OER Secondary Mathematics

EDITION 1

Grade 7

Family Guides

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Family Guide

FAMILY LETTER

Secondary Mathematics Edition 1
Grade 7

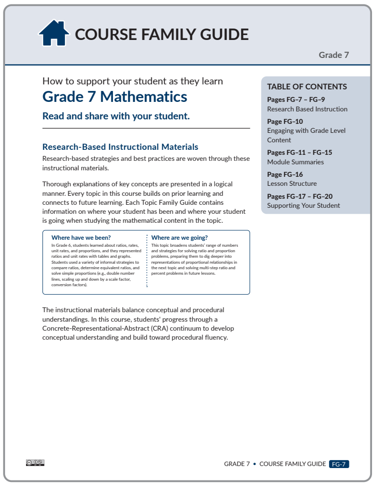
Dear Family,

We recognize that learning outside of the classroom is crucial to your student's success at school. This letter serves as an introduction to the resources designed to assist you as you talk to your student about what they are learning. Resources available to you include:

- Course Family Guide
- Topic Family Guides
- Topic Summaries
- Math Glossary

Course Family Guide

Following this letter, there is the Course Family Guide that will walk you though the research-based instructional approach, how the course is structured, how to bust math myths, using Talking Points from the Topic Family Guide, and using the TEKS mathematical process standards to initiate discussions.



COURSE FAMILY GUIDE

Grade 7

How to support your student as they learn

Grade 7 Mathematics

Read and share with your student.

Research-Based Instructional Materials

Research-based strategies and best practices are woven through these instructional materials.

Thorough explanations of key concepts are presented in a logical manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide contains information on where your student has been and where your student is going when studying the mathematical content in the topic.

Where have we been?
In Grade 6, students learned about ratios, rates, unit rates, and proportions, and they represented ratios and unit rates with tables and graphs. Students used a variety of informal strategies to compare ratios, determine equivalent ratios, and solve simple proportions (e.g., double number lines, scaling up and down by a scale factor, conversion factors).

Where are we going?
This topic broadens students' range of numbers and strategies for solving ratio and proportion problems, preparing them to dig deeper into representations of proportional relationships in the next topic and solving multi-step ratio and percent problems in future lessons.

The instructional materials balance conceptual and procedural understandings. In this course, students' progress through a Concrete-Representational-Abstract (CRA) continuum to develop conceptual understanding and build toward procedural fluency.

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GRADE 7 • COURSE FAMILY GUIDE FG-7

The Course Family Guide

Research and classroom experience guided course development, with the foundation being a scientific understanding of how people learn and a real-world understanding of how to apply that science to mathematics instructional materials. The instructional design elements presented in the Course Family Guide incorporate research-based strategies to develop conceptual understanding and creative problem solvers.

The Course Family Guide provides an overview of the structure of the course. The course consists of both a Learning Together component and a Learning Individually component. The teacher facilitates a collaborative learning experience during the Learning Together Days and uses data to target specific skills on the Learning Individually Days.

Next, the Course Family Guide includes Module Overviews of each module in the course, which include a detailed summary of what your student will be learning in each topic within the module. Below the topic summaries are facts and information that connect the concepts to the real world. Read and discuss the information below the topic summaries with your student and continue to come back to these pages as your student moves from one topic to the next within each module.

The Course Family Guide also highlights the lesson structure. Each lesson is structured the same way and includes four parts: Objectives & Essential Question, Getting Started, Activities, and the Talk the Talk.

Topic Family Guide

Each course is organized into modules. Each module consists of topics with corresponding Topic Family Guides. These guides all have the same structure. This consistency will allow you and your student to understand how to reference the content of each topic.

The Topic Family Guide begins with an overview of the content in the topic. This introduction includes a brief explanation of what your student will learn in the topic, the prior knowledge they will use to help them understand this topic, and a connection to future learning.

The next section of the Topic Family Guide is the Talking Points section. The Talking Points section provides questions you can ask as your student works through the math of the topic.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to reason using proportions.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Next, the Topic Family Guide lists all the new key terms of the topic and details some of the math strategies students will learn in the topic. Finally, each Topic Family Guide contains a Math Myth. Busting these Math Myths helps to build confidence and explain how math is accessible to everyone.

MYTH

"I'm not smart."

The word *smart* is tricky because it means different things to different people. For example, would you say a baby is "smart?" On the one hand, a baby is helpless and doesn't know anything. On the other hand, a baby is exceptionally smart because they are constantly learning new things every day.

This example is meant to demonstrate that *smart* can have two meanings. It can mean, "the knowledge that you have," or it can mean, "the capacity to learn from experience." When someone says they are "not smart," are they saying they do not have a lot of knowledge, or are they saying they lack the capacity to learn? If it's the first definition, then none of us are smart until we acquire information. If it's the second definition, then we know that is completely untrue because everyone has the capacity to grow as a result of new experiences.

So, if your student doesn't think that they are smart, encourage them to be patient. They have the capacity to learn new facts and skills. It might not be easy, and it will take some time and effort. But the brain is automatically wired to learn. *Smart* should not refer only to how much knowledge you currently have.

#mathmythbusted

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all new key terms of the topic and provides a summary of each lesson. Each lesson summary defines new key terms and reviews key concepts, strategies, and/or Worked Examples. The Topic Summary provides an opportunity for you and your student to discuss the key concepts from each lesson, review the examples, and do the math together.

LESSON **1**

Adding and Subtracting Rational Numbers

You can use what you know about adding and subtracting integers to solve problems with positive and negative fractions and decimals.

For example, yesterday Natalia was just \$23.75 below her fundraising goal. She got a check today for \$12.33 to put toward the fundraiser. Describe Natalia's progress toward the goal.

$-\$23.75 + \$12.33 = -\$11.42$

Natalia will still be below her goal because $-11.42 < 0$.

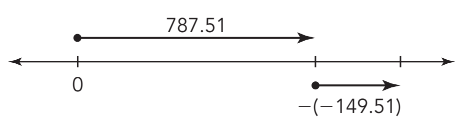
The difference between two numbers is a measure of the distance between the numbers.

For example, the freezing point of chlorine is −149.51°F. The freezing point of zinc is 787.51°F. How many more degrees is the freezing point of zinc than the freezing point of chlorine?

A model can help you estimate that the answer will be greater than 787.51.

$787.51 - (-149.51) = 937.02$

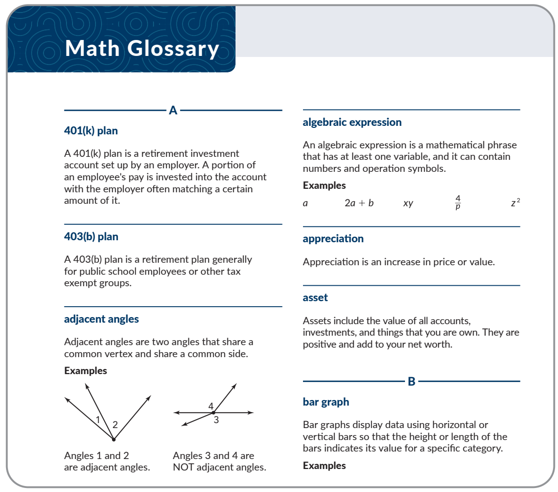
The freezing point of zinc is 937.02°F more than the freezing point of chlorine.



Evidence of the TEKS mathematical process standards are present in the Topic Summaries. Each lesson within the topic highlights one or more of the TEKS mathematical process standards. These processes will help your student develop effective communication and collaboration skills that are essential for becoming a successful learner. Discuss with your student the "I Can" statements associated with each of the TEKS mathematical process standards to help them develop their mathematical learning and understanding. The "I Can" statements for each of the TEKS mathematical process standards are included in the Course Family Guide. With your help, your student can develop the habits of a productive mathematical thinker.

Math Glossary

The Math Glossary for each course is a tool for your student to utilize and reference during their learning. Along with the definition of a term, the glossary provides examples to help further their understanding.


Math Glossary

A

401(k) plan

A 401(k) plan is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account with the employer often matching a certain amount of it.

403(b) plan

A 403(b) plan is a retirement plan generally for public school employees or other tax exempt groups.

adjacent angles

Adjacent angles are two angles that share a common vertex and share a common side.

Examples

Angles 1 and 2 are adjacent angles.

Angles 3 and 4 are NOT adjacent angles.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $\quad 2a + b$ $\quad xy$ $\quad \frac{4}{p}$ $\quad z^2$

appreciation

Appreciation is an increase in price or value.

asset

Assets include the value of all accounts, investments, and things that you are own. They are positive and add to your net worth.

B

bar graph

Bar graphs display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

Examples

We all have the same goal for your student: to become a successful problem solver and use mathematics efficiently and effectively in daily life. Encourage them to use the mathematics they already know when seeing new concepts and communicate their thinking while providing a critical ear to the thinking of others.

Thank you for supporting your student.

COURSE FAMILY GUIDE

Grade 7

How to support your student as they learn

Grade 7 Mathematics

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Research-Based Instruction

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<table>
<tr><th>Concrete</th><th>Representational</th><th>Abstract</th></tr>
<tr>
<td>Students deconstruct a circle and reconstruct it into a rectangle.

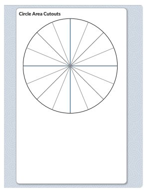
ACTIVITY 2.1 Deriving the Area Formula

In the last lesson, you derived formulas for the distance around a circle. In this lesson, you will investigate the space within a circle. Use the circle at the end of the lesson that is divided into 4, 8, and 16 equal parts.

1. Follow the steps to decompose the circle and then compose it into a new figure.

a. First, cut the circle into fourths and arrange the parts side by side so that they form a shape that looks like a parallelogram.

b. Then, cut the circle into eighths and then sixteenths. Each time, arrange the parts to form a parallelogram.


</td>
<td>Using the area of a rectangle, students substitute the radius for the height and half of the circumference for the base to develop the formula for the area of a circle.

2. Analyze the parallelogram you made each time.

a. How did the parallelogram change as you arranged it with the smaller equal parts of the same circle?

b. Suppose you built the parallelogram out of 40 equal circle sections. What would be the result? What about 100 equal circle sections?

c. Represent the approximate base length and height of the parallelogram in terms of the radius and circumference of the circle.

d. Use your answers to part (c) to determine the formula for the area of the parallelogram.

e. How does the area of the parallelogram compare to the area of the circle?

f. Write a formula for the area of a circle.</td>
<td>Students use the newly derived formula to calculate the area of circles given different measurements for a radius.

3. Use different representations for π to calculate the area of a circle.

a. Calculate the area of each circle with the given radius. Round your answers to the nearest ten-thousandths, when necessary.

<table>
<tr><th>Value for π</th><th>$r = 6$ units</th><th>$r = 1.5$ units</th><th>$r = \frac{1}{2}$ unit</th></tr>
<tr><td>π</td><td></td><td></td><td></td></tr>
<tr><td>Use the π key on a calculator</td><td></td><td></td><td></td></tr>
<tr><td>Use 3.14 for π</td><td></td><td></td><td></td></tr>
<tr><td>Use $\frac{22}{7}$ for π</td><td></td><td></td><td></td></tr>
</table>

b. Compare your area calculations for each circle. How do the different values of π affect your calculations?</td>
</tr>
</table>

Support is provided to students as they persevere in problem solving. These instructional materials include a problem-solving model, which includes questions your student can ask when productively engaging in real-world and mathematical problems. Prompts will encourage your student to use the problem-solving model throughout the course.

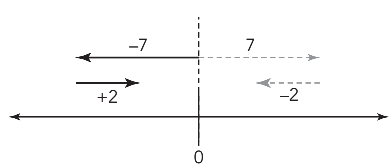
These instructional materials include features that support learners. Worked Examples throughout the product provide explicit instruction and provide a model your student can continually reference.

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connection between steps.

WORKED EXAMPLE

Consider the expression $-7 + 2$. When the model of $-7 + 2$ is reflected across 0 on the number line, the result is $7 - 2$.



So, $(-7 + 2)$ is the opposite of $(7 - 2)$.

This means that $-7 + 2 = -(7 - 2)$.

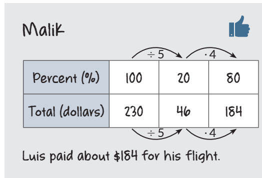
Thumbs Up, Thumbs Down, and Who's Correct Questions address your student's common misconceptions and provide opportunities for peer work analysis.

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connection between steps.

Ask Yourself

- Why is this method correct?
- Have I used this method before?


Malik

Percent (%)	100	20	80
Total (dollars)	230	46	184

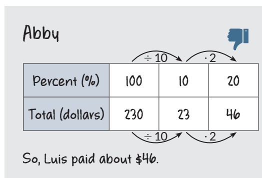
Luis paid about $184 for his flight.

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself

- Where is the error?
- Why is it an error?
- How can I correct it?


Abby

Percent (%)	100	10	20
Total (dollars)	230	23	46

So, Luis paid about $46.

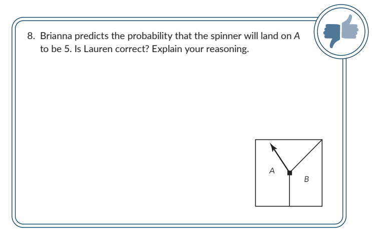
Who's Correct

When you see a Who's Correct icon:

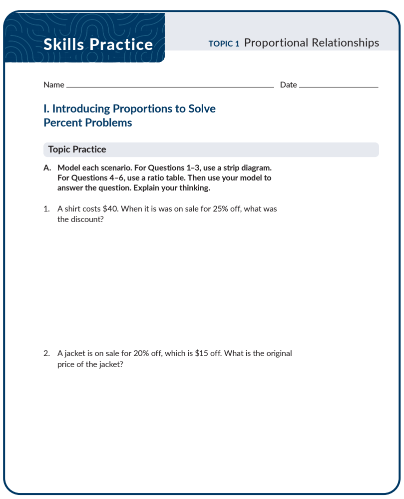
- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or incorrect

Ask Yourself

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?



Targeted Skills Practice provides support to your student as they work to gain proficiency of the course material. Spaced Practice provides a spaced retrieval of key concepts to your student. Extension opportunities provide challenges to accelerate your student's learning.


Skills Practice

TOPIC 1 Proportional Relationships

Name ______________________ Date __________

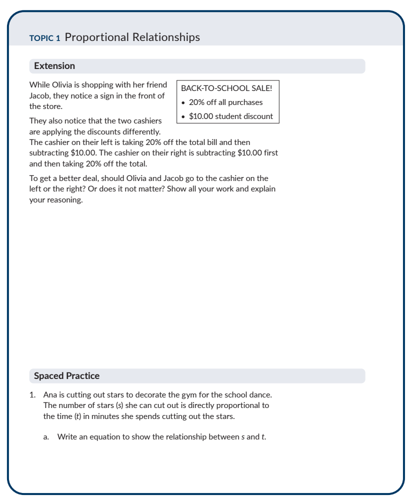
I. Introducing Proportions to Solve Percent Problems

Topic Practice

A. Model each scenario. For Questions 1–3, use a strip diagram. For Questions 4–6, use a ratio table. Then use your model to answer the question. Explain your thinking.

1. A shirt costs $40. When it is was on sale for 25% off, what was the discount?

2. A jacket is on sale for 20% off, which is $15 off. What is the original price of the jacket?


TOPIC 1 Proportional Relationships

Extension

While Olivia is shopping with her friend Jacob, they notice a sign in the front of the store.

BACK-TO-SCHOOL SALE!
- 20% off all purchases
- $10.00 student discount

They also notice that the two cashiers are applying the discounts differently. The cashier on their left is taking 20% off the total bill and then subtracting $10.00. The cashier on their right is subtracting $10.00 first and then taking 20% off the total.

To get a better deal, should Olivia and Jacob go to the cashier on the left or the right? Or does it not matter? Show all your work and explain your reasoning.

Spaced Practice

1. Ana is cutting out stars to decorate the gym for the school dance. The number of stars (s) she can cut out is directly proportional to the time (t) in minutes she spends cutting out the stars.

   a. Write an equation to show the relationship between s and t.

Each lesson features one or more ELPS (English Language Proficiency Standards) and provides the teacher with implementation strategies incorporating best practices for supporting language acquisition. In addition, students are provided with cognates for New Key Terms in the Topic Summaries and Topic Family Guides.

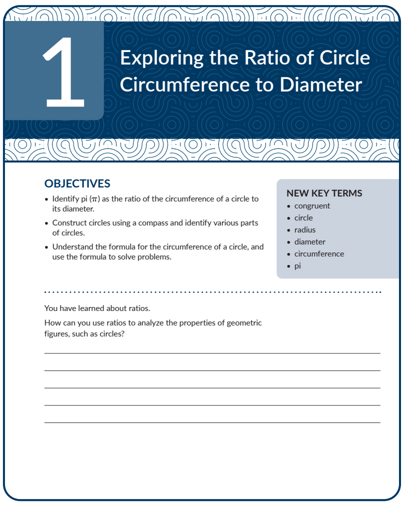
NEW KEY TERMS

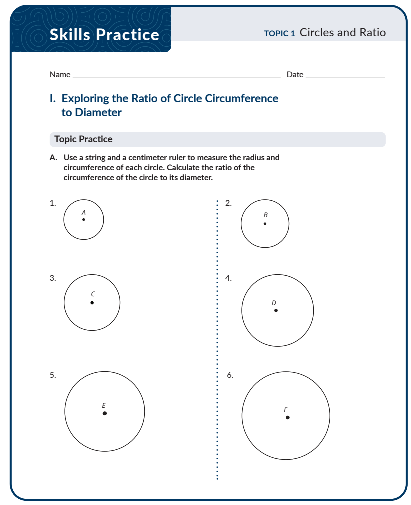
- congruent [congruente]
- circle [círculo]
- radius [radio]
- diameter [diámetro]
- circumference [circunferencia]
- pi [pi]
- unit rate
- composite figure [figura compuesta]

Engaging with Grade-Level Content

Your student will engage with grade-level content in multiple ways with the support of the teacher.

Learning Together	Learning Individually
The teacher facilitates active learning of lessons so that students feel confident in sharing ideas, listening to each other, and learning together. Students become creators of their mathematical knowledge.	Skills Practice provides students the opportunity to engage in additional skill building that aligns to each Learning Together lesson. The Learning Individually Days target discrete skills that may require additional practice to achieve proficiency.





At the end of each topic, your student will take an assessment aligned to the standards covered in the topic. This assessment consists of multiple-choice, multi-select, and open-ended questions designed for your student to demonstrate learning. Each assessment also includes a scoring guide for teachers to ensure consistent scoring. The scoring guide includes ways to support or challenge your student based on their responses to the questions on the assessment. The purpose of the assessment is for the teacher and student to reflect on the learning. Teachers will use your student's assessment results to target individual skills your student needs for proficiency or to accelerate and challenge your student.

Response to Student Performance

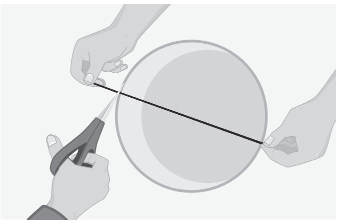
TEKS*	Question(s)	Recommendations
7.5B	3, 5	**To support students:** • Review the relationship between the radius, diameter, and circumference of a circle. • Use Skills Practice Set I.A and I.B for additional practice. • Review Lesson 1 Assignment Practice Questions 1 and 2.
7.9B	1, 6	**To support students:** • Review how to determine the circumference of circles. • Use Skills Practice Set I.B and I.C for additional practice. • Review Lesson 1 Assignment Practice Questions 1 and 2.
	2, 7, 9	**To support students:** • Review how to determine the area of circles. • Use Skills Practice Sets II.A, II.B, and II.C for additional practice. • Review Lesson 2 Assignment Practice Questions 1 and 2.
7.9C	4, 8	**To support students:** • Review how to determine the area of composite figures. • Use Skills Practice Sets III.A and III.B for additional practice. • Review Lesson 3 Assignment Practice Questions 1 and 2.

MODULE 1 **Thinking Proportionally**

In this module, your student will develop strategies for solving problems involving ratios and proportional relationships. There are three topics in this module: *Circles and Ratio*, *Fractional Rates*, and *Proportionality*. Your student will use what they already know about determining equivalent ratios in this module.

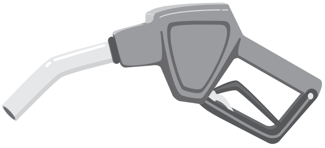
<table>
<tr><th>TOPIC 1
Circles and Ratio</th><th>TOPIC 2
Fractional Rates</th><th>TOPIC 3
Proportionality</th></tr>
<tr><td>Your student will develop formulas for the circumference and area of circles and will develop an understanding of pi (π).</td><td>Your student will write and use unit rates, including those with fractional values.</td><td>Your student will graph proportional relationships and determine the constant of proportionality.</td></tr>
<tr><td>Try this at home!

Cut a piece of string the length of a circle's diameter and then use that string to measure the circumference.



You should see that it takes a little more than 3 times the string's length to measure the circumference.</td><td>What in the world?

Unit rates are used in real life to determine which is the better deal. For example, would you rather pay \$3.05 per gallon of gas or \$2.97 per gallon of gas?



What is the unit rate if you pay \$32 for 10 gallons of gas?

[The unit rate for one gallon of gas is \$3.20.]</td><td>What in the world?

If you earn \$15 per hour, then the amount \$15 is the constant of proportionality. The amount of money you earn depends on the number of hours you work.

money earned = 15 · hours worked

What is the constant of proportionality if you earn \$160 for working an 8 hour day?

[The constant of proportionality is 20.]</td></tr>
</table>

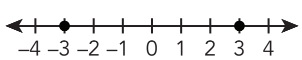
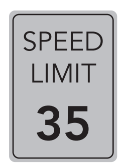
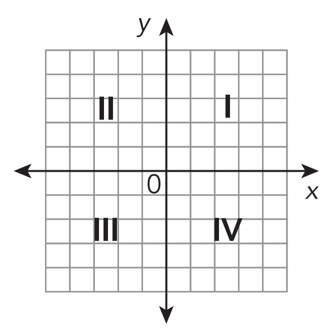
MODULE 2 **Applying Proportionality**

In this module, your student will deepen their understanding of financial literacy in real-world situations. There are two topics in this module: *Proportional Relationships* and *Financial Literacy: Interest and Budgets*. Your student will use what they already know about ratio and proportional relationships in this module.

TOPIC 1 Proportional Relationships	TOPIC 2 Financial Literacy: Interest and Budgets
Your student will use their knowledge of proportionality to solve real-world problems about money and scale drawings.	Your student will solve financial literacy problems about interest, budgets, and net worth.
What in the world? When a shop sells an item at a lower price than the original price, it is called a **markdown**.  How much would a $50 pair of shoes cost after a 10% markdown? [*After a 10% markdown of $5.00, it would cost $45.00.*]	Did you know that? Mansa Musa was a ruler of the Mali empire in Africa who lived about 700 years ago, and he is believed to be the most wealthy man that ever lived. Today, his **net worth** would be over $400 billion. 

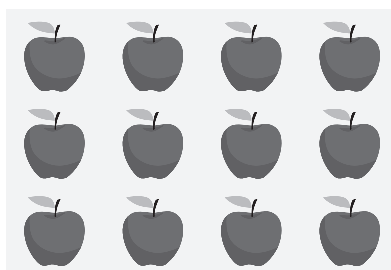
MODULE 3 Reasoning Algebraically

In this module, your student will deepen their understanding of formal algebra. There are three topics in this module: *Operating with Rational Numbers*, *Two-Step Equations and Inequalities*, and *Multiple Representations of Equations*. Your student will use what they already know about operating with the full set of rational numbers in this module.

<table>
<tr><th>TOPIC 1
Operating with
Rational Numbers</th><th>TOPIC 2
Two-Step Equations
and Inequalities</th><th>TOPIC 3
Multiple Representations
of Equations</th></tr>
<tr><td>Your student will build fluency operating with positive and negative rational numbers. This means they will solve problems more quickly and correctly.</td><td>Your student will develop an understanding of a solution to an equation or the solution set of an inequality.</td><td>Your student will write, analyze, and solve two-step equations using positive and negative numbers on four-quadrant graphs.</td></tr>
<tr><td>Try it!
When you multiply any expression by –1, the result is the opposite of that expression.
What is the opposite of the expression
$x + 3$?

$[-(x + 3) = -x - 3]$</td><td>What in the world?
Speed limits are one example in the real world that can be represented as inequalities.

If x is the speed that you are driving, then $x \le 35$.</td><td>Did you know?
The coordinate plane is split into four regions called quadrants.
These quadrants are numbered with Roman numerals from one (I) to four (IV).
</td></tr>
</table>

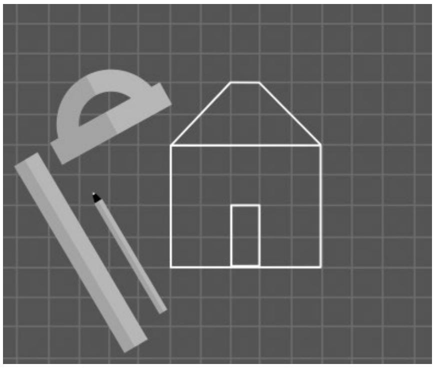
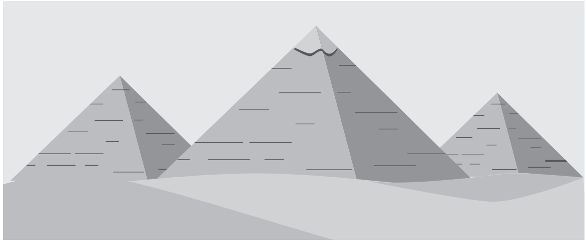
MODULE 4 Analyzing Populations and Probabilities

In this module, your student will deepen their understanding of probability. There are three topics in this module: *Introduction to Probability*, *Compound Probability*, and *Drawing Inferences*. Your student will use what they already know about data sets in this module.

TOPIC 1 Introduction to Probability	TOPIC 2 Compound Probability	TOPIC 3 Drawing Inferences
Your student will conduct simple experiments and determine theoretical and experimental probabilities of simple events.	Your student will use arrays and lists to organize the possible outcomes of an experiment that includes two simple events.	Your student will learn about samples, populations, censuses, parameters, and statistics.
Did you know?  If you flip a coin 100 times, there is an 8% chance of landing on each side exactly 50 times.	What in the world?  An array is a group of objects placed in rows and columns. They have many uses from computer science to advanced mathematics.	Birthday buddies!  In a room with 23 people, there is a more than 50% chance of two people having the same birthday, and in a group of 60 people, the probability is 99%!

MODULE 5 Constructing and Measuring

In this module, your student will deepen their understanding of angles and measures of two- and three-dimensional geometric objects. There are two topics in this module: *Angle Relationships* and *Area*, *Surface Area*, and *Volume*. Your student will use what they already know about two- and three-dimensional shapes in this module.

TOPIC 1 Angle Relationships	TOPIC 2 Area, Surface Area, and Volume
Your student will explore the relationships between 90° and 180° angles. They will write and solve equations involving the sum of the angles in a triangle and special angle relationships.	Your student will calculate the area, surface area, and volume of various two- and three-dimensional figures.
What in the world?  Angles are critical in the fields of architecture and construction. The corners of a standard door frame are exactly 90°, the most common roofs have a 90° design, and accurate angle measurements help to ensure that flooring fits perfectly along each wall in a building.	What in the world?  Pyramids were built all over the world by people thousands of years ago from Egypt to Mexico!

Lesson Structure

Each lesson in the course is laid out in the same way to develop deep understanding. Read through the parts of the lesson to learn more about your student's learning in their math classroom.

Objectives & Essential Question

Each lesson begins with objectives, listed to help students understand the focus of the lesson. Also included is an essential statement connecting students' learning with a question to ponder. The question is asked again at the end of each lesson to see how much your student understands.

Getting Started

The Getting Started engages your student in the learning. In the Getting Started, your student uses what they already know about the world, what they've already learned, and their intuition to get them thinking mathematically and prepare them for what's to come in the lesson.

Activities

In the Activities, students develop their mathematical knowledge and build a deep understanding of the math. These activities provide your student with the opportunity to communicate and work with others in their math classroom.

When your student is working through these activities, we encourage:

- It's not just about answer-getting. Doing the math and talking about it is important.
- Making mistakes is an important part of learning, so take risks.
- There is often more than one way to solve a problem.

Talk the Talk

The Talk the Talk gives your student an opportunity to reflect on the main ideas of the lesson and demonstrate their learning.

Lesson Assignment

The lesson assignment provides your students with practice to develop fluency and build proficiency. The lesson assignment also includes a section to help prepare students for the next lesson.

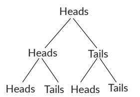
Key Concepts of the Lesson

At the end of the each topic, the Topic Summary provides a summary of each lesson in the topic. Encourage your student to use these as a tool to review and retrieve the key concepts of a lesson.

Supporting Your Student

Where are we now?

A **tree diagram** illustrates the possible outcomes of a given situation. It has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.



A **compound event** combines two or more events, using the word *and* or the word *or*.

Example:
Two friends are playing a game in which they each take turns rolling a six-sided number cube. To win, they must roll the same number twice in a row. In this case, winning is a compound event because it consists of two events that must occur.

In **Lesson 1: Using Arrays to Organize Outcomes**, students use arrays and lists to determine sample spaces and calculate probabilities.

Using a Number Array

To organize the outcomes for two events in a number array, list the outcomes for one event along one side and the outcomes for the other event along the other side. Combine the results in the intersections of each row and column.

MYTH

"I'm not smart."

The word *smart* is tricky because it means different things to different people. For example, would you say a baby is "smart?" On the one hand, a baby is helpless and doesn't know anything. On the other hand, a baby is exceptionally smart because they are constantly learning new things every day.

This example is meant to demonstrate that *smart* can have two meanings. It can mean, "the knowledge that you have," or it can mean, "the capacity to learn from experience." When someone says they are "not smart," are they saying they do not have a lot of knowledge, or are they saying they lack the capacity to learn? If it's the first definition, then none of us are smart until we acquire information. If it's the second definition, then we know that is completely untrue because everyone has the capacity to grow as a result of new experiences.

So, if your student doesn't think that they are smart, encourage them to be patient. They have the capacity to learn new facts and skills. It might not be easy, and it will take some time and effort. But the brain is automatically wired to learn. *Smart* should not refer only to how much knowledge you currently have.

#mathmythbusted

The Topic Family Guide

The Topic Family Guide provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides an example of a math model or strategy your student is learning in the topic, busting of a math myth, talking points to discuss and/or questions to ask your student, and all the new key terms your student will learn. You and your student can also use the Math Glossary to check terminology and definitions. Encourage your student to reference the new key terms in the Topic Family Guide and/or Math Glossary when completing math tasks.

Learning outside of the classroom is crucial for your student's success. While we don't expect you to be a math teacher, the Topic Family Guide can assist you as you talk to your student about the mathematical content of the course. The hope is that both you and your student will read and benefit from the guides.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning about probability for simple and compound events.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?

Math Myths

Math myths can lead students and adults to believe that math is too difficult for them, math is an unattainable skill, or that there is only one right way to do mathematics. The Math Myths section in the Topic Family Guides busts these myths and provides research-based explanations of why math is accessible to all students (and adults).

Examples of these myths include:
Myth Just give me the rule. If I know the rule, then I understand the math.

Memorize the following rule: *All quars are elos.* Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms.* How about now? Can you remember that? Of course you can because now it makes sense. Learning does not take place in a vacuum. It must be connected to what you already know. Otherwise, arbitrary rules will be forgotten.

Supporting Your Student

Myth There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. When one road is backed up, then you can always take a different route. When you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: Well, that's one way to do it. Is there another way? What are the pros and cons? That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work, or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

TEKS Mathematical Process Standards

Each module will focus on TEKS mathematical process standards that will help your student become a mathematical thinker. The TEKS mathematical process standards are listed below. Discuss with your student the "I Can" statements below the standards to help them develop their mathematical learning and understanding. With your help, your student can become a productive mathematical thinker.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Supporting Your Student

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:
- explain what a problem "means" in my own words.
- create a plan and change it when necessary.
- ask useful questions to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:
- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:
- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions when trying to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Create and use representations to organize, record, and communicate mathematical ideas.

I can:
- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Supporting Your Student

Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Reflecting on Learning and Progress

You can support your student by encouraging your student to reflect on the learning process. The instructional resources include a student Topic Self-Reflection for each topic. Encourage your student to accurately and frequently reflect on learning and progress throughout each topic. Talk about the specific concepts in the Topic Self-Reflection with your student and celebrate the progress from the beginning to the end of the topic. Remind your student to refer to the Topic Self-Reflection on Learning Individual Days after targeting specific skills and concepts. You can have your student explain concepts from the self-reflection using the topic summaries or lesson assignments to demonstrate understanding. In addition, encourage your student to reflect after taking an assessment. An Assessment Reflection is available to your student to assist with this process. Encourage your student to consider what went well and how to prepare for the next assessment. Ask your student how you can support them when preparing for the next assessment.

Supporting Your Student

TOPIC 2 SELF-REFLECTION

Name: ______________________

Compond Probability

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Compound Probability* topic by:

TOPIC 2: *Compound Probability*	Beginning of Topic	Middle of Topic	End of Topic
choosing the appropriate method, such as an organized list, a table, or a tree diagram, to represent sample spaces and create probability models for compound events.	○	○	○
representing the outcomes in the sample space that make up a compound event.	○	○	○
determining theoretical and experimental probabilities for compound events using data and sample spaces.	○	○	○
making predictions and determining solutions using experimental and theoretical probabilities for compound events.	○	○	○
designing and using a simulation to predict the probability of a compound event.	○	○	○

continued on the next page

Thanks!

Enjoy the fun mathematical adventure that is ahead for you and your student! Remember, the supports available to you and thank you for supporting your student's learning.

GRADE 7

MODULE 1

Thinking Proportionally

Family Guide

MODULE 1 Thinking Proportionally

Grade 7

TOPIC 1 Circles and Ratio

In this topic, students learn formulas for the circumference and area of circles and use those formulas to solve mathematical and real-world problems. To fully understand the formulas, students develop an understanding of the irrational number pi (π) as the ratio of a circle's circumference to its diameter. Throughout the topic, students practice applying the formulas for the circumference and area of a circle, often selecting the appropriate formula. Finally, students practice applying the formulas by using them to solve a variety of problems, including calculating the area of composite figures.

Where have we been?

Throughout elementary school, students used and labeled circles and determined the perimeters of shapes formed with straight lines. In Grade 6, students worked extensively with ratios and ratio reasoning. To begin this topic, students draw on these experiences as they use physical tools to investigate a constant ratio, pi.

Where are we going?

This early review of and experience with ratios prepares students for future lessons where they will move from concrete representations and reasoning about ratios and proportions to more abstract and symbolic work with solving proportions and representing proportional relationships. In future grades, students will use the circumference and area formulas of a circle to calculate surface areas and volumes of cylinders and composite three-dimensional shapes that include circles.

TALKING POINTS

Discuss With Your Student

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to think flexibly about mathematical relationships involving the constant ratio between a circle's circumference and its diameter, or pi (π), the circumference of a circle, and the area of a circle.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

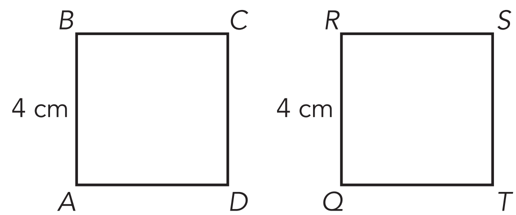
- congruent [congruente]
- circle [círculo]
- radius [radio]
- diameter [diámetro]
- circumference [circunferencia]
- pi [pi]
- unit rate
- composite figure [figura compuesta]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

Congruent means to have the same size, shape, and measurement.

Square *ABCD* is congruent to Square *QRST*.



A **unit rate** is a comparison of two different measurements in which the numerator or denominator has a value of one unit.

The speed 60 miles in 2 hours can be written as a unit rate:

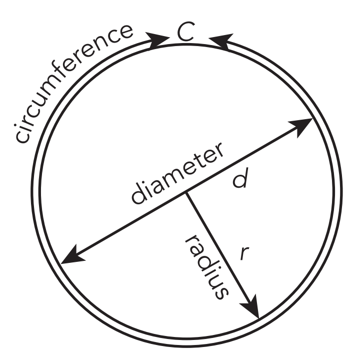
$$\frac{60 \text{ mi}}{2 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}}.$$

The unit rate is 30 miles per hour.

In **Lesson 1: Exploring the Ratio of Circle Circumference to Diameter** students learn formulas for the circumference and area of circles and use those formulas to solve mathematical and real-world problems.

Circles

To fully understand the formulas, students examine the number **pi** (π) as the ratio of a circle's **circumference** to its **diameter**.



circumference of the circle

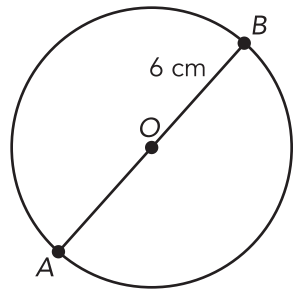
$$\pi = \frac{C}{d}$$

diameter of the circle

Circumference and Area

The distance around a circle is called the circumference of the circle and is calculated using the formula, $C = \pi d$, or $C = 2\pi r$. The formula used to determine the area of a circle is $A = \pi r^2$. Students need to choose the correct formula for a problem based on the information they know and the information they are trying to find.

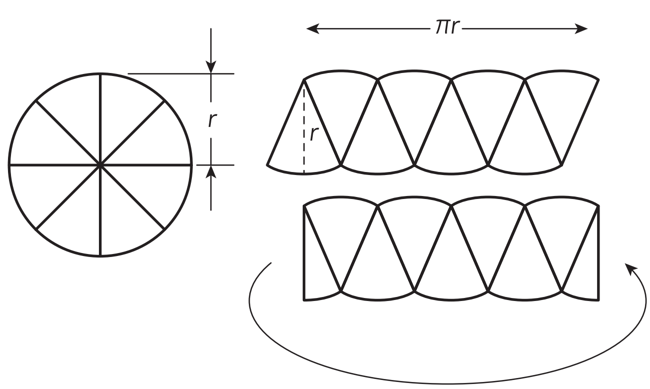
The diameter of Circle O is 12 centimeters. The circumference of Circle O is 12π centimeters. The area of Circle O is 36π square centimeters.



In **Lesson 2: Area of Circles**, students learn to see how the area of a circle relates to the area of a rectangle.

Modeling the Area of a Circle

You can divide a circle into a large number of equal-sized pieces. Laying these pieces as shown below, you can see that they almost make the shape of a rectangle. Notice the length of the rectangle and how it relates to what we know about the circle. The area of the rectangle is $\ell \cdot w = \pi r \cdot r = \pi r^2$. This helps students build the area formula for a circle, πr^2.



MYTH

"I don't have the math gene."

Let's be clear about something. There isn't **a** gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to our ability to reason mathematically. Some believe mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without any formal instruction. They can learn number sense and pattern recognition the same way.

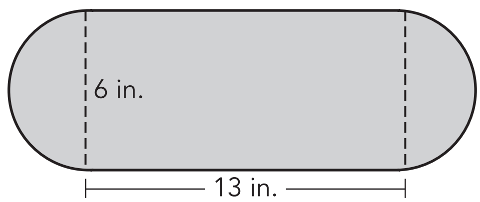
To further nurture your student's mathematical growth, attend to the learning environment. You can think of it as providing a nutritious mathematical diet that includes discussing math in the real world, offering the right kind of encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and giving them space for plenty of practice.

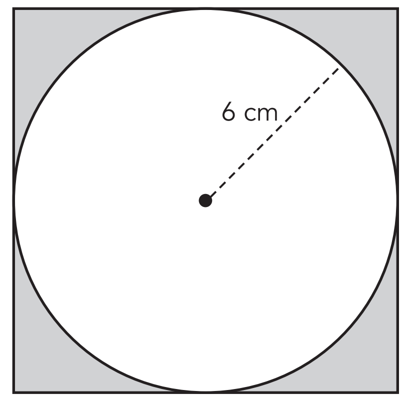
#mathmythbusted

In **Lesson 3: Solving Area and Circumference Problems**, students use the formulas to solve different kinds of problems, like calculating the area of **composite figures**.

Composite Figures

Students work with composite figures, which are made by putting together different shapes. They add or subtract to find the area of the light or dark part of the image.





TOPIC 2 Fractional Rates

In this topic, students extend their work with rates to include rates with fractional values. To begin the topic, students write, analyze, and use unit rates with whole numbers and fractions to solve problems. Next, students calculate and use unit rates from ratios of fractions. They use unit rates and proportions to convert between measurement systems. Finally, students review strategies for solving problems involving equivalent ratios and proportions.

Where have we been?

In Grade 6, students learned about ratios, rates, unit rates, and proportions, and they represented ratios and unit rates with tables and graphs. Students used a variety of informal strategies to compare ratios, determine equivalent ratios, and solve simple proportions (e.g., double number lines, scaling up and down by a scale factor, conversion factors).

Where are we going?

This topic broadens students' range of numbers and strategies for solving ratio and proportion problems, preparing them to dig deeper into representations of proportional relationships in the next topic and solving multi-step ratio and percent problems in future lessons.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to reason using fractional rates.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?

NEW KEY TERMS

- complex ratio [razón compleja]
- proportion [proporción]
- variable [variable]
- means [medios]
- extremes [extremos]
- solve a proportion [resolver una proporción]
- isolate the variable [aislar la variable]
- inverse operations [operaciones inversas]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **variable** is a letter or symbol used to represent a number.	$3x = 81$ $\frac{4}{p}$ z^2 variables
In a proportion written $a : b = c : d$, the two values on the outside, a and d, are the **extremes**. The two values on the inside, b and c, are the **means**.	$\underbrace{7\text{ books} : \overbrace{14\text{ days} = 3\text{ books}}^{\text{means}} : 6\text{ days}}_{\text{extremes}}$

In **Lesson 1: Unit Rate Representations** students extend their work with rates to include fractions.

Unit Rates

Students write, analyze, and use unit rates with whole numbers and fractions to solve problems.

In this example, the unit rate for traveling $\frac{1}{2}$ mile in $\frac{1}{4}$ hour is 2 miles per hour.

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1} \qquad \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$

$$\frac{2}{1} = 2 \qquad = \frac{1}{2} \cdot 4 = 2$$

In **Lesson 2: Solving Problems with Ratios of Fractions**, students calculate and use unit rates from ratios of fractions.

Converting Between Systems

They use unit rates and **proportions** to convert between measurement systems.

To convert between systems, you can scale up or scale down using ratios.

$\times 2.5$

$$\frac{1\text{ lb}}{0.45\text{ kg}} = \frac{2.5\text{ lb}}{1.125\text{ kg}}$$

$\times 2.5$

Write a ratio using the common conversion of 1 lb = 0.45 kg.

Scale up to calculate the number of kilograms in 2.5 pounds.

In **Lesson 3: Solving Proportions Using Means and Extremes**, students review strategies for solving problems involving equivalent ratios and proportions.

Means and Extremes

In the proportion $\frac{a}{b} = \frac{c}{d}$, the terms b and c are called the ***means***, and the terms a and d are called the ***extremes***.

extremes

$$3 : 4 = 9 : 12$$

means

$$(4)(9) = (3)(12)$$

or

$$\frac{3}{4} = \frac{9}{12}$$

means extremes

$$(4)(9) = (3)(12)$$

You can **solve a proportion** for an unknown variable using this method. First, identify the means and extremes. Then, set the product of the means equal to the product of the extremes. Finally, **isolate the variable** to solve for the unknown quantity.

$$\frac{\text{4 cups of granola}}{\text{1.5 cups of raisins}} = \frac{\text{18 cups of granola}}{x}$$

$$(1.5)(18) = (4)(x)$$

$$27 = 4x$$

$$\frac{27}{4} = \frac{4x}{4}$$

$$6.75 = x$$

There will be 6.75 cups of raisins used in 18 cups of granola.

MYTH

"If I can get the right answer, then I should not have to explain why."

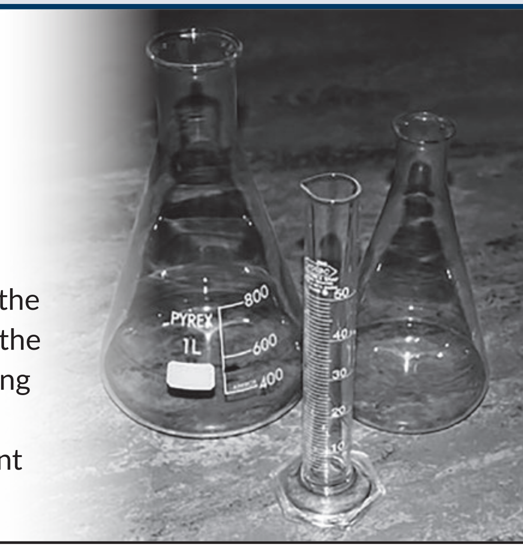
Sometimes you get the right answer for the wrong reasons. Suppose a student is asked, "What is 4 divided by 2?" and she confidently answers "2!" If she does not explain any further, then it might be assumed that she understands how to divide whole numbers. But, what if she used the following rule to solve that problem? "Subtract 2 from 4 one time." Even though she gave the right answer, she has an incomplete understanding of division.

However, if she is asked to explain her reasoning, either by drawing a picture, creating a model, or giving a different example, the teacher has a chance to remediate her flawed understanding. If teachers aren't exposed to their students' reasoning for both right and wrong answers, then they won't know about, or be able to address common misconceptions. This is important, because mathematics is cumulative in the sense that new lessons build upon previous understandings.

You should ask your student to explain their thinking, when possible, even if you don't know whether the explanation is correct. When children (and adults!) explain something to someone else, it helps them learn. Just the process of trying to explain is helpful.

#mathmythbusted

TOPIC 3 Proportionality



In this topic, students learn about the constant of proportionality: the ratio between the two quantities being compared. They recognize that the constant is determined by the order of the ratio elements, and they use proportions to write and analyze equations of directly proportional relationships. Students graph proportional relationships and determine the constant of proportionality from the graphs, interpreting this constant, the unit rate, in terms of the problem situation. Students practice determining if relationships are proportional, interpreting the meaning of linear proportional relationships, and determining and interpreting the constant of proportionality.

Where have we been?

In Grade 6, students developed a strong understanding of ratio and rate reasoning, including reasoning about equivalent ratios from graphs and tables. In the previous topic, students reviewed some of these basic ideas and developed a formal strategy for solving proportions.

Where are we going?

Students will continue to apply the constant of proportionality to solve multi-step ratio and percent problems in the next topic. They will solve percent problems using the constant of proportionality and directly proportional relationships and relate the constant of proportionality to the scale factor in scale drawings. The characteristics of proportional relationships, their graphs, and their equations provide the underpinnings of algebra and the study of functions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to reason using proportions.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- proportional relationship [relación proporcional]
- origin [origen]
- constant of proportionality [constante de proporcionalidad]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **proportional relationship** is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$ must represent the same constant.
A situation represents a **proportional relationship** if the ratio between the y-value and its corresponding x-value is constant for every point. You can say the quantities vary proportionally. If Isaiah earns \$8.25 per hour, then the amount he earns varies proportionally with the number of hours he works. The amount \$8.25 is the **constant of proportionality**.

In **Lesson 1: Proportional Relationships**, students learn about the constant of proportionality, which is the ratio between the two quantities being compared.

Comparing Two Quantities

They recognize that the constant is connected to how the numbers are placed in order, and they use proportions to write and analyze equations of proportional relationships.

Time (days)	Height of Bamboo (cm)
3	210
10.5	735
18	1260
25.5	1785

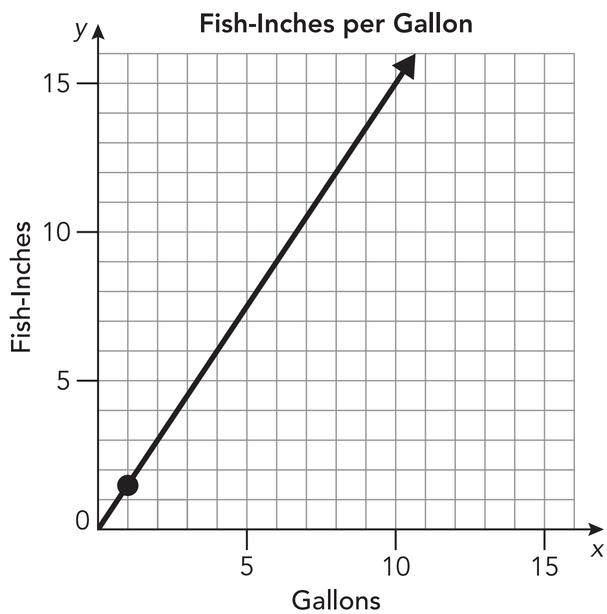
If it took 3 days for the bamboo to grow to 210 centimeters, then the constant of proportionality tells how tall it grew in one day.

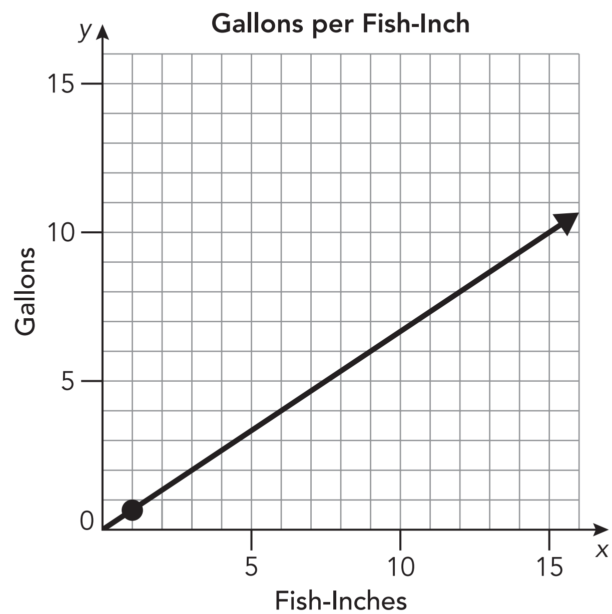
$$\frac{210\text{ cm}}{3\text{ days}} \quad \frac{210\text{ cm}}{3\text{ days}} \overset{\div 3}{\underset{\div 3}{=}} \frac{70\text{ cm}}{1\text{ day}}$$

The bamboo grew 70 centimeters in one day.

In **Lesson 3: Identifying the Constant of Proportionality in Graphs**, students graph proportional relationships and determine the constant of proportionality from the graphs in terms of the problem situation.

Interpreting the Constant of Proportionality



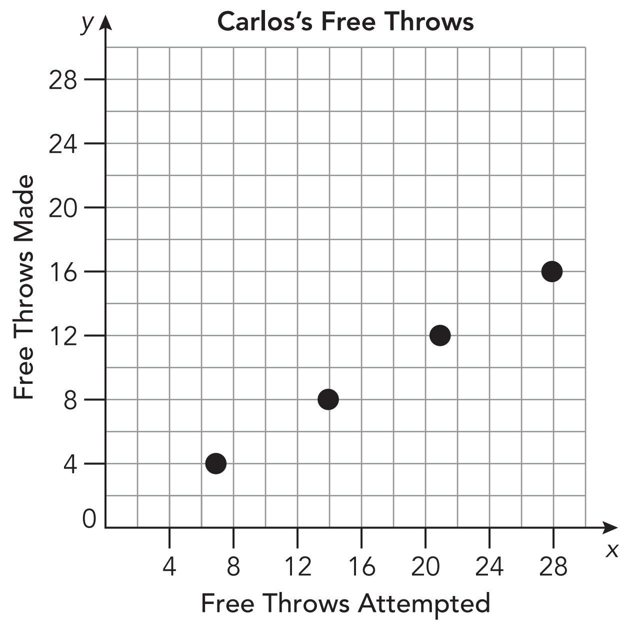


- The point $\left(1, 1\frac{1}{2}\right)$ on the Fish-Inches per Gallon graph represents the unit rate: $1\frac{1}{2}$ fish-inches per gallon.
- The point $\left(1, \frac{2}{3}\right)$ on the Gallons per Fish-Inch graph represents the unit rate: $\frac{2}{3}$ gallon per fish-inch.

In **Lesson 4: Constant of Proportionality in Multiple Representations**, students look at relationships in tables, graphs, words, and equations and decide if a relationship is proportional.

Relationships

If a relationship is proportional, students identify and explain the constant of proportionality.



The graph shows Carlos's total number of free-throw attempts and the total number of free throws made.

Explain how you know the graph represents a proportional relationship.

Determine the constant of proportionality and describe what it represents in the problem situation.

MYTH

Asking questions means you don't understand.

It is universally true that, for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. But there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more on a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, *everyone* should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

GRADE 7

MODULE
2

Applying Proportionality

Family Guide

MODULE 2 Applying Proportionality

Grade 7

TOPIC 1 Proportional Relationships

In *Proportional Relationships*, students use their knowledge of proportionality to solve real-world problems about money and scale drawings. They solve a wide variety of multi-step ratio and percent problems, including problems about tax, markups and markdowns, gratuities, simple interest, commissions, and scale factors and drawings. Students use percent models, proportions, and the constant of proportionality to solve markup and markdown problems. In addition to considering scenarios involving money, students calculate percent increase and decrease using geometric objects.

Where have we been?

In Grade 6, students used ratio strategies, including models and forming equivalent ratios, to solve percent problems involving determining the whole given a part and the percent. In previous lessons in this course, students learned and practiced solving proportions using means and extremes.

Where are we going?

Students learn financial literacy skills related to taxes and fees, commissions, markups and markdowns, including sales, rebates and coupons, tips, simple interest, and percent increase and decrease including depreciation. They learn how to use proportional reasoning to estimate, calculate, and judge the reasonableness of results of everyday percent problems they will encounter throughout their lives.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by discussing the importance of taking time to step back and consider a different strategy or approach that can help when they are stuck.

QUESTIONS TO ASK

- What strategy are you using?
- What is another way to solve the problem?
- Can you draw a model?
- Can you come back to this problem after doing some other problems?

NEW KEY TERMS

- markdown
- markup
- sale
- coupon [cupón]
- rebate
- percent equation [ecuación porcentual]
- simple interest [interés simple]
- commission [comisión]
- sales tax
- income tax
- percent increase
- percent decrease
- appreciation [apreciación]
- depreciation [depreciación]
- scale [escala]
- scale factor [factor de escala]
- corresponding [correspondiente]
- scale drawing
- similar figures [figuras semejantes/similares]

Refer to the Math Glossary for definitions of the New Key Terms.

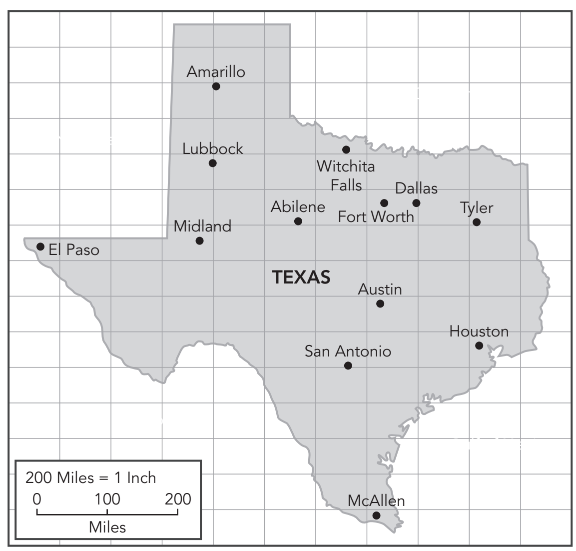
Where are we now?

Simple interest is a type of interest that is a fixed percent of the principal. Simple interest is paid over a specific period of time—either twice a year or once a year, for example. The formula for simple interest is $I = P \cdot r \cdot t$, where I represents the interest earned, P represents the amount of the principal, r represents the interest rate, and t represents the time that the money earns interest.

Kim deposits $300 into a savings account at a simple interest rate of 5% per year. The formula can be used to calculate the simple interest Kim will have earned at the end of 3 years.

$$\begin{aligned} \text{Interest} &= \text{Principal} \cdot \text{Rate} \cdot \text{Time} \\ \text{Interest} &= (300)(0.05)(3) \\ &= \$45 \end{aligned}$$

A **scale drawing** is a representation of a real object or place that is in proportion to the real object or place it represents.



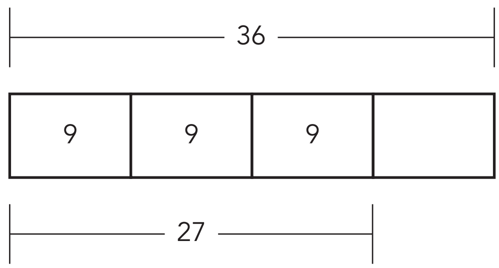
You can use the map scale to determine the distance between cities.

In **Lesson 1: Introducing Proportions to Solve Percent Problems**, students solve percent problems using proportions.

Markups and Markdowns

Students represent a percent scenario using a strip diagram.

For example, suppose a dress originally costs \$36. Daniela pays \$27 for the dress during a sale. What percent does Daniela save with the sale?



The amount Daniela saves is one-fourth the total amount of the dress, so she saves 25%. You can also write a percent as a proportion. For example, using the same scenario above, you can set up the proportion, $\frac{\text{part}}{\text{whole}} = \frac{\text{percent number}}{100}$.

$$\frac{x}{100} = \frac{27}{36}$$

$$36x = (100)(27)$$

$$\frac{36x}{36} = \frac{2700}{36}$$

$$x = 75$$

Daniela paid 75% of the cost, so she saved 25%.

To make money, businesses often buy products from a wholesaler or distributor for one amount and add to that amount to determine the price they use to sell the product to their customers. This increase in price is the **markup**. For example, a store marks up all of its prices by 25% to sell to its customers. If the store's cost for an item is \$16, what is the customer's cost?

$$\frac{25}{100} = \frac{x}{16}$$

$$x = 4$$

The customer's cost is \$16 + \$4 = \$20.

In **Lesson 2: Calculating Tips, Commissions, and Simple Interest**, students are introduced to percent equations.

Percent Equation

Students use a **percent equation** to determine things like calculating tips, commission, and sales or income tax. A percent equation can be written as *percent* · *whole* = *part*, where the percent is often written as a decimal.

MYTH

There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. If one road is backed up, then you can always take a different route. If you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: Well, that's one way to do it. Is there another way? What are the pros and cons? That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work, or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

#mathmythbusted

For example, suppose you want to leave a 15% tip on a restaurant bill of $45.

		percent	=	$\frac{\text{part}}{\text{whole}}$
percent	·	whole	=	part
(tip as a percent)	of	(total bill)	=	amount of tip
↑		↑		↑
$\frac{15}{100}$ or 0.15	·	45	=	t
		6.75	=	t

The amount of tip you should leave is $6.75.

In **Lesson 3: Sales Tax, Income Tax, and Fees,** students calculate sales tax, state income, and federal income tax.

Sales Tax

The amount of sales tax paid varies by state. However, the process of calculating sales tax is the same.

Sales Tax
Amir wants to buy a pair of shoes for $60. The sales tax in his state is 6%. What is the total price Rick will pay for the shoes, including sales tax?
Multiply the list price by (1 + 0.06). $60 · 1.06 = $63.60
Amir will pay $63.60 in toal.

In **Lesson 4: Percent Increase and Decrease**, students compute percent increase and percent decrease in several real-world situations.

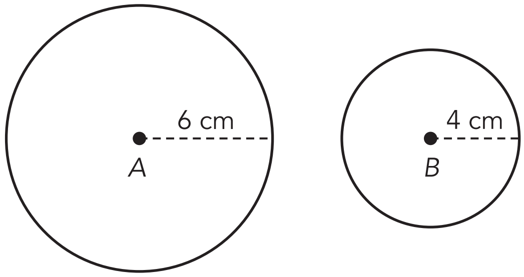
Percent Increase and Decrease

A **percent increase** occurs when the new amount is greater than the original amount, like when stores mark up, or increase, the price they pay for an item to make a greater profit. A percent increase is calculated as a ratio of the amount of increase to the original amount.

A **percent decrease** occurs when the new amount is less than the original amount. A percent decrease is calculated as a ratio of the amount of decrease to the original amount.

Generally, things such as homes and savings accounts gain value, or appreciate, over time. **Appreciation** is an increase in price or value. Other things, such as cars, depreciate every year. **Depreciation** is a decrease in price or value.

You can use percent increases and decreases when thinking geometrically. For example, what is the percent decrease in the area from Circle A to Circle B?



The area of Circle $A = 36\pi$ square units.
The area of Circle $B = 16\pi$ square units.

$$36\pi - 16\pi = 20\pi$$

$$\frac{20\pi}{36\pi} \approx 0.56$$

The percent decrease in the area is about 56%.

In **Lesson 5: Scale and Scale Drawings,** students use scale models to calculate measurements and enlarge and reduce the size of models.

Scale and Scale Factor

A **scale** is a ratio that compares two measures. When you multiply a measure by a scale to produce a reduced or enlarged measure, the scale is called a **scale factor.**

For example, Triangle A is an equilateral triangle with side lengths of 4 units. If Triangle A is reduced by 50% to create Triangle B, what are the side lengths of Triangle B? You can multiply each of Triangle A's lengths by the scale factor 1 : 2, or 50%, or $\frac{1}{2}$, to produce the side lengths for Triangle B.

Triangle *A*			Triangle *B*
side length $\longrightarrow$ 4 units	$\cdot$	1 : 2 or $\frac{1}{2}$ or 50%	=	2 units $\longleftarrow$ side length

$\uparrow$

scale,
scale factor

Family Guide

MODULE 2 Applying Proportionality Grade 7

TOPIC 2 Financial Literacy: Interest and Budgets

In *Financial Literacy: Interest and Budgets*, students focus on solving financial literacy problems about interest, budgets, and net worth. Students begin by calculating and comparing simple interest and compound interest earnings on investments. Next, students identify the different components of a personal budget and calculate the percent each category comprises of the total budget. They learn that net worth is determined by the difference in assets and liabilities. Then, students use a family budget estimator to determine the minimum household budget and wages needed for a family to meet its most basic needs.

Where have we been?

Students build off their understanding from Grade 6 of calculating the percent of a number in order to work with simple and compound interest. In the previous topic, *Proportional Relationships*, students were introduced to simple interest, but now they calculate compound interest and compare the process and earnings to simple interest.

Where are we going?

In Grade 8, students will build off of the work in Grade 7 on simple and compound interest, but this time, use it to investigate how interest rate and loan length affect the cost of credit. Students will calculate the total cost of repaying a loan, including credit cards, under various rates of interest over different periods of time. The focus on financial literacy right now is about understanding the benefits of financial responsibility.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning about the benefits of financial responsibility. Help them make connections between the work they are doing at school and the financial decisions you make at home.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- principal [principal]
- simple interest [interés simple]
- compound interest [interés compuesto]
- asset
- liability
- 401(k) plan [plan 401(k)]
- 403(b) plan [plan 403(b)]
- net worth
- personal budget
- fixed expenses
- variable expenses
- family budget estimator

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **401(k) plan** is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account with the employer often matching a certain amount of it.
A **family budget estimator** is a tool that people can use to determine the estimated cost of raising a family in a particular city.

In **Lesson 2: Net Worth Statements**, students are introduced to net worth statements, assets, and liabilities.

Assets and Liabilities

Assets include the value of all accounts, investments, and things that you own. They are positive and add to your net worth. **Liabilities** are financial obligations, or debts, that you must repay. They are negative and take away from your net worth.

Let's consider this example. Michael creates a list of all his accounts and obligations.

Accounts/Obligations

- Mortgage
- Credit Cards
- Savings Account
- 401(k) Plan
- School Loans
- Car

Michael's mortgage, credit cards, and school loans are liabilities because he must repay these. His savings account, 401(k) plan, and car are assets because they are things that he owns.

A net worth statement is a useful tool to measure your financial health from year to year. Your **net worth** is a calculation of the value of everything that you have minus the amount of money that you owe. A net worth statement includes this calculation as well as a detailed list of everything used to determine net worth.

For example, Olivia's accounts are shown:

Checking Account: $2876	Student Loan: $9560	Credit Card: $980
401(k) Account: $14,432	Car Loan: $18,680	Savings Account: $5500

Here is a list of Olivia's assets and liabilities:

Assets		**Liabilities**	
Type	**Amount**	**Type**	**Amount**
Checking Account	$2876	Student Loan	$9560
401(k)	$14,432	Credit Card:	$980
Savings Account	$5500	Car Loan	$18,680
Total:	$22,808	Total:	$29,220

Net worth = Assets − Liabilities
Net worth = $22,808 − $29,220 = −$6412.
Olivia's net worth is − $6412. The negative net worth means Olivia owes more than she owns.

In **Lesson 3: Personal Budgets**, students are introduced to the concept of a personal budget.

Budgets and Expenses

A **personal budget** is an estimate of the amount of money that a person or family will need for specific items. It includes current expenses as well as savings for future expenses. These expenses are often categorized as *fixed expenses* and *variable expenses*. **Fixed expenses** are expenses that don't change from month to month. **Variable expenses** are expenses that can be different from month to month. Each expense represents a percent of the total budget.

MYTH

Students only use 10% of their brains.

Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of some science fiction movies that ask the audience: *Imagine what you could accomplish if you could use 100% of your brain!*

Well, this isn't Hollywood. The good news is that you *do* use 100% of your brain. As you look around the room, your *visual cortex* is busy assembling images, your *motor cortex* is busy moving your neck, and all of the *associative areas* recognize the objects that you see. Meanwhile, the *corpus callosum*, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts . . . like mathematics!

#mathmythbusted

For example, Javier's family's monthly expenses are shown. Students can figure out the percent of each expense compared to the total amount of monthly expenses.

Fixed Expenses	Variable Expenses
• Mortgage: \$1150 • Utilities: \$320 • Savings: \$600	• Food: \$450 • Miscellaneous: \$250

Total expenses: $1150 + 320 + 600 + 450 + 250 = 2770$

Mortgage: $\frac{1150}{2770} \approx 0.42 = 42\%$

Utilities: $\frac{320}{2770} \approx 0.12 = 12\%$

Savings: $\frac{600}{2770} \approx 0.22 = 22\%$

Food: $\frac{450}{2770} \approx 0.16 = 16\%$

Miscellaneous: $\frac{250}{2770} \approx 0.09 = 9\%$

For Javier's family, their mortgage represents 42% of their budget, their utilities represent 12% of their budget, their savings represent 22% of their budget, their food represents 16% of their budget, and miscellaneous costs represent 9% of their budget.

In order to provide for yourself or your family, you must earn enough to cover the costs of all your expenses. It is important to remember that taxes are also taken out of your earnings, so you might need to earn slightly more than your expenses to maintain your budget.

Fernando's family's yearly expenses are about \$98,500. They pay 25% of their income in taxes.

$$98{,}500 = 0.75x$$
$$x = \frac{98{,}500}{0.75}$$
$$x = 131{,}333.3333$$

Fernando's family must earn at least \$131,333.33 to maintain their yearly expenses.

MODULE

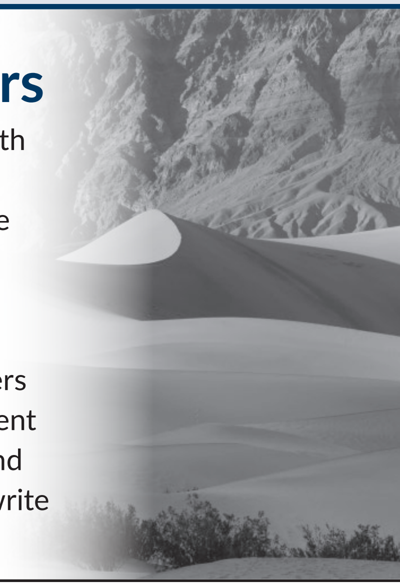
3

Reasoning Algebraically

Family Guide

MODULE 3 Reasoning Algebraically

Grade 7



TOPIC 1 Operating With Rational Numbers

In this course, students will extend previous understandings around operating with positive rational numbers and integers to now build fluency in operating with the full set of rational numbers. Students begin this topic by applying their knowledge of adding and subtracting positive and negative integers to the set of rational numbers. Next, students divide integers, resulting in rational numbers, given the divisor is not 0. They learn that the decimal form of quotients of integers always repeat or terminate. Next, they apply the rules for multiplying and dividing integers to the set of rational numbers in the context of problem solving. Students represent variable expressions on a number line and make connections between variable and numeric expressions. They then apply the distributive property as a strategy to write equivalent expressions and factor linear expressions in a variety of ways.

Where have we been?

In Grade 6, students represented integer operations with concrete models and connected the actions with the models to standardized algorithms. Then, they worked on building fluency in using all four operations with integers. In this course, they will build off of the foundation developed around operating with positive rational numbers and operating with integers, to now building fluency in operating with the full set of rational numbers. This topic combines students' knowledge of expressions and negative numbers on a number line to develop number line models for variable expressions.

Where are we going?

It is essential that students develop a strong conceptual foundation for operating with rational numbers, as a basis for manipulating and representing increasingly complex numeric and algebraic expressions. In future courses, students will focus more on expressions and equations than on numbers, including rational expressions, equations, and functions. Visualizing simple variable expressions on a number line will carry through the entire topic to help students develop a concrete idea relating expressions to each other and operating with algebraic expressions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is building fluency in operating with the full set of rational numbers.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- percent error [error porcentual]
- variable [variable]
- algebraic expression [expresión algebraica]
- linear expression [expresión lineal]
- constraint
- evaluate an algebraic expression [evaluar una expresión algebraica]
- factor [factor]
- coefficient [coeficiente]
- common factor [factor común]
- greatest common factor (GCF) [máximo común factor/divisor]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **variable** is a letter or symbol that is used to represent a number.	$3x = 81$ $\quad \frac{4}{p}$ $\quad z^2$ variables
A number that is multiplied by a variable in an algebraic expression is called a **coefficient**.	$14x$ $\quad \frac{1}{3}(g)$ $\quad \pi d$ $\quad w + 2.5$ coefficient The coefficient is 1 even though it is not shown.

In **Lesson 2: Quotients of Integers**, students divide integers, resulting in rational numbers, given the divisor is not 0.

Negative Rational Numbers

Students learn that a negative rational number can be represented using a negative sign before the fraction or in either the numerator or the denominator.

$$-\frac{2}{5} = \frac{-2}{5} = \frac{2}{-5}$$

A fraction with a negative sign in both the numerator and the denominator is positive.

$$\frac{-3}{-8} = \frac{3}{8}$$

In **Lesson 3: Simplifying Expressions to Solve Problems**, students use their knowledge of operating with rational numbers to solve real-world problems. They evaluate expressions with variables and learn about percent error.

Percent Error

Percent error is one way to tell the difference between estimated values and actual values.

$$\text{percent error} = \frac{\text{actual value} - \text{estimated value}}{\text{actual value}}$$

For example, an airline estimates that they will need an airplane that seats 416 passengers for the 8 a.m. flight from Austin to Orlando. Calculate the percent error if 380 actual passengers are booked.

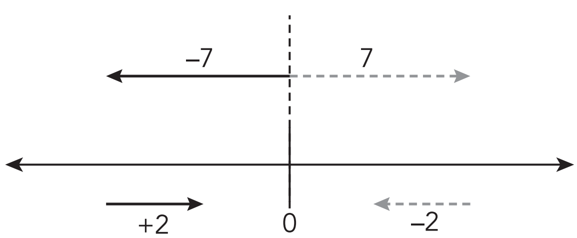
$$\frac{380 - 416}{380} = \frac{-36}{380} \approx -9.5\%$$

The airline had 9.5% fewer passengers than they expected.

In **Lesson 4: Using Number Properties to Interpret Expressions with Signed Numbers**, students interpret expressions and use reflections across 0 on the number line to determine the opposite of an expression.

Reflections of Expressions

Consider the expression $-7 + 2$. When the model of $-7 + 2$ is reflected across 0 on the number line, the result is $7 - 2$.



Therefore, $(-7 + 2)$ is the opposite of $(7 - 2)$.

This means that $-7 + 2 = -(7 - 2)$.

In **Lesson 5: Evaluating Algebraic Expressions**, students represent variable expressions on a number line and make connections between variable and numeric expressions. They use their previous knowledge of evaluating expressions to verify, or check, their reasoning.

MYTH

Just watch a video, and you will understand it.

Has this ever happened to you? Someone explains something, and it all makes sense at the time. You feel like you get it. But then, a day later when you try to do it on your own, you suddenly feel like something's missing? If that feeling is familiar, don't worry. It happens to us all. It's called the illusion of explanatory depth, and it frequently happens after watching a video.

How do you break this illusion? The first step is to try to make the video interactive. Don't treat it like a TV show. Instead, pause the video and try to explain it to yourself or to a friend. Alternatively, attempt the steps in the video on your own and rewatch it if you hit a wall. Remember, it's easy to confuse familiarity with understanding.

#mathmythbusted

Linear Expressions

A linear expression is a type of expression where each term is a constant number or the product of a constant number and a variable. Linear expression variables must be raised to the first power.

An example of a linear expression is $x + 1$.

Evaluate Expressions

To **evaluate an algebraic expression**, you replace each variable in the expression with a number or numeric expression and perform all possible mathematical operations.

For example, evaluate $\frac{1}{2}b + 2$ for $b = 8$.
Substitute the value for the variable. $\rightarrow$ $\frac{1}{2}(8) + 2$
Use the order of operations to simplify. $\rightarrow$ $4 + 2 = 6$

In **Lesson 6: Rewriting Expressions Using the Distributive Property**, students use the distributive property as a strategy to write equivalent expressions and factor linear expressions in different ways.

Factor Expressions

To **factor an expression** means to rewrite the expression as the product, or multiplication, of its factors.

For example, you can **factor** the expression $2x + 2$ and rewrite it as the product of two factors.

$$2x + 2 = 2(x + 1)$$

The Distributive Property

The **distributive property** states that if a, b, and c are real numbers, then $a(b + c) = ab + ac$. The property also applies if addition is replaced with subtraction: $a(b - c) = ab - ac$.

For example, use the distributive property to rewrite the expression $3(b + 2)$ in an equivalent form.

$$3(b + 2) = (3)(b) + (3)(2) = 3b + 6$$

Common Factors

A **common factor** is a number or an expression that is a factor of two or more numbers or algebraic expressions. In other words, if two numbers can be divided by the same number, that number is a common factor because it can be multiplied into both of the other numbers.

For example, in the expression $7(26) + 7(14)$, the number 7 is a common factor of both $7(26)$ and $7(14)$. The expression $7(26) + 7(14)$ can be factored and rewritten as $7(26 + 14)$.

Greatest Common Factor

The distributive property can also be used to factor algebraic expressions. When factoring algebraic expressions, you can factor out the greatest common factor from all the terms. The **greatest common factor (GCF)** is the largest factor that two or more numbers or terms have in common.

For example, consider the expression $12x + 42$. The greatest common factor of $12x$ and 42 is 6. Therefore, you can rewrite the expression as $6(2x + 7)$.

When factoring an expression, examine the structure of the expression first. If the expression contains a negative leading coefficient, you can include the negative with the value that you factor out.

For example, consider the expression $-2x + 8$. The greatest common factor is 2 and the leading coefficient it negative. So you can factor out -2.

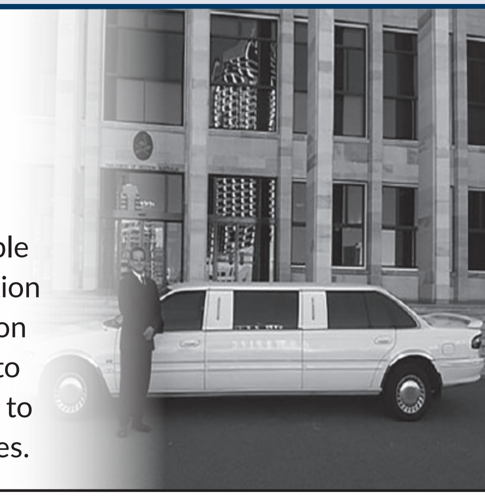
$$\begin{aligned} -2x + 8 &= (-2)x + (-2)(-4) \\ &= -2(x - 4) \end{aligned}$$

Family Guide

MODULE 3: Reasoning Algebraically

Grade 7

TOPIC 2 Two-Step Equations and Inequalities



Students begin this topic by reasoning with bar models and algebra tiles to write and solve equations. Next, they use a double number line with variable expressions. Throughout these reasoning exercises, the meaning of a solution to an equation is reinforced. Students check their solutions with substitution and write equations from solutions. Students then use inverse operations to solve equations. Students extend their understanding of solving equations to solving two-step inequalities and graphing the solution sets on number lines.

Where have we been?

Students encountered variable equations and used models to solve one-step equations and inequalities in a previous course. Work in this topic builds on students' knowledge of expressions and equations to introduce two-step equations and inequalities.

Where are we going?

In future courses, students will solve a wide variety of linear equations and inequalities, eventually using their knowledge of equations, inequalities, and solutions to solve non-linear equations and inequalities. The foundation of understanding developed in this course will enable students to work with a wider range of equations in the future, with a clear knowledge of what it means to solve an equation and an understanding of the reasoning behind those procedures for solving equations.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Your student is learning to solve two-step equations and inequalities. Encourage your student to take their time as they work through these problems. Suggest that they create a visual representation, such as a bar model or double number line. While this may seem time consuming, it can help solidify the connections between algebraic expressions.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- equation [ecuación]
- two-step equation

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

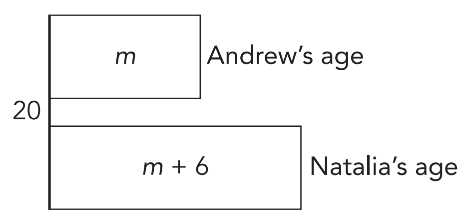
An **equation** is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.	$y = 2x + 4$ $6 = 3 + 3$ $2(8) = 26 - 10$ $\frac{1}{4} \cdot 4 = \frac{8}{4} - \frac{4}{4}$
A **two-step equation** requires that two inverse operations be performed to isolate the variable.	

In **Lesson 1: Modeling Equations as Equal Expressions**, students model real-world situations using picture algebra and define equations as representing equal expressions.

Models

You can create a model to represent equal expressions. For example, Natalia is 6 years older than Andrew. The sum of their ages is 20. You can represent the model you drew with a mathematical sentence using operations and an equals sign. An equation is a mathematical sentence created by placing an equal sign (=) between two expressions.

The equation that represents the model is
$20 = m + (m + 6)$, or $20 = 2m + 6$.



A **solution to an equation** is a value for the unknown that makes the equation true.

For example, the solution to the equation $20 = 2m + 6$ is $m = 7$.

$$20 = 2(7) + 6$$
$$20 = 14 + 6$$
$$20 = 20$$

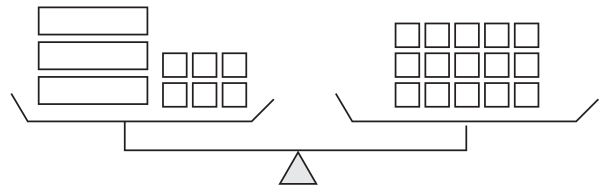
Andrew is 7 years old, and Natalia is 13 years old.

In **Lesson 2: Solving Equations Using Algebra Tiles**, students use algebra tiles and apply balance strategies to numeric equations containing a single variable.

Balancing Expressions

You can think of an equation as a balance of two equal math expressions.

For example, this balance shows 3 rectangles and 6 squares on the left side. This is equal to, or balanced with, 15 squares on the right side. What will balance one rectangle?



If you subtract 6 squares from both sides, you maintain balance. Then, each of the 3 rectangles on the left must be balanced with 3 squares on the right.

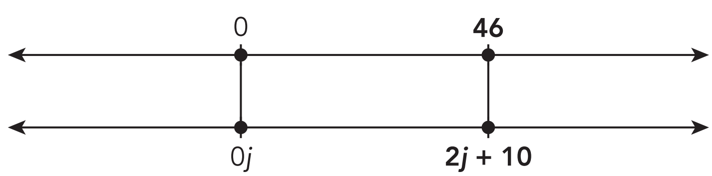
So, 3 squares balance 1 rectangle.

In **Lesson 3: Solving Equations on a Double Number Line**, students model contextual and mathematical situations using double number lines.

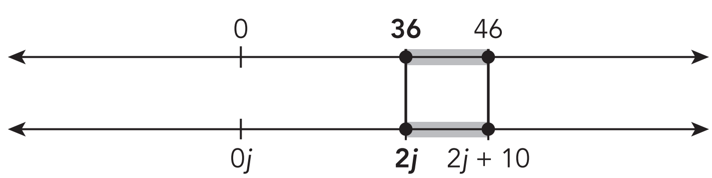
Double Number Lines

You can use double number lines to help you solve equations. When solving an equation, equality must be maintained. What is done to one expression must be done to the equivalent expression to maintain equality.

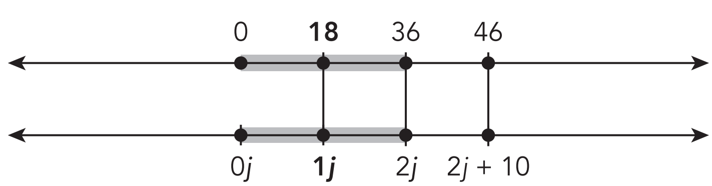
For example, solve the equation $2j + 10 = 46$. First, draw a model to set up the equation.



Next, start decomposing the variable expression. Place $2j$ in relation to $2j + 10$. The expression $2j$ is 10 to the left of $2j + 10$. To maintain equality, place a number 10 to the left of 46. So, $2j = 36$.



The expression $1j$, or j, is halfway between $0j$ and $2j$, and 18 is halfway between 0 and 36. So, $j = 18$.



MYTH

"Just give me the rule. If I know the rule, then I understand the math."

Memorize the following rule: *All quars are elos*. Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms*. How about now? Can you remember that? Of course you can, because now it makes sense.

Learning does not take place in a vacuum. It ***must be*** connected to what you already know. Otherwise, arbitrary rules will be forgotten.

#mathmythbusted

In **Lesson 4: Using Inverse Operations to Solve Equations**, students learn the formal strategies for solving two-step equations.

Two-Step Equations

A two-step equation requires two inverse operations, or applying two properties of equality, to isolate the variable.

For example, here is one way to solve the equation	$2x + 6 = 13$.
Subtract 6 from each side of the equation.	$2x + 6 - 6 = 13 - 6$
Divide both sides of the equation by 2.	$\frac{2x}{2} = \frac{7}{2}$

The solution is $x = 3\frac{1}{2}$.

In **Lesson 5: Using Inverse Operations to Solve Inequalities**, students learn the formal strategies for solving two-step inequalities.

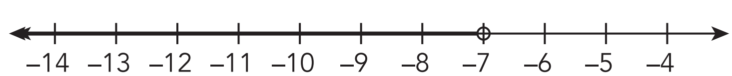
Solving Inequalities

To solve an inequality means to determine the values of the variable that make the inequality true. Solving two-step inequalities is similar to solving two-step equations, except for the fact that when you are solving an inequality and multiply or divide by a negative value, you must reverse the inequality symbol.

For example, solve the inequality $-3x + 7 > 28$.

$$-3x + 7 - 7 > 28 - 7$$
$$-3x > 21$$
$$\frac{-3x}{-3} < \frac{21}{-3}$$
$$x < -7$$

The solution to any inequality can be represented on a number line by a ray in which its starting point is an open or closed circle. For example, the solution $x < -7$ is represented by this number line. Notice that an open circle is used to represent that -7 is not included in the solution. If the inequality $x \leq -7$ was being represented, then a closed circle, or solid black dot, would be used to show that -7 is included in the solution.



TOPIC 3 Multiple Representations of Equations

This topic broadens students' perspective on solving and interpreting linear equations and inequalities through the use of tables and graphs. Students write and solve two-step equations using positive and negative numbers on four-quadrant graphs. Students then practice solving problems by writing equations and inequalities for problem situations, analyzing tables and graphs to solve the equations or inequalities, and interpreting the quantities in each problem situation.

Where have we been?

In previous courses, students used multiple representations to model and solve problems. They learned that quantities can vary in relation to each other and are often classified as independent and dependent quantities.

Where are we going?

Students' ability to use symbolic algebra can be supported through the use of visual representations. Using and connecting symbolic and graphical representations of equations and inequalities occurs throughout the study of functions in future mathematics courses.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to represent relationships involving the equivalence of values in a variety of ways.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERM

- unit rate of change

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

The **unit rate of change** describes the amount the dependent variable changes for every unit the independent variable changes.

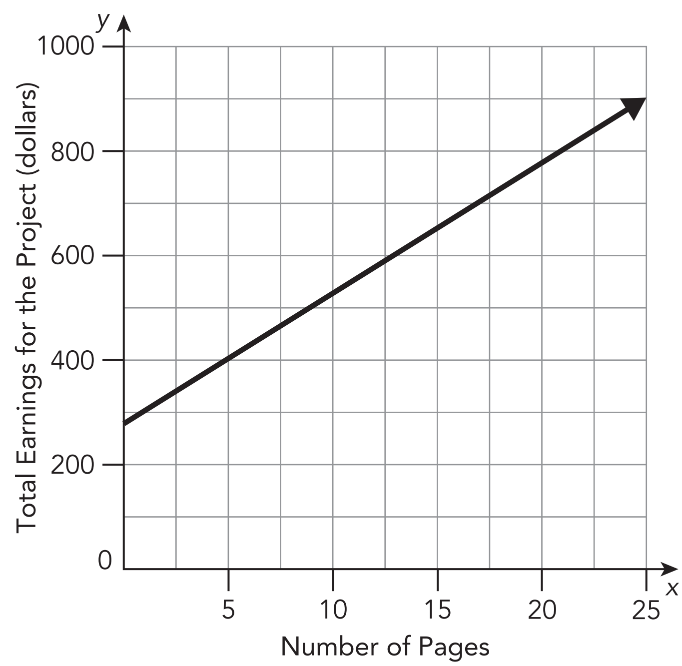
In **Lesson 1: Representing Equations with Tables and Graphs**, students analyze linear equations using tables and graphs.

Representing Problems

You can represent a problem situation in many ways. For example, Ms. Patel translates books for a living. Her earnings can be represented by a verbal description, table, graph, and equation.

Verbal description: Ms. Patel charges an initial fee of $275 to manage a project and $25 per page of translated text.

Equation: $y = 275 + 25x$



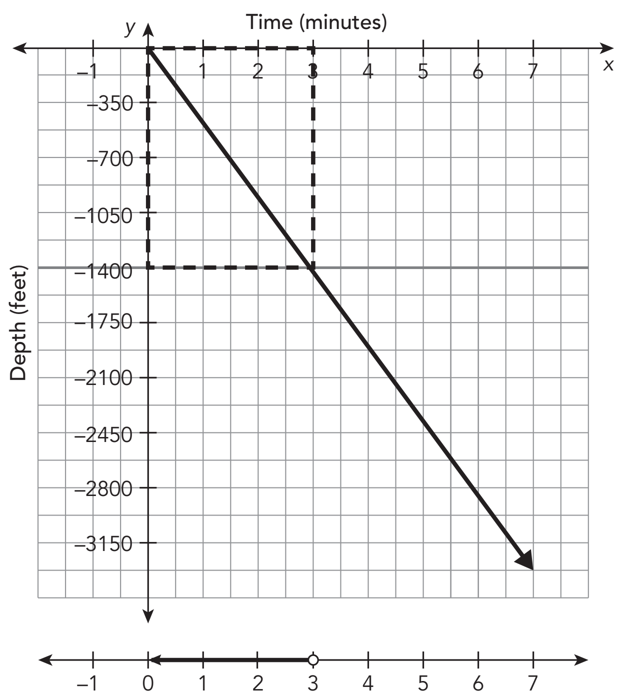
Number of Pages	Total Earnings for the Project (dollars)
1	300
3	350
10	525
25	900

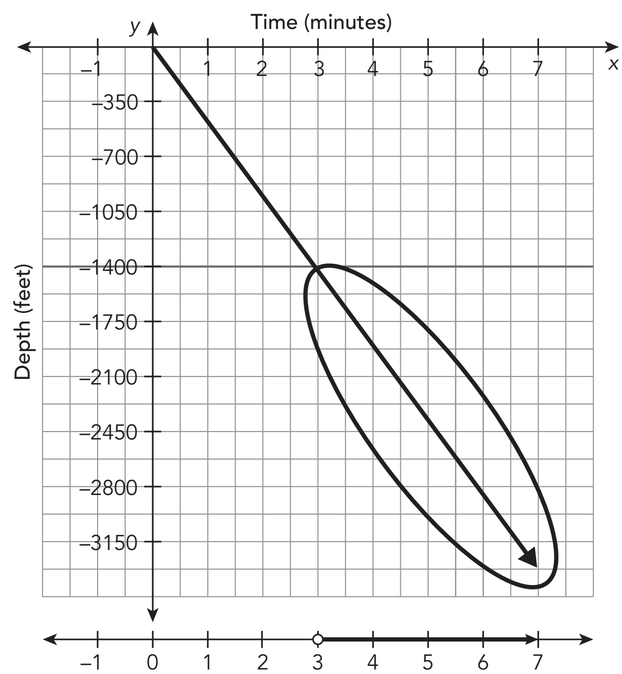
To solve a linear equation from a graph, locate the value of the given variable, independent or dependent, and determine the exact, if possible, or estimated point corresponding to that variable. For example, you can use the graph to determine that Ms. Patel will earn $400 if she translates 5 pages for a customer. She will earn approximately $775 for translating 20 pages.

In **Lesson 2: Building Inequalities and Equations to Solve Problems**, students work with a negative rate of change.

Unit Rate of Change

The unit rate of change is the amount that the dependent value changes for every one unit that the independent value changes. For example, suppose the submarine Deep Flight I is going to do a dive starting at sea level, descending 480 feet every minute. The unit rate of change is −480 feet per minute. You can use a graph to estimate solutions to inequality problems. Estimate the times Deep Flight I will be more than 1400 feet below sea level and the times Deep Flight I will be less than 1400 feet below sea level. Each of these graphs shows the relationship between the time in minutes and the depth of Deep Flight I. The rectangle on the left graph shows the set of all depths for Deep Flight I less than 1400 feet below sea level. The oval on the right graph shows the set of all depths for Deep Flight I more than 1400 feet below sea level.





Deep Flight I will be less than 1400 feet below sea level for times less than 3 minutes. The submarine will be more than 1400 feet below sea level for times greater than 3 minutes.

MYTH

Memory is like an audio or video recording.

Let's play a game. Memorize the following list of words: strawberry, grape, watermelon, banana, orange, peach, cherry, blueberry, raspberry. Got it? Good. Some believe that the brain stores memories in pristine form. Memories last for a long time and do not change—like a recording. Without looking back at the original list, was apple on it?

If you answered "yes," then go back and look at the list. You'll see that apple does not appear, even though it seems like it should. In other words, memory is an active, reconstructive process that takes additional information, like the category of words (e.g., fruit), and makes assumptions about the stored information.

This simple demonstration suggests memory is not like a recording. Instead, it is influenced by prior knowledge and decays over time. Therefore, students need to see and engage with the same information multiple times to minimize forgetting.

#mathmythbusted

In **Lesson 3: Using Multiple Representations to Solve Problems**, students put together all that they have learned about the different representations of a linear relationship.

Multiple Representations

Multiple representations, such as a table, an equation, and a graph, can be used to represent a problem situation. You may start with any of these representations to solve a problem and move from one to another by studying their forms and determining unit rates of change.

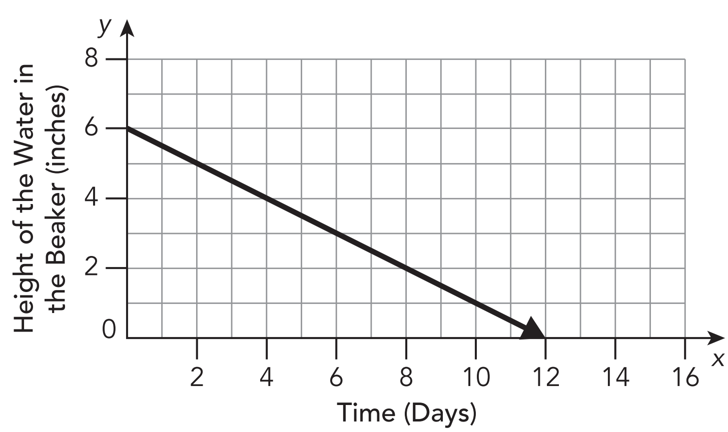
For example, suppose you are given this table of values.

You can use the values in the table to represent the problem situation with a graph, equation, and verbal description.

Equation: $y = 6 - 0.5x$

Verbal description: The height of the water in the beaker begins at 6 inches. The height of the water decreases by 0.5 inch each day.

Time	Height of the Water in the Beaker
Days	Inches
0	6
1	5.5
4	4
8	2

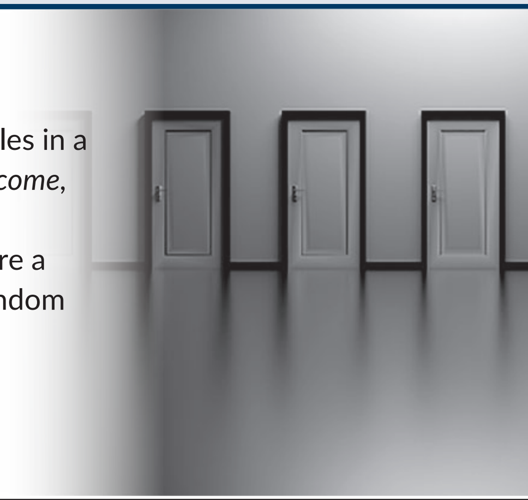


MODULE 4

Analyzing Populations and Probabilities

Family Guide

TOPIC 1 **Introduction to Probability**



In this topic, students use familiar objects, such as number cubes, marbles in a bag, and spinners, to learn the terminology of probability, including *outcome*, *experiment*, *sample space*, *event*, *simple event*, *probability*, *complementary events*, and *equally likely*. For real-world probability situations that require a large number of trials, students use simulation techniques, including random number tables, to simulate the results of experiments.

Where have we been?

This topic is students' formal introduction to probability, but they have encountered probability situations throughout their lives. The topic opens with asking students to interpret the meaning of a meteorologist's forecast. They use their intuition of the meaning of "chance of rain" and rewrite the percent as a fraction.

Where are we going?

In an upcoming topic, students will use probability and ideas of randomness to explore sampling and drawing inferences about data, which is the start of the formal study of statistical inference. The basic ideas developed in this topic will be used in the next topic on compound probability.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by resisting the urge, as long as possible, to get to the answer in a problem that your student is working on. Probability is a tricky concept. Students will need time and space to struggle with all the implications of thinking about events in terms of their probabilities. Practice asking good questions when your student is stuck.

QUESTIONS TO ASK

- Let's think about this. What are all the things you know?
- What do you need to find out?
- How can you model this problem?

NEW KEY TERMS

- outcome
- experiment [experimento]
- sample space
- event [evento]
- simple event [evento simple]
- probability [probabilidad]
- complementary events [eventos complementarios]
- equally likely
- probability model [modelo probabilístico]
- uniform probability model [modelo probabilístico uniforme]
- non-uniform probability model [modelo probabilístico no uniforme]
- theoretical probability [probabilidad teórica]
- experimental probability [probabilidad experimental]
- percent error [error porcentual]
- simulation [simulación]
- random number table

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **uniform probability model** occurs when all the probabilities in a probability model are equally likely to occur.

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

A **non-uniform probability model** occurs when all the probabilities in a probability model are not equal to each other.

Outcome	Red	Green	Blue
Probability	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{3}{8}$

In **Lesson 1: Defining and Representing Probability**, students calculate probabilities rolling number cubes, using spinners, and drawing marbles from a bag.

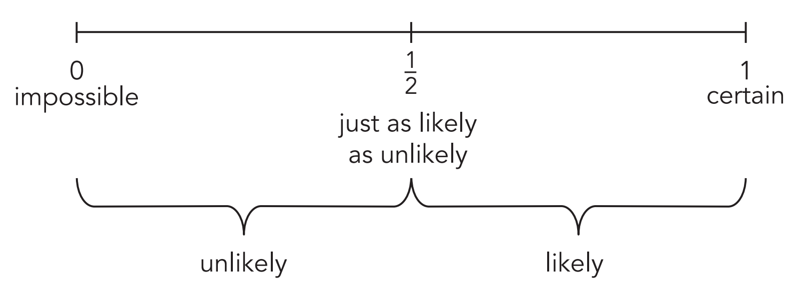
Probability

Probability is a measure of the likelihood that an event will occur. The probability of an event is often written as *P*(event).

In the number cube experiment, the probability of rolling a 5 could be written as *P*(5), and the probability of rolling an even number could be written as *P*(even).

The probability of an event occurring is a number between 0 and 1. If the event is certain to happen, then the probability is 1. If an event is impossible, then the probability is 0. If an event is just as likely to happen as not happen, then the probability is 0.5, or $\frac{1}{2}$.

The number line shown represents the probabilities, from 0 to 1, of any event occurring.



Complementary Events

Complementary events are events that together contain all of the outcomes in the sample space. If P(even) represents the probability of rolling an even number, then P(not even) is the complementary event. When the probabilities of all the outcomes of an experiment are equal, then the outcomes are called **equally likely**.

In **Lesson 3: Determining Experimental Probability of Simple Events**, students flip a coin multiple times to determine the probabilities of heads and tails based on the results of the experiment.

Theoretical and Experimental Probability

The **theoretical probability** of an event is the ratio of the number of desired outcomes to the total possible outcomes. **Experimental probability** is the ratio of the number of times an event occurs to the total number of trials performed.

$$\text{experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of trials performed}}$$

Proportional Reasoning

If you know the probability of an event, you can use proportional reasoning to predict the number of times the event will occur throughout an experiment.

For example, the probability that a spinner will land on blue is $\frac{2}{3}$. If you spin the spinner 60 times, you can set up and solve a proportion to predict the number of times the spinner will land on the blue section.

$$\begin{aligned} \frac{2}{3} &= \frac{x}{60} \\ 2(60) &= 3x \\ \frac{120}{3} &= \frac{3x}{3} \\ 40 &= x \end{aligned}$$

If you spin the spinner 60 times, you can expect it to land on blue 40 times.

MYTH

Cramming for an exam is just as good as spaced practice for long-term retention.

Everyone has been there. You have a big test tomorrow, but you've been so busy that you haven't had time to study. So, you had to learn it all in one night. You may have gotten a decent grade on the test. However, did you remember the material a week, a month, or a year later?

The honest answer is, "probably not." That's because long-term memory is designed to retain useful information. How does your brain know if a memory is "useful" or not? One way is the frequency in which you encounter a piece of information. If you only see something once (like during cramming), then your brain doesn't deem those memories as important. However, if you sporadically come across the same information over time, then it's probably important. To optimize retention, encourage your student to periodically study the same information over expanding intervals of time.

#mathmythbusted

In **Lesson 4: Simulating Simple Experiments**, students explore simulations and describe simulation models that fit each situation.

Simulation

A **simulation** is an experiment that models a real-world situation. When conducting a simulation, you must choose a model that has the same probability of the event. Using simulations to generate experimental probabilities is very useful in estimating the probability of an event for which the theoretical probability is hard, or impossible, to calculate. For example, one way to simulate the event of a litter of puppies being comprised of three females is to use 3 coin flips; let heads represent a female, and let tails represent a male. This is assuming that the theoretical probability of a female being born is equal to the theoretical probability of a male being born, which is $\frac{1}{2}$.

Random Number Tables

You can design and carry out a simulation for an experiment using a **random number table**. A random number table is a table that displays random digits. You assign a range of numbers to each outcome that models the same probability of an event and then choose any line from the table to perform a trial.

For example, in a five-question multiple-choice test, each question has five possible answer choices. How many questions can you expect to get correct simply by guessing?

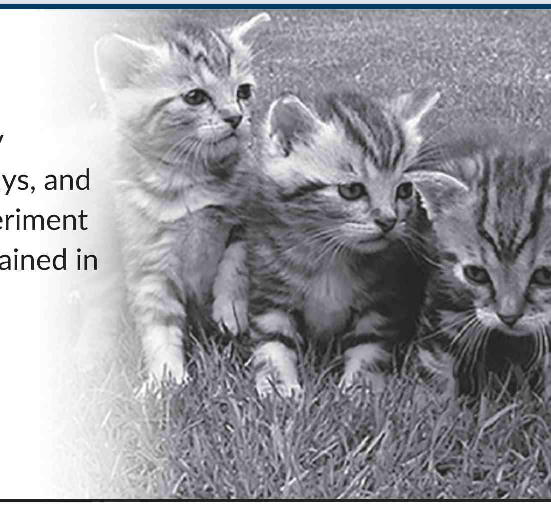
Each answer choice has a 20% chance of being selected, but only $\frac{1}{5}$ of the guesses are correct, while the others are incorrect. Let the numbers 00–19 represent correct guesses, and 20–99 represent incorrect guesses.

Line 4	12645	62000	61555	76404	86210
	11808	12841	45147	97438	60022

The numbers 12, 64, 56, 20, and 00 are chosen. This corresponds to correct, incorrect, incorrect, incorrect, correct. In this experiment, the correct answer was guessed 2 times out of 5.

TOPIC 2 Compound Probability

In this topic, students build on their understandings of the probability concepts from the previous topic. Students create tree diagrams, arrays, and lists to organize and represent the total possible outcomes of an experiment that includes two simple events. Students list outcomes that are contained in a compound event, distinguishing between *and* and *or* situations.



Where have we been?

Students have used arrays to represent relationships among numbers. Here, they apply their knowledge of arrays to represent outcomes from conducting two simple events simultaneously. Throughout this topic, they reinforce and deepen their understanding of probability concepts learned in the previous topic—simple events, experimental versus theoretical, making predictions, and simulation.

Where are we going?

Students will continue to further their understanding of probability concepts as they explore random sampling and draw inferences about a population. This topic provides students with the opportunity to build intuition about compound events. In future courses, students will engage with probability from a more formal and formula-driven perspective as they learn about mutually exclusive events and conditional probability.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning about probability for simple and compound events.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?

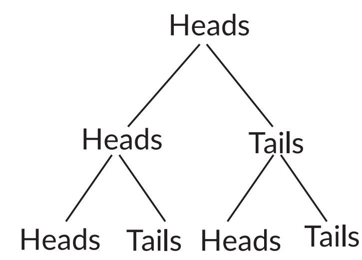
NEW KEY TERMS

- tree diagram
- compound event

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **tree diagram** illustrates the possible outcomes of a given situation. It has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.



A **compound event** combines two or more events, using the word *and* or the word *or.*

Example:
Two friends are playing a game in which they each take turns rolling a six-sided number cube. To win, they must roll the same number twice in a row. In this case, winning is a compound event because it consists of two events that must occur.

In **Lesson 1: Using Arrays to Organize Outcomes**, students use arrays and lists to determine sample spaces and calculate probabilities.

Using a Number Array

To organize the outcomes for two events in a number array, list the outcomes for one event along one side and the outcomes for the other event along the other side. Combine the results in the intersections of each row and column.

For example, this array shows the sample space—all the possible outcomes—for rolling two 6-sided number cubes and calculating the product of the numbers shown.

		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

The probability of the product being 6 is $\frac{4}{36}$, or $\frac{1}{9}$.

In **Lesson 2: Using Tree Diagrams**, students use tree diagrams as a method to determine the theoretical probability of an event.

Tree Diagrams

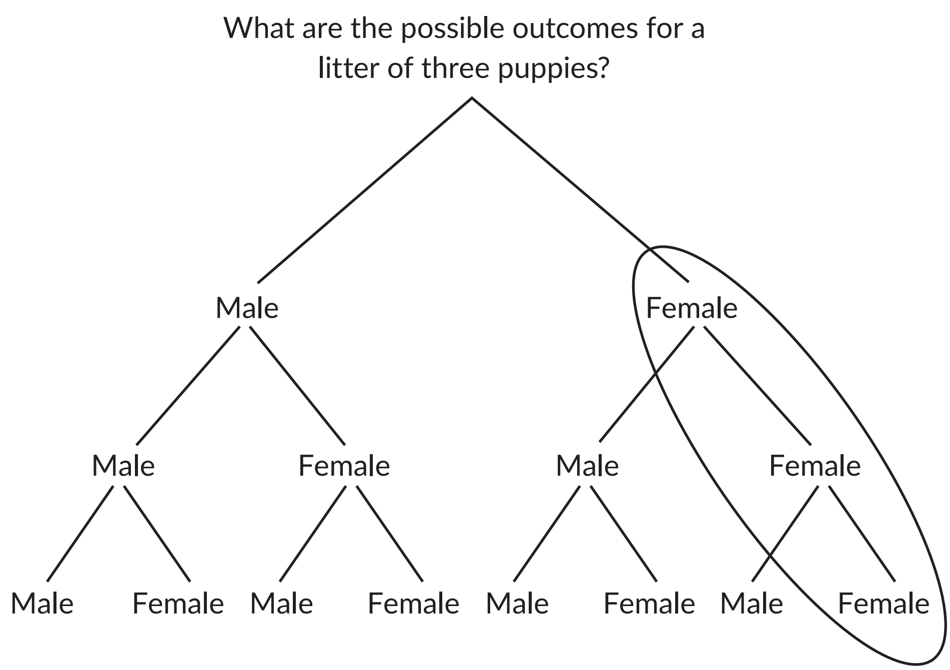
A **tree diagram** illustrates the possible outcomes of a given situation. Tree diagrams can be constructed vertically or horizontally. A tree diagram has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

For example, you can construct a tree diagram to show all the possible outcomes for a litter of three puppies.

List the possible outcomes of the 1st puppy.

List the possible outcomes of the 2nd puppy.

List the possible outcomes of the 3rd puppy.



You can use a tree diagram to determine the probability of an event. In the tree diagram above, there are 8 possible outcomes for a litter of three puppies. The probability for a litter having three female puppies is $\frac{1}{8}$.

MYTH

"I'm not smart."

The word *smart* is tricky because it means different things to different people. For example, would you say a baby is "smart?" On the one hand, a baby is helpless and doesn't know anything. On the other hand, a baby is exceptionally smart because they are constantly learning new things every day.

This example is meant to demonstrate that *smart* can have two meanings. It can mean, "the knowledge that you have," or it can mean, "the capacity to learn from experience." When someone says they are "not smart," are they saying they do not have a lot of knowledge, or are they saying they lack the capacity to learn? If it's the first definition, then none of us are smart until we acquire information. If it's the second definition, then we know that is completely untrue because everyone has the capacity to grow as a result of new experiences.

So, if your student doesn't think that they are smart, encourage them to be patient. They have the capacity to learn new facts and skills. It might not be easy, and it will take some time and effort. But the brain is automatically wired to learn. *Smart* should not refer only to how much knowledge you currently have.

#mathmythbusted

In **Lesson 3: Determining Compound Probability**, students explore compound events and compound probability.

Compound Events

Determining the probability of a compound event with the word *and* is different from determining the probability of a compound event with the word *or.*

The difference is that a compound event with the word *and* means that you are determining the probability that both events occur.

In **Lesson 4: Simulating Probability of Compound Events**, students design and conduct simulations that model three situations.

Using Random Numbers for Simulations

Many events involve very advanced rules for probability. In most cases, a simulation can be used to model the event. A random number table or technology can be used to simulate compound probability in many of these events.

For example, the preferences of customers who rent movies online are provided in the table below. You can design and conduct a simulation to predict the probability that out of the next 10 customers to rent a movie, at least 3 rent a comedy.

Movie Type	Comedy	Drama	Science Fiction	Documentary
Percent of Customers	31%	42%	22%	5%

You could use a random number table for this simulation. First, assign each movie type a range of two-digit numbers that correspond to the percent of customers who prefer that type.

- Comedy: 00–30
- Drama: 31–72
- Science Fiction: 73–94
- Documentary: 95–99

Then, run trials of the experiment. Each trial consists of selecting 10 two-digit numbers from the random number table. The experimental probability of at least 3 of the next 10 customers renting a comedy is the number of trials that contained at least 3 the numbers from 00 to 30 divided by the total number of trials.

Family Guide

MODULE 4 Analyzing Populations and Probabilities Grade 7

TOPIC 3 Drawing Inferences

In this topic, students continue developing their understanding of the statistical process. They use random sampling methods to learn about samples, populations, censuses, parameters, and statistics. Throughout this topic, students create data displays for data, such as bar graphs, circle graphs, dot plots, box plots, stem-and-leaf plots, and histograms. Students create data displays for two populations to compare the measures of center and the measures of variation. They make inferences and draw conclusions about two populations using generated random samples or provided data.

Where have we been?

Students have used aspects of the statistical problem-solving process: formulating questions, collecting data, analyzing data, and interpreting the results. They have also used numerical data displays, including both measures of center (mean, median, mode), and measures of variation (range and interquartile range). In the previous topic, students used a random number table to generate data.

Where are we going?

In future courses, students will learn about specific types of random sampling and the inherent bias in sampling techniques. They will continue analyzing and comparing random samples from populations using measures of center and measures of variation.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can support your student's learning by approaching problems slowly. Students may observe a classmate learning things very quickly, and they can easily come to believe that mathematics is about getting the right answer as quickly as possible. When this doesn't happen for them, future encounters with math can cause anxiety, making problem solving more difficult, and reinforcing a student's view of themselves as "not good at math." Slowing down is not the ultimate cure for math difficulties but it's a good first step for children who are struggling. You can reinforce the view that learning with understanding takes time, and that slow, deliberate work is the rule, not the exception.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- survey
- data [datos]
- population [población]
- census [censo]
- sample
- parameter [parámetro]
- statistic [estadística]
- random sample

Refer to the Math Glossary for definitions of the New Key Terms.

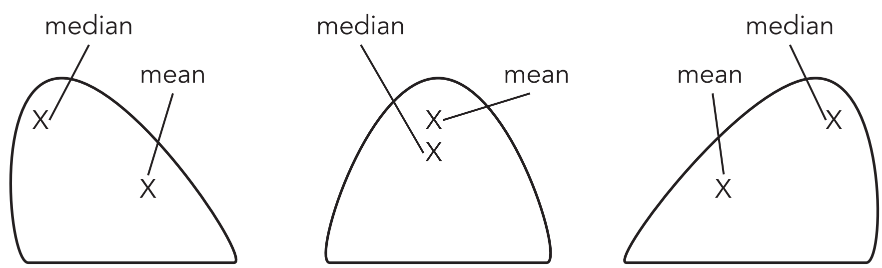
Where are we now?

A **sample** is a selection from a population. If you wanted to determine the average height of the students in your school, you could choose a certain number of students and measure their heights. The heights of the students in this group would be your sample.
A **census** is the data collected from every member of a population.

In **Lesson 2: Using Random Samples to Draw Inferences**, students use statistical information, gathered from a sample, to determine a parameter for a population.

Distributions of Data

There are three common distributions of data: skewed left, skewed right, and symmetric. The distribution of data can help you decide whether the mean or median is a better measure of center.

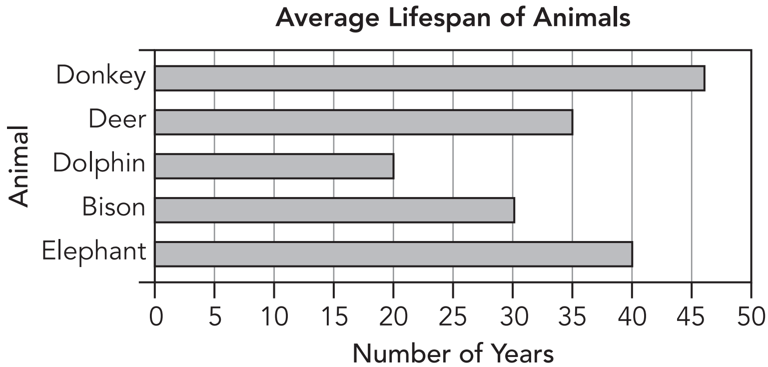


skewed right	**symmetric**	**skewed left**
The mean of a data set is greater than the median when the data are skewed to the right. The median is the better measure of center because the median is not affected by very large data values.	The mean and median are equal when the data are symmetric.	The mean of a data set is less than the median when the data are skewed to the left. The median is the better measure of center because the median is not affected by very small data values.

In **Lesson 3: Bar Graphs**, students analyze categorical data presented as single bar graphs, double bar graphs, stacked bar graphs, and circle graphs.

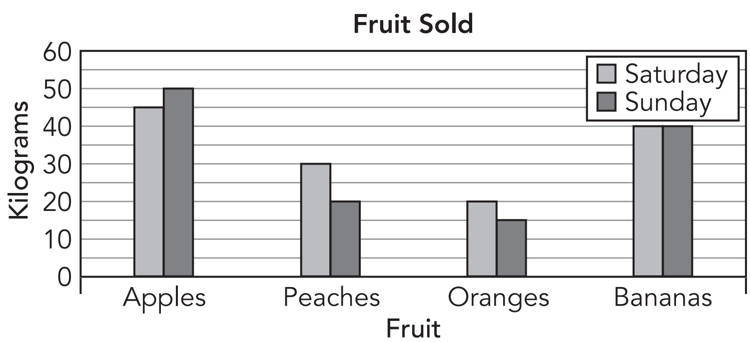
Bar Graphs

Some graphs are used to display data that consists of different categories. A bar graph displays data using horizontal or vertical bars, so that the height or length of the bars indicates the value for a specific category.



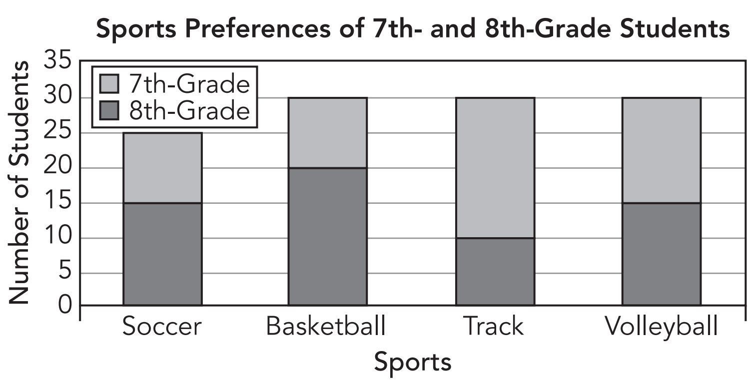
Double Bar Graphs

A double bar graph can be used when each category contains two different groups of data. The bars may be vertical or horizontal, and a key explains the colors or patterns for each group. The two bars representing the same category are side by side, and space is used to separate the categories.



Stacked Bar Graphs

A stacked bar graph is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole. Each bar represents a total for the whole category but still shows the data for each group within the entire category.



MYTH

Faster = smarter

In most cases, speed has nothing to do with how smart you are. Why is that? Because it largely depends on how familiar you are with a topic. For example, a bike mechanic can look at a bike for about 8 seconds and tell you details about the bike that you probably didn't even notice (e.g., the front tire is on backwards). Is that person smart? Sure! Suppose, instead, you show the same bike mechanic a car. Will they be able to recall the same amount of detail as for the bike? No!

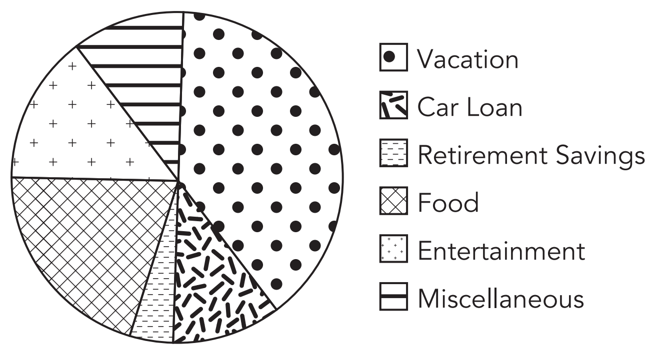
It's easy to confuse speed with understanding. Speed is associated with the memorization of facts. Understanding, on the other hand, is a methodical, time-consuming process. Understanding is the result of asking lots of questions and seeing connections between different ideas. Many mathematicians who won the Fields Medal (i.e., the Nobel prize for mathematics) describe themselves as extremely slow thinkers. That's because mathematical thinking requires understanding over memorization.

#mathmythbusted

Circle Graphs

You can also use circle graphs to represent the relationship between each part and the whole. For example, this circle graph shows the expenses for families in random cities across the state. As shown in the graph, families tend to spend the greatest amount of money on vacation and the least amount of money on retirement savings.

Average Family Expenses



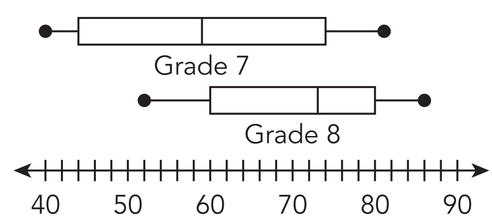
In **Lesson 4: Comparing Two Populations**, students calculate the measures of center and measures of variation for two different populations.

Comparing Populations

You can use the means to compare two **populations** with approximately symmetric data sets. Similarly, you can use the medians and the interquartile ranges to compare two populations with skewed data sets.

Grade 7 Leaf	Stem	Grade 8 Leaf
4 3 0	4	
9 6 4	5	2 7
6	6	0 1 9
6 4 3	7	6 7 9
1	8	5 6

Box Plot



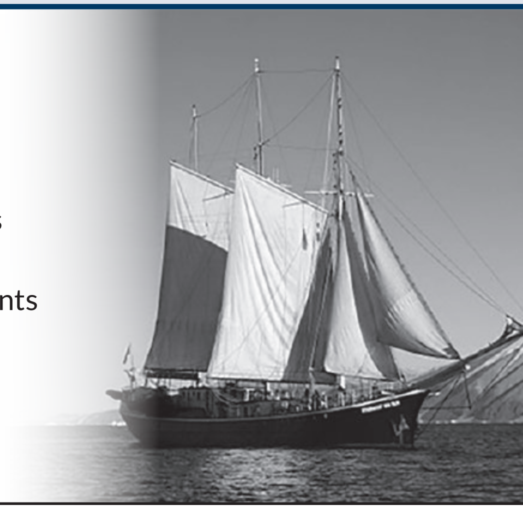
MODULE
5

Constructing and Measuring

Family Guide

MODULE 5 Constructing and Measuring | Grade 7

TOPIC 1 Angle Relationships



In this topic, students build on their existing knowledge of triangles and angles. They write and solve equations involving the sum of angles in a triangle, including isosceles triangles. Students explore the relationships between 90° and 180° angles. They use a protractor to explore the relationship between complementary and supplementary angles. Students explore different pairs of angles, including linear pairs, vertical angles, and adjacent angles. They use patty paper to create angles and explore relationships between them. Students then write and solve equations involving the sum of angles in angle relationships.

Where have we been?

Students have indentified angles as right, obtuse, and acute to classify triangles and to classify two-dimensional figures based on the presence or absence of specified angle measures. They have learned the Triangle Sum theorem. Students will use prior knowledge of angles and angle relationships as well as writing and solving single variable two-step equations from earlier in this course.

Where are we going?

In future courses, students will build from analyzing what happens when two lines intersect (e.g., vertical angles, supplementary angles) to analyzing what happens when more than two lines intersect (e.g., alternate interior angles, same side interior angles). They are expected to build a strong foundation with simple angle pairs to engage with the more complex relationships that result when more than two lines intersect.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is continuing to reason about geometric objects and angle relationships.

QUESTIONS TO ASK

- Can you use a diagram to model and solve this problem?
- How can you use what you know about geometric figures to help you solve this problem?
- Does your answer seem reasonable?

NEW KEY TERMS

- base angles [ángulos de la base]
- Base Angles theorem [teorema de los ángulos de la base]
- congruent sides
- congruent angles [ángulos congruentes]
- straight angle
- collinear [colineal]
- supplementary angles [ángulos suplementarios]
- complementary angles [ángulos complementarios]
- perpendicular [perpendicular]
- adjacent angles [ángulos adyacentes]
- linear pair [par lineal]
- vertical angles [ángulos verticales]

Refer to the Math Glossary for definitions of the New Key Terms.

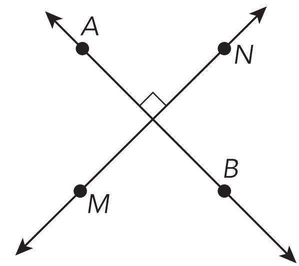
Where are we now?

When points lie on the same line or line segment, they are said to be **collinear**.

Points *C*, *A*, and *B* are collinear.

Two lines, line segments, or rays are **perpendicular** if they intersect to form 90° angles. The symbol for perpendicular is ⊥.

Line *AB* is perpendicular to line *MN*.



In **Lesson 1: Solving Equations Using the Triangle Sum Theorem**, students review the Triangle Sum theorem and use this information to write and solve equations for unknown values or unknown angle measurements.

Triangle Sum Theorem

The **Triangle Sum theorem** states that when you measure the interior angles of a triangle, they always add to 180 degrees. You can use this information, along with knowledge about angle relationships, to write equations and solve for an unknown value or unknown angle measure.

For example, write and solve an equation to determine the value of *x*. Then, determine each unknown angle measure.

$(6x - 2) + 4x + 42 = 180$

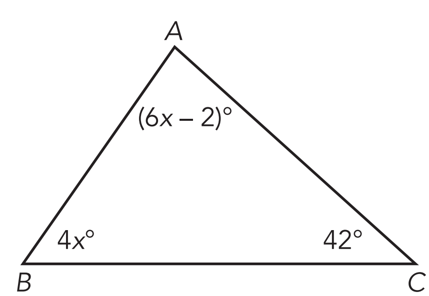
$10x + 40 = 180$

$10x = 140$

$x = 14$

$m\angle B = 4(14) = 56°$

$m\angle A = 6(14) - 2 = 82°$

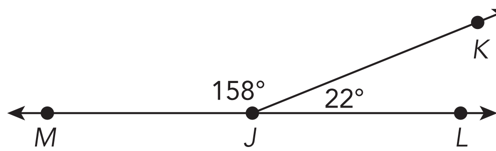


In **Lesson 2: Relationships Between 90° and 180° Angles**, students explore the relationships between 90° and 180° angles.

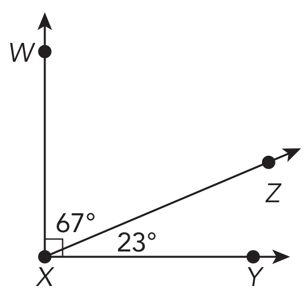
Angles

A **straight angle** is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex. When points lie on the same line or line segment, they are said to be **collinear**. For example, points *M*, *J*, and *L* are collinear.

Two angles are **supplementary angles** if the sum of their angle measures is equal to 180 degrees. For example, angles *MJK* and *KJL* are supplementary angles.



Two angles are **complementary angles** if the sum of their angle measures is equal to 90 degrees. For example, angles *WXZ* and *ZXY* are complementary angles.



MYTH

Some students are "right-brain" learners while other students are "left-brain" learners.

As you probably know, the brain is divided into two hemispheres: left and right. Some categorize people by their preferred or dominant mode of thinking. "Right-brain" thinkers are considered to be more intuitive, creative, and imaginative. "Left-brain" thinkers are more logical, verbal, and mathematical.

Another way to think about the brain is from the back to the front, where information goes from highly concrete to abstract. So, why don't we claim that some people are "back of the brain" thinkers, who are highly concrete, while others are "frontal" thinkers, who are more abstract?

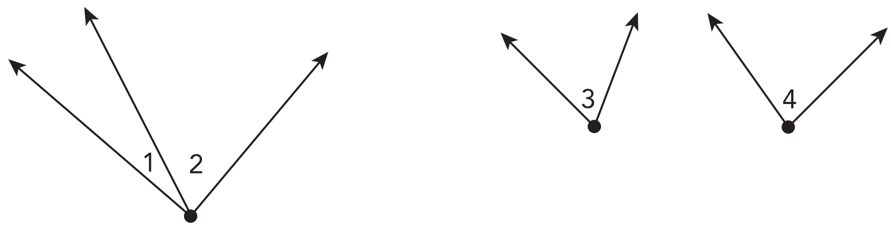
The brain is a highly interconnected organ. Each lobe hands off information to be processed by other lobes, and they are constantly talking to each other. So, it's time to dispense with the distinction between right- and left-brain thinkers. We are all *whole-brain* thinkers!

#mathmythbusted

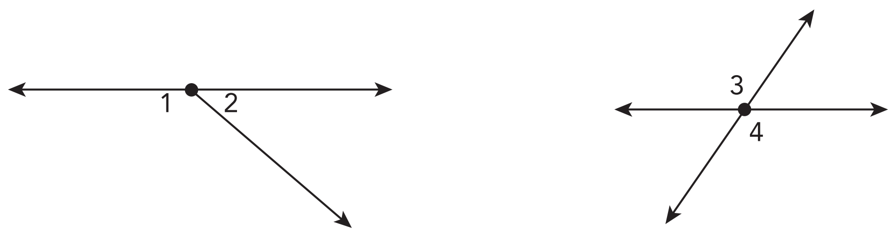
In **Lesson 3: Special Angle Relationships**, students explore the types of angles formed when two lines intersect.

Special Angle Relationships

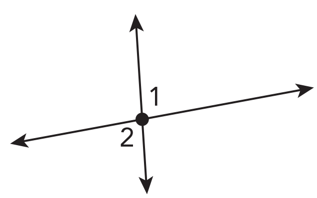
Adjacent angles are two angles that share a common vertex and a common side. $\angle 1$ and $\angle 2$ are adjacent angles. $\angle 3$ and $\angle 4$ are not adjacent angles.



A **linear pair** of angles is formed by two adjacent angles that have noncommon sides that form a line. Linear pairs are supplementary. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 3$ and $\angle 4$ do not form a linear pair.



Vertical angles are two nonadjacent angles that are formed by two intersecting lines. Vertical angles are congruent, meaning that they have the same size and shape.

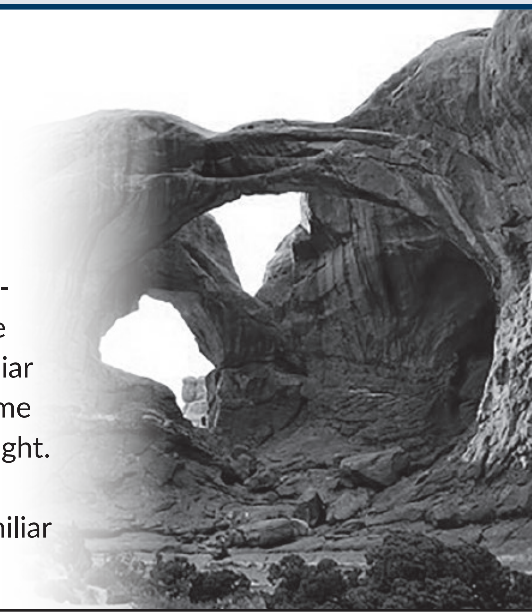


Family Guide

MODULE 5 Constructing and Measuring

Grade 7

TOPIC 2 Area, Surface Area, and Volume



In this topic, students use reasoning to develop different strategies for decomposing unfamiliar shapes into shapes with known area formulas to determine the areas of composite figures. Students use nets, or two-dimensional representations of three-dimensional figures, to determine the area of familiar faces that make up the solid. They use nets of familiar solids (right rectangular prisms and pyramids) to discover that the volume of a pyramid is one-third the volume of a prism with same base and height. Students practice using the volume formulas while solving real-world problems. They then calculate the total and lateral surface areas of familiar pyramids and prisms.

Where have we been?

Students have developed and applied area formulas for rectangles, triangles, parallelograms, and trapezoids to solve problems. They built upon this knowledge to determine the area of composite figures by decomposing them into shapes with known area formulas. Students determined the surface area of prisms and pyramids, as students calculate the areas of two-dimensional nets that represent three-dimensional solids.

Where are we going?

Students will use generalizations of the formulas for the volume of prisms and pyramids in future courses as they continue determining volumes of solids. They will relate the volume of a cone to the volume of a cylinder. Students will use Cavalieri's Principle to formally prove these familiar formulas in Geometry. They will build on the knowledge established in this topic to solve more complicated volume and surface area problems.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is continuing to reason about abstract geometric objects, such as three-dimensional pyramids and prisms.

QUESTIONS TO ASK

- How can you use what you know about geometric figures to help you solve this problem?
- Can you use a net to help you model and solve this problem?
- How can you verify your solution?

NEW KEY TERMS

- net
- surface area [área de (la) superficie / área superficial]
- pyramid [pirámide]
- slant height
- lateral surface area [área de la superficie lateral]

Refer to the Math Glossary for definitions of the New Key Terms.

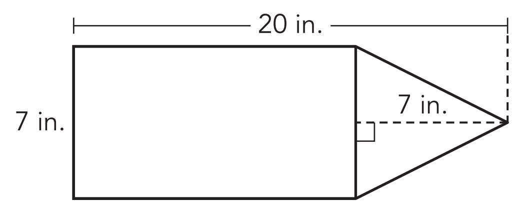
Where are we now?

The **lateral surface area** of a prism or pyramid is the sum of the areas of the lateral faces. To calculate the lateral surface area of a prism or pyramid, determine the total surface area of the figure and then subtract the area of the base(s).

In **Lesson 1: Composite Figures**, students calculate the area of complex figures. They compare two methods: decomposing a figure into familiar shapes and composing a figure into a rectangle.

Composite Figures

A composite figure is a figure that is made up of more than one geometric figure. The area of a composite figure can be determined by breaking it into familiar shapes and then adding the areas of those shapes together. For example, the composite figure shown is composed of a rectangle and a triangle.



$$\begin{aligned}\text{area of composite figure} &= \text{area of rectangle} + \text{area of triangle}\\ &= (7)(13) + \tfrac{1}{2}(7)(7)\\ &= 91 + 24\tfrac{1}{2}\\ &= 115\tfrac{1}{2}\end{aligned}$$

The area of the composite figure is $115\frac{1}{2}$ square inches.

In **Lesson 2: Total Surface Area of Prisms and Pyramids**, students apply mathematical and spatial reasoning to determine the surface areas of prisms and pyramids using nets, drawings, and measurements.

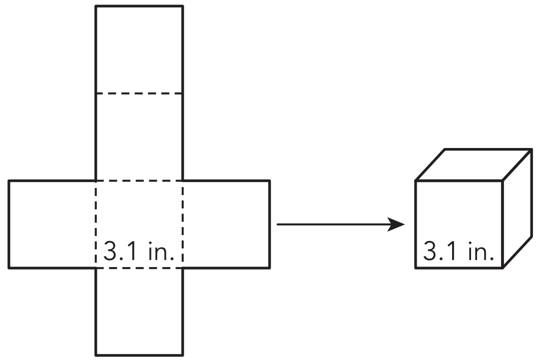
Nets

A **net** is a two-dimensional representation of a three-dimensional geometric figure.

A net has the following properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn so that they share common edges.

The surface area of a three-dimensional geometric figure is the total area of all of its two-dimensional faces.

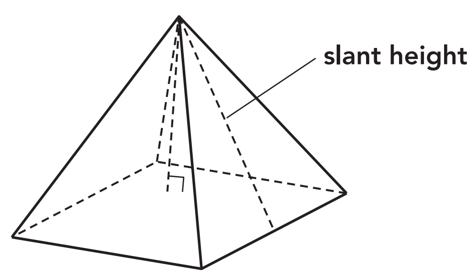


Pyramids

A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base.

The vertex of a pyramid is the point at which all the triangular faces intersect.

The **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint, or center, of the base of the triangular face.

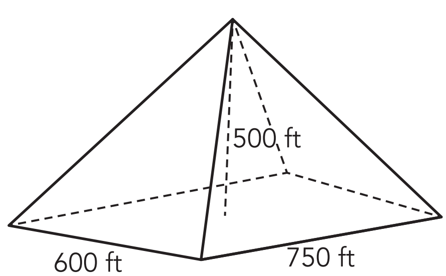


In **Lesson 3: Volume of Prisms and Pyramids**, students discover that the volume of the pyramid is one-third the volume of the prism and then write the formula for the volume of each.

Volume of a Pyramid

The volume of a pyramid is one-third the volume of a prism with the same base and height, so $V = \frac{1}{3}Bh$.

For example, calculate the volume of the rectangular pyramid shown.



$B = 750 \cdot 600$

$B = 450{,}000$

$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(450{,}000)(500)$

$V = 75{,}000{,}000$

The volume of the pyramid is 75,000,000 cubic feet.

MYTH

"Once I understand something, it has been learned."

Learning is tricky for three reasons.

First, even when we learn something, we don't always recognize when that knowledge is useful. For example, you know that there are four quarters in a dollar. But, if someone asks you, "What is 75 times 2?" you might not immediately recognize that is the same thing as having six quarters.

Second, when you learn something new, it's not as if the old way of thinking goes away. For example, some people think of north as straight ahead. But, have you ever been following directions on your phone and made a wrong turn only to catch yourself and think, "I know better than that!"?

The final reason that learning is tricky is that it is balanced by a different mental process: forgetting. Even when you learn something (e.g., your phone number), when you stop using it (e.g., when you move), it becomes extremely hard to remember.

There should always be an asterisk next to the word when we say we learned* something.

#mathmythbusted

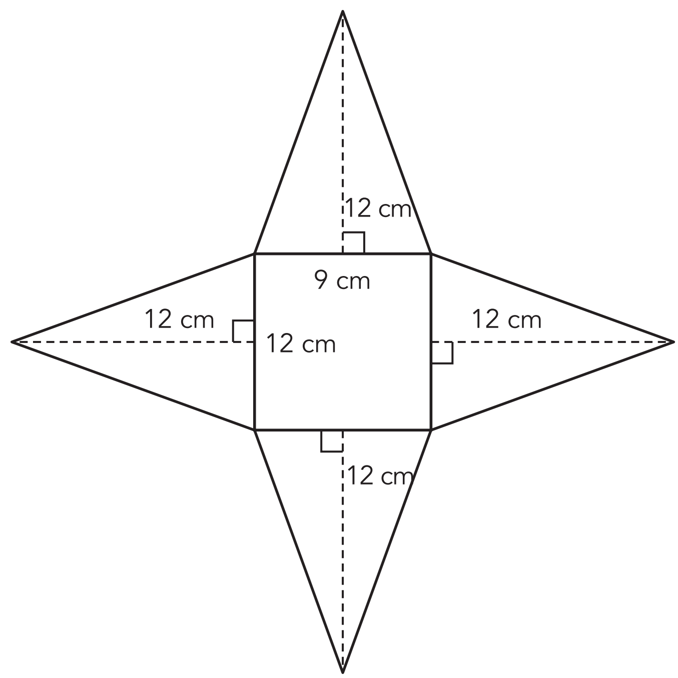
In **Lesson 4: Volume and Surface Area Problems with Prisms and Pyramids**, students are introduced to the term *lateral surface area*, and they compare the total and lateral surface areas.

Total and Lateral Surface Area of a Pyramid

The **total surface area** of a pyramid is the sum of the areas of all of its faces. The **lateral surface area** of a pyramid is the total surface area of the pyramid, except for the base.

All pyramids have a height and a slant height. The *slant height* is not the actual height of the pyramid. Rather, it is the height of an individual triangle that is a face of the pyramid.

For example, calculate the total and lateral surface areas of the pyramid whose net is shown.



$SA = 4 \cdot \frac{1}{2}(9)(12) + 9^2$
$SA = 4 \cdot 54 + 81$
$SA = 216 + 81$
$SA = 297$

$L = 4 \cdot \frac{1}{2}(9)(12)$
$L = 4 \cdot 54$
$L = 216$

The total surface area of the pyramid is 297 square centimeters. The lateral surface area of the pyramid is 216 square centimeters.

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