OER Secondary Mathematics

EDITION 1

Grade 7

Volume 1

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

If you have further product questions or to report an error, please email openeducationresources@tea.texas.gov.

OER Secondary Mathematics

EDITION 1

Grade 7

Course Guide

Welcome to the Course Guide for Secondary Mathematics, Grade 7

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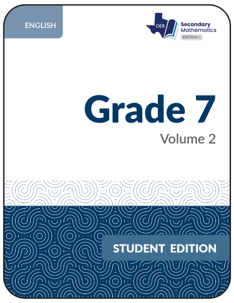
Instructional Design

The instructional materials help you learn math in different ways. There are two types of resources: Learning Together and Learning Individually. These resources provide various learning experiences to develop your understanding of mathematics.

Learning Together

On **Learning Together** days, you spend time engaging in active learning to build mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the materials for each lesson.



ENGLISH

Grade 7

Volume 2

STUDENT EDITION

STUDENT EDITION

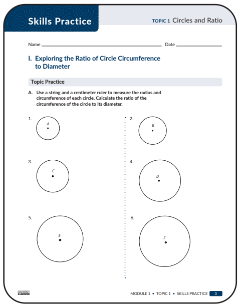
I am a record of your thinking, reasoning, and problem solving.

My lessons allow you to build new knowledge from prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On **Learning Individually** days, Skills Practice offers opportunities to engage with skills, concepts, and applications that you learn in each lesson. It also provides opportunities for interleaved practice, which encourages you to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning. This resource will help you build proficiency in specific skills based on your individual academic needs as indicated by monitoring your progress throughout the course.



Skills Practice

TOPIC 1 Circles and Ratio

Name ________ Date ________

I. Exploring the Ratio of Circle Circumference to Diameter

Topic Practice

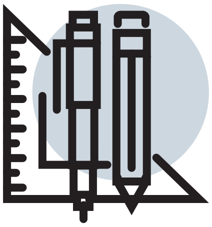
A. Use a string and a centimeter ruler to measure the radius and circumference of each circle. Calculate the ratio of the circumference of the circle to its diameter.

SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide problem sets for additional practice, enrichment, and extension.

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure you thoroughly understand the mathematical concepts. The instructional materials guide you to connect related ideas holistically, supporting the integration of your evolving mathematical understanding and developing proficiency with mathematical processes.



Intentional Mathematics Design

Mathematical Coherence: The path through the mathematics develops logically, building understanding by linking ideas within and across grades so you can learn concepts more deeply and apply what you've learned to more complex problems.

TEKS Mathematical Process Standards: The instructional materials support your development of the TEKS mathematical process standards. They encourage you to experiment, think creatively, and test various strategies. These mathematical processes empower you to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

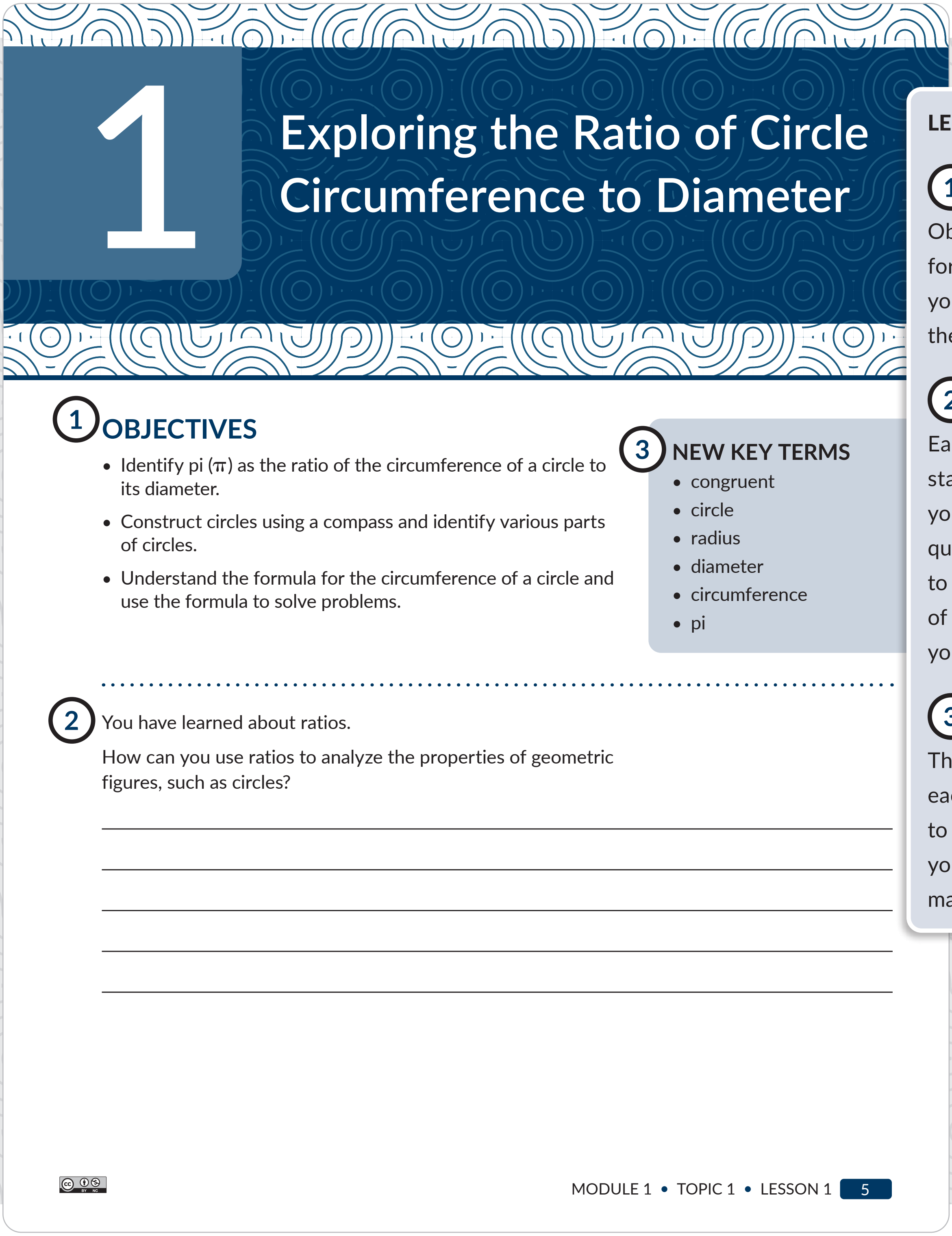
What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem-solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture your interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for you to build new understanding on the firm foundation of what you already know. You move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem-Solving: Solving problems is an essential life skill that you need to develop. The problem-solving model provides a structure to support you as you analyze and solve problems. It is a strategy you can continue to use as you solve problems in everyday life.



1 Exploring the Ratio of Circle Circumference to Diameter

① OBJECTIVES

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Understand the formula for the circumference of a circle and use the formula to solve problems.

③ NEW KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

② You have learned about ratios.

How can you use ratios to analyze the properties of geometric figures, such as circles?

MODULE 1 • TOPIC 1 • LESSON 1 5

LESSON STRUCTURE

① Objectives

Objectives are stated for each lesson to help you take ownership of the objectives.

② Essential Question

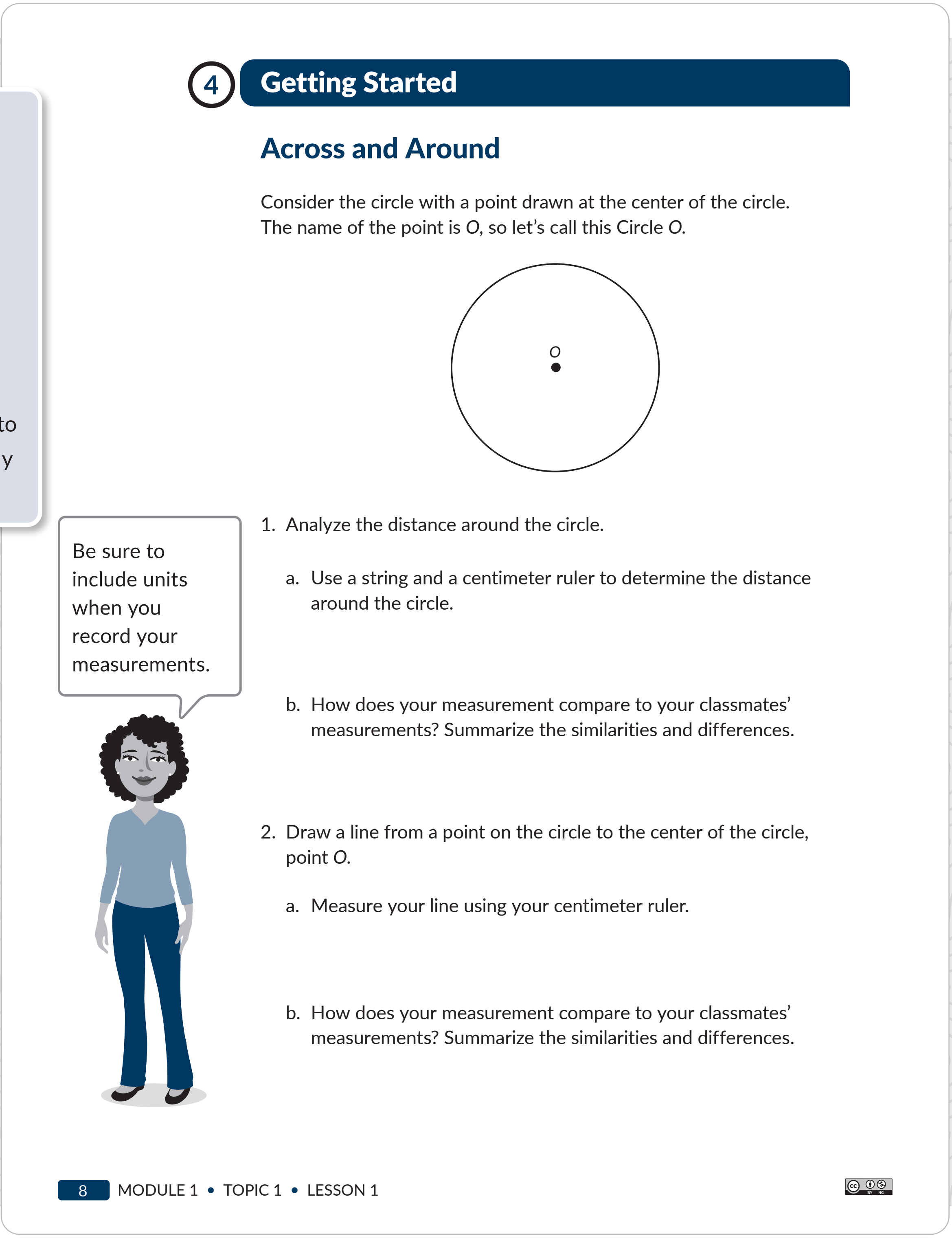
Each lesson begins with a statement connecting what you have learned with a question to ponder. Return to this question at the end of this lesson to gauge your understanding.

③ New Key Terms

The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.



4 Getting Started

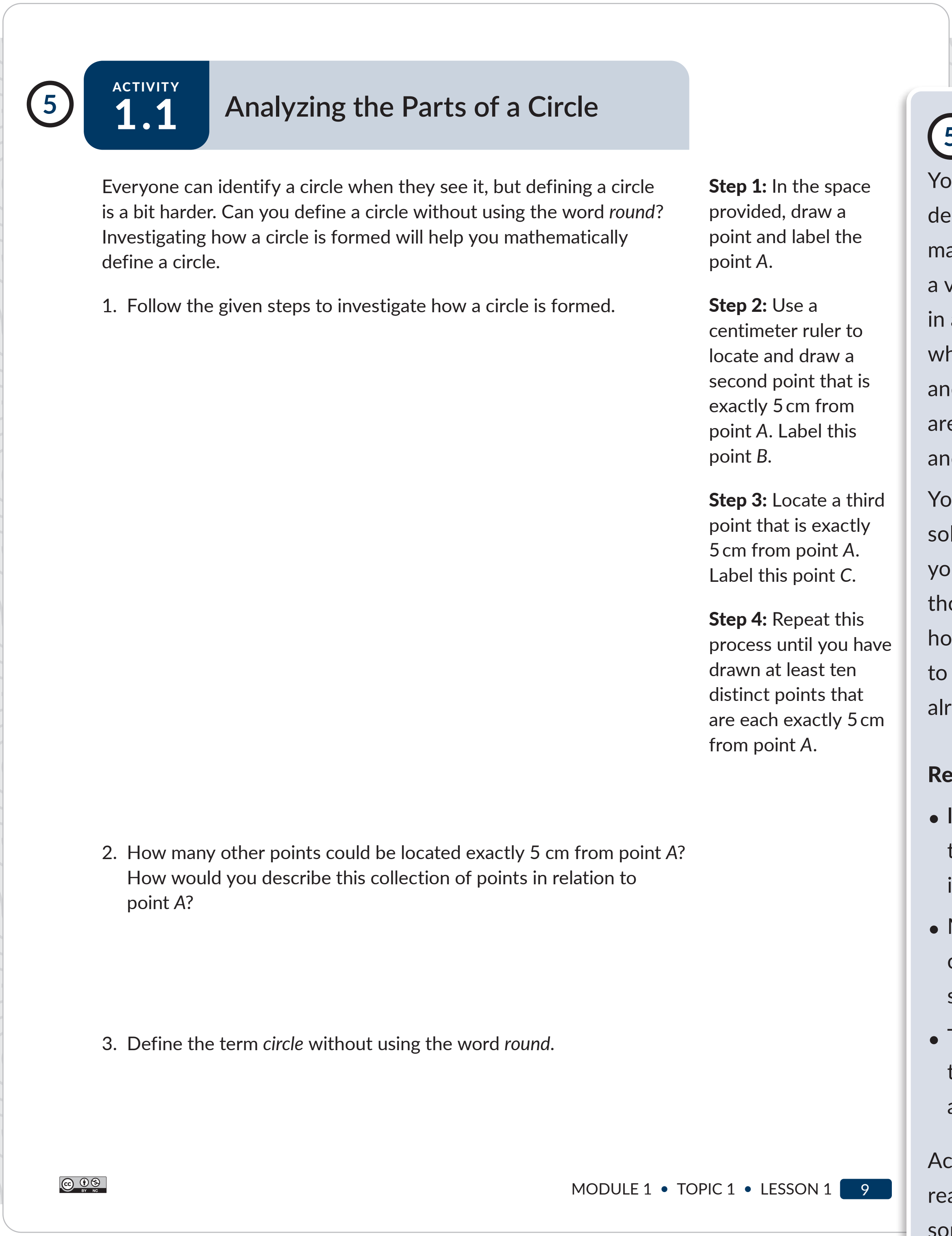
Across and Around

Consider the circle with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.

Be sure to include units when you record your measurements.

1. Analyze the distance around the circle.
   a. Use a string and a centimeter ruler to determine the distance around the circle.
   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.
2. Draw a line from a point on the circle to the center of the circle, point *O*.
   a. Measure your line using your centimeter ruler.
   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

8 MODULE 1 • TOPIC 1 • LESSON 1



⑤

ACTIVITY 1.1

Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a bit harder. Can you define a circle without using the word *round*? Investigating how a circle is formed will help you mathematically define a circle.

1. Follow the given steps to investigate how a circle is formed.

Step 1: In the space provided, draw a point and label the point *A*.

Step 2: Use a centimeter ruler to locate and draw a second point that is exactly 5 cm from point *A*. Label this point *B*.

Step 3: Locate a third point that is exactly 5 cm from point *A*. Label this point *C*.

Step 4: Repeat this process until you have drawn at least ten distinct points that are each exactly 5 cm from point *A*.

2. How many other points could be located exactly 5 cm from point *A*? How would you describe this collection of points in relation to point *A*?

3. Define the term *circle* without using the word *round*.

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⑤ Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about getting the answer. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

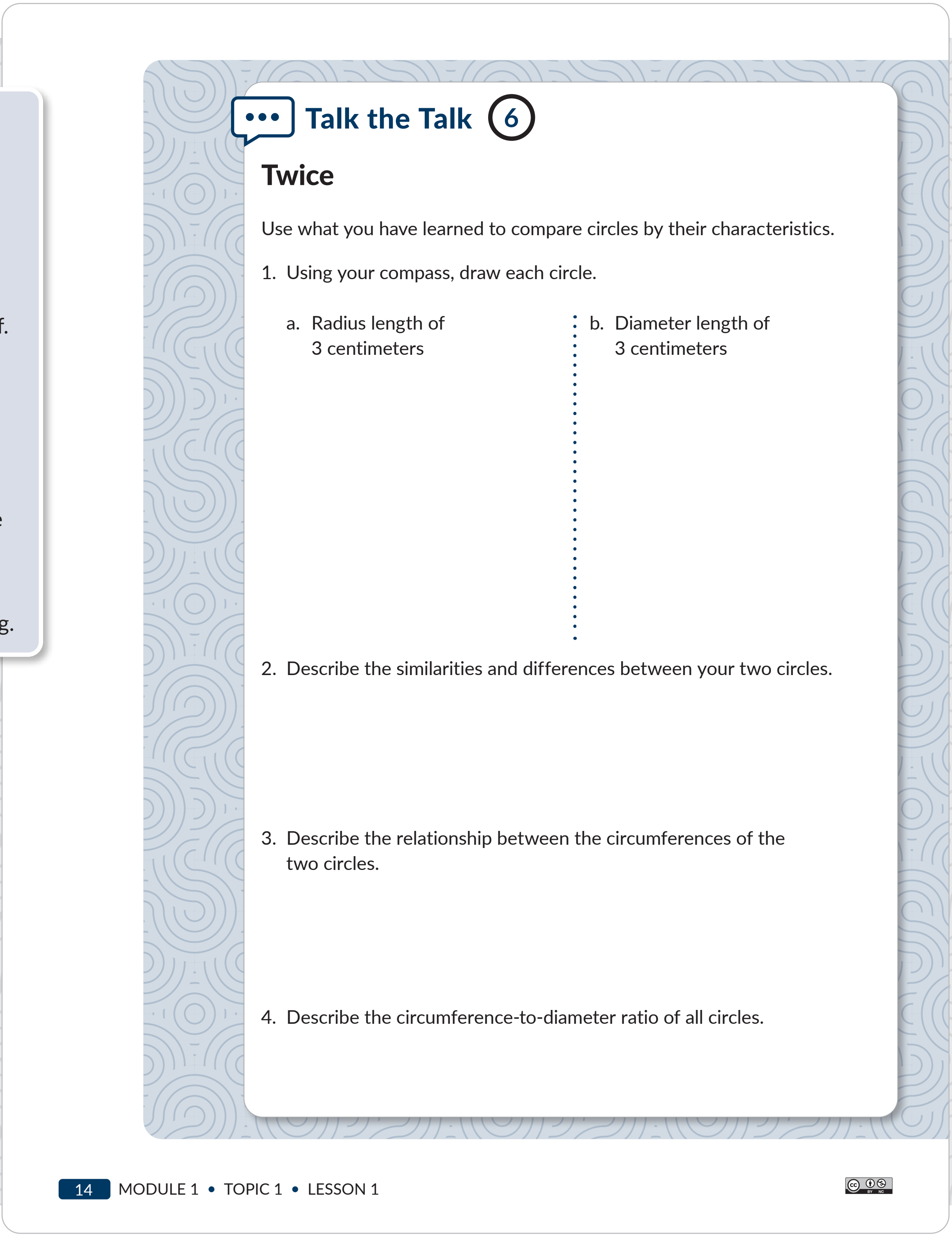
Be prepared to share your solutions and methods with your classmates.

⑥ Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.



Talk the Talk ⑥

Twice

Use what you have learned to compare circles by their characteristics.

1. Using your compass, draw each circle.

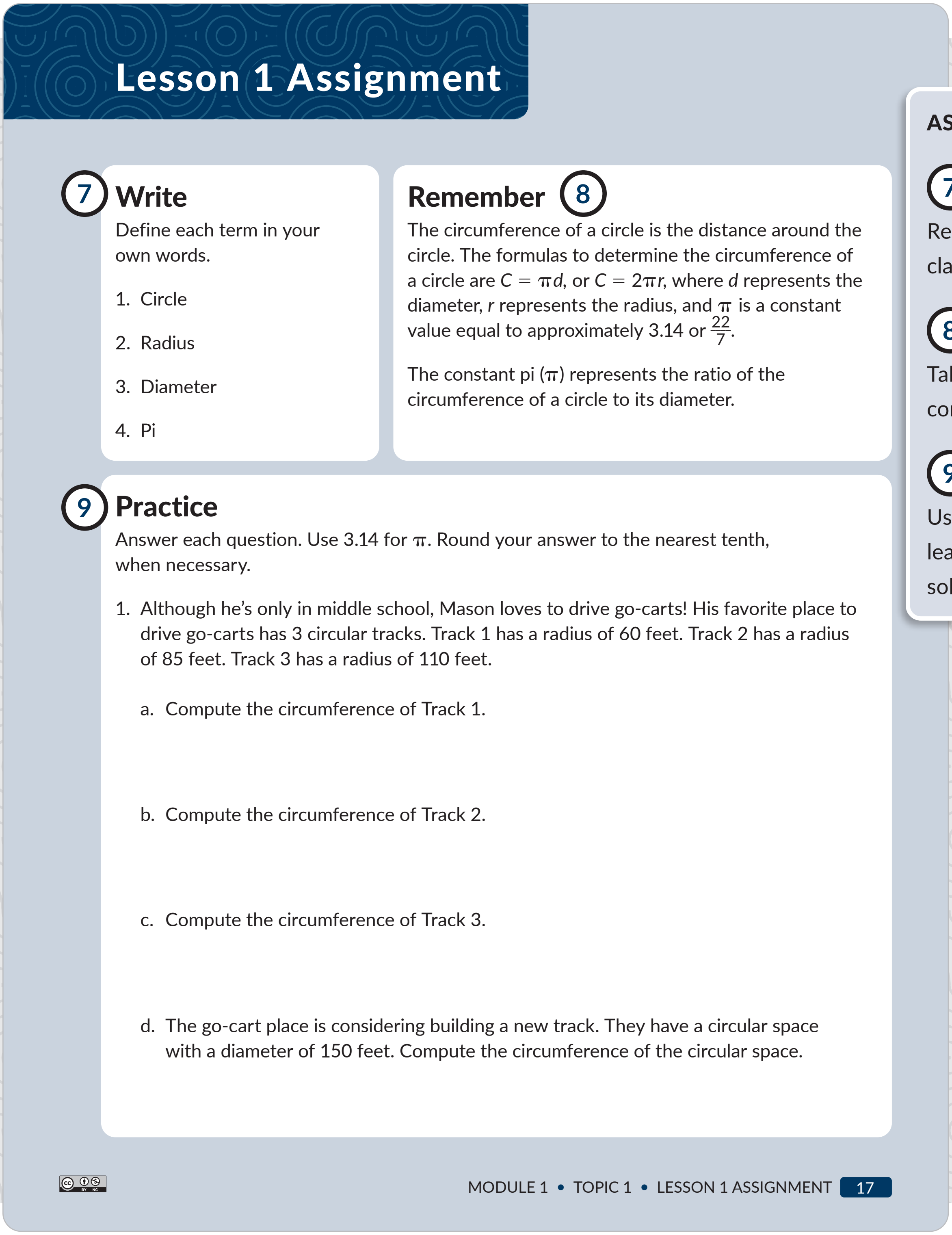
   a. Radius length of 3 centimeters

   b. Diameter length of 3 centimeters

2. Describe the similarities and differences between your two circles.

3. Describe the relationship between the circumferences of the two circles.

4. Describe the circumference-to-diameter ratio of all circles.



Lesson 1 Assignment

⑦ Write

Define each term in your own words.

1. Circle
2. Radius
3. Diameter
4. Pi

Remember ⑧

The circumference of a circle is the distance around the circle. The formulas to determine the circumference of a circle are $C = \pi d$, or $C = 2\pi r$, where d represents the diameter, r represents the radius, and π is a constant value equal to approximately 3.14 or $\frac{22}{7}$.

The constant pi (π) represents the ratio of the circumference of a circle to its diameter.

⑨ Practice

Answer each question. Use 3.14 for π. Round your answer to the nearest tenth, when necessary.

1. Although he's only in middle school, Mason loves to drive go-carts! His favorite place to drive go-carts has 3 circular tracks. Track 1 has a radius of 60 feet. Track 2 has a radius of 85 feet. Track 3 has a radius of 110 feet.
   a. Compute the circumference of Track 1.
   b. Compute the circumference of Track 2.
   c. Compute the circumference of Track 3.
   d. The go-cart place is considering building a new track. They have a circular space with a diameter of 150 feet. Compute the circumference of the circular space.

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ASSIGNMENT

⑦ Write

Reflect on your work and clarify your thinking.

⑧ Remember

Take note of the key concepts from the lesson.

⑨ Practice

Use the concepts learned in the lesson to solve problems.

Lesson 1 Assignment

2. Mason wants to build a circular go-cart track in his backyard. Use 3.14 for π.

   a. Suppose he wants the track to have a circumference of 157 feet. What does the radius of the track need to be?

   b. Suppose he wants the track to have a circumference of 314 feet. What does the radius of the track need to be?

   c. Suppose he wants the track to have a circumference of 471 feet. What does the diameter of the track need to be?

⑩ Prepare

Determine a unit rate for each situation.

1. $38.40 for 16 gallons of gas

2. 15 miles jogged in 3.75 hours

3. $26.99 for 15 pounds

ASSIGNMENT

⑩ **Prepare**

Get ready for the next lesson.

Research-Based Strategies

WORKED EXAMPLE

Isabella's first client of the day spent $150 to have her hair dyed and cut and gave Isabella a $30 tip.

Use a Proportion

$$\frac{t}{100} = \frac{30}{150}$$

$$t = \frac{(30)(100)}{150}$$

$$t = 20$$

Use a Percent Equation

$$(t)(150) = 30$$

$$150t = 30$$

$$\frac{150t}{150} = \frac{30}{150}$$

$$t = \frac{30}{150}$$

$$t = 0.2$$

Ask Yourself . . .

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Identify the error in Samuel's strategy.

Samuel 👎

$$2x + 6 = 14$$

$$\frac{2x}{2} + 6 = \frac{14}{2}$$

$$x + 6 = 7$$

$$-6 = -6$$

$$x = 1$$

Explain Ethan's process in solving the equation.

Ethan 👍

$$2x + 6 = 14$$

$$-6 = -6$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

Ask Yourself . . .

- Why is this method correct?
- Have I used this method before?

Ask Yourself . . .

- Where is the error?
- Why is it an error?
- How can I correct it?

WORKED EXAMPLE

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

THUMBS UP

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

THUMBS DOWN

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

WHO'S CORRECT?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine whether the work is correct or incorrect.

5. Alejandra says that the product being less than 10 and the product being more than 10 are complementary events. Hailey disagrees. Who is correct? Explain your reasoning.

Ask Yourself . . .

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I Can” expectations listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem solving process and reasonableness of the solution.

I CAN:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I CAN:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language, as appropriate.

I CAN:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Create and use representations to organize, record, and communicate mathematical ideas.

I CAN:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Analyze mathematical relationships to connect and communicate mathematical ideas.

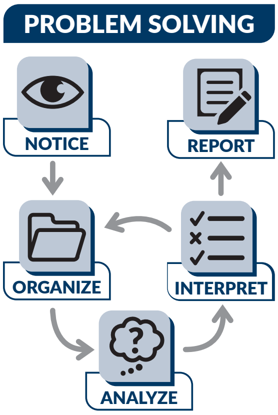
I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.



The Problem-Solving Model

Productive mathematical thinkers are problem-solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

Understanding the Problem-Solving Model

NOTICE

Notice | Wonder

Understand the situation by asking these questions.

- What do I know?
- What do I need to determine?
- What important information is given that I will need to determine a solution?
- What information is given that I do NOT need?
- Is there enough information given to solve the problem?

ORGANIZE

Organize | Mathematize

Devise a plan for your mathematical approach by asking these questions.

- What is a similar problem to this that I have solved before?
- What strategies may help to solve this problem using the given information?
- How can I represent this problem using a picture, diagram, symbols, graph or some other visual representation? Which representations make sense for this problem?

PREDICT

Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I show my math work using representations?
- Did I explain my mathematical solution in terms of the problem situation, when applicable?
- Did I describe how I arrived at my solution?
- Did I communicate my strategy and solution clearly using precise mathematical language as necessary?
- Can I make any predictions based on my work?

INTERPRET

Test | Interpret

Look back at your work and ask these questions.

- Does the solution answer the original question/problem?
- Does the reasoning and the solution make sense?
- How could I have used a different strategy to solve this problem? Would it have changed the outcome?

REPORT

Report

As you share your mathematical reasoning with others ask these questions.

- Did I share my solution with others?
- Do others understand the mathematics I communicated?

The Problem-Solving Model Graphic Organizer

NOTICE

Understand the Problem

ORGANIZE

Devise a Plan

PREDICT

Carry Out the Plan

INTERPRET

Look Back

REPORT

Report

Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition

Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Explain Your Reasoning

Definition

Give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

What is Productive Struggle?

Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process *productive struggle*. Productive struggle is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

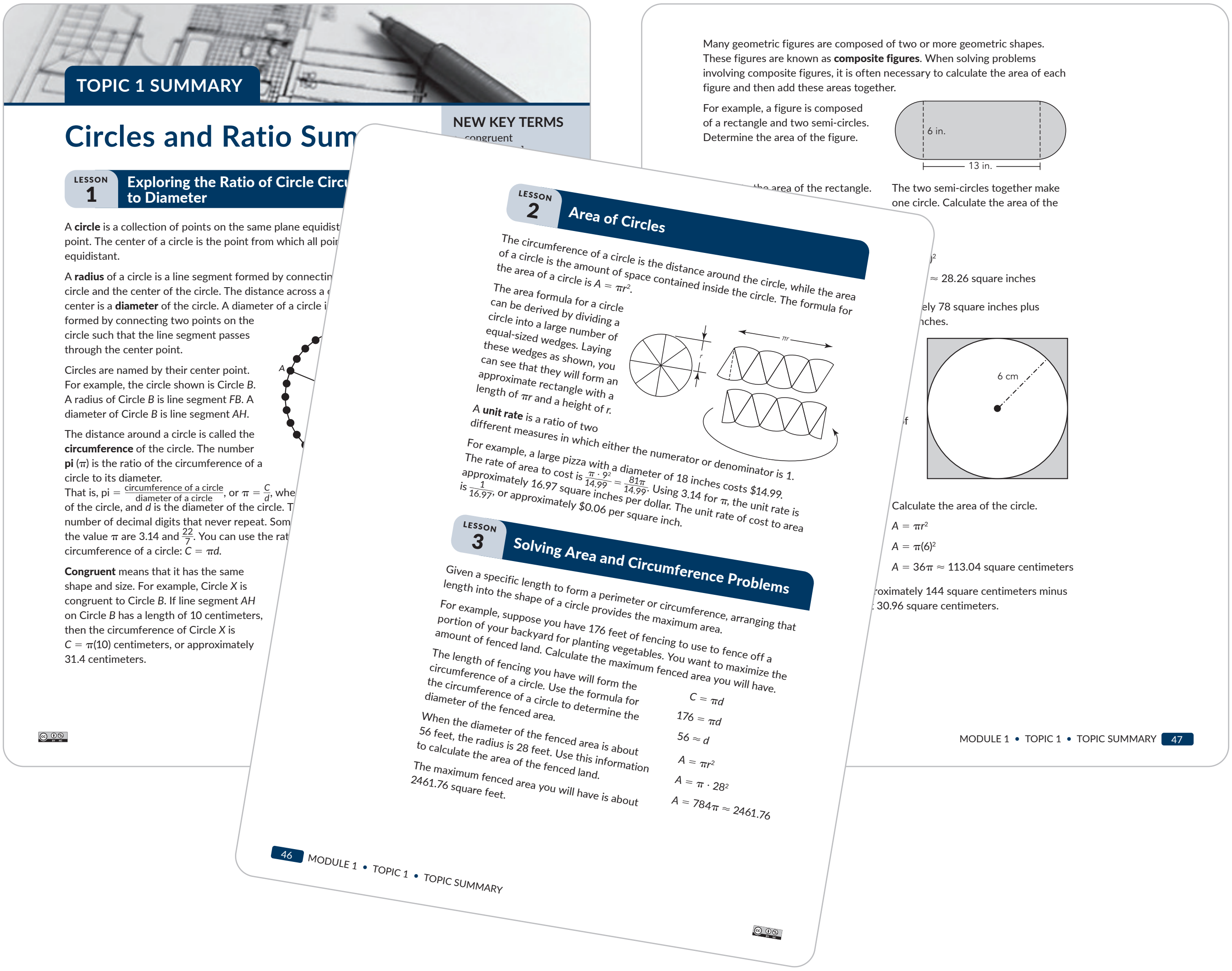
Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

Resources for Students and Families

Topic Summary

Each topic includes a Topic Summary. The Topic Summary contains a list of all new key terms addressed in the topic and a summary of each lesson, including worked examples and new key term definitions. Use the Topic Summary to review each lesson's major concepts and strategies as you complete assignments and/or share your learning outside of class.



TOPIC 1 SUMMARY

Circles and Ratio Sum

NEW KEY TERMS

congruent

LESSON 1 Exploring the Ratio of Circle Circ to Diameter

A **circle** is a collection of points on the same plane equidist point. The center of a circle is the point from which all poi equidistant.

A **radius** of a circle is a line segment formed by connectin circle and the center of the circle. The distance across a center is a **diameter** of the circle. A diameter of a circle i formed by connecting two points on the circle such that the line segment passes through the center point.

Circles are named by their center point. For example, the circle shown is Circle *B*. A radius of Circle *B* is line segment *FB*. A diameter of Circle *B* is line segment *AH*.

The distance around a circle is called the **circumference** of the circle. The number **pi** (π) is the ratio of the circumference of a circle to its diameter.
That is, pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, whe of the circle, and *d* is the diameter of the circle. T number of decimal digits that never repeat. Som the value π are 3.14 and $\frac{22}{7}$. You can use the rat circumference of a circle: $C = \pi d$.

Congruent means that it has the same shape and size. For example, Circle *X* is congruent to Circle *B*. If line segment *AH* on Circle *B* has a length of 10 centimeters, then the circumference of Circle *X* is $C = \pi(10)$ centimeters, or approximately 31.4 centimeters.

LESSON 2 Area of Circles

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. The formula for the area of a circle is $A = \pi r^2$.

The area formula for a circle can be derived by dividing a circle into a large number of equal-sized wedges. Laying these wedges as shown, you can see that they will form an approximate rectangle with a length of πr and a height of *r*.

A **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

For example, a large pizza with a diameter of 18 inches costs $14.99. The rate of area to cost is $\frac{\pi \cdot 9^2}{14.99} = \frac{81\pi}{14.99}$. Using 3.14 for π, the unit rate is approximately 16.97 square inches per dollar. The unit rate of cost to area is $\frac{1}{16.97}$, or approximately $0.06 per square inch.

LESSON 3 Solving Area and Circumference Problems

Given a specific length to form a perimeter or circumference, arranging that length into the shape of a circle provides the maximum area.

For example, suppose you have 176 feet of fencing to use to fence off a portion of your backyard for planting vegetables. You want to maximize the amount of fenced land. Calculate the maximum fenced area you will have.

The length of fencing you have will form the circumference of a circle. Use the formula for the circumference of a circle to determine the diameter of the fenced area.

$C = \pi d$

$176 = \pi d$

$56 \approx d$

When the diameter of the fenced area is about 56 feet, the radius is 28 feet. Use this information to calculate the area of the fenced land.

$A = \pi r^2$

$A = \pi \cdot 28^2$

$A = 784\pi \approx 2461.76$

The maximum fenced area you will have is about 2461.76 square feet.

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Many geometric figures are composed of two or more geometric shapes. These figures are known as **composite figures**. When solving problems involving composite figures, it is often necessary to calculate the area of each figure and then add these areas together.

For example, a figure is composed of a rectangle and two semi-circles. Determine the area of the figure.

the area of the rectangle.

The two semi-circles together make one circle. Calculate the area of the

)^2

$\approx$ 28.26 square inches

ely 78 square inches plus nches.

f

Calculate the area of the circle.

$A = \pi r^2$

$A = \pi(6)^2$

$A = 36\pi \approx 113.04$ square centimeters

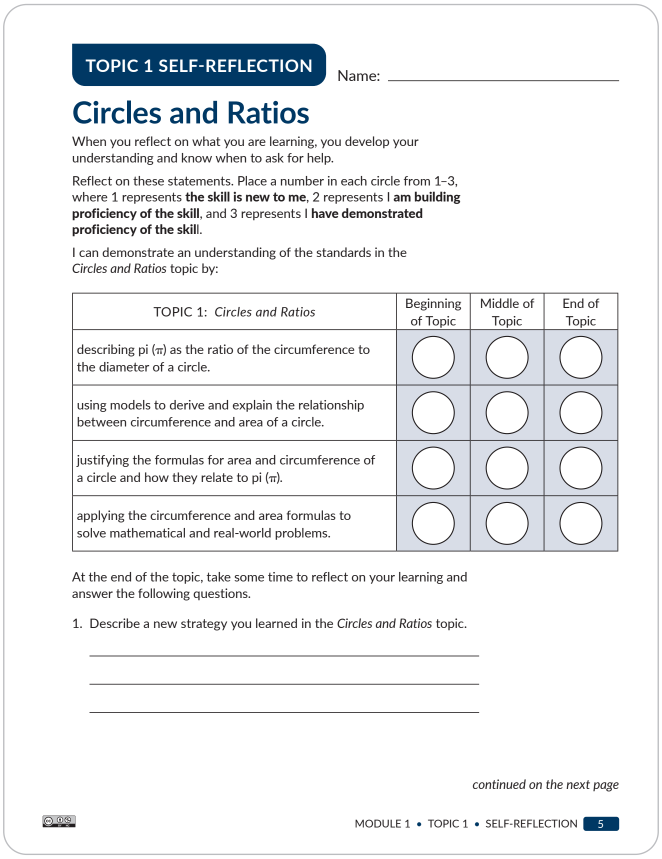
roximately 144 square centimeters minus 30.96 square centimeters.

MODULE 1 • TOPIC 1 • TOPIC SUMMARY 47

Topic Self-Reflection

The Topic Self-Reflection, provided at the end of each topic, empowers you to develop confidence in your mathematical understanding and monitor your own learning processes. Taking the time for self-reflection helps you identify your strengths and where you want to focus your efforts to improve.

Use the Topic Self-Reflection throughout the topic to monitor your progress toward the mathematical goals for the topic.



TOPIC 1 SELF-REFLECTION Name: ____________________

Circles and Ratios

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents I **am building proficiency of the skill**, and 3 represents I **have demonstrated proficiency of the skil**l.

I can demonstrate an understanding of the standards in the *Circles and Ratios* topic by:

TOPIC 1: *Circles and Ratios*	Beginning of Topic	Middle of Topic	End of Topic
describing pi (π) as the ratio of the circumference to the diameter of a circle.	○	○	○
using models to derive and explain the relationship between circumference and area of a circle.	○	○	○
justifying the formulas for area and circumference of a circle and how they relate to pi (π).	○	○	○
applying the circumference and area formulas to solve mathematical and real-world problems.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Circles and Ratios* topic.

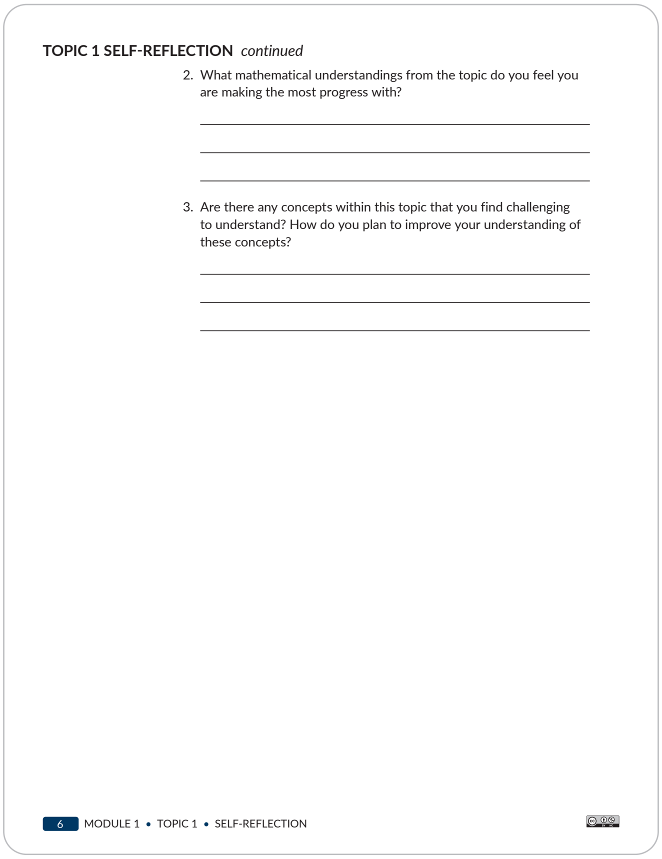
____

____

____

continued on the next page

MODULE 1 • TOPIC 1 • SELF-REFLECTION 5



TOPIC 1 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

____

____

____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

____

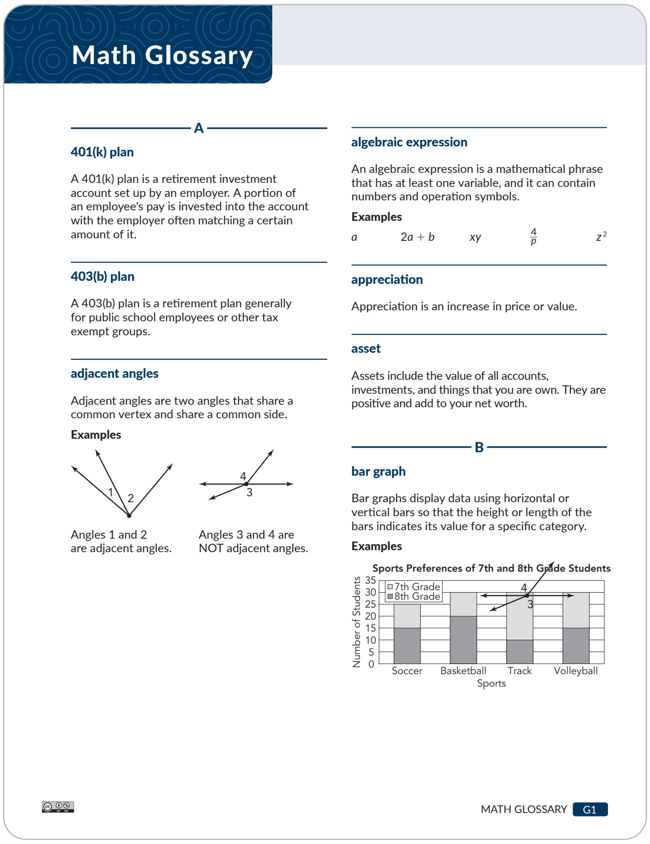
____

____

6 MODULE 1 • TOPIC 1 • SELF-REFLECTION

Math Glossary

A course-specific math glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.



Math Glossary

A

401(k) plan

A 401(k) plan is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account with the employer often matching a certain amount of it.

403(b) plan

A 403(b) plan is a retirement plan generally for public school employees or other tax exempt groups.

adjacent angles

Adjacent angles are two angles that share a common vertex and share a common side.

Examples

Angles 1 and 2 are adjacent angles.

Angles 3 and 4 are NOT adjacent angles.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $2a + b$ xy $\frac{4}{p}$ z^2

appreciation

Appreciation is an increase in price or value.

asset

Assets include the value of all accounts, investments, and things that you are own. They are positive and add to your net worth.

B

bar graph

Bar graphs display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

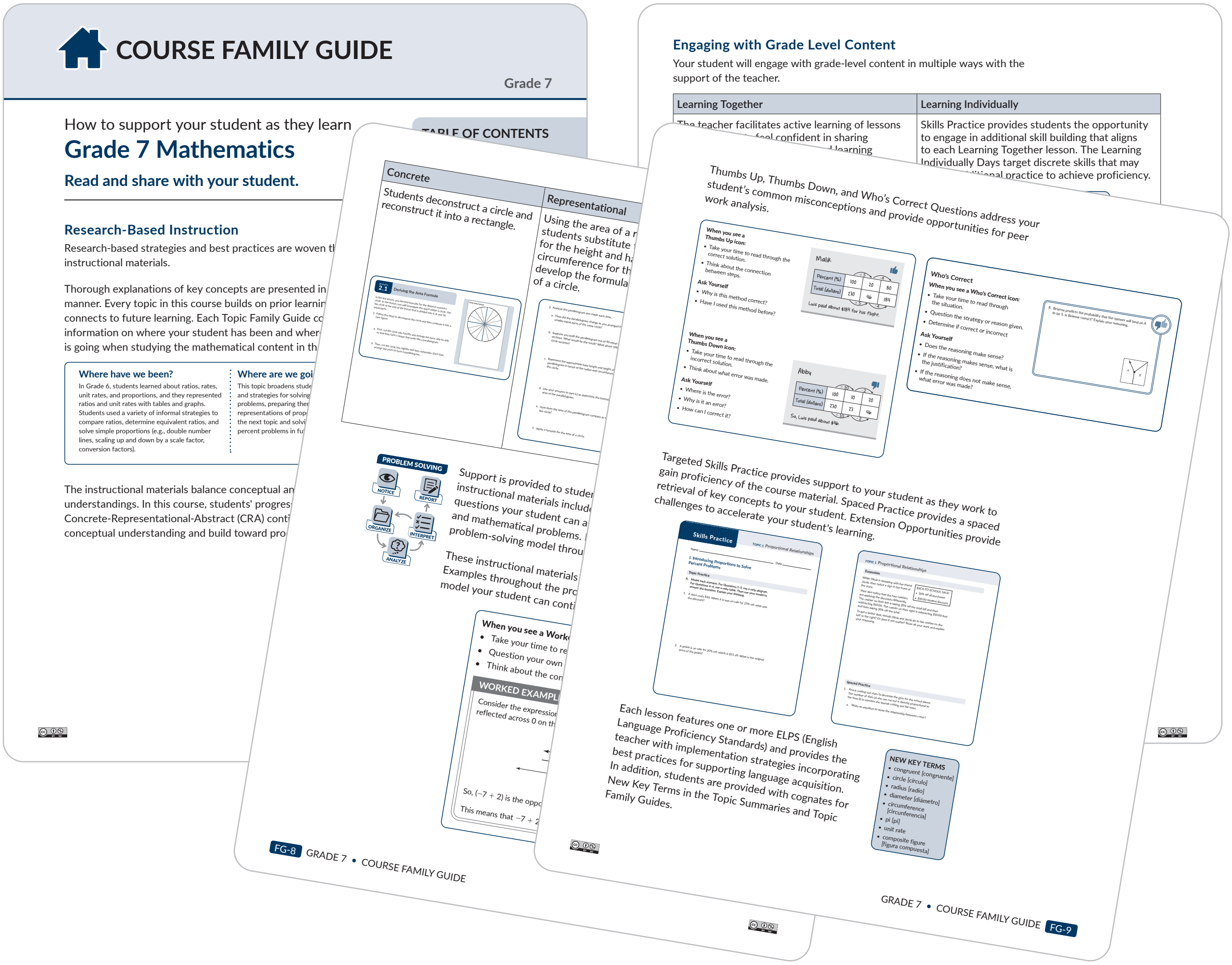
Examples

MATH GLOSSARY G1

Course Family Guide

The Course Family Guide provides you and your family an overview of the course design. The guide details the resources available to support your learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

The purpose of the Course Family Guides is to bridge your learning in the classroom to your learning at home. The goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together.



COURSE FAMILY GUIDE

Grade 7

How to support your student as they learn

Grade 7 Mathematics

Read and share with your student.

Engaging with Grade Level Content

Your student will engage with grade-level content in multiple ways with the support of the teacher.

Thumbs Up, Thumbs Down, and Who's Correct Questions address your student's common misconceptions and provide opportunities for peer work analysis.

Targeted Skills Practice provides support to your student as they work to gain proficiency of the course material. Spaced Practice provides a spaced retrieval of key concepts to your student. Extension Opportunities provide challenges to accelerate your student's learning.

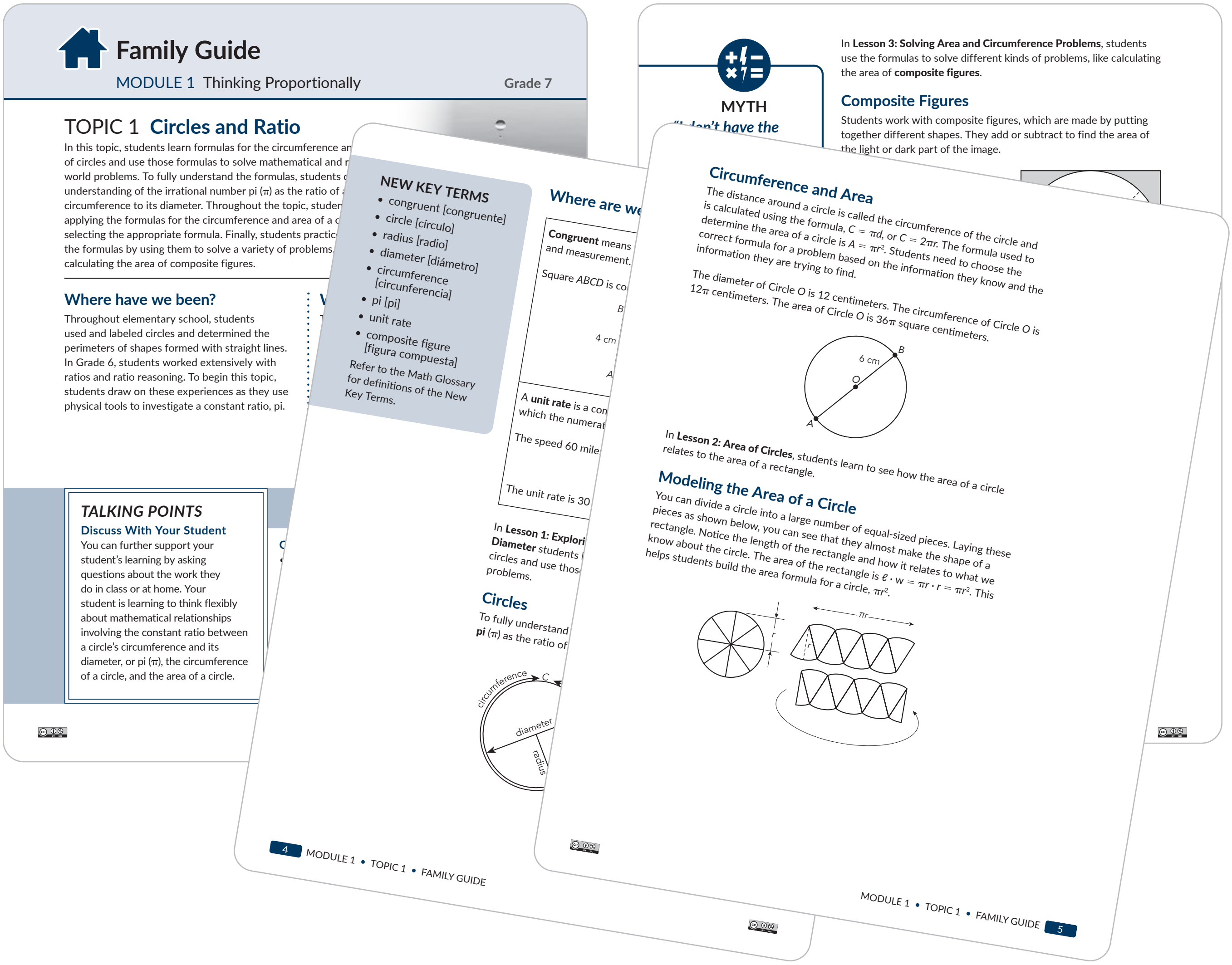
Each lesson features one or more ELPS (English Language Proficiency Standards) and provides the teacher with implementation strategies incorporating best practices for supporting language acquisition. In addition, students are provided with cognates for New Key Terms in the Topic Summaries and Topic Family Guides.

GRADE 7 • COURSE FAMILY GUIDE FG-9

Topic Family Guides

Each topic contains a Topic Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Topic Family Guide are an example of a math model or strategy taught in the topic, definitions of key terms, busting of a math myth, and questions family members can ask you to support your learning.

Learning outside of the classroom is crucial to student success at school. The Topic Family Guide serves to assist families in talking to students about the learning that is happening in the classroom.



Family Guide
MODULE 1 Thinking Proportionally
Grade 7

TOPIC 1 Circles and Ratio
In this topic, students learn formulas for the circumference an
of circles and use those formulas to solve mathematical and r
world problems. To fully understand the formulas, students
understanding of the irrational number pi (π) as the ratio of
circumference to its diameter. Throughout the topic, studen
applying the formulas for the circumference and area of a c
selecting the appropriate formula. Finally, students practic
the formulas by using them to solve a variety of problems
calculating the area of composite figures.

Where have we been?
Throughout elementary school, students used and labeled circles and determined the perimeters of shapes formed with straight lines. In Grade 6, students worked extensively with ratios and ratio reasoning. To begin this topic, students draw on these experiences as they use physical tools to investigate a constant ratio, pi.

TALKING POINTS
Discuss With Your Student
You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to think flexibly about mathematical relationships involving the constant ratio between a circle's circumference and its diameter, or pi (π), the circumference of a circle, and the area of a circle.

NEW KEY TERMS
- congruent [congruente]
- circle [círculo]
- radius [radio]
- diameter [diámetro]
- circumference [circunferencia]
- pi [pi]
- unit rate
- composite figure [figura compuesta]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we

Congruent means
and measurement.

Square ABCD is co

A unit rate is a com
which the numerat

The speed 60 mile

The unit rate is 30

In Lesson 1: Explori
Diameter students
circles and use thos
problems.

Circles
To fully understand
pi (π) as the ratio of

4 MODULE 1 • TOPIC 1 • FAMILY GUIDE

MYTH

In Lesson 3: Solving Area and Circumference Problems, students use the formulas to solve different kinds of problems, like calculating the area of composite figures.

Composite Figures
Students work with composite figures, which are made by putting together different shapes. They add or subtract to find the area of the light or dark part of the image.

Circumference and Area
The distance around a circle is called the circumference of the circle and is calculated using the formula, $C = \pi d$, or $C = 2\pi r$. The formula used to determine the area of a circle is $A = \pi r^2$. Students need to choose the correct formula for a problem based on the information they know and the information they are trying to find.

The diameter of Circle O is 12 centimeters. The circumference of Circle O is 12π centimeters. The area of Circle O is 36π square centimeters.

In Lesson 2: Area of Circles, students learn to see how the area of a circle relates to the area of a rectangle.

Modeling the Area of a Circle
You can divide a circle into a large number of equal-sized pieces. Laying these pieces as shown below, you can see that they almost make the shape of a rectangle. Notice the length of the rectangle and how it relates to what we know about the circle. The area of the rectangle is $\ell \cdot w = \pi r \cdot r = \pi r^2$. This helps students build the area formula for a circle, πr^2.

MODULE 1 • TOPIC 1 • FAMILY GUIDE 5

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Math Glossary G1

MODULE
1

Thinking Proportionally

TOPIC 1 Circles and Ratio 3
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GRADE 7

TOPIC
1

Dropping something into water causes a series of ripples to expand from the point of impact, forming concentric circles.

Circles and Ratio

INTRODUCTION LESSON
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INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources

OBJECTIVES

- Establish a community of learners.
- Discover learning resources available for this course.
- Apply the problem-solving model to a real-life situation.

In previous math classes, you have analyzed patterns and relationships, learned about numbers and operations in base ten and fractions, measurement and data, and geometry.

What resources are available in this course to help you extend your mathematical thinking?

Getting Started

You Already Know a Lot

Each lesson in this book begins with a Getting Started that gives you the opportunity to use what you know about the world, and what you have learned in previous math classes. You know a lot from a variety of learning experiences.

Think back to how you learn something new.

1. List three different skills that you recently learned. Then, describe why you wanted to learn that skill and the strategies that you used.

New Skill	Motivation to Learn the New Skill	Strategies I Used to Learn This Skill

Ask Yourself . . .
How do your strategies change based on what you are learning and what you already know about it?

One learning strategy is to talk with your peers. In this course, you will work with your classmates to solve problems, discuss strategies, and learn together.

Compare and discuss your list with a classmate.

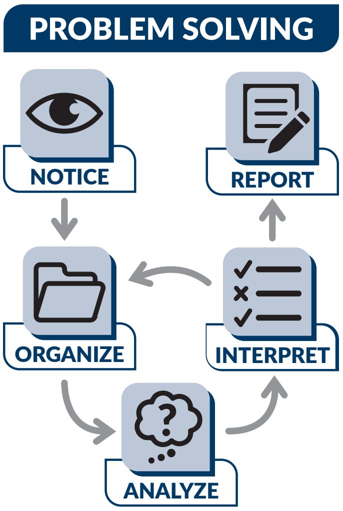
2. Which strategies do you have in common? Which strategies does your classmate have that you did not think of on your own?

Think About . . .
Listening well, cooperating with others, and appreciating different perspectives are essential life skills.

Be prepared to share your list of learning strategies with the class.

ACTIVITY 1.1

Learning Resources



In this course, you will learn new math concepts by exploring and investigating ideas, reading, writing, and talking to your classmates. You will even learn by making mistakes with concepts you haven't mastered yet.

Let's practice exploring and investigating. You do not need to answer the question yet. You will solve the question as you work through the problem-solving model.

A hockey team is awarded a penalty shot at the end of the game. The team wins the league championship when a player makes the penalty shot. The coach considers three players to attempt the penalty shot. Jasmine has attempted 4 penalty shots this season and made 2 of them. Luna has attempted 6 and made 4. Lucas has attempted 3 and made 1.

Which player would you recommend attempt the penalty shot?
Explain your reasoning.

The Academic Glossary is your guide as you engage with the kind of thinking you do as you are learning the content.

1. Locate the phrase **Explain Your Reasoning** in the Academic Glossary in the Course Guide. What questions should you ask yourself as you explain which player should attempt the penalty shot?

2. What is a related word or phrase for **explain your reasoning**?

The Academic Glossary provides definitions of terms you will see throughout the course as you think, reason, and communicate your ideas. The Math Glossary provides the definitions of new key terms in each lesson.

The problem-solving model provides a structure to help you become a better problem-solver.

NOTICE

Notice and Wonder

The first step in modeling a situation mathematically is to understand the problem, gather information, notice patterns, and formulate mathematical questions about what you notice.

Read through the *Questions to Ask* yourself for the first step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

3. What do you notice about the hockey players?

4. Why do you think the first step of the problem-solving model is important? How will it help you when you solve problems?

ORGANIZE

Organize and Mathematize

The second step in the problem-solving model is to devise a plan. When devising a plan, you will organize your information and begin to represent it using mathematical notation.

Read through the *Questions to Ask* yourself for the second step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

5. Describe your strategy for selecting a player.

6. Why do you think the second step of the problem-solving model process is important? How can you use the *Questions to Ask* yourself to help develop a strategy to solve the problem?

Predict and Analyze

PREDICT

The third step in the modeling process involves carrying out your plan. As you carry out the plan you will complete operations, analyze mathematical results, make predictions, and extend patterns.

Read through the *Questions to Ask* yourself for the third step of the problem-solving model. *Use the Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

7. Use your strategy to determine which player should attempt the penalty shot.

8. How can the *Questions to Ask* for the third step help you communicate your mathematical thinking?

Test and Interpret

INTERPRET

The fourth step in the modeling process is to look back, interpret your results, and test your mathematical predictions in the real world. When your predictions are incorrect, or your results are not reasonable, you can revisit your mathematical work and make adjustments—or start all over!

Read through the *Questions to Ask* yourself for the fourth step of the problem-solving model. Use the problem-solving model graphic organizer to answer the questions and show your thinking.

9. The coach decides to consider a fourth player. Nahimana has attempted 3 penalty shots and made 2. Can you apply the same reasoning to determine the player who should attempt the shot? Explain your reasoning.

10. How do the questions for the fourth step of the problem-solving model help you evaluate the reasonableness of your solution?

REPORT

Report

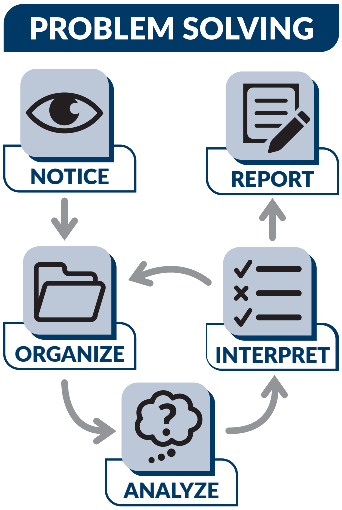
The final step in the modeling process is to share your results.

11. How does listening to others help you learn mathematics? Locate the TEKS mathematical process standards in the Course Guide.

12. Which TEKS mathematical process standard(s) did you use to determine which player to recommend?

Talk the Talk

The Problem-Solving Model



You will see this symbol throughout the course to remind you to use the problem-solving model.

In this lesson, you used a problem-solving model to solve a real-world problem. The basic steps of the problem-solving model are summarized in the diagram.

Summarize what is involved in each phase of this problem-solving model.

Notice and Wonder

Organize and Mathematize

Predict and Analyze

Test and Interpret

Report

1

Exploring the Ratio of Circle Circumference to Diameter

OBJECTIVES

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Understand the formula for the circumference of a circle and use the formula to solve problems.

NEW KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

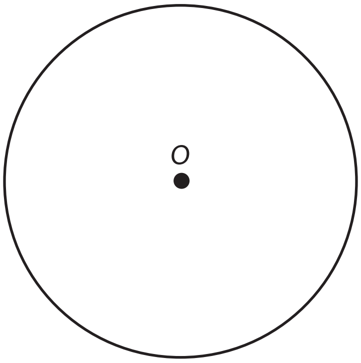
You have learned about ratios.

How can you use ratios to analyze the properties of geometric figures, such as circles?

Getting Started

Across and Around

Consider the circle with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.

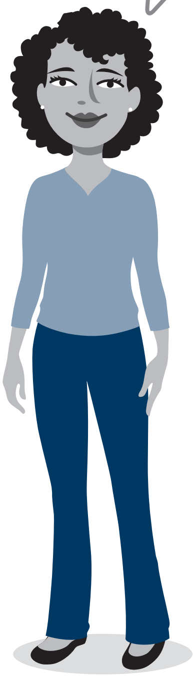


1. Analyze the distance around the circle.

   a. Use a string and a centimeter ruler to determine the distance around the circle.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

Be sure to include units when you record your measurements.



2. Draw a line from a point on the circle to the center of the circle, point *O*.

   a. Measure your line using your centimeter ruler.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

ACTIVITY

1.1

Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a bit harder. Can you define a circle without using the word *round*? Investigating how a circle is formed will help you mathematically define a circle.

1. Follow the given steps to investigate how a circle is formed.

Step 1: In the space provided, draw a point and label the point *A*.

Step 2: Use a centimeter ruler to locate and draw a second point that is exactly 5 cm from point *A*. Label this point *B*.

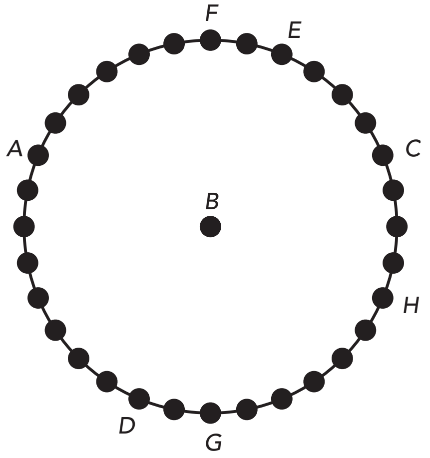
Step 3: Locate a third point that is exactly 5 cm from point *A*. Label this point *C*.

Step 4: Repeat this process until you have drawn at least ten distinct points that are each exactly 5 cm from point *A*.

2. How many other points could be located exactly 5 cm from point *A*? How would you describe this collection of points in relation to point *A*?

3. Define the term *circle* without using the word *round*.

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.



4. Use the circle shown to answer each question.

   a. Name the circle.

The **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is the diameter of the circle. The **diameter** of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point. The distance around a circle is called the **circumference** of the circle.

   b. Draw and identify a radius of the circle.

   c. Draw and identify a diameter of the circle.

The plural of *radius* is *radii*.

   d. Are all radii of this circle the same length? Explain your reasoning.

5. What is the relationship between the length of a radius and the length of a diameter?

ACTIVITY
1.2

Measuring the Distance Around a Circle

Let's explore circles. Use circles *A*, *B*, *D*, *E*, and *O* provided at the end of the lesson. Circle *O* is the same as the circle from the activity *Across and Around*.

1. Use a string and a centimeter ruler to measure the distance from a point on the circle to the center and the distance around each circle. Record your measurements in the table. In the last column, write the ratio of *Circumference* : *Diameter* in fractional form.

Circle	Circumference	Radius	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
Circle *A*				
Circle *B*				
Circle *O*				
Circle *D*				
Circle *E*				

2. Average the ratios recorded for $\frac{\text{Circumference}}{\text{Diameter}}$. What is the approximate ratio for the circumference to the diameter for the set of circles? Write the approximate ratio as a fraction and as a decimal.

3. How does your answer to Question 2 compare to your classmates' answers?

4. Average all of your classmates' answers to Question 2. Write the approximate ratio of circumference to the diameter as a fraction and as a decimal.

ACTIVITY
1.3

The Circumference Formula

The number **pi** (π) is the ratio of the circumference of a circle to its diameter. That is pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$.

1. Use this information to write a formula for the circumference of a circle, where d represents the diameter of a circle, and C represents the circumference of a circle.

2. Rewrite the formula for the circumference of a circle, where r represents the radius of a circle, and C represents the circumference of a circle.

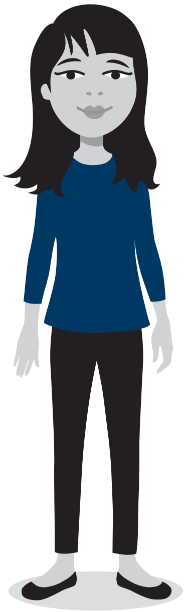
3. Use different representations for π to calculate the circumference of a circle.

   a. Calculate the circumference of a circle with a diameter of 4 centimeters and a circle with a radius of 6 inches. Round your answer to the nearest ten-thousandths, when necessary.

Value for π	$d = 4$ centimeters	$r = 6$ inches
π		
Use the π key on a calculator.		
Use 3.14 for π.		
Use $\frac{22}{7}$ for π.		

b. Compare your circumference calculations. How do the different values of π affect your calculations?

When you use 3.14 for pi, your answers are approximations. But an answer like 12π is exact.



4. Use the circumference of a circle formula to determine each unknown. Use 3.14 for π.

   a. Compute the diameter of the circle with a circumference of 65.94 feet.

   b. Compute the radius of the circle with a circumference of 109.9 millimeters.

5. What is the minimum amount of information needed to compute the circumference of a circle?

Talk the Talk

Twice

Use what you have learned to compare circles by their characteristics.

1. Using your compass, draw each circle.

   a. Radius length of 3 centimeters

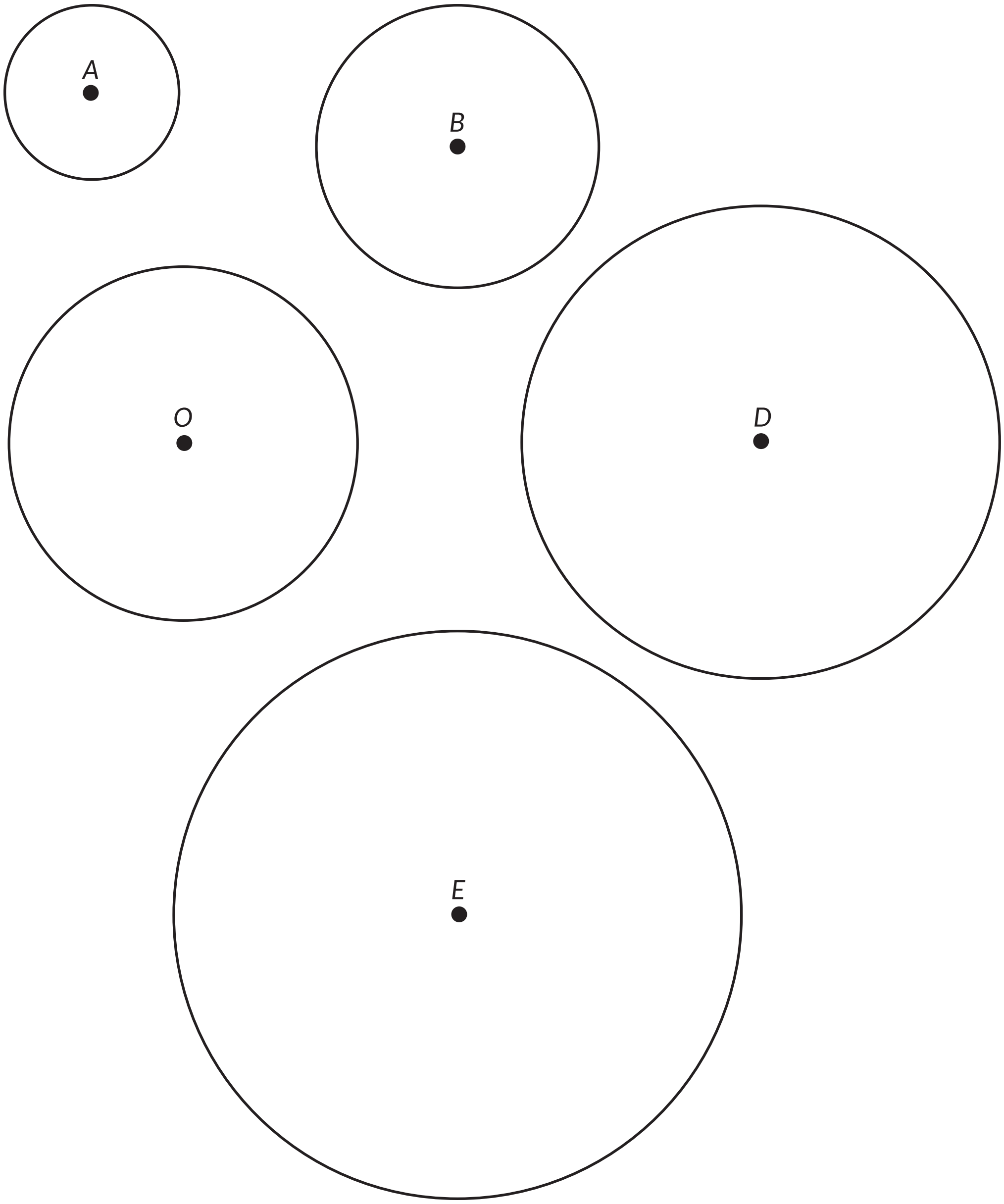
   b. Diameter length of 3 centimeters

2. Describe the similarities and differences between your two circles.

3. Describe the relationship between the circumferences of the two circles.

4. Describe the circumference-to-diameter ratio of all circles.

Measuring the Distance Around a Circle



Why is this page blank?

So you can remove the page with the circles on the other side

Lesson 1 Assignment

Write

Define each term in your own words.

1. Circle
2. Radius
3. Diameter
4. Pi

Remember

The circumference of a circle is the distance around the circle. The formulas to determine the circumference of a circle are $C = \pi d$, or $C = 2\pi r$, where d represents the diameter, r represents the radius, and π is a constant value equal to approximately 3.14 or $\frac{22}{7}$.

The constant pi (π) represents the ratio of the circumference of a circle to its diameter.

Practice

Answer each question. Use 3.14 for π. Round your answer to the nearest tenth, when necessary.

1. Although he's only in middle school, Mason loves to drive go-carts! His favorite place to drive go-carts has 3 circular tracks. Track 1 has a radius of 60 feet. Track 2 has a radius of 85 feet. Track 3 has a radius of 110 feet.

   a. Compute the circumference of Track 1.

   b. Compute the circumference of Track 2.

   c. Compute the circumference of Track 3.

   d. The go-cart place is considering building a new track. They have a circular space with a diameter of 150 feet. Compute the circumference of the circular space.

Lesson 1 Assignment

2. Mason wants to build a circular go-cart track in his backyard. Use 3.14 for π.

   a. Suppose he wants the track to have a circumference of 157 feet. What does the radius of the track need to be?

   b. Suppose he wants the track to have a circumference of 314 feet. What does the radius of the track need to be?

   c. Suppose he wants the track to have a circumference of 471 feet. What does the diameter of the track need to be?

Prepare

Determine a unit rate for each situation.

1. $38.40 for 16 gallons of gas

2. 15 miles jogged in 3.75 hours

3. $26.99 for 15 pounds

2

Area of Circles

OBJECTIVES

- Describe the relationship between the circumference and area of a circle and use the area formula to solve problems.
- Decide whether circumference or area is an appropriate measure for a problem situation.
- Calculate unit rates associated with circle areas.

NEW KEY TERM

- unit rate

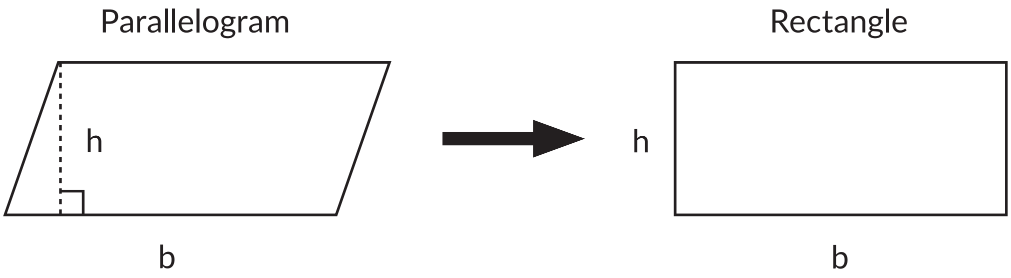
You have learned about the different parts and measures of a circle, including radius, diameter, and circumference.

How can you use the parts of a circle to determine the area of a circle?

Getting Started

What Changed? What Stayed the Same?

Consider the parallelogram and rectangle. The length of the base and height are the same.



1. How could you rearrange the parallelogram to create the rectangle?

2. What is the area of each figure?

ACTIVITY 2.1 Deriving the Area Formula

In the last lesson, you derived formulas for the distance around a circle. In this lesson, you will investigate the space within a circle. Use the circle at the end of the lesson that is divided into 4, 8, and 16 equal parts.

1. Follow the steps to decompose the circle and then compose it into a new figure.

   a. First, cut the circle into fourths and arrange the parts side by side so that they form a shape that looks like a parallelogram.

   b. Then, cut the circle into eighths and then sixteenths. Each time, arrange the parts to form a parallelogram.

2. Analyze the parallelogram you made each time.

   a. How did the parallelogram change as you arranged it with the smaller equal parts of the same circle?

   b. Suppose you built the parallelogram out of 40 equal circle sections. What would be the result? What about 100 equal circle sections?

   c. Represent the approximate base length and height of the parallelogram in terms of the radius and circumference of the circle.

d. Use your answers to part (c) to determine the formula for the area of the parallelogram.

e. How does the area of the parallelogram compare to the area of the circle?

f. Write a formula for the area of a circle.

3. Use different representations for π to calculate the area of a circle.

a. Calculate the area of each circle with the given radius. Round your answers to the nearest ten-thousandths, when necessary.

Value for π	$r = 6$ units	$r = 1.5$ units	$r = \frac{1}{2}$ unit
π			
Use the π key on a calculator			
Use 3.14 for π			
Use $\frac{22}{7}$ for π			

b. Compare your area calculations for each circle. How do the different values of π affect your calculations?

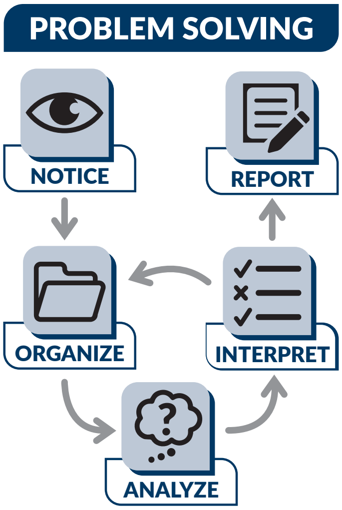
4. Suppose the ratio of radius lengths of two circles is 1 unit to 2 units.

   a. What is the ratio of areas of the circles? Experiment with various radius lengths to make a conclusion.

   b. When you double the length of the radius of a circle, what is the effect on the area?

ACTIVITY
2.2
Circumference or Area

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. When solving problems involving circles, it is important to think about what you are trying to determine.



INTERPRET

Use the problem-solving model whenever you see this icon.

1. A city park has a large circular garden with a path around it. The diameter of the garden is 60 feet.

   a. Sebastian likes to walk along the circular path during his lunch breaks. How far does Sebastian walk when he completes one rotation around the path?

Circle Formulas
$C = \pi d$, or $2\pi r$
$A = \pi r^2$

   b. Kayla works for the city's park department. She needs to spread plant food all over the garden. What is the area of the park she will cover with plant food?

2. Adriana is making a vegetable pizza. First, she presses the dough so that it fills a circular pan with a 16-inch diameter and covers it with sauce. What is the area of the pizza Adriana will cover with sauce?

3. Members of a community center have decided to paint a large circular mural in the middle of the parking lot. The radius of the mural is to be 11 yards. Before they begin painting the mural, they use rope to form the outline. How much rope will they need?

ACTIVITY
2.3

Unit Rates and Circle Area

A pizza place has a large variety of pizza sizes.

	Small	Medium	Large	X-Large	Enorme	Ginorme	Colossale
Diameter	10 in.	13 in.	16 in.	18 in.	24 in.	28 in.	36 in.
Slices	6	8	10	12	20	30	40
Cost	$7	$10	$13	$15	$23	$29	$55

Angelina and Mei are trying to decide whether to get two X-large pizzas or one Ginorme pizza. They ask themselves, "Which choice is the better buy?"

They each calculated a unit rate for the Ginorme pizza.

Recall that a unit rate is a ratio of two different measures in which either the numerator or denominator is 1.

Angelina

1 Ginorme: $\frac{\pi(14)^2}{29} = \frac{196\pi}{29} < 21$ square inches per dollar

The Ginorme gives you approximately 21 square inches of pizza per dollar.

Mei

1 Ginorme: $\frac{29}{14^2\pi} = \frac{29}{196\pi} \approx \$0.05$ per square inch

The Ginorme costs approximately $0.05 for each square inch of pizza.

1. Consider Angelina's and Mei's work.

   a. Explain why Angelina's and Mei's unit rates are different but still both correct.

   b. How would you decide which pizza was the better buy when you calculated the unit rate for each pizza using Angelina's method versus Mei's method?

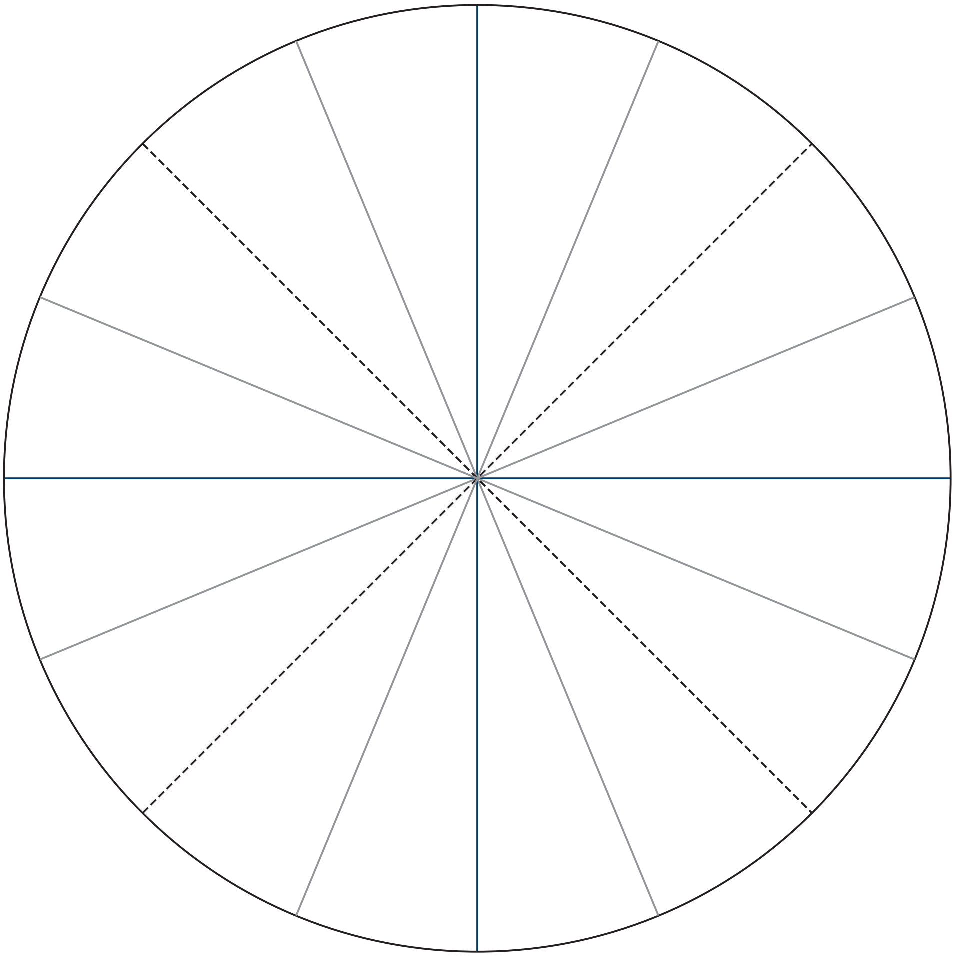
2. Which of the seven sizes of pizza from the pizza place is the best buy? Explain your answer.

Talk the Talk

Go With the Flow

1. Residents of a community are trying to determine which configuration would allow more water to flow through the pipe(s), one pipe with a radius of 8 cm or two pipes that each have a radius of 4 cm. Which configuration allows the most water to flow through the pipe(s), and what is the difference between the two configurations? Show your work and explain your reasoning.

Circle Area Cutouts



Why is this page blank?

So you can remove the page with the circle area on the other side

Lesson 2 Assignment

Write

Explain in your own words how to derive the formula for the area of a circle.

Remember

A formula for the area of a circle is $A = \pi r^2$.

Practice

Determine the area of the circle, given each measurement. Use 3.14 for π and round to the nearest hundredth.

1. Diameter: 8 in.

2. Radius: 10 in.

3. Radius: 1.5 ft

4. Diameter: 8.8 yd

5. Diameter: $1\frac{1}{2}$ in.

6. Radius: $2\frac{1}{2}$ cm

Determine which pizza is the better buy in each situation.

7. The 10-inch diameter pizza for $9 or the 6-inch diameter pizza for $5

Lesson 2 Assignment

8. The large 16-inch diameter pizza for $13 or the $26 X-large with a radius of 16 in

9. The 12-inch diameter pizza for $12.50 or the 20-inch diameter pizza for $17.50

10. The 4-inch radius pizza for $3 or the 8-inch radius pizza for $14

11. Two 12-inch diameter pizzas for $13 or one large 14-inch diameter pizza for $8

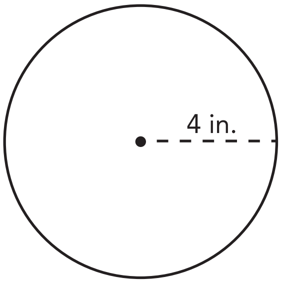
12. The 1-inch diameter pizza bite for $1 or the 10-inch diameter pizza for $10

Lesson 2 Assignment

Prepare

Determine the area of each circle. Use 3.14 for π.

1.



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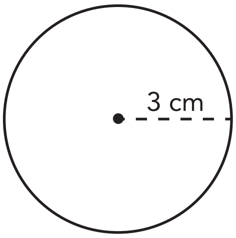

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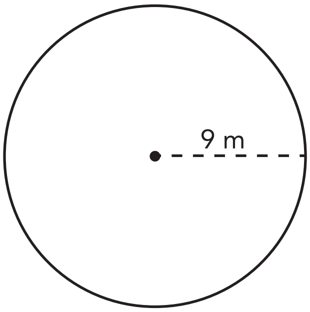

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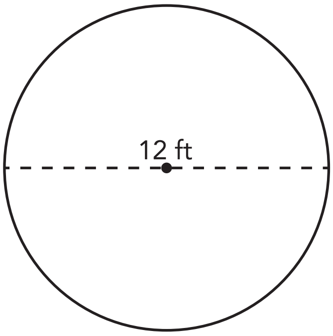


3

Solving Area and Circumference Problems

OBJECTIVES

- Solve problems using the area and circumference formulas for a circle.
- Calculate the areas of composite figures.

NEW KEY TERM

- composite figure

You encounter circles regularly in life.

Now that you know how to calculate the circumference and area of circles, what kind of problems can you solve?

Getting Started

A Winning Formula

Suppose that the circumference of a circle is approximately 157 centimeters.

1. Describe a strategy you can use to solve for the area of the circle.

When in doubt, use 3.14 for pi throughout this lesson.

2. Solve for the area of the circle. Use 3.14 for π.

ACTIVITY 3.1

A Maximum Area Problem

A friend gave you 120 feet of fencing. You decide to fence in a portion of the backyard for your dog. You want to maximize the amount of fenced land.

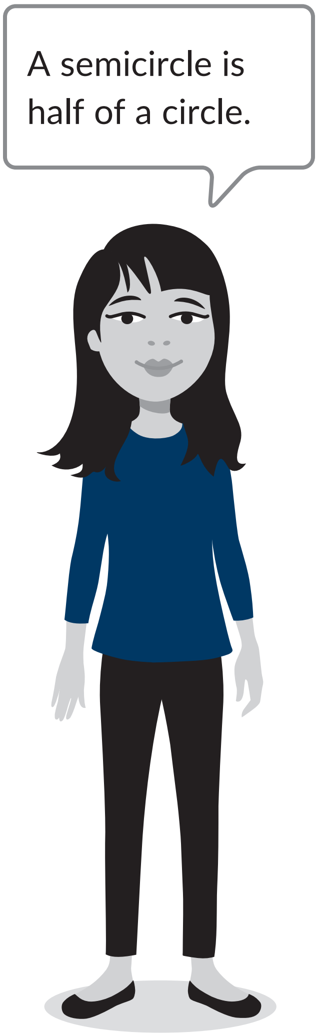
1. Draw a diagram, label the dimensions, and compute the maximum fenced area. Assume the fence is free-standing and you are not using any existing structure.

ACTIVITY

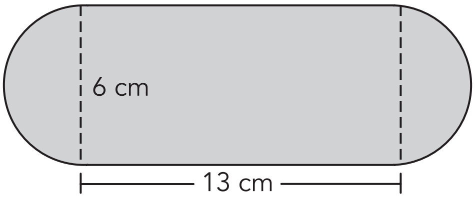
3.2 Composite Figure Problems

A composite figure is a figure that is made up of more than one geometric figure.

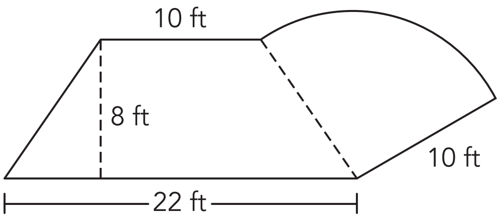
In previous grades, you worked with composite figures made up of triangles and various quadrilaterals. Now that you know the area of a circle, you can calculate the area of more interesting composite figures.



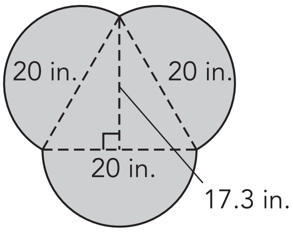
1. A figure is composed of a rectangle and two semicircles. Determine the area of the figure.



2. A figure is composed of a trapezoid and a quarter circle. Determine the area of the figure.

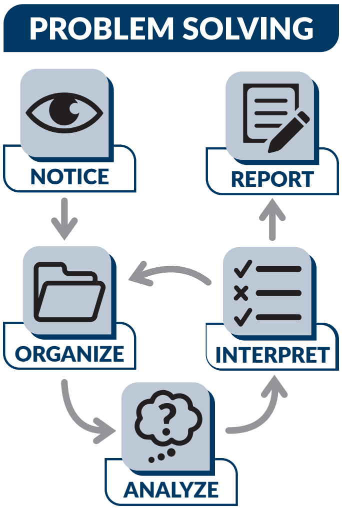


3. A figure is composed of a triangle and three semicircles. Determine the area of the figure to the nearest square inch.



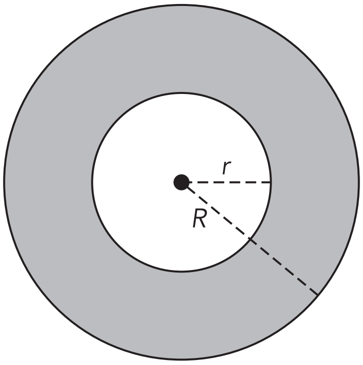
ACTIVITY
3.3

Shaded Region Problems



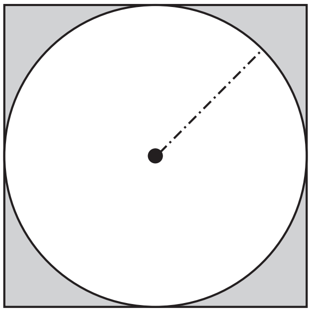
You have worked with composite figures by adding on areas.
Now, let's think about subtracting areas.

1. In the concentric circles shown, R represents the radius of the larger circle and r represents the radius of the smaller circle. Suppose that $R = 8$ centimeters and $r = 3$ centimeters. Calculate the area of the shaded region.



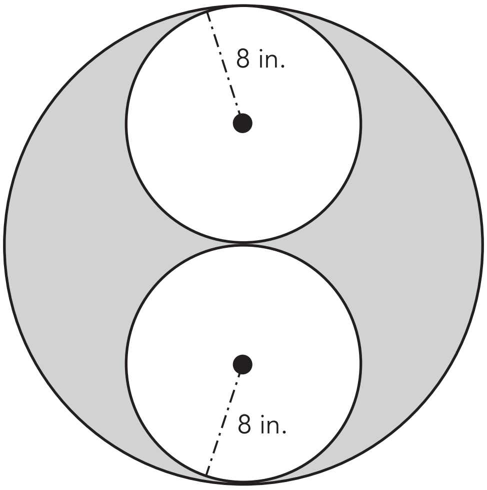
Concentric circles are circles with a common center. The region bounded by two concentric circles is called the annulus.

2. A circle is inscribed in a square. Use a centimeter ruler to determine the measurements required to determine the area of the shaded region. Then, determine the area of the shaded region.



When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.

3. Two small circles are drawn that touch each other, and both circles touch the large circle. Determine the area of the shaded region.



4. Gabriel and Diego each said the area of the shaded region is about 402 square inches. Compare their strategies.

Gabriel

Area of 1 small circle
$A \approx 3.14(8)^2$
$A \approx 3.14(64)$
$A \approx 200.96$

Area of 2 small circles
$A \approx 2(200.96)$
$A \approx 401.92$

Area of large circle
$A \approx (3.14)(16)^2$
$A \approx (3.14)(256)$
$A \approx 803.84$

Area of shaded region
$803.84 - 401.92 \approx 401.92$

The area of the shaded region is about 402 sq in.

Diego

Area of 1 small circle
$A = \pi(8)^2$
$A = 64\pi$

Area of 2 small circles
$A = 2(64\pi)$
$A = 128\pi$

Area of large circle
$A = \pi(16)^2$
$A = 256\pi$

Area of shaded region
$256\pi - 128\pi = 128\pi$
$A = 128\pi$
$A \approx 402.12$

This means the area of the shaded region is about 402 sq in.

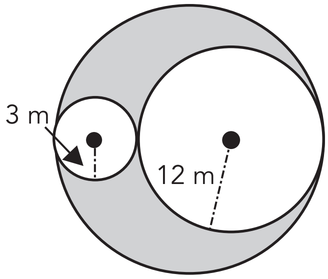
a. What did Gabriel and Diego do the same?

b. What was different about their strategies?

c. Which strategy do you prefer?

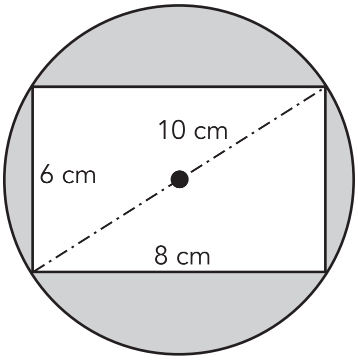
5. Determine the area of each shaded region.

a. One medium circle and one small circle touch each other, and each circle touches the large circle.



A rectangle is inscribed in a circle when all the vertices of the rectangle touch the circumference of the circle.

b. A rectangle is inscribed in a circle.



Talk the Talk

A Dog’s Leash

Valentina loves her dog. On sunny days, Valentina takes her dog to the park on a 12-foot leash. Valentina stands in one spot and lets her dog run around her to play.

1. Determine the diameter, the circumference, and the area of Rover’s play area. Use 3.14 for π.

2. Suppose Valentina wants to give her dog a little more room to play. She uses a 15-foot leash instead of the 12-foot leash. What is the area of her dog’s play area now? Use 3.14 for π.

Lesson 3 Assignment

Write

Write the area and circumference formulas for circles.

Describe pi in terms of the area and radius of a circle. Describe pi in terms of the circumference and radius of a circle.

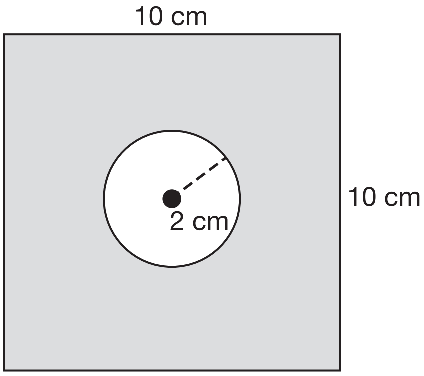
Remember

Given a specific length to form a perimeter or circumference, arranging that length into the shape of a circle provides the maximum area.

Practice

Calculate the area of the shaded region in each figure. Use 3.14 for π and round to the nearest tenth, when necessary.

1.



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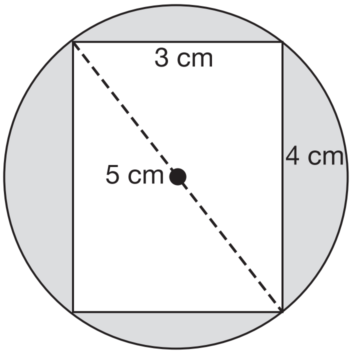

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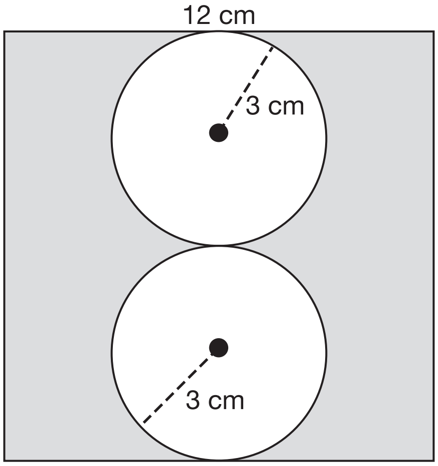

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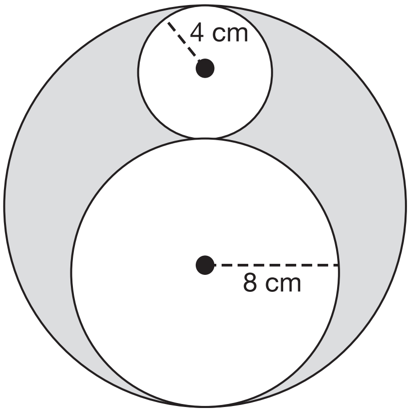


Lesson 3 Assignment

Prepare

Determine a unit rate in terms of each quantity for the given ratio.

1. 24 bracelets : 6 hours

2. 153 miles : 9 gallons

3. $48 : 3 pounds

4. 45 students : 3 teachers

TOPIC 1 SELF-REFLECTION

Name: ______________________________

Circles and Ratios

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents I **am building proficiency of the skill**, and 3 represents I **have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Circles and Ratios* topic by:

TOPIC 1: *Circles and Ratios*	Beginning of Topic	Middle of Topic	End of Topic
describing pi (π) as the ratio of the circumference to the diameter of a circle.	○	○	○
using models to derive and explain the relationship between circumference and area of a circle.	○	○	○
justifying the formulas for area and circumference of a circle and how they relate to pi (π).	○	○	○
applying the circumference and area formulas to solve mathematical and real-world problems.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Circles and Ratios* topic.

____

____

____

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

Circles and Ratio Summary

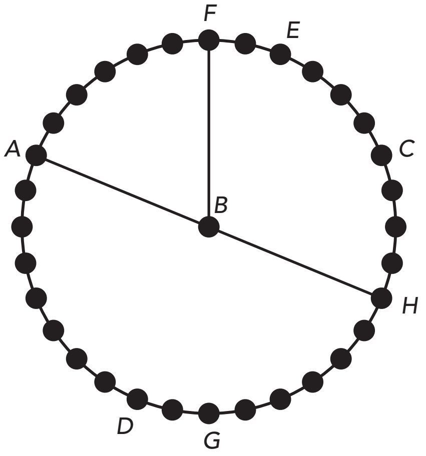
NEW KEY TERMS

- congruent [congruente]
- circle [círculo]
- radius [radio]
- diameter [diámetro]
- circumference [circunferencia]
- pi (π) [pi]
- unit rate
- composite figure [figura compuesta]

LESSON 1 Exploring the Ratio of Circle Circumference to Diameter

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant.

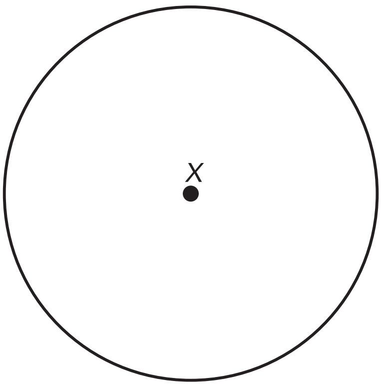
A **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is a **diameter** of the circle. A diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.



Circles are named by their center point. For example, the circle shown is Circle B. A radius of Circle B is line segment FB. A diameter of Circle B is line segment AH.

The distance around a circle is called the **circumference** of the circle. The number **pi** (π) is the ratio of the circumference of a circle to its diameter. That is, pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$. You can use the ratio to write a formula for the circumference of a circle: $C = \pi d$.

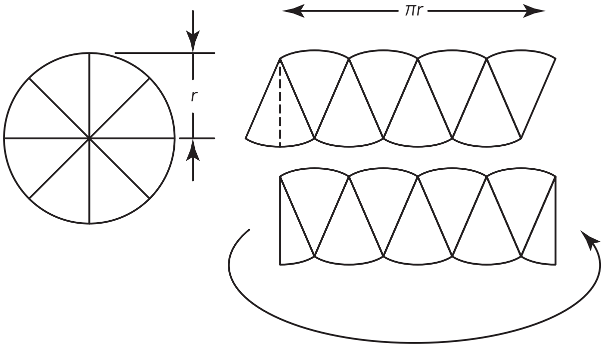
Congruent means that it has the same shape and size. For example, Circle X is congruent to Circle B. If line segment AH on Circle B has a length of 10 centimeters, then the circumference of Circle X is $C = \pi(10)$ centimeters, or approximately 31.4 centimeters.



LESSON 2 Area of Circles

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. The formula for the area of a circle is $A = \pi r^2$.

The area formula for a circle can be derived by dividing a circle into a large number of equal-sized wedges. Laying these wedges as shown, you can see that they will form an approximate rectangle with a length of πr and a height of r.



A **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

For example, a large pizza with a diameter of 18 inches costs \$14.99. The rate of area to cost is $\frac{\pi \cdot 9^2}{14.99} = \frac{81\pi}{14.99}$. Using 3.14 for π, the unit rate is approximately 16.97 square inches per dollar. The unit rate of cost to area is $\frac{1}{16.97}$, or approximately \$0.06 per square inch.

LESSON 3 Solving Area and Circumference Problems

Given a specific length to form a perimeter or circumference, arranging that length into the shape of a circle provides the maximum area.

For example, suppose you have 176 feet of fencing to use to fence off a portion of your backyard for planting vegetables. You want to maximize the amount of fenced land. Calculate the maximum fenced area you will have.

The length of fencing you have will form the circumference of a circle. Use the formula for the circumference of a circle to determine the diameter of the fenced area.

$$C = \pi d$$
$$176 = \pi d$$
$$56 \approx d$$

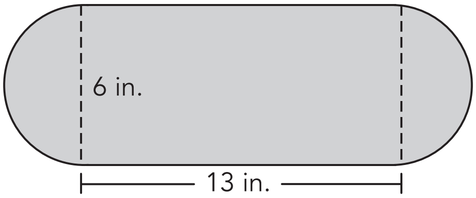
When the diameter of the fenced area is about 56 feet, the radius is 28 feet. Use this information to calculate the area of the fenced land.

$$A = \pi r^2$$
$$A = \pi \cdot 28^2$$
$$A = 784\pi \approx 2461.76$$

The maximum fenced area you will have is about 2461.76 square feet.

Many geometric figures are composed of two or more geometric shapes. These figures are known as **composite figures**. When solving problems involving composite figures, it is often necessary to calculate the area of each figure and then add these areas together.

For example, a figure is composed of a rectangle and two semi-circles. Determine the area of the figure.



Calculate the area of the rectangle.

$A = l \cdot w$

$A = (13)(6)$

$A = 78$ square inches

The two semi-circles together make one circle. Calculate the area of the circle.

$A = \pi r^2$

$A = \pi(3)^2$

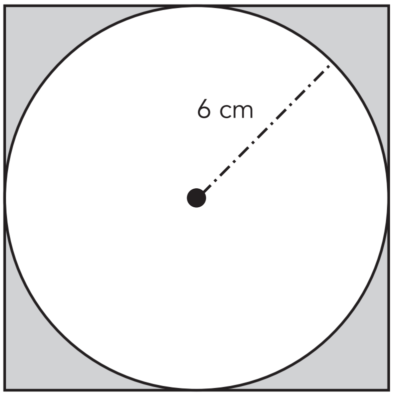
$A = 9\pi \approx 28.26$ square inches

The area of the composite figure is approximately 78 square inches plus 28.26 square inches, or about 106.28 square inches.

When determining the area of a shaded region of a figure, it is often necessary to calculate the area of a figure and subtract it from the area of a second figure.

For example, this figure shows a circle inscribed in a square. Determine the area of the shaded region.

When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.



Calculate the area of the square.

$A = s^2$

$A = 12^2$

$A = 144$ square centimeters

Calculate the area of the circle.

$A = \pi r^2$

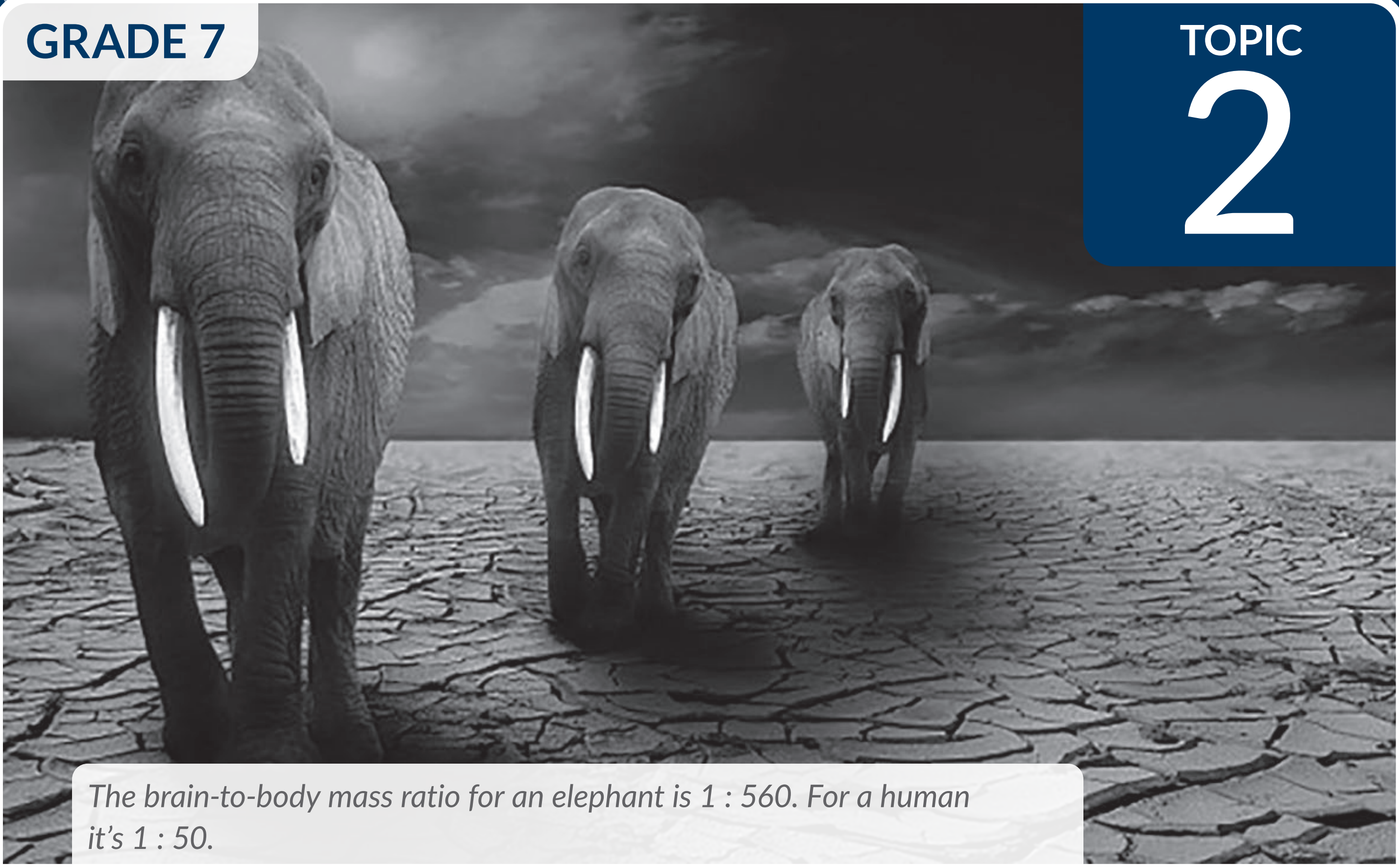
$A = \pi(6)^2$

$A = 36\pi \approx 113.04$ square centimeters

The area of the shaded region is approximately 144 square centimeters minus 113.04 square centimeters, or about 30.96 square centimeters.

GRADE 7

TOPIC 2



The brain-to-body mass ratio for an elephant is 1 : 560. For a human it's 1 : 50.

Fractional Rates

LESSON 1 Unit Rate Representations **51**

LESSON 2 Solving Problems with Ratios of Fractions **61**

LESSON 3 Solving Proportions Using Means and Extremes **75**

1

Unit Rate Representations

OBJECTIVES

- Compute unit rates associated with ratios of whole numbers and fractions.
- Represent unit rates using tables and graphs.
- Use unit rates to solve problems.

You have learned about ratios, rates, and unit rates.
How can you use tables and graphs to represent unit rates and solve problems?

Getting Started

The Pumpkin-iest!

Luna loves everything pumpkin: pumpkin waffles, pumpkin hand soap, pumpkin chili . . . Now, she's trying to make the perfect pumpkin smoothie. She is using this recipe:

Pumpkin Smoothie Recipe	
• 1 banana ($\frac{3}{4}$ cup)	• $\frac{1}{4}$ teaspoon pumpkin pie spice
• $\frac{1}{4}$ teaspoon cinnamon	• 1 cup ice
• 2 tablespoons maple syrup	• $\frac{2}{3}$ cup pumpkin puree
• $\frac{1}{2}$ cup milk	• $\frac{1}{2}$ cup vanilla yogurt

Luna wants to experiment with the given recipe.

1. What ingredients can Luna increase to make the smoothie more pumpkin-y? Less pumpkin-y?

2. What ingredients can Luna decrease to make the smoothie more pumpkin-y? Less pumpkin-y?

ACTIVITY 1.1

Determining Unit Rates

Four students share their recipes for lemon-lime punch. The class decides to analyze the recipes to determine which one will make the fruitiest tasting punch.

Mason's Recipe	Sebastian's Recipe
4 cups lemon-lime concentrate 8 cups club soda	3 cups lemon-lime concentrate 5 cups club soda
Gabriel's Recipe	Diego's Recipe
2 cups lemon-lime concentrate 3 cups club soda	1 cup lemon-lime concentrate 4 cups club soda

1. Which recipe has the strongest taste of lemon-lime? Show your work and explain your reasoning.

2. Which has the weakest taste of lemon-lime? Show your work and explain your reasoning.

Nahimana and Logan each used unit rates to compare Mason's and Sebastian's recipes.

Nahimana

Mason's recipe
4 cups lemon-lime : 8 cups club soda
The unit rate is $\frac{1}{2}$ cup lemon-lime per 1 cup of club soda.

Sebastian's recipe
3 cups lemon-lime : 5 cups club soda
The unit rate is $\frac{3}{5}$ cup lemon-lime per 1 cup of club soda.

$\frac{3}{5} > \frac{1}{2}$, so Sebastian's recipe has the stronger taste of lemon-lime.

Logan

Mason's recipe
4 cups lemon-lime : 12 cups total punch
The unit rate is $\frac{1}{3}$ cup lemon-lime per cup of punch.

Sebastian's recipe
3 cups lemon-lime : 8 cups total punch
The unit rate is $\frac{3}{8}$ cup lemon-lime per cup of punch.

$\frac{3}{8} > \frac{1}{3}$, so Sebastian's recipe has the stronger taste of lemon-lime.

3. Compare Logan's and Nahimana's strategies. In what ways are they different? How did they arrive at the same answer?

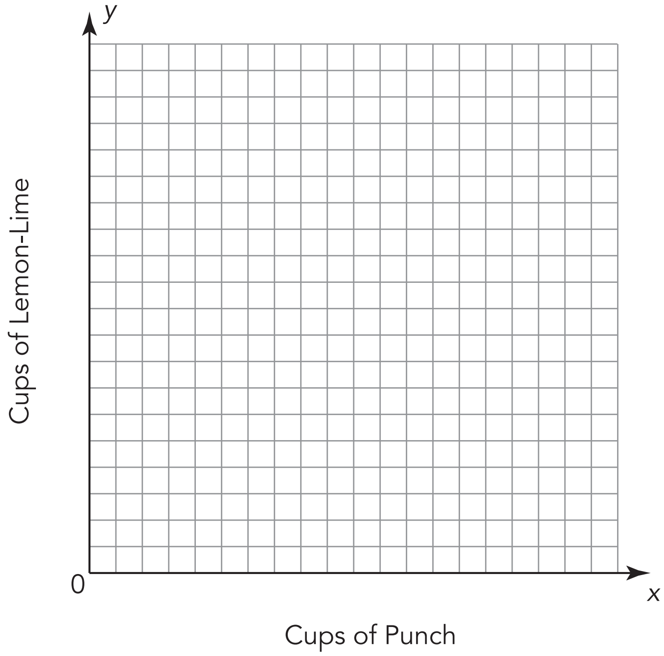
4. Complete each table and include the unit rate of lemon-lime for each cup of punch for each recipe. Then, draw a graph for each recipe on the coordinate plane. Label each graph with the person’s recipe and the unit rate.

Mason’s Recipe				
Lemon-Lime (c)				
Total Punch (c)	1			

Sebastian’s Recipe				
Lemon-Lime (c)				
Total Punch (c)	1			

Gabriel’s Recipe				
Lemon-Lime (c)				
Total Punch (c)	1			

Diego’s Recipe				
Lemon-Lime (c)				
Total Punch (c)	1			



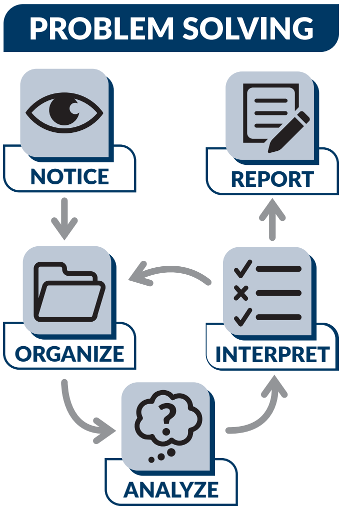
5. What does the steepness of each line represent?

6. How could you use the graphs to determine which recipe has the strongest lemon-lime taste?

The steepness of a graphed line is called its *slope*.

Look back at your process. Is there a more efficient strategy?

Talk the Talk



Getting Unit Rate-ier

Look back at the activity *The Pumpkin-iest!* Use the smoothie recipe to answer each question.

1. One teaspoon is approximately $\frac{1}{50}$ cup, and 1 tablespoon is equal to 3 teaspoons. Approximately how many cups of smoothie does Paige's recipe make?

2. How pumpkin-y are Luna's smoothies if she follows the recipe? Write a unit rate to represent the amount of pumpkin-y ingredients per cup of smoothie.

Lesson 1 Assignment

Write

How can unit rates be helpful when solving problems?

Remember

A *rate* is a ratio that compares quantities with different units. A *unit rate* is a rate in which the numerator or denominator (or both) is 1.

A unit rate is represented by the ordered pair (1, *r*) on a coordinate plane.

Practice

Complete each table and write a unit rate for the given situation.

1. General house cleaning (sweeping the floors, vacuuming, laundry, etc.) can burn 470 calories every 2 hours.

Time (hours)	1	2	3	4	5
Calories Burned					

2. A full strip plus another half of a strip of staples contains 315 staples.

Number of Strips	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
Number of Staples					

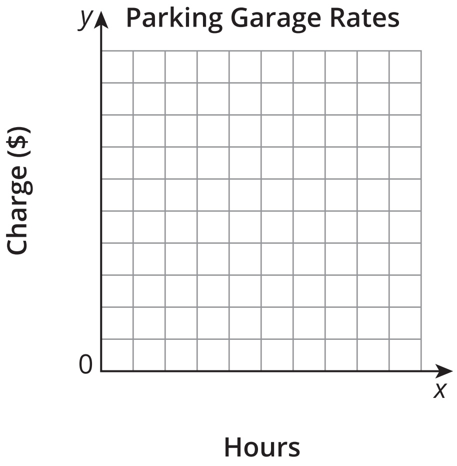
3. Kayla is making punch for a school party. The recipe she is using calls for 2 cups of lemonade to make 5 cups of punch.

Cups of Lemonade	1	2	10	12	60
Cups of Punch					

Lesson 1 Assignment

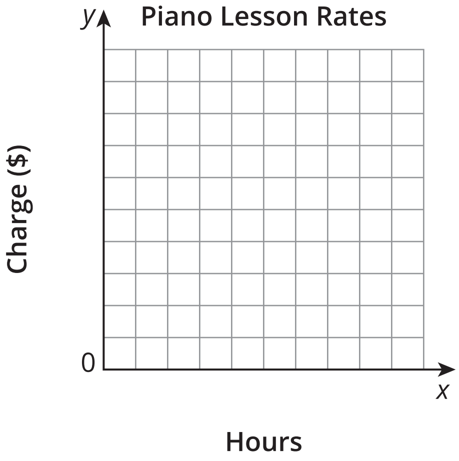
Complete each table and write a unit rate for the given situation. Then, graph the relationship on a coordinate plane.

4. The owner of a city parking garage uses a rate table so he can look up the parking charges quickly.



Hours	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
Charge	$2.25							

5. A piano teacher who provides lessons uses a rate table so that she can look up the charge for students' weekly lessons quickly.



Hours	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
Charge				$120.00				

Lesson 1 Assignment

Prepare

Determine each product or quotient.

1. $\frac{1}{2} \cdot \frac{3}{5}$

2. $\frac{5}{8} \cdot \frac{8}{5}$

3. $\frac{2}{3} \div \frac{3}{8}$

4. $\frac{3}{4} \div 1\frac{1}{2}$

2

Solving Problems with Ratios of Fractions

OBJECTIVES

- Compute unit rates from ratios of fractions, including ratios of lengths and areas.
- Interpret complex rates to solve real-world problems involving lengths and areas.

NEW KEY TERM

- complex ratio

You have learned about rates and unit rates. You have written unit rates from ratios of whole numbers.

How can you write ratios of fractions as unit rates in order to solve problems?

Getting Started

A Different Form, But Still the Same

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

Ratios can be written using any numbers. A ratio in which one or both of the quantities being compared are written as fractions is called a **complex ratio**.

For example, traveling $\frac{1}{3}$ mile in $\frac{1}{2}$ hour represents a ratio of fractions, or a complex ratio. It is also an example of a rate, since the units being compared are different.

You can write this ratio in fractional form: $\frac{\frac{1}{3}}{\frac{1}{2}}$

1. Rewrite each given rate as an equivalent ratio of fractions, or complex ratio, by converting one or both units of measure.

   a. $\frac{1}{2}$ inch of rain fell in 15 minutes.

   b. Avery ran 3520 feet in 20 minutes.

   c. The baby gained 6 ounces every week.

   d. Gas costs $2.50 per gallon.

Think about equivalent relationships. Fifteen minutes is what fraction of an hour?

ACTIVITY

2.1

Comparing Ratios of Fractions

The table shows the weights of four different adult birds and the weights of their eggs.

	Mother's Weight (oz)	Egg Weight (oz)
Pigeon	10	$\frac{3}{4}$
Chicken	80	2
Swan	352	11
Robin	$2\frac{1}{2}$	$\frac{1}{10}$

1. Compare the weights of the eggs. List the birds in order from the bird with the heaviest egg to the bird with the lightest egg.

2. Determine the ratio of egg weight to mother's weight for each bird.

The strategy to compare ratios is the same regardless of the types of numbers used.

3. Compare the ratios of egg weight to mother's weight. List the birds in order from the greatest to the least ratio.

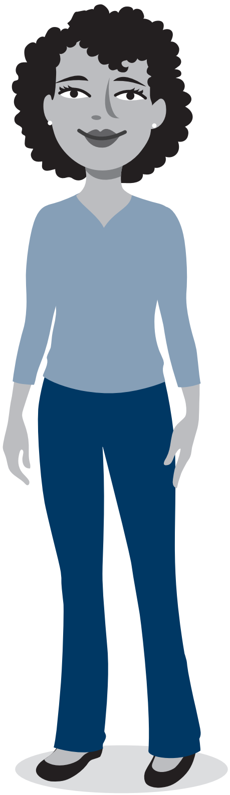
ACTIVITY 2.2 Determining Unit Rates from Ratios of Fractions

Although the ostrich is the largest living bird, it is also the fastest runner. The table shows distances that four birds ran and the amount of time it took each bird to run that distance.

Bird	Distance Covered	Time
Ostrich	22 miles	$\frac{1}{2}$ hour
Greater Roadrunner	300 yards	30 seconds
Quail	20 yards	$2\frac{1}{2}$ seconds
Pheasant	200 yards	50 seconds

Each row in the table shows a rate. The rate for each bird in this situation is the distance covered per the amount of time.

Remember, a rate is a ratio that compares two quantities that are measured in different units.



1. Write the rate for each bird as a complex rate.

   a. Ostrich

   b. Greater Roadrunner

   c. Quail

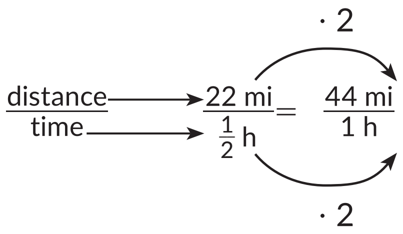
   d. Pheasant

The rates you wrote in Question 1 are each represented using different units of measure. To compare speeds, let's determine the unit rate in miles per hour for each bird. Consider the numbers and units of the original rate to choose a strategy. Analyze each Worked Example.

You know that the ostrich ran 22 miles in $\frac{1}{2}$ hour.

WORKED EXAMPLE

The rate of the ostrich is already measured in miles and hours. You can set up a proportion and scale the original rate up to 1 hour.



$$= \frac{44\text{ mi}}{1\text{ h}}$$

The ostrich's speed is 44 miles per hour.

2. Why was the scale factor of 2 used in this Worked Example?

There are 1760 yards in 1 mile.

You know that the Greater Roadrunner ran 300 yards in 30 seconds.

WORKED EXAMPLE

The rate of the Greater Roadrunner is written in yards per minute. You can use conversion rates to rewrite the rate in miles per hour.

$$\frac{300\text{ yd}}{30\text{ s}} \cdot \frac{60\text{ s}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ mi}}{1760\text{ yd}}$$

$$\frac{300\ \cancel{\text{yd}}}{30\ \cancel{\text{s}}} \cdot \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \cdot \frac{\cancel{60}\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ mi}}{1760\ \cancel{\text{yd}}}$$

$$\frac{300}{30} \cdot \frac{60}{1} \cdot \frac{60}{1} \cdot \frac{1}{1760}\,\frac{\text{mi}}{\text{hr}}$$

$$10 \cdot 60 \cdot 60 \cdot \frac{1}{1760}\,\frac{\text{mi}}{\text{hr}} \approx \frac{20.5\text{ mi}}{1\text{ hr}}$$

3. Why is the fractional representation of each conversion rate important?

4. Determine the quail's and pheasant's speeds in miles per hour.

   a. Quail's speed:

   b. Pheasant's speed:

5. Write the birds in order from the fastest rate to the slowest rate.

ACTIVITY

2.3 Converting Between Systems

In this activity, you will use the common conversions shown in the table to convert between customary and metric measurements.

Length	Mass	Capacity
1 in. = 2.54 cm	1 oz = 28.35 g	1 pt = 0.47 L
1 cm = 0.39 in.	1 g = 0.035 oz	1 L = 2.11 pint
1 ft = 30.48 cm	1 lb = 0.45 kg	1 qt = 0.95 L
1 m = 3.28 ft	1 kg = 2.2 lb	1 L = 1.06 qt
1 mi = 1.61 km		1 gal = 3.79 L
1 km = 0.62 mi		1 L = 0.26 gal
1 m = 39.37 in.		
1 in. = 0.0254 m		
1 m = 1.09 yd		

WORKED EXAMPLE

To convert between systems, you can scale up or scale down using ratios. Two methods are shown to determine how many kilograms are in 2.5 pounds.

$$\begin{tikzcd} 1\text{ lb} \arrow[d, bend right, "\cdot 2.5"'] & = & 0.45\text{ kg} \arrow[d, bend left, "\cdot 2.5"] \\ 2.5\text{ lb} & = & 1.125\text{ kg} \end{tikzcd}$$

Use the information from the chart.

Multiply to calculate the number of kilograms in 2.5 pounds.

$$\begin{tikzcd} \dfrac{1\text{ lb}}{0.45\text{ kg}} \arrow[rr, bend left, "\cdot 2.5"] \arrow[rr, bend right, "\cdot 2.5"'] & = & \dfrac{2.5\text{ lb}}{1.125\text{ kg}} \end{tikzcd}$$

Write a ratio using the information from the chart. Scale up to calculate the number of kilograms in 2.5 pounds.

The local zoo hosted a marathon to raise money to remodel the aviary. An aviary is a large enclosure for birds, which gives them more living space where they can fly, unlike confining them to birdcages.

1. To train for a marathon, which is 26.2 miles or approximately 42.2 km, runners build up their endurance by running shorter distances. Complete the table shown by writing the unknown measurements. Round to the nearest tenth.

Race	Kilometers	Miles
Short Distance	5	
Medium Distance	10	
Medium Distance	20	
Half Marathon		13.1
Ultramarathon	100	
Ironman Triathlon Swim		2.4
Ironman Triathlon Bike		112

2. The zoo earned the money that they needed to remodel the aviary! To figure out the amount of supplies needed, they will need to measure the space. The zookeeper realizes that she only has a meter stick, not a ruler or a yardstick. She measures the aviary but needs to know the dimensions in inches and feet in order to purchase the materials. She records the following measurements:
   - The length of the room is 5 meters.
   - The width of the room is 4 meters.
   - The height of the room is 2.5 meters.

   a. What is the length of the room in inches? In feet? Round to the nearest hundredth.

   b. What is the width of the room in inches? In feet? Round to the nearest hundredth.

   c. What is the height of the room in inches? In feet? Round to the nearest hundredth.

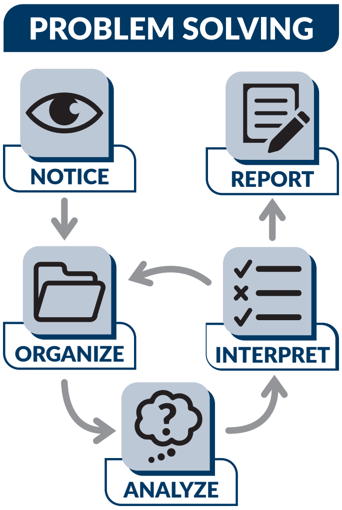
   d. There are 39.37 inches in a meter. Explain to a classmate how many feet are in a meter.

3. During cold conditions, pheasants fly 60 yards before taking to the ground for cover. In other parts of the year, they fly about 2 kilometers. Is the length of a pheasant's flight longer during cold conditions or other parts of the year? How do you know?

4. Adriana and Angelina volunteer at the zoo and are each taking care of a growing ostrich. Adriana says that her ostrich is 1.5 meters tall. Angelina's ostrich is 5 feet tall. Adriana says that her ostrich is taller, but Angelina disagrees. Who is correct? Explain your reasoning.

5. Carlos, Isaiah, Lucas, and Sebastian are also raising ostriches. Carlos's ostrich weighs 110 pounds, Isaiah's weighs 98 pounds, Lucas's weighs 42 kg, and Sebastian's weighs 52 kg. Place the students in order from the lightest weight of their ostrich to the heaviest weight using pounds and kilograms. Round to the nearest hundredth.

ACTIVITY
2.4 Solving Problems with Fractional Rates



1. Mei needs a rate table for her tutoring jobs so he can look up the charge quickly.

   a. Complete the rate table.

Lemon-Lime (c)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	$3\frac{1}{2}$	4
Charge ($)			37.50				

   b. How much would Mei charge for $7\frac{1}{2}$ hours of tutoring?

   c. Mei made $212.50 last weekend. How long did she tutor? Explain how you solved the problem.

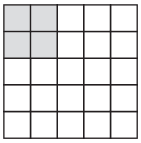
2. At a pizza restaurant, a new deal gives you $1\frac{1}{2}$ orders of wings for half the price of a single order. Without the deal, a single order of wings costs $12. What is the cost of a single order of wings with the deal?

3. Valentina uses $3\frac{3}{4}$ scoops of drink mix to make 10 cups of drink.

   a. How much drink mix would she need to use to make 1 cup of drink?

   b. She has $11\frac{1}{4}$ scoops of drink mix remaining. How many cups of drink can she make?

4. The square shown is composed of smaller equal-sized squares. The shaded section has an area of $\frac{9}{25}$ square inches. What is the area of the large square?



Talk the Talk

True, False, Example

Determine whether each statement is true or false. Provide one or more examples and an explanation to justify your answer.

1. To compute a unit rate associated with a ratio of fractions, multiply both the numerator and denominator by the reciprocal of the denominator. True False

2. Any ratio can be written as a complex ratio. True False

3. You never scale down to write a complex rate as a unit rate. True False

4. A statement with the word “per” is always a unit rate. True False

5. Dividing the numerator by the denominator is one way to convert a rate to a unit rate. True False

Lesson 2 Assignment

Write

Write a definition for *complex ratio*, provide an example, and show how your example can be converted into a unit rate.

Remember

To convert a complex rate to a unit rate, you can multiply the numerator and denominator by the reciprocal of the denominator, or you can use the definition of *division*.

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1}$$

$$= 2$$

$$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$

$$= \frac{1}{2} \cdot 4 = 2$$

Practice

1. The table shows the gallons filled in a pool over time.

Number of Hours	$\frac{1}{4}$	$\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{2}$
Gallons Filled		$637\frac{1}{2}$		

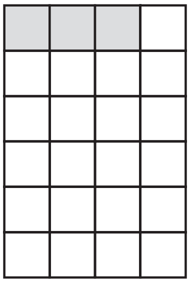
   a. Complete the table.

   b. Determine a unit rate for this situation.

   c. Use a unit rate to calculate the gallons filled in 5.5 hours.

   d. Use a unit rate to determine about how many minutes it will take to fill 100 gallons in the pool.

2. The rectangle shown is composed of smaller equal-sized squares. The shaded section has an area of $\frac{3}{16}$ square inches. Use a unit rate to determine the area of the larger rectangle.



Lesson 2 Assignment

Read each situation and answer each question.

3. Emily took a 5-mile walk at the park on Saturday. How many kilometers did Emily walk? Show your work.

4. Mr. Martinez pumped 12 gallons of gas into his car. How many liters of gas would that be? Show your work.

5. What is your height in centimeters? Show your work.

6. Use $>$, $<$, or $=$ to make each statement correct.

   a. 2 in. ________ 4 cm

   b. 2 kg ________ 4.4 lb

   c. 3 qt ________ 3 L

   d. 6 km ________ 3 mi

7. The school cafeteria has eight very large cans of tomato sauce for making pizza. Each can contains 2 gallons of sauce. Is there more or less than 50 L of sauce in these cans? Explain your reasoning.

Prepare

Solve each equation.

1. $w - 5 = 25$

2. $9x = 990$

3. $\frac{c}{12} = 48$

4. $1.15 + m = 10$

3

Solving Proportions Using Means and Extremes

OBJECTIVES

- Rewrite proportions to maintain equality.
- Represent proportional relationships by equations.
- Develop strategies to solve proportions.
- Use proportional relationships to solve multi-step problems.

NEW KEY TERMS

- proportion
- variable
- means
- extremes
- solve a proportion
- isolate the variable
- inverse operations

You have learned how to write proportions and calculate unknown values through scaling up and scaling down.

Is there a more efficient strategy that works for any unknown in any proportion?

Getting Started

Recall that a **proportion** is an equation that states that two ratios are equal.

Mix-N-Match

A proportion can be written several ways. Each example shows three proportions using the same four quantities.

	Example 1	Example 2
Proportion 1	$\frac{2}{3} = \frac{4}{6}$	$\frac{5}{7} = \frac{15}{21}$
Proportion 2	$\frac{6}{3} = \frac{4}{2}$	$\frac{21}{7} = \frac{15}{5}$
Proportion 3	$\frac{2}{4} = \frac{3}{6}$	$\frac{5}{15} = \frac{7}{21}$

1. In each example, use arrows to show how the numbers were rearranged from the:

   a. first proportion to the second proportion.

   b. first proportion to the third proportion.

ACTIVITY
3.1

Maintaining Equality with Proportions

Because it is impossible to count each individual animal, marine biologists use a method called the capture-recapture method to estimate the population of certain sea creatures. In certain areas of the world, biologists randomly catch and tag a given number of sharks. After a period of time, such as a month, they recapture a second sample of sharks and count the total number of sharks as well as the number of recaptured tagged sharks. Then, the biologists use proportions to estimate the population of sharks living in a certain area.

Biologists can set up a proportion to estimate the total number of sharks in an area:

$$\frac{\text{Original number of tagged sharks}}{\text{Total number of sharks in an area}} = \frac{\text{Number of recaptured tagged sharks}}{\text{Number of sharks caught in the second sample}}$$

Although capturing the sharks once is necessary for tagging, it is not necessary to recapture the sharks each time. At times, the tags can be observed through binoculars from a boat or at shore.

Biologists originally caught and tagged 24 sharks off the coast of Cape Cod, Massachusetts, and then released them back into the bay. The next month, they caught 80 sharks with 8 of the sharks already tagged. To estimate the shark population off the Cape Cod coast, biologists set up the following proportion:

$$\frac{24 \text{ tagged sharks}}{p \text{ total sharks}} = \frac{8 \text{ recaptured tagged sharks}}{80 \text{ total sharks}}$$

Notice the variable p in the proportion. In this proportion, let p represent the total shark population off the coast of Cape Cod.

A **variable** is a letter or symbol used to represent a number.

Ask Yourself . . .
What observation can you make?

1. Write three additional different proportions you could use to determine the total shark population off the coast of Cape Cod.

2. Estimate the total shark population using any of the proportions.

3. Did any of the proportions seem more efficient than the other proportions?

4. Wildlife biologists tag deer in wildlife refuges. They originally tagged 240 deer and released them back into the refuge. The next month, they observed 180 deer of which 30 deer were tagged. Approximately how many deer are in the refuge? Write a proportion and show your work to determine your answer.

A proportion of the form $\frac{a}{b} = \frac{c}{d}$ can be written in many different ways.

Another example is $\frac{d}{b} = \frac{c}{a}$ or $\frac{c}{a} = \frac{d}{b}$.

5. Show how the variables were rearranged from the proportion in the “if” statement to each proportion in the “then” statement to maintain equality.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{d}{b} = \frac{c}{a}$.

   If $\frac{d}{b} = \frac{c}{a}$, then $\frac{c}{a} = \frac{d}{b}$.

6. Write all the different ways you can rewrite the proportion $\frac{a}{b} = \frac{c}{d}$ and maintain equality.

ACTIVITY 3.2 Solving Proportions with Means and Extremes

A quality control department checks the product a company creates to ensure that the product is not defective.

The Ready Steady Battery Company tests batteries as they come through the assembly line and then uses a proportion to predict how many batteries in its total production might be defective.

On Friday, the quality controller tested every tenth battery and found that of the 320 batteries tested, 8 were defective. If the company shipped a total of 3200 batteries, how many might be defective?

Let's analyze a few methods.

Lucas

$$\frac{8 \text{ defective batteries}}{320 \text{ batteries}} = \frac{d \text{ defective batteries}}{3200 \text{ batteries}}$$

$$\underset{\cdot 10}{\overset{\cdot 10}{\frac{8}{320} = \frac{d}{3200}}}$$

$$d = 80$$

So, 80 batteries might be defective.

Mason

$$\cdot 10 \left(\begin{array}{l} 8 \text{ defective batteries} : 320 \text{ batteries} \\ \\ d \text{ defective batteries} : 3200 \text{ batteries} \end{array} \right) \cdot 10$$

$$d = 80$$

About 80 batteries will probably be defective.

1. How are Lucas's and Mason's methods similar?

Sarah

$$\frac{8 \text{ defective batteries}}{320 \text{ batteries}} \overset{\div 8}{\underset{\div 8}{=}} \frac{1 \text{ defective battery}}{40 \text{ batteries}} \overset{\cdot 80}{\underset{\cdot 80}{=}} \frac{80 \text{ defective batteries}}{3200 \text{ batteries}}$$

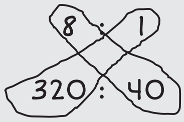
One out of every 40 batteries is defective. So, out of 3200 batteries, 80 batteries could be defective because 3200 ÷ 40 = 80.

2. Describe the strategy Sarah used.

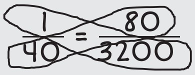
Camilla

When I write Sarah's ratios using colons like Lucas, I notice something about proportions . . .

$$8 : 320 = 1 : 40$$



$$1 : 40 = 80 : 3200$$



. . . the two middle numbers have the same product as the two outside numbers. So, I can solve any proportion by setting these two products equal to each other.

3. Verify that Camilla is correct.

4. Try the various proportion-solving methods on these proportions and determine the unknown value. Explain which method you used.

   a. $\frac{\text{3 granola bars}}{\text{420 calories}} = \frac{\textbf{\textit{g}}\text{ granola bars}}{\text{140 calories}}$

   b. 8 correct : 15 questions = 24 correct : q questions

   c. $\frac{\textbf{\textit{d}}\text{ dollars}}{\text{5 miles}} = \frac{\$9}{\text{7.5 miles}}$

Multiplying the means and extremes is like "cross-multiplying."

The relationship that Camilla noticed is between the *means* and *extremes*. In a proportion that is written a : b = c : d, the product of the two values in the middle (the **means**) equals the product of the two values on the outside (the **extremes**).

extremes

$a : b = c : d$

means

$bc = ad$

or

$\frac{a}{b} = \frac{c}{d}$

means extremes

$bc = ad$

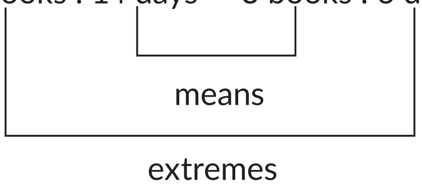
when $b \neq 0$, $d \neq 0$

To solve a proportion using this method, first identify the means and extremes. Then, set the product of the means equal to the product of the extremes and solve for the unknown quantity. To **solve a proportion** means to determine all the values of the variables that make the proportion true.

WORKED EXAMPLE

You can rewrite a proportion as the product of the means equal to the product of the extremes.

7 books : 14 days = 3 books : 6 days



$$(14)(3) = (7)(6)$$

$$42 = 42$$

$$\frac{7 \text{ books}}{14 \text{ days}} = \frac{3 \text{ books}}{6 \text{ days}}$$

$$(14)(3) = (7)(6)$$

$$42 = 42$$

5. You can rewrite the product of the means and extremes from the Worked Example as four different equations. Analyze each equation.

$$3 = \frac{(7)(6)}{14} \qquad 14 = \frac{(7)(6)}{3} \qquad \frac{(3)(14)}{7} = 6 \qquad \frac{(3)(14)}{6} = 7$$

   a. Why are these equations all true? Explain your reasoning.

   b. Compare these equations to the equation in the Worked Example showing the product of the means equal to the product of the extremes. How was the balance of the equation maintained in each?

6. Why is it important to maintain balance in equations?

WORKED EXAMPLE

In the proportion $\frac{a}{b} = \frac{c}{d}$, you can multiply both sides by b to isolate the variable a.

$$b \cdot \frac{a}{b} = b \cdot \frac{c}{d} \longrightarrow a = \frac{bc}{d}$$

When you **isolate the variable** in an equation, you perform an operation, or operations, to get the variable by itself on one side of the equals sign. Multiplication and division are *inverse operations*. **Inverse operations** are operations that "undo" each other.

WORKED EXAMPLE

Another strategy to isolate the variable a is to multiply the means and extremes and then isolate the variable by performing inverse operations.

$$\frac{a}{b} = \frac{c}{d}$$

Step 1: $ad = bc$

Step 2: $\frac{ad}{d} = \frac{bc}{d}$

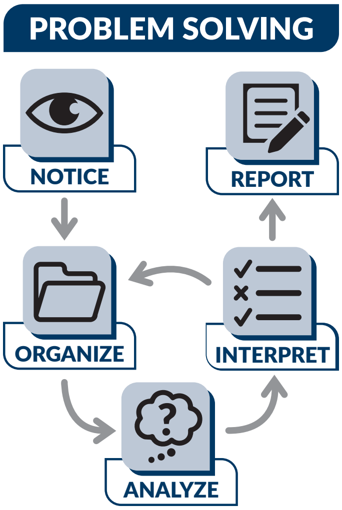
Step 3: $a = \frac{bc}{d}$

7. Describe each step shown.

8. Rewrite the proportion $\frac{a}{b} = \frac{c}{d}$ to isolate each of the other variables: b, c, and d. Explain the strategies you used to isolate each variable.

ACTIVITY
3.3

Solving Problems with Proportions



Write and solve proportions to solve each problem.

1. An astronaut who weighs 85 kilograms on Earth weighs 14.2 kilograms on the moon. How much would a person weigh on the moon if they weigh 95 kilograms on Earth? Round your answer to the nearest tenth.

2. Water goes over Niagara Falls at a rate of 180 million cubic feet every $\frac{1}{2}$ hour. How much water goes over the falls in 1 minute?

3. The value of the U.S. dollar in comparison to the value of foreign currency changes daily. Complete the table shown. Round to the nearest hundredth.

Euro	U.S. Dollar
1	1.07
	1.00
	6.00
6	
10	

Do you see how to set up proportions by using two different rows of the table?

4. To make 4.5 cups of fruity granola, the recipe calls for 1.5 cups of raisins, 1 cup of granola, and 2 cups of blueberries. If you want to make 18 cups of fruity granola, how much of each of the ingredients do you need?

Talk the Talk

Choose Your Own Proportion Adventure

Write a problem situation for each proportion. Show the solution.

1. $\frac{8}{3} = \frac{2}{n}$

2. $\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{h}{1}$

Lesson 3 Assignment

Write

Describe a situation in which you would use each.

1. Variable
2. Means and extremes
3. Inverse operations
4. Isolate the variable

Remember

To *solve a proportion* means to determine all the values of the variables that make the proportion true.

You can rewrite a proportion as the product of the means and extremes.

If $\frac{a}{b} = \frac{c}{d}$, then $bc = ad$.

Practice

Write and solve a proportion to answer each question.

1. Isaiah is making a strawberry drink. The recipe calls for 5 parts strawberry juice to 3 parts water. Isaiah would like to make 64 fluid ounces of the strawberry drink. How many fluid ounces of strawberry juice and water does Isaiah need?

2. Avery is making a grape drink. The recipe calls for 2 parts grape juice concentrate to 6 parts water. Avery would like to make 80 fluid ounces of the grape drink. How many fluid ounces of grape juice concentrate and water does Avery need?

3. Mei is making a trail mix. The recipe calls for 3 parts golden raisins to 2 parts cashews. Mei would like to make 30 cups of trail mix. How many cups of golden raisins and cashews does Mei need?

Lesson 3 Assignment

4. Jasmine is making a snack mix. The recipe calls for 6 parts of spicy tortilla chips to 3 parts of corn chips. Jasmine would like to make 45 cups of snack mix. How many cups of spicy tortilla chips and corn chips does Jasmine need?

5. Luna is making a bean salad. The recipe calls for 4 parts green beans to 3 parts yellow wax beans. Luna would like to make 56 ounces of bean salad. How many ounces of green beans and yellow wax beans does Luna need?

6. Diego is making smoothies. The recipe calls for 2 parts yogurt to 3 parts blueberries. Diego wants to make 10 cups of smoothie mix. How many cups of yogurt and blueberries does Diego need?

Prepare

1. A bus travels 18 miles in 15 minutes. At the same rate, what distance will the bus travel in 50 minutes?

2. A copy machine averages 210 copies in 5 minutes. At the same rate, how many copies can the machine make in 12 minutes?

TOPIC 2 SELF-REFLECTION

Name: ______________________________

Fractional Rates

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Fractional Rates* topic by:

TOPIC 2: *Fractional Rates*	Beginning of Topic	Middle of Topic	End of Topic
representing unit rates using tables and graphs.	○	○	○
computing unit rates associated with ratios of fractions.	○	○	○
solving problems with ratios of fractions.	○	○	○
using unit rates to convert between measurement systems.	○	○	○
using proportions to convert between measurement systems.	○	○	○
representing proportional relationships by writing equations.	○	○	○
solving proportions using formal strategies.	○	○	○
solving real-world problems that involve ratios, rates, unit rates, and proportions.	○	○	○

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Fractional Rates* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

Fractional Rates Summary

LESSON 1

Unit Rate Representations

A *rate* is a ratio that compares quantities with different units. A *unit rate* is a rate in which the numerator or denominator (or both) is 1. Two or more ratios or rates can be compared.

For example, two friends make lemon-lime punch using the following recipes.

Camilla's Recipe	Emily's Recipe
1 cup lemon-lime concentrate 3 cups club soda	2 cups lemon-lime concentrate 5 cups club soda

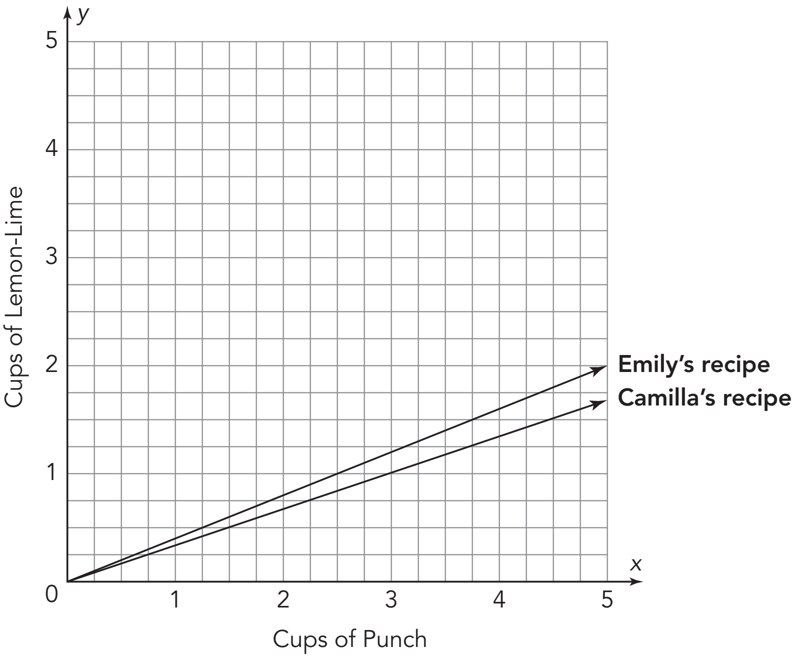
Determine which recipe has the stronger taste of lemon-lime.

For Camilla's recipe, the ratio of lemon-lime to club soda is 1 cup : 3 cups, so the unit rate is $\frac{1}{3}$ cup lemon-lime per 1 cup club soda. For Emily's recipe, the ratio of lemon-lime to club soda is 2 cups : 5 cups, so the unit rate is $\frac{2}{5}$ cup lemon-lime per 1 cup club soda.

$\frac{1}{3} < \frac{2}{5}$, so Emily's recipe has the stronger taste of lemon-lime.

NEW KEY TERMS

- complex ratio [razón compleja]
- proportion [proporción]
- variable [variable]
- means [medios]
- extremes [extremos]
- solve a proportion [resolver una proporción]
- isolate the variable [aislar la variable]
- inverse operations [operaciones inversas]



A unit rate can be represented by the ordered pair (1, *r*) on a coordinate plane.

For example, the graph shows the ratios of lemon-lime to club soda for Camilla's and Kim's recipes.

The point $\left(1, \frac{1}{3}\right)$ represents the unit rate for Camilla's recipe, and the point $\left(1, \frac{2}{5}\right)$ represents the unit rate for Emily's recipe.

The graph of Emily's line is steeper than the graph of Camilla's line, which shows that Emily's recipe has a stronger taste of lemon-lime.

LESSON

2

Solving Problems with Ratios of Fractions

A ratio in which one or both of the quantities being compared are written as fractions is called a **complex ratio**.

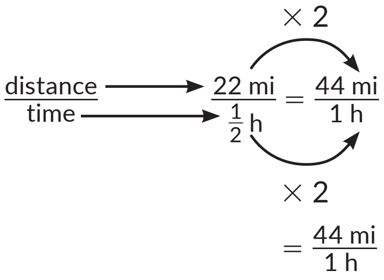
For example, traveling $\frac{1}{2}$ mile in $\frac{1}{4}$ hour represents a ratio of fractions, or a *complex ratio*. You can write this ratio, which is also a rate, in fractional form: $\frac{\frac{1}{2}}{\frac{1}{4}}$.

To convert a complex rate to a unit rate, you can multiply the numerator and denominator by the reciprocal of the denominator, or you can use the definition of division.

In this example, the unit rate for traveling $\frac{1}{2}$ mile in $\frac{1}{4}$ hour is 2 miles per hour.

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1} \qquad \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$

$$\frac{2}{1} = 2 \qquad = \frac{1}{2} \cdot 4 = 2$$



To compare unit rates that are given in different units, you can use proportions and conversion rates.

For example, an ostrich can run 22 miles in $\frac{1}{2}$ hour. The greater roadrunner can run 300 yards in $\frac{1}{2}$ minute. Which bird runs faster?

The rate of the ostrich is already measured in miles and hours. You can set up a proportion and scale the original rate up to 1 hour.

The ostrich's speed is 44 miles per hour.

The rate of the greater roadrunner is written in yards per minute. You can use conversion rates to rewrite the rate in miles per hour.

The Greater Roadrunner's speed is about 20.5 miles per hour.

The ostrich runs faster.

You can use common conversions to convert between measurement systems.

$$\frac{300 \text{ yd}}{\frac{1}{2} \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}}$$

$$\frac{300 \cancel{\text{yd}}}{\frac{1}{2} \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{1760 \cancel{\text{yd}}}$$

$$\frac{300}{\frac{1}{2}} \cdot \frac{60}{1} \cdot \frac{1 \text{ mi}}{1760 \text{ hr}}$$

$$600 \cdot \frac{60}{1} \cdot \frac{1 \text{ mi}}{1760 \text{ hr}} < \frac{20.5 \text{ mi}}{1 \text{ hr}}$$

To convert between systems, you can scale up or scale down using ratios.

Write a ratio using the common conversion of 1 lb = 0.45 kg.

Scale up to calculate the number of kilograms in 2.5 pounds.

$$\overset{\times 2.5}{\underset{\times 2.5}{\frac{1 \text{ lb}}{0.45 \text{ kg}} = \frac{2.5 \text{ lb}}{1.125 \text{ kg}}}}$$

LESSON
3

Solving Proportions Using Means and Extremes

A **proportion** is an equation that states that two ratios are equal. A **variable** is a letter or symbol used to represent a number. A proportion of the form $\frac{a}{b} = \frac{c}{d}$ can be written in many different ways. Another example is $\frac{d}{b} = \frac{c}{a}$ or $\frac{c}{a} = \frac{d}{b}$.

In a proportion that is written $a : b = c : d$, the product of the two values in the middle (the **means**) equals the product of the two values on the outside (the **extremes**). Multiplying the means and extremes is like "cross-multiplying.“

extremes

$a : b = c : d$

means

$bc = ad$

or

means $\frac{a}{b} = \frac{c}{d}$ extremes

$bc = ad$

when $b \neq 0, d \neq 0$

To solve a proportion using this method, first identify the means and extremes. Then, set the product of the means equal to the product of the extremes and solve for the unknown quantity. To **solve a proportion** means to determine all the values of the variables that make the proportion true.

For example, you can rewrite the proportion $\frac{7 \text{ books}}{14 \text{ days}} = \frac{3 \text{ books}}{6 \text{ days}}$ as the product of the means equal to the product of the extremes.

$$\underbrace{7\text{ books} : \underbrace{14\text{ days} = 3\text{ books}}_{\text{means}} : 6\text{ days}}_{\text{extremes}}$$

$$(14)(3) = (7)(6)$$

$$42 = 42$$

$$\frac{7\text{ books}}{14\text{ days}} = \frac{3\text{ books}}{6\text{ days}}$$

$$(14)(3) = (7)(6)$$

$$42 = 42$$

When you **isolate the variable** in an equation, you perform an operation, or operations, to get the variable by itself on one side of the equals sign. Multiplication and division are *inverse operations*. **Inverse operations** are operations that “undo” each other.

In the proportion $\frac{a}{b} = \frac{c}{d}$, you can multiply both sides by b to isolate the variable a.

$$b \cdot \frac{a}{b} = \frac{c}{d} \cdot b \longrightarrow a = \frac{cb}{d}$$

Another strategy is to multiply the means and extremes, and then isolate the variable by performing inverse operations.

$$\frac{a}{b} = \frac{c}{d}$$

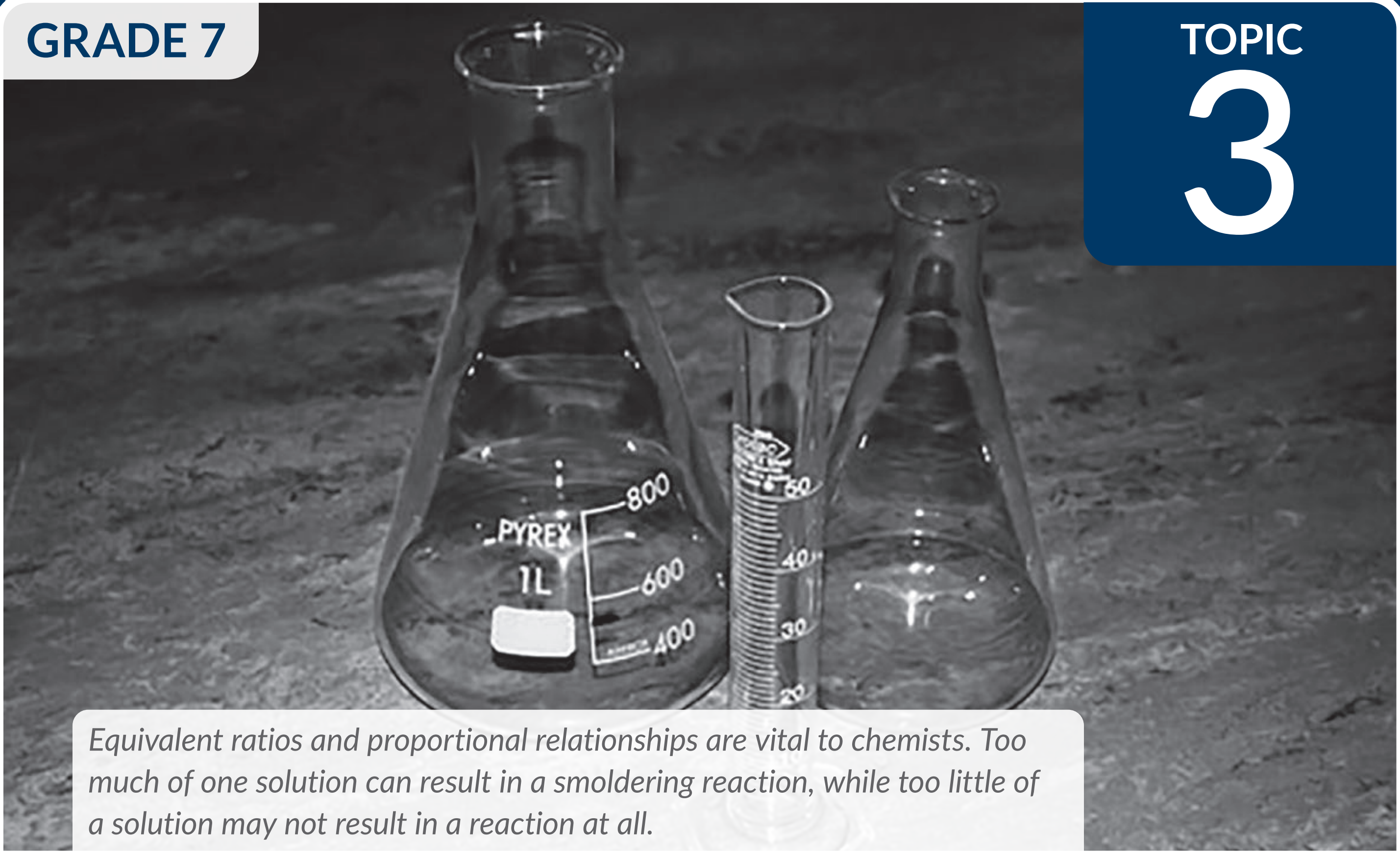
$$ad = bc$$

$$\frac{ad}{d} = \frac{bc}{d}$$

$$a = \frac{bc}{d}$$

GRADE 7

TOPIC
3



Equivalent ratios and proportional relationships are vital to chemists. Too much of one solution can result in a smoldering reaction, while too little of a solution may not result in a reaction at all.

Proportionality

LESSON 1 Proportional Relationships 97

LESSON 2 Constant of Proportionality 117

LESSON 3 Identifying the Constant of Proportionality in Graphs 135

LESSON 4 Constant of Proportionality in Multiple Representations 151

1

Proportional Relationships

OBJECTIVES

- Use tables and graphs to explore proportional relationships.
- Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table.
- Decide whether two quantities are in a proportional relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

NEW KEY TERMS

- origin
- proportional relationship

You have learned about the relationship between ratios, a comparison of two quantities, and proportions.

How can you determine whether a proportional relationship exists between two quantities?

Getting Started

Keep on Mixing!

The students in Mr. Park's art class created various quantities of bluish-green paint using pints of yellow and blue paint.

The table shows the different mixtures of paint, in pints, that the students made.

Amount of Bluish-Green Paint	Amount of Yellow Paint	Amount of Blue Paint
3 pt	1 pt	2 pt
5 pt	2 pt	3 pt
6 pt	2 pt	4 pt
12 pt	4 pt	8 pt
15 pt	6 pt	9 pt
20 pt	8 pt	12 pt

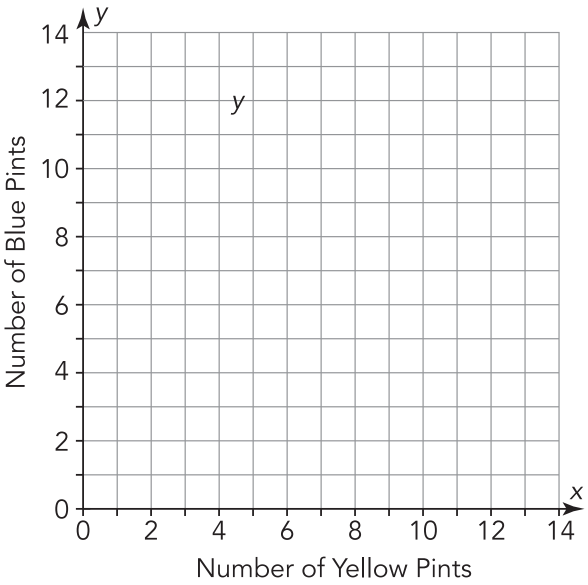
1. How many different shades of paint did the students make? How do you know?

2. Some of the shades of the paint are more yellow than others. Which mixture(s) are the most yellow? Explain your reasoning.

3. Plot an ordered pair for each bluish-green paint mixture. Draw a line connecting each point to the origin. What do you notice?

The **origin** is a point on a graph with the ordered pair (0, 0).

Bluish-Green Paint



ACTIVITY

1.1

Representations of Varying Quantities

The student government association at a local middle school is creating an urban garden at their school for use by their community. They divided up into groups to design different parts of the garden and were asked to (1) describe their project, (2) create an equation to model part of their design or to answer a question about their design, and (3) sketch a graph of their equation.

1. Parker, the president of the student government association, mixed up the representations of the projects after they were submitted to her. Help Parker match the scenarios, equations, and graphs.
    - Cut out the scenarios, equations, and graphs at the end of the lesson.
    - Sort the scenarios, equations, and graphs into corresponding groups.
    - Tape the representations into the table provided.

When you connect an equation to a graph, you are establishing a dependency between the quantities. Remember, the independent quantity is always represented on the *x*-axis.

Ask Yourself . . .
How does representing mathematics in multiple ways help communicate reasoning?

Scenario			
Equation			
Graph			

ACTIVITY

1.2 Defining Proportional Relationships

When looking over the submissions from the urban garden working groups, Isaac notices that there are two different types of graphical relationships represented: linear and non-linear.

1. Classify each group's graph as representing a linear or a non-linear relationship between quantities.

Parker notices that the linear graphs are slightly different, but she doesn't know why. She decides to analyze a table of values for each linear graph.

2. Create a table of at least four values for each linear relationship in the urban garden project.

x	y

x	y

Parker knows that simple equations can represent additive or multiplicative relationships between quantities.

3. Analyze the equations.

   a. Based on the equations, which graph represents an additive relationship between the variables and which represents a multiplicative relationship?

   b. Which variable represents the independent variable (input) and which represents the dependent variable (output)?

One special type of relationship that compares quantities using multiplicative reasoning is a ratio relationship. When two equivalent ratios are set equal to each other, they form a proportion. The quantities in the proportion are in a **proportional relationship**.

You can decide whether two quantities are in a proportional relationship by testing that all ratios, $\frac{y}{x}$ or $\frac{x}{y}$, in a table of values are equivalent.

For a relationship to illustrate a proportional relationship, all the ratios, $\frac{y}{x}$ or $\frac{x}{y}$, must represent the same constant.

4. Use your tables of values in Question 2 to determine whether any of the linear relationships illustrate a proportional relationship. Show the values of the ratios in each relationship.

Parker says the equation $y = \pi r^2$ represents a proportional relationship between y and r because it includes multiplication between a numerical coefficient and a variable expression.

5. Use a table of values and corresponding ratios in the form of $\frac{y}{x}$ to explain why Parker is incorrect.

6. Explain to Parker how the graphs in the urban garden design project are different. Include the terms *linear relationship* and *non-linear relationship*, *proportional relationship*, and *equivalent ratios*.

ACTIVITY 1.3

Proportional or Not?

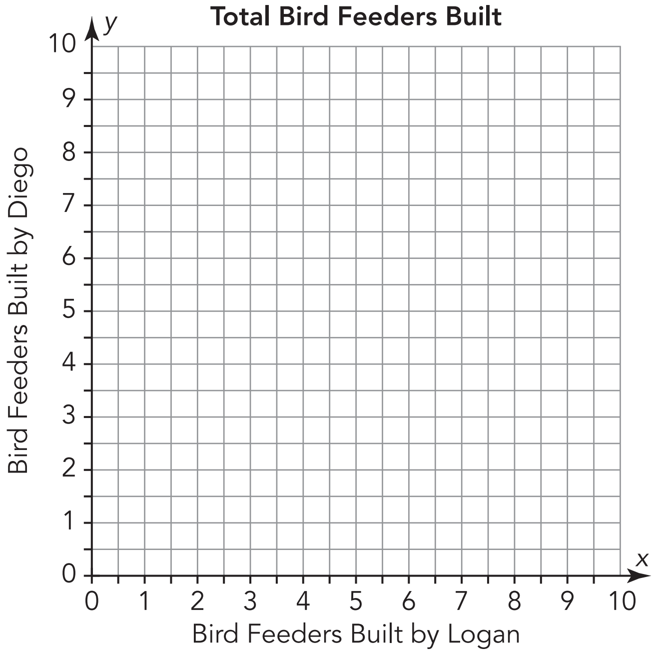
In this activity, you will analyze three different problem situations and then determine which represents a proportional relationship.

Logan and his little brother Diego want to build bird feeders to sell at a local farmers market. They have enough money to buy materials to build 10 bird feeders.

1. Complete a table of values by listing possible ways in which they can divide up the work. Assume that each brother only makes whole bird feeders. Then, complete the graph.

You can draw a line through your points to model the relationship. Then, decide whether all the points make sense in terms of the problem situation.

Bird Feeders Built by Logan	Bird Feeders Built by Diego



2. Describe how the number of bird feeders built by Logan affects the number of bird feeders Diego builds.

3. What is the ratio of bird feeders that Logan builds to the number of bird feeders that Diego builds? Explain your reasoning.

4. Gabriel claims that the number of bird feeders Logan builds is proportional to the number of bird feeders Diego builds. Do you agree with Gabriel claim? Explain your reasoning.

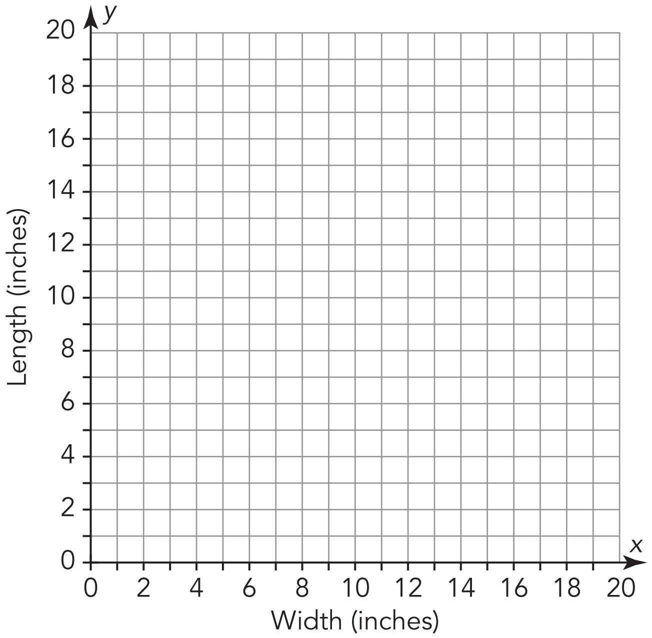
Camilla was given a math problem to determine how many different rectangles can be constructed with an area of 12 square inches.

5. Camilla thinks that there are only two: one with a width of 2 inches and a length of 6 inches and another with a width of 3 inches and a length of 4 inches.

   Is she correct? Explain your reasoning.

6. Complete a table of values for the width and length of a rectangle with an area of 12 square inches. Then, complete the graph.

Width of Rectangle (in.)	Length of Rectangle (in.)



7. Describe how the width of the rectangle affects the length of the rectangle.

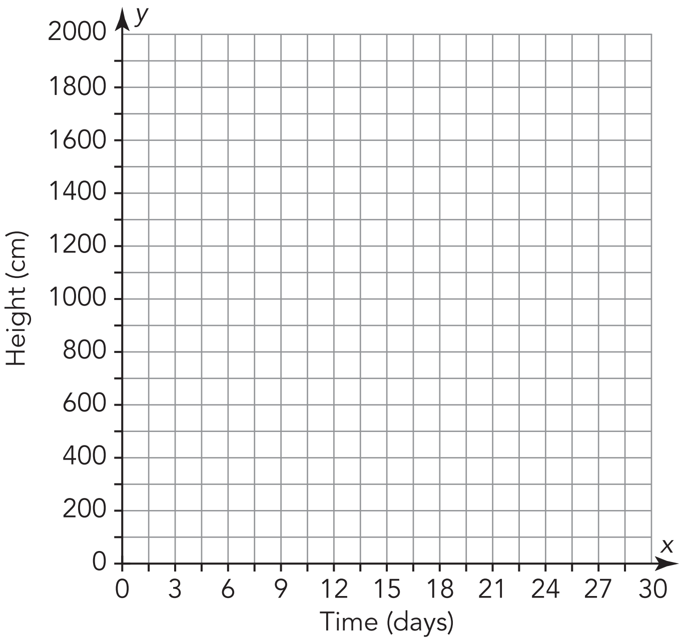
8. Do the width and length of a rectangle with an area of 12 square inches form a proportional relationship? Explain your reasoning.

One species of bamboo can grow at an average rate of 60 centimeters per day.

9. Complete a table of values using the given growth rate of the bamboo plant. Then, complete the graph.

Why do you think this problem says *average rate* instead of just *rate*?

Time (days)	Height of Bamboo (cm)



10. Describe how the time affects the height of the bamboo plant.

11. Is the number of days of growth proportional to the height of the bamboo plant? Explain your reasoning.

ACTIVITY

1.4 Graphs of Proportional Relationships

In the previous activity, the number of days of growth of the bamboo was proportional to the height of the bamboo plant.

The points of a proportional relationship on a graph form a straight line, and the line passes through the origin.

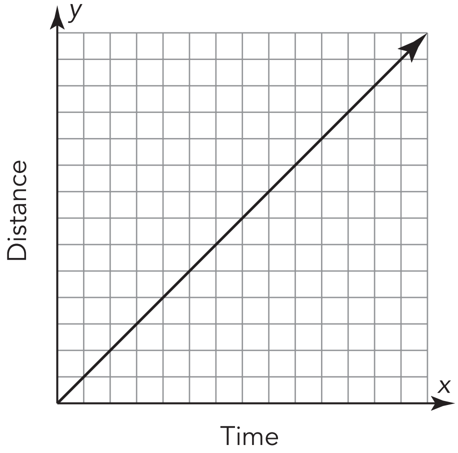
Examine the Worked Example.

For an object moving at a constant rate, the formula $d = rt$ relates the distance traveled, d, elapsed time, t, and rate, r.

WORKED EXAMPLE

A car driving at a constant rate of 60 miles per hour.

A sketch of a graph that represents y, the total distance driven in x hours is shown.



When you sketch a graph, be sure to include the labels for each axis. However, you don't *always* have to show values.

1. Explain how the situation in the Worked Example shows a proportional relationship.

Think about how the quantities relate to each other at any given point.

2. Explain how you can use each graph in the previous activity, *Proportional or Not?*, to determine which scenarios represent proportional relationships.

3. List another example of quantities that are proportional. Then, sketch a graph that could represent the relationship between the quantities.

Talk the Talk

Determining Proportionality from Tables and Graphs

Go back and examine the graphs in this lesson. Do you see a pattern?

1. How are all the graphs that display proportional relationships the same?

2. Sketch a graph that displays a proportional relationship.

3. Which tables display linear relationships? Which display proportional relationships? Explain your reasoning.

a.

x	y
1	10
2	11
4	13
5	14

b.

x	y
0	0
1	6
3	18
4	24

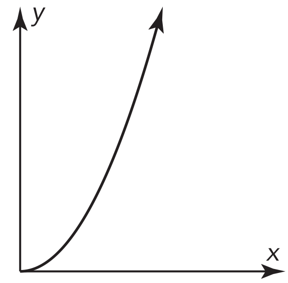
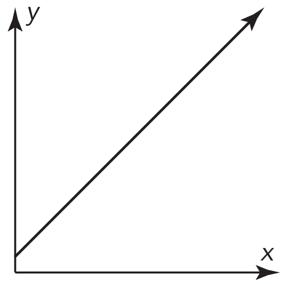
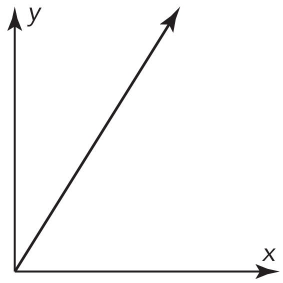
c.

x	y
0	4
1	8
2	12
3	16

d.

x	y
1	30
2	15
4	10
5	5

Cutouts for The Urban Garden Project

Group A is designing one section of the garden to include fresh herbs grown in circular beds. The group needs to determine the area of each herb bed given the radius of the bed.	Group B is developing a plan to landscape the perimeter of the urban garden. They could only locate a meter stick broken at 6 cm, so they reported the dimensions of the garden based on the measurements read off the meter stick. This group needs to determine the actual side lengths of the garden.	Group C is designing the vegetable patches and have decided that each rectangular vegetable patch will be 5 inches wide for every 8 inches long. This group needs to determine the possible dimensions for the lengths of the vegetable patches.
		
$y = 1.6x$	$y = \pi x^2$	$y = x + 6$

Why is this page blank?

So you can cut out the Urban Garden Project cards on the other side

Lesson 1 Assignment

Write

Explain how the following terms are related: *linear relationship*, *proportional relationship*, and *equivalent ratios*.

Remember

For a graph to represent a proportional relationship, the points of the graph must form a straight line and pass through the origin of the graph.

For a table of values to represent a proportional relationship, all the ratios of corresponding *x*- and *y*-values must be constant.

Practice

1. Analyze each table shown. Determine whether the relationship is proportional. When the relationship is proportional, state the constant ratio for the relationship.

   a. Of the 75 7th-graders at a school, 25 participate in at least one sport. Of the 120 8th-graders at this school, 30 participate in at least one sport.

School	Play Sports	Total
7th-Graders	25	75
8th-Graders	30	120

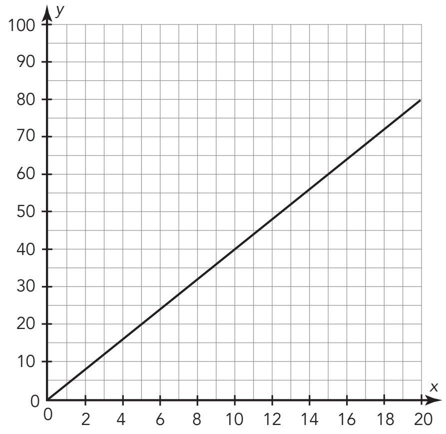
   b. Of the 210 7th-graders at another school, 190 have a cell phone. Of the 168 8th-graders at this school, 152 have a cell phone.

Another School	Cell Phones	Total
7th-Graders	190	210
8th-Graders	152	168

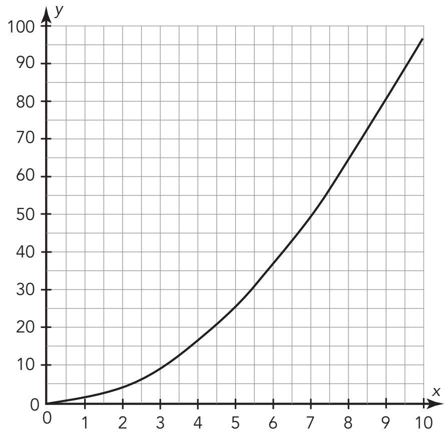
Lesson 1 Assignment

2. Match each graph with its scenario. Then, state whether the scenario represents a linear relationship. When it represents a linear relationship, state whether it represents a proportional relationship.

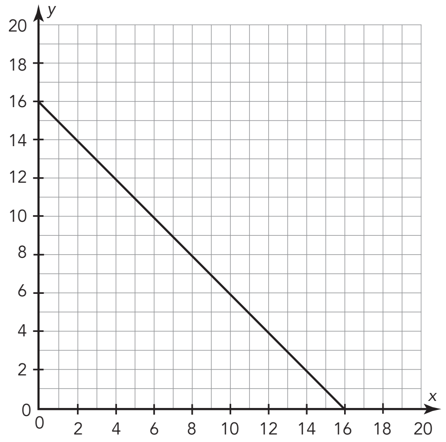
Graph A



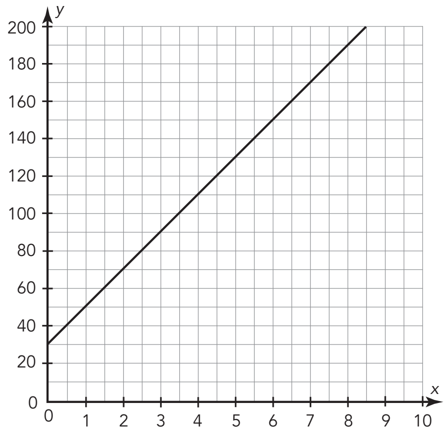
Graph B



Graph C



Graph D



a. Sarah and Emily must decide how to divide 16 marbles among themselves.

b. The perimeter of a square is 4 times the length of one side of the square.

c. The area of a square is calculated by squaring the length of one side of the square.

d. When Valentina, a nurse, works on Saturdays, she is paid a $30 bonus plus $20 per hour worked.

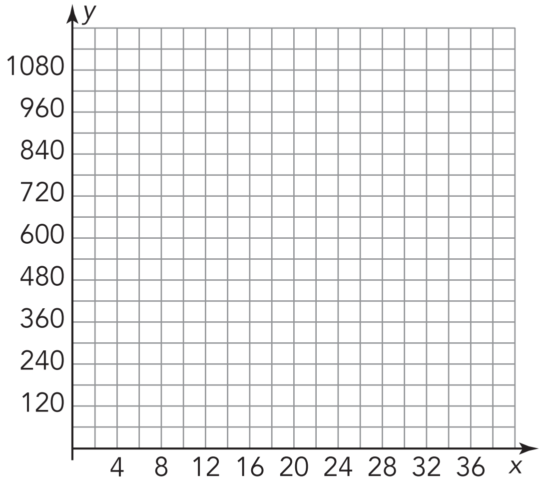
Lesson 1 Assignment

3. In 2010, a car company produced 12,194 sports cars. That means they made about 34 sports cars per day.

   a. Create a table to show the number of sports cars made in at least five different numbers of days.

   b. Use the table to determine if this situation represents a proportional relationship.

   c. Use the data in the table to create a graph of the total number of sports cars produced over time.



   d. Is the number of sports cars made proportional to the number of days?

Lesson 1 Assignment

Prepare

A local middle school collects canned food for a local community food bank. Last year, there were 180 students enrolled at the school, and they collected 100 cans of food.

1. Write the ratio representing the number of cans of food contributed to the total number of students in the school.

2. What is the unit rate of cans contributed per student?

3. This year, 243 students are enrolled in the school. Assume the number of cans of food contributed per student for both years is the same. How many cans of food should the school expect to be contributed this year?

2

Constant of Proportionality

OBJECTIVES

- Determine whether there is a constant ratio between two variables.
- Identify the constant of proportionality in proportional relationships.
- Identify the constant of proportionality in equations.
- Represent proportional relationships by equations.

NEW KEY TERM

- constant of proportionality

You know how to recognize proportional relationships from tables and graphs.

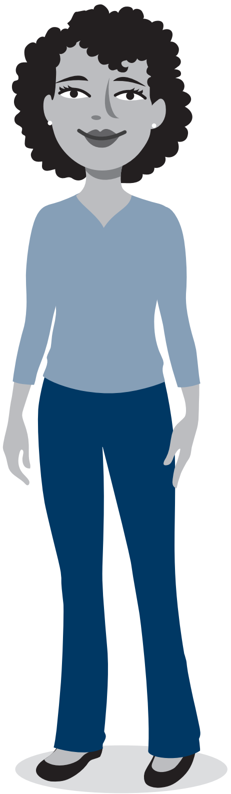
How do you represent proportional relationships with equations?

Getting Started

Is It Proportional?

Analyze each table to determine whether the relationship is proportional. When the table represents a proportional relationship, state the constant ratio that exists between corresponding values of the two quantities.

Does the order in which you write your ratios, $\frac{x}{y}$ or $\frac{y}{x}$, matter when determining whether a proportional relationship exists?



1. A 30-minute television show has 8 minutes of commercials and 22 minutes of the show. A 120-minute television movie has 32 minutes of commercials and 88 minutes of the movie.

Total Program Length (minutes)	Actual Show Length (minutes)	Commercial Length (minutes)
30	22	8
120	88	32

2. There are 250 students in 6th grade, and 75 are in the band. There are 200 students in 7th grade, and 60 are in the band.

Bower Middle School	Number of Students	Students in the Band
6th-graders	250	75
7th-graders	200	60

3. Commuters in two different cities either drive to work or take public transportation.

Commuters	Drive to Work	Public Transportation to Work
City 1	175	120
City 2	525	300

ACTIVITY 2.1

Defining the Constant of Proportionality

In a proportional relationship, the ratio of all *y*-values, or outputs, to their corresponding *x*-values, or inputs, is constant. This specific ratio, $\frac{y}{x}$, is called the **constant of proportionality**. Generally, the variable *k* is used to represent the constant of proportionality.

Let's revisit the television show scenario. This situation represents a proportional relationship.

Total Program Length (minutes)	Actual Show Length (minutes)	Commercial Length (minutes)
30	22	8
120	88	32

The input value is known. The output value is what you are trying to determine.

Suppose you want to determine the actual lengths of your favorite television shows, without commercials, when you know the total program length.

1. Identify the input and output quantities in this scenario.

To determine the length of a program, without commercials, you will need to multiply the total program length by a constant of proportionality.

Analyze the different ideas for determining the constant of proportionality.

Sebastian 👍

We want to know the actual show length, and we know the total program length, so

$k = \dfrac{22 \text{ minutes of show}}{30 \text{ minutes of total length}}$, or $k = \dfrac{11}{15}$.

Mei 👎

To determine whether a proportional relationship exists, the order of the ratio doesn't matter, so the constant of proportionality can be $k = \dfrac{15}{11}$ or $k = \dfrac{11}{15}$.

Angelina 👍

I think the constant of proportionality is

$k = \dfrac{22 \text{ minutes of show}}{8 \text{ minutes of commercials}}$, or $k = \dfrac{11}{4}$.

MASON 👎

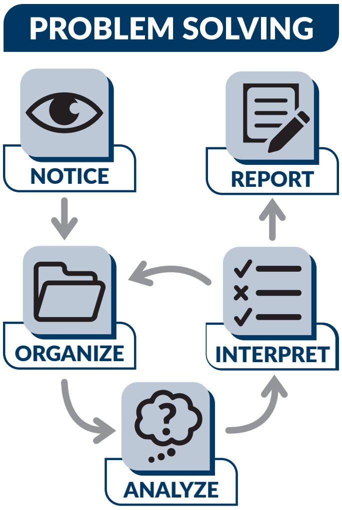
SEBASTIAN'S CORRECT ABOUT WHICH NUMBERS TO USE, BUT HE HAS THEM MIXED UP. THE CONSTANT OF PROPORTIONALITY IS

$K = \dfrac{30 \text{ MINUTES OF TOTAL LENGTH}}{22 \text{ MINUTES OF SHOW}}$, OR $K = \dfrac{15}{11}$.

2. Explain why Angelina's solution is incorrect.

3. Explain why Sebastian is correct but Mason and Mei are incorrect.

Ask Yourself . . .
How can you organize and record your mathematical reasoning?



ACTIVITY

2.2 The Meaning of the Constant of Proportionality

In a local school district, the number of middle school band students is proportional to the number of high school band students.

There are 5 middle school band students for every 6 high school band students enrolled in the local school district.

1. Set up proportions for each question. Then, solve each proportion to determine the unknown value. Use the information from the ratio given.

   a. When there are 300 high school band students, how many middle school band students are enrolled in the district?

What does proportional mean in the problem situation?

   b. When there are 325 middle school band students enrolled in the district, how many high school band students are enrolled in the district?

2. Define variables for the quantities that are changing in this situation.

3. Set up a proportion using your variables for the quantities to the ratio given for the enrollment of middle school band students to high school band students enrolled in the local school district.

4. Use your proportion.

   a. Write an equation to determine the number of middle school band students enrolled at the local school district when you know the number of high school band students enrolled.

   b. What is the constant of proportionality in this situation? Where is the constant of proportionality in the equation?

   c. What does the constant of proportionality mean in this problem situation?

5. Use your proportion.

   a. Write an equation to determine the number of high school band students enrolled at the local school district when you know the number of middle school band students enrolled.

   b. What is the constant of proportionality in this situation? Where do you see the constant of proportionality in the equation?

   c. What does the constant of proportionality mean in this problem situation?

6. What do you notice about the constant of proportionality in each situation?

7. Do you think each constant of proportionality makes sense in terms of the problem situation? Explain your thinking.

Sometimes, the constant of proportionality is not a whole number. The constant of proportionality can also be a decimal or a fraction. When the constant of proportionality involves whole items, like people, it may seem strange to think about the constant of proportionality in terms of a fraction. Instead, you can think of the constant of proportionality as a way to predict outcomes of a situation.

8. Use your equations and the information about the local school district to answer each question.

   a. When there are 79 high school band students enrolled, how many middle school band students are enrolled?

   b. When there are 119 middle school band students enrolled, how many high school band students are enrolled?

Did you use the constant of proportionality for the middle school band students or the high school band students? Does it matter which constant of proportionality you use?

ACTIVITY 2.3

Representing Proportional Relationships with Equations

At the local school district, 5 out of every 7 students play in the band. The guidance counselor, Ms. Williams, and the band director, Ms. Perez, are completing reports about the students at one of the district's middle schools.

Consider the information each person knows and use the constant of proportionality to write equations for each situation.

Guidance Counselor	**Band Director**
Ms. Williams knows the number of students on a given day, and she needs to be able to calculate the expected number of students who play in the band.	Ms. Perez knows the number of students participating in band, and she needs to be able to calculate the expected number of total students in the school.

1. Determine the constant of proportionality for each situation.

   a. Guidance Counselor

   b. Band Director

2. Define variables for the quantities that are changing in these situations.

3. Use the constants of proportionality to write equations to determine the information needed by each person.

   a. Guidance Counselor

   b. Band Director

In terms of proportionality, Ms. Williams could state that the band students is proportional to the number of total students in the school at a constant rate equal to the constant of proportionality.

4. Write Ms. Perez's situation using the language of proportionality and include the value for the constant of proportionality.

5. Consider the given equations, where y represents the dependent, or output, quantity and x represents the independent, or input, quantity.

$$\frac{y}{x} = k$$

$$\frac{y}{x} = \frac{k}{1}$$

$$y = kx$$

   a. Describe how the first equation represents the constant of proportionality.

   b. Explain how the second equation represents proportional relationships.

   c. Describe how the first equation was rewritten to create the third equation.

   d. Explain the meaning of the constant of proportionality, k, in the third equation.

6. Identify the constant of proportionality in each equation and describe its meaning.

   a. $d = 2r$, where d represents the diameter of a circle and r represents the radius of a circle.

   b. $P = 3s$, where P represents the perimeter and s represents the sides of an equilateral triangle.

ACTIVITY 2.4

Using the Constant of Proportionality to Solve Problems

A chemist must use a solution that is 30% of reagent and 70% of water for an experiment. A *solution* is a mixture of two or more liquids. A *reagent* is a substance used in a chemical reaction to produce other substances.

1. Define variables for the quantities that are changing in this problem situation.

2. Write an equation for the amount of water needed based on the amount of reagent. What is the constant of proportionality?

3. Use your equation from Question 2 to write an equation for the amount of reagent needed based on the amount of water. Explain your reasoning.

4. Use your equations to answer each question.

a. When the chemist uses 6 liters of reagent, how many liters of water will she need to make her 30% solution?

b. When the chemist uses 77 milliliters of water, how many milliliters of reagent will she need to make her 30% solution?

5. Write an equation to show that y is directly proportional to x using the constant of proportionality given. Then, solve for the unknown value.

a. $k = 0.7$ and $y = 4$

b. $k = \frac{3}{11}$ and $x = 9$

c. $k = 5$ and $x = 1\frac{1}{2}$

d. $k = \frac{1}{6}$ and $y = 3\frac{1}{3}$

Talk the Talk

Turning the Tables

Consider the equation $y = kx$. Use the value of the constant of proportionality assigned to you to answer the questions. You will present your work to your class.

1. Write a scenario for a proportional relationship that would be represented by the equation. Clearly define your variables and identify the direction of the proportional relationship.

2. Interpret the constant of proportionality in the context of your scenario.

3. Write and solve at least two questions that could be solved using your equation.

Lesson 2 Assignment

Write

Define *constant of proportionality* in your own words. Provide a specific example with your definition.

Remember

When y is directly proportional to x, the relationship can be represented by the equation $y = kx$, where k is the constant of proportionality.

Practice

1. Analyze each table or problem situation to determine whether the relationship is proportional. State a constant of proportionality when possible. Show your work.

a.

7th-Graders	8th-Graders
7	14
9	21
11	22

b. A baby blue whale weighed 5520 pounds at birth. After two days, the baby blue whale weighed 5710 pounds. After 14 days, the baby blue whale weighed 8180 pounds.

Lesson 2 Assignment

2. Adriana's construction company builds brick houses. The number of bricks her crew installs is proportional to the number of hours they work.

Hours Worked	Bricks Installed
8	1680
7	1470
6	1260

a. Define variables for the quantities that are changing in this problem situation.

b. Analyze the table to determine the constant of proportionality.

c. What does the constant of proportionality mean in this situation?

d. Write an equation to show the relationship between the number of hours worked, the number of bricks installed, and the constant of proportionality.

e. Use your equation to determine how many bricks Adriana's crew can install in 5.5 hours.

f. Use your equation to determine how many hours it will take Adriana's crew to install 840 bricks.

Lesson 2 Assignment

3. Given a value for the input variable, x, and the output variable, y, calculate the constant of proportionality.

   a. $x = 21$ and $y = 6$

   b. $x = 60$ and $y = 18$

   c. $x = 2\frac{2}{5}$ and $y = 7\frac{1}{2}$

   d. $x = 4\frac{8}{11}$ and $y = 3\frac{6}{11}$

4. The following is the recipe to make 6 cups of lemonade:

   - 1 cup sugar
   - 1 cup water (for the simple syrup)
   - 1 cup lemon juice
   - 4 cups cold water (to dilute)

   You want to analyze the relationship between the number of cups of sugar and the number of cups of lemonade.

   a. Define variables for the quantities that are changing in this problem situation.

   b. Set up a proportion using the ratio of cups of sugar to cups of lemonade.

   c. Use your proportion to write an equation for the number of cups of sugar based on the number of cups of lemonade.

   d. What is the constant of proportionality in this equation? What does it mean in this context?

   e. Use your equation in (c) to write an equation for the number of cups of lemonade based on the number of cups of sugar.

   f. What is the constant of proportionality in this equation? What does it mean in this context?

Lesson 2 Assignment

5. Lucas and Avery monitored the distance their pet turtle could walk in a certain amount of time. Their results are shown in the table. The table of values represents a proportional relationship.

Time (minutes)	Distance (inches)
5	14.5
14	40.6
19	55.1
25	72.5

a. Define variables for the quantities that are changing in this problem situation.

b. Write an equation for the distance traveled by the turtle based on the number of minutes.

c. What is the constant of proportionality in this equation? What does it mean in this context?

d. Use your equation in (b) to write an equation for the time it would take for the turtle to travel a given distance.

e. What is the constant of proportionality in this equation? What does it mean in this context?

Prepare

Solve each equation for the variable.

1. $\frac{1}{2}a = 5$

2. $p\left(\frac{1}{3}\right) = 2$

3. $3x = \frac{3}{2}$

4. $\frac{6}{z} = \frac{1}{6}$

3

Identifying the Constant of Proportionality in Graphs

OBJECTIVES

- Determine whether relationships represented in words, tables, equations, and graphs are proportional.
- Interpret the meaning of linear proportional relationships represented in words, tables, equations, and graphs.
- Identify and interpret the constant of proportionality for quantities that are proportional and represented in words, tables, equations, and graphs.
- Explain what a point on the graph of a proportional relationship means in terms of the problem situation.
- Explain what the points (0, 0) and (1, *r*) mean on the graph of a proportional relationship, where *r* is the unit rate.

You have determined the constant of proportionality in problem situations and from equations.

How can you represent the constant of proportionality in graphs?

Getting Started

The Fish-Inches System of Measurement

You are thinking of purchasing an aquarium. You contact the owner of an aquarium store. You need to know how many fish to purchase for an aquarium, but first you must determine how big the aquarium will be. The owner of the aquarium store tells you his rule of thumb is to purchase "a total length of fish of 3 inches for each 2 gallons of water in the aquarium."

1. How many gallons of water would you need when you had a 4-inch fish and a 2-inch fish? Draw a diagram to explain your reasoning.

2. Define variables for the quantities that are changing in this problem situation.

3. Write an equation for each:

   a. Fish-inches based on the gallons of water

   b. Gallons of water based on fish-inches

4. Use one of your equations to solve each problem.

   a. When an aquarium holds 10 gallons of water, how many fish-inches should you purchase?

   b. When you want to purchase a 5-inch fish, two 2-inch fish, and three 3-inch fish, how many gallons of water should the aquarium hold?

5. Determine the constant of proportionality given by each equation and explain what it means in context.

ACTIVITY

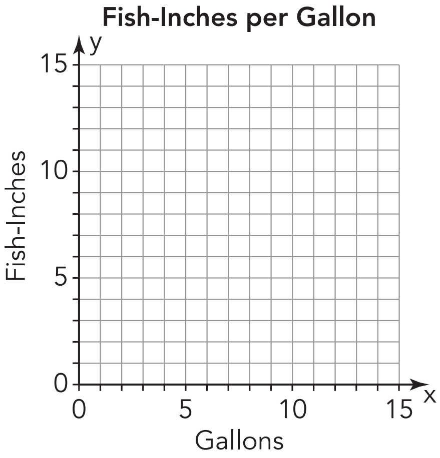
3.1 Graphs of Two Constants of Proportionality

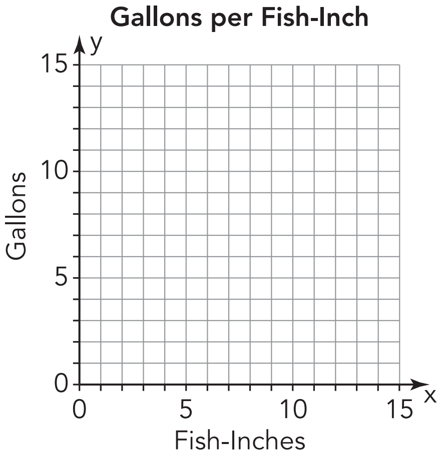
Let’s graph each equation you wrote in the previous activity.

1. Create a table of ordered pairs. Then, plot the ordered pairs to create a graph of each equation.

Fish-Inches							
Gallons							

Ask Yourself . . .
How can you use graphs in everyday life?





2. What does the point (0, 0) mean on each graph?

3. Determine the meaning of each point.

   a. What does the point (6, 9) on the Fish-Inches per Gallon graph represent?

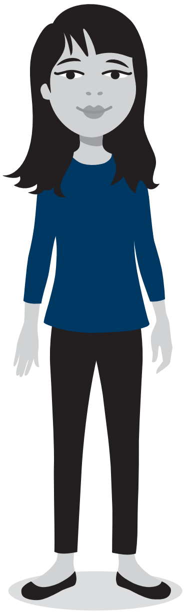
   b. What does the point (9, 6) on the Gallons per Fish-Inch graph represent?

c. What does the point $(1, 1\frac{1}{2})$ on the Fish-Inches per Gallon graph represent?

d. What does the point $(1, \frac{2}{3})$ on the Gallons per Fish-Inch graph represent?

4. What is the unit rate for each graph? Explain how you can determine the unit rate using the graph.

The constant of proportionality is represented on the graph, too. Can you locate it?



5. Use one of your graphs to determine each answer.

   a. How many fish-inches can fit into 10 gallons of water?

   b. How many gallons are needed for $7\frac{1}{2}$ fish-inches?

6. Use one of the graphs to estimate each answer. Explain how you used the graph to determine your estimate.

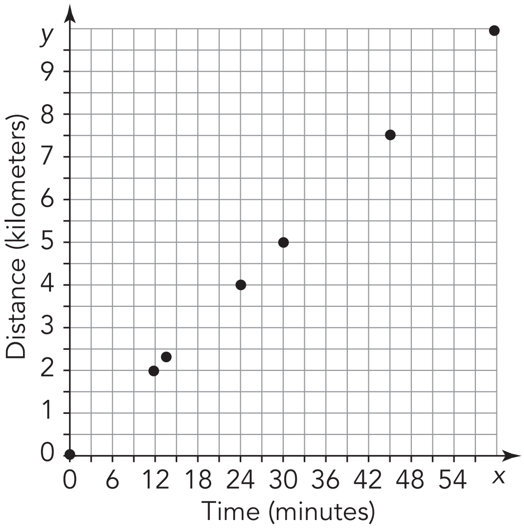
   a. How many gallons would be needed for 16 fish-inches?

   b. How many fish-inches would fit into 16 gallons?

ACTIVITY 3.2

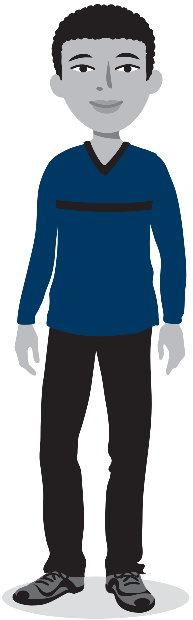
Constant of Proportionality from a Graph

The graph shown displays the relationship between the time and distance Kayla runs.



1. Define variables and write an equation to represent the relationship between Kayla's distance and time.

How can you determine the constant of proportionality from a graph?



2. Use your equation to answer each question.

   a. How far can Kayla run in 15 minutes?

b. How long does it take Kayla to run 7.5 kilometers?

c. How far can Kayla run in one hour?

3. Lucas and Nahimana each write an equation to model the distance Kayla runs.

Lucas	Nahimana
$y = 60x$	$d = 60t$

Who's correct? Explain your reasoning.

ACTIVITY

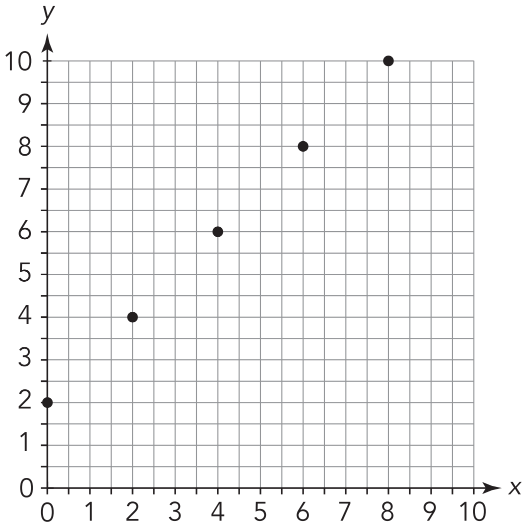
3.3

Determining Proportional Relationships from Graphs

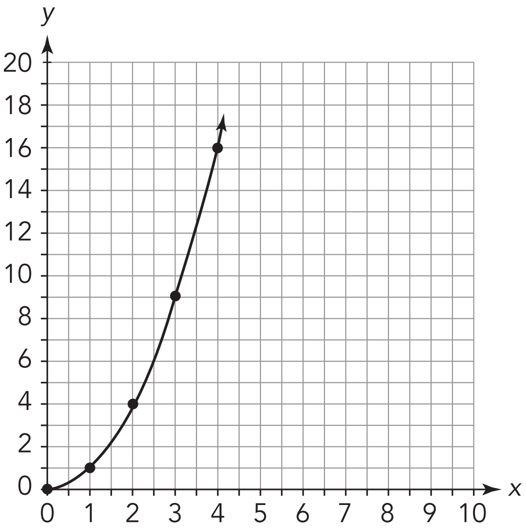
You have seen that proportional relationships can be represented on graphs and that the constant of proportionality can be identified from the graph.

1. Determine whether each graph shows a proportional relationship between *x* and *y*. When possible, determine the constant of proportionality. Explain how you determined your answer.

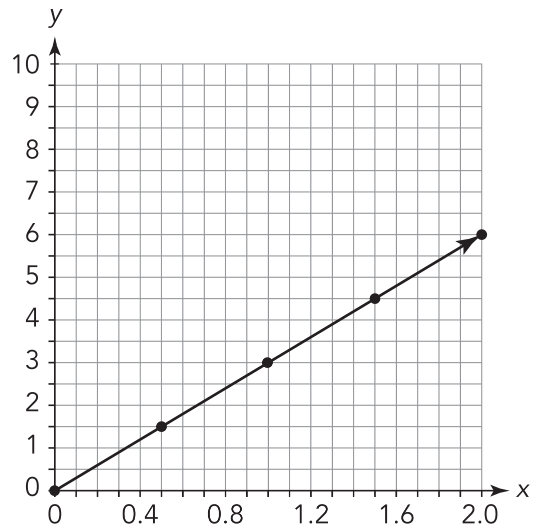
a.



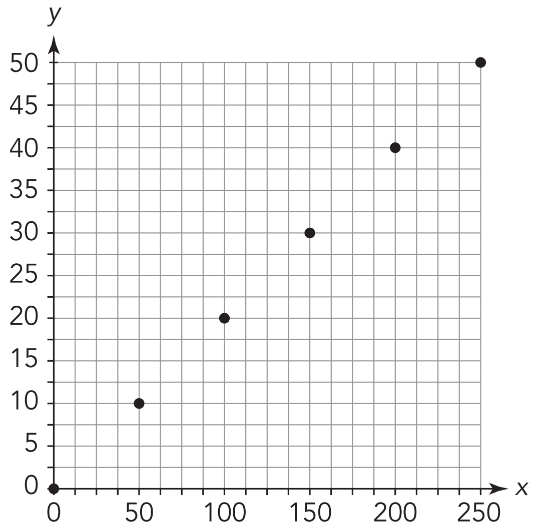
b.



c.


y
10
9
8
7
6
5
4
3
2
1
0
0
0.4
0.8
1.2
1.6
2.0
x

d.


y
50
45
40
35
30
25
20
15
10
5
0
0
50
100
150
200
250
x

Talk the Talk

How Do You Know?

Use examples to explain your answer to each question.

1. Given a graph of a relationship between two quantities, how do you know:

   a. whether the graph shows a proportional relationship?

   b. what the constant of proportionality is?

   c. what the unit rate is?

   d. what any ordered pair on the graph represents?

Lesson 3 Assignment

Write

Given a graph of a proportional relationship, the quotient of the *y*-value of any ordered pair divided by the *x*-value of that ordered pair is the constant of proportionality. Explain why.

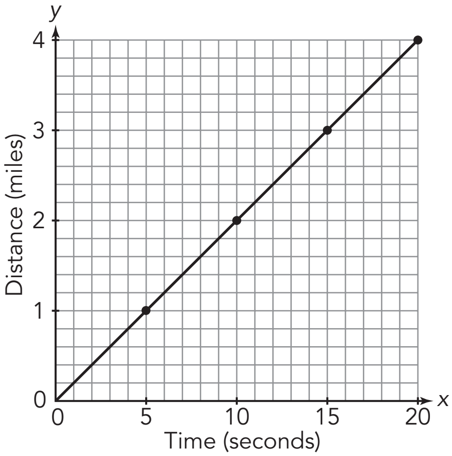
Remember

When the relationship between two quantities is proportional, the graph of the relationship is a straight line that passes through the origin. The point (1, *r*) represents the unit rate, and the ratio $\frac{r}{1}$ represents the constant of proportionality, *k*.

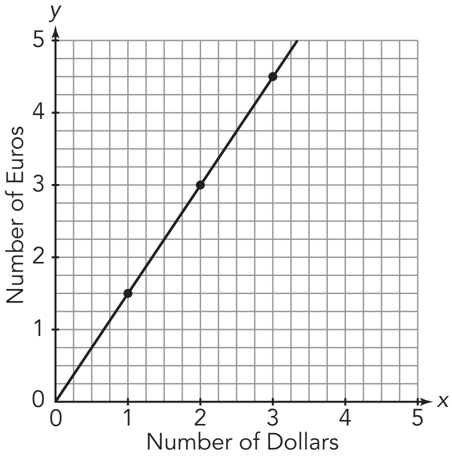
Practice

Determine the constant of proportionality *k* and interpret it in the context of each problem.

1. The graph shows the relationship between the distance in miles between you and a storm and the number of seconds between when you see lightning and when you hear thunder.

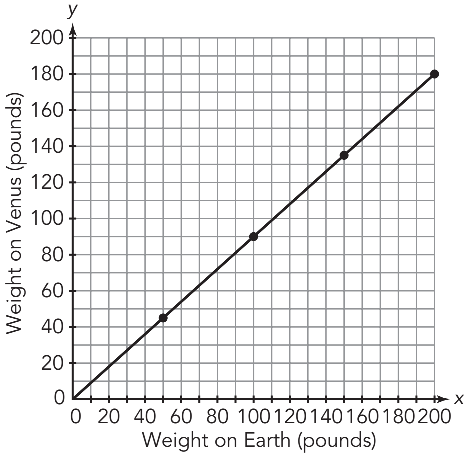


2. The graph shows the relationship between the number of euros Luna received and the number of dollars Luna exchanged during her trip to Spain.

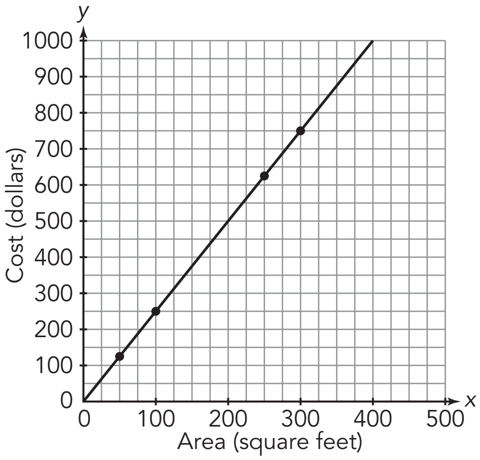


Lesson 3 Assignment

3. The graph shows the relationship between the weight of an object on Earth and the weight of the same object on Venus.

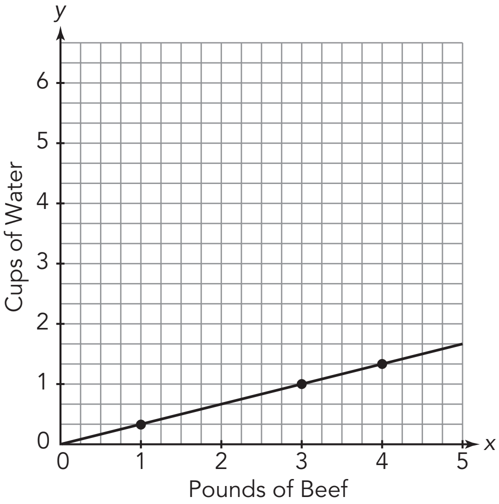


4. The graph shows the relationship between the area of a room in square feet and the cost of covering the floor with new tile.

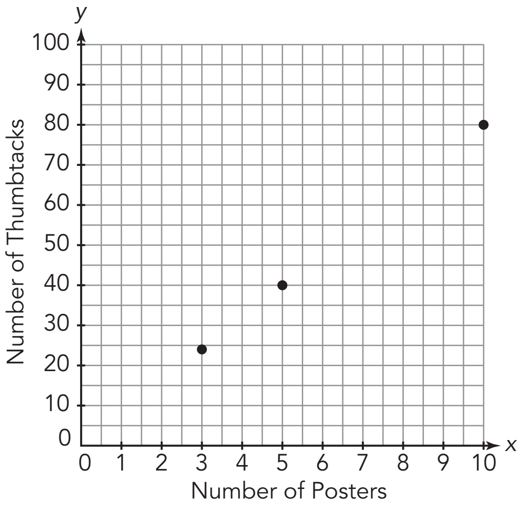


Lesson 3 Assignment

5. The graph shows the relationship between the cups of water and pounds of beef needed for a beef casserole.



6. The graph shows the relationship between the number of posters in a classroom and the number of thumbtacks used to hold them up.



Lesson 3 Assignment

Prepare

Solve each equation.

1. $5p = 2.5$

2. $\frac{1}{3}j = 9$

3. $0.12k = 10.08$

4. $8k = 0$

4

Constant of Proportionality in Multiple Representations

OBJECTIVES

- Determine whether relationships represented in words, tables, equations, or graphs are proportional.
- Interpret the meaning of linear proportional relationships represented in words, tables, equations, and graphs.
- Determine and interpret the constant of proportionality for quantities that are proportional and represented in words, tables, equations, and graphs.

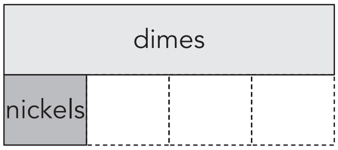
You have learned how to determine the constant of proportionality.

How can you solve problems using this constant in tables, graphs, and diagrams?

Getting Started

Jasmine's Nickels Are a Quarter of Her Dimes

Jasmine collects only nickels and dimes. She has one-quarter as many nickels as dimes.



A diagram can represent this problem situation. All together she has 40 coins. How much money does she have?

1. Explain how you can use the diagram to solve the problem. Determine the solution.

2. Write an equation to represent the proportional relationship between:

    a. the number of nickels and the number of dimes.

    b. the number of dimes and the total number of coins.

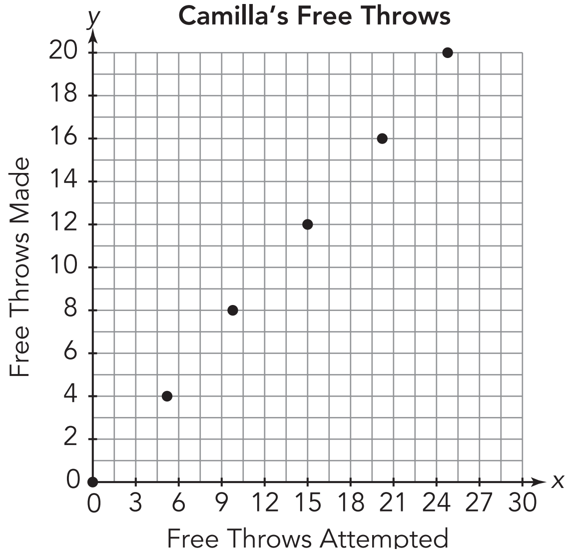
    c. the number of nickels and the total number of coins.

3. Identify the constant of proportionality in each proportional relationship described in Question 2.

ACTIVITY
4.1

Using a Graph to Write an Equation

The graph shows Camilla's total number of free throw attempts and the total number of free throws made.



Ask Yourself . . .
Did you justify your mathematical reasoning?

1. Explain how you know the graph represents a relationship that is proportional.

2. Determine the constant of proportionality and describe what it represents in this problem situation.

3. Suppose Camilla attempted 30 free throws. How many would you expect her to make? First, use your graph to estimate the answer. Then, verify your answer by using an equation.

ACTIVITY 4.2

Using an Equation to Create a Table

Another example of a proportional relationship is the relationship between the number of hours a worker works and their wages earned in dollars.

The amount of money (*m*) Isaiah earns is proportional to the number of hours (*h*) he works. The equation describing this relationship is $m = 9.25h$.

1. What does the constant of proportionality represent in this situation?

2. Complete the table based on the equation given. Include the constant of proportionality in the table.

Hours Worked	Earnings (dollars)
0	
	112.85
40	

During the summer, Avery works as a movie attendant. The number of hours he works varies each week.

3. Write an equation to represent this situation. Then, complete the table based on your equation and include the constant of proportionality.

Hours Worked	Earnings (dollars)
3	26.88

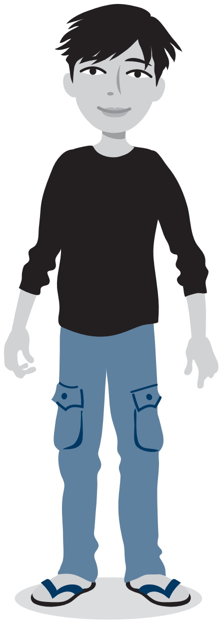
4. What is the constant of proportionality? What does it mean in this problem situation?

ACTIVITY
4.3 Using a Table to Create a Scenario

Analyze the given table.

Number of Windows	Amount of Window Cleaner (ounces)
0	0
2	16
3	24
4	32
5	40
6	48

How can you tell the table of values represents a proportional relationship?



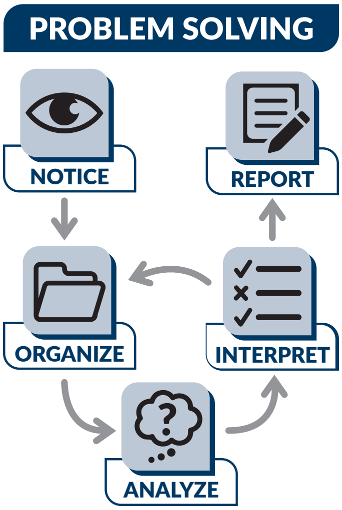
1. Describe one possible situation that could be represented by this table of values. Include how the quantities relate to each other.

2. What is the constant of proportionality and what does it represent in your situation?

3. When the table values were used to create a graph, how would the points appear?

ACTIVITY
4.4

Using a Scenario to Write an Equation



A baby elephant nurses for the first two years of its life. It drinks about 10 liters of milk every day.

1. Define variables and write an equation to represent the relationship between the amounts of milk the baby elephant consumes and the time it spends consuming the milk. Assume the elephant maintains the same rate of consumption.

2. Identify the constant of proportionality and describe what it means in this situation.

3. Create a graph to represent this situation.

The weight of an object on Earth is proportional to the weight of an object on the moon. A 150-pound object would weigh approximately 25 pounds on the moon.

4. Define variables and write an equation to represent the relationship between the weight of an object on Earth and the weight of the object on the moon.

5. Identify the constant of proportionality and describe what it means in this situation.

6. Create a graph to represent this situation.

ACTIVITY
4.5

Multiple Representations of Proportional Relationships

At a local grocery store, the price of coffee, *q*, is directly proportional to the number of pounds of coffee, *p*

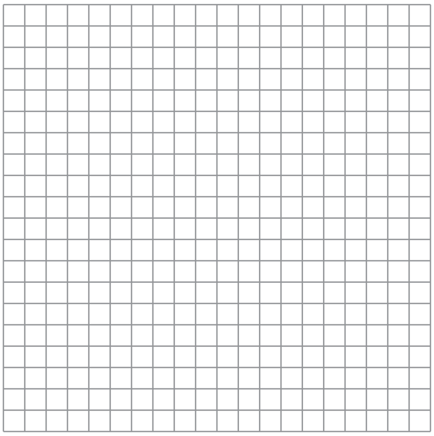
1. Complete the table for variables *p* and *q*.

p	*q*
0	
2	6
4	12
0.25	
	3
1.5	4.5

2. Write an equation to represent *q*, the price of coffee for *p* pounds.

3. Summarize how you can write the equation that represents the relationship between two variables when you are given a ratio table.

4. Graph your equation. Label your axes.



5. Summarize how to draw a graph from the equation representing the relationship between the two quantities.

6. Summarize how you can write the equation representing the relationship between two proportional quantities when you are given a graph.

Talk the Talk

Every Which Way

You have seen how to represent proportional relationships in scenarios, on graphs, in tables, and with equations.

1. Write an equation and sketch a graph to represent each relationship. Label your axes and identify the constant of proportionality on the graph.

   a. Suppose the quantity *p* is directly proportional to the quantity *q*.

   b. Suppose the quantity *q* is directly proportional to the quantity *p*.

2. Write a scenario that describes a proportional relationship between two quantities. Represent this relationship using an equation, a graph, and a table. For each model, identify the constant of proportionality and explain how the model shows that the relationship is proportional.

Scenario

Equation

A PROPORTIONAL RELATIONSHIP

Table

Graph

Lesson 4 Assignment

Write

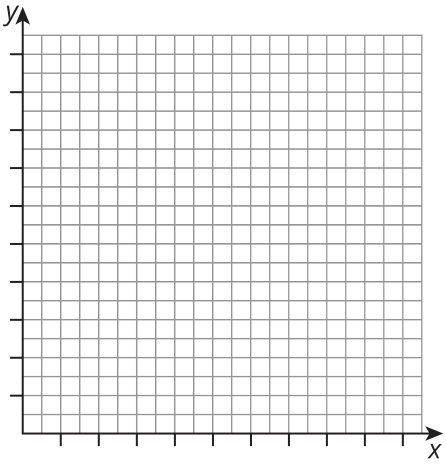
Describe how a table, an equation, and a graph can each represent a proportional relationship.

Remember

Proportional relationships can be represented in many of different ways, including diagrams, tables, equations, graphs, and in scenarios.

Practice

1. The constant of proportionality between the number of children (*c*) on a field trip and the number of teachers (*t*) on the trip is $\frac{14}{3}$.

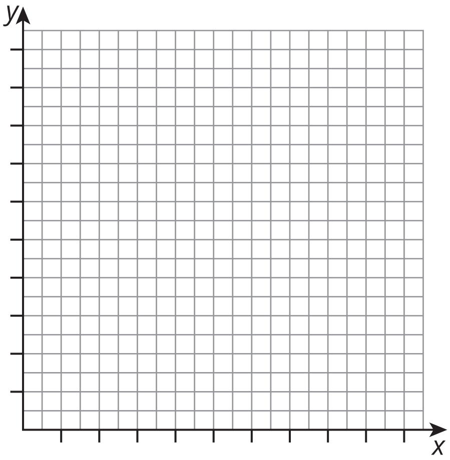


   a. Write an equation to represent this situation.

   b. Create a graph.

   c. When there are 70 children on a field trip. How many teachers are on the trip?

2. The constant of proportionality between the number of junior varsity players (*j*) on the track team and the number of varsity players (*v*) on the team is $\frac{2}{5}$.



   a. Write an equation to represent this situation.

   b. Create a graph.

   c. When there are 45 varsity players on the track team, how many junior varsity players are on the team?

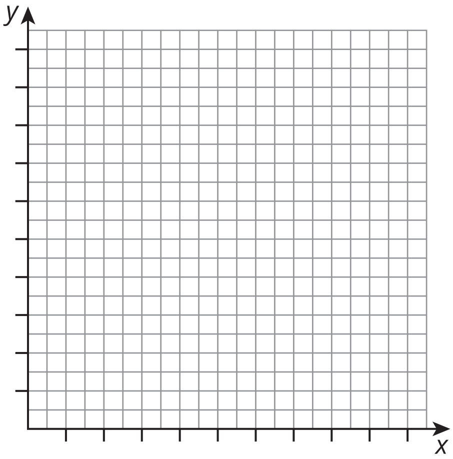
Lesson 4 Assignment

3. The constant of proportionality between the number of cats (c) in a pet shelter and the number of dogs (d) in the shelter is 3.

   a. Write an equation to represent this situation.

   b. Create a graph.

   c. When there are 27 cats in the shelter, how many dogs are in the shelter?



4. The constant of proportionality between the height of the water in a sink (h) in centimeters and the number of minutes it has been filling (m) is 0.95. The sink has been filling for 40 minutes. What is the height of the water in the sink?

5. The constant of proportionality between the number of fiction books (f) and the number of nonfiction books (n) in a library is $\frac{15}{22}$. There are 3498 nonfiction books in the library. How many fiction books are in the library?

6. The constant of proportionality between the number of markers (m) and the number of pencils (p) in an art room is $\frac{8}{3}$. There are 304 markers in the art room. How many pencils are in the art room?

Lesson 4 Assignment

Prepare

The regular price of a bathing suit is $89.99. Estimate the sale price of the bathing suit for each of the following sales.

1. 70% off regular price

2. 30% off regular price

3. 50% off regular price

4. 25% off regular price

TOPIC 3 SELF-REFLECTION

Name: ______________________________

Proportionality

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Proportionality* topic by:

TOPIC 3: *Proportionality*	Beginning of Topic	Middle of Topic	End of Topic
determining whether a proportional relationship exists between two quantities represented in tables, graphs, or equations.	○	○	○
representing proportional relationships with equations in the form of $y = kx$.	○	○	○
defining the constant of proportionality as a rate.	○	○	○
identifying the constant of proportionality (unit rate) from tables, graphs, equations, and verbal descriptions of proportional relationships.	○	○	○
interpreting the meaning of the constant of proportionality in real-world situations.	○	○	○
interpreting the meaning of any point on the graph of a proportional relationship.	○	○	○
interpreting the meaning of (0, 0) and (1, r) on the graph of a proportional relationship, where r is the unit rate.	○	○	○

continued on the next page

TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Proportionality* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 3 SUMMARY

Proportionality Summary

NEW KEY TERMS

- proportional relationship [relación proporcional]
- origin [origen]
- constant of proportionality [constante de proporcionalidad]

LESSON 1 Proportional Relationships

One special type of relationship that compares quantities using multiplicative reasoning is a *ratio relationship*. When two equivalent ratios are set equal to each other, they form a *proportion*. The quantities in the proportion are in a **proportional relationship**.

For a table of values to represent a proportional relationship, all the ratios of corresponding *x*- and *y*-values must be constant.

For example, the table displays the growth rate of a certain species of bamboo by comparing the time in days, *x*, to the height of the bamboo in centimeters, *y*. Does the table show a proportional relationship?

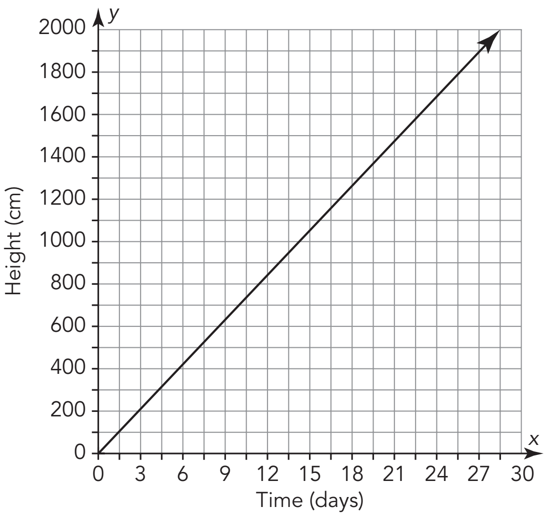
$$\frac{3}{210} = \frac{1}{70} \qquad \frac{10.5}{735} = \frac{1}{70} \qquad \frac{18}{1260} = \frac{1}{70} \qquad \frac{25.5}{1785} = \frac{1}{70}$$

All the values $\frac{x}{y}$ represent the same constant, $\frac{1}{70}$, so the table shows a proportional relationship.

Time (days)	Height of Bamboo (cm)
3	210
10.5	735
18	1260
25.5	1785

For a graph to represent a proportional relationship, the points of the graph must form a straight line and pass through the origin. The **origin** is the point with the ordered pair (0, 0).

For example, the values from the bamboo situation are graphed here. The graph represents a proportional relationship. A situation represents a **proportional relationship** when the ratio between the y-value and its corresponding x-value is constant for every point. The bamboo grows at a constant rate of 70 centimeters a day.



LESSON 2 Constant of Proportionality

If y is directly proportional to x, the relationship can be represented by the equation $y = kx$, where k is the **constant of proportionality.** This means that for any value of x, the value of y is x multiplied by k.

For example, the table of values represents a proportional relationship. Determine the constant of proportionality.

$$y = kx$$

$$6 = k \cdot 5$$

$$\frac{6}{5} = k$$

6th-grade students	7th-grade students
5	6
10	12
15	18

You can use the constant of proportionality to determine unknown values in proportions.

For example, using the constant of proportionality represented in the previous table, determine how many 6th-grade students. there are if there are 240 7th-grade students.

$$\frac{5}{6} = \frac{g}{240}$$

$$(5)(240) = 6g$$

$$1200 = 6g$$

$$200 = g$$

There are 200 6th-grade students.

Consider the same ratio of five 6th-grade students to six 7th-grade students. When you know the number of 7th-grade students and need to determine the number of 6th-grade students, you can use the equation $y = \frac{5}{6}x$, where x represents the number of 7th-grade students and y represents the number of 6th-grade students. In this equation, the $\frac{5}{6}$ represents the constant of proportionality. You can multiply the number of 7th-grade students by $\frac{5}{6}$ to determine the number of 6th-grade students. When you know the number of 6th-grade students and need to determine the number of 7th-grade students, you can use the equation $y = \frac{6}{5}x$, where x represents the number of 6th-grade students and y represents the number of 7th-grade students. In this scenario, the constant of proportionality is $\frac{6}{5}$. You can multiply the number of 6th-grade students by $\frac{6}{5}$ to determine the number of 7th-grade students.

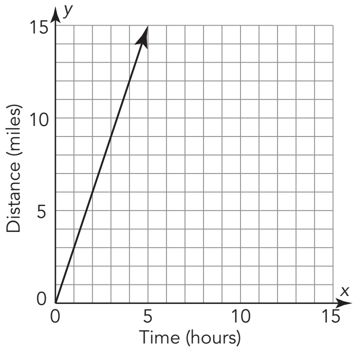
LESSON 3

Identifying the Constant of Proportionality in Graphs

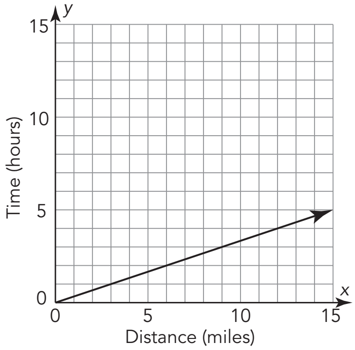
The constant of proportionality of a situation can be graphed in two different ways.

For example, Carlos walks at a constant rate of 3 miles per hour.

The first graph represents the ratio of distance to time. The point (1, 3) represents the unit rate. The ratio $\frac{3}{1}$ represents the constant of proportionality, $k = 3$.

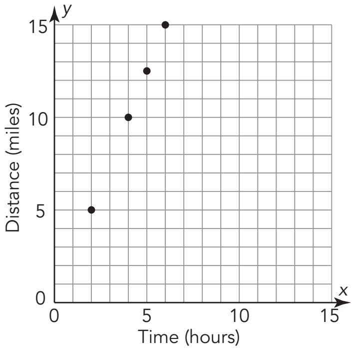


The second graph represents the ratio of time to distance. The point $\left(1, \frac{1}{3}\right)$ represents the unit rate. The ratio $\frac{\frac{1}{3}}{1}$ represents the constant of proportionality, $k = \frac{1}{3}$.



You can determine the constant of proportionality from a graph.

For example, the third graph displays the proportional relationship between the time and distance Adriana walks. Determine the constant of proportionality in miles per hour.



Choose an ordered pair from the graph: (4, 10).

$k = \frac{y}{x}$

$k = \frac{10}{4}$

$k = 2.5$

The constant of proportionality is 2.5.

LESSON 4

Constant of Proportionality in Multiple Representations

Proportional relationships can be represented in many different ways, including diagrams, tables, equations, and graphs. They can also be described in scenarios.

A graph can be used to write an equation.

For example, the graph shows Diego's total number of free throw attempts and the total number of free throws made. The graph represents a proportional relationship because it is a straight line that goes through the origin. Using the graph, you can determine that the constant of proportionality is $k = \frac{4}{7}$, which means that Diego made $\frac{4}{7}$ of the free throws he attempted. The equation that represents this is $y = \frac{4}{7}x$.

A scenario can also be used to write an equation.

For example, a blue whale eats 8000 pounds of food a day. Let x represent the independent quantity, the number of days, and let y represent the dependent quantity, the pounds of food eaten. The constant of proportionality is 8000, meaning that for every day that passes, the pounds of food that the blue whale has eaten increases by 8000. The equation that represents this is $y = 8000x$.

A table of values can be created using an equation.

For example, the equation $y = 8000x$ from the blue whale scenario can be used to create the table of values shown.

Number of Days	Pounds of Food
0	0
1	8000
4	32,000
9	72,000

MODULE
2

Applying Proportionality

TOPIC 1 Proportional Relationships 175

TOPIC 2 Financial Literacy: Interest and Budgets 273

GRADE 7

TOPIC
1

Did you get good service in a restaurant? If you did, it is common to leave a 20% tip for the server that waited on you.

Proportional Relationships

LESSON 1 Introducing Proportions to Solve Percent Problems 177

LESSON 2 Calculating Tips, Commissions, and Simple Interest 197

LESSON 3 Sales Tax, Income Tax, and Fees 219

LESSON 4 Percent Increase and Percent Decrease 233

LESSON 5 Scale and Scale Drawings 245

1

Introducing Proportions to Solve Percent Problems

OBJECTIVES

- Estimate and calculate the values of percents.
- Use percent models to solve percent problems.
- Use proportional relationships to solve for unknowns in multi-step percent problems.
- Solve percent problems using the constant of proportionality.

NEW KEY TERMS

- markdown
- markup
- sale
- coupon
- rebate

You have used ratio reasoning to solve percent problems.

How can you use proportional relationships to solve percent problems involving markdowns and markups?

Getting Started

Need New Kicks?

A marketing department is creating signs for an upcoming shoe sale.

HOLIDAY
SHOE SALE
35% OFF
the
Regular Price

HOLIDAY
SHOE SALE
65%
Of
Regular Price

1. Compare the two signs. What do you notice?

2. Which sign should the store use to advertise the shoe sale?
   Explain your reasoning.

Return to the Academic Glossary and read the meaning of **explain your reasoning**.

3. The regular price of a pair of shoes is $59.99.
   Estimate the sale price of the pair of shoes two different ways.

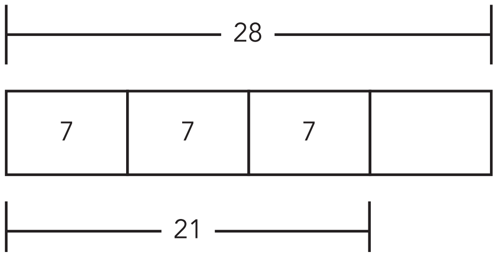
ACTIVITY 1.1

Percent Models

Three percent scenarios are shown. Match each strip diagram to the appropriate scenario.

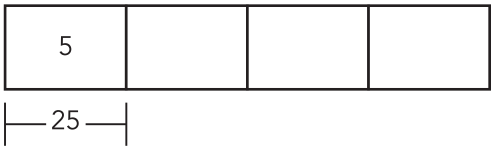
1. A shirt costs $20. When it was on sale for 75% off, what was the discount?

a.



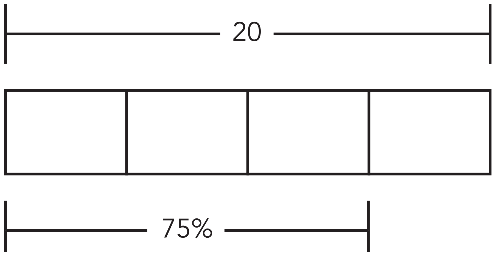
2. A skirt originally costs $28. Olivia pays $21 for the skirt during a sale. What percent does Olivia save with the sale?

b.



3. A ski hat is on sale for 25% off, which is $5 off. What is the original price of the ski hat?

c.



4. Use the appropriate strip diagram to solve each percent problem. Explain how you solved each.

Huyen

A shirt costs $20 and I am trying to determine the discount when it was on sale for 75% off.

÷ 2 ÷ 2

Percent (%)	100	50	25
Total (dollars)	20	10	5

÷ 2 ÷ 2

The discount was 75% off, so I can add the dollar amounts from the table for 50% and 25% to determine the discount. Since $10 + 5 = 15$, the discount on the shirt when it was on sale for 75% was $15.

5. Analyze Huyen's work to represent and answer Question 1. Explain her strategy.

6. Use Huyen's strategy to represent and answer Question 2.

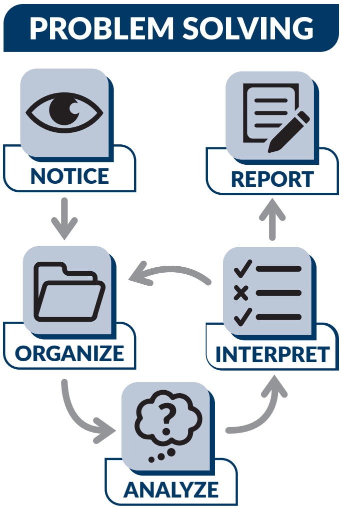
7. Use Huyen's strategy to represent and answer Question 3.

8. Destiny said that she had developed strategies to solve percent problems, depending on what was given.

    - Case A: When I want to calculate the percent, I can divide the part by the whole and rewrite the decimal as a percent.
    - Case B: When I want to calculate the part, or percent, of a number, I can multiply the percent, written in decimal form, by the number.
    - Case C: When I want to calculate the whole, I can divide the percent, written in decimal form, by the part.

Is Destiny correct?
Does this always work? Explain your thinking.

PROBLEM SOLVING



ACTIVITY
1.2
Using Proportions to Solve Percent Problems

Gabriela, Michael, and Hannah each set up a proportion to solve four different percent problems.

1. Read each percent problem and analyze the corresponding student's work.

A shirt costs $20. When it is on sale for 75% off, what is the discount?

Gabriela

$$\frac{75}{100} = \frac{x}{20}$$

$$(75)(20) = 100x$$

$$1500 = 100x$$

$$\frac{1500}{100} = \frac{100x}{100}$$

$$15 = x$$

The discount is $15.

a. How did Gabriela know where to place the 20 in her proportion?

A skirt originally cost $28. Olivia pays $21 for the skirt during a sale. What percent does Olivia save with the sale?

Michael

$$\frac{x}{100} = \frac{21}{28}$$

$$28x = (100)(21)$$

$$28x = 2100$$

$$\frac{28x}{28} = \frac{2100}{28}$$

$$x = 75$$

Olivia paid 75% of the cost, so Olivia saved 25%.

b. How did Michael decide Olivia saved 25%?

A ski hat is on sale for 25% off, which is $5 off. What is the original price of the ski hat?

Hannah 👎

$$\frac{25}{100} = \frac{x}{5}$$

$$(25)(5) = 100x$$

$$125 = 100x$$

$$\frac{125}{100} = \frac{100x}{100}$$

$$1.25 = x$$

c. Explain what Hannah did incorrectly. Then, set up and solve the problem correctly.

All of the problems in the previous activity involved a sale or a **markdown**. To make money, businesses often buy products from a wholesaler or distributor for one amount and add to that amount to determine the price they use to sell the product to their customers. This is called a **markup**. A markup is needed in order for the business to make a profit for the owner after paying expenses such as employee wages and facility costs.

Remember, you can write a percent as a proportion.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent number}}{100}$$

The school store is selling spirit wear. They mark up all of the prices by 20% to sell to the community and students. The school will use the profit from the school store to provide funds towards other needs, such as the purchase of band equipment.

2. When the store's cost for a sweatshirt is $25, what is the customer's cost? Analyze the four student responses shown.

Javier 👎

$$\frac{20}{100} = \frac{x}{25}$$

$$5 = x$$

The customer's cost is $5.

Fernando 👍

$$\frac{20}{100} = \frac{x}{25}$$

$$5 = x$$

The customer's cost is $5 + $25 = $30.

Alexander

The new "whole" is the total cost, which is 20% more than the original 100%. I can multiply the store cost by 120%, or 1.20.

$$25(1.20) = 30$$

The customer's cost is $30.

Jacob

The original cost is now the part.

$$\frac{100}{120} = \frac{25}{x}$$

$$30 = x$$

The customer's cost is $30.

a. Compare Javier's method to Fernando's method.

b. Compare Alexander's method to Jacob's method.

Can you see another proportion you can write to solve this problem?

3. Use the method(s) of your choice to complete the table of the store's cost and the customer's cost for the spirit wear.

	Store's Cost	Customer's Cost
T-shirt	$8	
Face Tattoos		$4.50

Solve each percent problem. Show your work.

4. The $199.99 game console Ana purchased was on sale for 10% off. How much did Ana pay?

5. A computer is normally $899 but is discounted to $799. What percent of the original price does Amir pay?

You know that the percent whole is always 100. So, as long as you know 2 of the other 3 values, you can solve the proportion.

6. When Koda paid $450 for a netbook that was 75% of the original price, what was the original price?

7. A store sells gaming cards for $9.99 but they pay only $6.75 per card. What is the percent markup?

8. William received 30% off when he purchased a rare book regularly priced at $96.50. How much did William pay?

9. Maria is selling car magnets. She purchases them for $7.50 each and marks up the price by 30%. How much is Maria planning to charge?

ACTIVITY 1.3

Answering the Question

Stores often try to entice shoppers by offering special deals and discounts. A percentage off the list price or a specific discount can save shoppers quite a bit of money. An understanding of percentages can help you locate the best deals.

Three common types of discounts that customers can benefit from are *sales*, *coupons*, and *rebates*. A **sale** is an event at which products are sold at reduced prices. Sales are typically held to clear outdated inventory. A **coupon** is a detachable part of a ticket or advertisement that entitles the holder to a discount. Coupons are commonly used to entice people to shop at certain stores. A **rebate** is a refund of part of the amount paid for an item. Generally, a customer completes and mails a form to a company after a purchase and a rebate check is mailed to the customer.

Angel is planning to purchase a washing machine. Four local appliance stores are offering the following discounts on the washer Noah wants.

- Store 1: Washers are 20% off
- Store 2: \$150 rebate off the price of a washer
- Store 3: Coupon for 10% off a washer
- Store 4: Coupon for $\frac{1}{2}$ off the purchase price of a washer

Store 1 (Sale)
The washer originally costs \$599.

$$\text{Sale price} = 599 - 599(0.20) = 479.2$$

The sale price of the washer is \$479.20.

Store 2 (Rebate)
The washer originally costs \$675.

$$\text{Price after rebate} = 675 - 150 = 525$$

The price of the washer after the rebate is \$525.

Store 3 (Coupon)
The washer originally costs \$505.

$$\text{Price with coupon} = 505 - 505(0.10) = \$454.50$$

The price of the washer with the coupon is \$454.50.

Store 4 (Coupon)
The washer originally costs \$975.
Price with coupon $= \frac{1}{2} \cdot 975 = 487.50$.
The price of the washer with the coupon is \$487.50.

1. Which store should Angel purchase the washer from? Explain your reasoning.

Luis is planning his vacation. The flight he selected was $229.99, but he got 20% off because he booked it online through a new travel website. What did he pay?

2. Explain why Abby's answer is incorrect. Then, determine the correct answer.

Abby 👎

$\xrightarrow{\div 10}$ $\xrightarrow{\cdot 2}$

Percent (%)	100	10	20
Total (dollars)	230	23	46

$\xrightarrow{\div 10}$ $\xrightarrow{\cdot 2}$

So, Luis paid about $46.

3. Explain why Malik's method worked.

Malik 👍

$\xrightarrow{\div 5}$ $\xrightarrow{\cdot 4}$

Percent (%)	100	20	80
Total (dollars)	230	46	184

$\xrightarrow{\div 5}$ $\xrightarrow{\cdot 4}$

Luis paid about $184 for his flight.

Solve each problem.

4. Olivia sold USB thumb drives for $4.95, which was a 10% markup from what she paid for each. How much did Olivia pay for the drives?

5. Games that usually sell for $36.40 are on sale for $27.30. What percent off are they?

6. Malik's new cell phone cost him $49.99 when he signed a 2-year plan, which was 75% off the original price. What was the original price?

7. Daniela is shopping for a game system. Two competing stores offer deals on the system that she wants to purchase.

Store A	Store B
• $375	• $399
• 15% off sale on all game systems	• $50 rebate after the purchase

Where should Daniela shop? Show all of your work and explain your reasoning.

8. Javier has been shopping around for a new mountain bike. He found two bikes that he likes equally—one is sold at Fernando's Bikes for $300, and the other is sold at Koda's Center for $275.

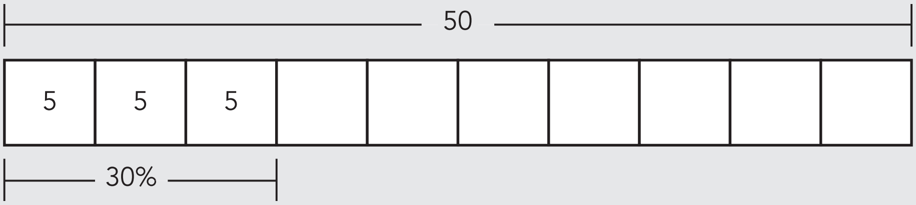
   Javier has a coupon for 25% off any bike at Fernando's Bikes. However, the manufacturer of the bike at Koda's Center has included a $40 rebate after the purchase of the bike.

   Where should Javier purchase his mountain bike? Show all of your work and explain your reasoning.

9. William is shopping for a new sweater that originally costs $50.00. The department store has all items marked down by 20%. He also has a coupon for an additional 10% off all purchases.

William

Since all items are marked down by 20%, and I have an additional 10% off, that means I get a 30% discount. I can determine 30% of 50 by using a strip diagram.



The discount is $5 + $5 + $5 = $15.
The cost of the sweater is $50.00 − $15.00 = $35.00

Describe William's mistake. Then, calculate how much William will pay for the sweater.

ACTIVITY
1.4

Percent and Direct Proportionality

A shoe store is holding a President's Day sale, with shoes priced at 80% of their original retail value. The chart hanging in the store displays the normal price of the shoes and the sale price.

Original Retail Value	Sale Price
\$20	\$16
\$25	\$20
\$30	\$24
\$35	\$28
\$40	\$32
\$50	\$40

Remember, proportions are formed by equivalent ratios.

Angel

👎

The sale prices are not directly proportional to the original retail value.

A \$20 pair of shoes is only \$4 cheaper on sale, while a \$50 pair of shoes is \$10 cheaper on sale.

1. Explain what is wrong with Angel's reasoning.

2. What is the constant of proportionality? Interpret the constant of proportionality for this problem situation.

3. Define the variables and write an equation to represent the relationship between the original retail value and sale price.

4. What is the sale price for a pair of shoes with an original retail value of $28? Explain your reasoning.

5. What is the original retail value for a pair of shoes that costs $15 on sale? Explain your reasoning.

Talk the Talk

Percent and Proportions

Demonstrate how to solve any type of percent problem with proportions.

1. Write each proportion, with a variable in the appropriate place, to calculate each specific unknown.

   a. Calculate the percent.

   b. Calculate the part.

   c. Calculate the whole.

Lesson 1 Assignment

Write

Explain how to use proportions to solve for the unknown in a percent problem.

Remember

The answer to the percent problem may or may not be the value of the unknown in your proportion. Reread the problem and ensure that you answer the question being asked.

Practice

1. The cable provider's "triple-play" package offers a landline, internet service, and cable TV for one fixed price. When you already subscribe to their cell phone service, they offer an additional 12% off the price of the triple-play package. When the discounted price of the triple-play is \$132, what is the price of the package without the discount?

2. A company sells juice in 1-gallon bottles. The current retail price of the juice is \$3.50 for 1 gallon. In order to remain competitive, the company will decrease the price to \$3.15.

    a. What percent of the original price will consumers pay?

    b. Suppose the company's cost per 1 gallon is \$2.70. What is the markup when they sell each gallon at \$3.19? What is the markup when they sell each gallon at \$3.51?

Lesson 1 Assignment

3. A company is trying to get schools in the state to sell their juice product.

   a. When the sales representative went to 300 schools and convinced 174 schools to sell their product, what percentage decided not to sell their product? Use two different strategies to calculate the answer.

   b. The sales representative made a deal with the schools for a discount on the individual juice bottles. The company usually sells the bottles to the distributors for $2.25, but they are selling them to the schools for 15% off. For what price will they sell each bottle to the schools?

   c. Suppose the schools pay $2.00 per bottle for the juice and sell it to community members for $2.50 per bottle. What percent markup are they charging?

Lesson 1 Assignment

4. Destiny is shopping for new school supplies. She sees a flyer in the newspaper for her favorite store. They are offering the following coupons.

School Supplies Sale!

All laptops—Buy now
and receive a $100 rebate
after purchase!

*cannot be used with any
other coupons

School Supplies Sale!

Receive 20% off any one item!

Destiny needs to buy a new laptop for the school year. The list price for the laptop is $479.99. Is it a better deal to use the coupon for the $100 rebate or the 20% off one item? Explain your reasoning.

Prepare

Express each percent as a decimal and as a fraction.

1. 47%
2. 3%
3. 12.5%
4. 0.25%
5. 4.99%

2

Calculating Tips, Commissions, and Simple Interest

OBJECTIVES

- Use percent equations to solve for unknowns in multi-step percent problems.
- Solve multi-step percent problems using the constant of proportionality.

NEW KEY TERMS

- percent equation
- simple interest
- commission

You have used proportions to solve percent problems involving markups and markdowns.

How can you use proportions and percent equations to solve for the unknown in different types of percent problems?

Getting Started

How to Tip Your Servers

In U.S. restaurants, most customers add a tip to the final bill to show their appreciation for the waitstaff. Usually, a customer will determine 15% to 20% of the bill and then add that amount to the total. Many times, customers will just round off the tip to the nearest dollar.

You can use benchmark percents to estimate the amount of any tip. Common benchmark percents used in calculating tips are 1%, 5%, 10%, and 25%.

WORKED EXAMPLE

One strategy to determine a 20% tip for a restaurant bill that is $38.95 is to first determine 10% of the total and then double that amount. Ten percent of $38.95 is $3.90, or approximately $4. So, a 20% tip should be about $8.

For each bill amount, use benchmark percents to estimate a 15% and 20% tip.

	15% Tip	20% Tip
$89.78		
$125.00		

ACTIVITY

2.1 Introduction to Percent Equations

Now that many people own phones with built-in calculators, some calculate the exact tip for their restaurant bill rather than use rounding and benchmark percents.

Suppose you want to determine the recommended 15% tip on a restaurant bill of $45.00. You can use a proportion to determine the amount of a tip based on the restaurant total.

1. Write a proportion that could be used to determine the amount of a tip based on the restaurant total.

You can also use a **percent equation** to determine the tip amount on a restaurant bill. A **percent equation** can be written in the form percent · whole = part, where the percent is often written as a decimal.

WORKED EXAMPLE

$$\begin{array}{ccccc}
 & & \text{percent} & = & \frac{\text{part}}{\text{whole}} \\
\text{percent as decimal} & \cdot & \text{whole} & = & \text{part} \\
\text{(tip as a percent)} & \text{of} & \text{(total bill)} & = & \text{amount of the tip} \\
\uparrow & & \uparrow & & \uparrow \\
\frac{15}{100} \text{ or } 0.15 & \cdot & 45 & = & t \\
 & & 6.75 & = & t
\end{array}$$

2. Analyze the Worked Example.

   a. Describe how the percent equation in the form $\text{percent} \cdot \text{whole} = \text{part}$ is equivalent to a proportion in the form $\text{percent} = \frac{\text{part}}{\text{whole}}$.

   b. Determine whether the variable is isolated. Then, describe how the tip amount is calculated using the percent equation.

3. Use proportions and percent equations to calculate the tip on the given restaurant bill. Isolate the variable first. Then, determine the tip amount. Finally, write your answer in a complete sentence.

Bill	Percent	Use a Proportion	Use a Percent Equation
\$30.00	15%		
Sentence			

4. Describe how the strategies you used to solve the proportion and the percent equation are similar.

ACTIVITY

2.2

Strategies for Calculating Percent

Restaurant servers are not the only people provided with tips for a job well done.

Gabriela is a hair stylist at a salon. Her clients pay their bills at the front desk but give her cash for her tip. She wondered what her typical tip as a percent was, so she calculated the tip as a percent received from each client on a specific day.

WORKED EXAMPLE

Gabriela's first client of the day spent \$150 to have her hair dyed and cut and gave Gabriela a \$30 tip.

Use a Proportion

$$\frac{t}{100} = \frac{30}{150}$$

$$t = \frac{(30)(100)}{150}$$

$$t = 20$$

Use a Percent Equation

$$(t)(150) = 30$$

$$150t = 30$$

$$\frac{150t}{150} = \frac{30}{150}$$

$$t = \frac{30}{150}$$

$$t = 0.2$$

1. Explain why Gabriela's methods result in different values for *t*. What tip as a percent did Gabriela receive from her client?

Ask Yourself: How can you use percents in everyday life?

2. Calculate the tip as a percent for Gabriela's next client. Use both proportions and percent equations in the table shown. Isolate the variable first. Then, calculate the answer. Finally, write your answer in a complete sentence.

Salon Bill	Tip Amount	Use a Proportion	Use a Percent Equation
$80	$15		
Sentence			

3. Describe the strategies you used to solve the proportion and the percent equation.

ACTIVITY

2.3

Strategies for Calculating the Whole

Tipping for services is not a universal standard, so some businesses add an automatic gratuity, or tip, onto every bill. Restaurants frequently add an 18% gratuity when the group includes 8 or more people. Some hotels, resorts, and service providers close to tourist areas often add an automatic 18% gratuity to the bill.

1. The esthetician and manicurist at a resort earn an automatic 18% gratuity on their services. To the nearest whole dollar, determine the value of the services each must provide in a day to earn the desired gratuity. Show your work and then write a sentence to explain your answer.

An *esthetician* is someone who is knowledgeable about skin care, particularly the face.

	Desired Gratuity	Use a Proportion	Use a Percent Equation
Esthetician	$270		
Sentence			
Manicurist	$180		
Sentence			

2. Hannah says that when she solved the proportions in Question 1, she could set up her proportions in any way she wanted because ratios can be written in any way. Do you agree with Hannah's statement? Explain your reasoning.

ACTIVITY

2.4 Tips and Proportionality

A room service waiter or waitress at a hotel receives an automatic gratuity that is directly proportional to the amount of the food bill.

1. Suppose a room service waitress at the resort receives gratuity represented by the equation $g = 0.15b$, where g represents the gratuity and b represents the food bill.

   a. What percent gratuity does the room service waitress receive? How do you know?

   b. When the food bill is $19, how much tip did she receive?

   c. When the room service waitress receives a $2.10 gratuity, how much is the food bill?

What does the form of the equation tell you?

2. Write an equation to represent the direct proportional relationship between the amount of gratuity (g) received by a room service waitress and the food bill (b). Let k represent the constant of proportionality. What does the constant of proportionality represent in this equation?

3. A gourmet restaurant has a policy of automatically adding an 18% tip to every restaurant bill.

   a. Write an equation to represent the relationship between the tip (*t*) and the restaurant bill (*b*).

   b. When a restaurant bill is $18, how much is the tip?

   c. How much would a restaurant bill be when it had a tip of $8.10 added to it?

ACTIVITY 2.5

Simple Interest

When you save money in a bank savings account, the bank pays you money each year and adds it to your account. This additional money is interest and it is added to bank accounts because banks routinely use your money for other financial projects. They then pay interest for the money they borrow from you.

An original amount of money in your account is called the *principal*. Interest is calculated as a percent of the principal. One type of interest is **simple interest**, which is a fixed percent of the principal. Simple interest is paid over a specific period of time—either twice a year or once a year, for example. The formula for simple interest is:

$$\underset{\substack{\uparrow\\\text{Interest earned}\\\text{(dollars)}}}{I} = \underset{\substack{\uparrow\\\text{Principal}\\\text{(dollars)}}}{P} \cdot \overset{\substack{\text{Interest rate (\%) as a decimal}\\\swarrow}}{r} \cdot \underset{\substack{\nwarrow\\\text{Time that the money earns interest}\\\text{(years)}}}{t}$$

WORKED EXAMPLE

For example, Madison deposits $300 into a savings account at a simple interest rate of 5% per year.

You can use the formula to calculate the interest she will have earned at the end of 3 years.

Interest = Principal · rate · time

Interest = (300)(0.05)(3)

= $45

Madison will have earned $45 in interest after 3 years.

1. Gabriela deposits $600 into a savings account at a simple interest rate of 3% per year. How much interest will Gabriela have earned at the end of 2 years?

2. Ana deposits $1500 into a bank account. She does not plan to make any deposits or withdraws. The account earns 5% simple interest. What will the account balance be after 2 years?

3. Luis deposits $800 into a bank account which earns simple interest. Luis does not make any deposits or withdraws. At the end of the year the account balance is $816. What is the interest rate?

In the same way that banks pay you interest when they use your money for financial projects, you pay interest as well.

4. When you borrow money from a bank, the amount you borrow is the principal and you pay the interest on that money to the bank. Paul borrows $16,000 from the bank. The loan has a simple interest rate of 8% per year. What is the amount of interest that Paul will pay at the end of the year.

ACTIVITY 2.6

Commissions and Proportionality

A car salesperson makes a 10% *commission* on each sale. A salesperson's commission is proportional to the cost of the products sold.

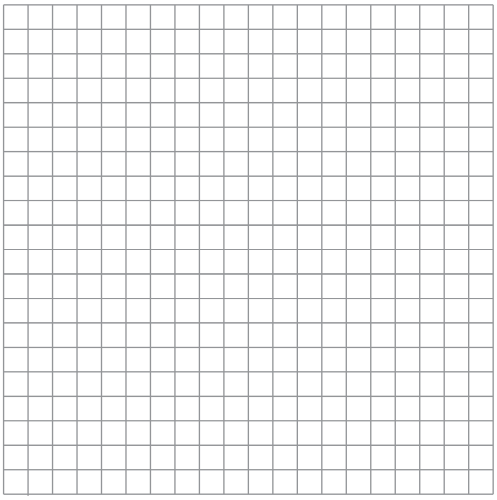
A **commission** is an amount of money a salesperson earns after selling a product. Many times, the commission is a certain percent of the product.

1. Complete the table to show the relationship between the price of a car and the commission the salesperson receives.

Price (dollars)	Commission (dollars)
0	
9000	
15,000	
	1000
18,000	
	2000

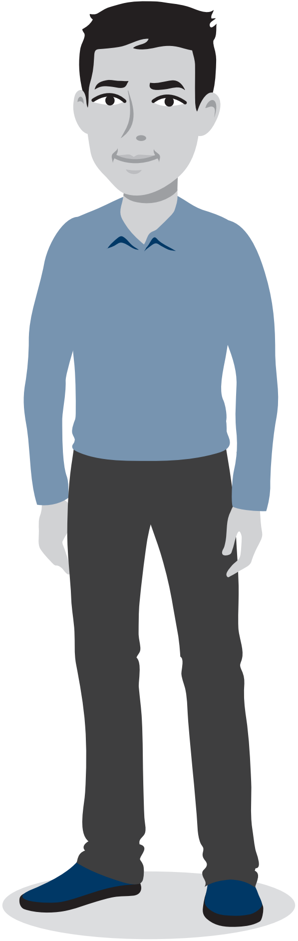
To help with sales goal-setting, the salesperson wants to create a visual to display in the office.

2. Graph the relationship between the price of a car and the commission received.



3. Explain how the graph illustrates that the relationship between car price and commission is proportional.

Because commission is proportional to the cost of products sold, you can make all kinds of predictions in these situations.



4. Define the variables and write the equation that represents the relationship between the price of a car and the commission received.

5. How much commission will the salesperson earn for selling a $25,000 car? Determine the commission using your equation.

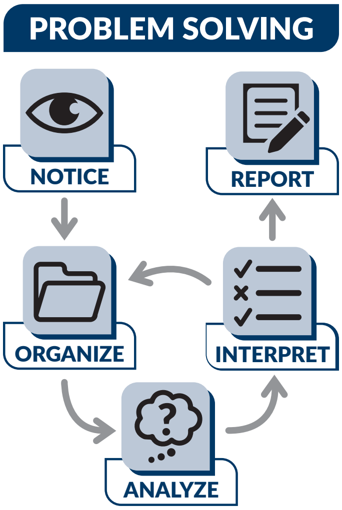
6. When the salesperson earned a $1250 commission, what was the price of the car? Determine the price of the car using your equation.

For each commission situation, define variables for the varying quantities, write a percent equation to represent the relationship and then answer the question.

7. A real estate agent earns 5% of the selling price of each house he sells. When he sells a home for $200,000, how much of a commission will he make?

8. A real estate agent made $18,000 on a $300,000 home sale. What was the percent of her commission?

Talk the Talk



Where's Malik?

Each of the restaurant receipts shown includes a unknown meal total of a guest who paid a specific percent of the total bill. The tip amount for each table is also provided. Use the receipts to answer the questions.

Table 1

Guest 1:
Guest 2: $18.00
Guest 3: $12.00
Tip: $8.00

Table 2

Guest 1: $34.00
Guest 2: $21.00
Guest 3:
Tip: $10.80

Table 1

Guest 1: $19.00
Guest 2:
Guest 1: $41.00
Tip: $12.00

1. Malik ate at a restaurant one night with 2 friends. The cost of his meal was 10% of the total food bill but not clearly shown on the receipt. He knows that his group tipped either 15% or 18%, but he's not sure which. At which table did Malik sit, and what was the cost of his meal? What percent did his table tip?

2. Assume, instead, that the mystery guest at each table paid 25% of the total bill. Determine the total bill and what tip as a percent each table gave their waitstaff.

Lesson 2 Assignment

Write

Explain how to solve for any unknown with a percent equation.

Remember

When a tip, a gratuity, or a commission is proportional to the cost of the product, you can represent the situation using the equation $t = kb$, where t represents the amount of tip, gratuity, or commission, k represents the tip as a percent, and b represents the total cost of the product.

Practice

1. Koda recently purchased a new home. In order to be able to afford the home, he came up with a budget that breaks down his monthly income as follows: 30% for housing, 20% for food, 22% for utilities, 10% for savings, 6% for entertainment, and 12% for personal items. Ben has a net income of $4000 per month. Calculations for budget are identical to calculations for tips and commissions. Use what you know about computing tips and commissions to answer each question. For each question,
   - set up and solve a proportion,
   - set up and solve a percent equation, and
   - write a sentence to explain your answer.

   a. Calculate the amount of money that Koda will have each month for his most essential budget items: housing, food, and utilities.

   b. Last month Koda spent $320 on food and $400 on personal items. What percent of his monthly income did he spend on these items?

Lesson 2 Assignment

c. After 5 years, Koda's income has undergone several increases. Last month, Koda put $600 in savings, which was 10% of his net income. What is Koda's new monthly net income?

d. Koda would like to purchase a new home. He figures out that his monthly housing costs for the new home will be $2100 per month. When he wants to keep the housing costs as 30% of his budget, what will his monthly net income need to be?

2. Maria receives a 4% commission on the merchandise she sells in a department store.

   a. Write an equation to represent the relationship between the total sales (s) and the commission (c) received.

   b. How much commission would Maria receive when the merchandise she sold totaled $140?

   c. How much would Maria have to sell to earn a commission of $100?

Lesson 2 Assignment

3. Abby deposits $1200 into a savings account that earns simple interest. The interest is applied to her account at the end of each year. Complete the table.

Year	Principal Balance	Simple Interest Rate	Simple Interest Earned
1	$1200	2%	
2	$1224	2%	
3		3%	

4. In the community of Del Valle, a local restaurant allows student-based clubs to host fundraising nights. The organizations must hand out flyers ahead of time, and the customers who come to dine at the restaurant must present the flyer with their order. The restaurant will donate 15% of the total sales to the organization.

   a. When the sales total $8000, how much will the organization make?

   b. Write an equation to represent the relationship between the dinner sales (d) and the amount (A) the organization will receive.

   c. What is the constant of proportionality? Interpret the constant of proportionality for this problem situation.

   d. What were the total dinner sales when the organization was given $885 by the restaurant? Explain your reasoning.

Lesson 2 Assignment

5. A restaurant also donates a percentage of its total sales for the entire day to a local soup kitchen. The table shows the total sales for different days and the amount that was donated to the soup kitchen.

Total Sales (dollars)	Amount Donated (dollars)
10,400	416
14,300	572
15,000	600
18,300	732

a. What percent of the total sales does the restaurant donate to the soup kitchen? How do you know?

b. Write an equation to represent the relationship between the total sales (*s*) and the amount donated to the soup kitchen (*d*).

c. What is the constant of proportionality? Interpret the constant of proportionality for this problem situation.

d. When the restaurant donated $700 to the soup kitchen, what were the total sales for the day?

Lesson 2 Assignment

Prepare

Use the distributive property to rewrite each product. Then, simplify the product. Round to the nearest penny.

1. \$7.80 (1 + 0.02)

2. \$36.00(1 + 0.045)

3. \$22(1 + 0.07)

3

Sales Tax, Income Tax, and Fees

OBJECTIVES

- Use proportional reasoning to calculate sales tax and income tax amounts.
- Solve sales tax and income tax problems by determining the tax amount, percent, or amount taxed.
- Use proportionality to solve multi-step ratio and percent problems involving taxes and fees by writing equations.

NEW KEY TERMS

- sales tax
- income tax

You have used proportionality and percent to solve problems involving price markups and markdowns, tips, and commissions.

How can you apply proportional reasoning to solve problems about taxes?

Getting Started

Pay the Piper

When you have ever bought something at the store, you may have noticed that the actual cost of the item ended up being more than what was listed. This is because sales tax is often added to the purchase price. **Sales tax** is a percentage of the selling price of a good or service that is added to the price.

Sales tax percents vary from state to state, though they are usually between 4% and 7%. The percents can also change over time. Sales tax is one of the largest sources of earnings for state governments. Sales taxes fund transportation and public schools.

1. The table shown lists 6 states and the different sales tax added to every dollar of a purchase as of January 2016. Complete the table by determining the tax amount added to each purchase amount. When necessary, round your answer to the nearest penny.

State	Sales Tax (percent)	Tax on \$10.00 (dollars)	Tax on \$100.00 (dollars)
Indiana	7		
Kansas	6.5		
Alabama	4		
Missouri	4.225		
Minnesota	6.875		
Oklahoma	4.5		

ACTIVITY 3.1

Sales Tax

Past sales tax information for California, Maine, and Texas is shown.

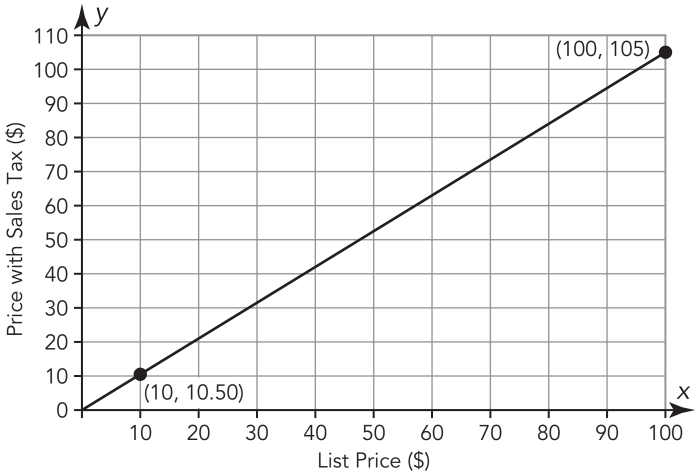
California

List Price of an Item ($)	Price Including Sales Tax ($)
117	126.65
200	216.50
250	270.63

Texas

$y = 1.0625x$, where x is the list price and y is the price including sales tax.

Maine



1. Consider the three state tax representations to answer each question.

   a. Determine the cost, including sales tax, of an item which has a list price of $150 for Texas and Maine. Show all your work and explain your reasoning.

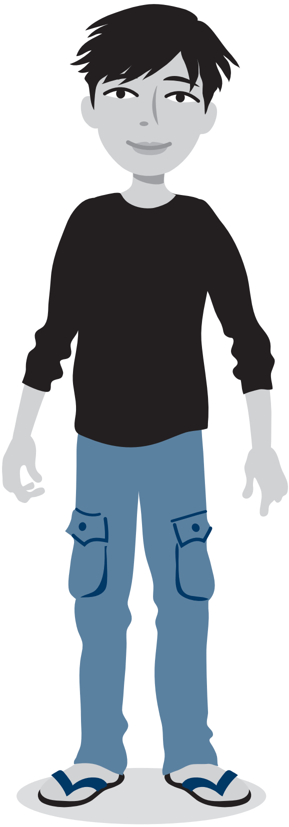
b. Determine the list price of an item that costs \$200, including sales tax for California and Texas. Show all your work and explain your reasoning.

c. Which of the three states had the highest sales tax? Explain your reasoning.

Do you have to calculate the sales tax to determine which state is the highest?

d. What was the sales tax percent in each of the three states? Explain your reasoning.

2. Consider the equation for the sales tax in Texas. Why is multiplying the list price by 1.0625 to determine the price including tax the same as calculating the sales tax and then adding it to the list price?



3. The sales tax in Pennsylvania is 6%.

   a. How much sales tax would there be on a \$750 flat screen TV? Show how you determined your answer.

   b. When the sales tax on a lawn mower is \$48, what is the cost of the mower?

ACTIVITY
3.2

Income Tax

Income tax is a percent of a person's or company's earnings that is collected by some states and the federal government. All U.S. citizens pay a federal income tax, but only 43 states also collect a state income tax. Currently, Texas does not collect a state income tax.

Sales tax and income tax are often called *rates*. This is because they are ratios of amount taxed per dollar spent or earned.

Let's first explore some scenarios where there is a state income tax.

1. When the state income tax rate is 4%, determine the amount of income tax paid on an annual income of $60,000.

2. Write an equation to describe the amount of income tax paid on an income of x dollars when the income tax rate is 4%.

3. Suppose the income rate is 6%. Determine the amount of money remaining after paying income tax on an income of $80,000.

4. Write an algebraic equation to describe the amount of money remaining after state income tax is paid on an income of x dollars when the income tax rate is 6%.

5. Suppose the state income tax rate is 5%. Determine a person's income before taxes when the amount of money taken out in taxes is $500.

6. Huyen just got her first job, which pays $1750 every 2 weeks. She pays 4% state income tax. How much does Huyen have remaining after state income tax is deducted from her paycheck?

Now let's explore some scenarios involving federal income tax.

Federal income tax is not a flat rate. The 2023 tax brackets and rates are listed in the table.

Tax Rate	For single Filers	For Married Individuals Filing Joint Returns	For Heads of Households
10%	\$0 to \$11,000	\$0 to \$22,000	\$0 to \$15,700
12%	\$11,000 to \$44,725	\$22,000 to \$89,450	\$15,700 to \$59,850
22%	\$44,725 to \$95,375	\$89,450 to \$190,750	\$59,850 to \$95,350
24%	\$95,375 to \$182,100	\$190,750 to \$364,200	\$95,350 to \$182,100
32%	\$182,100 to \$231,250	\$364,200 to \$462,500	\$182,100 to \$231,250
35%	\$231,250 to \$578,125	\$462,500 to \$693,750	\$231,250 to \$578,100
37%	\$578,125 or more	\$693,750 or more	\$578,100 or more

For federal income tax, you pay one rate on part of your income and a higher rate on income above a set amount based on how you file your taxes.

WORKED EXAMPLE

Olivia is filing single. Her total taxable income for 2023 is \$50,000. How much does Olivia pay in federal income tax?

Olivia's taxable income is in the 22% tax bracket.

She pays 10% income tax on the first \$11,000 of her income.	$\$11{,}000 \cdot 0.10 = \$1{,}100$
The next part of Olivia's income is taxed at 12%.	$\$44{,}725 - \$11{,}000 = \$33{,}725$ $\$33{,}725 \cdot 0.12 = \$4{,}047$
The last part of her income is taxed at 22%.	$\$50{,}000 - \$44{,}725 = \$5{,}275$ $\$5{,}275 \cdot 0.22 = \$1{,}160.50$

Her total federal income tax is \$1,100 + \$4,047 + \$1,160.50.

Olivia pays \$6,307.50 in federal income tax.

7. How is calculating federal income tax different from calculating state income tax?

Use the table of tax brackets and rates for 2023 to answer each question. Use a calculator to help with your calculations.

8. William files single. His taxable income is $36,000. How much does William pay in federal income tax?

9. Daniela and Angel are married and filing joint tax returns. Their combined taxable income is $180,000. How much do Daniela and Angel pay in federal income tax?

People often find their income tax to be lower than the calculated amount due to allowed federal deductions and tax credits.

ACTIVITY
3.3

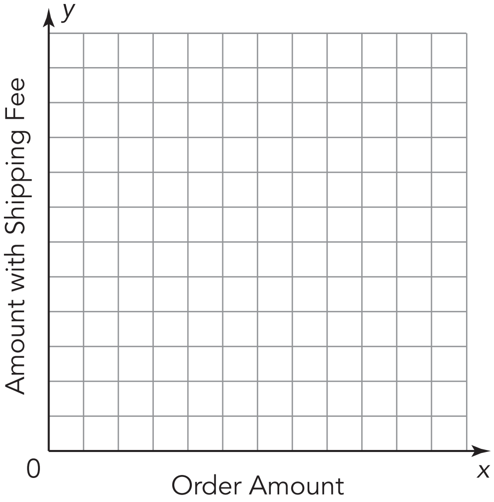
Graphs and Fees

Destiny orders yarn online to make hats for her crochet store. Because she orders her materials from out of the country, she pays a total of 20% in fees, including shipping, on each order.

1. Identify the constant of proportionality and interpret its meaning.

2. Complete the table to determine the amounts Tori paid on each of the four orders shown. Then, create a graph.

Order Amount	Amount with Shipping Fee
$10	
$15	
$20	
$5	



3. Write an equation to represent the relationship between the order amount and the total amount with shipping fees.

Talk the Talk

Life and Taxes

You have learned about markups and markdowns, tips, discounts, commissions, fees, and taxes.

1. What similarities and differences can you describe about all of these percent relationships?

2. Which of the percent relationships represent an increase from an original amount? Which represent a decrease? Explain your answers.

Lesson 3 Assignment

Write

Write a definition for each term in your own words. Provide an example for each definition.

1. *sales tax*
2. *income tax*

Remember

When calculating the total cost of a good or service that includes sales tax ($x\%$), multiplying the list price by $(1 + x\%)$ is the same as adding the sales tax ($x\%$) to the list price.

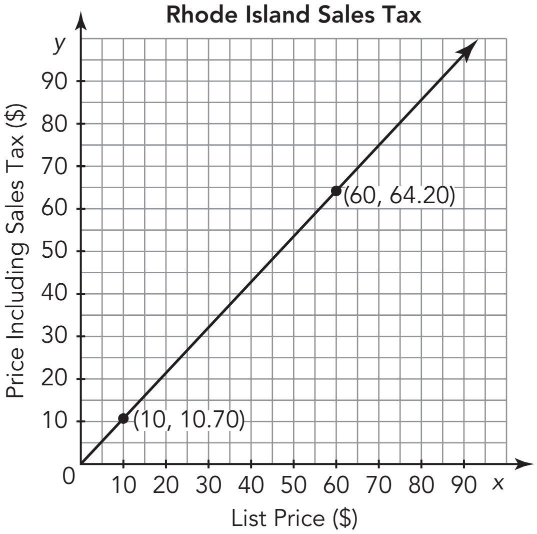
Practice

1. Three friends live in three different states. Gabriela lives in Rhode Island, Luis lives in Ohio, and Malik lives in Colorado.

Colorado Sales Tax	
List Price ($)	Price Including Sales Tax ($)
3.50	3.60
8.75	9.00
10.00	10.29

Ohio Sales Tax

$y = 1.055x$,
where x is the sales price in dollars and y is the price including sales tax in dollars.



a. Determine the sales tax rate in each state. Show your work.

b. All three friends want to purchase the same gaming system so they can play games together. The list price of the gaming system is $349.99. How much will each friend pay for the gaming system?

Lesson 3 Assignment

2. Michael earned a total of $75,000 this year. He lives in the District of Columbia, where the local income tax rate is 8.5% for all incomes over $40,000.

   a. How much will Michael pay in income tax?

   b. How much money will Michael have after paying his income tax?

The 2023 tax brackets and rates are listed in the table.

Tax Rate	For single Filers	For Married Individuals Filing Joint Returns	For Heads of Households
10%	$0 to $11,000	$0 to $22,000	$0 to $15,700
12%	$11,000 to $44,725	$22,000 to $89,450	$15,700 to $59,850
22%	$44,725 to $95,375	$89,450 to $190,750	$59,850 to $95,350

Use the table of tax brackets and rates for 2023 to answer each question.

3. Hannah files single. Her taxable income is $30,000. How much does Hannah pay in federal income tax?

4. Javier and Ana are married and filing joint tax returns. Their combined taxable income is $120,000. How much do Javier and Ana pay in federal income tax?

Lesson 3 Assignment

Prepare

Determine each product, rounding to the nearest penny.
Describe whether the product represents an increase or decrease.

1. $\$450 \cdot 1.5 = y$

2. $y = \$36 \cdot 0.9$

3. $\$100 \cdot 1.1 = y$

4. $\$250 \cdot 0.99 = y$

4

Percent Increase and Percent Decrease

OBJECTIVES

- Calculate a percent increase and decrease.
- Calculate the depreciation in a value.
- Use proportional reasoning to solve multi-step problems involving percent increases and decreases.
- Use proportional reasoning to solve multi-step problems involving area and volume of two- and three-dimensional objects.

NEW KEY TERMS

- percent increase
- percent decrease
- appreciation
- depreciation

You have used proportional reasoning to solve percent problems, including determining markups and markdowns.

How can you apply what you know to determine the percent a value has increased or decreased?

Getting Started

Up or Down?

Tell whether each situation represents an increase or a decrease in spending.

1. A $20 shirt is on sale for $16.

2. The cost of a movie ticket in the afternoon is $6.50, and it is $8.00 after 5:00 p.m.

3. A ride-all-day amusement park ticket last season was $27, and this season it is $35.

4. The cost of lunch yesterday was $5.50, and the cost of lunch today is $5.25.

ACTIVITY 4.1

Percent Increase and Decrease

You have used percents in many different situations. You can also use percents to describe a change in quantities.

A **percent increase** occurs when the new amount is greater than the original amount, such as when stores mark up the price they pay for an item to make a greater profit.

1. A jewelry store marks up its prices so it can maximize its profits. What is the percent increase for each item? Use the formula shown to complete the table.

$$\text{Percent Increase} = \frac{\text{Amount of Increase}}{\text{Original Amount}}$$

The Jewelry Store's Accounting Sheet				
Item	Cost (dollars)	Customer's Price (dollars)	Difference (dollars)	Percent Increase
Earrings	25	50		
Pin	36	45		

Evaporation is the process by which a liquid changes into a gas or vapor.

A **percent decrease** occurs when the new amount is less than the original amount. An example of a percent decrease is the amount water evaporates over time. Water evaporates at different rates, depending on the size and shape of the container it is in and the air temperature.

2. A science class is conducting an experiment to determine how fast water evaporates. They fill two differently shaped containers with water and measure the level once at the beginning of the experiment and once at the end. Use the formula shown to complete the table.

$$\text{Percent Decrease} = \frac{\text{Amount of Decrease}}{\text{Original Amount}}$$

Container	Starting Height (cm)	Ending Height (cm)	Difference (cm)	Percent Decrease
A	6	4.5		
B	2.5	1		

3. How do you know when the percent is a decrease or increase?

4. How would you describe a 50 percent decrease?

5. How would you describe a 100 percent increase?

Sometimes people incorrectly use “200% increase” to mean a 100% increase.

6. Fernando was doing a great job at work, so his boss gave him a 20 percent raise. Then, he started coming to work late and missing days, so his boss gave him a 20 percent pay cut. Fernando said, “That‘s okay. At least I‘m back to where I started.”

   Is Fernando correct in thinking he is making the same amount of money when he received a pay cut? When you agree, explain why he is correct. When you do not agree, explain to Fernando what is incorrect with his thinking and determine what percent of his original salary he is making now.

7. Analyze each student’s percent reasoning.

   Explain what Alexander did incorrectly. What expression correctly represents a 50% increase over a number n?

Alexander

When you multiply a number n by 0.5, that‘s a 50% increase because 0.5 = 50%.

Jacob

When you multiply a number n by 1.2, that’s a 20% increase:

$$n \cdot 1.2 = n(1 + 0.2)$$
$$= n + 0.2n$$

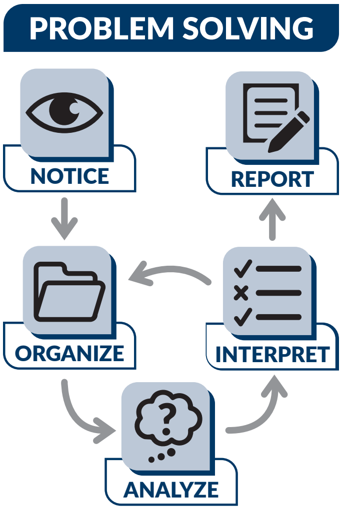
The expression $n + 0.2n$ means a 20% increase from n.

Maria

When you multiply a number n by 0.9, that’s a 10% decrease:

$$n \cdot 0.9 = n(1 - 0.1)$$
$$= n - 0.1n$$

The expression $n - 0.1n$ means a 10% decrease from n.



8. Use Jacob's and Maria's methods to determine the new price given each percent increase or decrease.

   a. A ticket originally costs \$20, but its price decreases by 20%.

   b. Abby is making \$12 per hour. She gets a 6% increase in pay.

ACTIVITY

4.2 Depreciation and Proportionality

Generally, things like homes and savings accounts gain value, or appreciate, over time. Other things, like cars, depreciate every year. New cars depreciate about 12% of their value each year.

Appreciation is an increase in price or value.

Depreciation is a decrease in price o value.

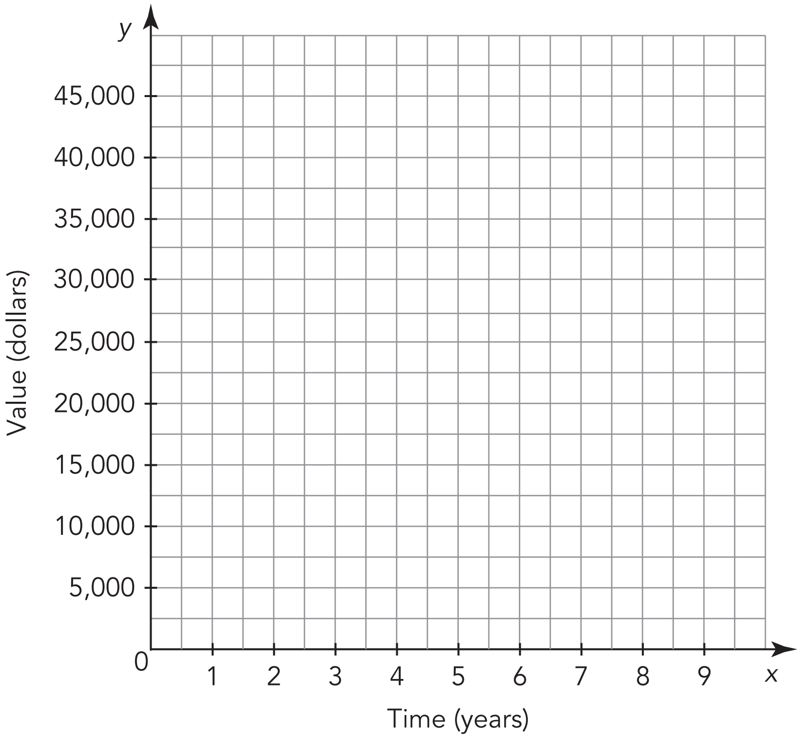
1. How much would a new car depreciate the first year when it costs $35,000?

2. When a car lost $3600 in depreciation in the first year, what is the original cost of this car?

3. Complete the table to record the value of a car that costs $50,000 and depreciates at the rate of 12% per year for the first four years.

Time (years)	Value of the Car (dollars)
0	50,000
1	
2	
3	
4	

4. Complete the graph.



5. Would it make sense to connect the points on the graph? When it makes sense, connect the points. Explain your reasoning.

6. Describe how the value of the car decreases over time.

7. Is the relationship between time and the car's value proportional? Explain how you know using the graph and the completed table of values.

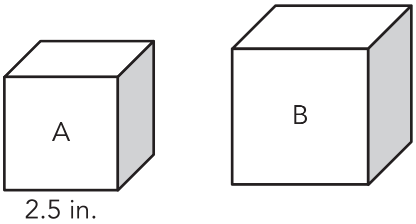
ACTIVITY

4.3

Percent Increase and Decrease with Geometry

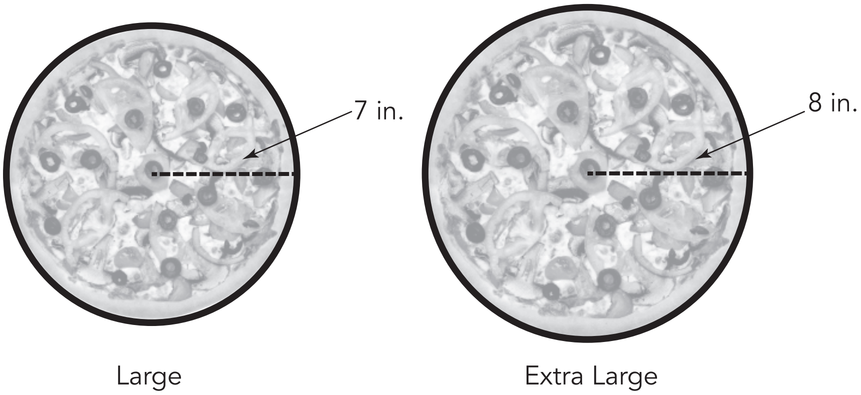
It is common to talk about percent increases and percent decreases when talking about money. But you can also use them when thinking geometrically.

1. Consider the cubes shown. The side lengths of Cube *B* are each 20% greater than the side lengths of Cube *A*.



Determine the volume of each cube. Show your work.

2. What is the percent decrease in the amount of pizza from the extra large to the large? Round to the nearest percent. Explain how you determined your answer.



Talk the Talk

Gas Prices

The table shows the average price of gasoline per gallon in the U.S. for the years 2019 through 2022.

1. Complete the table by determining each yearly percent increase or decrease in the price of gasoline. Round to the nearest percent.

Year	Price ($ per gallon)	% Increase or Decrease
2019	2.60	
2020	2.17	
2021	3.01	
2022	3.95	

2. Amir has $1800 dollars in a savings account to pay for gas. He withdraws $50 a month for 6 months for gas without depositing any money into the account. What is the percent decrease from the original amount in his savings account to the amount at the end of 6 months? Round to the nearest percent.

Lesson 4 Assignment

Write

Write a definition for *depreciation* in your own words. Use an example in your definition.

Remember

A *percent increase* is calculated as a ratio of the amount of increase to the original amount.
A *percent decrease* is calculated as a ratio of the amount of decrease to the original amount.

Practice

Calculate each percent increase or percent decrease. Round to the nearest whole percent when necessary.

1. Original amount: 30, new amount: 45
2. Original amount: 12, new amount: 16
3. Original amount: 17, new amount: 21
4. Original amount: 85, new amount: 56
5. Original amount: 48, new amount: 37
6. Original amount: 124, new amount: 76

Use the given information to answer each question.

7. A dress that normally sells for \$72 is on sale for \$45. What is the percent decrease in the price?
8. A home purchased for \$120,000 in 2012 is sold for \$156,000 in 2015. What is the percent increase in the price?
9. A music store purchases CDs for \$6 each and sells them for \$9 each. What is the percent increase in the price?

Lesson 4 Assignment

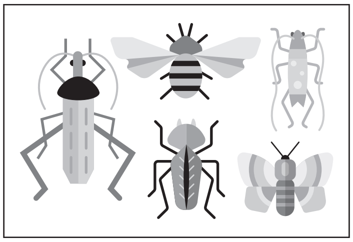
10. The music store is having a clearance sale. A CD player that originally sells for $60 is now priced at $36. What is the percent decrease in the price?

11. The local high school sold 1914 tickets this year to its spring musical. That was 174 more tickets sold than last year. What is the percent increase in the number of tickets sold?

12. Koda's heart rate went from 74 beats per minute while resting to 148 beats per minute while exercising. What is the percent increase in his heart rate?

Prepare

1. These bugs are not drawn to scale. What does that mean?



2. Is it possible that the leftmost bug is the smallest bug in real life? Explain your reasoning.

5

Scale and Scale Drawings

OBJECTIVES

- Identify the scale of a drawing as the ratio of drawing length to actual length.
- Compute actual lengths and areas given a scale and a scale drawing.
- Reproduce a scale drawing at a different scale.
- Use concepts of proportionality to determine the scale factor used to produce a scale drawing.

NEW KEY TERMS

- corresponding
- scale
- scale factor
- scale drawing
- similar figures

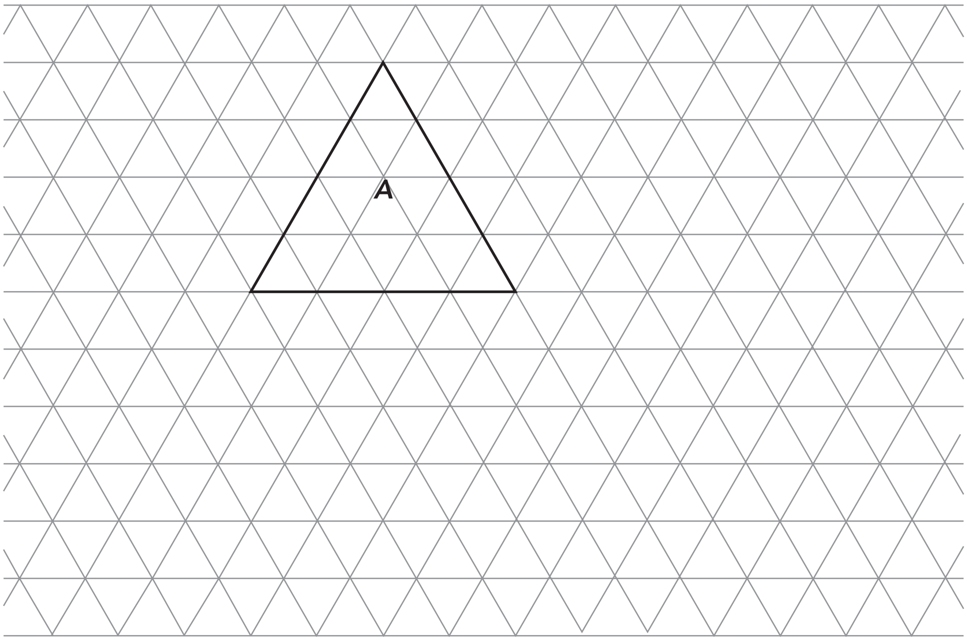
You have learned a lot about proportional relationships. You can use proportionality to solve problems with geometric figures too.

How can you use proportions to investigate real-world and mathematical objects at different scales?

Getting Started

Triangle Drawings

The triangle given, Triangle *A*, is an equilateral triangle—all of its sides are the same length. Each side has a length of 4 units.



1. Draw a new triangle, Triangle *B*, which has side lengths that are 50% the length of Triangle *A*'s side lengths.

2. Write each ratio to compare Triangle *B* to Triangle *A* in fraction form and as a percent. Describe the meaning of each ratio.

The bottom side of the original triangle and the bottom side of the new triangle are **corresponding** sides.

   a. Determine the ratio of corresponding side lengths for each side of the triangles.

   b. Determine the ratio of the triangles' perimeters.

ACTIVITY 5.1

Scales, Scale Factors, and Scale Drawings

A **scale** is a ratio that compares two measures. The ratios that you wrote to compare the measures of the triangles are scales.

When you multiply a measure by a scale to produce a reduced or enlarged measure, the scale is called a **scale factor**. You can multiply each of Triangle *A*'s lengths by the scale factor 1 : 2, or 50%, or $\frac{1}{2}$, to produce the side lengths for Triangle *B*.

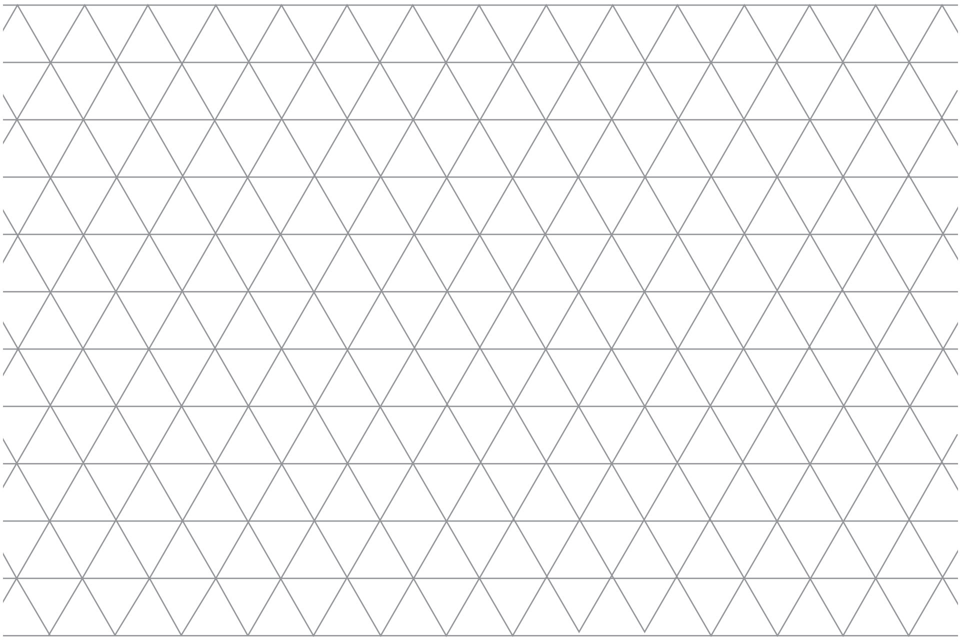
WORKED EXAMPLE

Triangle *A*				Triangle *B*
side length → 4 units	·	1 : 2, or $\frac{1}{2}$, or 50% ↑ scale, scale factor	=	2 units ← side length

1. What scale factor is used to produce Triangle *A*'s lengths, given Triangle *B*'s lengths? Explain your reasoning.

Triangle *A*'s side lengths are 200% of Triangle *B*'s side lengths. But they are 100% larger.

2. Draw a triangle, Triangle *C*, in which its side lengths are 150% of Triangle *B*’s side lengths.



3. What scale factor is used to produce:

   a. Triangle *C*’s lengths, given Triangle *B*’s lengths?

   b. Triangle *B*’s lengths, given Triangle *C*’s lengths?

   c. Triangle *C*’s lengths, given Triangle *A*’s lengths?

4. Compare the perimeters of Triangles *A*, *B*, and *C*. How did the scale factors affect the perimeters? Explain your reasoning.

A **scale drawing** is a representation of a real object or place that is in proportion to the real object or place it represents. The purpose of a scale drawing is to represent either a very large or very small object. The scales on scale drawings often use different units of measure.

WORKED EXAMPLE

The scale of a drawing might be written as:

$$\underset{\substack{\nearrow \\ \text{Drawing} \\ \text{Length}}}{1\text{ cm}} : \underset{\substack{\nwarrow \\ \text{Actual} \\ \text{Length}}}{4\text{ ft}}$$

This scale means that every 1 centimeter of length in the drawing represents 4 feet of the length of the actual object.

The scale of a map might look like this:

$$\underset{\substack{\nearrow \\ \text{Map} \\ \text{Distance}}}{1\text{ in.}} : \underset{\substack{\nwarrow \\ \text{Actual} \\ \text{Distance}}}{200\text{ mi}}$$

This scale means that every 1 inch of distance on the map represents 200 miles of actual distance.

5. Study the Worked Example. What scale factor is used to produce:

   a. the drawing length given the actual length?

   b. the map distance given the actual distance?

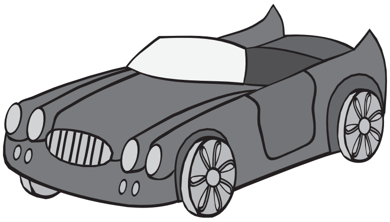
6. Write a sentence to describe the meaning of each.

   a. A scale on a map is 1 in. : 2 mi.

   b. A scale on a drawing is 2 in. : 1 in.

   c. A scale on a drawing is 1 cm : 1 cm.

7. The scale factor for a model car is 1 : 24. What does this mean?



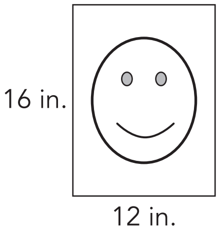
8. The scale factor for a model train is 1 : 87. What does this mean?

ACTIVITY
5.2

Similar Figures

Figures that are proportional in size or have proportional dimensions are called **similar figures**.

When a photo company prints photo packages, they include several sizes of photos that are all mathematically similar. The largest size is 12 in. · 16 in. This is read as “12 inches by 16 inches.” The first measure is the width of the photo, and the second measure is the height of the photo.



Huyen and William determined the width of a mathematically similar photo that has a height of 8 inches.

Huyen

To determine the unknown width of the smaller photo, I wrote ratios using the measurements from within each photo. The ratio on the left contains the measurements of the large photo. The ratio on the right contains the measurements of the smaller photo.

$$\frac{\text{width of larger photo}}{\text{height of larger photo}} = \frac{\text{width of smaller photo}}{\text{height of smaller photo}}$$

$$\frac{12 \text{ inches}}{16 \text{ inches}} = \frac{x \text{ inches}}{8 \text{ inches}}$$

$$(12)(8) = (16)(x)$$

$$96 = 16x$$

$$6 = x$$

I calculated that the width of the smaller photo is 6 inches.

William

To determine the unknown width of the smaller photo, I wrote ratios using the measurements between the two photos. The ratio on the left contains the width measurements of both photos. The ratio on the right contains the height measurements of both photos.

$$\frac{\text{width of larger photo}}{\text{width of smaller photo}} = \frac{\text{height of larger photo}}{\text{height of smaller photo}}$$

$$\frac{12 \text{ inches}}{x \text{ inches}} = \frac{16 \text{ inches}}{8 \text{ inches}}$$

$$(12)(8) = (x)(16)$$

$$96 = 16x$$

$$6 = x$$

I calculated that the width of the smaller photo is 6 inches.

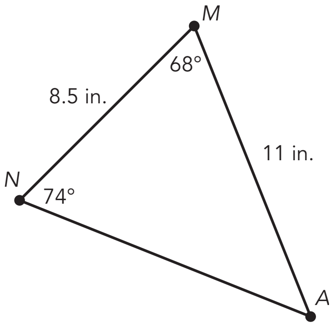
1. What is similar about the two solution methods? What is different about the two solution methods?

2. Determine other possible photo sizes that are mathematically similar.

a. 2 in. × ____________ b. 3 in. × ____________

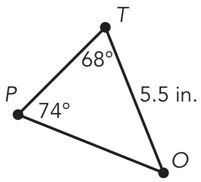
c. ____________ × 2 in. d. 4 in. × ____________

Given △*TOP* is similar to △*MAN* with *MA* = 11 inches, *MN* = 8.5 inches and *TO* = 5.5 inches.



3. What do you notice about the corresponding angle measures in the triangles?

When two figures are similar, the corresponding angles are congruent and the corresponding sides are proportional. You can use these properties to determine unknown measures in similar figures.



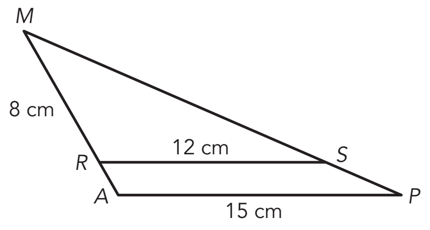
4. Use △*TOP* and △*MAN* to answer each question.

a. Identify the congruent corresponding angles and write ratios to identify the proportional sides of the triangle.

b. Do you have enough information to determine any side length(s)? Explain your reasoning. When you have enough information to determine a side length(s), write a proportion you can use to determine the unknown length(s).

c. Determine the lengths.

5. Given $\triangle MAP$ is similar to $\triangle MRS$ with $MR = 8$ centimeters, $RS = 12$ centimeters and $AP = 15$ centimeters.



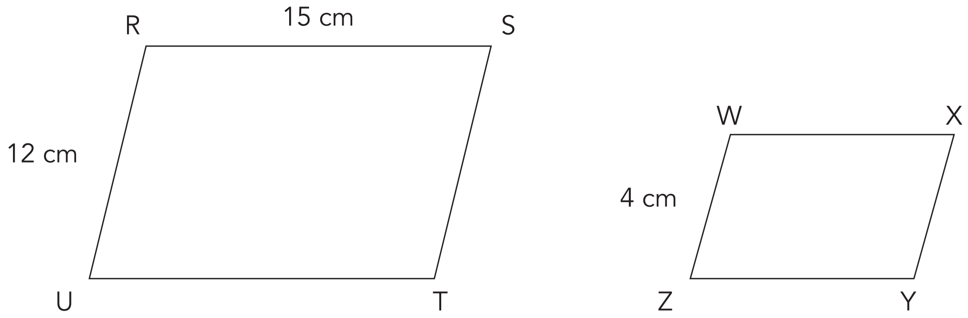
a. Identify the congruent corresponding angles and write ratios to identify the proportional sides of the triangles.

b. What other measurement(s) can you determine? Explain how you know.

c. Determine the measurement(s).

6. Mr. Jackson is creating a design for a website. The customer wants the design to include similar hexagons. How can the customer verify the hexagons in the design are similar?

7. Parallelogram *RSTU* is similar to Parallelogram *WXYZ*.



a. Write a proportion you can use to determine the length of $\overline{WX}$.

b. Determine the length of $\overline{WX}$.

ACTIVITY 5.3

Calculating with Map Scales

A map of the United States is shown.

In the scale drawing of a map of the United States 1 inch represents 1200 miles.

1. The distance on the map from San Francisco, California to Washington, D.C. is 2.25 inches. What is the approximate distance between the two cities in miles?

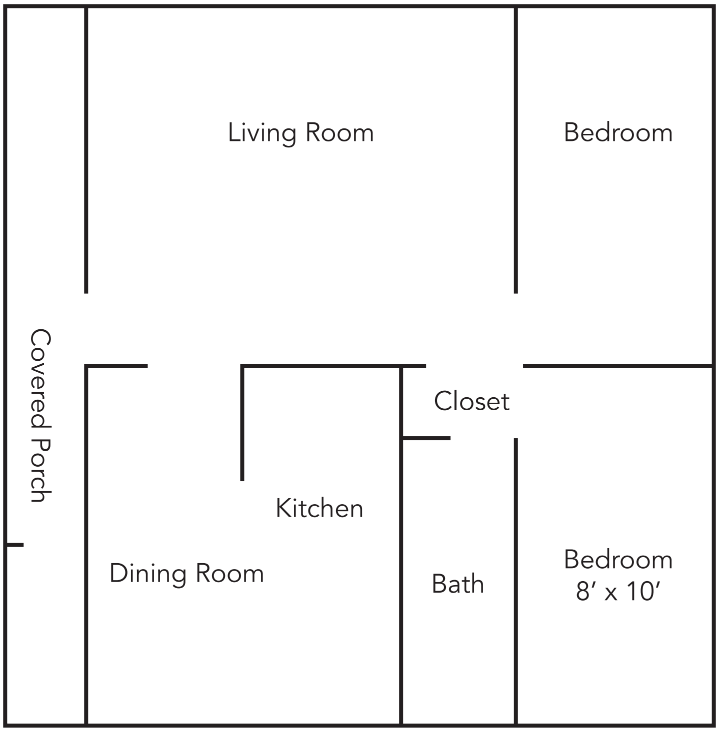
2. The distance on the map from Augusta, Maine, to Austin, Texas, is 1.75 inches. What is the approximate distance between the two cities in miles?

3. The distance from Chicago, Illinois, to Los Angeles, California, is approximately 2000 miles. What is the approximate distance between the two cities on the scale drawing of the map?

ACTIVITY 5.4

Calculating the Scale for a Blueprint

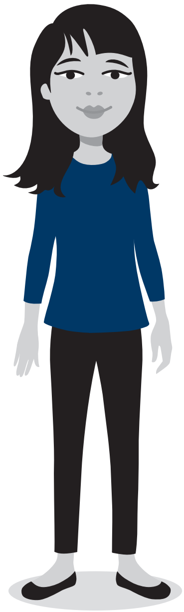
A blueprint is an example of a scale drawing that represents a larger structure. The blueprint shown represents a plan for a new house.



1. Use a centimeter ruler to determine the scale used to create the blueprint. Show your work.

2. Determine the approximate dimensions of the dining room, show your work.

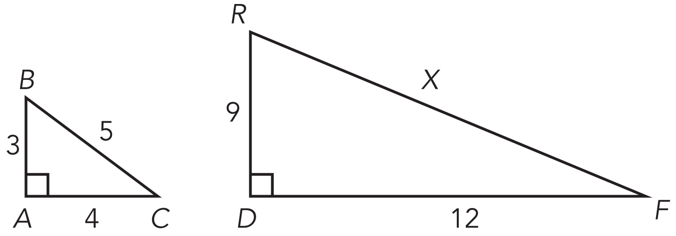
In blueprints, the ' symbol stands for feet. The " symbol is used for inches.



Talk the Talk

Interpreting Scales

1. $\triangle ABC \sim \triangle DRF$



a. Identify the corresponding congruent angles and write ratios that represent the proportional sides of the triangles.

b. Determine the unknown measure.

2. In a scale drawing, 2 inches represents 3 feet. The length of the bedroom is 14 feet. What is the length of the bedroom on the scale drawing in inches?

3. Which scale would produce the largest scale drawing of an object when compared to the actual object? Explain your reasoning.

1 in. : 25 in.
1 cm : 1 m
1 in. : 1 ft

Lesson 5 Assignment

Write

Write the term that best completes each sentence.

1. The purpose of a(n) ____________ is to create an accurate drawing of either a very large or very small object.
2. Two geometric figures that are proportional in size are ____________ figures.
3. The "bottom" sides of two similar triangles are ____________ sides.

Remember

A *scale* is a ratio that compares two measures.

When you multiply a measure by a scale to produce a reduced or enlarged measure, the scale is called a *scale factor.*

Practice

1. Madison created a scale drawing of the school cafeteria in his art class. In the scale drawing, the length of the cafeteria is 14 centimeters. The length of the actual cafeteria is 210 feet. What scale did Madison use to create the scale drawing of the school cafeteria? How many feet does 1 centimeter represent on the scale drawing?

2. Olivia is an architect and built a scale model of a new concert venue. In her model, 3 inches represents 50 feet. The height of the concert venue is 175 feet. What is the height of Olivia's scale model in inches?

3. The distance between two cities on a map is 5.5 centimeters. The map uses a scale in which 1 centimeter represents 30 miles. What is the actual distance between these two cities in miles?

Lesson 5 Assignment

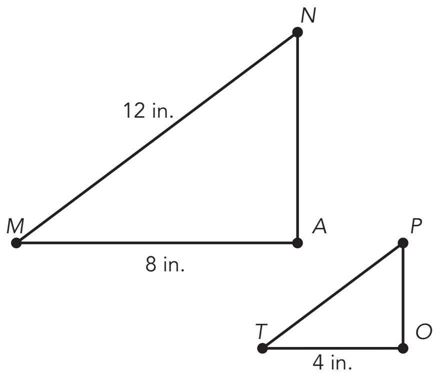
4. Javier drew a map of the Hawaiian Islands. In the scale Javier used for the map, 3 centimeters represents 18 kilometers. The actual distance between Oahu and Lanai is 108 kilometers. What is the distance in centimeters between Oahu and Lanai on Javier's map?

5. Fernando made a scale model of the U.S. Capitol building. The Capitol has an actual height of 288 feet. Fernando's model used a scale in which 1 inch represents 25 feet. What is the height of Fernando's model?

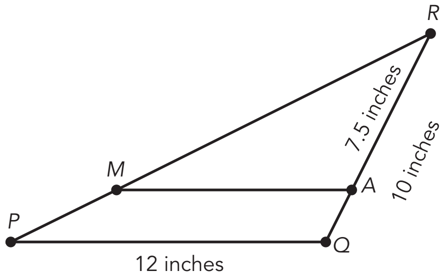
6. Daniela is using a map while on her family theme park vacation. The scale on the map uses $\frac{1}{2}$ cm to represent $\frac{1}{10}$ of a mile. On the map, the next attraction that Daniela wants to visit is $3\frac{1}{2}$ centimeters away. How far will Daniela have to walk to get to that attraction?

Lesson 5 Assignment

7. Given $\triangle TOP$ is similar to $\triangle MAN$ with $MA = 8$ inches, $MN = 12$ inches and $TO = 4$ inches, determine TP.



8. Given $\triangle PQR$ is similar to $\triangle MAR$ with $PQ = 12$ inches, $AR = 7.5$ inches and $QR = 10$ inches, determine MA.



Prepare

Evaluate each expression. Let $a = 3$ and $b = 5$.

1. a^2

2. $(2 + b)^2$

3. $2a + 2b$

TOPIC 1 SELF-REFLECTION

Name: ____________________

Proportional Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill.**

I can demonstrate an understanding of the standards in the *Proportional Relationships* topic by:

TOPIC 1: *Proportional Relationships*	Beginning of Topic	Middle of Topic	End of Topic
recognizing situations in which percentage proportional relationships apply.	○	○	○
using proportions to solve for unknowns in multi-step percent problems.	○	○	○
solving percent problems using the constant of proportionality.	○	○	○
using percent equations to solve for unknowns in percent problems.	○	○	○
using proportional reasoning to calculate sales tax and income tax amounts.	○	○	○
comparing monetary incentives such as sales, rebates, and coupons.	○	○	○
calculating simple interest.	○	○	○
using proportional reasoning to calculate a percent increase or decrease.	○	○	○
using proportional reasoning to solve problems involving area and volume of two- and three-dimensional objects.	○	○	○

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Proportional Relationships*	Beginning of Topic	Middle of Topic	End of Topic
identifying ratios within and between similar shapes.	○	○	○
describing the attributes of similar figures.	○	○	○
identifying the scale of a drawing as the ratio of drawing length to actual length.	○	○	○
determining actual lengths and areas given a scale and a scale drawing.	○	○	○
reproducing a scale drawing at a different scale.	○	○	○
using concepts of proportionality to determine the scale factor used to produce a scale drawing.	○	○	○
reproducing a scale drawing that is proportional to a given geometric figure using a different scale.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Proportional Relationships* topic.

   ____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   ____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   ____

TOPIC 1 SUMMARY

Proportional Relationships Summary

NEW KEY TERMS

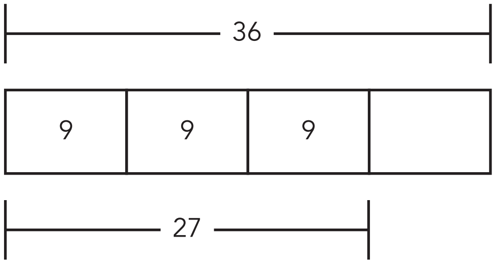
- markdown
- markup
- sale
- coupon [cupón]
- rebate
- percent equation [ecuación porcentual]
- simple interest [interés simple]
- commission [comisión]
- sales tax
- income tax
- percent increase
- percent decrease
- appreciation [apreciación]
- depreciation [depreciación]
- scale [escala]
- scale factor [factor de escala]
- corresponding [correspondiente]
- scale drawing
- similar figures [figuras semejantes/similares]

LESSON 1

Introducing Proportions to Solve Percent Problems

You can use a strip diagram to represent a percent scenario.

For example, suppose a dress originally costs \$36. Daniela pays \$27 for the dress during a sale. What percent does Daniela save with the sale?



The strip diagram represents the scenario. You can use the strip diagram to solve the problem. The amount Annika saves is one-fourth the total amount of the dress, so she saves 25%.

You can write a percent as a proportion.

For example, using the same scenario above, you can set up the proportion $\frac{\text{part}}{\text{whole}} = \frac{\text{percent number}}{100}$.

$$\frac{x}{100} = \frac{27}{36}$$

$$36x = (100)(27)$$

$$\frac{36x}{36} = \frac{2700}{36}$$

$$x = 75$$

Daniela paid 75% of the cost, so she saved 25%.

When businesses sell an item at a lower price than the original price, the reduction in price is called a **markdown**. To make money, businesses often buy products from a wholesaler or distributor for one amount and add to that amount to determine the price they use to sell the product to their customers. This increase in price is called a **markup**.

For example, a store marks up all of its prices by 25% to sell to its customers. If the store's cost for an item is \$16, what is the customer's cost?

$$\frac{25}{100} = \frac{x}{16}$$

$$x = 4$$

The customer's cost is \$16 + \$4 = \$20.

The answer to the percent problem may or may not be the value of the unknown in your proportion. Reread the problem and ensure that you answer the question being asked.

Three common types of discounts that customers can benefit from are sales, coupons, and rebates.

A **sale** is an event at which products are sold at reduced prices. Sales are typically held to clear outdated inventory.

A **coupon** is a detachable part of a ticket or advertisement that entitles the holder to a discount. Coupons are commonly used to entice people to shop at certain stores.

A **rebate** is a refund of part of the amount paid for an item. Generally, a customer completes and mails a form to a company after a purchase and a rebate check is mailed to the customer.

Let's consider the following scenario to determine the price of an item after applying discounts from a sale, a rebate, or a coupon.

Noah is planning to purchase a washing machine. Four local appliance stores in the area offer the following discounts for their customers.

- Washer/dryer sale in which all washers and dryers are 20% off
- \$150 rebate for the purchase of any washer
- Coupon for 10% off a washer
- Coupon for $\frac{1}{2}$ off the price of a washer

Sale	Rebate	Coupon	
Store 1 sells a washer that originally costs $599.	Store 2 sells a washer for $675.	Store 3 sells a washer that originally costs $505.	Store 4 sells a washer for $975.
Sale price $= 599 - 599(0.20)$ $= 479.2$	Price after rebate $= 675 - 150$ $= 525$	Price with coupon $= 505 - 505(0.10)$ $= 454.50$	Price with coupon $= \frac{1}{2} \cdot 975.00$ $= 487.50$
The sale price of the washer is $479.20.	The price of the washer after the rebate is $525.	The price of the vacuum with the coupon is $454.50.	The price of the washer with the coupon is $487.50

LESSON 2

Calculating Tips, Commissions, and Simple Interest

You can use a percent equation to determine the percent of a whole. A **percent equation** can be written in the form *percent* · *whole* = *part*, where the percent is often written as a decimal.

For example, suppose you want to leave a 15% tip on a restaurant bill of $45.

$$\begin{array}{ccccc} & & \text{percent} & = & \frac{\text{part}}{\text{whole}} \\ \text{percent} & \cdot & \text{whole} & = & \text{part} \\ \text{(tip as a percent)} & \text{of} & \text{(total bill)} & = & \text{amount of tip} \\ \uparrow & & \uparrow & & \uparrow \\ \frac{15}{100} \text{ or } 0.15 & \cdot & 45 & = & t \\ & & 6.75 & = & t \end{array}$$

The amount of tip you should leave is $6.75.

Simple interest is a fixed percent of an original amount of money called the *principal*. The formula for simple interest is:

$$\begin{array}{ccccccc} & & & & \text{Interest rate (\%)} & & \\ & & & & \swarrow & & \\ I & = & P & \cdot & r & \cdot & t \\ \nearrow & & \nearrow & & & & \nwarrow \\ \text{Interest earned} & & \text{Principal} & & \text{Time that the money earns interest} & & \\ \text{(dollars)} & & \text{(dollars)} & & \text{(years)} & & \end{array}$$

A **commission** is an amount of money a salesperson earns after selling a product. Many times, the commission is a certain percent of the product.

Price (dollars)	Commission (dollars)
0	0
7000	700
12,000	1200
23,000	2300

For example, a car salesperson's commission is directly proportional with the cost of the products sold. Determine the percent commission the salesperson makes.

In this scenario, the constant of proportionality is equal to the percent commission and $k = \frac{700}{7000} = 0.1$. The salesperson makes 10% commission.

When a tip, gratuity, or commission is directly proportional to the cost of the product, you can represent the situation using the equation $t = kb$, where t represents the amount of tip, gratuity, or commission, k represents the percent, and b represents the total cost of the product.

LESSON 3 Sales Tax, Income Tax, and Fees

A percent relationship can represent an increase or decrease from an original amount.

Sales tax is a percent of the selling price of a good or service that is added to the price. When calculating the total cost of a good or service that includes sales tax (x%), multiplying the list price by ($1 + x$%) is the same as adding the sales tax (x%) to the list price.

Income tax is a percent of a person's or company's earnings that is collected by the government. All U.S. citizens pay a federal income tax, but only 43 states also collect a state income tax. Currently, Texas does not collect a state income tax.

State income tax is defined and applied differently according to the state in which you live. For example, residents of Michigan pay 4.25% state income tax while residents of Texas do not pay state income tax.

When calculating the total income after state tax (x%) is taken out, multiplying the income by ($1 - x$%) is the same as subtracting the income tax (x%) from the income.

Sales Tax	State Income Tax
Amir wants to buy a pair of shoes for \$60. The sales tax in his state is 6%. What is the total price Rick will pay for the shoes, including sales tax?	Hannah earned an income of \$40,000 this year. The income tax rate in her state is 5%. How much will Gia keep after income tax is taken out?
Multiply the list price by $(1 + 0.06)$. $\$60 \cdot 1.06 = \$63.60$	Multiply the income by $(1 - 0.05\%)$. $\$40{,}000 \cdot 0.95 = \$38{,}000$
Amir will pay \$63.60 in total.	Hannah will keep \$38,000.

Federal income tax is not a flat rate. The 2023 tax brackets and rates are listed in the table.

Tax Rate	For Single Filers	For Married Individuals Filing Joint Returns	For Heads of Households
10%	$0 to $11,000	$0 to $22,000	$0 to $15,700
12%	$11,000 to $44, 725	$22,000 to $89,450	$15,700 to $59,850
22%	$44,725 to $95,375	$89,450 to $190,750	$59,850 to $95, 350
24%	$95,375 to $182,100	$190,750 to $364,200	$95,350 to $182, 100
32%	$182,100 to $231,250	$364,200 to $462,500	$182,100 to $231,250
35%	$231,250 to $578,125	$462, 500 to $693, 750	$231,250 to $578,100
37%	$578,125 or more	$693,750 or more	$578,100 or more

For federal income tax, you pay one rate on part of your income and a higher rate on income above a set amount based on how you file your taxes.

For example, Olivia is filing single. Her total taxable income for 2023 is $50,000. How much does Olivia pay in federal income tax?

Olivia's taxable income is in the 22% tax bracket.

She pays 10% income tax on the first $11,000 of her income.	$11,000 · 0.10 = $1,100
The next part of Olivia's income is taxed at 12%.	$44,725 – $11,000 = $33,725
	$33,725 · 0.12 = $4,047
The last part of her income is taxed at 22%.	$50,000 – $44,725 = $5,275
	$5,275 · 0.22 = $1,160.50

LESSON 4

Percent Increase and Percent Decrease

A **percent increase** occurs when the new amount is greater than the original amount, such as when stores mark up the price they pay for an item to make a greater profit. A percent increase is calculated as a ratio of the amount of increase to the original amount.

For example, when you multiply a whole amount n by 1.2, that is a 20% increase.

$n \cdot 1.2 = n(1 - 0.2) = n + 0.2n$

The expression $n + 0.2n$ means a 20% increase from n.

A **percent decrease** occurs when the new amount is less than the original amount. A percent decrease is calculated as a ratio of the amount of decrease to the original amount.

For example, when you multiply a whole amount n by 0.9, that is a 10% decrease.

$n \cdot 0.9 = n(1 - 0.1) = n - 0.1n$

The expression $n - 0.1n$ means a 10% decrease from n.

Generally, things such as homes and savings accounts gain value, or appreciate, over time. **Appreciation** is an increase in price or value. Other things, such as cars, depreciate every year. **Depreciation** is a decrease in price or value.

You can use percent increases and decreases when thinking geometrically.

For example, what is the percent decrease in the area from Circle A to Circle B?

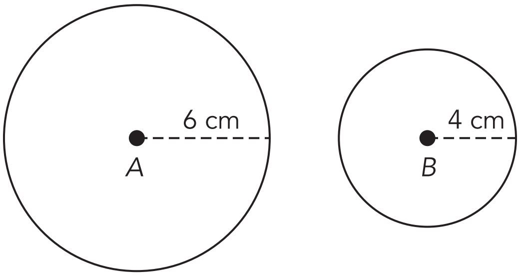
The area of Circle $A = 36\pi$ square units.

The area of Circle $B = 16\pi$ square units.

$36\pi - 16\pi = 20\pi$

$\frac{20\pi}{36\pi} \approx 0.56$

The percent decrease in the area is about 56%.



LESSON 5

Scale and Scale Drawings

A **scale** is a ratio that compares two measures. When you multiply a measure by a scale to produce a reduced or enlarged measure, the scale is called a **scale factor**.

For example, Triangle *A* is an equilateral triangle with side lengths of 4 units. If Triangle *A* is reduced by 50% to create Triangle *B*, what are the side lengths of Triangle *B*? You can multiply each of Triangle *A*'s lengths by the scale factor 1 : 2, or 50%, or $\frac{1}{2}$, to produce the side lengths for Triangle *B*.

Triangle *A*

Triangle *B*

$$\text{side length} \longrightarrow 4 \text{ units} \cdot \underset{\substack{\uparrow \\ \text{scale,} \\ \text{scale factor}}}{1 : 2 \text{ or } \frac{1}{2} \text{ or } 50\%} = 2 \text{ units} \longleftarrow \text{side length}$$

When you draw Triangle *A* and Triangle *B*, the bottom sides of the triangles are corresponding sides.

A scale drawing is a representation of a real object or place that is in proportion to the real object or place it represents. The scales on scale drawings often use different units of measure.

For example, you might write the scale of a drawing as: 1 cm : 4 ft. This scale means that every 1 centimeter of length in the drawing represents 4 feet of length of the actual object or place. The scale of a map is: 1 in. : 1200 mi. This scale means that every 1 inch of distance on the map represents 1200 miles of actual distance.

Figures that are proportional in size or have proportional dimensions, are called **similar figures**.

For example, a rectangle has a height of 20 inches and a width of 15 inches. Determine the width of a mathematically similar rectangle that has a height of 16 inches.

$$\frac{\text{height of larger rectangle}}{\text{height of smaller rectangle}} = \frac{\text{width of larger rectangle}}{\text{width of smaller rectangle}}$$

$$\begin{aligned} \frac{20 \text{ inches}}{16 \text{ inches}} &= \frac{15 \text{ inches}}{x \text{ inches}} \\ (20)(x) &= (15)(16) \\ 20x &= 240 \\ x &= 12 \end{aligned}$$

The width of the similar rectangle is 12 inches.

GRADE 7

TOPIC 2

Small amounts of money invested regularly, grow over time.

Financial Literacy: Interest and Budgets

LESSON 1 Simple and Compound Interest 275

LESSON 2 Net Worth Statements 289

LESSON 3 Personal Budgets 303

1

Simple and Compound Interest

OBJECTIVES

- Determine the difference between simple interest and compound interest.
- Calculate simple interest earned for a given balance and interest rate.
- Calculate the compound interest earned for a given balance and interest rate.
- Compare money earned from simple and compound interest.

NEW KEY TERMS

- principal
- simple interest
- compound interest

You have learned how to calculate simple interest.

What other types of interest are there, and how do they compare?

Getting Started

A Simple Gift

Recall that a *principal* is the term for an original amount of money on which interest is calculated. When you invest money, your principal can grow by earning interest. When you borrow money, interest is charged on the principal, which is the original amount you borrow. Simple interest is a percent of the principal that is added to the principal over time.

Your aunt gave you 5% of the money in her bank account as a gift when you started Kindergarten to assist you with your college savings. Your guardians put that money into a bank account for you so it's available when you graduate from high school.

1. When your aunt had a total of $60,000, how much money did she give you?

2. Use the formula for simple interest to calculate the total amount of money in your college savings account when you graduate (12 years later) when the bank offers 3% interest.

ACTIVITY
1.1

Comparing Simple and Compound Interest

Another common form of interest that lenders use is *compound interest*. **Compound interest** is a percentage of the principal *and* the interest that is added to the principal over time.

The definitions of *simple interest* and *compound interest* are very similar, but the small difference in the definitions can lead to a big difference in money.

Ask Yourself . . .
How does representing mathematics in multiple ways help to communicate reasoning?

1. Analyze the table that shows the total value over time of $1000 invested in accounts earning 4% interest.

Number of Years Invested	Total Value in a Simple Interest Account ($)	Total Value in a Compound Interest Account ($)
1	1040	1040.00
2	1080	1081.60
3	1120	1124.86
4	1160	1169.85
5	1200	1216.64
10	1400	1480.23
20	1800	2191.08
30	2200	3243.30

a. Describe the similarities and differences in the accounts.

b. Predict the amount of money in each account after 40 years. Explain your reasoning.

c. When graphing the total value of each account versus time, describe the shape of each graph. Explain your reasoning.

2. Determine whether you would prefer the principal of each account to earn simple interest (S) or compound interest (C).

______ **College Savings**	______ **Car Loan**	______ **Retirement Savings**
______ **Savings Account**	______ **Student Loan**	______ **Mortgage**

Explain your reasoning.

ACTIVITY

1.2 Simple Interest

The formula to determine simple interest is $I = Prt$, where I is the interest, P is the principal, r is the interest rate, and t is time.

Suppose you invest $1000 in an account that earns 4.00% simple interest. How much interest is earned after 15 years? What is the total value of the investment after 15 years?

WORKED EXAMPLE

Principal = $1000
Rate = 4.00%
Time = 15 years

$I = Prt$
$I = (1000)(0.04)(15)$
$I = 600$

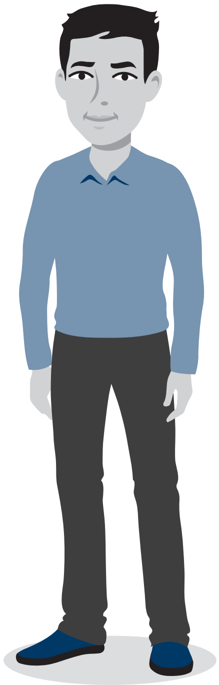
The amount of interest earned is $600.00.
The total value of the investment after 15 years is $1000.00 + $600.00 = $1600.00.

Remember to convert the interest rate to a decimal.

1. Analyze the Worked Example.

   a. Determine the total amount of the investment after 25 years.

   b. Determine the total value of the investment after 50 years.



2. Determine the total value of a $500 investment earning 3% simple interest over 7 years. Show all of your work and explain your reasoning.

3. Determine the total value of a $50,000 investment earning 3% simple interest over 15 years. Show all of your work and explain your reasoning.

4. When a $800 investment earns $32 in interest over 2 years, what is the simple interest rate on the account? Show all of your work and explain your reasoning.

ACTIVITY 1.3

Compound Interest

Suppose you invest $1000 in an account that earns 4% compound interest.

WORKED EXAMPLE

Principal at the Beginning of the Year	Interest ($I = Prt$)	Balance at the End of the Year
Year 1: $1000	$I = (\$1000)(0.04)(1)$ $= \$40.00$	$\$1000.00 + \$40.00$ $= \$1040.00$
Year 2: $1040	$I = (\$1040)(0.04)(1)$ $= \$41.60$	$\$1040.00 + \$41.60$ $= \$1081.60$
Year 3: $1081.60	$I = (\$1081.60)(0.04)(1)$ $= \$43.26$	$\$1081.60 + \$43.26$ $= \$1124.86$
Year 4: $1124.86		
Year 5:		

Ask Yourself . . .
What patterns do you notice?

1. Analyze the Worked Example.

   a. Determine the interest and balance for Years 4 and 5.

   b. Compare the interest earned each year for compound interest to the interest earned for simple interest.

   c. Explain how you think the differences in the way simple and compound interest are calculated.

To complete a table like the one in the Worked Example can be time-consuming and inefficient. Therefore, a formula is generally used to determine the final balance of an investment that earns compound interest. The compound interest formula is:

$$A = P(1 + r)^t$$

where *A* represents the final balance, *P* represents the original principal, *r* represents the annual rate, and *t* represents the time in years.

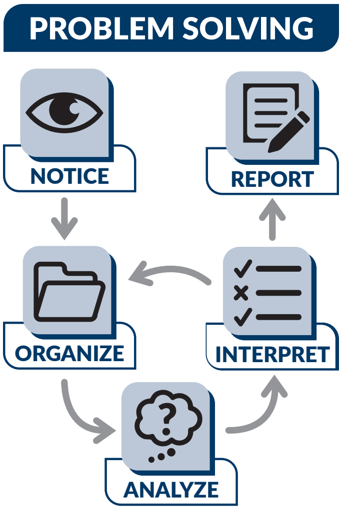
The United States has a free market economy based on supply and demand. As a result, interest rates change daily and vary at different institutions.

2. Determine the total value of a $500 investment earning 3% compound interest over 7 years.

3. Determine the total value of a $50,000 investment earning 3% compound interest over 15 years.

4. Compare your investments in Questions 2 and 3 to the same investments made in the previous activity. How much more money is earned through compound interest compared to simple interest for each investment?

5. Consider a $100.00 investment at a rate of 4%.



a. How long will it take the investment to double when the account earns simple interest?

b. How long will it take the investment to double when the account earns compound interest?

Be prepared to share your solutions and methods.

Talk the Talk

Not So Simple After All

Recall the scenario from the Getting Started. Suppose that instead of an account that earned simple interest, your guardians put the money from your aunt in an account that earned 3% compound interest for those 12 years.

a. Use the formula for compound interest to calculate the total value of the \$5000 investment at 3% interest for 12 years.

b. How much additional money is earned in interest by using the compound interest formula rather than the simple interest formula?

c. Why does an investment with compound interest increase so much more quickly than the same investment with simple interest?

Lesson 1 Assignment

Write

Suppose you deposit money into an account that earns interest and you also take out a loan that collects interest. In each instance, would you hope that simple or compound interest is being applied? Explain your reasoning.

Remember

Simple interest is a percent of the principal that is added to the investment over time. *Compound interest* is a percent of the principal and the interest that is already added to the investment over time.

Practice

Determine the simple interest for each principal over the given amount of time.

1. Principal: $2000
   Simple Interest Rate: 3%
   Time: 10 years

2. Principal: $4000
   Simple Interest Rate: 4.5%
   Time: 5 years

Determine the total value of each investment given the principal, simple interest rate, and time.

3. Principal: $200
   Simple Interest Rate: 3%
   Time: 5 years

4. Principal: $1800
   Simple Interest Rate: 2.5%
   Time: 20 years

Lesson 1 Assignment

Determine the simple interest rate for each given principal, time, and simple interest earned.

5. Principal: \$500
   Time: 20 years
   Simple Interest: \$250

6. Principal: \$850
   Time: 10 years
   Simple Interest: \$255

Determine the total value of each investment given the principal, the compound interest rate, and the time.

7. Principal: \$5000
   Compound Interest Rate: 4%
   Time: 10 years

8. Principal: \$850
   Compound Interest Rate: 5%
   Time: 8 years

Lesson 1 Assignment

9. The table shown represents the total value over time of a $2000 investment earning money in accounts with a 3% interest rate. Complete the last two rows of the table. Show your work.

Number of Years Invested	Total Value in a Simple Interest Account ($)	Total Value in a Compound Interest Account ($)
1	2060	2060.00
2	2120	2121.80
3	2180	2185.45
4	2240	2251.02
5	2300	2318.55
10	2600	2687.83
20		
30		

10. Alexander's grandparents are investing in his future. They invest $5000 into an account that earns 3% compound interest.

   a. Determine the total value of the account by the time Alexander graduates high school in 18 years.

Lesson 1 Assignment

b. Determine the amount of interest earned by the time Alexander graduates high school in 18 years.

Prepare

1. 24.7 + 8.3

2. 376.23 – 189.04

3. 74.3 + 12.2 – 18.753

4. 82.3 – 6.951 + 17.01

2

Net Worth Statements

OBJECTIVES

- Create and organize a financial assets and liabilities record.
- Construct a net worth statement.

NEW KEY TERMS

- net worth
- asset
- liability
- 401(k) plan
- 403(b) plan

As you grow older and purchase more expensive items, you may need to borrow money from a lender.

How will you compare what you own and what you owe in loans?

Getting Started

Pros and Cons

Sometimes when a person has to make an important decision, they will weigh the decision by listing the pros and cons, or advantages and disadvantages, for proceeding with the decision.

Daniela, a 7th-grader, is trying to decide whether to take a babysitting job. She would be watching her next door neighbor's 2 kids, ages 4 and 6. She would be babysitting every Wednesday evening and Saturday evening for 4 hours, from 6 PM to 10 PM, while her neighbor is at work. Her neighbor would pay her $6 per hour.

1. List at least two pros and two cons for Daniela accepting the babysitting job.

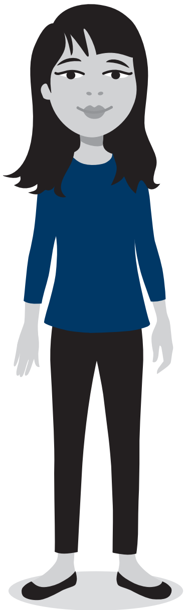
Pros	Cons

2. Based on the pros and cons you listed, do you think Daniela should take the job? Explain.

ACTIVITY
2.1

Assets and Liabilities

A mortgage is a loan obtained from a financial institution, like a bank, to purchase a property or house. The borrower then makes monthly payments back to the bank over a set period of time, typically including both principal and interest, until the loan is fully repaid.



Finances, even at the personal level, can often be complex. People sometimes have various investments, loans, bank accounts, mortgages, and so on, all at once. This complexity can make it difficult to determine how well a person is doing financially. Paying bills on time leads to a good credit score, but this score doesn't provide a full picture of financial health. A *net worth statement* is a useful tool to measure your financial health from year to year.

Your **net worth** is a calculation of the value of everything that you own minus the amount of money that you owe. A net worth statement includes this calculation as well as a detailed list of everything used to determine net worth. **Assets** include the value of all accounts, investments, and things that you own. They are positive and add to your net worth. **Liabilities** are financial obligations, or debts, that you must repay. They are negative and take away from your net worth.

1. Determine whether each item is an asset or a liability.

Money in checking	Credit cards	Savings
Student loans	Investments	Retirement money
Car loans	Mortgage	Home equity loan
Vehicles	Your home	Personal loans

Assets	Liabilities

Explain your reasoning.

2. Koda, Amir, and Destiny disagree about whether vehicles should be considered an asset or a liability.

Koda

A car is an asset. Even when you owe money on a car, it always has value. Therefore, it is an asset.

Amir

A car is a liability. When you are making payments on a car, it is often worth less than you owe.

Destiny

A car can be an asset or a liability. It all depends on what the car is worth and how much you owe on it.

Who is correct? Explain your reasoning.

3. Describe a situation where a home is an asset.

4. Describe a situation where a home is a liability.

ACTIVITY 2.2

Understanding Retirement Accounts

The complexity of personal finances is often seen in the number of loans and investments a single person or family may have. Retirement accounts are important assets to consider when determining a person's financial health. Many people have retirement plans, such as a *401(k)* or *403(b)*.

A **401(k) plan** is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account, with the employer often matching a certain amount of it. A **403(b) plan** is a retirement plan generally for public school employees or other tax-exempt groups.

1. Gabriela is curious about her financial health, so she compiles a list of all her accounts.

Checking account: $4210	Student loan: $34,980	Certificate of deposit: $10,000
401(k) account at work: $60,432	Car loan: $17,680	Savings account: $10,500
Mortgage: $140,000	403(b) account from previous job: $14,953	Home equity loan: $9750
Home value: $175,000	Credit card: $2350	Other investments: $2275
Truck loan: $29,750	Stamp collection: $1230	Medical bills: $1250

   a. Analyze each of Gabriela's accounts. Create a net worth statement to organize her financial information into assets and liabilities.

Ask Yourself . . .
How can you organize and record your mathematical ideas?

Assets		Liabilities	
Type	Amount	Type	Amount
Total:		Total:	

b. Determine Gabriela's net worth. Show all your work and explain your reasoning.

2. When a person's asset-to-liability ratio is 1 : 1, what is their net worth? Explain your reasoning.

Ask Yourself . . .
Did you justify your mathematical reasoning?

3. A family's gross income is $65,000 for a given year, with 20% taken out in taxes. They acquire no other assets for the year. Is their net worth positive or negative when the sum of their liabilities for that year is $48,000? Show all your work and explain your reasoning.

Talk the Talk

Show Me the Money!

Complete all entries in the net worth statement so that the person has a net worth of −$10,000.

Assets		Liabilities	
Type	Amount	Type	Amount
Total:		Total:	

Net Worth Calculation:

Lesson 2 Assignment

Write

Explain the difference between a 401(k) plan and a 403(b) plan in your own words.

Remember

Assets include the value of all accounts, investments, and things that a person owns. Assets are positive and add to a person's worth. *Liabilities* are financial obligations, or debts, that a person must repay. Liabilities are negative and take away from a person's worth. *Net worth* is a calculation of the value of everything that a person owns minus the amount of money a person owes. So, to determine a person's net worth, you can subtract the total of their liabilities from the total of their assets.

Practice

Identify each item as a liability or asset. Explain your reasoning.

1. The mortgage on your house
2. Money in your savings account
3. A car that is paid off
4. A loan used to pay for college

Lesson 2 Assignment

Analyze each set of accounts. Create a net worth statement, then determine each person's net worth.

5. Hannah's accounts:
   - School loans: $15,755
   - Savings account: $3986.75
   - Car: $17,600
   - 401(k) account: $45,390
   - Credit card: $4324.98

Assets		Liabilities	
Type	Amount	Type	Amount
Total:		Total:	

Lesson 2 Assignment

6. Madison's accounts:
   - Checking account: $2275.16
   - Medical bills: $567.50
   - Home: $180,000
   - Mortgage: $135,000
   - Coin collection: $1400
   - Certificate of deposit: $10,000
   - Car loan: $26,500

Assets		Liabilities	
Type	Amount	Type	Amount
Total:		Total:	

Lesson 2 Assignment

7. William's accounts:

- Video game collection: $800
- School loans: $8130
- Car: $10,540
- Medical bills: $1675
- Credit cards: $1535

Assets		Liabilities	
Type	Amount	Type	Amount
Total:		Total:	

Lesson 2 Assignment

Ana is assessing his financial health. Her accounts are shown.

• Medical bills: \$4520	• Bank accounts: \$16,850	• Car: \$19,750
• 401(k) account: \$52,198	• Home value: \$230,000	• Home equity loan: \$12,150
• IRA (retirement fund): \$154,320	• Student loan: \$34,980	• Certificate of deposit: \$12,000
• Car loan: \$7680	• Mortgage: \$195,000	• Credit card: \$3158

8. Create a net worth statement of Ana's accounts.

Assets		Liabilities	
Type	Amount	Type	Amount
Total:		Total:	

9. Determine Ana's net worth.

Lesson 2 Assignment

Prepare

Maria estimated the number of hours she spends on several activities on a typical weekday.

1. Calculate the percent of the 24 hours per day Maria spends on each activity. Round to the nearest whole number.

   Your percents may not add up to 100% exactly due to rounding.

   8 hours: ________________% sleep

   8 hours: ________________% school

   4 hours: ________________% spending time with family

   2 hours: ________________% spending time with friends or on the phone

   2 hours: ________________% other

3

Personal Budgets

OBJECTIVES

- Identify the components of a personal budget.
- Calculate the percent of each category within a total budget.
- Estimate the minimum household budget and average hourly wage needed for a typical family to meet its basic needs.

NEW KEY TERMS

- personal budget
- fixed expenses
- variable expenses
- family budget estimator

You know about earning an income and have learned about assets and liabilities.

How can you estimate the amount of money needed to pay for your liabilities and other expenses?

Getting Started

Stick to the Budget

A **personal budget** is an estimate of the amount of money that a person or family will need for specific financial items. It generally includes current expenses as well as savings for anticipated future expenses.

A family of four wants to create a personal budget.

1. Create a list of the current expenses that a family should include in their budget. Explain your reasoning.

ACTIVITY
3.1

Fixed and Variable Expenses

Two types of expenses are *fixed expenses* and *variable expenses*. **Fixed expenses** are expenses that don't change from month to month. **Variable expenses** are expenses that can be different from month to month.

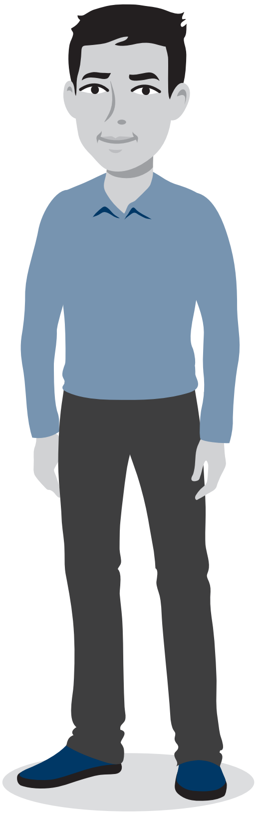
1. In the Getting Started, you created a list of expenses for a family of four to include in their budget. Consider the family's expenses.

   a. Sort the expenses you listed into the categories of fixed expenses and variable expenses. Explain your reasoning.

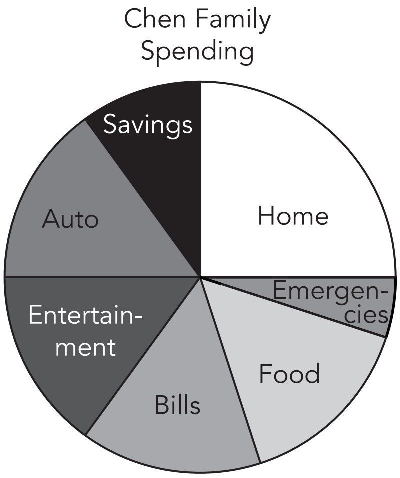
   b. Which expenses do you believe will make up the largest part of the budget? Explain your reasoning.

2. Create a list of future expenses that a family should save for. Rank them in order of importance. Explain your reasoning.

You may be wondering about other expenses, such as taxes and costs due to emergencies. Taxes would fit into the "Bills" category, and emergencies would fit into the "Savings" category.



3. The Chen family received their monthly statement from the bank that showed how they spent and saved their $3500 in earnings during the month. Analyze their expenses represented in the circle graph.



a. Identify the fixed and variable expenses in the Chen family's budget.

b. What conclusions can you make about the family's expenses for the month?

Review the definition of **estimate** in the Academic Glossary.

c. Estimate the amount of money in each category. Show all your work and explain your reasoning.

d. The Chen family's financial planner recommends saving approximately 20% of their earnings for college and retirement. Describe a specific plan for cutting expenses from one category and moving the money to savings. Include estimated dollar amounts as well as revised percentages in your plan.

e. The Chen family's gross income before taxes is $4269.00 per month. Estimate the percent of their gross income that is taken out in taxes. Show all your work and explain your reasoning.

ACTIVITY

3.2 Creating Circle Graphs to Display Budgets

WORKED EXAMPLE

You can create a circle graph to represent a family's anticipated expenses for the month.

- Household Expenses: $750.00
- Living Expenses: $1375.00
- Savings: $250.00
- Emergencies: $125.00

Determine the sum of the total expenses.

Total Expenses = 750 + 1375 + 250 + 125 = 2500.00

Calculate the percentage of the total monthly budget represented by each expense.

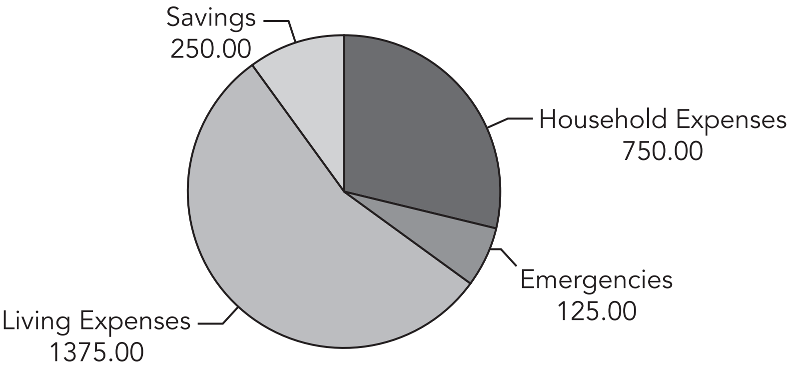
Household Expenses: $\frac{750.00}{2500.00} = 0.30 = 30\%$

Living Expenses: $\frac{1375.00}{2500.00} = 0.55 = 55\%$

Savings: $\frac{250.00}{2500.00} = 0.10 = 10\%$

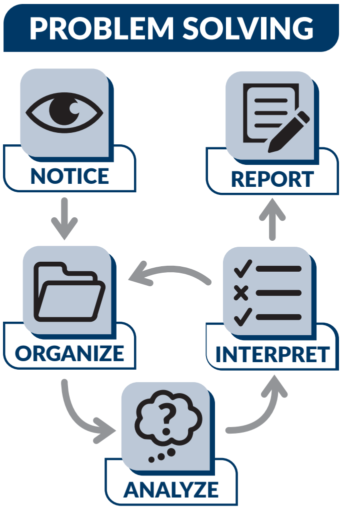
Emergencies: $\frac{125.00}{2500.00} = 0.05 = 5\%$

Living expenses make up slightly more than half of the circle, while household expenses make up about a third of the circle. The remaining portion of the circle represents savings.



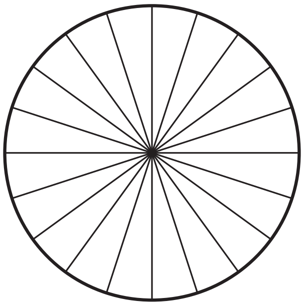
1. The Jackson family creates a list of anticipated expenses for the month.
   - Mortgage: $1000.00
   - Food: $800.00
   - Utility bills: $400.00
   - Entertainment: $400.00
   - College savings: $400.00
   - Retirement savings: $400.00
   - Emergencies: $200.00
   - Miscellaneous (shoes, clothing, supplies, etc): $400.00

   a. Determine the percentage of the total expenses represented by each category in the Jackson family's list. Show all your work and explain your reasoning.



Ask Yourself . . .
Did you complete all the steps in the problem-solving model?

b. Create a circle graph to represent each category of the Jackson family's expenses.



Each division = 5% of circle

c. Mr. Jackson is interested in switching careers. Determine his minimum monthly salary in order to maintain the family's current budget.

d. Assume that 20% of his gross income is taken out in federal income tax. How much money must he make before taxes in order to maintain the family's current budget?

ACTIVITY

3.3 Family Budget Estimator

When researching careers, it is important to have an understanding of how much money you will need to provide for you and your family. Although you should consider fixed and variable expenses, even your fixed expenses may vary depending on factors such as where you live and the size of your family. A **family budget estimator** is a tool that people can use to determine the estimated cost of raising a family in a particular city.

1. Use the family budget estimator to determine the minimum household budget needed for a family of four to meet its basic needs in your region of the state.

2. Determine the average hourly wage necessary to earn enough money to support a family of four in your region of the state.

3. Use the family budget estimator to determine the minimum household budget needed for a family of six to meet its basic needs in your region of the state.

4. Determine the average hourly wage necessary to earn enough money to support a family of six in your region of the state.

Talk the Talk

Bills, Bills, Bills

Review the major categories and percents from the Jackson family budget in Activity 3.2, Question 1.

Mortgage	25%
Food	20%
Utility bills	10%
Entertainment	10%
College savings	10%
Retirement savings	10%
Emergencies	5%
Miscellaneous	10%

- Consider the minimum household budget needed for a family of four to meet its basic needs in your region of the state.
- Use that budget amount to calculate the amount of money that would be spent in each of the major categories for one month.

Mortgage $________________

Food $________________

Utility bills $________________

Entertainment $________________

College savings $________________

Retirement savings $________________

Emergencies $________________

Miscellaneous $________________

Lesson 3 Assignment

Write

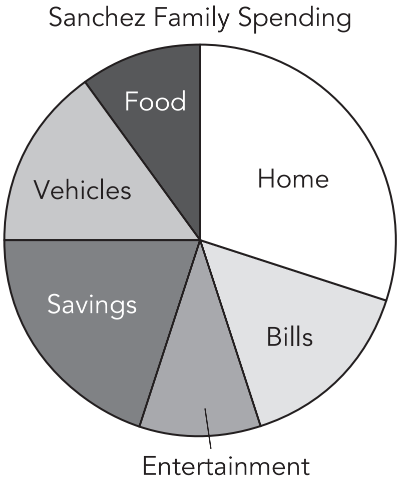
Describe the information a *family budget estimator* provides and the benefit of its use.

Remember

A *personal budget* is an estimate of the costs that a person or family will need for specific financial items. It generally includes current expenses as well as savings for anticipated future expenses. When budgeting for expenses, taxes must be considered so that a gross income that can support a person's or family's budget can be determined.

Practice

The circle graph represents how the Sanchez family spent and saved its $98,000 in earnings this year. Use the graph to answer each question. Explain your reasoning.



1. Which category did the Sanchez family spend the most on?

2. Which categories did the Sanchez family spend the least on?

3. Estimate the amount of money the Sanchez family spent on their vehicles this past year.

Lesson 3 Assignment

4. Estimate the amount of money the Sanchez family put into their savings this past year.

5. How much more did the Sanchez family spend on their home than on entertainment this year?

6. The Sanchez family's gross income for the year is $108,600 before taxes. Estimate the percent of the family's gross income that is taken out in taxes.

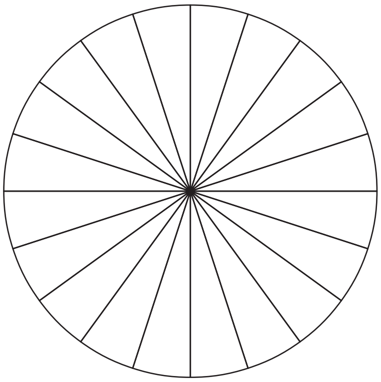
The Rodriguez family is concerned about their personal monthly budget. They make a list of their estimated monthly expenses.

- Mortgage: $2000
- Utilities: $500
- Vacation fund: $500
- Credit cards: $1000
- College savings: $1000
- Food: $500
- Entertainment: $500
- Miscellaneous: $500
- Vehicle loans: $500
- Monthly medical bills: $500
- Retirement savings: $2000
- Emergency expenditures: $500

Lesson 3 Assignment

7. Identify each expense as a current expense or future expense. Explain your reasoning.

8. Determine the percent of each expense compared to the total expenses. Then, use these percentages to make a circle graph of the expenses.



Lesson 3 Assignment

9. The Rodriguez family pays 22% of income in taxes. What must the family's minimum monthly income be in order to maintain their monthly expenses?

Prepare

Determine each sum.

1. $\frac{1}{6} + \frac{1}{3}$

2. $\frac{2}{7} + \frac{2}{5}$

3. $\frac{1}{2} + \frac{3}{5}$

4. $\frac{1}{3} + \frac{4}{5}$

TOPIC 2 SELF-REFLECTION

Name: ____________________

Financial Literacy: Interest and Budgets

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Financial Literacy: Interest and Budgets* topic by:

TOPIC 2: *Financial Literacy: Interest and Budgets*	Beginning of Topic	Middle of Topic	End of Topic
determining the difference between simple interest and compound interest.	○	○	○
calculating simple interest earned for a given balance and interest rate.	○	○	○
calculating the compound interest earned for a given balance and interest rate.	○	○	○
comparing money earned from simple and compound interest.	○	○	○
creating and organizing a financial assets and liabilities record.	○	○	○
constructing a net worth statement.	○	○	○
identifying the components of a personal budget.	○	○	○

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Financial Literacy: Interest and Budgets*	Beginning of Topic	Middle of Topic	End of Topic
calculating the percent of each category within a total budget.	○	○	○
estimating the minimum household budget and average hourly wage needed for a typical family to meet its basic needs.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Financial Literacy: Interest and Budgets* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

Financial Literacy: Interest and Budgets Summary

LESSON
1

Simple and Compound Interest

Principal is the original amount of money on which interest is calculated. When you invest money, your principal can grow by earning interest. When you borrow money, interest is charged on the principal, which is the original amount you borrow. Depending on the type of investment or loan, the principal may earn simple interest or compound interest over time.

Simple interest is a percent of the principal that is added to the principal over time. The formula to determine simple interest is $I = Prt$, where I is the interest, P is the principal, r is the interest rate, and t is time.

Let's consider this example. Michael invests $600 in an account with a 2.5% simple interest rate. The account is open for 10 years.

$$I = Prt$$

$$I = (600)(0.025)(10)$$

$$I = 150$$

$600 + 150 = 750$

The amount of interest earned is $150. After 10 years, the total value is $750.

Compound interest is a percent of the principal and the interest that is added to the principal over time. The formula for compound interest is $A = P(1 + r)^t$, where A represents the final balance, P represents the original principal, r represents the annual rate, and t represents the time in years.

NEW KEY TERMS

- principal [principal]
- simple interest [interés simple]
- compound interest [interés compuesto]
- asset
- liability
- 401(k) plan [plan 401(k)]
- 403(b) plan [plan 403(b)]
- net worth
- personal budget
- fixed expenses
- variable expenses
- family budget estimator

For example, Daniela invests $850 in an account with a 3% compound interest rate. The account is open for 8 years.

$$A = 850(1 + 0.03)^8$$
$$A = 850(1.03)^8$$
$$A = 1076.75$$

After 8 years, there is now $1076.75 in Daniela's account.

The definitions of *simple interest* and *compound interest* are very similar, but the small difference in the definitions can lead to a big difference in money.

LESSON 2 Net Worth

Assets include the value of all accounts, investments, and things that you own. They are positive and add to your net worth. **Liabilities** are financial obligations, or debts, that you must repay. They are negative and take away from your net worth.

Let's consider this example. Michael creates a list of all his accounts and obligations.

- Mortgage
- Credit cards
- Savings account
- 401(k) plan
- School loans
- Car

Michael's mortgage, credit cards, and school loans are liabilities because he must repay these. His savings account, 401(k) plan, and car are assets because they are things that he owns.

Retirement accounts, such as 401(k) or 403(b) plans, are important assets to consider when determining a person's financial health. A **401(k) plan** is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account, with the employer often matching a certain amount of it. A **403(b) plan** is a retirement plan generally for public school employees or other tax-exempt groups.

A net worth statement is a useful tool to measure your financial health from year to year. Your **net worth** is basically a calculation of the value of everything that you have minus the amount of money that you owe. A net worth statement includes this calculation as well as a detailed list of everything used to determine net worth.

For example, Abby's accounts are shown:

Checking account: $2876	Student loan: $9560	Credit card: $980
401(k) account: $14,432	Car loan: $18,680	Savings account: $5500

Here is a list of Abby's assets and liabilities.

Assets		**Liabilities**	
Type	**Amount**	**Type**	**Amount**
Checking account	$2876	Student loan	$9560
401(k)	$14,432	Credit card:	$980
Savings account	$5500	Car loan	$18,680
Total:	**$22,808**	Total:	**$29,220**

Net worth = Assets − Liabilities

Net worth = $22,808 − $29,220 = −$6412.

Abby's net worth is −$6412. The negative net worth means Abby owes more than she owns.

LESSON
3

Personal Budgets

A **personal budget** is an estimate of the amount of money that a person or family will need for specific financial items. It generally includes current expenses as well as savings for anticipated future expenses. These expenses are often categorized as *fixed expenses* and *variable expenses*. **Fixed expenses** are expenses that don't change from month to month. **Variable expenses** are expenses that can be different from month to month. Each expense represents a percent of the total budget.

For example, Javier's family's monthly expenses are shown.

Fixed Expenses	Variable Expenses
• Mortgage: \$1150 • Utilities: \$320 • Savings: \$600	• Food: \$450 • Miscellaneous: \$250

Total expenses: $1150 + 320 + 600 + 450 + 250 = 2770$

Mortgage: $\frac{1150}{2770} \approx 0.42 = 42\%$

Utilities: $\frac{320}{2770} \approx 0.12 = 12\%$

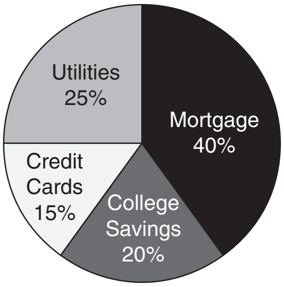
Savings: $\frac{600}{2770} \approx 0.22 = 22\%$

Food: $\frac{450}{2770} \approx 0.16 = 16\%$

Miscellaneous: $\frac{250}{2770} \approx 0.09 = 9\%$

For Javier's family, their mortgage represents 42% of their budget, their utilities represent 12% of their budget, their savings represent 22% of their budget, their food represents 16% of their budget, and miscellaneous costs represent 9% of their budget.

A circle graph can be used to represent a personal budget. Each expense is represented by one section of the circle graph.



For example, the circle graph represents the Chen family's yearly expenses.

In the Chen family's budget, their mortgage represents 40% of their budget, college savings represents 20% of their budget, credit card payments represent 15% of their budget, and utilities represent 25% of their budget.

In order to provide for yourself or your family, you must earn enough to cover the costs of all your expenses. It is important to remember that taxes are also taken out of your earnings, so you might need to earn slightly more than your expenses to maintain your budget.

For example, Fernando's family's yearly expenses are about \$98,500. They pay 20% of their income in taxes.

$$98{,}500 = 0.80x$$
$$x = \frac{98{,}500}{0.80}$$
$$x = 123{,}125$$

Fernando's family must earn at least \$123,125 to maintain their yearly expenses.

A **family budget estimator** is a tool that people can use to determine the estimated cost of raising a family in a particular city.

MODULE

3

Reasoning Algebraically

TOPIC 1 Operating with Rational Numbers 325

TOPIC 2 Two-Step Equations and Inequalities 411

TOPIC 3 Multiple Representations of Equations 513

GRADE 7

TOPIC 1

Death Valley has some very high and very low elevations: Badwater Basin is the point of the lowest elevation in North America, at 282 feet below sea level, while Telescope Peak in the Panamint Range has an elevation of 11,043 feet.

Operating With Rational Numbers

LESSON 1 Adding and Subtracting Rational Numbers **327**

LESSON 2 Quotients of Integers **339**

LESSON 3 Simplifying Expressions to Solve Problems **349**

LESSON 4 Using Number Properties to Interpret Expressions with Signed Numbers **363**

LESSON 5 Evaluating Algebraic Expressions **373**

LESSON 6 Rewriting Expressions Using the Distributive Property . . **385**

1

Adding and Subtracting Rational Numbers

OBJECTIVES

- Interpret and determine sums and differences of rational numbers in real-world contexts.
- Represent and apply the additive inverse in real-world contexts.
- Determine distance as the absolute value of the difference between two signed numbers in real-world contexts.
- Solve real-world and mathematical problems involving operations with rational numbers.

You have learned how to add and subtract with signed numbers using models and rules.

How can you solve real-world problems by adding and subtracting signed numbers?

Getting Started

Seeing Things Rationally

The set of integers is a subset of the rational numbers. The set of rational numbers also includes mixed numbers, fractions, and their decimal equivalents.

Remember that the set of rational numbers are all numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

1. Sketch a Venn diagram to show all of the sets and subsets of numbers that you know.

2. Use your Venn diagram to decide whether each statement is true or false. Explain your reasoning.

   a. An integer is sometimes a rational number.

   b. A rational number is always an integer.

ACTIVITY
1.1

Sum of Rational Numbers Problems

You can use what you know about adding and subtracting positive and negative integers to solve problems with rational numbers.

Yesterday, Natalia was just $23.75 below her fundraising goal. She got a check today for $12.33 to put toward the fundraiser. Describe Natalia's progress toward the goal.

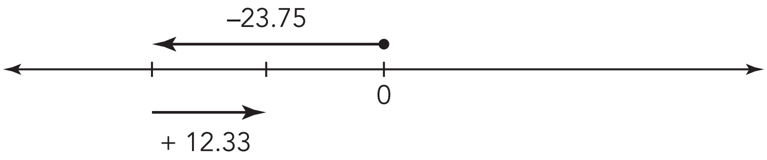
WORKED EXAMPLE

You can model this situation using addition:

$$\underset{\substack{\text{currently below}\\ \text{the goal}}}{\underset{\uparrow}{-\$23.75}} \quad + \quad \underset{\substack{\text{got a check for}\\ \text{this amount}}}{\underset{\uparrow}{\$12.33}}$$

- Estimate.

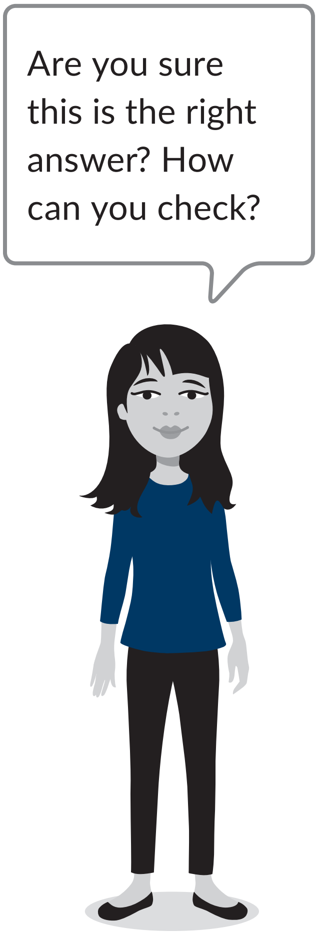
  Natalia will still be below her goal, because $-23.75 + 12.33 < 0$.



- Determine the sum.

  $-23.75 + 12.33 = -11.42$

1. Explain how the final sum was calculated.



2. What does the sum mean in terms of the problem situation?

3. Explain how you can know that addition is the correct operation to use to solve this problem.

Sketch a model to estimate each sum or difference. Then, determine each solution and write an equation.

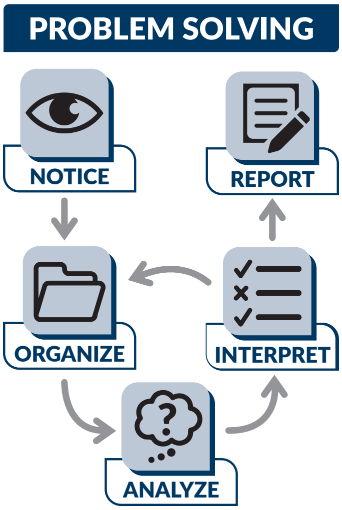
4. The table shows the freezing points of some of the elements in the periodic table.

a. Kaya and Mario are trying to figure out how much the temperature would have to increase from the freezing point of hydrogen to reach the freezing point of phosphorus. Kaya says the temperature would have to increase 545.7°, and Mario says the temperature would have to increase 322.3°. Who is correct?

b. Trung and Samantha are trying to figure out how much the temperature would have to increase from the freezing point of nitrogen to reach the freezing point of mercury. Trung says the temperature would have to increase $308\frac{1}{20}$°, and Samantha says the temperature would have to increase $383\frac{9}{20}$°. Who is correct?

Element	Freezing Point (°F)
Helium	-458
Hydrogen	-434
Oxygen	-368.77
Nitrogen	$-345\frac{3}{4}$
Chlorine	-149.51
Mercury	$-37\frac{7}{10}$
Phosphorus	111.7

5. A drilling crew dug to a height of $-45\frac{1}{4}$ feet during their first day of drilling. On the second day, the crew dug down $9\frac{1}{3}$ feet more than on the first day. What is the total distance that the drilling crew dug at the end of the second day?



6. The ancient Babylonians were writing fractions in 1800 BC. But they did not have a concept of zero until about 1489 years later. In what year did the Babylonians develop the concept of zero?

7. Andrew purchased lunch today at the school cafeteria for \$2.75. Before today, Andrew owed \$9.15 on his lunch account. What is the status of his lunch account after today?

8. The highest mountain in the world is Mt. Everest, whose peak is 29,029 feet above sea level. But the tallest mountain is Mauna Kea. The base of Mauna Kea is 19,669 feet below sea level, and its peak is 33,465 feet above its base. How much higher above sea level is Mt. Everest than Mauna Kea?

ACTIVITY 1.2

Rational Number Difference Problems

The freezing point of chlorine is −149.51° Fahrenheit. The element zinc freezes at much higher temperatures. The freezing point of zinc is 787.51° Fahrenheit. How many more degrees is the freezing point of zinc than the freezing point of chlorine?

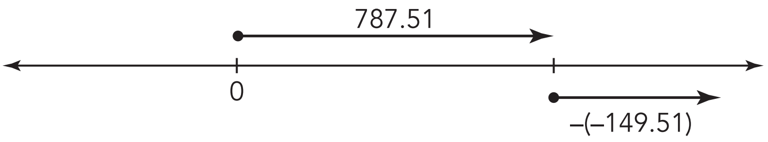
WORKED EXAMPLE

You can model this situation using subtraction:

$$\underset{\substack{\text{freezing point}\\ \text{of zinc}}}{\underset{\uparrow}{787.51}} \quad - \quad \underset{\substack{\text{freezing point}\\ \text{of chlorine}}}{\underset{\uparrow}{(-149.51)}}$$

- Estimate.

  The answer is greater than 787.51, because
  $787.51 - (-149.51) = 787.51 + 149.51$.



- Determine the difference. Write an equation.

  $787.51-(-149.51) = 937.02$

1. What does the difference mean in terms of the problem situation?

2. Explain how you can check the answer.

Sketch a model to estimate. Then determine each solution and write an equation.

3. The temperature in Wichita, Kansas, is −3°C. The temperature in Alejandro's hometown is 18° colder than that. What is the temperature in Alejandro's hometown?

4. To qualify to compete in the high jump finals, athletes must jump a certain height in the semi-finals. Paola jumped $2\frac{3}{8}$ inches below the qualifying height, but her friend Elena made it to $1\frac{5}{6}$ inches over the qualifying height. How much higher was Elena's semi-final jump compared with Paola's?

5. The Down Under roller coaster rises up to 65.8 feet above the ground before dropping 90 feet into an underground cavern. Describe the height of the roller coaster at the bottom of the cavern.

ACTIVITY
1.3

Adding and Subtracting Rational Numbers

Determine each sum or difference.

1. $4.7 + (-3.65)$

2. $-\frac{2}{3} + \frac{5}{8}$

3. $3.95 + (-6.792)$

4. $2\frac{5}{7} + \left(-1\frac{1}{3}\right)$

5. $-\frac{3}{4} + \frac{5}{8}$

6. $-7.38 - (-6.2)$

7. $-\frac{3}{4} - \frac{5}{8}$

8. $-2\frac{5}{6} + 1\frac{3}{8}$

9. $-\frac{7}{12} - \frac{5}{6}$

10. $-37.27 + (-13.2)$

11. $-0.8 - (-0.6)$

12. $2\frac{3}{7} + \left(-1\frac{3}{4}\right)$

13. $0.67 + (-0.33)$

14. $-42.65 - (-16.3)$

15. $-7300 + 2100$

16. $-3\frac{5}{8} - \left(-2\frac{1}{3}\right)$

17. $-4.7 + 3.16$

18. $26.9 - (-3.1)$

19. $-325 + (-775)$

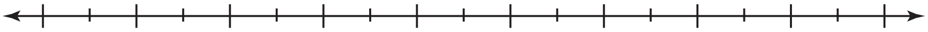
20. $-2\frac{1}{5} - 1\frac{3}{10}$

Talk the Talk

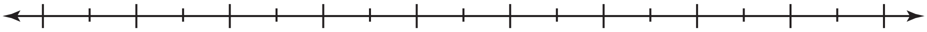
Mixing Up the Sums

Represent each number as the sum of two rational numbers. Use a number line to explain your answer.

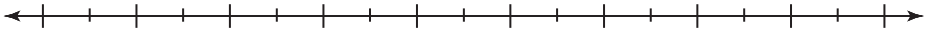
1. -2.1



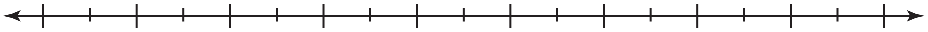
2. $-5\frac{2}{3}$



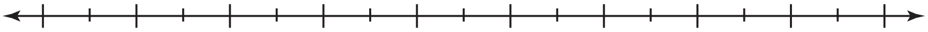
3. $4\frac{7}{9}$



4. 5.8



5. $-1\frac{4}{7}$



Lesson 1 Assignment

Write

Explain in your own words how adding and subtracting positive and negative numbers with fractions and decimals is different from and similar to adding and subtracting with whole numbers.

Remember

The opposite of a number is called the additive inverse of the number. The absolute value of the difference between two numbers is a measure of the distance between the numbers.

Practice

Calculate each sum. Be sure to estimate first.

1. $12\frac{2}{5} + \left(-3\frac{1}{4}\right)$

2. $5.3 + (-7.45)$

3. $-\frac{5}{8} + 8\frac{3}{8}$

Calculate each difference. Estimate before calculating.

4. $-8.38 - 11.29$

5. $7\frac{2}{3} - \left(-4\frac{1}{4}\right)$

6. $-4\frac{5}{6} - 6\frac{2}{3}$

7. $-1.3 - (-2.4)$

Lesson 1 Assignment

Prepare

Classify each number into as many categories as it belongs: natural number, whole number, integer, rational number.

1. -3

2. $\frac{1}{2}$

3. 0

4. 5

2

Quotients of Integers

OBJECTIVES

- Determine that every quotient of integers is a rational number, provided the divisor is not zero, and use long division to represent those quotients.
- Write equivalent forms of signed rational numbers.
- Solve real-world problems using operations with signed numbers.

You have learned the rule to determine the sign of a quotient.

Does a quotient change if the negative sign is on the divisor instead of the dividend?

Getting Started

Status Quotient

Recall that a quotient is the result of dividing two numbers or expressions. So, the quotient of a and b is $\frac{a}{b}$.

1. Rewrite each quotient as a fraction using two integers. Then divide to write the quotient in decimal form.

   a. $5 \div 8$

   b. $2 \div \frac{5}{4}$

   c. $\frac{7}{5} \div 2$

   d. $6 \div 2$

   e. $9 \div 1.2$

   f. $2.5 \div 50$

2. What types of numbers are the quotients in Question 1? Use the definitions of the different number classifications to explain why this makes sense.

ACTIVITY
2.1

Equivalent Rational Numbers

You can use multiple representations to express equivalent rational numbers.

1. Write a new equivalent fraction representation and write an equivalent decimal representation.

a. $-\frac{4}{5} = \frac{-4}{5}$ = ☐ = ☐

b. $\frac{-2}{3} = \frac{2}{-3}$ = ☐ = ☐

c. $\frac{11}{-4} = -\frac{11}{4}$ = ☐ = ☐

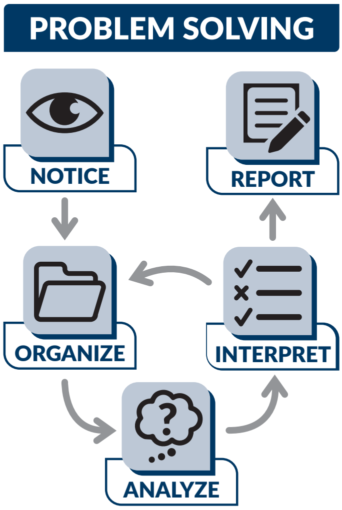
d. $-\frac{39}{60} = \frac{39}{-60}$ = ☐ = ☐

e. $\frac{-7}{22} = \frac{7}{-22}$ = ☐ = ☐

Think about how you determine the sign of a quotient. What is special about each of these representations?

2. Does it make a difference where you write the negative sign for negative fractions? Explain your reasoning.

Consider how you can use positive and negative signs to write an equivalent form of $\frac{3}{5}$.



ACTIVITY

2.2

Solving Problems with Rational Numbers

You already know how to operate with integers and positive rational numbers. You can use what you know to operate with all rational numbers.

Solve each problem.

1. Natalia and Mario both borrowed money from Kaya. Natalia owes Kaya $25.00. Mario owes Kaya $\frac{3}{4}$ the amount that Natalia owes. How much does Mario owe Kaya?

2. In 4 hours in direct sunlight the water level in a bucket changes $-\frac{1}{8}$ inch. How fast is the water level changing per hour?

3. The temperature changes $-1\frac{3}{4}^{\circ}$ per hour from 10:00 P.M. to 5:30 A.M. What is the total expected temperature change over the time period?

4. Andrew is riding his bicycle a total of 15 miles around a $3\frac{3}{4}$-mile loop. If it takes him $12\frac{1}{2}$ minutes to ride each loop, how long does it take him to complete his ride?

5. Alejandro ordered 6 pizzas for the 17 players on his soccer team, and some of the coaches. He thinks that each person will eat $\frac{5}{16}$ of a pizza. How many coaches does Alejandro think he can feed?

6. Trung earns \$17.50 per hour working at her job delivering packages. While she is working, she pays her neighbor \$3.10 per hour to watch her dog. How much money does Trung keep after working a $5\frac{1}{2}$-hour shift?

Talk the Talk

Be Rational

Determine the value of each expression.

1. $-\frac{2}{5}(2.5)$

2. $38.25 \div -4\frac{1}{2}$

3. $-112.6 \cdot \left(-1\frac{3}{4}\right)$

4. $31.5 \div \frac{7}{8}$

5. $2\frac{3}{4} + \left(-3\frac{1}{2}\right) - \frac{5}{8}$

Lesson 2 Assignment

Write

Explain how the three different fractional representations of a rational number are related to determining the sign of the quotient of two integers.

Remember

The sign of a negative rational number in fractional form can be placed in front of the fraction, in the numerator of the fraction, or in the denominator of the fraction.

Practice

Write each rational number as an equivalent fraction by adding or changing the placement of the negative sign(s).

1. $-\frac{4}{7}$

2. $\frac{-5}{3}$

3. $\frac{1}{2}$

4. $\frac{9}{-2}$

Solve each problem.

5. A submarine starts at a naval base at sea level. On the first day, it descends $\frac{1}{25}$ mile. Each day after that, it descends $\frac{2}{25}$ mile per day. What is the elevation of the submarine after 9 days?

Lesson 2 Assignment

6. A recipe for banana nut bread that makes 12 slices calls for $1\frac{1}{2}$ cups of flour. One serving is half a slice. How much flour is in each serving?

Lesson 2 Assignment

Prepare

Simplify each expression.

1. $-20 \div 2\left(7\frac{2}{3}\right)$

2. $-20 - 2\left(-7\frac{2}{3}\right)$

3. $-7\left(\dfrac{-\frac{3}{4}}{-\frac{2}{3}}\right)$

3

Simplifying Expressions to Solve Problems

OBJECTIVES

- Model situations using expressions with rational numbers.
- Evaluate expressions with rational numbers and variables.
- Solve real-world problems using operations with signed rational numbers.

NEW KEY TERM

- percent error

You have learned how to operate with signed numbers, including integers and other rational numbers.

How can you use what you know to solve problems?

Getting Started

Orville and Wilbur

In the middle of December 1903, two brothers—Orville and Wilbur Wright—became the first two people to make a controlled flight in a powered plane. They made four flights on December 17, the longest covering only 852 feet and lasting just 59 seconds.

The table shows information about the flights made that day.

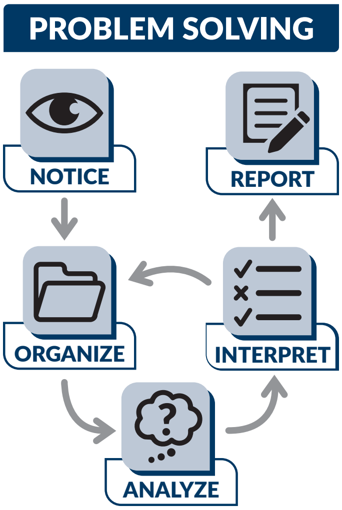
Flight	Pilot	Flight Time (s)	Distance (ft)
A	Orville	12	120
B	Wilbur	13	175
C	Orville	15	200
D	Wilbur	59	852

1. Determine the approximate speed of all four flights, in feet per second.

Human flight progressed amazingly quickly after those first flights. In the year before Orville Wright died, Chuck Yeager had already piloted the first flight that went faster than the speed of sound: 767.269 miles per hour or approximately 1125.33 feet per second!

ACTIVITY 3.1

Operating with Rational Numbers to Solve Problems



In order to build a balsa wood model of the Wright brothers' plane, you would need to cut long lengths of wood spindles into shorter lengths for the wing stays, the vertical poles that support and connect the two wings. Each stay for the main wings of the model needs to be cut $3\frac{1}{4}$ inches long.

Show your work and explain your reasoning.

1. When the wood spindles are each 10 inches long, how many stays could you cut from one spindle?

2. How many inches of the spindle would be left over?

3. When the wood spindles are each 12 inches long, how many stays could you cut from one spindle?

4. How many inches of the spindle would be left over?

You also need to cut vertical stays for the smaller wing that are each $1\frac{5}{8}$ inches long.

5. When the wood spindles are each 10 inches long, how many of these stays could you cut from one spindle?

6. How many inches of the spindle would be left over?

7. When the wood spindles are each 12 inches long, how many stays could you cut from one spindle?

8. How many inches of the spindle would be left over?

9. Which length of spindle should be used to cut each of the different stays so that there is the least amount wasted?

ACTIVITY
3.2

Calculating Percent Error

Airline travel has come a long way since the days of Orville and Wilbur Wright. In 2015, there were approximately 9.1 million flights that took off from U.S. airports carrying approximately 895.5 million passengers. To transport this many passengers to and from their destinations, airlines have to make good estimations about the number of flights passengers will book, the size of the airplanes to use for a given route, and the approximate arrival time for each flight.

Tracking the accuracy of these estimations is important for airlines. Calculating **percent error** is one way to compare an estimated value to an actual value. To compute percent error, determine the difference between the estimated and actual values and then divide by the actual value.

$$\text{percent error} = \frac{\text{actual value} - \text{estimated value}}{\text{actual value}}$$

When planning which airplanes to use for a given route, airlines have to estimate how many people they think will book that particular flight. They want to be able to have enough seating to meet the demand but not have too big of a plane and waste the extra fuel needed.

1. An airline estimates that they will need an airplane that seats 224 passengers for the 6 A.M. flight from Washington, D.C., to Boston. Calculate the percent error for each number of actual passengers booked. Show your work.

   a. 186 booked tickets

   b. 250 booked tickets

Airlines use historical data on how long the flight has taken in the past, but these estimates are often impacted by weather issues, airport traffic, and earlier flight delays.

Another challenge is accurately estimating the travel time for each flight. Having minimal error in these estimations allows airlines to keep their schedules accurate and passengers happy.

2. An airline estimates that the flight from Washington, D.C., to Boston takes 1 hour and 27 minutes. Calculate the percent error for each actual flight time. Show your work.

   a. 1 hour and 11 minutes

   b. 2 hours

3. What does a negative value for percent error indicate?

4. Ashley is told that the D.C. to Boston flight took 10 minutes longer than estimated. She calculated the percent error and got 10.3%. She later learns that she had been given the wrong information. The flight took 10 minutes *less* than estimated. Ashley thinks that the percent error should just be −10.3%. Is she correct? Explain why or why not.

ACTIVITY
3.3
Evaluating Expressions with Rational Numbers and Variables

Recall that to evaluate an expression with a variable, substitute the value for the variable, and then perform the operations.

WORKED EXAMPLE

Evaluate the expression $-12\frac{1}{2} - 3v$ for $v = -5$.

Estimate:

$$-12 - 3(-5) = -12 + 15 = 3$$

Substitute -5 for v and solve:

$$\begin{aligned} -12\tfrac{1}{2} - 3(-5) &= -12\tfrac{1}{2} - (-15) \\ &= -12\tfrac{1}{2} + 15 \\ &= 2\tfrac{1}{2} \end{aligned}$$

1. Evaluate the expression for $v = -\frac{6}{7}$.

Evaluate each expression for the given value. Write your answers as a fraction and a decimal.

2. Evaluate $-3.25 - 2.75z$ for $z = -4$.

3. Evaluate $\left(-1\frac{1}{4}\right)x - 8\frac{7}{8}$ for $x = -\frac{2}{5}$.

Remember to check your work. How can you verify that your answers are correct?

4. Evaluate $-0.75(p - 1.2)$ for $p = 2$.

5. Evaluate $\frac{m}{-\frac{6}{5}}$ for $m = 6\frac{3}{4}$.

Talk the Talk

Rational Thinking

Evaluate the expression for the given value.

1. $-25v$ for $v = \frac{3}{4}$

Write each problem as a product or quotient of rational numbers and then solve. Show your work.

2. Therese is measuring how fast water evaporates in a bucket in her backyard. In 6 hours in direct sunlight, the water level changes $-\frac{3}{8}$ inch. How fast is the water level changing per hour?

3. A meteorologist forecasts that the temperature is going to change $-1\frac{1}{2}°$ per hour from 11:00 P.M. to 7:00 A.M. What is the total expected temperature change over the time period?

Lesson 3 Assignment

Write

Write the steps you would follow to evaluate an expression for a variable. Use an example in your description.

Remember

Percent error is one way to report the difference between estimated values and actual values.

$$\text{percent error} = \frac{\text{actual value} - \text{estimated value}}{\text{actual value}}$$

Practice

Write an expression with rational numbers to represent each situation and then solve. Show your work.

1. Eduardo's start-up business makes a profit of $450 during the first month. However, the company records a profit of −$60 per month for the next four months and profit of $125 for the final month. What is the total profit for the first six months of Eduardo's business?

2. A diver is exploring the waters of the Great Barrier Reef.

   a. She is currently −5 feet from the surface of the water and plans to explore a shipwreck that is at −75 feet from the surface. When she moves at a rate of −8 feet per minute, how many minutes does it take the diver to reach the shipwreck?

   b. When she is done exploring the reef, she ascends at a rate of 5 feet per minute. Once she reaches a height of −30 feet, she must rest for 15 minutes to allow her body to adapt to the changing water pressure. She then continues to the surface at the same rate. How long will it take the diver to reach the surface?

Lesson 3 Assignment

3. The drain in your 45-gallon bathtub is partially clogged, but you need to take a shower. The showerhead had a flow rate of 2.25 gallons per minute, but the bathtub only drains at a rate of −0.5 gallon per minute. What is the longest shower you can take?

4. Mariana withdrew $22.75 each week for four weeks from her savings account to pay for her piano lessons. By how much did these lessons change her savings account balance?

Calculate the percent error.

5. Chris estimated that 30 people would attend the dinner event, but only 25 people attended.

6. Jorge estimated the length of the fence to be 150 feet, but the actual measurement was 142 feet.

Lesson 3 Assignment

Evaluate each expression for the given value.

7. $\frac{5}{6}x$ for $x = -8$

8. $9\frac{1}{3} - m$ for $m = -1\frac{2}{3}$

9. $t \div \frac{3}{4}$ for $t = 9\frac{3}{4}$

10. $\frac{2}{5}k - 3\frac{1}{2}$ for $k = 15$

Prepare

Use the order of operations to simplify each expression.

1. $18 + 6 \cdot (-3) - 4$

2. $5 \div (1 - 6) \cdot 10$

3. $8 - (-3) \cdot 9 \cdot 0$

4

Using Number Properties to Interpret Expressions with Signed Numbers

OBJECTIVES

- Use the commutative, associative, and distributive properties, additive and multiplicative inverses, identity, and zero properties to rewrite numeric expressions with signed numbers in order to interpret their meanings and solve problems.
- Apply the properties of operations to add, subtract, multiply, and divide with rational numbers.
- Use number properties to solve mathematical problems involving signed numbers and other rational numbers more efficiently.

You have learned how to add, subtract, multiply, and divide with signed numbers and other rational numbers.

How can you use number properties with rational numbers to solve problems

Getting Started

All in Your Head

You have used mental math before to solve problems without calculating on paper. Now try it with signed numbers!

1. Determine each sum or difference using mental math.

   a. $-8 + 5 + 8$

   b. $-\frac{1}{2} + \frac{3}{5} + -\frac{1}{2}$

   c. $\frac{3}{8} + \left(\frac{5}{8} + \left(-\frac{5}{6}\right)\right)$

The *commutative property* says that you can add or multiply in any order without changing the sum or product.
The *associative property* says that you can group addends or factors without changing the sum or product.

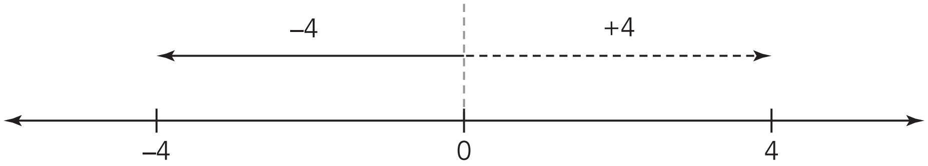
2. Explain how you can use the commutative and associative properties to help you solve the problems in your head.

ACTIVITY

4.1 Distributing and Factoring with −1

When first learning about negative numbers, you reflected a positive value across 0 to determine the opposite of the value.

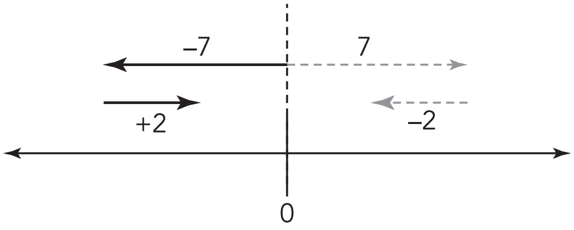
This illustrates that the opposite of 4 is −4, or (−1)(4) = −4.



In the same way, you can use reflections across 0 on the number line to determine the opposite of an expression.

WORKED EXAMPLE

Consider the expression −7 + 2. When the model of −7 + 2 is reflected across 0 on the number line, the result is 7 − 2.



So, (−7 + 2) is the opposite of (7 − 2).

This means that −7 + 2 = −(7 − 2).

1. Draw models like the one in the Worked Example to show the opposite of each expression. Rewrite each as the opposite of the expression shown.

a. −1 − 6

b. 2 + (−3)

c. −4 + 5

How would your answer be different when the expression were −4 · 5?

Jaylen

To reflect an expression across 0 on the number line, multiply the expression by -1.

$$\begin{aligned} -1(2 + 3) &= (-1)(2) + (-1)(3) \\ &= -2 + -3 \end{aligned}$$

2. What property did Jaylen use to show his reasoning?

3. Does Jaylen's expression, $-1(2 + 3)$, mean the same thing as $-(2 + 3)$? Draw a model and explain your reasoning.

Rewriting an expression as a product with -1 is also called *factoring out a* -1.

4. Rewrite each expression as an addition or subtraction expression using a factor of -1.

a. $-2 + (-4) = -1(\underline{\qquad\qquad})$

b. $-5 - 8 = -1(\underline{\qquad\qquad})$

c. $-9 - (-9) = -1(\underline{\qquad\qquad})$

5. Use the distributive property to show that your expressions in Question 4 are correct.

ACTIVITY

4.2 Subtraction as Adding the Opposite

You know that subtracting a number is the same as adding the opposite of that number. Rewriting subtraction as addition allows you to apply the commutative property to any expression involving addition and subtraction.

For example, $-4.5 - 3 + 1.5 = -4.5 + 1.5 + (-3)$. Rewriting expressions helps you to see patterns and use mental math to make solving simpler.

You can use what you know about adding opposites to help you solve problems more efficiently.

1. Simplify each expression.

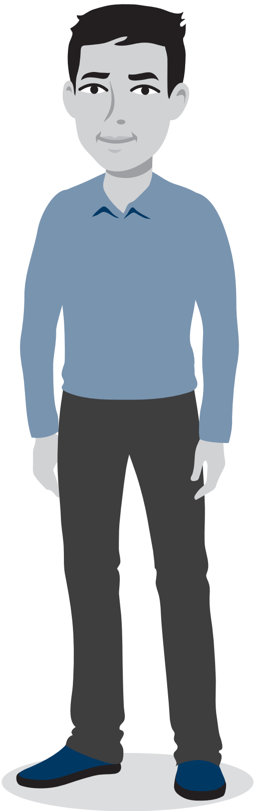
   a. $10.5 + 6 + 2 - 0.5$

   b. $-\frac{1}{2} + \left(\frac{1}{2} - \frac{4}{5}\right)$

   c. $3\frac{7}{8} - 4\frac{1}{2}$

2. Explain how you can use the commutative, associative, and distributive properties to help you simplify the expressions in Question 1.

You can rewrite a mixed number as the sum of a whole number and a fraction. $7\frac{1}{2} = 7 + \frac{1}{2}$ How can you rewrite $-7\frac{1}{2}$?



ACTIVITY

4.3 Practice with the Properties

1. For each equation, identify the number property used.

Equation	Number Property
a. $-3\frac{1}{2} + 5 = 5 + \left(-3\frac{1}{2}\right)$	
b. $\left(3\frac{1}{2}\right)\left(2\frac{1}{5}\right)5 = 3\frac{1}{2}\left(2\frac{1}{5}\right)(5)$	
c. $-3\frac{1}{2} + \left(-2\frac{1}{2} + 5\right) = \left(-3\frac{1}{2} + \left(-2\frac{1}{2}\right)\right) + 5$	
d. $-\left(-3\frac{1}{2} + 2\frac{1}{4}\right) = -1\left(-3\frac{1}{2}\right) + -1\left(2\frac{1}{4}\right)$	
e. $\frac{-3\frac{1}{2} - 2\frac{1}{4}}{4} = \frac{-3\frac{1}{2}}{4} - \frac{2\frac{1}{4}}{4}$	
f. $(-7.02)(-3.42) = (-3.42)(-7.02)$	

Evaluate each expression. Describe your strategy.

2. $-2\left(2\frac{1}{4}\right) + \left(-2\left(-\frac{3}{4}\right)\right)$

3. $\left(-3\frac{1}{4} - 2\frac{1}{5}\right) + \left(-6\frac{3}{5}\right)$

4. $\frac{7}{8}\left(-\frac{4}{5}\right)\left(-\frac{8}{7}\right)$

5. $\dfrac{\frac{8}{9} + \left(-\frac{4}{5}\right)}{4}$

6. $(-11.4)\ (6.4) + (-11.4)(-12.4)$

Talk the Talk

What's It All About?

When you rewrite addition and subtraction expressions using a factor of -1, you are "factoring out" a -1. Here are some other examples.

$-8 + 5 = -1(8 - 5)$ $-2 - 9 = -1(2 + 9)$ $3 - (-4) = -1(-3 - 4)$

1. Describe how you can factor out a -1 from any addition or subtraction expression.

2. How is factoring out a negative 1 from an addition or subtraction expression different from factoring out a negative 1 from a multiplication or division expression?

3. Demonstrate using words and models why the product of -1 and any expression is the opposite of that expression.

Lesson 4 Assignment

Write

Describe in your own words how to factor a -1 out of an addition or subtraction expression.

Remember

When you multiply any expression by -1, the result is the opposite of that expression.

Practice

Factor out a negative 1 from each expression.

1. $7 + (-6)$

2. $-4 - (5 + 3)$

3. $-9 - 1$

4. Use the distributive property to show that your answers to Questions 1 through 3 are correct.

Lesson 4 Assignment

Use a number property to solve each problem efficiently. Show your work and list the property or properties used.

5. $-9.9 + 5.2 + 3.9 + 1$

6. $-\frac{3}{5} + \left(\frac{1}{5} - \frac{3}{2} + 0\right)$

Prepare

Perform each operation.

1. $(-3)(6.6)$

2. $-3 + 6.6$

3. $-3 - 6.6$

4. $6.6 \div (-3)$

5

Evaluating Algebraic Expressions

OBJECTIVES

- Compare unknown quantities on a number line.
- Define linear expressions.
- Evaluate algebraic expressions.
- Solve real-world and mathematical problems using algebraic expressions.

NEW KEY TERMS

- variable
- algebraic expression
- linear expression
- constraint
- evaluate an algebraic expression

You have written and evaluated algebraic expressions with positive rational numbers.

How do you evaluate algebraic expressions over the set of rational numbers?

Getting Started

The Empty Number Line

In algebra, a **variable** is a letter or symbol that is used to represent an unknown quantity.

Consider the list of six *variable* expressions:

x $2x$ $3x$ $\frac{1}{2}x$ $-x$ $-\frac{1}{2}x$

1. With your partner, think about where you would place each expression and sketch your conjecture.

2. Compare your number line with another group's number line. What is the same? What is different?

3. Your teacher will select students to place an index card representing each expression on the number line on the board. Record the locations agreed upon by the class.

ACTIVITY

5.1

Algebraic Expressions

In this lesson, you will explore the relationship between unknown quantities by writing and evaluating *algebraic* expressions. An **algebraic expression** is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Each of the expressions in the *Empty Number Line* activity is an algebraic expression. They are also *linear expressions*. A **linear expression** is any expression in which each term is either a constant or the product of a constant and a single variable raised to the first power.

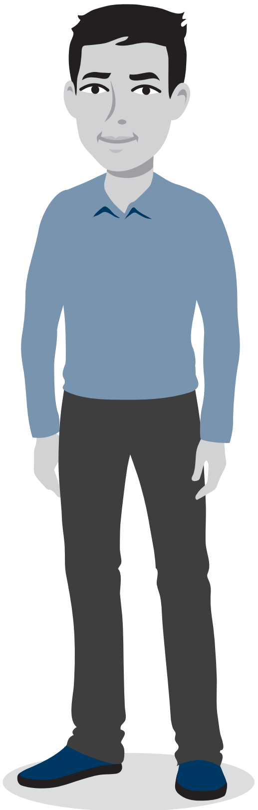
Additional examples of linear expressions include:

$\frac{1}{2}x + 2$, $-3 + 12.5x$, $-1 + 3x + \frac{5}{2}x - \frac{4}{3}$, or $4y$.

The expressions $3x^2 + 5$ and $-\frac{1}{2}xy$ are examples of expressions that are not linear expressions.

1. Provide a reason why each expression does not represent a linear expression.

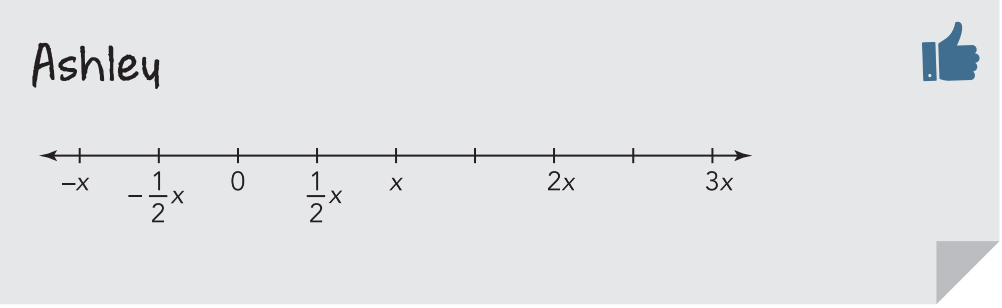
How could you verify the placement of the expressions on the number line?

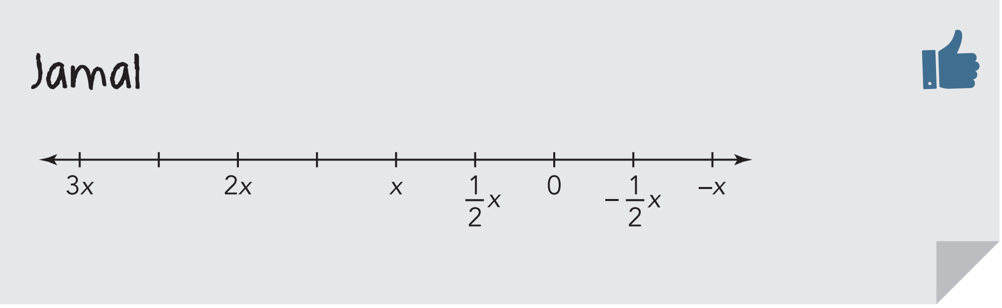


Let's revisit how you may have plotted the expressions in the previous activity. The directions did not specify the possible values for *x*. When you graphed each expression, did you think about the set of all possible values of *x* or just the set of positive *x*-values?

In mathematics, it is sometimes necessary to set *constraints* on values. A **constraint** is a condition that a solution or problem must satisfy. A constraint can be a restriction set in advance of solving a problem or a limit placed on a solution or graph so the answer makes sense in terms of a real-world scenario.

Analyze the number lines created by Ashley and Jamal using the expressions from the *Empty Number Line* activity.





2. Compare and contrast each representation.

   a. What are the constraints on each representation? Identify the set of *x*-values that make each number line true.

   b. Select a value for *x* from your set of possible values and substitute that value for *x* in each expression to verify the plotted locations are correct.

   c. Compare your values from part (b) with your classmates. Do you have the same values? When not, what does that mean?

One strategy to verify your placement of the cards is to substitute values for the variable *x* into each expression.

ACTIVITY 5.2

Substitution with Rational Numbers

To earn money for a summer trip, Matthew is working to help out around his neighborhood. Matthew has been hired to build a wooden fence. He plans to use a post hole digger to dig the holes for the posts.

Matthew starts the project on Saturday morning but because of the type of soil, he only starts the holes, fills them with water, and then plans to return Sunday to finish the job. When Matthew starts on Sunday, each hole is 3 inches deep. Each time he uses the post hole diggers, he extracts 2 inches of soil. The height of the soil in the hole with respect to ground level can be modeled by the linear expression $-3 - 2n$, where n is the number of times Matthew extracted soil with the post hole diggers.

A depth of 3 inches is equivalent to a height of -3 inches.

1. Determine the height of the soil in the hole as Matthew works.

Number of Soil Extractions	Height of the Soil (inches)
0	
1	
5	
10	
15	

Use the order of operations to evaluate the expressions.

2. From his research about digging post holes, Matthew knows that each pole must be placed at a depth that is 2 feet below the frost level, and the frost level is 16 inches beneath ground level.

   a. How deep must Matthew dig each hole?

   b. Determine the minimum number of soil extractions for each hole.

Cable drilling, also known as percussion drilling, is a method used to drill a borehole.

Matthew's mom, Tiara, uses a cable tool rig to dig wells. Her rig can dig 12.4 meters of hard rock per day. When Tiara starts working on one well, the hole is already 33 meters deep.

3. Write a linear expression for the height of the hole with respect to ground level for the number of days that Tiara runs the rig.

4. Use your expression to determine the height of the hole after each number of days.

   a. 5 days after Tiara starts

   b. 2 days before Tiara started

ACTIVITY 5.3

Evaluating Expressions

Previously, you evaluated algebraic expressions with positive rational numbers. Now, you can evaluate expressions with negative rational numbers. To **evaluate an algebraic expression**, you replace each variable in the expression with a number or numeric expression and then perform all possible mathematical operations.

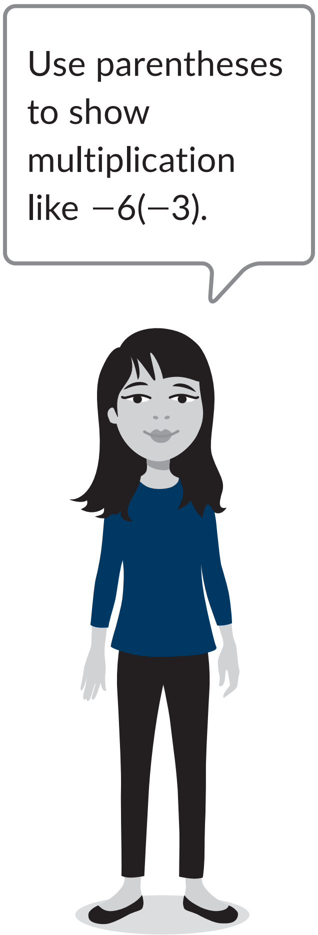
1. Evaluate each algebraic expression.

   a. $-6y$

   - for $y = -3$
   - for $y = 7$

   b. $-1.6 + 5.3n$

   - for $n = -5$
   - for $n = 0$



Sometimes, it is more convenient to use a table to record the results when evaluating the same expression with multiple values.

2. Complete each table.

a.

h	$-2h - 7$
-1	
8	
-7	

b.

a	-12	-4	0
$\frac{a}{4} + 6$			

Which of these algebraic expressions are also linear expressions?

c.

x	$x^2 - 5$
1	
3	
-2	

3. Evaluate each algebraic expression for $x = 2, -3, 0.5$, and $-2\frac{1}{3}$.

a. $-3x$

b. $5x + 10$

c. $6 - 3x$

d. $8x + 75$

4. Evaluate each algebraic expression for $x = 23.76$ and $-21\frac{5}{6}$.

a. $2.67x - 31.85$

b. $11\frac{3}{4x} + 56\frac{3}{8}$

How can you use estimation and number sense to judge the reasonableness of your answers?

Talk the Talk

Strategies

Summarize the lesson by addressing the questions.

1. Describe your basic strategy for evaluating any algebraic expression.

2. Complete the table.

t	$2 - \frac{2}{3}t$
2	
-6	
$\frac{24}{5}$	

Lesson 5 Assignment

Write

Explain the difference between an *algebraic expression* and a *linear expression*.

Remember

To *evaluate an algebraic expression*, replace each variable in the expression with a number or numeric expression and then perform all possible mathematical operations.

Practice

Evaluate each algebraic expression.

1. $64 - 9p$ for $p = 4, 9, -3$

2. $-w + 8.5$ for $w = 12, -1.5, 5.3$

3. $46 + (-2k)$ for $k = 3, 23, -2$

Complete each table.

4.

b	$3b + 14$
-5	
-3	
0	
4	

Lesson 5 Assignment

5.

v	1	2	5	−3.25
$6.75 - 6v$				

6.

f	4	8	−12	−1
$\frac{f}{4} + 3$				

Evaluate each algebraic expression for the given quantity.

7. $-6x + 1.4, x = -9.3$

8. $3\frac{1}{2} - 5\frac{1}{3}x, x = \frac{2}{5}$

Prepare

Write a numeric expression for the opposite of each given expression.

1. $-7 - 2$

2. $3 - 9$

3. $-3 + 2$

4. $3 - (-7)$

6

Rewriting Expressions Using the Distributive Property

OBJECTIVES

- Write and use the distributive property.
- Apply the distributive property to expand expressions with rational coefficients.
- Apply the distributive property to factor linear expressions with rational coefficients.

NEW KEY TERMS

- factor
- coefficient
- common factor
- greatest common factor (GCF)

You have used the distributive property to expand and factor algebraic expressions with positive numbers.

How can you apply the property to all rational numbers?

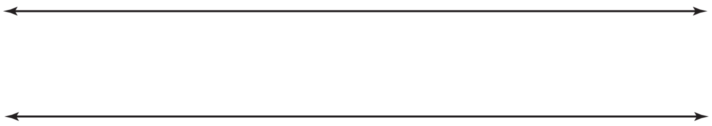
Getting Started

Where Are They?

Consider the list of linear expressions.

$x + 1$　　$2x + 2$　　$3x + 3$　　$4x + 4$

1. On the empty number line, plot each algebraic expression by estimating its location.



2. Explain your strategy. How did you decide where to plot each expression?

3. What assumptions did you make to plot the expressions? Does everyone's number line look the same? Why or why not?

ACTIVITY
6.1

Algebraic Expressions on the Number Line

Consider the four expressions plotted in the previous activity.
How can you prove that you are correct?

Josh

I can use an example by evaluating all four expressions at the same value of x and plot the values.

Let $x = 4$.

$x + 1 = 4 + 1 = 5$

$2x + 2 = 2(4) + 2 = 10$

$3x + 3 = 3(4) + 3 = 15$

$4x + 4 = 4(4) + 4 = 20$

I can plot the expressions at 5, 10, 15, and 20.

Lizzie

The expressions look similar.
I can factor out the coefficient of each expression.

$x + 1$

$2x + 2 = 2(x + 1)$

$3x + 3 = 3(x + 1)$

$4x + 4 = 4(x + 1)$

So, I can plot $x + 1$ and use that expression to plot the other expressions.

1. Use Josh's strategy with a different positive value for x to accurately plot the four expressions.

2. Use Josh's strategy with a negative value for x to accurately plot the four expressions. How is your number line different from the number line in Question 1?

To **factor** an expression means to rewrite the expression as a product of factors.

A **coefficient** is a number that is multiplied by a variable in an algebraic expression.

Often, writing an expression in a different form reveals the structure of the expression. Lizzie saw that each expression could be rewritten as a product of two factors.

Lizzie's expressions

$x + 1$

$2x + 2 = 2(x + 1)$

$3x + 3 = 3(x + 1)$

$4x + 4 = 4(x + 1)$

3. What are the two factors in each of Lizzie's expressions? What is common about the factors of each expression?

4. Use Lizzie's work to accurately plot the four expressions. Explain your strategy.

5. Lizzie noticed that the expressions formed a sequence. Write and plot the next two terms in the sequence. Explain your strategy.

When a variable has no coefficient, the understood coefficient is 1.

6. What property did Lizzie use when she factored out the coefficient of the expressions?

ACTIVITY
6.2

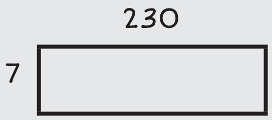
Applying the Distributive Property

Recall that the distributive property states that when a, b, and c are any real numbers, then $a(b + c) = ab + ac$. The property also holds when addition is replaced with subtraction, then $a(b - c) = ab - ac$.

Alyssa remembers that the distributive property can be modeled with a rectangle. She illustrates with this numeric example.

Alyssa

Calculating $230 \cdot 7$ is the same as determining the area of a rectangle by multiplying the length by the width.



But, I can also decompose the rectangle into two smaller rectangles and calculate the area of each. I can then add the two areas to get the total.

$1400 + 210 = 1610$

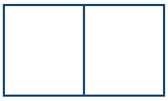
So, $7(230) = 1610$.

1. Write Alyssa's problem in terms of the distributive property.

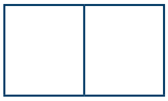
You can also use area models with algebraic expressions.

2. Draw a model for each expression, and then rewrite the expression with no parentheses.

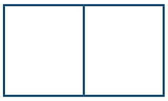
a. $6(x + 9)$



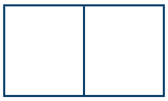
b. $7(2b - 5)$



c. $-2(4a + 1)$



d. $\frac{x + 15}{5}$



3. Use the distributive property to rewrite each expression in an equivalent form.

a. $3(4y + 2)$

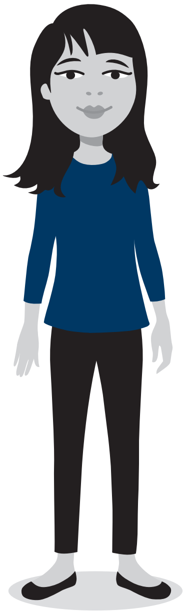
b. $12(x + 3)$

c. $-4(3b - 5)$

d. $-7(2x + 9 - 3x)$

e. $\frac{6m + 12}{-2}$

f. $\frac{22 - 4x}{2}$



Be careful with the signs of the products and quotients. Don't forget to multiply or divide both terms of the binomial by the constant.

4. Simplify each expression. Show your work.

a. $-6(3 + (-4y)) + 22$

b. $-4(-3x - 8) - 34$

c. $\dfrac{-7.2 - 6.4x}{-0.8}$

d. $\left(-2\frac{1}{2}\right)\left(3\frac{1}{4}\right) + \left(-2\frac{1}{2}\right)\left(-2\frac{1}{4}\right)$

e. $\dfrac{\left(-7\frac{1}{2}\right) + 5y}{2\frac{1}{2}}$

5. Evaluate each expression for the given value. Then, use properties to simplify the original expression. Finally, evaluate the simplified expression.

   a. $\frac{1}{2}(-3x + 7)$ for $x = -1\frac{2}{3}$

   b. $\frac{4.2x - 7}{1.4}$ for $x = 1.26$

   c. Which form—simplified or not simplified—did you prefer to evaluate? Why?

6. A student submitted the following quiz. Grade the paper by marking each correct item with a √ or incorrect item with an X. Correct any mistakes.

Name Natalia

Distributive Property Quiz

a. $2(x + 5) = 2x + 10$

b. $2(3x - 6) = 6x - 6$

c. $-3(4y - 10) = -12y - 30$

d. $2.5(3 + 2y) = 7.5 + 4.5y$

e. $\frac{15x + 10}{5} = 3x + 2$

f. $\frac{8x - 4}{4} = 2x + 1$

g. $12x + 4 = 3(4x + 1)$

h. $-2x + 8 = -2(x - 4)$

ACTIVITY 6.3 Factoring Linear Expressions

You can use the distributive property to expand expressions, as you did in the previous activity, and to factor linear expressions, as Lizzie did. Consider the expression:

$$7(26) + 7(14)$$

Since both 26 and 14 are being multiplied by the same number, 7, the distributive property says you can add 26 and 14 together first, and then multiply their sum by 7 just once.

$$7(26) + 7(14) = 7(26 + 14)$$

You have factored the original expression.

The number 7 is a *common factor* of both 7(26) and 7(14).

A **common factor** is a number or an algebraic expression that is a factor of two or more numbers or algebraic expressions.

1. Factor each expression using the distributive property.

   a. $4(33) - 4(28)$

   b. $16(17) + 16(13)$

The distributive property can also be used to factor algebraic expressions. For example, the expression $3x + 15$ can be written as $3(x) + 3(5)$, or $3(x + 5)$. The factor, 3, is the *greatest common factor* to both terms.

The **greatest common factor (GCF)** is the largest factor that two or more numbers or terms have in common.

When factoring algebraic expressions, you can factor out the greatest common factor from all the terms.

WORKED EXAMPLE

Consider the expression $12x + 42$.

The greatest common factor of $12x$ and 42 is 6. Therefore, you can rewrite the expression as $6(2x + 7)$.

How can you check to make sure you factored correctly?

It is important to pay attention to negative numbers. When factoring an expression that contains a negative leading coefficient, *it* is preferred to factor out the negative sign.

WORKED EXAMPLE

Consider the expression $-2x + 8$. You can think about the greatest common factor as being the coefficient of -2.

$$\begin{aligned} -2x + 8 &= (-2)x + (-2)(-4) \\ &= -2(x - 4) \end{aligned}$$

2. Rewrite each expression by factoring out the greatest common factor.

a. $7x + 14$

b. $9x - 27$

c. $10y - 25$

d. $8n + 28$

e. $-3x - 27$

f. $-6x + 30$

So, when you factor out a negative number, all the signs will change.

Often, especially in future math courses, you will need to factor out the coefficient of the variable, so that the variable has a coefficient of 1.

3. Rewrite each expression by factoring out the coefficient of the variable.

a. $10x - 45$

b. $-2x + 3$

c. $-x + 4$

d. $-x - 19$

4. Rewrite each expression by factoring out the GCF.

a. $-24x + 16$

b. $-4.4 - 1.21z$

c. $-27x - 33$

d. $-2 - 9y$

e. $28x + (-35) - 21x$

5. Evaluate each expression for the given value. Then, factor the expression and evaluate the factored expression for the given value.

a. $-4x + 16$ for $x = 2\frac{1}{2}$

b. $30x - 140$ for $x = 5.63$

c. Which form—simplified or not simplified—did you prefer to evaluate? Why?

Talk the Talk

Flexible Expressions

As you have seen, you can rewrite expressions by factoring out a GCF or by factoring out the coefficient of the variable. You can also rewrite expressions by factoring out any value. For example, some of the ways $6x + 8$ can be rewritten are provided.

$2(3x + 4)$ $6\left(x + \frac{4}{3}\right)$ $-2(-3x - 4)$

$-6\left(-x - \frac{4}{3}\right)$ $\frac{1}{2}(12x + 16)$ $-\frac{1}{2}(-12x - 16)$

Rewrite each expression in as many ways as you can by factoring the same value from each term.

1. $4x - 12$

2. $-3x + 15$

3. $10 - 20y$

4. $-8y + 9$

Lesson 6 Assignment

Write

Match each term to the correct example.

1. factor	a. the 6 in $6(x) + 6(3)$
2. coefficient	b. $-6x - 18 = -6(x + 3)$
3. common factor	c. the 4 in $4x + 3$

Remember

The distributive property states that when a, b, and c are any real numbers, then $a(b + c) = ab + ac$.

The distributive property makes it possible to write numeric and algebraic expressions in equivalent forms by expanding and factoring expressions.

Practice

Use the distributive property to rewrite each expression in its equivalent form.

1. $4(x + 3)$

2. $-7(4 - y)$

3. $6(3x - 4 + 5x)$

4. $\dfrac{9a - 3}{3}$

5. $\dfrac{0.4(0.3m + 0.6)}{1.2}$

6. $-9\frac{2}{3}\left(-2\frac{1}{4}a + 8\frac{1}{4}\right)$

Lesson 6 Assignment

Rewrite each linear expression by factoring out the greatest common factor.

7. $64x + 24$

8. $-5y - 35$

9. $36 - 8z$

10. $54n - 81$

Rewrite each linear expression by factoring out the coefficient of the variable.

11. $-2x + 5$

12. $3x - 8$

13. $\frac{-1}{2}x + 6$

14. $-x - 10$

Lesson 6 Assignment

Prepare

Write each phrase as a mathematical expression.

1. the sum of 6 less than a number and 3

2. the distance between a number and 2 on the number line

3. half as many as 7 more than a number

4. an amount, shared equally among 5 people

TOPIC 1 SELF-REFLECTION

Name: ______________________________

Operating with Rational Numbers

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill.**

I can demonstrate an understanding of the standards in the *Operating with Rational Numbers* topic by:

TOPIC 1: *Operating with Rational Numbers*	Beginning of Topic	Middle of Topic	End of Topic
explaining why the quotient of two integers, except when the divisor is 0, is always a rational number.	○	○	○
converting a fraction to a decimal and explaining how the decimal form of a rational number terminates or repeats.	○	○	○
applying properties of operations to calculate numbers in any form and convert among numeric forms when necessary.	○	○	○
solving multi-step real-world and mathematical problems with numbers in any form (whole numbers, fractions, and decimals), using tools strategically.	○	○	○
solving real-world problems by adding, subtracting, multiplying, and dividing rational numbers.	○	○	○
using properties of operations to add, subtract, multiply, and divide rational numbers.	○	○	○
applying proportional reasoning to solve percent error problems.	○	○	○

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Operating with Rational Numbers*	Beginning of Topic	Middle of Topic	End of Topic
evaluating algebraic expressions with rational coefficients.	○	○	○
using properties of operations to write equivalent expressions.	○	○	○
factoring and expanding linear expressions with rational coefficients using the distributive property.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Operating with Rational Numbers* topic.

____

____

____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

____

____

____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

____

____

____

TOPIC 1 SUMMARY

Operating with Rational Numbers Summary

LESSON 1 Adding and Subtracting Rational Numbers

You can use what you know about adding and subtracting integers to solve problems with positive and negative fractions and decimals.

For example, yesterday Natalia was just \$23.75 below her fundraising goal. She got a check today for \$12.33 to put toward the fundraiser. Describe Natalia's progress toward the goal.

$$-\$23.75 + \$12.33 = -\$11.42$$

Natalia will still be below her goal because $-11.42 < 0$.

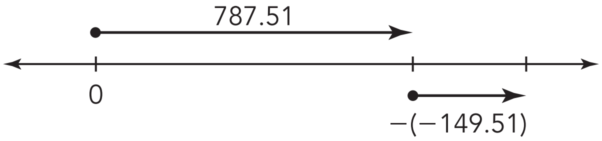
The difference between two numbers is a measure of the distance between the numbers.

For example, the freezing point of chlorine is −149.51°F. The freezing point of zinc is 787.51°F. How many more degrees is the freezing point of zinc than the freezing point of chlorine?

A model can help you estimate that the answer will be greater than 787.51.

$$787.51 - (-149.51) = 937.02$$

The freezing point of zinc is 937.02°F more than the freezing point of chlorine.



NEW KEY TERMS

- percent error [error porcentual]
- variable [variable]
- algebraic expression [expresión algebraica]
- linear expression [expresión lineal]
- constraint
- evaluate an algebraic expression [evaluar una expresión algebraica]
- factor [factor]
- coefficient [coeficiente]
- common factor [factor común]
- greatest common factor (GCF) [máximo común factor/divisor]

LESSON 2 Quotients of Integers

The sign of a negative rational number in fractional form can be placed in front of the fraction, in the numerator of the fraction, or in the denominator of the fraction.

$$-\frac{3}{5} = \frac{-3}{5} = \frac{3}{-5}$$

You can use what you know about integers and rational numbers to solve real-world problems involving operations with real numbers.

In 4 hours in direct sunlight the water level in a bucket changes $-\frac{1}{8}$ inch. How fast is the water level changing per hour?

$$-\frac{1}{8} \div 4 = -\frac{1}{32}$$

The water level changes $-\frac{1}{32}$ inch per hour.

LESSON 3 Simplifying Expressions to Solve Problems

You can use the rules you have learned to operate with rational numbers in order to solve problems and equations.

Percent error is one way to report the difference between estimated values and actual values.

$$\text{percent error} = \frac{\text{actual value} - \text{estimated value}}{\text{actual value}}$$

For example, an airline estimates that they will need an airplane that seats 416 passengers for the 8 A.M. flight from Austin to Orlando. Calculate the percent error if 380 actual passengers are booked.

$$\frac{380-416}{380} = \frac{-36}{380} \approx -9.5\%$$

The airline served 9.5% fewer passengers than they expected.

Expressions with variables can be evaluated for rational numbers.

For example, evaluate the expression $-12\frac{1}{2} - 3v$ for $v = -5$.

Substitute -5 for v and solve:

$$\begin{aligned} -12\tfrac{1}{2} - 3(-5) &= -12\tfrac{1}{2} - (-15) \\ &= -12\tfrac{1}{2} + 15 \\ &= 2\tfrac{1}{2} \end{aligned}$$

When $v = -5$, the value of the expression is $2\frac{1}{2}$.

LESSON
4

Using Number Properties to Interpret Expressions with Signed Numbers

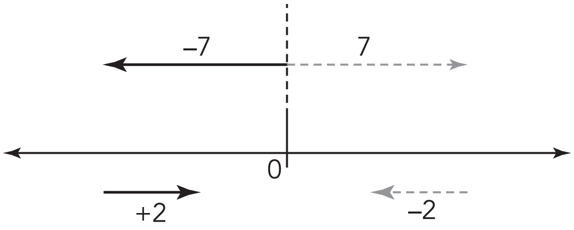
You can use reflections across 0 on the number line to determine the opposite of an expression.

For example, consider the expression $-7 + 2$. When the model of $-7 + 2$ is reflected across 0 on the number line, the result is $7 - 2$.

Therefore, $(-7 + 2)$ is the opposite of $(7 - 2)$.

This means that $-7 + 2 = -(7 - 2)$.

When you multiply any expression by -1, the result is the opposite of that expression. Rewriting an expression as a product with -1 is also called factoring out -1.



Using number properties can help you to solve problems involving signed numbers more efficiently.

Rewriting subtraction as addition allows you apply the commutative property to any expression involving addition and subtraction. For example, $-4.5 - 3 + 1.5 = -4.5 + 1.5 + (-3)$.

You can apply what you know about the zero property of addition and the associative property to expressions with positive and negative integers. For example, $-\frac{3}{4} + \left(\frac{3}{4} - \frac{1}{5}\right) = \left(-\frac{3}{4} + \frac{3}{4}\right) - \frac{1}{5} = 0 - \frac{1}{5} = -\frac{1}{5}$.

You can use the distributive property to find the opposite of an expression. For example, the opposite of the expression $-3 - 7$ would be $-(-3 - 7)$, which is the same as $-1(-3 - 7)$. $-1(-3 - 7) = (-1)(-3) - (-1)(7) = 3 - (-7) = 3 + 7$. Therefore, $3 + 7$ is the opposite of $-3 - 7$.

LESSON 5 Evaluating Algebraic Expressions

In algebra, a **variable** is a letter or symbol that is used to represent an unknown quantity. An **algebraic expression** is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols. A **linear expression** is any expression in which each term is either a constant or the product of a constant and a single variable raised to the first power.

Examples of linear expressions include $\frac{1}{2}x + 2$, $-3 + 12.5x$, $-1 + 3x + \frac{5}{2}x - \frac{4}{3}$, and $4y$.

In mathematics, it is sometimes necessary to set *constraints* on values. A **constraint** is a condition that a solution or problem must satisfy. A constraint can be a restriction set in advance of solving a problem or a limit placed on a solution or graph so the answer makes sense in terms of a real-world scenario.

To **evaluate an algebraic expression**, you replace each variable in the expression with a number or numeric expression and perform all possible mathematical operations.

For example, evaluate $-\frac{1}{2}b + 2$ for $b = -8$.

Substitute the value for the variable. → $-\frac{1}{2}(-8) + 2$

Use the order of operations to simplify. → $4 + 2 = 6$

LESSON 6 Rewriting Expressions Using the Distributive Property

To **factor** an expression means to rewrite the expression as a product of factors. A **coefficient** is a number that is multiplied by a variable in an algebraic expression. When a variable has no coefficient, the understood coefficient is 1.

For example, you can factor the expression $2x + 2$ and rewrite it as the product of two factors.

$$2x + 2 = 2(x + 1)$$

The distributive property states that if a, b, and c are real numbers, then $a(b + c) = ab + ac$. The property also holds if addition is replaced with subtraction: $a(b - c) = ab - ac$.

For example, use the distributive property to rewrite the expression $-3(5b - 2)$ in an equivalent form.

$$-3(5b - 2) = (-3)(5b) - (-3)(2) = -15b + 6$$

You can use the distributive property to expand expressions and to factor linear expressions. For example, in the expression $7(26) + 7(14)$, the number 7 is a *common factor* of both $7(26)$ and $7(14)$. A **common factor** is a number or an expression that is a factor of two or more numbers or algebraic expressions. The expression $7(26) + 7(14)$ can be factored and rewritten as $7(26 + 14)$.

The distributive property can also be used to factor algebraic expressions. When factoring algebraic expressions, you can factor out the *greatest common factor* from all the terms. The **greatest common factor (GCF)** is the largest factor that two or more numbers or terms have in common.

For example, consider the expression $12x + 42$. The greatest common factor of $12x$ and 42 is 6. Therefore, you can rewrite the expression as $6(2x + 7)$.

When factoring an expression, examine the structure of the expression first. If the expression contains a negative leading coefficient, you can factor out the negative factor.

For example, consider the expression $-2x + 8$. You can think about the greatest common factor as being -2.

$$\begin{aligned} -2x + 8 &= (-2)x + (-2)(-4) \\ &= -2(x - 4) \end{aligned}$$

TOPIC

2

For many kinds of rentals—e.g., limos or taxis—there is a fixed charge plus a variable charge, which is an amount charged per mile.

Two-Step Equations and Inequalities

LESSON 1 Modeling Equations as Equal Expressions **413**

LESSON 2 Solving Equations Using Algebra Tiles **427**

LESSON 3 Solving Equations on a Double Number Line **455**

LESSON 4 Using Inverse Operations to Solve Equations **467**

LESSON 5 Using Inverse Operations to Solve Inequalities **487**

1

Modeling Equations as Equal Expressions

OBJECTIVES

- Create and interpret pictorial models to represent equal expressions.
- Write an equation to represent a situation and interpret the parts of the equation.
- Solve word problems leading to equations of the form $ax + b = c$.

NEW KEY TERM

- equation

You have learned about both numeric expressions and algebraic expressions.

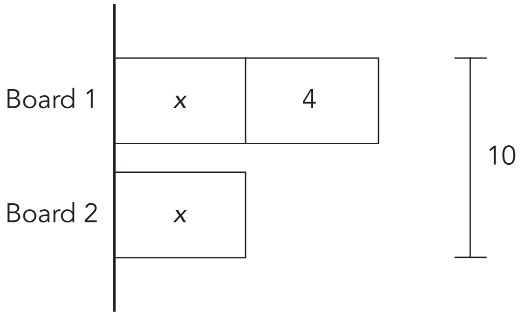
How can you model situations using equal algebraic expressions?

Getting Started

You can draw a bar model to represent a situation like this.

In the Dog House

To build a dog house, you and your friends cut a 10-foot board into two boards. One of the boards is 4 feet longer than the other. How long is each board?



1. Explain what each part of the model represents in terms of this situation.

    a. What does the number 4 represent?

    b. What does the variable x represent?

    c. What does the number 10 represent?

2. Use the model to explain what each expression means in terms of this situation.

   a. What does the expression $2x$ represent?

   b. What does the expression $2x + 4$ represent?

3. How long is each board?

ACTIVITY 1.1

Creating a Model to Represent Equal Expressions

Fido and Jet are two small dogs. Fido weighs exactly 10 pounds more than Jet. Together, they weigh exactly 46 pounds.

1. Draw a bar model to represent this situation. Let j equal Jet's weight.

2. Use your model to explain what each expression represents in terms of the situation.

   a. What does the expression 46 represent?

   b. What does the expression $2j$ represent?

c. What does the expression $j + 10$ represent?

d. What does the expression $2j + 10$ represent?

3. How much does each dog weigh? Use the model to help you solve the problem.

ACTIVITY 1.2

Creating a Model to Solve an Equation

Ask Yourself . . .
Which tool or strategy is most efficient?

You and your friends Alejandro and Kaya decide to make some money during summer vacation by building and selling dog houses. To get the business started, Alejandro contributes $25.55, and Kaya contributes $34.45 to buy equipment and materials. You all agree that each person will earn the same amount of money after Alejandro and Kaya get back what they invested. Your business earns a total of $450.

1. Draw a bar model to represent this situation.

2. Compare your models with your classmates' models.

   a. What unknown quantity or quantities are represented in the model?

   b. What algebraic expressions can you write to represent different parts of the situation?

You can represent the model you drew as a mathematical sentence using operations and an equals sign. An **equation** is a mathematical sentence created by placing an equal sign (=) between two expressions.

3. Write an equation to show that the total amount that you, Kaya, and Alejandro earn, including the amounts Kaya and Alejandro invested, is equal to $450.

4. Describe how the different parts of the equation are represented in the model and in the situation.

5. How much money does each person get at the end of the summer? Use your model to solve the problem.

6. Explain how the solution is represented in the equation.

Remember . . .
The solution to an equation is a value for the unknown that makes the equation true.

ACTIVITY

1.3

Solving Addition Equations

In a small town, there are two main sections called the Hill Section and the Lake Section. The town has a population of 3496. The number of people who live in the Hill Section is 295 more than twice the number of people who live in the Lake Section.

1. Draw a bar model to represent this situation.

2. Use your model to write an equation that represents the situation.

3. How many people live in each section of town? Use your model to help you solve the problem.

4. Explain how the solution is represented in the equation.

The members of a small town's local arts council are selling raffle tickets. The art council decides that the top three raffle ticket sellers will share a portion of the profits. The second-place seller will receive twice as much as the third-place seller. The first-place seller will receive $20 more than the second-place seller. The profit portion they will share is $200.

5. Draw a bar model to represent this situation.

6. Use your model to write an equation that represents the situation.

7. How much will each of the top three sellers receive? Use your model to help you solve the problem.

8. Explain how the solution is represented in the equation.

ACTIVITY
1.4

Solving a Subtraction Equation

Trung is 3 years younger than his brother, Eduardo. The sum of the brothers’ ages is 21.

1. Draw a bar model to represent this situation.

2. Use your model to write an equation that represents the situation.

3. How old are Eduardo and Trung? Use your model to help you solve the problem.

4. Explain how the solution is represented in the equation.

Talk the Talk

Consider the Possibilities!

Think about all the equations you modeled and solved in this lesson.

- $2x + 4 = 10$
- $2j + 10 = 46$
- $3x + 60 = 450$
- $3p + 295 = 3496$
- $5p + 20 = 200$
- $2j - 3 = 21$

1. How are all of these equations similar in structure?

2. What does it mean to solve an equation?

3. Sasha and Tango are cats. Sasha weighs exactly 2 pounds more than Tango. Together, they weigh exactly 16 pounds.
Draw a bar model to represent this situation. Then, write an equation and determine Tango's and Sasha's weights.

Lesson 1 Assignment

Write

Write a definition for *equation* in your own words. Use an example to illustrate your definition.

Remember

The solution to an equation is a value for the unknown that makes the equation true.

Practice

1. The local aquatic club recently held a fundraiser to raise money for a local charity. The swimmers received money for each lap that they swam during a one-week period. The three swimmers who raised the most money were Samantha, Chris, and Jorge. Together they swam a total of 2125 laps. Chris swam three times as many laps as Samantha, and Jorge swam 25 more laps than Chris.

   How many laps did each swimmer swim?

   a. Draw a picture to represent the situation. Label the unknown parts with variables and the known parts with their values.

   b. Determine the number of laps each person swam using the picture you created. Explain your reasoning.

Lesson 1 Assignment

c. Write an expression for the number of laps each person swam. Let L represent the number of laps swam by Samantha.

d. Write an equation to represent this situation.

e. When the swimmers received \$2 for every lap they swam, how much did each swimmer earn for charity?

Prepare

Solve each equation.

1. $2x = 10$

2. $-3x = 45$

3. $x + 8 = -10$

4. $x - 2 = 12$

2

Solving Equations Using Algebra Tiles

OBJECTIVES

- Model and solve equations.
- Represent solutions for equations on number lines.
- Verify solutions to equations.

You have modeled situations using equal expressions.

How do you maintain equality when solving equations?

Getting Started

Drawing Conclusions

1. When ☺☺ = ☆☆, what can you conclude?

2. When ☺☺☺☺ = ☆☆, what can you conclude?

3. When ☺☺☺☺☆ = ☆☆, what can you conclude?

4. When ☺☺☺☺ = ☺☆, what can you conclude?

5. How did you determine the answer to Question 1?

6. How did you determine the answer to Question 2?

7. How did you determine the answer to Question 3?

8. How did you determine the answer to Question 4?

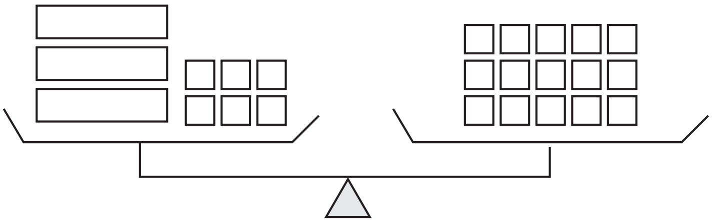
ACTIVITY 2.1

Maintaining Balance

1. Each representation shows a balance. What will balance one rectangle ▭ in each problem? Adjustments can be made in each pan as long as the balance is maintained. Describe your strategies.

The diagrams show pan balances. The two pans should be balanced on a center point. In the diagrams, the center balance point is represented by a triangle.

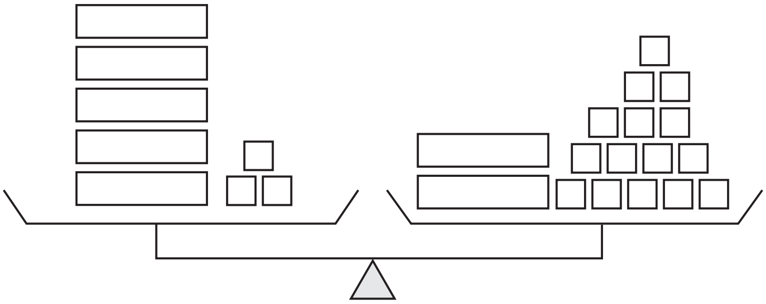
a.



Strategies:

What will balance one rectangle?

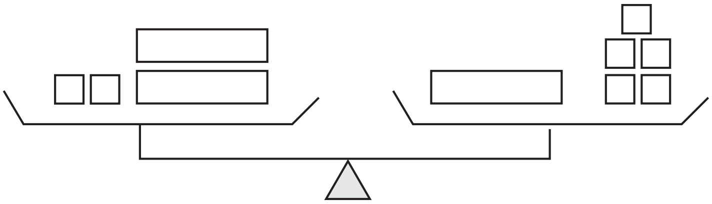
b.



Strategies:

What will balance one rectangle?

c.



Strategies:

What will balance one rectangle?

2. Generalize the strategies for maintaining balance. Complete each sentence.

   a. To maintain balance when you subtract a quantity from one side, you must

   ____

   ____

   b. To maintain balance when you add a quantity to one side, you must

   ____

   ____

   c. To maintain balance when you multiply a quantity by one side, you must

   ____

   ____

   d. To maintain balance when you divide one side by a quantity, you must

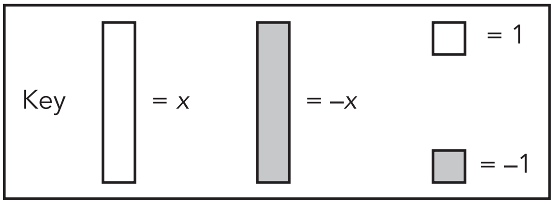
   ____

   ____

ACTIVITY 2.2

Two Steps Back

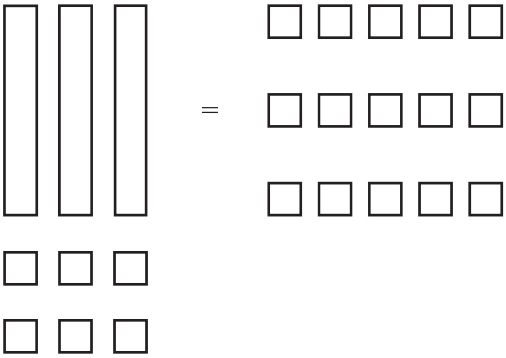
You can rewrite an equation using numbers and variables. The Key represents the value of different algebra tiles.



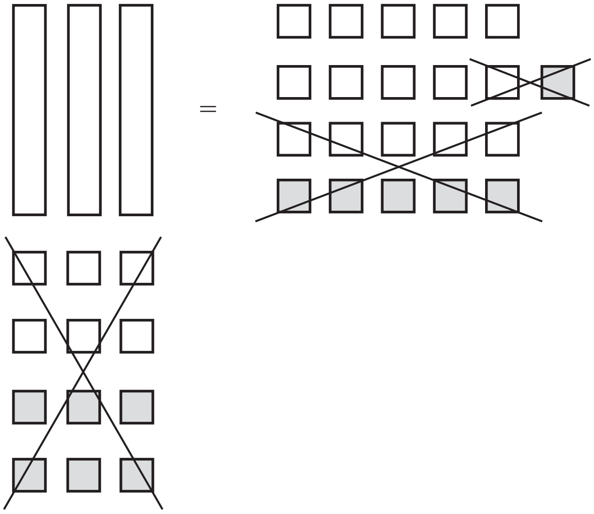
WORKED EXAMPLE

You can use the same strategies you used when maintaining balance to solve an equation with algebra tiles.

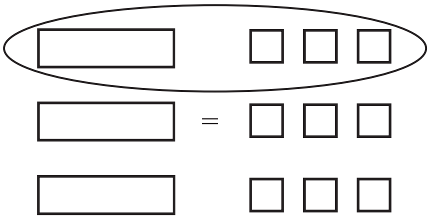
The algebra tiles represent the equation $3x + 6 = 15$.



Step 1: Subtract 6 units from each side, which creates a zero pair on the left side of the equation.



Step 2: Divide the 9 units remaining into 3 equal groups to determine the value of one rectangle.



1. a. What is the value of *x*?

   b. Substitute the value of *x* back into the original equation $3x + 6 = 15$. Does the value of *x* maintain balance in the original equation? Show your work.

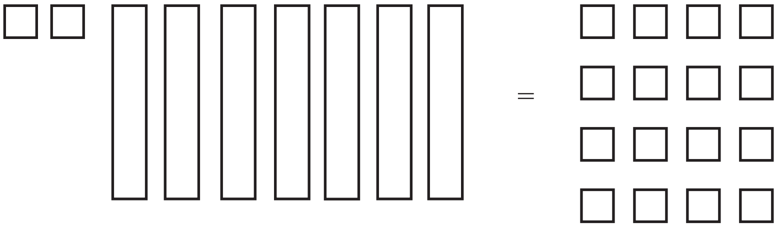
   c. Describe the order of operations used in the original equation.

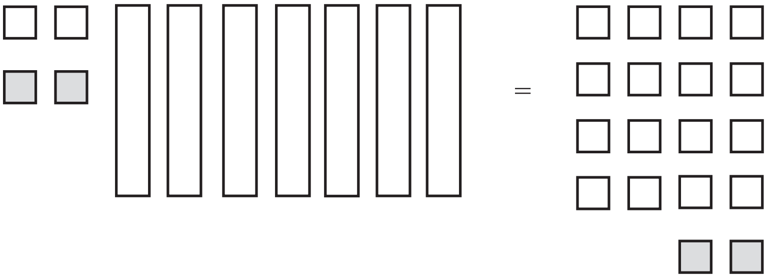
   d. Compare the steps used to solve the equation to the order of the operations in the original equation.

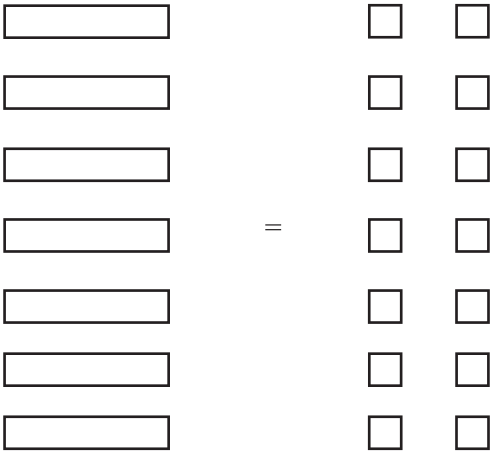
When you "undo" operations, it is often simpler to undo them in the reverse of the order of operations. In other words, the operation that comes *last* in the original equation should be undone *first*.

2. Write a sentence to describe how to undo the operations to solve each equation. Then, use algebra tiles to solve each equation and verify your solution. Graph each solution on a number line.

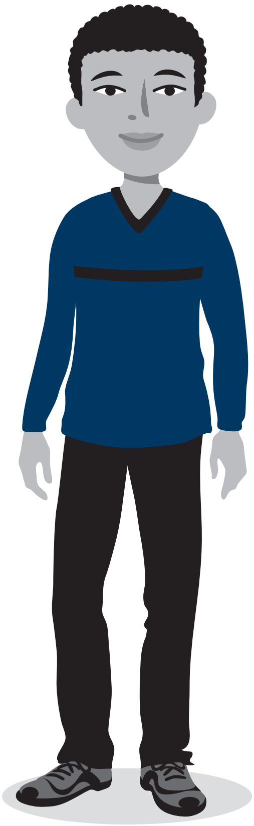
a. $2 + 7x = 16$

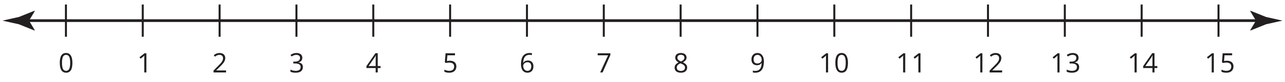






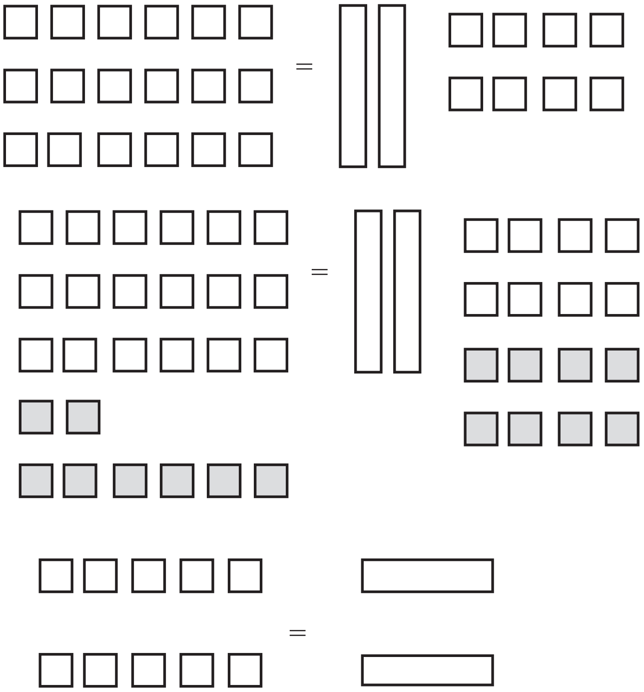
You can represent solutions to equations algebraically, and you can also represent them on a number line by drawing a number line and plotting a point representing the solution on the number line.

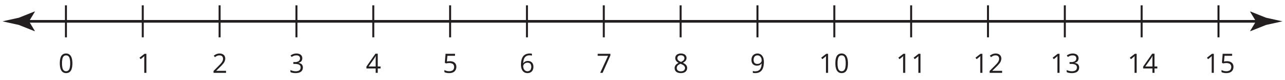




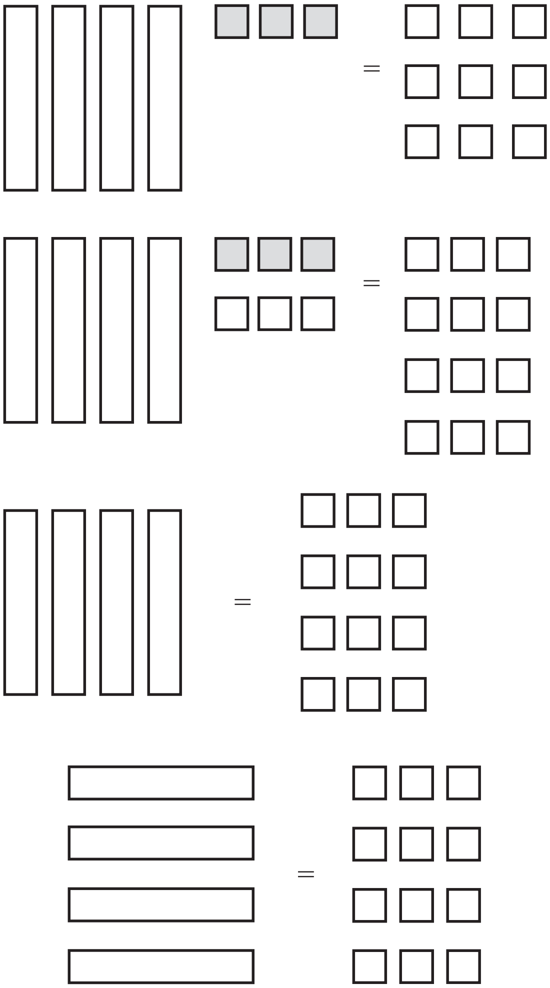
Ask Yourself . . .
Which tool or strategy is most efficient?

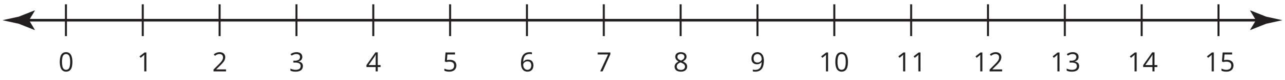
b. $18 = 2x + 8$





c. $4x - 3 = 9$

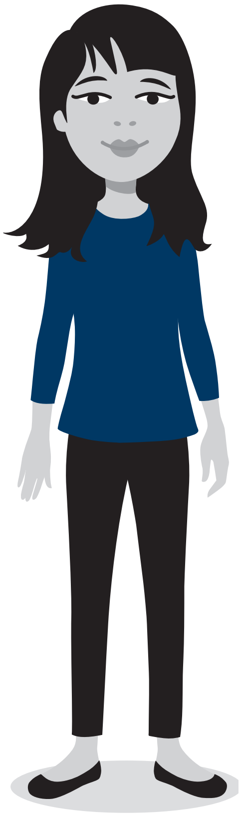




3. Determine whether each value is a solution to the equation. Explain your reasoning.

   a. Is $q = 9$ a solution to the equation $2q - 6 = 24$?

Remember . . .
a value is a solution when that value substituted into the equation makes the equation true.



   b. Is $b = 4$ a solution to the equation $12 = \frac{b}{2} + 10$?

   c. Is $n = 11$ a solution to the equation $10 + 3n = 43$?

   d. Is $x = 5$ a solution to the equation $10 - \frac{x}{5} = 5$?

ACTIVITY
2.3

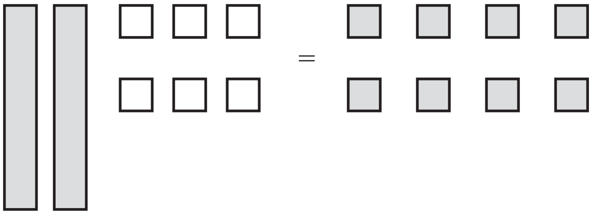
Equations with Negatives

You can use algebra tiles to solve equations with negative coefficients.

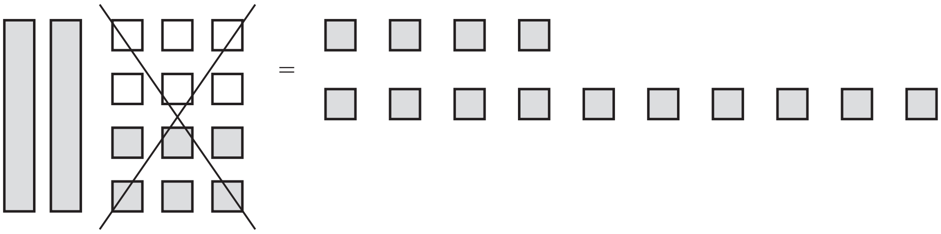
WORKED EXAMPLE

$-2x + 6 = -8$

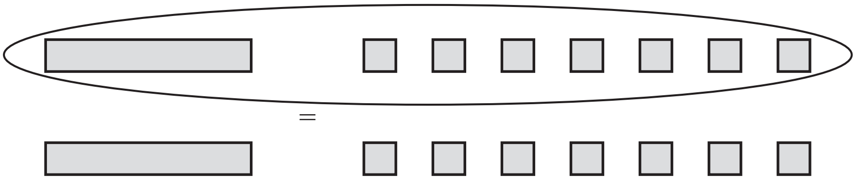
Step 1: Model the equation with algebra tiles.



Step 2: Subtract 6 units from each side, which creates a zero pair on the left side of the equation.



Step 3: Divide the tiles into 2 equal groups to determine the value of one rectangle.

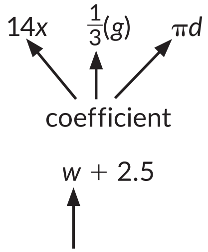


Step 4: Since one rectangle represents -*x* or the opposite of *x* change the sign of the algebra tiles on both sides of the equal sign to determine the value of *x*.

$x = 7$

Remember . . .
A number that is multiplied by a variable in an algebraic expression is called a coefficient.

Examples



The coefficient is 1 even though it is not shown.

1. Use algebra tiles and explain the steps you used to solve each equation. Then, plot the solution on a number line.

   a. $-3x + 4 = -14$

b. $-2x - 6 = 10$

c. $-3x + 2 = -10$

d. $5x - 2 = -12$

e. $-2x - 6 = 3x + 9$

Talk the Talk

Algebra Tiles and Equations

Three equations are displayed when you walk into class:

$\frac{2}{3}x + 4 = 7$

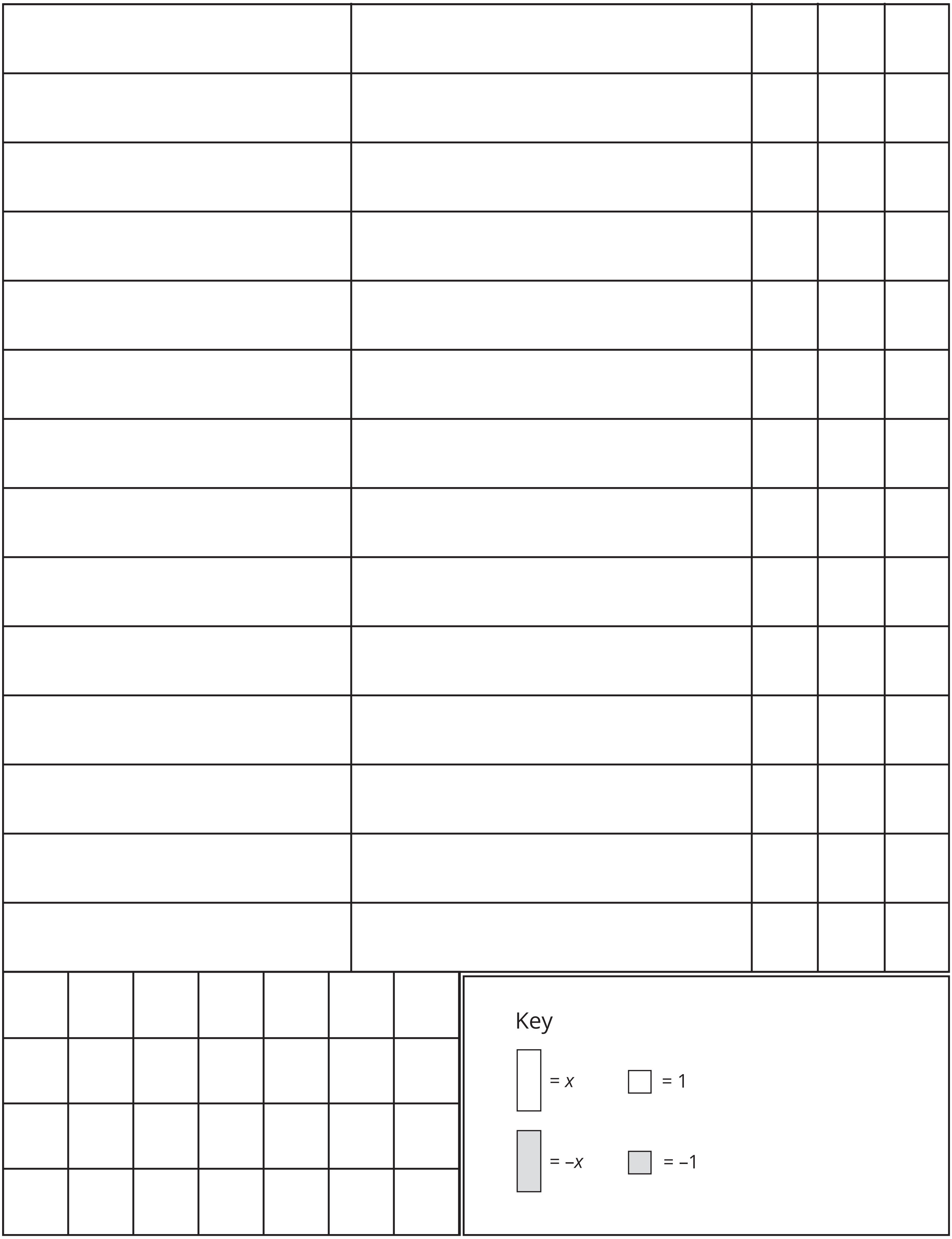
$18x + 63 = 135$

$-2x + 6 = -12$

1. Which of the equations would you model and solve using algebra tiles? Explain your reasoning.

2. Model and solve the equation $-5x + 8 = -2$.

Algebra Tiles



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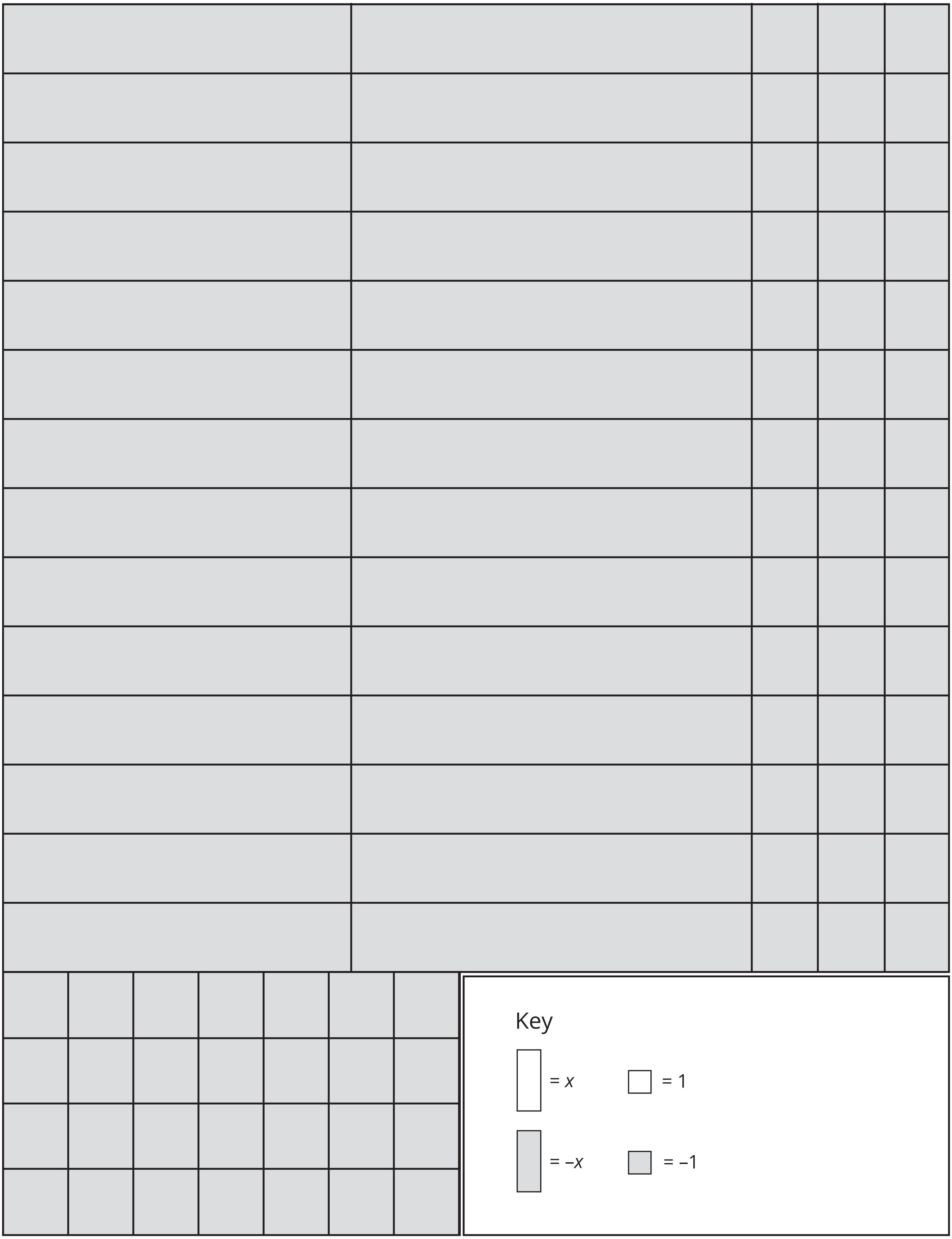
So you can cut out the Algebra Tiles on the other side.

Algebra Tiles



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So you can cut out the Algebra Tiles on the other side.

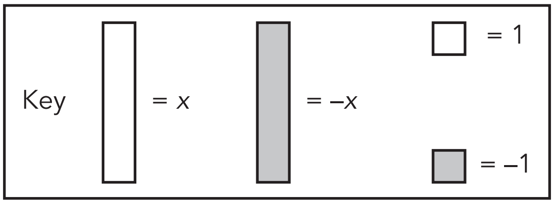
Lesson 2 Assignment

Write

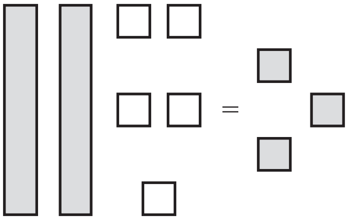
Explain what it means to maintain equality when solving equations.

Remember

You can use algebra tiles to model and solve equations. Use the following key which represents the value of different algebra tiles.



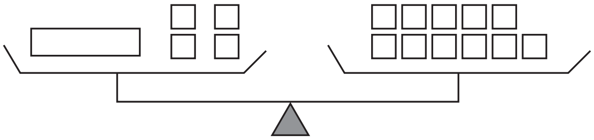
To model the equation $-2x + 5 = -3$, use the following algebra tiles.



Practice

1. A local middle school has a Math and Science Club that holds meetings after school. The club has decided to enter a two-day competition that involves different math and science challenges. The first day of competition involves solving multi-step math problems. Teams will receive one point for every problem they get correct. Halfway through the day, the middle school team has 4 points. After a dinner break, the team does more problems and is able to finish the day with 11 points.

   The representation shows a balance for this situation. The left side of the balance represents the 4 points the team had at midday plus the additional points they got after dinner. The right side of the balance represents the total points they had at the end of the day. What will balance 1 rectangle in this representation? Describe your strategy.



Lesson 2 Assignment

Strategy:

What will balance 1 rectangle?

How many problems did they get correct after dinner?

Model and solve each equation using algebra tiles.

2. $-4x + 3 = -5$

Lesson 2 Assignment

3. $2x - 8 = 6$

4. $3x + 8 = -7$

5. $-5x - 9 = 11$

Lesson 2 Assignment

Determine whether each value is a solution to the equation. Explain your reasoning. When the value is a solution to the equation, graph the solution on a number line.

6. Is $t = 2$ a solution to the equation $8t - 8 = 24$?

7. Is $f = -7$ a solution to the equation $2f - 21 = -35$?

8. Is $w = 8$ a solution to the equation $20 = \frac{w}{2} + 16$?

Lesson 2 Assignment

Prepare

Explain whether or not Expression B is equivalent to Expression A. When the expressions are not equivalent, determine an expression equivalent to Expression A.

1. A: $2(x - 5)$ B: $2x - 5$

2. A: $8 - 2(n + 3)$ B: $6(n + 3)$

3. A: $-(x - 4)$ B: $-x + 4$

3

Solving Equations on a Double Number Line

OBJECTIVES

- Identify relationships between expressions.
- Decompose an equation to isolate the unknown.
- Model and solve equations using a double number line.
- Construct equations to solve problems by reasoning about the quantities.

Previously, you have identified expressions that are equivalent. In this lesson, you will model situations as equal expressions on a double number line.

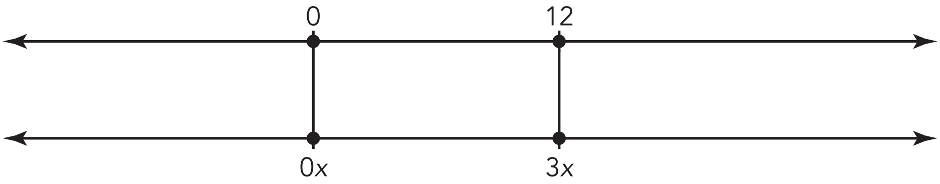
How can you maintain equality to determine the unknown quantities in linear equations?

Getting Started

Play Together, Stay Together

A double number line diagram is a model used to show equivalent relationships.

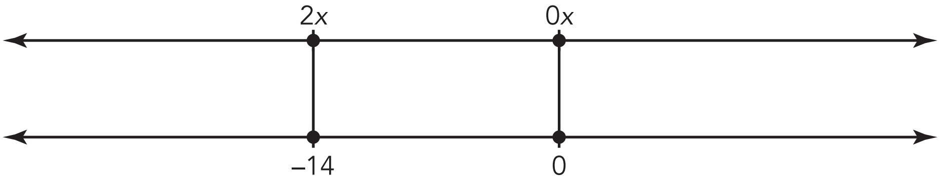
Consider this double number line. The expressions 12 and 3*x* have the same location, so they have the same value.



1. Write an equation to show that 3*x* and 12 have the same value.

2. Extend each number line in both directions by identifying and labeling additional equivalent relationships. Explain the reasoning you used to place each relationship.

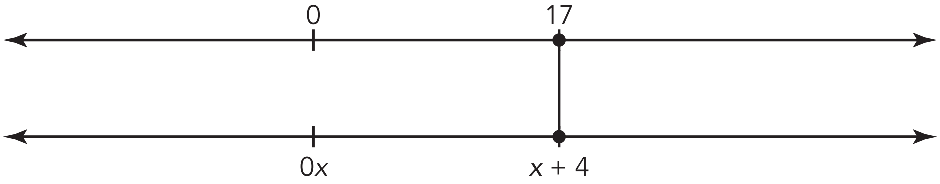
3. Consider this double number line.



   a. Write an equation to show that −14 and 2*x* have the same value.

   b. Extend each number line in both directions by identifying and labeling additional equivalent relationships. Explain the reasoning you used to place each relationship.

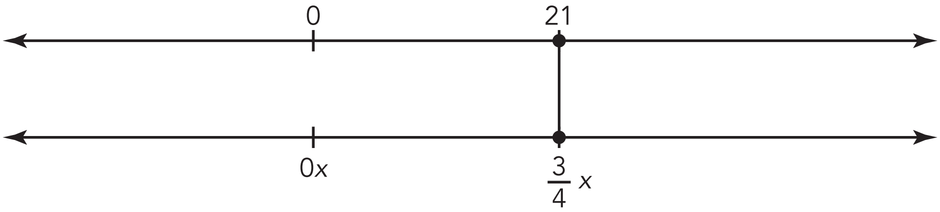
4. Consider this double number line.



a. Write an equation to show that $x + 4$ and 17 have the same value.

b. Extend each number line in both directions by identifying and labeling additional equivalent relationships. Explain the reasoning you used to place each relationship.

5. Consider this double number line.



a. Write an equation to show that $\frac{3}{4}x$ and 21 have the same value.

b. Extend each number line in both directions by identifying and labeling additional equivalent relationships. Explain the reasoning you used to place each relationship.

ACTIVITY

3.1

Solving an Equation

In the previous lesson, you modeled a problem in which Fido and Jet are two small dogs. Fido weighs exactly 10 pounds more than Jet. Together, they weigh exactly 46 pounds.

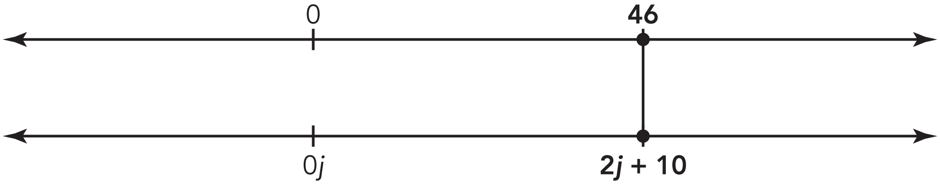
This situation can be represented by the equation $2j + 10 = 46$, where j represents Jet's weight. You can also represent this situation and solve the equation using a double number line.

Think about how to transform the equation to isolate the variable.

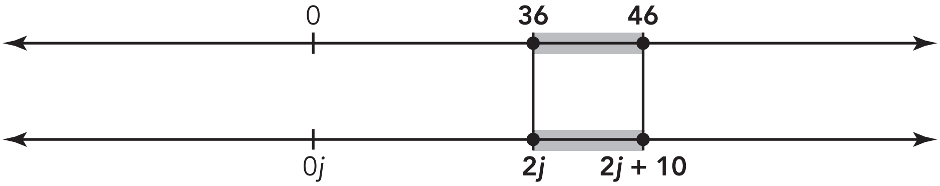
WORKED EXAMPLE

Solve the equation $2j + 10 = 46$.

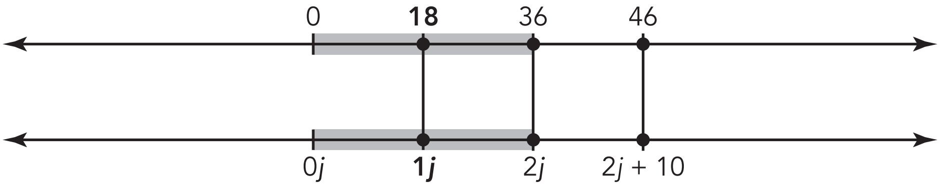
First, draw a model to set up the equation.



Next, start decomposing the variable expression. Place $2j$ in relationship to $2j + 10$. The expression $2j$ is 10 to the left of $2j + 10$. To maintain equality, place a number that is 10 to the left of 46. So, $2j = 36$.



The expression $1j$, or j, is halfway between $0j$ and $2j$. And 18 is halfway between 0 and 36. So, $j = 18$.



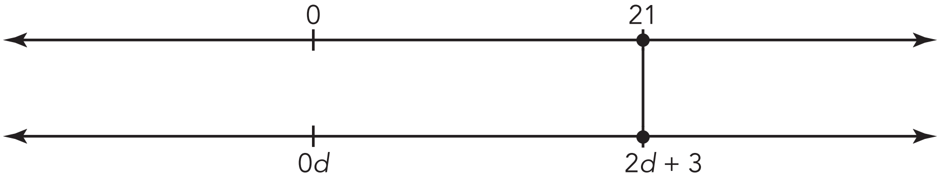
1. How can you check to see whether $j = 18$ is the solution to the original equation?

2. What does the solution $j = 18$ represent in terms of this problem situation?

3. What operation is used in each step to move toward the solution?

Is there another way to use the double number line to determine the value of j?

4. Use the double number line shown to solve the equation $21 = 2d + 3$. Describe the steps you use, including the operations represented at each step.



ACTIVITY
3.2

Practice Solving Equations

In this activity, you will use double number lines to solve equations.

1. Model each equation on the double number line given. Then, use the model to solve the equation and verify your solution. Describe the steps and operations you used and explain your reasoning.

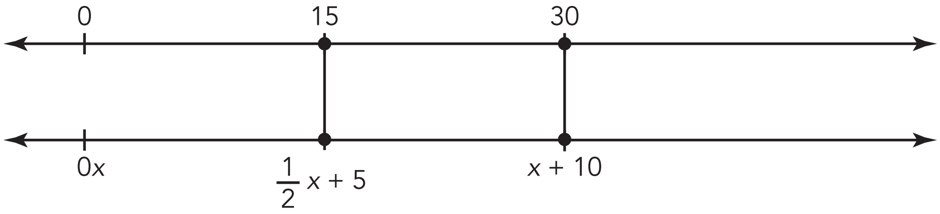
   a. $\frac{1}{2}x + 5 = 15$

   b. $52.5 = t - 3.1$

c. $4b + 4 = 20$

d. $m + 4.5 = -10$

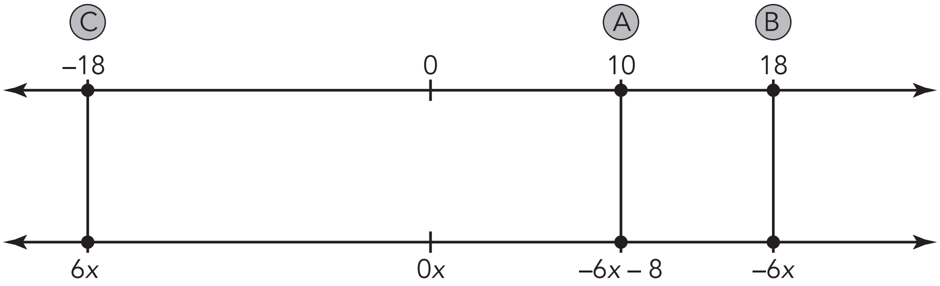
2. Jaylen showed how he started to solve the equation $\frac{1}{2}x + 5 = 15$. Describe his method. Then, complete his process to solve the equation.



ACTIVITY
3.3

Reasoning with Negatives to Solve Equations

The double number line shows one way to start to solve the equation $-6x - 8 = 10$. A through C represent the order in which the steps were completed.



1. Describe each step used to solve the equation. List the operation used at each step.
   From Step A to Step B:

   From Step B to Step C:

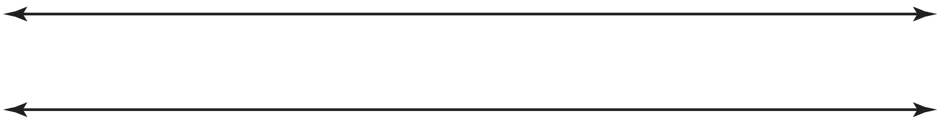
2. Between which two steps is there a reflection across 0 on the number line? What operation is used to accomplish this reflection?

3. What is the solution to the equation?

Think About . . .
How is using the opposite of the order of operations represented on the double number lines?

4. Solve each equation using a double number line. Describe your solution steps.

   a. $-x + 10 = 40$



b. $-\frac{3}{4}x - 4 = 11$

c. $-3x + 4 = 10$

Talk the Talk

Keeping It Together

You have solved a lot of different equations in this lesson and the previous lesson. Now it's your turn.

1. Start with a solution and create an equation by either multiplying both sides by a constant and then add or subtract a different constant. Describe the process you use to compose your equation. Then, give your equation to a classmate to solve.

2. Record the steps your partner uses to solve your equation.

3. Compare the steps you used to create the equation with the steps your classmate used to solve your equation. What do you notice?

4. How do you keep the expressions equal as you solve the equation?

Lesson 3 Assignment

Write

Describe what it means to solve an equation.

Remember

When solving an equation, equality must be maintained. What is done to one expression must be done to the equivalent expression to maintain equality.

Practice

Solve each equation using a double number line.

1. $4x + 12 = 24$

2. $-8x + 25 = -15$

3. $-5x - 12 = 18$

4. $20x + 55 = 495$

5. $-8 = 2x - 14$

6. $11x + 13 = -9$

Lesson 3 Assignment

Prepare

Solve each equation.

1. $2.3p = -11.73$

2. $\frac{3}{4}r = 10$

3. $y + 5.92 = 1.63$

4. $7\frac{2}{5} + t = 3\frac{1}{4}$

4

Using Inverse Operations to Solve Equations

OBJECTIVES

- Use properties of equality to solve equations.
- Write two-step equations.
- Solve two-step equations of the form $ax + b = c$ with efficiency.
- Check solutions to equations algebraically.

NEW KEY TERM

- two-step equation

You have solved equations using double number lines.

How can you use the properties of equality and inverse operations to solve equations?

Getting Started

How Does That Work?

Recall that to solve an equation means to determine the value or values for a variable that make the equation true. In the process of solving equations, you must always maintain equality, using the properties of equality.

Properties of Equality	For all numbers a, b, and c . . .
addition property of equality	If $a = b$, then $a + c = b + c$.
subtraction property of equality	If $a = b$, then $a - c = b - c$.
multiplication property of equality	If $a = b$, then $ac = bc$.
division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

1. Solve $2x + 6 = 14$ using whatever strategy you choose.

2. Explain which properties of equality you used in the process of solving the equation.

ACTIVITY
4.1

Strategies for Applying Inverse Operations

Throughout this topic, you have written and solved *two-step equations*. A **two-step equation** requires two inverse operations, or applying two properties of equality, to isolate the variable.

Josh, Matthew, and Jamal each solved $2x + 6 = 14$ in a different way. Analyze their solution strategies.

Josh 👍

$$\begin{array}{r} 2x + 6 = 14 \\ \frac{2x + 6}{2} = \frac{14}{2} \\ x + 3 = 7 \\ -3 = -3 \\ \hline x = 4 \end{array}$$

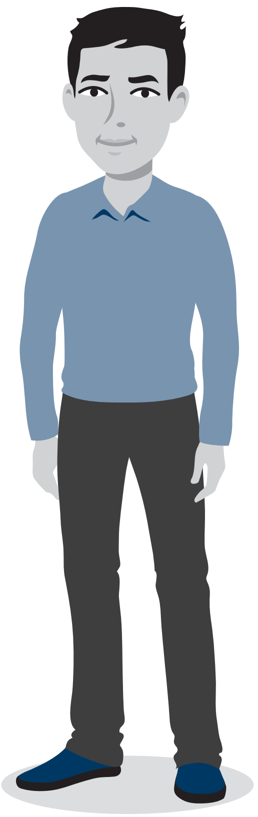
Matthew 👎

$$\begin{array}{r} 2x + 6 = 14 \\ \frac{2x}{2} + 6 = \frac{14}{2} \\ x + 6 = 7 \\ -6 = -6 \\ \hline x = 1 \end{array}$$

Jamal 👍

$$\begin{array}{r} 2x + 6 = 14 \\ -6 = -6 \\ \hline \frac{2x}{2} = \frac{8}{2} \\ x = 4 \end{array}$$

What operation is the inverse of addition? What operation is the inverse of multiplication?



1. Compare the strategies used by Josh and Matthew.

2. Compare the strategies used by Josh and Jamal.

To make the addition and subtraction simpler, you can leave fractions in improper form. They already have a common denominator.

3. Solve each equation by first applying either the addition or subtraction property of equality.

   a. $56 = -10 + 2x$

   b. $13 + \frac{x}{3} = 35$

4. Solve each equation by first applying either the multiplication or division property of equality.

   a. $56 = -10 + 2x$

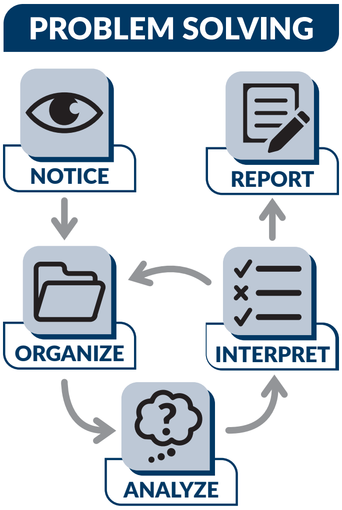
   b. $13 + \frac{x}{3} = 35$

Consider the equations and your solutions in Questions 3 and 4.

5. Do you prefer one order over the other? Why? When your preference changes depending on the equation, explain why.

ACTIVITY 4.2

Writing and Solving Two-Step Equations



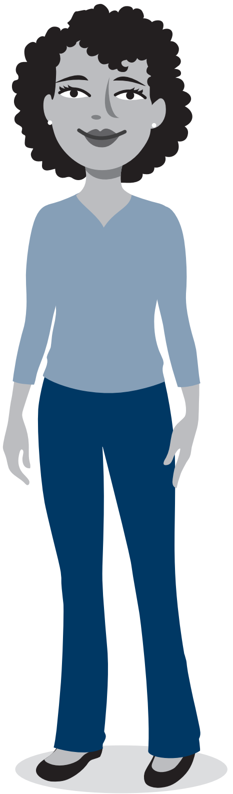
1. Paola is throwing a graduation party. She is sending invitations to her friends and family. She finds a company that charges $6 for a 10-pack of personalized invitations, plus a $5 shipping fee for the entire order, no matter how many 10-packs are ordered. Paola wants to calculate the cost of an order, based on the number of packs of invitations she orders.

   a. Define variables for the two quantities that are changing in this scenario.

   b. Write an equation that represents the total cost of any order based on the number of packs of invitations.

   c. Use your equation to determine how many packs of invitations are ordered when the total is $53. What about when the total is $29?

Remember to check each solution and determine if it is reasonable in terms of the scenario.



2. Ashley's Pet Grooming charges $15 for each dog washed and groomed on the weekend. The cost of the dog shampoo and grooming materials for a weekend's worth of grooming is $23.76. Ashley is interested in her weekend profits.

   a. Define variables for the two quantities that are changing in this scenario.

   b. Write an equation that represents the total profits based on the number of dogs groomed.

   c. Use your equation to determine how many dogs Ashley groomed when her profits were $261.24.

3. Andrew works as a pet sitter all week long but he is more in demand on some days than others. He posts his rates as $12 per visit plus a surcharge, which depends on the day. On his busiest days, Andrew can serve 8 houses for pet sitting. He is interested in his daily profits.

   a. Define variables for the two quantities that are changing in this scenario.

   b. Write an expression to represent the surcharge for 8 days.

c. Use the distributive property to simplify your expression.

d. Write an equation that represents the maximum total profits based on the surcharge for that day.

e. Determine the Saturday surcharge when Andrew's total profit is $142 by solving the equation you wrote in part (d). What is the total fee Andrew charges per house for pet sitting on a Saturday?

4. What real-world situation can you think of to model the equation $12x - 5 = 36$?

ACTIVITY
4.3

Solving Equations with Efficiency

A savvy mathematician (you!) can look at an equation, see the structure of the equation, and look for the most efficient solution strategy.

As you have seen, there are multiple ways to solve equations. Sometimes an efficient strategy involves changing the numbers in the equation—in mathematically appropriate ways!

1. Analyze each correct solution strategy to the equation $1.1x + 4.3 = 6.2$.

Mariana

$$1.1x + 4.3 = 6.2$$
$$1.1x + 4.3 - 4.3 = 6.2 - 4.3$$
$$1.1x = 1.9$$
$$x = \frac{1.9}{1.1}$$
$$x = \frac{19}{11}$$

Elena

$$1.1x + 4.3 = 6.2$$
$$11x + 43 = 62$$
$$11x + 43 - 43 = 62 - 43$$
$$11x = 19$$
$$x = \frac{19}{11}$$

a. Explain how the two solutions strategies are alike and how they are different.

b. What property of equality did Elena apply before she started solving the equation?

Remember . . .
to maintain equality, any operation applied to one side of the equation must be applied to the other side of the equation.

2. Mario used Elena's strategy to solve the equation $2.6x - 1.4 = 38$. Identify his mistake and then determine the correct solution.

Mario

$2.6x - 1.4 = 38$

$26x - 14 = 38$

$26x = 52$

$x = 2$

3. Use Elena's strategy to solve each equation. Then check your solution in the original equation.

a. $-9.6x + 1.8 = -12.3$

b. $2.99x - 1.4 = 13.55$

Now let's consider strategies to solve two different equations that contain fractions.

WORKED EXAMPLE

	$\frac{11}{3}x + 5 = \frac{17}{3}$	$\frac{1}{2}x + \frac{3}{4} = 2$
Step 1:	$3\left(\frac{11}{3}x + 5\right) = 3\left(\frac{17}{3}\right)$	$4\left(\frac{1}{2}x + \frac{3}{4}\right) = 4(2)$
Step 2:	$11x + 15 = 17$	$2x + 3 = 8$
Step 3:	$x = \frac{17 - 15}{11}$	$x = \frac{8 - 3}{2}$
	$= \frac{2}{11}$	$= \frac{5}{2}$

You should be fluent in operating with decimals and fractions, but these strategies can ease the difficulty of the calculations when solving equations.

4. Answer each question about the strategies used to solve each equation in the Worked Example.

   a. Explain Step 1. Why might this strategy improve your efficiency with solving equations?

b. What property was applied in Step 2?

c. Explain Step 3.

5. Lizzie used the strategy from the Worked Example to solve $3 = \frac{1}{4}x - \frac{1}{4}$. Identify her mistake and determine the correct solution.

Lizzie

$3 = \frac{1}{4}x - \frac{1}{4}$

$3 = 4\left(\frac{1}{4}x - \frac{1}{4}\right)$

$3 = x - 1$

$4 = x$

6. Use the strategy from the Worked Example to solve $\frac{2}{3}x + \frac{4}{5} = \frac{5}{3}$. Check your solution in the original equation.

Consider the solution strategies used to solve two more equations.

WORKED EXAMPLE

	$\mathbf{-20x + 80 = 230}$	$\mathbf{-38 = -6x - 14}$
Step 1:	$10(-2x + 8) = 10(23)$	$-2(19) = -2(3x + 7)$
Step 2:	$-2x + 8 = 23$	$19 = 3x + 7$
Step 3:	$x = \frac{23 - 8}{-2} = -\frac{15}{2}$	$\frac{19 - 7}{3} = x$
		$4 = x$

7. Answer each question about the strategies used to solve each equation in the Worked Example.

   a. How is the strategy used in this pair of examples different from the strategies used in Questions 1 and 2?

   b. When might you want to use this strategy?

   c. Use the strategy from the Worked Example to solve $44x - 24 = 216$. Check your solution in the original equation.

ACTIVITY 4.4 Solving More Equations

Number riddles are popular types of problems to solve using two-step equations.

1. Solve each number riddle by writing and solving an equation.

   a. What is a number that when you multiply it by 3 and subtract 5 from the product, you get 28?

   b. What is a number that when you multiply it by 4 and add 15 to the product, you get 79?

   c. Make a number riddle for a partner to solve.

Ask Yourself . . . How else can you represent this information?

2. Solve each equation. Check your solutions.

   a. $-17 = 2x - 8$

   b. $-\frac{1}{4} - \frac{1}{2}x = -\frac{19}{4}$

   c. $-6 = -3x - 33$

   d. $2\frac{1}{2} - \frac{1}{2}x = \frac{1}{4}$

   e. $6.4 = 4.8 + 2.4x$

Remember . . . all the strategies you learned in this lesson.

Talk the Talk

Get Creative

1. Any equation in the form $ax + b = c$ can be solved in two steps, but do you need to write out both steps to solve the equation?

   a. Isolate the variable x so that it has a coefficient of 1.

   b. Use your answer from part (a) to solve $4x + 5 = 61$.

2. Write a real-world situation that can be modeled by each equation.

a. $3b - 5 = 22$

b. $19 = 2.5 + 4.5n$

c. $\frac{1}{2}t + 2 = 16$

Lesson 4 Assignment

Write

Explain the process of solving a two-step linear equation.

Remember

You can use the properties of equality to rewrite equations and increase your efficiency with solving equations.

- When the equation contains fractions, you can multiply both sides of the equation by the least common denominator.
- When the equation contains decimals, you can multiply both sides of the equation by a multiple of 10.
- When the equation contains large values, you can divide both sides of the equation by a common factor.

Practice

1. A middle school has a Math and Science Club that holds meetings after school. The club has decided to enter a two-day competition that involves different math and science challenges. The first day of competition involves solving multi-step math problems. Teams will receive three points for every problem they get correct, and an additional 12 points for winning the competition for the day.

    a. Write an equation to represent the number of points received by the winning team on one day of the competition. Remember to define your variable(s).

b. The winning team finishes the day with 39 points. Solve the equation and interpret the solution in the context of the problem.

c. The second day of the competition was the science portion, involving hands-on science problems. When the winning team for the day finished with 27 points, how many science problems did they get correct? Write and solve an equation to answer the question.

2. Employees at an electronics store earn a base salary plus a 20% commission on their total sales for the year. Suppose the base salary is $40,000.

   a. Write an equation to represent the total earnings of an employee. Remember to define your variable(s).

   b. Eduardo wants to make $65,000 this year. How much must he make in sales to achieve this salary? Solve the equation and interpret the solution in the context of the problem.

c. Describe the equation $52{,}000 + 0.3s = 82{,}000$ in terms of the problem situation.

3. The manager of a home store is buying lawn chairs to sell at his store. Each pack of chairs contains 10 chairs. The manager will sell each chair at a markup of 20% of the wholesale cost, plus a \$2.50 stocking fee.

   a. Write an equation that represents the retail price of a chair, r, in terms of the wholesale price, w.

   b. Use your equation to calculate the retail price of the chair when the wholesale price is \$8.40.

   c. Use your equation to calculate the wholesale price when the retail price is \$13.30.

Lesson 4 Assignment

4. What is a number that when you multiply it by 0.9 and subtract 6.3 from the product, you get 4.5? Write and solve an equation to solve the riddle.

5. Chris and four of his friends had a car wash to earn some extra money. They split the profits and Chris got an extra $18 to repay his parents for the car wash supplies. When Chris got $32, how much total money did they split among themselves? Write and solve an equation to answer the question.

6. Kaya bought a laptop for $500. It was marked $50 off because it was out of the box and slightly scratched. She also got a 20% student discount, which was taken off the original price. What was the original price of the laptop? Write and solve an equation to answer the question.

Lesson 4 Assignment

7. Solve each equation. Check your solution.

a. $1 = 3x - 11$

b. $7x + 2 = -12$

c. $9 = \frac{y}{4} - 2$

d. $13 - \frac{a}{7} = 6$

e. $-5b - 12 = 18$

f. $-8 = 2h - 14$

g. $-6x - 21 = 18$

h. $-14 = -10 + 2x$

i. $45.99c - 50 = 133.96$

j. $1.1x + 2.35 = -8.1$

Lesson 4 Assignment

Prepare

Solve each inequality.

1. $2.3p < -13.8$

2. $\frac{1}{4}r > 10$

3. $y + 1.2 \leq 0.5$

4. $-2 + t > +1\frac{1}{2}$

5

Using Inverse Operations to Solve Inequalities

OBJECTIVES

- Solve and graph two-step inequalities.
- Solve word problems leading to inequalities of the form $ax + b > c$ and $ax + b < c$.
- Graph the solution sets of inequalities and interpret the solutions in context.

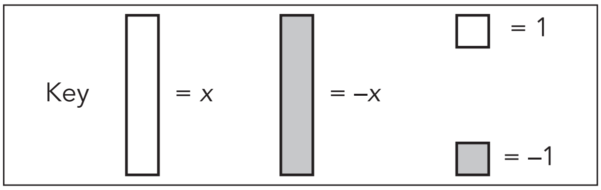
You have used properties of equality and inverse operations to solve equations.

How can you use what you know about equations to solve inequalities?

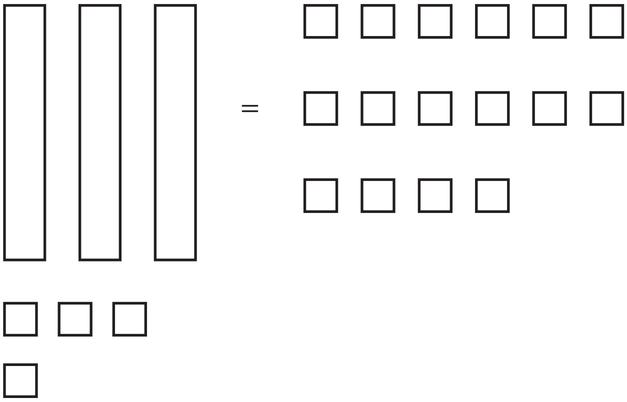
Getting Started

Solving Inequalities with Algebra Tiles

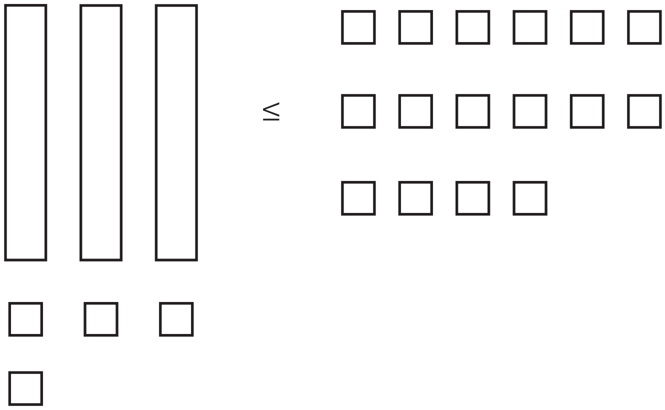
You can use algebra tiles to solve inequalities.



1. Describe how you can use the algebra tiles to solve the equation $3x + 4 = 16$.



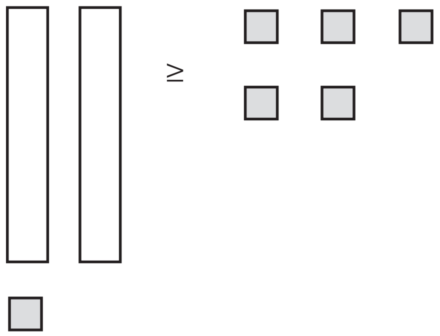
2. Write the inequality represented by the tiles.



3. Can you use the same reasoning to solve the inequality $3x + 4 \leq 16$? Explain.

4. What is the solution to the inequality $3x + 4 \leq 16$?

5. Describe how to use the algebra tiles to solve the equation.

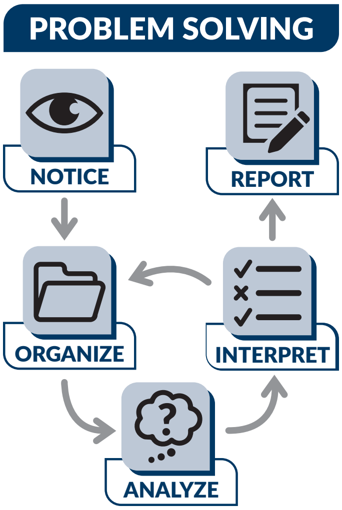


6. Write the inequality represented by the tiles.

7. What is the solution to the inequality?

ACTIVITY
5.1

Solving Two-Step Inequalities



Jorge wants to buy new football pads that cost $55.00 an online store. The online store charges $11 for shipping on orders less than $75. He also wants to buy some packages of eyeblack strips for $4 each, but he does not want to pay more than the $11.00 shipping fee.

1. Write and solve an inequality that describes the possible number of packages of eyeblack strips Jorge can purchase and still remain in the $11.00 shipping fee category. Let p represent the number of packages of eyeblack strips. Explain your solution in terms of the problem situation.

You just solved a problem that involved setting up and solving a two-step inequality.

2. Write a real-world situation that can be modeled by the inequality $4d + 10 \le 38$.

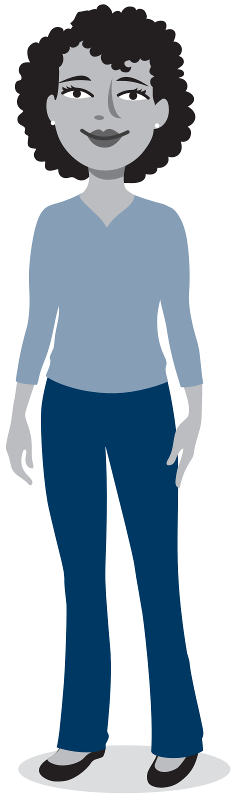
3. A set of possible solutions for each inequality is shown. Circle the solutions that make the inequality true. Then, list three additional solutions to the inequality.

   a. $3x - 2 \geq 7$
      $\{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$

   b. $3x \geq 9$
      $\{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$

   c. $x \geq 3$
      $\{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$

To determine whether a value is a solution to an inequality, use substitution. When the resulting inequality is true, then it's a solution!

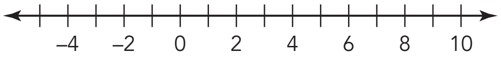


4. What do you notice about the solutions you circled in Question 3, parts (a) through (c)?

5. What do you notice about the three additional solutions you wrote for each inequality?

6. Compare the sequence of the three inequalities to the steps you used to solve the equation in Question 2. What do you notice? Explain your reasoning.

7. Graph the solution set for $3x - 2 \geq 7$.



You can check your solution to an inequality by choosing a value that is in your solution set and substituting it into the original inequality. When that substituted value makes the inequality true, then you have verified a correct solution.

8. Choose a value from the solution set of the inequality $3x - 2 \geq 7$ and verify that it is a solution.

9. Analyze the solution strategy and solution for each inequality.

Paola

$$-\frac{1}{2}x + \frac{3}{4} < 2$$

$$-4\left(-\frac{1}{2}x + \frac{3}{4} < 2\right)$$

$$2x - 3 > -8$$

$$2x > -5$$

$$x > \frac{-5}{2}$$

$$x > -2.5$$

Jaylen

$$-12x + 20 < 32$$

$$\frac{-12x + 20}{-4} < \frac{32}{-4}$$

$$3x - 5 < -8$$

$$3x < -3$$

$$x < -1$$

Describe the strategy that Paola used correctly.

Identify the error in Jaylen's strategy and determine the correct solution.

10. Solve each inequality or equation, and show your work. Then, graph the solution set on a number line.

a. $2x + 5 < -17$

b. $6.5x - 1.1 > 6.9$

c. $97 \leq -8x + 1$

d. $10 < 2x - \frac{3}{5}$

e. $18 \geq -x + 7$

Talk the Talk

Getting Creative with Inequalities

1. Describe the steps you would take to model and solve the inequality $3x - 3 \leq 12$. Then, solve the inequality.

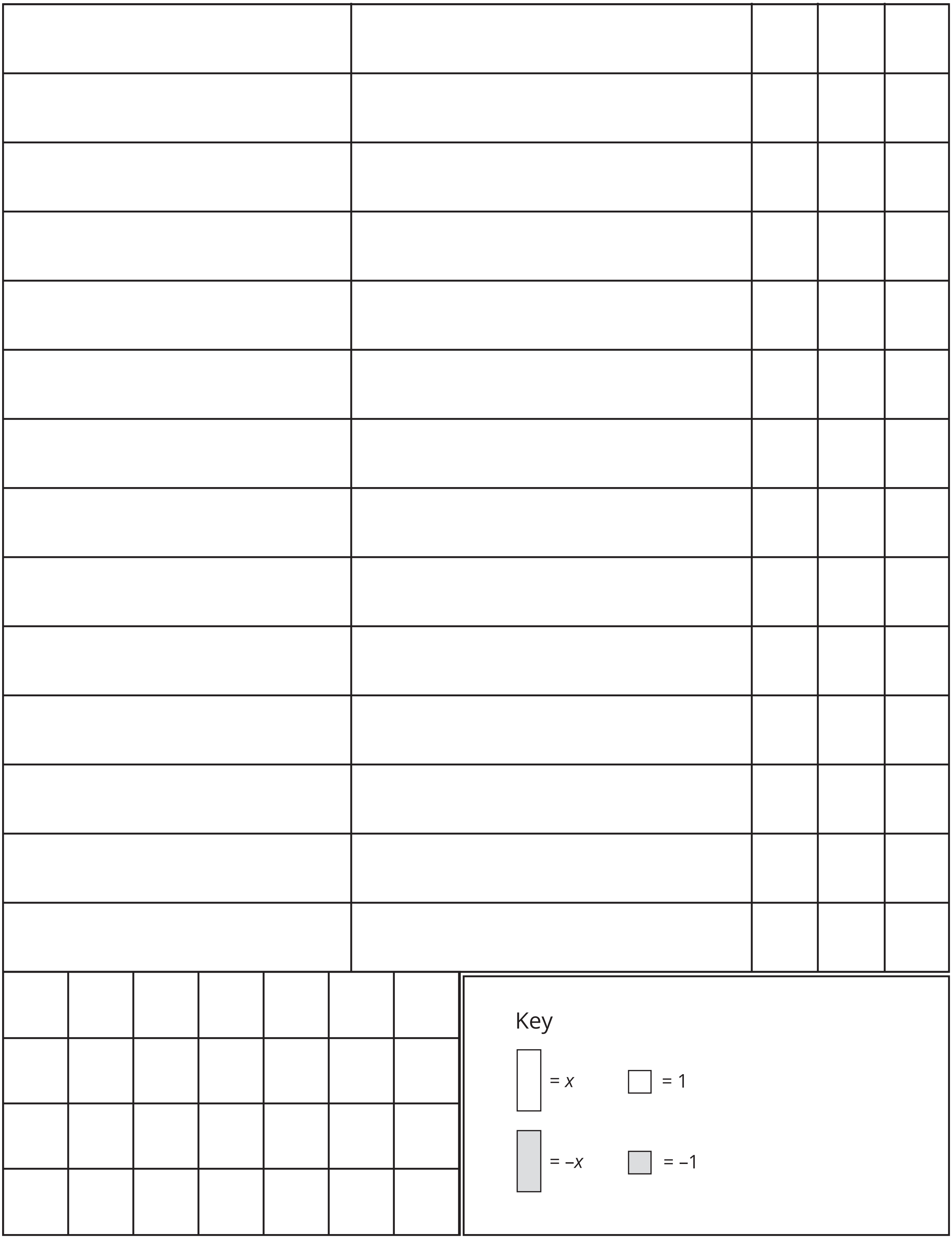
Ask Yourself . . .
How can you use inequalities in everyday life?

2. Write a real-world situation that can be modeled by each inequality.

   a. $20 > 4.50x - 4$

   b. $\frac{1}{2}t + 6 \geq 24$

Algebra Tiles



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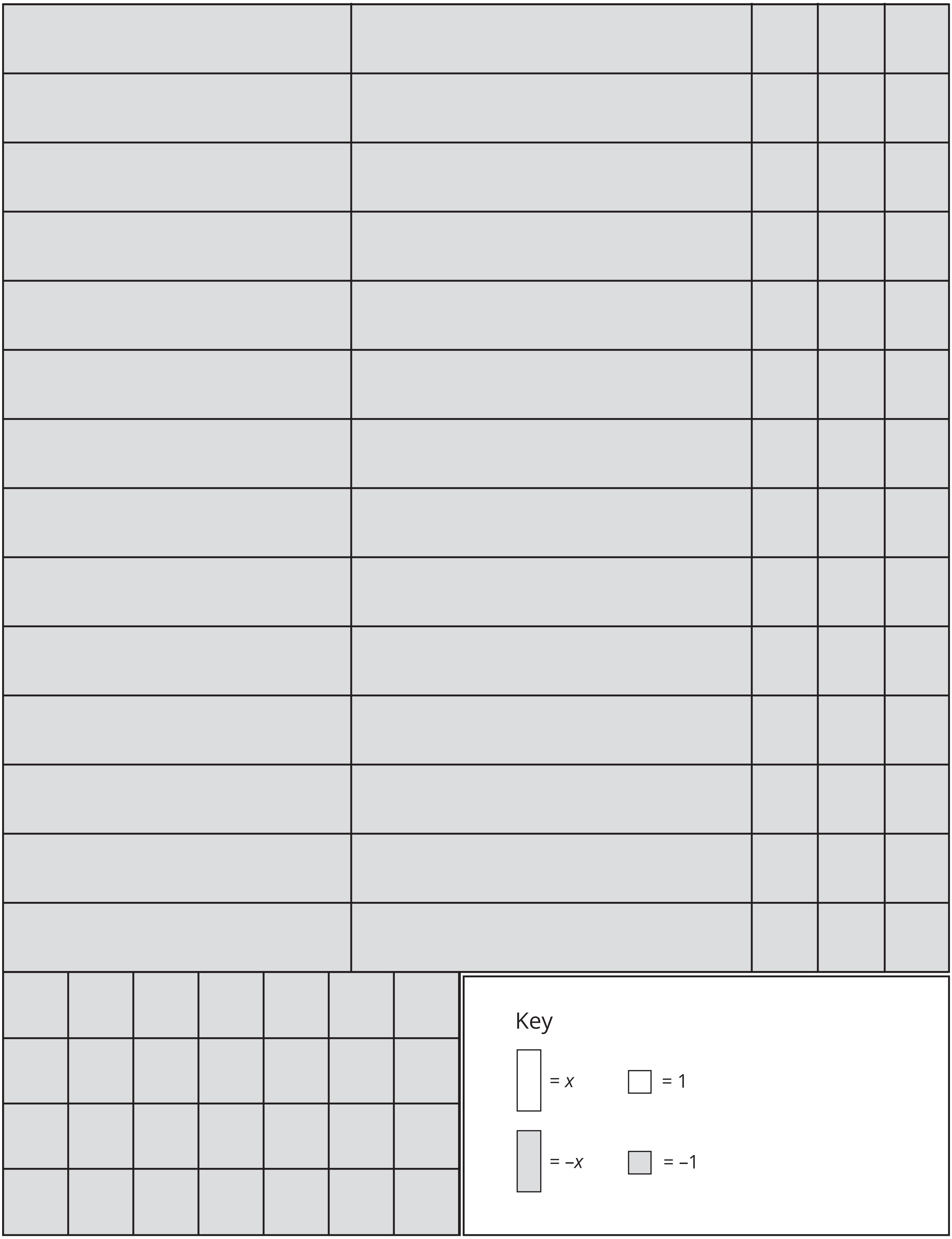
So you can cut out the Algebra Tiles on the other side.

Algebra Tiles



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So you can cut out the Algebra Tiles on the other side.

Lesson 5 Assignment

Write

Explain the process of solving a two-step linear inequality.

Remember

To solve an inequality means to determine what value or values will replace the variable to make the inequality true.

Practice

1. Match each inequality with the correct solution.

a. $x < -2$	i. $4x + 12 < 20$
b. $x < 2$	ii. $55 < 35 + 10x$
c. $x > -2$	iii. $-\frac{3}{2}x + 12 > 15$
d. $x > 2$	iv. $-8x < 16$

2. Solve each one-step inequality and graph the solution set on a number line.

a. $x + 7 \geq 13$

b. $-4 > x - 3$

c. $\frac{x}{4} \leq \frac{5}{2}$

d. $18.3 > 6.1x$

e. $3 < -\frac{x}{8}$

f. $-10x \geq 45$

Lesson 5 Assignment

3. Solve each two-step inequality and graph the solution set on a number line.

   a. $-17 < 3 - 5x$

   b. $21 - 9x \geq -6$

   c. $-500 \leq 11x - 60$

   d. $-x + 38 < 59$

Lesson 5 Assignment

Prepare

Mariana charges $35 an hour for tutoring services plus a $5 travel fee if she has to go to the student's house.

1. Name the quantities that are changing in this problem situation.

2. Name the quantities that remain constant.

3. Write an equation for the amount Mariana charges, assuming she must travel to the student's house.

4. When Mariana made $75, how many hours did she tutor?

TOPIC 2 SELF-REFLECTION

Name: ______________________________

Two-Step Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Two-Step Equations and Inequalities* topic by:

TOPIC 2: *Two-Step Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
modeling and solving one-variable two-step equations using bar models, algebra tiles, and double number lines.	○	○	○
connecting bar models, algebra tiles, and double number lines to the algorithm for solving two-step equations.	○	○	○
writing and fluently solving word problems leading to equations in the form $ax + b = c$.	○	○	○
identifying the properties of equality used to solve an algebraic equation of the form $ax + b = c$.	○	○	○
writing and fluently solving word problems leading to inequalities in the form $ax + b > c$ and $ax + b < c$.	○	○	○
graphing and explaining the solution set of an inequality in the context of the problem.	○	○	○
writing a corresponding real-world problem for a given two-step equation or inequality.	○	○	○

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Two-Step Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
determining whether given values make a two-step equation or inequality true.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Two-Step Equations and Inequalities*

   ____

   ____

   ____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   ____

   ____

   ____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   ____

   ____

   ____

TOPIC 2 SUMMARY

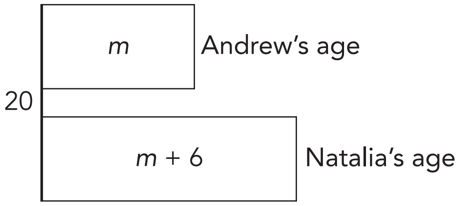
Two-Step Equations and Inequalities Summary

KEY TERMS
- equation [ecuación]
- two-step equation

LESSON 1 Modeling Equations as Equal Expressions

You can create a model to represent equal expressions.

For example, Natalia is 6 years older than Andrew. The sum of their ages is 20.



You can represent the model you drew with a mathematical sentence using operations and an equals sign. An **equation** is a mathematical sentence created by placing an equal sign (=) between two expressions.

The equation that represents the model above is $20 = m + (m + 6)$, or $20 = 2m + 6$.

A solution to an equation is a value for the unknown that makes the equation true.

For example, the solution to the equation $20 = 2m + 6$ is $m = 7$.

$20 = 2(7) + 6$

$20 = 14 + 6$

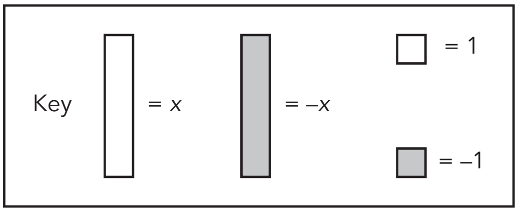
$20 = 20$

Andrew is 7 years old, and Natalia is 13 years old.

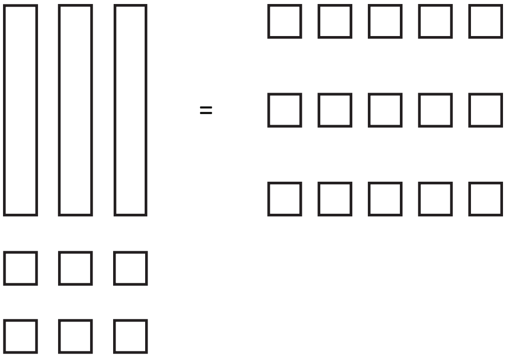
LESSON 2 Solving Equations Using Algebra Tiles

You can use algebra tiles to model and solve equations.

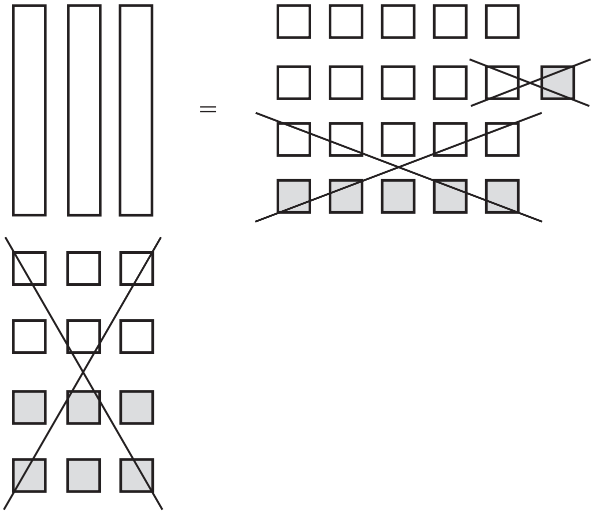
The Key represents the value of different algebra tiles.



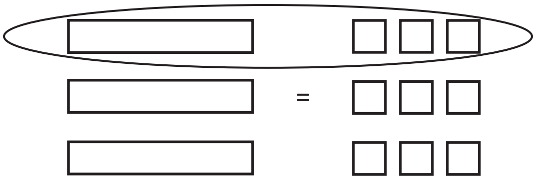
The algebra tiles represent the equation $3x + 6 = 15$.



Step 1: Subtract 6 units from each side, which creates a zero pair on the left side of the equation.



Step 2: Divide the 9 units remaining into 3 equal groups to determine the value of one rectangle.

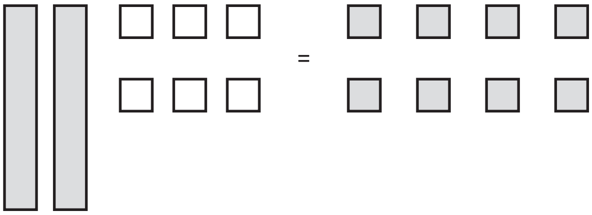


$x = 3$

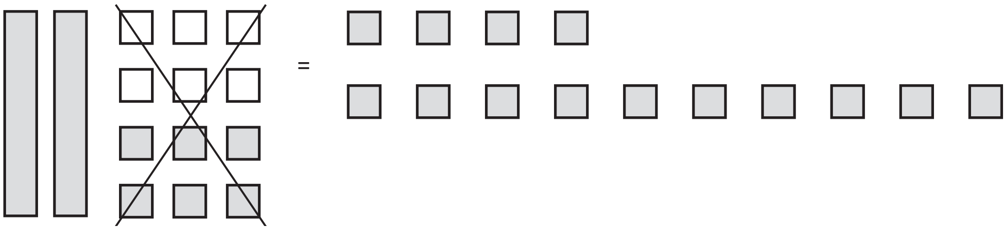
You can use algebra tiles to model and solve equations with negative coefficients.

$-2x + 6 = -8$

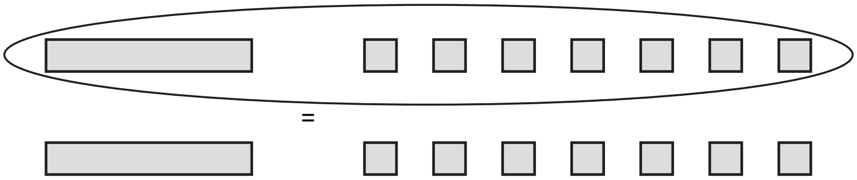
Step 1: Model the equation with algebra tiles.



Step 2: Subtract 6 units from each side, which creates a zero pair on the left side of the equation.



Step 3: Divide the tiles into 2 equal groups to determine the value of one rectangle.



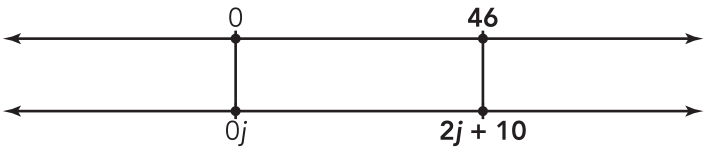
Step 4: Since one rectangle represents $-x$ or the opposite of x, change the sign of the algebra tiles on both sides of the equal sign to determine the value of x.

$x = 7$

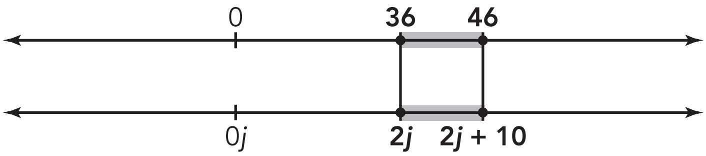
LESSON 3 Solving Equations on a Double Number Line

You can use double number lines to help you solve equations. When solving an equation, equality must be maintained. What is done to one expression must be done to the equivalent expression to maintain equality.

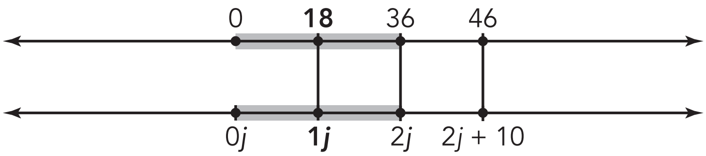
For example, solve the equation $2j - 10 = 46$. First, draw a model to set up the equation.

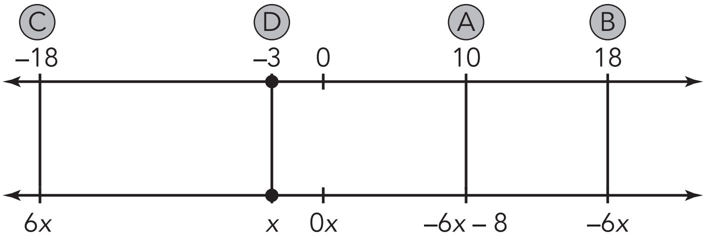


Next, start decomposing the variable expression. Place $2j$ in relationship to $2j + 10$. The expression $2j$ is 10 to the left of $2j + 10$. To maintain equality, place a number 10 to the left of 46. So, $2j = 36$.



The expression $1j$, or j, is halfway between $0j$ and $2j$, and 18 is halfway between 0 and 36. So, $j = 18$.





You may need to reason with negatives to solve equations.

For example, the double number line shows one way to solve the equation $-6x - 8 = 10$. A through D represent the order in which the steps were completed.

A. Set $-6x - 8$ equal to 10.

B. $-6x$ is 8 units to the right of $-6x - 8$, and 18 is 8 units to the right of 10.

C. $6x$ is the reflection of $-6x$ across 0, and -18 is the reflection of 18 across 0.

D. The location of x is one-sixth the distance from 0 to $6x$, and the location of -3 is one-sixth the distance from 0 to -18.

LESSON 4

Using Inverse Operations to Solve Equations

A **two-step equation** requires two inverse operations, or applying two properties of equality, to isolate the variable.

For example, here is one way to solve the equation $2x + 6 = 13$.

Subtract 6 from each side of the equation. $2x + 6 - 6 = 13 - 6$

Divide both sides of the equation by 2. $\frac{2x}{2} = \frac{7}{2}$

The solution is $x = 3\frac{1}{2}$.

You can use the properties of equality to rewrite equations and increase your efficiency with solving equations. Analyze the structure of the equation to determine the most efficient solution strategy.

- If the equation contains fractions, you can multiply both sides of the equation by the least common denominator.
- If the equation contains decimals, you can multiply both sides of the equation by a power of 10.
- If the equation contains large values, you can divide both sides of the equation by a common factor.

LESSON
5

Using Inverse Operations to Solve Inequalities

To solve an inequality means to determine the values of the variable that make the inequality true. Solving two-step inequalities is similar to solving two-step equations, except for the fact that when you are solving an inequality and multiply or divide by a negative value, you must reverse the inequality symbol.

For example, solve the inequality $-3x + 7 > 28$.

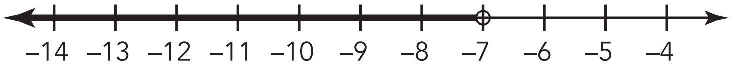
$$-3x + 7 - 7 > 28 - 7$$

$$-3x > 21$$

$$\frac{-3x}{-3} > \frac{21}{-3}$$

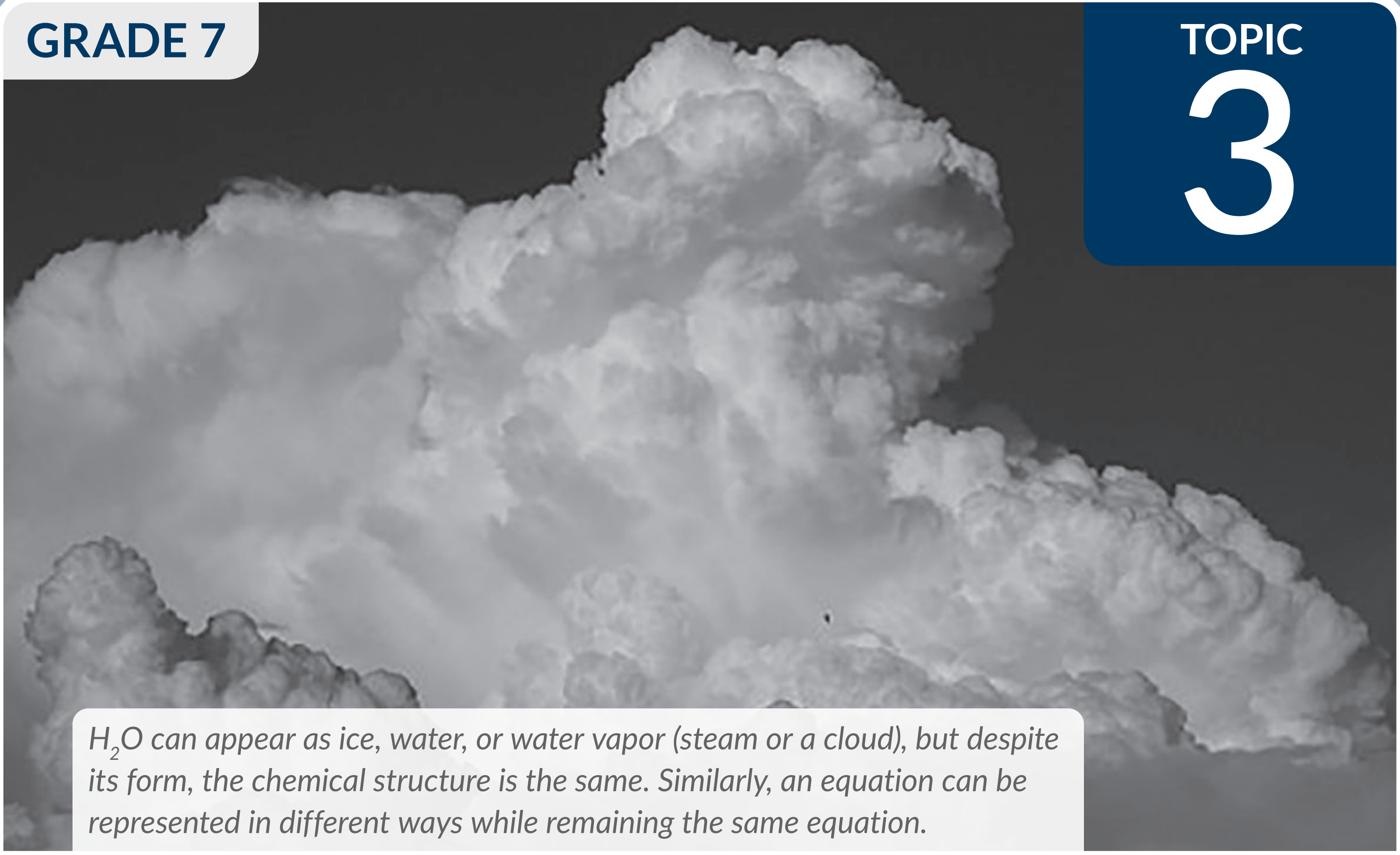
$$x < -7$$

The solution to any inequality is represented on a number line by a ray in which its starting point is an open or closed circle. For example, the solution $x < -7$ is represented by this number line.



GRADE 7

TOPIC 3



H_2O can appear as ice, water, or water vapor (steam or a cloud), but despite its form, the chemical structure is the same. Similarly, an equation can be represented in different ways while remaining the same equation.

Multiple Representations of Equations

LESSON 1 Representing Equations with Tables and Graphs **515**

LESSON 2 Building Inequalities and Equations to Solve Problems .. **541**

LESSON 3 Using Multiple Representations to Solve Problems **559**

1

Representing Equations with Tables and Graphs

OBJECTIVES

- Write and solve two-step equations to solve real-world problems.
- Use multiple representations to reason about quantities and analyze problem situations.
- Identify independent and dependent variables.
- Interpret negative solutions to problem situations.

You have solved two-step equations algebraically.

How can graphs of linear equations be used to solve equations?

Getting Started

It's All Greek to Me

Ms. Patel translates books for a living. She decides to change her fees to keep up with the cost of living. She will charge an initial fee of $325 to manage each project and $25 per page of translated text. Ms. Patel does not consider partial pages in her fees.

1. Name the quantities that change in this problem situation.

2. Name the quantities that remain constant.

3. Which quantity depends on the other?

ACTIVITY

1.1 Graphing Linear Equations

You can represent a problem situation in many ways. You used verbal descriptions to represent the relationship between the number of pages Ms. Patel translates and her total fees. Let's consider how to represent the relationship with tables and graphs.

1. Complete the table that shows the various projects that Ms. Patel has managed recently.

Number of Pages	Total Fees for the Project (dollars)
1	
2	
	400
	425
10	
	1150
	2100
92	

2. What is the least number of pages that Ms. Patel could translate? What is the greatest number of pages that Ms. Patel has translated recently?

3. What are the least and greatest amounts of money that Ms. Patel has earned?

The interval is the number you are counting by on a given axis.

Sometimes, you have to adjust your bounds based on the interval you choose. Just make sure that your data remains visible within the bounds that you choose.

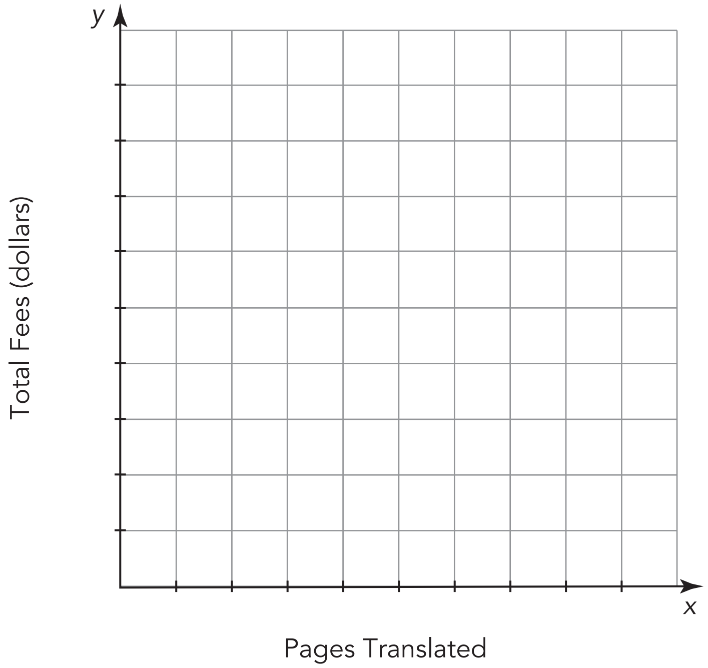
When representing a relationship as a graph, you need to ensure that the bounds of your graph are an appropriate size to surround the data from the table.

4. Consider the ranges of values in the table to choose maximum and minimum values for the *x*- and *y*-axes. Write these values in the table shown for each quantity.

Variable Quantity	Minimum Value	Maximum Value	Interval
Pages Translated			
Total Fees (dollars)			

5. Calculate the difference between the maximum and minimum values for each quantity and the number of tick marks that you have on each axis. Then, choose an appropriate interval for each axis and write these in the table.

6. Use the maximum and minimum values and intervals to label each axis. Then, create a graph of the data from the table.



7. Do all of the points on the line make sense for this problem situation? Explain your reasoning.

8. Describe the relationship between the two quantities represented in the graph.

Review the definition of **describe** in the Academic Glossary.

9. Write a linear equation to represent this situation. Make sure to define your variables.

When you first analyzed this situation, you listed two quantities that remain constant in this scenario: $325 and $25 per page.

10. Refer to your equation and graph to answer each question.

    a. Where is $325 represented in the graph and in the equation?

Draw a line through the points on your graph to model the relationship.

b. Where is $25 per page represented in the graph and in the equation?

11. Is there a proportional relationship between the number of pages translated and Ms. Patel's earnings? Justify your answer using the table, equation, and graph.

12. Use the graph to answer each question. Explain your reasoning.

a. Approximately how much money would Ms. Patel earn when she translated 57 pages?

How would a solution to the equation appear on the graph?

b. Approximately how many pages would Ms. Patel need to translate to earn $750?

13. For each translating project Ms. Patel completed this month, determine whether her pay was correct. When it is not, state the amount she should have received. Explain each answer in terms of the equation and the graph.

    a. Ms. Patel translated a 23-page technical manual for Technicians Reference Guide Inc. She received a check for $900.

    b. Ms. Patel translated a 42-page year-end report for Sanchez and Johnson Law Office. She received a check for $1050.

    c. Ms. Patel translated a 35-page product specification document for Storage Pros. She received a check for $2075.

You have represented the situation with Ms. Patel's book translation business multiple ways: as a scenario, in a table, with an equation, and on a graph. These representations are useful for analyzing the situation in different ways.

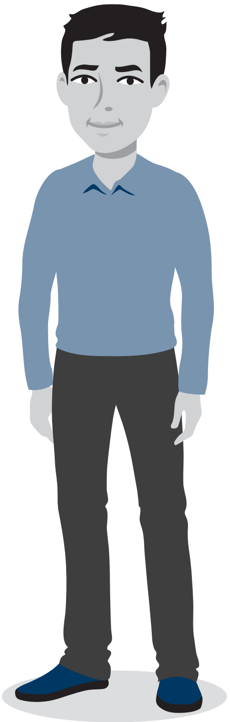
ACTIVITY
1.2

Interpreting Situations in More Than One Quadrant

El Capitan is a 3000-foot vertical rock formation in Yosemite National Park in California. The granite cliff is one of the most popular challenges for experienced rock climbers. On July 3, 2008, Hans Florine and Yuji Hirayama scaled El Capitan in a record time of 2 hours 43 minutes and 33 seconds.

1. On average, about how fast in feet per minute did the record holders climb?

What are the two quantities that are changing in this problem situation?



Two new climbers want to attempt to break the record by climbing El Capitan in 2 hours and 30 minutes.

2. If these climbers are to reach their goal, on average, how fast in feet per minute will they have to climb?

You want to watch the climbers attempt to break the record for climbing El Capitan. On the morning of the climb, you arrive late at 11:30 A.M. When you arrive, the climbers are exactly halfway to the top.

3. How many feet high are the climbers?

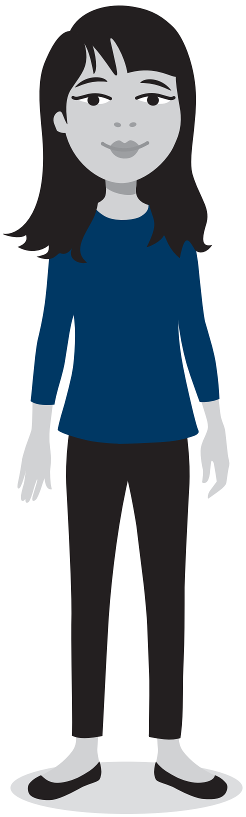
4. Assuming they are climbing at the average rate needed, how many feet up the cliff will the climbers be:

   a. in two more minutes?

   b. in a quarter of an hour?

   c. in one hour?

Pay attention to the units of measure!



5. Consider the quantities in this scenario.

   a. Which quantity depends on the other?

b. Identify and define the independent and dependent variables with their units of measure for this situation.

c. Write an equation for calculating the value of the dependent variable when the value of the independent variable is given.

Use your equation from Question 5 part (c) to answer Questions 6–9.

6. Determine how long after 11:30 A.M. it will take the climbers to reach the top at 3000 feet.

   a. Rewrite the equation.

   b. Solve the equation.

   c. What time would the climbers reach the top?

7. Determine when the climbers are 1400 feet up the cliff.

   a. Rewrite the equation.

   b. Solve the equation.

   c. What does this answer mean in terms of the problem situation?

8. Determine how high up the cliff the climbers were:

   a. Two minutes before 11:30 A.M.

   b. A half hour before 11:30 A.M.

9. Determine how many minutes before 11:30 A.M. the climbers started to climb. What time of day was that?

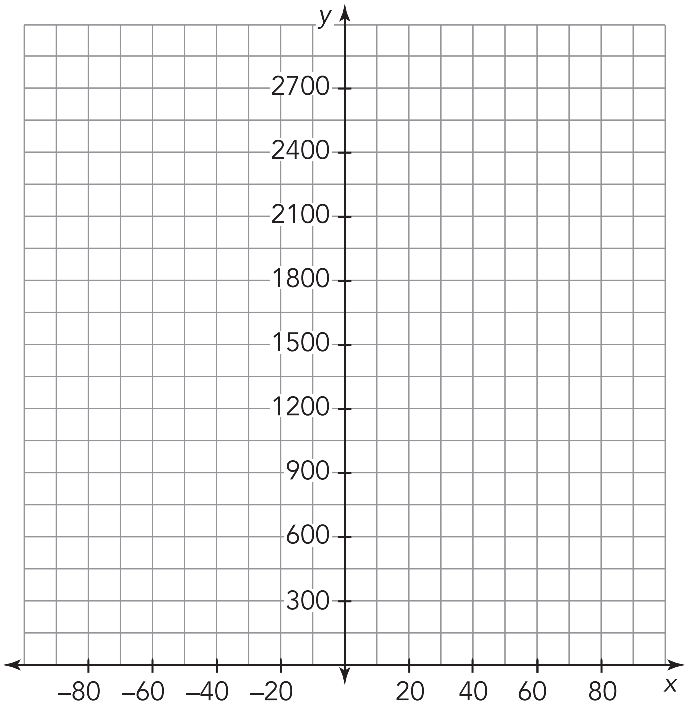
Now that you have represented the situation with an equation, represent it with a table and a graph.

10. Complete the table for this situation.

Quantities	Time	Height
Units of Measure		
Variables		
	0	
	2	
	15	
	60	

This graph displays Quadrants I and II. Why doesn't it include Quadrants III and IV as well?

11. Plot the points from the table on the coordinate plane shown. Label the axes and draw the graph of your equation.



12. What should be the leftmost point on your graph? Explain your reasoning.

13. What should be the rightmost point on your graph? Explain your reasoning.

14. Locate the point with an *x*-coordinate of -60.

    a. What is the height of the climbers at this point?

    b. Write this point as an ordered pair and interpret the meaning in terms of the problem situation.

15. This analysis of the climbers' progress assumes that the climbers would climb at a steady rate of 20 feet per minute. In reality, would the climbers be able to do this during the whole climb? If not, how might the graph reflect this?

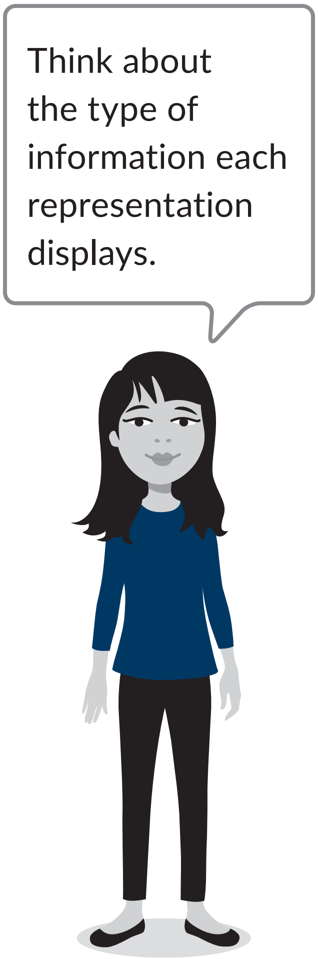
16. Explain what the negative values of time represent.

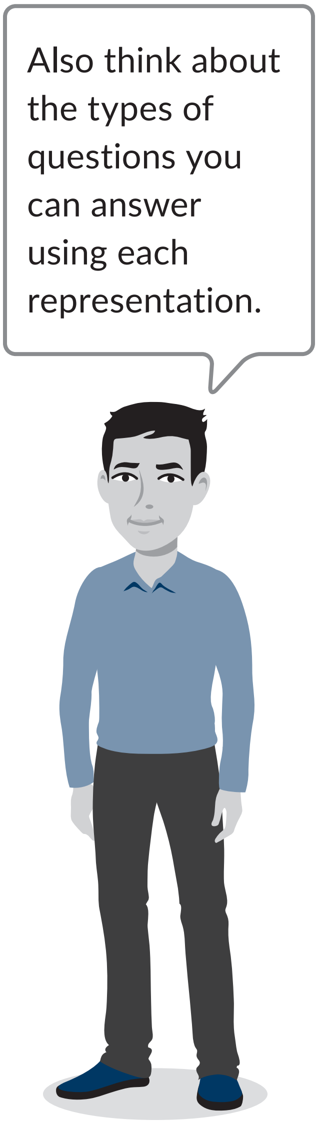
Talk the Talk

Take Advantage of the Situation

You have represented two situations in four different ways: as a sentence, as a table, as a graph, and as an equation.

1. Complete the graphic organizer to explain the advantages and disadvantages of each representation.





SENTENCE

Advantages

Disadvantages

TABLE

Advantages

Disadvantages

MULTIPLE REPRESENTATIONS

Advantages

Disadvantages

GRAPH

Advantages

Disadvantages

EQUATION

Lesson 1 Assignment

Write

Explain how the constant term and the coefficient of the variable of $y = 2x + 5$ would be represented in the graph of the equation.

Remember

A table provides specific values for a given problem situation. A graph is a visual representation of the data related to a problem situation. An equation generalizes a problem situation.

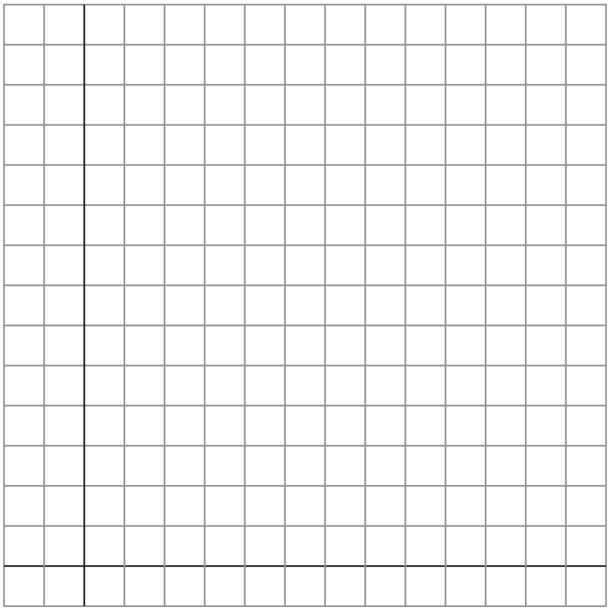
Practice

1. Mario joins a book club. He pays $12 for each book and $5 for shipping and handling charges for each order.

   a. Name the quantities that change in this problem situation and the quantities that remain constant. Determine which quantity is independent and which quantity is dependent.

b. Create a table of values to represent the total cost when Mario orders 1 or 2 books or spends \$41, \$65, or \$125.

c. Create a graph of the data from the table. Carefully select the maximum values, minimum values, and intervals. Remember to label the axes and the intervals.



d. Describe the relationship between the two quantities.

e. Mario said that he spent exactly $80 on a book order. Use your graph to determine whether Mario is correct.

f. Write an algebraic equation to represent the situation. Define your variables.

g. Use the equation, table, and graph to explain whether this situation represents a proportional relationship.

2. Mr. Davis is a rare coin collector. He recently bought a coin valued at $5400. It has been determined that the coin will increase in value by $30 each month. Mr. Davis plans to sell the coin within 5 years.

   a. Name the quantities that change in this problem situation and the quantities that remain constant. Determine which quantity is independent and which quantity is dependent.

Lesson 1 Assignment

b. Create a table of values that represents a variety of different number of months for which Mr. Davis could own the coin and the total value of the coin.

c. Create a graph of the data from the table. Carefully select the maximum values, minimum values, and intervals. Remember to label the axes and the intervals.

d. Describe the relationship between the two quantities.

e. Use the graph to determine the approximate worth of the coin when Mr. Davis owns it for 3 years.

f. Use the graph to determine approximately when the coin will be worth $6600.

g. Write an algebraic equation to represent the situation. Define your variables.

h. After owning the coin for 3 years, Mr. Davis wants to sell the coin. He tells a potential buyer it is worth \$6480. The buyer disagrees and says it is worth \$5490. Who is correct? Explain your reasoning in terms of the equation.

3. Andrew is looking for a new car. He has particular interest in an expensive sports car with a list price of \$32,500. Andrew knows that the minute he drives the car off the lot, it will start to lose value, or depreciate. He finds out that the car will depreciate to a scrap value of \$1000 in 15 years.

   a. What is the total change in value of the car in 15 years?

b. What is the average amount of depreciation per year?

c. When the car is exactly 7.5 years old, Andrew decides to sell it to his friend Natalia. What is the value of the car when Natalia buys it?

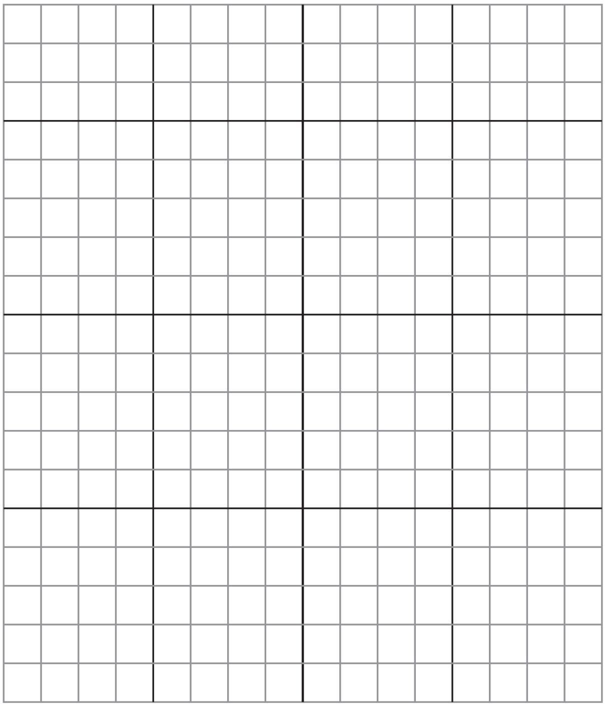
d. What are the two quantities that are changing in part (c)? Define and identify the independent and dependent variables for the quantities you defined with their units of measure.

Lesson 1 Assignment

e. Write an equation to calculate the value of the car given the number of years Natalia has owned the car.

f. Create a table of values that includes when Natalia has owned the car 0 years, 6 months, and two and a half years. Also, include when the value of the car was $1000, $22,000, $25,150, $30,400, and $32,500.

g. Create a graph of the data from the table. Carefully select the maximum values, minimum values, and intervals. Remember to label the axes and the intervals.



h. Locate the point where the value of the independent quantity is −5. What is the value of the dependent quantity at this point? Write the point as an ordered pair. What does the ordered pair mean in the context of the problem?

Lesson 1 Assignment

Prepare

Determine the parts of the solution set that make each inequality true.
Solution set: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

1. $x > 6$

2. $x + 2 > 6$

3. $2x + 2 > 6$

4. $6 < 2(x + 2) - 4$

2

Building Inequalities and Equations to Solve Problems

OBJECTIVES

- Solve word problems by building and interpreting inequalities of the form $ax + b < c$ and $ax + b > c$.
- Graph the solution sets of inequalities in order to solve problems.
- Calculate and interpret the unit rate of change in a problem situation.

NEW KEY TERM

- unit rate of change

You have graphed equations to solve problems.

How can you use the graphs of inequalities to help you solve problems?

Getting Started

Lemonade at the Pool

The concession stand at a local swimming pool sells small and large glasses of freshly squeezed lemonade. This weekend, they made more than $250 selling glasses of lemonade. A large glass of lemonade sells for $4.00, and the total sales generated from selling small glasses of lemonade was $65.

1. Write an inequality to represent the relationship between the amount they made and the number of large glasses they sold.

2. Solve the inequality. Interpret the solution in terms of the problem situation.

3. Graph the solution set on the number line.

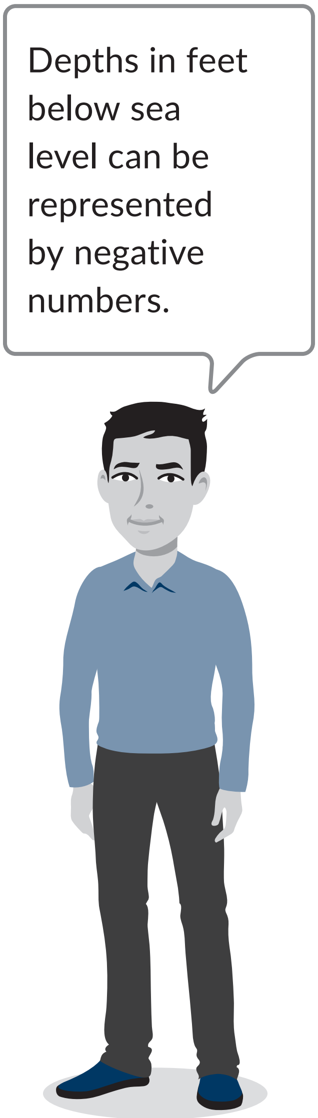
ACTIVITY 2.1 Unit Rate of Change

To explore one of the last unknown regions on our planet, companies are starting to produce single-person, submersible deep-sea submarines like the Deep Flight I. Suppose the submarine Deep Flight I is going to do a dive starting at sea level, descending 480 feet every minute.

1. Identify the independent and dependent quantities and their units of measure, and define variables for these quantities. Then, write an equation to represent Deep Flight I's depth.

2. Use your equation to complete the table shown for this problem situation. Do not forget to define the quantities, units of measures, and variables for this situation.

	Independent Quantity	Dependent Quantity
Quantities		
Units of Measure		
Variables		
	0	
	1	
	2	
	3	
	4	
	5	
	6	



3. Consider the possible values for time and depth.

   a. What do you think are all the possible values for time in terms of this situation? Write an inequality to express your answer.

   b. What do you think are all the possible values for depth in terms of this situation? Write an inequality to express your answer.

4. Examine your table. What do you notice about each depth value in relation to the one before and the one after?

Ask Yourself:
What patterns do you notice?

The **unit rate of change** describes the amount that the dependent variable changes for every one unit that the independent variable changes.

5. In this problem, what is the unit rate of change?

6. How deep would the submarine be after:

   a. 2.5 minutes?

   b. 90 seconds?

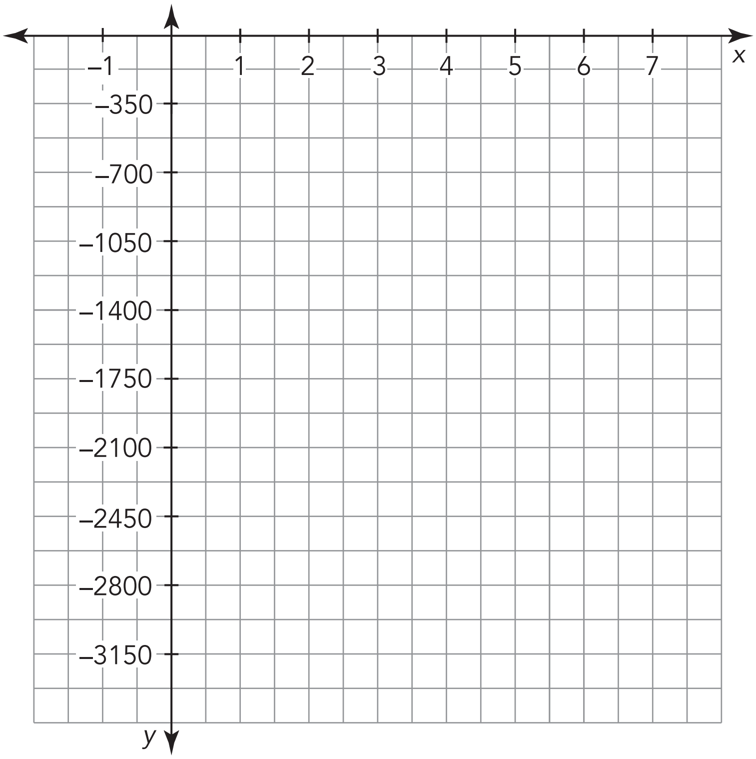
c. 45 seconds?

7. How many minutes would it take Deep Flight I to be:

a. 1400 feet below sea level?

b. 2100 feet below sea level?

8. Construct a graph of this problem situation. Label the units on each axis. Then, plot all the points from the table and from Questions 6 and 7. Finally, draw the graph to represent the problem situation.



ACTIVITY

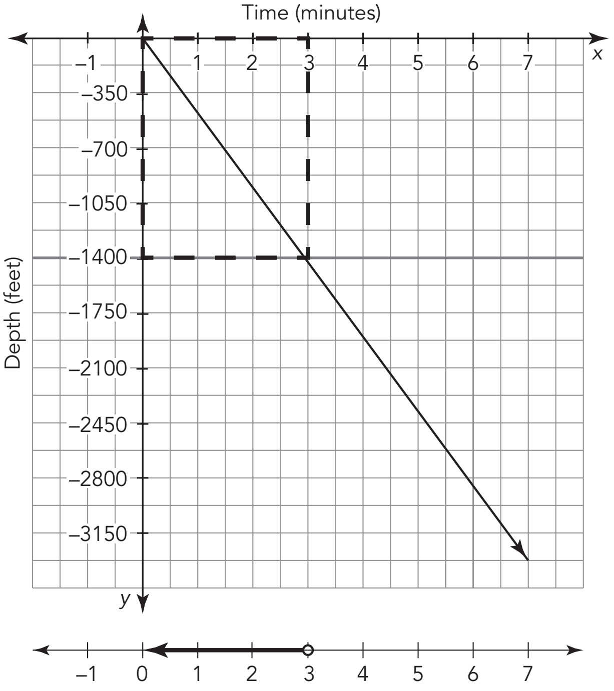
2.2 Visual Representations of Inequalities

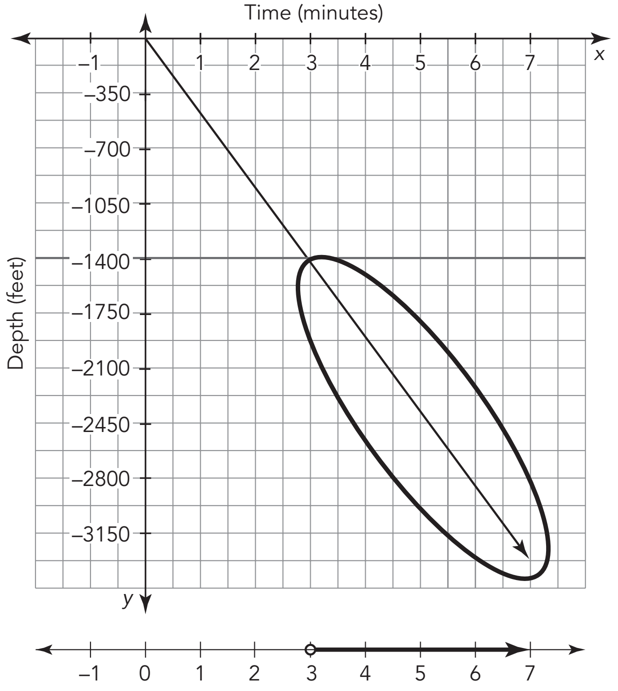
You can use your graph to estimate solutions to inequality problems. Use the graph to estimate the times Deep Flight I will be more than 1400 feet below sea level and the times Deep Flight I will be less than 1400 feet below sea level.

WORKED EXAMPLE

Each of these graphs shows the relationship between the time in minutes and the depth of Deep Flight I.

The rectangle on the left graph shows the set of all depths for Deep Flight I less than 1400 feet below sea level. The oval on the right graph shows the set of all depths for Deep Flight I more than 1400 feet below sea level.





Deep Flight I will be greater than –1400 feet for times less than 3 minutes. Deep Flight I will be less than –1400 feet for times greater than 3 minutes.

1. Use the graph to estimate the times Deep Flight I will be:

   a. Less than 2100 feet below sea level.

   b. More than 2100 feet below sea level.

2. Write an inequality, solve it, and graph its solution on a number line to determine the time Deep Flight I is:

   a. Less than 2100 feet below sea level.

   b. More than 2100 feet below sea level.

How do the graph and the inequality model the starting depth?

3. How do your answers using the graph compare to those when you wrote and solved inequalities?

ACTIVITY

2.3 Graphs of Equations and Inequalities

Deep Flight I can dive to a depth of 3300 feet below sea level and can ascend to the surface at a rate of 650 feet per minute.

1. Suppose Deep Flight I is going to ascend to sea level starting at its maximum depth of 3300 feet below sea level. Identify the independent and dependent quantities, define variables for these quantities, and write an equation to represent Deep Flight I's depth.

2. Use your equation to complete the table shown for this problem situation.

	Independent Quantity	Dependent Quantity
Quantities		
Units of Measure		
Variables		
	0	
	1	
	2	
	3	
	4	
	5	

3. Why does the table end at 5 minutes for this problem situation?

4. Consider the possible values for time and depth.

   a. What do you think are all the possible values for time in terms of this situation? Write inequalities to express your answer.

   b. What do you think are all the possible values for depth in terms of this situation? Write inequalities to express your answer.

5. Examine your table. What do you notice about each depth value in relation to the one before and the one after?

6. In this problem, what is the unit rate of change?

7. How deep would the submarine be after ascending for:

   a. 2.5 minutes?

   b. 90 seconds?

   c. 45 seconds?

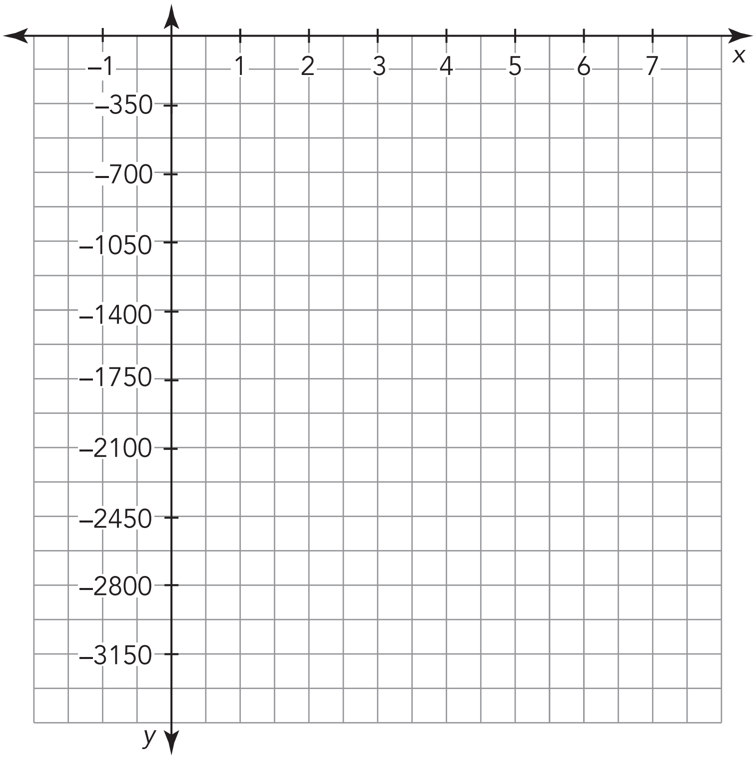
8. How many minutes would it take Deep Flight I to ascend to:

   a. 1000 feet below sea level?

   b. 2100 feet below sea level?

   c. Sea level?

9. Use your information to construct a graph of this problem situation. First, label the units of measure on each axis. Then, plot all the points from the table and from Questions 7 and 8. Finally, draw the graph to represent the problem situation.



10. Draw a box or oval on the graph to estimate each.

a. The time Deep Flight I is above 1000 feet below sea level

b. The time Deep Flight I is below 2000 feet below sea level

11. Write an inequality, solve it, and graph its solution on a number line to determine the time Deep Flight I is above 1000 feet below sea level.

12. Write an inequality, solve it, and graph its solution on a number line to determine the time Deep Flight I is below 2000 feet below sea level.

Talk the Talk

Digging to China

Did you ever hear the saying, “If you dig deep enough, you will dig to China?” You would have to live in South America, possibly Argentina, for this to happen. If you live in the United States, chances are you would pop out on the other side of the Earth in the Indian Ocean! Technically speaking it would be impossible to dig a hole to the other side of the Earth, but let’s pretend.

Suppose you were digging at a rate of 10 feet a day. Assume you are at sea level when you begin digging.

1. Identify the independent and dependent quantities and their units of measure and define variables for these quantities.

2. Write an equation to represent the depth of the hole, where d represents depth in feet and t represents the time in days.

3. If there are 365 days in a year, write an equation to represent the depth of the hole, where d represents depth in feet and t represents the time in years.

4. Use your equation to complete the table for this problem situation.

Time (Years)	Depth (Feet)
t	d
0	
1	
2	
3	
4	
5	

5. Write an inequality, solve it, and graph its solution on a number line to determine when the hole is more than one mile deep. There are 5280 feet in a mile.

6. Write an inequality and solve it to determine the number of years before the hole has reached to the other side of the Earth. The approximate diameter of the Earth is 7917.5 miles.

Lesson 2 Assignment

Write

In your own words, describe how to estimate an inequality using the graph of an equation. Include an example in your description.

Remember

The *unit rate of change* is the amount that the dependent value changes for every one unit that the independent value changes.

Practice

A tire manufacturing company produces all types of tires at its factory. Due to fixed costs associated with running the factory, the company starts with a loss of $200,000, or a profit of −$200,000, at the beginning of each month. The first major hurdle the company faces each month is to break even, or reach the point at which the profit is zero. The tires are sold in batches of 1000. The company earns $40,000 for each batch of tires they manufacture and sell.

1. Identify the two quantities that are changing in this situation, identify the independent and dependent quantities, and define the variables for these quantities. Then, write an equation to represent the profit the company will make when they manufacture and sell batches of tires.

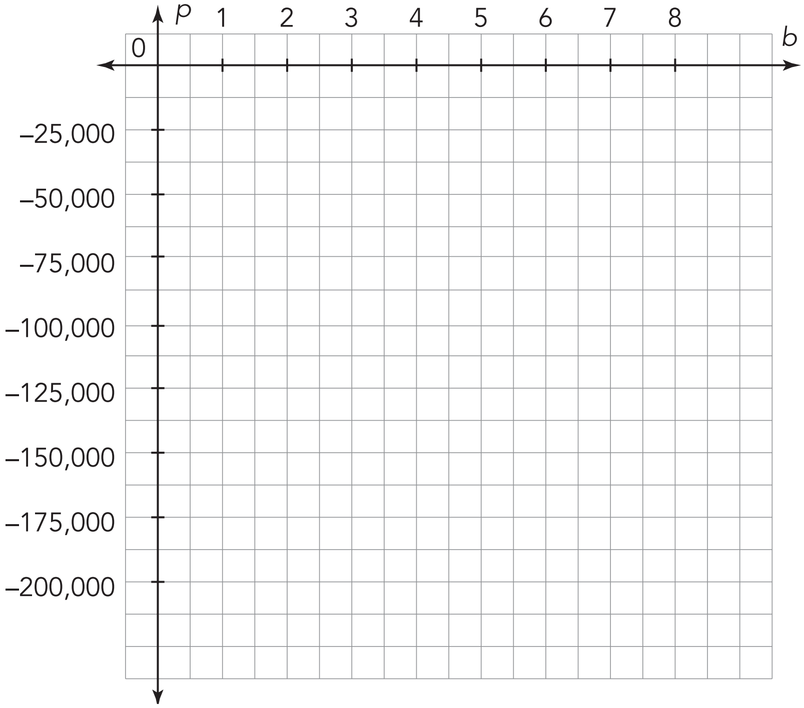
Lesson 2 Assignment

2. Use your equation to complete a table for this problem situation.

3. What is the unit rate of change in this problem?

Lesson 2 Assignment

4. Construct a graph for the problem situation.



For Questions 5 and 6, write an inequality. Then, solve and graph the inequality to answer the question.

5. How many batches of tires does the company have to manufacture and sell to have a profit of more than −$60,000?

Lesson 2 Assignment

6. How many batches of tires does the company have to manufacture and sell to have a profit of more than –$190,000?

Prepare

1. How many minutes are in a quarter of an hour?

2. How many minutes are in an hour and a half?

3. How is five and a half minutes written as a decimal?

3

Using Multiple Representations to Solve Problems

OBJECTIVES

- Use multiple representations to analyze and interpret problem situations.
- Use tables, graphs, and equations to represent and solve word problems by reasoning about quantities.

Different representations of a problem can give you different insights into possible solutions.

How can you use a variety of representations to help you solve problems?

Getting Started

Matching Game

Four equations are given. Match each equation to a graph or table and explain your reasoning. Then, complete the table and graph for the unmatched equation.

Equations

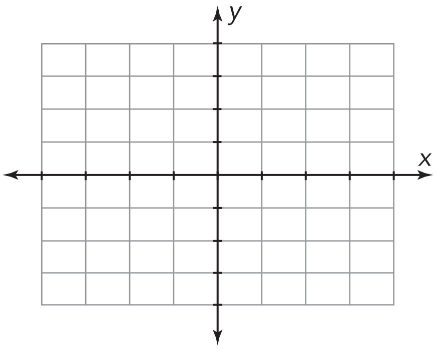
1. $y = 6$

2. $y = \frac{1}{6}x$

3. $y = -x + 6$

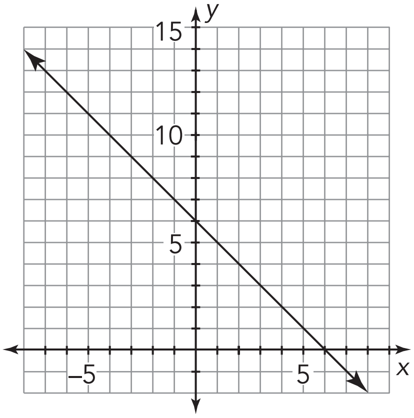
4. $2x + y = 6$

x	y



Tables and Graphs

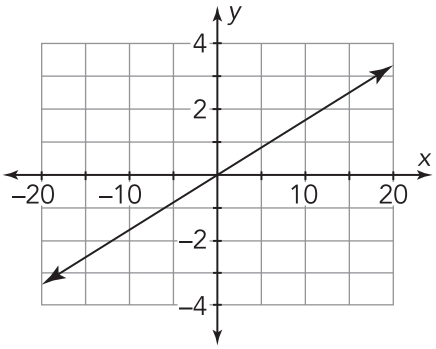
a.



b.

x	y
$\frac{1}{6}$	6
0	6
1	6

c.



ACTIVITY 3.1

Starting with a Verbal Description to Solve a Problem

A tank that currently contains 2500 gallons of oil is being emptied at a rate of 25 gallons per minute. The capacity of this tank is 3000 gallons.

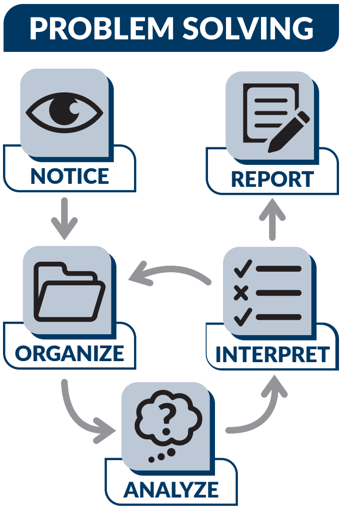
1. How many gallons are currently in the tank?

2. How fast is the tank being emptied?

3. What are the two quantities that are changing? Define variables for these quantities and identify which is the independent variable and which is the dependent variable.

4. What is the unit rate of change in this situation? Explain your reasoning.

5. Write an equation that relates the two quantities.



6. How many gallons will be in the tank after:

   a. A quarter of an hour?

   b. Five and a half minutes?

   c. An hour and a half?

7. When will the tank be:

   a. Half full?

   b. Empty?

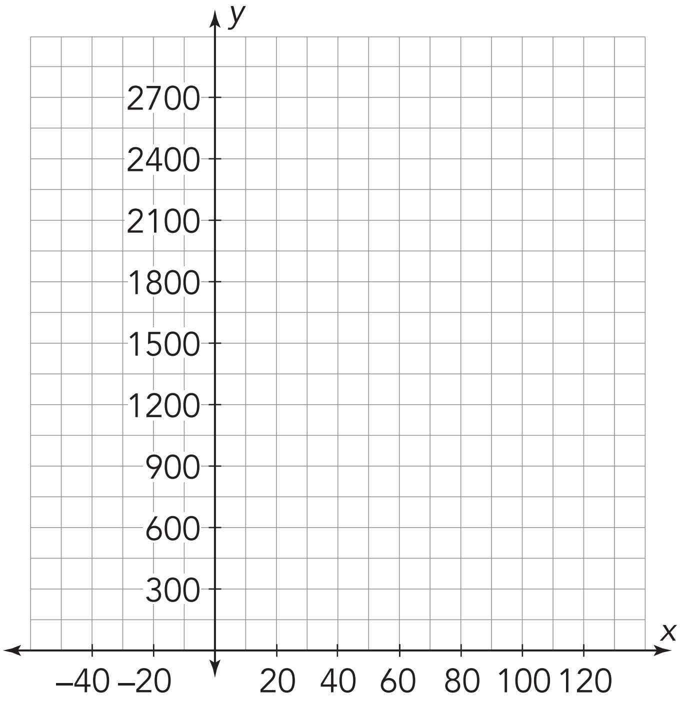
8. How long ago did the tank contain 2600 gallons?

9. How long ago was the tank full?

10. Complete the table for this problem situation.

	Independent Quantity	Dependent Quantity
Quantities		
Units of Measure		
Variables		

11. Label the units of measure on each axis and plot all the points from the table. Then, graph the equation for this situation. Make sure to label the units on the axes.



ACTIVITY
3.2

Starting with an Equation to Solve a Problem

The equation that converts a temperature in degrees Celsius to a temperature in degrees Fahrenheit is $F = \frac{9}{5}C + 32$, where F is the temperature in degrees Fahrenheit, and C is the temperature in degrees Celsius.

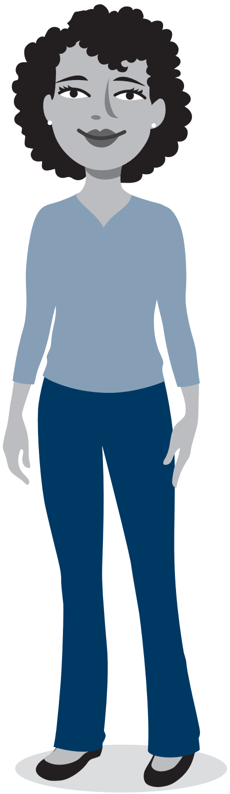
Ask Yourself . . .
Can others follow and understand your process and reasoning?

1. What is the temperature in degrees Fahrenheit when the temperature is:

   a. 36°C ?

   b. −20°C ?

You can also write this formula as $F = 1.8C + 32$.



2. What is the temperature in degrees Celsius when the temperature is:

   a. 32°F ?

   b. 212°F ?

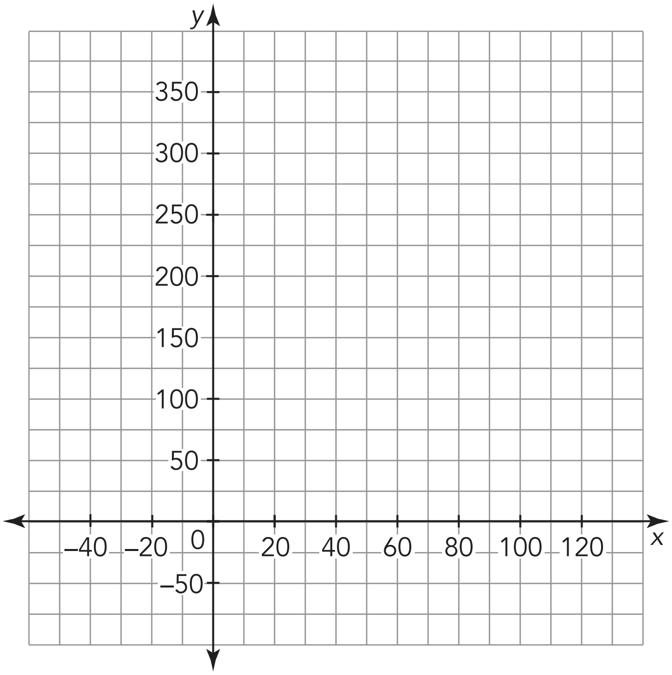
3. What is the unit rate of change? Explain your reasoning.

4. At what temperature are both the Fahrenheit and Celsius temperature values equal? Show your work.

5. Complete the table with the information you calculated in Question 1 through Question 4.

	Independent Quantity	Dependent Quantity
Quantities		
Units of Measure		
Variables		

6. Label the units of measure on each axis and plot all the points from the table. Then, graph the equation for this situation.



ACTIVITY 3.3

Starting with a Table to Solve a Problem

Alejandro and Trung found this table. The bottom three entries in the second column were smudged, and the boys couldn't read them.

Let's see whether you can calculate the unknown values.

Time (minutes)	Total Cost (dollars)
0	20
1	23
2	26
3	29
5	35
10	
20	
50	

1. What is the unit rate of change shown in the table? Explain your reasoning.

2. Define variables for the quantities in the table and write an equation that relates the two quantities.

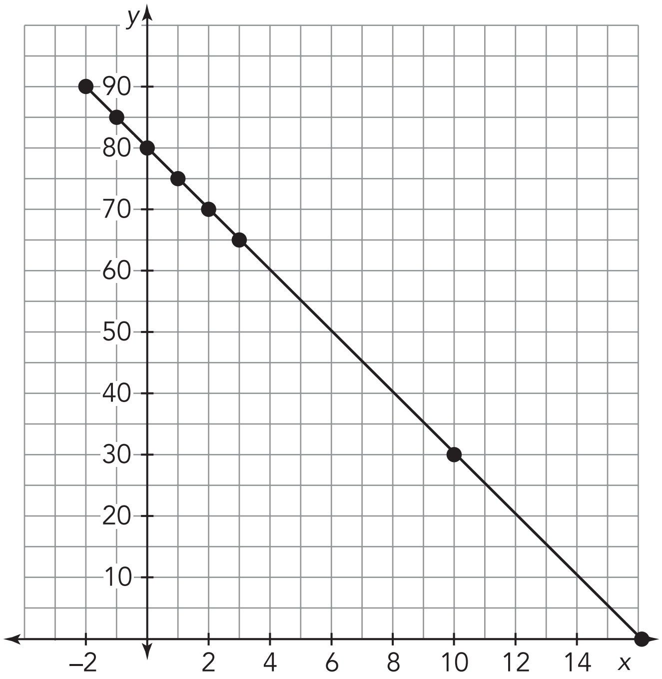
3. Use your equation to complete the table. Show your work.

4. Use your completed table to construct a graph.

ACTIVITY
3.4

Starting with a Graph to Solve a Problem

This graph shows the relationship between two quantities.



1. Complete the table using the information in the graph.

Independent Variable	0							
Dependent Variable	80							

2. Write an equation for this relationship.

WORKED EXAMPLE

You can write a situation to represent an equation.
To write a situation to represent $y = 80 - 5x$:

Step 1: Determine what you know from the equation.
y represents the independent variable.
x represents the dependent variable.
80 represents the y-intercept or the starting value when $x = 0$.
-5 represents the unit rate of change: for every 5 units y decreases, x increases one unit.

Step 2: Choose units for the independent and dependent variables.
For example,
independent variable: height
dependent variable: time

Step 3: Use the start value. Remember this is the value of the independent variable when $x = 0$.
For example, a hot air balloon is currently 80 meters in the air.

Step 4: Use the rate of change.
Since the unit rate of change is $-\frac{5}{1}$, you can say that the balloon descends 5 meters every minute.
A problem situation that represents the equation is:
A hot air balloon is currently at a height of 80 meters. It has been descending at a rate of 5 meters every minute.

3. Write a problem situation to represent the equation.

   $y = 3x + 6$

4. Write a problem situation to represent the equation.

   $y = -\frac{1}{2}x - 2$

Talk the Talk

Equivalent Representations

Ms. Hernandez wrote the table shown on the board.

She asked her students to complete the table, write the unit rate of change, and finally, write an equation for this relationship.

x	y
0	−5
1	−3
2	−1
3	1
4	
5	

1. Complete the table of values and identify the unit rate of change.

2. Two of Ms. Hernandez's students wrote equations to represent this relationship.

Kaya: $y = 2x - 5$

Eduardo: $y = \frac{1}{2}(4x - 10)$

Who is correct? Explain your reasoning.

3. Create a problem situation that might fit this equation.

Lesson 3 Assignment

Write

Which representation of equal expressions—tables, graphs, equations, or verbal descriptions—do you prefer? Explain why you prefer this representation and provide an example.

Remember

Multiple representations, such as a table, a graph, an equation, and verbal description can be used to represent a problem situation.

Practice

The Department of Transportation in each state is responsible for the improvements and repairs of that state's roads. One important job is to repaint the road lines that have worn away or faded. A painting crew is painting a 24-mile stretch of road. They have already completed a total of 9.5 miles of the road. The crew has been painting at a rate of 0.25 mile per hour and continues to paint at the same rate.

1. Identify the two quantities that are changing in this situation, identify the independent and dependent quantities, and define the variables for these quantities. Then, write an equation that relates the two quantities.

2. What is the unit rate of change in this situation? Explain your reasoning.

Lesson 3 Assignment

3. How many total miles of the road will be completed when the crew works for another 2 hours?

4. How many more hours does the crew need to work to complete half of the job?

5. Complete the table and then construct a graph.

	Independent Quantity	Dependent Quantity
Quantities		
Units of Measure		
Variables		
	0	
	2	
	5	
	6.5	
		12
		24
		8
		0

Lesson 3 Assignment

Prepare

Convert each fraction to a decimal and percent.

1. $\frac{4}{5}$

2. $\frac{7}{100}$

3. $\frac{5}{12}$

4. $\frac{24}{25}$

5. $\frac{21}{40}$

TOPIC 3 SELF-REFLECTION

Name: ____________________

Multiple Representations of Equations

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1-3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Multiple Representations of Equations* topic by:

TOPIC 3: *Multiple Representations of Equations*	Beginning of Topic	Middle of Topic	End of Topic
using variables to represent numbers in real-world or mathematical problems.	○	○	○
writing and solving two-step equations that represent real-world problems.	○	○	○
writing and solving two-step inequalities that represent real-world problems.	○	○	○
representing linear relationships using verbal descriptions, tables, graphs, and equations.	○	○	○
making connections among verbal descriptions, tables, graphs, and equations of problem situations.	○	○	○
representing constant rates of change in mathematical and real-world problems given verbal descriptions, tables, graphs, and equations.	○	○	○
using a graph to solve a two-step equation or inequality with rational coefficients.	○	○	○

continued on the next page

TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Multiple Representations of Equations* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

Multiple Representations of Equations Summary

NEW KEY TERM
- unit rate of change

LESSON 1 Representing Equations with Tables and Graphs

You can represent a problem situation in many ways.

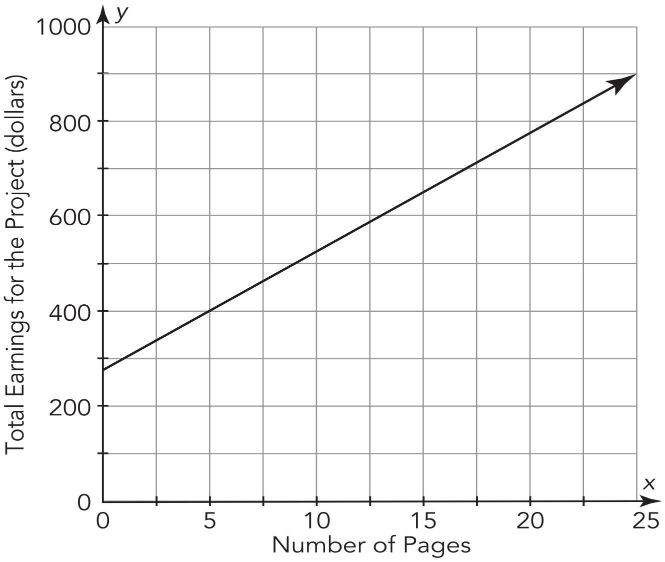
For example, Ms. Patel translates books for a living. Her earnings can be represented by a verbal description, table, graph, and equation.

Verbal description: Ms. Patel charges an initial fee of $275 to manage a project and $25 per page of translated text.

Table:

Number of Pages	Total Earnings for the Project (dollars)
1	300
3	350
10	525
25	900

Graph:



Equation: $y = 275 + 25x$

To solve a linear equation from a graph, locate the value of the given variable, independent or dependent, and determine the exact, if possible, or estimated point corresponding to that variable.

For example, you can use the graph to determine that Ms. Patel will earn $400 if she translates 5 pages for a customers. She will earn approximately $775 for translating 20 pages.

LESSON 2

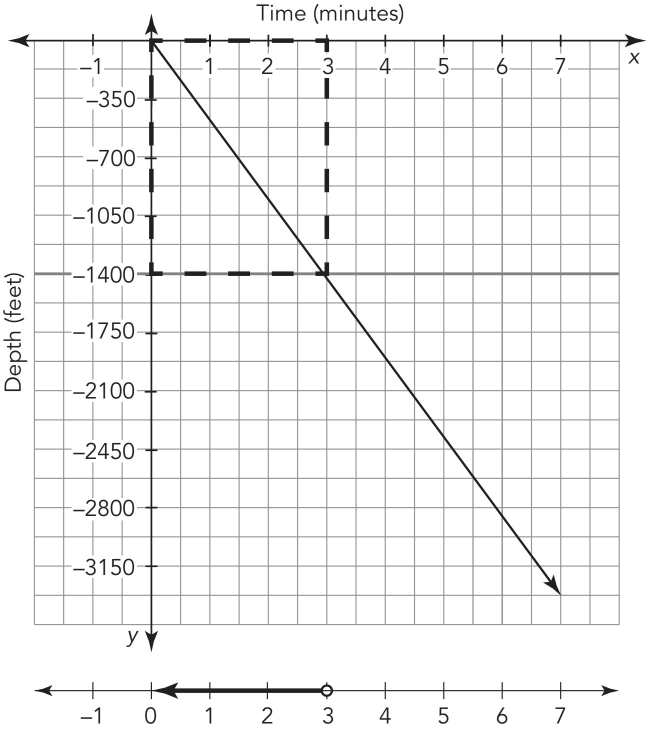
Building Inequalities and Equations to Solve Problems

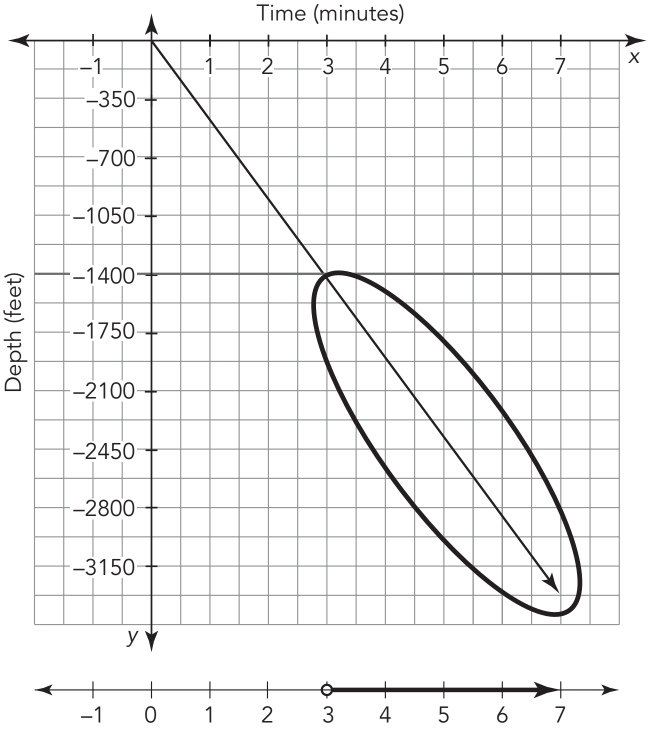
The **unit rate of change** is the amount that the dependent value changes for every one unit that the independent value changes.

For example, suppose the submarine Deep Flight I is going to do a dive starting at sea level, descending 480 feet every minute. The unit rate of change is −480 feet per minute. You can use a graph to estimate solutions to inequality problems.

Estimate the times Deep Flight I will be more than 1400 feet below sea level and the times Deep Flight I will be less than 1400 feet below sea level.

Each of these graphs shows the relationship between the time in minutes and the depth of Deep Flight I. The rectangle on the left graph shows the set of all depths for Deep Flight I less than 1400 feet below sea level. The oval on the right graph shows the set of all depths for Deep Flight I more than 1400 feet below sea level.





Deep Flight I will be less than 1400 feet below sea level for times less than 3 minutes. The submarine will be more than 1400 feet below sea level for times greater than 3 minutes.

LESSON
3

Using Multiple Representations to Solve Problems

Multiple representations, such as a table, an equation, and a graph, can be used to represent a problem situation. You may start with any of these representations to solve a problem and move from one to another by studying their forms and determining unit rates of change.

For example, suppose you are given this table of values.

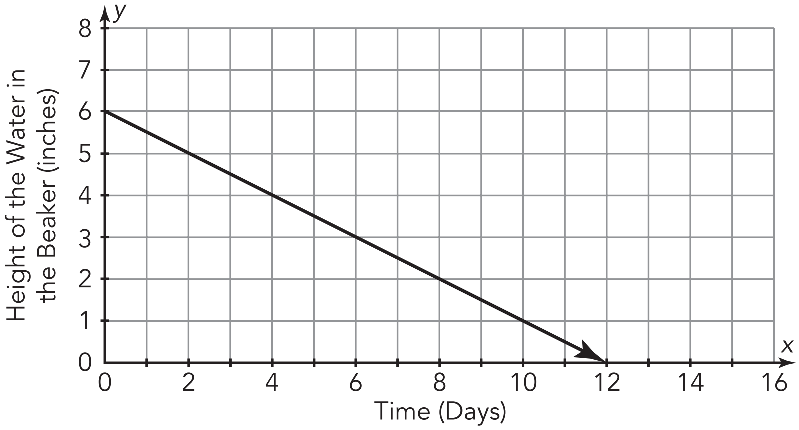
Time	Height of the Water in the Beaker
Days	Inches
0	6
1	5.5
4	4
8	2

You can use the values in the table to represent the problem situation with a graph, equation, and verbal description.

Equation:
$y = 6 - 0.5x$

Verbal description:
The height of the water in the beaker begins at 6 inches. The height of the water decreases by 0.5 inches each day.

Graph:



Math Glossary

A

401(k) plan

A 401(k) plan is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account with the employer often matching a certain amount of it.

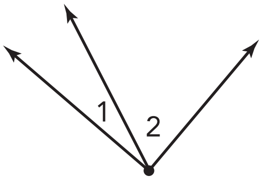
403(b) plan

A 403(b) plan is a retirement plan generally for public school employees or other tax exempt groups.

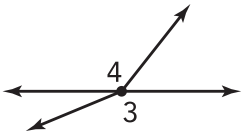
adjacent angles

Adjacent angles are two angles that share a common vertex and share a common side.

Examples



Angles 1 and 2 are adjacent angles.



Angles 3 and 4 are NOT adjacent angles.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $2a + b$ xy $\frac{4}{p}$ z^2

appreciation

Appreciation is an increase in price or value.

asset

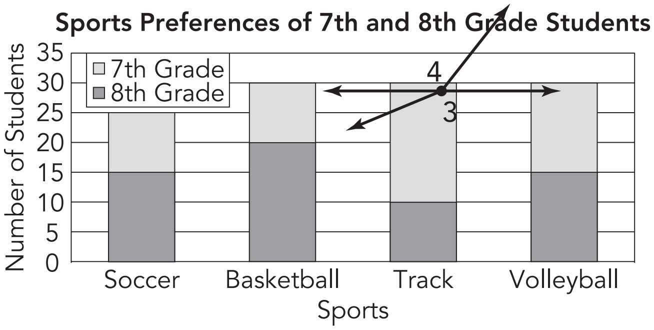
Assets include the value of all accounts, investments, and things that you are own. They are positive and add to your net worth.

B

bar graph

Bar graphs display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

Examples

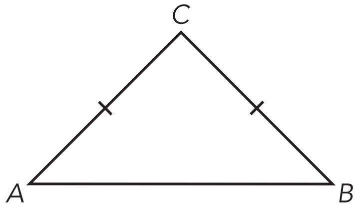


base angles

The angles opposite the two sides that have the same length in an isosceles triangle are called base angles.

Example

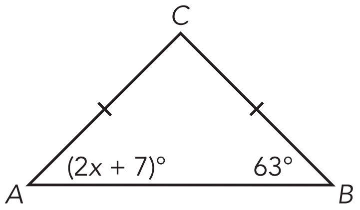
$\angle A$ and $\angle B$ are base angles of $\triangle ABC$



Base Angles theorem

The Base Angles theorem states the angles opposite the congruent sides of an isosceles triangle are congruent.

Example



$\triangle ABC$ is an isosceles triangle. Since, $\angle A$ and $\angle B$ are base angles, their measures are the same.

$2x + 7 = 63$

C

census

A census is the data collected from every member of a population.

Example

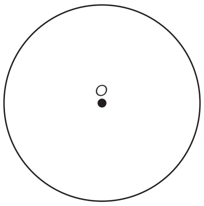
The U.S. Census is taken every 10 years. The U.S. government counts every member of the population every 10 years.

circle

A circle is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.

Example

The circle shown is Circle *O*.

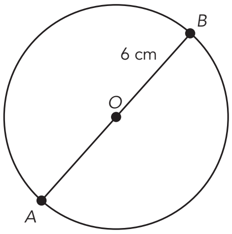


circumference

The distance around a circle is called the circumference of the circle. The circumference is calculated by the formula: $C = \pi(d)$.

Example

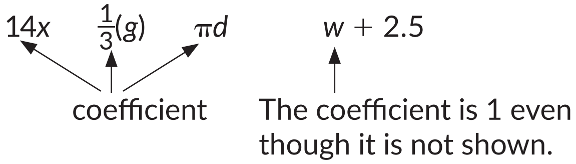
The diameter of Circle *O* is 12 centimeters. The circumference of Circle *O* is 12π.



coefficient

A number that is multiplied by a variable in an algebraic expression is called a coefficient.

Examples



collinear

When points lie on the same line or line segment, they are said to be collinear.

Example

Points *C*, *A*, and *B* are collinear.

commission

A commission is an amount of money a salesperson earns after selling a product. Many times, the commission is a certain percent of the product.

Example

5% commission on $350

0.05 × 350 = $17.50 ⟵ commission

common factor

A common factor is a number that is a factor of two or more numbers.

Example

Factors of 60: **1**, **2**, **3**, **4**, 5, **6**, 10, **12**, 15, 20, 30, 60

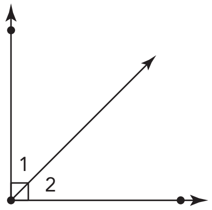
Factors of 24: **1**, **2**, **3**, **4**, **6**, 8, **12**, 24

Common factors of 60 and 24: 1, 2, 3, 4, 6, and 12

complementary angles

Two angles are complementary angles if the sum of their angle measures is equal to 90°.

Example



Angles 1 and 2 are complementary angles.

complementary events

Complementary events are events that together contain all of the outcomes in the sample space.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the event of rolling an even number and the event of rolling an odd number (not even) are complementary events.

complex ratio

A ratio in which one or both of the quantities being compared are written as fractions is a complex ratio.

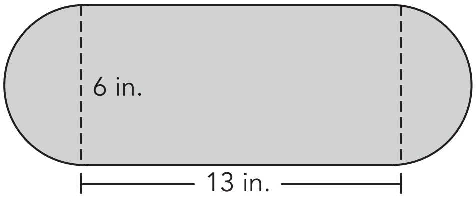
Example

Traveling $\frac{1}{3}$ mile in $\frac{1}{2}$ hour represents a ratio of fractions, or a complex ratio.

composite figure

Composite figures are geometric figures composed of two or more geometric shapes.

Example



The composite figure is composed of a rectangle and two semi-circles.

compound event

A compound event combines two or more events, using the word “and” or the word “or.”

compound interest

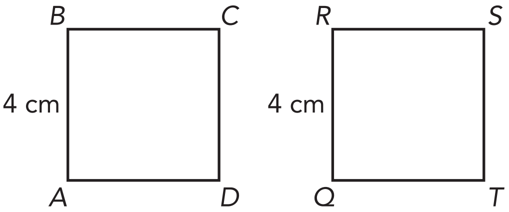
Compound interest is a percentage of the principal and the interest that is added to the principal over time.

congruent

Congruent means to have the same size, shape, and measure.

Example

Square *ABCD* is congruent to Square *QRST*.



congruent angles

Congruent angles are angles that have the same measure.

congruent sides

Congruent sides are sides that have the same length.

constant of proportionality

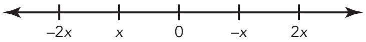
In a proportional relationship, the ratio of all *y*-values to their corresponding *x*-values is constant. This specific ratio, $\frac{y}{x}$, is called the constant of proportionality. Generally, the variable *k* is used to represent the constant of proportionality.

constraint

A constraint is a condition that a solution or problem must satisfy. A constraint can be a restriction set in advance of solving a problem or a limit placed on a solution or graph so the answer makes sense in terms of a real-world scenario.

Example

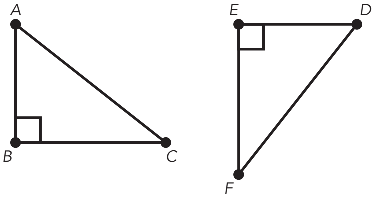
The expressions 0, x, $2x$, $-x$, and $-2x$ are graphed on a number line using the constraint $x < 0$.



corresponding

Corresponding means to have the same relative position in geometric figures, usually referring to sides and angles.

Example



Sides *AB* and *DE* are corresponding sides.

Angle *B* and Angle *E* are corresponding angles.

coupon

A coupon is a detachable part of a ticket or advertisement that entitles the holder to a discount.

D

data

Data are categories, numbers, or observations gathered in response to a statistical question.

Examples

Favorite foods of sixth-graders
Heights of different animals at the zoo

depreciation

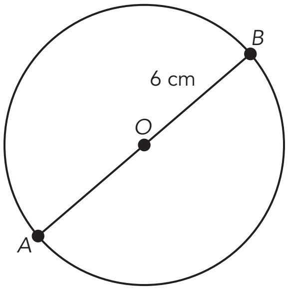
Depreciation is a decrease in price or value.

diameter

The diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.

Example

In Circle O, segment AB is a diameter. The length of diameter AB is two times the length of radius OA. The length of radius OA is 6 centimeters, so the length of diameter AB is 12 centimeters.



direct variation

A situation represents a direct variation if the ratio between the y-value and its corresponding x-value is constant for every point. The quantities are said to vary directly.

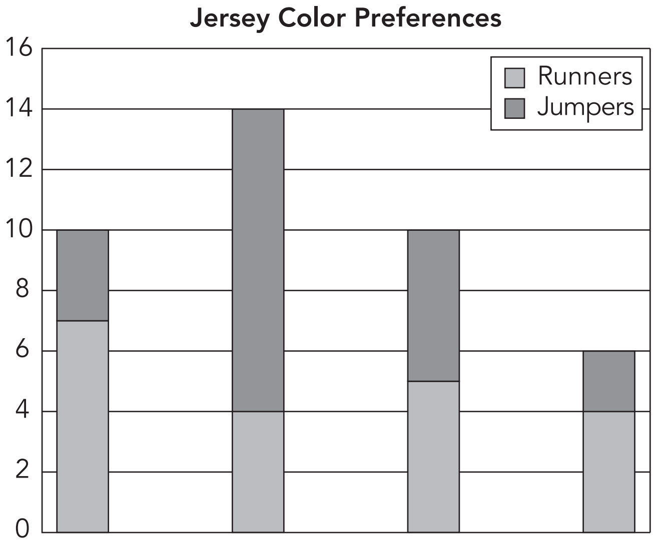
Example

If Melissa earns $8.25 per hour, then the amount she earns is in direct variation with the number of hours she works. The amount $8.25 is the constant of proportionality.

double bar graph

A double graph is used when each category contains two different groups of data.

Example



E

equally likely

When the probabilities of all the outcomes of an experiment are equal, the outcomes are called equally likely.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the probability of rolling each number from 1 through 6 is equally likely.

equation

An equation is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

Examples

$$y = 2x + 4$$

$$6 = 3 + 3$$

$$2(8) = 26 - 10$$

$$\frac{1}{4} \cdot 4 = \frac{8}{4} - \frac{4}{4}$$

evaluate an algebraic expression

To evaluate an algebraic expression means to determine the value of the expression for a given value of each variable.

Example

Evaluate the expression $\frac{4x + (2^3 - y)}{p}$ for $x = 2.5$, $y = 8$, and $p = 2$.

- First replace the variables with numbers: $\frac{4(2.5) + (2^3 - 8)}{2}$.
- Then, calculate the value of the expression: $\frac{10 + 0}{2} = \frac{10}{2} = 5$.

event

An event is one possible outcome or a group of possible outcomes for a given situation.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, an event could be rolling an even number.

experiment

An experiment is a situation involving chance that leads to results, or outcomes.

Example

Rolling a six-sided number cube is an experiment.

experimental probability

Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. You draw the blue marble 20 times out of 50 trials. The experimental probability, P_E(blue), is $\frac{20}{25}$ or $\frac{2}{5}$.

extremes

In a proportion that is written $a : b = c : d$, the two values on the outside, a and d, are the extremes.

Example

$$\underbrace{7 \text{ books} : 14 \text{ days} = 3 \text{ books} : 6 \text{ days}}_{\text{extremes}}$$

F

Family Budget Estimator

The Family Budget Estimator is a tool that people can use to determine the estimated cost of raising a family in a particular city.

fixed expenses

Fixed expenses are expenses that don't change from month to month.

factor

To factor an expression means to rewrite the expression as a product of factors.

Example

$5(12) + 5(9) = 5(12 + 9)$

G

greatest common factor (GCF)

The greatest common factor, or GCF, is the largest factor two or more numbers have in common.

Example

Factors of 16: **1**, **2**, **4**, 8, 16

Factors of 12: **1**, **2**, 3, **4**, 6, 12

Common factors: 1, 2, 4

Greatest common factor: 4

I

income tax

Income tax is a percentage of a person's or company's earnings that is collected by the government.

Example

If a person earns $90,000 in one year and has to pay an income tax rate of 28%, then that person owes 90,000 × 0.28 or $25,200 in income tax to the government.

inverse operations

Inverse operations are pairs of operations that reverse the effects of each other.

Examples

Addition and subtraction are inverse operations: $351 + 25 - 25 = 351$.

Multiplication and division are inverse operations: $351 \cdot 25 \div 25 = 351$.

isolate the variable

When you isolate the variable in an equation, you perform an operation, or operations, to get the variable by itself on one side of the equals sign.

Example

In the equation $\frac{a}{b} = \frac{c}{d}$, you can multiply both sides by b to isolate the variable a.

$$b \cdot \frac{a}{b} = b \cdot \frac{c}{d} \longrightarrow a = \frac{bc}{d}$$

L

lateral surface area

The lateral surface area of a prism or pyramid is the sum of the areas of the lateral faces.

liability

A liability is a financial obligation, or debt, that you must repay. It is negative and take away from your net worth.

like terms

In an algebraic expression, like terms are two or more terms that have the same variable raised to the same power.

Examples

like terms

$4x + 3p + x + 2 = 5x + 3p + 2$

like terms

$24a^2 + 2a - 9a^2 = 13a^2 + 2a$

no like terms

$m + m^2 - x = x^3$

linear expression

A linear expression is any expression in which each term is either a constant or the product of a constant and a single variable raised to the first power.

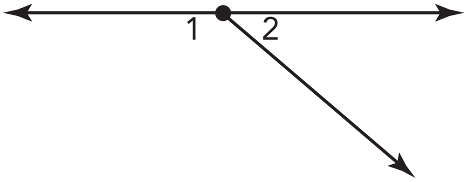
Examples

$\frac{1}{2}x + 2$, $-3 + 12.5x$, $-1 + 3x + \frac{5}{2}x - \frac{4}{3}$

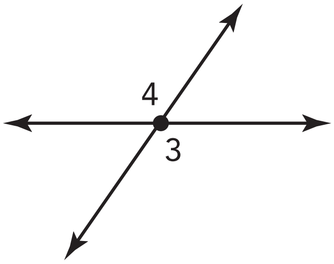
linear pair

A linear pair of angles is formed by two adjacent angles that have noncommon sides that form a line.

Examples



Angles 1 and 2 form a linear pair.



Angles 3 and 4 do NOT form a linear pair.

literal equation

A literal equation is an equation in which the variables represent specific measures.

Examples

$A = \ell w \qquad A = \frac{1}{2}bh \qquad d = rt$

M

markdown

When businesses sell an item at a lower price than the original price, it is called a markdown.

markup

To make money, businesses often buy products from a wholesaler or distributor for one amount and add to that amount to determine the price they use to sell the product to their customers. This is called a markup.

means

In a proportion that is written $a : b = c : d$, the two values in the middle, b and c, are the means.

Example

$7 \text{ books} : \underbrace{14 \text{ days} = 3 \text{ books}}_{\text{means}} : 6 \text{ days}$

N

net worth

Your net worth is basically a calculation of the value of everything that you own minus the amount of money that you owe.

non-uniform probability model

A non-uniform probability model occurs when all the probabilities in a probability model are not equal to each other.

Example

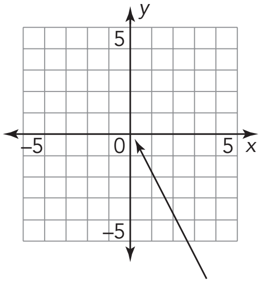
Outcome	Red	Green	Blue
Probability	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{3}{8}$

O

origin

The origin is a point on a graph with the ordered pair (0, 0).

Example



The origin is at (0,0)

outcome

An outcome is the result of a single trial of a probability experiment.

Example

The numbers on the faces of a six-sided number cube are the outcomes that can occur when rolling a six-sided number cube.

P

parameter

When data are gathered from a population, the characteristic used to describe the population is called a parameter.

Example

If you wanted to determine the average height of the students at your school, and you measured every student at the school, the characteristic “average height” would be a parameter.

percent decrease

A percent decrease occurs when the new amount is less than the original amount. It is a ratio of the amount of decrease to the original amount.

Example

The price of a $12 shirt has decreased to $8.

$$\frac{12 - 8}{12} = \frac{4}{12} = 0.3 = 33.3\%$$

The percent decrease is 33.3%

percent equation

A percent equation can be written in the form percent × whole = part, where the percent is often written as a decimal.

Example

40% of 25 = 10

$(0.40)\ (25) = 10$

↑ ↑ ↑

Percent Part

Whole

percent error (estimation)

Calculating percent error is one way to compare an estimated value to an actual value. To compute percent error, determine the difference between the estimated and actual values and then divide by the actual value.

Example

An airline estimates that they will need an airplane that sits 320 passengers for a flight. An actual 300 tickets were booked for the flight.

$$\text{Percent Error} = \frac{300 - 320}{300} = \frac{-20}{300} \approx -6.7\%$$

percent error (probability)

In probability, the percent error describes how far off the experimental probability is from the theoretical probability as a percent ratio.

Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. You draw the blue marble 20 times out of 50 trials.

The experimental probability, P_E(blue), is $\frac{20}{50}$ or $\frac{2}{5}$. The theoretical probability, P_T(blue), is $\frac{1}{4}$.

The percent error is $\dfrac{\frac{2}{5} - \frac{1}{4}}{\frac{1}{4}} = \dfrac{\frac{3}{20}}{\frac{1}{4}} = \dfrac{3}{5}$

$= 0.6 = 60\%$

percent increase

A percent increase occurs when the new amount is greater than the original amount. It is a ratio of the amount of increase to the original amount.

Example

The price of a $12 shirt has increased to $13.20.

$$\frac{13.20 - 12}{12} = \frac{1.20}{12} = 0.1 = 10\%$$

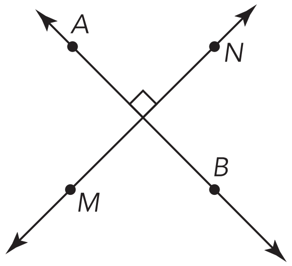
The percent increase is 10%.

perpendicular

Two lines, line segments, or rays are perpendicular if they intersect to form 90° angles. The symbol for perpendicular is ⊥.

Example

Line *AB* is perpendicular to line *MN*



personal budget

A personal budget is an estimate of the amount of money that a person or family will need for specific financial items. It generally includes current expenses as well as savings for anticipated future expenses.

principal

A principal is the term for an original amount of money on which interest is calculated.

pi

The number pi (π) is the ratio of the circumference of a circle to its diameter. That is $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle.

population

A population is an entire set of items from which data are collected.

Example

If you wanted to determine the average height of the students at your school, the number of students at the school would be the population.

probability

Probability is the measure of the likelihood that an event will occur. It is a way of assigning a numerical value to the chance that an event will occur by dividing the number of times an event can occur by the number of possible outcomes.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the probability of rolling a 5, or $P(5)$, is $\frac{1}{6}$.

probability model

A probability model is a list of each possible outcome along with its probability, often shown in a table.

Example

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

This is a probability model for rolling a six-sided number cube with the numbers 1 through 6 on each face.

proportion

A proportion is an equation that states that two ratios are equal.

Example

$\frac{1}{2} = \frac{4.5}{9}$

proportional relationship

A proportional relationship is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$ must represent the same constant.

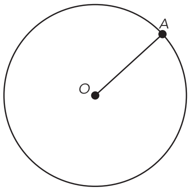
R

radius

The radius of a circle is a line segment formed by connecting a point on the circle and the center of the circle.

Example

In the circle, *O* is the center and segment *OA* is the radius.



random number table

A random number table is a table that displays random digits. These tables can contain hundreds of digits.

Example

Line 7	54621	62117	55516	40467

random sample

A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

Example

If you wanted to determine the average height of the students at your school, you could choose just a certain number of students randomly and measure their heights. This group of students would be a random sample.

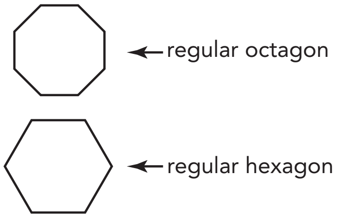
rebate

A rebate is a refund of part of the amount paid for an item.

regular polygon

A regular polygon is a polygon with all sides congruent and all angles congruent.

Examples



S

sale

A sale is an event at which products are sold at reduced prices.

sales tax

Sales tax is a percentage of the selling price of a good or service which is added to the price.

Example

You want to purchase an item for $8.00 in a state where the sales tax is 6.25%, therefore you will pay 8×0.0625 or $0.50 in sales tax. You will pay a total of $8.50 for the item.

sample

A sample is a selection from a population.

Example

If you wanted to determine the average height of the students in your school, you could choose a certain number of students and measure their heights. The heights of the students in this group would be your sample.

sample space

A list of all possible outcomes of an experiment is called a sample space.

Example

When rolling a six-sided number cube that has one number, from 1 through 6, on each face, the sample space is {1, 2, 3, 4, 5, 6}.

scale

A scale is a ratio that compares two measures.

Example

1 cm : 4 cm

scale drawing

A scale drawing is a representation of a real object or place that is in proportion to the real object or place it represents.

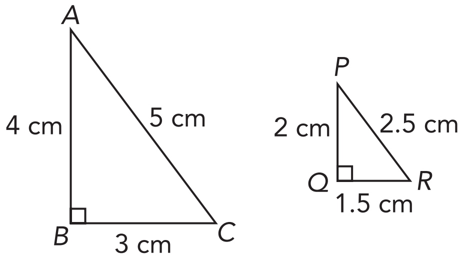
Examples

A map or a blueprint is an example of a scale drawing.

scale factor

When you multiply a measure by a scale to produce a reduced or enlarged measure, the scale is called a scale factor.

Example

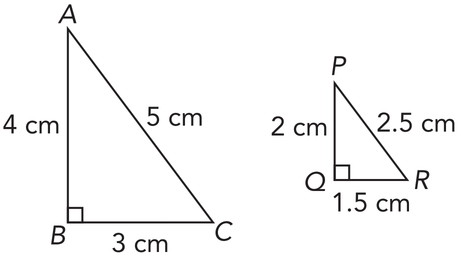


The scale factor from Triangle *ABC* to Triangle *PQR* is $\frac{1}{2}$.

similar figures

Figures that are proportional in size, or that have proportional dimensions, are called similar figures.

Example



Triangle *ABC* and Triangle *PQR* are similar figures.

simple event

A simple event is an event consisting of one outcome.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, rolling a 5 is a simple event.

simple interest

Simple interest is a type of interest that is a fixed percent of the principal. Simple interest is paid over a specific period of time—either twice a year or once a year, for example. The formula for simple interest is $I = P \cdot r \cdot t$, where I represents the interest earned, P represents the amount of the principal, r represents the interest rate, and t represents the time that the money earns interest.

Example

Kim deposits $300 into a savings account at a simple interest rate of 5% per year. The formula can be used to calculate the simple interest Kim will have earned at the end of 3 years.

Interest = Principal · rate · time

Interest = (300)(0.05)(3)

= $45

simulation

A simulation is an experiment that models a real-life situation.

solve a proportion

To solve a proportion means to determine all the values of the variables that make the proportion true.

stacked bar graph

A stacked bar graph is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole.

statistic

When data are gathered from a sample, the characteristic used to describe the sample is called a statistic.

Example

If you wanted to determine the average height of the students in your school, and you chose just a certain number of students randomly and measured their heights, the characteristic “average height” would be called a statistic.

straight angle

A straight angle is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

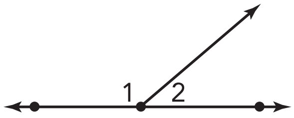
Example

Angle *CAB* is a straight angle.

supplementary angles

Two angles are supplementary angles if the sum of their angle measures is equal to 180°.

Example



Angles 1 and 2 are supplementary angles.

surface area

The **surface area** of a prism or pyramid is the total area of all its two-dimensional faces.

survey

A survey is one method of collecting data in which people are asked one or more questions.

Example

A restaurant may ask its customers to complete a survey with the following question:

On a scale of 1–10, with 1 meaning "poor" and 10 meaning "excellent," how would you rate the food you ate?

T

theoretical probability

The theoretical probability of an event is the ratio of the number of desired outcomes to the total possible outcomes.

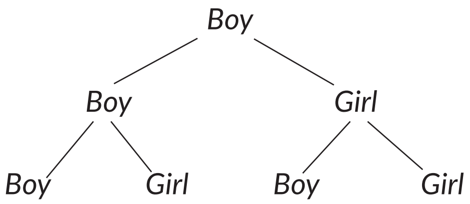
Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. The theoretical probability of drawing a blue marble, P_T(blue), is $\frac{1}{4}$.

tree diagram

A tree diagram illustrates the possible outcomes of a given situation. It has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

Example



two-step equation

A two-step equation requires that two inverse operations be performed to isolate the variable.

U

uniform probability model

A uniform probability model occurs when all the probabilities in a probability model are equally likely to occur.

Example

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

unit rate

A unit rate is a comparison of two different measurements in which the numerator or denominator has a value of one unit.

Example

The speed 60 miles in 2 hours can be written as a unit rate:

$\frac{60\text{ mi}}{2\text{ h}} = \frac{30\text{ mi}}{1\text{ h}}$.

The unit rate is 30 miles per hour.

unit rate of change

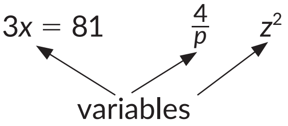
The unit rate of change describes the amount the dependent variable changes for every unit the independent variable changes.

V

variable

A variable is a letter or symbol that is used to represent a number.

Examples



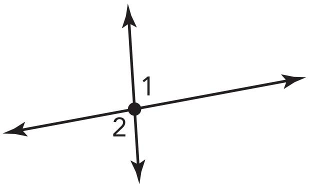
variable expenses

Variable expenses are expenses that can be different from month to month.

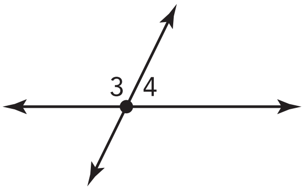
vertical angles

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

Examples



Angles 1 and 2 are vertical angles.



Angles 3 and 4 are NOT vertical angles.

Z

zero pair

A positive counter and a negative counter together make a zero pair since the total value of the pair is zero.

Example

⊕ + ⊖ = 0

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