a court, and the *validity* of an article. Reread the sentence and clarify any remaining misunderstandings about the use of *validity* in the context of the sentence.

There is often more than one way to prove a theorem. Suppose that point *R* is rotated 90° clockwise using point *O* as the center of rotation.

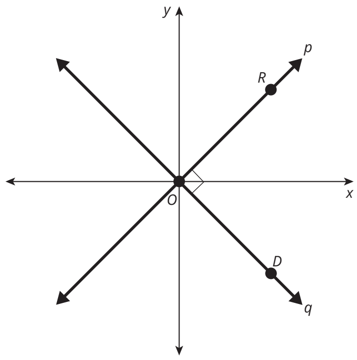
3. Rewrite the proof using the clockwise rotation of point *R*.

   Since the rotation of point *R* clockwise maps onto line *q* as *D* (*b*, −*a*), the slope of line *p* is $m_1 = \frac{b}{a}$ and the slope of line *q* is $m_2 = \frac{-a}{b}$.

   Using these slopes, you can demonstrate that

   $m_1 = -\frac{1}{m_2}$.

   $$\frac{b}{a} = \frac{-1}{\frac{-a}{b}}$$
   $$= -1 \cdot \frac{b}{-a}$$
   $$= \frac{b}{a}$$



4. Line *j* and line *k* are perpendicular. Given each slope of line *j*, determine the slope of line *k*.

   a. $m = \frac{2}{3}$ For line *j*, $m = -\frac{3}{2}$.

   b. $m = -\frac{4}{5}$ For line *j*, $m = \frac{5}{4}$.

   c. $m = -3$ For line *j*, $m = \frac{1}{3}$.

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MODULE 2 • TOPIC 2 • LESSON 3 309

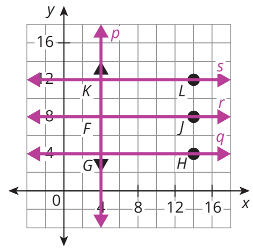
Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete Questions 1–5.
- Check in and share.
- Group students to complete Questions 6–10.
- Share and summarize.

ACTIVITY 3.2 Horizontal and Vertical Lines

Consider the graph shown.

1. Use a straightedge to extend $\overline{GK}$ to create line p, extend $\overline{GH}$ to create line q, extend $\overline{FJ}$ to create line r, and extend $\overline{KL}$ to create line s.



2. Consider the three horizontal lines you drew for Question 1. For any horizontal line, if x increases by one unit, by how many units does y change?

   The value of y does not change.

3. Describe the slope of any horizontal line. Explain your reasoning.

   The slope of any horizontal line is zero because as x increases, the value of y stays the same, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{x_2 - x_1} = 0$.

4. Consider the vertical line you drew in Question 1. Suppose that y increases by one unit. By how many units does x change?

   The value of x does not change.

5. Describe the slope of any vertical line. Explain your reasoning.

   The slope of any vertical line is undefined because as y increases, the value of x stays the same, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{0} = $ undefined.

EB STUDENT TIP

For all proficiency levels

Determine whether students are familiar with the term *extend* and its Spanish cognate *extender*. If not, state the two definitions of *extend* as *to make longer or wider* and *to hold something out toward someone*. Discuss real-world examples of the term *extend*, such as *extending* a roadway, *extending* a deadline, *extending* the range of acceptable answers on a test, and *extending* a hand for someone to shake. Ensure students' understanding of the context of *extend* in Question 1, as *to make the given line segment longer.*

310 MODULE 2 • TOPIC 2 • LESSON 3

6. Determine whether each of the given statements is always, sometimes, or never true. Explain your reasoning.

   a. All vertical lines are parallel.

      The slopes of all vertical lines are undefined, and therefore, the same. So, all vertical lines are parallel.

   b. All horizontal lines are parallel.

      The slopes of all horizontal lines are zero, and therefore, the same. So, all horizontal lines are parallel.

7. Describe the relationship between any vertical line and any horizontal line.

   Any vertical line and any horizontal line are perpendicular to each other.

8. Write an equation for a horizontal line and an equation for a vertical line that pass through the point (2, −1).

   Horizontal line: $y = -1$

   Vertical line: $x = 2$

9. Write an equation for a line that is perpendicular to the line given by $x = 5$ and passes through the point (1, 0).

   $y = 0$

10. Write an equation for a line that is perpendicular to the line given by $y = -2$ and passes through the point (5, 6).

    $x = 5$

Questions 8 through 10 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing the equations of vertical and horizontal lines, assign Skills Practice Set A for this lesson.

CC BY NC

MODULE 2 • TOPIC 2 • LESSON 3 311

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Remember . . .
You can use point-slope form to write an equation for any line when you know its slope and one point on that line.

ACTIVITY 3.3

Writing Equations of Perpendicular Lines

In a previous lesson you wrote the equation of a line parallel to a given line that passes through a given point. You can write the equation of a perpendicular line using what you know about the slope of that line and any point on that line.

1. Write the equation of the line perpendicular to $y = 2x + 1$ that passes through the point (6, 2).

   The slope of a line of a line perpendicular to a line with a slope of 2 is $-\frac{1}{2}$.
   $y - y_1 = m(x - x_1)$
   $y - 2 = -\frac{1}{2}(x - 6)$
   $y - 2 = -\frac{1}{2}x + 3$
   $y = -\frac{1}{2}x + 5$

2. Write the equation of the line perpendicular to $y = -\frac{3}{4}x$ that passes through the point (3, −8).

   The slope of a line perpendicular to a line with a slope of $\frac{3}{4}$ is $\frac{4}{3}$.
   $y - y_1 = m(x - x_1)$
   $y - (-8) = \frac{4}{3}(x - 3)$
   $y + 8 = \frac{4}{3}x - 4$
   $y = \frac{4}{3}x - 12$

3. Write the equation of the line that passes through the point (6, 2) and is perpendicular to a line that passes through the points (−5, 3) and (−1, −9).

   $m = \frac{y_2 - y_1}{y_2 - y_1} = \frac{-9-3}{-1-(-5)} = \frac{-9-3}{-1-(-5)} = \frac{-12}{4} = -3$
   The slope of a line perpendicular to a line with a slope of −3 is $\frac{1}{3}$.
   $y - y_1 = m(x - x_1)$
   $y - 2 = \frac{1}{3}(x - 6)$
   $y - 2 = \frac{1}{3}x - 2$
   $y = \frac{1}{3}x$

312 MODULE 2 • TOPIC 2 • LESSON 3

4. Write the equation of a line that passes through the point (−2, 7) and is perpendicular to a line that passes through the points (−6, 1) and (0, 4).

$m = \frac{y_2 - y_1}{y_2 - y_1} = \frac{4-1}{0-(-6)} = \frac{3}{6} = \frac{1}{2}$

The slope of a line perpendicular to a line with a slope of $\frac{1}{2}$ is −2.

$y - y_1 = m(x - x_1)$

$y - 7 = -2(x - (-2))$

$y - 7 = -2x - 4$

$y = -2x + 3$

5. A pair of perpendicular lines intersect at the point (5, 9). Write the equation of the line that is perpendicular to the line that also passes through point (−4, 4).

$m = \frac{y_2 - y_1}{y_2 - y_1} = \frac{4-9}{-4-5} = \frac{-5}{-9} = \frac{5}{9}$

The slope of a line perpendicular to a line with a slope of $\frac{5}{9}$ is $-\frac{9}{5}$.

$y - y_1 = m(x - x_1)$

$y - 9 = -\frac{9}{5}(x - 5)$

$y - 9 = -\frac{9}{5}x + 9$

$y = -\frac{9}{5}x + 18$

Activity 3.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing the equation of a line that is perpendicular to a given line and that passes through a given point, assign Skills Practice Set B for this lesson.

MODULE 2 • TOPIC 2 • LESSON 3 313

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

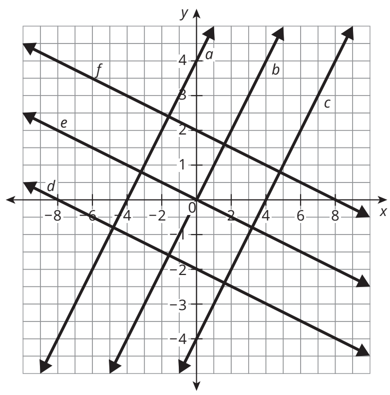
Modeling Moment

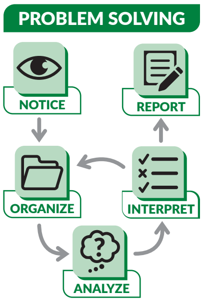
- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

Talk the Talk

Things Aren't Always What They Seem

Consider the graphs of six linear equations shown.





1. Linh says lines *a*, *b*, and *c* are parallel to each other and perpendicular to lines *d*, *e*, and *f*. Jackson agrees that lines *a*, *b*, and *c* are parallel to each other but says they are not perpendicular to lines *d*, *e*, and *f*. Who is correct? Justify your reasoning.

   Jackson is correct. I used two points on each line to determine the slopes of lines *a*, *b*, and *c* are all equal to 1. Since lines *a*, *b*, and *c* have equal slopes, they are parallel. I then used two points on each line to determine the slopes of lines *d*, *e*, and *f* are all equal to $-\frac{1}{4}$.

   Since the slopes of the lines are not negative reciprocals, the lines are not perpendicular to each other.

314 MODULE 2 • TOPIC 2 • LESSON 3

Lesson 3 Assignment

Write

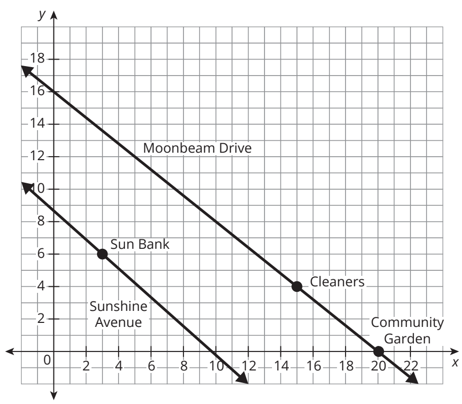
Explain in your own words why the slope of a vertical line is undefined.

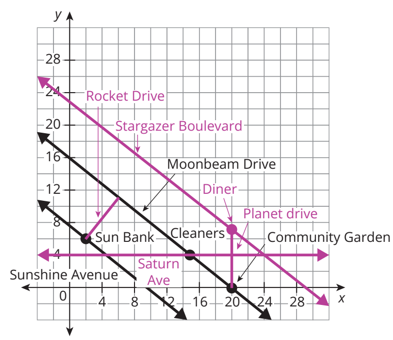
Remember

The slopes of perpendicular lines are negative reciprocals. Any vertical line is perpendicular to any horizontal line.

Practice

Nakota is a developer and plans to build a new development. Use the grid to help Nakota create a map for his development. Each gridline represents one block.





1. There are two main roads that pass through the development, Moonbeam Drive and Sunshine Avenue. Are these two roads parallel to each other? Explain your reasoning.

   Yes, the two roads are parallel because they have the same slope.

   Slope of Moonbeam Dr.: $m = \frac{6-2}{20-15} = -\frac{4}{5}$

   Slope of Sunshine Ave.: $m = \frac{6-2}{2-7} = -\frac{4}{5}$

Write

Sample answer:

The slope of a line is the ratio of the change in y-values to the change in x-values between any two points on the line. For a vertical line, the x-values do not change at all; therefore, the change in x-values is 0. Since you cannot divide by 0, the value of the slope is undefined.

MODULE 2 • TOPIC 2 • LESSON 3 ASSIGNMENT 315

Lesson 3 Assignment

2. Nakota wants to build a road named Stargazer Boulevard that will be parallel to Moonbeam Drive. On this road, he will build a diner located seven blocks north of the community garden. Determine the equation of the line that represents Stargazer Boulevard. Show your work. Then, draw and label Stargazer Boulevard on the coordinate plane.

   The slope of Stargazer Blvd. is the same as the slope of Moonbeam Dr. because the two lines are parallel. The location of the diner is at (20, 7).

   $y - 7 = -\frac{4}{5}(x - 20)$

   $y = -\frac{4}{5}x + 23$

   The equation of the line representing Stargazer Blvd. is $y = -\frac{4}{5}x + 23$. See graph.

3. Nakota wants to build a road named Rocket Drive that connects Sun Bank to Moonbeam Drive. He wants this road to be as short as possible. Determine the equation of the line that represents Rocket Drive. Show your work. Then, draw and label Rocket Drive on the coordinate plane.

   The slope of Moonbeam Dr. is $-\frac{4}{5}$. So, I know the slope of Rocket Dr. is $\frac{5}{4}$. The coordinates of Sun Bank are (2, 6).

   $y - 6 = \frac{5}{4}(x - 2)$

   $y = \frac{5}{4}x + \frac{7}{2}$

   The equation of the line representing Rocket Drive is $y = \frac{5}{4}x + \frac{7}{2}$. See graph.

4. Two office buildings are to be located at the points (8, 4) and (12, 10). Would the shortest road between the two office buildings be a line that is perpendicular to Moonbeam Drive? Explain your reasoning.

   Slope of line joining two office buildings:

   $m = \frac{10 - 4}{12 - 8} = \frac{6}{4} = \frac{3}{2}$.

   The slope of Moonbeam Dr. is $-\frac{4}{5}$.

   So, a line joining the two office buildings would not be perpendicular to Moonbeam Dr.

316 MODULE 2 • TOPIC 2 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

5. A straight road named Planet Drive is planned that will connect the diner and the community garden. What is the equation of the line that represents Planet Drive? Show your work. Draw and label Planet Drive on the coordinate plane.

   The line representing the road connecting the community garden and the diner is a vertical line, so the slope is undefined. The equation of the line representing Planet Dr. is $x = 20$. See graph.

6. Nakota decides that another road to be named Saturn Avenue is needed that will go past the cleaners and be perpendicular to Planet Drive. Determine the equation of the line that represents Saturn Avenue. Show your work. Draw and label Saturn Avenue on the coordinate plane.

   The line perpendicular to Planet Dr. is a horizontal line because Planet Dr. is a vertical line. Therefore the slope of the line representing Saturn Ave. is 0. The line goes past the cleaners, so it passes through the point (15, 4). The equation of the line representing Saturn Ave. is $y = 4$.

Prepare

Determine whether each set of ordered pairs represents a function. Explain your reasoning.

1. {(−1, −1), (0, 0) (1, 1), (2, 2)}

   Function; each input maps to exactly one output

2. {(−1, −2), (0, 0) (1, 2), (2, 4)}

   Function; each input maps to exactly one output

3. {(−1, −1), (0, −1) (1, −1), (2, −1)}

   Function; each input maps to exactly one output

4. {(−1, −1), (−1, 0) (−1, 1), (−1, 2)}

   Not a function; −1 maps to 4 different values

MODULE 2 • TOPIC 2 • LESSON 3 ASSIGNMENT 317

318 MODULE 2 • TOPIC 2 • LESSON 3 ASSIGNMENT

4 Comparing Linear Functions in Different Forms

MATERIALS

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students analyze functions represented as tables, graphs, equations, and verbal descriptions. They explore slope with particular attention to parallelism and perpendicularity in different representations. Students compare properties, such as slope, *y*-intercept, and the units for independent and dependent quantities, all in terms of the situations they represent. Students also identify the scale and origin on the graph of a function given a situation description. Finally, they generate and compare their own linear functions using tables, graphs, and equations.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Linear Functions, Equations, and Inequalities

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3A determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.

A.3C graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems.

(TEKS continued on next page)

ELPS

(1) Learning Strategies

The student is expected to:

(D) speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution (conveying ideas by defining or describing when exact English words are not known).

(2) Listening

The student is expected to:

(G) understand the general meaning, main points, and important details of spoken language ranging from situations in which topics, language, and contexts are familiar to unfamiliar.

(4) Reading

The student is expected to:

(A) learn relationships between sounds and letters of the English language and decode (sound out) words using a combination of skills such as recognizing sound-letter relationships and identifying cognates, affixes, roots and base words.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

The student is expected to:

A.12B evaluate functions, expressed in function notation, given one or more elements in their domains.

ESSENTIAL IDEAS

- Functions can be represented using tables, equations, graphs, and with verbal descriptions.
- Features of linear functions, such as *y*-intercepts, slope, independent quantities, and dependent quantities, can be determined from different representations of functions.
- Lines that are parallel have the same slope. Lines that are perpendicular have slopes that are negative reciprocals.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Odd One Out 5–10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students are presented with four different linear functions in four different representations: a table, a graph, an equation, and a verbal description. Students choose a representation that doesn't belong with the others and justify their choice. Each representation is different from the others in at least one way, including decreasing across the entire domain, a slope between 0 and 1, a nonzero *y*-intercept, and a domain of whole numbers only.

DEVELOP

Activity 4.1: Slopes of Linear Relationships 15–20 minutes

MATHEMATICAL PROBLEM SOLVING

Students are presented with several linear relationships modeled by an equation, table, graph, or context. They identify whether the graph of each linear relationship is parallel or perpendicular to the graph of a given line expressed as an equation.

Activity 4.2: Comparing Tables and Graphs 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students are provided a scenario in which two recycling centers pay out different amounts per ton of cardboard. One recycling center represents its payout amounts in a table, and another advertises its payout amounts in a graph. Students compare the functions in each representation, interpreting the units for the independent and dependent quantities as well as the *y*-intercepts and slopes.

DAY 2

Activity 4.3: Comparing Equations and Tables 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students are provided a scenario in which two friends who live in different countries must convert the temperatures given by the other friend from degrees Celsius to degrees Fahrenheit or vice versa. One friend represents temperature conversion with a table, and the other friend with an equation. Students compare the functions in each representation, interpreting the units for the independent and dependent quantities as well as the *y*-intercepts and slopes.

Activity 4.4: Comparing Descriptions and Graphs 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students are provided a scenario in which two students deposit money into bank accounts opened at the same time. One account balance is represented with a graph, and the other with a verbal description. Students compare the functions in each representation, interpreting the units for the independent and dependent quantities as well as the *y*-intercepts and slopes. Students also use the problem situation to determine and interpret the scale and origin on the graph of one of the functions.

DEMONSTRATE

Talk the Talk: Function Maker Space 10–15 minutes

EXIT TICKET PROCEDURE

Students create a situation that represents a given table of values. They use this situation to write a function equation with a different slope less than the one in their situation. Students then compare the two functions from their slopes and describe the strategies they used to compare the linear functions.

Getting Started

ENGAGE

Odd One Out

Facilitation Notes

In this activity, students are presented with four different linear functions in four different representations: a table, a graph, an equation, and a verbal description. Students are asked to choose a representation that doesn't belong with the others and justify their choice.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Is there more than one correct answer? Explain. • What would a graph of this function look like? • How do the slopes of the functions compare? • Compare the *y*-intercepts of the functions. • What is different about the functions' domains?

Summary

STAMP THE LEARNING

Characteristics of functions can be determined using a variety of different function representations.

ACTIVITY 4.1

Slopes of Linear Relationships

DEVELOP

Facilitation Notes

Students are presented with several linear relationships modeled by an equation, table, graph, or context. They identify whether the graph of each linear relationship is parallel or perpendicular to the graph of a given line expressed as an equation.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Errors determining whether lines are perpendicular by considering reciprocal slopes, but not considering their signs must be opposite of one another.
- Different interpretations of the elevator scenario.
- Unnecessary steps when considering the tables and graph, such as determining the equation for the line rather than just calculating the slope. If this occurs, discuss the more efficient method with students.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How can you use the slopes of two lines to determine whether the graphs will be parallel or perpendicular?
Probing	• Do you need to calculate the slope between each pair of points in the table? Explain your reasoning. • What sign did you use for the slope of the elevator? Why?

STAMP THE LEARNING

Summary

The slopes of different representations of lines can be compared to determine whether the graphs of their lines are parallel or perpendicular.

ACTIVITY 4.2

Comparing Tables and Graphs

Facilitation Notes

In this activity, students are given a scenario which includes two functions. One function is described using a table while the second function is described using a graph. Students compare the functions by identifying the units for the independent and dependent quantities as well as the *y*-intercepts and slopes.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Methods of comparison. Did they compare values using the different representations, or did they change one representation to be the same as the other?
- Reasoning to compare values when $x = 3$, since it is not in the table or on the graph.

DIFFERENTIATION STRATEGIES

Access for All

- Have students make a table for the graph so that they have the same representations to compare.

Just in Time Support

Materials Needed: Colored Pencils

- Suggest students use a colored pencil to graph the data from Travis County Recycling on the graph with Hays County Recycling. This provides students with the same representations to compare. Ensure students share other strategies they could use to compare the recycling centers.

Challenge Opportunity

- Ask students to compare the two recycling centers using equations.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How many pounds are in a ton?
Probing	• How can you tell the independent quantity and dependent quantity from the scenario? Table? Graph? • How did you determine the values for $j(0.5)$ and $j(1)$? • Did you need to calculate $w(3)$ and $j(3)$ to complete the statement? Explain your thinking. • How can you tell that each county consistently used the same payout rate regardless of the number of tons of cardboard? • How did you determine each county's payout rate? • How could each county explain their payout rate as an equation? In a statement? • Which representation do you think citizens would prefer? Explain your thinking.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Properties of functions represented in tables and graphs can be compared.

STAMP THE LEARNING

ACTIVITY 4.3

Comparing Equations and Tables

Facilitation Notes

In this activity, students are given a scenario which involves converting degrees Celsius to degrees Fahrenheit and vice versa. The functions are described using different representations, and students identify the units for the independent and dependent quantities as well as the y-intercepts and slopes. Students will revisit this context when they solve literal equations.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Reasoning to determine whether the table and equation convert from °C to °F or °F to °C.

DIFFERENTIATION STRATEGIES

Access for All

Materials Needed: Thermometer

- Display a thermometer with both °F and °C to verify responses.

Challenge Opportunity

- Have students represent both conversions using graphs and compare key characteristics.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How could knowing that 0° F = 32° F help you make sense of the representations? • How did you determine the slope from Joey's table? • How can you tell the y-intercept from each representation? • For this situation, do you think a table or equation is more helpful?
Seeing structure	• What is the relationship between the two rates of change? Why do you think this is the case?

STAMP THE LEARNING

Summary

Properties of functions represented in tables and equations can be compared.

ACTIVITY 4.4

Comparing Descriptions and Graphs

Facilitation Notes

In this activity, students are given a scenario that involves bank accounts. The functions are described using different representations, and students identify the scale of each axis, the meaning of the origin, and the *y*-intercepts and slopes. Note that the growth of the deposits are the focus, interest is not being considered.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

- To extend the activity, ask students to compare the two bank accounts using equations.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you tell the independent quantity and dependent quantity from the scenario? • How did you use the given information to label and scale the axes? • What are the units for each slope? • Did you use the situation or graph to determine Miguel's savings rate? Explain your method. • How did you identify Lauren's savings rate? • Which representation is more straightforward in this context? Why?

Summary

Properties of functions represented in verbal descriptions and graphs can be compared.

STAMP THE LEARNING

Talk the Talk

DEMONSTRATE

FUNCTION MAKER SPACE

Facilitation Notes

In this activity, students create a scenario using a given table of values. They then write an equation containing a slope that is less steep than the relationship shown in the table of values. Next, students compare a graph to the given function.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

continued

COMMON MISCONCEPTION

Students commonly reverse the *x*-values and *y*-values from the table because they have frequently dealt with situations where the *x*-values begin at 0.

For students who transpose the values, encourage them to determine the *y*-value when $x = 0$. Have them consider this point as a starting place for creating a situation.

AS STUDENTS WORK, LOOK FOR

- Confusion as to how to make the $x = -10$ value fit their situation.
- Whether or not to use the *y*-intercept in writing their scenario.

DIFFERENTIATION STRATEGIES

Access for All

- Suggest quantities, such as time and money, for the independent variables. Discuss how the units of these quantities can be changed to determine a situation that is reasonable.
- Ask students for the *y*-value when $x = 0$. Have them consider this point as a starting place for creating a situation.
- When comparing the slopes, suggest using decimal values rather than fractional values.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What information did you derive from the table to create your situation? • Does your situation model a linear relationship? • Does your situation model an increasing or decreasing function? • How does your situation encompass the entire domain? • Did you change the sign of the slope? Why or why not? • How did you determine the slope of the graphed line?
Seeing structure	• Can you tell whether a slope is increasing or decreasing more easily from a table or graph? • What is the advantage of calculating the slope of a line from a table?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

A variety of strategies can be used to compare linear functions presented in different representations.

4 Comparing Linear Functions in Different Forms

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Compare linear functions represented algebraically, graphically, in tables, or with verbal descriptions.
- Choose and interpret appropriate units to represent independent and dependent quantities in situations modeled by linear functions.
- Choose and interpret appropriate scales and origins for graphs of linear functions.

You have represented linear functions in a variety of different ways. How does each linear function representation compare with the others?

Sample answer:

You can represent a linear function using an equation, a table, a graph, or a verbal description.

You can understand characteristics of linear functions, such as slope, *y*-intercepts, and independent and dependent quantities from different representations of functions.

MODULE 2 • TOPIC 2 • LESSON 4 319

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.

Getting Started

Odd One Out

1. Choose the function that does not belong with the others and justify your choice.

Function A

x	y
−5	10
−1.5	3
0	0
2.5	−5

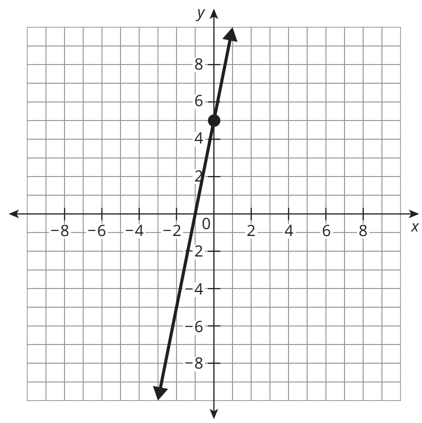
Function B

During one year at a high school, the ratio of 9th-graders to 10th-graders was 1 : 1.

Function C

$c(x) = \frac{1}{5}x$

Function D



Sample answer:

The table is the only one which shows a decreasing function. The graph is the only one which shows a function that has a y-intercept not equal to 0. The equation is the only one that has a slope between 0 and 1. The verbal description is the only one that has a domain that is whole numbers only.

ACTIVITY 4.1 Slopes of Linear Relationships

Consider the line that represents the equation $y = 4x - 3$.

1. Determine whether the graph of each linear relationship is parallel, perpendicular, or neither parallel nor perpendicular to the graph of $y = 4x - 3$.

a.

x	y
-2.5	-6
0	4
0.5	6
3	16

The slope is 4. It is parallel to $y = 4x - 3$.

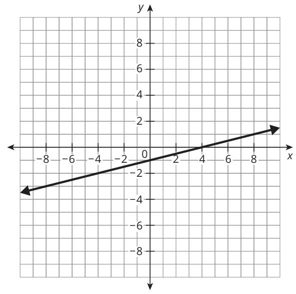
b. An elevator descends 4 feet every second.

Sample answer:
The slope of the elevator moving is -4. It is neither parallel nor perpendicular to $y = 4x - 3$.

c. $y = \frac{1}{4}x + 5$

The slope is $\frac{1}{4}$. It is neither parallel nor perpendicular to $y = 4x - 3$.

d.



The slope is $\frac{1}{4}$. It is neither parallel nor perpendicular to $y = 4x - 3$.

e.

x	y
-4	3
0	2
2	1.5
8	0

The slope is $-\frac{1}{4}$. It is perpendicular to $y = 4x - 3$.

f. For every hour Chiletso bakes, she makes 4 dozen cookies.

The slope is 4. It is parallel to $y = 4x - 3$.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

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MODULE 2 • TOPIC 2 • LESSON 4 321

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

PROBLEM SOLVING

Ask Yourself . . .
How else can you represent this information?

Ask Yourself . . .
Did you complete all the steps in the problem-solving model?

ACTIVITY **4.2**

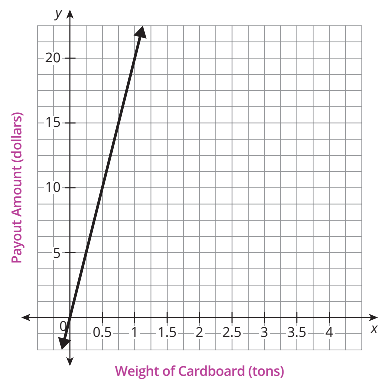
Comparing Tables and Graphs

Large recycling centers pay out different amounts per ton of cardboard recycled. Travis County Recycling Center lists its payout amounts in a table, and Hays County uses a graph to advertise its payout amounts. The two centers' payout amounts are close but not equal.

Travis County Recycling

Weight of Cardboard (tons)	Payout Amount (dollars)
0.5	\$7.50
1	\$15
1.5	\$22.50
2	\$30

Hays County Recycling



1. Each representation shows a functional relationship between quantities. Label the quantities and their units in the table and on the graph.

2. Let $w(x)$ represent the function for Travis County, and let $j(x)$ represent the function for Hays County. Use $>$, $<$, or $=$ to complete each statement.

   a. $w(0.5)$ __$<$__ $j(0.5)$ b. $w(1)$ __$<$__ $j(1)$ c. $w(3)$ __$<$__ $j(3)$

3. Compare the y-intercepts of each relationship. What does each y-intercept mean in terms of the relationship between the quantities?

   For each function, the y-intercept is 0. This means that \$0 is paid out for 0 tons of cardboard.

4. Which recycling center would you choose? Provide evidence to justify your choice.

   I would choose the Hays County Recycling Center because its payout rate is \$20 per ton, while Travis County Recycling Center's payout rate is only \$15 per ton.

EB STUDENT TIP

For all proficiency levels

The term *recycling center* is used in the problem for this activity. Discuss how the prefix *re-* means *to repeat*. Provide other examples, such as *retype* and *retrace*. In this case, *recycle* means *to cycle back and reuse* rather than to *discard*.

ACTIVITY 4.3 Comparing Equations and Tables

Xavier and Joey are friends who live in different countries. When they chat online and talk about the weather, one of them always uses temperature in degrees Celsius while the other talks about temperature using degrees Fahrenheit.

Xavier uses an equation to convert Joey's temperatures, and Joey looks at a table to convert Xavier's temperatures. The equation and part of the table are shown.

Joey's Table

Degrees Fahrenheit	Degrees Celsius
10	−12.2
30	−1.1
50	10
75	23.9

Xavier's Equation

$J(x) = \frac{9}{5}x + 32$

Remember . . .
0°C = 32°F

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

1. Identify quantities in the table and equation. Who is converting from °C to °F, and who is converting from °F to °C? Explain your reasoning.

   In Xavier's equation, the *x* represents the degrees Celsius and *j*(*x*) represents degrees Fahrenheit.

   Joey is converting from degrees Fahrenheit to degrees Celsius. Xavier is converting from degrees Celsius to degrees Fahrenheit.

2. Compare the given characteristics for each function in terms of the quantities.

   a. slope

   The slope for Joey's function is $\frac{5}{9}$. For every 9 degrees increase in degrees Fahrenheit, there is 5 degrees of increase in degrees Celsius. The rate of change for Xavier's function is $\frac{9}{5}$. For every 5 degrees increase in degrees Celsius, there is 9 degrees of increase in degrees Fahrenheit.

   b. y-intercept

   The *y*-intercept for Joey's function is approximately −17.75. This means that 0°F is approximately −17.75°C. The *y*-intercept for Xavier's function is 32. This means that 0°C is 32°F.

Think About . . .
The units of the quantities are very important.

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MODULE 2 • TOPIC 2 • LESSON 4 323

Chunking the Activity

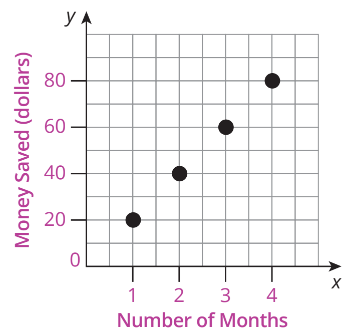
- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Activity 4.4 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice comparing rates of change and *y*- intercepts for functions represented in tables and graphs, and representing domain and range of each function with inequalities, assign Skills Practice Set A for this lesson.

ACTIVITY 4.4 Comparing Descriptions and Graphs

Lauren and Miguel both opened bank accounts at the same time. Miguel's account balance is shown in the graph. He started with $0 and deposited the same amount each month. In the 4th month, he had $80 saved. Michelle opened her bank account on September 1st with $25 and continues to deposit $25 each month.

Miguel



Lauren

Lauren opened her bank account on September 1st with $25 and continues to deposit $25 each month.

1. Use what you know about Miguel's account to determine the scale of each axis and interpret the origin on the graph. Explain your reasoning.

   The origin represents (0 months, 0 dollars saved).

2. Compare the given characteristics for each function in terms of the quantities.

   a. slope

   The rate of change for Miguel's bank account is 20. This means he saves $20 every month. The rate of change for Lauren's bank account is 25. This means she saves $25 every month.

   b. *y*-intercept

   The *y*-intercept for Miguel's bank account is 0. This means he started his bank account with $0. The *y*-intercept for Lauren's bank account is 25. This means she started her bank account with $25.

Ask Yourself . . .
How can you use comparing graphs, tables, and descriptions in everyday life?

EB STUDENT TIP

For all proficiency levels

Students may be unfamiliar with terms such as *bank account*, *account balance*, and *deposit*. Spanish-speakers may benefit from connecting the terms *bank* and *deposit* to their Spanish cognates *banco and depósito*. Assess students' prior knowledge of these terms by asking them to share definitions in their own words and then clarify any discrepancies.

324 MODULE 2 • TOPIC 2 • LESSON 4

Talk the Talk

Function Maker Space

Consider the table of values.

x	−10	10	20	25
y	0	5	7.5	8.75

1. Create a situation to represent the table of values shown.

   Sample answer:

   Alejandra is reading a book. Currently she is at the middle of page 2. Every 10 minutes, she reads another 2.5 pages.

2. Write an equation that has a slope that is less steep than the relationship in the table.

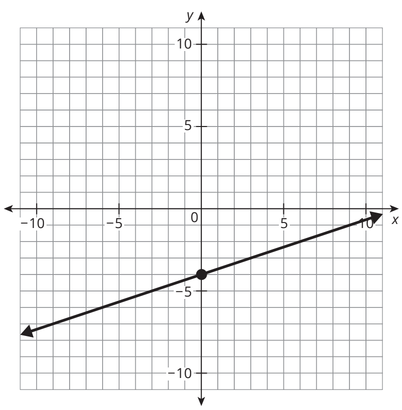
   Sample answer:

   $y = 0.2x + 30$

3. How does the slope of the graph shown compare to the slope from the table of values and your equation?

   Sample answer:

   The slope of the graphed line is 0.333... or $\frac{1}{3}$. It is steeper than the slope from the table and the slope in my equation.



4. What strategies did you use to create your linear functions and to compare the slopes?

   Answers will vary.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

MODULE 2 • TOPIC 2 • LESSON 4 325

326 MODULE 2 • TOPIC 2 • LESSON 4

Lesson 4 Assignment

Write

Describe how to compare the slopes and *y*-intercepts of two linear functions if one is represented as a graph and one is represented as a table.

Remember

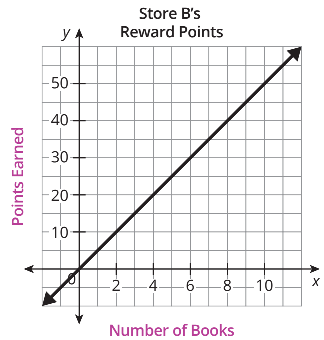
A linear function can be represented using an equation, a table, a graph, or with a verbal description. Characteristics of linear functions, such as slope, *y*-intercepts, and independent and dependent quantities, can be understood from different representations of functions.

Practice

1. Bookstores specializing in selling used books award different amounts of points to customers who supply them with used books. The points are used toward the purchase of other books in the store. Store A lists its point values in a table, and Store B uses a graph to post its point values.

Store A's Reward Points

Number of Books	Points Earned
2	12
4	24
6	36
8	48



a. Label each column of values in the table and label the *x*-axis and *y*-axis on the graph with the appropriate variable quantities.

b. Compare the slope of each function and explain what each represents in context.

The rate of change for Store A is 6. This means they award 6 points for every book traded. The rate of change for Store B is 5. This means they award 5 points for every book traded.

Write

Sample answer:

A linear function is any function that has a straight line for its graph. I can represent linear functions algebraically, graphically, in tables, or with verbal descriptions.

MODULE 2 • TOPIC 2 • LESSON 4 ASSIGNMENT 327

Lesson 4 Assignment

c. Compare the y-intercepts of each function and explain what each represents in context.

For each function, the y-intercept is 0. This means that 0 points are awarded when 0 books are traded.

2. Hailey and Ben live in different cities. They are planning to meet in Nashville, but they will both need to drive several days to get there. They have each calculated the distance to Nashville from their homes, but one calculated the distance in miles and the other calculated the distance in kilometers.

Hailey uses an equation to convert the distances Ben plans to drive each day, and Ben uses a table to convert the distances Sherry plans to drive each day. The equation and part of the table are shown.

Ben's Table

Distance in Miles	Distance in Kilometers
300	482.80
382	614.77
426	685.58
475	764.44

Hailey's Equation

$y = 0.6214x$

a. Label each column of quantities in Ben's table and identify the meaning of x and y in Hailey's equation. Who is converting from miles to kilometers and who is converting from kilometers to miles? Explain your reasoning.

In Hailey's equation, the x represents the distance in kilometers and the y represents the distance in miles.

Ben is converting from miles to kilometers. Hailey is converting from kilometers to miles.

328 MODULE 2 • TOPIC 2 • LESSON 4 ASSIGNMENT

Lesson 4 Assignment

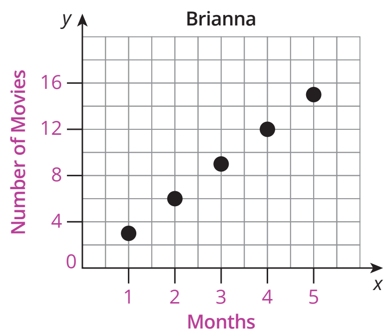
b. Compare the slope for each function. Explain what each slope represents in context.

The rate of change for Ben's function is 1.6094, or 1.6094 kilometers equals 1 mile. The rate of change for Hailey's function is 0.6214, or every 0.6214 mile equals 1 kilometer.

c. Compare the y-intercepts of each function and explain what each y-intercept represents in context.

For each function, the y-intercept is 0. This means that 0 miles is equivalent to 0 kilometers.

3. Antonio and Brianna collect movies. Brianna's movie collection is shown in the graph. She started with 0 movies and added the same number of movies to her collection each month. In the 5th month, she had 15 movies. Antonio's movie collection is given by a description.



Antonio

Antonio started his collection with 27 movies he inherited from his uncle and continues to buy 2 movies each month.

a. Use what you know about Brianna's movie collection to determine the scale of her graph. Label the x- and y-axis, the origin, and the intervals on both axes. Explain your reasoning.

Brianna's graph must show that she collected movies for 5 months and that the size of her collection is 15 movies after 5 months. Since there are only 10 intervals on the y-axis, they must be labeled by 2s.

MODULE 2 • TOPIC 2 • LESSON 4 ASSIGNMENT 329

Lesson 4 Assignment

b. Compare the slope for each function. Explain what each represents in context.

The rate of change for Brianna's movie collection is 3. This means she buys 3 movies every month. The rate of change for Antonio's movie collection is 2. This means he buys 2 movies every month.

c. Compare the *y*-intercepts of each function and explain what each represents in context.

The *y*-intercept for Brianna's movie collection is 0. This means her movie collection started with 0 movies. The *y*-intercept for Antonio's movie collection is 27. This means he started his movie collection with 27 movies.

Prepare

Solve each equation for *x*.

1. $\frac{1}{3}x = 8$

   $x = 24$

2. $5 + x = 12.7$

   $x = 7.7$

3. $2x - 9 = 6$

   $x = 7\frac{1}{2}$

4. $12 + 2x = 3x - 1$

   $x = 13$

330 MODULE 2 • TOPIC 2 • LESSON 4 ASSIGNMENT

TOPIC 2 SELF-REFLECTION

Name: ______________________

Transforming and Comparing Linear Functions

Notes

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Transforming and Comparing Linear Functions* topic by:

TOPIC 2: *Transforming and Comparing Linear Functions*	Beginning of Topic	Middle of Topic	End of Topic
recognizing that the parent function for linear functions is $f(x) = x$.	○	○	○
determining the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $a \cdot f(x)$, $f(x) + d$, $f(x - c)$ and $f(bx)$.	○	○	○
describing the transformations performed on a parent function to create a new function in transformation form $a \cdot f(b(x - c)) + d$.	○	○	○
writing the equation of a line that contains a given point and is parallel or perpendicular to a given line.	○	○	○

continued on the next page

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MODULE 2 • TOPIC 2 • SELF-REFLECTION 331

Notes

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Transforming and Comparing Linear Functions* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

332 MODULE 2 • TOPIC 2 • SELF-REFLECTION

TOPIC 2 SUMMARY

Transforming and Comparing Linear Functions Summary

NEW KEY TERM
- parent function

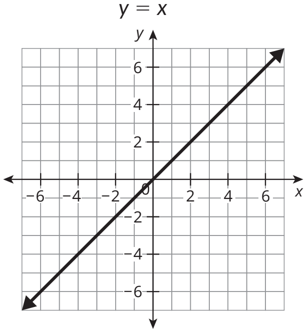
LESSON 1 Transforming Linear Functions

A **parent function** is the simplest function of its type. For example, $f(x) = x$ is the simplest linear function. It is in the form $f(x) = ax + b$, where $a = 1$ and $b = 0$.

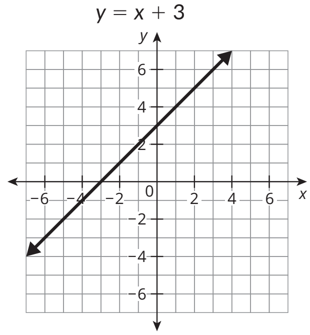
For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values of the function. For $d > 0$, the graph vertically shifts up. For $d < 0$, the graph vertically shifts down. The amount of shift is given by $|d|$.

For example, the function $y = x + 3$ translates the graph of $y = x$ vertically up 3 units.

$y = x$



$y = x + 3$



You can algebraically prove that a line and its translation are parallel to each other.

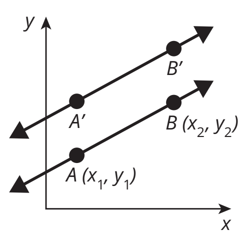
Line AB was translated a units up to create line $A'B'$.

The coordinates of point A' are $(x_1, y_1 + a)$ and the coordinates of point B' are $(x_2, y_2 + a)$.

slope of line $AB = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

slope of line $A'B' = \frac{[(y_2 + a) - (y_1 + a)]}{(x_2 - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

Since the slope of line AB is equal to the slope of line $A'B'$, the lines are parallel.



Notes

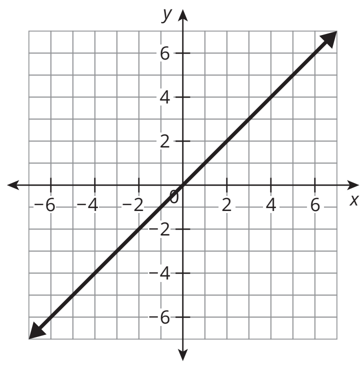
MODULE 2 • TOPIC 2 • TOPIC SUMMARY 333

Notes

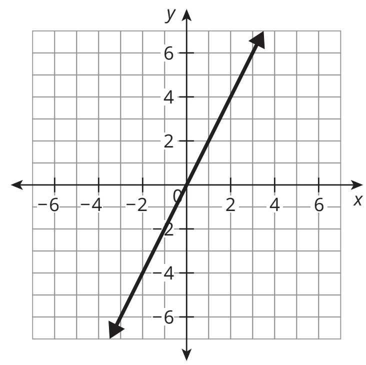
For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ affects the output values of the function. For $|a| > 1$, the graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the graph vertically compresses by a factor of a units. For $a < 0$, the graph reflects across the x-axis.

For example, the function $y = 2x$ dilates the graph of $y = x$ by a factor of 2.

$y = x$



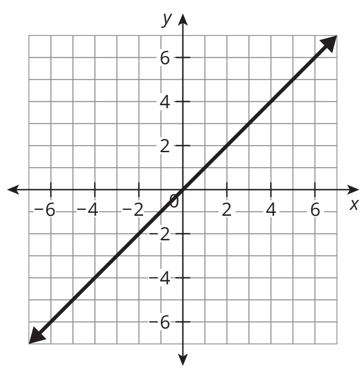
$y = 2x$



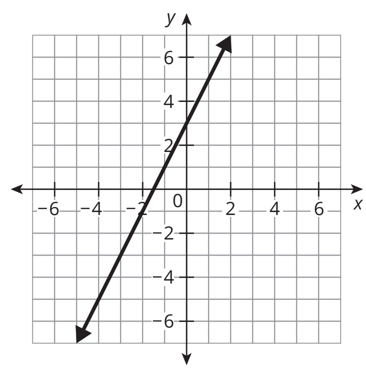
When a function is both translated and vertically dilated, the resulting function can be written in the form $a \cdot f(x) + d$, where d represents the vertical translation of $f(x)$ and a represents the vertical dilation of $f(x)$.

For example, the graph of $y = 2x + 3$ represents both a vertical translation of 3 units and vertical dilation by a factor of 2.

$y = x$



$y = 2x + 3$



334 MODULE 2 • TOPIC 2 • TOPIC SUMMARY

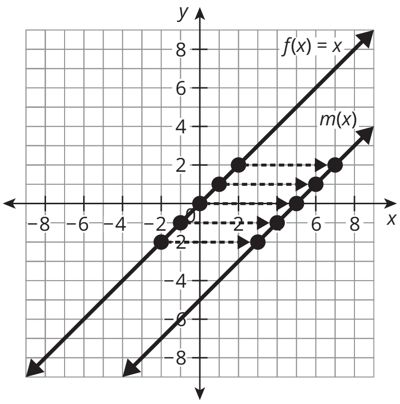
LESSON 2 Vertical and Horizontal Transformations of Linear Functions

For the parent function $f(x) = x$, the transformed function $y = f(x - c)$ affects the input values, or x-values, of the function. For $c > 0$, the resulting graph horizontally shifts right c units. For $c < 0$, the resulting graph horizontally shifts left c units.

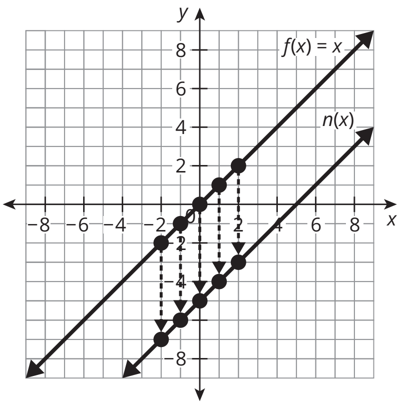
For the parent function $f(x) = x$, the transformed function $y = f(x) + d$ affects the output values, or y-values, of the function. For $d > 0$, the resulting graph vertically shifts up d units. For $d < 0$, the resulting graph vertically shifts down d units.

You have determined that for the parent function $f(x) = x$, a horizontal translation right n units produces the same graph as a vertical translation down n units. In addition, a horizontal translation left n units produces the same graph as a vertical translation up n units.

For example, you can translate the graph of $f(x) = x$ to the right 5 units by moving each point 5 units to the right. The transformed graph is labeled as $m(x)$. $m(x) = f(x - 5)$ which is $m(x) = x - 5$. You can translate the graph of $f(x) = x$ down 5 units by moving each point 5 units down. The transformed graph is labeled as $n(x)$. $n(x) = f(x) + (-5)$ which is $n(x) = x - 5$, thus producing the same equation and graph as $m(x)$.



Original Graph	
x	**f(x)**
−2	−2
−1	−1
0	0
1	1
2	2



Transformed Graph	
x	**m(x)**
3	−2
4	−1
5	0
6	1
7	2

Transformed Graph	
x	**n(x)**
−2	−7
−1	−6
0	−5
1	−4
2	−3

Notes

MODULE 2 • TOPIC 2 • TOPIC SUMMARY 335

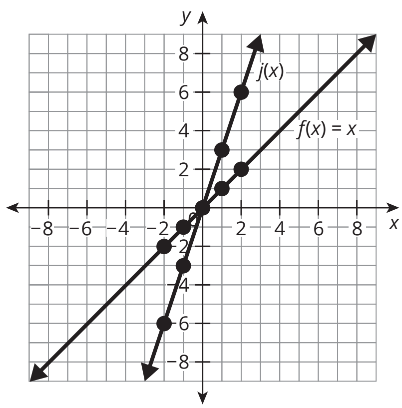
Notes

For the parent function $f(x) = x$, the transformed function $y = a \cdot f(x)$ affects the output values, or y-values, of the function. For $|a| > 1$, the resulting graph vertically stretches by a factor of a units. For $0 < |a| < 1$, the resulting graph vertically compresses by a factor of a units. For $a < 0$, the resulting graph reflects across the x-axis.

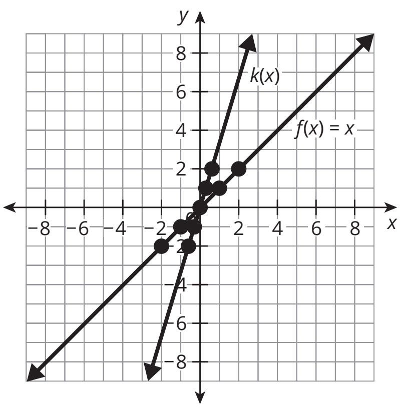
For the parent function $f(x) = x$, the transformed function $y = f(bx)$ affects the input values, or x-values, of the function. For $|b| > 1$, the resulting graph horizontally compresses by a factor of $\frac{1}{|b|}$ units. For $0 < |b| < 1$, the resulting graph horizontally stretches by a factor of $\frac{1}{|b|}$ units. For $b < 0$, the resulting graph reflects across the y-axis.

You have determined that for the parent function $f(x) = x$, a vertical dilation by a factor of n units produces the same graph as a horizontal dilation by a factor of $\frac{1}{n}$ units. In addition, a horizontal dilation by a factor of n units produces the same graph as a vertical dilation by a factor of $\frac{1}{n}$ units.

For example, you can vertically dilate the graph of $f(x) = x$ by a scale factor of 3. The transformed graph, $j(x)$, is a result of vertically stretching the graph of $f(x)$ by a factor of 3 units; $j(x) = 3f(x)$ which is $j(x) = 3x$. You can horizontally dilate the graph of $f(x) = x$ by a scale factor of $\frac{1}{3}$. The transformed graph, $k(x)$, is a result of horizontally compressing the graph $f(x)$ by a factor of $\frac{1}{|b|}$ units or 3 units in this case. $k(x) = f\left(\frac{1}{3}x\right)$,which is $k(x) = 3x$.



Original Graph	
x	**f(x)**
−2	−2
−1	−1
0	0
1	1
2	2



Transformed Graph	
x	**j(x)**
−2	−6
−1	−3
0	0
1	3
2	6

Transformed Graph	
x	**k(x)**
$-\frac{2}{3}$	−2
$-\frac{1}{3}$	−1
0	0
$\frac{1}{3}$	1
$\frac{2}{3}$	2

336 MODULE 2 • TOPIC 2 • TOPIC SUMMARY

LESSON 3

Determining Slopes of Perpendicular Lines

Notes

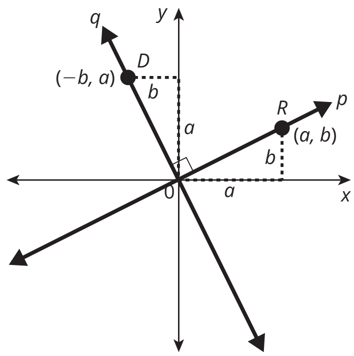
Perpendicular lines or line segments form a right angle at the point of intersection. You can think of perpendicular lines as a line and its rotation 90° about a point, which is also the point of intersection.

The product of the slopes of perpendicular lines is −1.

For example, consider the theorem.

Theorem: If two lines are perpendicular, their slopes are negative reciprocals.

Use the graph and the proof shown to analyze the validity of the theorem.



Given: $p \perp q$

Let m_1 = slope of line p and let m_2 = slope of line q.

Point R lies on line p.

Prove: $m_1 = -\frac{1}{m_2}$

Rotate point R 90° counterclockwise using point O as the center of rotation. Since p and q are perpendicular, the image (point D) will lie on line q under this 90° rotation.

Since this rotation maps the positive x-axis to the positive y-axis and the positive y-axis to the negative x-axis, then the coordinates of $R(a, b)$ are transformed into the coordinates of $D(-b, a)$. Graphically, you can see the movement of lengths a and b under the rotation.

The graph shows the slope of line p, $m_1 = \frac{b}{a}$, and the slope of line q, $m_2 = \frac{a}{-b}$.

Using these slopes, we can demonstrate that $m_1 = \frac{-1}{m_2}$.

$$\frac{b}{a} = \frac{-1}{\frac{a}{-b}}$$
$$= -1 \cdot \frac{-b}{a}$$
$$= \frac{b}{a}$$

The slope of line q is the negative reciprocal of the slope of line p.

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MODULE 2 • TOPIC 2 • TOPIC SUMMARY 337

Notes

The slope of any horizontal line is 0 since no matter the change for x, there is 0 change for y.

$$\frac{0}{x_2 - x_1} = 0$$

The slope of any vertical line is undefined since no matter the change for y, there is 0 change for x.

$\frac{y_2 - y_1}{0}$ is undefined since a value cannot be divided by zero.

All horizontal lines are parallel to each other since their slopes are equal and all vertical lines are parallel since their slopes are equal. A horizontal and a vertical line are always perpendicular to each other.

For example, to write an equation for a line that passes through the point $(-4, -2)$ and is perpendicular to the line $y = 3$, first determine that the line given by $y = 3$ is a horizontal line. Therefore, a line that is perpendicular to $y = 3$ is a vertical line. A vertical line that passes through the point $(-4, -2)$ has the equation $x = -4$.

You can use what you know about the slopes of perpendicular lines and slope-intercept form to write the equation of a perpendicular line.

For example, consider the line $y = 4x - 1$. Write the equation of the line that passes through the point $(-4, 2)$ and is perpendicular to $y = 4x - 1$.

The slope of $y = 4x - 1$ is 4. The slope of the line perpendicular to $y = 4x - 1$ must have a slope of $-\frac{1}{4}$, since $4 \cdot -\frac{1}{4} = -1$.

Using the given point and slope-intercept form, you can set up an equation to solve for b, the y-intercept.

$$2 = -\frac{1}{4}(-4) + b$$
$$2 = 1 + b$$
$$b = 1$$

Therefore, the equation of the line that passes through the point $(-4, 2)$ and is perpendicular to $y = 4x - 1$ is $y = -\frac{1}{4}x + 1$.

338 MODULE 2 • TOPIC 2 • TOPIC SUMMARY

LESSON 4

Comparing Linear Functions in Different Forms

Notes

Functions can be represented using tables, equations, graphs, and with verbal descriptions. Features of linear functions, such as y-intercepts, slopes, and independent and dependent quantities, can be determined from different representations of functions.

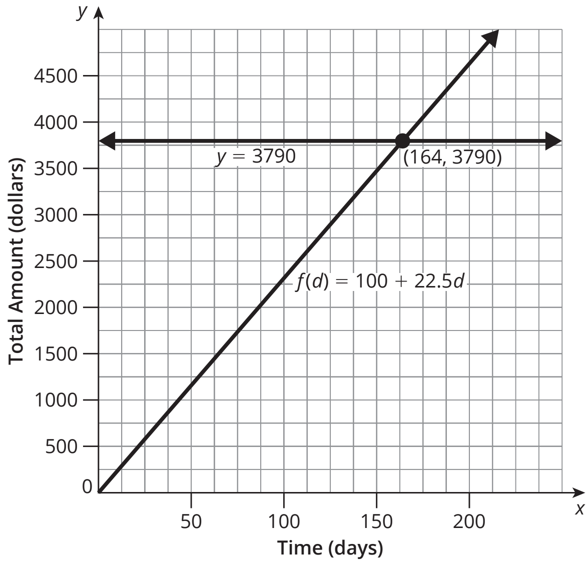
A table can help you calculate solutions given a few specific input values. A graph can help you determine exact solutions if the graph of the function crosses the grid lines exactly. A function can be solved for any value, so any and all solutions can be determined. Technology can allow for more accuracy when using a graph to determine a solution.

For example, suppose Omar had $100 in his car fund. He earns $7.50 per hour at his after-school job. He works 3 hours each day, including weekends. Omar puts all of his earned money in his car fund. How many days will it take him to have enough money to buy a car that costs $3790?

A table can be used to estimate that it will take between 100 and 175 days to buy the car. A graph can be used to estimate that it will take about 160 days to buy the car. A function will give an exact solution. It will take exactly 164 days to buy a car that costs $3790.

d	$100 + 22.50d$
0	100
10	325
20	550
50	1225
100	2350
175	4037.5

$$f(d) = 100 + 22.50d$$
$$3790 = 100 + 22.50d$$
$$3690 = 22.50d$$
$$\frac{3690}{22.50} = \frac{22.50d}{22.50}$$
$$164 = d$$



MODULE 2 • TOPIC 2 • TOPIC SUMMARY 339

Notes

340 MODULE 2 • TOPIC 2 • TOPIC SUMMARY

MODULE 3

Modeling Linear Equations and Inequalities

MODULE 3 OVERVIEW

TEKS* Addressed:
A.2A, **A.2C**, A.2H, **A.2I**, A.3A, **A.3D**, A.3F, A.3G, A.3H, **A.5A**, A.5B, **A.5C**, A.12E

*Bold TEKS = Readiness Standard

Modeling Linear Equations and Inequalities

Sessions: **33**

Why is this module named *Modeling Linear Equations and Inequalities*?

Modeling Linear Equations and Inequalities extends what students know about linear functions by adding more formal notation, writing and solving more complex equations, introducing new strategies for solving systems of equations, and graphing and solving inequalities.

Students review equation-solving strategies, apply those strategies as they write expressions and equations to model a scenario, and then solve the equations to answer questions. *Modeling Linear Equations and Inequalities* teaches equation solving as a tool to solve real-world and mathematical problems involving systems of equations and inequalities. This application allows students to see the purpose of solving equations with variables on both sides, helps them judge the reasonableness of their answers, and prevents them from seeing equation solving as a meaningless algorithm.

The Research Shows . . .

"It is advantageous for students to develop fluency in the use of multiple strategies for solving equations and to develop the ability to select the most appropriate strategy for a given problem."

—*Developing Essential Understanding of Expressions, Equations, and Functions: Grades 6–8,* | Page 71

What is the mathematics of *Modeling Linear Equations and Inequalities*?

Modeling Linear Equations and Inequalities contains two topics: *Linear Equations and Inequalities* and *Systems of Linear Equations* and *Inequalities*. Students explore graphing and solving both linear equations and inequalities. They then apply what they have learned to systems of linear equations and systems of linear inequalities.

1 DAY PACING = 45-MINUTE SESSION

11 SESSIONS
10 LEARNING • 1 ASSESSMENT

TOPIC 1 *Linear Equations and Inequalities*

Learning Together: 7 Sessions

TEKS: **A.2C**, A.3A, **A.5A**, A.5B, A.12E

Students use the properties of equality to justify the steps to solve one-variable equations.

- Students solve more complex equations in one variable.
- Students solve literal equations for variables of interest.
- Students solve linear inequalities in one variable.

Learning Individually: 3 Sessions

Targeted Skills Practice for *Linear Equations and Inequalities*

- Students solve linear equations with variables on both sides involving the distributive property.
- Students write and solve linear equations and inequalities.
- Rewrite linear equations in different forms.
- Students solve literal equations for variables of interest.

22 SESSIONS
21 LEARNING • 1 ASSESSMENT

TOPIC 2 *Systems of Linear Equations and Inequalities*

Learning Together: 15 Sessions

TEKS: **A.2A**, **A.2C**, A.2H, **A.2I**, **A.3D**, A.3F, A.3G, A.3H, **A.5C**

Students build on their current tools for solving systems of equations.

- Students solve systems of equations using graphs, substitution, and linear combinations.
- Students solve systems of linear inequalities using graphs.

Learning Individually: 6 Sessions

Targeted Skills Practice for *Systems of Linear Equations and Inequalities*

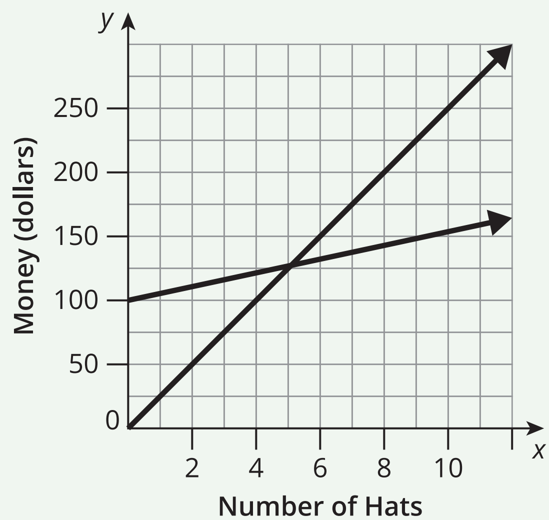
- Students solve systems of linear equations using graphs, substitution, and linear combinations in mathematical and real-world problems.
- Students write systems of inequalities given a table of values.
- Students graph linear inequalities in two variables.
- Students solve systems of linear inequalities in two variables.

How is *Modeling Linear Equations and Inequalities* connected to prior learning?

Students have solved a wide variety of equations and understand the need to maintain balance in an equation. In *Modeling Linear Equations and Inequalities*, they begin to make connections between solutions of equations and the zeros of a function. Previously, students solved systems using graphing, and their understanding of the structure of equations now prepares them to add substitution and linear combinations to their repertoire of strategies.

Math Representation

Jenna plans to sell hats that she knits at the local craft fair. The cost for a booth at the fair is $100, and each hat costs her $5 to make. She plans to sell each hat for $25. Jenna's total cost for selling the hats can be represented by the equation $y \cdot 100 + 5x$, while her income can be represented by the equation $y \cdot 25x$. The graph of each equation is shown. The point of intersection is (5, 125). This point represents the break-even point because it will cost Jenna $125 to make 5 hats, and her income from selling 5 hats will be $125.

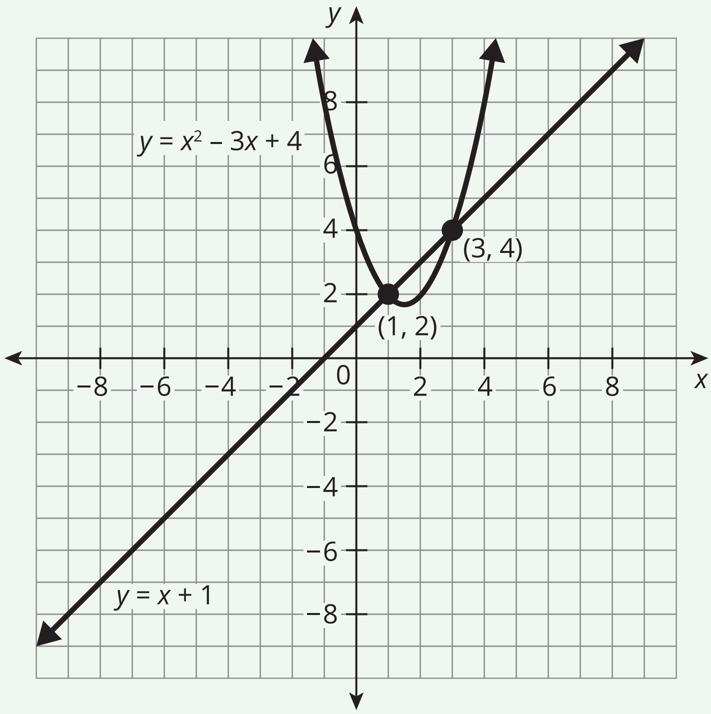


When will students use knowledge from *Modeling Linear Equations and Inequalities* in future learning?

Students will use what they know about solving linear equations to solve more complicated equations later in this course and in future courses. They will apply substitution to solve systems of three linear equations in three variables, and systems of linear and quadratic equations in a future course.

Math Representation

You can solve the following system of two equations in two variables algebraically and then verify the solution graphically.



$$\begin{cases} y = x + 1 \\ y = x^2 - 3x + 4 \end{cases}$$

$x^2 - 3x + 4 = x + 1$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$ or $x = 1$

Substitute $x = 3$ into the linear equation.

$y = 3 + 1 = 4$

Substitute $x = 1$ into the linear equation.

$y = 1 + 1 = 2$

The solutions to the system are (3, 4) and (1, 2).

3 Modeling Linear Equations and Inequalities

MODULE 3: Assessment Summary

Topic	Topic Title	Name	Administered	TEKS*
1	Linear Equations and Inequalities	End of Topic Assessment	After Topic 1	**A.2C** **A.5A** A.5B A.12E
2	Systems of Linear Equations and Inequalities	End of Topic Assessment	After Topic 2	A.2H **A.2I** **A.3D** A.3F A.3G A.3H **A.5C**

*Bold TEKS = Readiness Standard

TOPIC 1

A region can be described as above, below, to the left, or to the right of a line.

Linear Equations and Inequalities

Linear Equations and Inequalities

How are the key concepts of *Linear Equations and Inequalities* organized?

Linear Equations and Inequalities has students focus on the algebraic representation of a linear function, a linear equation, or inequality. Students solve equations in one variable, examining the structure of each to predict whether the equation has one solution, no solution, or infinite solutions. They use the properties of equalities and basic number properties to construct a viable argument to justify a solution method. Students play Tic-Tac-Bingo to practice solving equations, specifically trying to build equations with different types and numbers of solutions. They generalize their knowledge of solving equations in one variable to solve literal equations for given variables, enabling them to connect the meaning of the variables in the standard form and the slope-intercept form of a linear equation.

Math Representation

Isabella and Ethan each convert the formula $C = \frac{5}{9}(F - 32)$ to degrees Fahrenheit.

Isabella

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$\frac{9C}{5} + \frac{160}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 32 = F$$

First, Isabella distributed the $\frac{5}{9}$ to the F and the 32. Next, she multiplied each side by 9. Then, she added 160 to both sides. Finally, she isolated F by dividing by 5.

Ethan

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - 32$$
$$9(C) = 9\left(\frac{5}{9}F - 32\right)$$
$$9C = 5F - 288$$
$$9C + 288 = 5F$$
$$\frac{9C}{5} + \frac{288}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 57.6 = F$$

Ethan's error occurs in the second line. He did not distribute the $\frac{5}{9}$ to the 32.

Students then explore linear inequalities. They graph linear inequalities and explore solving an inequality with a negative slope, which affects the sign of the inequality. They use a horizontal line to solve an inequality for a given constraint and graph the solution set on a number line.

Math Representation

Analyze Samuel's solution strategy.

Samuel

$$-\frac{1}{2}x + \frac{3}{4} < 2$$

$$-4\left(-\frac{1}{2}x + \frac{3}{4} < 2\right)$$

$$2x - 3 > -8$$

$$2x > -5$$

$$x > \frac{-5}{2}$$

$$x > -2.5$$

Samuel multiplied by −4 on both sides to eliminate the fractions. When he did this, he reversed the inequality symbol. Then, he solved the equation similar to solving any two-step equation.

What is the entry point for students?

Coming into this course, students have solved two-step equations with variables on both sides. They understand the underpinnings of solving equations by maintaining equality. From this intuitive understanding, students learn to use properties to justify each step in the equation-solving process. They also use this understanding to solve literal equations for given variables, connecting back to the formulas they have used for area, volume, and surface area to solve for unknown values. Previously, students solved two-step inequalities and graphed the solutions on a number line. Students build from their knowledge in this topic when they solve more complex inequalities in one variable.

Throughout the topic, students start by examining graphical representations of each concept and using what they know about the structure of graphs to solve problems and make connections to the algebraic solution process.

Why is *Linear Equations and Inequalities* important?

In previous courses, students developed fluency in solving linear equations in one variable. In this topic, students formalize their reasoning for each step of the solution process, and they extend their understanding of equations to solve linear inequalities. By recognizing the connections between algebraic and graphical solutions to an equation or inequality, students are better prepared to solve more complicated equations. Additionally, maintaining equivalence in two expressions is an important skill that students will use repeatedly throughout higher level mathematics.

Math Representation

Consider this linear absolute value equation.

$$|a| = 6$$

There are two points that are 6 units away from zero on the number line—one to the right of zero, and one to the left of zero.

$$+(a) = 6 \quad \text{or} \quad -(a) = 6$$
$$a = 6 \quad \text{or} \quad a = -6$$

Now, consider the case where $a = x - 1$.

$$|x - 1| = 6$$

If you know that you can write $|a| = 6$ as two separate equations, you can rewrite any absolute value equation.

$$+(x - 1) = 6 \quad \text{or} \quad -(x - 1) = 6$$
$$(x - 1) = 6 \quad \text{or} \quad (x - 1) = -6$$

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Linear Equations and Inequalities* when they can:

- Solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.
- Construct an argument to justify a solution process for a one-variable equation.
- Use and interpret units when solving literal equations for a given variable.
- Solve literal equations for a specified variable.
- Recognize linear relationships from problem situations and write equations or inequalities to model linear relationships.
- Solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.
- Interpret solutions in the context of a problem situation and determine whether they are reasonable.
- Represent constraints to a problem situation as an inequality.

NEW KEY TERMS

- solution [solución]
- infinite solutions [soluciones infinitas]
- no solution [sin solución]
- literal equation [ecuación literal]
- linear inequality [desigualdad lineal]

How do the activities in *Linear Equations and Inequalities* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards are evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Students reason quantitatively as they decontextualize problem situations to write linear equations and inequalities, and they then contextualize the solutions (A.1A). They examine the structure of equations to predict the number and type of solutions (A.1A). They construct viable arguments to justify solution methods, explaining each step as following from the equality of numbers asserted at the previous step (A.1G). Students model problem scenarios with graphs, equations, and inequalities, and they connect the solutions in each representation (A.1D).

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Use graphic organizers like charts, diagrams, and concept maps, along with icons, drawings, and visuals, to bridge cognates and their shared meanings together. Visual aids and organizers can help anchor new learning as students build meaning and synthesize new mathematical concepts.

3 Modeling Linear Equations and Inequalities

TOPIC 1: Linear Equations and Inequalities

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1D, A.1F, A.1G

ELPS: 1.H, 2.B, 2.C, 2.H, 3.A, 4.B, 4.D, 4.J, 5.D, 5.E

Topic Pacing: 11 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Solving Linear Equations	Students start with a simple solution statement and create more complex equations by performing the same operation on each side of the equation. They then analyze different equations created by two students and reason about how to verify that the equations have the same solution as the original equation. The properties of equality and some basic number properties are reviewed before students practice solving linear equations and justifying their steps. They also compare the different properties two students used to solve the same equation. Next, students investigate a mathematical statement that is always true and a mathematical statement that is always false. The terms *no solution* and *infinite solutions* are defined. Finally, students play Tic-Tac-Bingo as they work together to create equations with given solution types from assigned expressions. They then summarize strategies for determining if an equation has no solution or infinite solutions. **Materials needed:** Expressions Cards	**A.5A**	2
2	Literal Equations	Students begin with a perimeter problem in context to address solving formulas for different variables. They then identify the slope, *x*-intercept, and y-intercept of linear equations in slope-intercept, point-slope, and standard form and consider which form is most efficient in determining these characteristics. Next, the term *literal equation* is defined. The common literal equation for converting degrees Fahrenheit to degrees Celsius is provided. Students rewrite the formula to convert degrees Celsius to degrees Fahrenheit, identify errors in student work when rewriting the formula, and interpret equivalent equations written in standard form. The lesson concludes by having students solve various literal equations for specific variables. **Materials needed:** None	A.2B A.3A A.12E	2

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	**Modeling Linear Inequalities**	Students begin with a scenario and table that can be modeled by a *linear inequality* with a positive rate of change. They then analyze a graph that models the situation. Students use that graph to solve inequalities and graph the solution set on a number line. Next, the term *solve an inequality* is defined, and students write and solve inequalities algebraically, taking into account the context of the problem situation. They then analyze an inequality with a negative rate of change to make sense of how the sign of the solution to the inequality is affected. Lastly, students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation, and ones that require the distributive property. **Materials needed:** None	**A.2C** A.5B	3
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				3

*Bold TEKS = Readiness Standard

MODULE 3, TOPIC 1 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: **A.5A** **LESSON 1** Solving Linear Equations **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 1** continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.3A, A.12E **LESSON 2** Literal Equations **GETTING STARTED** **ACTIVITY 1**	**LESSON 2** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.2C**, A.5B **LESSON 3** Modeling Linear Inequalities **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 3** continued **ACTIVITY 4** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 11				
END OF TOPIC ASSESSMENT				

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are three Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Solving Linear Equations

MATERIALS

Expression Cards (located at the end of these Facilitation Notes)

LESSON OVERVIEW

Students start with a simple solution statement and create more complex equations by performing the same operation on each side of the equation. They then analyze different equations created by two students and reason about how to verify that the equations have the same solution as the original equation. The properties of equality and some basic number properties are reviewed before students practice solving linear equations and justifying their steps. They also compare the different properties two students used to solve the same equation. Next, students investigate a mathematical statement that is always true and a mathematical statement that is always false. The terms *no solution* and *infinite solutions* are defined. Finally, students play Tic-Tac-Bingo as they work together to create equations with given solution types from assigned expressions. They then summarize strategies for determining whether an equation has no solution or infinite solutions.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions.

The student is expected to:

A.5A solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ELPS

(1) Learning Strategies

The student is expected to:

(H) develop and expand repertoire of learning strategies such as reasoning inductively or deductively, looking for patterns in language, and analyzing sayings and expressions commensurate with grade-level learning expectations.

(2) Listening

The student is expected to:

(B) recognize elements of the English sound system in newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters.

(H) understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations.

(4) Reading

The student is expected to:

(B) recognize directionality of English reading such as left to right and top to bottom.

ESSENTIAL IDEAS

- A solution to an equation is any variable value that makes that equation true.
- Solving equations requires the use of number properties and the properties of equality.
- The properties of equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality.
- When the properties of equality are applied to an equation, the transformed equation will have the same solution as the original equation.
- Equations with infinite solutions are created by equating two equivalent expressions.
- Equations with no solution are created by equating expressions of the form $mx + b$ with the same value for m and different values for b.
- Equations with a solution $x = 0$ are created by equating expressions of the form $mx + b$ with different values for m and the same value for b.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Equation Creation 15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students start with a simple solution statement and create more complex equations by performing the same operation on each side of the equation. They then exchange the equations they created with a partner to verify correct solutions.

DEVELOP

Activity 1.1: Using Properties to Justify Solutions 15–20 minutes

PEER WORK ANALYSIS, MATHEMATICAL PROBLEM SOLVING

Students use peer work analysis to compare two different equations built from the same solution statement and explain how to verify that both equations are equivalent to the solution statement. They review the properties of equality and some basic number properties before practicing solving linear equations and justifying each step. Students compare the different properties used by two students to solve the same equation and conclude that there is more than one way to determine a solution.

Activity 1.2: Solutions to Linear Equations 10–15 minutes

INVESTIGATION, MATHEMATICAL PROBLEM SOLVING

Students are given a mathematical sentence that is always true and one that is always false. They choose any variable or constant and use the properties of equality to investigate ways to change the outcome of the given number sentences. Students reason that the mathematical sentence that is always true is still always true and that one that is false is still false. The terms *no solution* and *infinite solutions* are defined, and students relate the terms back to the equations they previously created.

DAY 2

Activity 1.3: Tic-Tac-Bingo 35–40 minutes

MATHEMATICAL PROBLEM SOLVING

Students fill in their Tic-Tac-Bingo board with possible solution types, similar to a Bingo game. After being provided with an expression, they work with other students to create and solve equations. Students compete to be the first person to fill in a row, column, or diagonal on their board. They then summarize strategies for determining whether an equation has no solution or infinite solutions.

DEMONSTRATE

Talk the Talk: One Step at a Time 5–10 minutes

EXIT TICKET PROCEDURES

Students give the properties that justify each step taken to solve an equation.

Getting Started

ENGAGE

Equation Creation

Facilitation Notes

In this activity, students start with a simple solution statement and create more complex equations by performing the same operation on each side of the equation. They then exchange the equations they created with a partner to verify correct solutions.

Ask a student to read the introduction and definition. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Errors when dividing each side of the equation by the same constant. Sometimes, students divide only the variable terms by the constant rather than all terms.
- Confusion when the resulting equation has fractions. While it is acceptable to have equations with fractions, some students may want to avoid fractions. If that is the case, take advantage of the opportunity to demonstrate how multiplying all terms of the equation by the denominator eliminates the fraction. This practice is another strategy to solve Sofia's equation in the next activity.

COMMON MISCONCEPTION

Students may confuse multiplying to create an equation with their experiences solving equations and only multiply x by the constant, instead of multiplying the entire equation.

Help students understand why they need to multiply each term by relating the process of creating multiple groups of expressions.

DIFFERENTIATION STRATEGY

Just in Time Support

Materials Needed: Algebra Tiles

Have students build their equations with algebra tiles so they can visually see what is happening as they build their equation.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Which property did you use when you multiplied by a constant on the side of the equation with two terms?
Probing	• What is another possible equation? • Demonstrate how you divided each side of the equation by a constant.

Summary

An equation created by performing the same operation on each side of a solution statement will have the same solution as the starting statement.

STAMP THE LEARNING

ACTIVITY 1.1

Using Properties to Justify Solutions

DEVELOP

Facilitation Notes

In this activity, the properties of equality and some basic number properties are reviewed. Students verify the accuracy of two different equations built from the same solution statement using substitution and properties of equality. They solve equations and analyze student work demonstrating different properties used to solve equations.

DIFFERENTIATION STRATEGIES

Assistance for All

Materials needed: Poster Board or Butcher Paper

Begin the lesson by analyzing both tables in Activity 1.1: the properties of equality and the number properties.

- Have students create posters of the properties as reference for the next several lessons. These lessons involve solving linear equations, literal equations, linear inequalities, and compound inequalities.
- While the focus is on using acceptable strategies when solving equations, students may be confused as to why there are two separate lists, one for properties of equality and one for number properties. Explain that properties of equality involve bringing additional terms and operations into an equation, while number properties provide rules for rewriting already existing expressions.

Just in Time Support

- Review key terms from previous courses such as *expression*, *constant*, and *operation*.

Ask a student to read the introduction aloud and review the properties of equality as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain what each property means in your own words.
Probing	• Show how you applied each property in the Getting Started activity. • When using substitution, what result will verify that 2 is the solution? • Explain the steps you used to verify the solution using the properties of equality. • What is another way to apply the properties of equality and arrive at the same solution?

DIFFERENTIATION STRATEGIES

Assistance for All

Using Sofia's strategy, model how to rewrite the equation with integer coefficients by scaling up the fraction.

$$\begin{gathered} 2x = 5 - \tfrac{1}{2}x \\ 2\left(2x = 5 - \tfrac{1}{2}x\right) \\ 4x = 10 - x \end{gathered}$$

Have students continue the process to verify the equation is equivalent to $x = 2$.

Just in Time Support

Materials Needed: Algebra Tiles

To assist students in solving Harper's equation, model solving the equation with algebra tiles. Discuss how the properties of equality are represented with the tiles. Then, discuss why this strategy does not work for Sofia's strategy to emphasize the importance of moving beyond the model.

Ask a student to read the information following Question 1 aloud and review the number properties as a class.

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- The result of $\frac{1}{4}$ rather than $-\frac{1}{4}$ for Question 2 part (a), often signifying a strategy error rather than just a sign error.

Correct

$$\begin{array}{rcl} 24x + 7 & = & -4x \\ +\,4x & & +\,4x \\ \hline 28x + 7 & = & 0 \\ -7 & & -7 \\ \hline \dfrac{28x}{28} & = & \dfrac{-7}{28} \\ x & = & \dfrac{-1}{4} \end{array}$$

Incorrect

$$\begin{array}{rcl} 24x + 7 & = & -4x \\ +\,4x & & +\,4x \\ \hline \dfrac{28x}{28} & = & \dfrac{7}{28} \\ x & = & \dfrac{1}{4} \end{array}$$

Students skip writing $= 0$ and move the equals sign between the remaining terms.

- A variety of correct solution paths.

DIFFERENTIATION STRATEGIES

Assistance for All

Have students write the word *example* above the Thumbs Up strategy as an indicator that it is a correct solution to reference later. As students read through the strategy and think about the connections, suggest they ask themselves:

- Why is this method correct?
- Have I used this method before?

Just In Time Support

To scaffold support, add intermediate steps in Daniel's and Chloe's strategies.

COMMON MISCONCEPTION

Students' first step might be to use the addition property of equality to rewrite the equation as $28x + 7 = 0$. This step is mathematically correct but may lead students to incorrectly rewrite the equation as $28x = 7$.

Remind students that to create equivalent equations, they must apply the properties of equality. Have students reflect on the properties that they used and whether they applied them correctly.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Create an example with numbers for each number property. • What is the difference between the commutative and associative properties?
Probing	• What will your steps look like if you add 4x to both sides of the equation in part (a)? • What strategy did you apply to address the fraction in part (c)? • Why might someone multiply both sides of the equation by 2? • Why do you think Chloe chose 4 as the divisor? • Whose method do you prefer? Why?

Summary

STAMP THE LEARNING

The properties of equality as well as basic number properties are useful in solving equations and justifying the solution.

ACTIVITY 1.2

Solutions to Linear Equations

Facilitation Notes

In this activity, students are given a mathematical sentence that is always true and one that is always false. They choose any variable or constant and use the properties of equality to investigate ways to change the outcome of the given number sentences. Students reason that the mathematical sentence that is true is always true and that one that is false is always false. The terms *no solution* and *infinite solutions* are defined, and students relate the terms to the equations they previously created.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

COMMON MISCONCEPTION

Students may misinterpret statements, such as $8 = 8$ and $4 = 4$, as meaning $x = 8$ and $x = 4$ rather than understanding that these equalities demonstrate the substituted value is a solution. Discuss explicitly what students may be thinking and clarify any inaccuracies in their thoughts.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Demonstrate the results of applying each property of equality.
Seeing structure	• Explain what this question demonstrates.

Ask a student to read the information following Question 2 aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Provide examples to support your conclusion. • Use substitution to verify your solution.

DIFFERENTIATION STRATEGY

Challenge Opportunity

To extend the activity, ask students to create equations with 0, 1, or infinite solutions.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

A linear equation can have one solution, no solution, or infinite solutions.

ACTIVITY 1.3

Tic-Tac-Bingo

Facilitation Notes

In this activity, students fill in their Tic-Tac-Bingo board with possible solution types, similar to a Bingo game. After each student is provided with one expression, they pair with other students to create equations, determine the solution type, and mark that solution type on their game card. Students compete to be the first person to fill in a row, column, or diagonal on their board. They then summarize strategies for determining if an equation has no solution or infinite solutions.

In advance of the lesson, cut out the expression cards located at the end of the facilitation notes for this activity, or write the expressions on index cards so they can be reused for multiple classes.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have a student read the directions to the game.

- Have each student prepare a game board in ink. Check that all game boards are completed according to directions.
- Ask students to explain the directions in their own words. If there is confusion, model how the game is played by having 2 students complete a mini-game with this information:

  Student 1: $2x + 3$
  Student 2: $-2x + 5$

  If Student 1 and Student 2 pair up, their equation would be $2x + 3 = -2x + 5$. The solution would be $x = \frac{1}{2}$. Student 1 and Student 2 would document this exchange on their game cards in an appropriate box:

Positive rational solution
Equation: $2x + 3 = -2x + 5$
Solution: $x = \frac{1}{2}$

- Determine ground rules. Questions to consider: Are you giving a time limit? Do you want students to complete the game in silence? Will you stop play when the first student wins to have the class check their card and then resume play?

Distribute the expression cards and play the game.

- Distribute an expression card to each student. Do not allow them to look at their cards until everyone has a card.
- Have everyone stand up, move around the classroom, and create equations!

AS STUDENTS WORK, LOOK FOR

Students that are strategically completing their cards using the fact that integers are also considered positive or negative rational numbers.

DIFFERENTIATION STRATEGY

Just in Time Support

Provide students with a subset of the expressions. Have them write six equations, one with each type of solution: positive rational solution, negative rational solution, nonzero integer solution, a solution that is not positive or negative, no solution, and infinite solutions.

Complete Question 1 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is an example of a nonzero integer solution? A positive rational solution that is not an integer?
Probing	• What strategy did you use to locate matches for your game card efficiently?
Seeing structure	• What type of solution will each of these equations have? Why? $2x + 5 = 2x + 5$ $2x + 5 = 2x + 7$ $2x + 5 = 3x + 5$ $2x + 5 = 3x + 7$ • How can you generalize the structure of these equation types and their type of solution?

DIFFERENTIATION STRATEGY

Just in Time Support

Provide students with examples of the solution types, such as infinite solutions, no solutions, one solution, etc. Have students identify the equation type by comparing coefficients and constants.

STAMP THE LEARNING

Summary

A linear equation can have one solution, no solution, or infinite solutions.

DEMONSTRATE

Talk the Talk

ONE STEP AT A TIME

Facilitation Notes

In this activity, students identify the properties that justify each step taken to solve an equation.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know when to apply the distributive property? • When did you use the division property of equality? • Did everyone use the same properties regardless of how they solved the equation? Explain why or why not.

Have students read and answer the Essential Question on the lesson opener page.

Summary

The properties of equality as well as basic number properties are useful in solving equations and justifying the solution.

STAMP THE LEARNING

Expression Cards

	1	2	3	4	5	6
A	$x + 3(x + 1)$	$2x + 2(x + 2) - 5$	$-2(3x - 1) + 3$	$2(1 - 3x)$	$8x - 3(2x - 1)$	$\frac{(2x - 7)}{2} + \frac{(2x - 3)}{2}$
B	$3(x - 2) + 2$	$9x - 2(3x + 1) + 7$	$-4(2x - 2) + 2(2x - 3)$	$-2(2x + 3) + 1$	$2(x - 2) + 3(x + 1)$	$5(x - 1) + 1$
a	$\frac{(12x + 9)}{3}$	$3(x - 1) + (x + 2)$	$3(2x + 2) - 4\left(3x + \frac{1}{4}\right)$	$2x - 2(4x - 1)$	$\frac{4x + 5}{2} + \frac{1}{2}$	$3 + 2(x - 4)$
b	$x + 2(x - 2)$	$2(4x + 5) - 5(x + 1)$	$-2(2x - 1)$	$(4x + 1) - 2(4x + 3)$	$5\left(x - \frac{1}{5}\right)$	$-2(x + 2) + 7x$

Why is this page blank?

So you can cut out the Expression Cards on the other side

1 Solving Linear Equations

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Write equivalent equations using properties of equality.
- Use properties of equality to solve linear equations and justify a solution method.
- Determine whether an equation has one solution, no solution, or infinite solutions.
- Solve linear equations with variables on both sides.

NEW KEY TERMS

- solution
- no solution
- infinite solutions

You know that equations are one way to represent a linear function, and you have used equations to evaluate linear functions for a given input value.

How can you use equations to solve for unknown input values of a function?

Sample answer:

I can substitute a known output value into an equation representing a linear function.

Then, I can solve the equation to find the unknown input value.

MODULE 3 • TOPIC 1 • LESSON 1 345

Chunking the Activity

- Read and discuss the introduction and definition.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The paragraph and definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Getting Started

Equation Creation

An *equation* is a mathematical sentence that uses an equals sign to show that two expressions are equivalent. When one of those expressions contains a variable, you can solve the equation.

A **solution** to an equation is a value for the variable that makes the equation a true statement.

Consider the equation $x = 2$. You can substitute the value 2 for x to create the true statement $2 = 2$. Because this is the only value that makes the statement true, 2 is the only solution to the equation $x = 2$.

By performing the same operation on each side of an equation, you can create more complex equations that have the same solution.

1. Consider the equation $x = 2$.

   a. Choose any constant. Add that constant to each side of the equation and simplify.

   Sample answer:

   $x + 3 = 5$

   b. Choose any constant other than 0. Multiply each side of the equation you created in part (a) by that constant and simplify.

   Sample answer:

   $4x + 12 = 20$

   c. Choose any number other than 0 to represent a in the expression ax. Subtract the term ax from each side of the equation you created in part (b) and simplify.

   Sample answer:

   $2x + 12 = -2x + 20$

   d. Choose any constant other than 0. Divide each side of the equation you created in part (c) by that constant and simplify.

   Sample answer:

   $x + 6 = 10 - x$

Think about . . .
What strategies can you use to verify a solution?

2. Have a partner solve the equation you created in Question 1 to verify that $x = 2$ is the solution.

   Students should either substitute the value 2 for the variable in their partner's equation to verify it has the solution $x = 2$ or use what they know about solving equations to isolate the variable and see that $x = 2$.

346 MODULE 3 • TOPIC 1 • LESSON 1

ACTIVITY **1.1**

Using Properties to Justify Solutions

Recall that the properties of equality are rules that allow you to maintain balance and rewrite equations to isolate the variable.

Properties of Equality	For all numbers ***a***, ***b***, and ***c***
addition property of equality	If $a = b$, then $a + c = b + c$.
subtraction property of equality	If $a = b$, then $a - c = b - c$.
multiplication property of equality	If $a = b$, then $ac = bc$.
division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Harper and Sofia both created new equations starting from the solution statement $x = 2$.

Harper

$-3x + 14 = 18 - 5x$

Sofia

$2x = 5 - \frac{1}{2}x$

1. Verify that both equations are equivalent to $x = 2$ using the given strategy.

   a. substitution

   Harper

   $-3(2) + 14 = 18 - 5(2)$

   $-6 + 14 = 18 - 10$

   $8 = 8$

   Sofia

   $2(2) = 5 - \frac{1}{2}(2)$

   $4 = 5 - 1$

   $4 = 4$

   b. properties of equality

   Harper

   $-3x + 14 = 18 - 5x$

   $-3x + 14 - 14 = 18 - 5x - 14$

   $-3x + 5x = 4 - 5x + 5x$

   $2x = 4$

   $\frac{2x}{2} = \frac{4}{2}$

   $x = 2$

   Sofia

   $2x = 5 - \frac{1}{2}x$

   $2x + \frac{1}{2}x = 5 - \frac{1}{2}x + \frac{1}{2}x$

   $\frac{5}{2}x = 5$

   $\frac{2}{5}\left(\frac{5}{2}x\right) = \frac{2}{5}(5)$

   $x = 2$

Chunking the Activity

- Read and discuss the properties of equality.
- Group students to complete Question 1.
- Check in and share.
- Read and discuss the number properties.
- Group students to complete Questions 2 and 3.
- Share and summarize.

STAMP THE LEARNING

The table of properties provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For all proficiency levels

Discuss the term *substitute*. Discuss how in math, numbers are *substituted* for variables to determine the value of an expression. Relate this to how a *substitute* teacher replaces the regular teacher when they are absent.

STAMP THE LEARNING

The table of properties provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 2 and 3 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice solving equations, assign Skills Practice Set A for this lesson.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1F:

- Can students identify and explain mathematical relationships?

There are also basic number properties that you are already familiar with that can be used to justify your steps when solving equations.

Number Properties	For all numbers a, b, and c
commutative property	$a + b = b + a$ $ab = ba$
associative property	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$
distributive property	$a(b + c) = ab + ac$

Justifying your steps with the properties of equality will better prepare you for writing geometrical proofs in future courses.

2. Solve each equation and check your solution. Write the properties that justify each step of your solving strategy.

a. $24x + 7 = -4x$

Sample answer:

$24x + 7 = -4x$

$24x - 24x + 7 = -4x - 24x$ subtraction property of equality

$\frac{7}{-28} = \frac{-24x}{-28}$ division property of equality

$x = -\frac{1}{4}$

b. $0.6(2x + 1) = 4x + 6.2$

Sample answer:

$0.6(2x + 1) = 4x + 6.2$

$1.2x + 0.6 = 4x + 6.2$ distribution property

$1.2x - 4x + 0.6 = 4x - 4x + 6.2$ subtraction property of equality

$-2.8x + 0.6 - 0.6 = 6.2 - 0.6$ subtraction property of equality

$\frac{-2.8x}{-2.8} = \frac{5.6}{-2.8}$ division property of equality

$x = -2$

c. $\frac{1}{2}x - 6 = 2 + (2x + 1)$

Sample answer:

$\frac{1}{2}x - 6 = 2 + (2x + 1)$

$\frac{1}{2}x - 6 = 2 + (1 + 2x)$ commutative property

$\frac{1}{2}x - 6 = (2 + 1) + 2x$ associative property

$\frac{1}{2}x - \frac{1}{2}x - 6 = 3 + 2x - \frac{1}{2}x$ subtraction property of equality

$-6 - 3 = 3 - 3 + \frac{3}{2}x$ subtraction property of equality

$\frac{2}{3}(-9) = \frac{2}{3}\left(\frac{3}{2}x\right)$ multiplication property of equality

$x = -6$

SELF-MONITORING STRATEGIES

Using self-motivation and self-discipline to persevere in problem-solving. Refer to the Course and Implementation Guide for further details on these look-fors.

3. Compare the solution strategies used by Daniel and Chloe. What do you notice about the properties each student used?

Daniel	Chloe
$4(3x+2)=8x+4$	$4(3x+2)=8x+4$
$12x+8=8x+4$	$\frac{4(3x+2)}{4}=\frac{8x+4}{4}$
$4x=-4$	$3x+2=2x+1$
$x=-1$	$x=-1$

Sample answer:
Daniel first used the distributive property, and Chloe first used the division property of equality. Both students used the subtraction and division properties of equality to finish solving the equation.

ACTIVITY 1.2 Solutions to Linear Equations

In the Getting Started, you built equations from $x = 2$. In this activity, you will build equations from two mathematical sentences and compare your results.

1. Consider the mathematical sentence $2 = 2$.
   Is there any variable or constant that you could add, subtract, multiply, or divide to both sides using the properties of equality to make this true sentence false? Choose variables and constants to create new mathematical sentences to justify your conclusion.

   Sample answer:

   $2x = 2x$

   $2x + 5 = 2x + 5$

   There is no variable or constant that I can add, subtract, multiply, or divide to both sides to make the sentence false. If I follow the properties of equality, the sentence is always true.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Read and discuss the information.
- Group students to complete Questions 3 and 4.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining the number of solutions to an equation, assign Skills Practice Set B for this lesson.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Discuss the terms *justify* and *verify*. The terms are synonyms meaning *to demonstrate* or *prove to be right or reasonable*. Discuss what is required when *justifying* or *verifying* an answer.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Question 3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining whether equations have one solution, no solution or infinitely many solutions, assign Skills Practice Set B for this lesson.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1G:

- Do students defend their mathematical reasoning?
- Do students use precise mathematical language when communicating?

2. Consider the mathematical sentence $2 = 3$.
   Is there any variable or constant that you could add, subtract, multiply, or divide to both sides using the properties of equality to make this false sentence true? Choose variables and constants to create new mathematical sentences to justify your conclusion.

   Sample answer:

   $2 + x = 3 + x$

   $5(2 + x) = 5(3 + x)$

   There is no variable or constant that I can add, subtract, multiply or divide to both sides to make the sentence true. If I follow the properties of equality, the sentence is always false.

A linear equation can have *one solution*, *no solution*, or *infinite solutions*. The equations you solved in the previous activity are examples of linear equations with one solution. A linear equation with **no solution** means that there is no value for the variable that makes the equation true. A linear equation with **infinite solutions** means that any value for the variable makes the equation true.

3. Consider the equations you created in this activity.

   a. Explain whether the equation(s) you created in Question 1 have one solution, no solution, or infinite solutions.
   Infinite solutions

   b. Explain whether the equation(s) you created in Question 2 have one solution, no solution, or infinite solutions.
   No solution

4. Consider the equation $2x = 3x$. Does this equation have one solution, no solution, or infinite solutions? Explain your reasoning.

   One solution; $x = 0$

   The equation has one solution, $x = 0$. If I have the same variable with different coefficients set equal to each other, the solution will be 0 because any number multiplied by 0 is equal to 0, and $0 = 0$.

EB STUDENT TIP

For all proficiency levels

Materials needed: Sticky Notes

Have students practice using the mathematical language *rational number* and *integer.*

Beginning: Display a number line that includes negative and positive whole numbers and a Venn diagram with the category *rational numbers* encompassing the category *integers*. Have students move labeled sticky notes from the number line and move these to the Venn diagram. Students can then practice saying or pointing

(continued on next page)

ACTIVITY 1.3

Tic-Tac-Bingo

In this activity, you are going to play a game called Tic-Tac-Bingo. The object of the game is to match two expressions to create an equation with specific solution types. Use the Tic-Tac-Bingo sheet located at the end of the lesson.

Prepare the board.

The board has 9 spaces. Three spaces are already designated. Fill each remaining space with one of the solution types listed. Each option must be used at least once.

Solution Types
- positive rational solution
- negative rational solution
- non-zero integer solution

Play the game.

Your teacher will assign you an expression. When you and a classmate have created an equation with one of the solution types, write your equation in the corresponding box.

Try to be the first person to fill three spaces in a row. Then, try to be the first person to completely fill your board with equations.

1. Reflect on the equations you created.

   a. How can you look at an equation and determine that it has no solution?
   If, when rewritten in the fewest terms, the coefficients of x on both sides of the equation are the same but the constants are different, there will be no solution.

   b. How can you look at an equation and determine that it has infinite solutions?
   If, when rewritten in the fewest terms, the coefficients of x on both sides of the equation are the same and the constants are also the same, meaning that the expressions are equivalent, there will be infinite solutions.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the game directions.
- Distribute Expression Cards and play the game.
- Complete Question 1 as a class.

Optimizing Learning

This activity provides options that supports students to vary the methods for response and navigation.

EB STUDENT TIP *(continued)*

to whether a number is a rational number or both an integer and a rational number.

Intermediate: Provide sentence frames to use with the Venn diagram activity mentioned in the previous section, such as: The number ____ is both an integer and a rational number because ____ and: The number ____ is a rational number but not an integer because ____.

Advanced/Advanced High: Ask students to give examples of the previously mentioned categories and ask if they can come up with other categories or examples of numbers that fit neither of the existing two categories. Ask them to justify their examples and answers using precise mathematical language.

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing and solve equations with variables on both sides, assign Skills Practice Set C for this lesson.

Talk the Talk

One Step at a Time

1. Rectangle A has a width of $(5 + 2x)$ inches and a length of 6 inches. Rectangle B has a width of 8 inches and a length of $(x + 5)$ inches. Write and solve an equation to determine the length of x when both rectangles have the same area. Write the property that justifies each step to solve the equation.

$6(5 + 2x) = 8(x + 5)$	
$30 + 12x = 8x + 40$	distributive property
$30 + 12x - 30 = 8x + 40 - 30$	subtraction property of equality
$12x - 8x = 8x - 8x + 10$	subtraction property of equality
$\frac{4x}{4} = \frac{10}{4}$	division property of equality
$x = 2.5$	

352 MODULE 3 • TOPIC 1 • LESSON 1

Tic-Tac-Bingo Board

Equation: Solution:	**Solution is neither positive nor negative** Equation: Solution:	Equation: Solution:
No solution Equation: Solution:	**FREE SPACE**	Equation: Solution:
Equation: Solution:	Equation: Solution:	**Infinite solutions** Equation: Solution:

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MODULE 3 • TOPIC 1 • LESSON 1 353

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So you can cut out the Bingo Board on the other side

354 MODULE 3 • TOPIC 1 • LESSON 1

Answers for Tic-Tac-Bingo

			A1	A2	A3	A4	A5	A6	B1	B2	B3	B4	B5	B6
			$x + 3(x + 1)$	$2x + 2(x + 2) - 5$	$-2(3x - 1) + 3$	$2(1 - 3x)$	$8x - 3(2x - 1)$	$\frac{(2x - 7)}{2} + \frac{(2x - 3)}{2}$	$3(x - 2) + 2$	$9x - 2(3x + 1) + 7$	$-4(2x - 2) + 2(2x - 3)$	$-2(2x + 3) + 1$	$2(x - 2) + 3(x + 1)$	$5(x - 1) + 1$
		Equivalent Expression	$4x + 3$	$4x - 1$	$-6x + 5$	$-6x + 2$	$2x + 3$	$2x - 5$	$3x - 4$	$3x + 5$	$-4x + 2$	$-4x - 5$	$5x - 1$	$5x - 4$
a1	$\frac{(12x + 9)}{3}$	$4x + 3$	infinite	no	$x = \frac{1}{5}$	$x = -\frac{1}{10}$	$x = 0$	$x = -4$	$x = -7$	$x = 2$	$x = -\frac{1}{8}$	$x = -1$	$x = 4$	$x = 7$
a2	$3(x - 1) + (x + 2)$	$4x - 1$	no	infinite	$x = \frac{3}{5}$	$x = \frac{3}{10}$	$x = 2$	$x = -2$	$x = -3$	$x = 6$	$x = \frac{3}{8}$	$x = -\frac{1}{2}$	$x = 0$	$x = 3$
a3	$3(2x + 2) - 4(3x + \frac{1}{4})$	$-6x + 5$	$x = \frac{1}{5}$	$x = \frac{3}{5}$	infinite	no	$x = \frac{1}{4}$	$x = \frac{5}{4}$	$x = 1$	$x = 0$	$x = \frac{3}{2}$	$x = 5$	$x = \frac{6}{11}$	$x = \frac{9}{11}$
a4	$2x - 2(4x - 1)$	$-6x + 2$	$x = -\frac{1}{10}$	$x = \frac{3}{10}$	no	infinite	$x = -\frac{1}{8}$	$x = \frac{7}{8}$	$x = \frac{2}{3}$	$x = -\frac{1}{3}$	$x = 0$	$x = \frac{7}{2}$	$x = \frac{3}{11}$	$x = \frac{6}{11}$
a5	$\frac{4x + 5}{2} + \frac{1}{2}$	$2x + 3$	$x = 0$	$x = 2$	$x = \frac{1}{4}$	$x = -\frac{1}{8}$	infinite	no	$x = 7$	$x = -2$	$x = -\frac{1}{6}$	$x = -\frac{4}{3}$	$x = \frac{4}{3}$	$x = \frac{7}{3}$
a6	$3 + 2(x - 4)$	$2x - 5$	$x = -4$	$x = -2$	$x = \frac{5}{4}$	$x = \frac{7}{8}$	no	infinite	$x = -1$	$x = -10$	$x = \frac{7}{6}$	$x = 0$	$x = -\frac{4}{3}$	$x = -\frac{1}{3}$
b1	$x + 2(x - 2)$	$3x - 4$	$x = -7$	$x = -3$	$x = 1$	$x = \frac{2}{3}$	$x = 7$	$x = -1$	infinite	no	$x = \frac{6}{7}$	$x = -\frac{1}{7}$	$x = -\frac{3}{2}$	$x = 0$
b2	$2(4x + 5) - 5(x + 1)$	$3x + 5$	$x = 2$	$x = 6$	$x = 0$	$x = -\frac{1}{3}$	$x = -2$	$x = -10$	no	infinite	$x = -\frac{3}{7}$	$x = -\frac{10}{7}$	$x = 3$	$x = \frac{9}{2}$
b3	$-2(2x - 1)$	$-4x + 2$	$x = -\frac{1}{8}$	$x = \frac{3}{8}$	$x = \frac{3}{2}$	$x = 0$	$x = -\frac{1}{6}$	$x = \frac{7}{6}$	$x = \frac{6}{7}$	$x = -\frac{3}{7}$	infinite	no	$x = \frac{1}{3}$	$x = \frac{2}{3}$
b4	$(4x + 1) - 2(4x + 3)$	$-4x - 5$	$x = -1$	$x = -\frac{1}{2}$	$x = 5$	$x = \frac{7}{2}$	$x = -\frac{4}{3}$	$x = 0$	$x = -\frac{1}{7}$	$x = -\frac{10}{7}$	no	infinite	$x = -\frac{4}{9}$	$x = -\frac{1}{9}$
b5	$5(x - \frac{1}{5})$	$5x - 1$	$x = 4$	$x = 0$	$x = \frac{6}{11}$	$x = \frac{3}{11}$	$x = \frac{4}{3}$	$x = -\frac{4}{3}$	$x = -\frac{3}{2}$	$x = 3$	$x = \frac{1}{3}$	$x = -\frac{4}{9}$	infinite	no
b6	$-2(x + 2) + 7x$	$5x - 4$	$x = 7$	$x = 3$	$x = \frac{9}{11}$	$x = \frac{6}{11}$	$x = \frac{7}{3}$	$x = -\frac{1}{3}$	$x = 0$	$x = \frac{9}{2}$	$x = \frac{2}{3}$	$x = -\frac{1}{9}$	no	infinite

Why is this page blank?

So you can cut out the Bingo Answers on the other side

Lesson 1 Assignment

Write

Explain how you know when an equation has no solution and when it has infinite solutions.

Remember

To solve an equation, use the properties of equality to isolate the variable. A linear equation can have one solution, no solution, or infinite solutions.

Practice

1. Solve each equation. Write the properties that justify each step in the solution method.

a. $3x - 8 = -7x + 18$

$3x - 8 = -7x + 18$	
$3x = -7x + 26$	addition property of equality
$10x = 26$	addition property of equality
$x = \frac{13}{5}$	division property of equality

b. $-2(4 - x) = 12x - 3$

$-2(4 - x) = 12x - 3$	
$-8 + 2x = 12x - 3$	distributive property
$2x = 12x + 5$	addition property of equality
$-10x = 5$	subtraction property of equality
$x = -\frac{1}{2}$	division property of equality

c. $\frac{1}{2}(-10x + 4) = -4(-3 + 2x) + 8$

$\frac{1}{2}(-10x + 4) = -4(-3 + 2x) + 8$	
$-5x + 2 = 12 - 8x + 8$	distributive property
$-5x + 2 = 20 - 8x$	associative property
$3x + 2 = 20$	addition property of equality
$3x = 18$	subtraction property of equality
$x = 6$	division property of equality

d. $\frac{(-2x - 4)}{5} + \frac{8}{5} = 3(x - 1)$

$\frac{(-2x - 4)}{5} + \frac{8}{5} = 3(x - 1)$	
$-2x - 4 + 8 = 15(x - 1)$	multiplication property of equality
$-2x - 4 + 8 = 15x - 15$	distributive property
$-2x + 4 = 15x - 15$	associative property
$-17x + 4 = -15$	subtraction property of equality
$-17x = -19$	subtraction property of equality
$x = \frac{19}{17}$	division property of equality

Write

Sample answer:
An equation has no solution when there is no value for the variable that makes the equation true. An equation has infinite solutions when any value for the variable makes the equation true.

MODULE 3 • TOPIC 1 • LESSON 1 ASSIGNMENT 355

Lesson 1 Assignment

e. $\frac{4}{3}x + 2\left(9 - \frac{1}{3}x\right) = -\frac{7}{3}x + 9$

$\frac{4}{3}x + 2\left(9 - \frac{1}{3}x\right) = -\frac{7}{3}x + 9$

$4x + 6\left(9 - \frac{1}{3}x\right) = -7x + 27$ multiplication property of equality

$4x + 54 - 2x = -7x + 27$ distributive property

$2x + 54 = -7x + 27$ associative property

$9x + 54 = 27$ addition property of equality

$9x = -27$ subtraction property of equality

$x = -3$ division property of equality

2. Determine whether each equation has one solution, no solution, or infinite solutions. Explain your reasoning.

a. $-2(x + 5) = -6x + 4(x - 2)$

No solution; $-10 \neq -8$

b. $4(0.2x - 1.2) = -0.5x + 4.3$

One solution; $x = 7$

c. $\dfrac{\left(\frac{1}{2}x - 5\right)}{2} = -3x + 4$

One solution; $x = 2$

d. $2(x - 4) + x = 3(x - 2) - 2$

Infinite solutions; $-8 = -8$

e. $3 - \frac{2}{5}x - \frac{12}{5} = \frac{10 - 2x}{5}$

No solution; $3 \neq 10$

f. $6(x - 1) + 21 = 6x + 15$

Infinite solutions; $15 = 15$

Prepare

The formula for the circumference of a circle is $C = 2\pi r$. Determine the radius for each circle with the given circumference. Use 3.14 for π and round to the nearest tenth of a unit, if necessary.

1. C = 62.8 in.

$r = 10$ in.

2. C = 10 cm

$r = 1.6$ cm

3. C = 15.7 ft

$r = 2.5$ ft

4. C = 48 mm

$r = 7.6$ mm

356 MODULE 3 • TOPIC 1 • LESSON 1 ASSIGNMENT

2 Literal Equations

MATERIALS

None

LESSON OVERVIEW

Students begin with a perimeter problem in context to address solving formulas for different variables. They then identify the slope, x-intercept, and y-intercept of linear equations in slope-intercept, point-slope, and standard form and consider which form is most efficient in determining these characteristics. Next, the term *literal equation* is defined. The common literal equation for converting degrees Fahrenheit to degrees Celsius is provided. Students rewrite the formula to convert degrees Celsius to degrees Fahrenheit, identify errors in student work when rewriting the formula, and interpret equivalent equations written in standard form. The lesson concludes by having students solve various literal equations for specific variables.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.

The student is expected to:

A.2B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

(TEKS continued on next page)

ELPS

(3) Speaking

The student is expected to:

(A) practice producing sounds of newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters to pronounce English words in a manner that is increasingly comprehensible.

(4) Reading

The student is expected to:

(D) use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text.

(J) demonstrate English comprehension and expand reading skills by employing inferential skills such as predicting, making connections between ideas, drawing inferences and conclusions from text and graphic sources, and finding supporting text evidence commensurate with content area needs.

ALGEBRA I TEKS *(TEKS continued from previous page)*

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3A determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$

Number and Algebraic Methods

(12) The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

The student is expected to:

A.12E solve mathematic and scientific formulas, and other literal equations, for a specified variable.

ESSENTIAL IDEAS

- The slope-intercept form of a linear equation is $y = mx + b$, where m and b are real numbers; m represents the slope, and b represents the y-intercept.
- The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m represents the slope, and (x_1, y_1), represents a point on the line.
- The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers, and both A and $B \neq 0$. It can be rewritten in slope-intercept form as $y = -\frac{A}{B}x + \frac{C}{B}$; $-\frac{A}{B}$ represents the slope, $\frac{C}{B}$ represents the y-intercept, and $\frac{C}{A}$ represents the x-intercept.
- Slope-intercept form of a linear equation is the most useful form to identify the slope and y-intercept. Point-slope form of a linear equation is the most useful form to identify the slope and a point on the line.
- Literal equations can be rewritten to highlight a specific variable.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Perimeter Perspectives 15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students think about a scenario from two different perspectives to write two equations that can each be used to determine the specific information needed. They then show that the two equations are equivalent.

DEVELOP

Activity 2.1: Equations in Different Forms 25–30 minutes

INVESTIGATION, MATHEMATICAL PROBLEM SOLVING

Students analyze three forms of a linear equation: slope-intercept, point-slope, and standard form. They consider which form is most efficient for determining the slope, x-intercept, and y-intercept. Students then rewrite linear equations in different forms.

DAY 2

Activity 2.2: Rewriting Literal Equations 15–20 minutes

PEER WORK ANALYSIS

The term *literal equation* is defined. Students are given the equation for converting degrees Fahrenheit to degrees Celsius. They rewrite the formula to convert degrees Celsius to degrees Fahrenheit, identify errors in student work when rewriting the formula, and interpret equivalent equations written in standard form.

Activity 2.3: Solving Literal Equations for Specific Variables 15-20 minutes

REAL-WORLD PROBLEM SOLVING, MATHEMATICAL PROBLEM SOLVING

Students solve different literal equations for given variables. They use the converted equations to solve for unknown values.

DEMONSTRATE

Talk the Talk: Rearrange It 5–10 minutes

EXIT TICKET PROCEDURES

Students convert the formulas for the lateral surface area and surface area of a cylinder to solve for the height. The converted equations and given values are then used to determine the height.

Getting Started

ENGAGE

Perimeter Perspectives

Facilitation Notes

In this activity, students think about a scenario from two different perspectives, write two equations, then show that the two equations are equivalent.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Assign students either Juan's or Mia's situation to focus on as they rewrite the formula for the perimeter.
- Help students develop a strategy for rewriting the formula to determine the perimeter by first providing a few possible values for the perimeter, such as 500 feet or 1000 feet. Ask students to determine the possible amounts of fencing Juan and Mia would need. Then, have them rewrite the perimeter formula based on Juan's or Mia's situation.
- Confirm the fact that the solution process is complete once the variable they are solving for is isolated on one side of the equation. Students may be uncertain when they are finished solving the equation because the solution is not a numeric answer.
- When answering Question 2, provide additional direction. To prove the equations are equivalent, students must start with both equations and use properties of equality and number properties to get them to be exactly the same; suggest that students transform each equation back to the perimeter formula.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why does Juan need to determine the park's width? • Was it more difficult to solve this equation than previous equations? Why or why not?
Probing	• Explain your steps to isolate the variable w. • What properties did you apply to solve for w? • How can you show both equations are equivalent to the perimeter equation?
Seeing structure	• How does the expression $\frac{1}{2}P - l$ relate to the diagram?
Reflecting and justifying	• Why does it make sense to show both equations are equivalent to the perimeter equation?

Summary

A formula involving more than one variable can be rewritten in an equivalent form to solve a problem situation.

STAMP THE LEARNING

ACTIVITY 2.1 Equations in Different Forms

DEVELOP

Facilitation Notes

In this activity, students identify the intercepts and slope given different forms of a linear equation. They then answer questions that focus on when it is most efficient to use each of these forms and rewrite linear equations into different forms.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Encourage students who confuse the *x*-intercept and *y*-intercept to make a quick sketch and label the coordinate pair (x, 0) for the *x*-intercept and (0, y) for the *y*-intercept.

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 1

Probing	• Which characteristics of the linear function were apparent from the equation? • How did you determine the *x*-intercept? How was your strategy related to the meaning of the *x*-intercept?

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 2

Probing	• Explain how you determined the value of each characteristic.
Seeing structure	• Use substitution to show why the *x*-intercept is (2, 0). • How can you use the structure of the equation to identify the *x*-intercept?

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 3

Probing	• How did you identify the intercepts? The slope? • Which characteristics of the graph are obvious when the equation is written in this form?

Ask a student to read the information following Question 3. Complete Question 4 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What do slope-intercept form and point-slope form have in common? • Which form can you also refer to as the slope-intercept form? Why? • How can you easily identify the point-slope form of a linear equation? • Describe the structure of a linear equation in standard form.

COMMON MISCONCEPTION

Students may question the usefulness of standard form. Consider these points:

- One advantage of standard form is that both horizontal lines and vertical lines can be written in standard form. For example, the equation for the vertical line $x = 5$ is considered to be written in standard form, where $B = 0$; it is impossible to write the equation for a vertical line in slope-intercept form or point-slope form.
- Sometimes it is easier to represent a context in standard form. For example, write an equation modeling the number of hamburgers and orders of french fries that can be purchased with $20 if hamburgers cost $4 each and french fries cost $2 each. The equation in standard form, $4h + 2f = 20$, is the most efficient way to represent this context.

Have students work with a partner or in a group to complete Questions 5 through 10. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Show how you applied the same thinking you used in Question 3 to solve for these characteristics. • How can you identify the slope and x-intercept from point-slope form?
Reflecting and justifying	• For any of these purposes, are there multiple forms of the equation that are equally efficient? Explain your reasoning. • Which form do you prefer to start with when rewriting an equation in slope-intercept form? point-slope form? standard form?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Play "Guess My Function" with the students. Start by giving them one characteristic of a linear function, for example, "My function has a positive slope," and ask them to write an equation that contains this criteria. Then add a second, third, and fourth characteristic, each time asking them to adjust their equation. Have students share their final equations and check for correctness. Discuss the form in which each final equation was written and ask students why they chose that form.

To close Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

Slope-intercept form of a linear equation is most useful form to identify the slope and y-intercept. Point-slope form of a linear equation is the most useful form to identify the slope and a point on the line.

ACTIVITY 2.2 Rewriting Literal Equations

Facilitation Notes

In this activity, the term *literal equation* is defined and students are given the literal equation for converting degrees Fahrenheit to degrees Celsius. They rewrite the equation to convert degrees Celsius to degrees Fahrenheit and use the formulas to make conversions. They then analyze student work to identify errors when rewriting the literal equation and to compare equivalent equations written without the fraction.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the definition and introduction aloud. Discuss as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What part of the world measures temperatures using degrees Celsius? Degrees Fahrenheit?
Seeing structure	• How can you tell whether an equation is set up to convert from Celsius to Fahrenheit or Fahrenheit to Celsius?

DIFFERENTIATION STRATEGIES

Assistance for All

- Ask students to convert the freezing point and boiling point for water from degrees Fahrenheit to degrees Celsius.

Just in Time Support

Materials Needed: Thermometer

- Have a thermometer available so that students can make sense of the scales and check their answers.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Different first steps used when solving for *F*.
- Errors in dealing with the fraction when solving for *F*.
- Errors in using the distributive property, if that process is used, when solving for *F*.
- Students making connections between the terms in the two versions of the formulas.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how you solved for *F*. What is another way? • What would Jayden's steps be if he substituted 77 in the original formula? • Which method do you prefer, substituting the 77 in the original formula and solving the equation or isolating *F* first and using substitution? Why? • Why did Isabella multiply both sides by 9? • Which term did Isabella add to both sides? • Compare your strategy to Isabella's strategy. • Explain the difference in the errors Ethan and Minh made when using the distributive property.
Seeing structure	• Explain how the rewritten formula relates to the original formula.
Gathering	• Which property did Isabella apply first?

DIFFERENTIATION STRATEGY

Assistance for All

Address the efficiency of different strategies when solving the literal equation for *F*. Consider these two approaches.

$$C = \frac{5}{9}(F - 32) \qquad\qquad C = \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = \left(\frac{9}{5}\right)\frac{5}{9}(F - 32) \qquad\qquad C = \frac{5}{9}F - \frac{160}{9}$$

COMMON MISCONCEPTION

Students may wonder why Jayden must solve the literal equation first and then substitute the value in the formula rather than just substitute the degrees Celsius and solve the resulting equation. The second method is a viable option, especially if only one conversion is being completed. However, if Jayden is going to be converting degrees often, by solving the literal equation first, he only has to solve the equation one time, and the remaining times he just has to simplify the expression on the right side of the equation. Refer to the warm-up as an example.

Discuss the Worked Example as a class. Have students work with a partner or in a group to complete Question 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Determine the forms of the equations. • What is the form of this equation?
Probing	• Are the equations in the Worked Example and what you wrote in Question 4 equivalent?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Ask students to write a literal equation involving a formula for the area of any polygon, such as $A = \frac{1}{2}bh$, then solve the equation for one of the variables.

Summary

STAMP THE LEARNING

A formula that includes more than one variable may be rewritten to isolate a different variable in the formula for efficiency in solving problems.

ACTIVITY 2.3 Solving Literal Equations for Specific Variables

Facilitation Notes

In this activity, students solve literal equations for specified variables. The converted equations are then used to solve for unknown values.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

COMMON MISCONCEPTION

Students may think they can always substitute 3.14 for π. For accuracy, π should not be replaced with an approximation when rewriting a literal equation; however, π may be replaced with an approximation when calculating.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Suggest students draw diagrams and label their dimensions visualizing each context.
- Help students see that 75, the area of the base, should be substituted for the expression ℓw rather than a single variable.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how you solved for *h*. • What variables represent the area of the base? • How did you solve for *h*? • What is the result of $\frac{\text{units}^3}{\text{units}^2}$?
Gathering	• What are the units for the volume?

Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Different first steps used when solving for *h*.

COMMON MISCONCEPTION

Students may only have experience dividing by a single term and may not know that they can divide by $(b_1 + b_2)$ because it has two terms. Discuss that $(b_1 + b_2)$ represents a quantity and help students record their steps for converting the formula for the height.

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 3

Probing	• Did you apply the distributive property to solve for *h*? • How could you solve for *h* without applying the distributive property?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Discuss why distributing *h* to two terms is not an efficient strategy when solving for *h*.
- Have students check their solution for $h = 3$ by substituting all three dimensions into the area formula.

Challenge Opportunity

- Have students solve for b_1 or b_2 and compare it to the process they used to solve for *h*.

QUESTIONS TO SUPPORT DISCOURSE FOR QUESTION 4

Probing	• What is another way to express the equation in part (a)? • What operation did you use so that *D* was no longer in the denominator?

STAMP THE LEARNING

Summary

A formula that includes more than one variable may be rewritten to isolate any variable contained in the formula by applying properties of equality and number properties.

DEMONSTRATE

Talk the Talk

REARRANGE IT

Facilitation Notes

In this activity, students convert the formulas for the lateral surface area and surface area of a cylinder to solve for the height. The converted equations and given values are then used to determine the height.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Confusion between the concepts of volume and surface area. Discuss the two concepts and the measures of units for each.
- Incorrect dividing out of terms in the surface area formula.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the most efficient way to solve for *h*? • Why is it easier to use your response in Question 1 rather than the original formula to calculate the value of *h*?
Seeing structure	• What is the relationship between your steps to solve the equation in Question 3 and those to solve a two-step equation? • Would you rather substitute the value of *h* in the original formula and then solve the resulting equation or solve the literal equation first and then evaluate the resulting expression? Why?

Have students read and answer the Essential Question on the lesson opener page.

Summary

A formula that includes more than one variable may be rewritten to isolate any variable contained in the formula by applying properties of equality and number properties.

STAMP THE LEARNING

2 Literal Equations

OBJECTIVES

- Rewrite linear equations in different forms.
- Analyze the structure of different forms of linear equations.
- Recognize and use literal equations.
- Rearrange literal equations to highlight quantities of interest.

NEW KEY TERM

- literal equation

Setting the Stage

- Communicate the objectives and the new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

You have used different properties to solve linear equations to determine which value makes the equation true.

How can you use those same properties to solve for one specific variable in an equation that has multiple variables?

Sample answer:

I can use the properties to isolate one specific variable on one side of the equation with all other terms on the other side.

MODULE 3 • TOPIC 1 • LESSON 2 357

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Student Look-Fors

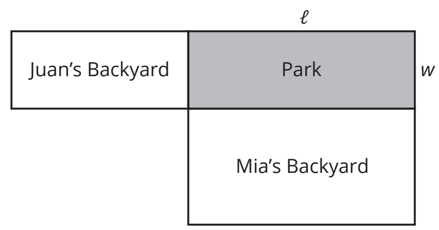
Whether students are demonstrating proficiencies related to TEKS A.1D:

- Do students represent their mathematical reasoning with appropriate representations?
- Can students communicate how a chosen representation demonstrates their reasoning?
- Can students identify the advantages and disadvantages of different representations?

Getting Started

Perimeter Perspectives

Juan and Mia have rectangular backyards that each share a side with a neighborhood park. The sides shared with the park are adjacent to each other as shown.



Remember . . .
The formula for the perimeter of a rectangle is $P = 2\ell + 2w$.

Juan and Mia are each responsible for constructing their own fence to separate their yard from the park. The city gives them the measure of the perimeter of the park in feet. So that each knows how much fencing to buy, Juan needs to determine the width of the park, and Mia needs to determine the length of the park.

1. How can you write an equation for the perimeter of the park from each person's perspective to help them determine the information they need?

   Juan needs to know the width of the park, so he can use the formula $w = \frac{P}{2} - \ell$.

   Mia needs to know the length of the park, so she can use the formula $\ell = \frac{P}{2} - w$.

2. Show that the two equations are equivalent.

   $w = \frac{P}{2} - \ell$
   $\ell = \frac{P}{2} - w$
   $2w = 2\left(\frac{P}{2} - \ell\right)$
   $2\ell = 2\left(\frac{P}{2} - w\right)$
   $2w = P - 2\ell$
   $2\ell = P - 2w$
   $2w + 2\ell = P$
   $2w + 2\ell = P$

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Assess students' prior knowledge of the term *adjacent*. Ask students to provide examples of non-mathematical or mathematical contexts in which they have heard the term *adjacent* being used. Some examples may include *adjacent* angles, *adjacent* sides, and *adjacent* classrooms. Sketch examples of *adjacent* angles and of common shapes with *adjacent* sides labeled.

358 MODULE 3 • TOPIC 1 • LESSON 2

ACTIVITY 2.1 Equations in Different Forms

The slope, *x*-intercept, and *y*-intercept are each important characteristics of linear functions. The structure of equations can reveal these characteristics of a function.

1. Consider the equation $y = -\frac{2}{3}x + 4$. Determine each characteristic and then explain your work.

   a. Slope
   $-\frac{2}{3}$

   b. *y*-intercept
   (0, 4)

   c. *x*-intercept
   (6, 0)

   d. How did you determine each characteristic?
   Sample answer:
   I identified the slope and *y*-intercept directly from the equation. I substituted 0 for *y* and solved the equation for *x* to determine the *x*-intercept.

2. Consider the equation $y = 4(x - 2)$. Determine each characteristic and then explain your work.

   a. Slope
   4

   b. *y*-intercept
   (0, −8)

   c. *x*-intercept
   (2, 0)

   d. How did you determine each characteristic?
   Sample answer:
   I identified the slope and *x*-intercept directly from the equation. I used the distributive property to rewrite the equation in slope-intercept form to determine the *y*-intercept.

Remember . . .

- You can calculate slope using $\frac{y_2 - y_1}{x_2 - x_1}$.
- You can determine the *y*-intercept by substituting 0 for *x* and solving for *y*.
- You can determine the *x*-intercept by substituting 0 for *y* and solving for *x*.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the forms.
- Complete Question 4 as a class.
- Group students to complete Questions 5–10.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Materials Needed: Poster Paper

Create a chart titled *Key Characteristics of Linear Functions* and label the columns with the terms *x-intercept*, *y-intercept*, and *slope*. Have students contribute definitions, illustrations, and examples on the chart for each term. Discuss how to identify each characteristic from an equation or graph.

STAMP THE LEARNING

The forms of linear equations provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 4–6 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice converting linear equations from standard form to slope-intercept form, assign Skills Practice Set A for this lesson. To provide additional practice converting linear equations from point-slope form to slope-intercept and standard form, assign Skills Practice Set B for this lesson. To provide additional practice converting linear equations from standard to slope-intercept form, assign Skills Practice Set C for this lesson.

3. Consider the equation $3x - 2y = 8$. Determine each characteristic and then explain your work.

   a. Slope
   $\frac{3}{2}$

   b. y-intercept
   $(0, -4)$

   c. x-intercept
   $\left(\frac{8}{3}, 0\right)$

   d. How did you determine each characteristic?
   Sample answer:
   I rewrote the equation in slope-intercept form to get the slope and y-intercept. I substituted 0 for y and solved the equation for x to determine the x-intercept.

In slope-intercept and point-slope form, the values of m, x_1, and y_1 can be any real numbers. In standard form, however, there are constraints on the variables: A, B, and C must be integers, and A and B cannot both be 0.

Recall that are three useful forms of linear equations.

Slope-Intercept Form	Point-Slope Form	Standard Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$

4. Identify the form for each equation in Questions 1 through 3.
   $y = -\frac{2}{3}x + 4$, slope-intercept form
   $y = 4(x - 2)$, point-slope form
   $3x - 2y = 8$, standard form

Consider how the structure of the standard form of a linear function reveals its key characteristics.

5. For the equation $Ax + By = C$, determine the slope, x-intercept, and y-intercept.
   Slope: $-\frac{A}{B}$
   x-intercept: $\left(\frac{C}{A}, 0\right)$
   y-intercept: $\left(0, \frac{C}{B}\right)$

360 MODULE 3 • TOPIC 1 • LESSON 2

6. Which form of a linear equation is more efficient for determining each characteristic?

   a. Slope
      Slope-intercept form or point-slope form

   b. a point on the line
      point-slope form

   c. y-intercept
      Slope-intercept form

7. If you want to graph an equation using technology, which form is more efficient? Explain your reasoning.
   Slope-intercept form

8. Rewrite each equation in slope-intercept form.

   a. $y - 2 = 5(x - 7)$
      $y = 5x - 33$

   b. $8x - 2y = 12$
      $y = 4x - 6$

9. Rewrite each equation in standard form.

   a. $y + 7 = 0.5(x - 1)$
      $x - 2y = 15$

   b. $y = -4x + 3$
      $4x + y = 3$

10. Write the equation for a line in standard form that passes through the given point with the identified slope.

    a. Slope $= \frac{1}{2}$
       Passes through $(-3, 5)$
       $x - 2y = -7$

    b. Slope $= -4$
       Passes through $(2, 1)$
       $4x + y = 7$

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MODULE 3 • TOPIC 1 • LESSON 2 361

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the definition and situation.
- Group students to complete Questions 1–3.
- Check in and share.
- Discuss the Worked Example as a class.
- Group students to complete Question 4.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 2.2 Rewriting Literal Equations

Literal equations are equations in which the variables represent specific measures. You most often see literal equations when you study formulas. These literal equations can be manipulated in order to allow you to solve for one specific variable.

You have used a common literal equation, the formula for converting degrees Fahrenheit to degrees Celsius.

$$C = \frac{5}{9}(F - 32)$$

Jayden is talking on the phone to his friend Mateo who lives in Europe. Mateo is used to describing temperatures in °C, while Jayden is used to describing temperatures in °F. Mateo can use the formula above to quickly convert the temperatures Jayden describes to °C.

1. How can you rewrite the formula so that Jayden can quickly convert the temperatures that Mateo describes to °F? Justify your solution method.

$C = \frac{5}{9}(F - 32)$

$\frac{9}{5}C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$

$\frac{9}{5}C = F - 32$

$F = \frac{9}{5}C + 32$

2. Jayden tells Mateo that the temperature where he lives is currently 77°F. Mateo tells Jayden that the temperature where he lives is currently 30°C. Jayden says it is warmer where he lives and Mateo says it is warmer where he lives. Who is correct? Explain your reasoning.

Mateo is correct. I can either convert Mateo's temperature to degrees Fahrenheit or Jayden's temperature to degrees Celsius to compare.

$F = \frac{9}{5}(30) + 32$

$F = 54 + 32$

$F = 86$

$C = \frac{5}{9}(77 - 32)$

$C = \frac{5}{9}(45)$

$C = 25$

It is 77°F where Jayden lives and 86°F where Mateo lives. It is 25°C where Jayden lives and 30°C where Mateo lives.

362 MODULE 3 • TOPIC 1 • LESSON 2

3. Isabella, Ethan, and Minh each convert the given formula to degrees Fahrenheit.

Isabella

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 5F - 160$$
$$9C + 160 = 5F$$
$$\frac{9C}{5} + \frac{160}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 32 = F$$

a. Explain Isabella's reasoning.
First, Isabella distributed the $\frac{5}{9}$ to the F and the 32. Next, she multiplied both sides by 9. Then, she added 160 to both sides. Finally, she isolated F by dividing by 5.

Ethan

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - 32$$
$$9(C) = 9\left(\frac{5}{9}F - 32\right)$$
$$9C = 5F - 288$$
$$9C + 288 = 5F$$
$$\frac{9C}{5} + \frac{288}{5} = \frac{5F}{5}$$
$$\frac{9}{5}C + 57.6 = F$$

b. Explain the error in Ethan's work.
Ethan's error occurs in the second line. He did not distribute the $\frac{5}{9}$ to the 32.

Minh

$$C = \frac{5}{9}(F - 32)$$
$$C = \frac{5}{9}F - \frac{160}{9}$$
$$9(C) = 9\left(\frac{5}{9}F - \frac{160}{9}\right)$$
$$9C = 45F - 1440$$
$$9C + 1440 = 45F$$
$$\frac{9C}{45} + \frac{1440}{45} = \frac{45F}{45}$$
$$\frac{1}{5}C + 32 = F$$

c. Explain the error in Minh's work.
Minh's error occurs in the fourth line. He did not distribute the 9 to the $\frac{5}{9}F$ and to the $-\frac{160}{9}$ correctly.

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MODULE 3 • TOPIC 1 • LESSON 2 363

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

WORKED EXAMPLE

You can use multiplication to rewrite an equation without fractions. Consider the final equation from Isabella's work.

$\frac{9}{5}C + 32 = F$

$5\left(\frac{9}{5}C + 32\right) = (5)F$ — Multiply both sides of the equation by 5 to eliminate the fraction.

$9C + 160 = 5F$ — Subtract $9C$ from both sides of the equation.

$160 = 5F - 9C$

4. Use the strategy from the worked Example to rewrite Isabella's original equation $C = \frac{5}{9}(F - 32)$ without fractions.

$C = \frac{5}{9}(F - 32)$

$(9)C = (9)(\frac{5}{9}(F - 32))$

$9C = 5(F - 32)$

$9C = 5F - 160$

$9C - 5F = -160$

364 MODULE 3 • TOPIC 1 • LESSON 2

ACTIVITY 2.3 Solving Literal Equations for Specific Variables

Convert each literal equation to solve for the given variable.

1. Think Inside the Box is manufacturing new boxes for Pack-n-Ship. Pack-n-Ship told Think Inside the Box that the boxes must have a specific volume and area of the base. However, Pack-n-Ship did not specify a height for the boxes.

   a. Write a literal equation to calculate the volume of a box. Then, convert the volume formula to solve for height.
   $V = Bh$, $h = \frac{V}{B}$

   b. Pack-n-Ship specified the volume of the box must be 450 cubic inches and the area of the base must be 75 square inches. Use your formula to determine the height of the new boxes.
   $h = 6$ in.

2. The volume of an ice cream cone is the measure of how much ice cream fits inside the cone. An ice cream cone company wants to make an ice cream cone with a greater height that still holds the same amount of ice cream.

   a. Write an equation to calculate the volume of a cone. Then, convert the equation to solve for the height.
   $V = \frac{1}{3}\pi r^2 h$, $h = \frac{3V}{\pi r^2}$

   b. Explain how your equation determines a linear measurement when the original equation determined a cubic measurement.
   The area of the base of the cone is in square units since the radius is multiplied by itself to calculate it. Dividing the volume, which is a measurement in cubic units, by a measurement in square units, gives you a linear measurement of the height.

Remember . . .
Volume is measured in cubic units since it's calculated using three dimensions. Height measures only one dimension.

The formula for the volume of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$, where B is the area of the base of the cone, r is the radius and h is the height.

Chunking the Activity

- Group students to complete Questions 1 and 2.
- Check in and share.
- Group students to complete Questions 3 and 4.
- Share and summarize.

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MODULE 3 • TOPIC 1 • LESSON 2 365

Questions 1–4 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice solving literal equations for an indicated variable, assign Skills Practice Set D for this lesson.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1A:

- Do students persevere in real-world problem solving?
- Do students recognize they can use literal equations outside of the math classroom?

3. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h is the height and b_1 and b_2 are the lengths of each base.

   a. Convert the area formula to solve for the height.

   $h = \frac{2A}{b_1 + b_2}$

   b. Use your formula to determine the height of a trapezoid with an area of 24 square centimeters and base lengths of 9 cm and 7 cm.

   $h = 3\,\text{cm}$

4. For the given literal equation $Z = \frac{A}{B} + \frac{C}{D}$, solve for each variable given. Justify your solution method.

   a. A

   $A = BZ - \frac{BC}{D}$

   I multiplied both sides of the equation by B. Then I isolated A by subtracting $\frac{BC}{D}$ from both sides.

   b. D

   $D = \frac{BC}{(BZ - A)}$

   I multiplied both sides of the equation by D and B to eliminate the fractions. Then, I moved all the terms with a D to the left side of the equation and factored out D. Finally, I divided both sides by $BZ - A$.

SELF-MONITORING STRATEGIES

Engaging in productive struggle as they use their reasoning to solve problems. Refer to the Course and Implementation Guide for further details on these look-fors.

Talk the Talk

Rearrange It

1. The formula for the lateral surface area of a cylinder is $S = 2\pi rh$, where S is the lateral surface area, r is the length of the radius of the base, and h is the height. Convert the formula to solve for the height.

   $h = \frac{S}{2\pi r}$

2. The lateral surface area of a can of soup is 37.68 square inches, and the length of the radius of the base of the can is 1.5 inches. Use your formula to determine the height of the can of soup. Use 3.14 for π.

   $h = 4$ in.

3. The formula for the surface area of a cylinder is $S = 2\pi r^2 + 2\pi rh$, where S is the surface area, r is the length of the radius of the base, and h is the height. Convert the formula to solve for the height.

   $h = \frac{S - 2\pi r^2}{2\pi r}$

4. The surface area of a can of soup is 51.81 square inches, and the length of the radius of the base is 1.5 inches. Use your formula to determine the height of the can of soup. Use 3.14 for π.

   $h = 4$ in.

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Optimizing Learning

This activity optimizes student supports to promote understanding across languages.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Some geometric terms included in this lesson are *perimeter*, *area*, *volume*, and *surface area*. Define each term in words and illustrations. Connect the meanings of *area* and *surface area*. Discuss which types of measurement are used for two-dimensional shapes and which apply to three-dimensional shapes. Sketch examples of two- and three- dimensional shapes and ask students to identify what dimensions are necessary for each type of measurement.

Lesson 2 Assignment

Write

Define the term *literal equation* in your own words.

Remember

Literal equations can be rewritten using properties of equality to allow you to solve for one specific variable.

Practice

1. Rewrite each equation in the given form.

   a. $y = \frac{3}{4}x - 8$ in standard form

   $3x - 4y = 32$

   b. $y - 12 = -3(x - 1)$ in slope-intercept form

   $y = -3x + 15$

   c. $5x - 2y = -18$ in slope-intercept form

   $y = \frac{5}{2}x + 9$

   d. $y + 4 = 0.5(x - 14)$ in standard form

   $x - 2y = 22$

Write

Sample answer:

Literal equations are equations in which the variables all represent specific measures, like height, weight, or temperature.

MODULE 3 • TOPIC 1 • LESSON 2 ASSIGNMENT 369

Lesson 2 Assignment

2. Shoe boxes are often used in other capacities once the shopper has bought the shoes. Sometimes the boxes are used to hold other items, so it is helpful to know the volume of the box.

   a. Write the equation to solve for the volume of the shoe box.
   $V = Bh$

   b. If the area of the base of the box is 112 square inches and the height is 3.5 inches, what is the volume of the box?
   The volume of the box is 392 cubic inches.

   c. Rewrite the equation to solve for height.
   $h = \frac{V}{B}$

   d. A box has a volume of 456 cubic inches, with a length of 1 foot and a height of 4 inches. Determine the width of the box.
   9.5 in.

3. Solve each equation for the specified variable.

   a. $V = \frac{1}{3}Bh$ for B
   $B = \frac{(3V)}{h}$

   b. $I = prt$ for r
   $r = \frac{I}{(pt)}$

   c. $\frac{x + y}{3} = 6$ for y
   $y = -x + 18$

   d. $A + B + C = 180$ for C
   $C = 180 - (A + B)$

Prepare

Solve each statement for x.

1. $x + 1 = 6$
   $x = 5$

2. $x - 2 > 5$
   $x > 7$

3. $\frac{x}{3} \leq -1$
   $x \leq -3$

4. $-x > 4$
   $x < -4$

370 MODULE 3 • TOPIC 1 • LESSON 2 ASSIGNMENT

3 Modeling Linear Inequalities

MATERIALS

None

LESSON OVERVIEW

Students begin with a scenario and table that can be modeled by a linear inequality with a positive rate of change. They then analyze a graph that models the situation. Students use that graph to solve inequalities and graph the solution set on a number line. Next, the term *solve an inequality* is defined, and students write and solve inequalities algebraically, taking into account the context of the problem situation. They then analyze an inequality with a negative rate of change to make sense of how the sign of the solution to the inequality is affected. Lastly, students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation and ones that require the distributive property.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

(TEKS continued on next page)

ELPS

(2) Listening
The student is expected to:

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

(5) Writing
The student is expected to:

(D) edit writing for standard grammar and usage, including subject-verb agreement, pronoun agreement, and appropriate verb tenses commensurate with grade-level expectations as more English is acquired.

(E) employ increasingly complex grammatical structures in content area writing commensurate with grade level expectations such as (i) using correct verbs, tenses, and pronouns/antecedents; (ii) using possessive case (apostrophe -s) correctly; and, (iii) using negatives and contractions correctly.

ALGEBRA I TEKS *(TEKS continued from previous page)*

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions.

The student is expected to:

A.5B solve linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.

ESSENTIAL IDEAS

- A linear inequality context can be modeled with a table of values, a graph on a coordinate plane, a graph on a number line, and with an inequality statement.
- Solutions to linear inequalities can be determined both graphically and algebraically; they can be expressed using a number line or inequality statement.
- The steps to solving a linear inequality algebraically are the same steps to solve a linear equation, except that when solving a linear inequality with a negative rate of change, the inequality sign of the solution must be reversed to accurately reflect the relationship.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Fundraising Function 10–15 minutes

ESTABLISHING A SITUATION

A scenario is given that can be represented by a function in the form $f(x) = mx + b$. Students write and analyze the function that describes the scenario, identifying the independent and dependent quantities, the rate of change, and the *y*-intercept.

DEVELOP

Activity 3.1: Modeling Linear Inequalities 25–30 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

Students continue working with the Fundraising Function scenario. They analyze a graph that models the situation and use it to solve inequalities. Students graph the solution set for each inequality on a number line.

DAY 2

Activity 3.2: Solving Two-Step Linear Inequalities 15–20 minutes

WORKED EXAMPLE

The term *solve an inequality* is defined. A Worked Example demonstrates how to solve an inequality algebraically. Students practice writing and solving inequalities and choosing the most accurate answer in the context of the problem situation. They then determine that the method to solve an inequality does not change, regardless of the form of the inequality.

Activity 3.3: Reversing Inequality Signs 20–25 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

A scenario is provided that can be represented by an inequality with a negative slope. Students write a function that describes the scenario and identify key characteristics of the function in terms of the problem situation. They use the graph of the function to solve the inequality before solving the inequality algebraically. They compare the graphic solution to the algebraic solution. Students complete a table of inequalities relating $h(m)$ and m to demonstrate that multiplication or division of a negative number reverses the inequality symbol when solving an inequality.

DAY 3

Activity 3.4: Solving Other Linear Inequalities 25–30 minutes

PEER WORK ANALYSIS

Students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation, and ones that require the distributive property.

DEMONSTRATE

Talk the Talk: It's About Solutions, More or Less 10–15 minutes

EXIT TICKET PROCEDURES

Students solve four inequalities and graph each solution on a number line. They also identify constraints that make different inequality statements true.

Getting Started

ENGAGE

Fundraising Function

Facilitation Notes

In this activity, a scenario is given that can be represented by a function in the form $f(x) = mx + b$. Students write and analyze the function that describes the scenario, identifying the independent and dependent quantities, the rate of change, and the y-intercept.

Ask a student to read the introduction aloud. Discuss as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain the scenario in your own words.
Seeing structure	• Which type of function models this scenario? Why?

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How are the key features of the function identifiable in the scenario given?
Gathering	• How did you use function notation to represent the independent and dependent quantities? • What are the units for the slope?

Summary

STAMP THE LEARNING

A scenario can be represented by a function that describes the rate of change and the y-intercept.

Facilitation Notes

In this activity, students continue working with the Fundraising Function scenario. They analyze a graph that models the situation and use it to solve inequalities. Students graph the solution set for each inequality on a number line.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

Suggest students get in the habit of extending their arrows to the end of the number line. This will help them list other solutions to the inequality represented by the shaded region. It will also be important in the lesson on compound inequalities when students determine overlapping and non-overlapping shaded regions.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why does this situation require only Quadrant I of the coordinate plane? • How does the number line in part (c) relate to the graph? • Why doesn't your inequality symbol include an equals sign? • Mark the described region on the graph. • What is the inequality in terms of the dependent variable? • How did you know what inequality symbol to use? • What is the inequality in terms of the independent variable?
Seeing structure	• How is your response on the number line related to the information on the coordinate plane? • Why do only positive integers make sense for this situation?

AS STUDENTS WORK, LOOK FOR

Confusion between the discrete and continuous aspects of the graphical representations of the problem situation on both the coordinate plane and the number line. The domain of the situation can only be positive integers; whereas, the domains in both graphical representations appear to include all real numbers.

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain what the phrase *at least* means.
Probing	• How did you modify the original function to write the inequality? • How can you identify the values of the independent variable that apply to this question?
Seeing structure	• How is part (c) different than parts (a) and (b)? How is this difference reflected in your response?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Regions of a graphed function can be represented on a number line as solutions to an inequality.

STAMP THE LEARNING

ACTIVITY 3.2

Solving Two-Step Linear Inequalities

Facilitation Notes

In this activity, the term *solve an inequality* is defined. A Worked Example demonstrates how to solve an inequality algebraically. Students write and solve inequalities and choose the most accurate answer in the context of the problem situation. They conclude that an inequality, regardless of its form, is solved in the same way as an equation, by using properties of equality and number properties.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction. Discuss the Worked Example and answer Question 1 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how the inequality makes sense for the situation in the Worked Example. • Since the situation involves whole boxes of popcorn only, how did you know whether the solution is $b \geq 286$ or $b \geq 287$?
Seeing structure	• What is the relationship between the steps to solve an inequality and the steps to solve an equation?

DIFFERENTIATION STRATEGY

Access for All

Suggest students use a strategy similar to the Worked Example. Encourage them to write the algebraic expression with the variable placed to the left of the inequality symbol and the $f(x)$-value to the right. This way, the inequality symbol required matches the wording in the question.

For example: Elijah's total sales, $f(b)$, need to be at least \$1100.

At least means $\geq$.

Correct: $3.75b + 25 \geq 1100$ Incorrect: $1100 \geq 3.75b + 25$

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Model for students how to ask themselves whether values greater or less than \$600 and \$1500 make sense in the context. Guide them to write inequalities based on their decisions.
- Have students check their answers by graphing each solution on a number line.
- Have students create a graphic organizer listing $>$, $\geq$, $<$, $\leq$, and phrases that imply the use of these inequality symbols.

AS STUDENTS WORK, LOOK FOR

- Incorrect selection of inequality symbols. Sometimes the choice of $>$ or $<$ and whether $=$ should be used is counterintuitive to students based upon the wording of the problem.
- Confusion regarding whether or not to round up or down to the nearest integer based upon the context.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you know what inequality symbol to use for this situation? • How did you solve the inequality? • How do you know whether to round up or down to the nearest integer?
Seeing structure	• Consider the inequality $3.75b + 25 \geq 1500$. How would you rewrite this inequality if you wanted 1500 to be on the left side of the inequality?

STAMP THE LEARNING

Summary

Problems in context can be represented and solved using inequalities. Solving an inequality is similar to solving an equation; both involve using properties of equality and number properties.

ACTIVITY 3.3

Reversing Inequality Signs

Facilitation Notes

In this activity, a scenario is provided that can be represented by an inequality with a negative slope. Students write a function that describes the scenario and identify key characteristics of the function in terms of the problem situation. They then use a graph of the function to solve an inequality. The remainder of the activity may flow two different ways, depending upon whether students remember from previous courses that the inequality symbol must be reversed when multiplying or dividing an inequality by a negative value. If students know this rule, the algebraic solution to the inequality verifies the graphical solution. They then complete a table with inequality statements that justify this rule. If students do not know this rule, the algebraic solution to the inequality contradicts the graphical representation. They then complete a table with inequality statements based upon the correct graphical approach and use the table results to make sense of why multiplication and division by a negative value requires reversing the inequality symbol.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Which type of function models this scenario? Why? • Why might someone prefer to write the function as $h(m) = 4800 - 20m$ rather than $h(m) = -20m + 4800$?
Probing	• Why is the coefficient of the variable a negative number? • How can you determine the *x*-intercept from the equation? • Explain how you identified the domain.

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Placement of the oval on the graph.
- Estimates that are close to 80.
- Algebraic solutions of $m < 80$ that conflict with the graphical solution of $m > 80$.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Have students sketch a picture or annotate the graph to help make sense of the situation.
- Help them determine the correct graphical solution based on the context and direct them to use those results to complete the table in Question 7. Address errors in the algebraic process when discussing Question 7.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you use the graph to determine the smallest independent value that makes sense in this situation? • Explain your steps to solve the inequality. • Are your algebraic and graphical solutions the same? Explain.

Complete Question 6 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why do you think you reverse the inequality symbols in this scenario but not in Elijah's popcorn sales scenario? What is different about the scenarios? • How do you know when to reverse the inequality symbol and when to keep it as is?

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

When both sides of an inequality are multiplied or divided by a negative number, the sign of the resulting inequality is reversed.

ACTIVITY 3.4

Solving Other Linear Inequalities

Facilitation Notes

In this activity, students analyze methods to solve more complex linear inequalities: ones with the variable on both sides of the equation, and ones that require the distributive property. The methods for solving inequalities are compared with those for solving equations.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Whose process do you prefer? Why?
Probing	• How are Victoria's and Kai's processes the same? Different?
Seeing structure	• Why do you think the rule about reversing the signs applies to both division and multiplication?

COMMON MISCONCEPTION

Students sometimes need support in remembering when to reverse the inequality symbol. Refer students back to Activity 3.3 to help them make sense of reversing the inequality symbol in context.

DIFFERENTIATION STRATEGIES

Just in Time Support

Materials Needed: Algebra Tiles

- Have students use algebra tiles to model and solve the inequality using both Kai's and Victoria's strategies. Discuss why this strategy would not be useful for solving in Elijah's situation in the previous activities.
- Have students rewrite Victoria's and Kai's methods so that the difference in their first steps is more explicit.

Victoria	Kai
$\begin{array}{rcl} 5x + 2 & \geq & 3x - 10 \\ -3x & & -3x \\ \hline 2x + 2 & \geq & -10 \end{array}$	$\begin{array}{rcl} 5x + 2 & \geq & 3x - 10 \\ -5x & & -5x \\ \hline 2 & \geq & -2x - 10 \end{array}$

Challenge Opportunity

- Graph the solution inequality on a number line, identify a value in the solution set, then substitute that value in the original inequality to verify that it results in a true inequality statement.
- Suggest students always deal with $x = 0$ to check their answer. Refer to the solution inequality to determine whether zero is a solution or not. If zero is a solution, then substituting in zero should result in a true inequality statement. This method will be expanded on when working in the coordinate plane, where students can use the origin to check accuracy of solutions.

COMMON MISCONCEPTION

Students may incorrectly rewrite Kai's inequality solution, $-6 \leq x$, as $x \leq -6$ because they overgeneralize the process used for equations, where $6 = x$ can be rewritten as $x = 6$.

To correct this error, suggest students read the entire inequality statement, including the inequality symbol, from right to left. In this case, $-6 \leq x$ would be read as x is greater than or equal to 26 and is written as $x \geq -6$. Be cautious of the phrase *reversing the inequality symbol* since it references the procedure for multiplying or dividing an inequality by a negative value. In this instance, students are simply rewriting an already correct inequality statement.

Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why did Ava reverse the inequality symbol earlier in his process?
Seeing structure	• Why didn't Aaliyah reverse the inequality symbol when he used the distributive property to multiply by a negative number?

Summary

STAMP THE LEARNING

There are multiple solution paths when solving more complex linear inequalities. All correct solution paths involve the same steps as solving a linear equation as well as reversing the sign of the inequality symbol when both sides of the inequality are multiplied or divided by a negative number.

DEMONSTRATE

Talk the Talk

IT'S ABOUT SOLUTIONS, MORE OR LESS

Facilitation Notes

In this activity, students solve four inequalities and graph each solution on a number line. They also identify the constraints required to make true inequality statements.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Confusion dealing with the negative sign with the fraction.
- Errors regarding inequality sign changes when multiplying or dividing by a negative number.
- Issues dealing with the variable on the right side of the inequality symbol.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Suggest a strategy for abstract problems such as these. First, have students use values such as $A = 3$ and $B = 4$. Then, suggest they try positive numbers, negative numbers, 0, 1, -1 and fractions for C.
- Suggest a range and interval to number the number lines.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Did you rewrite the inequality in Question 2 before you solved it so that the variable was on the left side of the inequality symbol? If so, explain your process. • If you didn't write the inequality in Question 2, explain how you graphed the solution $-9 < x$. • For what inequalities did you have to apply the rule to reverse the inequality symbol?
Seeing structure	• For 5b, why is the equation true only for specific values of C? • How did you apply the rules to solve inequalities to complete these statements?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

There are multiple solution paths when solving more complex linear inequalities. All correct solution paths involve the same steps as solving a linear equation as well as reversing the sign of the inequality symbol when both sides of the inequality are multiplied or divided by a negative number.

3 Modeling Linear Inequalities

Setting the Stage

- Communicate the objectives and new key term.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Write and solve inequalities.
- Analyze a graph on a coordinate plane to solve problems involving inequalities.
- Interpret how a negative rate affects how to solve an inequality.

NEW KEY TERM

- linear inequality

You have used horizontal lines on a graph and properties of equality with equations to solve problems that can be modeled with linear equations.

Can you use these same methods to solve problems involving linear inequalities?

Sample answer:

I can use the same methods as long as I reverse the direction of the inequality symbol when multiplying or dividing both sides by a negative number.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Before students begin the lesson, review the four basic inequality symbols and ensure students' understanding of each sign. Create a chart on the board and list the four symbols as the column headers: $>$, $<$, $\geq$, and $\leq$. Ask students to provide several words that represent each symbol. For example, for *less than or equal to*, students may use *fewer*, *under*, *no more than*, and *at most*. Provide examples of inequalities using the four symbols and ask students to translate each inequality into words.

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Fundraising Function

Ask yourself . . .
How will you represent the $25 credit in your function?

Elijah's camping troop is selling popcorn to earn money for an upcoming camping trip. Each camper starts with a credit of $25 toward his sales, and each box of popcorn sells for $3.75.

1. Write a function, $f(b)$, to show Elijah's total sales as a function of the number of boxes of popcorn he sells, b.

   $f(b) = 3.75b + 25$

2. Analyze the function you wrote.

   a. Identify the independent and dependent quantities and their units.

   The independent quantity is the number of boxes sold, and the dependent quantity is the total sales, in dollars.

   b. What is the slope of the function? What does it represent in this problem situation?

   The slope is 3.75. It represents the cost $3.75 for each box of popcorn.

   c. What is the y-intercept? What does it represent in this problem situation?

   The y-intercept is 25. This represents the $25 credit toward the total sales each troop member receives.

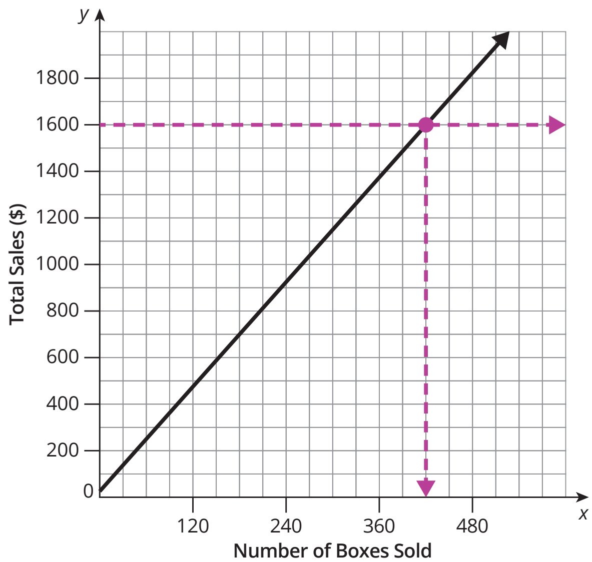
EB STUDENT TIPS

For "Intermediate" and higher proficiency levels

Review the terms *independent quantity* and *dependent quantity*. Ask students to define the terms *independent* and *dependent* in a non-mathematical context and then ask them to connect the terms in the context of the quantities in the given function. Review the problem in the section and clarify any additional misunderstandings.

ACTIVITY 3.1 Modeling Linear Inequalities

The graph shown represents the function you wrote in the previous activity, $f(b) = 3.75b + 25$.



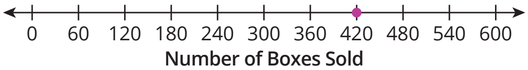
1. Suppose Elijah has a sales goal of $1600.

   a. Draw a horizontal line from the *y*-axis until it intersects with the graphed function to determine the point on the graph that represents $1600 in total sales.

   b. How many boxes would Elijah have to sell to make $1600 in total sales? Explain your reasoning.

   420 boxes; I drew a vertical line down from where the horizontal line intersected the graph and saw that it intersected the *x*-axis at 420.

   c. Use the number line to represent the number of boxes sold if the total sales is equal to $1600.



Ask yourself . . .
Will the points you graph on the number line be open or closed?

Chunking the Activity

- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Questions 4 and 5.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

EB STUDENT TIPS

For all proficiency levels

To help students stay focused on the activity's goal, ensure they understand the meaning of *camping troop*. Explain that it is a group that goes camping together.

MODULE 3 • TOPIC 1 • LESSON 3 373

A **linear inequality** is a non-equal comparison of two linear expressions.

2. Analyze the region of the graph that lies below the horizontal line you drew, up to and including the point intersected by the line.

   a. What does this region of the graph represent?

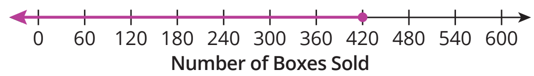
   It represents all the numbers of boxes sold, *b*, that would earn Elijah \$1600 or less.

   b. Write an inequality to represent this region.

   $f(b) \leq 1600$ or $b \leq 420$

Remember . . .
There are five inequality signs, less than (<), greater than (>), less than or equal (≤), greater than or equal (≥) and not equal (≠). The solution set of an inequality is all the values that make the inequality statement true.

   c. Use the number line to represent the number of boxes that are solutions to the inequality you wrote.



   d. Do all the solutions make sense in context of the problem? Explain your reasoning.

   No, only positive integers less than or equal to 420 make sense in the problem situation because Elijah cannot sell negative or fractional boxes of popcorn.

3. Analyze the region of the graph that lies above the horizontal line you drew, not including the point intersected by the line.

   a. What does this region of the graph represent?

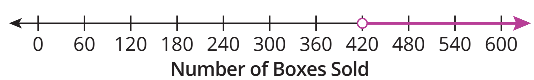
   It represents all the numbers of boxes sold, *b*, that would earn Elijah more than \$1600.

   b. Write an inequality to represent this region.

   $f(b) > 1600$ or $b > 420$

Graphing the solutions to inequalities on number lines will prepare you for graphing linear inequalities in two variables.

   c. Use the number line to represent the number of boxes that are solutions to the inequality you wrote.



   d. Do all the solutions make sense in context of the problem? Explain your reasoning.

   No, only positive integers greater than 420 make sense in the problem situation because Elijah cannot sell fractional boxes of popcorn.

EB STUDENT TIP

For all proficiency levels

Students may be familiar with the term *region* in the context of countries or parts of the world. Ensure they understand that *region* refers to a location on the graph. Allow students to connect their understanding to the meaning in this context.

374 MODULE 3 • TOPIC 1 • LESSON 3

4. Explain the difference between the open and closed circles on your number lines.

   The open circle means that the point is not included in the solution.

   The closed circle means that the point is included in the solution.

Ask Yourself . . .
How does determining the intersection point help you determine your answers?

5. Use the graph to answer each question. Write an equation or inequality statement for each.

   a. How many boxes would Elijah have to sell to earn at least \$925?

      Elijah would have to sell at least 240 boxes.
      $b \geq 240$ or $3.75b + 25 \geq 925$

   b. How many boxes would Elijah have to sell to earn less than \$2050?

      Elijah would have to sell fewer than 540 boxes.
      $b < 540$ or $3.75b + 25 < 2050$

   c. How many boxes would Elijah have to sell to earn exactly \$700?

      Elijah would have to sell exactly 180 boxes.
      $b = 180$ or $3.75b + 25 = 700$

Questions 1–5 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice of writing and solving a linear inequality graphically, assign Skills Practice Set A for this lesson. To provide additional practice representing a solution to an inequality on a number line, assign Skills Practice Set B for this lesson.

MODULE 3 • TOPIC 1 • LESSON 3 375

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction and Worked Example.
- Answer Question 1 as a class.
- Group students to complete Questions 2 and 3.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY **3.2** Solving Two-Step Linear Inequalities

To solve an inequality means to determine the values of the variable that make the inequality true.

Another way to determine the solution set of any inequality is to solve it algebraically. The objective when solving an inequality is similar to the objective when solving an equation: You want to isolate the variable on one side of the inequality symbol.

To make the first deposit on the trip, Elijah's total sales, $f(b)$, need to be at least \$1100.

WORKED EXAMPLE

You can set up an inequality and solve it to determine the number of boxes Elijah needs to sell.

$$f(b) \geq 1100$$
$$3.75b + 25 \geq 1100$$

Solve the inequality in the same way you would solve an equation.

$$3.75b + 25 \geq 1100$$
$$3.75b + 25 - 25 \geq 1100 - 25$$
$$3.75b \geq 1075$$
$$\frac{3.75b}{3.75} \geq \frac{1075}{3.75}$$
$$b \geq 286.\overline{6}$$

Think about . . .
How accurate does your answer need to be?

1. How many boxes of popcorn does Elijah need to sell to make the first deposit? Explain your reasoning.

   Elijah needs to sell at least 287 boxes of popcorn. Elijah can't sell partial boxes of popcorn. If he sold 286 boxes, that would not be enough to earn at least \$1100.

376 MODULE 3 • TOPIC 1 • LESSON 3

2. Write and solve an inequality for each. Show your work.

   a. What is the greatest number of boxes Elijah could sell and still not have made \$600 in sales?

   $3.75b + 25 < 600$

   $3.75b + 25 - 25 < 600 - 25$

   $3.75b < 575$

   $\frac{3.75b}{3.75} < \frac{575}{3.75}$

   $b < 153.33...$

   Elijah could sell at most 153 boxes of popcorn.

   b. At least how many boxes would Elijah have to sell to make \$1500 in sales?

   $3.75b + 25 \geq 1500$

   $3.75b + 25 - 25 \geq 1500 - 25$

   $3.75b \geq 1475$

   $\frac{3.75b}{3.75} \geq \frac{1475}{3.75}$

   $b \geq 393.33...$

   Elijah would need to sell at least 394 boxes.

3. The Worked Example showed how to solve an inequality of the form, $y = mx + b$. How does your method change if the inequality is in a different form?

   Because an inequality is solved in the same way as an equation, by using properties of equality and number properties, the method to solve an inequality does not change if it is written in a different form.

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MODULE 3 • TOPIC 1 • LESSON 3 377

Chunking the Activity

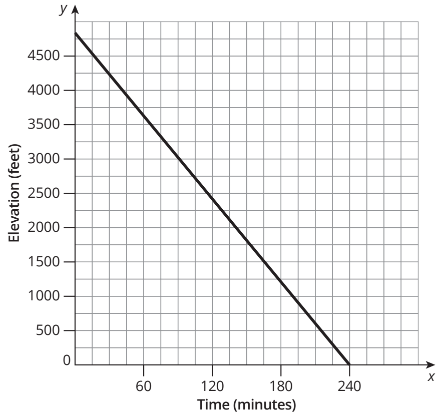
- Group students to complete Questions 1 and 2.
- Check in and share.
- Group students to complete Questions 3–5.
- Check in and share.
- Complete Question 6 as a class and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY **3.3**

Reversing Inequality Signs

Elijah's camping troop hikes down from their campsite at an elevation of 4800 feet to the base of the mountain, which is at sea level. They hike down at a rate of 20 feet per minute.

Sea level has an elevation of 0 feet.



1. Write a function, *h*(*m*), to show the troop's elevation as a function of time in minutes. Label the function on the coordinate plane.

   $h(m) = -20m + 4800$

Ask yourself . . . What are the independent and dependent quantities and their units?

2. Analyze the function. Identify each characteristic and explain what it means in terms of this problem situation.

   a. Slope

   The slope is −20. This represents a descent of 20 feet every minute.

   b. *y*-intercept

   The *y*-intercept is 4800. The troop starts their hike at an elevation of 4800 feet.

   c. *x*-intercept

   The *x*-intercept is 240. The hikers reach the bottom of the mountain in 240 minutes, or 4 hours.

   d. Domain of the function

   The domain is $0 \le x \le 240$. They begin their descent at 0 minutes and end it 240 minutes later.

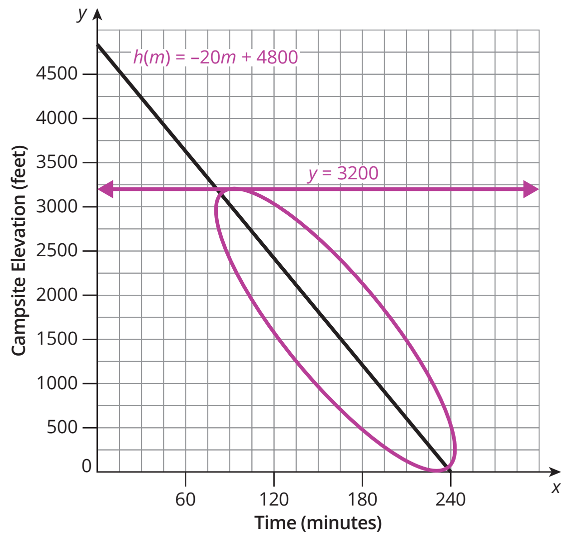
EB STUDENT TIPS

For all proficiency levels

Have students circle the verb or verb statement at the beginning of each question in the activity. Discuss what each verb means and have students suggest examples of wording they could use when answering the question.

3. Use the graph to determine how many minutes passed if the troop is below 3200 feet. Draw an oval on the graph to represent this part of the function and write the corresponding inequality statement.

   It appears from the graph that about 80 or more minutes have passed if the troop is below 3200 feet; $m > 80$



4. Write and solve an inequality to verify the solution set you interpreted from the graph.

   $-20m + 4800 < 3200$

   $m > 80$

5. Compare and contrast the solution sets you wrote using the graph and the function. What do you notice?

   The solution set should be the same, regardless of the strategy used.

6. Analyze the relationship between the inequality statements representing $h(m)$ and m.

   a. Complete the table by writing the corresponding inequality statement that represents the number of minutes for each height.

   b. Compare each row in the table shown. What do you notice about the inequality signs?

   The inequality signs are reversed in each row.

$h(m)$	m
$h(m) > 3200$	$m < 80$
$h(m) \geq 3200$	$m \leq 80$
$h(m) = 3200$	$m = 80$
$h(m) < 3200$	$m > 80$
$h(m) \leq 3200$	$m \geq 80$

   c. Explain your answer from part (b). Use what you know about solving inequalities when you have to multiply or divide by a negative number.

   The function includes a negative coefficient for x. When I divide by the negative coefficient to solve the inequality, the sign reverses. That is why the inequality symbol is reversed in every row.

Questions 1–7 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice of writing and solving a linear inequality algebraically, assign Skills Practice Set C for this lesson.

EB STUDENT TIPS

For all proficiency levels

Ensure students understand the meaning of terms such as *elevation* and *base* in the context of the situation. Encourage students to explain their understanding of the terms before clarifying any possible misconceptions about the situation.

MODULE 3 • TOPIC 1 • LESSON 3 379

Chunking the Activity

- Read the Essential Question and activity summaries from Session 2.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Group students to complete Questions 3 and 4.
- Share and summarize.

ACTIVITY 3.4

Solving Other Linear Inequalities

The inequalities that you have solved in this lesson so far have all been two-step inequalities. Let's consider inequalities in different forms.

Victoria and Kai each solved the inequality $5x + 2 \geq 3x - 10$.

You can rewrite Kai's solution to show it is equivalent to Victoria's. It is sometimes preferred to have the variable on the left side of the inequality. So, $-6 \leq x$ is equivalent to $x \geq -6$.

Victoria

$$5x + 2 \geq 3x - 10$$
$$5x - 3x + 2 \geq 3x - 3x - 10$$
$$2x + 2 \geq -10$$
$$2x + 2 - 2 \geq -10 - 2$$
$$2x \geq -12$$
$$\frac{2x}{2} \geq \frac{-12}{2}$$
$$x \geq -6$$

Kai

$$5x + 2 \geq 3x - 10$$
$$5x - 5x + 2 \geq 3x - 5x - 10$$
$$2 \geq -2x - 10$$
$$2 + 10 \geq -2x - 10 + 10$$
$$12 \geq -2x$$
$$\frac{12}{-2} \leq \frac{-2x}{-2}$$
$$-6 \leq x$$

Remember. . .
When multiplying or dividing by a negative number while solving inequalities, the inequality sign reverses.

1. Describe the process each student used to solve the inequality.

   a. Victoria

   First, the expressions containing variables were moved to the left side of the inequality. Next, the constants were moved to the right side of the inequality. Finally, the variable was isolated by dividing by 2.

   b. Kai

   First, the expression containing variables was moved to the right side of the inequality. Next, the constants were moved to the left side of the inequality. Finally, the variable was isolated by dividing by -2 which reversed the sign of the inequality.

2. How does the process of solving an inequality with variables on both sides compare to the process of solving an equation with variables on both sides?

   The process for solving is basically the same. In both, I have to move all the variables to one side and all the constants to the other side, and then isolate the variable. When I solve an inequality I have to pay attention to the inequality symbol. If I multiply or divide by a negative value, the inequality symbol is reversed.

EB STUDENT TIPS

For "Intermediate" and higher proficiency levels

Ask students to list the types of properties that can be used when solving equations and inequalities. Discuss how the *distributive property* relates to the term *distribute*. Create a list of synonyms for *distribute*, such as *to give out*, *hand out*, and *dish out*. Ask students to demonstrate their understanding of the *distributive property* by explaining the steps in a simplified example, such as $2(3 + 4) = 2(3) + 2(4)$.

Aaliyah and Ava each solved the inequality $-4(x - 6) < 22$.

Aaliyah

$$-4(x - 6) < 22$$
$$-4x + 24 < 22$$
$$-4x + 24 - 24 < 22 - 24$$
$$-4x < -2$$
$$\frac{-4x}{-4} > \frac{-2}{-4}$$
$$x > \frac{1}{2}$$

Ava

$$-4(x - 6) < 22$$
$$\frac{-4(x - 6)}{-4} > \frac{22}{-4}$$
$$x - 6 > -5\frac{1}{2}$$
$$x > \frac{1}{2}$$

3. Describe the process each student used to solve the inequality.

   a. Aaliyah

   First, the distributive property was used to remove the parentheses. Next, the constants were moved to the right side of the inequality. Finally the variable was isolated by dividing by -4, which reversed the inequality symbol.

   b. Ava

   First, both sides of the inequality were divided by -4, which reversed the inequality symbol. Next, the constants were moved to the right side of the inequality, isolating the variable.

4. How does the process of solving an inequality using the distributive property compare to the process of solving an equation using the distributive property?

   The process of using the distributive property to remove parentheses from either an inequality or an equation is the same.

Questions 1–4 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice of solving a linear inequality algebraically, assign Skills Practice Set D for this lesson.

EB STUDENT TIPS

For "Intermediate" and higher proficiency levels

Encourage students to annotate the strategies, identifying each step, as an alternate way of responding to Questions 1 and 3.

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

It's About Solutions, More or Less

1. Solve each inequality and graph the solution on the number line.

   a. $-\frac{2}{3}x \geq 7$

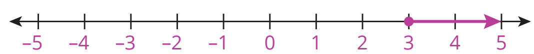
   $x \leq -\frac{21}{2}$

   b. $32 > 23 - x$

   $x > -9$

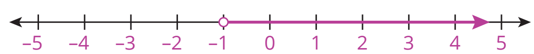
   c. $15 - 4x \leq 3x - 6$

   $x \geq 3$



   d. $2(x + 6) < 14 + 4x$

   $x > -1$



2. If $A < B$, identify the constraints to make each statement true.

   a. $A + C < B + C$

   The statement is true for all values of A, B, and C.

   b. $AC > BC$

   The statement is only true for values of $C < 0$.

   c. $-A < -B$

   The statement is never true for any values of A or B.

382 MODULE 3 • TOPIC 1 • LESSON 3

Lesson 3 Assignment

Write

Describe how to solve an inequality in your own words.

Remember

The methods for solving linear inequalities are similar to the methods for solving linear equations. Be sure to reverse the direction of the inequality symbol when multiplying or dividing both sides by a negative number.

Practice

1. Noah is going on a trip to visit some friends from summer camp. He will use $40 for food and entertainment. He will also need money to cover the cost of gas. The price of gas at the time of his trip is $3.25 per gallon.

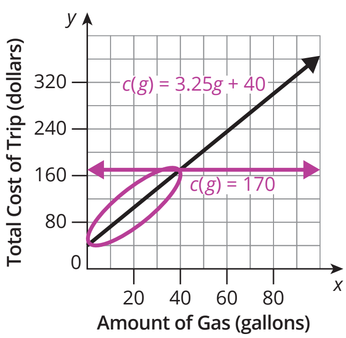
   a. Write a function to represent the total cost of the trip as a function of the number of gallons used.

   $C(g) = 3.25g + 40$

   b. Identify the rate of change and the *y*-intercept. Explain their meanings in terms of the problem situation.

   The rate of change is 3.25. This represents the cost for each gallon of gas. The *y*-intercept is 40. This represents the cost of the trip if no gas is used.

   c. Graph the function representing this situation on a coordinate plane.



   d. Use the graph to determine how many gallons of gas Noah can buy if he has $170 saved for the trip. Draw an oval on the graph to represent the solution. Then, write your answer in words and as an inequality.

   See graph in part (c).
   Noah can buy no more than 40 gallons of gas on the trip.
   $g \leq 40$

Write

Sample answer:

To solve an inequality means to isolate the variable on one side of the inequality sign to determine what values of the variable will make the inequality a true statement. All correct solution paths involve the same steps as solving a linear equation as well as reversing the sign of the inequality symbol when both sides of the inequality are multiplied or divided by a negative number.

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MODULE 3 • TOPIC 1 • LESSON 3 ASSIGNMENT 383

Lesson 3 Assignment

e. Verify the solution set you interpreted from the graph.

$3.25g + 40 \leq 170$

$3.25g \leq 130$

$g \leq 40$

f. Noah's mom gives him some money for his trip. He now has a total of $220 saved for the trip. What is the greatest number of gallons of gas he can buy before he runs out of money? Show your work and graph your solution on a number line.

$3.25g + 40 \leq 220$

$3.25g \leq 180$

$g \leq 55.38$

Noah can buy no more than 55.38 gallons of gas.

g. If Noah spent more than $92 on his trip, how much gas could he have bought? Show your work and graph your solution on a number line.

$3.25g + 40 > 92$

$3.25g > 52$

$g > 16$

Noah bought more than 16 gallons of gas if he spent more than $92.

2. Noah is on his way to visit his friends at camp. Halfway to his destination, he realizes there is a slow leak in one of the tires. He checks the pressure and it is at 26 psi. It appears to be losing 0.1 psi per minute.

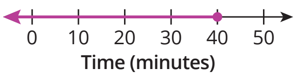
a. Write a function to represent the tire's pressure as a function of time in minutes.

$p(t) = 26 - 0.1t$

384 MODULE 3 • TOPIC 1 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

b. Noah knows that if the pressure in a tire goes below 22 psi, it may cause a tire blowout. What is the greatest amount of time that he can drive before the tire pressure hits 22 psi? Show your work and graph the solution.



$$26 - 0.1t \geq 22$$
$$-0.1t \geq -4$$
$$t \leq 40$$

Noah can drive a maximum of 40 minutes before the pressure goes below 22 psi.

3. Solve each inequality for the unknown value.

a. $13 + 4x > 9$

$x > -1$

b. $3(4 - 5x) > 8x - 149$

$x < 7$

c. $99 - 5d \geq 4d$

$d \leq 11$

d. $3k - 9 \leq -6k - 225$

$k \leq -24$

Prepare

Determine an ordered pair that represents a solution to each equation.

1. $4x + 7y = 24$

Sample answer:

(6, 0)

2. $5x - 2y = -6$

Sample answer:

(0, 3)

3. $\frac{1}{2}x + \frac{3}{4}y = 10$

Sample answer:

(20, 0)

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MODULE 3 • TOPIC 1 • LESSON 3 ASSIGNMENT 385

TOPIC 1 SELF-REFLECTION

Name: ____________________

Linear Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Linear Equations and Inequalities* topic by:

TOPIC 1: *Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
solving linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.	◯	◯	◯
constructing an argument to justify a solution process for a one-variable equation.	◯	◯	◯
using and interpreting units when solving literal equations for a given variable.	◯	◯	◯
solving literal equations for a specified variable.	◯	◯	◯
recognizing linear relationships from problem situations and writing equations or inequalities to model linear relationships.	◯	◯	◯
solving linear inequalities in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.	◯	◯	◯
interpreting solutions in the context of a problem situations and determining whether they are reasonable.	◯	◯	◯
representing constraints to a problem situation as an inequality.	◯	◯	◯

continued on the next page

Notes

MODULE 3 • TOPIC 1 • SELF-REFLECTION 387

Notes

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Linear Equations and Inequalities* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

388 MODULE 3 • TOPIC 1 • SELF-REFLECTION

TOPIC 1 SUMMARY

Linear Equations and Inequalities Summary

NEW KEY TERMS

- solution [solución]
- infinite solutions [soluciones infinitas]
- no solution [sin solución]
- literal equation [ecuación literal]
- linear inequality [desigualdad lineal]

LESSON 1 Solving Linear Equations

A **solution** to an equation is any variable value that makes the equation true. Solving equations requires the use of number properties and the properties of equality. The properties of equality state that if an operation is performed on both sides of the equation, to all terms of the equation, the equation maintains its equality. When the properties of equality are applied to an equation, the transformed equation will have the same solution as the original equation.

A linear equation in the form $y = mx + b$ can have *one solution*, *infinite solutions*, or *no solution*.

Linear equations with **infinite solutions** are created by equating two equivalent expressions.

$$\begin{aligned} 2x + 6 &= 2(x + 3) \\ &= (2 \cdot x) + (2 \cdot 3) \\ &= 2x + 6 \end{aligned}$$

Linear equations with **no solution** are created by equating expressions of the form $mx + b$, with the same value for m and different values for b.

$$\begin{aligned} 4x + 2 &= 4x + 5 \\ 4x - 4x + 2 &= 4x - 4x + 5 \\ 2 &\neq 5 \end{aligned}$$

Linear equations with the solution $x = 0$ are created by equating expressions of the form $mx + b$, with different values for m and the same value for b.

$$\begin{aligned} 2x + 3 &= 5x + 3 \\ 2x - 2x + 3 &= 5x - 2x + 3 \\ 3 &= 3x + 3 \\ 3 - 3 &= 3x + 3 - 3 \\ 0 &= 3x \\ 0 &= x \end{aligned}$$

LESSON 2 Literal Equations

There are three useful forms of linear equations.

Slope-Intercept	Point-Slope Form	Standard Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By = C$

Notes

MODULE 3 • TOPIC 1 • TOPIC SUMMARY 389

Notes

You can use any real numbers for the values of m, b, x_1, *and* y_1 in slope-intercept and point-slope from. In standard form, however, there are constraints on the variables: A, B, and C must be integers, and A and B cannot both be 0.

It is often necessary to change between forms, as the structure of each form reveals different key characteristics of the equation, such as the slope and x- and y-intercepts. Remember, to convert to slope-intercept form from standard form, solve for y. To convert to standard form from slope-intercept form, isolate both variables on the same side of the equation and the constant on the other side of the equation.

A **literal equation** is an equation in which the variables represent specific measures. Literal equations are most often seen when studying formulas. These literal equations can be manipulated in order to solve for one specific variable.

For example, a common literal equation is the formula for converting degrees Fahrenheit to degrees Celsius, $C = \frac{5}{9}(F - 32)$. You can use the properties of equality to rewrite the equation to convert degrees Celsius to degrees Fahrenheit, $F = \frac{9}{5}(C + 32)$.

LESSON 3 Modeling Linear Inequalities

To solve a **linear inequality**, first write a function to represent the problem situation. Then, write the function as an inequality based on the independent quantity. To determine the solution, identify the values of the variable that make the inequality true. The objective when solving an inequality is similar to the objective when solving an equation: isolate the variable on one side of the inequality symbol. Finally, interpret the meaning of the solution.

For example, consider the situation in which Diego has $25 in his gift fund that he is going to use to buy graduation gifts. Graduation is 9 weeks away. He would like to have at least $70 to buy gifts, how much should he save each week?

The function is $f(x) = 25 + 9x$, so the inequality is $25 + 9x \geq 70$.

$$
\begin{aligned}
25 + 9x &\geq 70 \\
25 + 9x - 25 &\geq 70 - 25 \\
9x &\geq 45 \\
\frac{9x}{9} &\geq \frac{45}{9} \\
x &\geq 5
\end{aligned}
$$

Diego needs to save at least $5 each week to meet his goal.

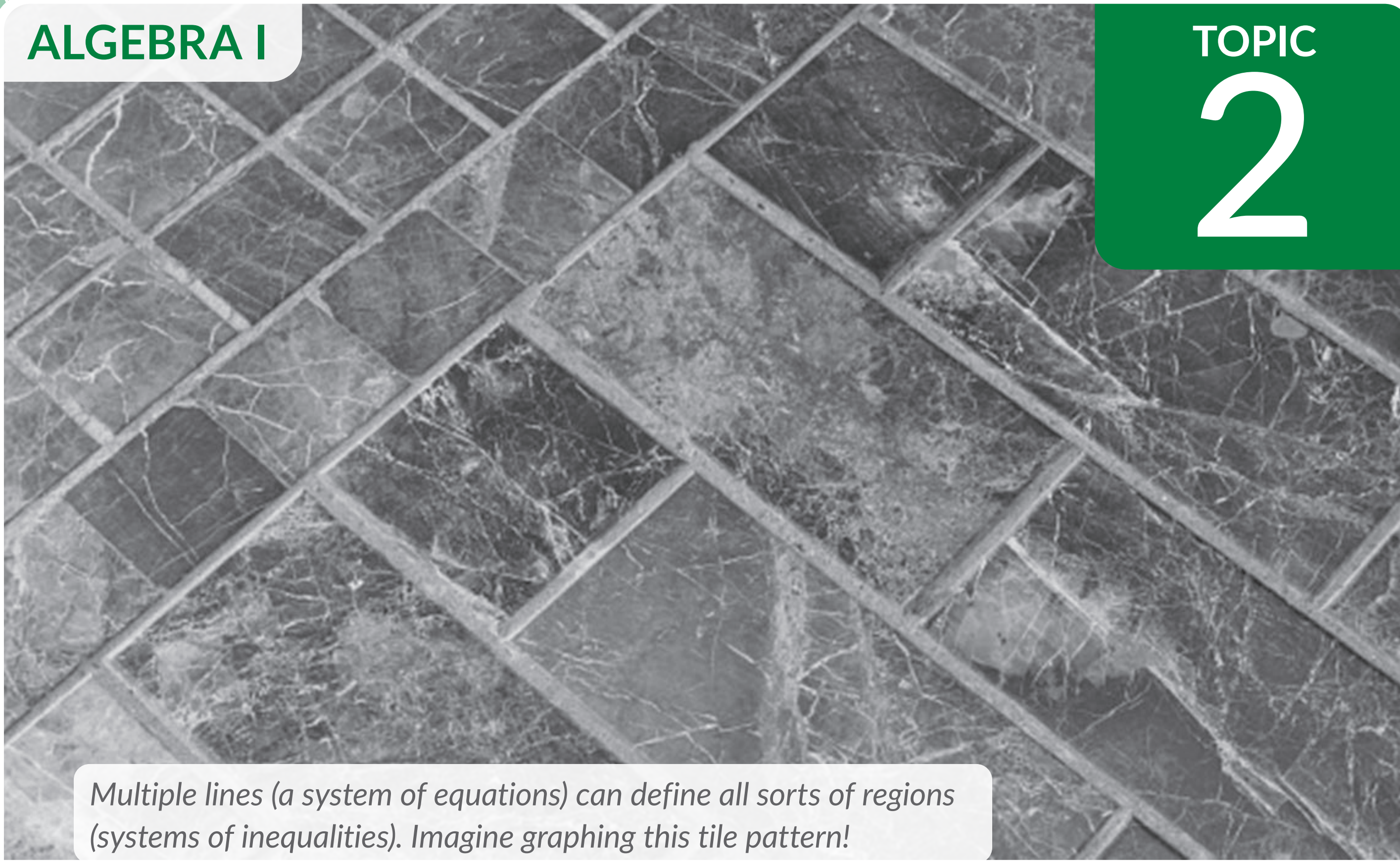
When you divide each side of an inequality by a negative number, the inequality sign reverses. For example, consider the inequality $250 - 9.25x < 398$.

$$
\begin{aligned}
250 - 9.25x &< 398 \\
250 - 9.25x - 250 &< 398 - 250 \\
-9.25x &< 148 \\
\frac{-9.25x}{-9.25} &< \frac{148}{-9.25} \\
x &> -16
\end{aligned}
$$

390 MODULE 3 • TOPIC 1 • TOPIC SUMMARY

ALGEBRA I

TOPIC 2



Multiple lines (a system of equations) can define all sorts of regions (systems of inequalities). Imagine graphing this tile pattern!

Systems of Linear Equations and Inequalities

Systems of Linear Equations and Inequalities

How are the key concepts of *Systems of Equations and Inequalities* organized?

In *Systems of Linear Equations and Inequalities*, students begin with writing a system of linear equations to represent scenarios and learning to solve systems graphically. Next, students write systems of linear equations and solve them algebraically using substitution. Students have experience with this concept from Grade 8, and this topic reminds them of what they already know. Next, they work with a system of linear equations written in standard form. This builds upon what they learned about standard form in previous topics and prepares them to solve systems using the linear combinations method. They analyze consistent systems of equations, which have either one solution or infinite solutions, and inconsistent systems, which have no solution. Students then move on to solve systems of linear equations using the linear combinations method. They first analyze and solve systems where two equations share a variable whose coefficient in one equation is the additive inverse of its coefficient in the other; then, students learn to transform one or both equations to use this method. Many of the systems are in context, and students are asked to verify and interpret their solution in terms of the problem situation.

Math Representation

You can use linear combinations to solve systems of equations.

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

$3(4x + 2y) = 3(3)$ $-2(5x + 3y) = -2(4)$	Multiply each equation by a constant that results in coefficients that are additive inverses for one of the variables.
$12x + 6y = 9$ $-10x - 6y = -8$	You can now add these equations together to isolate the variable x.

Students then consider linear inequalities in two variables. They extend their knowledge of solutions to understand that the solution set of an inequality in two variables is half of a plane. They then graph two linear inequalities on the same plane and identify the solution set as the intersection of the corresponding half-planes.

Finally, students synthesize their understanding of systems by encountering several problems that can be solved using either a system of linear equations or a system of linear inequalities. Students decide which type of system is required and choose a reasonable and efficient solution strategy.

What is the entry point for students?

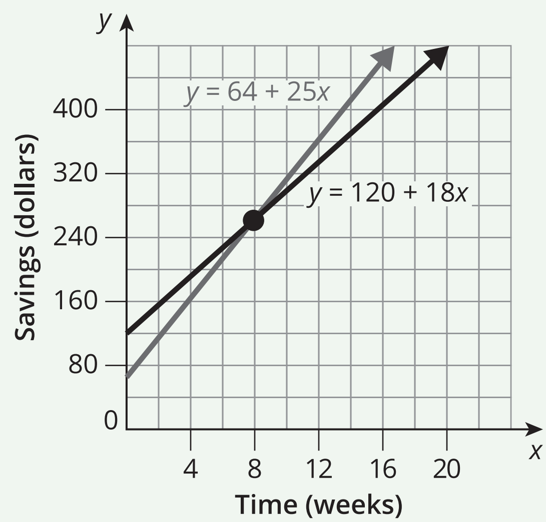
In previous courses, students learned that the point of intersection of two graphs provides x- and y-values that make both equations true. Students have written systems of linear equations and have solved them graphically. That knowledge is a springboard for this topic.

Math Representation

You can write a system of linear equations with a brace.

$$\begin{cases} y = 120 + 18x \\ y = 64 + 25x \end{cases}$$

You can determine the solution to this system by graphing the equations. The point of intersection is the solution to the system.



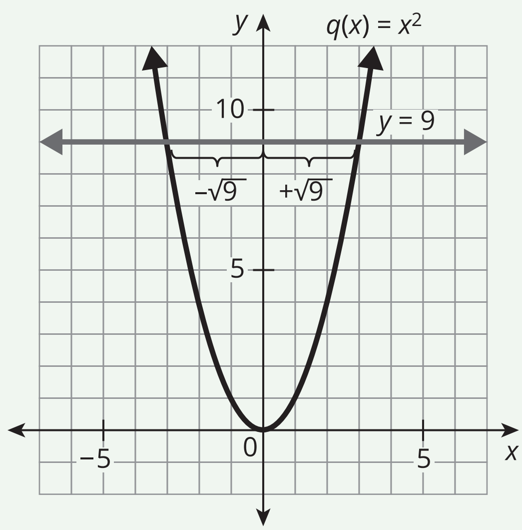
Students have also solved one-variable inequalities and graphed their solutions on a number line. Their work with two-variable inequalities in this topic will connect to their understanding of inequalities in one variable and their knowledge of graphing a line on the coordinate plane.

Why is *Systems of Equations and Inequalities* important?

Systems of equations and inequalities provide students with a way to compare and contrast similar real-world situations. They allow students to consider and represent constraints on a real-world situation. Knowing how to solve systems of linear equations prepares students to solve systems that include nonlinear equations. In future courses, students may encounter more advanced methods, such as matrices or Cramer's Rule, to solve systems of equations with more than two variables and more than two equations.

Math Representation

Solving $x^2 = 9$ on a graph means that you are looking for the points of intersection between $y = x^2$ and $y = 9$.



How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in *Systems of Equations and Inequalities* when they can:

- Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane.
- Write an equation, an inequality, or a system of linear equations or inequalities that best models a problem.
- Graph systems of equations on a coordinate grid with appropriate labels and scales.
- Solve a system of two linear equations in two variables approximately (graphically) and exactly (algebraically).
- Interpret the meaning of the solution to a system of linear equations in terms of a problem situation.
- Identify linear systems that have no solution and explain why they have no solution.
- Identify linear systems that have infinite solutions and explain why they have infinite solutions.
- Write and graph an inequality and a system of inequalities in two variables on a coordinate plane.
- Understand that the solution of a system of linear inequalities is the intersection of the shaded regions of both inequalities.
- Interpret the solution to an inequality in two variables graphed on a coordinate plane.

How do the activities in *Systems of Equations and Inequalities* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so TEKS mathematical process standards are evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Throughout this topic, students model real-world situations using equations and inequalities (A.1A). They reason about the coefficients as they rewrite systems to use substitution or the linear combinations methods to solve (A.1D). Student examine the structure of the equations to recognize when systems represent problems with one solution, no solution, or infinite solutions (A.1E). Students use the structure of inequalities to determine which half-plane represents the solution set (A.1F). Finally, students reason about situations to determine which type of system each represents and which solution strategy is the most efficient to use (A.1C).

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Use familiar and realistic scenarios in both languages when you discuss new mathematical vocabulary. This helps students see the practical application of mathematics and helps them contextualize vocabulary and concepts that are often abstract.

NEW KEY TERMS

- system of linear equations [sistema de ecuaciones lineales]
- consistent systems [sistemas consistentes]
- inconsistent systems [sistemas inconsistentes]
- standard form of a linear equation [forma estándar/genera de una ecuación lineal]
- substitution method [método de sustitución]
- linear combinations method [método de combinaciones lineales]
- half-plane
- boundary line
- constraints
- solution of a system of linear inequalities [solución de un sistema de desigualdades lineales]

3 Modeling Linear Equations and Inequalities

TOPIC 2: Systems of Linear Equations and Inequalities

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: A.1A, A.1B, A.1C, A.1D, A.1E, A.1F, A.1G

ELPS: 1.D, 2.B, 2.D, 2.H, 2.I, 3.A, 3.B, 3.C, 3.F, 4.A, 4.B, 4.G, 5.E

Topic Pacing: 22 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Using Graphing to Solve Systems of Equations	Students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They graph the linear equation using intercepts and then analyze a second graph with the independent and dependent variables reversed. A new relationship between the quantities is then provided, and students write the equation expressing the relationship. Finally, they graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario, creating a system of linear equations. Students solve the system both graphically and using technology, checking the solution by substituting the values back in to the original equations. Next, they are provided three related scenarios in which they write systems of equations in slope-intercept form and solve the systems graphically. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions. The related terms *consistent systems* and *inconsistent systems* are defined. **Materials Needed:** Graphing Technology	**A.2A** **A.2C** **A.2I** A.3F A.3G **A.5C**	2
2	Using Substitution to Solve Linear Systems	Students use the substitution method to solve systems of linear equations. They use substitution to solve systems of linear equations, including those with no solution or those with infinite solutions. Students define variables, write systems of equations, solve systems, and interpret the meaning of the solution in terms of the problem context. In the last activity, they are given four systems of linear equations and solve each system using the substitution method. **Materials Needed:** Problem-Solving Model Graphic Organizer	**A.2I** A.3F A.3G **A.5C**	3

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	Using Linear Combinations to Solve a System of Linear Equations	Students are given a problem scenario and use reasoning to determine the two unknowns. They then write a system of linear equations in standard form to represent a problem situation. Students analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They practice the linear combinations method with systems in which the coefficients of one variable are additive inverses. Next, worked examples guide students to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system and then they solve two problems in context, one with fractional coefficients. The lesson concludes with students addressing when it is appropriate to use the graphing, substitution, or linear combinations methods. **Materials Needed:** Problem-Solving Model Graphic Organizer	**A.2I** **A.5C**	3
4	Graphing Inequalities in Two Variables	Students explore a linear inequality in two variables through a scenario. They write an inequality, complete a table of values, graph the coordinate pairs from the table, and determine which parts of the graph are solutions to the inequality. Students then formalize the process of graphing inequalities through practice without context; they graph the corresponding equation of an inequality as a boundary line, determine whether the line should be solid or dashed, and identify which half plane to shade by testing the point (0, 0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs. They then solve a problem in context where they use a table of values to write and graph a linear inequality and refer to the inequality and/or its graph to respond to questions. Finally, students summarize the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities. **Materials Needed:** None	A.2H **A.3D**	2
5	Systems of Linear Inequalities	Students represent a scenario with a system of linear inequalities and graph the system. Overlapping shaded regions identify the possible solutions to the system. Students then practice graphing several systems of inequalities and representing the solution set. A different scenario is given that students model with a system of linear inequalities. They then graph the system, determine two different solutions, and algebraically prove that the solutions satisfy both constraints defined by the system. Finally, students match systems, graphs, and possible solutions of systems that have identical terms with different inequality symbols. **Materials Needed:** None	A.2H **A.3D** A.3H	3

*Bold TEKS = Readiness Standard

Lesson	Lesson Title	Highlights	TEKS*	Pacing
6	Solving Systems of Equations and Inequalities	Students solve problems in context requiring a system of linear equations. While most problems can be modeled by a system of two equations, they are guided through the process of solving a system of four equations, and another context can be modeled by a system of three equations. Students have the opportunity to solve the systems using any method and sometimes must respond in the format of an email or proposal. Solutions involve making a decision based upon inputs that lie before or after the point of intersection, thus requiring solutions written as inequalities. **Materials Needed:** Problem-Solving Model Graphic Organizer	A.2H **A.2I** **A.3D** A.3H **A.5C**	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				6

*Bold TEKS = Readiness Standard

MODULE 3, TOPIC 2 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: **A.2A, A.2C, A.2I,** A.3F, A.3G, **A.5C** **LESSON 1** Using Graphing to Solve Systems of Equations **Getting Started** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 1** continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.2I**, A.3F, A.3G, **A.5C** **LESSON 2** Using Substitution to Solve Systems of Equations **GETTING STARTED** **ACTIVITY 1**	**LESSON 2** continued **ACTIVITY 2**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 2 continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **A.2I**, **A.5C** **LESSON 3** Using Linear Combinations to Solve a System of Linear Equations **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 3** continued **ACTIVITY 4** **TALK THE TALK**
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.2H, **A.3D** **LESSON 4** Graphing Inequalities in Two Variables **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 4** continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.2H, **A.3D**, A.3H **LESSON 5** Systems of Linear Inequalities **GETTING STARTED** **ACTIVITY 1**

*Bold TEKS = Readiness Standard

Day 16	Day 17	Day 18	Day 19	Day 20
LESSON 5 continued ACTIVITY 2 ACTIVITY 3	**LESSON 5** continued ACTIVITY 4 TALK THE TALK	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: A.2H, **A.2I**, **A.3D**, A.3H, **A.5C** **LESSON 6** Solving Systems of Equations and Inequalities GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 6** continued ACTIVITY 3 ACTIVITY 4 TALK THE TALK

Day 21	Day 22
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are six Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Using Graphing to Solve Systems of Equations

LESSON OVERVIEW

Students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They graph the linear equation using intercepts and then analyze a second graph with the independent and dependent variables reversed. A new relationship between the quantities is then provided, and students write the equation expressing the relationship. Finally, they graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario, creating a system of linear equations. Students solve the system both graphically and using technology, checking the solution by substituting the values back in to the original equations. Next, they are provided three related scenarios in which they write systems of equations in slope-intercept form and solve the systems graphically. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions. The related terms *consistent systems* and *inconsistent systems* are defined.

MATERIALS

Graphing Technology

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

(TEKS continued on next page)

ELPS

(2) Listening

The student is expected to:

(B) recognize elements of the English sound system in newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters.

(3) Speaking

The student is expected to:

(C) speak using a variety of grammatical structures, sentence lengths, sentence types, and connecting words with increasing accuracy and ease as more English is acquired.

(4) Reading

The student is expected to:

(B) recognize directionality of English reading such as left to right and top to bottom.

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2A determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities.

A.2C write linear equations in two variables given a table of values, a graph, and a verbal description.

A.2I write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.
The student is expected to:

A.3F graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

A.3G estimate graphically the solutions to systems of two linear equations with two variables in real-world problems.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions.
The student is expected to:

A.5C solve systems of two linear equations with two variables for mathematical and real-world problems.

ESSENTIAL IDEAS

- The standard form of a linear equation is $Ax + By = C$ where A, B, and C are integers and A and B are not both zero. Linear functions written in standard form can be graphed using the x- and y-intercepts.
- Understand that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane.
- A linear system of equations is two or more linear equations that define a relationship between quantities. The solution of a linear system is an ordered pair that makes both equations in the system true.
- Lines that do not intersect describe a system of equations in which each linear equation has the same slope and there is no solution.
- Lines intersecting at a single point describe a system of equations in which each linear equation has a different slope and there is one solution.
- Lines intersecting at an infinite number of points describe a system of equations in which each linear equation is the same equation and there are an infinite number of solutions.
- Consistent systems of equations are systems that have one or many solutions. Inconsistent systems of equations are systems that have no solutions.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Ticket Tabulation 5–10 minutes

ESTABLISH A SITUATION

Students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They explain why there is more than one possible solution.

DEVELOP

Activity 1.1: Analyzing the Graph of an Equation in Standard Form 20–25 minutes

INVESTIGATION, PEER WORK ANALYSIS

Students graph the equation from the previous activity using intercepts. They analyze a second graph with the independent and dependent variables reversed.

Activity 1.2: Determining the Solution to a System of Linear Equations 10–15 minutes

INVESTIGATION

Students are given additional information about the scenario from the previous activity, and they write an equation to represent the new relationship. They then graph the new equation on two separate coordinate planes showing the graphed lines from the original scenario creating a system of linear equations. Students then approximate the solution to the system of equations using the graph they drew and determine a solution using technology.

DAY 2

Activity 1.3: Systems with No Solution, One Solution, or an Infinite Number of Solutions 30–35 minutes

INVESTIGATION, PEER WORK ANALYSIS, REAL-WORLD PROBLEM SOLVING

Students are provided three related scenarios in which they write systems of equations in slope-intercept form and solve the systems graphically. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions.

DEMONSTRATE

Talk the Talk: Beating the System 5–10 minutes

GENERALIZATION

The related terms *consistent systems* and *inconsistent systems* are defined. Students contrast consistent and inconsistent systems of linear equations in terms of their *y*-intercepts, number of solutions, and description of the graph.

Getting Started

ENGAGE

Ticket Tabulation

Facilitation Notes

In this activity, students write an equation in standard form to represent a scenario and determine a solution to the scenario and equation. They explain why there is more than one possible solution.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Variables used to represent number of students and number of adults.
- Various methods to determine a solution to the equation.
- Negative values presented as solutions to the situation.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why does it make sense to write your equation in standard form? • How can you tell from your equation that there is more than one solution?
Probing	• How did you determine your solution to Question 2? • If the Athletic Association sold 400 student tickets, how many adult tickets do they need to sell? • What is another possible solution?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials needed: Poster or Butcher Paper, Sticker Dots

Create a poster with a coordinate plane and have students post sticker dots as their answers. Discuss how solutions can be written as ordered pairs. Analyze characteristics of the line formed.

STAMP THE LEARNING

Summary

Some situations are better represented by an equation in standard form than an equation in slope-intercept form.

ACTIVITY 1.1

Analyzing the Graph of an Equation in Standard Form

DEVELOP

Facilitation Notes

In this activity, students graph the equation from the previous activity using intercepts. They then analyze a second graph with the independent and dependent variables reversed.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Have students replace the *x* at the end of the *x*-axis with *s*, or the variable used for students, and *y* at the end of the *y*-axis with *a*, or the variable used for adults.

AS STUDENTS WORK, LOOK FOR

- Confusion about which variable represents the independent quantity and which represents the dependent quantity.
- Mental math to determine the intercepts.
- Identification of the slope from the graph or by changing the form of the equation.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are the independent and dependent variables according to the graph?
Probing	• How did you use substitution to calculate the *x*-intercept? The *y*-intercept? • Why is the slope negative? • Are the domain and range always all real numbers for linear functions? Why or why not? • What type of numbers make sense in this situation? • How did you determine the slope of the line? • To rewrite the equation in slope-intercept form, how do you know whether to solve for *s* or *a*?
Seeing structure	• Show how the combinations you listed in the Getting Started lie on the graph of the line.

COMMON MISCONCEPTIONS

- Students may think the domain and range of the real-world situation include all positive real numbers. Address the fact that the number of tickets must be whole numbers, and that there is an upper limit to the number of each type of ticket because the total sales must be exactly $3000.
- Students may think the dependent variable is the amount of money raised. Clarify this common misconception by addressing the units of coordinate pairs, and explain that the graph represents the fact that as the number of one type of ticket increases, the other decreases.

Have students work with a partner or in a group to complete Questions 6 through 12. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why is the slope also negative on Ethan's graph? • How did you calculate the slope? • Why does it make sense that the slope of this line is also negative? • How can you select a point on the line to demonstrate your thinking?
Seeing structure	• Why can Ethan reverse the independent and dependent variables?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials needed: Poster or Butcher Paper, Sticker Dots

Create a poster with a coordinate plane and have students post sticker dots representing other points on Ethan's graph. Discuss how their process is different from the last graph they created.

STAMP THE LEARNING

Summary

When a line is written in standard form, either quantity can be considered the independent or dependent variable. Identifying and graphing the x- and y-intercepts is an efficient method of graphing a line written in standard form.

ACTIVITY 1.2 Determining the Solution to a System of Linear Equations

Facilitation Notes

In this activity, students are given additional information about the scenario from the previous activity, and they write an equation to represent the new relationship. They then graph the new equation on two separate coordinate planes, showing the graphed lines from the original scenario creating a system of linear equations. Students then approximate the solution to the system of equations using the graph they drew and determine a solution using technology.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

Have students place the coefficient of 1 in front of the variables. This allows them to see that the structure of this equation is the same as the structure of the previous equation.

Challenge Opportunity

Materials Needed: Poster or Butcher Paper, Sticker Dots

Have students graph this line using sticker dots on the poster graphs from the previous activities.

COMMON MISCONCEPTIONS

- With the addition of another equation, students may confuse what the variables and their coefficients represent. In the equations $s + a = 450$ or $1s + 1a = 450$, s and a continue to represent the number of student and adult tickets respectively. Each coefficient of 1 keeps a count of the number of tickets, in other words, each ticket counts as 1.
- When interpreting the point of intersection, students often note only one of the quantities or one of the equations. Stress the importance of a complete response, such as, "When 300 student tickets and 150 adult tickets are sold, you reach the goal of $3000 raised with 450 tickets sold."

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Compare what this question represents in contrast to the equation in Activity 1.
Seeing structure	• Why doesn't the graph of the line appear the same in both graphs? • Why is there only one solution in this case when you identified many possible solutions in the Getting Started?
Probing	• How did you determine the solution?
Reflecting and justifying	• Verify that your solution makes sense for both criteria, the number of tickets sold and the dollar amount of sales.

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Sign errors when rewriting the equations in standard form to slope-intercept form.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What are the advantages and disadvantages of using standard form? • Is it better to use the original equations in standard form or the equations rewritten in slope-intercept form to check the accuracy of your solution?
Reflecting and justifying	• How do you know if your answer is correct after you complete the substitution?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Discuss an alternative method in which students complete a linear regression using the two intercepts to write the equation in slope-intercept form.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Two linear equations that share a relationship between quantities are a system of linear equations. If the graphs of the two linear equations intersect, their point of intersection is a solution to the system of linear equations.

ACTIVITY 1.3 Systems with No Solution, One Solution, or an Infinite Number of Solutions

Facilitation Notes

In this activity, students are provided three related scenarios in which they write systems of equations in slope-intercept form and solve the systems graphically. This activity demonstrates that a system of two linear equations may have no solution, one solution, or an infinite number of solutions.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

- After they determine the equations for Samuel and Diego, write the equations on the board for reference throughout the activity. Add Isabella's equation once it is encountered for comparison purposes.

Challenge Opportunity

- Have students use the substitution method to solve the system algebraically.

  Method 1: To determine when the functions have the same amount of money, some students may understand that the expressions representing the money must be set equal to one another.

$$10x + 25 = 10x + 40$$

  Method 2: For other students, the term *substitute* will resonate them, and it may make more sense for them to write one equation first and then substitute an equivalent expression for y as a second step.

$$y = 10x + 40$$
$$10x + 25 = 10x + 40$$

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why does it make sense to write your equations in slope-intercept form? • When the slopes of two linear equations are equal, how would you describe the behaviors of their graphs?
Probing	• Use the situation to support your response using the equations. • How can you tell which line in the graph represents Diego's savings? • When the point of intersection is the solution to the system of equations, what does it mean if there is no point of intersection?

Have students work with a partner or in a group to complete Question 7. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Is (0, 40) a point of intersection, even though it is at the start of the shown graphs? Explain.
Probing	• Which characteristic do both equations have in common? How is this reflected in their graphs? • How can you tell which line in the graph represents Tonya's savings?

Have students work with a partner or in a group to complete Questions 8 through 11. Share responses as a class

COMMON MISCONCEPTIONS

- Students may think that the graph of Isabella's situation is no longer a line since there are two different rates. Remind students that they are interested in the total amount saved per week, not the amount deposited. So, students should combine Isabella's two deposits to determine one weekly rate.

- Students may confuse *infinite solutions* with *all real numbers*. Ask students to identify coordinate pairs that are solutions and coordinate pairs that are not solutions. Discuss the fact that all the points that lie on the line are solutions, but all coordinate pairs on the coordinate grid are not solutions.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How does DIego's equation change? How does Isabella's equation change?
Probing	• Why is there a single line on the graph? What does it represent? • Is there a single point of intersection? Why or why not?
Seeing structure	• When two lines are described by the same equation, what are the graphical implications? • Is Diego saving more money than Isabella after the shopping spree? How do you know?

DIFFERENTIATION STRATEGIES

Access for All

- Suggest students use double arrows to show that there are two of the same lines graphed.
- Have students identify several coordinate pair answers when there are infinite solutions to solidify the fact that all of the infinite solutions lie on the line.

STAMP THE LEARNING

Summary

A system of linear equations may have no solution, one solution, or an infinite number of solutions.

DEMONSTRATE

Talk the Talk

BEATING THE SYSTEM

Facilitation Notes

In this activity, the related terms *consistent systems* and *inconsistent systems* are defined. Students contrast consistent and inconsistent systems of linear equations in terms of their y-intercepts, number of solutions, and descriptions of their graphs.

Have students work with a partner or in a group to complete Questions 1 through 2. Share responses as a class.

COMMON MISCONCEPTION

Since systems with no solution are called *inconsistent systems*, students might mistakenly relate *consistent systems* with only infinite solutions. Emphasize to students that *consistent systems* can have one solution, many solutions, or infinite solutions. Remind them of the examples of one solution and infinite solutions they worked with. Challenge students to draw a system of two functions with more than one solution and do not have infinite solutions. Some possible examples include a system with a quadratic and linear function or a system with an exponential and linear function.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Do you apply the same properties of equality to solve an equation and a system of equations? Explain. • What are the different possible number of solutions when solving an equation? A system of equations? • How do you express one solution to an equation? A system of equations?
Probing	• Which scenario in Activity 2.3 is an example of this type of system? • How can you use algebra to verify the solution?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials Needed: Index Cards

Provide each student two index cards and have each of them create an example of a consistent system of linear equations and inconsistent system of linear equations. Next, have each student exchange index cards with a partner and identify the solution type for each system. Follow up by having the titles *inconsistent systems* and *consistent systems* on the board, and have each student post their examples under the appropriate title. Discuss patterns, then separate the consistent systems into two types: one solution or infinite solutions. Have students provide an additional row on the table to provide examples of each type of solution.

Have students read and answer the Essential Question on the lesson opener page.

Summary

Consistent systems of equations have one solution or many solutions, and inconsistent systems of equations have no solutions. A system of two linear equations has one solution, infinite solutions, or no solution.

STAMP THE LEARNING

1 Using Graphing to Solve Systems of Equations

Setting the Stage

- Communicate the objectives and the new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Write equations in standard form.
- Determine the intercepts of an equation in standard form.
- Use intercepts to graph an equation.
- Write a system of equations to represent a problem context.
- Use a graph to solve a system of linear equations.
- Interpret the solution to a system of equations in terms of a problem situation.
- Use the slope and the *y*-intercept to determine whether the system of two linear equations has one solution, no solution, or infinite solutions.

NEW KEY TERMS

- system of linear equations
- consistent systems
- inconsistent systems

You have examined different linear functions and solved for unknown values.

How can you solve problems that require two linear functions? How many solutions exist when you consider two functions at the same time?

I can find the intersection of a graphed system of linear equations, which is the solution to both equations. A system of linear equations can have one solution, no solution, or infinite solutions.

MODULE 3 • TOPIC 2 • LESSON 1 393

Chunking the Activity

- Read and discuss the scenario.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Ticket Tabulation

A high school sells tickets for its basketball games. Students pay $5, and adults pay $10 for a ticket. The high school needs to raise $3000 selling tickets to send the team to an invitational tournament.

1. Write an equation to represent this situation.

   $5s + 10a = 3000$

   s = number of students

   a = number of adults

2. What combination of student and adult ticket sales would achieve the high school's goal?

   Sample answer:

   The high school can sell 400 student tickets and 100 adult tickets.

3. Compare your combination of ticket sales with your classmates'. Did you all get the same answer? Explain why or why not.

   Sample answer:

   No, there is more than one possible combination of student/adult tickets sold that will add up to $3000.

394 MODULE 3 • TOPIC 2 • LESSON 1

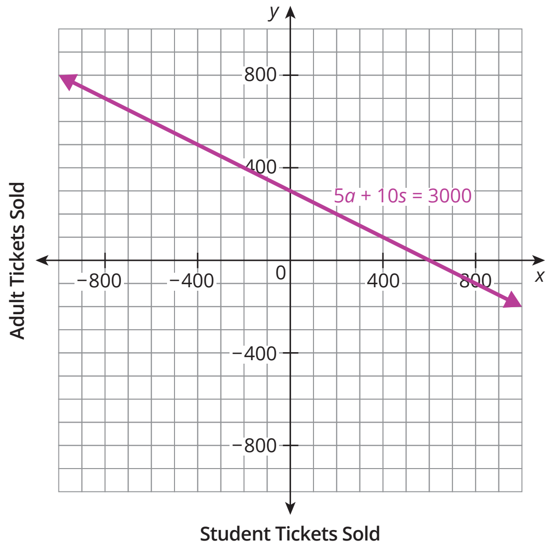
ACTIVITY 1.1

Analyzing the Graph of an Equation in Standard Form

Consider the goal of the athletic association described in the previous activity. Let *s* represent the number of student tickets sold, and let *a* represent the number of adult tickets sold. Written in standard form, the equation that represents the situation is $5s + 10a = 3000$.

One efficient way to graph a linear function in standard form is to use *x*- and *y*-intercepts. You can calculate the *x*-intercept by substituting $y = 0$ and solving for *x*. You can calculate the *y*-intercept by substituting $x = 0$ and solving for *y*.

1. Use the *x*- and *y*-intercepts to graph the equation.



2. Determine the domain and range of each.

   a. The mathematical function

   The domain is all real numbers.

   The range is all real numbers.

   b. The function modeling the real-world situation

   The domain is all whole numbers such that $0 \leq s \leq 600$.

   The range is all whole numbers such that $0 \leq a \leq 300$.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–5.
- Check in and share.
- Group students to complete Questions 6–12.
- Share and summarize.

Ask Yourself . . .
Which quantity is represented on each axis?

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MODULE 3 • TOPIC 2 • LESSON 1 395

3. Explain what each intercept means in terms of the problem situation.

The x-intercept is (600, 0). The x-intercept represents the number of student tickets that must be sold to reach \$3000 if no adult tickets are sold.

The y-intercept is (0, 300). The y-intercept represents the number of adult tickets that must be sold to reach \$3000 if no student tickets are sold.

4. Identify the slope of the function. Interpret its meaning in terms of the problem situation.

The slope is $-\frac{1}{2}$. This means that for every 1 adult ticket sold, 2 less student tickets need to be sold.

Ask Yourself . . .
What does each point on the graph of an equation represent?

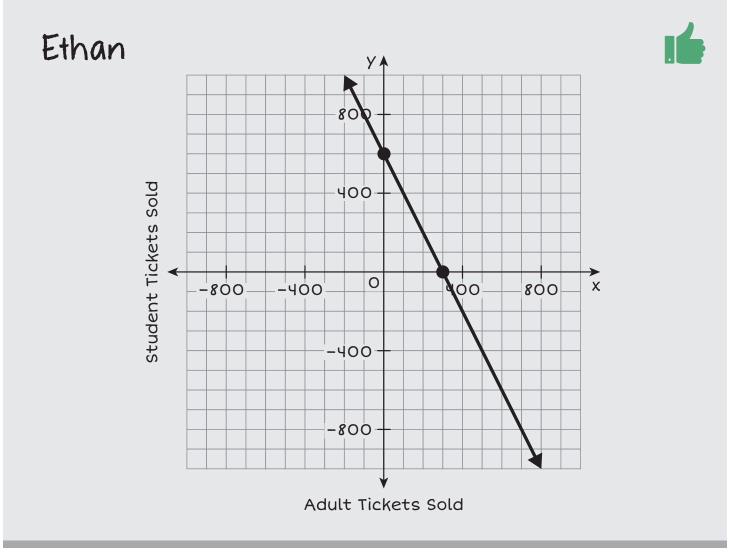
5. How can you use the graph to determine a combination of ticket sales to meet the goal of \$3000?

I can use any point on the graphed line within the domain and range that has an x- and y-value that are whole numbers to determine a possible combination of ticket sales that will meet the goal of \$3000.

6. Ethan graphed the equation $5s + 10a = 3000$ in a different way. Explain why Ethan's graph is correct.

Ethan's graph is correct. He reversed the independent and dependent variables, letting the x-axis represent the number of adult tickets sold and the y-axis represent the number of student tickets sold, and he reversed the x- and y-intercepts when graphing the line.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?



396 MODULE 3 • TOPIC 2 • LESSON 1

7. Use Ethan's graph to describe the domain and range of each.

   a. The mathematical function

      The domain of the mathematical function is all real numbers.

      The range of the mathematical function is all real numbers.

   b. The function modeling the real-world situation

      The domain of the function modeling the real-world situation is all whole numbers such that $0 \leq a \leq 300$.

      The range of the function modeling the real-world situation is all whole numbers such that $0 \leq s \leq 600$.

8. Explain what each intercept means in terms of the problem situation.

   The x-intercept is (300, 0). The x-intercept represents the number of adult tickets that must be sold to reach $3000 if no student tickets are sold.

   The y-intercept is (0, 600). The y-intercept represents the number of student tickets that must be sold to reach $3000 if no adult tickets are sold.

9. Identify the slope of the function. Interpret its meaning in terms of the problem situation.

   The slope is -2. This means that for every two student tickets sold, 1 less adult ticket needs to be sold.

10. Compare the domain and range of the two functions that model the real-world situation. What do you notice?

    The domain and range are switched. The domain of the first function that models the real-world situation is the range of the second function that models the real-world situation, and the range of the first function that models the real-world situation is the domain of the second function that models the real-world situation.

11. Compare the x-intercepts and the y-intercepts of the two graphs. What do you notice?

    The intercepts are switched. The x-intercept of the first graph is the y-intercept of the second graph, and the y-intercept of the first graph is the x-intercept of the second graph.

12. Is there a way to determine the total amount of money collected from either graph? Explain why or why not.

    Yes. The group raises $3000 for any ordered pair that lies on either graph.

MODULE 3 • TOPIC 2 • LESSON 1 397

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Questions 4 and 5.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 1.2 Determining the Solution to a System of Linear Equations

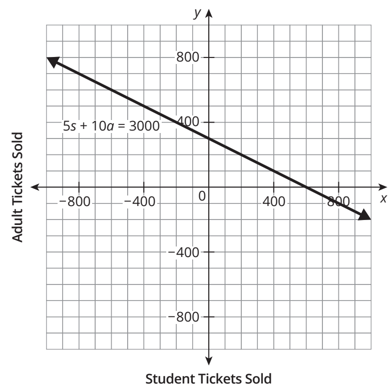
When two or more linear equations define a relationship between quantities, they form a **system of linear equations**.

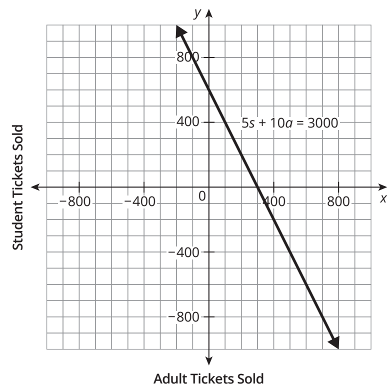
The athletic director of the high school says that 450 total tickets were sold to the home game.

1. Write an equation that represents this situation. Let s represent the number of student tickets sold, and let a represent the number of adult tickets sold.

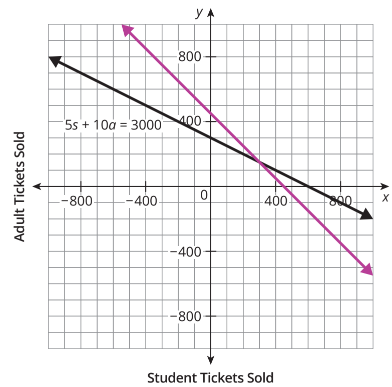
   $s + a = 450$

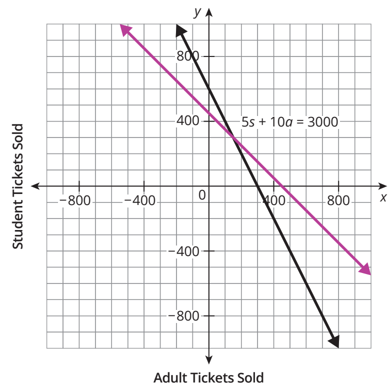
The coordinate planes shown already contain the function that models the earnings from ticket sales.





2. Use x- and y-intercepts to graph the function modeling the total number of tickets sold on each coordinate plane.





398 MODULE 3 • TOPIC 2 • LESSON 1

3. When the high school reached its goal of \$3000 in ticket sales, how many of each type of ticket was sold? Is there more than one solution?

   There should be 300 student tickets and 150 adult tickets sold. There is only one solution because there is only one point of intersection between the graphed lines.

4. Use technology to locate the exact point of intersection. Explain the process you used.

   Sample answer:
   Place the equations of the lines in slope-intercept form. Enter them into the graphing technology. Follow the prompts to determine a point of intersection.

5. Algebraically justify that your solution is correct.

$$\begin{aligned} 5s + 10a &= 3000 \\ 5(300) + 10(150) &= 3000 \\ 1500 + 1500 &= 3000 \\ 3000 &= 3000 \end{aligned}$$

$$\begin{aligned} s + a &= 450 \\ 300 + 150 &= 450 \\ 450 &= 450 \end{aligned}$$

Remember . . .
According to the situation, 450 tickets were sold to the game.

This activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice of writing and solving a system of linear equations graphically, assign Skills Practice Set A for this lesson.

Optimizing Learning

This activity provides options to use multiple tools for construction and compostion.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete Questions 1–6.
- Check in and share.
- Group students to complete Question 7.
- Check in and share.
- Group students to complete Questions 8–11.
- Share and summarize.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Ensure students are familiar with contextual terms, such as *deposit* and *allowance*. As students read and discuss the situation, encourage them to annotate which elements are related to Marcus and which are related to Phillip.

ACTIVITY 1.3

Systems with No Solution, One Solution, or an Infinite Number of Solutions

Samuel and Diego are in the Robotics Club. They are both saving money to buy materials to build a new robot.

Samuel opens a new bank account. He deposits \$25 that he won at a robotics competition. He also plans on depositing \$10 a week that he earns from tutoring. Diego decides he wants to save money as well. He already has \$40 saved from mowing lawns over the summer. He plans to also save \$10 a week from his allowance.

1. Write equations to represent the amount of money Samuel saves and the amount of money Diego saves.

   Let x represent the time in weeks.

   Let y represent the amount of money saved in dollars.

   $$\begin{cases} y = 25 + 10x \\ y = 40 + 10x \end{cases}$$

2. Use your equations to predict when Samuel and Diego will have the same amount of money saved.

   Marcus and Phillip will never have the same amount of money saved.

You can prove your prediction by solving and graphing a system of linear equations.

3. Analyze the equations in your system.

   a. How do the slopes compare? Describe what this means in terms of this problem situation.

   The slopes are the same. This means that both friends save the same amount each week, which is \$10.

   b. How do the y-intercepts compare? Describe what this means in terms of this problem situation.

   The y-intercept in Diego's equation is greater than the y-intercept in Samuel's equation. This means that Diego started with more money than Samuel.

EB STUDENT TIP

For all proficiency levels

Materials Needed: Colored Paper or String

Beginning: Explain that a *linear system* is like having two recipes that use the same ingredients but in different amounts. On a large graph, use two strips of colored paper or strings to represent the two "recipes," explaining that sometimes these lines can cross at a point (one solution), overlap completely (infinite solutions), or never touch (no solutions). Prompt students to create each kind of linear system by manipulating the lines and optionally saying "one solution," "no solution," or "infinite solutions," as appropriate.

Intermediate: Ask students to graph two simple equations that form a linear system. Encourage

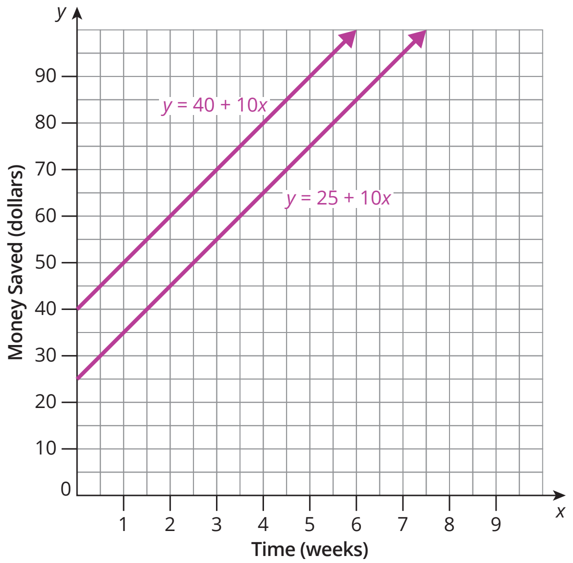
(continued on next page)

4. Determine the solution of the system of linear equations graphically.

   a. Predict what the graph of this system will look like. Explain your reasoning.

      The graph will be two parallel lines.

   b. Graph both equations on the coordinate plane.



5. Analyze the graph you created.

   a. Describe the relationship between the graphs.

      The graphs are the same distance apart from one another, which means they are parallel.

   b. Does this linear system have a solution? Explain your reasoning.

      No. Sample explantion: The graphs will never intersect because they are parallel, so there is no solution.

EB STUDENT TIP *(continued)*

students to discuss with a partner when they think a system will have one solution, no solution, or infinite solutions, based on the slopes and y-intercepts of the lines they graph, using this sentence frame: This linear system has ________ (one solution / no solution / infinite solutions) because ________.

Advanced/Advanced High: Use the same activity from the Intermediate section, but do not provide a sentence frame. Ask students to write their findings, emphasizing the mathematical reasoning behind the existence of one solution, no solution, or infinitely many solutions and how these concepts can connect to real-world issues.

MODULE 3 • TOPIC 2 • LESSON 1 401

6. Was your prediction in Question 2 correct? Explain how you algebraically and graphically proved your prediction.

   Sample answer:
   Yes. My prediction was correct because Samuel will never have as much money as Diego. Algebraically, the solution of $25 \neq 40$ proved that there is no solution. Graphically, the two parallel lines proved there is no solution.

Isabella is also in the Robotics Club and has heard about Samuel's and Diego's savings plans. She wants to be able to buy her new materials before Diego, so she opens her own bank account. She is able to deposit $40 in her account that she has saved from her job as a waitress. Each week, she also deposits $4 from her tips.

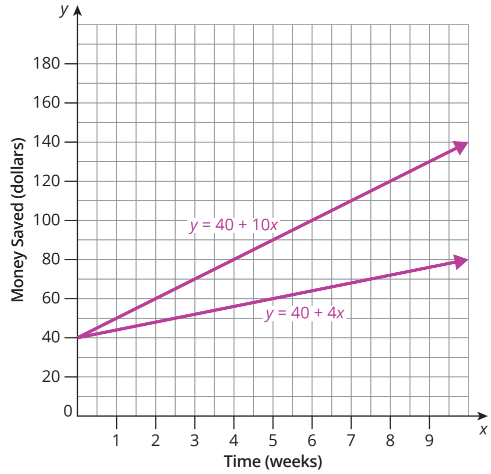
Remember . . .
Don't forget to define your variables!

7. Use equations and graphs to determine when Isabella and Diego have saved the same amount of money.

   a. Write a linear system to represent the total amount of money Isabella and Phillip have saved after a certain amount of time.

   $$\begin{cases} y = 40 + 4x \\ y = 40 + 10x \end{cases}$$

   b. Graph the linear system on the coordinate plane.



   c. Do the graphs intersect? If so, describe the meaning in terms of this problem situation.

   Yes. The graphs intersect at (0, 40). This means that they both start out with the same amount, which is $40.

402 MODULE 3 • TOPIC 2 • LESSON 1

Diego and Isabella went on a shopping spree this weekend and spent all their savings except for $40 each. Diego is still saving $10 a week from his allowance. Isabella now deposits her tips twice a week. On Tuesdays, she deposits $4, and on Saturdays, she deposits $6.

8. Diego claims he is still saving more each week than Isabella.

   a. Do you think DIego's claim is true? Explain your reasoning.

      No. I do not think Diego's claim is true because Phillip and Isabella are now saving the same amount each week.

   b. How can you prove your prediction?

      I can prove my prediction by writing a new system of linear equations and solving them graphically.

9. Prove your prediction graphically.

   a. Write a new linear system to represent the total amount of money each friend has after a certain amount of time.

      Isabella's equation is now $y = 40 + 6x + 4x$ because of her extra savings per week. Diego's equation is the same.

$$\begin{cases} y = 4 + 10x \\ y = 40 + 6x + 4x \end{cases}$$

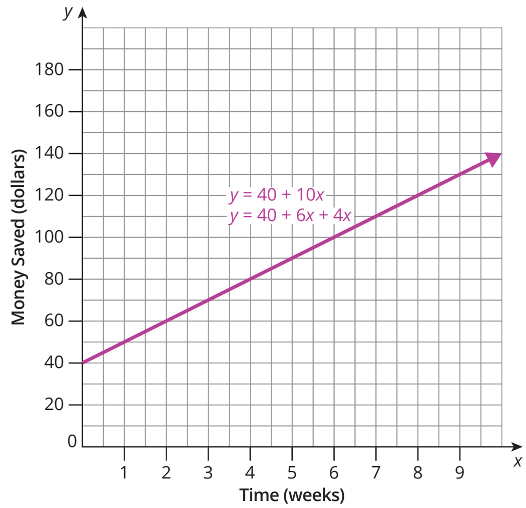
EB STUDENT TIP

For all proficiency levels

Students may not be familiar with the term *shopping spree*. Define the word *spree* as a sustained period of unrestrained activity. Discuss other examples of *sprees* besides *shopping sprees*, such as an eating *spree*, a laughing *spree*, or a scoring *spree*. Finally, review the problem scenario for Question 8 and ensure students' understanding of *shopping spree* in the context of the problem.

This activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice of solving a system of linear equations graphically and determine the number of solutions, assign Skills Practice Set B for this lesson.

b. Graph the linear system on the coordinate plane.



10. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

The graphs are the same. This means that Isabella and Diego will always have the same amount of money.

11. Was Diego's claim that he is still saving more than Isabella a true statement? Explain why or why not.

No. Diego is not saving more than Isabella. He is saving the same amount as Isabella.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Ask students to identify what the prefix *pre-* means in the word *predict*. Follow up with additional examples of words with the prefix *pre-*, including *pretest*, *preview*, and *precooked*. Define these words and then ask students to explain why *predict* means to *state what should happen in the future* in the context of making predictions about the outcome of the problems based on the given information.

Talk the Talk

Beating the System

A system of equations may have one unique solution, no solution, or infinite solutions. Systems that have one solution or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.

1. Complete the table.

System of Two Linear Equations	Consistent Systems		Inconsistent Systems
Description of y-Intercepts	Same or different y-intercepts	Same y-intercepts	Different y-intercepts
Number of Solutions	One solution	Infinite solutions	No solutions
Description of Graph	The lines intersect.	The lines are the same.	The lines are parallel.

2. Explain why the x- and y-coordinates of the points where the graphs of a system intersect are solutions.

   The x- and y-values of the point of intersection of the two graphs makes both equations true.

Ask Yourself . . .
What patterns do you notice?

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

CC BY NC
MODULE 3 • TOPIC 2 • LESSON 1 405

Lesson 1 Assignment

Write

Define each term in your own words.

1. Consistent systems
2. Inconsistent systems

Remember

When two or more equations define a relationship between quantities, they form a system of linear equations. The point of intersection of a graphed system of linear equations is the solution to both equations. A system of linear equations can have one solution, no solution, or infinite solutions.

Practice

1. Mr. Nguyen gives his class 50-question multiple choice tests. Each correct answer is worth 2 points, while a half point is deducted for each incorrect answer. If the student does not answer a question, that question does not get any points.

   a. A student needs to earn 80 points on the test in order to keep an A grade for the semester. Write an equation in standard form that represents the situation. Identify three combinations of correct and incorrect answers that satisfy the equation.

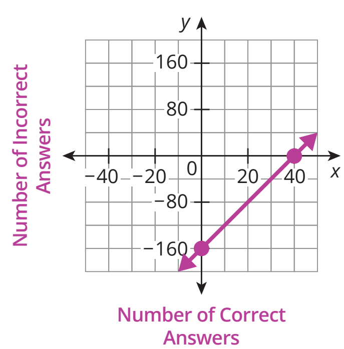
   Let *c* represent the number of correct answers, and let *w* represent the number of incorrect answers.

   $2c - \frac{1}{2}w = 80$

   (40, 0), (41, 4), (42, 8)

   b. Determine the *x*- and *y*-intercepts of the equation and use them to graph the equation. Explain what each intercept means in terms of the problem situation.

   The *x*-intercept is (40, 0), and the *y*-intercept is (0, −160). The *x*-intercept represents the number of answers the students must get correct to get an 80 if no answers are incorrect. The *y*-intercept is −160. There cannot be a negative number of incorrect answers, so this indicates that it is impossible to get an 80 on the test if there are 0 correct answers.



Write

1. A system of linear equations is a consistent system if there is at least one solution that satisfies both equations. A consistent system may have 1 or many solutions.
2. A system of linear equations is an inconsistent system if there is no solution that satisfies both equations.

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MODULE 3 • TOPIC 2 • LESSON 1 ASSIGNMENT 407

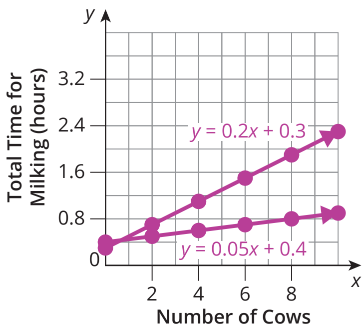
Lesson 1 Assignment

2. Harper owns a dairy farm. In the morning, it takes her 0.3 hour to set up for milking the cows. Once she has set up, it takes Harper 0.2 hour to milk each cow by hand. She is contemplating purchasing a milking machine in hopes that it will speed up the milking process. The milking machine she is considering will take 0.4 hour to set up each morning and takes 0.05 hour to milk each cow.

   a. Write a system of linear equations that represents the total amount of time Harper will spend milking the cows using the two different methods.

   $$\begin{cases} y = 0.2x + 0.3 \\ y = 0.05x + 0.4 \end{cases}$$

   b. Graph both equations on a coordinate plane.



   c. Estimate the point of intersection. Explain how you determined your answer.

   The lines intersect at about (0.7, 0.4).

   d. What does the point of intersection represent in this problem situation?

   The intersection point represents when both methods take the same amount of time for the same number of cows.

   e. Does the solution make sense in terms of this problem situation? Explain your reasoning.

   No. Fractional answers do not make sense for the number of cows.

   f. Is this system of equations consistent or inconsistent? Explain your reasoning.

   This system is consistent because it has one unique solution.

408 MODULE 3 • TOPIC 2 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

3. Identify whether each system is consistent or inconsistent. Explain your reasoning.

a. $\begin{cases} -3x = 4y = 3 \\ -12x + 16y = 8 \end{cases}$

Inconsistent; no solutions

b. $\begin{cases} 7x + 3y = 0 \\ -14x + 6y = 0 \end{cases}$

Consistent; infinite solutions

c. $\begin{cases} 6x + y = 1 \\ -6x - 4y = -4 \end{cases}$

Consistent; one solution: (0, 1)

Prepare

Analyze each system of equations. What can you conclude about the value of *y* in each?

1. $\begin{cases} x = 12 \\ y = x + 22 \end{cases}$

   $y = 34$

2. $\begin{cases} x = 0 \\ y = x - 45 \end{cases}$

   $y = -45$

3. $\begin{cases} x = y \\ y = 2x - 10 \end{cases}$

   $y = 10$

4. $\begin{cases} x = y + 3 \\ y = 2x - 10 \end{cases}$

   $y = 4$

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MODULE 3 • TOPIC 2 • LESSON 1 ASSIGNMENT 409

2 Using Substitution to Solve Linear Systems

MATERIALS

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students use the substitution method to solve systems of linear equations. They use substitution to solve systems of linear equations, including those with no solution or those with infinite solutions. Students define variables, write systems of equations, solve systems, and interpret the meaning of the solution in terms of the problem context. In the last activity, they are given four systems of linear equations and solve each system using the substitution method.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2I write systems of two linear equations given a table of values, a graph, and a verbal description.

(TEKS continued on next page)

ELPS

(2) Listening
The student is expected to:

(D) monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed.

(3) Speaking
The student is expected to:

(A) practice producing sounds of newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters to pronounce English words in a manner that is increasingly comprehensible.

(4) Reading
The student is expected to:

(A) learn relationships between sounds and letters of the English language and decode (sound out) words using a combination of skills such as recognizing sound-letter relationships and identifying cognates, affixes, roots and base words.

ALGEBRA I TEKS *(TEKS continued from previous page)*

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3F graph systems of two linear equations in two variables on the coordinate plane and determine the solutions if they exist.

A.3G estimate graphically the solutions to systems of two linear equations with two variables in real-world problems.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions.

The student is expected to:

A.5C solve systems of two linear equations with two variables for mathematical and real-world problems.

ESSENTIAL IDEAS

- The substitution method is a process for solving a system of equations. It is an alternative method to graphing, especially when the solution is difficult to read from a graph.
- To use the substitution method, it is useful when at least one equation is written in slope-intercept form.
- When a system has no solution, the equation resulting from the substitution step has no solution.
- When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.
- Problem situations can be expressed using systems of equations and solved for unknown quantities using substitution methods.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Goats, Chickens, and Pigs 45 minutes

ESTABLISH A SITUATION

Students are given a context with three unknowns. Because they have not yet encountered three-variable problems, they must employ problem-solving strategies and reasoning skills in addition to algebraic methods, such as defining variables and writing equations. The problem-solving strategies required to solve this problem hint to the informal use of substitution, which is the strategy for solving systems of two equations that will be addressed in this lesson.

DAY 2

DEVELOP

Activity 2.1: Introduction to Substitution 20–25 minutes

WORKED EXAMPLE, REAL-WORLD PROBLEM SOLVING

Students learn to solve a system of linear equations using substitution. They write and graph a system of linear equations that has one equation written in standard form and the other in slope-intercept form. Because the decimal solution cannot be read accurately from the graph, students use substitution to solve the system.

Activity 2.2: Substitution with Special Systems 20–25 minutes

REAL-WORLD PROBLEM SOLVING

In this activity, students write and use substitution to solve a system of linear equations with no solution and a system of linear equations with infinitely many solutions. Connections are made between solving these systems graphically and solving them algebraically.

DAY 3

Activity 2.3: Solving Systems by Substitution 20–35 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

Students write systems of linear equations to represent real-world situations, use substitution to solve them, and interpret the solutions.

DEMONSTRATE

Talk the Talk: The Substitution Train 10–15 minutes

EXIT TICKET PROCEDURES

In this activity, students solve systems of linear equations using the substitution method and check their answers algebraically.

Getting Started

ENGAGE

Goats, Chickens, and Pigs

Facilitation Notes

In this activity, students are given a context with three unknowns. Because they have not yet encountered three-variable problems, they must employ problem-solving strategies and reasoning skills in addition to algebraic methods, such as defining variables and writing equations. The problem-solving strategies required to solve this problem hint to the informal use of substitution, which is the strategy for solving systems of two equations that will be addressed in this lesson.

Ask a student to read the scenario aloud. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- General problem-solving strategies, such as drawing diagrams, creating notation to show trades, or remembering steps.
- Representing the trades using equations and using substitution to make comparisons.

DIFFERENTIATION STRATEGY

Access for All

Have students present their solution strategies, being selective in the order they are presented. Begin with diagram models and build to more abstract algebraic models. Ask questions to make connections among the models.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How many different variables did you use in this situation?
Probing	• Explain how you could use substitution to solve this problem. • How can you tell from your equations if it is a fair deal? • Suppose one farmer trades 4 goats for 5 chickens and a second farmer trades 3 goats for 5 chickens. Is this fair? How could you change the trade to make it fair?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Systems of equations can involve more than two variables and can be solved using different reasoning strategies.

ACTIVITY **2.1** Introduction to Substitution

DEVELOP

Facilitation Notes

In this activity, students learn to solve a system of linear equations using substitution. They write and graph a system of linear equations that has one equation written in standard form and the other in slope-intercept form. Because the decimal solution cannot be read accurately from the graph, students use substitution to solve the system.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction and the given scenario aloud. Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know whether to use the equation $y = 8x$ or $x = 8y$? • What is the result when (1, 8) is substituted into the equation in standard form?
Seeing structure	• Why did it make sense to write the cost equation in standard form?

Ask a student to read the information and definition aloud. Analyze and discuss the Worked Example and complete Questions 7 through 9 as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

To scaffold support, have students interact with the Worked Example.

- Have them label the equations first and second equation according to the Worked Example.
- For Step 1, have them draw a box around $8x$ in the equation $y = 8x$ to make it explicit what they are substituting for y.
- After going through the Worked Example, have them redo the steps in the margin by following the directions to solidify the process in their minds.
- Have students write the answer as an ordered pair. This step is automatic for students when reading an answer from a graph. They should be comfortable writing answers in ordered pair notation, regardless of the method used to solve the system of equations.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why does it make sense to use $y = 8x$, rather than the equation in standard form, for the substitution? • How did you know where to substitute $8x$ in the equation in standard form? • How do you know which equation to use to calculate the value of y?
Seeing structure	• How are the forms of these two equations different from the previous Worked Example? • Why isn't it necessary to check the solution in both equations?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Discuss the flexibility and power of the multiplication property of equality. Consider the equation $1.25x + 1.05y = 30$. This equation can be multiplied by 100 to rewrite the decimals as integers, leading to $125x + 105y = 3000$. Discuss these facts:

- The graph of the original line and the new line are the same. Have students demonstrate this through showing that both equations have the same x- and y-intercepts or that they have the same slope and y-intercept.
- Although the equation is multiplied by 100, the other equation in the system does not need to be multiplied by 100. Because we demonstrated that multiplying the equation by 100 on both sides creates an equivalent equation, there is no need to change the other equation in the system.

STAMP THE LEARNING

Summary

You can use the substitution method to solve a system of equations. It is an alternative method to graphing, especially when the solution is difficult to read from a graph.

ACTIVITY 2.2

Substitution with Special Systems

Facilitation Notes

In this activity, students write and use substitution to solve a system of linear equations with no solution and a system of linear equations with infinitely many solutions. Connections are made between solving these systems graphically and solving them algebraically.

Have students work with a partner or in a group to complete Questions 1 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do the variables in your equations represent? • What does the solution to this system represent? • Do parallel lines have the same slope? The same y-intercept? • How many points of intersection does a pair of parallel lines have?
Probing	• Explain your steps to solve this equation. • Why does it make sense that there is no solution to this situation? • Describe the graphs of these equations.
Seeing structure	• Why isn't it enough to consider just the slope when determining the type of solution? • How would your response be different if the y-intercepts were also the same? Explain your thinking.

Have students work with a partner or in a group to complete Questions 10 through 16. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do the variables in your equations represent? • What does the solution to this system represent?
Probing	• What is the equation, in standard form, for the number of tickets purchased? The total cost? • Explain your steps to rewrite each equation in slope-intercept form. • What is another way to solve this system algebraically? • What are some solutions to this system of equations? To the situation? • Describe the graphs of these equations.

COMMON MISCONCEPTION

Students sometimes misinterpret *infinitely many solutions* with the idea that any or all numbers are solutions to a system. Clarify this misunderstanding by having students provide ordered pairs that satisfy the equations $x + y = 5$ and $4x + 4y = 20$. Have them graph the ordered pairs to see that they all lie on the same line. Contrast this with ordered pairs that are not solutions to the system and do not lie on the line.

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

When a system has no solution, the equation resulting from the substitution step has no solution. When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

STAMP THE LEARNING

ACTIVITY **2.3** Writing and Solving Systems

Facilitation Notes

In this activity, students write systems of linear equations to represent graphs, tables, and real-world situations, use substitution to solve them, and interpret the solutions.

To begin the Day 3 session, have a student read the Essential Question aloud.

DIFFERENTIATION STRATEGY

Access for All

This lesson includes five problems. Create five stations in the classroom, with each station providing the information for one problem. Have students cycle through the stations, completing each problem within given time constraints.

Have students work with a partner or in a group to complete Questions 1–3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What information in the problem situation helped you write equations in the system? • How can you use the graph to write the equations in the system? • How can you use the tables to write the equations for the systems?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you use substitution to solve this system of equations? • How would you write this solution as an ordered pair?

STAMP THE LEARNING

Summary

The substitution method can be used to solve systems of linear equations that represent real-world situations.

DEMONSTRATE

Talk the Talk

THE SUBSTITUTION TRAIN

Facilitation Notes

In this activity, students solve systems of linear equations using the substitution method and check their answers algebraically.

Have students complete Question 1 individually. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How is setting the equations equal the same as a substitution? • How do you know which equation to substitute into the other equation? • How did you get the second value of your solution? • How can you express your solution as an ordered pair?
Reflecting and justifying	• How can you check that your solutions are correct?

Have students read and answer the Essential Question on the lesson opener page.

Summary

The substitution method can be used to solve systems of linear equations, regardless of whether they are written in standard form or slope-intercept form.

STAMP THE LEARNING

2 Using Substitution to Solve Linear Systems

Setting the Stage

- Communicate the objectives and new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Write a system of equations to represent a problem context.
- Algebraically solve a system of equations using substitution.
- Interpret a solution to a system of linear equations in terms of the problem situation.
- Solve real-world and mathematical problems with two linear equations in two variables.

NEW KEY TERM

- substitution method

You have graphed systems of equations.

Suppose you graph a system of equations, but the point of intersection is not clear from the graph. How can you determine the solution to the system?

Sample answer:

I can use algebra to determine values that make both equations true, which represents the solution to the system.

MODULE 3 • TOPIC 2 • LESSON 2 411

Chunking the Activity

- Read and discuss the scenario.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

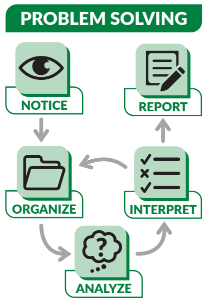
Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1B:

- Are students referring to the problem-solving model?
- Do students evaluate the reasonableness of their solution?
- Do students adapt their plan as needed?



Getting Started

Goats, Chickens, and Pigs

At the county fair, farmers bring some of their animals to trade with other farmers. To make all trades fair, a master of trade oversees all trades. Assume all chickens are of equal value, all goats are of equal value, and all pigs are of equal value.

- In the first trade of the day, 4 goats were traded for 5 chickens.
- In the second trade, 1 pig was traded for 2 chickens and 1 goat.
- In the third trade, Farmer Lyndi put up 3 chickens and 1 pig against Farmer Simpson's 4 goats.

1. Is this a fair trade? If not, whose animals are worth more? How could this be made into a fair trade?

   This is not a fair trade. Farmer Lyndi's animals are worth more than Farmer Simpson's animals. To make this a fair trade, Farmer Simpson should include 1 more goat.

412 MODULE 3 • TOPIC 2 • LESSON 2

ACTIVITY 2.1

Introduction to Substitution

Sofia is helping her mother make potato salad for the county fair and is asked to go to the market to buy fresh potatoes and onions. Sweet onions cost \$1.25 per pound, and potatoes cost \$1.05 per pound. Her mother told her to use the \$30 she gave her to buy these two items.

The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

1. Write an equation that relates the number of pounds of potatoes and the number of pounds of onions that Sofia can buy for \$30. Use x to represent the number of pounds of onions and y to represent the number of pounds of potatoes that Sofia can buy. Then, rewrite your equation in standard form.

   The equation is $1.25x + 1.05y = 30$. The equation in standard form is $25x + 21y = 600$.

2. Sofia's mother told her that the number of pounds of potatoes should be 8 times greater than the number of pounds of onions in the salad. Write an equation in x and y that represents this situation.

   $y = 8x$

3. Will 1 pound of onions and 8 pounds of potatoes satisfy both equations? Explain your reasoning.

   No. One pound of onions and eight pounds of potatoes does not satisfy both equations. Substituting the ordered pair (1, 8) into both equations makes the first equation false and the second equation true.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction and scenario.
- Group students to complete Questions 1–6.
- Check in and share.
- Complete Questions 7–9 as a class.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

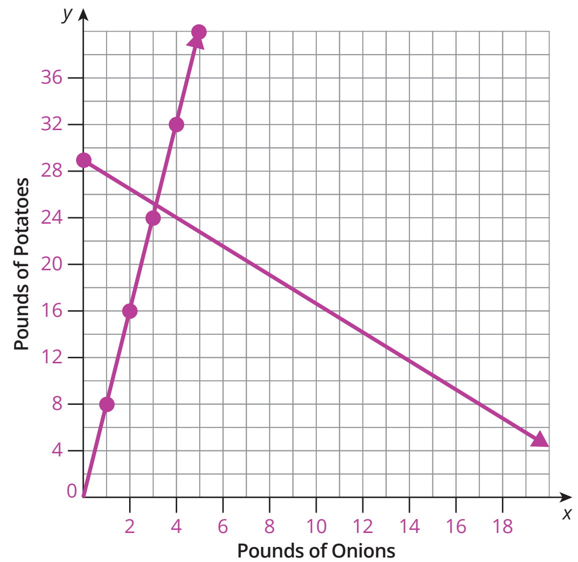
MODULE 3 • TOPIC 2 • LESSON 2 413

This activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice solving systems of equations using graphing, assign Skills Practice Set A for this lesson.

4. Create graphs of both equations. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval
Pounds of Onions	0	20	2
Pounds of Potatoes	0	40	2

See table below.



5. Can you determine the exact solution of this linear system from your graph? Explain your reasoning.

No. The answer cannot be determined from the graph because the point of intersection does not fall exactly on the grid lines.

6. Estimate the point of intersection from your graph.

An estimate of the solution might be (3, 25).

414 MODULE 3 • TOPIC 2 • LESSON 2

In many systems, it is difficult to determine the solution from the graph. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

WORKED EXAMPLE

Let's consider the system you wrote.

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Step 1: To use the substitution method, begin by choosing one equation and isolating one variable. Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation.

Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$.

Write the new equation.

$$1.25x + 1.05y = 30$$
$$1.25x + 1.05(8x) = 30$$

You have just created a new equation with only one unknown.

Step 3: Solve the new equation.

$$1.25x + 8.40x = 30$$
$$9.65x = 30$$
$$x \approx 3.1$$

Therefore, Sofia should buy approximately 3.1 pounds of onions.

Now, substitute the value for x into $y = 8x$ to determine the value of y.

$$y = 8(3.1) = 24.8$$

Therefore, Sofia should buy approximately 24.8 pounds of potatoes.

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept of the line.

Ask Yourself . . .
Keep in mind what the value represents.

STAMP THE LEARNING

The definition and Worked Example provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 3 • TOPIC 2 • LESSON 2 415

7. Check that the solution is correct. Show your work.

$1.25(3.1) + 1.05(24.8) \approx 30$
$3.875 + 26.04 \approx 30$
$29.915 \approx 30$
$24.8 = 8(3.1)$
$24.8 = 24.8$

8. What is the solution to the system? What does it represent in terms of the problem situation?

The solution is (3.1, 24.8). For $30, Sofia should purchase about 3.1 pounds of onions and 24.8 pounds of potatoes. Rounded to the nearest pound, Sofia should purchase 3 pounds of onions and 25 pounds of potatoes.

9. Compare your solution using the substitution method to the solution on your graph. What do you notice?

The two solutions are approximately equal.

416 MODULE 3 • TOPIC 2 • LESSON 2

ACTIVITY 2.2 Substitution with Special Systems

Daniel and Chloe are helping to set up the booths at the fair. They are each paid \$7 per hour to carry the wood that is needed to build the various booths. Daniel arrives at 7:00 A.M. and begins working immediately. Chloe arrives 90 minutes later and starts working.

1. Write an equation that gives the amount of money that Daniel will earn, y, in terms of the number of hours he works, x.

   $y = 7x$

2. How much money will Daniel earn after 90 minutes of work?

   Daniel will earn \$10.50 after 90 minutes of working.

3. Write an equation that gives the amount of money Chloe will earn, y, in terms of the number of hours since Daniel started working, x.

   $y = 7(x - 1.5) = 7x - 10.5$

4. How much money will each student earn by noon?

   Daniel will earn \$35 by noon, and Chloe will earn \$24.50.

Chunking the Activity

- Read and discuss the situation.
- Group students to complete Questions 1–9.
- Check in and share.
- Group students to complete Questions 10–16.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

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MODULE 3 • TOPIC 2 • LESSON 2 417

5. Will Chloe ever earn as much money as Daniel? Explain your reasoning.

   No. Chloe will not earn as much money as Daniel because she started 90 minutes later. No matter how many hours they work, Chloe will always be working 1.5 hours less.

6. Write a system of linear equations for this problem situation.

   $y = 7x$

   $y = 7x - 10.5$

Think About . . .
How is this similar to solving linear equations with no solution or with infinite solutions?

7. Analyze the system of linear equations. What do you know about the solution of the system by observing the equations? Explain your reasoning.

   Because the slopes are the same, the lines are parallel. This means that the system has no solution because the lines will not intersect.

Let's see what happens when we solve the system algebraically.

8. Since both equations are written in slope-intercept form as expressions for y in terms of x, substitute the expression from the first equation into the second equation.

   a. Write the new equation.

      $7x = 7x - 10.5$

   b. Solve the equation for x.

      $0 = -10.5$

   c. Does your result for x make sense? Explain your reasoning.

      No. Zero does not equal (-10.5).

418 MODULE 3 • TOPIC 2 • LESSON 2

9. What is the result when you algebraically solve a linear system that contains parallel lines?

   The result is a false mathematical statement, such as $0 = -10.5$.

On Monday night, the fair is running a special for the the local schools: if tickets are purchased from the school, you can buy student tickets for $4 and adult tickets for $4. You buy 5 tickets and spend $20.

10. Write an equation that relates the number of student tickets, *x*, and the number of adult tickets, *y*, to the total amount spent.

    $4x + 4y = 20$

11. Write an equation that relates the number of student tickets, *x*, and the number of adult tickets, *y*, to the total number of tickets purchased.

    $x + y = 5$

12. Write both equations in slope-intercept form.

    $y = 5 - x$
    $y = 5 - x$

13. Analyze the system of linear equations. What do you know about the solution of the system by looking at the equations?

    The equations are the same, so there will be infinite solutions.

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MODULE 3 • TOPIC 2 • LESSON 2 419

Let's see what happens when you solve the system algebraically.

14. Since both equations are now written in slope-intercept form as expressions for y in terms of x, substitute the expression from the first equation into the second equation.

    a. Write the new equation and solve the equation for x.

    $5 - x = 5 - x$

    $0 = 0$

    b. Does your result for x make sense? Explain your reasoning.

    Yes. This makes sense. No matter the value of x, the mathematical sentence will always be true. So, x can be any number.

15. How many student tickets and adult tickets did you purchase?

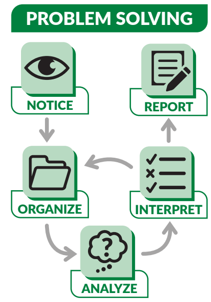
    It is not possible to determine the number of each ticket purchase; any combination of student and adult tickets that sum to 5 is possible.

16. What is the result when you algebraically solve a linear system that contains two lines that are actually the same line?

    The result is a mathematical statement that is always true, such as $x = x$ or $0 = 0$.

420 MODULE 3 • TOPIC 2 • LESSON 2

ACTIVITY
2.3 Writing and Solving Systems



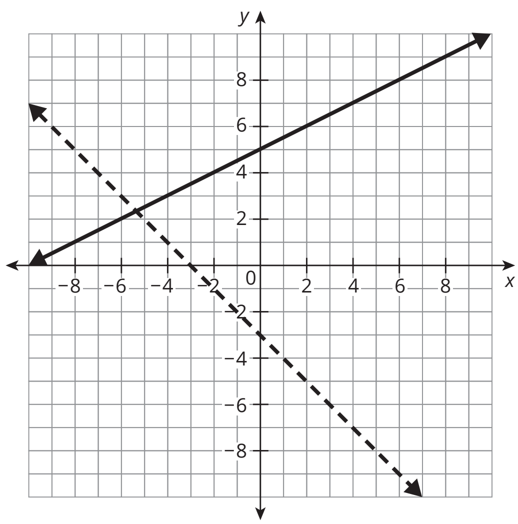
Write a system of equations for each graph, table, or description.

1. The admission fee for a fair is \$7 for an adult and \$5 for children under the age of 5. There are 449 people at the fair. They collect \$2768 in admission fees. How many adults, x, and children, y, are at the fair?

   $x + y = 449$

   $7x + 4y = 2768$

2. Write a system of equations from the graph.



   Sample answer:

   $y = \frac{1}{2}x + 5$

   $y = -x - 3$

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the directions.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Questions 4 and 5.
- Share and summarize.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing and solving systems of equations that model problem situations, assign Skills Practice Set A for this lesson.

Questions 2 and 3 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing a system of equations from graphs and tables, assign Skills Practice Set B for this lesson.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Questions 2 through 5, have students work individually or with a partner to complete the graphic organizer.

MODULE 3 • TOPIC 2 • LESSON 2 421

3. Write a system of linear equations from the tables provided.

Line *A*	
x	*y*
−2	−20
0	−15
2	−10
4	−5

Line *B*	
x	*y*
−4	16
−1	10
2	4
5	−2

$y = 2.5x - 15$

$y = -2x + 3$

Write and solve a system of equations to solve each problem.

4. The business manager for a band must make $236,000 from ticket sales to cover costs and make a reasonable profit. The auditorium where the band will play has 4000 seats, with 2800 seats on the main level and 1200 on the upper level. Attendees will pay $20 more for main-level seats.

   a. Write a system of equations with *x* representing the cost of the main-level seating and *y* representing the cost of the upper-level seating.

   $2800x + 1200y = 236{,}000$

   $x = y + 20$

   b. Without solving the system of linear equations, interpret the solution.

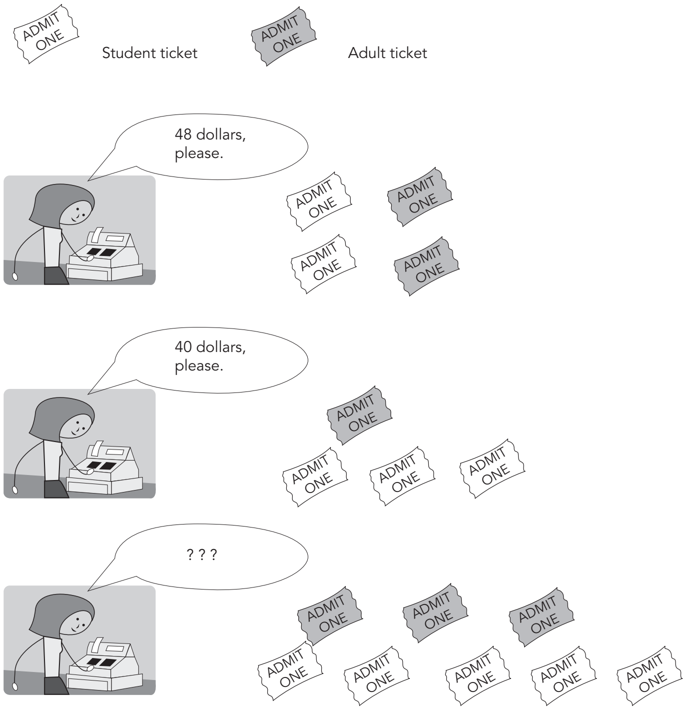
   The solution will represent the cost, in dollars, of the main-level tickets and the upper-level tickets needed to make the targeted total sales of $236,000.

   c. Solve the system of equations using the substitution method. Then, interpret the solution of the system in terms of the problem situation.

   (65, 45); In order to make the targeted total sales, the cost of main-level seating will be $65, and the cost of upper-level seating will be $45.

422 MODULE 3 • TOPIC 2 • LESSON 2

5. Mia is working as a cashier at the sports arena. What should she tell the next person in line?



Write and solve a system of equations that represents the problem situation. Define the variables. Then, determine the cost of each type of ticket. Finally, state the amount Mia charges the third person.

Let x be the cost of student tickets, and let y be the cost of adult tickets.

$2x + 2y = 48$

$3x + y = 40$

(8, 16); Student tickets cost \$8 each, and adult tickets cost \$16 each.

The total cost of three adult and five student tickets will be \$88.

MODULE 3 • TOPIC 2 • LESSON 2 423

Chunking the Activity

- Have students complete the activity individually.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

This activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice solving systems of equations by substitution and determining if they are consistent or inconsistent, assign Skills Practice Set C for this lesson.

Talk the Talk

The Substitution Train

1. Determine the solution to each linear system by using the substitution method. Check your answers algebraically.

a. $\begin{cases} 2x + 3y = 34 \\ y = 5x \end{cases}$

$2x + 3(5x) = 34$

$2x + 15x = 34$

$17x = 34$

$x = 2$

$y = 5(2)$

$y = 10$

Check: $2(2) + 3(10) = 34$

$4 + 30 = 34$

$34 = 34$ ✓

The solution is (2, 10).

b. $\begin{cases} y = 4x + 2 \\ y = 3x - 2 \end{cases}$

$4x + 2 = 3x - 2$

$x + 2 = -2$

$x = -4$

$y = 4(-4) + 2$

$y = -16 + 2$

$y = -14$

Check: $-14 = 3(-4) - 2$

$-14 = -12 - 2$

$-14 = -14$ ✓

The solution is (−4, −14).

424 MODULE 3 • TOPIC 2 • LESSON 2

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c. $\begin{cases} 3x + 2y = 4 \\ 2x - y = 5 \end{cases}$

$3x + 2(2x - 5) = 4$

$3x = 4x - 10 = 4$

$7x - 10 = 4$

$7x = 14$

$x = 2$

$y = 2x - 5$

$y = 2(2) - 5$

$y = 4 - 5$

$y = -1$

Check: $3(2) + 2(-1) = 4$

$6 + (-2) = 4$

$4 = 4$ ✓

$2(2) - (-1) = 5$

$4 + 1 = 5$

$5 = 5$ ✓

The solution is $(2, -1)$.

d. $\begin{cases} 3x + y = 8 \\ 6x + 2y = 10 \end{cases}$

$y = -3x + 8$

$6x + 2(-3x + 8) = 10$

$6x + (-6x) + 16 = 10$

$16 \neq 10$

There is no solution.

MODULE 3 • TOPIC 2 • LESSON 2 425

Lesson 2 Assignment

Write

Explain how to use the substitution method to solve systems of linear equations.

Remember

When a system has no solution, the equation resulting from the substitution step has no solution.

When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

Practice

1. Victoria is trying to become more environmentally conscious by making her own cleaning products. She researches different cleaners and decides to make furniture polish using olive oil and lemon juice. She wants to make enough to fill two 24-fluid-ounce bottles.

   a. Write an equation in standard form that relates the amount of olive oil and lemon juice to the total amount of mixture Victoria wants to make. Use x to represent the amount of lemon juice and y to represent the amount of olive oil.

   $x + y = 48$

   b. The recommendation for the mixture is that the amount of olive oil be twice the amount of lemon juice. Write an equation in terms of x and y as defined in part (a) that represents this situation.

   $y = 2x$

   c. Use substitution to solve the system of equations. Check your answer.

   The solution is (16, 32).

   d. What does the solution of the system represent in terms of the mixture?

   Victoria will need to use 16 fluid ounces of lemon juice and 32 fluid ounces of olive oil to make 48 fluid ounces of the furniture polish mixture. The mixture will contain twice as much olive oil as lemon juice.

Write

Sample answer:

The substitution method is a process of solving a system of equations by substituting an equal expression for a variable in one equation. To use the substitution method, begin by choosing one equation and isolating one variable. Substitute the expression equal to the isolated variable into the second equation to create a new equation with only one unknown. Solve the new equation and substitute that value into the first equation to solve for the second variable.

MODULE 3 • TOPIC 2 • LESSON 2 ASSIGNMENT 427

Lesson 2 Assignment

e. The best price Victoria can find for lemon juice is $0.25 per fluid ounce. The best price she can find for olive oil is $0.39 per fluid ounce. She buys a total of 84 fluid ounces of lemon juice and olive oil and spends $29.40. Write equations in standard form for this situation. Use *x* to represent the amount of lemon juice she buys, and use *y* to represent the amount of olive oil she buys.

Amount: $x + y = 84$
Price: $0.25x + 0.39y = 29.40$

f. Solve the system of equations you wrote using the substitution method. Check your answer. Describe the solution in terms of the problem situation.

The solution is (24, 60). Serena bought 24 fluid ounce of lemon juice and 60 fluid ounce of olive oil. She spent $29.40.

2. In an effort to eat healthier, Juan is tracking his food intake by using an application on his phone. He records what he eats, then the application indicates how many calories he has consumed.

One day, Juan eats 10 medium strawberries and 8 vanilla wafer cookies as an after-school snack. The caloric intake from these items is 192 calories. The next day, she eats 20 medium strawberries and 1 vanilla wafer cookie as an after-school snack. The caloric intake from these items is 99 calories.

a. Write a system of equations for this problem situation. Define your variables.

Let *x* represent the number of calories in one strawberry and *y* represent the number of calories in one vanilla wafer cookie.
$10x + 8y = 192$
$20x + y = 99$

b. Without solving the system of linear equations, interpret the solution.

The solution represents the number of calories in one strawberry and the number of calories in one vanilla wafer cookie.

428 MODULE 3 • TOPIC 2 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

c. Solve the system of equations using the substitution method. Check your work.

The solution is (4, 19).

d. Interpret the solution of the system in terms of the problem situation.

Each strawberry has 4 calories, and each vanilla wafer cookie has 19 calories.

e. Juan's friend Jayden also has a calorie counting application on his phone. The two friends decide to compare the two programs. Juan eats 1 banana and 5 pretzel rods, and his application tells him he consumed 657 calories. Jayden eats 1 banana and 5 pretzel rods, and his application tells him he consumed 656 calories. The boys want to know how many calories are in each food. Write a system of equations for this problem. Define your variables.

Let x represent the number of calories in a banana, and let y represent the number of calories in a pretzel rod.

$x + 5y = 657$

$x + 5y = 656$

f. Solve the system of equations using the substitution method. Interpret your answer in terms of the problem.

There is no solution to this system.

The applications must have a different number of calories allotted to a banana, a pretzel rod, or both.

3. Write a system of linear equations for each graph, table, or description.

a. Mateo has 13 coins. The coins are nickels and quarters. The coins have a total value of $2.05. Let n represent the number of nickels, and let q represent the number of quarters.

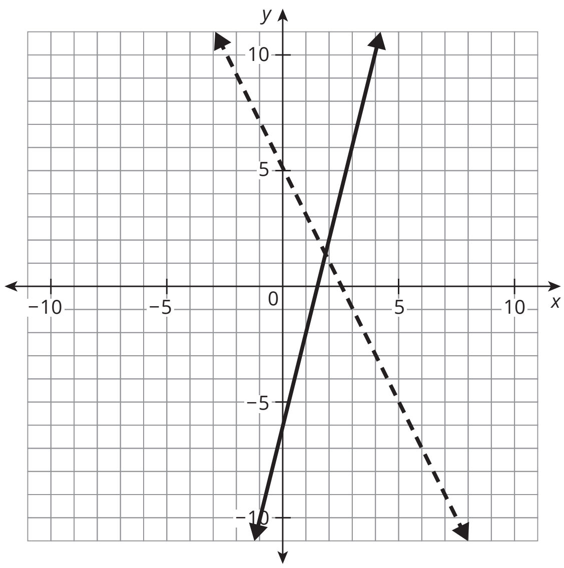
$n + q = 13$

$0.05n + 0.25q = 2.05$

MODULE 3 • TOPIC 2 • LESSON 2 ASSIGNMENT 429

Lesson 2 Assignment

b.



Sample answer:

$y = -2x + 5$

$y = 4x - 6$

c.

Line *A*	
x	y
−3	−15
0	−3
3	9
6	15

Line *B*	
x	y
−4	12
−2	5
2	−9
6	−23

$y = 4x - 3$

$y = -3.5x - 2$

430 MODULE 3 • TOPIC 2 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

4. Use the substitution method to determine the solution of each system of linear equations. Check your solutions.

a. $\begin{cases} 9x + y = 16 \\ y = 7x \end{cases}$

(1, 7)

b. $\begin{cases} 3x + \frac{1}{2}y = -3.5 \\ y = -6x + 11 \end{cases}$

No solution

c. $y = -5x$
$21x - 7y = 28$

$\left(\frac{1}{2}, -\frac{2}{5}\right)$

d. $2x + 4y = -32$
$y = -\frac{1}{2}x - 8$

Infinite solutions

Prepare

Determine the additive inverse for each expression.

1. 4

   -4

2. x

   $-x$

3. $20x$

   $-20x$

4. $-9x$

   $9x$

5. $78.5x$

   $-78.5x$

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MODULE 3 • TOPIC 2 • LESSON 2 ASSIGNMENT 431

3 Using Linear Combinations to Solve a System of Linear Equations

MATERIALS

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students are given a problem scenario and use reasoning to determine the two unknowns. They then write a system of linear equations in standard form to represent a problem situation. Students analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They practice the linear combinations method with systems in which the coefficients of one variable are additive inverses. Next, Worked Examples guide students to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system and then they solve two problems in context, one with fractional coefficients. The lesson concludes with students addressing when it is appropriate to use the graphing, substitution, or linear combinations methods.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

(TEKS continued on next page)

ELPS

(2) Listening

The student is expected to:

(H) understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations.

(3) Speaking

The student is expected to:

(C) speak using a variety of grammatical structures, sentence lengths, sentence types, and connecting words with increasing accuracy and ease as more English is acquired.

(4) Reading

The student is expected to:

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2I write systems of two linear equations given a table of values, a graph, and a verbal description.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions.
The student is expected to:

A.5C solve systems of two linear equations with two variables for mathematical and real-world problems.

ESSENTIAL IDEAS

- The linear combinations method is a process to solve a system of linear equations by adding two equations together, resulting in an equation in one variable.
- When using the linear combinations method, it is often necessary to multiply one or both equations by a constant to create two equations in which the coefficients of one of the variables are additive inverses.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Coding Class 10 minutes

ESTABLISH A SITUATION

Students are given a problem scenario and use reasoning to determine the two unknowns.

DEVELOP

Activity 3.1: Combining Linear Systems 30–35 minutes

PEER WORK ANALYSIS, MATHEMATICAL PROBLEM SOLVING

Students write a system of linear equations in standard form to represent the Coding Class problem. They analyze two solution paths, one using substitution and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They then solve two systems of linear equations using the linear combinations method where the coefficients of one of the variables are additive inverses.

DAY 2

Activity 3.2: Using Additive Inverses to Combine Linear Systems 20–25 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

Students are provided Worked Examples to guide them to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system of equations.

Activity 3.3: Applying the Linear Combinations Method 15–20 minutes

REAL-WORLD PROBLEM SOLVING

Students solve a problem in context by writing a system of equations in standard form, applying the linear combinations method, and interpreting the solution in terms of the context.

DAY 3

Activity 3.4: Fractions and Linear Combinations 30–35 minutes

PEER WORK ANALYSIS, REAL-WORLD PROBLEM SOLVING

Students solve a problem in context by writing a system of equations in standard form with fractional coefficients. They are provided two strategies to solve the system, one that rewrites the given equations without fractional coefficients, and the other that uses the linear combinations method with fractional coefficients.

DEMONSTRATE

Talk the Talk: There's a Method in My Madness 10 minutes

GENERALIZATION

Students summarize when it is appropriate to use the graphing, substitution, or linear combinations methods.

Getting Started

ENGAGE

Coding Class

Facilitation Notes

In this activity, students are given a problem scenario and use reasoning to determine the two unknowns.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Levels of sophistication used to solve the problem, from guessing and checking, to graphing, to writing and solving a system of equations using substitution.
- Reference to the context to make sure their answers make sense.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is meant by, "The number of 9th-grade students outnumber the number of 10th-grade students by 34"? • What is the total number of 9th-grade and 10th-grade students in the class? • What is the difference between the number of 9th-graders in the coding class and the number of 10th-graders in the coding class?
Probing	• How did you solve this problem? What is another way? • For 5(b), how did you rewrite an equation to use this method? • Why does it make more sense to use the linear combinations method rather than substitution when solving these systems?
Seeing structure	• Why do you have to check your solution against two criteria? • How can you tell that you can model this situation with a system of equations?

STAMP THE LEARNING

Summary

Some problems in context involve two different relationships between two variables that can be represented using a system of equations. There is more than one method to solve a system of equations.

ACTIVITY
3.1 Combining Linear Systems

DEVELOP

Facilitation Notes

In this activity, students write a system of linear equations in standard form to represent the Coding Class problem. They analyze two solution paths, one using substitution, and one using the *linear combinations method* in its most basic form prior to its formal definition later in the activity. They then solve two systems of linear equations using the linear combinations method where the coefficients of one of the variables are additive inverses.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Suggest that students use the variables f and m rather than x and y to write and interpret their equations.
- Have students interact with Mateo's work and Minh's work by:
  - Inserting additional steps within the shown student work.
  - Numbering the steps.
  - Visualizing the vertical orientation of Minh's work by drawing vertical arrows to make it explicit that the third line of Minh's work is determined by summing like terms in the first two lines of her work; aligning the sum of the terms, $2x$, in the same column as the x's.

$$\begin{array}{rcrcl} x & + & y & = & 324 \\ x & - & y & = & 34 \\ 2x & & & = & 358 \end{array}$$

AS STUDENTS WORK, LOOK FOR

Errors translating the context into an equation, such as $f + 34 = m$, rather than $f = m + 34$.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did Mateo derive the equation $(y + 34) + y = 324$? • Why did Mateo isolate x in the second line of his work? • How did Mateo determine the value of x? • What is the name of the method that Mateo used? Why does it have that name? • Explain how Minh derived the equation $2x = 358$. • How did Minh determine the value of y?
Seeing structure	• If the coefficients of y are not additive inverses, will Minh's strategy still work? Explain your thinking.

Ask a student to read the information following Question 4 aloud. Discuss as a class.

Have students practice the linear combinations method with Question 5. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Forgetting to solve for the second variable.
- Substitution of the solution of the first variable in the wrong place when solving for the second variable.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why does the name *linear combinations method* make sense?
Probing	• How can you tell in advance which variable will be eliminated from the system of equations? • Is your strategy any different if the coefficients are fractions? Explain. • For 5(b), how did you rewrite an equation to use this method? • Why does it make more sense to use the linear combinations method rather than substitution when solving these systems?
Seeing structure	• How could you express your solution to the system of equations using coordinate notation?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Address the fact that the linear combinations method can be applied to other forms of equations as well. For example, have students solve this system using the linear combinations method.

$$y = 4x + 6$$
$$y = -4x + 22$$

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

The linear combinations method is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. For the linear combinations method to work, the coefficients of one of the variables must be additive inverses.

ACTIVITY 3.2 Using Additive Inverses to Combine Linear Systems

Facilitation Notes

Students are provided Worked Examples to guide them to multiply one, and then both, equations by a constant to create equations in which a variable has coefficients that are additive inverses. Students concentrate on creating coefficients that are additive inverses with several systems without entirely solving the system of equations.

To begin the Day 2 session, have a student read the Essential Question aloud.

Analyze the Worked Examples and complete Questions 1 through 3 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why do you think that some people refer to the linear combinations method as the elimination method?
Probing	• Which variable adds to zero in this Worked Example? • Why does it make sense to multiply the second equation by -2? • Why is it necessary that the resulting equation has one variable only?
Seeing structure	• Do you see another way to create additive inverses?
Reflecting and justifying	• Which property of equality supports $-2(4x + y) = -2(15)$?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

COMMON MISCONCEPTION

Students may incorrectly multiply only the side of the equation with the variables by a constant instead of multiplying both sides by a constant. Facilitate a discussion about the importance of maintaining equality by multiplying both sides of the equation by the same constant.

AS STUDENTS WORK, LOOK FOR

Interpretation of $0 = 0$ as a system with no solution, rather than an infinite number of solutions. Emphasize that any true statement, such as $0 = 0$, $1 = 1$, or $2 = 2$, implies an infinite number of solutions.

DIFFERENTIATION STRATEGY

Just in Time Support

Model the thinking to create additive inverses. Suggest that students write this pre-work before solving the system.

Question 4 part (b):

$$\begin{array}{rcl} -\,6 & \text{think: } \frac{-6}{3} = -2 & \\ x + 3y = 15 & & -2(x + 3y) = (15)(-2) \\ & \Longrightarrow & \\ 5x + 2y = 7 & & 3(5x + 2y) = (7)3 \\ +\,6 & \text{think: } \frac{6}{2} = 3 & \end{array}$$

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Is it more efficient to eliminate a specific variable? Explain your reasoning. • What is another possible first step? • How did you decide which equation to rewrite? • How do you know when to multiply one or two equations by a value? • When multiplying a system by a value, how do you know which product will cause a variable to add or subtract out of the system? • How did you decide which equation to rewrite?
Seeing structure	• How do you know, algebraically or graphically, that a system has an infinite number of solutions? • How do you know, algebraically or graphically, that a system has no solutions?

STAMP THE LEARNING

Summary

Sometimes, one or both equations in a linear system must be multiplied by a constant to create additive inverses so the resulting new equation contains one variable.

ACTIVITY 3.3 Applying the Linear Combinations Method

Facilitation Notes

In this activity, students solve a problem in context by writing a system of equations in standard form, applying the linear combinations method, and interpreting the solution in terms of the context.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What expression represents two nights of lodging and four meals? • What expression represents three nights of lodging and eight meals? • Looking at both equations, what are the *n*-coefficients? *m*-coefficients?
Probing	• Will adding the equations together result in an equation with the *n*- or the *m*-variable? Why or why not? • Is it more efficient to eliminate a specific variable? Why or why not? • How did you solve for the first variable? The second variable?
Seeing structure	• How do you express your solution to the system of equations using coordinate notation?
Reflecting and justifying	• To check your solution algebraically, what do you need to do?

DIFFERENTIATION STRATEGIES

Challenge Opportunity

- Have students solve the system of equations a second time, this time solving for the other variable first. Compare this process and solution to the first time solving the system.
- Discuss naming conventions when writing ordered pairs for variables other than (x, y). Typically, variables are listed in alphabetical order. In this case, the solution would be written as (m, n).

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

After writing a system of equations to represent a real-world problem, you can use the linear combinations method to solve the system and interpret the solution in terms of the problem.

ACTIVITY 3.4 Fractions and Linear Combinations

Facilitation Notes

In this activity, students solve a problem in context by writing a system of equations in standard form with fractional coefficients. They are provided two strategies to solve the system, one that rewrites the given equations without fractional coefficients, and the other that uses the linear combinations method with fractional coefficients.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What expression represents one-half the number of 5-inch bracelets plus three-fourths the number of 7-inch bracelets? • Looking at both equations, what are the *x*-coefficients? *y*-coefficients? • Whose method do you prefer? Why?
Probing	• How is this system of linear equations different from other systems of linear equations you have previously solved? • Describe how each equation relates to the context. • What is the purpose of Aaliyah's method? Liam's method? • Which value did you multiply both equations by to make additive inverses? • How could you solve for the other variable first?
Seeing structure	• How do you express your solution to the system of equations using coordinate notation?
Reflecting and justifying	• To check your solution algebraically, what do you need to do?

STAMP THE LEARNING

Summary

The linear combinations method can be used to solve systems of equations with any real number coefficients.

DEMONSTRATE

Talk the Talk

THERE'S A METHOD IN MY MADNESS

Facilitation Notes

In this activity, students summarize when it is appropriate to use the graphing, substitution, or linear combinations methods.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Which method always provides an exact answer? • Is one method more efficient than the others? Explain. • What characteristic of the graph identifies the solution to a system of linear equations? • Which method requires first isolating the variable in one of the linear equations? • When using the linear combinations method, how do you know that one or both equations must be rewritten? • Which method can easily be used to make predictions? • Which method does not always provide an exact answer? • Which method is best to use when the equations are both written in standard form? • Which method requires at least one of the equations to be written in slope-intercept form?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials Needed: Poster or Butcher Paper

Have students make posters solving the same system of linear equations using all three methods. Provide systems of equations written in various forms (2 in general form, 2 in standard form, 1 in general form, and 1 in standard form). Follow-up by having a class discussion listing pros and cons of each method.

Have students read and answer the Essential Question on the lesson opener page.

Summary

The graphing method, the substitution method, and the linear combinations method can be used to solve a system of linear equations. Sometimes, one method is more efficient than the others based upon the forms of the equations.

STAMP THE LEARNING

3 Using Linear Combinations to Solve a System of Linear Equations

Setting the Stage

- Communicate the objectives and new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using linear combinations.
- Interpret the solution to a system of equations in terms of a problem situation.

NEW KEY TERM

- linear combinations method

You have solved a system of linear equations graphically and algebraically, using the substitution method.

What are other algebraic strategies for solving systems of equations?

Sample answer:

I can also use an elimination method by adding or subtracting combinations of the equations in the system until I have the solution.

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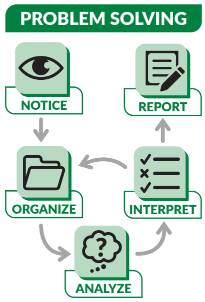
MODULE 3 • TOPIC 2 • LESSON 3 433

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.
- See the Facilitating Productive Struggle section in the Course and Implementation Guide for additional guidance on supporting students through problem-solving activities.



Getting Started

Coding Class

There are a total of 324 students taking a coding class. The number of 9th-grade students outnumber the number of 10th-grade students by 34.

1. Use reasoning to determine the number of 9th- and 10th-grade students who joined the coding class.

   There are 179 9th-grade students and 145 10th-grade students in the class.

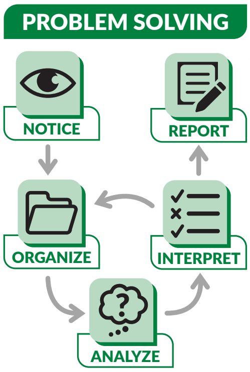
2. How can you check that your solution is correct?

   I can make sure that the total number of students adds to 324 and that the difference between the number of 9th-grade students and the number of 10th-grade students is 34.

434 MODULE 3 • TOPIC 2 • LESSON 3

ACTIVITY **3.1**

Combining Linear Systems



Consider the scenario from the Getting Started. Let's explore an algebraic strategy to determine a solution.

1. Write the system that represents the problem situation. Use x to represent the 9th-grade students in the class, and use y to represent the 10th-grade students in the class. Write the equations in standard form.

$$\begin{cases} x + y = 324 \\ x - y = 34 \end{cases}$$

Mateo and Minh used different strategies to solve the system of equations in a similar way. Analyze each student's reasoning.

Mateo

I can use the substitution method to solve this system.

$$\begin{aligned} x + y &= 324 \\ x - y &= 34 \rightarrow x = y + 34 \\ (y + 34) + y &= 324 \\ 2y + 34 &= 324 \\ 2y &= 290 \\ y &= 145 \end{aligned}$$

$$\begin{aligned} x &= 145 + 34 \\ x &= 179 \end{aligned}$$

Minh

You can eliminate one of the quantities by adding the two equations together.

$$\begin{array}{r} x + y = 324 \\ + \ x - y = 34 \\ \hline 2x = 358 \\ x = 179 \end{array}$$

$$\begin{aligned} 179 + y &= 324 \\ y &= 145 \end{aligned}$$

Remember . . .

As long as you maintain equality you can rewrite equations any way you want.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–4.
- Check in and share.
- Read and discuss the information.
- Group students to complete Question 5.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.
- See the Facilitating Productive Struggle section in the Course and Implementation Guide for additional guidance on supporting students through problem-solving activities.

MODULE 3 • TOPIC 2 • LESSON 3 435

2. Explain why each student is correct in their reasoning.

Mateo is correct because substituting an equivalent expression for x in one equation results in an equation in one variable, y, that can be solved. Then, by substituting the value of y into one of the original equations, the value of x can be calculated.

Minh is correct because the y-variables have coefficients that are additive inverses, so when the equations are added together, the y-variables are eliminated from the equation, resulting in an equation in one variable, x, that can be solved. Then, by substituting the value of x into one of the original equations, the value of y can be calculated.

3. Examine the structure of the system. What characteristic of the system made Minh's strategy efficient?

Both equations are in standard form, and one variable has coefficients that are additive inverses.

4. Identify the solution to the linear system as an ordered pair. Then, interpret the solution in terms of this problem situation.

(179, 145)

179 9th-grade students and 145 10th-graders are in the class.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

The algebraic method used by Minh to solve the linear system is called the *linear combinations method*. The **linear combinations method** is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

5. Solve each system of equations using the linear combinations method.

a. $\begin{cases} -4x + y = 10 \\ -2x - y = -1 \end{cases}$

(−1.5, 4)

b. $\begin{cases} -\frac{1}{2}x + 5y = -6 \\ \frac{1}{2}x + y = -6 \end{cases}$

(−8, −2)

436 MODULE 3 • TOPIC 2 • LESSON 3

ACTIVITY **3.2**

Using Additive Inverses to Combine Linear Systems

In the system of equations from the previous activity, one of the variables in both equations has coefficients that are additive inverses. What if a system doesn't have variables that are additive inverses? Let's use the strategy of linear combinations to solve other systems.

WORKED EXAMPLE

Consider this system of equations: $\begin{cases} 7x + 2y = 24 \\ 4x + y = 15 \end{cases}.$

$$\begin{aligned} 7x + 2y &= 24 \\ -2(4x + y) &= -2(15) \end{aligned}$$

Multiply the second equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$\begin{array}{rrcl} & 7x + 2y &=& 24 \\ + & -8x - 2y &=& -30 \\ \hline & -x &=& -6 \\ & x &=& 6 \end{array}$$

Now that the y-values are additive inverses, you can solve this linear system for x.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ 2y &= -18 \\ y &= -9 \end{aligned}$$

Substitute the value for x into one of the equations to determine the value for y.

The solution to the system of linear equations is $(6, -9)$.

Remember . . .
Two numbers with a sum of zero are called *additive inverses*.

1. In the Worked Example, only one equation needs to be rewritten to solve using the linear combinations method. Why?

   If I multiply the equation $4x + y = 15$ by (-2), then the coefficients of the y-terms will be additive inverses.

Ask Yourself . . .
How can you solve the system of equations by transforming the first equation instead of the second?

Chunking the Activity

- Read and discuss the introduction and the Worked Examples.
- Complete Questions 1–3 as a class.
- Group students to complete Questions 4 and 5.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 3 • TOPIC 2 • LESSON 3 437

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 2–4 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice solving linear systems by linear combination, assign Skills Practice Set A for this lesson.

Now, let's consider a system where both equations need to be rewritten.

Ask Yourself . . .
If you multiply both sides of an equation by the same number, is the equation still true?

WORKED EXAMPLE

$$\begin{cases} 4x + 2y = 3 \\ 5x + 3y = 4 \end{cases}$$

$$3(4x + 2y) = 3(3)$$
$$-2(5x + 3y) = -2(4)$$

Multiply each equation by a constant that results in coefficients that are additive inverses for one of the variables.

$$12x + 6y = 9$$
$$-10x - 6y = -8$$

2. Determine the solution for the linear system shown in the second Worked Example.

$\left(\frac{1}{2}, \frac{1}{2}\right)$

3. How could you have solved this system by creating *x*-values that are additive inverses?

Sample answer:

Multiply the equation $4x + 2y = 3$ by 5 and the equation $5x + 3y = 4$ by -4. The coefficients of x would be additive inverses, 20 and -20. Then, solve for y. Substitute $y = \frac{1}{2}$ into one of the equations and solve for x, $x = \frac{1}{2}$.

4. Describe the first step needed to solve each system using the linear combinations method. Identify the variable that will be solved for when you add the equations.

a. $\begin{cases} 5x + 2y = 10 \\ 3x + 2y = 6 \end{cases}$

Sample answer:

Multiply the second equation by -1, then add the equations to get a resulting equation to determine the *x*-value.

b. $\begin{cases} x + 3y = 15 \\ 5x + 2y = 7 \end{cases}$

Sample answer:

Multiply the first equation by -5, then add the equations to get a resulting equation to determine the *y*-value.

438 MODULE 3 • TOPIC 2 • LESSON 3

c. $\begin{cases} 4x + 3y = 12 \\ 3x + 2y = 4 \end{cases}$

Sample answer:

Multiply the first equation by 3 and the second equation by −4, then add the equations to get a resulting equation to determine the *y*-value.

5. Analyze each system. How would you rewrite the system to solve for one variable? Explain your reasoning.

a. $\begin{cases} \frac{1}{2}x - 5y = -45 \\ -\frac{1}{2}x + 10y = -20 \end{cases}$

Sample answer:

Do not rewrite either equation. The coefficients of the *x*-terms are already additive inverses.

b. $\begin{cases} 4x + 3y = 24 \\ 3x + y = -2 \end{cases}$

Sample answer:

Multiply the second equation by −3, then add the equations to get a resulting equation to determine the *x*-value.

c. $\begin{cases} 3x + 5y = 17 \\ 2x + 3y = 11 \end{cases}$

Sample answer:

Multiply the first equation by 2 and the second equation by −3, then add the equations to get a resulting equation to determine the *y*-value.

d. $\begin{cases} 6x + 3y = 5 \\ 2x + y = 1 \end{cases}$

Sample answer:

Multiply the second equation by −3, then add the equations to get a resulting equation with no variable terms and is a false statement.

e. $\begin{cases} x + 2y = -6 \\ 2x + 4y = -12 \end{cases}$

Sample answer:

Multiply the first equation by −2, both the *x*-terms and *y*-terms will be eliminated, resulting in a true statement.

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MODULE 3 • TOPIC 2 • LESSON 3 439

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Questions 1–3 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing and solving linear systems by linear combination, assign Skills Practice Set B for this lesson.

ACTIVITY 3.3 Applying the Linear Combinations Method

A snow resort offers two winter specials: the Get-Away Special and the Extended Stay Special. The Get-Away Special offers two nights of lodging and four meals for $270. The Extended Stay Special offers three nights of lodging and eight meals for $435. Determine what Let It Snow charges per night of lodging and per meal.

1. Write the system of linear questions that represents the problem situation. Let n represent the cost for one night of lodging at the resort and m represent the cost for each meal. Write the equations in standard form.

   $\begin{cases} 2n + 4m = 270 \\ 3n + 8m = 435 \end{cases}$

2. How are these equations the same? How are these equations different?

   Both equations are written in standard form.

   The coefficients of n and m are different.

3. Solve the system comparing the two winter specials.

   $n = 105, m = 15$

440 MODULE 3 • TOPIC 2 • LESSON 3

4. Interpret the solution of the linear system in the problem situation.

   Let It Snow Resort charges $105 per night of lodging and $15 per meal.

5. Check your solution algebraically.

$$\begin{aligned} 2n + 4m &= 270 \\ 2(105) + 4(15) &= 270 \\ 210 + 60 &= 270 \\ 270 &= 270 \end{aligned}$$

$$\begin{aligned} 3n + 8m &= 435 \\ 3(105) + 8(15) &= 435 \\ 315 + 120 &= 435 \\ 435 &= 435 \end{aligned}$$

6. Is the Extended Stay Special the better deal? Explain why or why not.

   No. Let It Snow Resort charges the same amount for meals and lodging for both specials.

MODULE 3 • TOPIC 2 • LESSON 3 441

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

ACTIVITY 3.4 Fractions and Linear Combinations

The School Spirit Club is making beaded friendship bracelets with the school colors to sell in the school store. The bracelets are black and orange and come in two lengths: 5 inches and 7 inches. The club has enough beads to make a total of 84 bracelets. So far, they have made 49 bracelets, which represents $\frac{1}{2}$ the number of 5-inch bracelets plus $\frac{3}{4}$ the number of 7-inch bracelets they plan to make and sell. Determine how many 5-inch and 7-inch bracelets the club plans to make.

1. Let x represent the number of 5-inch bracelets, and let y represent the number of 7-inch bracelets. Write a system of equations in standard form to represent this problem situation.

   $$\begin{cases} x + y = 84 \\ \frac{1}{2}x + \frac{3}{4}y = 49 \end{cases}$$

2. Aaliyah says that the first step to solve this system is to multiply the second equation by the least common denominator (LCD) of the fractions. Liam says that the first step is to multiply the first equation by $-\frac{1}{2}$. Who is correct? Explain your reasoning.

   Both students are correct. Aaliyah chose to multiply by the LCD to rewrite the second equation without fractions, then she will have to multiply one of the equations by a constant to create additive inverses for either the x- or y-term. Liam chose to work with the fractions in the second equation; by multiplying by $-\frac{1}{2}$, when he adds the equations, the resulting equation determines the y-value.

EB STUDENT TIP

For all proficiency levels

Ensure students understand the acronym LCD (*least common denominator*). Ask students to demonstrate their understanding of the term by identifying the *LCD* of several fraction pairs.

3. Rewrite the equation containing fractions as an equivalent equation without fractions.

   $2x + 3y = 196$

4. Determine the solution to the system of equations by using linear combinations and check your answer.

   $(56, 28)$

   $$x + y = 84$$
   $$56 + 28 = 84$$
   $$\frac{1}{2}x + \frac{3}{4}y = 49$$
   $$\frac{1}{2}(56) + \frac{3}{4}(28) = 49$$
   $$28 + 21 = 49$$

5. Interpret the solution of the linear system in terms of this problem situation.

   The School Spirit Club plans to make a total of 56 five-inch and 28 seven-inch friendship bracelets.

MODULE 3 • TOPIC 2 • LESSON 3 443

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1C:

- Are students considering which strategies or tools to use?

Talk the Talk

There's a Method in My Madness

You have used three different methods for solving systems of equations: graphing, substitution, and linear combinations.

1. Describe how to use each method and the characteristics of the system that makes this method most appropriate.

   a. Graphing method:

   Graph the two lines and identify the intersection point; however, sometimes only an estimate of the intersection point is possible. This method is most appropriate when the equations are both written in slope-intercept form and the values are reasonable to graph.

   b. Substitution method:

   Choose one equation and isolate one variable; this will be considered the first equation. Then, substitute the expression equal to the isolated variable into the second equation. Solve the new equation with only one variable. Use substitution to solve for the second variable. This method is most appropriate when one of the equations is written in slope-intercept form.

   c. Linear combinations method:

   Rewrite one or both of the equations so that the coefficients of one of the variables are additive inverses. Add the two equations, resulting in an equation in one variable. Solve the new equation with only one variable. Use substitution to solve for the second variable. This method is most appropriate when both equations are written in standard form.

444 MODULE 3 • TOPIC 2 • LESSON 3

Lesson 3 Assignment

Write

Explain how you would combine the two equations to solve for x and y. Use the following terms in your explanation: *linear combination* and *additive inverses.*

$$3x + 2y = -25$$
$$x - 4y = 5$$

Remember

You can use the linear combinations method to solve a system of equations by adding two equations together, resulting in an equation with one variable. You can then determine the value of that variable and use it to determine the value of the other variable.

Practice

1. Two high schools are taking field trips to the state capital. A total of 408 students from East High will be going in 3 vans and 6 buses. A total of 516 students from West High will be going in 6 vans and 7 buses. Each van has the same number of passengers, and each bus has the same number of passengers.

   a. Write a system of equations that represents this problem situation. Let x represent the number of students in each van, and let y represent the number of students in each bus.

   $$\begin{cases} 3x + 6y = 408 \\ 6x + 7y = 516 \end{cases}$$

   b. How are the equations in the system the same? How are they different?

   Both equations are written in standard form. The coefficients of x and y are different, as are the constant terms.

   c. Describe the first step needed to solve the system using the linear combinations method. Identify the variable that will be eliminated as well as the variable that will be solved for when you add the equations.

   Answers may vary.

   Multiply the first equation by -2. Then, when the two equations are added, the resulting equation can be solved for y.

Write

Sample answer:

I would apply the linear combinations method and create additive inverses for the coefficients of the x terms. Then, I would add the two equations and solve for y. I would then substitute the value for y and solve for x.

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MODULE 3 • TOPIC 2 • LESSON 3 ASSIGNMENT 445

Lesson 3 Assignment

d. Solve the system of equations using the linear combinations method. Show your work.

$x = 16$; $y = 60$

e. Interpret the solution of the linear system in terms of the problem situation.

The solution (16, 60) means that each van is carrying 16 students and each bus is carrying 60 students.

f. Check your solution algebraically.

$$\begin{aligned} 3(16) + 6(60) &= 408 \\ 48 + 360 &= 408 \\ 408 &= 408 \end{aligned}$$

$$\begin{aligned} 6(16) + 7(60) &= 516 \\ 96 + 420 &= 516 \\ 516 &= 516 \end{aligned}$$

2. Solve each system of linear equations.

a. $\begin{cases} 3x + y = 9 \\ 7x + y = 32 \end{cases}$

(5.75, −8.25)

b. $\begin{cases} \frac{2}{3}x + \frac{1}{4}y = 18 \\ \frac{1}{6}x - \frac{3}{8}y = -6 \end{cases}$

(18, 24)

446 MODULE 3 • TOPIC 2 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

c. $\begin{cases} 5x - 8y = 25 \\ -x + 4y = -8 \end{cases}$

$(3, -1.25)$

d. $\begin{cases} 5x + 4y = -14 \\ 3x + 6y = 6 \end{cases}$

$(-6, 4)$

Prepare

Determine if each point is a solution to $y > x$, $y < x$, or $y = x$.

1. $(8, -2)$

   $y < x$

2. $(0, 7)$

   $y > x$

3. $(-1, -1)$

   $y = x$

4. $(-4, -3)$

   $y > x$

5. $(9, 9)$

   $y = x$

6. $(-3, -10)$

   $y < x$

MODULE 3 • TOPIC 2 • LESSON 3 ASSIGNMENT 447

4 Graphing Inequalities in Two Variables

MATERIALS

None

LESSON OVERVIEW

Students explore a linear inequality in two variables through a scenario. They write an inequality, complete a table of values, graph the coordinate pairs from the table, and determine which parts of the graph are solutions to the inequality. Students then formalize the process of graphing inequalities through practice without context; they graph the corresponding equation of an inequality as a boundary line, determine whether the line should be solid or dashed, and identify which half-plane to shade by testing the point (0, 0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs. They then solve a problem in context where they use a table of values to write and graph a linear inequality and refer to the inequality and/or its graph to respond to questions. Finally, students summarize the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2H write linear inequalities in two variables given a table of values, a graph, and a verbal description.

(TEKS continued on next page)

ELPS

(1) Learning Strategies
The student is expected to:

(D) speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution (conveying ideas by defining or describing when exact English words are not known).

(2) Listening
The student is expected to:

(I) demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.

(5) Writing
The student is expected to:

(E) employ increasingly complex grammatical structures in content area writing commensurate with grade-level expectations such as (i) using correct verbs, tenses, and pronouns/antecedents; (ii) using possessive case (apostrophe -s) correctly; and, (iii) using negatives and contractions correctly.

ALGEBRA I TEKS *(TEKS continued from previous page)*

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3D graph the solution set of linear inequalities in two variables on the coordinate plane.

ESSENTIAL IDEAS

- The graph of a linear inequality is a half-plane, or half of a coordinate plane.
- Shading is used to indicate which half-plane describes the solution set of the inequality.
- Dashed and solid lines are used to indicate if the line itself is included in the solution set of an inequality.
- Linear inequalities and their graphs can be used to represent and solve problems in context.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Making a Statement 5–10 minutes

ESTABLISH A SITUATION

Students compare five different solution statements that relate the value of *x* to the constant 2. Next, they write a scenario that can be represented by one of the statements and then modify the scenario so that it can be represented by one of the different statements.

DEVELOP

Activity 4.1: Linear Inequalities in Two Variables 30–35 minutes

PEER WORK ANALYSIS, MATHEMATICAL PROBLEM SOLVING

Students explore a linear inequality in two variables through a scenario. They write an inequality, complete a table of values, graph the values from the table, and determine which parts of the graph are solutions to the inequality.

DAY 2

Activity 4.2: Determining the Graphs of Linear Inequalities 25 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

Students graph inequalities by graphing each inequality's corresponding equation as a boundary line, determining whether it should be solid or dashed, and identifying which half-plane to shade by testing the point (0, 0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs.

Activity 4.3: Interpreting the Graph of a Linear Inequality 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students use a table of values to write and graph a linear inequality to model a context. They then use the inequality and/or its graph to respond to questions provided in context. Students also interpret the meaning of points on the line, above the line, or below the line.

DEMONSTRATE

Talk the Talk: There's a Fine Line 5–10 minutes

GENERALIZATION

Students demonstrate an understanding of the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.

Getting Started

ENGAGE

Making a Statement

Facilitation Notes

In this activity, students compare five different solution statements that relate the value of *x* to the constant 2. Next, they write a scenario that can be represented by one of the statements and then modify the scenario so that it can be represented by one of the different statements.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

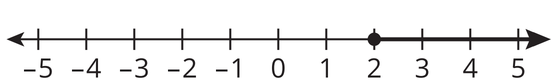
QUESTIONS TO SUPPORT DISCOURSE

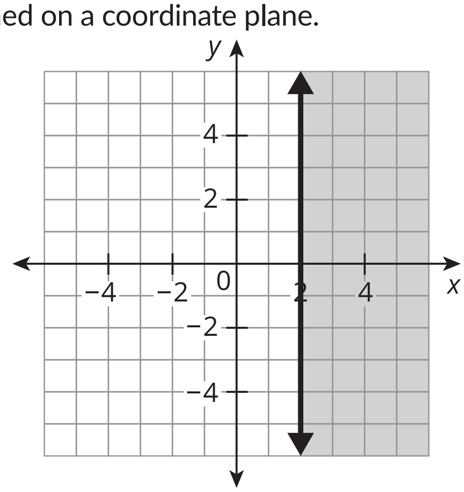
Gathering	• Which statements have an infinite number of solutions? • Which solution statements include negative numbers? • Which solution statements include 2?
Probing	• Describe a scenario that included 2, then how you modified it to exclude 2, or vice versa. • Describe a scenario that included less than, then how you modified it to include greater than, or vice versa. • Did you modify the entire scenario or just particular words to change the interpretation?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Ask students to graph the solution set $x \geq 2$. Do not provide additional guidance to see if students choose a number line or coordinate plane to sketch the graph. Then, discuss the appropriateness of each representation and the possible *y*-values when the solution statement is graphed on a coordinate plane.





STAMP THE LEARNING

Summary

When a solution statement compares a variable to a constant, the constant may or may not be included in the solution set.

ACTIVITY 4.1

Linear Inequalities in Two Variables

DEVELOP

Facilitation Notes

In this activity, students write an inequality to represent a given scenario. They use the inequality to complete a table of values, graph the data from the table, then determine which parts of the graph are solutions to the inequality.

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials Needed: Poster or Butcher Paper

Have students work in groups using large poster-sized coordinate planes. Make the activity more open-ended by eliminating many of the scaffolding questions, and have students complete only Question 1 and Question 5. Extend Question 5 so each student is responsible for creating a table with at least 10 different sets of values. Have them use green and red dot stickers to plot combinations of points that meet/exceed or do not meet/exceed Kai's points-per-game average, respectively. Have all groups display their graphs. Lead a discussion by asking students to explain patterns they observed, noting the boundary line and pulling out the mathematics from the remaining questions. When students are making the graphs, allow them to choose which axis represents the number of 2-point and the number of 3-point shots; however, suggest that they scale their axes by an interval of 1.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do your variables represent? • According to your graph, which is the independent variable, and which is the dependent variable? • Does it make a difference what variable is dependent? Explain.
Probing	• What is another combination of shots in which Kai earns exactly 20 points? • How did you graph your line? Why did you choose that method? • How did you know which inequality symbol to use?

Have students work with a partner or in a group to complete Questions 5 through 11. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do points on the line represent?
Probing	• How did you calculate the number of total points scored? • Should your shading include the actual line? Explain. • What other types of numbers don't make sense in this situation?
Seeing structure	• How do you know that all these points contain a combination of shots with a total above 20 points?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

The solutions to a linear equality are points either above or below the graph of the line and may or may not include the points on the line.

ACTIVITY 4.2

Determining the Graphs of Linear Inequalities

Facilitation Notes

In this activity, the graph of a linear equality is described as a half-plane. Students graph inequalities by graphing each inequality's corresponding equation as a boundary line, determining whether it should be solid or dashed, and identifying which half-plane to shade by testing the point (0, 0) in the original inequality. Students also match inequalities to graphs and write inequalities presented as graphs.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Different methods to graph the boundary line, such as slope and y-intercept, a table of values, or transformations.
- Errors when using a table of values to graph the boundary line. If this occurs, it may be because students used the inequality, rather than the equation, to complete the table.

COMMON MISCONCEPTION

Some students may be confused by language such as shading above or below the line, especially when the line appears nearly vertical to them and *shading left or right* seems to make more sense. Explain that the inequality is written with the variable y isolated, so they are looking for (x, y) pairs, where the

y-value is greater than $4x - 6$. If students are confused, have them focus on the y-intercept of the boundary line, where *above* and *below* may have a more obvious correspondence to the terms *greater than* and *less than*.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• Show that a point in the shaded region creates a true statement when substituted in the inequality.
Probing	• Why do you think the origin is a good test point? • Can you test a different point instead of the origin?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Whether students are using a test point or the inequality symbol to determine which half-plane to shade.
- Whether students keep Question 5, part (c), written in standard form to graph or if they rewrite it in slope-intercept form.

COMMON MISCONCEPTION

Students may overgeneralize and think that when the inequality symbol of an inequality written in standard form is greater than, it always means to shade above the boundary line and when the inequality symbol is less than, it always means to shade below the boundary line. This is not always correct. This is only correct when the y-variable is isolated and written in front of the expression. Address this common misconception when discussing Question 5, part (c).

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What does the shaded portion of the graph represent?
Seeing structure	• What strategy did you use to match the inequalities and graphs? • How did you use the solid and dashed lines to make your matches? • How did you graph the boundary line? • How did you know whether to make the boundary line solid or dashed? • How did the results of the test point help you determine where to shade the graph?

Analyze the Worked Example following Question 5 as a class.

Have students work with a partner or in a group to complete Question 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What steps did you use to determine the inequality that represents the graph? • How did you determine the equation of the boundary line? • How is the dashed boundary line represented in your inequality? • How did you know whether to use the greater than or less than symbol?
Reflecting and justifying	• How can you check whether your inequality is correct?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Provide students with a linear equation and ask them to use the equation to draw a graph of each inequality situation. For example, given $y = -3x + 5$, ask students to graph $y > -3x + 5$, $y \leq -3x + 5$, $y < -3x + 5$, and $y \geq -3x + 5$.

STAMP THE LEARNING

Summary

When graphing linear inequalities, dashed lines are associated with the symbols $>$ and $<$, whereas solid lines are associated with the symbols $\geq$ and $\leq$. The shaded region above or below the graphed line includes all the points that are solutions to the inequality.

ACTIVITY 4.3 Interpreting the Graph of a Linear Inequality

Facilitation Notes

In this activity, students use a table of values to write and graph a linear inequality to model a context. They then use the inequality and/or its graph to respond to questions provided in context. Students also interpret the meaning of points on the line, above the line, or below the line.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Whether students use the table or graph to write the equation of the boundary line.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the equation represented by the table of values? • Did you write your equation in slope-intercept form or standard form? Why did you choose that form? • How did you determine which inequality symbol to use? • How did you determine which portion of the graph to shade? • Why is the solution in the first quadrant only?
Gathering	• If the combination of quantities of time expend his prepaid card, does that mean they use exactly $50?

Have students work with a partner or in a group to complete Questions 3 through 6. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Whether students use the inequality or graph to respond to the questions in context.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Did you use your inequality or graph to solve this problem? Why or why not? • Why could you use your graph to solve this problem? • How could you solve this problem algebraically?

Summary

STAMP THE LEARNING

A real-world problem can be modeled by a linear inequality, and both the inequality and its graph can be used to solve problems in context.

DEMONSTRATE

Talk the Talk

THERE'S A FINE LINE

Facilitation Notes

In this activity, students demonstrate an understanding of the difference between the graphs of linear equations and linear inequalities and compare the solution sets of linear equations and linear inequalities.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know whether to use a solid or dashed line? • How do you determine which portion of the plane is the solution to the inequality? • If a line passes through the point (0, 0), how do you decide which other point to use as the test point?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

The graph of an inequality uses the same line as the graph of the equation. The inequality symbol determines whether or not the line is included in the solution set and which half-plane contains the rest of the solution set.

4 Graphing Inequalities in Two Variables

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Write an inequality in two variables.
- Graph an inequality in two variables on a coordinate plane.
- Determine whether a solid or dashed boundary line is used to graph an inequality on a coordinate plane.
- Interpret the solutions of inequalities mathematically and in the context of real-world problems.

NEW KEY TERMS

- half-plane
- boundary line

You have graphed linear inequalities in one variable.

What does the graph of a linear inequality in two variables look like? How does it compare to the graph of a linear equation?

Sample answer:

The graph of a linear inequality is a solid or dashed line, with shading on one side of the line.

The graph of a linear equation is the graph of a solid line.

Chunking the Activity

- Group students to complete the activity.
- Share and Summarize.

Getting Started

Making a Statement

Consider each solution statement.

$$x = 2$$

$$x < 2 \qquad x \leq 2$$

$$x > 2 \qquad x \geq 2$$

1. Compare the solution statements. What does each one mean?

   Each solution statement compares the variable x to the constant 2. The value 2 is the only solution to $x = 2$. Values greater than 2 are included in the solution sets of $x > 2$ and $x \geq 2$, and values less than 2 are included in the solution sets of $x < 2$ and $x \leq 2$. The value 2 is also included in the solution set of $x \geq 2$ and $x \leq 2$.

2. Choose a solution statement and write a scenario to represent it. Then, modify the scenario so the resulting interpretation is one of the other four solution statements.

   Answers will vary.

ACTIVITY 4.1 Linear Inequalities in Two Variables

1. Coach Garcia is analyzing the scoring patterns of players on his basketball team. Kai is averaging 20 points per game from scoring on two-point and three-point shots.

   a. If she scores 6 two-point shots and 2 three-point shots, will Kai meet her points-per-game average?

   $12 + 6 = 18$. No, Kai will not meet her points-per-game average.

   b. If she scores 7 two-point shots and 2 three-point shots, will Kai meet her points-per-game average?

   $14 + 6 = 20$. Yes, Kai will meet her points-per-game average.

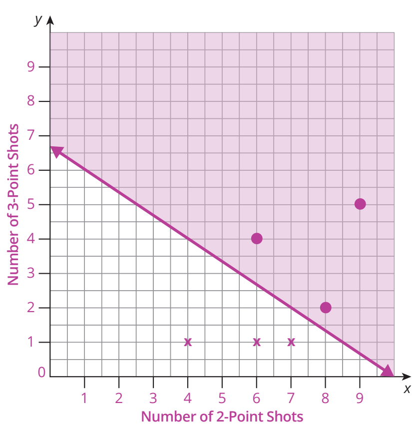
   c. If she scores 7 two-point shots and 4 three-point shots, will Kai meet her points-per-game average?

   $14 + 12 = 26$. Kai will exceed her points-per-game average.

2. Write an equation to represent the number of two-point shots and the number of three-point shots that total 20 points.

   Let x represent the number of two-point shots, and let y represent the number of three-point shots.
   $2x + 3y = 20$

3. Graph the equation you wrote on the coordinate plane.



Chunking the Activity

- Group students to complete Questions 1–4.
- Check in and share.
- Group students to complete Questions 5–11.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Ask Yourself . . .
How should you label the graph?

EB STUDENT TIP

For all proficiency levels

Ask students to share experiences of any sports activities they have been involved in. Prior to Question 4, discuss the idea of sports teams and ask if they are familiar with the term *district playoffs*. Explain how a *district* is *a particular area, or unit, such as in a school system*. Also discuss how sports teams will often compete in *playoffs*, which are *additional games played to determine the teams that will compete in a championship game*.

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MODULE 3 • TOPIC 2 • LESSON 4 451

4. Coach Garcia believes that the team can win the district playoffs if Kai scores at least 20 points per game.

   a. How can you rewrite the equation you wrote in Question 2 to represent that Kai must score at least 20 points per game?
   The equation must be rewritten to show that the 2-point baskets and 3-point baskets Kai scores must be equal to or greater than Kai's points-per-game average.

Remember. . .
An inequality is a statement formed by placing an inequality symbol ($>$, $\geq$, $<$, $\leq$,) between two expressions.

   b. Write an inequality in two variables that represents this problem situation.
   $2x + 3y \geq 20$

5. Complete the table of values. Then, add the ordered pairs in the table to the graph in Question 3. If the number of total points scored does not meet or exceed Kai's points-per-game average, use an "x" to plot the point. If the number of total points scored meets or exceeds Kai's points-per-game average, use a dot to plot the point.

Number of Two-Point Shots Scored	Number of Three-Point Shots Scored	Number of Total-Points Scored
4	1	11
6	1	15
7	1	17
8	2	22
6	4	24
9	5	33

6. What do you notice about your graph?
   All the points that exceed Kai's average are above the line, and the points that are do not exceed Kai's average are below the line.

7. What can you interpret about the solutions of the inequality from the graph?
   Sample answer:
   The line is the boundary of points that are and are not solutions to the inequality.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Determine whether students are familiar with the word *exceed*. If not, create a list of synonyms for *exceed*, such as *surpass*, *outdo*, *to go past a limit*, and *is greater than*. Review the basketball team problem scenario and ask students to explain how *exceeds* is used in the context of the scenario in reference to scoring points.

8. Choose a different ordered pair located above the line and a different ordered pair that is located below the line. How do these points confirm your interpretation of the situation? Explain your reasoning.

   Answers will vary.

9. Shade the side of the graph that contains the combinations of shots that are greater than or equal to Kai's points-per-game average.

   See graph in Question 3.

10. How do the solutions of the linear equation $2x + 3y = 20$ differ from the solutions of the linear inequality $2x + 3y \geq 20$?

    The solutions of the linear equation are the points on the line. The solutions of the linear inequality include half of the coordinate plane.

11. Does the ordered pair (6.5, 5.5) make sense as a solution in the context of this problem situation? Explain why or why not.

    No. Kai cannot score any partial baskets so there can only be whole numbers of two-point and three-point shots.

Like linear equations, linear inequalities take different forms. Each of the linear inequalities in two variables shown represents a different relationship between the variables.

$$ax + by < c \qquad ax + by > c$$
$$ax + by \leq c \qquad ax + by \geq c$$

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MODULE 3 • TOPIC 2 • LESSON 4 453

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the definitions.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Questions 4 and 5.
- Check in and share.
- Read and discuss the Worked Example.
- Group students to complete Question 6.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 1 and 2 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining whether the boundary line should be solid or dashed, assign Skills Practice Set A for this lesson.

ACTIVITY **4.2**

Determining the Graphs of Linear Inequalities

If the inequality symbol is $\geq$ or $\leq$, the boundary line is a solid line because all points on the line are part of the solution set. If the symbol is $>$ or $<$, the boundary line is a dashed line because no point on that line is a solution.

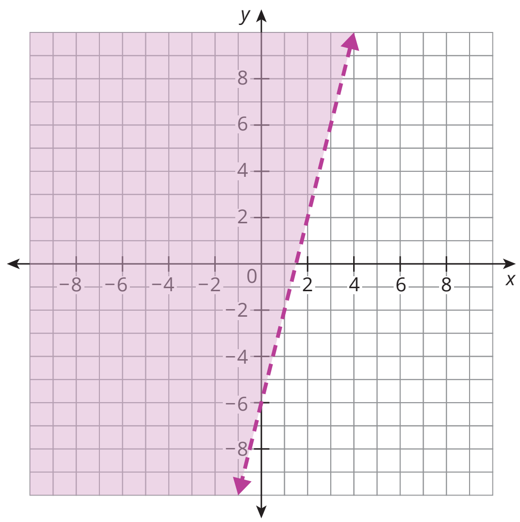
The graph of a linear inequality in two variables is a **half-plane**, or half of a coordinate plane. A **boundary line**, determined by the inequality, divides the plane into two half-planes, and the inequality symbol indicates which half-plane contains all the solutions. These solutions are represented by shading the appropriate half-plane.

Consider the linear inequality $y > 4x - 6$. The boundary line that divides the plane is determined by the equation $y = 4x - 6$.

1. Should the boundary line in this graph be a solid line or a dashed line? Explain your reasoning.

   Because the inequality symbol is $>$, the line is not included in the graph. Therefore, it should be represented by a dashed line.

2. Graph the boundary line on the coordinate plane shown.



EB STUDENT TIP

For all proficiency levels

The term *boundary line* is used throughout the lesson. Discuss the non-mathematical use of the term *boundary line*, such as a line used to separate states and countries, or *boundary lines* on a sports field or court. Discuss the difference between the non-mathematical examples and the application of a *boundary line* when graphing the solution of a linear inequality.

After you graph the inequality with either a solid or a dashed boundary line, you need to decide which half-plane to shade. To make your decision, consider the point (0, 0). If (0, 0) is a solution, then the half-plane that contains (0, 0) contains the solutions and should be shaded. If (0, 0) is *not* a solution, then the half-plane that does not contain (0, 0) contains the solutions and should be shaded.

3. Decide which half-plane to shade.

   a. Is (0, 0) a solution? Explain your reasoning.

      $0 > 4(0) - 6 \qquad 0 > -6$

      The ordered pair (0, 0) is a solution of the inequality.

   b. Shade the correct half-plane on the coordinate plane.

      See answer to Question 2.

Think About . . .

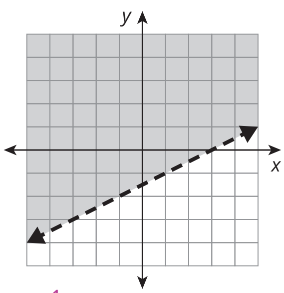
It's a good idea to check points in both half-planes to verify your solution.

4. Match each graph to one of the inequalities given. In part (d), graph the inequality that was not graphed in parts (a) through (c).

$$y \geq \frac{1}{2}x - 3 \qquad y \leq \frac{1}{2}x - 3$$

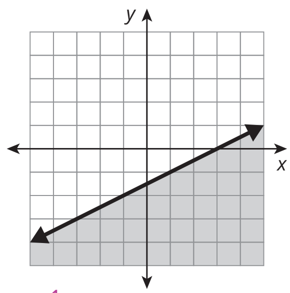
$$y > \frac{1}{2}x - 3 \qquad y < \frac{1}{2}x - 3$$

a.



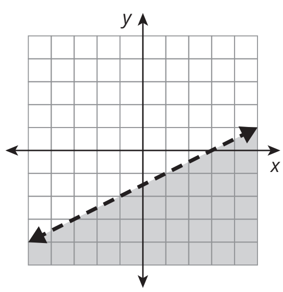
$y > \frac{1}{2}x - 3$

b.



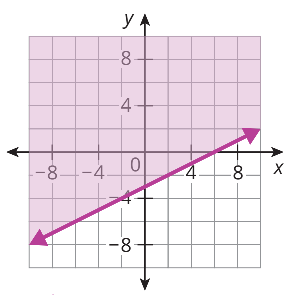
$y \leq \frac{1}{2}x - 3$

c.



$y < \frac{1}{2}x - 3$

d.



$y \geq \frac{1}{2}x - 3$

Questions 3 and 4 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice testing points, assign Skills Practice Set B for this lesson.

MODULE 3 • TOPIC 2 • LESSON 4 455

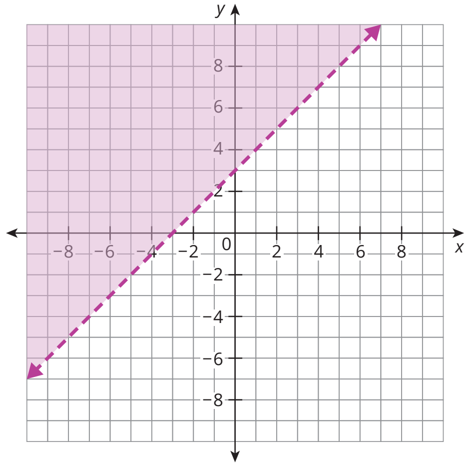
Question 5 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing linear inequalities, assign Skills Practice Set C for this lesson.

Think About . . .

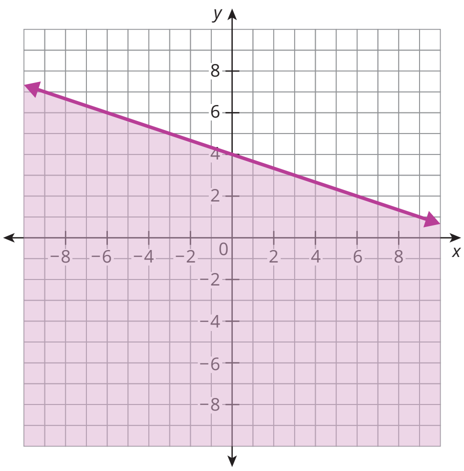
Consider the inequality symbol and which half-plane will be shaded before you test any points.

5. Graph each linear inequality.

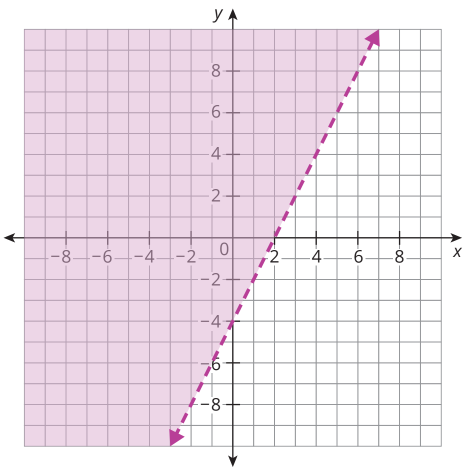
a. $y > x + 3$



b. $y \leq -\frac{1}{3}x + 4$



c. $2x - y < 4$



456 MODULE 3 • TOPIC 2 • LESSON 4

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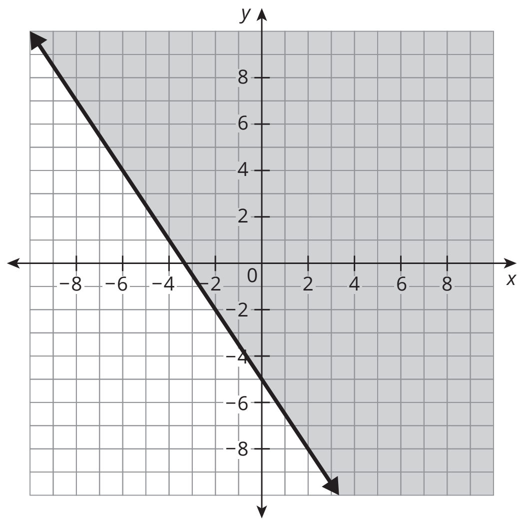
Previously, you have written a linear equation given various representations, including two points, one point and the slope, a table of values, or a graph. You can use a similar approach when writing a linear inequality.

WORKED EXAMPLE

Write a linear equality for the graph.



You can use what you have previously learned about the graphs of linear equations to determine that the boundary line is represented by the equation $y = -\frac{3}{2}x - 5$. Now, you must decide which inequality symbol should replace the equals sign in the equation.

Since the graph shows a solid boundary line and the half-plane above the line is shaded, use the symbol $\geq$.

$y \geq -\frac{3}{2}x - 5$

Test a point in the solution set to check the linear inequality.

Test the point (0, 0):

$0 \overset{?}{\geq} -\frac{3}{2}(0) - 5$

$0 \geq -5$ ✓

Remember. . .
The point (0, 0) can be used as a test point, unless the boundary line passes through (0, 0).

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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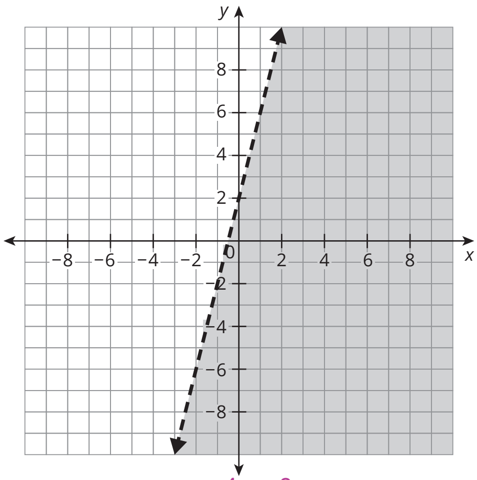
MODULE 3 • TOPIC 2 • LESSON 4 457

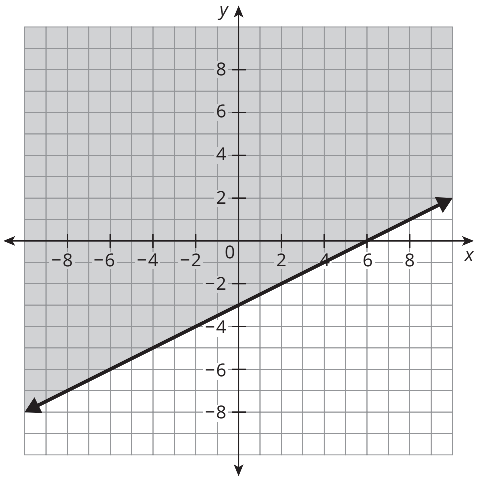
6. Write a linear inequality for each graph.

a.



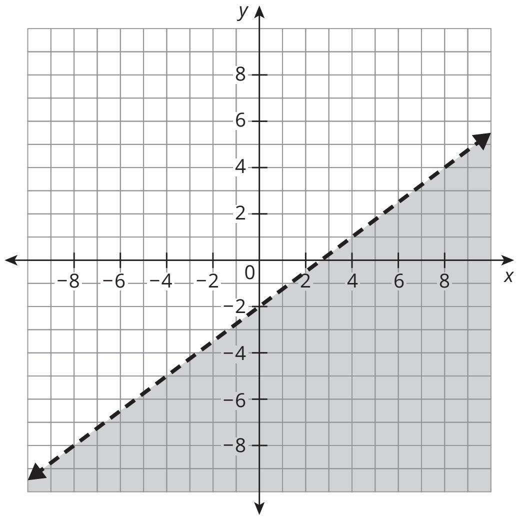
$y < 4x + 2$

b.



$y \geq \frac{1}{2}x - 3$

c.



$y < \frac{3}{4}x - 2$

458 MODULE 3 • TOPIC 2 • LESSON 4

ACTIVITY 4.3 Interpreting the Graph of a Linear Inequality

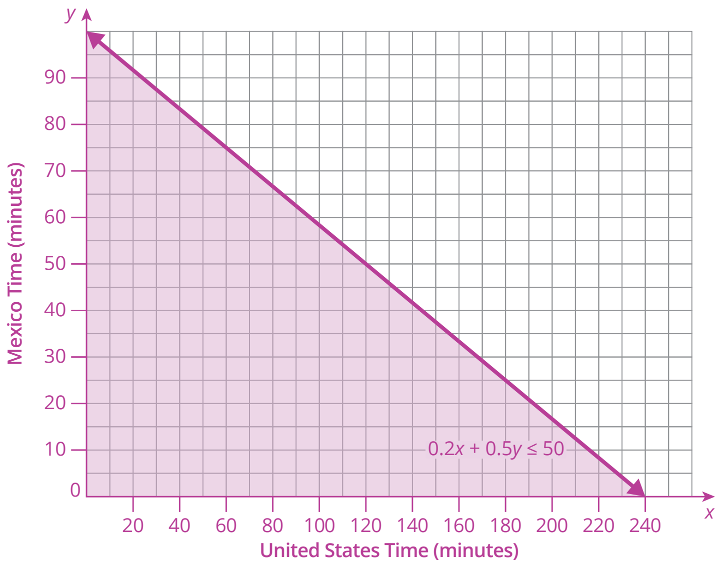
César has relatives living in both the United States and Mexico. He is given a prepaid phone card worth $50 for his birthday. The table of values shows combinations of minutes for calls within the United States, *x*, and calls to Mexico, *y*, that expend his $50 prepaid phone card.

Length of Calls Within United States (minutes)	Length of Calls to Mexico (minutes)
0	100
50	80
140	44
200	20
240	4

1. Write an inequality modeling the number of minutes César can use for calls within the United States and for calls to Mexico.

   $0.2x + 0.5y \le 50$

2. Graph your inequality on the given coordinate grid. Be sure to label your axes.



Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Group students to complete Questions 3–6.
- Share and summarize.

CC BY NC

MODULE 3 • TOPIC 2 • LESSON 4 459

This activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing and graphing linear inequalities for problem situations, assign Skills Practice Set D for this lesson.

Ask Yourself . . .
How can you use linear inequalities in everyday life?

3. If César speaks with his aunt in Guadalajara, Mexico, for 70 minutes using his phone card, how long can he speak with his cousin in New York using the same card?

   César can speak with his cousin in New York for 75 or fewer minutes using his prepaid phone card.

4. Can César call his uncle in San Antonio for 100 minutes and also call his grandmother in Juárez, Mexico, for 80 minutes using his phone card? Explain your reasoning.

   No. The point (100, 80) is not in the solution set of the linear inequality.

5. Can César call his brother in Mexico City, Mexico, for 55 minutes and also call his sister in Denver for 90 minutes using his phone card? Explain your reasoning.

   Yes. The point (55, 90) is in the solution set of the linear inequality.

6. Interpret the meaning of each.

   a. Points on the line

      The points on the line represent the maximum number of minutes César can spend on a call to Mexico given the number of minutes spent on a call within the United States using his prepaid phone card.

   b. Points above the line

      The points above the line represent times in minutes that are impossible for César to spend on calls either to Mexico or within the United States using his prepaid phone card.

   c. Points below the line

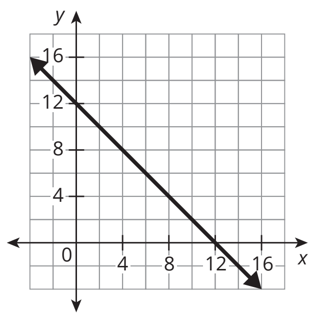
      The points below the line represent all the combinations of minutes César can spend on calls to Mexico and within the United States using his prepaid phone card.

460 MODULE 3 • TOPIC 2 • LESSON 4

Talk the Talk

There's a Fine Line

Consider the graph of the linear equation $x + y = 12$.



Use the graph to answer each question.

1. Describe how to graph $x + y < 12$ and choose a point to test this region.

   I would change the graph to a dashed line, and I would shade the half-plane below the line.
   I can use the point (0, 0) to test this region: $0 + 0 < 12$

2. Describe how to graph $x + y \leq 12$ and choose a point to test this region.

   I would keep the graphed line, and I would shade the half-plane below the line.
   I can use the point (0, 0) to test this region: $0 + 0 \leq 12$

3. Describe how to graph $x + y > 12$ and choose a point to test this region.

   I would change the graph to a dashed line, and I would shade the half-plane above the line.
   I can use the point (10, 10) to test this region: $10 + 10 > 12$

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

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MODULE 3 • TOPIC 2 • LESSON 4 461

4. Complete the table.

Equation or Inequality	Description of the Solution Set
$x + y = 0$	All points that lie on the graphed line $y = -x$
$x + y \geq 0$	All points that lie on the graphed line $y = -x$ and all points contained in the half-plane above the graphed line
$x + y \leq 0$	All points that lie on the graphed line $y = -x$ and all points contained in the half-plane below the graphed line
$x + y > 0$	All points contained in the half-plane above the graphed line $y = -x$
$x + y < 0$	All points contained in the half-plane below the graphed line $y = -x$

462 MODULE 3 • TOPIC 2 • LESSON 4

Lesson 4 Assignment

Write

Describe a half-plane in your own words.

Remember

The graph of a linear inequality in two variables is the half-plane that contains all the solutions. If the inequality symbol is $\leq$ or $\geq$, the graph shows a solid boundary line because the line is part of the solution set. If the symbol is $<$ or $>$, the boundary line is a dashed line because no point on the line is a solution.

Practice

1. Elijah is working two jobs to save money for his college education. He makes \$8 per hour working for his uncle at a pizzeria bussing tables and \$10 per hour tutoring peers after school in math. His goal is to make \$160 per week.

   a. If Elijah works 8 hours at the pizzeria and tutors 11 hours during the week, does he reach his goal?

   Yes.

   b. Write an expression to represent the total amount of money Elijah makes in a week from working both jobs. Let x represent the number of hours he works at the pizzeria and y represent the number of hours he tutors.

   $8x + 10y$

   c. After researching the costs of colleges, Elijah decides he needs to make more than \$160 each week. Write an inequality with two variables to represent the amount of money Elijah needs to make.

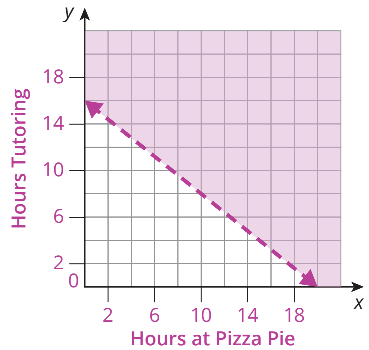
   $8x + 10y > 160$

Write

Sample answer: A half-plane is the half of the coordinate plane that contains the solutions to an inequality. The coordinate plane is separated by a line created from the inequality, and the half of the plane that contains the solutions to the inequality is shaded.

Lesson 4 Assignment

d. Graph the inequality from part (c).



e. Is the point (0, 0) in the shaded region of the graph? Explain why or why not.

No. The point (0, 0) is not in the shaded region because it is not a solution to the inequality.

f. According to the graph, if Elijah works 5 hours at the pizzeria and tutors for 10 hours, will he make more than \$160? Explain why or why not.

No. The point (5, 10) on the graph is not in the shaded solution region.

g. Due to days off from school, Elijah will only be tutoring for 6 hours this week. Use the graph to determine the least amount of full hours he must work at the pizzeria to still reach his goal. Then, show that your result satisfies the inequality.

Elijah must work at least 13 hours at the pizzeria to make more than 160 dollars for the week. $8(13) + 10(6) > 160$; $164 > 160$

464 MODULE 3 • TOPIC 2 • LESSON 4 ASSIGNMENT

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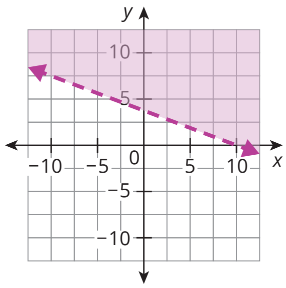
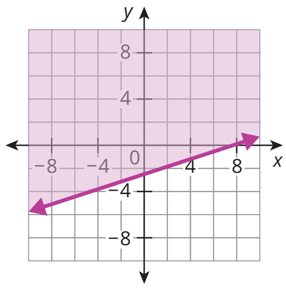
Lesson 4 Assignment

2. Graph each inequality on a coordinate plane.

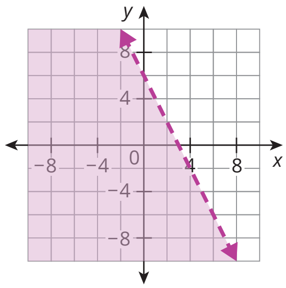
a. $x + 3y > 9$



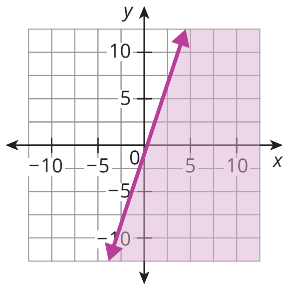
b. $2x - 6y \leq 15$



c. $2x + y < 6$



d. $3x - y \geq 1$



Prepare

Determine an ordered pair (x, y) that satisfies each inequality.

Sample answers shown.

1. $x + y < 18$

   (0, 0)

2. $x - y > -7$

   (0, 0)

3. $2x + 3y \leq -5$

   (−1, −2)

4. $-5x - 2y \geq 10$

   (0, −5)

MODULE 3 • TOPIC 2 • LESSON 4 ASSIGNMENT 465

5 Systems of Linear Inequalities

MATERIALS

None

LESSON OVERVIEW

Students represent a scenario with a system of linear inequalities and graph the system. Overlapping shaded regions identify the possible solutions to the system. Students then practice graphing several systems of inequalities and representing the solution set. A different scenario is given that students model with a system of linear inequalities. They then graph the system, determine two different solutions, and algebraically prove that the solutions satisfy both constraints defined by the system. Finally, students match systems, graphs, and possible solutions of systems that have identical terms with different inequality symbols.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1A apply mathematics to problems arising in everyday life, society, and the workplace.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.
The student is expected to:

A.2H write linear inequalities in two variables given a table of values, a graph, and a verbal description.

(TEKS continued on next page)

ELPS

(2) Listening
The student is expected to:

(H) understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations.

(3) Speaking
The student is expected to:

(F) ask and give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments.

(4) Reading
The student is expected to:

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

ALGEBRA I TEKS *(TEKS continued from previous page)*

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3D graph the solution set of linear inequalities in two variables on the coordinate plane.

A.3H graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

ESSENTIAL IDEAS

- In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are constrained to lie within a certain region.
- The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersecting region satisfies all inequalities in the system.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: A River Runs Through It 10–15 minutes

ESTABLISH A SITUATION

Students are presented with a scenario and write two inequalities to represent each of the defined relationships before writing a system of linear inequalities to represent the entire problem situation.

DEVELOP

Activity 5.1: Determining Solutions to Systems of Linear Inequalities 30 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

Students graph the inequalities they wrote in the Getting Started. The definition of a *solution of a system of linear inequalities* as the intersection of the solutions to each inequality is provided, and students apply that definition to determine the solution graphically. They discuss the number of possible solutions as well as interpret coordinate pairs that are and are not solutions in terms of the context.

DAY 2

Activity 5.2: Analyzing Graphs of Systems of Linear Inequalities 20–25 minutes

PEER WORK ANALYSIS, MATHEMATICAL PROBLEM SOLVING

Students graph a system of linear inequalities. They then use algebraic methods to determine when a point is a solution to the system. Students explain why the point of intersection of two linear inequalities is not always included in the solution to a system of inequalities. Finally, they solve two systems of linear inequalities that include parallel lines.

Activity 5.3: Applying Systems of Linear Inequalities 15–20 minutes

REAL-WORLD PROBLEM SOLVING

Students solve a problem in context requiring a system of linear inequalities. They interpret solutions in terms of the problem situation and use algebra to demonstrate that the solutions satisfy both constraints. Because of the decimal coefficients in this problem, students are encouraged to use graphing technology.

DAY 3

Activity 5.4: Identifying Systems of Linear Inequalities 20–25 minutes

MATHEMATICAL PROBLEM SOLVING

Students match each of four systems of linear inequalities to their graph and a solution expressed as a coordinate pair. The systems differ only by their combinations of inequality symbols.

DEMONSTRATE

Talk the Talk: Get to Know the Region 15–20 minutes

GENERALIZATION

Students determine the region of the graph of the solution set for different systems of linear inequalities. Next, they compare the solution of a system of linear inequalities to the solution of a system of linear equations.

Getting Started

ENGAGE

A River Runs Through It

Facilitation Notes

In this activity, students are presented with a scenario and write two inequalities to represent each of the defined relationships before writing a system of linear inequalities to represent the entire problem situation.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Whether or not Noah was counted as one of the adults on the raft. Noah will be on the raft, so his weight must be considered, but he will not be paying for the experience.
- Different inequalities that students may possibly write involving money, where *c* represents the number of children and *a* represents the number of adults.

$$50c + 75(a - 1 \geq 150$$
$$50c + 75a - 75 \geq 150$$
$$50c + 75a \geq 225$$

Discuss their equivalence, what each term represents, and what inequality makes most sense for the problem situation.

DIFFERENTIATION STRATEGIES

Just in Time Support

Suggest students label each inequality so they can refer to the units in the problem to help connect coefficients to the correct variables in the appropriate inequality.

$$\text{Weight (pounds): } 100c + 200a \leq 800$$
$$\text{Money Collected (dollars): } 50c + 75(a - 1) \geq 150$$

Challenge Opportunity

Have students modify their system of equations if *a* represents the number of adults not including Noah.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Is Noah on the raft with his customers? • Does Noah's weight need to be included as part of the 800 pounds?
Probing	• Explain how your inequality relates to the context. • How did you adjust the term representing the amount of money collected for the adults not to include Noah?

STAMP THE LEARNING

Summary

Real-world problems can be modeled using a system of linear inequalities. The same variables must be used and defined the same way for all inequalities in the system.

ACTIVITY **5.1**

Determining Solutions to Systems of Linear Inequalities

DEVELOP

Facilitation Notes

In this activity, students graph the inequalities they wrote in the Getting Started activity. The definition of a *solution of a system of linear inequalities* as the intersection of the solutions to each inequality is provided, and students apply that definition to determine the solution graphically. They discuss the number of possible solutions and interpret coordinate pairs that are and are not solutions in terms of the context.

Ask a student to read the definition aloud. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

COMMON MISCONCEPTION

Students may think that satisfying only one goal, either the weight goal or the financial goal, qualifies it as part of the solution to the system.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are the two constraints for this situation?
Probing	• How can you use your inequalities to respond to these questions? • Which method did you use to graph each line? • How did you determine which part of the plane to shade for each inequality? • What does the region where the shading overlaps represent? • Which portion of the overlapping region is a solution to the problem situation? • Identify a solution to the system of linear inequalities that is also a solution to the problem situation.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

The solution to a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

ACTIVITY
5.2
Analyzing Graphs of Systems of Linear Inequalities

Facilitation Notes

In this activity, students graph a system of linear inequalities. They first determine the solution graphically and are then guided through a process that includes algebraic methods. Students explain why the point of intersection of two linear inequalities is not always included in the solution to a system of inequalities. Finally, they solve two systems of linear inequalities that include parallel lines.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Different methods to graph the boundary line.

COMMON MISCONCEPTIONS

- Students may graph both lines and then test the (0, 0) point one time, rather than testing (0, 0) and shading after each line is graphed. Address why testing (0, 0) and shading must occur each time a new line is graphed and the coordinate plane is separated into two different half-planes.
- Because the point of intersection is the solution to a system of linear equations, students may assume it is always included as part of the solution to a system of linear inequalities. Question 3 addresses this common misconception.

DIFFERENTIATION STRATEGIES

Access for All

Materials Needed: Colored Pencils

- Insist students extend all lines the entire length and width of the coordinate plane.
- Have students use a different color pencil to graph each inequality, with each boundary line and its corresponding half-plane having the same color.
- Make the two methods of determining the solution to a system of inequalities explicit: 1) shade after each line is graphed, and the solution to the system is the intersection of the solutions to each inequality, and 2) graph all lines with or without shading, then test a point from each portion of the graph in the algebraic representation of each inequality in the system. If the point value creates true statements for each inequality, then that portion of the graph is a solution to the system. The entire solution to the system is the sum of all portions of the graph that are solutions.

Just in Time Support

- Provide a Worked Example with numbered steps for reference.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How many regions do you create when graphing two intersecting lines? • How can you tell from the graph whether the points in each region satisfy both constraints, one constraint, or no constraints in the situation?
Probing	• How will the results from your substitution demonstrate whether a point is a solution to both equations?
Seeing structure	• Is the intersection point to this system of inequalities included in the solution? Why or why not? • How can you use the fact that the lines are solid or dashed to identify whether to include the intersection point in the solution? • Is an intersection point formed by two solid lines always part of the solution to the inequalities? Part of the solution to the problem situation?

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

COMMON MISCONCEPTION

Because parallel lines imply no solution for a system of linear equations, students may assume that parallel boundary lines always imply there is no solution for this type of system of linear inequalities. Question 4, part (b) disproves this misconception. To provide an example using non-vertical lines, make both inequality symbols the same for Question 4, part (a).

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• For a system of inequalities to have no solution, do the lines have to be parallel? Explain. • If a system of inequalities includes parallel lines, is there always no solution? Explain your thinking.

STAMP THE LEARNING

Summary

A system of inequalities can be solved graphically or by using a combination of graphing and algebraic methods. For a system of two linear equations, the point of intersection is the solution; however, for a system of linear inequalities, the point of intersection of two boundary lines may or may not be included in the solution. For a system of two linear equations, when the lines are parallel, there is no solution; however, for a system of linear inequalities, when the lines are parallel, the system may or may not have solutions.

ACTIVITY 5.3

Applying Systems of Linear Inequalities

Facilitation Notes

In this activity, students solve a problem in context requiring a system of linear inequalities. They interpret solutions in terms of the problem situation and use algebra to demonstrate the solutions satisfy both constraints. Because of the decimal coefficients in this problem, students are encouraged to use graphing technology.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Confusion about what coefficients to use for the equation representing time.
- Errors sketching the graph of the equation for calories by hand due to the decimal values.
- Errors rewriting the equation in slope-intercept form to enter it in a graphing calculator.
- Difficulty transferring the graph created by technology to the coordinate plane on paper.

COMMON MISCONCEPTION

Because information about the context is provided in table form, students may think that the dependent variable is calories burned per minute because it is the title of the second column. They may also assume the independent quantity is time because it often used as a label for the *x*-axis.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What constraints did Emma set for herself?
Probing	• Explain how your inequalities address Emma's constraints. • How did you determine what coefficients to use for the variables? • How did you determine which variable is dependent? Did anyone think differently? Explain.
Seeing structure	• Is the point of intersection part of the solution? Why or why not? • How did you select solutions from the graph?

DIFFERENTIATION STRATEGIES

Access for All

Materials Needed: Graphing Technology

- Suggest students use technology to graph the boundary lines prior to creating the graphs of inequalities on paper.

- Use technology to determine the point of intersection. Because the point of intersection must be visible on the graph, this is helpful to know prior to scaling the axes.
- Use the table feature to plot points on the graph. This may be more accurate than trying to use the slope and *y*-intercept that are decimal values on a coordinate plane scaled with large intervals.

Challenge Opportunity

- Have students solve similar problems using any two exercise entries from the first column of the table.

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Technology is a useful tool when solving problems with non-integer values.

ACTIVITY 5.4 Identifying Systems of Linear Inequalities

Facilitation Notes

To begin the Day 3 session, have a student read the Essential Question aloud.

In this activity, students match each of four systems of linear inequalities to their graph and a solution expressed as a coordinate pair. The systems differ only by their combinations of inequality symbols.

Ask a student to read the introduction. Discuss Question 1 as a class.

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

Suggest students number the top inequality in each system as 1 and the bottom inequality as 2. That way, they can label the boundary lines with their corresponding inequality for reference purposes.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What strategy did you use to match the systems and their graphs efficiently? • How does the fact that one line is increasing and the other is decreasing help you? • Is it easier to match the solutions to the systems or the graphs? Why?

Summary

The solution sets of systems of linear inequalities that differ by inequality symbols also differ graphically by intersecting regions.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

GET TO KNOW THE REGION

Facilitation Notes

In this activity, students determine which region of the graph represents the solution set for each given system of linear inequalities. They then compare the solution of a system of linear inequalities to the solution of a system of linear equations.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Incorrect interpretations of the inequality symbol when *y* is on the right of the inequality symbol.
- Rewriting the inequality so that *y* is on the left of the inequality symbol.
- Sketches to determine the solutions to each inequality.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Did you sketch the graphs to identify the solution region? • Were you able to identify the solution region from the inequalities? If so, explain your strategy. • If you identified the region to shade based on the inequality symbol, did the fact that *y* was to the left or right of the inequality symbol impact your thinking? Explain.
Seeing structure	• How many solutions can each type of system have? • Is the point of intersection included in the solution set for both systems? Explain.

Have students read and answer the Essential Question on the lesson opener page.

Summary

The inequality symbols in a system of linear inequalities determine the location of the intersecting region.

STAMP THE LEARNING

5 Systems of Linear Inequalities

OBJECTIVES

- Represent constraints in a problem situation with systems of inequalities.
- Write and graph systems of linear inequalities.
- Graph the solutions to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
- Verify solutions to systems of linear inequalities algebraically.

NEW KEY TERMS

- constraints
- solution of a system of linear inequalities

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

You have graphed a linear inequality in two variables and interpreted the solutions.

What does the graph of a system of linear inequalities look like, and how can you describe the solution set?

Sample answer:

The graph of a system of linear inequalities is a plane that shows two lines, one or both of which could be dashed lines, and shading in one or more regions defined by the intersection of the two lines.

MODULE 3 • TOPIC 2 • LESSON 5 467

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

This activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing systems of linear inequalites, assign Skills Practice Set A for this lesson.

Getting Started

A River Runs Through It

Noah is an experienced whitewater rafter who guides groups of adults and children out on the water for amazing adventures. The raft he uses can hold 800 pounds of weight. Any weight greater than 800 pounds can cause the raft to sink, hit more rocks, and/or maneuver more slowly.

Noah estimates the weight of each adult as approximately 200 pounds and the weight of each child as approximately 100 pounds. Noah charges adults \$75 and children \$50 to ride down the river with him. His goal is to earn at least \$150 each rafting trip.

Think About . . .
Does Chase count himself when determining the weight and the cost?

1. Write an inequality to represent the most weight Noah can carry in terms of rafters. Define your variables.

   Let a represent the number of adult rafters, and let c represent the number of children under age sixteen.

   $200a + 100c \leq 800$

2. Write an inequality to represent the minimum amount of money Noah wants to collect for each rafting trip.

   $75(a - 1) + 50c \geq 150$

3. Write a system of linear inequalities to represent the maximum weight of the raft and the minimum amount of money Noah wants to earn per trip.

   $200a + 100c \leq 800$

   $75(a - 1) + 50c \geq 150$

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Students may not be familiar with the terms *whitewater rafting* and *maneuver.* Discuss what is meant by *whitewater* and what a *raft* is. Provide a list of synonyms for *maneuver*, such as *move*, *steer*, *navigate*, and *guide*. Relate the scenario of the trip to creating linear equations that represent the number of children and adults and their estimated weights for the rafts.

ACTIVITY 5.1

Determining Solutions to Systems of Linear Inequalities

In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region on the graph.

1. Let's consider two trips that Noah guides. Determine whether each combination of rafters is a solution of the system of linear inequalities. Then, describe the meaning of the solution in terms of this problem situation.

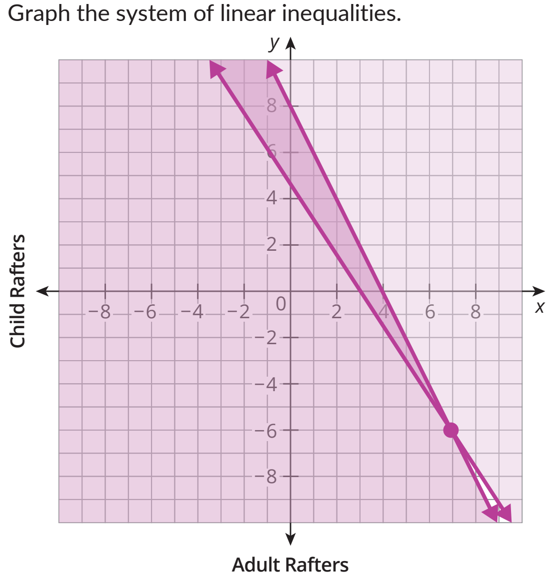
   a. First Trip: Noah guides 2 adults and 2 children.

   Yes. This is a solution of this system of linear inequalities because the number of adults and children results in a true statement for both inequalities. If 2 adults and 2 children go with Noah, they are at the weight requirements of the raft and he will earn at least $150.

   b. Second Trip: Noah guides 5 adults.

   No. This is not a solution to the system of equations because it does not satisfy the inequality representing the weight requirement.

2. Graph the system of linear inequalities.



Shade the half-plane of each inequality differently. You can use colored pencils or simply vertical and horizontal lines.

Chunking the Activity

- Read and discuss the definition.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 3 • TOPIC 2 • LESSON 5 469

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

The **solution of a system of linear inequalities** is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

3. Analyze your graph.

   a. Describe the possible number of solutions for a system of linear inequalities.

   A system of linear inequalities can have many solutions as long as the half-planes overlap.

   b. Is the intersection point a solution to this system of inequalities? Why or why not?

   While the intersection point is a solution to this system of inequalities, it does not make sense in the problem situation. The intersection point (7, −6) would mean that there were 7 adults and −6 children, but there cannot be a negative number of children.

   c. Identify three different solutions of the system of linear inequalities you graphed. What do the solutions represent in terms of the problem situation?

   Answers will vary.

   d. Determine one combination of adults and children that is not a solution for this system of linear inequalities. Explain your reasoning.

   Answers will vary.

   The point (2, 1) does not represent a solution. Although Noah, 1 other adult, and 1 child are within the weight limit for the raft, the money earned is less than $150.

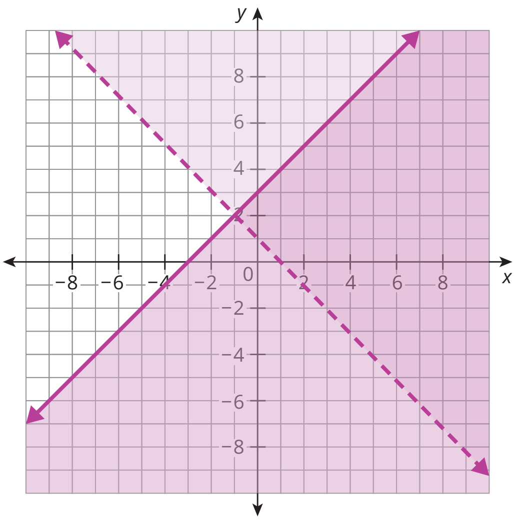
470 MODULE 3 • TOPIC 2 • LESSON 5

ACTIVITY 5.2 Analyzing Graphs of Systems of Linear Inequalities

Determine the solution set of the given system of linear inequalities.

$$\begin{cases} x + y > 1 \\ -x + y \leq 3 \end{cases}$$

1. Graph the system of linear inequalities.



Think About . . .
Notice the inequality symbols. How does this affect your graph?

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the directions.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Question 4.
- Share and summarize.

MODULE 3 • TOPIC 2 • LESSON 5 471

Question 2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining whether a given point is a solution of a system of linear inequalites, assign Skills Practice Set B for this lesson.

2. Choose a point in each shaded region of the graph. Determine whether each point is a solution of the system. Then, describe how the shaded region represents the solution.

Answers will vary.

Point	$x + y > 1$	$-x + y \leq 3$	Description of location
$(-8, 2)$	$-8 + 2 > 1$ $-6 > 1$ ✗	$-(-8) + 2 \leq 3$ $10 \leq 3$ ✗	The point is not a solution to either inequality and it is located in the region that is not shaded by either inequality.
$(2, 8)$	$2 + 8 > 1$ $10 > 1$ ✓	$-2 + 8 \leq 3$ $6 \leq 3$ ✗	The point is a solution of the first inequality, but not the second. It is located in the region shaded by the first inequality.
$(8, 2)$	$8 + 2 > 1$ $10 > 1$ ✓	$-8 + 2 \leq 3$ $-6 \leq 3$ ✓	The point is a solution for both inequalities and it is located in the region shaded by both inequalities.
$(-2, -8)$	$-2 + (-8) > 1$ $-10 > 1$ ✗	$-(-2) + (-8) \leq 3$ $-6 \leq 3$ ✓	The point is a solution of the second inequality but not the first. It is located in the region shaded by the second inequality.

3. Ava makes the statement about the intersection point of a system of inequalities. Explain why Ava's statement is incorrect.

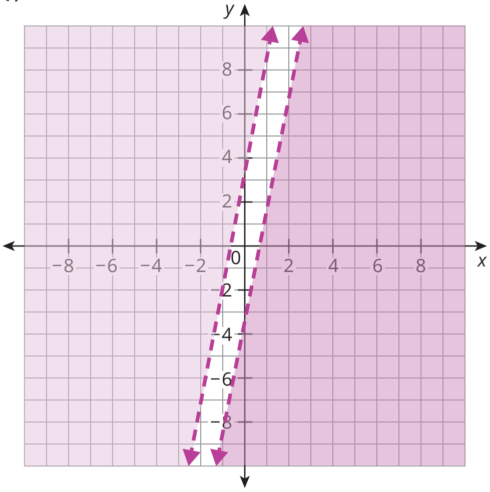
> Ava
>
> The intersection point is always a solution to a system of inequalities because that is where the two lines meet.

Ava is incorrect because the intersection point is not always a solution to the system of linear inequalities. The intersection point for this system, $(-1, 2)$, only works for one of the inequalities, not both, which means it is not a solution. If the inequality symbols are not both "or equal to," then the intersection point is not a solution.

472 MODULE 3 • TOPIC 2 • LESSON 5

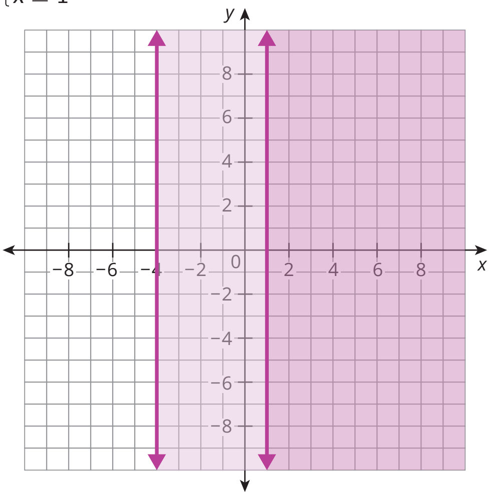
4. Solve each system of linear inequalities by graphing the solution set. Then, identify two points that are solutions of the system.

a. $\begin{cases} y > 5x + 3 \\ y < 5x - 3 \end{cases}$



The solutions of each inequality in the system do not intersect. Therefore, there are no solutions.

b. $\begin{cases} x \geq -5 \\ x \geq 1 \end{cases}$



Ask Yourself . . .
Did you share your solution(s) with others?

Answers will vary.

Two possible solutions are (2, 0) and (5, 5).

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MODULE 3 • TOPIC 2 • LESSON 5 473

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 5.3

Applying Systems of Linear Inequalities

Emma and a group of friends decide to use the fitness room after school. A sign on the wall provides the information shown.

Exercise	Calories Burned per Minute
Treadmill—light effort	7.6
Treadmill—vigorous effort	12.4
Stair Stepper—light effort	6.9
Stair Stepper—vigorous effort	10.4
Stationary Bike—light effort	5.5
Stationary Bike—vigorous effort	11.1

Emma decides to use the stair stepper. She has, at most, 45 minutes to exercise and he wants to burn at least 400 calories.

1. Write a system of linear inequalities to represent Emma's workout. Define your variables.

   Let x represent the number of minutes spent on the stair stepper with light effort, and let y represent the number of minutes spent on the stair stepper with vigorous effort.

   $$\begin{cases} x + y \leq 45 \\ 6.9y + 10.4x \geq 400 \end{cases}$$

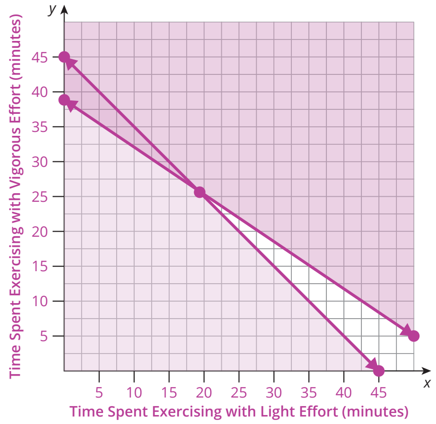
EB STUDENT TIP

For all proficiency levels

Some non-mathematical terms that appear in this activity are *fitness room*, *treadmill*, *stair stepper*, and *stationary bike*. Discuss these terms so that students may engage more fully in the activity.

474 MODULE 3 • TOPIC 2 • LESSON 5

2. Graph the system of inequalities from Question 1 on the coordinate plane. Be sure to label your axes.



Use technology to graph your inequalities and check your answer.

3. Analyze your graph.

a. Identify two different solutions of the system of inequalities.

Two possible solutions are (0, 45) and (5, 40).

b. Interpret your solutions in terms of Emma's workout.

The solution (0, 45) means that Emma can exercise with light effort for 0 minutes and exercise with vigorous effort for 45 minutes and burn at least 400 calories in 45 minutes at most.

The solution (5, 40) means that Emma can exercise with light effort for 5 minutes and exercise with vigorous effort for 40 minutes and burn at least 400 calories in 45 minutes at most.

c. Algebraically prove that your solutions satisfy the system of linear inequalities.

$0 + 45 \leq 45$

$45 = 45$

$6.9(0) + 10.4(45) \geq 400$

$468 \geq 400$

The solution (0, 45) satisfies the system.

$5 + 40 \leq 45$

$45 = 45$

$6.9(5) + 10.4(40) \geq 400$

$450.4 \geq 400$

The solution (5, 40) satisfies the system.

MODULE 3 • TOPIC 2 • LESSON 5 475

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the introduction.
- Complete Question 1 as a class.
- Group students to complete Question 2.
- Share and summarize.

Question 2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice graphing systems of linear inequalites, assign Skills Practice Set C for this lesson.

ACTIVITY 5.4 Identifying Systems of Linear Inequalities

Consider the four systems shown.

System A	System B	System C	System D
$\begin{cases} y < \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y > \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y > \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$	$\begin{cases} y < \frac{3}{5}x + 3 \\ y < -\frac{3}{5}x + 3 \end{cases}$

1. Analyze Isabella's statement and explain why it is incorrect.

> **Isabella**
> Since the equations in each system are the same, the graphs and solutions should all be identical.

Sample answer:
While it is true that the equations in each system are the same, this only defines the boundary lines. The inequalities are not the same. Each system uses a different combination of inequality symbols. While the graphs of the lines are the same, the different inequality symbols means that the intersecting regions will be different, so the solutions will not be identical.

2. Match a graph and possible solution to each given system of linear inequalities. Complete the blank graph and partial solution set to make four complete sets.

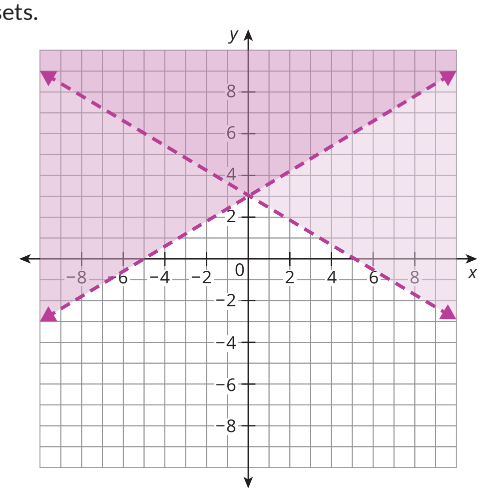
System A: Graph C; answers will vary for Possible Solutions D.

Sample answers: (6, 2) and (8, 0)

System B: See Graph D and Possible Solutions B.

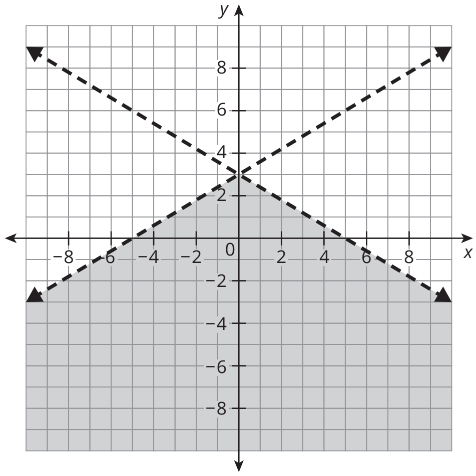
System C: See Graph B and Possible Solutions C.

System D: See Graph A and Possible Solutions A.

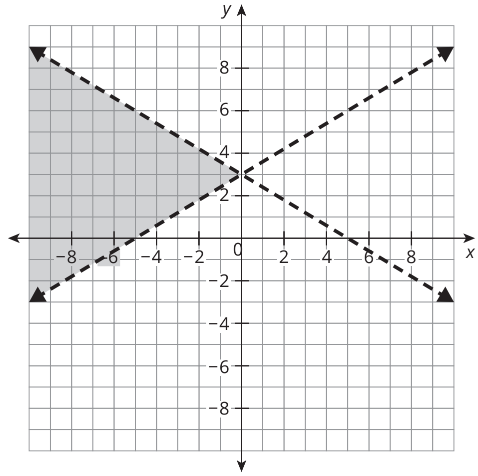


476 MODULE 3 • TOPIC 2 • LESSON 5

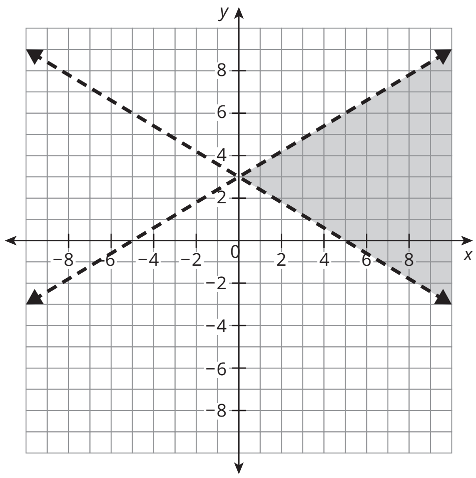
Graph A



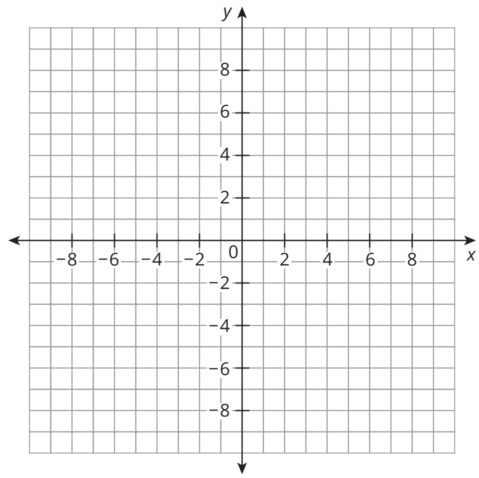
Graph B



Graph C



Graph D



Possible Solutions A	Possible Solutions B	Possible Solutions C	Possible Solutions D
(0, −6) and (4, −4)	(0, 7.5) and (−4, 10)	(−6, 4) and (−10, 8)	_____and_____

MODULE 3 • TOPIC 2 • LESSON 5 477

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

Get to Know the Region

The solution set to a system of inequalities can be any of four regions on the coordinate plane.

1. Consider each system of linear inequalities and decide which region represents the solution set. Explain your reasoning.
   - A region above both lines.
   - A region below both lines.
   - A region between the lines.
   - No solution.

   a. $y < 8 + 2x$
   $3 + 2x > y$
   A region below both lines

   b. $5 + x < y$
   $y > 7 + x$
   A region above both lines

   c. $y > 12 - 3x$
   $10 - 3x > y$
   No solution

   d. $6 - x < y$
   $y < 9 - x$
   A region between the lines

2. How is the solution to a system of linear inequalities the same as or different from the solution to a system of linear equations?

   Sample answers:
   The solution to a system of linear inequalities and a system of linear equations will satisfy both inequalities or both equations. There may be no solutions to a system of linear inequalities or to a system of linear equations. When two equations in a system of linear equations intersect, there is one unique solution. When the graphs of two inequalities in a system of linear inequalities intersect, there are multiple solutions.

478 MODULE 3 • TOPIC 2 • LESSON 5

Lesson 5 Assignment

Write

Describe how you know which region, if any, represents the solution to a system of linear inequalities.

Remember

The solution of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

Practice

1. Samuel is remodeling his basement. One part of the planning involves the flooring. He knows that he would like both carpet and hardwood, but he isn't sure how much of each he will use. The most amount of flooring area he can cover is 2000 square feet. The carpet is \$4.50 per square foot, and the hardwood is \$8.25 per square foot. Both prices include labor costs. Samuel has budgeted \$10,000 for the flooring.

   a. Write a system of inequalities to represent the maximum amount of flooring needed and the maximum amount of money Samuel wants to spend.

   $$\begin{cases} x + y \leq 2000 \\ 4.50x + 8.25y \leq 10{,}000 \end{cases}$$

   b. One idea Samuel has is to make two rooms—one having 400 square feet of carpeting and the other having 1200 square feet of hardwood. Determine whether this amount of carpeting and hardwood are solutions to the system of inequalities. Explain your reasoning in terms of the problem situation.

   No. This is not a solution to this system of inequalities because this amount of carpet and hardwood only results in a true statement for one of the inequalities. This means that although it will not exceed the total amount of square footage available to finish in the basement, the amount of each puts the total cost for flooring over the maximum budget of \$10,000.

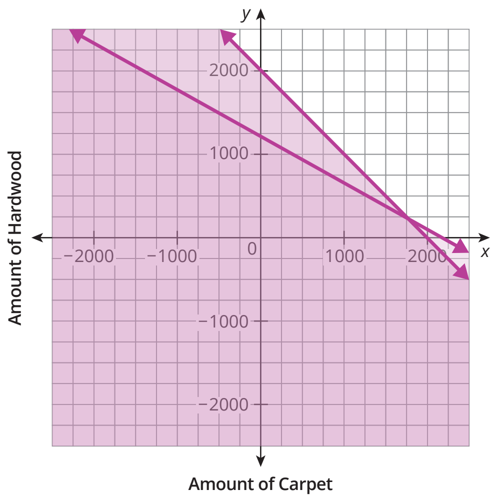
Write

Sample answer: To solve a system of linear inequalities, I graph each inequality individually. I then identify which region represents a solution to both inequalities—the region that has been shaded twice. If there is no region with overlapping shading, then there are no solutions to the system.

MODULE 3 • TOPIC 2 • LESSON 5 ASSIGNMENT 479

Lesson 5 Assignment

c. Graph this system of inequalities.



d. Determine the intersection point of the two lines. Is this a solution to this system of inequalities in terms of the problem situation?

The intersection point of this system of inequalities is $(1733\frac{1}{3}, 266\frac{2}{3})$. Although it does make sense that there can be $1733\frac{1}{3}$ square feet of carpet and $266\frac{2}{3}$ square feet of hardwood, Samuel will most likely have to buy the flooring in whole number values of square feet.

e. Identify two different solutions to the system of inequalities. Explain what the solutions represent in terms of the problem situation.

Sample answer:

(1500, 250); this solution means that Samuel can put down 1500 square feet of carpet and 250 square feet of hardwood and not have too much flooring while not going over the budget. (0, 1000); this solution means that Samuel can put down no carpeting and 1000 square feet of hardwood and not have too much flooring while not going over the budget.

480 MODULE 3 • TOPIC 2 • LESSON 5 ASSIGNMENT

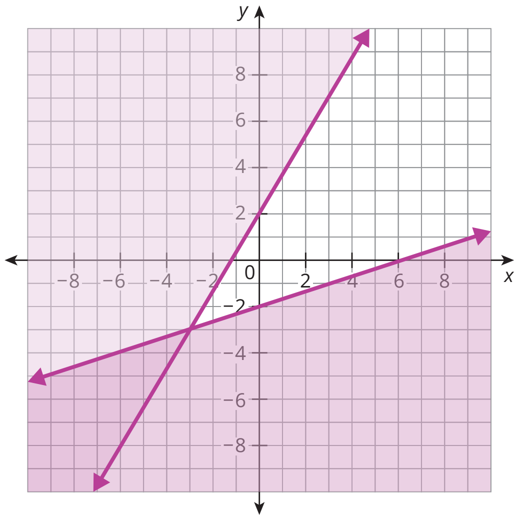
Lesson 5 Assignment

f. Determine one combination of amounts of carpet and hardwood that is not a solution for the system of inequalities. Explain your reasoning.

Sample answer:
The point (500, 1000) does not represent a solution. Although Samuel would not have too much flooring, he would go over budget if he bought 500 square feet of carpet and 1000 square feet of hardwood.

2. Solve each system of linear inequalities.

a. $\begin{cases} -x + 3y \leq -6 \\ -5x + 3y \geq 6 \end{cases}$



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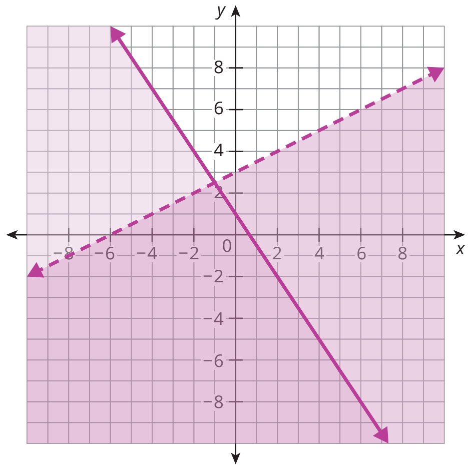
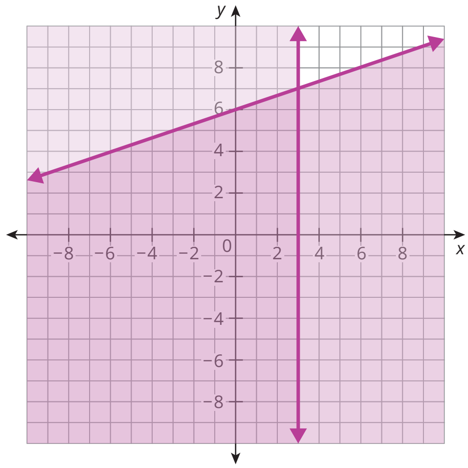
MODULE 3 • TOPIC 2 • LESSON 5 ASSIGNMENT 481

Lesson 5 Assignment

b. $\begin{cases} -x + 2y < 6 \\ 3x + 2y \leq 2 \end{cases}$



c. $\begin{cases} -x + 3y \leq 18 \\ x \leq 3 \end{cases}$



482 MODULE 3 • TOPIC 2 • LESSON 5 ASSIGNMENT

Lesson 5 Assignment

Prepare

Determine whether the point (1, 7) is a solution to each system.

1. $\begin{cases} 4x - y = -3 \\ -2x + y = 5 \end{cases}$

   The point is a solution to the system.

2. $\begin{cases} x + y > 4 \\ 4x - y < -4 \end{cases}$

   The point is not a solution to the system.

3. $\begin{cases} y = -3.5x - 2 \\ y = 4.5x - 10 \end{cases}$

   The point is not a solution to the system.

4. $\begin{cases} -2x + y < 8 \\ x - y > -8 \end{cases}$

   The point is a solution to the system.

MODULE 3 • TOPIC 2 • LESSON 5 ASSIGNMENT 483

6 Solving Systems of Equations and Inequalities

MATERIALS

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students solve problems in context requiring a system of linear equations. While most problems can be modeled by a system of two equations, they are guided through the process of solving a system of four equations, and another context can be modeled by a system of three equations. Students have the opportunity to solve the systems using any method and sometimes must respond in the format of an email or proposal. Solutions involve making a decision based upon inputs that lie before or after the point of intersection, thus requiring solutions written as inequalities.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

A.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

A.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

A.1E create and use representations to organize, record, and communicate mathematical ideas.

(TEKS continued on next page)

ELPS

(2) Listening
The student is expected to:

(D) monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed.

(3) Speaking
The student is expected to:

(B) expand and internalize initial English vocabulary by learning and using high-frequency English words necessary for identifying and describing people, places, and objects, by retelling simple stories and basic information represented or supported by pictures, and by learning and using routine language needed for classroom communication.

(5) Writing
The student is expected to:

(E) employ increasingly complex grammatical structures in content area writing commensurate with grade-level expectations such as (i) using correct verbs, tenses, and pronouns/antecedents; (ii) using possessive case (apostrophe -s) correctly; and, (iii) using negatives and contractions correctly.

ALGEBRA I TEKS *(TEKS continued from previous page)*

Linear Functions, Equations, and Inequalities

(2) The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.

The student is expected to:

A.2H write linear inequalities in two variables given a table of values, a graph, and a verbal description.

A.2I write systems of two linear equations given a table of values, a graph, and a verbal description.

(3) The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

The student is expected to:

A.3D graph the solution set of linear inequalities in two variables on the coordinate plane.

A.3H graph the solution set of systems of two linear inequalities in two variables on the coordinate plane.

(5) The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions.

The student is expected to:

A.5C solve systems of two linear equations with two variables for mathematical and real-world problems.

ESSENTIAL IDEAS

- Contexts about choosing between two options can sometimes be modeled by a system of linear equations or inequalities.
- The point of intersection of two lines separates the input values, with x-values less than and x-values greater than the x-value of the point of intersection. The solution to a problem in context may be dependent upon where the input values lie relative to the point of intersection.
- Based upon a context, the solution of a system may be represented by inequalities rather than a single coordinate pair.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Systems of Summer Savings 15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students write a system of linear equations and a system of linear inequalities to determine the solution to a real-world problem.

DEVELOP

Activity 6.1: Determining the Better Deal 15 minutes

REAL-WORLD PROBLEM SOLVING

Students write a system of linear equations, with each equation in the form $y = mx + b$, to model a scenario. After determining the point of intersection of the system, students must interpret the meaning of the system and its point of intersection to determine the better deal. Students write a proposal with the better deal, including evidence from their analysis.

Activity 6.2: Determining the Better Buy 15 minutes

REAL-WORLD PROBLEM SOLVING

Students write a system of linear equations to model a scenario. The best buy must be interpreted using inequalities based upon the input values.

DAY 2

Activity 6.3: Solving a System of Linear Inequalities with Four Constraints 20–25 minutes

PEER WORK ANALYSIS, REAL-WORLD PROBLEM SOLVING

Students explore a context that can be modeled using a system of linear equations with four constraints. They write two of the constraints in terms of the sale price given the percent reduction and represent the other two constraints as vertical lines. Students graph the system and use the graph and algebra to answer questions in context.

Activity 6.4: Determining the Better Job Offer 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students explore a context that can be modeled with several different systems of linear equations. Possible systems include equations based upon years, equations based upon months, as well as a system of three equations. Once again, input values must be considered to determine the better job offer.

DEMONSTRATE

Talk the Talk: Which Cab Is More Fab? 10–15 minutes

EXIT TICKET APPLICATION

Students write a system of linear equations, with each equation in the form $y = mx + b$, to model a scenario. They make a decision as to what is the best option, with their response expressed as inequalities based upon the input values.

Getting Started

ENGAGE

Systems of Summer Savings

Facilitation Notes

In this activity, students write a system of linear equations and a system of linear inequalities to determine the solution to a real-world problem.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Different interpretations for the system of inequalities:
  1. Using x to represent the number of times Harper uses the pool, $x > 25$ and $2x + 100 \leq 200$, with solution $25 < x \leq 50$
  2. Using a to represent the number of times Harper uses the pool in July and b as the number of times Harper uses the pool in August, $a + b > 25$ and $a + b \leq 50$, with solution $25 < a + b \leq 50$
- Confusion with variable definitions for the equation and inequality

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are the constraints in this situation? • How can you write your solution as a compound inequality?
Probing	• What information did you gain by solving the system of equations? • How did you determine the maximum number of times Harper can go to the pool?
Reflecting and justifying	• Show how your solution satisfies this situation's constraints.

STAMP THE LEARNING

Summary

A scenario can be represented by a system of linear equations and a system of linear inequalities. The solution to each system can then be interpreted in terms of the problem situation.

ACTIVITY 6.1

Determining the Better Deal

DEVELOP

Facilitation Notes

In this activity, students write a system of linear equations, with each equation in the form $y = mx + b$, to model a scenario. After determining the point of intersection of the system, students must interpret the meaning of the system and its point of intersection to determine the better deal. Students write a proposal with the better deal, including evidence from their analysis.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

Provide a template (with space for defining variables, writing the system of equations, solving the system, and interpreting the solution) to help students organize their work in this activity and in Activities 2 and 4. Provide as little scaffolding as possible so that students can experience the modeling process.

AS STUDENTS WORK, LOOK FOR

- Various methods to solve the system of equations.
- Use of technology to graph the system and determine the point of intersection.
- Failure to extend the solution beyond the point of intersection.
- Questions about what makes a better deal because that is not made explicit.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why did you use a system of equations to solve this problem? • Explain how each equation relates to the situation. • Which method did you use to solve the system of equations? • What does the solution to the system of equations represent? • How did you interpret the solution to offer a recommendation? • How can you represent your proposal using inequalities?

Summary

STAMP THE LEARNING

In a real-world context involving determining the better deal between two relationships modeled by a system of linear equations, the intersection point of the lines and the possible input values are helpful in making decisions.

ACTIVITY 6.2

Determining the Better Buy

Facilitation Notes

In this activity, students write a system of linear equations to model a scenario. The best buy must be interpreted using inequalities based upon the input values.

Ask a student to read the introduction aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Discuss the context prior to having students write the equations. Provide examples with different numbers of MB of data and have students solve them using arithmetic to make sense of the context.

Challenge Opportunity

- Have students graph the system of equations and interpret the graphs in terms of the context.

AS STUDENTS WORK, LOOK FOR

- Difficulty expressing additional MB of data in an algebraic expression.
- Use of technology to graph the system and determine the point of intersection.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the significance of the point of intersection with respect to the problem situation? • What does the solution to this system of equations tell you about the problem situation? • What will the graph of the system of equations look like? • What does your recommendation depend upon?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

In a real-world context involving determining the better buy between two relationships modeled by a system of linear equations, the intersection point of the lines and the possible input values are helpful in making decisions.

ACTIVITY 6.3 Solving a System of Linear Inequalities with Four Constraints

Facilitation Notes

In this activity, students explore a context that can be modeled using a system of linear inequalities with four constraints. They write two of the constraints in terms of the sale price given the percent reduction and represent the other two constraints as vertical lines. Students graph the system and use the graph and algebra to answer questions in context.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Expressions for the percent of the discount rather than the sale price.
- Two different correct equations for the same context, such as $s = 0.40r$ and $s = r - 0.60r$.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why does your inequality include two variables? • How did you determine the coefficients for r to reflect the reduced price? • Explain why each expression represents a sale price. • What do you think the graph of this system looks like?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

Materials Needed: Poster Paper

- To support all students, have groups of students graph the system on large poster paper grids for comparison and reference during the class discussion.

Just in Time Support

Materials Needed: Colored Pencils

- To scaffold support, have students use the same color pencil to graph each boundary line and shade the half-plane that is the solution to its corresponding inequality.

COMMON MISCONCEPTION

Students may shade a region which is does not represent the solution to the system of inequalities for Question 4. Guide students through the process of selecting a test point in the shaded region and substituting it into each inequality to verify it satisfies each inequality.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine what variable should be the dependent variable? • Is the graph what you predicted it would look like? Explain. • What do the two vertical lines represent in the situation? • How did you determine what region of the graph to shade?
Seeing structure	• Select a point in the shaded region and interpret it in the context of the problem.

Have students work with a partner or in a group to complete Questions 6 through 8. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Express your response as a compact inequality. • Do you prefer a graph or equation to solve these problems? Why or why not?
Probing	• How did you use the graph to determine the range of sale prices? • How did you know which equation to use to solve this problem?
Seeing structure	• How are questions 6 and 7 alike and different?

STAMP THE LEARNING

Summary

A real-world context can be represented by a system of linear inequalities that has more than two inequalities. The solution is all ordered pairs that satisfy all inequalities in the system.

ACTIVITY 6.4 Determining the Better Job Offer

Facilitation Notes

In this activity, students explore a more complex context that can be modeled with several different systems of linear equations. Possible systems include equations based upon years, equations based upon months, as well as a system of three equations. Once again, input values must be considered to determine the better job offer.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Confusion whether x represents number of weeks or dollars in sales per week.
- Equations in terms of earnings per year or earnings per week.
- In equations in terms of earnings per year, the variable x representing sales per week or sales per year.
- An error in equations in terms of earnings per year where x represents sales per week, where the commission is expressed for a week, $(0.09)x$, instead of for a year, $52(0.09)x$.
- Different methods of including $2000 in the solution process: either solving the system of two equations and using $2000 in the interpretation phase, or including $2000 as an equation in the system of equations and graphs.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What do the variables in your system of equations represent? • How did you address the fact that you measure a salary in years but sales in weeks? • Which method did you use to solve this system? • How did you use the point of intersection and Juan's sales prediction to make a recommendation?

Summary

STAMP THE LEARNING

A real-world context can be interpreted by different systems of equations, depending upon the units used. The results of the different systems should all lead to the same solution.

DEMONSTRATE

Talk the Talk

WHICH CAB IS MORE FAB?

Facilitation Notes

In this activity, students write a system of linear equations, with each equation in the form $y = mx + b$, to model a scenario. They make a decision as to what is the best option, with their response expressed as inequalities based upon the input values.

Have students read and answer the Essential Question on the lesson opener page.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why did you use a system of equations to solve the problem? • Explain how each equation relates to the situation. • Which method did you use to solve the system of equations? • What does the solution to the system of equations represent? • Use an inequality to describe how Emma should make her choice.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

In a real-world context involving determining the better deal between two relationships modeled by a system of linear equations, both the intersection point of the lines and the possible input values are sometimes necessary in making decisions.

6 Solving Systems of Equations and Inequalities

Setting the Stage

- Communicate the objectives to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Use various methods of solving systems of linear equations to determine the better buy or the better job offer.
- Solve systems of linear inequalities with more than two inequalities.
- Graph the solutions to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

You have solved systems of linear equations by graphing, by using substitution, and by using linear combinations. You have also graphed systems of linear inequalities to determine possible solutions.

How can you use these various methods to reason about real-world problems?

Sample answer:

I can write inequalities to represent a real-world situation and then solve the real-world problem using whichever method best fits the problem and the forms of the inequalities that I wrote.

MODULE 3 • TOPIC 2 • LESSON 6 485

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1E:

- Do students create representations to present and organize their mathematical reasoning?

Getting Started

Systems for Summer Savings

A neighborhood pool club offers two membership plans. Plan A includes a seasonal sign-up fee of $100 and charges $2 each time you use the pool. Plan B has no sign-up fee but charges $6 each time you use the pool. Harper chooses Plan A. She has a budget of $200 to spend on pool fees during the months of July and August.

Harper wants to be sure she uses the pool enough times so that the plan she chooses works out to be the better deal between the two plans, but she does not want to go over her budget.

1. Use a system of linear equations and a system of linear inequalities to make a recommendation to Harper as to how often she should use the pool in July and August.

   The system of linear equations is

   $$\begin{cases} y = 2x + 100 \\ y = 6x. \end{cases}$$

   The point of intersection for this system is (25, 150), so the two plans cost the same for 25 uses of the pool. This means that Harper will need to use the pool more than 25 times to get the better deal between plans.

   The system of linear inequalities is

   $$\begin{cases} a + b > 25 \\ a + b \leq 50, \end{cases}$$

   where a is the number of times she uses the pool in July and b is the number of times she uses the pool in August.

   Any combination of numbers of trips to the pool in July and August should add to more than 25 and no more than 50 to get the best deal and stay under budget.

486 MODULE 3 • TOPIC 2 • LESSON 6

ACTIVITY 6.1 Determining the Better Deal

PROBLEM SOLVING

A bicycle company is planning to make a low price ultra-light bicycle. There are two different plans being considered for building this bicycle. The first plan includes a cost of $125,000 to design and build a prototype bicycle. The combined materials and labor costs for each bike made under the first plan will be $225. The second plan includes a cost of $100,000 to design and build the prototype. The combined materials and labor costs for each bike made under the second plan will be $275.

1. You recently got a job at the bicycle company as a financial analyst. Analyze the costs for each proposed bicycle prototype and determine which plan they should follow. Provide evidence for your proposal.

   Students' methods will vary.

   The system of linear equations that represents the problem situation is

   $$\begin{cases} y = 125,000 + 225x \\ y = 100,000 + 275x. \end{cases}$$

   Using any method, students should determine that $x = 500$. The better plan depends on the number of bicycles made. If the company makes more than 500 bicycles, they should use Plan A. If the company makes fewer than 500 bicycles, they should use Plan B.

Ask Yourself . . .
Can others follow and understand your process and reasoning?

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Students may be unfamiliar with the terms *prototype* and *labor costs*. Explain the meaning of these terms so the scenario makes sense to students.

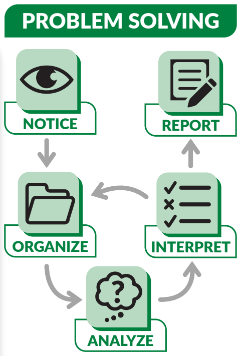
MODULE 3 • TOPIC 2 • LESSON 6 487

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.



ACTIVITY 6.2 Determining the Better Buy

Diego is in search of a new cell phone plan. He is considering two different cell phone services from two different providers.

One plan is for a phone with GPS capabilities that has a monthly fee of \$99.99 and 2000 MB of data per month. Once a user exceeds the free monthly data allowance, each additional MB of data used is \$0.05. The other plan is for a phone without GPS capabilities for a monthly fee of \$79.99 and 1500 MB of data per month. Once a user exceeds the free monthly data allowance, each additional MB of data used is \$0.08. Diego is unsure which plan to choose. He wasn't very careful with his last contract and paid a lot of extra money in charges for data.

1. Write an email to advise Diego which plan to choose. Provide evidence in your response.

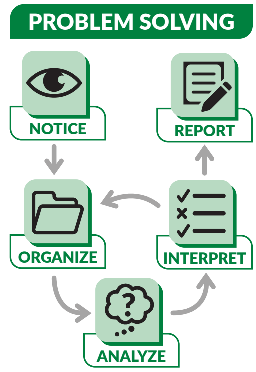
   Students' methods will vary.
   The system of linear equations that represents the problem situation is

   $$\begin{cases} y = 99.00 + 0.05(t - 2000) \\ y = 79.99 + 0.08(t - 1500). \end{cases}$$

   Using any method, students should determine that $t = 1333.33$. The better plan depends on the amount of data that Diego uses each month. If Diego uses more than 1333.33 MB of data per month, he should choose the GPS capable cell service. If he uses less than 1333.33 MB of data per month, he should choose the plan that does not have the GPS capable phone plan.

488 MODULE 3 • TOPIC 2 • LESSON 6

ACTIVITY 6.3 Solving a System of Linear Inequalities with Four Constraints



Daniel's eye doctor informed him that he needs glasses. Luckily, the local vision store is having a sale on all eyeglass frames. The advertisement in the window is as shown.

Previously, you solved a system containing two linear inequalities. However, systems can consist of more than two linear inequalities.

Save 60% to 75% on All Frames
Regularly Priced at
\$120–\$360

1. Use the advertisement to write two inequalities that represent the regular price of eyeglass frames. Let *r* represent the regular price of the frames.

   $r \geq 120$ and $r \leq 360$

2. Use the same advertisement to write two inequalities that represent the reduced price of the eyeglass frames. Let *s* represent the sales price of the frames in terms of *r*.

   $s \geq 0.25r$ and $s \leq 0.4r$

3. Sofia wrote this system of linear inequalities for the problem situation. Explain why it is incorrect.

   Sofia

   $$\begin{cases} r \geq 120 \\ r \leq 360 \\ s \leq 0.6r \\ s \geq 0.75r \end{cases}$$

   Sofia correctly wrote the inequalities for the regular price, but she incorrectly wrote the inequalities for the percent off the regular price. Her inequalities mistakenly indicate that the sale price of the eyeglasses is between 25% and 40% off the regular price.

Remember . . .
When an item is 20% off the regular price, you can think of that item costing 80% of the regular price.

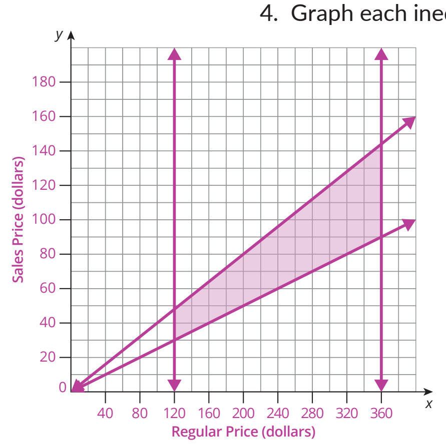
Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete the Questions 1–3.
- Check in and share.
- Group students to complete the Questions 4 and 5.
- Check in and share.
- Group students to complete the Questions 6–8.
- Share and summarize.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Questions 2 through 8 have students work individually or with a partner to complete the graphic organizer.

MODULE 3 • TOPIC 2 • LESSON 6 489



4. Graph each inequality on the grid.

5. Shade the portion of the graph that satisfies the system of linear inequalities. What shape does the solution region resemble?
   See graph. The solution region resembles a trapezoid.

6. About how much would Daniel expect to spend if he purchases eyeglasses that are regularly priced at $320?
   Daniel should expect to spend between about $80 and $130 if he purchases glasses that are regularly priced at $320.

Remember . . .
When graphing a system of linear inequalities, you must determine the portion of the graph that satisfies all the inequalities in the system.

7. Daniel is definitely going to purchase a pair of eyeglasses that are on sale. What is the least amount of money Daniel can expect to spend? What is the greatest amount he can expect to spend?
   The least amount of money Daniel can expect to spend is $30 if he purchases eyeglasses that are regularly priced at $120. The greatest amount of money Daniel can expect to spend is a little more than $140 if he purchases eyeglasses that are regularly priced at $360.

8. Daniel decides on a pair of eyeglasses that are regularly priced at $240.
   a. Can Daniel expect to save more or less than $140 off the purchase price of this pair of eyeglasses? Use your graph to determine an approximate answer.
      According to my graph, Daniel can expect to spend between $60 and $96 for eyeglasses that are regularly priced at $240. This would be a savings of between $144 and $180, which is more than a savings of $140.

The graph shows you the sale price of the eyeglasses, but how can you determine how much he will save?

   b. Use algebra to determine the greatest amount of money Daniel can save by purchasing eyeglasses that are regularly priced at $240.
      I can substitute the value of *r* as 240 in the greater discount, 75%, and solve for *s*.

$$\begin{cases} s = 0.25(240) \\ s = 60 \end{cases}$$

      The least amount Daniel can spend on the purchase of eyeglasses that regularly cost $240 is $60. So the greatest amount of money Daniel can save is $240 − $60, or $180.

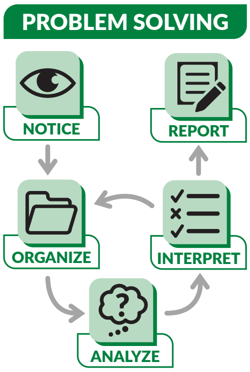
EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Discuss the definition of *resemble* to assist students in answering Question 5. If necessary, provide a word bank of the names of quadrilaterals that students may choose from.

490 MODULE 3 • TOPIC 2 • LESSON 6

ACTIVITY
6.4 Determining the Better Job Offer



Juan interviewed for two different sales positions at competing companies. Company A has offered Juan a salary of $31,200 per year, plus a 9% commission on his total sales. Company B will offer him $26,000 per year, plus a 15% commission on his total sales.

Juan isn't sure which offer to accept. He's great at making a sale, but he's just not sure which job will be better in terms of his pay. He is confident that he can make at least $2000 worth of sales each week.

1. Write an email to Juan with your recommendation of which job offers better compensation. Provide evidence in your response.

   Sample answer:

   The system of linear equations that represents the problem situation is

   $$\begin{cases} y = 31,200 + 52(0.09s) \\ y = 26,000 + 52(0.15s). \end{cases}$$

   Using any method, students should determine that $s \approx 1666.67$. The better job will depend on Juan's sales each week. If he sells more than $1666.67 each week, he should choose the job with Company B. If he sells less than $1666.67 each week, he should choose the job with Company A. Since Juan states that he is confident he can make $2000 worth of sales each week, then he should choose the job with Company B.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice solving problems using systems of linear equations and inequalites, assign Skills Practice Set A for this lesson.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Discuss the meaning of the terms *commission* and *compensation* in the context of business so that students may engage more fully in the activity.

Chunking the Activity

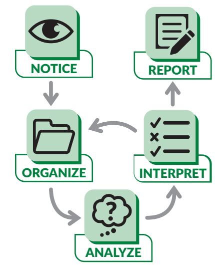
- Read and discuss the scenario.
- Group students to complete the activity.
- Have students answer the Essential Question on the lesson opener.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

Talk the Talk

PROBLEM SOLVING



Which Cab Is More Fab?

Cab Company A charges $3.50 upon entry and an additional $1.25 per mile driven. Cab Company B charges $5 upon entry and an additional $1.15 per mile driven.

1. Emma needs to take a cab to the airport. Which company should she use if she wants to minimize the cost? Use any method to solve.

   Students' methods will vary.
   The system of linear equations that represents the problem situation is

   $$\begin{cases} y = 1.25x + 3.50 \\ y = 1.15x + 5.00 \end{cases}$$

   Using any method, students should determine that $x = 15$. If the number of miles to the airport is 15, the cost will be the same. If the number of miles to the airport is less than 15, using Cab Company A is less expensive. If the number of miles to the airport is more than 15, using Cab Company B is less expensive.

492 MODULE 3 • TOPIC 2 • LESSON 6

Lesson 6 Assignment

Write

You have used three methods to solve systems: graphing, substitution, and linear combinations. Describe the characteristics you would look for when determining which method to use.

Remember

The solution set to a system of inequalities with more than two constraints can be described as the region where all the graphs overlap.

Practice

1. Jayden wants to subscribe to a service that will allow him to rent DVDs and stream movies online. Company A offers a subscription for \$14.25 a month. With this subscription, Antonio can check out as many DVDs as he wants each month and must pay \$1.40 for each movie he streams online. Company B offers a subscription for \$8.50 a month. With this subscription, Jayden can checkout as many DVDs as he wants each month and must pay \$3.25 for each movie he streams online.

   a. Write a system of linear equations to represent this problem situation.

   $$\begin{cases} y = 14.25 + 1.40x \\ y = 8.50 + 3.25x \end{cases}$$

   b. Analyze the two subscription plans and determine which one is the better deal. Use any or all of the methods you have learned to determine your answer.

   If Antonio streams fewer than 3 movies a month, then the Company B subscription is a better deal. If he streams more than 3 movies a month, then the Company A subscription is a better deal.

   c. Write a short paragraph recommending which subscription Antonio should choose.

   I would recommend that Jayden subscribe to Company B if he anticipates streaming 3 or fewer movies a month. If he plans to stream more than 3 movies a month, I would recommend that he subscribe to Company A.

Write

Sample answer:

If a system has only smaller whole numbers, I would use graphing. If a system has an equation where you could easily get a variable by itself, I would use substitution. And if a system has numbers that can be manipulated by multiplication to create coefficients that are additive inverses, I would use linear combinations.

MODULE 3 • TOPIC 2 • LESSON 6 ASSIGNMENT 493

Lesson 6 Assignment

d. Which method do you think provides the quickest way to analyze a system of equations to determine which one is the better deal? Explain your reasoning.

Answers will vary.

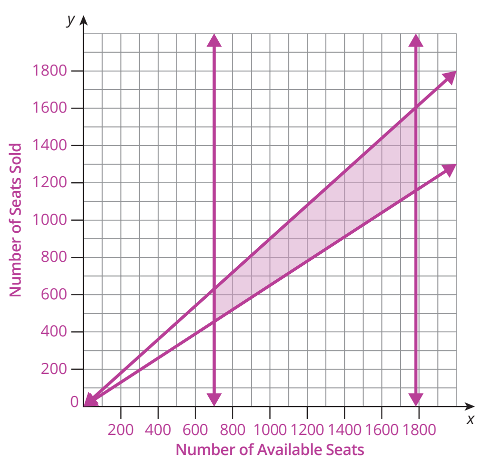
2. A ballet company needs to rent a venue for their production. There are a number of arenas they are considering. The arenas have seating capacities that range from 800 to 1776 seats. The management of the company knows the ticket sales may not be good this year, but their goal is to sell between 65% and 90% of the available seats. Whichever arena they choose, 100 seats must be set aside for the company's donors.

   a. Write a system of inequalities that represents the problem situation. Define your variables.

   Let a represent the number of available seats and s represent the number of seats sold.

$$\begin{cases} a \geq 700 \\ a \leq 1776 \\ s \leq 0.90a \\ s \geq 0.65a \end{cases}$$

   b. Graph each inequality on a coordinate plane.



494 MODULE 3 • TOPIC 2 • LESSON 6 ASSIGNMENT

Lesson 6 Assignment

c. One of the arenas they are considering has 1200 available seats. Determine the minimum and maximum number of seats they would need to sell for management to reach their goal.

The minimum number of seats to sell is 780.

The maximum number of seats to sell is 1080.

d. If the company sells 900 seats, what is the range of seating capacities for the arenas they may rent?

If they sell 900 tickets, then the minimum number of seats in the arena is 1000 and the maximum number of seats is 1384.

e. If they rent an arena that has a 1300-seat capacity and sell 800 tickets, would management reach their goal? Explain your reasoning.

No. Management would not reach their goal. The point (1300, 800) does not fall in the solution region, so they would sell at least 65% of the seats.

Prepare

Simplify each expression.

1. (−10)(−10)(−10)

   −1000

2. (−10)(−10)(−10)(−10)

   10,000

3. (−1)(2)(−3)(4)(−5)

   −120

4. (−2)(−3)(−4)(−5)

   120

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MODULE 3 • TOPIC 2 • LESSON 6 ASSIGNMENT 495

TOPIC 2 SELF-REFLECTION

Name: ______________________________

Notes

Systems of Linear Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Systems of Linear Equations and Inequalities* topic by:

TOPIC 2: *Systems of Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
understanding that the graph of an equation in two variables is the set of all its solutions plotted on the coordinate plane.	○	○	○
writing an equation, an inequality, or a system of linear equations or inequalities that best models a problem.	○	○	○
graphing systems of equations on a coordinate grid with appropriate labels and scales.	○	○	○
solving a system of two linear equations in two variables approximately (graphically) and exactly (algebraically).	○	○	○
interpreting the meaning of the solution to a system of linear equations in terms of a problem situation.	○	○	○
identifying linear systems that have no solution and explain why they have no solution.	○	○	○
identifying linear systems that have infinite solutions and explain why they have infinite solutions.	○	○	○

continued on the next page

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MODULE 3 • TOPIC 2 • SELF-REFLECTION 497

Notes

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Systems of Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
writing and graphing an inequality and a system of inequalities in two variables on a coordinate plane.	◯	◯	◯
understanding that the solution of a system of linear inequalities is the intersection of the shaded regions of both inequalities.	◯	◯	◯
interpreting the solution to an inequality in two variables graphed on a coordinate plane.	◯	◯	◯

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Systems of Linear Equations and Inequalities* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

continued on the next page

498 MODULE 3 • TOPIC 2 • SELF-REFLECTION

TOPIC 2 SELF-REFLECTION *continued*

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

Notes

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MODULE 3 • TOPIC 2 • SELF-REFLECTION 499

TOPIC 2 SUMMARY

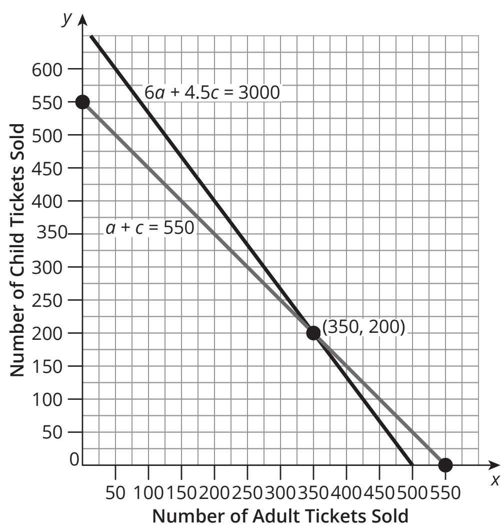
Systems of Equations and Inequalities Summary

LESSON 1 Using Graphing to Solve Systems of Equations

When two or more linear equations define a relationship between quantities, they form a **system of linear equations**. The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. One way to predict the solution to a system of equations is to graph both equations and identify the point at which the two graphs intersect.

For example, suppose a fundraiser sells 550 tickets for a spaghetti dinner to raise \$3000 for charity. An adult ticket costs \$6, and a child ticket costs \$4.50. To determine how many adult tickets and child tickets were sold, let a represent the number of adult tickets purchased, and let c represent the number of child tickets purchased.

$$\begin{cases} a + c = 550 \\ 6a + 4.5c = 3000 \end{cases}$$



NEW KEY TERMS

- system of linear equations [sistema de ecuaciones lineales]
- consistent systems [sistemas consistentes]
- inconsistent systems [sistemas inconsistentes]
- standard form of a linear equation [forma estándar/genera de una ecuación lineal]
- substitution method [método de sustitución]
- linear combinations method [método de combinaciones lineales]
- half-plane
- boundary line
- constraints
- solution of a system of linear inequalities [solución de un sistema de desigualdades lineales]

Notes

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MODULE 3 • TOPIC 2 • TOPIC SUMMARY 501

Notes

You can use x- and y-intercepts to graph each of the two equations to determine how many of each type of tickets were sold.

The intersection point appears to be (350, 200). There were 350 adult tickets and 200 child tickets sold.

A system of equations may have one unique solution, no solution, or infinite solutions. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.

LESSON 2 Using Substitution to Solve Linear Systems

The **standard form of a linear equation** can be written as $Ax + By = C$, where A, B, and C are integers, and A and B are not both zero.

Sometimes, a system of equations may not be accurately solved using graphs. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

For example, consider this system of equations.

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Step 1: To use the substitution method, begin by choosing one equation and isolating the variable. This will be considered the first equation. Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation. Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$. Write the new equation: $1.25x + 1.05(8x) = 30$.

Step 3: Solve the new equation.

$$\begin{aligned} 1.25x + 8.40x &= 30 \\ 9.65x &= 30 \\ x &\approx 3.1 \end{aligned}$$

Now, substitute the value for x into $y = 8x$ to determine the value for y.

$y \approx 8(3.1) \approx 24.8$

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

When a system has no solution, the equation resulting from the substitution step has no solution. When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

502 MODULE 3 • TOPIC 2 • TOPIC SUMMARY

LESSON 3

Using Linear Combinations to Solve a System of Linear Equations

The **linear combinations method** is a process used to solve a system of equations by adding two equations so they result in an equation with one variable. Once the value of this variable is determined, it can be used to calculate the value of the other variable. In many cases, one or both of the equations may need to be multiplied by a constant so that the coefficients of the term containing either x or y are opposites. Then, when the equations are added, the result is an equation in one variable.

For example, consider this system of linear equations.

$$\begin{cases} 7x + 2y = 24 \\ x + y = 15 \end{cases}$$

$$\begin{aligned} 7x + 2y &= 24 \\ -2(4x + y) &= -2(15) \end{aligned}$$

Multiply the bottom equation by a constant that results in opposite coefficients for one of the variables.

$$\begin{aligned} 7x + 2y &= 24 \\ +\ -8x - 2y &= -30 \\ \hline -x &= -6 \\ x &= 6 \end{aligned}$$

Now that the y-values are opposite, you can add the equations and solve this linear system.

$$\begin{aligned} 7(6) + 2y &= 24 \\ 42 + 2y &= 24 \\ -2y &= -18 \\ y &= -9 \end{aligned}$$

Substitute the value for x into one of the equations to calculate the value for y.

The solution to the system of linear equation is $(6, -9)$.

LESSON 4

Graphing Inequalities in Two Variables

The graph of a linear inequality in two variables is a **half-plane**, or half of a coordinate plane. A **boundary line**, determined by the inequality, divides the plane into two half-planes. The inequality symbol identifies which half-plane contains the solutions. When the symbol is $\leq$ or $\geq$, the boundary line is part of the solution and is solid. When the symbol is $<$ or $>$, the boundary line is not part of the solution and is represented with a dashed line. Use (0, 0) as a test point to determine which half of the plane is the solution of the inequality and should, therefore, be shaded.

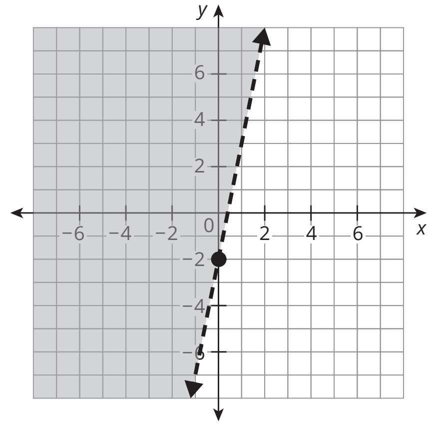
Notes

MODULE 3 • TOPIC 2 • TOPIC SUMMARY 503

Notes

For example, the graph shows the solution to the inequality $y < 5x - 2$. Since the inequality symbol in the solution is $<$, the shaded half-plane does not include points on the line. Since (0, 0) is not a solution, then the region to the right of the dashed line is the solution set.

$$y < 5x - 2$$
$$0 \overset{?}{<} 5(0) - 2$$
$$0 \overset{?}{<} -2$$



LESSON 5 Systems of Linear Inequalities

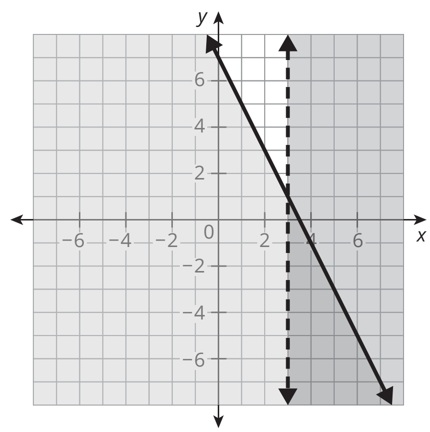
In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region.

The **solution of a system of linear inequalities** is the intersection of the solutions of each inequality. Every point in the intersection region satisfies the system. To determine the solution set of the system, graph each inequality on the same coordinate plane. The region that overlaps is the solution to the system.

For example, the overlapping region of the graphs of the inequalities $2x + y \le 7$ and $x > 3$ is the solution to the system.

$$\begin{cases} 2x + y \le 7 \\ x > 3 \end{cases}$$

Two points that are solutions of the system are (4, −5) and (5, −8).



504 MODULE 3 • TOPIC 2 • TOPIC SUMMARY

LESSON 6
Solving Systems of Equations and Inequalities

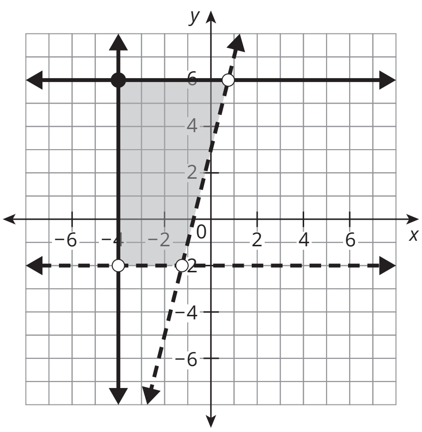
Substitution, linear combinations, and graphing are three methods for determining the solution of a linear system of equations. Use substitution when a variable in one equation can easily be isolated or when the equations are in slope-intercept form. Use the linear combinations method when the coefficients of like terms are opposites or can be easily made into opposites by multiplication. Use the graphing method when the numbers are convenient to graph or when an exact solution is not needed.

To solve a system of linear inequalities with more than two inequalities, graph each linear inequality and shade the correct region that contains the solution sets that satisfy all the inequalities in the system. Also, determine all the points of intersection for the boundary lines that make the vertices of the solution region. Use a closed point when the point is part of the solution region, and use an open dot when the point is not part of the solution region but is a point of intersection.

The graphs show the solution to a system of four inequalities.

$$\begin{cases} x \geq -4 \\ y > -2 \\ y \leq 6 \\ y > 4x + 3 \end{cases}$$

One solution of the system of inequalities is (−2, 2).



Notes

MODULE 3 • TOPIC 2 • TOPIC SUMMARY 505

Math Glossary

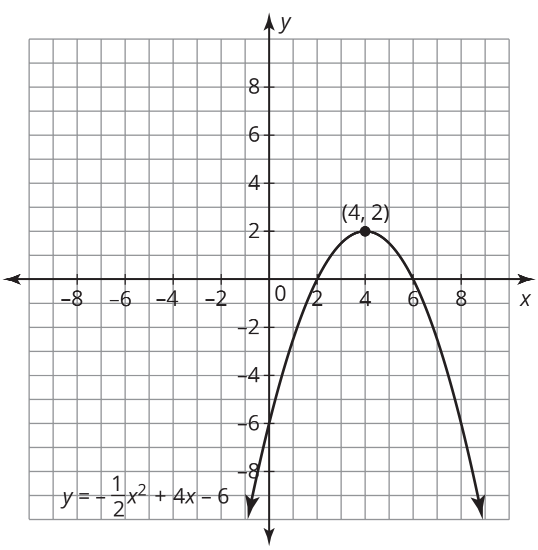
A

absolute maximum

A function has an absolute maximum if there is a point that has a *y*-coordinate that is greater than the *y*-coordinates of every other point on the graph.

Example

The ordered pair (4, 2) is the absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$.

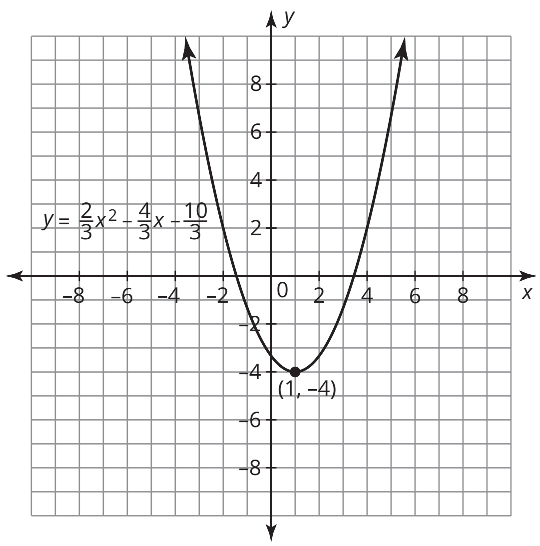


absolute minimum

A function has an absolute minimum if there is a point that has a *y*-coordinate that is less than the *y*-coordinates of every other point on the graph.

Example

The ordered pair (1, −4) is the absolute minimum of the graph of the function $y = \frac{2}{3}x^2 - \frac{4}{3}x - \frac{10}{3}$.



argument of a function

The argument of a function is the variable on which the function operates.

Example

In the function $f(x + 5) = 32$, the argument is $x + 5$.

arithmetic sequence

An arithmetic sequence is a sequence of numbers in which the difference between any two consecutive terms is a constant.

Example

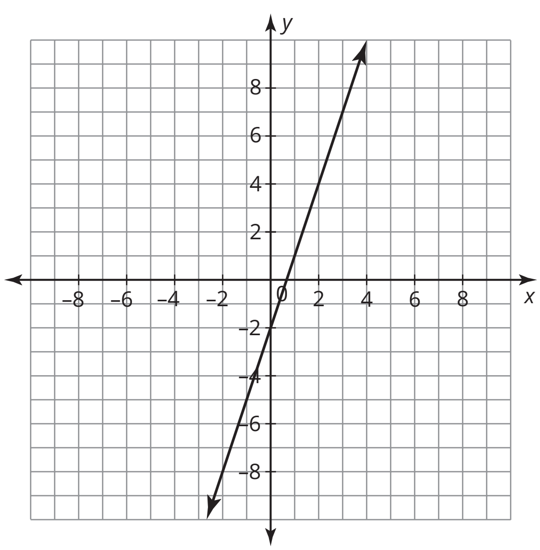
The sequence 1, 3, 5, 7 is an arithmetic sequence with a common difference of 2.

average rate of change

Another name for the slope of a linear function is average rate of change. The formula for the average rate of change is $\frac{f(t) - f(s)}{t - s}$.

Example

The average rate of change of the function shown is 3.

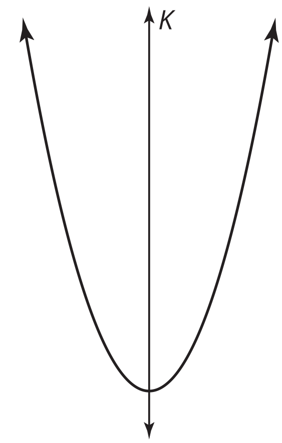


axis of symmetry

The axis of symmetry of a parabola is the vertical line that passes through the vertex and divides the parabola into two mirror images.

Example

Line K is the axis of symmetry of this parabola.



B

base

The base of a power is the expression that is used as a factor in the repeated multiplication.

binomial

Polynomials with exactly two terms are binomials.

Example

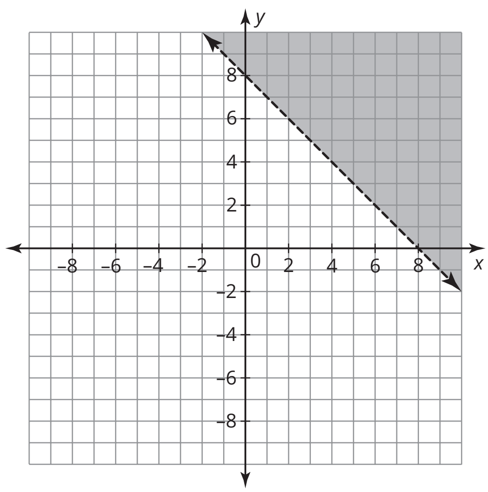
The polynomial $3x + 5$ is a binomial.

boundary line

A boundary line, determined by the inequality in a linear inequality, divides the plane into two half-planes and the inequality symbol indicates which half-plane contains all the solutions.

Example

For the linear inequality $y > -x + 8$, the boundary line is a dashed line because no point on that line is a solution.



C

causation

Causation is when one event affects the outcome of a second event.

centroid

The centroid is a point in which *x*-value is the mean of all the *x*-values of the points on the scatterplot and its *y*-value is the mean of all the *y*-values of the points on the scatterplot.

Example

For the data points (1, 3), (1, 7), (2, 6), (3, 5), and (3, 4), the centroid is (2, 5).

closed (closure)

When an operation is performed on any of the numbers in a set and the result is a number that is also in the same set, the set is said to be closed (or to have closure) under that operation.

Example

The set of whole numbers is closed under addition. The sum of any two whole numbers is always another whole number.

coefficient of determination

The coefficient of determination measures how well the graph of a regression fits the data. It is calculated by squaring the correlation coefficient and represents the percentage of variation of the observed values of the data points from their predicted values.

Example

The correlation coefficient for a data set is −0.9935. The coefficient of determination for the same data set is approximately 0.987, which means 98.7% of the data values should fall on the graph.

common difference

The difference between any two consecutive terms in an arithmetic sequence is called the common difference. It is typically represented by the variable *d*.

Example

The sequence 1, 3, 5, 7 is an arithmetic sequence with a common difference of 2.

common ratio

The ratio between any two consecutive terms in a geometric sequence is called the common ratio. It is typically represented by the variable *r*.

Example

The sequence 2, 4, 8, 16 is a geometric sequence with a common ratio of 2.

common response

A common response is when a variable other than the ones measured cause the same result as the one observed in the experiment.

completing the square

Completing the square is a process for writing a quadratic expression in vertex form which then allows you to solve for the zeros.

compound interest

In a compound interest account, the balance is multiplied by the same amount at each interval.

Example

Sonya opens a savings account with $100. She earns $4 in compound interest the first year. The compound interest y is found by using the equation $y = 100(1 + 0.04)^t$, where t is the time in years.

concave down

A graph that opens downward is identified as being concave down.

concave up

A graph that opens upward is identified as being concave up.

confounding variable

A confounding variable is when there are other variables in an experiment that are unknown or unobserved.

conjecture

A conjecture is a mathematical statement that appears to be true but has not been formally proven.

consistent systems

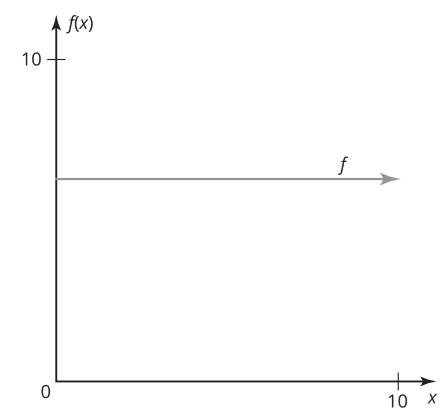
Systems that have one or many solutions are called consistent systems.

constant function

If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a constant function.

Example

The function shown is a constant function.



constraints

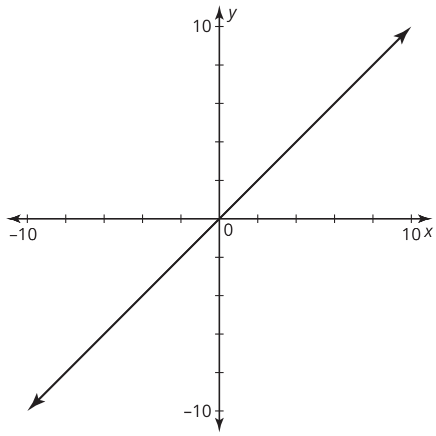
In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are "constrained" to lie within a certain region on the graph.

continuous graph

A continuous graph is a graph of points that are connected by a line or smooth curve on the graph. Continuous graphs have no breaks.

Example

The graph shown is a continuous graph.



correlation

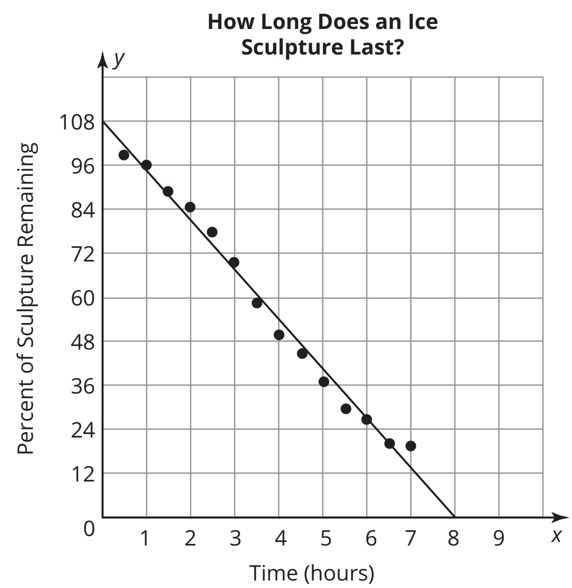
A measure of how well a regression fits a set of data is called a correlation.

correlation coefficient

The correlation coefficient is a value between −1 and 1, which indicates how close the data are to the graph of the regression function. The closer the correlation coefficient is to −1 or 1, the stronger the relationship is between the two variables. The variable *r* is used to represent the correlation coefficient.

Example

The correlation coefficient for these data is −0.9935. The value is negative because the equation has a negative slope. The value is close to −1 because the data are very close to the graph of the equation of the line.



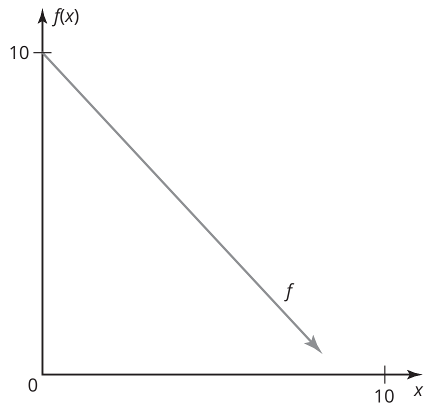
D

decreasing function

If a function decreases across the entire domain, then the function is called a decreasing function.

Example

The function shown is a decreasing function.



degree

The degree of a polynomial is the greatest variable exponent in the expression.

degree of a polynomial

The greatest exponent for any variable term in a polynomial determines the degree of the polynomial.

Example

The polynomial $2x^3 + 5x^2 - 6x + 1$ has a degree of 3.

dependent quantity

When one quantity is determined by another in a problem situation, it is said to be the dependent quantity.

Example

In the relationship between driving time and distance traveled, distance is the dependent quantity, because distance depends on the driving time.

difference of two squares

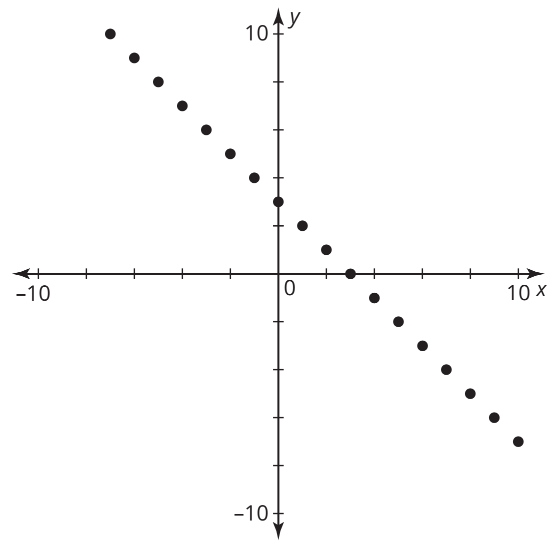
The difference of two squares is an expression in the form $a^2 - b^2$ that can be factored as $(a + b)(a - b)$.

discrete graph

A discrete graph is a graph of isolated points.

Example

The graph shown is a discrete graph.



discriminant

The discriminant is the radicand expression in the quadratic formula which “discriminates” the number of real roots of a quadratic equation.

Example

The discriminant in the quadratic formula is the expression $b^2 - 4ac$.

domain

The domain is the set of input values in a relation.

Example

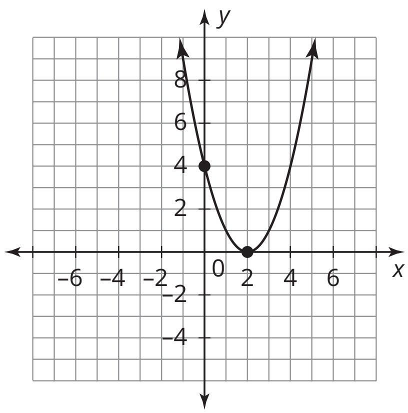
The domain of the function $y = 2x$ is the set of all real numbers.

double root

The root of an equation indicates where the graph of the equation crosses the x-axis.
A double root occurs when the graph just touches the x-axis but does not cross it.

Example

The quadratic equation $y = (x - 2)^2$ has a double root at $x = 2$.



E

explicit formula

An explicit formula of a sequence is a formula for calculating the value of each term of a sequence using the term's position in the sequence. The explicit formula for an arithmetic sequence is $a_n = a_1 + d(n - 1)$. The explicit formula for a geometric sequence is
$g_n = g_1 \cdot r^{n-1}$.

Example

The sequence 1, 3, 5, 7, 9, . . . can be described by the rule $a_n = 2n - 1$ where n is the position of the term. The fourth term of the sequence
a_4 is $2(4) - 1$, or 7.

exponent

The exponent of a power is the number of times that the base is used as a factor in the repeated multiplication.

exponential decay function

An exponential decay function is an exponential function with a b-value greater than 0 and less than 1 and is of the form $y = a(1 - r)^x$, where r is the rate of decay.

Example

Greenville has a population of 7000. Its population is decreasing at a rate of 1.75%. The exponential decay function that models this situation is $f(x) = 7000 \cdot 0.9825^x$.

exponential functions

The family of exponential functions includes functions of the form $f(x) = ab^x$, where a and b are real numbers, and b is greater than 0 but is not equal to 1.

Example

The function $f(x) = 2^x$ is an exponential function.

exponential growth function

An exponential growth function is an exponential function with a b-value greater than 1 and is of the form $y = a(1 + r)^x$, where r is the rate of growth.

Example

Blueville has a population of 7000. Its population is increasing at a rate of 1.4%. The exponential growth function that models this situation is $f(x) = 7000 \cdot 1.014^x$.

extract the square root

To extract a square root, solve an equation of the form $a^2 = b$ for a.

extrapolation

To make predictions for values of x that are outside of the data set is called extrapolation.

F

Factor theorem

The Factor theorem states that a polynomial function $p(x)$ has $x - r$ as a factor if and only if the value of the function at r is 0, or $p(r) = 0$.

factored form

A quadratic function written in factored form is in the form $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$.

Example

The function $h(x) = x^2 - 8x + 12$ written in factored form is $(x - 6)(x - 2)$.

finite sequence

If a sequence terminates, it is called a finite sequence.

Example

The sequence 22, 26, 30 is a finite sequence.

first differences

First differences are the values determined by subtracting consecutive output values in a table when the input values have an interval of 1.

Example

	Time (minutes)	Height (feet)	First Differences
	0	0	
$1 - 0 = 1$	1	1800	$1800 - 0 = 1800$
$2 - 1 = 1$	2	3600	$3600 - 1800 = 1800$
$3 - 2 = 1$	3	5400	$5400 - 3600 = 1800$

function

A function is a relation that assigns to each element of the domain exactly one element of the range.

Example

The equation $y = 2x$ is a function. Every value of x has exactly one corresponding y-value.

function notation

Function notation is a way of representing functions algebraically.

Example

In the function $f(x) = 0.75x$, f is the name of the function, x represents the domain, and $f(x)$ represents the range.

function family

A function family is a group of functions that share certain characteristics.

Example

Linear functions and exponential functions are examples of function families.

G

geometric sequence

A geometric sequence is a sequence of numbers in which the ratio between any two consecutive terms is a constant.

Example

The sequence 2, 4, 8, 16 is a geometric sequence with a common ratio of 2.

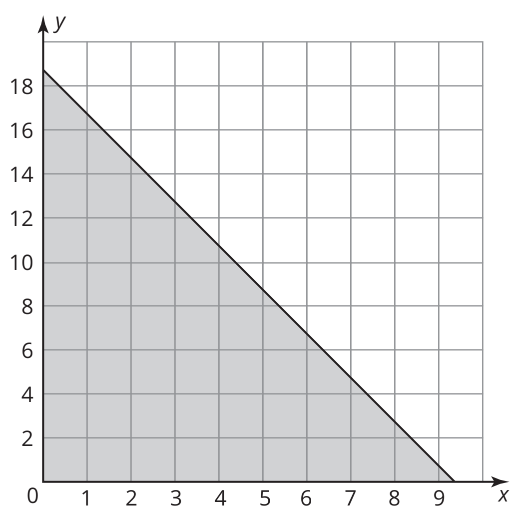
H

half-plane

The graph of a linear inequality is a half-plane, or half of a coordinate plane.

Example

The shaded portion of the graph is a half-plane.

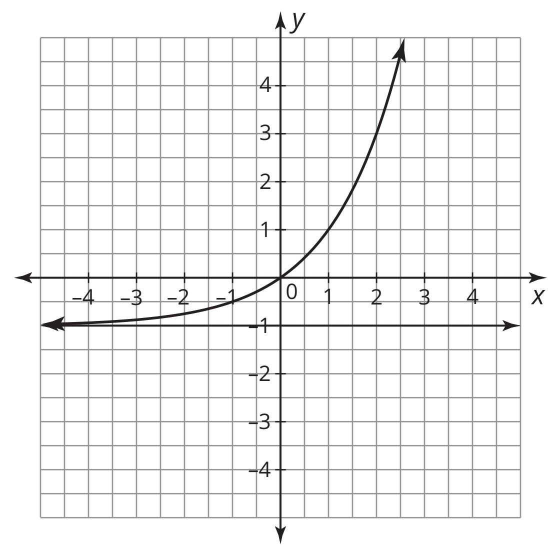


horizontal asymptote

A horizontal asymptote is a horizontal line that a function gets closer and closer to but never intersects.

Example

The graph shows a horizontal asymptote at $y = -1$.



I

inconsistent systems

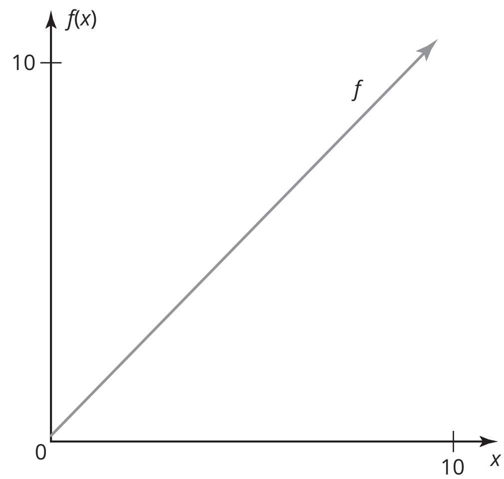
Systems with no solution are called inconsistent systems.

increasing function

If a function increases across the entire domain, then the function is called an increasing function.

Example

The function shown is an increasing function.



independent quantity

The quantity that the dependent quantity depends upon is called the independent quantity.

Example

In the relationship between driving time and distance traveled, driving time is the independent quantity, because it does not depend on any other quantity.

infinite sequence

If a sequence continues on forever, it is called an infinite sequence.

Example

The sequence 22, 26, 30, 34 . . . is an infinite sequence.

infinite solutions

An equation with infinite solutions means that any value for the variable makes the equation true.

Example

The equation $2x + 1 = 2x + 1$ has infinite solutions.

interpolation

Using a linear regression to make predictions within the data set is called interpolation.

L

leading coefficient

The leading coefficient of a polynomial is the numeric coefficient of the term with the greatest power.

Example

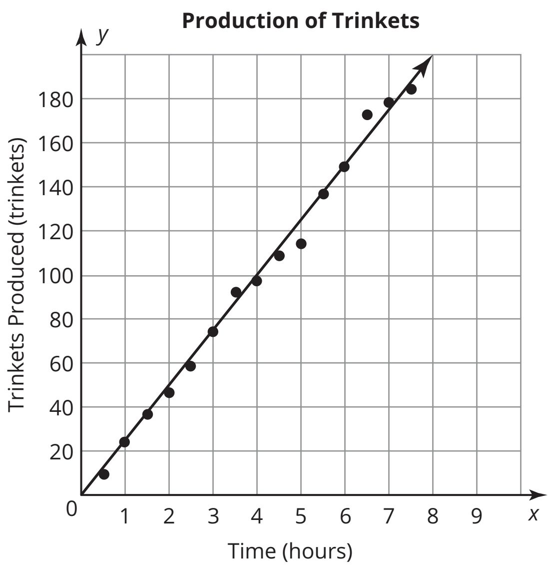
In the polynomial $-7x^2 + x + 25$, the value -7 is the leading coefficient.

Least Squares Method

The Least Squares Method is a method that creates a regression line for a scatterplot that has two basic requirements: 1) the line must contain the centroid of the data set, and 2) the sum of the squares of the vertical distances from each given data point is at a minimum with the line.

Example

The regression line shown was created using the Least Squares Method.



linear combinations method

The linear combinations method is a process used to solve a system of equations by adding two equations together, resulting in an equation with one variable.

Example

Solve the following system of equations by using the linear combinations method:

$$\begin{cases} 6x - 5y = 3 \\ 2x + 2y = 12 \end{cases}$$

First, multiply the second equation by -3. Then, add the equations and solve for the remaining variable. Finally, substitute $y = 3$ into the first equation and solve for x. The solution of the system is (3, 3).

linear functions

The family of linear functions includes functions of the form $f(x) = ax + b$, where a and b are real numbers.

Example

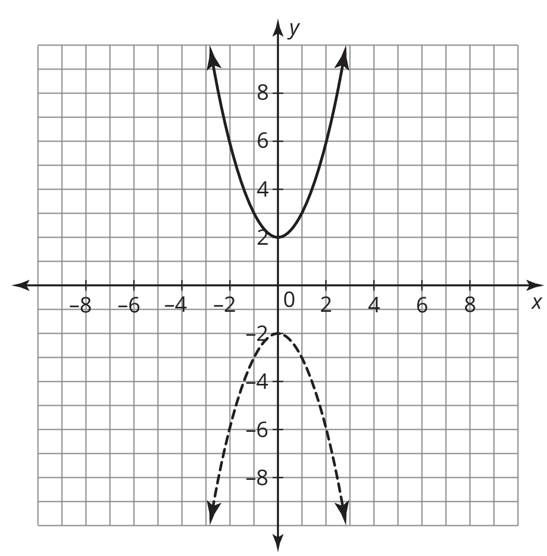
The function $f(x) = 3x + 2$ is a linear function.

line of reflection

A line of reflection is the line that the graph is reflected across.

Example

The graph of $y = x^2 + 2$ was reflected across the line of reflection, $y = 0$.



literal equation

Literal equations are equations in which the variables represent specific measures.

Example

The equations $I = Prt$ and $A = lw$ are literal equations.

M

mathematical modeling

Mathematical modeling is explaining patterns in the real world based on mathematical ideas.

monomial

Polynomials with only one term are monomials.

Example

The expressions $5x$, 7, $-2xy$, and $13x^3$ are monomials.

N

necessary condition

A correlation is a necessary condition for causation, meaning that for one variable to cause another, they must be correlated.

no solution

An equation with no solution means that there is no value for the variable that makes the equation true.

Example

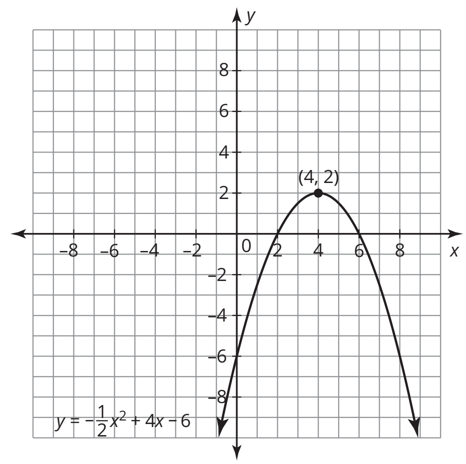
The equation $2x + 1 = 2x + 3$ has no solution.

P

parabola

The shape that a quadratic function forms when graphed is called a parabola. A parabola is a smooth curve with reflectional symmetry.

Example



parent function

A parent function is the simplest function of its type.

Examples

The parent linear function is $f(x) = x$.
The parent exponential function is $g(x) = 2^x$.
The parent quadratic function is $h(x) = x^2$.

perfect square trinomial

A perfect square trinomial is an expression in the form $a^2 + 2ab + b^2$ or in the form $a^2 - 2ab + b^2$.

point-slope form

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line.

polynomial

A polynomial is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients.

Example

The expression $3x^3 + 5x - 6x + 1$ is a polynomial.

polynomial long division

Polynomial long division is an algorithm for dividing one polynomial by another of equal or lesser degree. The process is similar to integer long division.

Example

Polynomial Long Division	
$(2x^2 + 5x - 12) \div (x + 4)$ or $\frac{2x^2 + 5x - 12}{x + 4}$	
$\begin{array}{r} \text{A}\quad \text{D}\quad \\ 2x - 3 \\ x + 4 \overline{)\, 2x^2 + 5x - 12} \\ \text{B}\ \underline{-(2x^2 + 8x)} \downarrow \ \text{C} \\ -3x - 12 \\ \underline{-(-3x - 12)} \\ \text{Remainder } 0 \end{array}$	A. Divide $\frac{2x^2}{x} = 2x$. B. Multiply $2x(x + 4)$, and then subtract. C. Bring down -12. D. Divide $\frac{-3x}{x} = -3$. E. Multiply $-3(x + 4)$, and then subtract.

power

A power has a *base* and an *exponent*.

principal square root

The principal square root is a positive square root of a number.

Q

quadratic formula

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and can be used to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c$, where a, b, and c represent real numbers and $a \neq 0$.

quadratic functions

The family of quadratic functions includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

Examples

The equations $y = x^2 + 2x + 5$ and $y = -4x^2 - 7x + 1$ are quadratic functions.

R

range

The range is the set of output values in a relation.

Example

The range of the function $y = x^2$ is the set of all numbers greater than or equal to zero.

recursive formula

A recursive formula expresses each new term of a sequence based on the preceding term in the sequence. The recursive formula for an arithmetic sequence is $a_n = a_{n-1} + d$. The recursive formula for a geometric sequence is $g_n = g_{n-1} \cdot r$.

Example

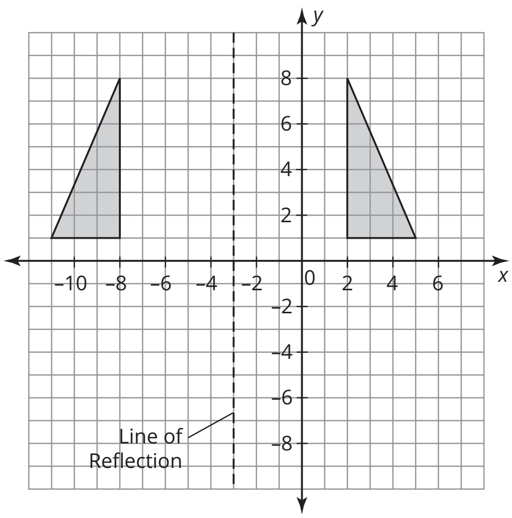
The formula $a_n = a_{n-1} + 2$ is a recursive formula. Each successive term is calculated by adding 2 to the previous term. If $a_1 = 1$, then $a_2 = 1 + 2 = 3$.

reflection

A reflection of a graph is a mirror image of the graph about a line of reflection.

Example

The triangle on the right is a reflection of the triangle on the left.



regression function

On a scatterplot, a regression function is a mathematical model that can be used to predict the values of a dependent variable based upon the values of an independent variable.

relation

A relation is the mapping between a set of input values called the domain and a set of output values called the range.

Example

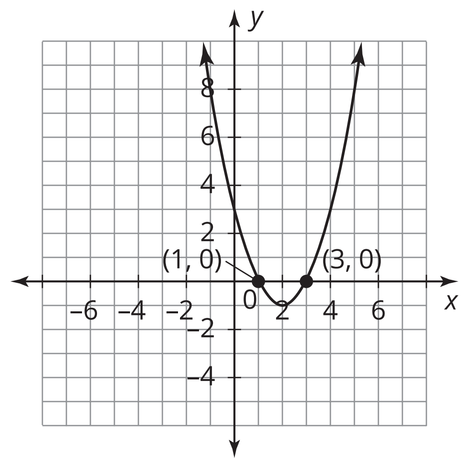
The set of points {(0, 1), (1, 8), (2, 5), (3, 7)} is a relation.

root (roots)

The root or roots of an equation indicate where the graph of the equation crosses the *x*-axis.

Example

The roots of the quadratic equation $x^2 - 4x + 3 = 0$ are $x = 3$ and $x = 1$.



S

second differences

Second differences are the differences between consecutive values of the first differences.

Example

x	y
−3	−5
−2	0
−1	3
0	4
1	3
2	0
3	−5

First Differences
5
3
1
−1
−3
−5

Second Differences
−2
−2
−2
−2
−2

sequence

A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.

Example

The numbers 1, 1, 2, 3, 5, 8, 13 form a sequence.

simple interest

In a simple interest account, the interest earned at the end of each interval is a percent of the starting balance (also known as the principal).

Example

Tonya deposits $200 in a 3-year certificate of deposit that earns 4% simple interest. The amount of interest that Tonya earns can be found using the simple interest formula.

$$I = (200)(0.04)(3)$$
$$I = 24$$

Tonya earns $24 in interest.

solution

The solution to an equation is any value for the variable that makes the equation a true statement.

Example

The solution of the equation $3x + 4 = 25$ is 7 because 7 makes the equation true: $3(7) + 4 = 25$, or $25 = 25$.

solution set of a system of linear inequalities

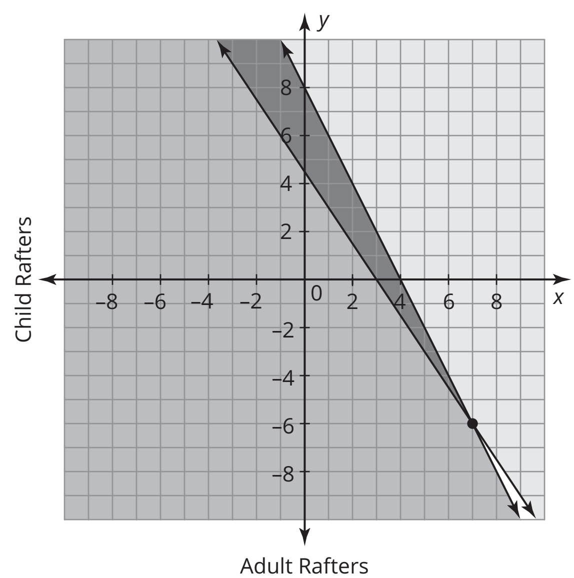
The solution set of a system of linear inequalities is the intersection of the solutions to each inequality. Every point in the intersection region satisfies all inequalities in the system.

Example

The solution set of this system of linear inequalities …

$$\begin{cases} 200a + 100c \leq 800 \\ 75(a - 1) + 50c \geq 150 \end{cases}$$

… is shown by the shaded region, which represents the intersection of the solutions to each inequality.



solve an inequality

To solve an inequality means to determine the values of the variable that make the inequality true.

Example

The inequality $x + 5 > 6$ can be solved by subtracting 5 from each side of the inequality. The solution is $x > 1$. Any number greater than 1 will make the inequality $x + 5 > 6$ true.

standard form of a linear equation

The standard form of a linear equation is $Ax + By = C$, where A, B, and C are integers and A and B are not both zero.

standard form of a quadratic function

A quadratic function written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$, is in standard form.

Example

The function $f(x) = -5x^2 - 10x + 1$ is written in standard form.

sufficient condition

A correlation is not a sufficient condition for causation, meaning that a correlation between two variables is not enough to establish that one variables causes another.

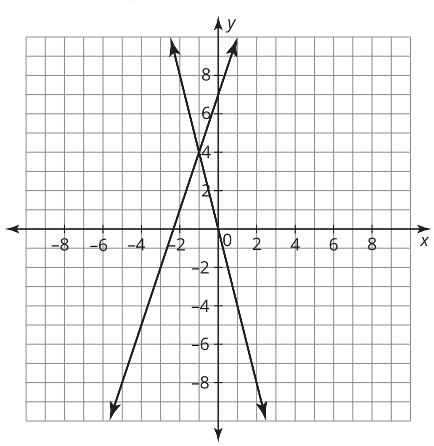
system of linear equations

When two or more linear equations define a relationship between quantities, they form a system of linear equations.

Example

The equations $y = 3x + 7$ and $y = -4x$ are a system of linear equations.

$$\begin{cases} y = 3x + 7 \\ y = -4x \end{cases}$$



T

term of a sequence

A term of a sequence is an individual number, figure, or letter in the sequence.

Example

In the sequence 2, 4, 6, 8, 10, the first term is 2, the second term is 4, and the third term is 6.

trinomial

Polynomials with exactly three terms are trinomials.

Example

The polynomial $5x^2 - 6x + 9$ is a trinomial.

V

vertex form

A quadratic function written in vertex form is in the form $f(x) = a(x - h)^2 + k$, where $a \neq 0$.

Example

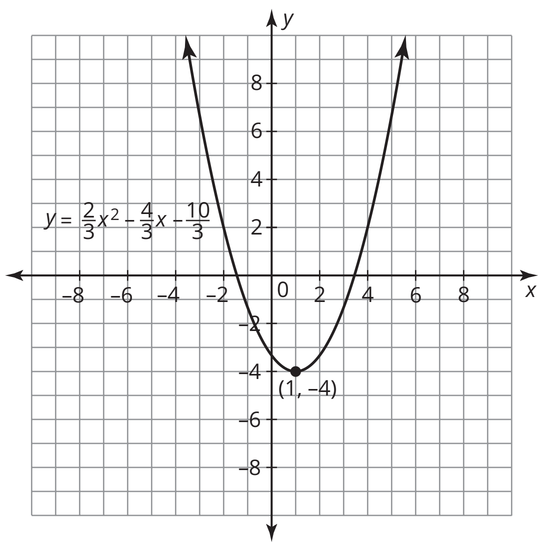
The quadratic equation $y = 2(x - 5)^2 + 10$ is written in vertex form. The vertex of the graph is the point (5, 10).

vertex of a parabola

The vertex of a parabola is the lowest or highest point on the graph of the quadratic function.

Example

The vertex of the graph of $y = \frac{2}{3}x^2 - \frac{4}{5}x - \frac{10}{3}$ is the point $(1, -4)$, the absolute minimum of the parabola.

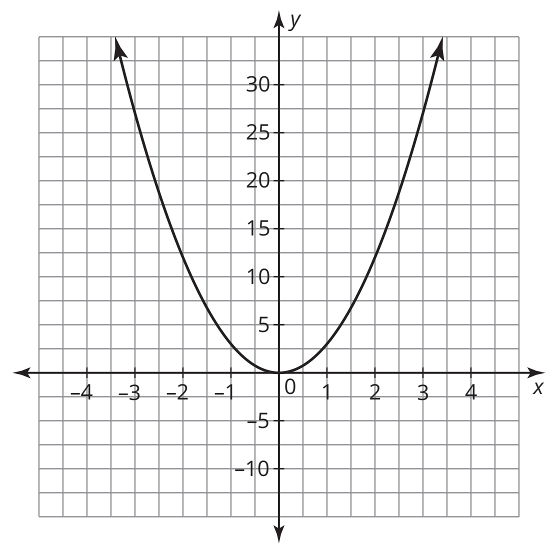


Vertical Line Test

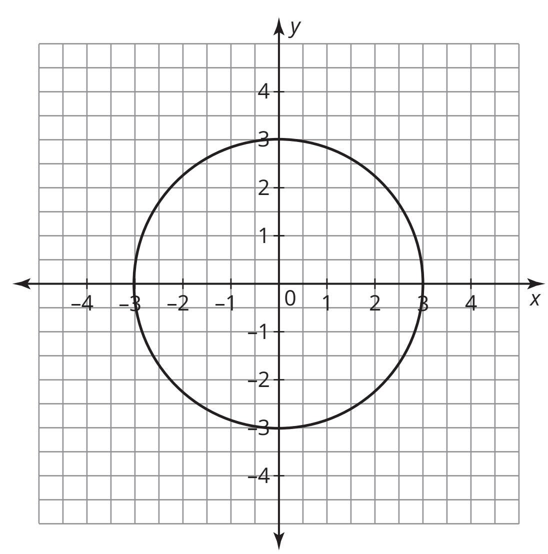
The Vertical Line Test is a visual method used to determine whether a relation represented as a graph is a function.

Example

The equation $y = 3x^2$ is a function. The graph passes the Vertical Line Test because there are no vertical lines that can be drawn that would intersect the graph at more than one point.



The equation $x^2 + y^2 = 9$ is not a function. The graph fails the Vertical Line Test because a vertical line can be drawn that intersects the graph at more than one point.



vertical motion model

A vertical motion model is a quadratic equation that models the height of an object at a given time. The equation is of the form $g(t) = -16t^2 + v_0t + h_0$, where $g(t)$ represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial velocity (speed) of the object in feet per second, and h_0 represents the initial height of the object in feet.

Example

A rock is thrown in the air at a velocity of 10 feet per second from a cliff that is 100 feet high. The height of the rock is modeled by the equation $y = -16t^2 + 10t + 100$.

X

x-intercept

The point where a graph crosses the x-axis is the x-intercept.

Y

y-intercept

The point where a graph crosses the y-axis is the y-intercept.

Z

zero of a function

A zero of a function is a real number that makes the value of the function equal to zero, or $f(x) = 0$.

Example

The zero of the linear function $f(x) = 2(x - 4)$ is (4, 0).

The zeros of the quadratic function $f(x) = -2x^2 + 4x$ are (0, 0) and (2, 0).

zero product property

The zero product property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero.

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