OER Secondary Mathematics

EDITION 1

Grade 7

Volume 2

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

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Secondary Mathematics

EDITION 1

Grade 7

Course Guide

Welcome to the Course Guide for Secondary Mathematics, Grade 7

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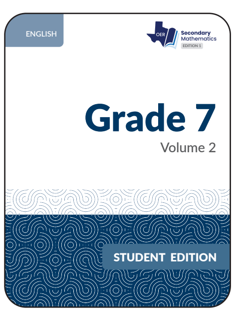
Instructional Design

The instructional materials help you learn math in different ways. There are two types of resources: Learning Together and Learning Individually. These resources provide various learning experiences to develop your understanding of mathematics.

Learning Together

On **Learning Together** days, you spend time engaging in active learning to build mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the materials for each lesson.



STUDENT EDITION

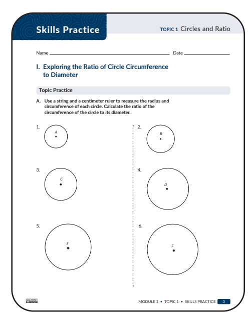
I am a record of your thinking, reasoning, and problem solving.

My lessons allow you to build new knowledge from prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On **Learning Individually** days, Skills Practice offers opportunities to engage with skills, concepts, and applications that you learn in each lesson. It also provides opportunities for interleaved practice, which encourages you to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning. This resource will help you build proficiency in specific skills based on your individual academic needs as indicated by monitoring your progress throughout the course.

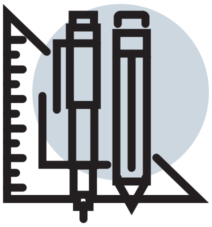


SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide problem sets for additional practice, enrichment, and extension.

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure you thoroughly understand the mathematical concepts. The instructional materials guide you to connect related ideas holistically, supporting the integration of your evolving mathematical understanding and developing proficiency with mathematical processes.



Intentional Mathematics Design

Mathematical Coherence: The path through the mathematics develops logically, building understanding by linking ideas within and across grades so you can learn concepts more deeply and apply what you've learned to more complex problems.

TEKS Mathematical Process Standards: The instructional materials support your development of the TEKS mathematical process standards. They encourage you to experiment, think creatively, and test various strategies. These mathematical processes empower you to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem-solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture your interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for you to build new understanding on the firm foundation of what you already know. You move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem-Solving: Solving problems is an essential life skill that you need to develop. The problem-solving model provides a structure to support you as you analyze and solve problems. It is a strategy you can continue to use as you solve problems in everyday life.

1 Exploring the Ratio of Circle Circumference to Diameter

① OBJECTIVES

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Understand the formula for the circumference of a circle and use the formula to solve problems.

③ NEW KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

② You have learned about ratios.

How can you use ratios to analyze the properties of geometric figures, such as circles?

LESSON STRUCTURE

① Objectives

Objectives are stated for each lesson to help you take ownership of the objectives.

② Essential Question

Each lesson begins with a statement connecting what you have learned with a question to ponder. Return to this question at the end of this lesson to gauge your understanding.

③ New Key Terms

The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

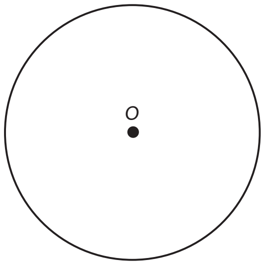
4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

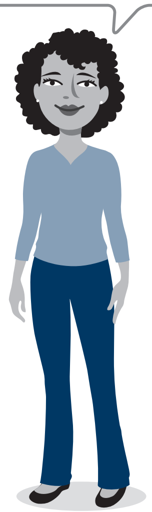
4 Getting Started

Across and Around

Consider the circle with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.



Be sure to include units when you record your measurements.



1. Analyze the distance around the circle.

   a. Use a string and a centimeter ruler to determine the distance around the circle.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

2. Draw a line from a point on the circle to the center of the circle, point *O*.

   a. Measure your line using your centimeter ruler.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

5

ACTIVITY 1.1

Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a bit harder. Can you define a circle without using the word *round*? Investigating how a circle is formed will help you mathematically define a circle.

1. Follow the given steps to investigate how a circle is formed.

Step 1: In the space provided, draw a point and label the point *A*.

Step 2: Use a centimeter ruler to locate and draw a second point that is exactly 5 cm from point *A*. Label this point *B*.

Step 3: Locate a third point that is exactly 5 cm from point *A*. Label this point *C*.

Step 4: Repeat this process until you have drawn at least ten distinct points that are each exactly 5 cm from point *A*.

2. How many other points could be located exactly 5 cm from point *A*? How would you describe this collection of points in relation to point *A*?

3. Define the term *circle* without using the word *round*.

5 Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about getting the answer. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Talk the Talk 6

Twice

Use what you have learned to compare circles by their characteristics.

1. Using your compass, draw each circle.

   a. Radius length of 3 centimeters

   b. Diameter length of 3 centimeters

2. Describe the similarities and differences between your two circles.

3. Describe the relationship between the circumferences of the two circles.

4. Describe the circumference-to-diameter ratio of all circles.

Lesson 1 Assignment

7 Write

Define each term in your own words.

1. Circle
2. Radius
3. Diameter
4. Pi

Remember 8

The circumference of a circle is the distance around the circle. The formulas to determine the circumference of a circle are $C = \pi d$, or $C = 2\pi r$, where d represents the diameter, r represents the radius, and π is a constant value equal to approximately 3.14 or $\frac{22}{7}$.

The constant pi (π) represents the ratio of the circumference of a circle to its diameter.

9 Practice

Answer each question. Use 3.14 for π. Round your answer to the nearest tenth, when necessary.

1. Although he's only in middle school, Mason loves to drive go-carts! His favorite place to drive go-carts has 3 circular tracks. Track 1 has a radius of 60 feet. Track 2 has a radius of 85 feet. Track 3 has a radius of 110 feet.
   a. Compute the circumference of Track 1.
   b. Compute the circumference of Track 2.
   c. Compute the circumference of Track 3.
   d. The go-cart place is considering building a new track. They have a circular space with a diameter of 150 feet. Compute the circumference of the circular space.

ASSIGNMENT

7 Write
Reflect on your work and clarify your thinking.

8 Remember
Take note of the key concepts from the lesson.

9 Practice
Use the concepts learned in the lesson to solve problems.

Lesson 1 Assignment

2. Mason wants to build a circular go-cart track in his backyard. Use 3.14 for π.

   a. Suppose he wants the track to have a circumference of 157 feet. What does the radius of the track need to be?

   b. Suppose he wants the track to have a circumference of 314 feet. What does the radius of the track need to be?

   c. Suppose he wants the track to have a circumference of 471 feet. What does the diameter of the track need to be?

10 **Prepare**

Determine a unit rate for each situation.

1. $38.40 for 16 gallons of gas

2. 15 miles jogged in 3.75 hours

3. $26.99 for 15 pounds

ASSIGNMENT

10 **Prepare**

Get ready for the next lesson.

Research-Based Strategies

WORKED EXAMPLE

Isabella's first client of the day spent $150 to have her hair dyed and cut and gave Isabella a $30 tip.

Use a Proportion

$$\frac{t}{100} = \frac{30}{150}$$

$$t = \frac{(30)(100)}{150}$$

$$t = 20$$

Use a Percent Equation

$$(t)(150) = 30$$

$$150t = 30$$

$$\frac{150t}{150} = \frac{30}{150}$$

$$t = \frac{30}{150}$$

$$t = 0.2$$

Ask Yourself . . .

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Identify the error in Samuel's strategy.

Samuel

$$2x + 6 = 14$$

$$\frac{2x}{2} + 6 = \frac{14}{2}$$

$$x + 6 = 7$$

$$-6 = -6$$

$$x = 1$$

Ask Yourself . . .

- Where is the error?
- Why is it an error?
- How can I correct it?

Explain Ethan's process in solving the equation.

Ethan

$$2x + 6 = 14$$

$$-6 = -6$$

$$2x = 8$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

Ask Yourself . . .

- Why is this method correct?
- Have I used this method before?

WORKED EXAMPLE

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

THUMBS UP

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

THUMBS DOWN

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

WHO'S CORRECT?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine whether the work is correct or incorrect.

5. Alejandra says that the product being less than 10 and the product being more than 10 are complementary events. Hailey disagrees. Who is correct? Explain your reasoning.

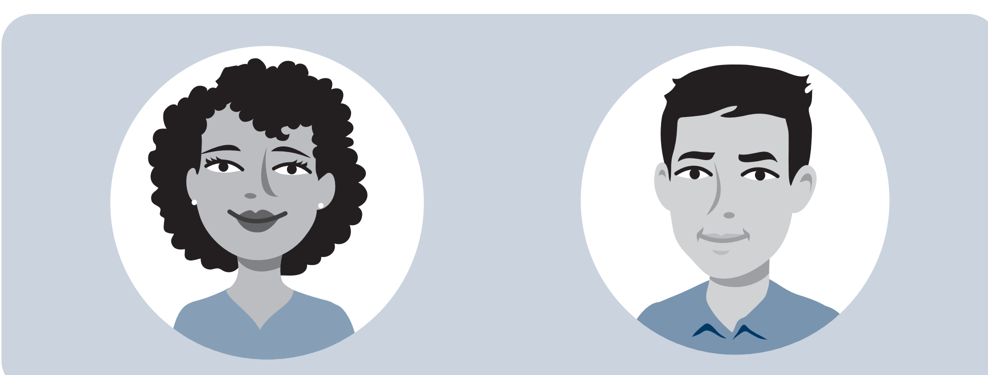
Ask Yourself . . .

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.



TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I Can” expectations listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem solving process and reasonableness of the solution.

I CAN:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I CAN:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language, as appropriate.

I CAN:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Create and use representations to organize, record, and communicate mathematical ideas.

I CAN:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Analyze mathematical relationships to connect and communicate mathematical ideas.

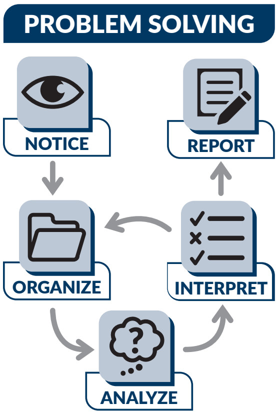
I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.



The Problem-Solving Model

Productive mathematical thinkers are problem-solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

Understanding the Problem-Solving Model

Notice | Wonder

Understand the situation by asking these questions.

- What do I know?
- What do I need to determine?
- What important information is given that I will need to determine a solution?
- What information is given that I do NOT need?
- Is there enough information given to solve the problem?

Organize | Mathematize

Devise a plan for your mathematical approach by asking these questions.

- What is a similar problem to this that I have solved before?
- What strategies may help to solve this problem using the given information?
- How can I represent this problem using a picture, diagram, symbols, graph or some other visual representation? Which representations make sense for this problem?

Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I show my math work using representations?
- Did I explain my mathematical solution in terms of the problem situation, when applicable?
- Did I describe how I arrived at my solution?
- Did I communicate my strategy and solution clearly using precise mathematical language as necessary?
- Can I make any predictions based on my work?

Test | Interpret

Look back at your work and ask these questions.

- Does the solution answer the original question/problem?
- Does the reasoning and the solution make sense?
- How could I have used a different strategy to solve this problem? Would it have changed the outcome?

Report

As you share your mathematical reasoning with others ask these questions.

- Did I share my solution with others?
- Do others understand the mathematics I communicated?

The Problem-Solving Model Graphic Organizer

NOTICE **Understand the Problem**	ORGANIZE **Devise a Plan**
PREDICT **Carry Out the Plan**	
INTERPRET **Look Back**	
REPORT **Report**	

Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition

Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Explain Your Reasoning

Definition

Give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

What is Productive Struggle?

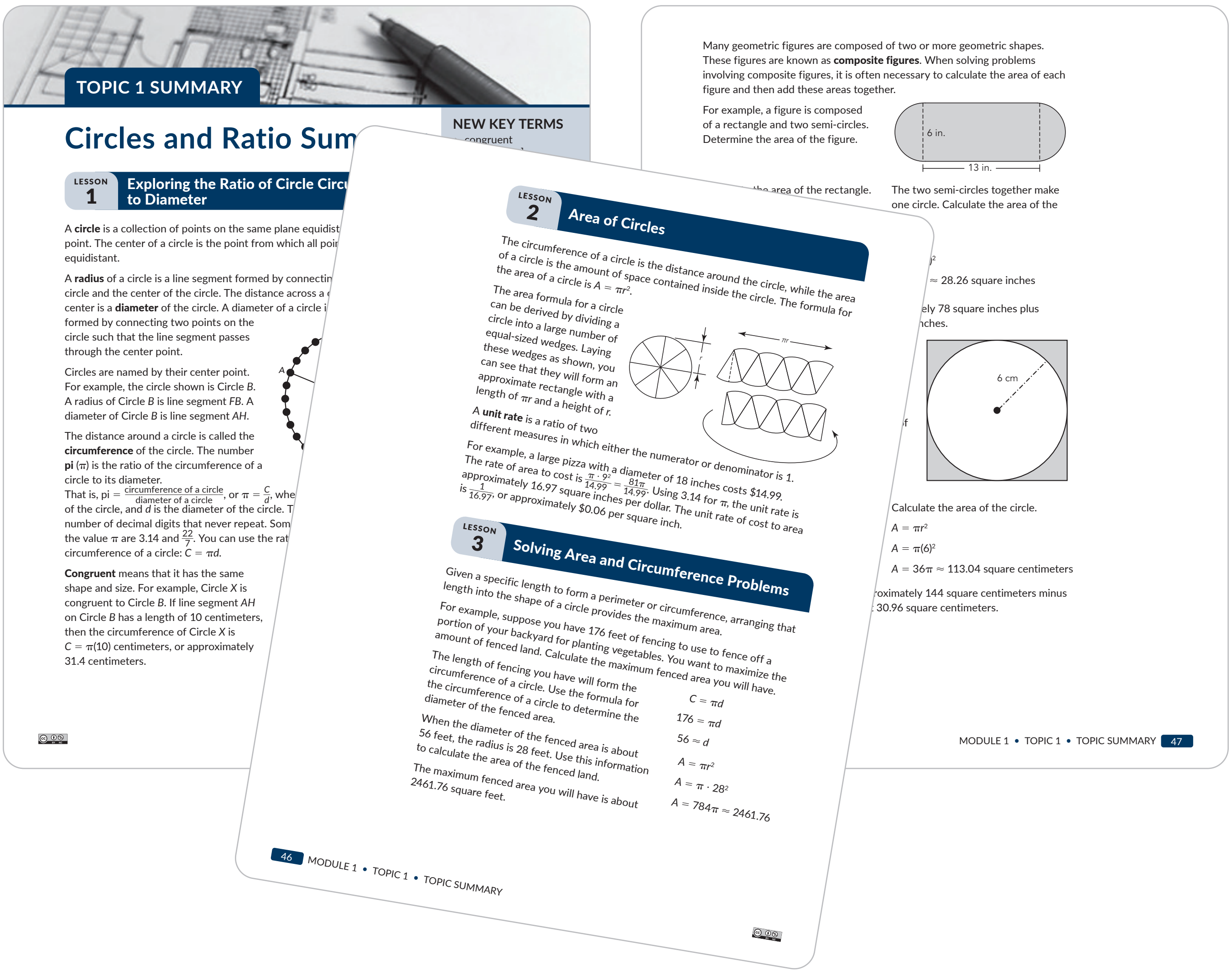
Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process *productive struggle*. Productive struggle is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

Topic Summary

Each topic includes a Topic Summary. The Topic Summary contains a list of all new key terms addressed in the topic and a summary of each lesson, including worked examples and new key term definitions. Use the Topic Summary to review each lesson's major concepts and strategies as you complete assignments and/or share your learning outside of class.



Topic Self-Reflection

The Topic Self-Reflection, provided at the end of each topic, empowers you to develop confidence in your mathematical understanding and monitor your own learning processes. Taking the time for self-reflection helps you identify your strengths and where you want to focus your efforts to improve.

Use the Topic Self-Reflection throughout the topic to monitor your progress toward the mathematical goals for the topic.

TOPIC 1 SELF-REFLECTION Name: ______________________

Circles and Ratios

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents I **am building proficiency of the skill**, and 3 represents I **have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Circles and Ratios* topic by:

TOPIC 1: *Circles and Ratios*	Beginning of Topic	Middle of Topic	End of Topic
describing pi (π) as the ratio of the circumference to the diameter of a circle.	◯	◯	◯
using models to derive and explain the relationship between circumference and area of a circle.	◯	◯	◯
justifying the formulas for area and circumference of a circle and how they relate to pi (π).	◯	◯	◯
applying the circumference and area formulas to solve mathematical and real-world problems.	◯	◯	◯

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Circles and Ratios* topic.

____

____

____

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

____

____

____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

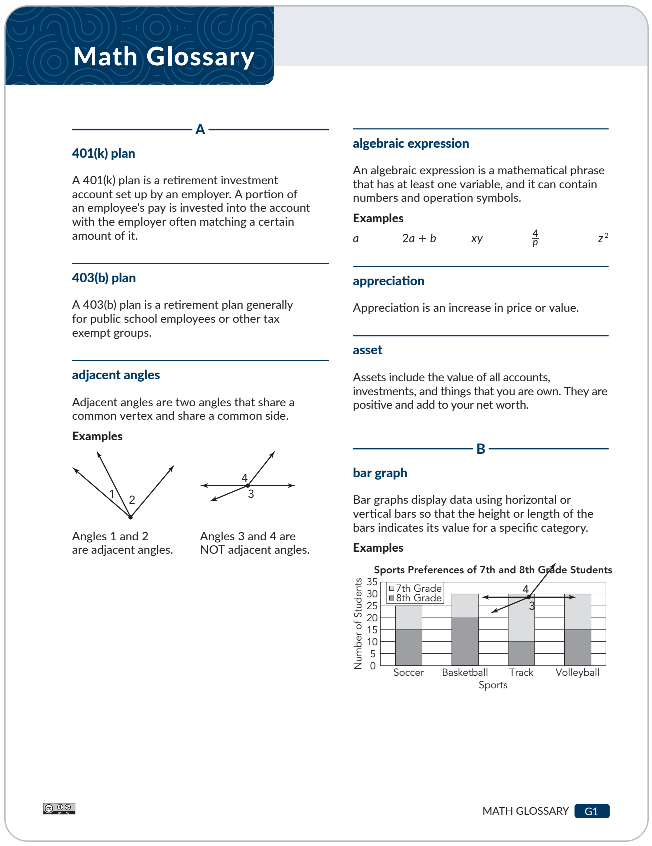
____

____

____

Math Glossary

A course-specific math glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.



Math Glossary

A

401(k) plan

A 401(k) plan is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account with the employer often matching a certain amount of it.

403(b) plan

A 403(b) plan is a retirement plan generally for public school employees or other tax exempt groups.

adjacent angles

Adjacent angles are two angles that share a common vertex and share a common side.

Examples

Angles 1 and 2 are adjacent angles.

Angles 3 and 4 are NOT adjacent angles.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $2a + b$ xy $\frac{4}{p}$ z^2

appreciation

Appreciation is an increase in price or value.

asset

Assets include the value of all accounts, investments, and things that you are own. They are positive and add to your net worth.

B

bar graph

Bar graphs display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

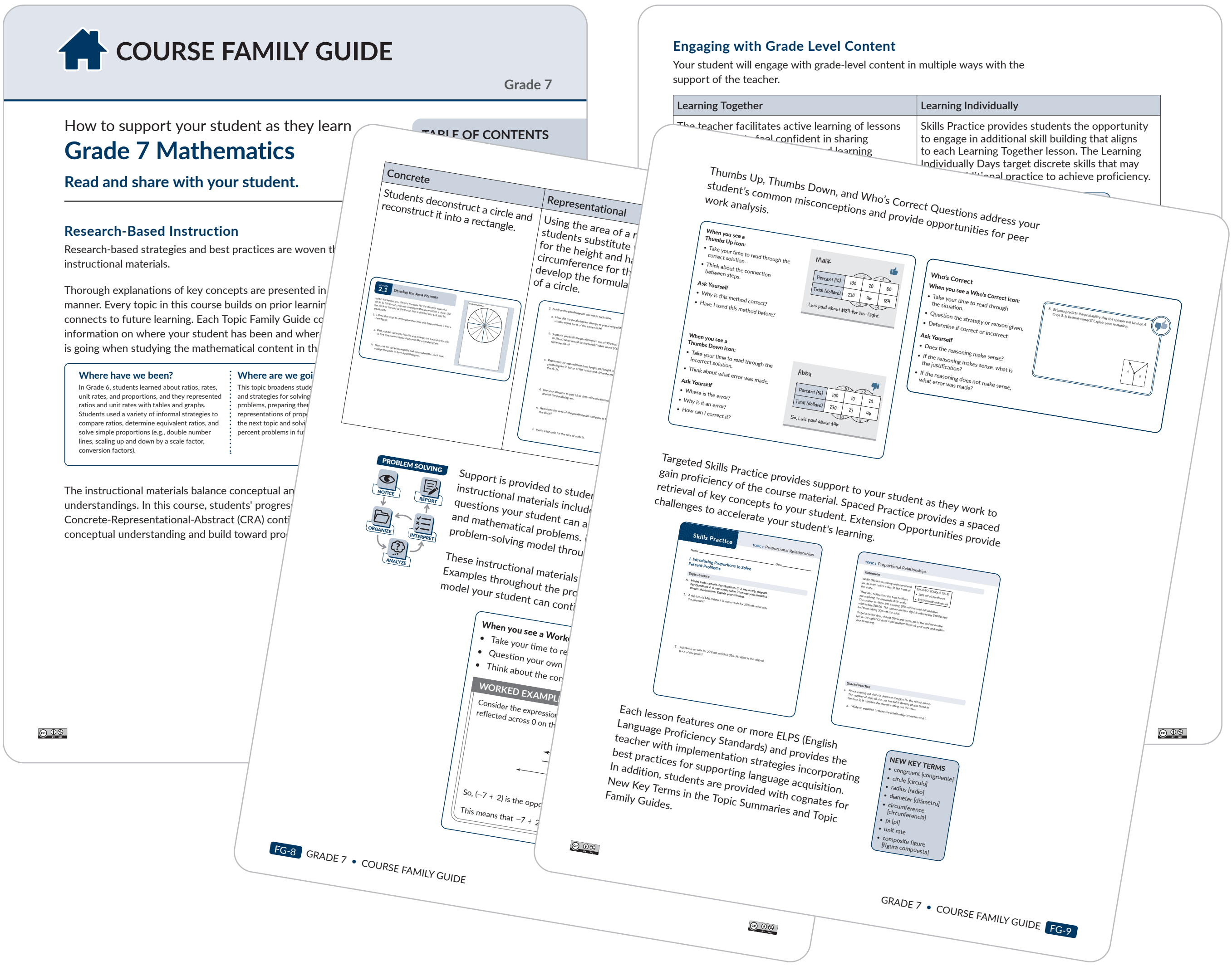
Examples

MATH GLOSSARY G1

Course Family Guide

The Course Family Guide provides you and your family an overview of the course design. The guide details the resources available to support your learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

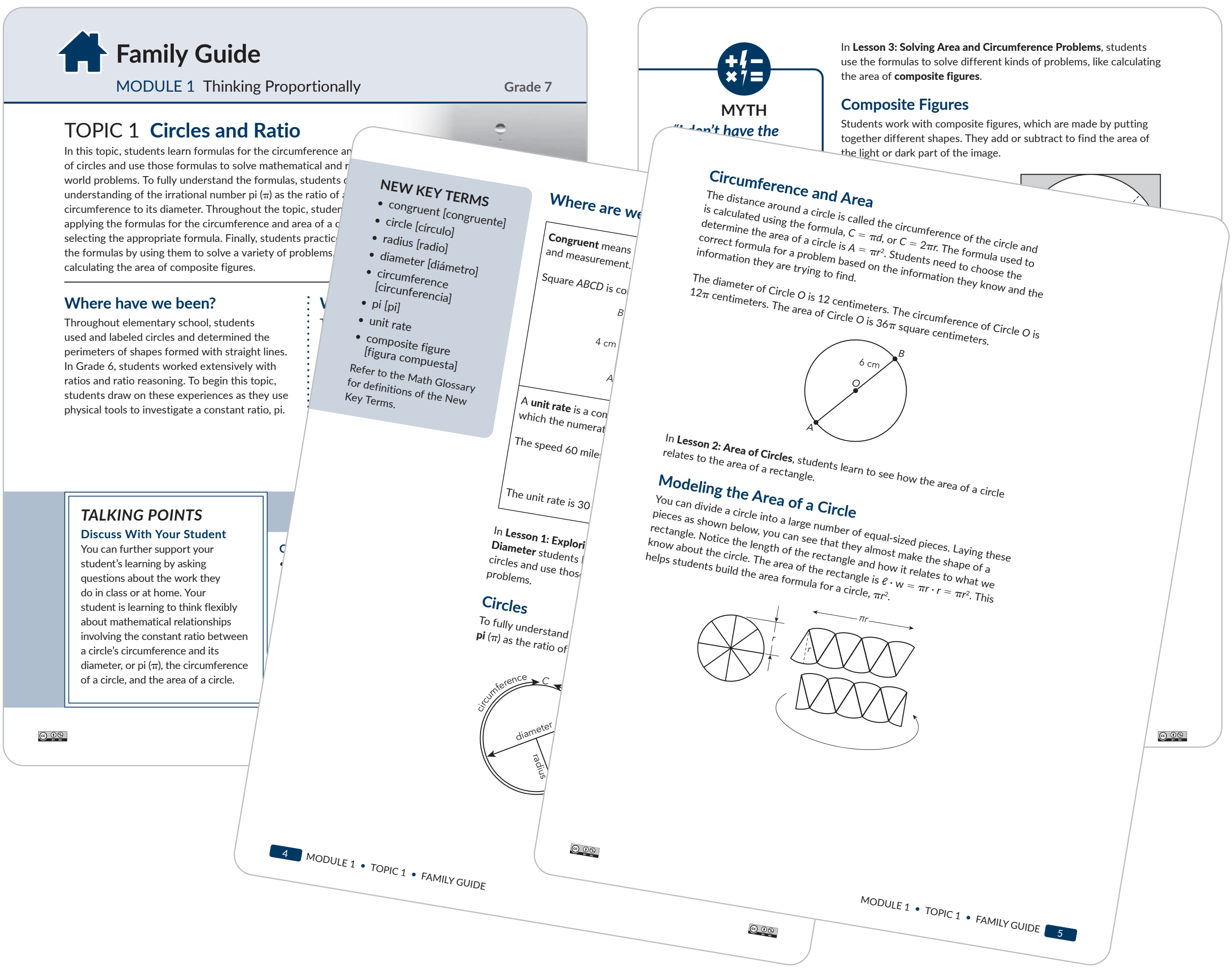
The purpose of the Course Family Guides is to bridge your learning in the classroom to your learning at home. The goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together.



Topic Family Guides

Each topic contains a Topic Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Topic Family Guide are an example of a math model or strategy taught in the topic, definitions of key terms, busting of a math myth, and questions family members can ask you to support your learning.

Learning outside of the classroom is crucial to student success at school. The Topic Family Guide serves to assist families in talking to students about the learning that is happening in the classroom.



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GRADE 7

MODULE
4

Analyzing Populations and Probabilities

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GRADE 7

TOPIC 1

Behind two of the doors are delicious cakes. Behind one of the doors is a delicious piece of chocolate. If you can only pick a door once, what are your chances of eating a cake versus snacking on a piece of chocolate?

Introduction to Probability

LESSON 1 Defining and Representing Probability **587**

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1 Defining and Representing Probability

OBJECTIVES

- Differentiate between an outcome and an event for an experiment.
- List the sample space for an experiment.
- Determine the probability for an event and the complement of an event.
- Identify the probability of an event as a number between 0 and 1, which can be expressed as a fraction, decimal, or percent.
- Determine that the sum of the probabilities of the outcomes of an experiment is always 1.

NEW KEY TERMS

- outcome
- experiment
- sample space
- event
- simple event
- probability
- complementary events
- equally likely

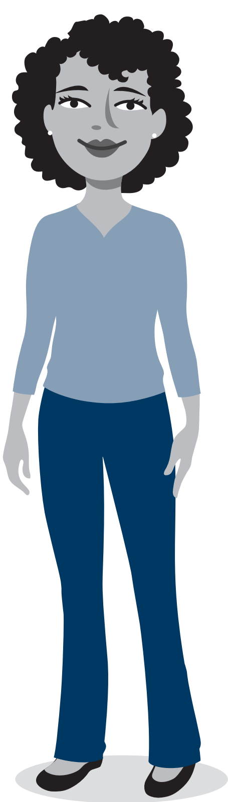
You have used ratios to represent relationships between two quantities.

How can you use ratios to represent the likelihood of an event?

Getting Started

What Are the Chances?

Can a meteorologist predict the future? A meteorologist uses lots of science and math to provide you with the probability of what may happen with the weather.



Consider each statement.

1. The local weatherman broadcasted that there is a 40% chance of rain today.

   a. In your own words, explain what the statement means.

   b. Rewrite Question 1 using an equivalent fraction in place of the percent. Explain what it means.

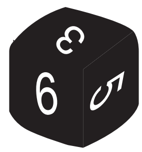
2. For a multiple-choice question with four answer choices, the likelihood of guessing the correct answer is $\frac{1}{4}$.

   a. In your own words, explain what the statement means.

   b. Rewrite Question 2 using an equivalent percent in place of the fraction. Explain what it means.

ACTIVITY 1.1 Calculating Simple Probabilities

A six-sided number cube has one number, from 1 through 6, on each face. Number cubes are often used when playing board games.



1. Create a list of all the possible numbers that can appear on the top face if you roll a six-sided number cube.

Remember. . .
A cube is a three-dimensional figure with 6 sides called faces.

The numbers on the faces of a six-sided number cube are the *outcomes* that can occur when rolling a six-sided number cube. An **outcome** is the result of a single trial of a probability *experiment*. An **experiment** is a situation involving chance that leads to results, or outcomes. A list of all possible outcomes of an experiment is called a **sample space**.

A sample space is typically enclosed in braces, { }, with commas between the outcomes.

2. List the sample space for the experiment of rolling a six-sided number cube.

An **event** is one possible outcome or a group of possible outcomes for a given situation. A **simple event** is an event consisting of one outcome. For example, in the number cube experiment, an event could be rolling an even number. However, rolling a 5 is a simple event.

An event is a subset of the sample space.

Probability is a measure of the likelihood that an event will occur. It is a way of assigning a numerical value to the chance that an event will occur. The probability of an event is often written as *P*(event). For example, in the number cube experiment, the probability of rolling a 5 could be written as *P*(5). The probability of rolling an even number could be written as *P*(even).

There is a formula to determine the probability of an event.

$$\text{probability} = \frac{\text{number of times an event can occur}}{\text{number of possible outcomes}}$$

WORKED EXAMPLE

To determine the probability of rolling an odd number, or P(odd), follow these steps.

Step 1: First, list all the possible outcomes in the event.
The possible odd numbers that can be rolled are 1, 3, and 5.

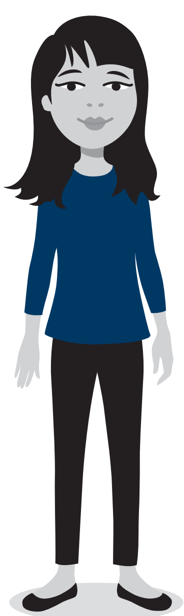
Step 2: Add the number of outcomes.
There are 3 possible outcomes of rolling an odd number.

Step 3: Use the equation to determine the probability of rolling an odd number.

$$P(\text{odd}) = \frac{\text{number of times an odd can occur}}{\text{number of possible outcomes}} = \frac{\text{3 possible odd numbers}}{\text{6 possible numbers}}$$

The probability of rolling an odd number is $\frac{3}{6}$ or $\frac{1}{2}$.

So, to determine P(4) consider how many times a 4 can occur when you roll a six-sided number cube.



3. What is the probability of rolling a 4, or P(4)?
   Explain your reasoning.

4. What is the probability of rolling a 6, or P(6)?
   Explain your reasoning.

5. Determine the probability of rolling an even number.

   a. Which outcome or outcomes make up the event of rolling an even number?

   b. Calculate the probability of rolling an even number.

6. Determine the probability of rolling a number that is *not* even.

    a. Which outcome or outcomes make up the event of rolling a number that is *not* even?

    b. Calculate the probability of rolling a number that is *not* even.

7. Determine the probability of rolling a number greater than 4.

    a. Which outcome or outcomes make up the event of rolling a number greater than 4?

    b. Calculate the probability of rolling a number greater than 4.

8. Determine the probability of rolling a number that is *not* greater than 4.

    a. Which outcome or outcomes make up the event of rolling a number that is *not* greater than 4?

    b. Calculate the probability of rolling a number that is *not* greater than 4.

ACTIVITY 1.2

Complementary Events

In the previous activity, you calculated the probability of *complementary events*. **Complementary events** are events that together contain all of the outcomes in the sample space.

One way to notate the complement of an event A is *not A*. For instance, suppose that *P*(even) represents the probability of rolling an even number on a number cube. Then, *P*(not even) represents the probability of rolling a number that is *not* even on a number cube.

1. Consider the events from Activity 1.1, Questions 5 and 6—“rolling an even number” and “rolling a number that is not even.” What do you notice about the sum of the probabilities of these two complementary events?

2. Consider the events from Activity 1.1, Questions 7 and 8—“rolling a number greater than 4” and “rolling a number that is not greater than 4.” What do you notice about the sum of the probabilities of these two complementary events?

3. What is the sum of the probabilities of *any* two complementary events? Explain why your answer makes sense.

4. The probability of rolling a 5 or less on a number cube is $P(\text{5 or less}) = \frac{5}{6}$. Catalina and Linh calculated $P(\text{not 5 or less})$. Their work is shown.

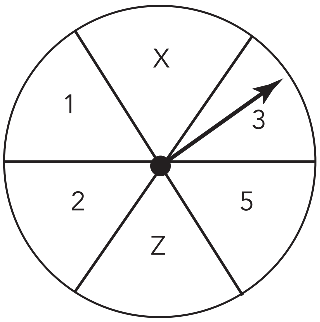
Catalina

$$P(\text{not 5 or less}) = \frac{\text{number of possible outcomes}}{\text{total number of outcomes}} = \frac{1}{6}$$

Linh

$$P(\text{not 5 or less}) = 1 - P(\text{5 or less}) = 1 - \frac{5}{6} = \frac{1}{6}$$

Explain the difference between Catalina's strategy and Linh's strategy.



ACTIVITY
1.3 Simple Probability and Spinners

Consider the spinner shown. All sections of the spinner are the same size. An experiment consists of spinning the spinner one time.

1. How many possible outcomes are there in the experiment? How did you determine your answer?

2. List the sample space for the experiment.

When the probabilities of all the outcomes of an experiment are equal, the outcomes are called **equally likely**.

3. Are the possible outcomes of the experiment equally likely. Explain your reasoning.

4. Determine the probability that the spinner lands on a letter.

   a. Describe the event and the possible outcomes of the event.

   b. Calculate *P*(letter).

c. Describe the complement of this event and the possible outcomes of the complement.

d. Calculate P(not a letter).

e. Are these complementary events equally likely? Explain your reasoning.

5. Determine the probability that the spinner lands on an odd number.

a. Describe the event and the possible outcomes of the event.

b. Calculate P(odd number).

c. Describe the complement of this event and the possible outcomes of the complement.

d. Calculate P(not an odd number).

e. Are these complimentary events equally likely? Explain your reasoning.

6. Determine the probability that the spinner lands on a vowel.

a. Describe the event and the possible outcomes of the event.

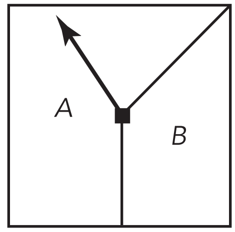
b. Calculate *P*(vowel).

c. Describe the complement of this event and the possible outcomes of the complement.

d. Calculate *P*(not a vowel).

e. Are these complementary events equally likely?

The Spinning Square Game is a game that consists of spinning the square spinner. If a player takes a spin and the spinner lands on *B*, the player wins a prize. If the spinner lands on *A*, the player does not receive a prize.



7. Predict each probability.

   a. $P(A) =$

   b. $P(B) =$

8. Brianna predicts the probability that the spinner will land on *A* to be 5. Is Brianna correct? Explain your reasoning.

ACTIVITY 1.4

Probabilities from 0 to 1

1. What is the greatest possible probability in any experiment? Explain your reasoning.

2. What is the least possible probability in any experiment? Explain your reasoning.

3. What is the probability of an event that is just as likely to occur as not occur? Explain your reasoning.

Probabilities can be expressed as fractions, decimals, or percents.

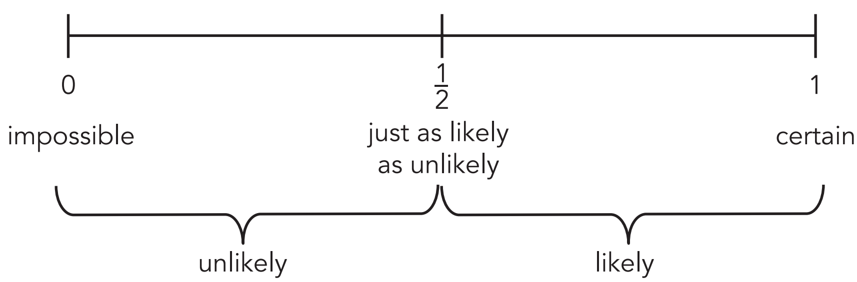
The probability of an event occurring is a number between 0 and 1. If the event is certain to happen, then the probability is 1. If an event is impossible to happen, then the probability is 0. If an event is just as likely to happen as not happen, then the probability is 0.5, or $\frac{1}{2}$.

Complete the chart representing the different probabilities.

4.

	Fraction	Decimal	Percent
P(certain event)			
P(event that is just as likely as unlikely to occur)			
P(impossible event)			

The number line shown represents the probabilities, from 0 to 1, of any event occurring.

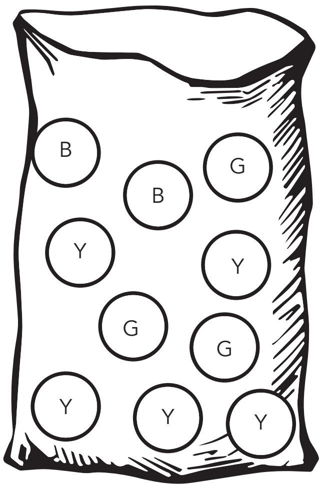


5. Estimate the probability of each event occurring. Then, place the letter corresponding to the estimated probability of the event on the number line.

	Fraction	Decimal	Percent
a. A card selected from a standard 52-card deck is red.	________	________	________
b. Your neighbors will get a pet dinosaur.	________	________	________
c. You will have a test in one of your classes this month.	________	________	________
d. A seventh-grader is more than 6 feet tall.	________	________	________

ACTIVITY
1.5 A Simple Probability Experiment

Suppose there are 2 blue, 3 green, and 5 yellow marbles in a bag. One marble will be drawn from the bag.



1. List the sample space for the experiment.

2. Calculate each probability.

   a. $P(B) =$

   b. $P(G) =$

   c. $P(Y) =$

3. Are the outcomes in the marble experiment equally likely? Explain your reasoning.

4. Determine the sum of all the probabilities.

   $P(B) + P(G) + P(Y) =$

5. Determine the sum of the probabilities for all the outcomes of the first spinner in Activity 1.3.

   $P(1) + P(2) + P(3) + P(5) + P(X) + P(Z) =$

6. Do you think the sum of the probabilities for all outcomes of any experiment will always be 1? Explain your reasoning.

James surveys some members of his class at random and asks about their favorite color. Each student chooses 1 color. The results of his survey are in the table below.

Color	Number
Red	4
Orange	9
Yellow	6
Green	3
Blue	10
Purple	4

Given that one of the surveyed students is chosen at random, answer each question.

7. What is the probability the chosen student's favorite color is blue?

8. What is the probability the chosen student's favorite color is not purple?

9. How many times more likely is it that the chosen student favorite color is yellow than green?

10. Which colors are equally likely to the be the favorite color of the chosen student?

11. What is the sum of *P*(red) + *P*(orange) + *P*(yellow) + *P*(green) + *P*(blue) + *P*(purple)? Does the sum match your prediction in Question 6? Explain your reasoning.

Talk the Talk

Absolutely/Never

1. Can the sum of the probabilities for all outcomes of some experiment ever be greater than 1? Explain your reasoning.

2. Write an event that has a probability of 1.

3. Write an event that has a probability of 0.

4. If $P(\text{not event } X) = 1$, what is $P(\text{event } X)$?

5. If $P(\text{not event } Y) = 0$, what is $P(\text{event } Y)$?

Lesson 1 Assignment

Write

Complete each statement using a term from the word box.

experiment	*probability*	*event*	*equally likely*
outcome	*sample space*	*simple event*	*complementary events*

1. A(n) ________________ is one or a group of possible outcomes for a given situation.
2. A list of all possible outcomes of an experiment is called a(n) ________________.
3. A(n) ________________ is a situation involving chance that leads to results.
4. The measure of the likelihood that an event will occur is its ________________.
5. The result of an experiment is a(n) ________________.
6. An event consisting of one outcome is a(n) ________________.
7. When the probability of all the outcomes of an experiment are equal, then the probabilities are called ________________.
8. Two events that together contain all of the outcomes in the same space is/are ________________.

Remember

Probability is a measure of the likelihood that an event will occur. To calculate the probability of an event, or *P*(event), determine the ratio of the number of times the event occurs to the total number of outcomes.

Lesson 1 Assignment

Practice

1. Ricardo is getting dressed in the dark. He reaches into his sock drawer to get a pair of socks. He knows that his sock drawer contains 6 pairs of socks, and each pair is a different color. Each pair of socks is folded together. The pairs of socks in the drawer are red, brown, green, white, black, and blue.

   a. How many possible outcomes are there in the experiment?

   b. What are the possible outcomes of the experiment?

   c. List the sample space for the experiment.

   d. Calculate P(blue).

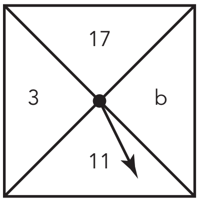
   e. Calculate P(green).

   f. Calculate P(not red).

   g. Calculate P(not purple).

Lesson 1 Assignment

2. Consider the square spinner shown and assume all sections are the same size. An experiment consists of spinning the spinner one time.



a. How many possible outcomes are there in the experiment?

b. What are the possible outcomes of the experiment?

c. List the sample space for the experiment.

d. Calculate *P*(*b*).

e. Calculate *P*(number).

f. Calculate *P*(not a number greater than 10).

g. Calculate P(number less than 2).

3. Determine whether each event is certain to occur, just as likely to occur as not to occur, or impossible to occur. Then, write the probability.

   a. A coin is flipped and the coin lands heads up. Express the probability as a fraction.

   b. Tuesday follows Monday in the week. Express the probability as a percent.

   c. You have only white shirts in your closet. Express the probability of reaching into your closet and choosing a red shirt as a fraction.

   d. A box contains 2 green balls and 2 yellow balls. You reach into the box and grab a yellow ball. Express the probability as a decimal.

Lesson 1 Assignment

Prepare

Determine each ratio.

1. Number of heads to number of sides of coin

2. Number of 6s to number of faces on a six-sided number cube

3. Number of day names ending in “day” to total number of day names

2 Probability Models

OBJECTIVES

- Develop a probability model for an experiment and use it to determine probabilities of events.
- Develop and interpret a uniform probability model assigning equal probability to all outcomes.
- Construct and interpret a non-uniform probability model.

NEW KEY TERMS

- probability model
- uniform probability model
- non-uniform probability model

You now know about simple probabilities and how to write them.

How can you use models to represent probabilities and solve problems?

Getting Started

Pocket Probabilities

Alejandra has 3 pennies, 4 nickels, 3 dimes, and 2 quarters in her pocket. She takes one coin out of her pocket and hides it in the palm of her hand. She wants her best friend to guess which coin she is holding.

1. Complete the table. Use a fraction to represent each probability.

Outcome	Penny	Nickel	Dime	Quarter
Probability				

2. Determine the sum of the probabilities.

3. What coin would you advise Alejandra's best friend to guess? Explain your reasoning.

ACTIVITY
2.1 Using Probability Models

Jackson and Nakota are training to be magicians and are working on a card trick. They have 5 cards of different colors—red, blue, yellow, green, and purple. They are discussing the likelihood of an audience member picking each color, assuming that they can't see the card's color.

1. List the sample space for choosing a card.

2. What is the probability of selecting each card? Write the probability for each in the table.

Outcome	*R*	*B*	*Y*	*G*	*P*
Probability					

When solving a probability problem, it is helpful to construct a *probability model*. A **probability model** is a list of each possible outcome along with its probability. Probability models are often shown in a table.

A probability model will list all the outcomes. The probability of each outcome will be greater than 0 but less than 1. The sum of all the probabilities for the outcomes will always be 1.

3. Why is the sum of the probabilities in a probability model always 1?

4. Nakota claims, “In any probability model, the probabilities for all outcomes are equal to each other.” Jackson disagrees and says, “The sum of all the probabilities is 1 for any probability model, but that does not mean the probabilities of all the outcomes are equal.” Who is correct? Explain your reasoning.

Ask Yourself . . .
Did you justify your mathematical reasoning?

A **uniform probability model** occurs when all the probabilities in a probability model are equally likely to occur. Each color card in the magicians’ trick had the same probability of being chosen.

When all probabilities in a probability model are not equal to each other, it is called a **non-uniform probability model**. An example would be a weather forecast that states there is a 30 percent chance of rain. This means that there is a 70 percent chance that it will not rain. The sum of these two probabilities is 1, but the outcomes do not have the same probability.

ACTIVITY 2.2

Constructing and Interpreting Probability Models

An eight-sided polyhedron is like a six-sided number cube, but instead it has 8 faces.

1. Construct a probability model for each situation. Explain how you constructed the model. Then, determine whether or not the probability model is a uniform probability model.

   a. Rolling an 8-sided polyhedron with the numbers 1 through 8:

Outcome								
Probability								

   b. Choosing a marble from a bag of marbles containing 1 green marble, 2 red marbles, and 7 blue marbles:

Outcome			
Probability			

c. Selecting a member of the chess club whose members are Omar, Hailey, Gracie, Ben, Nicky, Xavier, Joey:

Outcome							
Probability							

d. Selecting a member of the chess club with a three-syllable name out of Brianna, Hailey, Gracie, Nia, Nicky, Xavier, Lucia:

Outcome		
Probability		

2. Use the probability model to calculate each probability.

Outcome	2	3	4	5	6	7
Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

a. $P(3) =$

b. $P(8) =$

c. $P(\text{number less than } 8) =$

d. $P(\text{prime number}) =$

e. $P(\text{even number}) =$

ACTIVITY 2.3

More with Probability Models

When Mr. Flores receives his homeroom list for this year, he cannot believe it—all of the last names in his homeroom start with one of 5 letters! The table shows how many students in Mr. Flores's homeroom have last names beginning with the letters listed.

1. How many students are in Mr. Flores's homeroom?

Letter	Number of Students
A	7
B	4
M	5
O	2
S	12

2. Create a probability model for selecting a student from Mr. Flores's homeroom.

Outcome					
Probability					

3. Use the probability model to determine each probability.

   a. Choosing a student at random with a last name that begins with a vowel

   b. Choosing a student at random with a last name that does not begin with S

   c. Choosing a student at random with a last name that begins with *C*.

Talk the Talk

Compare and Contrast

Your best friend is not in school today and misses this lesson. Help your friend understand what you have learned. Write 1 or 2 paragraphs that explain the answers to these two questions.

1. What is the difference between a uniform probability model and a non-uniform probability model?

2. Will the sum of all the probabilities of the outcomes of an event be equal to 1 for both uniform and non-uniform probability models?

Lesson 2 Assignment

Write

Complete each sentence.

1. When all probabilities in a probability model are the same, it is called a(n) ______________________.

2. When all probabilities in a probability model are not the same, it is called a(n) ______________________.

3. A(n) ________________________ is a list of each possible outcome along with its probability.

Remember

When the probabilities of all the outcomes in a situation are represented in a probability model, the sum of the probabilities is 1.

Practice

1. Use the probability model to determine each probability.

Outcome	1	2	3	4	5	6	7	8
Probability	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{1}{25}$	$\frac{5}{25}$	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{3}{25}$	$\frac{4}{25}$

a. $P(4)$

b. $P(7)$

c. $P(\text{less than } 7)$

Lesson 2 Assignment

d. P(greater than 8)

e. P(odd number)

f. P(less than 10)

g. Is this a uniform probability model? Explain your answer.

2. Use the probability model to determine each probability.

Outcome	A	B	C	D	E	F
Probability	$\frac{2}{20}$	$\frac{5}{20}$	$\frac{2}{20}$	$\frac{7}{20}$	$\frac{1}{20}$	$\frac{3}{20}$

a. P(B)

b. P(F)

c. *P*(not C)

d. *P*(consonant)

e. *P*(not A)

f. *P*(vowel)

g. Is this a uniform probability model? Explain your answer.

Lesson 2 Assignment

Prepare

Write each fraction as a percent.

1. $\frac{3}{10}$

2. $\frac{1}{4}$

3. $\frac{9}{20}$

4. $\frac{2}{5}$

3 Determining Experimental Probability of Simple Events

OBJECTIVES

- Predict the experimental probability of a chance event by collecting data in a probability experiment.
- Develop a probability model using frequencies in data generated from a probability experiment.
- Use proportional reasoning to determine the theoretical probability of random events.
- Calculate the difference between a theoretical prediction and the experimental results as a percent error ratio.

NEW KEY TERMS

- theoretical probability
- experimental probability
- percent error

You can create probability models of simple events.

Do these models always reflect what actually happens when you toss a coin, roll a number cube, or spin a spinner? How are the probability models related to these actions in the real world?

Getting Started

Flip It!

1. With a partner, flip a coin 30 times, record the results, and then calculate the probability of each event.

Outcome	Tally	Total	Probability
Heads			
Tails			

For this probability experiment, the probability is the number of times the outcome occurred over the total number of flips.

2. Combine the results of your experiment with those of your classmates. Then, calculate the probability of each event.

3. What is the actual probability of flipping heads?

4. How does the actual probability compare to the calculated probability in Question 2?

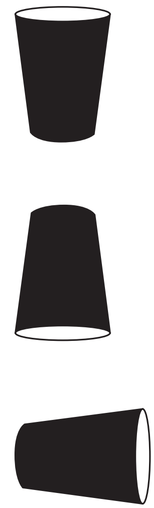
ACTIVITY 3.1

Determining Experimental Probability

Two friends are designing a game.

The game is played between two players. To play the game, a paper or plastic cup is needed. To start the game, the cup is tossed in the air.

- When the cup lands on its bottom, Player 1 wins a point.
- When the cup lands on its top, Player 2 wins a point.
- When the cup lands on its side, neither player receives a point.



1. Predict the probability for each position in which the cup can land.

2. List the sample space for the game.

3. Can you use the sample space to determine the probability that the cup lands on its top, bottom, or side? Explain why or why not.

4. Do you think all the outcomes are equally likely? Explain your reasoning.

5. Play the game 25 times with a partner. Decide who will be Player 1 and who will be Player 2.

Result	Tally Marks	Total
Player 1 (lands on bottom)		
Player 2 (lands on top)		
Side		

a. Record your results in the table using tally marks. Then, write your and your opponent's total score and write the number of times the cup landed on its side.

b. Summarize your results.

The **theoretical probability** of an event is the ratio of the number of desired outcomes to the total number of possible outcomes. This is the kind of probability you have been working with in previous lessons.

Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

$$\text{Experimental Probability} = \frac{\text{number of times the event occurs}}{\text{total number of trials performed}}$$

6. What is the experimental probability of the cup landing:

   a. On its bottom?

   b. On its top?

   c. On its side?

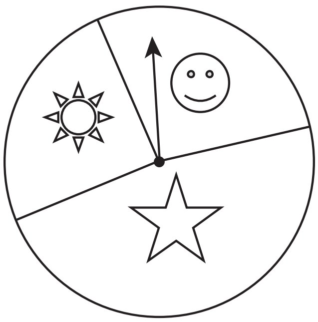
7. Do you think this is a fair game to play? Why or why not?

8. Is it possible to determine the exact probability of the cup landing on its top, bottom, or side? Explain your reasoning.

ACTIVITY 3.2

Predicting Theoretical Probability Using Experimental Probability

Examine the spinner shown.



1. List the sample space.

2. Can you use the sample space to determine the probabilities of the spinner landing on each symbol? Explain why or why not.

3. What section of the spinner represents the greatest probability? What section of the spinner represents the least probability?

4. Predict the probability of the spinner landing on each symbol.

   a. $P($☺$) =$

   b. $P($☼$) =$

   c. $P($☆$) =$

5. Miguel and Juliana make the following predictions for the spinner landing on each symbol. Explain why each student is incorrect.

Miguel

P(☺) = $\frac{1}{4}$

P(☼) = $\frac{1}{4}$

P(☆) = $\frac{2}{5}$

Juliana

P(☺) = $\frac{1}{3}$

P(☼) = $\frac{1}{3}$

P(☆) = $\frac{1}{3}$

6. Is there a way to determine the exact probabilities of landing on each of the shapes? Explain your reasoning.

Let's determine the experimental probability of the spinner landing on each of the symbols. Use a paper clip as the arrow part of the spinner. Place a pencil point through the paper clip, and then on the center of the circle. Working with a partner, one person will spin the spinner and the other person will record the result of each spin.

7. Spin the spinner 50 times and record the data using tally marks. Then, complete the table.

Shape	Tally	Total	Probability
☺			
☼			
☆			

8. Calculate the experimental probabilities using your data.

   a. $P($ ☺ $) =$

   b. $P($ ☼ $) =$

   c. $P($ ☆ $) =$

9. Compare the experimental probabilities with your predictions from Question 4. What do you notice? Why did this happen?

ACTIVITY 3.3 Comparing Probabilities Using Proportional Reasoning

If you know the probability of an event, then you can use proportional reasoning to predict the number of times that event will occur throughout an experiment.

WORKED EXAMPLE

The probability that a spinner will land on is $\frac{7}{20}$.

If you spin the spinner 60 times, you can predict the number of times the spinner will land on the section.

Step 1: Set up a proportion.

$$\frac{\text{number of times an event will occur}}{\text{number of possible outcomes}} \qquad \frac{7}{20} = \frac{x}{60}$$

Step 2: Solve the proportion.

$$\frac{7}{20} = \frac{x}{60}$$

$$7(60) = 20x$$

$$\frac{420}{20} = \frac{20x}{20}$$

$$21 = x$$

If you spin the spinner 60 times, you can expect it to land on the 21 times.

1. Suppose these are the probabilities for the symbols on the spinner.

   $P(☺) = \frac{7}{20}$ $\quad$ $P(☼) = \frac{2}{5}$ $\quad$ $P(☆) = \frac{1}{4}$

   a. If you spin the spinner 40 times, predict the number of times the spinner would land on each symbol.

   ☺ ☼ ☆

   b. If you spin the spinner 100 times, predict the number of times you would land on each symbol.

   ☺ ☼ ☆

   c. How did you predict the number of times the spinner would land on each symbol for the given number of times the spinner would be spun?

   d. Do you think you would land on the symbol exactly the number of times you calculated in parts (a) and (b) if you spin the spinner? Why or why not?

To compare the theoretical and experimental probabilities, you can use a measure called *percent error*. In probability, the **percent error** describes how far off the experimental probability is from the theoretical probability as a percent ratio.

WORKED EXAMPLE

Suppose you spun the  20 times out of 50 trials. The experimental probability, P_E (), is $\frac{20}{50}$ or $\frac{2}{5}$. The theoretical probability, P_T (), is $\frac{7}{20}$.

The percent error can be calculated using this formula:

$$\frac{P_E - P_T}{P_T} \cdot 100 = \frac{\frac{2}{5} - \frac{7}{20}}{\frac{7}{20}} \cdot 100$$

$$= \frac{\frac{8}{20} - \frac{7}{20}}{\frac{7}{20}} \cdot 100$$

$$= \frac{\frac{1}{20}}{\frac{7}{20}} \cdot 100 \approx 14.3\%$$

2. Write a sentence to interpret the result in the Worked Example.

3. What would it mean to have a 0% error? Use an example with the formula to explain your reasoning.

4. What would it mean to have a negative percent error? Use an example with the formula to explain your reasoning.

5. Use the spinner results from the previous activity to calculate the percent errors for *P*(☼) and *P*(☆) and describe their meaning.

ACTIVITY 3.4 Practice with Probability and Percent Error

As the number of trials increases, the experimental probability tends to get closer and closer to the theoretical probability. What do you think will happen to the percent error as the number of trials increases?

1. Flip a coin 10 times. Record each measure.

Outcome	Tally
Heads	
Tails	

a. Experimental probability of flipping heads

b. Theoretical probability of flipping heads

c. Percent error for *P*(heads)

d. Interpret the percent error in the context of your outcomes.

2. Combine your results with a partner. Describe how the experimental probability, theoretical probability, and percent error changed.

3. Combine your results with the class. Describe how the experimental probability, theoretical probability, and percent error changed.

4. What seems to happen to the percent error as the number of trials increases? Why do you think this happens?

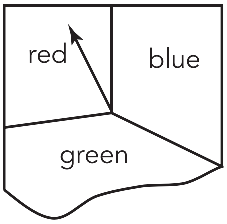
Talk the Talk

Theoretical or Experimental?

1. Explain the difference between experimental and theoretical probability.

2. Determine whether each probability of the scenario can be calculated experimentally, theoretically, or both. Explain your reasoning.

   a. The probability of the spinner landing on red.



A four-sided polyhedron is like a six-sided number cube. It's just that it has four faces instead of six! What do you think a four-sided polyhedron looks like?

   b. P(sum of 4) when a four-sided number polyhedron with numbers 1 through 4 is rolled twice.

c. Probability a particular medicine will cure a disease.

d. Probability a car will last for more than 100,000 miles.

Lesson 3 Assignment

Write

Define each term in your own words.

1. *experimental probability*
2. *theoretical probability*

Remember

The *percent error* describes how far off the experimental probability is from the theoretical probability.

The percent error is the ratio $\frac{P_E - P_T}{P_T} \cdot 100$.

- If $P_E > P_T$, then the percent error is positive.
- If $P_E < P_T$, then the percent error is negative.
- If $P_E = P_T$, then the percent error is 0.

Practice

1. Suppose the probabilities for the letters on a spinner are known to be:

   $P(A) = \frac{1}{4}$ $\quad$ $P(B) = \frac{1}{3}$ $\quad$ $P(C) = \frac{5}{12}$

Predict the number of times you would land on each letter if you were to spin the spinner the number of times given.

a. You spin the spinner 12 times.

   $P(A)$: $\quad$ $P(B)$: $\quad$ $P(C)$:

b. You spin the spinner 96 times.

   $P(A)$: $\quad$ $P(B)$: $\quad$ $P(C)$:

c. You spin the spinner 6000 times.

   $P(A)$: $\quad$ $P(B)$: $\quad$ $P(C)$:

Lesson 3 Assignment

2. A six-sided number cube was rolled 30 times. Use the results listed in the table to answer each question.

Outcome	Tally
1	~~||||~~ |||
2	||
3	~~||||~~ |||
4	||||
5	|||
6	~~||||~~

a. What is the theoretical probability of rolling a number less than 4?

b. What is the experimental probability of rolling a number less than 4?

c. What is the percent error for *P*(less than 4)? Interpret your results.

d. What is the theoretical probability of rolling the cube once with the outcome being a 2?

Lesson 3 Assignment

e. What is the experimental probability of rolling a 2?

f. What is the percent error for $P(2)$? Interpret your results.

g. How might you adjust the experiment to decrease the percent error?

Lesson 3 Assignment

Prepare

Solve each proportion.

1. $\frac{3}{x} = \frac{1}{24}$

2. $\frac{4}{9} = \frac{10}{y}$

3. $\frac{3}{5} = \frac{p}{212}$

4 Simulating Simple Experiments

OBJECTIVES

- Conduct trials using a simulation to determine probability.
- Conduct a large number of trials and observe the long-run relative frequency of outcomes to demonstrate that experimental probability approaches theoretical probability.

NEW KEY TERMS

- simulation
- random number table

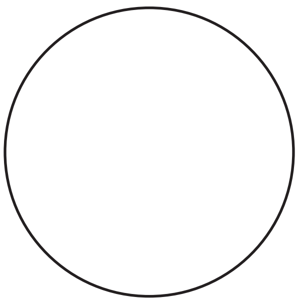
Conducting probability experiments are helpful when you are rolling a number cube, flipping a coin, spinning a spinner, etc.

What about more complicated experiments? How can you use models to run simulations of experiments when it's not possible to do the experiments in real life?

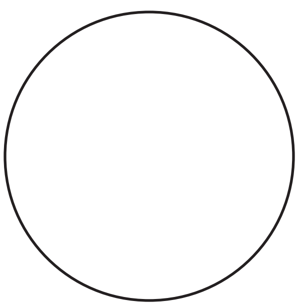
Getting Started

Designing Sims

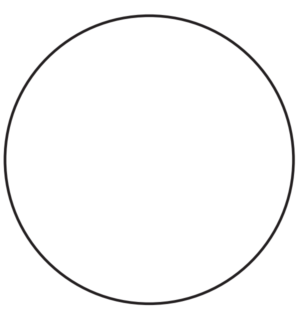
1. Design a spinner that could be used to generate the experimental probability of guessing the correct answer on a true or false test question. Explain your design.



2. Design a spinner that could be used to generate the experimental probability of guessing the correct answer on a multiple-choice test where each question has three possible answers. Explain your design.



3. Design a spinner that could be used to generate the experimental probability of guessing the correct answer on a multiple-choice test where each question has four possible answers. Explain your design.



ACTIVITY 4.1 Simulating Chicken Hatchings

What percent of chickens hatching are female? One way to answer the question is to perform a *simulation*. A **simulation** is an experiment that models a real-world situation. When conducting a simulation, you must choose a model that has the same probability of the event.

1. What model might be appropriate for creating a simulation of this event?

2. Describe the sample space for this situation.

3. What is the event you are trying to determine?

4. Suppose the probability of a chicken hatching a female chicken is $\frac{1}{2}$. What percent of all chickens would you expect to be female?

Let's use the toss of a coin as the model. Heads will represent the hatching of a female, and tails will represent the hatching of a male.

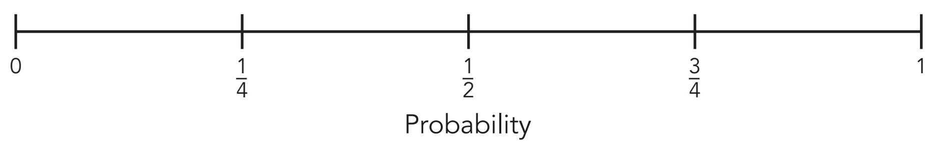
5. If you toss a coin twice, list all the possible outcomes from this simulation.

6. Toss your coin 2 times.

   a. How many of the coin tosses resulted in heads?

   b. According to your simulation, what is the experimental probability of a female chicken being hatched?

   c. Share your results with your classmates. Then, create a dot plot of all the experimental probabilities. Did everyone end up with the same results as the theoretical probability of a female chicken being hatched?

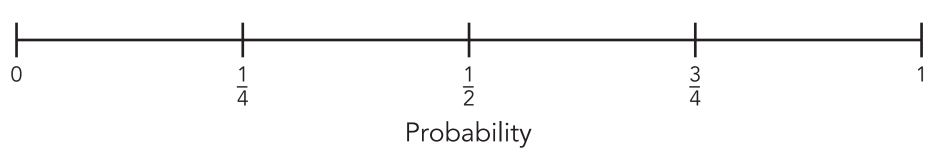


7. Toss your coin 8 times.

   a. How many of the coin tosses resulted in heads?

   b. According to your simulation, what is the experimental probability of a female chicken being hatched?

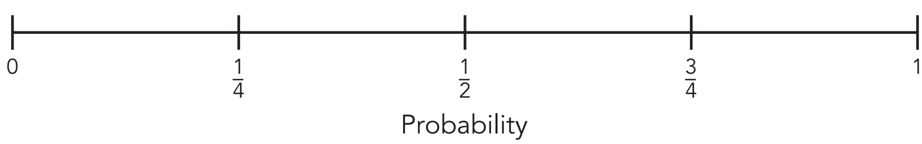
   c. Share your results with your classmates. Then, create a dot plot of all the experimental probabilities. How do these results compare to the results from Question 6?



8. Conduct 50 trials of the simulation. Record the results in the table using tally marks.

Result	Tally	Total	Percent
Heads			
Tails			

9. Share your results with your classmates. Then, create a dot plot of all the probabilities. What do you notice?



10. What can you conclude about the experimental probability of an event and its theoretical probability as the number of trials of the simulation increases?

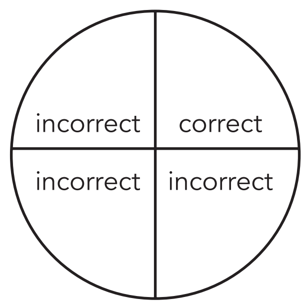
Using simulations to generate experimental probabilities is very useful in estimating the probability of an event for which the theoretical probability is difficult, or impossible, to calculate. As you have investigated, the experimental probability of an event approaches the theoretical probability when the number of trials increases.

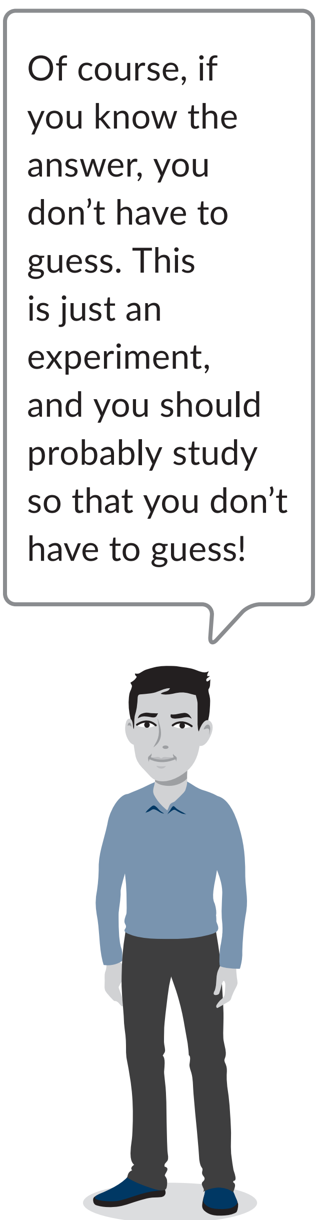
ACTIVITY 4.2 Guessing on Tests

Mr. Garcia, your history teacher, is giving a five-question multiple-choice test. Each question has 4 possible answers. How many questions can you expect to get correct simply by guessing?

1. Estimate the number of questions you expect to get correct by guessing.

One model that you could use to simulate this problem situation is a spinner divided into 4 sections that are the same size.





2. According to the spinner shown, what is the theoretical probability of correctly guessing the answer to one question?

3. Describe one trial of the experiment if you want to simulate guessing on every question of the test.

4. Will one trial provide a good estimate of how many questions you should expect to get correct? Explain your reasoning.

5. How can you use technology to simulate the guessing on every question of the test?

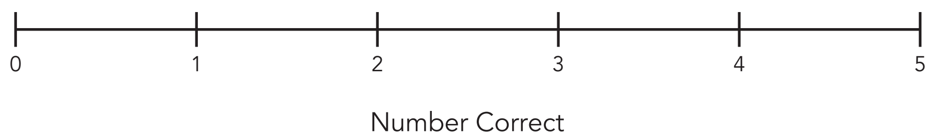
6. Conduct 50 trials of the simulation using technology. Record your results.

Trial Number	Number Correct
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	

Trial Number	Number Correct
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	
31	
32	
33	
34	

Trial Number	Number Correct
35	
36	
37	
38	
39	
40	
41	
42	
43	
44	
45	
46	
47	
48	
49	
50	

7. Graph your results on the dot plot.



8. According to your simulation, about how many questions would you expect to get correct on the test by only guessing?

9. Do you think that guessing on a five-question multiple-choice test will result in a good grade on the test?

ACTIVITY 4.3 Simulation Using Random Numbers

What if each multiple choice question had 5 answer choices instead of 4? How does this change the probability of guessing correct answers?

Instead of creating a new spinner for this simulation, you can design and carry out a simulation for the five-question test guessing experiment using a random *number table*. A **random number table** is a table that displays random digits.

Choose any line in the random number table and let the numbers 00–19 represent a correct guess and 20–99 represent incorrect guesses since each answer choice in a question has a 20% probability of being selected.

The digits of a random number table are often displayed with a space after every 5 digits so that the digits are easier to read. Some random number tables also place a space after every 5 lines for the same reason.

WORKED EXAMPLE

This line of a random number table shows one trial of the simulation. The numbers 12, 64, 56, 20, and 00 are chosen. This corresponds to correct, incorrect, incorrect, incorrect, correct.

Line 4	12645	62000	61555	76404	86210	11808	12841	45147	97438	60022

In this trial, the correct answer was guessed two times.

Use the random number table at the end of the lesson to select random two-digit numbers as a simulation.

1. Conduct one trial of the experiment. List and interpret the results of your first trial.

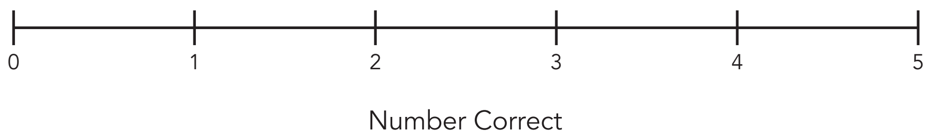
You can also use technology to generate random numbers.

2. Conduct a total of 25 trials. Record your results.

Trial Number	Number Correct
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

Trial Number	Number Correct
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	

3. Represent the results from your table on the dot plot shown.



4. What is the experimental probability of guessing:

a. 0 questions correctly?

b. 1 question correctly?

c. 2 questions correctly?

d. 3 questions correctly?

e. 4 questions correctly?

f. all 5 questions correctly?

5. How do the results from this experiment with 5 answer choices for each question compare to the results from the experiment with 4 answer choices for each question?

Talk the Talk

Mind Readers

Describe a simulation to model each situation and then describe one trial and the experiment. Conduct the simulation and answer the question.

1. How many questions would you get correct on a ten-question true or false test simply by guessing?

   Simulation without technology:

   Trial:

   Experiment:

   Conduct the simulation one time:

2. A board game requires you to roll a 6 on a number cube before you can begin playing. How many times would you expect to roll the cube before you can begin the game?

   Simulation with technology:

Trial:

Experiment:

Conduct the simulation 10 times and calculate the mean of your answers:

3. James claims he can read your mind. He gives you four cards: a red 1, a blue 2, a blue 3, and a red 4. You draw one of the cards and look at it without showing James.

   a. If you ask James to guess the color, what percent of the time could he guess correctly?

      Simulation without technology:

      Trial:

      Experiment:

      Conduct the simulation 10 times and count the number of times your card is selected:

b. If you ask James to guess the numbers, what percent of the time should he guess correctly?

Simulation without technology:

Trial:

Experiment:

Conduct the simulation 10 times and count the number of times your card is selected:

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61404	43516	17020
Line 2	17264	57327	38224	29301	18164	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86214	11808	12840	55147	97438	60222	12645	62090
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19107	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	11415	36457
Line 8	18975	95428	33226	55901	31605	43816	22259	00317	46999	98571
Line 9	59138	39542	71168	57609	91510	27904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

Lesson 4 Assignment

Write

1. Each time you repeat an experiment, it is called a(n) ______________________________.

2. A(n) ____________ is an experiment that models a real-world situation.

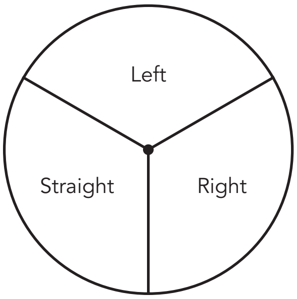
Remember

As the number of trials increases, the experimental probability gets closer and closer to the theoretical probability.

Practice

Conduct each experiment as described and record your results in a table. Use your results to determine the experimental probability.

1. At the first intersection of a corn maze, a person can go left, right, or straight. Use the spinner to model the person choosing the direction they will go. Use a paper clip as the arrow part of the spinner. Place a pencil point through the paper clip and then on the center of the circle. Perform 30 trials of the experiment. Record the results in a table using tally marks.



   What is your experimental probability that the person turns right?

2. A theater audience is made up of half adults and half children. One person is chosen at random to volunteer on stage. Toss a coin to model the person being chosen from the audience. Perform 40 trials of the experiment. Record the results in a table using tally marks.

   What is your experimental probability that the volunteer is a child?

3. Two-thirds of the fish in a lake are trout. A fisherman catches 1 fish. Roll a number cube to model the fisherman catching the fish. Perform 25 trials of the experiment. Record the results in a table using tally marks.

   What is your experimental probability that the fisherman catches a fish that is not a trout?

Lesson 4 Assignment

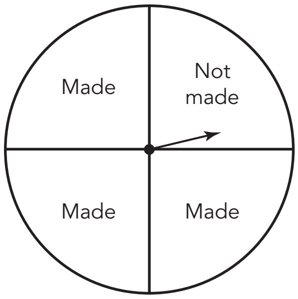
4. A drawer contains 10 white socks and 10 brown socks. The socks are mixed up. Catalina chooses 1 sock without looking. Use a number cube to model Catalina choosing the sock. Perform 30 trials of the experiment. Record the results in a table using tally marks.

   What is your experimental probability that Catalina chooses a brown sock?

5. A multiple-choice quiz has 4 questions. Each question has 3 possible answers. You guess the answer to each question. Use 3 slips of paper, one labeled correct, one labeled incorrect, and another labeled incorrect, to model guessing the answer to one question. Perform 10 trials of the experiment, where each trial consists of pulling a slip of paper from a bag without looking 4 times. Be sure to return the paper you chose back into the bag before choosing again. Record the results in a table.

   What is your experimental probability that you get at least 2 questions correct?

6. A basketball player makes a foul shot 75% of the time. He is given the chance to make 2 foul shots. Use the spinner to model the player attempting a foul shot. Perform 20 trials of the experiment, where each trial consists of spinning the spinner 2 times. Record the results in a table.



   What is your experimental probability that the player makes both foul shots?

Lesson 4 Assignment

Prepare

Solve each proportion for the unknown value.

1. $\frac{x}{150} = \frac{1}{12}$

2. $\frac{12}{x} = \frac{1}{36}$

3. $\frac{1}{8} = \frac{x}{100}$

4. $\frac{x}{16} = \frac{1}{4}$

TOPIC 1 SELF-REFLECTION

Name: ______________________________

Introduction to Probability

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Introduction to Probability* topic by:

TOPIC 1: *Introduction to Probability*	Beginning of Topic	Middle of Topic	End of Topic
identifying the sample space of an experiment.	◯	◯	◯
recognizing and explaining that the probability of a chance event is a number between 0 and 1 that expresses how likely an event is to occur.	◯	◯	◯
determining the probabilities of a simple event and its complement and describing the relationship between the two.	◯	◯	◯
calculating experimental probability as the number of times an outcome occurs divided by the total number of times the experiment is completed.	◯	◯	◯
calculating theoretical probability as the number of favorable outcomes in the sample space divided by the total number of outcomes in the sample space.	◯	◯	◯
making predictions and determining solutions using experimental and theoretical probability for simple events.	◯	◯	◯
determining experimental and theoretical probabilities related to simple events using data and sample spaces.	◯	◯	◯

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Introduction to Probability*	Beginning of Topic	Middle of Topic	End of Topic
using probability models to determine the probability of events.	◯	◯	◯
designing and using a simulation to determine experimental probabilities of simple events.	◯	◯	◯

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Introduction to Probability* topic.

____

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____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

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3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

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TOPIC 1 SUMMARY

Introduction to Probability Summary

LESSON 1 **Analyzing Populations and Probabilities**

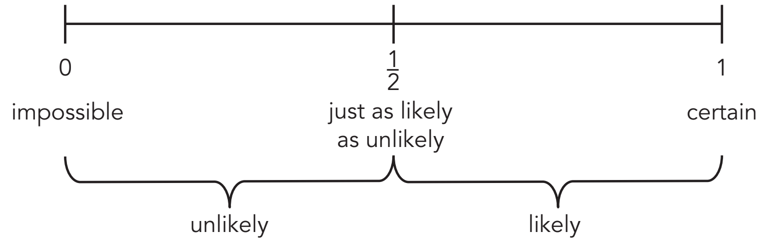
A six-sided number cube that has one number, from 1 through 6, on each face is rolled.

The numbers on the faces of the cube are the *outcomes* that can occur. An **outcome** is the result of a single trial of a probability *experiment*. An **experiment** is a situation involving chance that leads to results, or outcomes. The experiment is rolling the number cube. A list of all possible outcomes of an experiment is called a **sample space**. For example, the sample space for rolling the six-sided number cube is {1, 2, 3, 4, 5, 6}. An **event** is one possible outcome or a group of possible outcomes for a given situation. It is a subset of the sample space. A **simple event** is an event consisting of one outcome. In the number cube experiment, an event could be rolling an even number, while rolling a 5 is a simple event.

Probability is a measure of the likelihood that an event will occur. The probability of an event is often written as *P*(event). In the number cube experiment, the probability of rolling a 5 could be written as *P*(5), and the probability of rolling an even number could be written as *P*(even).

The probability of an event occurring is a number between 0 and 1. If the event is certain to happen, then the probability is 1. If an event is impossible, then the probability is 0. If an event is just as likely to happen as not happen, then the probability is 0.5, or $\frac{1}{2}$.

The number line shown represents the probabilities, from 0 to 1, of any event occurring.



NEW KEY TERMS

- outcome
- experiment [experimento]
- sample space
- event [evento]
- simple event [evento simple]
- probability [probabilidad]
- complementary events [eventos complementarios]
- equally likely
- probability model [modelo probabilístico]
- uniform probability model [modelo probabilístico uniforme]
- non-uniform probability model [modelo probabilístico no uniforme]
- theoretical probability [probabilidad teórica]
- experimental probability [probabilidad experimental]
- percent error [error porcentual]
- simulation [simulación]
- random number table

Probability is the ratio of the number of times an event can occur to the number of possible outcomes.

$$\text{probability} = \frac{\text{number of times an event can occur}}{\text{number of possible outcomes}}$$

To determine the probability of rolling an odd number on a six-sided number cube, or $P(\text{odd})$, follow these steps:

- First, list all the possible outcomes. The possible odd numbers that can be rolled are 1, 3, and 5.
- Next, add the number of outcomes. There are 3 possible outcomes of rolling an odd number.
- Then, use the equation to determine the probability of rolling an odd number.

$$P(\text{odd}) = \frac{3 \text{ possible odd numbers}}{6 \text{ possible numbers}}$$

The probability of rolling an odd number is $\frac{3}{6}$, or $\frac{1}{2}$.

Complementary events are events that together contain all of the outcomes in the sample space. If $P(\text{even})$ represents the probability of rolling an even number, then $P(\text{not even})$ is the complementary event.

When the probabilities of all the outcomes of an experiment are equal, then the outcomes are called **equally likely**.

LESSON 2 Probability Models

When solving a probability problem, it is helpful to construct a *probability model*. A **probability model** is a list of each possible outcome along with its probability. Probability models are often shown in a table.

For example, the probability model for rolling a six-sided number cube is shown.

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

A probability model lists all the outcomes. The probability of each outcome will be greater than 0 but less than 1. The sum of all the probabilities for the outcomes will always be 1.

A **uniform probability model** occurs when all the probabilities in a probability model are equally likely to occur. When all the probabilities in a probability model are not equal to each other, it is called a **non-uniform probability model**.

LESSON 3 Determining Experimental Probability of Simple Events

The **theoretical probability** of an event is the ratio of the number of desired outcomes to the total possible outcomes. **Experimental probability** is the ratio of the number of times an event occurs to the total number of trials performed.

$$\text{experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of trials performed}}$$

When you know the probability of an event, you can use proportional reasoning to predict the number of times the event will occur throughout an experiment.

For example, the probability that a spinner will land on blue is $\frac{2}{3}$. When you spin the spinner 60 times, you can set up and solve a proportion to predict the number of times the spinner will land on the blue section.

$$\frac{2}{3} = \frac{x}{60}$$

$$2(60) = 3x$$

$$\frac{120}{3} = \frac{3x}{3}$$

$$40 = x$$

When you spin the spinner 60 times, you can expect it to land on blue 40 times.

To compare theoretical and experimental probabilities, you can use a measure called *percent error*. In probability, the **percent error** describes how far off the experimental probability is from the theoretical probability as a percent ratio.

For example, suppose you spin blue 30 times out of 40 trials. The experimental probability, P_E(blue), is $\frac{30}{40}$ or $\frac{3}{4}$. The theoretical probability, P_T(blue), is $\frac{2}{3}$.

The percent error can be calculated using this formula:

$$\frac{P_E - P_T}{P_T} \cdot 100 = \frac{\frac{3}{4} - \frac{2}{3}}{\frac{2}{3}} \cdot 100$$

$$= \frac{\frac{9}{12} - \frac{8}{12}}{\frac{8}{12}} \cdot 100$$

$$= \frac{\frac{1}{12}}{\frac{8}{12}} \cdot 100 = 12.5\%$$

LESSON 4

Simulating Simple Experiments

A **simulation** is an experiment that models a real-world situation. When conducting a simulation, you must choose a model that has the same probability of the event. Using simulations to generate experimental probabilities is very useful in estimating the probability of an event for which the theoretical probability is difficult, or impossible, to calculate.

You can design and carry out a simulation for an experiment using technology or a *random number table*. A **random number table** is a table that displays random digits. You assign a range of numbers to each outcome that models the same probability of an event and then choose any line from the table to perform a trial.

For example, in a five-question multiple-choice test, each question has five possible answer choices. How many questions can you expect to get correct simply by guessing?

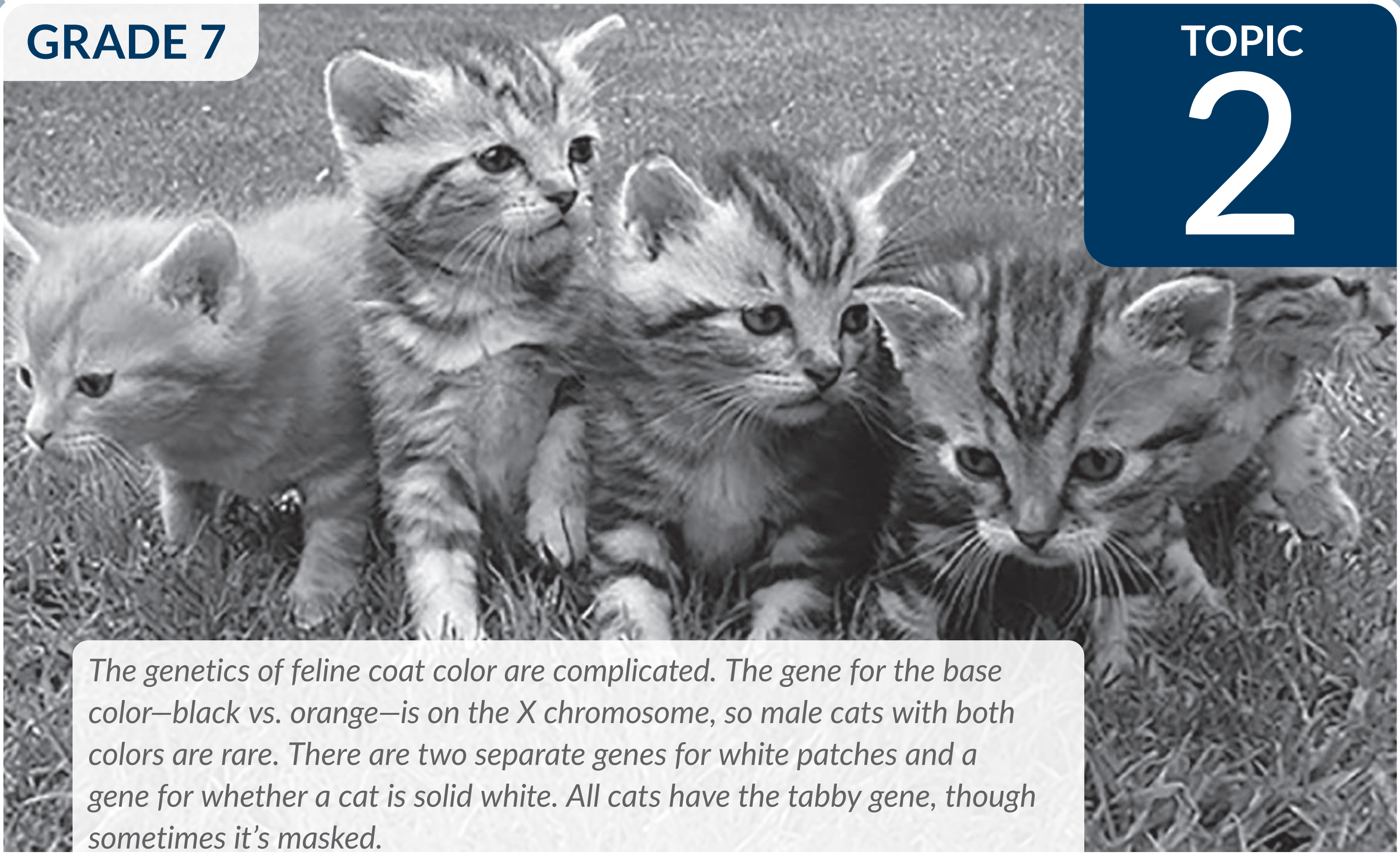
Each answer choice has a 20% chance of being selected, but only $\frac{1}{5}$ of the guesses are correct, while the others are incorrect. Let the numbers 00–19 represent correct guesses and 20–99 represent incorrect guesses.

This line of a random number table shows one trial of the simulation.

Line 4	12645	62000	61555	76404	86210	11808	12841	45147	97438	60022

The numbers 12, 64, 56, 20, and 00 are chosen. This corresponds to correct, incorrect, incorrect, incorrect, correct. In this trial, the correct answer was guessed 2 times out of 5.

You need to run multiple trials of this simulation to predict the number of questions you can expect to get correct simply by guessing.



The genetics of feline coat color are complicated. The gene for the base color—black vs. orange—is on the X chromosome, so male cats with both colors are rare. There are two separate genes for white patches and a gene for whether a cat is solid white. All cats have the tabby gene, though sometimes it's masked.

Compound Probability

LESSON 1 Using Arrays to Organize Outcomes **675**

LESSON 2 Using Tree Diagrams **695**

LESSON 3 Determining Compound Probability **711**

LESSON 4 Simulating Probability of Compound Events **729**

1 Using Arrays to Organize Outcomes

OBJECTIVES

- Conduct trials of an experiment.
- Predict the theoretical probability of an event using the results from the trials of an experiment.
- Represent and organize sample spaces for compound events using arrays, lists, and tables.
- Identify the outcomes in the sample space that compose a compound event.
- Calculate the experimental and theoretical probabilities of an experiment.
- Calculate probabilities of multiple outcomes using theoretical probabilities.
- Use proportional reasoning to predict the probability of random events.

You have calculated experimental and theoretical probabilities of simple events.

How can you organize the outcomes of multiple events to determine theoretical probabilities?

Getting Started

Tossing Coins

Suppose you and a partner each flip a coin at the same time.
What are the different outcomes that could occur?

1. Use an array to list the possible outcomes of tossing two coins into the air at the same time.

		Student 1	
		Heads	Tails
Student 2	Heads		
	Tails		

2. Use your array to write the sample space for tossing two coins at the same time.

3. Conduct this experiment for 30 trials and record the results.

   a. Calculate the experimental probability of each outcome in the table.

Outcome Student 1	Outcome Student 2	Tally	Total	Probability	Class Probability
Heads	Heads				
Heads	Tails				
Tails	Heads				
Tails	Tails				

b. Combine the results of your experiment with those of your classmates. Then, calculate the experimental probability of each outcome and record the class probabilities in the table.

4. Which probability is more accurate: the probabilities from the experiment with your partner or the class probabilities? Explain your reasoning.

5. Use your table to predict the actual probabilities of each outcome. Create a probability model table to display your predictions.

ACTIVITY 1.1 Experimental Probability of Sums

James and Ricardo are playing a game using two 6-sided number cubes. The number cubes are rolled, and the sum of the 2 numbers shown is calculated. If the sum is even, James wins a point. If the sum is odd, Ricardo wins a point.

1. Make a list of all possible outcomes when rolling two 6-sided number cubes.

2. Use your list to create the sample space for the possible sums in the game.

3. Do you think each of the outcomes in the sample space is equally likely? Explain your reasoning.

4. Predict who has a better chance of winning this game. Explain your reasoning.

5. Play the game 25 times with a partner.

   a. List each possible outcome in the first column. Tally your results in the second column.

   b. Calculate the experimental probability for each outcome. Express your probability as a fraction and as a percent. Record your probabilities in the third column of the table.

Outcome	Tally	Probability

6. According to your experiment, who would win the game between James and Ricardo? Explain your reasoning.

7. Does the outcome of the game match your prediction of the winner? Explain your reasoning.

8. Suppose you wanted to calculate the theoretical probability of each sum. Explain how you might proceed.

Make a plan, but do not carry it out. You will calculate the theoretical probabilities of this experiment in the next activity.

ACTIVITY 1.2 Theoretical Probability of Sums

Recall the game James and Ricardo are playing with two 6-sided number cubes. Now, you will calculate the theoretical probability of each sum.

By playing the game in the experiment, you can see that this game has a non-uniform probability model. However, calculating the theoretical probabilities of each outcome is difficult without knowing them all. One method to determine the probabilities of the outcomes is to make a list of all the possible outcomes.

1. How many different outcomes are there when rolling two 6-sided number cubes?

When there are a large number of possible outcomes, an array like the one you used before is useful in organizing the outcomes. The entries in the array should be related to the experiment.

2. The array shown has two numbers filled in: 2 and 7.

		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	2					
	2						
	3						
	4			7			
	5						
	6						

a. What does the 2 represent in the array?

b. What does the 7 represent in the array?

3. Complete the array.

		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	2					
	2						
	3						
	4			7			
	5						
	6						

4. How many different outcomes are in the number array?

Ask Yourself . . .
How can you organize and record your mathematical ideas?

5. Does it appear that the list of all the outcomes when rolling 2 number cubes has the same number of outcomes as the number array?

6. Using the array, determine the number of times each sum appears and the theoretical probability, in fraction form, of each sum. Record your answers in the table.

Outcome	Number of Times	Probability

7. Is there an equally likely chance for each outcome to occur from rolling 2 number cubes according to the theoretical probabilities?

8. According to the theoretical probabilities, who should win the game? Explain your reasoning.

9. Did the experimental probability of your experiment in the previous activity match the theoretical probability of the game? If not, why do you think the results of the experimental and theoretical probabilities were different?

10. Calculate the percent error between the experimental and theoretical probabilities of Ricardo winning the game.

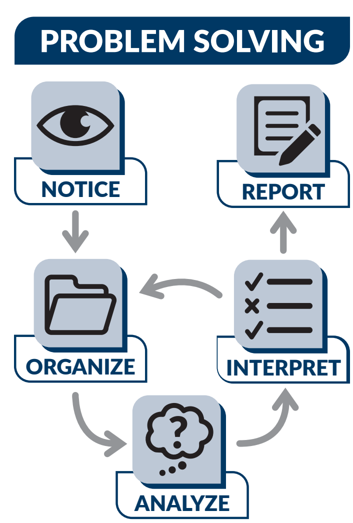
11. Calculate each probability when rolling 2 number cubes and summing the resulting numbers. Explain your calculations.

a. *P*(prime number) b. *P*(greater than 7) c. *P*(1)

12. If the number cubes are tossed 180 times, how many times do you predict the following sums would occur?

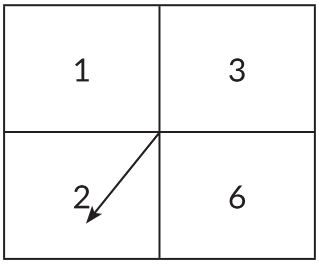
a. 1 b. 4 c. 9

d. 10 e. 12 f. prime number



13. Catalina claims that the probability of getting a sum of 4 when rolling two number cubes should be 1 out of 11 since there are 11 possible outcomes. Is Catalina correct? If not, how could you convince Catalina that her thinking is incorrect?

ACTIVITY 1.3 Probabilities of Products



The square spinner shown is spun twice, and the results of the two spins are multiplied together to produce a product that is recorded.

1. Determine all the outcomes for obtaining the products using a list.

2. Complete the array to determine the possible products.

		Spin 1			
Spin 2					

3. List the sample space.

4. Are all outcomes equally likely? Explain your calculations.

5. Calculate the probability of each product. Record your answers in the table.

Outcome	Probability

6. Did you calculate the experimental or theoretical probability? Explain how you know.

7. Calculate each probability shown for the experiment with the square spinner.

   a. *P*(even product)

   b. *P*(odd product)

   c. *P*(spin results in a multiple of 3)

d. *P*(perfect square)

e. *P*(less than 50)

Ask Yourself . . .
Did you make a plan to solve the problem?

8. If this experiment is conducted 200 times, how many times do you predict you would get each product?

a. 2

b. 3

c. 6

d. 36

e. An even product

f. An odd product

Talk the Talk

Tossing More Coins

Think back to the experiment you conducted in the *Tossing Coins* activity. You listed the sample space and made an array of outcomes for tossing two coins.

Suppose you are now interested in the number of heads when you toss two coins.

1. Create an array to display the sample space.

		First Coin	
		Heads	Tails
Second Coin	Heads		
	Tails		

2. List the sample space for the number of heads when you toss two coins.

3. Using the array, determine the theoretical probability of each outcome. Record your answers in the table.

Outcome	Probability

4. Determine the probability of tossing at least 1 head.

5. If you were to conduct 100 trials of the experiment, how many times would you expect to toss 2 heads?

Lesson 1 Assignment

Write

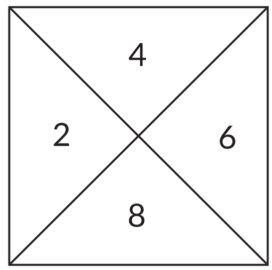
Describe the advantages of using an array to determine the sample space for an experiment.

Remember

To organize the outcomes in a number array, list the outcomes for one trial along one side and the outcomes for the other trial along the other side of an array. Combine the results in the intersections of each row and column.

Practice

1. Linh received a dart board for his birthday. The rule book says that two darts are to be thrown and that an individual's score is the sum of the two numbers.



   a. Complete an array to determine all the outcomes for obtaining the sums.

   b. List the sample space for the sum of the numbers.

   c. Are all outcomes equally likely? Explain your reasoning.

Lesson 1 Assignment

d. How many outcomes are in the array?

e. Use the array to help create a probability model listing the theoretical probabilities for each sum.

f. Calculate each probability of an even sum, a sum greater than 8, and an odd sum.

g. If two darts are thrown 80 times, how many times do you predict each of the following sums would occur? 8? 10? 14?

2. Jackson writes the numbers 1, 2, and 3 on papers and puts them in a bag. He chooses one paper without looking, writes the number down, returns it to the bag, and chooses another paper without looking. He calculates the absolute value of the difference between the two numbers.

   a. Complete an array to determine all the outcomes for obtaining the differences.

   b. List the sample space for the difference of the numbers.

   c. Are all outcomes equally likely? Explain your reasoning.

   d. How many outcomes are in the array?

e. Use the array to help create a probability model listing the theoretical probabilities for each difference.

f. If Jackson repeats this experiment 100 times, how many times do you predict he would get a difference of 2?

Prepare

Lauren's treat bag has 5 chocolate bars, 7 peanut butter cups, and 8 sour gummies. If she selects 1 piece of candy at random, determine each probability.

1. *P*(chocolate bar)

2. *P*(peanut butter cups)

3. *P*(sour gummy)

4. *P*(not sour gummy)

2 Using Tree Diagrams

OBJECTIVES

- Develop a probability model and use it to determine probabilities.
- Construct a tree diagram to determine the theoretical probability of an event.
- Construct and interpret a non-uniform probability model.

NEW KEY TERM

- tree diagram

Organized lists and arrays are two strategies for analyzing experiments that have a number of different outcomes.

How can you use tree diagrams to display outcomes and their probability of occuring?

Getting Started

Three Puppies, Three Females

What is the probability that if a dog has a litter of 3 puppies, those 3 puppies are females? Let's say that the theoretical probability of a female being born is equal to the theoretical probability of a male being born, which is $\frac{1}{2}$.

Let's simulate the event of a litter of 3 puppies being 3 females.

1. Choose an appropriate model to simulate the probability of a litter of 3 puppies being all females. Explain how you will represent females and males in your model.

2. Conduct 25 trials of the simulation. Record the results in the table shown.

Trial	Results
1	
2	
3	
4	
5	
6	
7	
8	
9	

Trial	Results
10	
11	
12	
13	
14	
15	
16	
17	
18	

Trial	Results
19	
20	
21	
22	
23	
24	
25	

3. List all possible outcomes for the number of females among a litter of 3 puppies.

4. Use the results from your simulation to construct a probability model.

Outcome	0 females	1 female	2 females	3 females
Probability				

5. What is the experimental probability that a litter of 3 puppies is all females according to your probability model?

ACTIVITY
2.1 Introduction to Tree Diagrams

In the previous simulation, your probability model was based on experimental probabilities. In some cases, this is the only method of constructing a probability model. For example, in an earlier lesson, you determined the experimental probabilities of a cup landing on its top, bottom, or side when it was tossed. It would be difficult or impossible to determine the theoretical probabilities for the cup toss. However, it is possible to determine the theoretical probability for a litter of 3 puppies being 3 females.

Ask Yourself . . .
Did you justify your mathematical reasoning?

One method to calculate the theoretical probability for a litter of puppies being 3 females is to list all of the possible outcomes for a litter of 3 puppies. You can then determine how many of those outcomes include 3 females.

1. Nakota says, "I think that the probability of a litter of puppies being 3 females is 1 out of 3 because there is only one outcome that has all 3 puppies being female. There are only two other outcomes."

   Joey says, "I don't think that's correct. I think the probability is much lower since there are many combinations of males and females in a litter of three puppies."

   Who's correct? Explain your reasoning.

2. List all of the possible outcomes for having 3 puppies in a litter, using *F* to represent females and *M* to represent males.

3. What does the outcome MFF represent?

4. Complete the probability model using all possible outcomes.

Outcome	0 females	1 female	2 females	3 females
Probability				

5. What is the theoretical probability that a litter of 3 puppies is comprised of 3 females?

A tree diagram has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

Another method of determining the theoretical probability of an event is to construct a *tree diagram*. A **tree diagram** illustrates the possible outcomes of a given situation. Tree diagrams can be constructed vertically or horizontally.

You can construct a tree diagram to show all the possible outcomes for a litter of 3 puppies.

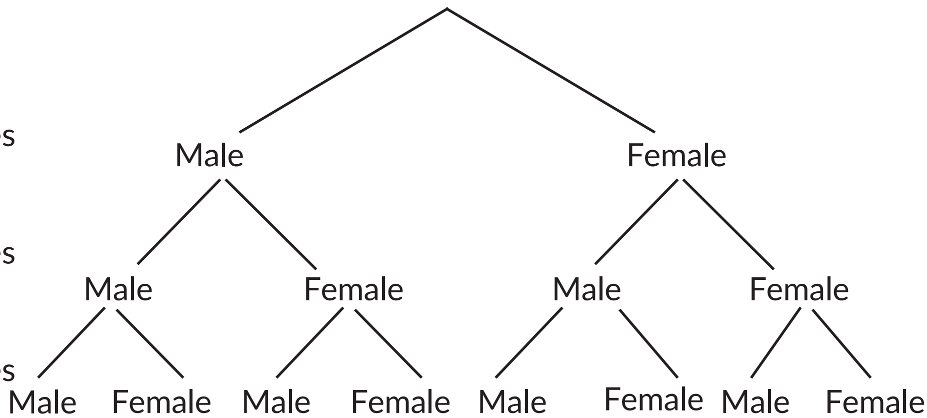
WORKED EXAMPLE

What are the possible outcomes for a litter of 3 puppies?

List the possible outcomes for the 1st puppy.

List the possible outcomes for the 2nd puppy.

List the possible outcomes for the 3rd puppy.



6. How would this tree diagram change if you were trying to determine the possible outcomes for a litter of 4 puppies?

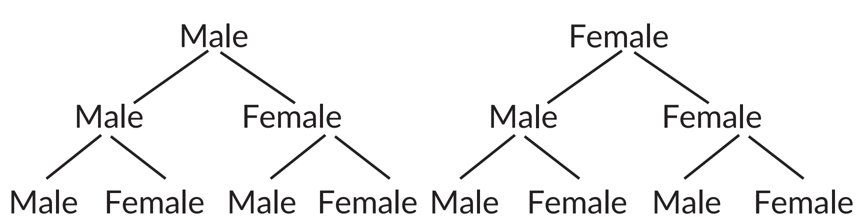
7. How does the tree diagram in the Worked Example compare to the list you made in Question 2?

8. Circle the outcome(s) of a litter of three puppies that are all females on the tree diagrams shown.

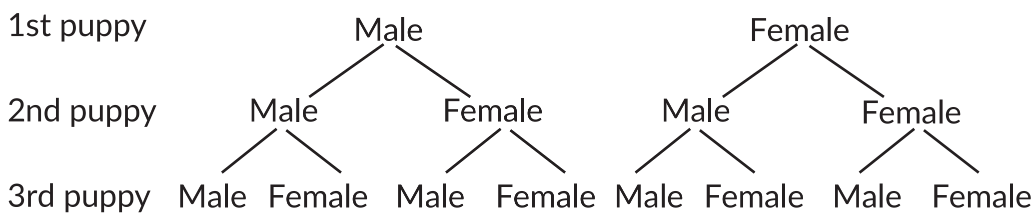
1st puppy

2nd puppy

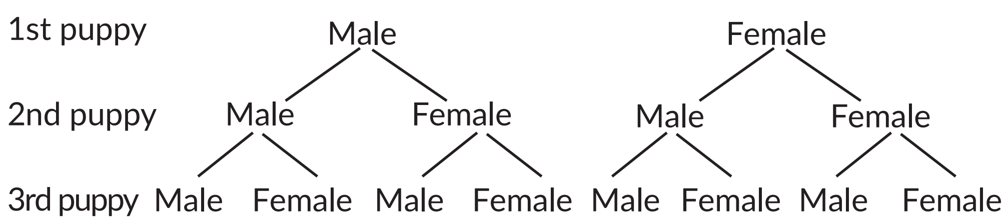
3rd puppy



9. Circle the outcome(s) of a litter of three puppies in which two of the puppies are females in the tree diagrams shown.



10. Circle the outcome MMF in the tree diagrams shown.



11. Complete the probability model shown with the information from the tree diagrams.

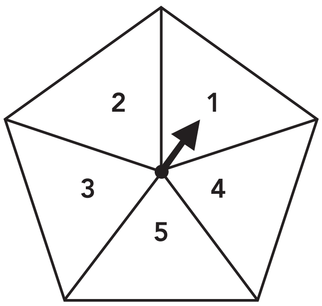
Outcome	0 females	1 female	2 females	3 females
Probability				

12. Is there a difference in the theoretical probability of each outcome between the list of outcomes you wrote and the tree diagrams you analyzed?

ACTIVITY 2.2

Using Tree Diagrams to Determine Probabilities

The five-number spinner is spun twice and a product is calculated.



1. Construct a tree diagram to determine all the possible outcomes. Then, list the product at the end of each branch of the tree.

Product	Probability
1	
2	
3	
4	
5	
6	
8	
9	
10	
12	
15	
16	
20	
25	

2. Construct a probability model for spinning the spinner twice and recording the product.

3. Use the probability model you created to calculate the probability for each event shown.

a. *P*(10)

b. *P*(less than 10)

c. *P*(multiple of 5)

d. *P*(not a multiple of 5)

4. Which events from Question 3 represent complementary events? Explain your reasoning.

5. Alejandra says that the product being less than 10 and the product being more than 10 are complementary events. Hailey disagrees. Who is correct? Explain your reasoning.

6. What event is complementary to the event that the product is an even number? Determine the probability of both events.

7. What is the sum of the probabilities of two complementary events? Explain why your answer makes sense.

Talk the Talk

True-False Trees

1. Construct a tree diagram to determine all the possible outcomes for correctly guessing the answers to a three-question true-or-false quiz.

Ask Yourself . . .
How can you use tree diagrams in everyday life?

2. Construct a probability model for the tree diagram you completed in Question 1.

3. Calculate the probabilities:

   a. P(3 questions correct) =

   b. P(1 or 2 questions correct) =

   c. P(0 questions correct) =

Lesson 2 Assignment

Write

How are tree diagrams useful when constructing probability models?

Remember

A *tree diagram* illustrates the possible outcomes of a given situation.

Practice

1. Brianna's dog gives birth to a litter of 4 puppies. The birth order of the puppies is F, F, M, F. Create a tree diagram to list all the possible birth orders of a litter of 4 puppies. Then, determine the probability of the puppies' birth order.

Lesson 2 Assignment

2. Gracie is learning probability in middle school while her little brother, Nicky, is learning arithmetic in first grade. Gracie uses a six-sided number cube to help Nicky learn how to add one-digit numbers. She rolls two cubes, numbered 1 through 6, and Nicky adds up the two numbers on the faces.

   a. Construct a tree diagram to determine all the possible outcomes. List the sum at the end of each branch of the tree.

   b. Construct a probability model for rolling two 6-sided number cubes and determining the sum of the faces.

   c. What is the probability that the sum is 7?

   d. What is the probability that the sum is 11?

e. Calculate the probability that the sum is an even number.

f. Calculate the probability that the sum is greater than 5.

g. What event would be complementary to the event that the sum is greater than 5? Explain your reasoning.

3. When Gracie and Nicky finish their math homework, they go outside to shoot some hoops. On average, Gracie makes one-half of all of the shots she takes.

   a. She shoots the basketball 4 times. Construct a tree diagram for all possible outcomes of the 4 shots.

b. Construct the probability model.

c. What is the probability she makes all 4 shots?

d. Calculate the probability she makes 3 or more shots.

e. Calculate the probability she makes 2 or more shots.

Prepare

Miguel has 6 crayons. The crayons are blue, red, green, purple, brown, and yellow. He chooses one crayon at random.

1. What is the probability that the crayon Miguel chooses will be red or purple?

2. What is the complementary event?

3. What is the probability of the complementary event?

3 Determining Compound Probability

OBJECTIVES

- Calculate probabilities of compound events using tables.
- Represent the sample space for compound events and calculate probabilities of compound events using tree diagrams.

NEW KEY TERM

- compound event

You know how to calculate the probabilities of simple events. You have used organized lists, arrays, and tree diagrams to calculate the probabilities of more complicated events.

How do you define a probability event that combines multiple simple events? How can you calculate the probability of these events?

Getting Started

And Or What?!

What's the difference between the words *and* and *or*? You've been using them your whole life, so you can imagine that you are pretty sure of the difference. However, these words have a very specific meaning in mathematics, and it is important to understand the difference.

For each situation, identify how many of your classmates are in each category.

1. Cat owners vs. dog owners

    a. Cat owners

    b. Dog owners

    c. Cat or dog owners

    d. Cat and dog owners

2. Even birth dates vs. odd birth dates

    a. Even birthdays

    b. Odd birthdays

    c. Even or odd birthdays

    d. Even and odd birthdays

3. Athletes vs. musicians

    a. Athletes

    b. Musicians

    c. Athletes or musicians

    d. Athletes and musicians

What is the difference between *and* and *or*?

The word *or* is used to identify a union—the set of items that are in either set or both. The word *and* is used to identify an intersection—the set of items that are in both sets.

4. If you know the answers to parts (a) and (b) from Questions 1–3, can you determine the answers to the other two situations? Explain why or why not.

   a. Part (c)?

   b. Part (d)?

You can calculate the probability of multiple events, say events *A* and *B*. Sometimes, you will want to know the probability of *A* or *B* occurring. Other times, you will want to know the probability of *A* and *B* occurring.

ACTIVITY 3.1

Compound Events

A pet shelter has the following animals available for adoption. You will complete a probability model and use it to determine the probability that the next pet chosen at random is a dog or a cat.

Pets	Number Available
Cat	4
Dog	7
Snake	1
Rabbit	3
Bird	5

1. How many pets are available for adoption?

2. Complete the probability model for adopting a pet from a pet shelter, at random.

Outcome	Cat	Dog	Snake	Rabbit	Bird
Probability					

3. Is the model a uniform or a non-uniform probability model? Explain your reasoning.

4. What is the probability the next pet adopted is:

a. a dog?

b. a snake?

c. a cat?

5. Determine the probability that the next pet adopted is a dog or a cat.

   a. How many of the pets are dogs or cats?

   b. How many total number of pets are there?

   c. What is the probability the next pet adopted at random is a dog or a cat?

Examples of students' methods to solve Question 5 are shown.

Nia (thumbs up)

I added the number of cats and dogs and got a total of 11. 4 + 7 = 11

That's 11 pets out of 20.

$\frac{\text{desired}}{\text{total}} = \frac{11}{20}$

Xavier (thumbs up)

Don't want 9 out of 20. So, what's left is 11 out of 20.

Ben (thumbs up)

CATS $\frac{4}{20}$

DOGS $+\frac{7}{20}$

$\frac{11}{20}$

Antonio (thumbs down)

$\frac{2}{5}$

Cats and dogs

That's 2 animals listed out of 5

Omar (thumbs down)

Probability of a cat $\frac{4}{20}$ or $\frac{1}{5}$

Probability of a dog $\frac{7}{20}$

$\frac{1}{5} + \frac{7}{20} = \frac{8}{20}$

Review the definition of **analyze** in the Academic Glossary.

6. Analyze each student's responses to the question.

   a. Describe the strategy Xavier used.

   b. Describe the strategy Ben used.

   c. What is the difference between Nia's work and Ben's work?

   d. Explain to both Antonio and Omar why their methods are incorrect for determining the probability that the next pet adopted will be a cat or dog.

A **compound event** combines two or more events, using the word *and* or the word *or.*

Determining the probability of a compound event with the word *and* is different from a compound event with *or.*

The difference is that a compound event with the word *and* means that you are determining the probability that both events occur.

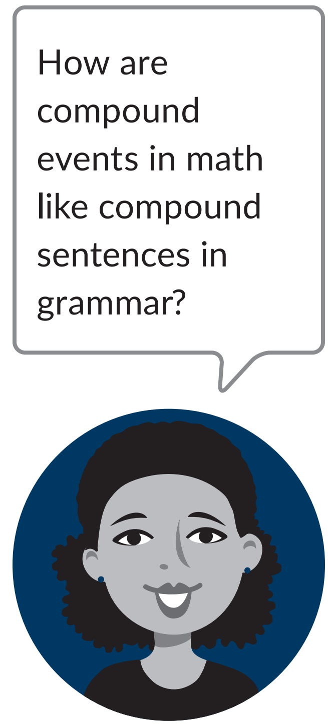
When determining a compound event with *or*, you are determining the probability that one, the other, or both outcomes occur.

7. What is the probability that the next pet adopted is a four-legged animal?

   a. What events make up "the next pet adopted being a four-legged animal"?

   b. For how many events are you determining the probability? How do you know?

   c. Rewrite Question 7 using the events you wrote in part (a).

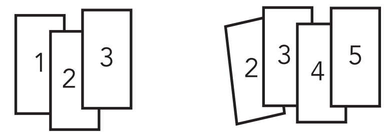


Can you write a question that would have a probability of 1?

d. Determine the probability that the next pet adopted is a four-legged animal. Show how you determined your answer.

ACTIVITY 3.2 Tree Diagrams and Compound Events

The cards shown were placed facedown in two piles so that they can be chosen at random.



1. If one card is chosen at random from each pile, determine the probability of choosing a matching pair at random.

   a. What are the possible outcomes for choosing one card from each pile at random? Make sure you show your work by either creating an organized list or constructing a tree diagram.

   b. How many possible outcomes are there?

   c. What events make up choosing a matching pair at random?

   d. Rewrite Question 1 using the events you wrote in part (c).

e. Determine the probability of choosing a pair at random.

2. Determine the probability of choosing two cards at random that have a sum of 5. Show your work.

3. The class is asked to determine the probability of choosing 2 odd cards at random.. Lucy says, “The probability of drawing 2 odd cards is 4 out of 7 because there are 7 cards and 4 of them are odd.” Do you agree with Lucy’s statement? If not, explain to Lucy why her reasoning is not correct.

4. Write a problem using the number cards for which Lucy’s answer would be correct.

5. What outcomes make up the event of “choosing 2 odd cards” given one card is chosen from each pile?

6. Determine the probability choosing 2 odd cards at random.

7. Determine the probability of selecting one card from each pile where the first card is a 2 and the second card is odd.

   a. List the event(s) for determining the probability.

   b. List the outcome(s) for the event(s).

   c. Determine the probability of selecting one card from each pile where the first card is a 2 and the second card is odd.

Remember to carefully read the event in the scenario. There is a big difference between determining a compound event with *and*, and a compound event with *or*.

8. Determine the probability that the first card is a 2 or the second card is odd.

   a. List the event(s) for determining the probability.

   b. Determine the outcome(s) for the event(s).

   c. How many outcomes are listed? Are any of the outcomes listed in both events?

When there are two or more events for which you are determining the probability, an outcome might occur for both events. When this occurs and you are determining the number of outcomes, count the repeated outcome only once.

d. How many unique outcomes are there for the two events?

e. Determine the probability that the first card is a 2 or the second card is odd.

f. Why would you not count all of the outcomes when calculating the probability?

9. Explain the difference between the events in Questions 7 and 8.

Talk the Talk

Evens and Odds

You and a friend are playing a game in which you take turns rolling a six-sided number cube and spinning a spinner with 4 congruent sections numbered 1–4.

You and your friend take turns.

- If your friend rolls an odd number and spins an even number, he or she wins.
- If you roll an even number and spin an odd number, you win.

1. List the possible outcomes for playing the game.

2. Explain why the events of your friend winning or of you winning are compound events.

3. List the outcomes for the event of your friend winning the game and for the event of you winning the game. Then, calculate each probability and record in the table.

	Outcomes of the Event	Probability
Your Friend Wins		
You Win		

4. Do you and your friend have equally likely chances of winning the game? Explain your reasoning.

5. List the outcomes for and determine the probability of each statement.

   a. *P*(even number on the number cube and even number on the spinner)

   b. *P*(even number on the number cube or even number on the spinner)

Lesson 3 Assignment

Write

Explain how to determine the probability of a compound event. Be sure to include both types of compound events.

Remember

A *compound event* combines two or more events using the word *and* or the word *or*.

Practice

1. Lucia is selecting colored tiles out of a bag to use for an art project. The table shows the number of tiles of each color that are in the bag. Lucia selects one tile at random from the bag.

Color	Number of Tiles
Blue	10
Yellow	12
Pink	6
Green	3
Purple	9

   a. How many tiles are in her bag?

   b. Complete the probability model for selecting tiles from the bag.

Outcome	Blue	Yellow	Pink	Green	Purple
Probability					

   c. What is the probability that Lucia selects a green or purple tile?

   d. What is the probability that Lucia selects a pink, green, or purple tile?

   e. What is the probability that Lucia selects a pink and purple tile in one draw?

Lesson 3 Assignment

2. Once Lucia finishes placing the tiles in her art project, she needs to determine the color of the grout that goes in between the tiles and the color of the frame around the project. She flips a coin to decide if she is going to use blue or yellow grout. She assigns heads to blue grout and tails to yellow grout. She puts a yellow, green, blue, and purple tile in a bag and pulls one out to determine the frame color.

   a. Determine the possible outcomes for flipping a coin once and selecting one tile out of the bag at random.

   b. How many possible outcomes are there?

   c. What events make up the outcome of having the same color for grout and the frame at random?

   d. Determine the probability of the outcome where the same color for grout and the frame are chosen at random.

Lesson 3 Assignment

e. What events make up selecting blue for the grout or the frame?

f. Determine the probability of selecting blue for the grout or the frame.

Prepare

1. Explain how to use a six-sided number cube to simulate whether someone prefers gymnastics, soccer, or baseball, given that these outcomes are equally likely.

2. Explain how to use a random number table to simulate whether an evenly divided spinner lands on red, blue, green, or yellow.

4 Simulating Probability of Compound Events

OBJECTIVES

- Use simulations to estimate compound probabilities.
- Design and use a simulation to generate frequencies for compound events.
- Determine probabilities of compound events using simulation.

Calculating the theoretical probability of many compound events requires advanced rules and formulas.

How can you simulate real-world compound events using tools you already have?

Getting Started

Shoot Out

James is the star player on his basketball team. So far this season, he has made 5 out of 6 free throws.

Use your knowledge of probability to answer each question.

1. Out of the next 6 free throws, how many would you expect him to make?

2. Out of the next 10 free throws, how many would you expect him to make?

3. Predict the likelihood that James will make his next free throw.

4. Predict the likelihood that he will make his next 5 free throws.

5. Suppose James made 5 free throws in a row. Predict what will happen on his next free throw.

ACTIVITY
4.1 Simulating Free Throws

James is also the team's top 3-point shooter. Suppose James is fouled while attempting a 3-point shot; he then attempts 3 free throws.

1. What might be a good simulation tool for James attempting the free throws?

2. Explain what constitutes a success, or James making a free throw, and what constitutes a failure, or James missing a free throw, using your tool.

3. James's coach wants to know the probability that James will make all 3 shots. Use a simulation to determine the probability.

    a. Describe one trial of the simulation.

How is calculating the mean of the probabilities similar to compiling all of the class data and then computing the probability?

b. Conduct 10 trials of the simulation and record your results in the table.

Trial Number	Number of Free Throws Made
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

c. Count the number of times that your simulation resulted in James making all 3 free throws.

d. According to your simulation, what is the probability that James makes all 3 free throws?

e. Calculate the mean of the probabilities from the simulations conducted by your classmates. What does this tell you?

James has been on a "hot streak" lately!

4. Design and conduct a simulation to model the number of times James would shoot before missing a shot. That is, on what shot number would you expect James to miss?

   a. Describe one trial of the simulation.

   b. Conduct 10 trials of the simulation and record the results in the table.

Trial Number	Number of Shots For First Miss
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

c. Calculate the mean number of shots it takes for James to miss a shot.

d. Compile your data with your classmates' data. Calculate the mean number of shots it takes for James to miss a shot.

e. How was this simulation different from the first simulation?

ACTIVITY 4.2

Using Random Numbers to Simulate Compound Probability

Overall, the percent of people in the U.S. having each blood group is given in the table. The percents have been rounded to the nearest whole number percent.

Blood Groups	A	B	O	AB
Percent of Population	42%	10%	44%	4%

Suppose the Red Cross is having a blood drive at the community center.

1. What is the probability that the next person who enters the community center to donate blood has Group A blood?

2. What is the probability that the next person who enters the community center to donate blood has Group A or Group O blood?

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

3. Determine the probability that out of the next 5 people to donate blood, at least 1 person has Group AB blood.

   a. Describe different strategies with and without technology you could use to simulate the blood groups of people who give blood.

A random number table is provided at the end of the lesson.

   b. Because of the values of the percents, use a random number table or technology for this simulation. How could you assign numbers to people to account for the different blood groups?

   c. Describe one trial of the simulation.

   d. Conduct 20 trials of the simulation and record your results in the table.

Trial Number	Number of AB Blood Donors
1	
2	
3	
4	
5	
6	

Trial Number	Number of AB Blood Donors
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

e. Out of the 20 trials, how many had at least one number that represented an AB blood donor?

f. According to your simulation, what is the probability that out of the next 5 people to donate blood, at least one of them has Group AB blood?

4. How many people would be expected to donate blood before a person with Group B blood would donate blood?

   a. Describe one trial of the simulation.

   b. Conduct 20 trials of the simulation and record your results in the table.

Trial Number	Number of Donors Until a Group B
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

Trial Number	Number of Donors Until a Group B
13	
14	
15	
16	
17	
18	
19	
20	

c. Calculate the average for your 20 trials.

d. About how many people would be expected to donate blood before a person with Group B blood enters?

Talk the Talk

It's Good!

The kicker of the high school football team makes 7 out of every 10 extra point attempts.

1. Design and conduct a simulation with or without technology for each situation. Conduct 20 trials of your simulation and prepare to present your results to your classmates.

   a. Determine the probability that the kicker will successfully make 3 out of the next 4 extra points he attempts.

   b. Determine the number of extra point attempts it takes for his first miss.

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61404	43516	17020
Line 2	17264	57327	38224	29301	18164	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86214	11808	12840	55147	97438	60222	12645	62090
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19107	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	11415	36457
Line 8	18975	95428	33226	55901	31605	43816	22259	00317	46999	98571
Line 9	59138	39542	71168	57609	91510	27904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

Lesson 4 Assignment

Write

Explain the difference between designing and conducting a simulation that asks for the probability of a specific number of successes in a given number of observations and a simulation that asks for the number of observations until the first success.

Remember

Many events involve very advanced rules for probability. In most cases, a simulation can be used to model the event.

Practice

1. In 1900, half of the babies born in America were born with blue eyes. What is the probability that 3 out of 4 babies born had blue eyes?

   a. What might be a good model for simulating the probability of a baby being born with blue eyes in 1900?

   b. Describe how you would assign outcomes and then describe one trial of the simulation.

c. Conduct 20 trials of the simulation and record your results in a table.

Trial Number	Outcome	Number of Heads from the 4 Coins

Trial Number	Outcome	Number of Heads from the 4 Coins

d. According to your simulation, what is the probability that 3 out of 4 babies born have blue eyes?

Lesson 4 Assignment

2. By the start of the 21st century, only 1 in 6 babies in America was born with blue eyes.

   What is the probability that at least 1 out of 2 babies has blue eyes?

   a. What might be a good model for simulating the probability of a baby being born with blue eyes in 2001?

   b. Describe how you would assign outcomes and then describe one trial of the simulation.

   c. Conduct 20 trials of the simulation and record your results in a table.

Trial Number	Outcome	Number of Times the Cube Showed Number 1

Lesson 4 Assignment

Trial Number	Outcome	Number of Times the Cube Showed Number 1

d. According to your simulation, what is the probability that at least 1 out of 2 babies born in 2001 has blue eyes?

3. The preferences of customers who rent movies online are given in the table. Design and conduct a simulation for each question. Be sure to describe how you would assign outcomes and what makes up one trial. Conduct 10 trials for each.

Movie Type	Comedy	Drama	Science Fiction	Documentary
Percent of Customers	31%	42%	22%	5%

a. Determine the probability that out of the next 5 customers to rent a movie, at least 1 rents a science fiction movie.

Trial Number	Numbers	Number of Customers Who Rent Science Fiction

Trial Number	Numbers	Number of Customers Who Rent Science Fiction

b. Determine the number of customers you would expect to rent movies until someone rents a science fiction movie.

Trial Number	Numbers	Number of Customers Until One Rents Science Fiction

Lesson 4 Assignment

Trial Number	Numbers	Number of Customers Until One Rents Science Fiction

Lesson 4 Assignment

Prepare

A light bulb company tested 24 of the bulbs it just produced and found that 3 of them were defective. Use proportions to predict how many light bulbs would be defective in shipments of each size.

1. 100 light bulbs

2. 400 light bulbs

3. 750 light bulbs

TOPIC 2 SELF-REFLECTION

Name: ______________________________

Compond Probability

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Compound Probability* topic by:

TOPIC 2: *Compound Probability*	Beginning of Topic	Middle of Topic	End of Topic
choosing the appropriate method, such as an organized list, a table, or a tree diagram, to represent sample spaces and create probability models for compound events.	◯	◯	◯
representing the outcomes in the sample space that make up a compound event.	◯	◯	◯
determining theoretical and experimental probabilities for compound events using data and sample spaces.	◯	◯	◯
making predictions and determining solutions using experimental and theoretical probabilities for compound events.	◯	◯	◯
designing and using a simulation to predict the probability of a compound event.	◯	◯	◯

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Compound Probability* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

Compound Probability Summary

NEW KEY TERMS
- tree diagram
- compound event

LESSON 1 Using Arrays to Organize Outcomes

To organize the outcomes for two events in a number array, list the outcomes for one event along one side and the outcomes for the other event along the other side. Combine the results in the intersections of each row and column.

For example, this array shows the sample space—all the possible outcomes—for rolling two 6-sided number cubes and calculating the product of the numbers shown.

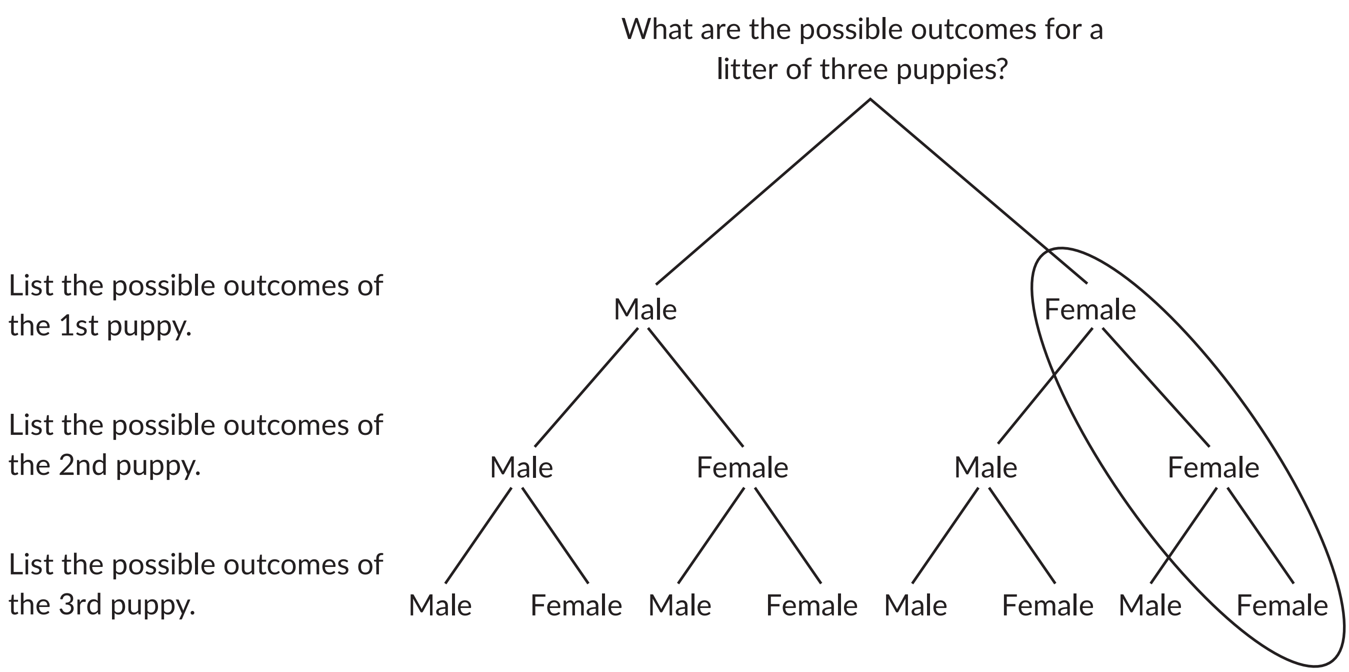
		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

The probability of the product being 6 is $\frac{4}{36}$, or $\frac{1}{9}$.

LESSON 2 Using Tree Diagrams

One method of determining the theoretical probability of an event is to construct a *tree diagram*. A **tree diagram** illustrates the possible outcomes of a given situation. Tree diagrams can be constructed vertically or horizontally. A tree diagram has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

For example, you can construct a tree diagram to show all the possible outcomes for a litter of three puppies.



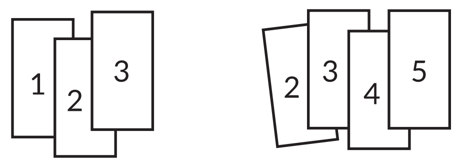
You can use a tree diagram to determine the probability of an event. In the tree diagram above, there are 8 possible outcomes for a litter of three puppies. The probability for a litter having three female puppies is $\frac{1}{8}$.

LESSON 3 Determining Compound Probability

A **compound event** combines two or more events using the word *and* or the word *or*. Determining the probability of a compound event with the word *and* is different from determining the probability of a compound event with the word *or*.

The difference is that a compound event with the word *and* means that you are determining the probability that both events occur.

For example, the cards shown were placed facedown in two piles so that they could be chosen at random.



The possible outcomes for choosing one card at random from each pile are (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 2), (3, 3), (3, 4), and (3, 5).

Determine the probability that the first card is a 1 and the second card is even.

The possible desired outcomes are (1, 2) and (1, 4). The probability that the first card is a 1 and the second card is even is $\frac{2}{12}$, or $\frac{1}{6}$.

When determining the probability of a compound event with the word *or*, you are determining the probability that one or the other or both outcomes occur.

For example, using the same experiment above, determine the probability that the first card is a 1 or the second card is even.

The possible desired outcomes are (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 4), (3, 2), and (3, 4).

The probability that the first card is a 1 or the second card is even is $\frac{8}{12}$, or $\frac{2}{3}$.

LESSON 4 Simulating Probability of Compound Events

Many events involve very advanced rules for probability. In most cases, a simulation can be used to model the event. A random number table or technology can be used to simulate compound probability in many of these events.

For example, the preferences of customers who rent movies online are provided in the table below. You can design and conduct a simulation to predict the probability that out of the next 10 customers to rent a movie, at least 3 rent a comedy.

Movie Type	Comedy	Drama	Science Fiction	Documentary
Percent of Customers	31%	42%	22%	5%

You could use a random number table for this simulation. First, assign each movie type a range of two-digit numbers that correspond to the percent of customers who prefer that type.

- Comedy: 00–30
- Drama: 31–72
- Science Fiction: 73–94
- Documentary: 95–99

Then, run trials of the experiment. Each trial consists of selecting 10 two-digit numbers from the random number table. The experimental probability of at least 3 of the next 10 customers renting a comedy is the number of trials that contained at least 3 the numbers from 00 to 30 divided by the total number of trials.

GRADE 7

TOPIC 3

Choosing a sample randomly from a population is a good way to select items that are representative of the entire population.

Drawing Inferences

LESSON 1 Collecting Random Samples 759

LESSON 2 Using Random Samples to Draw Inferences 783

LESSON 3 Bar Graphs 807

LESSON 4 Comparing Two Populations 835

LESSON 5 Using Random Samples from Two Populations to Draw Conclusions 851

1 Collecting Random Samples

OBJECTIVES

- Differentiate between a population and a sample.
- Differentiate between a parameter and a statistic.
- Differentiate between a random sample and a non-random sample.
- Identify the benefits of random sampling of a population, including supporting valid statistical inferences and generalizations.
- Use several methods to select a random sample.

NEW KEY TERMS

- survey
- data
- population
- census
- sample
- parameter
- statistic
- random sample

The statistical process is a structure for answering questions about real-world phenomena.

How can you make sure that the data you collect accurately answer your statistical questions?

Getting Started

Reviewing the Statistical Process

There are four components of the statistical process:

- Formulating a statistical question.
- Collecting appropriate data.
- Analyzing the data graphically and numerically.
- Interpreting the results of the analysis.

1. Summarize each of the four components. You may want to use examples to support your answers.

How could you describe the students in your classroom? How do the students in your classroom compare to other groups of students in your school or to other seventh-graders in the United States?

2. Formulate a statistical question about your classmates. How might you collect the information to answer your question?

One data collection strategy you can use is a *survey*. A **survey** is a method of collecting information about a certain group of people. It involves asking a question or a set of questions to those people. When information is collected, the facts or numbers gathered are called **data**.

3. Answer each question in the survey shown. You will use the results in the next activity.

   a. What is your approximate height? ______________________________

   b. Do you have a cell phone? Yes _________ No _________

   c. About how many text messages do you send each day? _________

ACTIVITY 1.1 Representative Samples

Some examples of populations include:

- every person in the United States
- every person in your math class
- every person in your school
- all the apples in your supermarket
- all the apples in the world

The **population** is the entire set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects.

A **census** is the data collected from every member of a population.

1. Use your survey to answer each question.

    a. Besides you, who else took the math class survey?

    b. What is the population in your class survey?

    c. Are the data collected in the class survey a census? Explain your reasoning.

Ever since 1790, the United States has taken a census every 10 years to collect data about population and state resources. The original purposes of the census were to decide the number of representatives a state could send to the U.S. House of Representatives and to determine the federal tax burden.

2. Describe the population for the United States census.

3. Why do you think this collection of data is called "the census"?

In most cases, it is not possible or logical to collect data from every member of the population. When data are collected from a part of the population, the data are called a **sample**. A sample should be representative of the population. In other words, it should have characteristics that are similar to the entire population.

When data are gathered from a population, the characteristic used to describe the population is called a **parameter**.

When data are gathered from a sample, the characteristic used to describe the sample is called a **statistic**. A statistic is used to make an estimate about the parameter.

At a particular middle school, 29% of students had perfect attendance last year. This is a parameter because you can count every single student at the school.

Nationally, 16% of all teachers miss 3 or fewer days per year. This characteristic is a statistic because the population is too big to survey every single teacher.

4. After the 2000 census, the United States Census Bureau reported that 7.4% of Georgia residents were between the ages of 10 and 14. Was a parameter or a statistic reported? Explain your reasoning.

5. A recent survey of 1000 teenagers from across the United States shows that 4 out of 5 carry a cell phone with them.

   a. What is the population in the survey?

   b. Were the data collected in the survey a census? Why or why not?

c. Does the given statement represent a parameter or a statistic? Explain how you determined your answer.

d. Of those 1000 teenagers surveyed, how many carry a cell phone? How many do not carry a cell phone?

6. Use your math class survey, or the data from a sample class survey provided at the end of this lesson, to answer each question. Use a complete sentence to justify each answer.

   a. How many students in the class have a cell phone?

   b. What percent of the students in the class have a cell phone?

   c. Does the percent of students in the class that have a cell phone represent a parameter or a statistic? Explain how you determined your answer.

7. Suppose you only want to survey a sample of the class about whether they have a cell phone. Discuss whether or not these samples would provide an accurate representation of all students in a class. Use complete sentences to justify your answers.

   a. the selection of all of the 12-year-olds for the sample

   b. the selection of the students in the first seat of every row

   c. the selection of every fourth student alphabetically

   d. the selection of the first 10 students to enter the classroom

8. Suppose you wanted to determine the number of students who have a cell phone across the entire seventh grade.

    a. What is the population?

    b. Suggest and justify a method of surveying students in the seventh grade to obtain a representative sample.

When information is collected from a sample to describe a characteristic about the population, it is important that such a sample be as representative of the population as possible. A **random sample** is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

ACTIVITY **1.2**

Selecting a Random Sample

Ms. Flores is purchasing standing desks for her classroom. She wants to set up the desks to minimize the amount of desk height adjusting by the students and decides to use the mean height of students in her math class as a guide. Rather than using the heights of all students in her class, she decides to collect a random sample of students in her class.

1. What is the population for this problem?

2. Ms. Flores received two suggestions to randomly sample her class. Decide if each strategy represents a random sample. If not, explain why not.

    a. Ms. Flores chooses the 12-year-olds in the class.

    b. Ms. Flores chooses all of the students wearing white sneakers.

Review the definitions of **select** and **estimate** in the Academic Glossary.

3. Ms. Flores decides to select a random sample of five students in her class and then calculate the mean height. She assigns each student in her class a different number. Then, she randomly selects 5 numbers.

   a. Explain why Ms. Flores's method of taking a sample is a random sample.

   b. Do you think randomly selecting five students will accurately represent the population of her class? If not, do you think she should pick more or fewer students?

   c. James hopes Ms. Flores will assign him the number 7 because it will have a better chance of being selected for the sample. Do you agree or disagree with James? Explain your reasoning.

   b. Catalina claims Ms. Flores must begin with the number 1 when assigning numbers to students. Ricardo says she can start with any number as long as she assigns every student a different number. Who is correct? Explain your reasoning.

One way to select the students is to write the numbers (or student names) on equal-sized pieces of paper, put the papers in a bag, draw out a piece of paper, and record the result. To create a true random sample, the papers should be returned to the bag after each draw.

Help Ms. Flores randomly select five students from her class.

4. With your partner, create a bag with 30 numbers from which to select your sample.

   a. Draw five numbers and record your results.

   b. Compare your sample with the samples of your classmates. What do you notice?

5. Suppose Ms. Flores starts with the number 15 when she assigns each of the 30 students in her class a number. How can you change your selection process to accommodate the list beginning at 15?

6. Suppose that the five numbers selected from your bag resulted in five 12-year-olds.

   a. Is the sample still a random sample? Explain your reasoning.

   b. How is this outcome different from choosing all 12-year-olds to represent the sample in Question 2?

ACTIVITY 1.3

Using Random Number Tables to Select a Sample

The standing desks improved student motivation, attendance, and achievement so much in Ms. Flores's class that the principal, Ms. Brown, has decided to order standing work desks for every seventh-grade class in the school.

The school has 450 seventh-grade students, and Ms. Brown would like to take a random sample of 20 seventh-graders, determine their heights, and use their mean height for the initial set-up of the standing desks. However, Ms. Brown does not want to write the 450 names on slips of paper.

Technology, such as spreadsheets, graphing technology, and random number generator applications, can be used to generate random numbers.

There are other ways to select a random sample. One way to select a random sample is to use a random number table like you used previously to simulate events.

You can use a random number table to choose a number that has any number of digits in it. For example, if you are choosing 6 three-digit random numbers and begin with Line 7, the first 6 three-digit random numbers would be: 242, 166, 344, 421, 283, and 070.

Random Number Table										
Line 6	62490	99215	84987	28759	19177	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	10415	36457
Line 8	16975	95428	33226	55903	31605	43817	22250	03918	46999	98501
Line 9	59138	39542	71168	57609	91510	77904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825

1. What number does “070” represent when choosing a three-digit random number? Why are the zeros in the number included? Explain your reasoning.

Do you think Ms. Flores’s class was a representative sample of all seventh-graders?

2. If selecting a three-digit random number, how would the number 5 be displayed in the table?

3. Begin on Line 10 and select 5 three-digit random numbers.

4. Explain how to assign numbers to the 450 seventh-grade students so that Ms. Brown can take a random sample.

5. Use Line 6 as a starting place to generate a random sample of 20 students.

   a. What is the first number that appears?

b. What do you think Ms. Brown should do with that number?

c. Continuing on Line 6, what are the 20 three-digit numbers to be used to select Ms. Brown's sample?

d. What should you do if a three-digit number appears twice in the random number table?

e. Will choosing a line number affect whether Ms. Brown's sample is random?

f. Will choosing a line number affect who will be chosen for the sample?

Talk the Talk

Lunching with Ms. Brown

Ms. Brown wishes to randomly select 10 students for a lunch meeting to discuss ways to improve school spirit. There are 1500 students in the school.

1. What is the population for this problem?

2. What is the sample for this problem?

3. Ms. Brown selects three to four student council members from each grade to participate. Does this sample represent all of the students in the school? Explain your answer.

4. Ms. Flores recommended that Ms. Brown use a random number table to select her sample of 10 students. How would you recommend Ms. Brown assign numbers and select her random sample?

Results from a Sample Class Survey			
Student	1. What is your approximate height?	2. Do you carry a cell phone with you?	3. About how many text messages do you send in one day?
1-Catalina	60 in.	Yes	75
2-James	68 in.	Yes	5
3-Ricardo	63 in.	No	0
4-Linh	65 in.	Yes	20
5-Jackson	62 in.	Yes	50
6-Lauren	68 in.	Yes	100
7-Nakota	56 in.	Yes	60
8-Alejandra	70 in.	No	0
9-Hailey	69 in.	Yes	50
10-Joey	66 in.	Yes	100
11-Brianna	61 in.	Yes	0
12-Gracie	68 in.	Yes	60
13-Nicky	64 in.	Yes	50
14-Nia	66 in.	Yes	40
15-Miguel	60 in.	Yes	90

Results from a Sample Class Survey			
Student	1. What is your approximate height?	2. Do you carry a cell phone with you?	3. About how many text messages do you send in one day?
16-Lucia	67 in.	Yes	10
17-Xavier	64 in.	Yes	0
18-Ben	65 in.	Yes	25
19-Juliana	63 in.	Yes	30
20-Omar	58 in.	Yes	80

Lesson 1 Assignment

Write

Match each definition to the corresponding term.

1. the facts or numbers gathered by a survey
2. the characteristic used to describe a sample
3. the collection of data from every member of a population
4. a method of collecting information about a certain group of people by asking a question or set of questions
5. a sample that is selected from the population in such a way that every member of the population has the same chance of being selected
6. the characteristic used to describe a population
7. the entire set of items from which data can be selected
8. the data collected from part of a population

a. census

b. data

c. parameter

d. population

e. sample

f. statistic

g. survey

h. random sample

Remember

Statistics obtained from data collected through a random sample are more likely to be representative of the population than those statistics obtained from data collected through non-random samples.

Lesson 1 Assignment

Practice

1. Explain which sampling method is more representative of the population.

   a. Lauren and Linh live in Springfield, RI, and are interested in the average number of skateboarders who use their town's skate park in one week. Lauren recorded the number of skateboarders who used the park each week in June, July, and August. Linh recorded the number of skateboarders who used the park the second week of each of the 12 months.

   b. Nakota and Alejandra want to determine the most popular lunch choice in the school cafeteria among seventh-graders. They decide to collect data from a sample of seventh-graders at school. Nakota surveys 20 seventh-graders that are in line at the cafeteria. Alejandra surveys 20 seventh-graders whose student ID numbers end in 9.

2. The coach of the soccer team is asked to select 5 students to represent the team in the Homecoming Parade. The coach decides to randomly select the 5 students out of the 38 members of the team.

   a. What is the population for this problem?

Lesson 1 Assignment

b. What is the sample for this problem?

c. Suggest a method for selecting the random sample of 5 students.

3. Consider the population of integers from 8 to 48.

   a. Select a sample of 6 numbers. Is this a random sample? Explain your reasoning.

   b. How can you assign random numbers to select a sample using a random number table?

   c. Use the random number table to choose 6 numbers from this population.

Lesson 1 Assignment

d. Use a different line of the random number table to choose 6 numbers from this population.

e. Compare the results from each sample. Do the results surprise you? Explain.

4. The manager of the local mall wants to know the mean age of the people who shop at the mall and the stores in which they typically shop. Jackson has been put in charge of collecting data for the mall. He decides to interview 100 people one Saturday because it is the mall's busiest shopping day.

   a. What is the population for this situation?

   b. What is the sample?

c. When Jackson calculates the mean age of the people who shop at the mall, will he be calculating a parameter or a statistic? Explain your reasoning.

d. Describe three different ways Jackson can take a sample. Describe how any of these three possible samples may cause the results of Jackson's survey to inaccurately reflect the average age of shoppers at the mall.

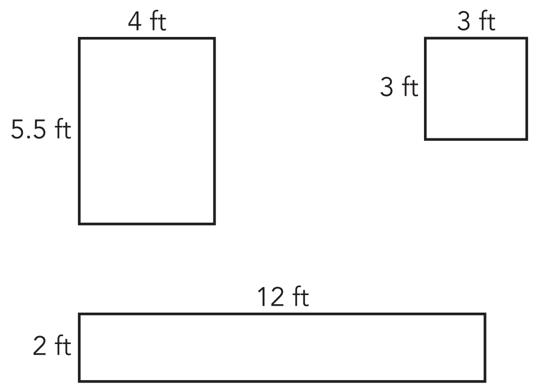
e. Jackson decides to use a random number table to choose the next 10 people to interview. Explain how to choose 10 two-digit numbers between 1 and 80 from a random number table.

f. Record the numbers of the people who would be interviewed. Be sure to specify which line you used to generate your list.

g. Suppose Jackson uses a random number table to generate his sample, resulting in Jackson interviewing 10 people who all go into the gaming store. Does this mean the sample is not random? Explain.

Prepare

Mr. Lopez has three bulletin boards in his classroom. What is the average amount of space per bulletin board?



2 Using Random Samples to Draw Inferences

OBJECTIVES

- Investigate how results from a random sample are more reliable for representing the population than results from a sample that is not random.
- Use data from a random sample and proportional reasoning to draw inferences about a population parameter.
- Calculate the percent error between population parameters and sample statistics.

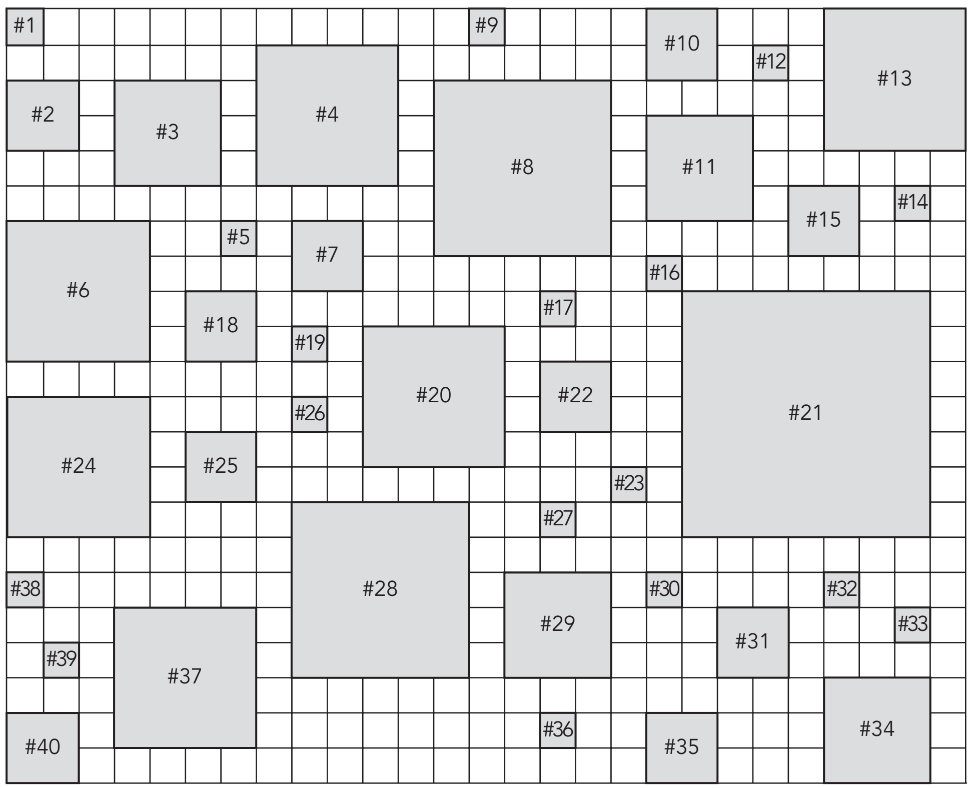
You know how to generate random samples.

How can you use random samples to make inferences about population parameters?

Getting Started

Selecting Squares

The Art Club created a design on the floor of the art room. Each of the 40 numbered squares on the floor will have colored tiles. The club needs to calculate how many colored tiles they must buy. Each small grid square represents a square that is one foot long and one foot wide.



The Art Club needs to complete the art room floor plan project by tomorrow! Because they are short of time, they decide they do not have enough time to measure all 40 squares.

1. Suggest a method for the Art Club to use to sample the 40 squares. What would they do once they collect their sample?

ACTIVITY 2.1

Using Multiple Samples to Make Predictions

I wonder why Hailey picked those squares?

Hailey says, "Since we don't have a lot of time, why don't we just select 5 squares and calculate their total area? That should be good enough for us to estimate the total area of all the squares that will need colored tiles."

1. Do you think Hailey's idea could work to estimate the total area for all the squares that need colored tiles? Explain your reasoning.

2. What is the population for this problem?

WORKED EXAMPLE

As you learned, you can select a sample to estimate parameters of a population. In this problem situation, the Art Club is going to set up a ratio using the sample of squares they select to the total area of those sample squares.

They will use the ratio:
number of squares in the sample : total area of the sample squares.

Hailey decides to select the following squares: 1, 15, 21, 37, and 40.

Total area

#1: $1 \cdot 1 = 1$ square foot

#15: $2 \cdot 2 = 4$ square feet

#21: $7 \cdot 7 = 49$ square feet

#37: $4 \cdot 4 = 16$ square feet

#40: $2 \cdot 2 = 4$ square feet

Ratio

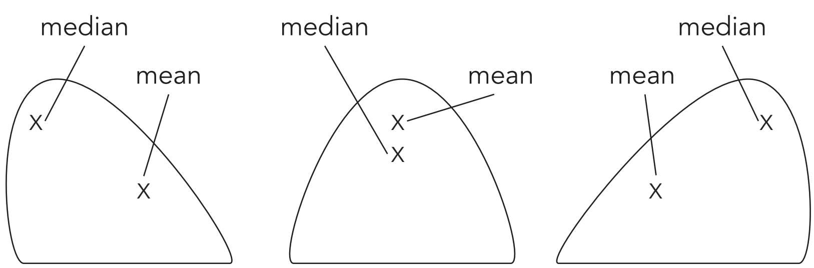
number of squares in the sample	:	total area of the sample squares
5 squares	:	74 square feet

So, the total area of these 5 squares is 74 square feet.

3. Select 5 numbered squares that you think best represent the 40 squares that need colored tiles. Record the numbers of the squares you selected.

4. Calculate the total area of the 5 numbered squares you selected.

Now that you have collected your data, you need to analyze the data. Remember, there are three common distributions of data: skewed left, skewed right, and symmetric. The distribution of data can help you determine whether the mean or median is a better measure of center. Examine the diagrams shown.



Skewed Right

The mean of a data set is greater than the median when the data are skewed to the right.

The median is the best measure of center because the median is not affected by very large data values.

Symmetric

The mean and median are equal when the data are symmetric.

Skewed Left

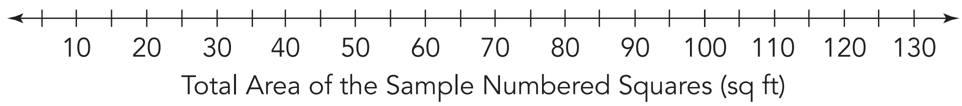
The mean of a data set is less than the median when the data are skewed to the left.

The median is the best measure of center because the median is not affected by very small data values.

The median is not affected by very large or very small data values, but the mean is affected by these large and small values.

5. Compare the total area of your sample to the total areas of your classmates' samples.

   a. Record the total area you calculated for your sample on the dot plot shown. Then, record the total areas your classmates calculated on the same dot plot.



   b. Describe the distribution of the dot plot.

   c. Estimate the total area for a sample of 5 squares using the data values in the dot plot.

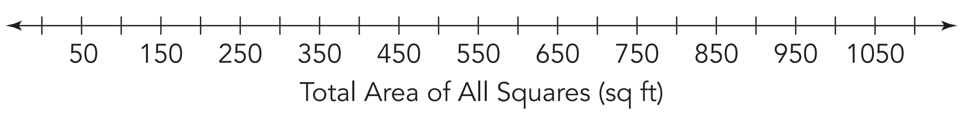
You can set up a ratio of the sample of 5 squares to the total area of those 5 sample squares, as Hailey did, and you can then set up a proportion to estimate the total area of those 40 squares in the Art Club's floor plan design. In doing so, you are scaling up from your sample to the population of the squares.

6. Write a ratio of the number of squares in your sample to the total area of the squares in your sample.

7. Estimate the total area of all 40 squares on the floor plan using proportional reasoning.

8. Compare the estimated total area of the 40 squares on the floor plan with your classmates' estimated total areas.

   a. Record the estimated total area of the 40 squares on the floor plan on the dot plot shown. Then, record your classmates' estimates of the total area of the 40 squares.



   b. Look at the distribution of the dot plot shown in section a, above. Then, look at the distribution of the dot plot shown in Question 5. What are some similarities or some differences you see between this dot plot and the one pictured on Question 5? What inferences can be made about the means and medians of the data sets based on these dot plots?.

   c. Estimate the total area of the squares in the floor plan using data values in the dot plot.

ACTIVITY 2.2 Using Random Samples to Make Predictions

Samples chosen by looking at the squares and trying to pick certain squares will probably contain many more of the larger squares in the floor plan. Most of the squares actually have small areas (17 of the 40 squares have an area of 1 square foot, and 10 of the 40 squares have an area of 4 square feet); therefore, you need to use another method to randomly choose squares.

1. How might you randomly choose 5 squares for your sample?

2. Brianna has a suggestion on how to randomly select the numbered squares. She says, “I can cut out the squares from the floor plan and then I can put these squares in a bag. That will help me randomly select squares.” Will Brianna’s method result in a random sample? Explain your reasoning or suggest a way to modify her strategy.

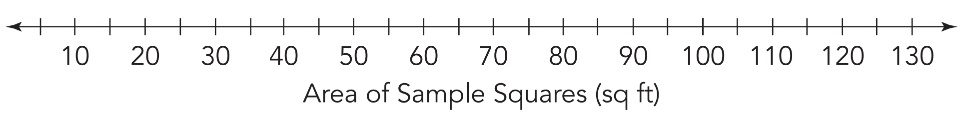
3. You have used random number tables to generate a random sample. You can also use technology, in the form of a random number generator, to select a random sample. How can you use a random number generator to choose 5 numbered squares for your sample?

4. Use a random number generator to choose 5 numbered squares using two-digit numbers ranging between 1 and 40. Record the square numbers.

5. Calculate the total area of the 5 numbered squares you selected.

6. Compare the total area of your sample to the total areas your classmates calculated from their random samples.

   a. Record the total areas your classmates calculated and the total area you calculated on the dot plot shown.



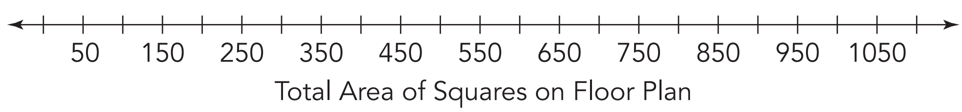
How does using a random number generator compare to using a random number table?

   b. How do the values plotted on this dot plot compare to the values plotted in Activity 2.1, Question 5? Compare the shapes and the centers of the data values for both dot plots.

7. Using proportional reasoning, estimate the total area of all 40 squares on the floor plan using the area you calculated from the random sample.

8. Compare your estimated total area from the random sample for all 40 squares with your classmates' total area estimates.

   a. Record your estimated total area of the 40 squares on the dot plot shown. Then, record your classmates' estimates of the total area of the 40 squares on the dot plot.



   b. Estimate the total area of the squares in the floor plan using data values in the dot plot.

   c. How do the values plotted on this dot plot compare to the values plotted in Activity 2.1, Question 8? Compare the distributions and the centers of the data values for both dot plots.

9. The actual total area of the 40 numbered squares is 288 square feet.

   a. Is 288 a parameter or a statistic? Explain your reasoning.

Percent error is the absolute value of the ratio of the difference between the statistic and parameter to the parameter.

$$\left| \frac{\text{Statistic - Parameter}}{\text{Parameter}} \right|$$

   b. Locate 288 on each dot plot you created in the previous activity and this activity. What do you notice?

   c. Calculate the percent error for the parameter and your statistics from this activity and the previous activity for the total sum of the areas of the squares.

   d. Based on your percent error, which sample is more accurate? Is this what you expected? Explain your reasoning.

ACTIVITY
2.3 Using Samples to Justify Predictions

Ask Yourself . . .
Can you restate the problem in your own words?

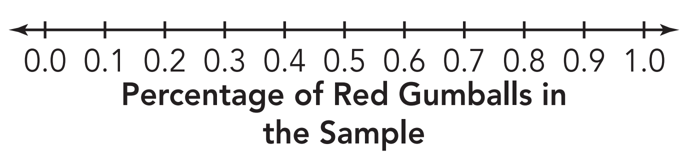
The student council holds regular fundraisers to raise money for community service projects. To raise money for Back-to-School Backpacks for the local homeless shelter, they hold a Gumball Guessing Competition. They place differently colored gumballs in a large, clear gumball machine. Students pay $1.00 to predict the percent of red gumballs in the machine. Any students who predict within 5% of the actual percent win a $5.00 credit at the school store and a share of the gumballs.

To make their predictions, students take a sample of 25 gumballs (and then return the gumballs to the machine), and they use the percent of red gumballs in the sample to make their guess. The results from the first 100 students' samples are provided in the table.

Percent of Red Gumballs in the Sample	Number of Samples
12%	5
16%	8
20%	13
24%	13
28%	16
32%	18
36%	13
40%	10
44%	3
48%	1

1. What do you notice about the data in the table?

2. Create a dot plot of the results. Be sure to label your dot plot.



3. Use the results to predict the likely percent of gumballs that are red. Explain your reasoning.

4. How many of the students obtained a sample that was less than 25% red gumballs?

5. The gumball machine holds 10,000 gumballs, and there are 2936 red gumballs in the machine.

   a. How many students will split the gumballs? How many gumballs will each student receive?

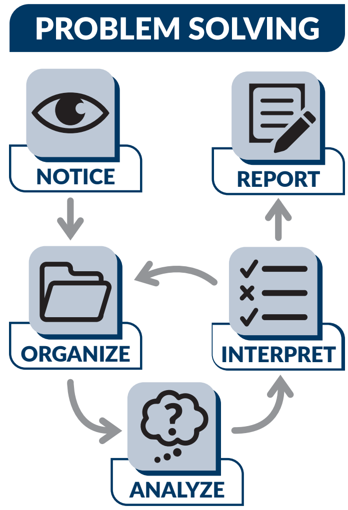
   b. Is it reasonable that none of the estimates were equal to the actual percent of red gumballs? Explain your reasoning.

c. Suppose a disgruntled student argued that there must be at least 40% red gumballs. Use the analysis to explain why this is unlikely.

d. The principal did not take a random sample to create his estimate. Instead, he based his estimate on a visual inspection of the gumball machine. His guess was 35%. Calculate the percent error of the principal's guess from the true percent of red gumballs.

Talk the Talk

Pumpkin Patch



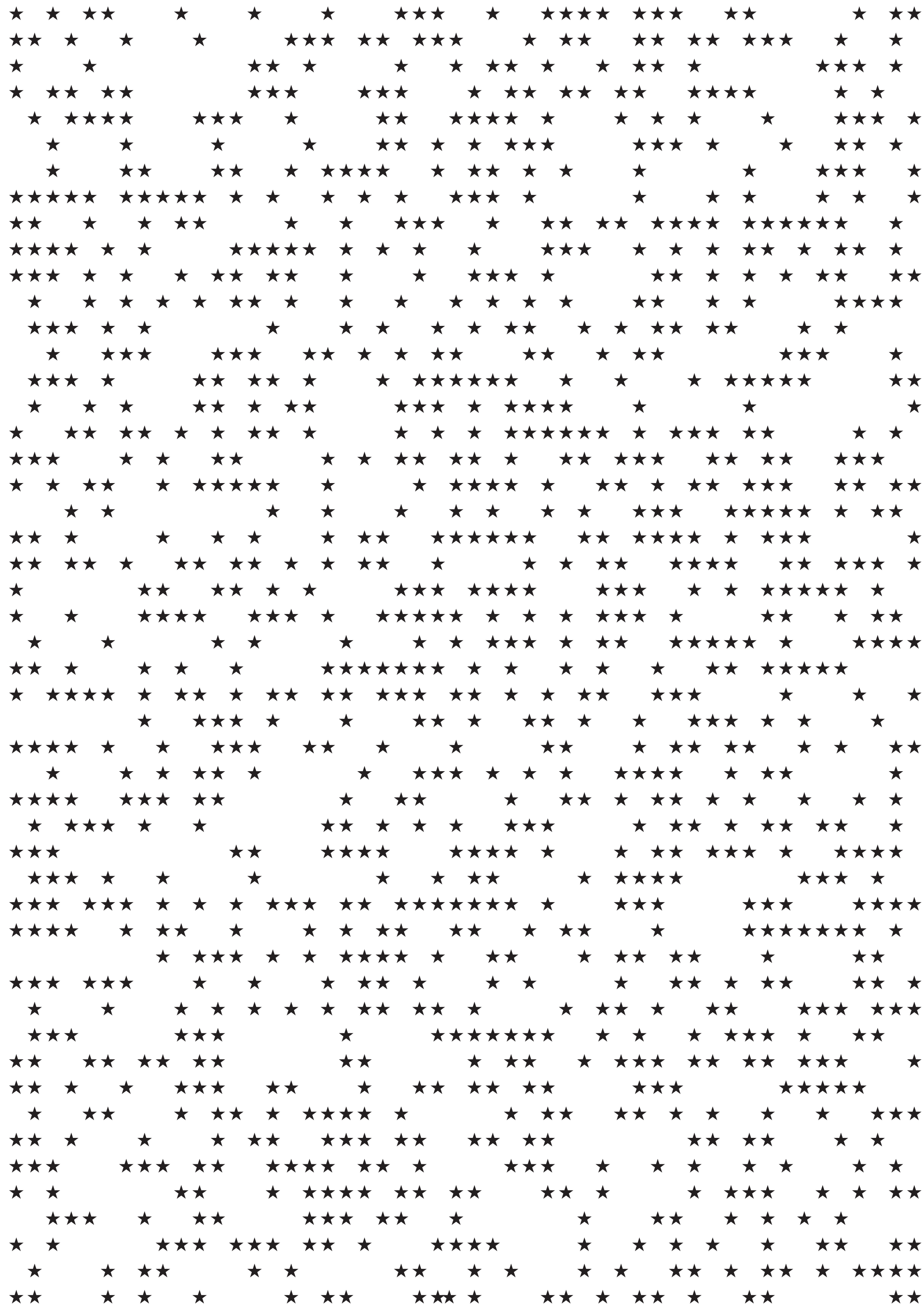
Right before pumpkin picking season, you are hired to work at a Pumpkin Patch. Your first task is to determine the number of pumpkins available for picking. In addition to growing pumpkins in the pick-your-own field, the Pumpkin Patch also grows gourds.

The diagram on the next page shows the field that contains the pumpkins and the gourds. The stars represent the gourds. Notice that there are also gaps in the field.

You and your manager agree that it would take too long to count all the pumpkins in the field.

1. Design and carry out a method to estimate the total number of pumpkins in the field without counting all the shapes. Then, prepare a presentation for your classmates that includes an explanation of your method, your results, and a justification of your estimate.

Pumpkins and Gourds



Lesson 2 Assignment

Write

Explain the purpose and process of taking a sample when you are interested in a characteristic of a population.

Remember

When you are estimating a parameter that is a count, rather than a percentage, you can use proportional reasoning to scale up from the ratio of the number of observations in your sample to the statistic.

Practice

The table at the end of this assignment shows the names and ages at inauguration of 46 Presidents of the United States.

1. You want to determine the mean age of the U.S. Presidents at their inaugurations. Instead of calculating the mean using the 46 Presidents' ages, you will take a sample.

   a. What is the population for this situation?

   b. Select 10 presidents whose ages best represent the mean age of a U.S. President at inauguration.

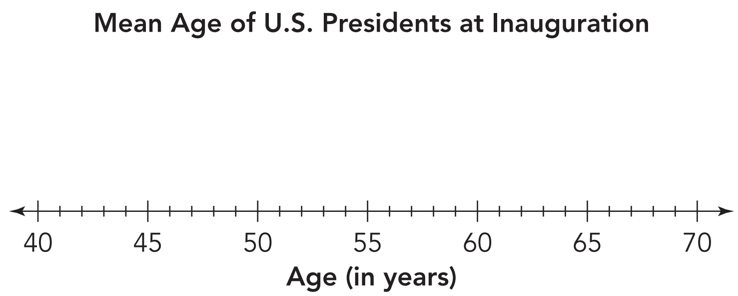
   c. Record the ages of these presidents.

d. Explain why you chose these Presidents.

e. Is this a random sample? Explain your reasoning.

f. Calculate the mean age of the presidents you selected. Round to the nearest year.

g. Record the mean age you calculated and the mean age your classmates calculated on a dot plot.



h. Describe the distribution of the dot plot in part (g).

2. You decide to use another method to choose presidents.

   a. Randomly select 10 presidents. Record the ages of these presidents.

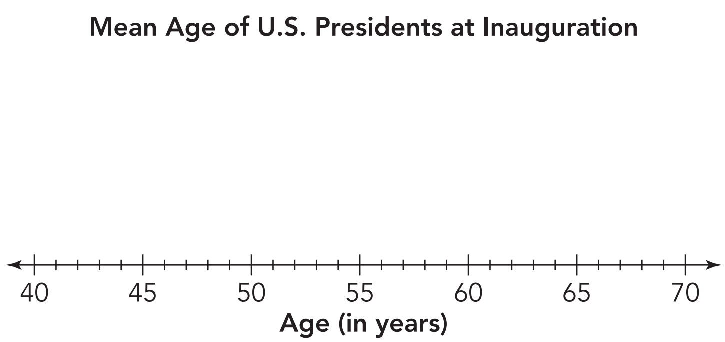
   b. Is this a random sample? Explain your reasoning.

   c. Calculate the mean age of the presidents you selected. Round to the nearest year.

   d. Is the mean age of the 10 presidents you selected a statistic or parameter? Explain your reasoning.

Lesson 2 Assignment

e. Record the mean age you calculated and the mean age your classmates calculated on a dot plot.



f. Describe the distribution of the dot plot in part (e).

g. Based on the distribution of the data in your dot plot, what is the relationship between the median and the mean?

h. Calculate the actual mean age at inauguration of the 46 Presidents. Round to the nearest year. Plot this age with an M on the dot plot in part (e).

i. Calculate the percent error between the statistic from your random sample and the true mean age.

3. How would the dot plot change if you had purposefully selected 10 presidents who were older in age as your sample?

Lesson 2 Assignment

Prepare

Determine each measure given the data provided. Show your work.

47, 35, 37, 32, 39, 38, 34, 36, 35

1. Mean

2. Median

3. Mode

4. Range

Lesson 2 Assignment

Presidents of the United States			
President	Age at Inauguration	President	Age at Inauguration
George Washington	57	Grover Cleveland	55
John Adams	61	William McKinley	54
Thomas Jefferson	57	Theordore Roosevelt	42
James Madison	57	William Howard Taft	51
James Monroe	58	Woodrow Wilson	56
John Quincy Adams	57	Warren G. Harding	55
Andrew Jackson	61	Calvin Coolidge	51
Martin Van Buren	54	Herbert Hoover	54
William Henry Harrison	68	Franklin D. Roosevelt	51
John Tyler	51	Harry S. Truman	60
James K. Polk	49	Dwight D. Eisenhower	62
Zachary Taylor	64	John F. Kennedy	43
Millard Fillmore	50	Lyndon B. Johnson	55
Franklin Pierce	48	Richard Nixon	56
James Buchanan	65	Gerald Ford	61

Lesson 2 Assignment

Presidents of the United States			
President	Age at Inauguration	President	Age at Inauguration
Abraham Lincoln	52	Jimmy Carter	52
Andrew Johnson	56	Ronald Reagan	69
Ulysses S. Grant	46	George H.W. Bush	64
Rutherford B. Hayes	54	Bill Clinton	46
James A. Garfield	49	George W. Bush	54
Chester A. Arthur	51	Barack Obama	47
Grover Cleveland	47	Donald Trump	70
Benjamin Harrison	55	Joe Biden	78

3 Bar Graphs

OBJECTIVES

- Organize data into single bar graphs, double bar graphs, stacked bar graphs, and circle graphs.
- Analyze data and interpret results of data analysis from single bar graphs, double bar graphs, stacked bar graphs, and cirlce graphs.

You have learned about various data displays including dot plots, histograms, and stem-and-leaf plots.

How can bar graphs help you organize data? What information can you interpret from their different forms?

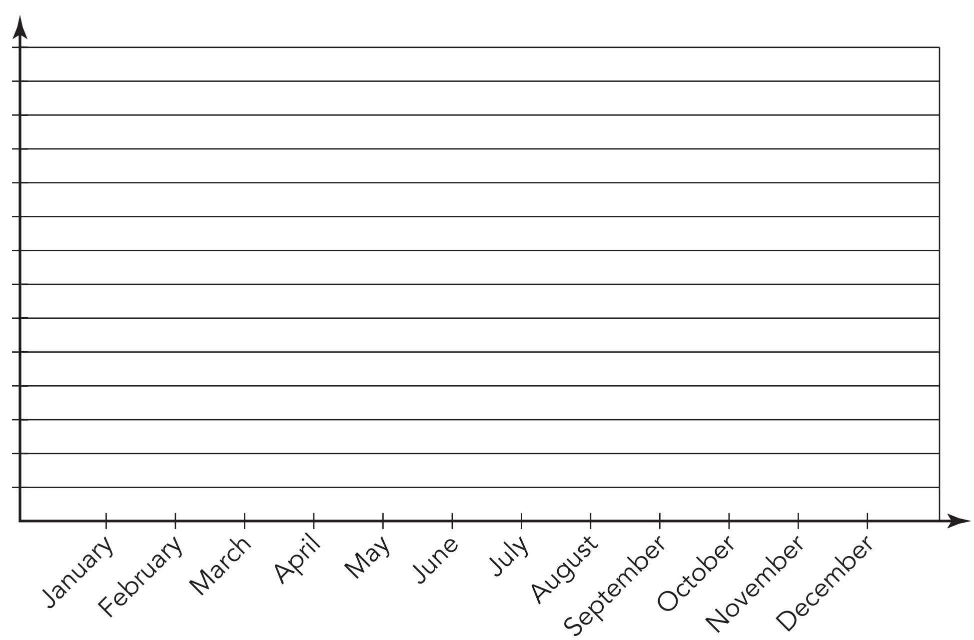
Getting Started

The Birthday Paradox

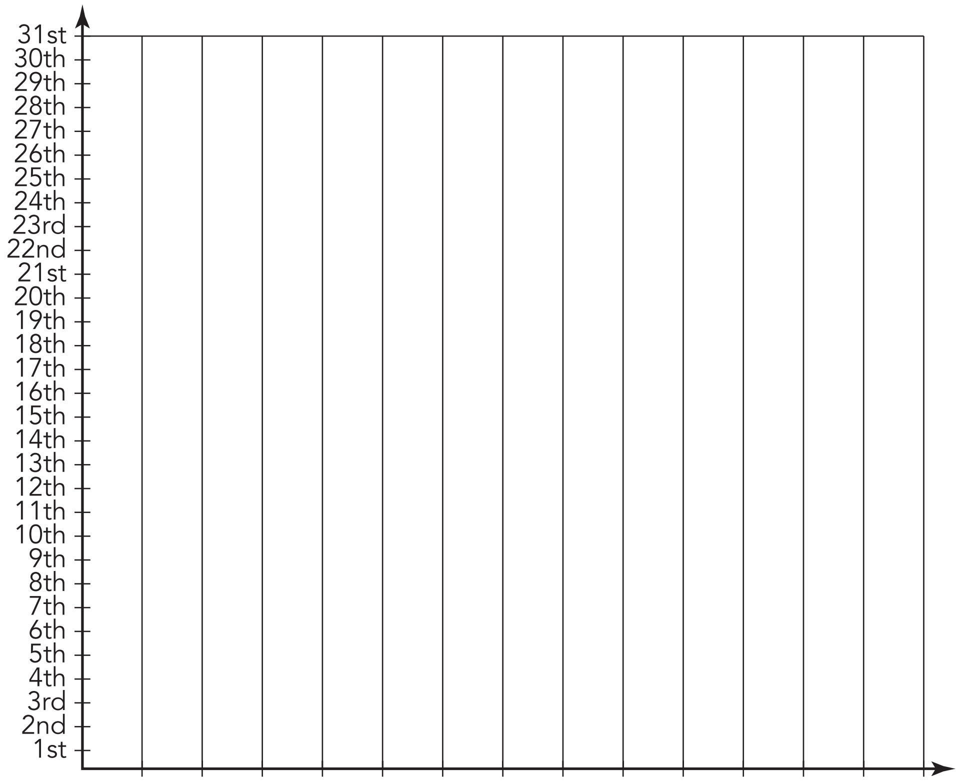
In a room of just 23 people, there is a 50-50 chance of at least two people having the same birthday. If there are 75 people, there is a 99.9% chance of at least two people sharing the same birthday. This set of facts is known as the Birthday Paradox.

1. Survey everyone in your classroom, including your teacher, and record the month and day of their birthday.

2. Use the *x*- and *y*-axes below to create a data display to show the months in which everyone in your classroom was born.



3. Use the *x*- and *y*-axes below to create a data display to show the day of the month on which everyone in your classroom was born.

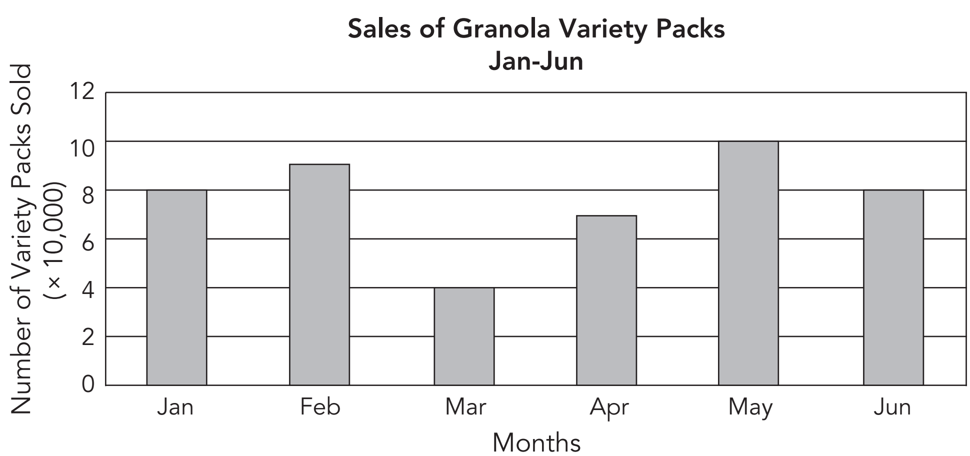


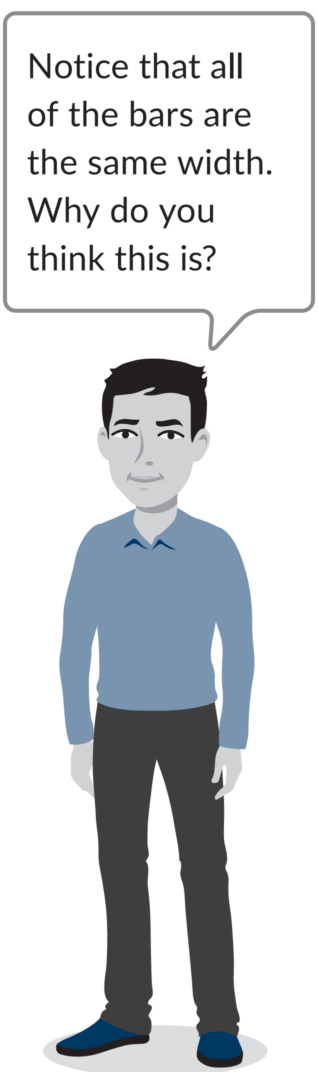
4. Does the number of people in your classroom fit the criteria of the Birthday Paradox? Does anyone in your classroom have the same birthday as someone else?

ACTIVITY 3.1 Bar Graphs

Some graphs are used to display data that consists of different categories. *Bar graphs* display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

1. The bar graph shows the number of granola variety packs sold in the first six months of the year. The bars are vertical.



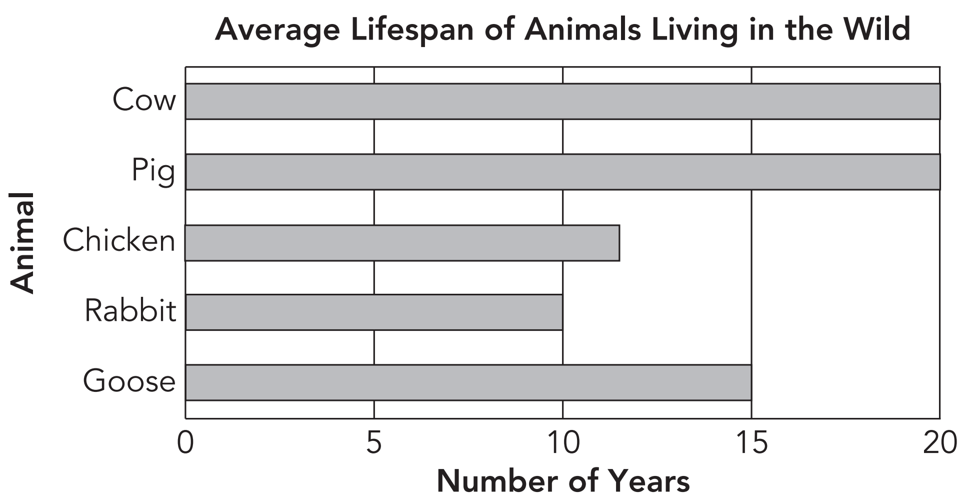


a. What does each bar represent?

b. Which month has the most sales of granola variety packs? How many were sold?

c. How many granola variety packs were sold in the first half of the year (January through June) according to the data in the bar graph?

2. The graph titled "Average Lifespan of Animals Living in the Wild" is an example of a bar graph where the bars are horizontal.



a. Which animal has the shortest lifespan according to the graph?

b. Which animal has, on average, the longest lifespan according to the graph?

c. How long does the average chicken live? How did you calculate your answer from the graph?

d. What interval is used to record the number of years on the bar graph?

e. Why do you think the bar graph includes “average” in its title? Explain your reasoning.

f. What other name could you use to replace “average” in the title?

Ask Yourself . . .
Did you justify your mathematical reasoning?

3. Compare the “Average Lifespan of Animals Living in the Wild” bar graph with the “Sales of Granola Variety Packs” bar graph.

a. Gracie says that the graphs are similar because in both graphs, the frequency of the data in each interval is represented by the height or length of the bar. Joey says that the graphs are similar because in both graphs, the intervals are the same.

Who is correct? Explain your reasoning.

b. In what other ways are the bar graphs similar? Name at least two similarities in your answer.

c. How are the bar graphs different? Name at least two differences in your answer.

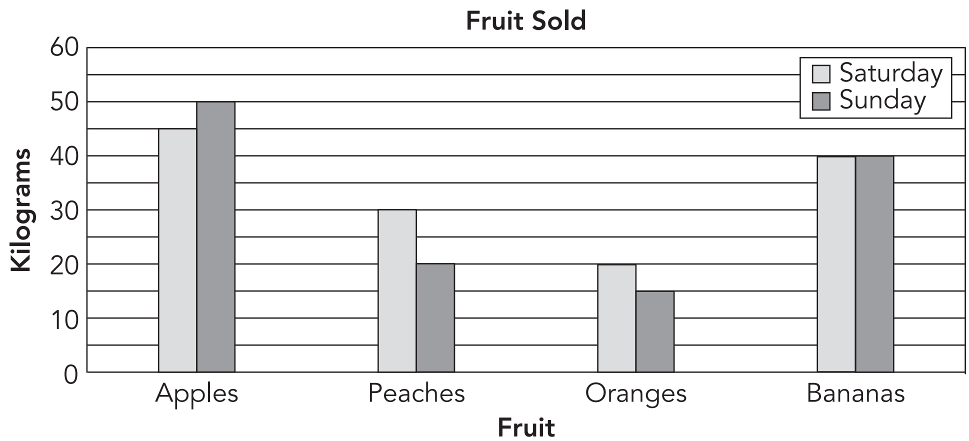
4. Use the data in the table shown to create a bar graph with vertical or horizontal bars.

Austin Motor Vehicle Administration Registration Report 2023	
Month	Approximate Number of Vehicles Registered
January	17,500
February	15,000
March	17,500
April	20,000
May	20,000
June	20,000
July	27,500
August	25,000
September	27,500
October	20,000
November	17,500
December	20,000

ACTIVITY 3.2

Double Bar Graphs

Miguel owns a fruit stand at a local farmers market that he only opens on Saturdays and Sundays. Miguel recorded the amount of fruit he sold last weekend.



Miguel displayed his data in a *double bar graph*, which is used when each category contains two different groups of data. The bars may be vertical or horizontal. A *key* explains the colors or patterns for each group. The two bars representing the same category are side-by-side. Space is used to separate the categories.

1. Use the double bar graph to answer each question.

   a. What does the key tell you?

   b. How many kilograms of apples were sold on Sunday?

c. Which fruit showed the greatest difference in sales over the two days?

d. What was the difference between Saturday's sales and Sunday's sales of your answer to part (c)?

e. Which fruit showed the smallest difference in sales over the two days?

f. What was the difference between Saturday's sales and Sunday's sales of your answer to part (e)?

2. In many countries, reports are kept for all sorts of statistics. In addition to recording the number of births in the United States, different research companies compile the same information in different ways. The table below shows the number males and females born in the United States between 2000 and 2006. You will create your own double bar graph using the information shown in the table.

U.S. Male and Female Births 2000–2006		
Year	Male	Female
2000	2,076,969	1,981,845
2001	2,057,922	1,968,011
2002	2,057,979	1,963,747
2003	2,093,535	1,996,415
2004	2,150,228	1,961,794
2005	2,118,982	2,019,367
2006	2,184,237	2,081,318

a. What will your key show?

b. Create a name for your double bar graph.

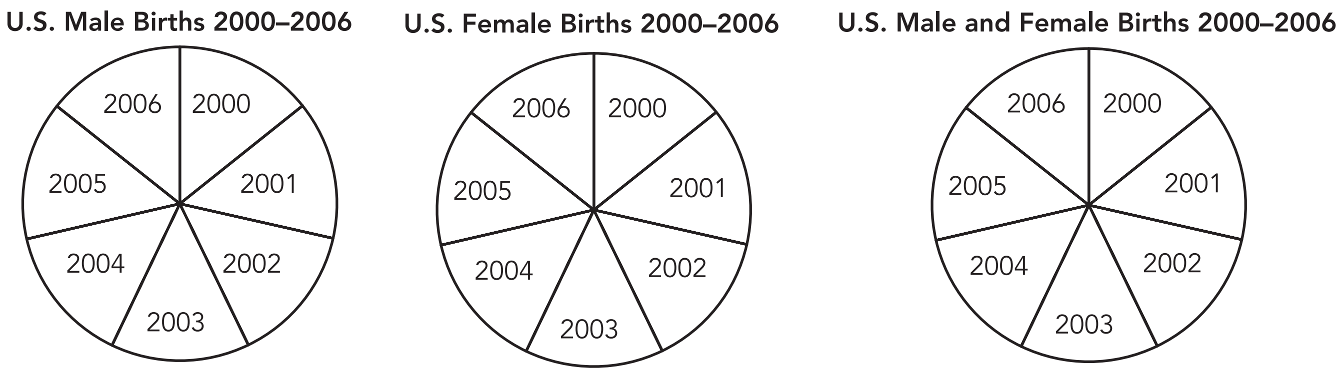
c. What are your categories?

d. How will you number the axes of your graph? State the intervals you will use.

e. Create a double bar graph.

3. When is a double bar graph a better graph choice than a single bar graph?

4. You can also display the data using *circle graphs* to represent the relationship between each part and the whole.



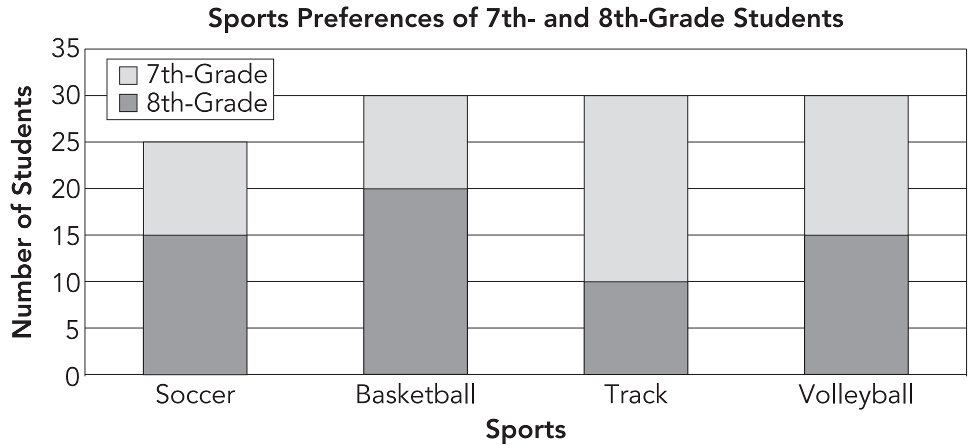
a. By analyzing the U.S. Female Births circle graph, can you determine whether more females were born in 2004 or 2005? Explain.

b. The 2002 category for the U.S. Male Births circle graph is about the same size as the 2002 category for the U.S. Male and Female Births circle graph. Does that mean that in 2002 the actual number of male births is about the same as the actual number of male and female births? Explain.

c. Is it more meaningful to display the data using a double bar graph or circle graphs? Explain.

ACTIVITY 3.3 Stacked Bar Graphs

Seventh- and eighth-grade students in an after-school program were asked which sport they preferred. Their answers are recorded in the graph below.



The "Sports Preferences of 7th- and 8th-Grade Students" display is a *stacked bar graph*. A *stacked bar graph* is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole. Each bar represents a total for the whole category but still shows how many data pieces make up each group within the entire category.

1. Use the stacked bar graph to answer each question.

   a. What does the key indicate?

   b. How many total students prefer basketball over the other choices?

c. Write a ratio for each group of students that prefers basketball. Then, determine which group of students has the higher preference.

d. Which sport is preferred equally by the 7th- and 8th-grade students in the after-school program? How can you tell?

2. Do you know what blood type you have? Do you know anyone in your class who has your same blood type? There are eight different human blood types and four blood groups. (O, A, B, and AB) These groups indicate what types of blood a person can receive should they need a transfusion. Some blood types are more common than others.

Blood Type	Percent of Americans with Blood Type
O+	37
O−	6
A+	34
A−	6
B+	10
B−	2
AB+	4
AB−	1

a. Create a stacked bar graph to represent the data listed in the table.

b. Using the information in your graph, what is the most common blood type found in Americans?

c. What is the least common blood group, when its positive and negative carriers are combined?

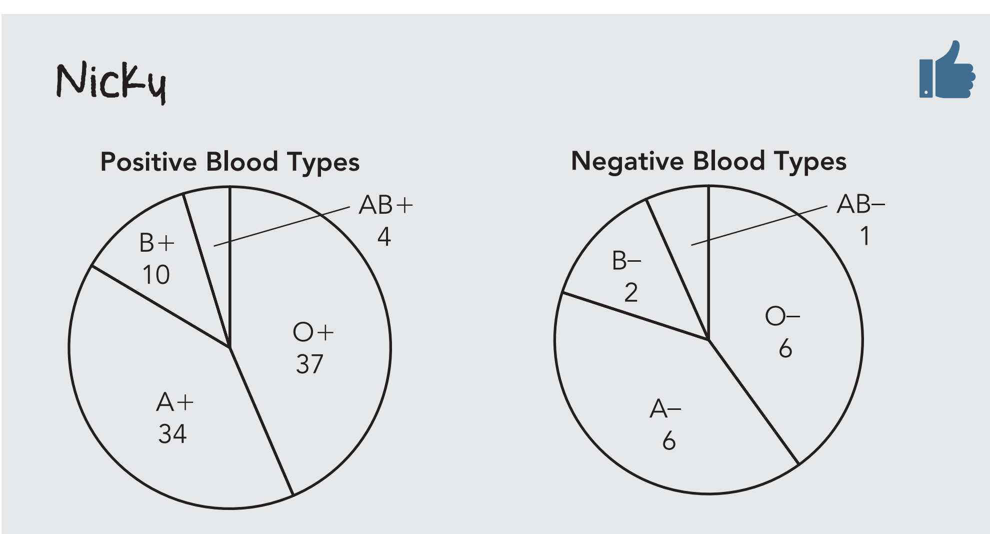
d. What percent of Americans have Group B blood with either a positive or negative factor?

3. How is a stacked bar graph the same as other bar graphs?

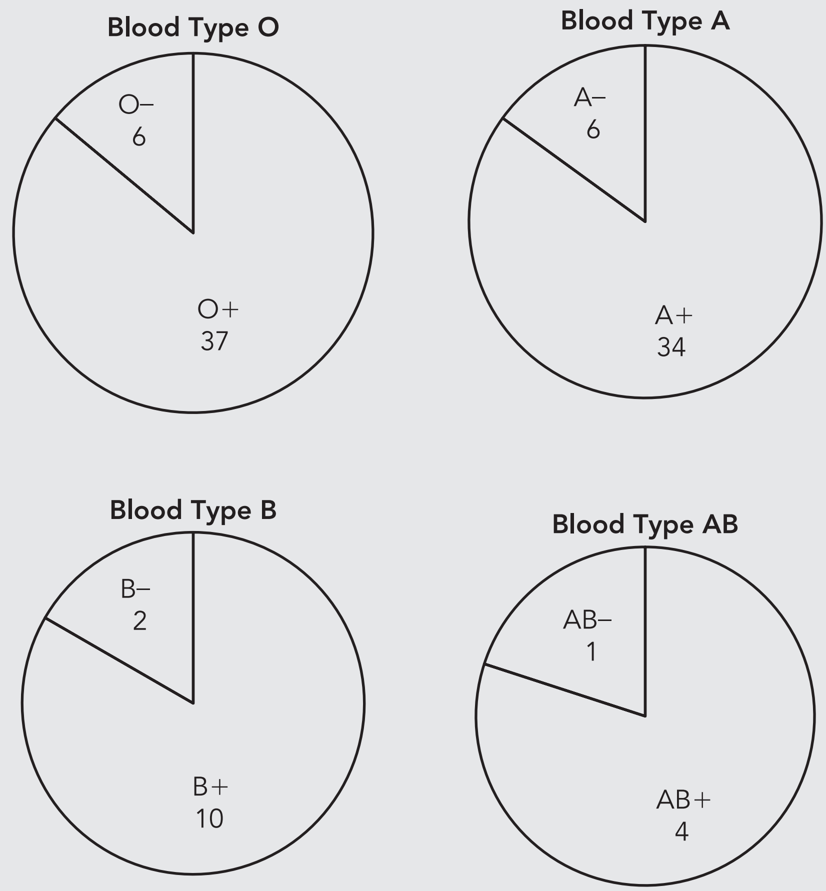
4. How is a stacked bar graph different from other bar graphs?

5. When would you use a stacked bar graph?

6. Nicky and Lucia created circle graphs using the blood type data.



Lucia

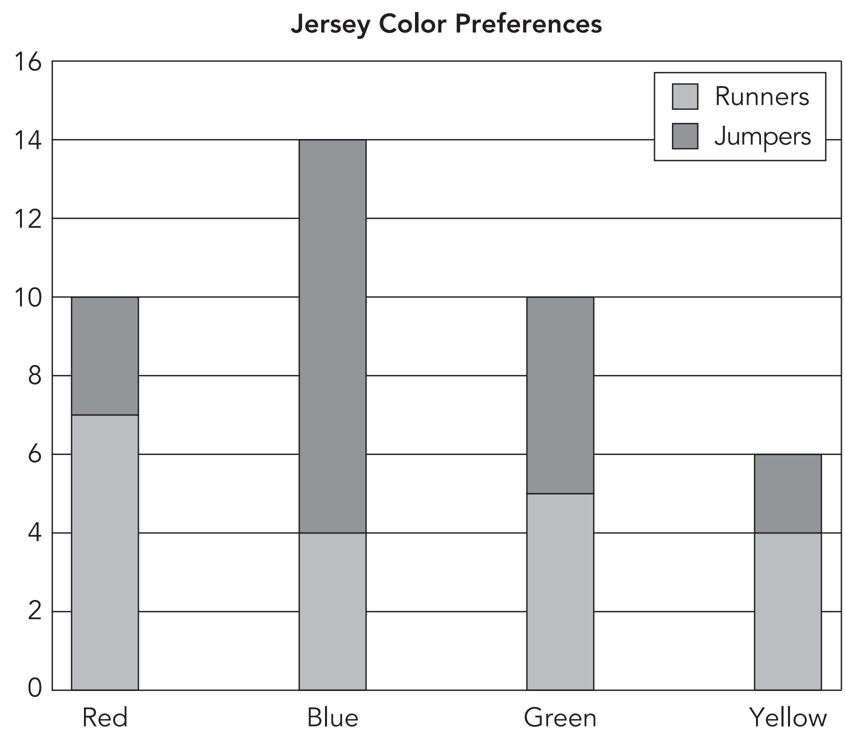


Who is correct? Explain your reasoning.

Talk the Talk

Runners and Jumpers and Jerseys, oh My!

A group of 7th-graders selected the color jersey they wanted to wear for the school track and field games. The results of their selections are shown in the stacked bar graph.



1. Create a double bar graph that displays the same information as the stacked bar graph.

2. Answer each regarding the students' color choice. Then, write which graph, double bar graph or stacked bar graph, displays the information better to answer each question.

   a. How many more jumpers than runners preferred blue?

   b. How many more runners than jumpers preferred red?

c. What fraction of students who preferred green were runners?

d. What fraction of students who preferred yellow were runners?

e. What type of question is best answered by a double bar graph? Explain.

f. What type of question is best answered by a stacked bar graph? Explain.

g. What ratio represents the number of jumpers that preferred red compared to the number of runners that preferred red? Which type of graph is a more meaningful way to display the information?

Lesson 3 Assignment

Write

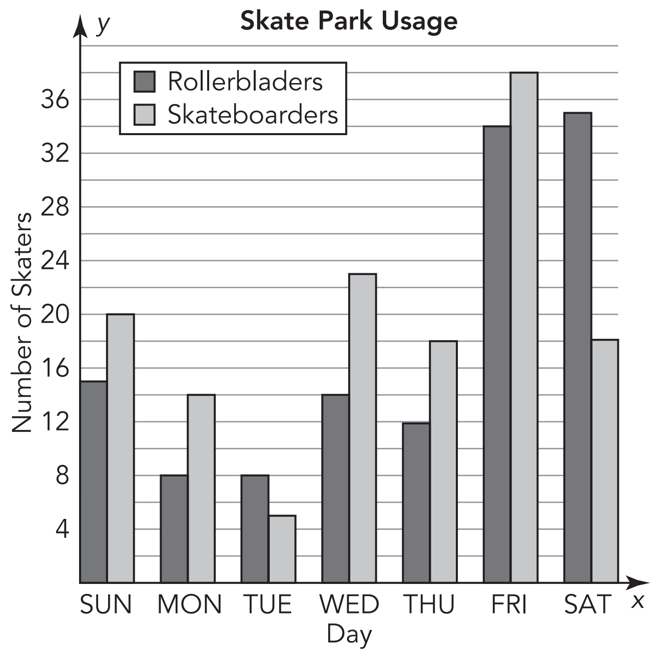
Explain the similarities and differences between bar graphs, double bar graphs, and stacked bar graphs.

Remember

Bar graphs display the frequency of categorical data using horizontal and vertical bars.

Practice

The double bar graph shows the number of skaters who used a skate park in one week. Use the double bar graph to answer the questions.



1. What does the key tell you?

Lesson 3 Assignment

2. How many rollerbladers used the park during the week? Explain how you determined your answer.

3. What is the difference between the number of skateboarders that used the park on Friday and the number of skateboarders that used it on Saturday? Show your work.

4. On how many days were there more skateboarders using the park than rollerbladers? Explain how you determined your answer.

Lesson 3 Assignment

5. Create a stacked bar graph using the data represented in the double bar graph.

6. How many total skaters used the park on Wednesday?

7. How many rollerbladers and how many skateboarders used the park on Tuesday?

Lesson 3 Assignment

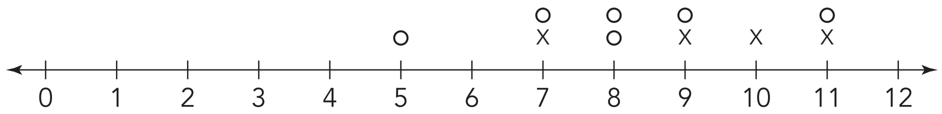
8. On which day did the most skaters use the park? Explain how you determined your answer.

9. The city would like to close the park one day a week for maintenance. Which day would you recommend they close the park based on the data shown in the bar graph? Explain your reasoning.

Lesson 3 Assignment

Prepare

The dot plot shows the number of football games 7th- and 8th-grade students attended. The “o” represents 7th-graders’ responses, and the “x” represents 8th-graders’ responses.



1. Estimate the mean number of games the 7th-grade students attended.

2. Estimate the mean number of games the 8th-grade students attended.

3. What observations can you make from your estimations of the data?

4 Comparing Two Populations

OBJECTIVES

- Calculate the measures of center and measures of variation for two populations.
- Construct comparative dot plots for two populations.
- Compare two populations of data using measures of center and measures of variation.

You have used measures of center and measures of variation to analyze single data sets.

How can you use statistics to compare two different data sets in terms of their measures of center and measures of variation?

Getting Started

Couch Potatoes

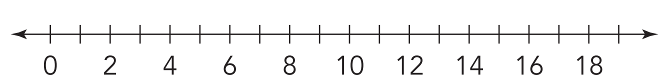
Several teenagers were surveyed to determine the number of hours they spend watching TV during a typical weekend. Another group was surveyed about the number of hours they spend playing outside. Eight surveys were randomly chosen from each group.

Survey Number	Hours Spent Watching TV (Per Weekend)
1	5
2	3
3	10
4	6
5	15
6	9
7	8
8	3

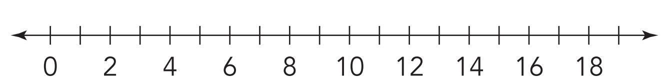
Survey Number	Hours Spent Playing Outside (Per Weekend)
1	1
2	2
3	0
4	3
5	8
6	2
7	3
8	3

1. Create a dot plot for each data set.

Hours Spent Watching TV



Hours Spent Playing Outside



2. Calculate the mean, median, and interquartile range (IQR) for each data set.

The IQR is the difference between the third quartile and the first quartile.

3. What do these measures of center and measures of variation tell you about the data from the surveys?

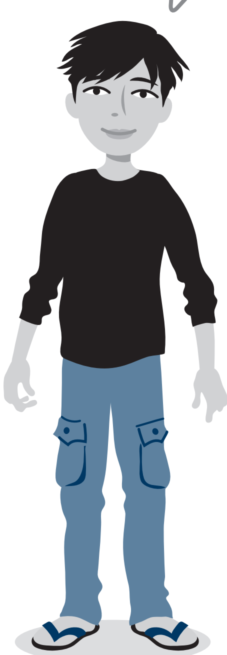
ACTIVITY 4.1

Comparing Measures of Center and Measures of Variation

Nia has just opened a new restaurant, which serves nothing but chocolate milk. She is experimenting with two new flavors— a spicy chocolate milk and a dark chocolate milk.

Nia has asked you to provide a report analyzing customer feedback about the new flavors. You have conducted a survey of 20 random customers, asking each customer to rate a flavor on a scale of 0 to 100.

The 4 steps of the statistical process are:
1. Formulate a statistical question.
2. Collect data.
3. Analyze the data.
4. Interpret the data.

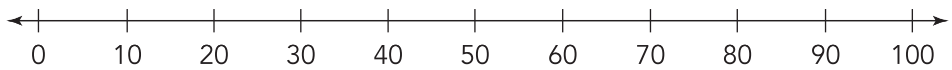


Flavor	Rating	Flavor	Rating
spicy	50	spicy	70
dark	20	dark	30
dark	30	dark	40
spicy	100	spicy	70
spicy	60	dark	20
spicy	80	dark	60
spicy	60	spicy	80
dark	10	dark	20
dark	30	spicy	60
dark	40	spicy	70

1. Display the results on a dot plot. Use an "x" to represent the spicy flavor responses and an "o" to represent the dark flavor responses.

Review the definition of **represent** in the Academic Glossary.

Flavor Ratings



2. What observations can you make from your dot plot?

3. Describe the distribution of data values for the spicy flavor and for the dark flavor.

4. Analyze the data values on your dot plot for spicy and dark.

Remember, you can describe distributions as skewed right, symmetric, or skewed left.

a. Estimate the mean rating for the spicy flavor. Mark the mean on your dot plot with an "s." Explain how you determined your estimate.

b. Estimate the mean rating for the dark flavor. Mark the mean on your dot plot with a "d." Explain how you determined your estimate.

5. Calculate the actual mean rating for the spicy flavor.

6. Calculate the actual mean rating for the dark flavor.

7. What observations can you make about the spread of the two data sets?

8. Can you report on which flavor has a more consistent rating? Explain your reasoning.

WORKED EXAMPLE

Comparing the difference of means with the variation in each data set can be an important way of determining just how different two data sets are.

Consider these data sets.

5, 3, 4, 5, 10	5, 3, 100, 5, 10
Mean = 5.4	Mean = 24.6

The difference in their means is 19.2. Depending on what you are measuring, that can be a big difference.

9. For the spicy flavor and dark flavor data, compare the difference in the means. What observations can you make?

10. What recommendation would you give to Nia about the two new flavors?

ACTIVITY 4.2

Comparing Distributions

In 2022, a middle school's football team won only 2 games. The school decided to give the coach another chance at improving the team the next year. In 2023, the team won 7 games. Did scoring points have something to do with the team improving their record? The stem-and-leaf plot shows the number of points scored in each game by the football team in 2022 and 2023.

Middle School Football Team

Points scored 2022					Stem	Points scored 2023			
		7	3	0	0				
7	4	4	2	0	1	4	7	7	
			9	8	2	4	7	8	8
				5	3	5	8		
					4	2	5		

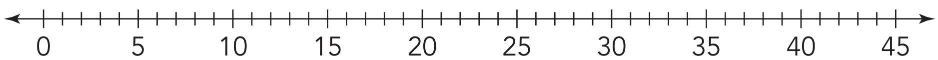
Key = 0|1|4 means 10 and 14

1. In which year did the football team score more points?

2. Describe the distribution of the stem-and-leaf plots for each year.

3. Create a combined dot plot to represent the data. Use the letters "x" and "o" to represent data from the different years.

Points Scored by the Football Team, 2022 and 2023



a. How does the shape of the stem-and-leaf plot distribution compare with the shape of the dot plot distributions?

4. Determine the five-number summary and IQR for each data set. Then, complete the table.

	2022	2023
Minimum		
Q1		
Median		
Q3		
Maximum		
IQR		

5. To calculate the five-number summary and IQR, did you use the data from the stem-and-leaf plot or the dot plots? Explain.

6. Compare the median and the IQR for the two data sets.

7. How does the difference in the medians compare to the IQR?

The final step in the statistical process is to interpret the data.

8. Do you think that scoring more points may have been one reason the football team improved its record?

Talk the Talk

Summarize

Write 1–2 paragraphs to summarize this lesson. Be sure to answer the question below in your response.

How can you use the five-number summary to interpret and compare the data of two populations?

Lesson 4 Assignment

Write

How can dot plots and stem-and-leaf plots help you to compare two sets of data?

Remember

Data for two populations may overlap. Comparing the measures of center and measures of variation for the two populations can help you interpret the differences between the two populations.

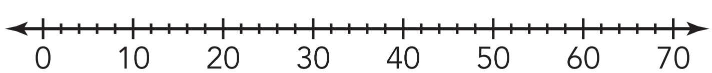
Practice

The head librarian at the local public library is investigating the current trends in technology and the effects of computers and electronic books on the loaning of books. She thinks that the users at the library on the computers are generally younger than the people who actually check out books. She asks the ages of a sample of both computer users and book borrowers. The results are shown in the table.

Patron		Age
Computer user	Book borrower	
C		27
C		16
	B	57
C		20
	B	55
	B	60
C		22
	B	59
	B	63
C		20
C		24
	B	63
	B	60
C		22
C		20
C		17
	B	66
	B	60
	B	55
C		25

1. Display the results on a dot plot. Use an “o” to represent the computer users’ ages and an “x” to represent the book borrowers’ ages from the information in the table.

Ages of Patrons



Lesson 4 Assignment

2. Display the results using a stem-and-leaf plot. Be sure to include a key.

3. Describe the distribution of data values for the computer users and the book borrowers.

4. Calculate the mean age of the computer users and the mean age of the book borrowers.

Lesson 4 Assignment

5. Determine the five-number summary for the computer users and the book borrowers.

6. Calculate, interpret, and compare the IQR for both the computer users and the book borrowers.

7. What can you say about these two populations?

Lesson 4 Assignment

Prepare

Determine the five-number summary, mean, and IQR of the data set represented in the stem-and-leaf plot.

0	1 2 4
1	0 1
2	3
3	1 2 3 5
4	
5	5 6

Key: 3|1 = 31

5 Using Random Samples from Two Populations to Draw Conclusions

OBJECTIVES

- Compare the measures of variation for random samples from two populations.
- Express the difference between the centers of two data distributions as a multiple of a measure of variation.
- Use measures of variation to draw conclusions about two populations.

You have learned about measures of variation and have calculated the variations of different data sets.

How can you compare data sets in terms of their variation to solve problems?

Getting Started

Downloading Podcasts

Square Roots is a radio show that airs 10 times a week on local radio station WMTH.

WMTH is trying to raise its commercial airtime rates during *Square Roots*. The station claims that while this music show is listened to by hundreds of middle school students via the radio, there are actually a greater number of middle school students who listen to the show by regularly downloading the podcast. Advertisers disagree with WMTH's claim. Advertisers want the station to verify its claim that there are more students listening to downloaded podcasts than listening to live radio. To do so, WMTH and the advertisers choose the local middle school to collect data. They send out a survey and ask the following two questions:

- Do you listen to *Square Roots* on the radio or download the podcast?
- How often do you listen to *Square Roots* per week?

All 389 students at the local middle school who listen to *Square Roots* responded to the survey.

1. What are the two populations for the *Square Roots* survey WMTH is conducting?

WMTH decides to select a random sample for each population.

2. There are 180 regular radio listeners and 209 podcast listeners at the local middle school. Describe how WMTH and the advertisers can randomly select students for their sample.

ACTIVITY 5.1

Simulating Random Samples

Let's use a random number table to simulate random samples from the data. The random number table and data are at the end of the lesson.

1. Use the random number table and the list of radio and podcast listeners at the local middle school at the end of the lesson to help WMTH randomly select a sample.

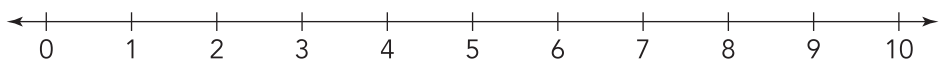
   a. Randomly select 10 radio listeners. Record each student's last name. Then, use the list to record the number of times each student listened to *Square Roots* during the week.

   b. Randomly select 10 podcast listeners. Record each student's last name. Then, use the list to record the number of podcasts each student downloaded in one week.

When you are assigning each student a number, each number should have the maximum number of digits in the largest number of a population. Therefore, if there are 300 people in a population, each number assigned should have three digits.

2. Construct a combined dot plot for the two groups. What conclusions can you draw?

Radio Shows and Podcasts (per week)



Review the definition of **describe** in the Academic Glossary.

3. Describe the distribution for each graph. Describe any clusters or gaps in the data values in each graph.

4. Estimate the mean for each dot plot. Explain how you determined your estimate.

5. Calculate the mean number of radio shows listened to in a week and the mean number of podcasts downloaded in a week.

6. Compare the two samples. Are more shows listened to on the radio, or are more podcasts downloaded?

7. Compare the measures of center and measures of variation.

8. Xavier says,“If we had started on a different line number in the random number table, our results would have been the same.” Is Xavier correct? Explain your reasoning.

The third step in the statistical process is to interpret the data.

9. Combine your data with the data from other classmates. Calculate measures of center and measures of variation for the two combined random samples and interpret your results.

10. Determine the difference of means for the two samples and describe this difference as a multiple of the measure of variation.

ACTIVITY 5.2

Comparing Random Samples

Juliana graduated from college and now has a choice of two jobs. One of the jobs is in Ashland, and the other job is in Belsano. Since Juliana enjoys mild weather and average temperatures in the 60s (°F), she decides to compare the monthly average temperatures of the two cities. She gathered the following sample of average monthly temperatures for a previous year for the two cities, as shown in the table.

Month	Ashland Average Monthly Temperatures (°F)	Belsano Average Monthly Temperatures (°F)
January	56	48
February	58	55
March	60	59
April	61	62
May	65	66
June	70	69
July	75	78
August	82	88
September	73	82
October	68	69
November	60	59
December	56	49

1. What are some ways Juliana could analyze the data to determine which city is warmer?

2. Juliana decides to calculate the mean and median temperature for each city to determine which city is warmer overall. Calculate the mean and median temperature for both cities. What do you notice?

3. Ben says that since Ashland and Belsano have very similar mean and median temperatures, Juliana could choose either city to live in because they both have mild temperatures. Do you agree or disagree? Explain your reasoning.

4. Construct a back-to-back stem-and-leaf plot for the average monthly temperature for each city.

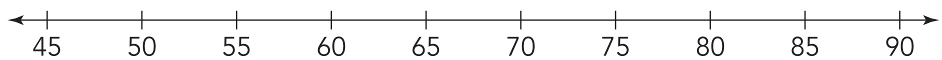
Ashland		Belsano

5. What conclusions can you draw from the plot?

6. Determine and interpret the five-number summary and IQR for each data set. Then, describe some observations from the data of each five-number summary.

	Ashland	Belsano
Minimum:		
Q1:		
Median:		
Q3:		
Maximum:		
IQR:		

7. Construct and label box plots for each data set using the same number line for both. What conclusions can you make?

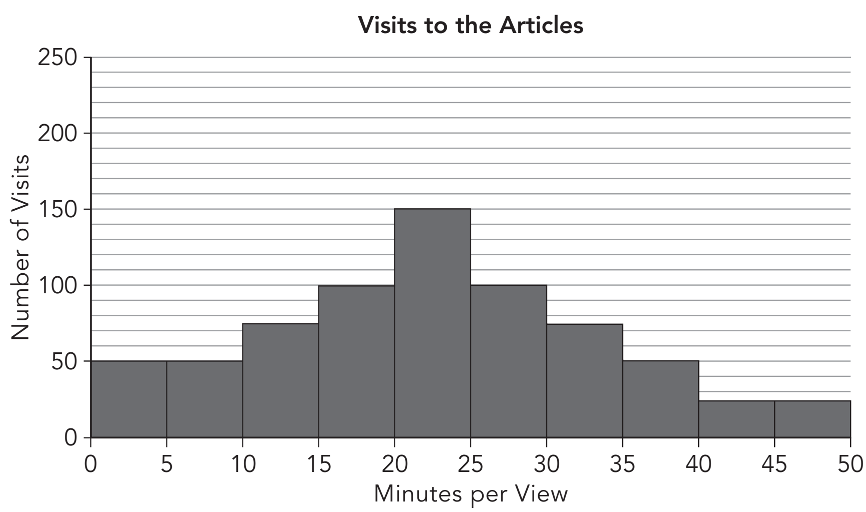


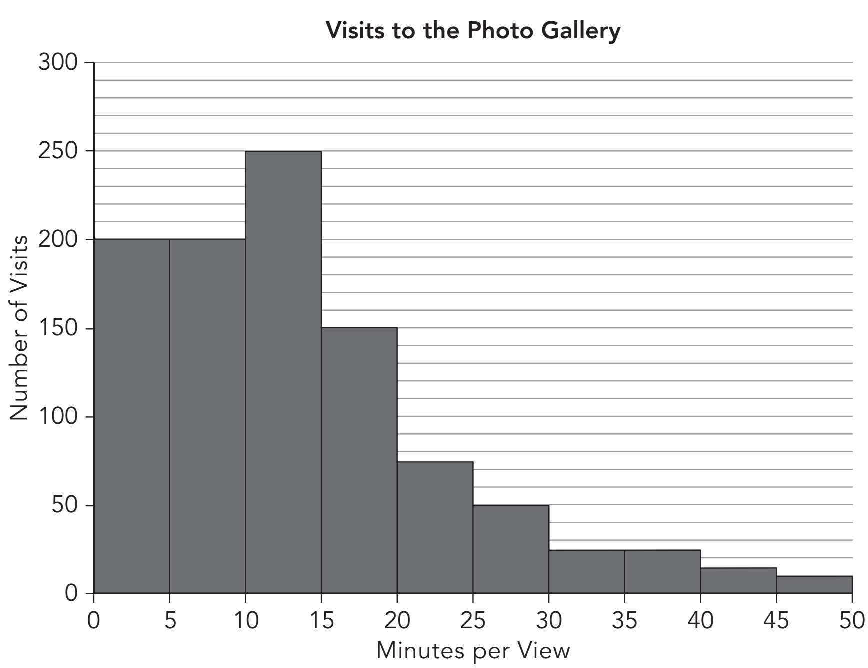
8. Compare the mean and variation of the samples. If you were Juliana, which city would you choose to live in? Explain your reasoning.

ACTIVITY 5.3 Analyzing Displays of Data from Random Samples

Websites often analyze customer visits to see if there are patterns or trends. Gaining information about patterns helps companies display the information users want.

A sample of customer visits to a news and opinion website is shown in the histograms. The histograms display the number of visits customers made to the Articles and Photo Gallery sections of the website, along with how long each customer spent viewing the content in each section.





1. Describe the populations and samples for this problem.

2. What do the intervals along the *x*-axis and *y*-axis represent for each histogram?

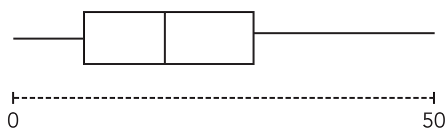
3. About how many visits were made to the Articles sample? The Photo Gallery sample? Explain how you determined your answer.

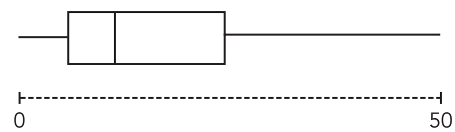
4. Antonio claims that the variation in the length of views of each section in terms of the range is about the same for both sections. Do you agree or disagree?

5. Describe the shape of each histogram and explain what this means in terms of the number of minutes per view.

6. Determine whether the mean or the median is greater for each section. Then, explain why that measure of center is greater in value for each section.

7. The box plots shown represent the minutes per view for the Photo Gallery and Articles sections. Using the information you know from the histograms for each section, which box plot do you think represents the minutes per view for the Articles, and which represents the minutes per view for the Photo Gallery? Explain your choice.

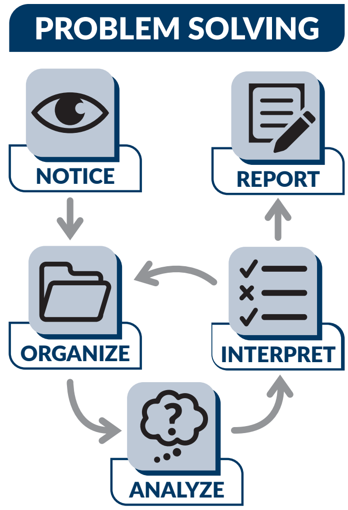




8. In terms of the box plots, which section has more variation in the minutes per view? Explain your reasoning.

Talk the Talk

Into Each Life Some Rain Must Fall



Omar lives in Seattle, Washington, and he says it seems like it rains all the time. Ricardo lives in Washington, DC, and he says it seems like it doesn't rain very much.

The table contains the average monthly rainfall for both cities over the past 30 years.

1. Use any method you want to determine the validity of Omar's and Ricardo's statements.

Month	Seattle, Washington Average Monthly Rainfall (inches)	Washington, DC. Average Monthly Rainfall (inches)
January	5.24	3.21
February	4.09	2.63
March	3.92	3.60
April	2.75	2.77
May	2.03	3.82
June	1.55	3.13
July	0.93	3.66
August	1.16	3.44
September	1.61	3.79
October	3.24	3.22
November	5.67	3.03
December	6.06	3.05

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)
1	Abunto	1	21	D'Ambrosio	0	41	Granger	0
2	Adler	3	22	Datz	4	42	Guca	2
3	Aizawa	3	23	Delecroix	2	43	Haag	8
4	Alescio	4	24	Difiore	6	44	Heese	5
5	Almasy	8	25	Dobrich	7	45	Hilson	1
6	Ansari	6	26	Donoghy	1	46	Holihan	1
7	Aro	7	27	Donaldson	5	47	Hudack	1
8	Aung	2	28	Dreher	2	48	Ianuzzi	3
9	Baehr	7	29	Dubinsky	1	49	Islamov	4
10	Bellmer	1	30	Dytko	8	50	Jacobsen	5
11	Bilski	4	31	Fabry	7	51	Jessell	4
12	Blinn	6	32	Fetcher	1	52	Ji	1
13	Bonetto	3	33	Fontes	5	53	Johnson	2
14	Breznai	1	34	Frick	3	54	Jomisko	1
15	Cabot	3	35	Furmanek	5	55	Jones	6
16	Chacalos	0	36	Gadgil	4	56	Joy	5
17	Cioc	0	37	Gavlak	4	57	Jumba	1
18	Cole	3	38	Gibbs	0	58	Juth	7
19	Creighan	4	39	Gloninger	2	59	Jyoti	6
20	Cuthbert	6	40	Goff	1	60	Kachur	2

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)
61	Kanai	0	81	McNary	7	101	Nzuyen	2
62	Keller	2	82	Meadows	0	102	O'Bryon	0
63	Khaing	7	83	Merks	8	103	Obitz	3
64	Kindler	5	84	Mickler	5	104	Oglesby	8
65	Kneiss	5	85	Minniti	4	105	Ono	1
66	Kolc	1	86	Mohr	3	106	Paclawski	0
67	Kuisis	2	87	Mordecki	5	107	Pappis	6
68	Labas	1	88	Mueser	3	108	Peery	3
69	Lasek	8	89	Musati	3	109	Phillips	5
70	Leeds	5	90	Myron	2	110	Potter	3
71	Lin	0	91	Nadzam	4	111	Pribanic	7
72	Litsko	2	92	Nazif	7	112	Pwono	5
73	Lodi	3	93	Newby	0	113	Quinn	2
74	Lookman	2	94	Ng	1	114	Rabel	5
75	Lucini	1	95	Nino	2	115	Rayl	2
76	Lykos	0	96	Northcutt	4	116	Rea	4
77	MacAllister	1	97	Novi	3	117	Reynolds	5
78	Magliocca	6	98	Null	8	118	Rhor	8
79	Marchick	5	99	New	5	119	Rielly	8
80	McGuire	1	100	Nyiri	1	120	Risa	7

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)
121	Robinson	8	141	Stevens	4	161	Volk	5
122	Roethlein	1	142	Tabor	0	162	Vyra	4
123	Romanski	7	143	Tevis	5	163	Wadhwani	5
124	Rouce	7	144	Thomas	0	164	Warnaby	0
125	Rubio	7	145	Thompson	1	165	Weasley	2
126	Rutland	3	146	Thorne	1	166	Weidt	8
127	Rychcik	1	147	Tiani	0	167	Whitelow	0
128	Sabhnani	6	148	Tokay	6	168	Wilson	4
129	Sandroni	2	149	Toomey	6	169	Woller	1
130	Saxon	3	150	Trax	1	170	Woo	0
131	Scalo	4	151	Truong	8	171	Wunderlich	6
132	Schessler	1	152	Tunstall	1	172	Wycoff	4
133	Seeley	0	153	Twiss	6	173	Xander	8
134	Shanahan	3	154	Tyler	0	174	Yahya	0
135	Siejk	1	155	Ueki	0	175	Yezovich	4
136	Skaro	3	156	Uriah	6	176	Youse	0
137	Slonaker	8	157	Vagnelli	6	177	Yuzon	8
138	Sobr	2	158	Van Dine	5	178	Za Khai	3
139	Spatz	4	159	Vella	1	179	Ziff	2
140	Sramac	2	160	Vidnic	0	180	Zuk	5

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
1	Aaronson	2	21	Chang	4	41	Frena	1
2	Abati	0	22	Clarke	9	42	Galdi	8
3	Ackerman	4	23	Crnkovich	0	43	Gansberger	3
4	Aderholt	2	24	Dahl	0	44	Gianni	1
5	Akat	7	25	Dax	7	45	Glencer	1
6	Aleck	9	26	Defoe	1	46	Godec	7
7	Alessandro	5	27	Dengler	4	47	Goldstein	0
8	Allen	3	28	Di Minno	4	48	Graef	6
9	Ansil	1	29	Dilla	5	49	Gula	1
10	Archer	5	30	Draus	5	50	Hagen	2
11	Badgett	9	31	Duffy	0	51	Haupt	8
12	Bartle	2	32	Ecoff	3	52	Herc	4
13	Bibby	9	33	Esparra	7	53	Hnat	9
14	Bilich	5	34	Fakiro	7	54	Hodak	3
15	Bloom	3	35	Ferlan	4	55	Hoyt	2
16	Boccio	5	36	Fetherman	2	56	Huang	3
17	Bracht	3	37	Fillipelli	6	57	Iannotta	4
18	Bujak	7	38	Fisher	2	58	Irwin	5
19	Caliari	9	39	Folino	9	59	Jackson	7
20	Cerminara	8	40	Forrester	9	60	Jamil	1

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
61	Jessop	1	81	Ling	6	101	Moorey	6
62	Johnson	9	82	Loch	4	102	Mox	7
63	Joos	9	83	Lorenzo	4	103	Mrkali	7
64	Joseph	5	84	Lovejoy	5	104	Mu	0
65	Jubic	3	85	Luba	8	105	Muller	3
66	Juhl	7	86	Lukitsch	4	106	Murphy	2
67	Jung	9	87	Luzzi	8	107	Mwambazi	6
68	Jurgensen	4	88	Lyman	5	108	Myers	4
69	Jyoti	0	89	MacIntyre	8	109	Nangle	3
70	Kaib	5	90	Maddex	5	110	Neilan	7
71	Kapoor	6	91	Marai	2	111	Nicolay	5
72	Kennedy	2	92	Mato	9	112	Niehl	6
73	Kimel	5	93	McCaffrey	0	113	Nix	2
74	Klaas	4	94	McElroy	5	114	Noga	3
75	Ko	9	95	McMillan	3	115	Nowatzki	7
76	Krabb	1	96	Meng	9	116	Nuescheler	5
77	Ladley	9	97	Michelini	5	117	Nye	6
78	Lawson	1	98	Misra	0	118	Nytra	6
79	Lemieux	7	99	Miller	8	119	O'Carrol	6
80	Lewan	6	100	Modecki	7	120	Obedi	7

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)
121	Oehrle	8
122	Olds	5
123	Oleary	0
124	Ondrey	1
125	Owusu	9
126	Palamides	9
127	Pappas	0
128	Pecori	3
129	Pennix	4
130	Pendleton	1
131	Phillippi	2
132	Pieton	6
133	Ploeger	2
134	Pressman	4
135	Puzzini	1
136	Qu	4
137	Qutyan	8
138	Raab	5
139	Raeff	0
140	Rav	1

Student Number	Student Name	Podcasts Downloaded (per week)
141	Rea	5
142	Renard	7
143	Rex	7
144	Richards	7
145	Ridout	7
146	Rivera	6
147	Roberts	4
148	Rodwich	0
149	Roney	7
150	Ross	6
151	Rothering	0
152	Rua	4
153	Russo	8
154	Ryer	8
155	Sagi	8
156	Sallinger	6
157	Sau	7
158	Sbragia	7
159	Schaier	5
160	Schmit	2

Student Number	Student Name	Podcasts Downloaded (per week)
161	Scopaz	4
162	Sebula	4
163	Shah	1
164	Sidor	6
165	Skraly	6
166	Sokolowski	5
167	Speer	6
168	T'Ung	9
169	Tamar	9
170	Tebelius	1
171	Tesla	4
172	Thuma	0
173	Tibi	2
174	Tobkes	9
175	Torelli	8
176	Tozzi	0
177	Traut	6
178	Trax	0
179	Tu	8
180	Tumicki	2

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
181	Tyson	1	191	Wallace	2	201	Wulandana	8
182	Uansa	0	192	Webb	9	202	Wysor	9
183	Ulan	4	193	Weisenfeld	0	203	Xiao	4
184	Urbano	7	194	Whalen	7	204	Yee	6
185	Uzonyi	7	195	Wiley	9	205	Yost	1
186	Vaezi	8	196	Williams	5	206	Young	7
187	Vinay	6	197	Williamson	6	207	Yuros	6
188	Vu	8	198	Witek	9	208	Zaki	0
189	Wallee	0	199	Wojcik	3	209	Zimmerman	1
190	Waldock	4	200	Woollett	6			

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61004	43516	17020
Line 2	17264	57327	38224	29301	31381	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86210	11808	12841	45147	97438	60022	12645	62000
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19177	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	10415	36457
Line 8	16975	95428	33226	55903	31605	43817	22250	03918	46999	98501
Line 9	59138	39542	71168	57609	91510	77904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

Lesson 5 Assignment

Write

Explain how to use measures of center and measures of variation to compare two populations.

Remember

You can use the means to compare two populations with approximately symmetric data sets. You can use the medians and the interquartile range to compare two populations with skewed data sets.

Practice

1. Repeat the sampling procedure you used with the data in Activity 5.1. Choose a different line number in the random number table.

   a. Record the results for 20 students.

   b. Construct a combined dot plot for the two groups, using the same scale.

   c. Describe the distribution of the data for each group.

   d. Estimate and calculate the mean for the data in each group.

   e. Compare your new results with your results from Activity 4.1.

Lesson 5 Assignment

2. Ratings can be used to show people's opinions of hotels and restaurants. One of the things that can help a restaurant get good ratings is the time it takes to be seated (the wait time) at a restaurant without reservations. A steak house and a pizza parlor both claim to have the shortest wait times in town. To check out the claims, Omar a restaurant reviewer, records the time it takes to be seated without reservations. The results of wait times from 8 visits to each restaurant are shown in the table below.

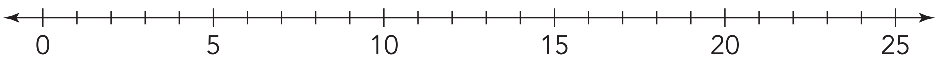
Steak House Wait Times (minutes)	Pizza Parlor Wait Times (minutes)
5	11
13	19
22	14
7	14
20	15
21	20
20	10
12	17

a. Describe the populations and samples for this problem.

b. Calculate the mean wait times for the two restaurants. Which restaurant seems to have faster service?

c. Complete the dot plot of the times. What do you notice?

Wait Times



O = Steak House

X = Pizza Parlor

d. Determine the median wait time for each restaurant.

e. Explain the difference in median times for the two restaurants.

f. Which measure of center would you use if:

- you are the steak house and want to claim you have the shortest wait time?

- you are the pizza parlor and want to claim you have the shortest wait time?

- you are a customer and want the shortest wait time?

g. Suppose another restaurant reviewer records wait times at each restaurant several times. Do you think it is possible that the wait times might be different from Omar's wait times? Explain your reasoning.

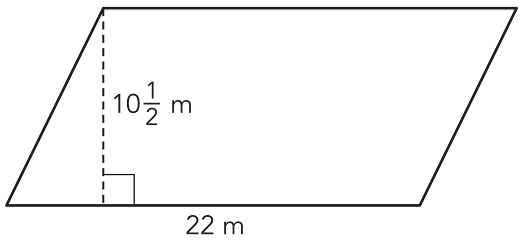
h. How could we be more certain which restaurant has the shortest wait time?

Lesson 5 Assignment

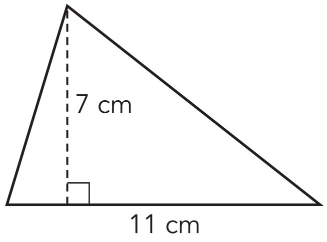
Prepare

Use a formula to determine the area of each figure.

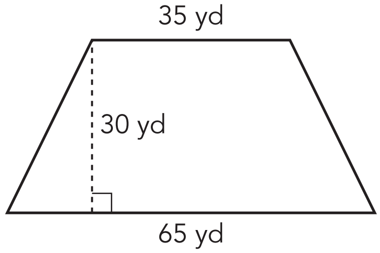
1.



2.



3.



TOPIC 3 SELF-REFLECTION

Name: ______________________________

Drawing Inferences

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Drawing Inferences* topic by:

TOPIC 3: *Drawing Inferences*	Beginning of Topic	Middle of Topic	End of Topic
applying knowledge of probability and proportional relationships to analyze data and make inferences about a population.	○	○	○
applying knowledge of the statistical process to analyze data and make inferences about a population.	○	○	○
using tools to generate a random sample that represents the population.	○	○	○
creating and interpreting data displays, such as bar graphs, dot plots and circle graphs, including part-to-part and part-to-whole comparisons.	○	○	○
generating random samples for two populations to identify their similarities and differences.	○	○	○
comparing and drawing informal inferences about two populations based on their measures of center and measures of variation. (mean, median, range, interquartile range).	○	○	○

continued on the next page

TOPIC 3 SELF-REFLECTION *continued*

TOPIC 3: *Drawing Inferences*	Beginning of Topic	Middle of Topic	End of Topic
comparing two numerical data distributions on a graph, such as a comparative dot plot, stem-and-leaf plot, histogram, or box plot, by visually comparing the shapes, centers, and spreads of the data displays.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Drawing Inferences* topic.

____

____

____

____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

____

____

____

____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

____

____

____

____

TOPIC 3 SUMMARY

Drawing Inferences Summary

NEW KEY TERMS

- survey
- data [datos]
- population [población]
- census [censo]
- sample
- parameter [parámetro]
- statistic [estadistica]
- random sample

LESSON 1 Collecting Random Samples

There are four components of the statistical process:

- Formulating a statistical question.
- Collecting appropriate data.
- Analyzing the data graphically and numerically.
- Interpreting the results of the analysis.

One data collection strategy you can use is a survey. A **survey** is a method of collecting information about a certain group of people. It involves asking a question or a set of questions to those people. When information is collected, the facts or numbers gathered are called **data**.

The **population** is the entire set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects. A **census** is the data collected from every member of a population.

Often, it is not possible or logical to collect data from every member of the population. When data are collected from a part of the population, the data are called a **sample**.

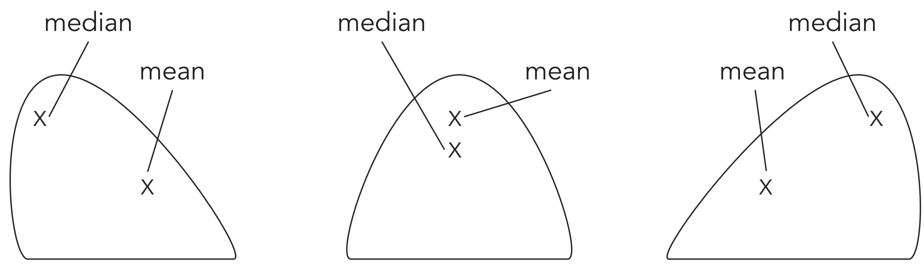
When data are gathered from a population, the characteristic used to describe the population is called a **parameter**. When data are gathered from a sample, the characteristic used to describe the sample is called a **statistic**. A statistic is used to make an estimate about the parameter.

When information is collected from a sample to describe a characteristic about the population, it is important that such a sample be as representative of the population as possible. A **random sample** is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

LESSON 2 Using Random Samples to Draw Inferences

When you are estimating a parameter that is a count, rather than a percentage, you can use proportional reasoning to scale up from the ratio of the number of observations in your sample to the statistic.

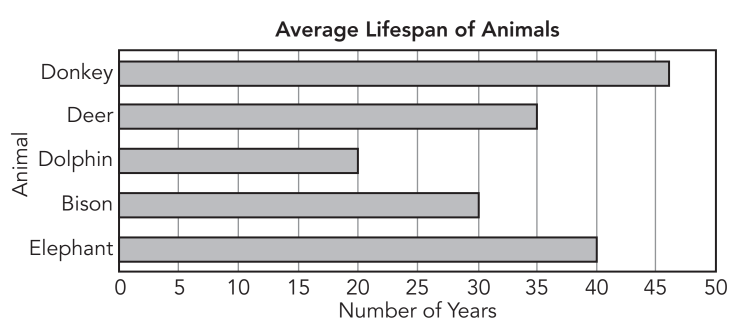
There are three common distributions of data: skewed left, skewed right, and symmetric. The distribution of data can help you determine whether the mean or median is a better measure of center.



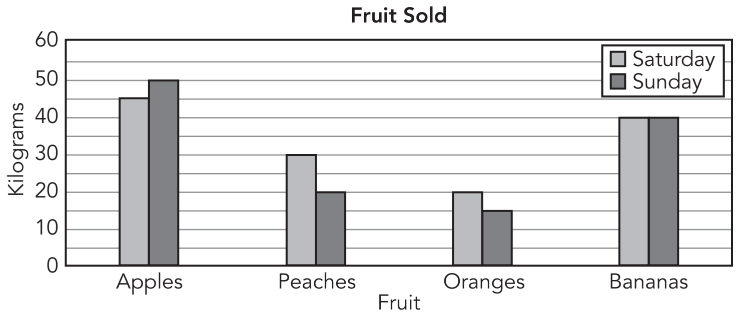
Skewed Right	Symmetric	Skewed Left
The mean of a data set is greater than the median when the data are skewed to the right.	The mean and median are equal when the data are symmetric.	The mean of a data set is less than the median when the data are skewed to the left.
The median is the better measure of center because the median is not affected by very large data values.		The median is the better measure of center because the median is not affected by very small data values.

LESSON 3 Bar Graphs

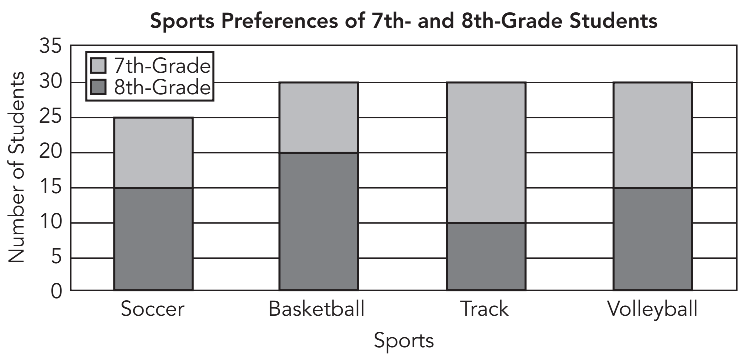
Some graphs are used to display data that consists of different categories. A bar graph displays data using horizontal or vertical bars, so that the height or length of the bars indicates its value for a specific category.



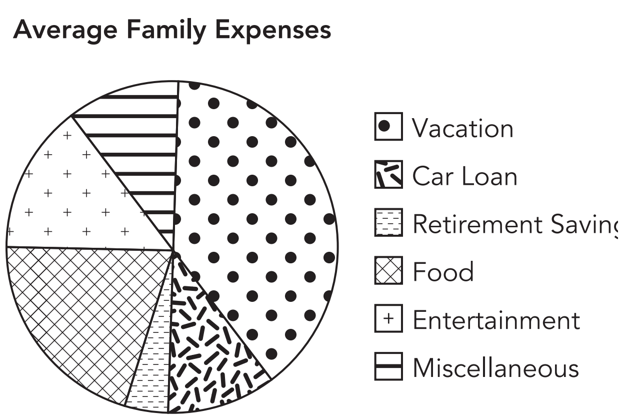
A *double bar graph* can be used when each category contains two different groups of data. The bars may be vertical or horizontal, and a key explains the colors or patterns for each group. The two bars representing the same category are side by side, and space is used to separate the categories.



A *stacked bar graph* is a graph that stacks the frequencies of two different groups for a given category on top of one another, so that you can compare the parts to the whole. Each bar represents a total for the whole category but still shows the data for each group within the entire category.



You can also use *circle graphs* to represent the relationship between each part and the whole. For example, this circle graph shows the expenses for families in random cities across the state. As shown in the graph, families tend to spend the greatest amount of money on vacation and the least amount of money on retirement savings.



LESSON 4 Comparing Two Populations

Data for two populations may overlap. Comparing the measures of center and measures of variation for the two populations can help you interpret the differences between the two populations.

Comparing the difference of means with the variation in each data set can be an important way of determining just how different two data sets are.

For example, consider these data sets.

5, 3, 4, 5, 10	5, 3, 100, 5, 10
Mean = 5.4	Mean = 24.6

The difference in their means is 19.2. Depending on what you are measuring, this could be a big difference. You can create a comparative dot plot to visually represent and observe the difference in the data sets. Upon investigating the values in each data set, it is apparent that the data sets overlap. The right data set is the same as the left one, with the exception of one number. The data value 100 in the right data set is skewing the data to the right and causing the difference of 19.2 between the means of the two data sets.

LESSON 5 Using Random Samples from Two Populations to Draw Conclusions

You can use the means to compare two populations with approximately symmetric data sets. You can use the medians and the interquartile ranges to compare two populations with skewed data sets.

For example, suppose Antonio wanted to compare the number of text messages sent each day by seventh-grade students to the number of text messages sent each day by eighth-grade students. He randomly selected and surveyed 10 students from each grade. The data sets provided represent the number of text messages each student said they sent per day.

Grade 7: 44, 40, 43, 73, 74, 76, 56, 59, 66, 54

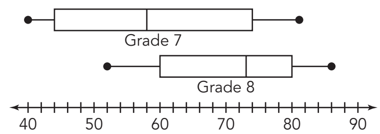
Grade 8: 77, 86, 69, 52, 76, 79, 60, 61, 57, 85

Antonio calculated the mean of each data set. He determined that the mean number of texts sent per day is 58.5 for seventh-graders and 70.2 for eighth-graders.

Antonio created a back-to-back stem-and-leaf plot as well as a box plot, to display the data.

Grade 7 Leaf	Stem	Grade 8 Leaf
4 3 0	4	
9 6 4	5	2 7
6	6	0 1 9
6 4 3	7	6 7 9
	8	5 6

Box Plot



Antonio used his data displays to determine the five-number summary and IQR for each data set.

	Grade 7	Grade 8
Minimum	40	52
Q1:	44	60
Median	57.5	72.5
Q3:	73	79
Maximum	76	86
IQR	29	19

Antonio compared and analyzed the data for the two populations. He used the measures of center to conclude that on average, eighth-graders sent a greater number of texts per day, and he used the measures of variation to conclude that seventh-graders had more variation in the number of texts sent per day.

GRADE 7

MODULE
5

Constructing and Measuring

TOPIC 1 Angle Relationships **891**

TOPIC 2 Area, Surface Area, and Volume **947**

GRADE 7

TOPIC 1

A jib is a triangular sail that sits ahead of the foremast of a sailing ship. Its most crucial function is as an airfoil, increasing performance and overall stability by reducing turbulence on the leeward side of the mainsail.

Angle Relationships

LESSON 1 Solving Equations Using the Triangle Sum Theorem **893**

LESSON 2 Relationships Between 90° and 180° Angles **905**

LESSON 3 Special Angle Relationships **921**

1 Solving Equations Using the Triangle Sum Theorem

OBJECTIVES

- Use facts about isosceles triangles to write and solve equations for unknown angles.
- Apply the Triangle Sum theorem to determine unknown values or angle measures in a triangle.
- Apply the formula for the perimeter of a geometric figure to determine unknown values or side lengths.

NEW KEY TERMS

- base angles
- Base Angles theorem
- congruent sides
- congruent angles

You already know a lot about triangles. In previous grades, you classified triangles by side lengths and angle measures.

How can you apply the Triangle Sum theorem to write and solve equations?

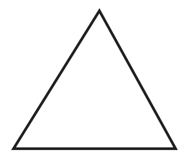
Getting Started

Remember:
An isosceles triangle is a triangle with two sides that are the same length. An equilateral triangle is a triangle with three angles that are the same measure.

Triangles, Triangles, Triangles

You have previously studied triangles. Use what you know about triangles to answer the questions.

1. Triangle *ABC* is an isosceles triangle. The measure of angle *B* is 58° and the measure of angle *C* is 64°. Determine the measure of angle *A*.



Triangle *DEF* is an equilateral triangle.

2. What do you notice about the measure of $\angle A$ and the measure of $\angle B$?

3. All three angles in $\triangle DEF$ have the same measure. What is the measure of each angle in the triangle? Explain your reasoning.

Remember:
A right triangle has one angle that measures 90°. An obtuse triangle has one obtuse angle.

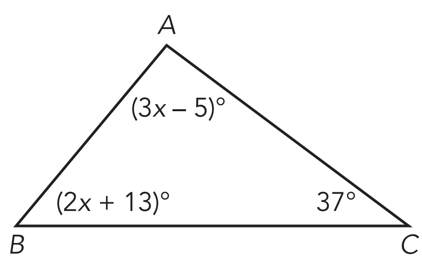
4. How many right angles can a triangle have? Explain your reasoning.

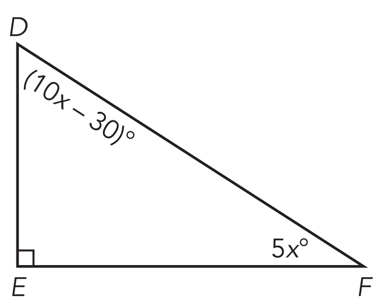
5. Can two angles in a right triangle have the same measure? Explain your reasoning.

ACTIVITY
1.1 Applying the Triangle Sum Theorem

You have learned about the Triangle Sum theorem, which states that the sum of the measures of the interior angles of a triangle is 180°. You can use that information to write equations and solve for an unknown value.

Write and solve an equation to determine the measure of each unknown angle.

1. 

2. 

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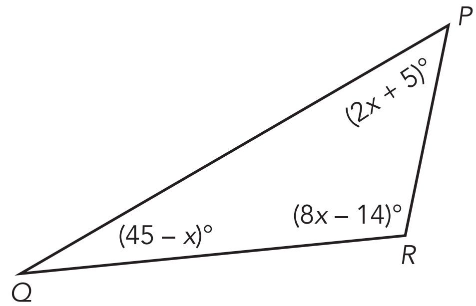

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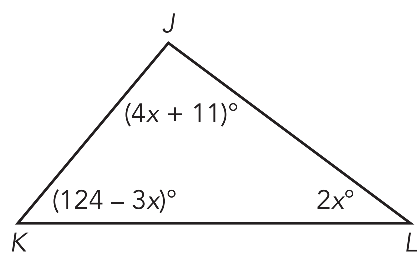
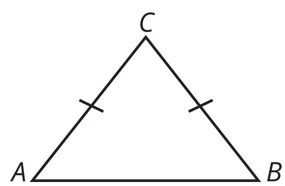


ACTIVITY 1.2 Isosceles Triangles

An isosceles triangle is a triangle with two sides that are the same length. The angles opposite these sides are called **base angles**. The **Base Angles theorem** states that the angles opposite the congruent sides of an isosceles triangle are congruent. **Congruent sides** are sides that have the same length. **Congruent angles** are angles that have the same measure.

The two sides of the triangle are each marked with one mark. Since the sides have the same marks, the sides are congruent.



1. Which sides in $\triangle ABC$ have the same length?

Remember:
$m\angle A$ is a symbol. The notation means the measure of angle A.

2. Which angles in the $\triangle ABC$ have the same measure?

3. Write and solve equation to determine the value of x when $m\angle A = (2x + 7)°$ and $m\angle B = 63°$.

4. $\triangle DEF$ is an isosceles triangle. $m\angle D = m\angle E$, $m\angle E = (4x - 11)°$, and $m\angle F = 98°$. Write and solve an equation to determine the value of x.

5. a. The two base angles of an isosceles triangle each measure $(5z + 19)°$. The third angle of the triangle measures 122°. Write and solve an equation to determine the value of z.

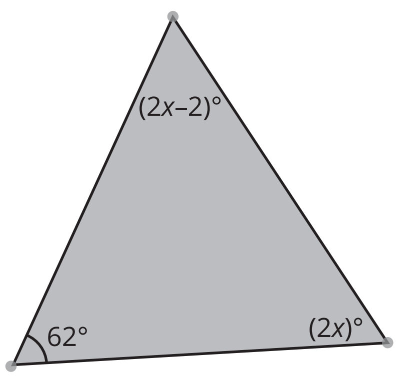
   b. What is the measure of each base angle in the triangle?

Talk the Talk

Determining Unknowns

1. An isosceles triangle has base angles that measure 58°. Write an equation to determine the measure of the third angle.

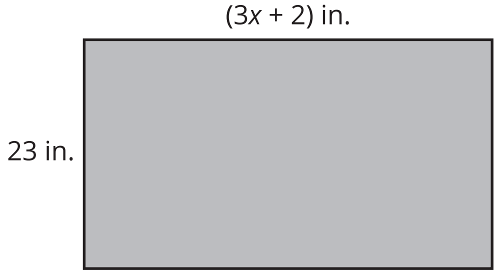
2. The angle measures of a triangle are shown in the diagram.



Write an equation to determine the value of x. Then, solve for x.

Remember:
Perimeter is the distance around a figure.

3. The perimeter of the rectangle shown is 128 inches.



Write an equation to determine the value of x. Then, solve for x.

Lesson 1 Assignment

Write

In your own words, explain the Base Angles theorem.

Remember

The sum of the measures of the interior angles of a triangle is 180°.

You can use what you know about solving equations and the Triangle Sum theorem to determine unknown values for the interior angles of a triangle.

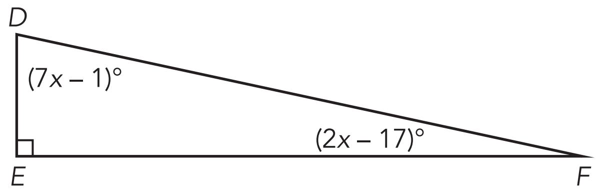
Practice

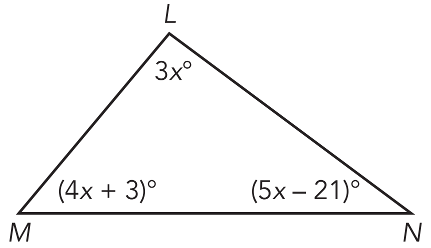
1. Write and solve an equation to determine the measure of the third angle when the base angle of an isosceles triangle is 52°.

2. Write and solve an equation to determine the measure of the base angles of an isosceles triangle when the non-base angle is 88°.

Lesson 1 Assignment

3. Determine the measure of each unknown angle.

a. 

b. 

Lesson 1 Assignment

4. The perimeter of a rectangle is 58 inches. The rectangle has a width of 15 inches and a length of $3.5x$. Write an equation and solve for the value of x.

Prepare

1. $90 = 5x + x$

2. $x + 3x = 180$

3. $(180 - x) + (90 - x) = 210$

2 Relationships Between 90° and 180° Angles

OBJECTIVES

- Calculate the supplement of an angle.
- Calculate the complement of an angle.
- Use facts about supplementary and complementary angles to write and solve simple equations for unknown angles.

NEW KEY TERMS

- straight angle
- supplementary angles
- complementary angles
- perpendicular
- collinear

You know how to write and solve equations using the Triangle Sum theorem. Now consider other angle relationships.

What types of special relationships exist between 90° and 180° angles?

Getting Started

Splitting Up

A **straight angle** is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

Let's use a straightedge and a protractor to investigate some special relationships between angles.

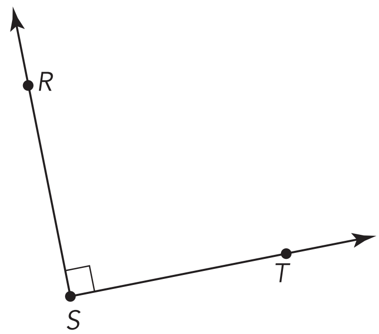
1. Draw a *straight angle*, $\angle ABC$, and label each point.

   a. Draw a point *D* above line *AC* and use your straightedge to draw a ray, $\overrightarrow{BD}$. Next, use your protractor to measure the angles formed by $\overrightarrow{BD}$ and $\overleftrightarrow{AC}$.

   b. Draw a point *E* different from point *D* below line *AC* and use your straightedge to draw another ray, $\overrightarrow{BE}$. Next, use your protractor to measure the angles formed by $\overrightarrow{BE}$ and $\overleftrightarrow{AC}$.

   c. What do you notice about the measures of the angles formed by a single ray coming off of a line?

2. Use the right angle shown to answer each question.



a. Draw a point V in the interior of $\angle RST$ and use your straightedge to draw a ray, $\overrightarrow{SV}$. Next, use your protractor to measure the angles formed by $\overrightarrow{SV}$ and $\angle RST$.

b. Draw a point Z in the interior of $\angle RST$ and use your straightedge to draw another ray, $\overrightarrow{SZ}$. Next, use your protractor to measure the angles formed by $\overrightarrow{SZ}$ and $\angle RST$.

c. What do you notice about the measures of the angles formed by a single ray that divides a right angle into two angles?

ACTIVITY
2.1 Supplements and Complements

Two angles are **supplementary angles** if the sum of their angle measures is equal to 180°.

Two angles are **complementary angles** if the sum of their angle measures is equal to 90°.

In the previous activity, you created *supplementary angles* and *complementary angles*.

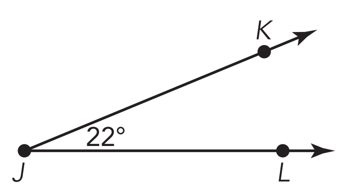
Let's create sets of supplementary angles.

1. Use a protractor to draw a pair of supplementary angles that share a side. What is the measure of each angle?

2. Use a protractor to draw a pair of supplementary angles that do not share a side. What is the measure of each angle?

To "draw" means you can use your measurement tools . . . so, get out your protractor and straightedge.

3. Calculate the measure of an angle that is supplementary to $\angle KJL$.

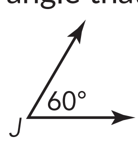


Now, let's create sets of complementary angles.

4. Use a protractor to draw a pair of complementary angles that share a side. What is the measure of each angle?

5. Use a protractor to draw a pair of complementary angles that do not share a side. What is the measure of each angle?

6. Calculate the measure of an angle that is complementary to $\angle J$.



7. Given each statement, write and solve an equation to determine the measure of each angle in the angle pair.

   a. Two angles are both congruent and supplementary.

   b. Two angles are both congruent and complementary.

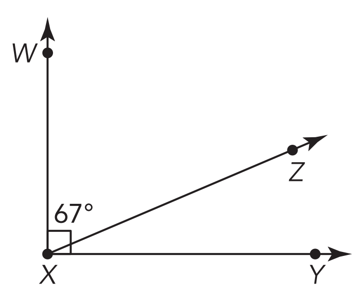
   c. The supplement of an angle is half the measure of the angle itself.

   d. The supplement of an angle is 20° more than the measure of the angle itself.

e. Angles 1 and 2 are complementary. The measure of Angle 2 is 10° larger than the measure of Angle 1.

f. Angles 1 and 2 are supplementary. The measure of Angle 1 is three degrees less than twice the measure of Angle 2.

8. Use the figure to determine the measure of ∠ZXY.



ACTIVITY 2.2
Perpendicular Lines

Two lines, line segments, or rays are **perpendicular** if they intersect to form 90° angles. The symbol for perpendicular is $\perp$.

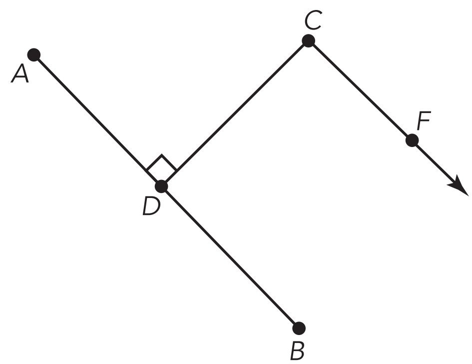
Let's explore angles formed by *perpendicular* lines.

1. Draw and label $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point *E*. How many right angles are formed?

2. Draw and label $\overleftrightarrow{BC} \perp \overleftrightarrow{AB}$ at point *B*. How many right angles are formed?

Compare your drawings with your partner's drawings. What do you notice?

3. Name all angles that you know are right angles in the figure shown. Note: Points *A*, *D*, and *B* lie on the same line segment.



When points lie on the same line or line segment, they are said to be **collinear**.

You can use patty paper to create special angle pairs, just as you can use a protractor and straightedge.

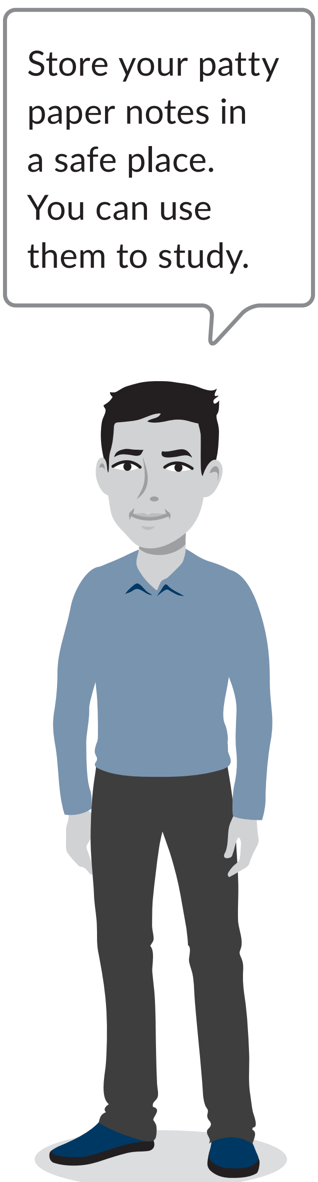
4. Draw a figure on a sheet of patty paper. What do you notice when you write on the paper? What about when you fold the paper?

Patty paper is traditionally a food service product, used to separate hamburger patties, meats, cheeses, and pastries.

Read each question in its entirety before using your patty paper to illustrate the angles. You should use one sheet of patty paper for each question.

5. Draw a straight angle on your patty paper. Then *fold* your patty paper to create a pair of supplementary angles that are not congruent. Label the patty paper "Supplementary Angles."

6. Draw a straight angle on your patty paper. Then *fold* your patty paper to create a pair of supplementary angles that are congruent. What do you know about the angles? What do you know about the straight angle and the line you created with your fold? Label the patty paper accordingly.



7. Draw a right angle on your patty paper. Then *fold* your patty paper to create a pair of complementary angles that are not congruent. Label the patty paper "Complementary Angles."

Talk the Talk

More Than One Way

Angle Q and angle J are complementary angles. The measure of angle Q is 32°. The measure of angle J is $(3.5x + 2)°$.

1. Use what you know about complementary angles to write and solve an equation to determine the value of x and the measure of angle J.

2. Is there another equation that you can write and solve to determine the value of x?

Lesson 2 Assignment

Write

Draw an example of each term. Provide an explanation when necessary.

1. supplementary angles
2. complementary angles
3. perpendicular lines

Remember

Two angles are *supplementary angles* when the sum of their angle measures is equal to 180°. Two angles are *complementary angles* when the sum of their angle measures is equal to 90°.

Practice

1. Draw a pair of supplementary angles. One of the angles should have a measure of 85°. Label each angle with its measure.

2. Draw a pair of complementary angles. One of the angles should have a measure of 62°. Label each angle with its measure.

Lesson 2 Assignment

Write and solve an equation to determine the measure of each unknown angle.

3. Angles *C* and *D* are complementary. When $m\angle D$ is 25° is greater than $m\angle C$, what is the measure of each angle?

4. When the supplement of an angle is 30 degrees more than the measure of the angle, what is the measure of the angle?

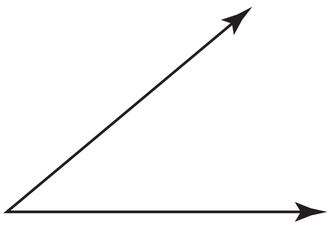
5. When the supplement of an angle is 12 degrees less than twice the measure of the angle, what is the measure of the angle?

Lesson 2 Assignment

Prepare

Classify each angle.

1.



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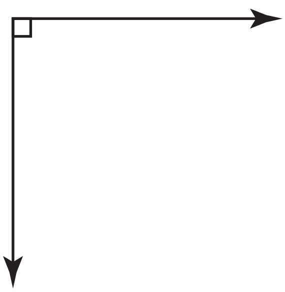

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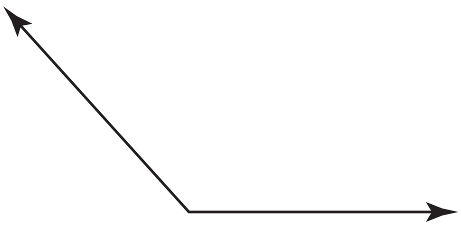


3 Special Angle Relationships

OBJECTIVES

- Classify adjacent angles, linear pairs, and vertical angles.
- Use facts about adjacent angles, linear pairs, and vertical angles in multi-step problems to write and solve simple equations for unknown angles.

NEW KEY TERMS

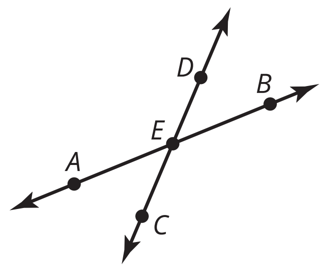
- adjacent angles
- linear pair
- vertical angles

You know how to classify individual angles based on their measure.

What types of special relationships exist between pairs of angles?

Getting Started

Name Those Angles!



1. How many different angles are formed by the intersection of $\overline{AB}$ and $\overline{CD}$?

2. Name all of the angles less than 180° formed by the intersection of $\overline{AB}$ and $\overline{CD}$.

3. Describe the location of all of the angles equal to 180° formed by the intersection of $\overline{AB}$ and $\overline{CD}$.

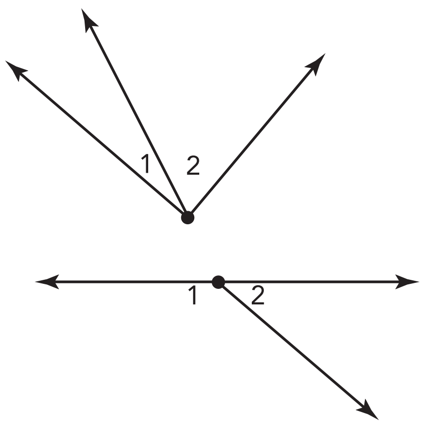
ACTIVITY 3.1

Adjacent Angles

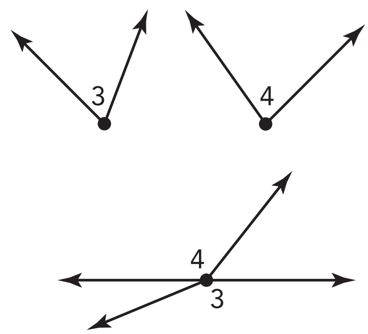
In each of the next three activities you will explore special angle pairs.

WORKED EXAMPLE

$\angle 1$ and $\angle 2$ are adjacent angles.

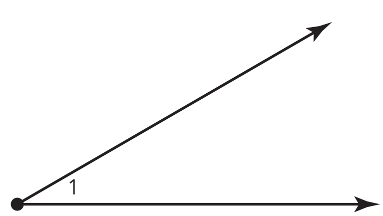


$\angle 3$ and $\angle 4$ are *not* adjacent angles.

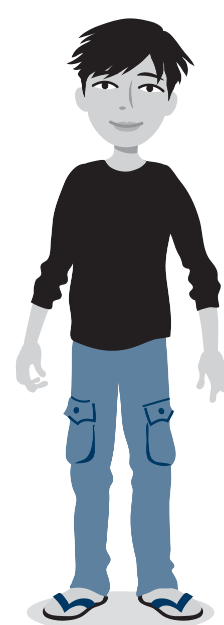


1. Use the figures in the Worked Example to define *adjacent angles* in your own words.

2. Draw and label $\angle 2$ so that it is adjacent to $\angle 1$.



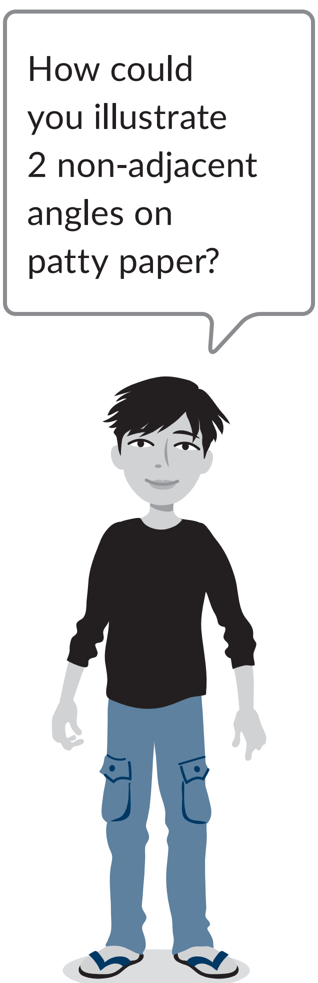
Can you think of other ways to draw $\angle 2$ so that it is adjacent to $\angle 1$?



Adjacent angles are two angles that share a common vertex and share a common side.

3. Is it possible to draw two angles that share a vertex but do not share a common side? If so, draw an example. Would the angles be adjacent angles? If not, explain your reasoning.

4. Is it possible to draw two angles that share a side but do not share a vertex? If so, draw an example. If not, explain your reasoning.



5. Use one sheet of patty paper to create a set of *congruent* adjacent angles. Describe your process. Label the patty paper "Congruent Adjacent Angles."

6. Use one sheet of patty paper to create a set of *non-congruent* adjacent angles. Describe your process. Label the patty paper "Adjacent Angles."

7. Are all adjacent angles supplementary? Explain your reasoning.

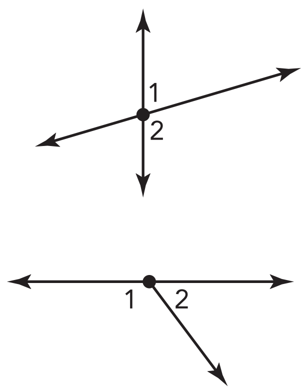
8. Are all supplementary angles adjacent? Explain your reasoning.

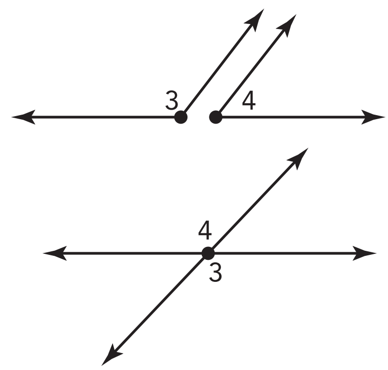
ACTIVITY 3.2
Linear Pairs

Let's explore a different angle relationship.

WORKED EXAMPLE

∠1 and ∠2 form a linear pair. ∠3 and ∠4 are do *not* form a linear pair.

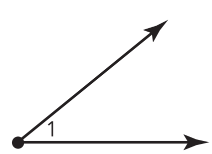




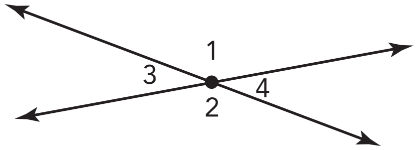
1. Use the figures in the Worked Example to define a *linear pair of angles* in your own words.

A **linear pair** of angles is formed by two adjacent angles that have noncommon sides that form a line.

2. Draw ∠2 so that it forms a linear pair with ∠1. Use one sheet of patty paper and record your response. Label your patty paper "Linear Pair."



3. Name all linear pairs in the figure shown.

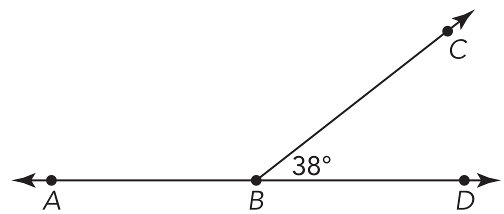


4. If the angles that form a linear pair are congruent, what can you conclude?

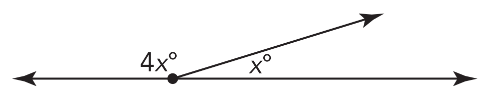
5. What is the difference between a linear pair of angles and supplementary angles that share a common side?

6. What is the difference between a linear pair of angles and supplementary angles that do not share a common side?

7. Angle *ABC* and angle *CBD* form a linear pair. Write and solve an equation to determine the measure of $\angle ABC$.



8. The angles shown are a linear pair of angles. Solve for *x*.



9. Write and complete the sentence on your Linear Pairs patty paper.

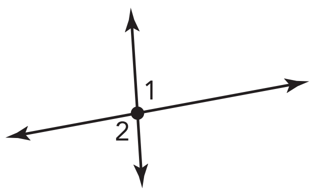
If two angles form a linear pair, then the sum of the measures of the linear pair of angles is ______________________.

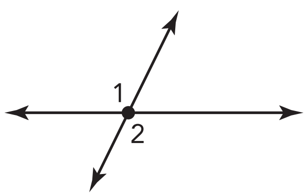
ACTIVITY
3.3 Vertical Angles

Let's explore one more special angle relationship.

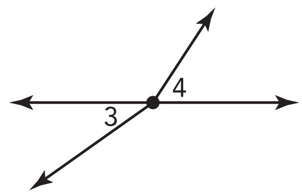
WORKED EXAMPLE

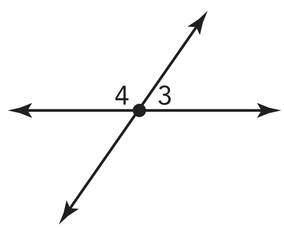
∠1 and ∠2 are vertical angles.





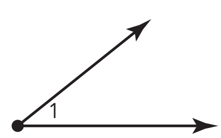
∠3 and ∠4 are *not* vertical angles.





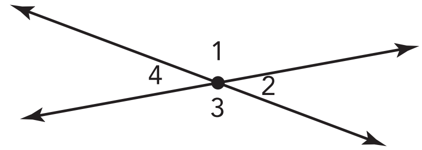
1. Use the figures in the Worked Example to define *vertical angles* in your own words.

2. Draw ∠2 so that it forms a vertical angle pair with ∠1.



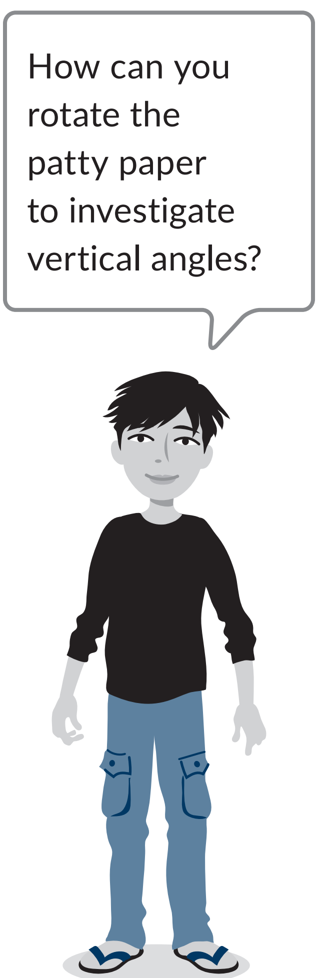
Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

3. Name all vertical angle pairs in the diagram shown.



4. Trace the figure in Question 3 on two different sheets of patty paper. Be sure you number the angles. Use the patty paper to investigate the measures of vertical angles. What do you notice?

5. Use your protractor to measure each angle in Question 3. What do you notice?



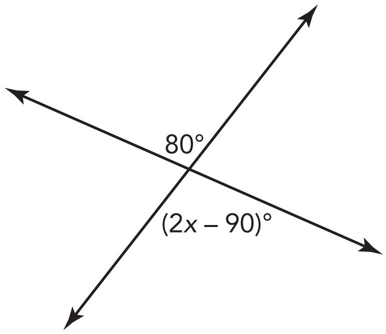
6. Use what you know about supplementary angles and linear pairs to justify your investigations in Questions 4 and 5.

7. Label one sheet of your patty paper "Vertical Angles" and write and complete the sentence.

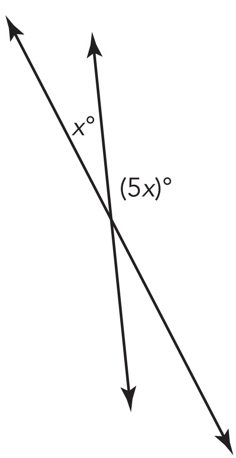
When two lines intersect to form vertical angles, each pair of vertical angles ______________________________.

8. Write and solve an equation to determine the measures of all four angles in each diagram.

a.



b.



Talk the Talk

Perfect Pairs

Answer each question. Draw a figure to justify your response.

1. Two intersecting lines form how many

   a. Pairs of supplementary angles?

   b. Pairs of complementary angles?

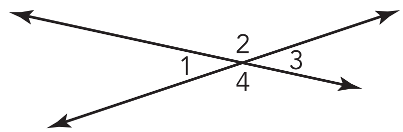
   c. Pairs of adjacent angles?

   d. Linear pairs of angles?

   e. Pairs of vertical angles?

2. Suppose two lines intersect. When you are given the measure of one angle, can you determine the measures of the remaining angles without using a protractor? Explain your reasoning.

3. When two lines intersect, four different angles are formed as shown.



   a. Describe the relationship between vertical angles.

   b. Describe the relationship between adjacent angles.

4. Draw and label a diagram that includes at least one of each relationship. Then, identify the angles that satisfy each description.

   - Complementary angles
   - Supplementary angles
   - Perpendicular lines
   - Adjacent angles
   - Linear pair
   - Vertical angles

Lesson 3 Assignment

Write

Draw an example of each term. Provide an explanation when necessary.

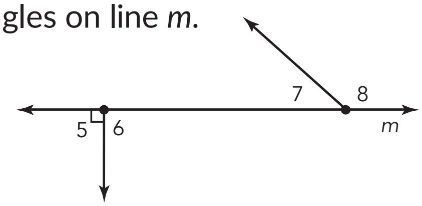
1. Adjacent angles
2. Linear pair
3. Vertical angles

Remember

Many geometric figures contain a mixture of special angle pairs. Understanding the relationships between special angle pairs will help you understand more complex geometric diagrams.

Practice

1. Use the diagram to identify the specified angles on line *m*.



   a. All adjacent angles

   b. All linear pairs

   c. All vertical angles

   d. All right angles

   e. All supplementary angles

Lesson 3 Assignment

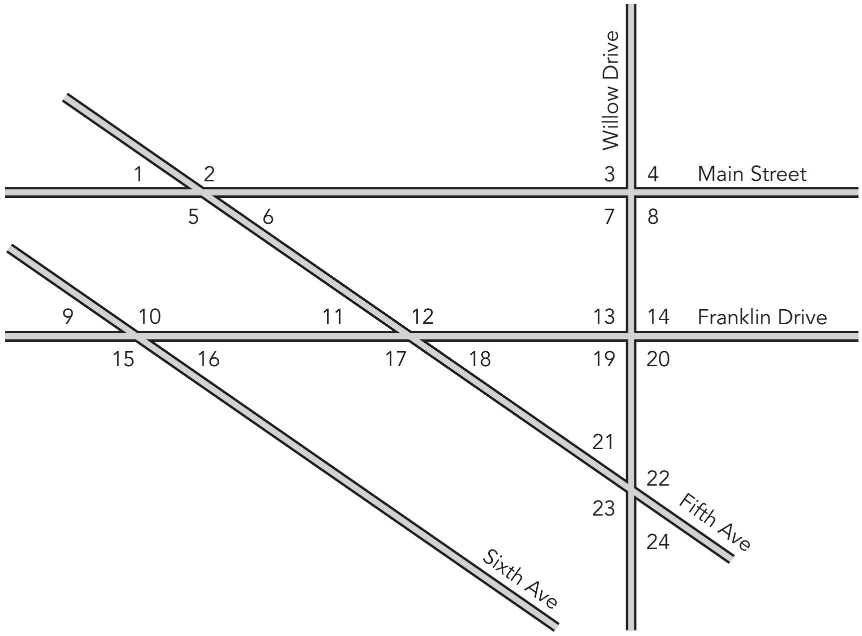
2. Draw the angle pairs described and answer each question. Label each angle with its measure.

   a. Draw a linear pair of angles. One of the angles should have a measure of 123°.

   b. Draw two angles with the same measures as those in part (a), such that they are not a linear pair.

   c. Explain the differences and similarities between linear pairs and supplementary angles.

3. Suppose each street in the map shown represents a line. Provide an example of each angle relationship.



a. Vertical angles

d. Adjacent angles

b. Supplementary angles

e. Congruent angles

c. Linear pair

Lesson 3 Assignment

Write and solve an equation to determine the measure of each unknown angle.

4. When two angles form a linear pair and the measure of the first angle is one-fifth the measure of the second angle, what is the measure of each angle?

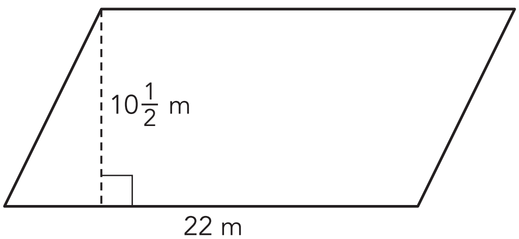
5. When two angles form a linear pair and the measure of the first angle is three times the measure of the second angle, what is the measure of each angle?

Lesson 3 Assignment

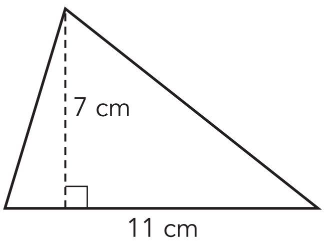
Prepare

Use a formula to determine the area of each figure.

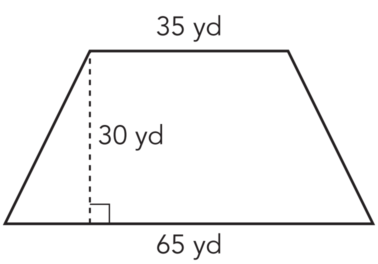
1.



2.



3.



TOPIC 1 SELF-REFLECTION

Name: ______________________

Angle Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Angle Relationships* topic by:

TOPIC 1: *Angle Relationships*	Beginning of Topic	Middle of Topic	End of Topic
determining unknown angle measures by writing and solving algebraic equations using the Triangle Sum theorem.	○	○	○
recognizing the special angle pairs formed when two lines intersect: supplementary angles, complementary angles, vertical angles, adjacent angles, and linear pairs.	○	○	○
determining unknown angle measures by writing and solving algebraic equations based on relationships between angles.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Angle Relationships* topic.

____

____

____

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

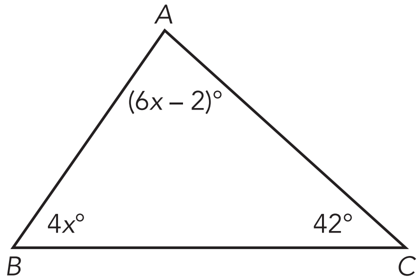
Angle Relationships Summary

LESSON 1 Solving Equations Using the Triangle Sum Theorem

You have learned about the Triangle Sum theorem, which states that the sum of the measures of the interior angles of a triangle is 180°. You can use this information, along with knowledge about angle relationships, to write equations and solve for an unknown value or unknown angle measure.

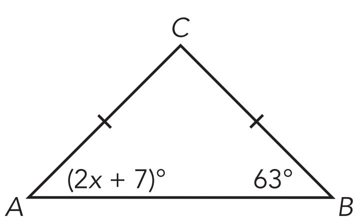
For example, write and solve an equation to determine the value of x. Then, determine each unknown angle measure.

$$(6x - 2) + 4x + 42 = 180$$
$$10x + 40 = 180$$
$$10x = 140$$
$$x = 14$$
$$m\angle B = 4(14) = 56°$$
$$m\angle A = 6(14) - 2 = 82°$$



An isosceles triangle is a triangle with two sides that are the same length. The angles opposite these sides are called **base angles**. The **Base Angles theorem** states that theangles opposite the congruent sides of an isosceles triangle are congruent. **Congruent sides** are sides that have the same length. **Congruent angles** are angles that have the same measure.

For example, △ABC is an isosceles triangle. You can write and solve an equation to determine the value of x. Since ∠A and ∠B are the base angles, their measures are the same.



$$2x + 7 = 63$$
$$2x = 56$$
$$x = 28$$

NEW KEY TERMS

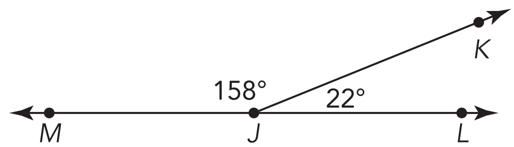
- base angles [ángulos de la base]
- Base Angles theorem [teorema de los ángulos de la base]
- congruent sides
- congruent angles [ángulos congruentes]
- straight angle
- collinear [colineal]
- supplementary angles [ángulos suplementarios]
- complementary angles [ángulos complementarios]
- perpendicular [perpendicular]
- adjacent angles [ángulos adyacentes]
- linear pair [par lineal]
- vertical angles [ángulos verticales]

LESSON 2 Relationships Between 90° and 180° Angles

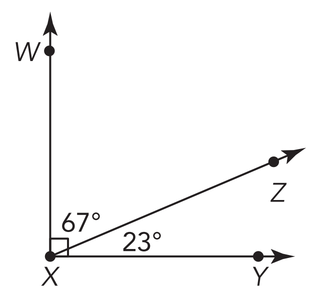
A **straight angle** is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

When points lie on the same line or line segment, they are said to be **collinear**. For example points *M*, *J*, and *L* are collinear.

Two angles are **supplementary angles** if the sum of their angle measures is equal to 180 degrees. For example, Angles *MJK* and *KJL* are supplementary angles.



Two angles are **complementary angles** if the sum of their angle measures is equal to 90 degrees. For example, Angles *WXZ* and *ZXY* are complementary angles.



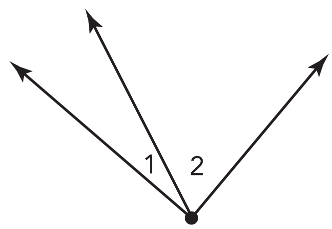
Two lines, line segments, or rays are **perpendicular** if they intersect to form 90 degree angles. The symbol for perpendicular is ⊥.

LESSON 3

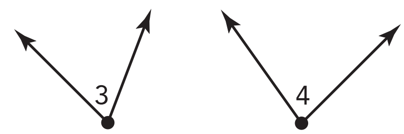
Special Angle Relationships

Adjacent angles are two angles that share a common vertex and a common side.

∠1 and ∠2 are adjacent angles.

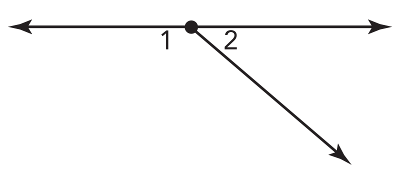


∠3 and ∠4 are *not* adjacent angles.

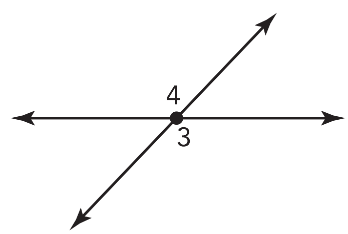


A **linear pair** of angles is formed by two adjacent angles that have noncommon sides that form a line. Linear pairs are supplementary.

∠1 and ∠2 form a linear pair.

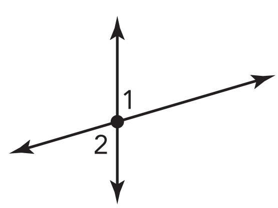


∠3 and ∠4 do *not* form a linear pair.

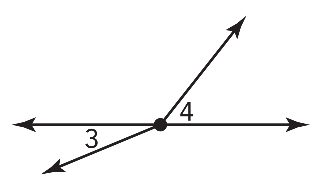


Vertical angles are two nonadjacent angles that are formed by two intersecting lines. Vertical angles are congruent.

∠1 and ∠2 are vertical angles.

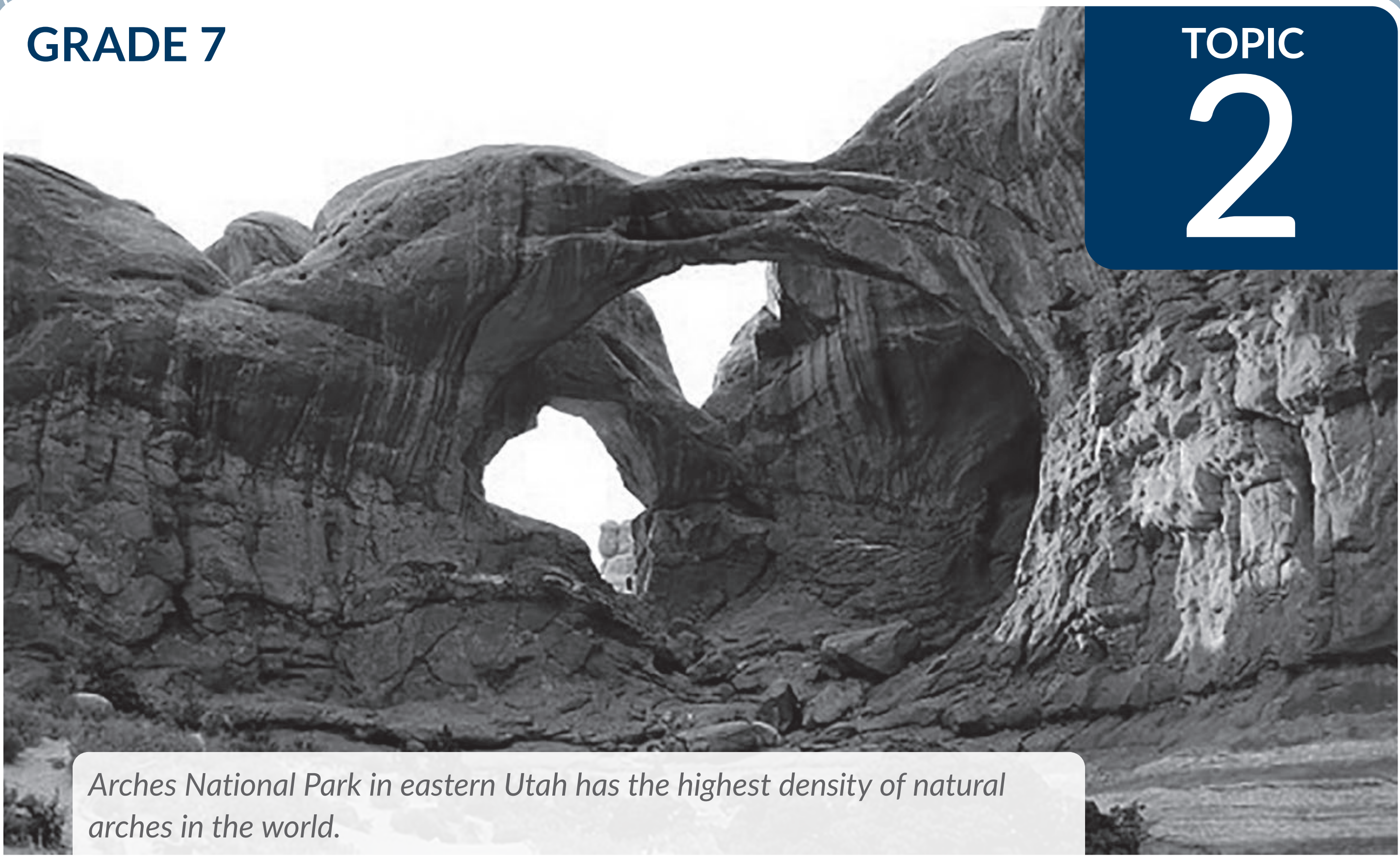


∠3 and ∠4 are *not* vertical angles.



GRADE 7

TOPIC 2



Arches National Park in eastern Utah has the highest density of natural arches in the world.

Area, Surface Area, and Volume

LESSON 1 Composite Figures **949**

LESSON 2 Total Surface Area of Prisms and Pyramids **959**

LESSON 3 Volume of Prisms and Pyramids **981**

LESSON 4 Volume and Surface Area Problems with Prisms and Pyramids **1001**

1 Composite Figures

OBJECTIVES

- Decompose composite geometric figures into rectangles, parallelograms, and/or triangles to determine their areas.
- Solve real-world problems by composing and decomposing shapes into triangles and rectangles.

You know how to calculate the area of triangles, rectangles, parallelograms, and trapezoids.

How can you use what you know about the areas of these shapes to determine areas of more complex shapes?

Getting Started

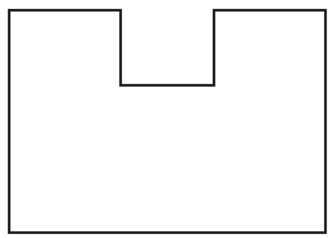
Remember:
A composite figure is a figure that is made up of more than one geometric figure.

Compose or Decompose?

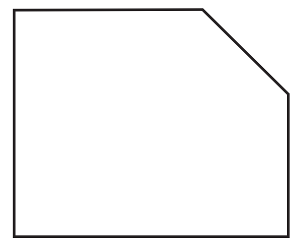
Consider each composite figure shown.

1. Show how you could decompose each figure into two or more familiar figures. Describe the shapes that make up the composite figure.

a.

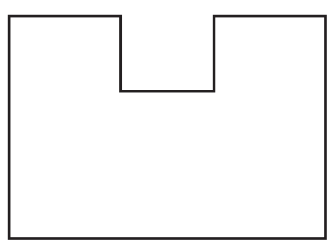


b.



2. Show how you could compose each figure into a larger, familiar figure. Describe the shapes that make up the composite figure.

a.



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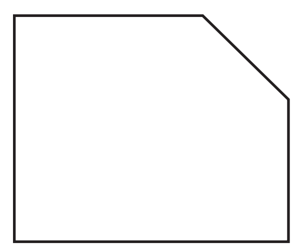
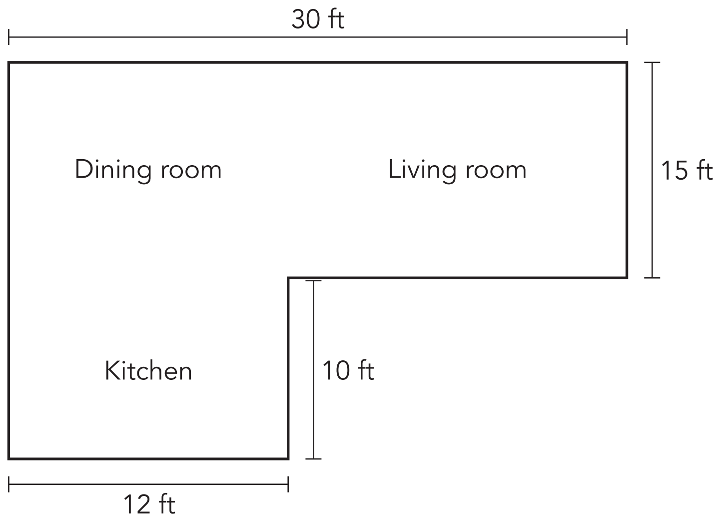


Consider the blueprint of a floor plan for a kitchen, dining room, and living room combination.



3. Suppose the homeowner wants to replace the floors in all the rooms.

a. Isabella says she can determine the total area of flooring needed by decomposing the blueprint into two rectangles. Ethan says he can determine the total area of flooring needed by composing the blueprint into one large rectangle and then subtracting the unused area. Who is correct?

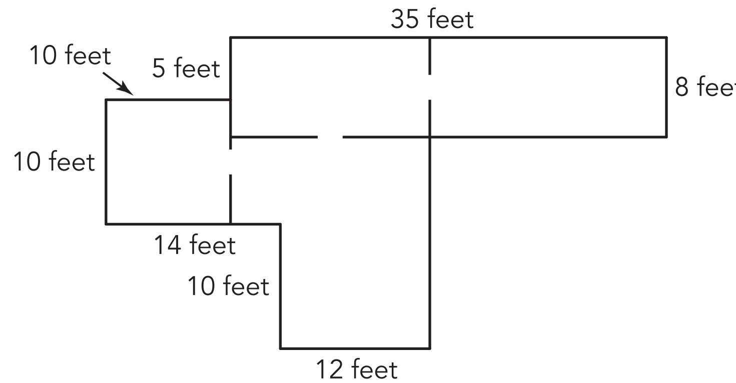
b. Show how to calculate the amount of flooring needed using Isabella's method.

c. Show how to calculate the amount of flooring needed using Ethan's method.

ACTIVITY 1.1 Area of Composite Figures

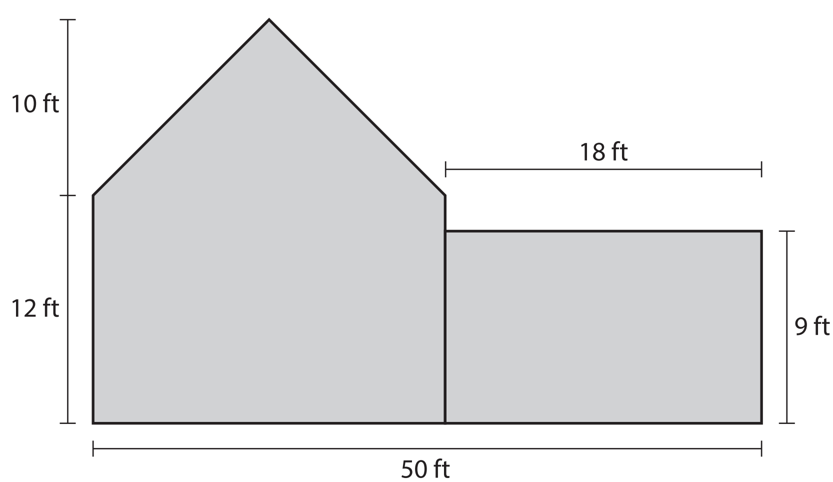
Solve each problem. Show your work.

1. Suppose that carpeting costs $1.20 per square foot. How much would it cost to carpet every room in this house, except the kitchen?

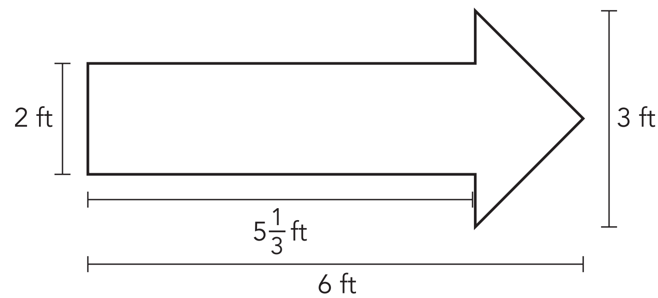


Be sure that the simpler figures you draw do not have overlapping areas.

2. Suppose a gallon of paint covers about 400 square feet. How much paint would you need to paint the entire back of this house?

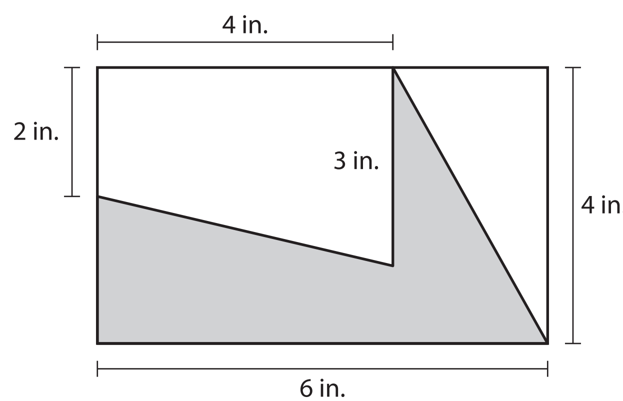


3. Samuel is spray painting an arrow on the side of a building to point to the entrance of his store. The can of gold spray paint he wants to use covers up to 12 square feet. Does Samuel have enough spray paint for his arrow?

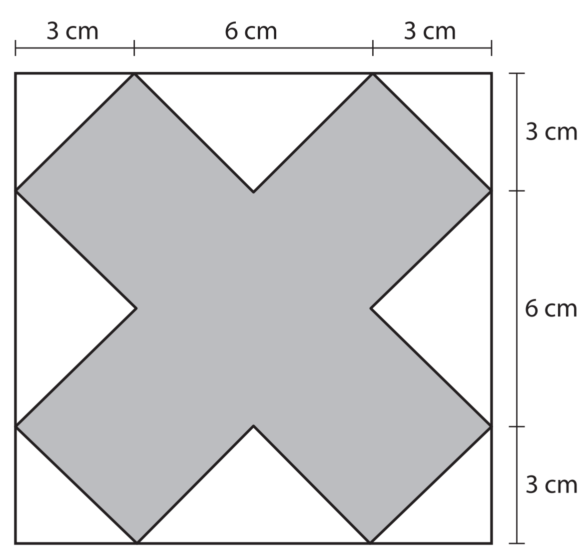


Determine the area of the shaded region in each figure. Show your work.

4. A right triangle and a trapezoid are drawn within a rectangle to create the shaded region in the figure.



5. Eight triangles are drawn within a square to create the shaded region in the figure.

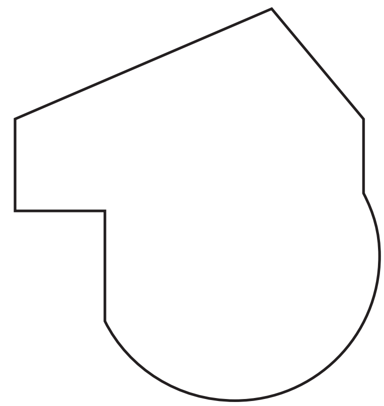


ACTIVITY **1.2**

Area of Complex Figures

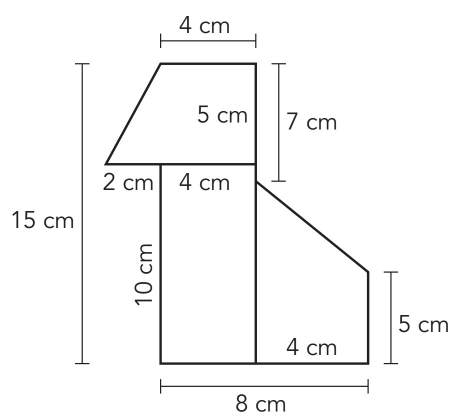
You can divide more complex figures with oddly-figured regions into smaller, familiar regions to calculate the approximate area.

1. Draw lines in the figure to divide the figure into smaller, familiar figures. Then, name the familiar figures that make up the total figure.

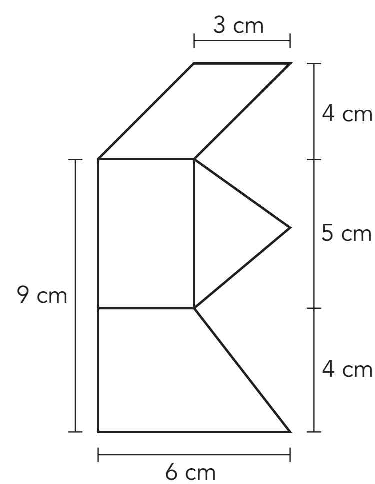


2. Determine the area of each complex figure.

a.



b.

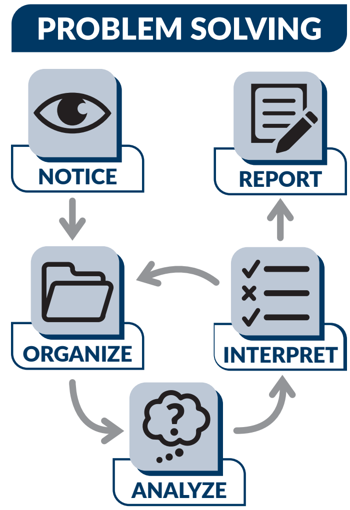


3. Estimate the area of France.

4. Estimate the area of Namibia.

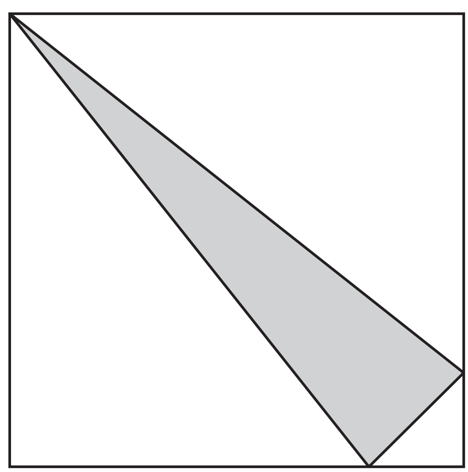
Talk the Talk

Use Your Powers of Mathematical Reasoning



Sometimes, a figure you are already familiar with is divided into smaller figures. You can use what you know about the areas of these figures to determine the area of a specific region.

1. Use a centimeter ruler to determine the measurements required to calculate the area of the shaded triangle inside the square. Then, determine the area. Explain your strategy.



Create a presentation of your solution strategy for the class.

Lesson 1 Assignment

Write

Define *composite figure* and draw an example.

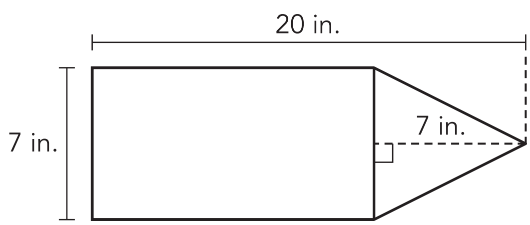
Remember

The area of a composite figure can be determined by decomposing it into familiar shapes and then adding the areas of those shapes together.

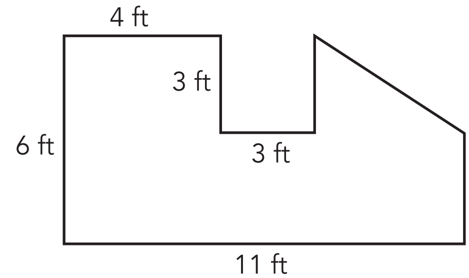
Practice

1. Calculate the area of the composite figure.

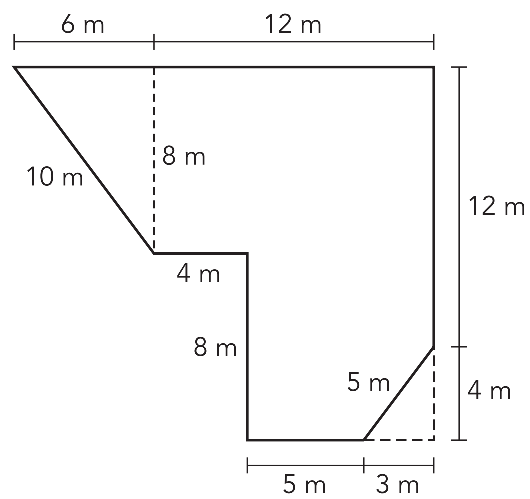
a.



b.

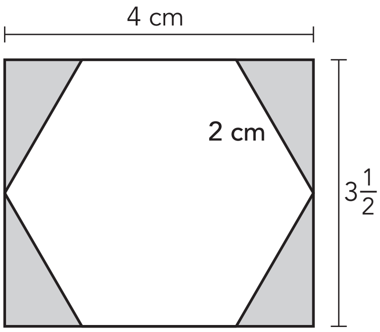


2. A city wants to create a garden according to the plan below. Calculate the area of the garden.

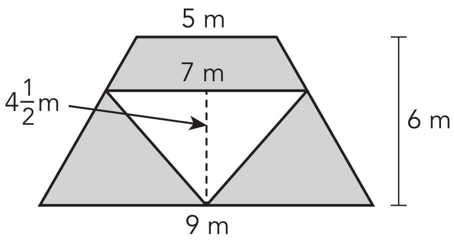


Lesson 1 Assignment

3. The figure shown is composed of a rectangle and a hexagon. The length of each side of the hexagon is 2 centimeters. Determine the area of the shaded region.



4. The figure shown is composed of a triangle within a trapezoid. Determine the area of the shaded region.



Prepare

Solve each equation.

1. $90 = 5x + x$

2. $x + 3x = 180$

3. $(180 - x) + (90 - x) = 210$

2 Total Surface Area of Prisms and Pyramids

OBJECTIVES

- Represent solid figures using two-dimensional nets made up of rectangles and triangles.
- Use nets of solid figures to determine the surface areas of the figures.
- Solve real-world and mathematical problems involving surface area.
- Fluently multiply and divide multi-digit decimals using standard algorithms.

NEW KEY TERMS

- net
- surface area
- pyramid
- slant height

You know how to determine how many cubic units fill a rectangular prism.

How can you calculate the number of square units it takes to cover the outside of a prism or pyramid?

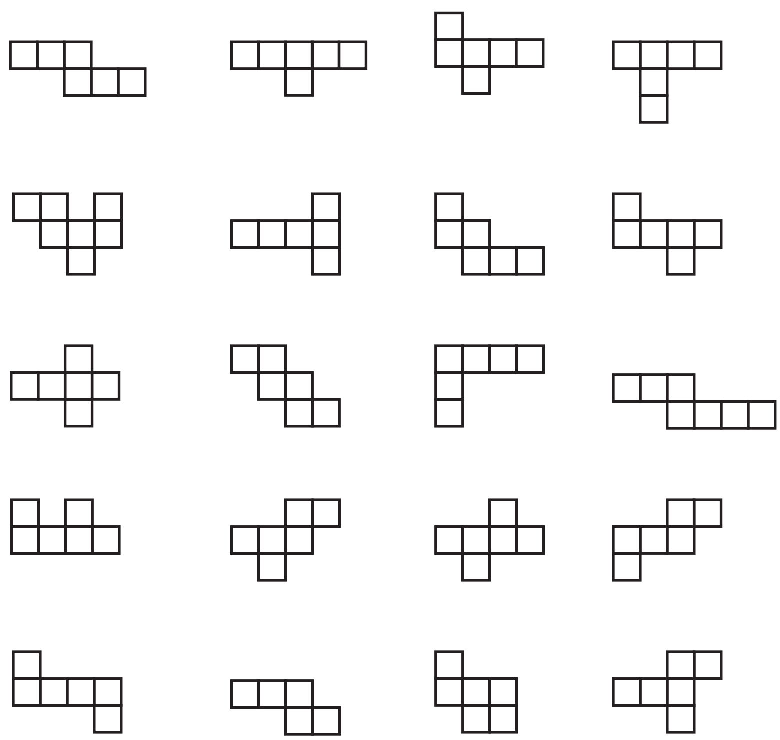
Getting Started

Breaking Down a Cube

A **net** is a two-dimensional representation of a three-dimensional geometric figure. A net is cut out, folded, and taped to create a model of a geometric solid.

1. Cut, fold, and tape the cube net found at the end of the lesson.

2. Are there other nets that form a cube? Circle the 11 cutouts that can form a cube.



3. How did you determine which are nets of cubes?

4. What do all of the nets for a cube have in common? Consider the number of faces, edges, and vertices in your explanation.

ACTIVITY 2.1

Nets of Rectangular Prisms

A net has all these properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn such that they share common edges.

The **surface area** of a polyhedron is the total area of all its two-dimensional faces.

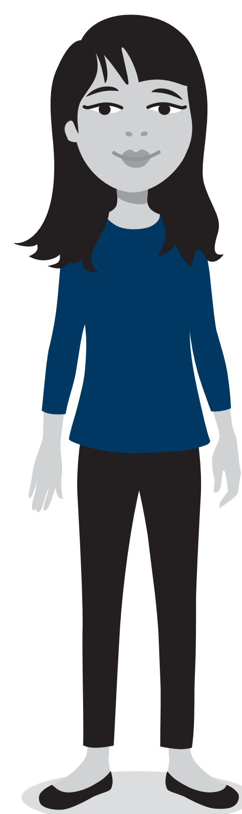
Consider the cube you created.

1. How is the area of a face of a cube measured? Analyze the two responses and explain why Harper is incorrect in her reasoning.

Harper
This is a 3D figure, which means that its measurements are cubic units.

Diego
Surface area is still measuring area, which is always measured in square units.

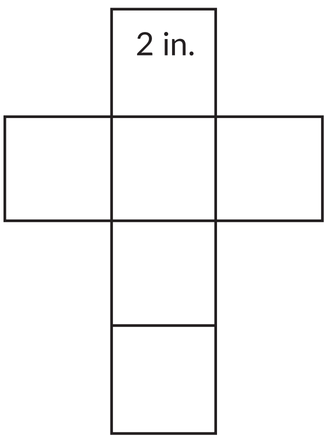
You can think about surface area as the total area covered by the net of the solid.



2. Describe a strategy that you can use to determine the surface area of a cube.

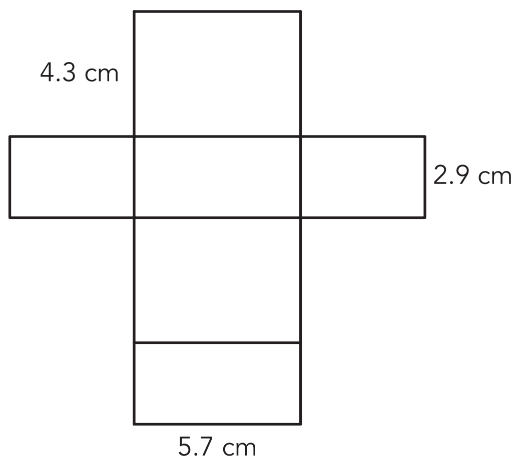
3. Use a ruler to determine the surface area of the cube you created.

4. Consider the cube net shown. Calculate the surface area.



5. Let's consider a different rectangular prism.

   a. Use the net to estimate the surface area of the right rectangular prism.

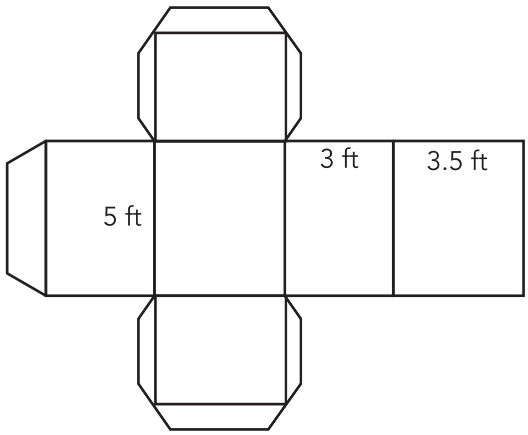


   b. Calculate the surface area of the right rectangular prism. Explain your calculation.

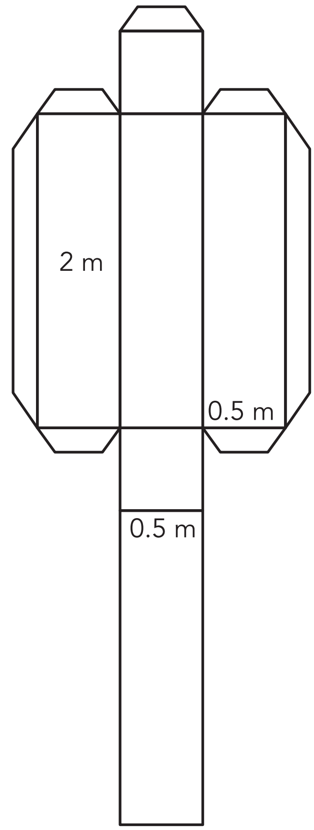
Review the definitions of **consider, describe, explain your calculation,** and related phrases in the Academic Glossary.

6. Calculate the surface area of the solid figure represented by each net.

a.

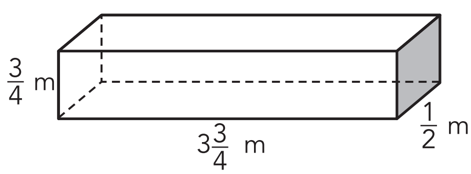


b.

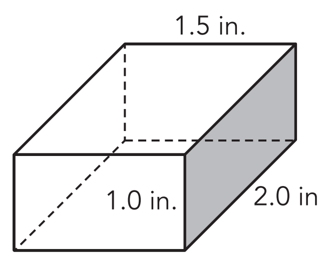


7. Draw a net to represent each solid figure. Label each net with measurements and then calculate the surface area of the solid figure.

a.

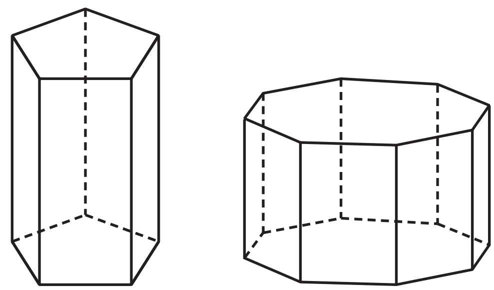


b.



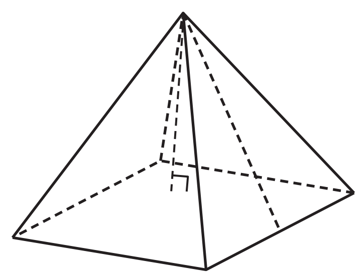
ACTIVITY
2.2 Prisms and Pyramids

The base of a prism does not have to be rectangular. The base of a prism can be a triangle, a pentagon, a hexagon, and so on.



A **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint of an edge of the base.

A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The vertex of a pyramid is the point at which all the triangular faces intersect.



1. Analyze the figures. Then, complete the table using the figures.

Figure *A*

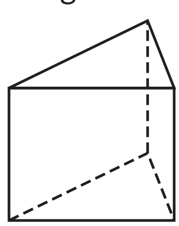


Figure *B*

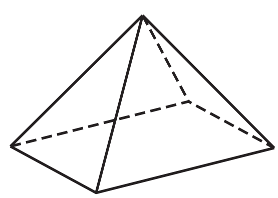


Figure *C*

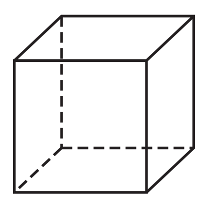


Figure *D*

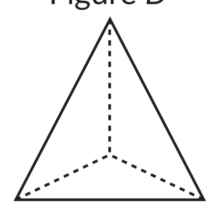
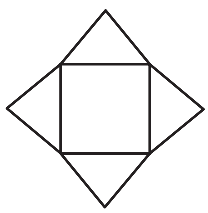


Figure	Is it a Prism or Pyramid?	Shape of Base	Number of Faces	Number of Vertices	Number of Edges
A					
B					
C					
D					

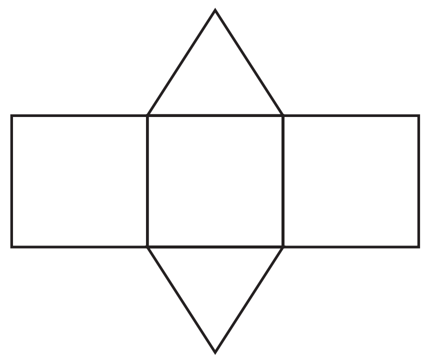
2. Write the names of Figures *A*, *B*, *C*, and *D* from your completed table.

3. Label each net with the name of the solid figure it forms.

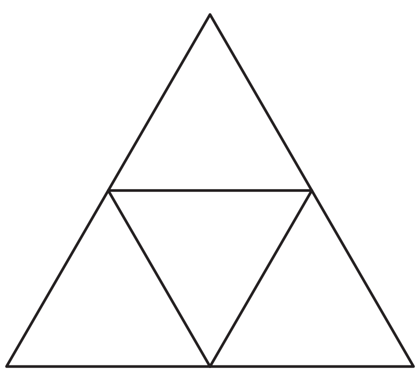
a.



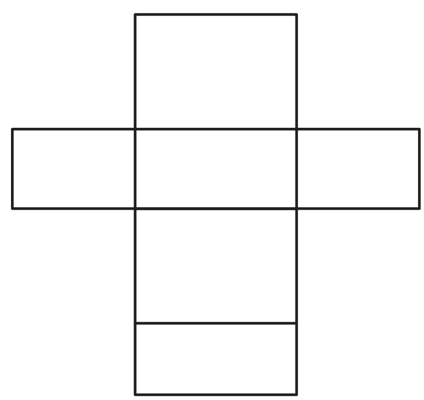
b.



c.



d.



ACTIVITY
2.3 Nets of Other Solids

1. Locate the nets for the triangular prism and triangular pyramid at the end of the lesson.

   a. Measure the edge lengths of each net with a centimeter ruler. Label the lengths.

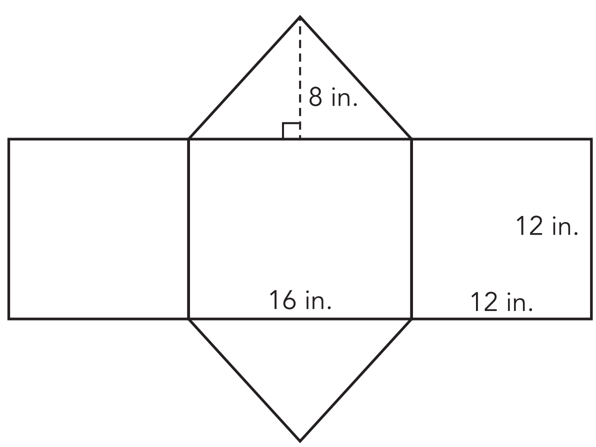
   b. Calculate the surface area of each solid figure.

   c. Cut out, fold, and tape each net.

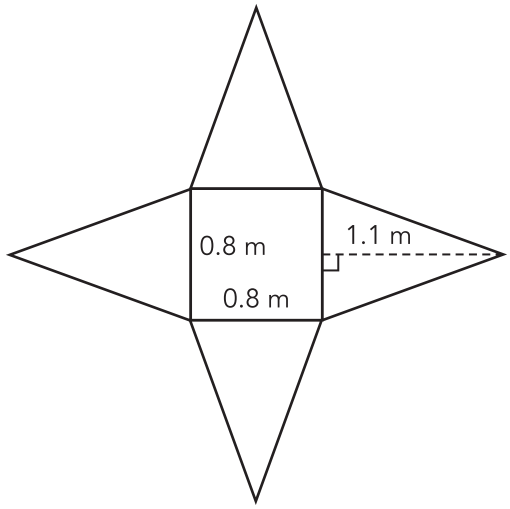
   d. Name each solid figure.

2. Calculate the surface area of the solid figure represented by each net.

   a.



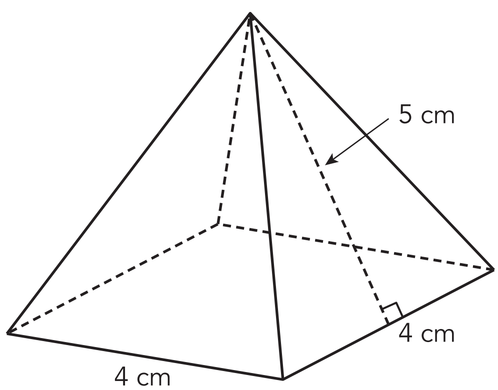
b.



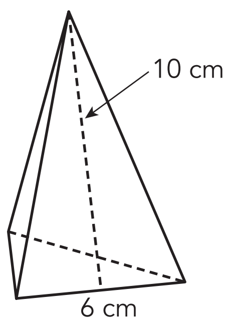
3. Draw a net to represent each solid figure. Label each net with measurements and then calculate the surface area of the solid figure.

Review the definition of **represent** in the Academic Glossary.

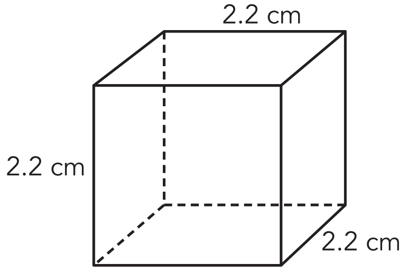
a.

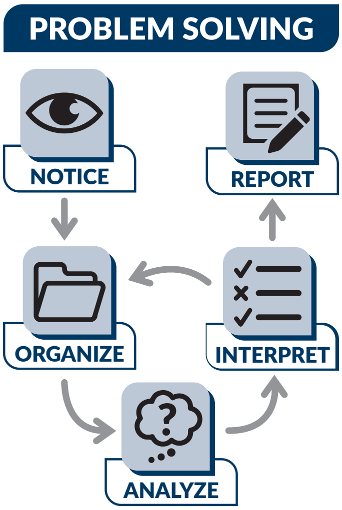


b. The slant heights are all equal. The height of the base is 5.2 cm.



c.



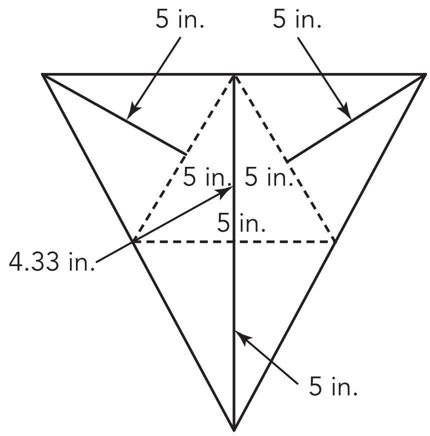


ACTIVITY 2.4

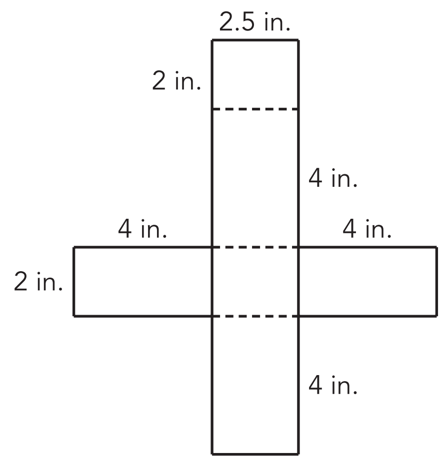
Surface Area Problems

A company produces candles in a variety of shapes. To produce each candle, the company first creates a mold, then pours hot wax into the mold. When the hot wax cools and solidifies, the mold is removed.

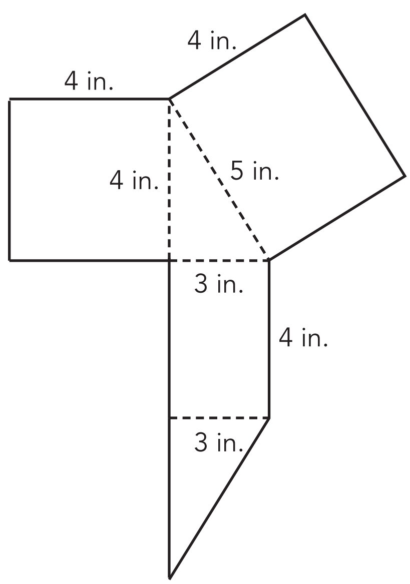
Candle Mold *A*



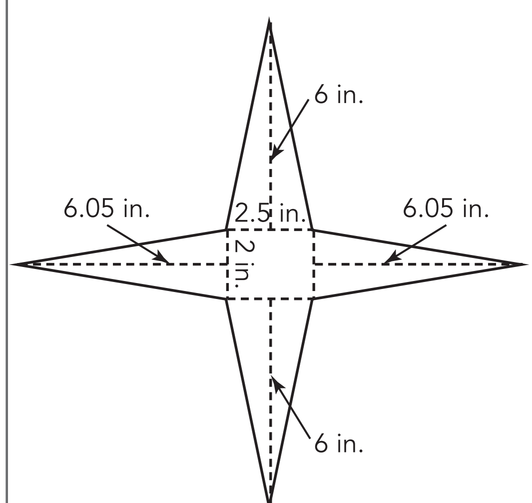
Candle Mold *B*



Candle Mold *C*



Candle Mold *D*



1. Classify the shape of each candle, based on the candle mold.

    a. Candle Mold *A*

    b. Candle Mold *B*

    c. Candle Mold *C*

    d. Candle Mold *D*

2. Use each candle mold to answer each question.

    a. Calculate the surface area of each candle.

    b. How could the company use the surface area of the candles to determine how to price each candle?

Talk the Talk

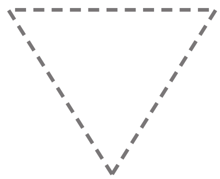
Nothing but Net

1. A rectangular prism has a height of 6 feet, a length of 7.5 feet, and a width of 5 feet.

   a. Draw a net of the rectangular prism and label its measurements.

   b. Calculate the surface area of the prism.

2. Consider the net of the triangular pyramid shown. The net is composed of four equilateral triangles, each with a side length of 4 meters and a height of approximately 3.5 meters.



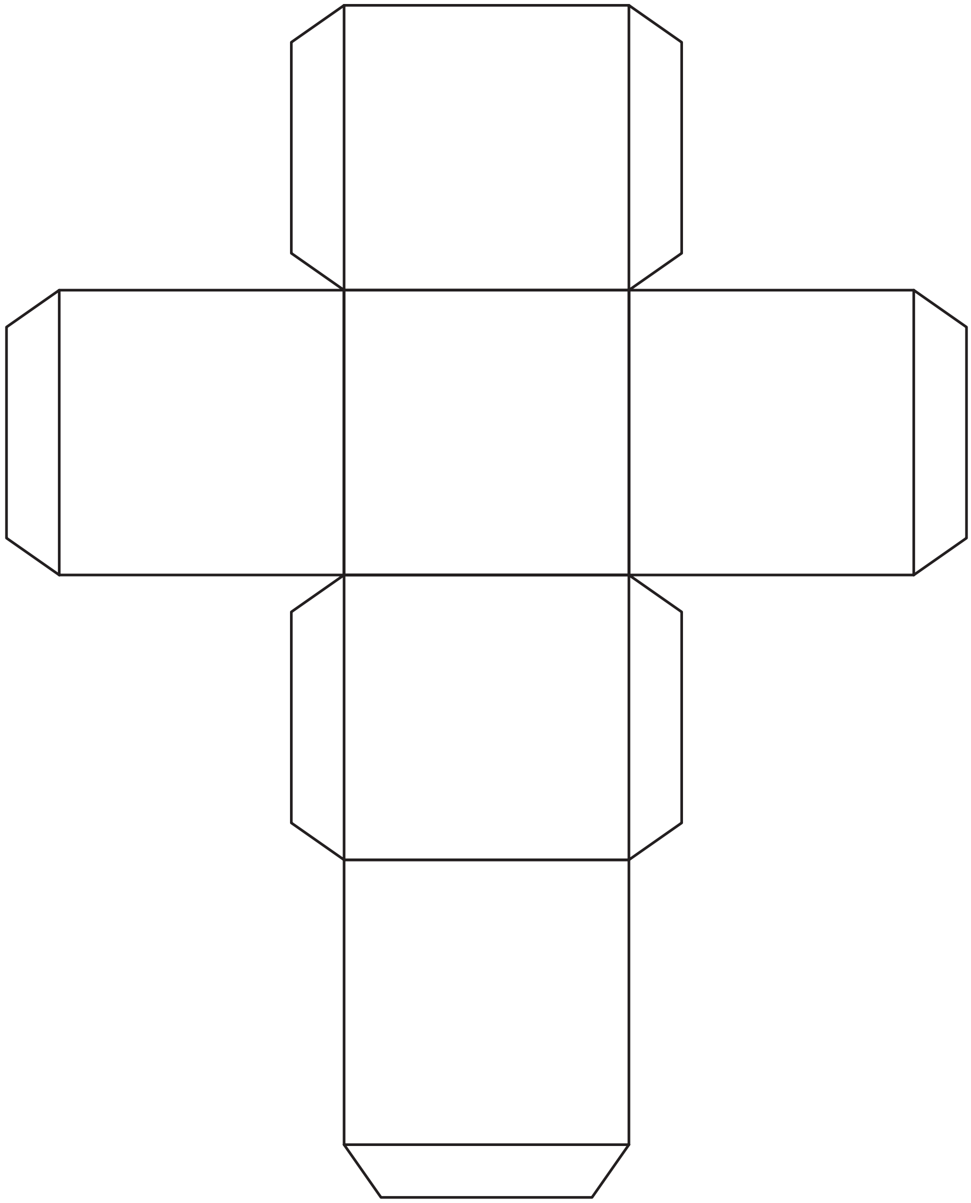
   a. Label the pyramid with its measurements.

   b. Calculate the surface area of the pyramid.

3. Explain in your own words how to determine the surface area of a pyramid.

Ask Yourself . . .
Did you justify your mathematical reasoning?

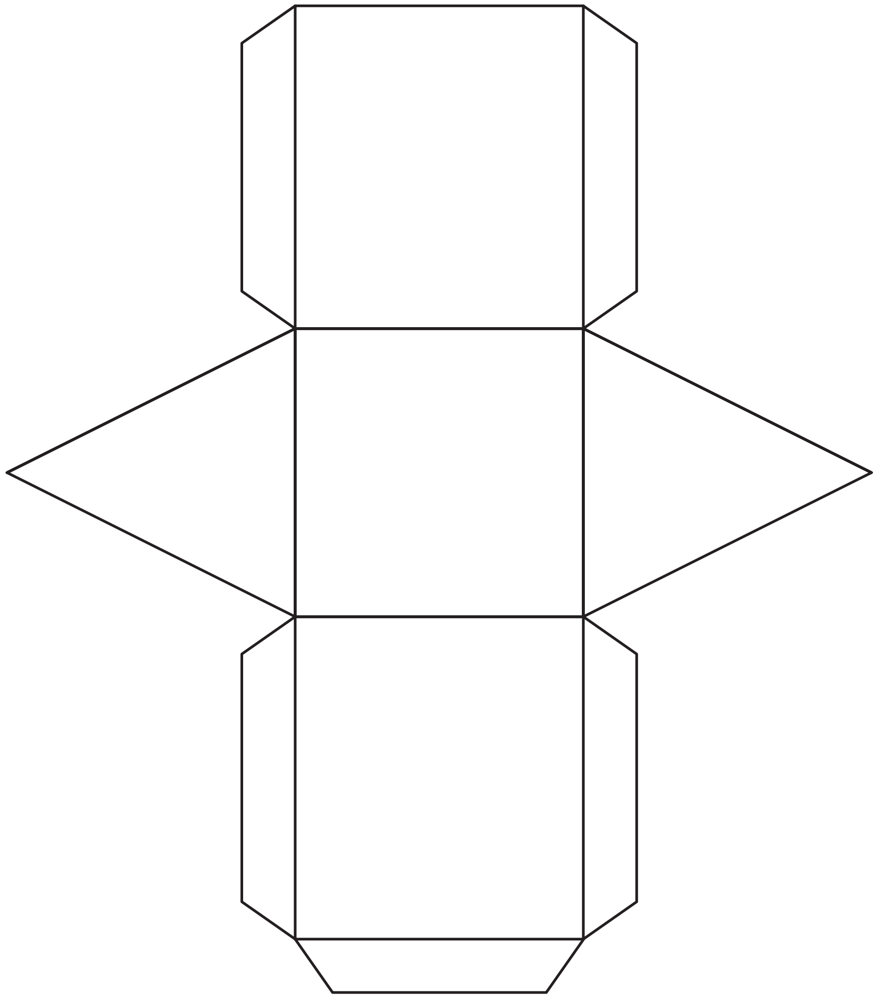
Cube Net



Why is this page blank?

So you can cut out the net on the other side.

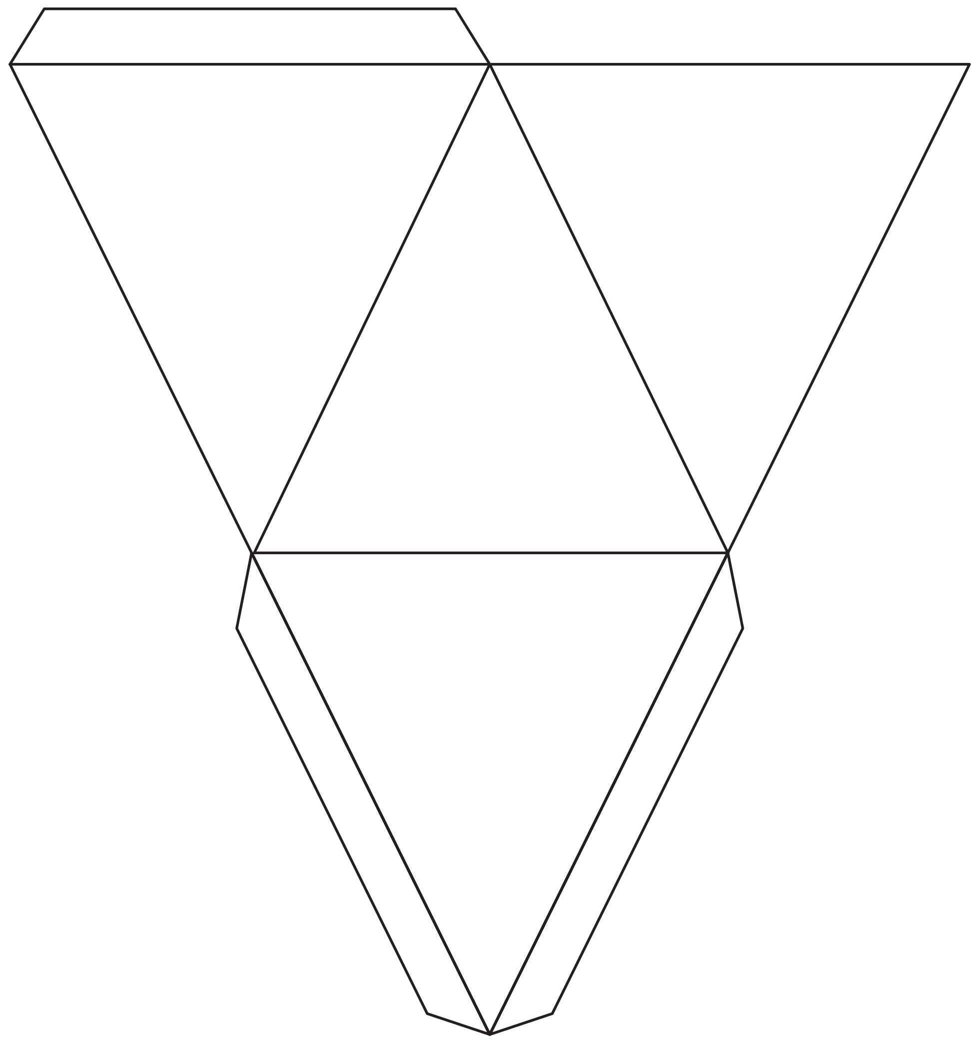
Triangular Prism Net



Why is this page blank?

So you can cut out the net on the other side.

Triangular Pyramid Net



Why is this page blank?

So you can cut out the net on the other side.

Lesson 2 Assignment

Write

Match each definition to its corresponding term.

1. The amount of space occupied by an object
2. A regular polyhedron whose six faces are congruent squares
3. The total area of the two-dimensional surfaces that make up a three-dimensional object
4. The distance across a circle through its center
5. A geometric solid that is made up of polygons
6. The intersection of two faces of a polyhedron
7. A closed figure formed by three or more line segments
8. A two-dimensional representation of a three-dimensional geometric figure
9. A cube that is one unit in length, one unit in width, and one unit in height
10. A bounded three-dimensional geometric figure
11. A portion of a line that includes two points and all the points in between those two points
12. The polygons that make up a polyhedron
13. A location in space
14. The point where the edges of a polyhedron meet

a. *polygon*
b. *polyhedron*
c. *cube*
d. *unit cube*
e. *surface area*
f. *volume*
g. *point*
h. *line segment*
i. *geometric solid*
j. *faces of a polyhedron*
k. *edge of a polyhedron*
l. *vertex of a polyhedron*
m. *net*
n. *diameter*

Remember

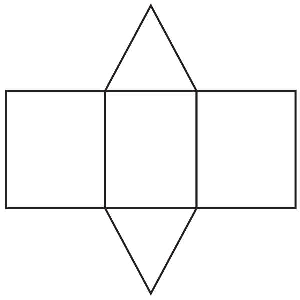
The surface area of a polyhedron is the sum of all the areas of the faces of the polyhedron.

Lesson 2 Assignment

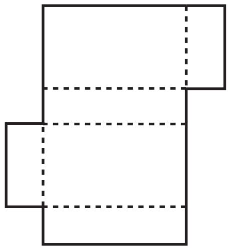
Practice

1. Name the solid figure formed by each net.

   a.

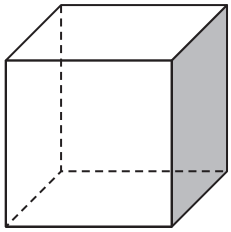


   b.

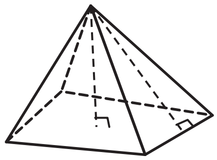


2. Draw a net that will form each solid figure.

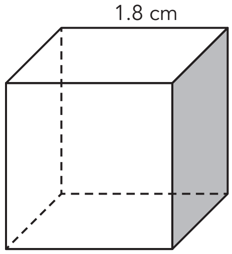
   a.



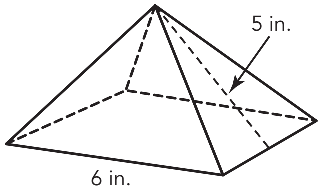
b.



3. Calculate the surface area of the cube.



4. The pyramid shown has a square base and congruent triangular faces. Calculate the surface area of the pyramid.

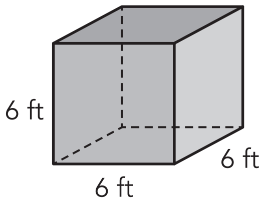


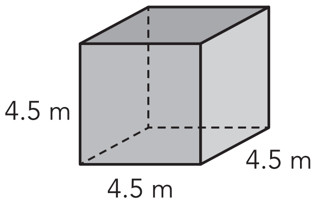
5. Estimate and calculate the surface area of a rectangular prism with a length of 9.06 ft, a width of 4.11 ft, and a height of 6.2 ft.

Lesson 2 Assignment

Prepare

Calculate the volume of each cube.

1. 

2. 

3. A cube that has a side length of 150 millimeters

3 Volume of Prisms and Pyramids

OBJECTIVES

- Determine the volume of pyramids.
- Determine how prisms.
- Solve mathematical and real-world problems involving volumes of pyramids and objects composed of solid figures.

You have learned about the volume of rectangular prisms.

How does the volume of pyramids compare?

Getting Started

You're a Real Cut-Up

Cut out Figures *A* and *B* at the end of the lesson and fold along the dashed lines to form solid figures. Tape the sides together so that the sides do not overlap.

1. Name each solid figure.

2. Compare the bases of the figures. What do you notice?

3. Compare the heights of the figures. What do you notice?

4. Which figure appears to have a greater volume? Explain your reasoning.

5. Use a ruler to measure the dimensions the of the rectangular prism. Then, calculate the volume of the rectangular prism.

ACTIVITY
3.1 Rectangular Prism Volume vs. Rectangular Pyramid Volume

Use the pyramid you built in the previous activity. Fill the pyramid with birdseed. Then, dump the birdseed from the pyramid into the prism. Repeat this process until the prism is full.

1. How many times did you fill the pyramid?

2. Compare the volume of a rectangular pyramid to the volume of a rectangular prism, when the bases are congruent polygons and the heights are of equal measure.

3. In the Getting Started, you determined the volume of the rectangular prism. Based on you exploration, approximate the volume of the pyramid based on the volume of the prism.

4. Write an equation that shows the relationship between the volume of a rectangular prism and the volume of a rectangular pyramid that have congruent bases and heights of equal measure. Let V_{prism} represent the volume of the prism, and let V_{pyramid} represent the volume of the pyramid.

5. Use a separate piece of paper to create your own nets for a rectangular prism and a rectangular pyramid that have congruent bases and heights of equal measure. What do you notice about their volumes?

6. Write formulas for both the volume of a rectangular prism and the volume of a rectangular pyramid. Use V for the volume, B for the area of the base, and h for the height.

   $V_{\text{prism}} =$ ____________________

   $V_{\text{pyramid}} =$ ____________________

ACTIVITY 3.2

Triangular Prism Volume vs. Triangular Pyramid Volume

Cut out Figures *C* and *D* at the end of the lesson and fold along the dashed lines to form a triangular prism and a triangular pyramid. Tape the sides together so that the sides do not overlap. Leave one side of your figure untaped so that you can fill it with birdseed.

1. Fill the pyramid with birdseed. Then, dump the birdseed from the pyramid into the prism. Repeat this process until the prism is full. How many times did you fill the pyramid?

2. Compare the volume of a triangular pyramid to the volume of a triangular prism, when the bases are congruent polygons and the heights are of equal measure.

Ask Yourself . . .
What observations can you make?

3. Write an equation that shows the relationship between the volume of a triangular prism and the volume of a triangular pyramid that have congruent bases and heights of equal measure. Let V_{prism} represent the volume of the prism, and let $V_{pyramid}$ represent the volume of the pyramid.

4. Use a separate piece of paper to create your own nets for a triangular prism and triangular pyramid that have congruent bases and heights of equal measure. What do you notice about their volumes?

5. Write formulas for the volume of a triangular prism and the volume of a triangular pyramid. Use *V* for the volume, *B* for the area of the base, and *h* for the height.

$V_{\text{prism}} =$ ____________________

$V_{\text{pyramid}} =$ ____________________

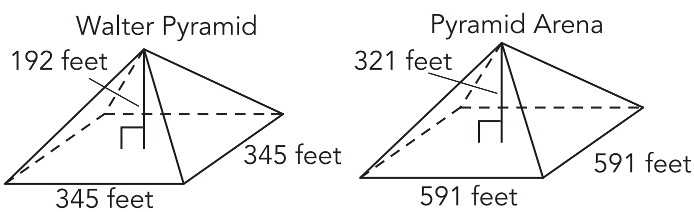
6. Compare your formulas for the volume of a triangular prism and the volume of a triangular pyramid to the formulas for the volume of a rectangular prism and the volume of a rectangular pyramid. What do you notice?

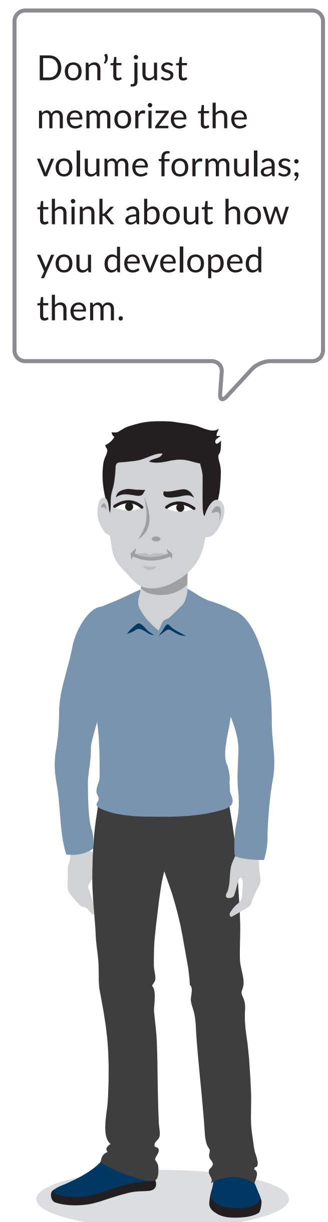
ACTIVITY 3.3

Applying the Volume of Pyramids and Prisms

Now, apply what you know about pyramid and prism volume to solve mathematical and real-world problems.

1. Models of the Walter Pyramid and the Pyramid Arena are shown. Calculate the volume of each pyramid. Which is larger?

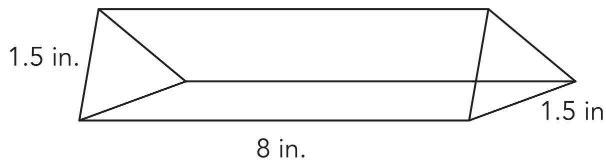




2. A pyramid designed by I.M. Pei, a well-known Chinese-American architect, sits in front of the Louvre art museum in Paris. This square pyramid has a base that has a side length of 115 feet and a height of about 70 feet. Calculate the volume of this pyramid.

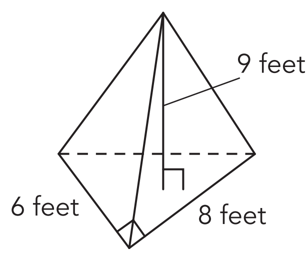
3. A storage box is the shape of a rectangular prism. The box has a side length of 2.5 feet and a height of 3 feet. What is the volume of the storage box?

4. A type of candy comes in a box the shape of a triangular prism. Determine the volume of the box when the height is 2 in., the width is 1.5 in., and the length is 8 in.

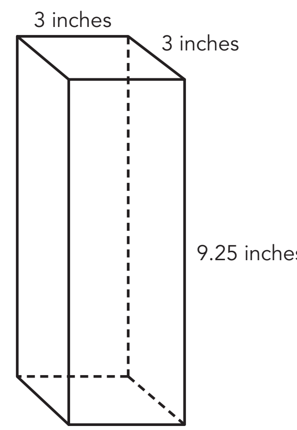


5. Calculate the volume of each prism or pyramid.

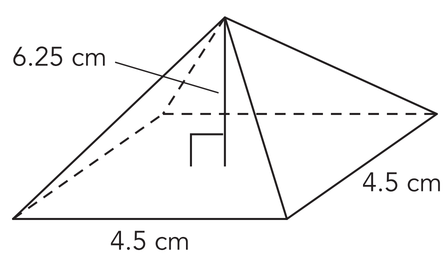
a.



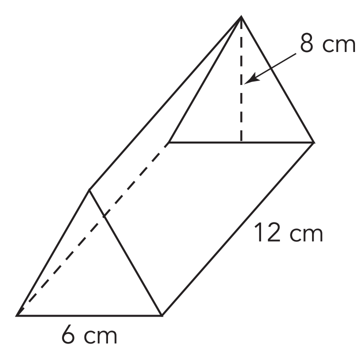
b.



c.



d.

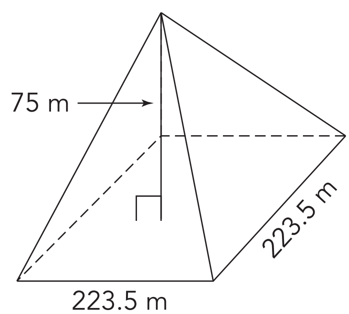


6. The volume of a rectangular pyramid is 520 cubic meters. The area of the base of the pyramid is 156 square meters. Determine the height of the pyramid.

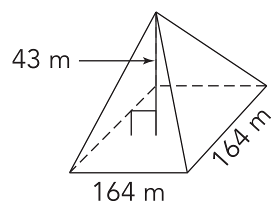
Talk the Talk

The Sun and the Moon

Most people know about Egypt's Great Pyramids at Giza. However, there are a number of pyramids located in the Americas. The Mayan, Aztec, and Inca civilizations all built pyramids to bury their kings. Two of these pyramids are the Pyramid of the Sun and the Pyramid of the Moon, located in Teotihuacán, Mexico.



Pyramid of the Sun

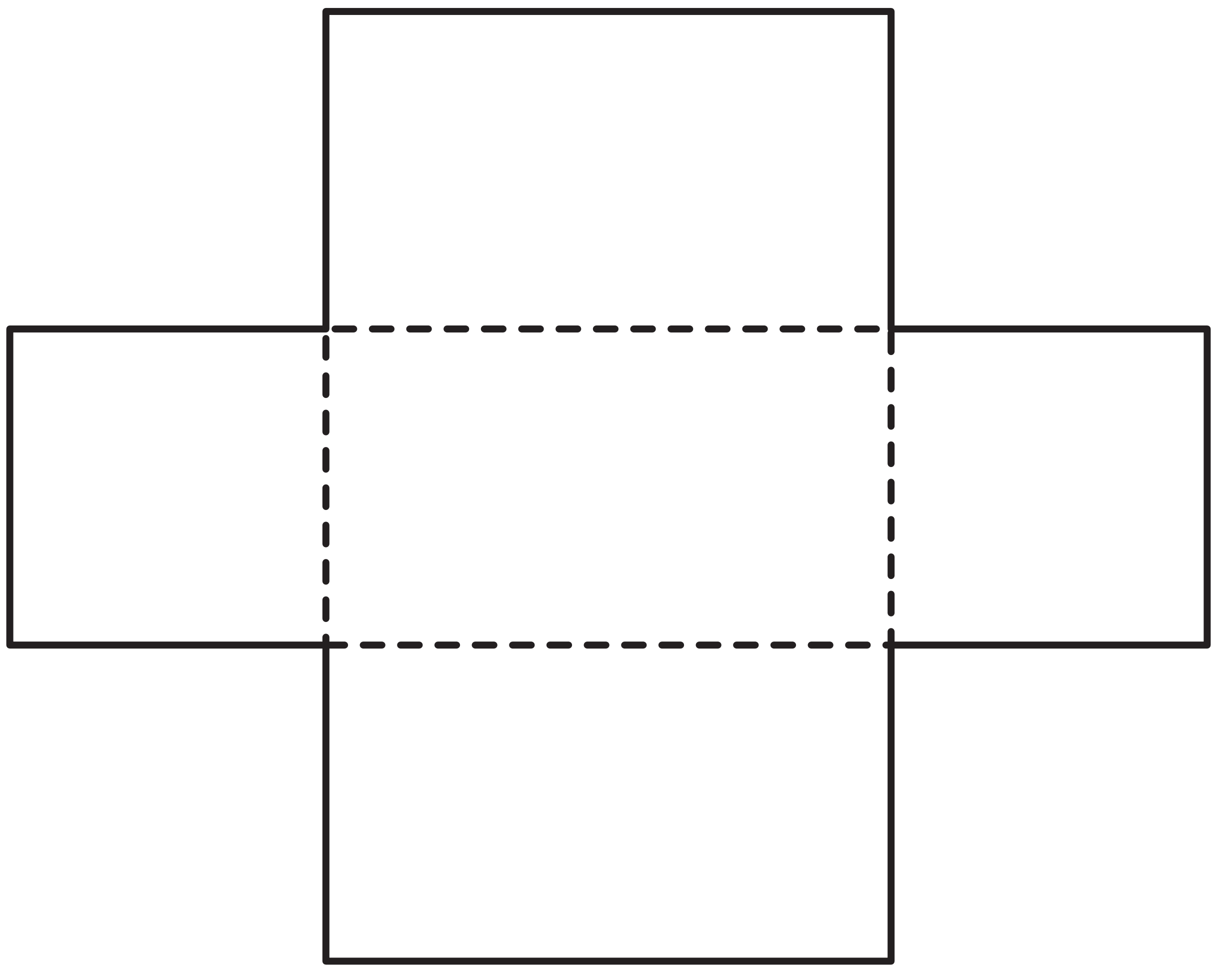


Pyramid of the Moon

1. Determine the volume of each pyramid. Show your work.

Ask Yourself . . .
Did you make a plan to solve the problem?

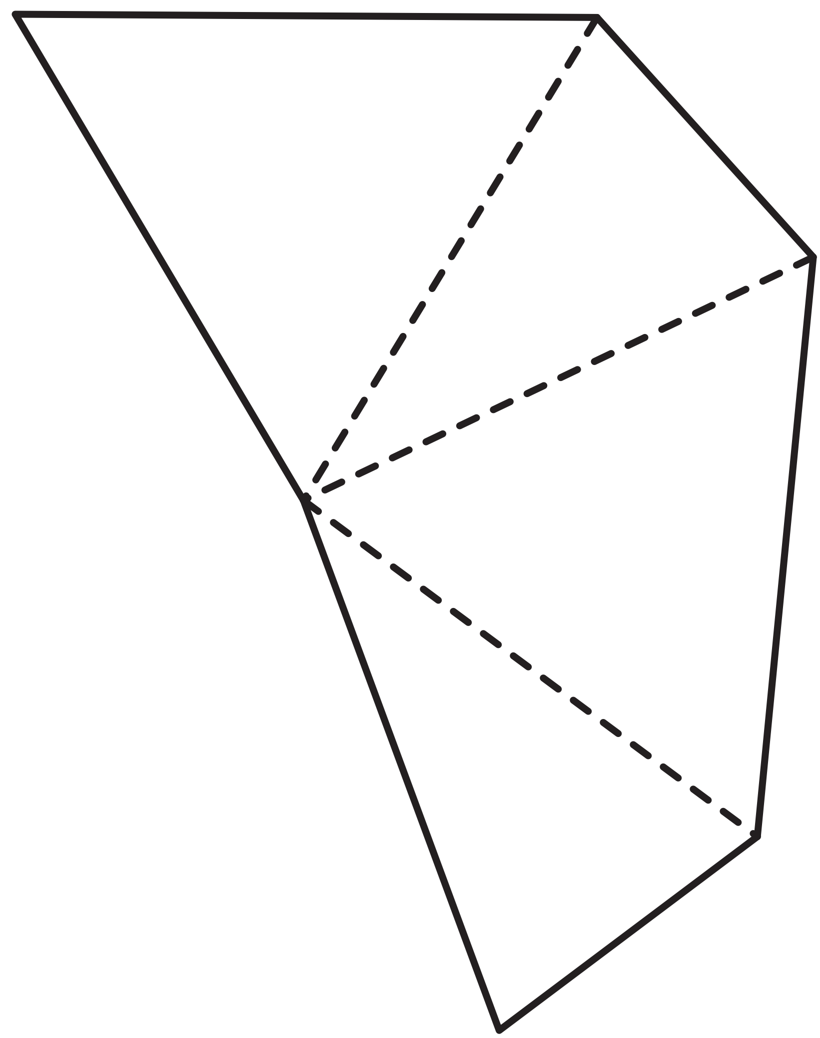
Figure *A*



Why is this page blank?

So you can cut out the figure on the other side.

Figure *B*



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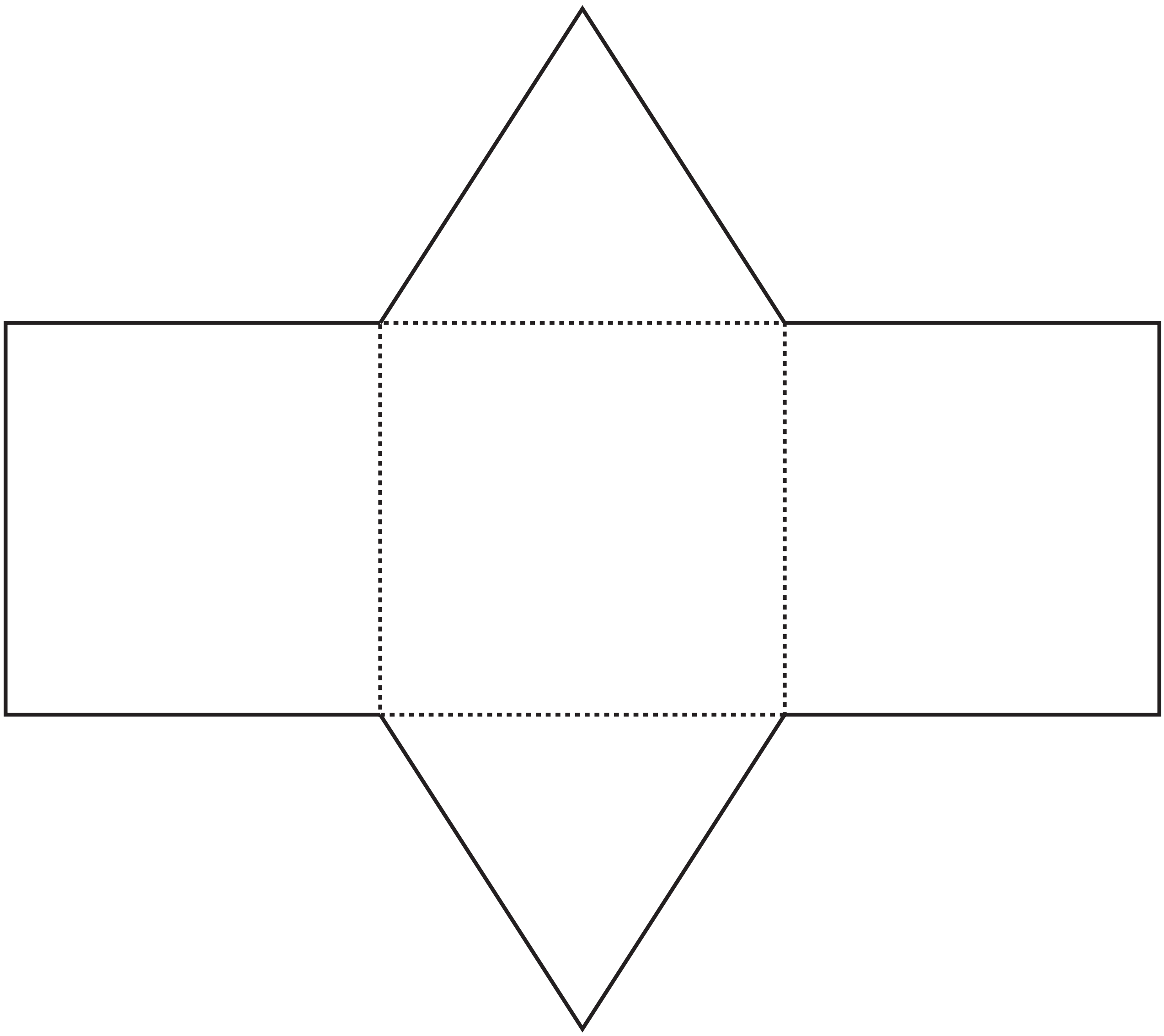
So you can cut out the figure on the other side.

Figure C



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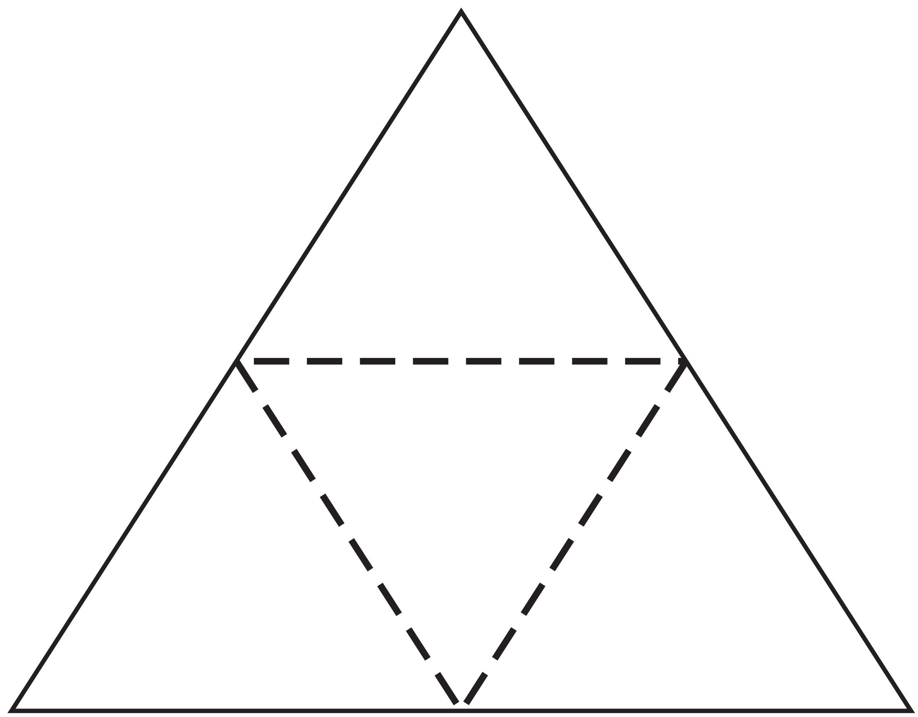
So you can cut out the figure on the other side.

Figure *D*



Why is this page blank?

So you can cut out the figure on the other side.

Lesson 3 Assignment

Write

Describe the differences between pyramids and prisms. Use examples to help illustrate your description.

Remember

The volume of a pyramid is one-third the volume of a prism with the same base and height.

$$V = \frac{1}{3}Bh$$

Practice

Pyramid tents were popular for a time in the 19th century. Although their popularity declined during the 20th century, they have recently begun to regain popularity. The design is ideal for shaping canvas, and it only requires one pole and some stakes to secure it. Joe wants to make a right square pyramid tent and is considering two different sizes. He will either make one with a base that is 10 feet by 10 feet and has a height of 12 feet, or he will make one with a base that is 12 feet by 12 feet and has a height of 8 feet.

1. Sketch the two pyramid designs Joe is considering and label them with the given measurements.

2. How can you determine which pyramid tent will have the most interior space?

Lesson 3 Assignment

3. Calculate the volume of each proposed pyramid tent. Show your work.

4. Which tent would you recommend Joe make? Explain your reasoning.

Prepare

Calculate the volume of each right rectangular prism or pyramid.

1. Prism: length $= \frac{4}{3}$ ft; width $= 1.8$ ft; height $= 3$ ft

2. Prism: area of the base $= 26\ \text{cm}^2$; height $= 5$ cm

3. Pyramid: length $= \frac{4}{3}$ ft; width $= 1.8$ ft; height $= 3$ ft

4. Pyramid: area of the base $= 26\ \text{cm}^2$; height $= 5$ cm

4 Volume and Surface Area Problems with Prisms and Pyramids

OBJECTIVES

- Compare and contrast the lateral and total surface areas of geometric solids.
- Apply volume and surface area concepts to solve real-world and mathematical problems involving surface areas of pyramids and composite solids.

NEW KEY TERM

- lateral surface area

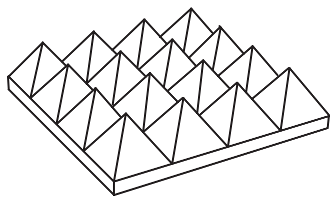
You have determined the surface area of prisms and pyramids.

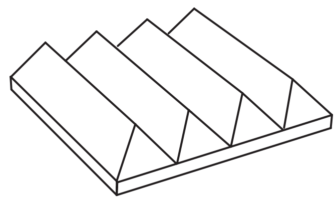
How can you use the lateral and total surface areas of pyramids or prisms to solve problems?

Getting Started

Noise-Canceling Geometry

In small recording studios and large auditoriums, acoustical foam is placed on walls and ceilings to control sound and correct sound problems. The size and shape of the foam that is used depends on the sound problem. Two different acoustical foam shapes are shown.


Pyramidal foam


Wedge foam

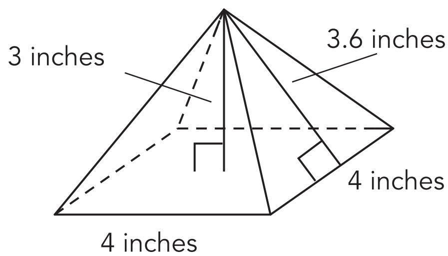
1. Describe the polyhedrons used in each type of acoustical foam board.

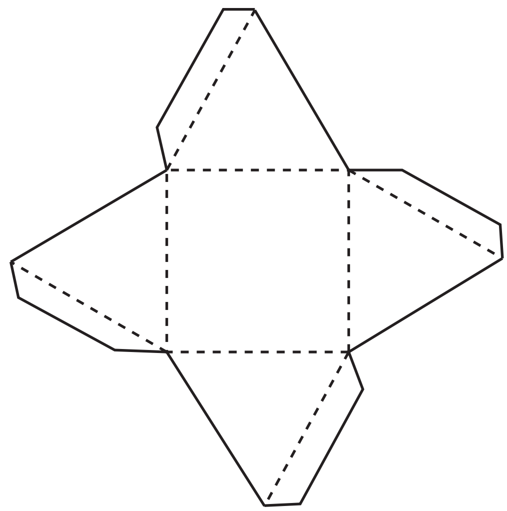
2. Compare the top surface areas of each piece of foam board.

ACTIVITY
4.1

Lateral and Total Surface Area of a Pyramid

Consider the square pyramid and it's net.





All pyramids have a height and a slant height. The slant height is not the actual height of the pyramid. Rather, it is the height of an individual triangle that is a face of the pyramid.

1. Label all known dimensions on the net shown.

2. Calculate the total surface area of the pyramid.

The **lateral surface area** of a prism or pyramid is the sum of the areas of the lateral faces. You can calculate the lateral surface area of a prism or pyramid by determining the total surface area of the figure and then subtracting the area of the base(s).

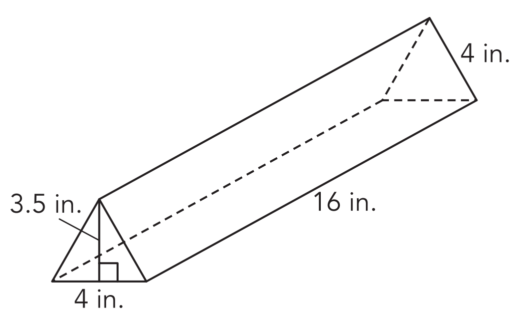
3. Label the base and shade the lateral faces of the net.

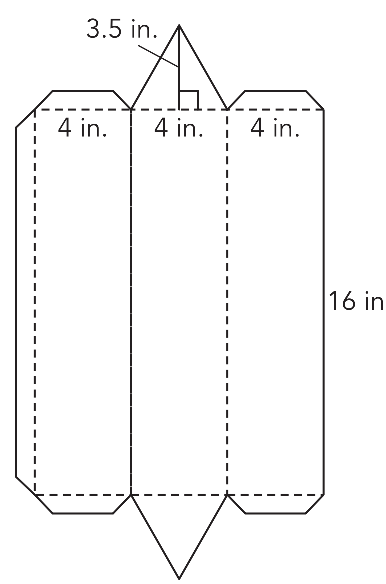
4. Compare the lateral surface area of the pyramid to the total surface area of the pyramid.

5. Calculate the lateral surface area of the pyramid.

6. Consider the original piece of pyramidal foam board. Could you use the lateral surface area of a pyramid to determine the surface area of the foam board? Explain your reasoning.

One triangular prism from the wedge foam board is shown. A net of the triangular prism and its dimensions are also shown.



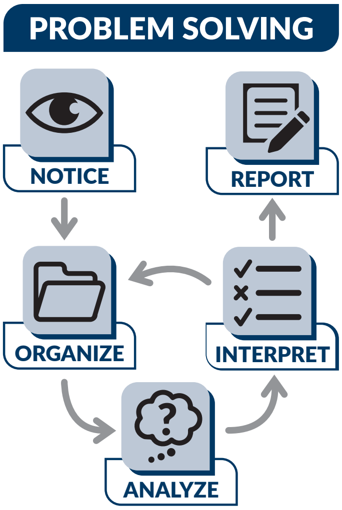


7. Calculate the total surface area of the prism.

8. Compare the lateral surface area of the prism to the total surface area of the prism.

9. Calculate the lateral surface area of the prism.

10. Consider the original piece of wedge foam board.Could you use the lateral surface area of a triangular prism to determine the surface area of the foam board? Explain your reasoning.

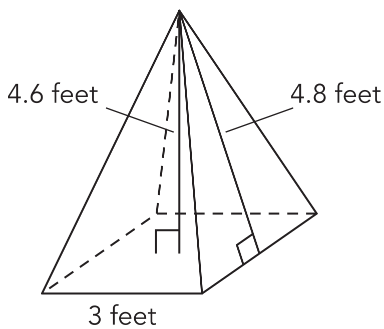


ACTIVITY 4.2

Surface Area Problems

When solving a problem, it may not be obvious whether you should determine the volume or surface area of a solid. So, it is important to think about the context of the problem situation before making a decision about how to answer the question.

1. Consider the square pyramid shown.

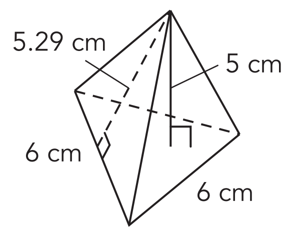


   a. Draw a net to represent the square pyramid. Label the dimensions.

   b. Suppose that the base of the pyramid is made of wood, and the triangular faces are made of plastic covered in blue felt. How much blue felt do you need?

c. Suppose that the pyramid is filled with rice. How much rice is needed?

2. Consider the triangular pyramid shown. The base is an equilateral triangle. The area of the base is 15.59 square centimeters.



a. Draw a net to represent the triangular pyramid. Label the dimensions.

b. Suppose Sofia fills the triangular pyramid with sand, how much sand will she need?

c. Suppose Sofia covers the triangular pyramid with foil, how much foil will she need?

d. Suppose Sofia covers the entire pyramid, except for the base, with foil. How much foil will she need?

Ask Yourself . . .
How can you use surface area and volume in everyday life?

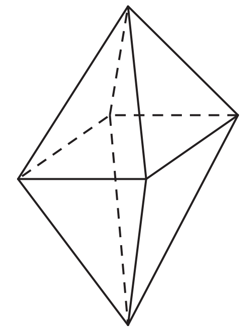
3. Daniel has a picnic table that has two wooden seats. Each seat is a rectangular prism that is 72 inches long, 18 inches wide, and 1 inch thick.

   a. Draw a net that represents one seat from the picnic table. Label the dimensions.

b. Daniel wants to cover the seats with a weather-proofing stain. How much stain will he need?

c. Suppose that Daniel decides he is not going to stain the sides that are 18 inches by 1 inch. How much stain will he need?

4. Juan is creating an ornament by gluing the bases of two square pyramid blocks together, as shown.



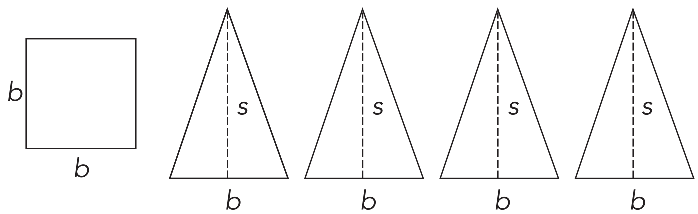
The square base of each pyramid has a side length of 1 inch, the height of each pyramid is 1.5 inches, and the slant height of each pyramid is 1.6 inches.

a. Draw a net to represent one of the square pyramids. Label the dimensions.

b. After gluing the pyramids together, Juan decides to cover the ornament with metallic paint. How much of the ornament will be covered in paint?

ACTIVITY
4.3 Building a Surface Area Strategy

The five faces of a square pyramid are shown.

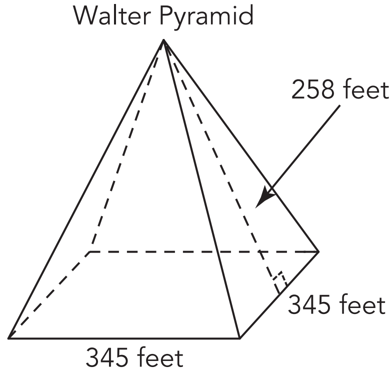


The variable *s* represents the height of each triangular face of the pyramid.

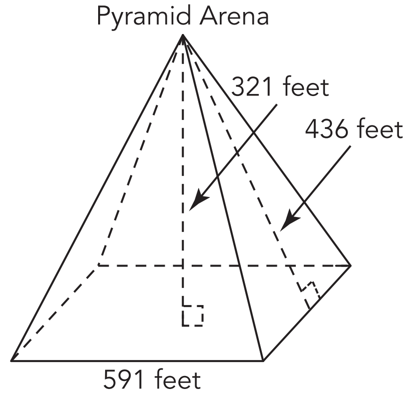
1. Write equations to represent the lateral and total surface area of a square pyramid. Explain how the equations describe each value.

2. Use your equations to calculate the lateral and total surface area of each square pyramid.

a.



b.



Talk the Talk

The Information You Need

1. What information do you need to determine the volume of a pyramid? Provide an example to explain your reasoning.

2. What information do you need to determine the lateral surface area of a pyramid? What additional information do you need to determine the total surface area? Provide an example to explain your reasoning.

Lesson 4 Assignment

Write

In your own words, explain how to determine the total surface area of any square pyramid. Use an example to illustrate your explanation.

Remember

The *total surface area* of a figure is the sum of the areas of all of its faces, including the base(s). The *lateral surface area* of a figure is the total surface area of the figure, excluding the base(s).

Practice

Calculate the volume, total surface area, and lateral surface area of each solid described.

1. The base of a pyramid is an equilateral triangle with a side length of 12 inches. The height of the base is 10.4 inches. The height of the pyramid is 10 inches, and its slant height is 10.6 inches.

   a. Volume:

   b. Total surface area:

   c. Lateral surface area:

2. A square pyramid has a base length of 1.5 meters, a height of 2.3 meters, and a slant height of 2.4 meters.

   a. Volume:

   b. Total surface area:

   c. Lateral surface area:

Lesson 4 Assignment

3. The base of a prism is a square with a side length of 2.5 centimeters. The height of the prism is 16 centimeters.

   a. Volume:

   b. Total surface area:

   c. Lateral surface area:

TOPIC 2 SELF-REFLECTION

Name: ____________________

Area, Surface Area, and Volume

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1-3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Area, Surface Area, and Volume* topic by:

TOPIC 2: *Area, Surface Area, and Volume*	Beginning of Topic	Middle of Topic	End of Topic
applying the techniques of composing and/or decomposing to determine the area of composite figures to solve mathematical and real-world problems.	◯	◯	◯
determining the areas of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, and triangles.	◯	◯	◯
representing three-dimensional figures using nets made up of rectangles and triangles.	◯	◯	◯
combining the areas for rectangles and triangles in the net to calculate the total surface area of a three-dimensional figure.	◯	◯	◯
solving real-world and mathematical problems involving total surface area using nets for rectangular and triangular prisms and pyramids.	◯	◯	◯
modeling and explaining the relationship between the volume of a rectangular prism and a rectangular pyramid having congruent bases and heights and connecting to the formulas.	◯	◯	◯

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Area, Surface Area, and Volume*	Beginning of Topic	Middle of Topic	End of Topic
explaining how the volume of a triangular prism relates to the volume of a triangular pyramid with both congruent bases and heights, and connects to their formulas.	◯	◯	◯
solving problems involving the volumes of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids.	◯	◯	◯

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Area, Surface Area, and Volume* topic.

   ____

   ____

   ____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   ____

   ____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   ____

   ____

   ____

TOPIC 2 SUMMARY

Area, Surface Area, and Volume Summary

NEW KEY TERMS

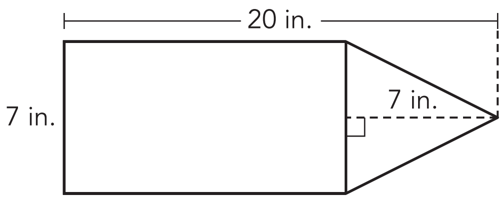
- net
- surface area [área de (la) superficie / área superficial]
- pyramid [pirámide]
- slant height
- lateral surface area [área de la superficie lateral]

LESSON 1 Composite Figures

A **composite figure** is a figure that is made up of more than one geometric figure.

Area is additive. The area of a composite figure can be determined by decomposing it into familiar shapes and then adding the areas of those shapes together.

For example, the composite figure shown is composed of a rectangle and a triangle.



$$\begin{aligned}\text{Area of composite figure} &= \text{Area of Rectangle} + \text{Area of Triangle}\\ &= (7)(13) + \tfrac{1}{2}(7)(7)\\ &= 91 + 24\tfrac{1}{2}\\ &= 115\tfrac{1}{2}\end{aligned}$$

The area of the composite figure is $115\frac{1}{2}$ square inches.

LESSON 2 Total Surface Area of Prisms and Pyramids

A **net** is a two-dimensional representation of a three-dimensional geometric figure.

A net has the following properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn so that they share common edges.

The **surface area** of a three-dimensional geometric figure is the total area of all of its two-dimensional faces.

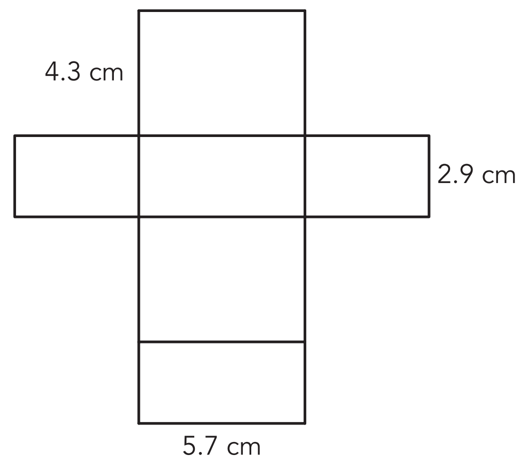
For example, you can use the net to calculate the surface area of the right rectangular prism.

Determine the area of each unique face.

$4.3 \text{ cm} \cdot 5.7 \text{ cm} = 24.51 \text{ cm}^2$

$2.9 \text{ cm} \cdot 5.7 \text{ cm} = 16.53 \text{ cm}^2$

$4.3 \text{ cm} \cdot 2.9 \text{ cm} = 12.47 \text{ cm}^2$

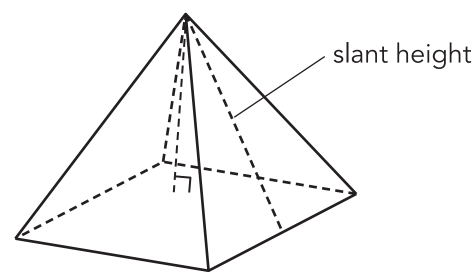


Determine the sum of all faces of the right rectangular prism.

$2(24.51) + 2(16.53) + 2(12.47)$
$= 49.02 + 33.06 + 24.94$
$= 107.02$

The surface area of the right rectangular prism is 107.02 cm^2.

A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The vertex of a pyramid is the point at which all the triangular faces intersect. A **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint, or center, of the base.



LESSON 3 Volume of Prisms and Pyramids

The volume of a pyramid is one-third the volume of a prism with the same base and height, so $V = \frac{1}{3}Bh$.

For example, calculate the volume of the rectangular pyramid shown.

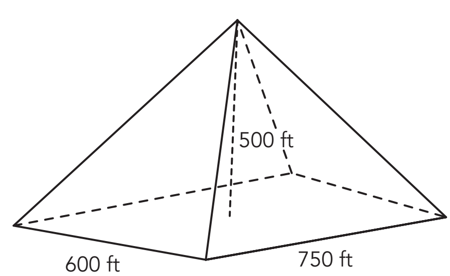
$B = 750 \cdot 600$

$B = 450{,}000$

$V = \frac{1}{3}\text{Bh}$

$V = \frac{1}{3}(450{,}000)(500)$

$V = 75{,}000{,}000$



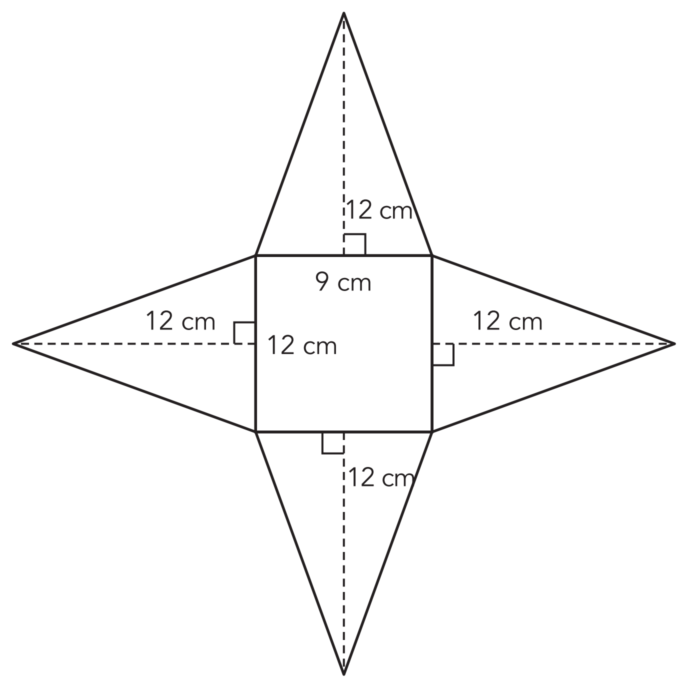
The volume of the pyramid is 75,000,000 cubic feet.

LESSON 4 Volume and Surface Area Problems with Prisms and Pyramids

The total surface area of a pyramid is the sum of the areas of all of its faces. The **lateral surface** area of a pyramid is the total surface area of the pyramid, excluding the base. The dimensions of a polyhedron are used to calculate the lateral surface area in a real-world problem situation.

All pyramids have a height and a slant height. The slant height is not the actual height of the pyramid. Rather, it is the height of an individual triangle that is a face of the pyramid.

For example, calculate the total and lateral surface areas of the pyramid whose net is shown.



$SA = 4 \cdot \frac{1}{2}(9)(12) + 9^2$ $\quad$ $L = 4 \cdot \frac{1}{2}(9)(12)$

$SA = 4 \cdot 54 + 81$ $\quad$ $L = 4 \cdot 54$

$SA = 216 + 81$ $\quad$ $L = 216$

$SA = 297$

The total surface area of the pyramid is 297 square centimeters. The lateral surface area of the pyramid is 216 square centimeters.

Similarly, the total surface area of a prism is the sum of the area of all of its faces. The *lateral surface* area of a prism is the total surface area of the prism excluding the area of the base.

When solving a problem, it may not be obvious whether you should determine the volume, total surface area, or lateral surface area of a solid. Therefore, it is important to think about the context of the problem situation before making a decision about how to answer the question.

Math Glossary

A

401(k) plan

A 401(k) plan is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account with the employer often matching a certain amount of it.

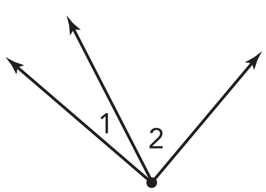
403(b) plan

A 403(b) plan is a retirement plan generally for public school employees or other tax exempt groups.

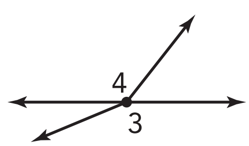
adjacent angles

Adjacent angles are two angles that share a common vertex and share a common side.

Examples



Angles 1 and 2 are adjacent angles.



Angles 3 and 4 are NOT adjacent angles.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $\quad 2a + b$ $\quad xy$ $\quad \frac{4}{p}$ $\quad z^2$

appreciation

Appreciation is an increase in price or value.

asset

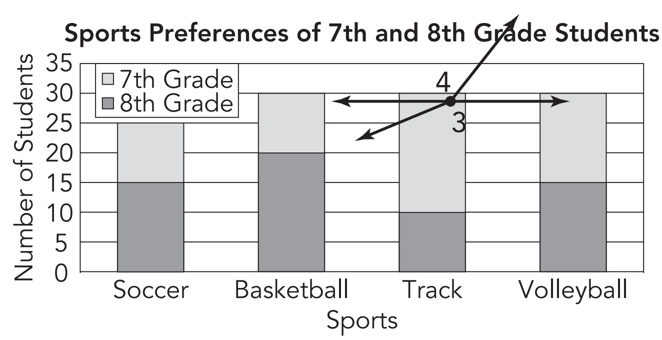
Assets include the value of all accounts, investments, and things that you are own. They are positive and add to your net worth.

B

bar graph

Bar graphs display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

Examples

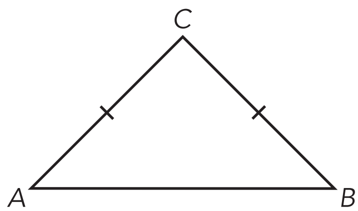


base angles

The angles opposite the two sides that have the same length in an isosceles triangle are called base angles.

Example

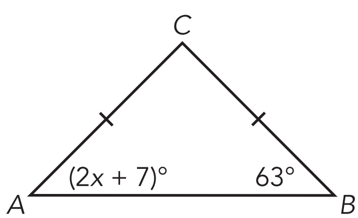
$\angle A$ and $\angle B$ are base angles of $\triangle ABC$



Base Angles theorem

The Base Angles theorem states the angles opposite the congruent sides of an isosceles triangle are congruent.

Example



$\triangle ABC$ is an isosceles triangle. Since, $\angle A$ and $\angle B$ are base angles, their measures are the same.

$2x + 7 = 63$

C

census

A census is the data collected from every member of a population.

Example

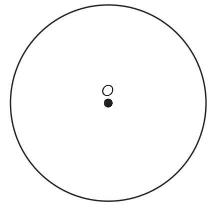
The U.S. Census is taken every 10 years. The U.S. government counts every member of the population every 10 years.

circle

A circle is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.

Example

The circle shown is Circle *O*.

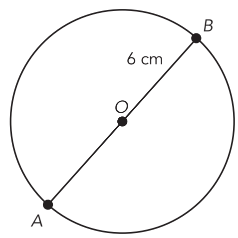


circumference

The distance around a circle is called the circumference of the circle. The circumference is calculated by the formula: $C = \pi(d)$.

Example

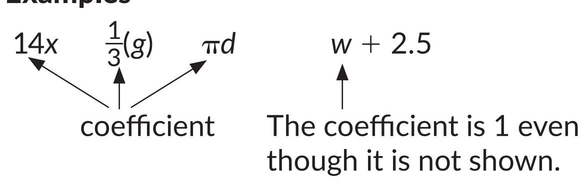
The diameter of Circle *O* is 12 centimeters. The circumference of Circle *O* is 12π.



coefficient

A number that is multiplied by a variable in an algebraic expression is called a coefficient.

Examples



collinear

When points lie on the same line or line segment, they are said to be collinear.

Example

Points C, A, and B are collinear.

commission

A commission is an amount of money a salesperson earns after selling a product. Many times, the commission is a certain percent of the product.

Example

5% commission on \$350

$0.05 \times 350 = \$17.50 \leftarrow$ commission

common factor

A common factor is a number that is a factor of two or more numbers.

Example

Factors of 60: **1**, **2**, **3**, **4**, 5, **6**, 10, **12**, 15, 20, 30, 60

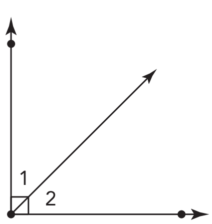
Factors of 24: **1**, **2**, **3**, **4**, **6**, 8, **12**, 24

Common factors of 60 and 24: 1, 2, 3, 4, 6, and 12

complementary angles

Two angles are complementary angles if the sum of their angle measures is equal to 90°.

Example



Angles 1 and 2 are complementary angles.

complementary events

Complementary events are events that together contain all of the outcomes in the sample space.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the event of rolling an even number and the event of rolling an odd number (not even) are complementary events.

complex ratio

A ratio in which one or both of the quantities being compared are written as fractions is a complex ratio.

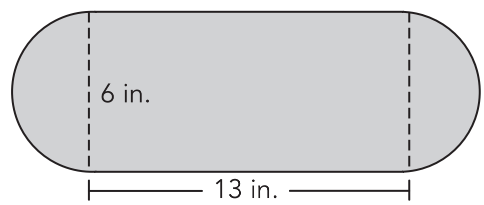
Example

Traveling $\frac{1}{3}$ mile in $\frac{1}{2}$ hour represents a ratio of fractions, or a complex ratio.

composite figure

Composite figures are geometric figures composed of two or more geometric shapes.

Example



The composite figure is composed of a rectangle and two semi-circles.

compound event

A compound event combines two or more events, using the word "and" or the word "or."

compound interest

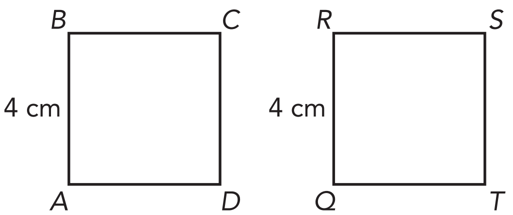
Compound interest is a percentage of the principal and the interest that is added to the principal over time.

congruent

Congruent means to have the same size, shape, and measure.

Example

Square *ABCD* is congruent to Square *QRST*.



congruent angles

Congruent angles are angles that have the same measure.

congruent sides

Congruent sides are sides that have the same length.

constant of proportionality

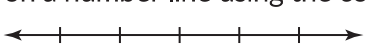
In a proportional relationship, the ratio of all *y*-values to their corresponding *x*-values is constant. This specific ratio, $\frac{y}{x}$, is called the constant of proportionality. Generally, the variable *k* is used to represent the constant of proportionality.

constraint

A constraint is a condition that a solution or problem must satisfy. A constraint can be a restriction set in advance of solving a problem or a limit placed on a solution or graph so the answer makes sense in terms of a real-world scenario.

Example

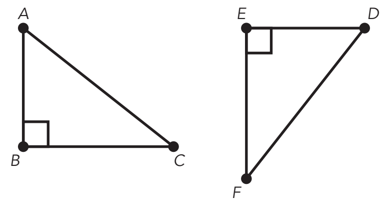
The expressions 0, x, $2x$, $-x$, and $-2x$ are graphed on a number line using the constraint $x < 0$.



corresponding

Corresponding means to have the same relative position in geometric figures, usually referring to sides and angles.

Example



Sides *AB* and *DE* are corresponding sides.

Angle *B* and Angle *E* are corresponding angles.

coupon

A coupon is a detachable part of a ticket or advertisement that entitles the holder to a discount.

D

data

Data are categories, numbers, or observations gathered in response to a statistical question.

Examples

Favorite foods of sixth-graders
Heights of different animals at the zoo

depreciation

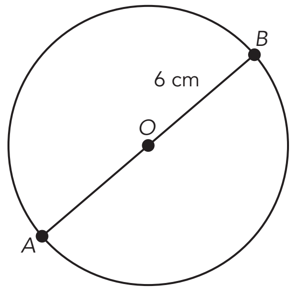
Depreciation is a decrease in price or value.

diameter

The diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.

Example

In Circle O, segment AB is a diameter. The length of diameter AB is two times the length of radius OA. The length of radius OA is 6 centimeters, so the length of diameter AB is 12 centimeters.



direct variation

A situation represents a direct variation if the ratio between the y-value and its corresponding x-value is constant for every point. The quantities are said to vary directly.

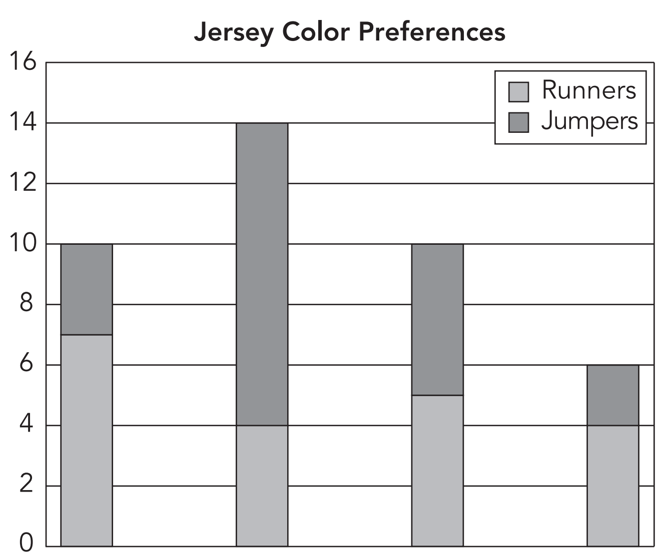
Example

If Melissa earns $8.25 per hour, then the amount she earns is in direct variation with the number of hours she works. The amount $8.25 is the constant of proportionality.

double bar graph

A double graph is used when each category contains two different groups of data.

Example



E

equally likely

When the probabilities of all the outcomes of an experiment are equal, the outcomes are called equally likely.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the probability of rolling each number from 1 through 6 is equally likely.

equation

An equation is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

Examples

$$y = 2x + 4$$

$$6 = 3 + 3$$

$$2(8) = 26 - 10$$

$$\frac{1}{4} \cdot 4 = \frac{8}{4} - \frac{4}{4}$$

evaluate an algebraic expression

To evaluate an algebraic expression means to determine the value of the expression for a given value of each variable.

Example

Evaluate the expression $\frac{4x + (2^3 - y)}{p}$ for $x = 2.5$, $y = 8$, and $p = 2$.

- First replace the variables with numbers: $\frac{4(2.5) + (2^3 - 8)}{2}$.
- Then, calculate the value of the expression: $\frac{10 + 0}{2} = \frac{10}{2} = 5$.

event

An event is one possible outcome or a group of possible outcomes for a given situation.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, an event could be rolling an even number.

experiment

An experiment is a situation involving chance that leads to results, or outcomes.

Example

Rolling a six-sided number cube is an experiment.

experimental probability

Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. You draw the blue marble 20 times out of 50 trials. The experimental probability, P_E(blue), is $\frac{20}{25}$ or $\frac{2}{5}$.

extremes

In a proportion that is written $a : b = c : d$, the two values on the outside, a and d, are the extremes.

Example

7 books : 14 days = 3 books : 6 days

extremes

F

Family Budget Estimator

The Family Budget Estimator is a tool that people can use to determine the estimated cost of raising a family in a particular city.

fixed expenses

Fixed expenses are expenses that don't change from month to month.

factor

To factor an expression means to rewrite the expression as a product of factors.

Example

$5(12) + 5(9) = 5(12 + 9)$

G

greatest common factor (GCF)

The greatest common factor, or GCF, is the largest factor two or more numbers have in common.

Example

Factors of 16: **1**, **2**, **4**, 8, 16

Factors of 12: **1**, **2**, 3, **4**, 6, 12

Common factors: 1, 2, 4

Greatest common factor: 4

I

income tax

Income tax is a percentage of a person's or company's earnings that is collected by the government.

Example

If a person earns $90,000 in one year and has to pay an income tax rate of 28%, then that person owes $90{,}000 \times 0.28$ or $25,200 in income tax to the government.

inverse operations

Inverse operations are pairs of operations that reverse the effects of each other.

Examples

Addition and subtraction are inverse operations: $351 + 25 - 25 = 351$.

Multiplication and division are inverse operations: $351 \cdot 25 \div 25 = 351$.

isolate the variable

When you isolate the variable in an equation, you perform an operation, or operations, to get the variable by itself on one side of the equals sign.

Example

In the equation $\frac{a}{b} = \frac{c}{d}$, you can multiply both sides by b to isolate the variable a.

$b \cdot \frac{a}{b} = b \cdot \frac{c}{d} \longrightarrow a = \frac{bc}{d}$

L

lateral surface area

The lateral surface area of a prism or pyramid is the sum of the areas of the lateral faces.

liability

A liability is a financial obligation, or debt, that you must repay. It is negative and take away from your net worth.

like terms

In an algebraic expression, like terms are two or more terms that have the same variable raised to the same power.

Examples

like terms

$4x + 3p + x + 2 = 5x + 3p + 2$

like terms

$24a^2 + 2a - 9a^2 = 13a^2 + 2a$

no like terms

$m + m^2 - x = x^3$

linear expression

A linear expression is any expression in which each term is either a constant or the product of a constant and a single variable raised to the first power.

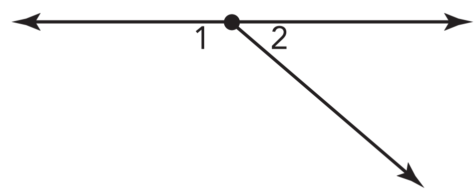
Examples

$\frac{1}{2}x + 2,\ -3 + 12.5x,\ -1 + 3x + \frac{5}{2}x - \frac{4}{3}$

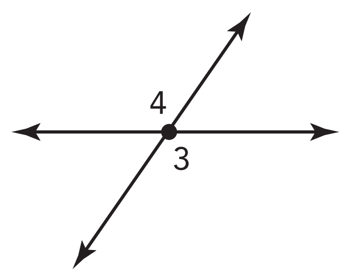
linear pair

A linear pair of angles is formed by two adjacent angles that have noncommon sides that form a line.

Examples



Angles 1 and 2 form a linear pair.



Angles 3 and 4 do NOT form a linear pair.

literal equation

A literal equation is an equation in which the variables represent specific measures.

Examples

$A = \ell w \qquad A = \frac{1}{2}bh \qquad d = rt$

M

markdown

When businesses sell an item at a lower price than the original price, it is called a markdown.

markup

To make money, businesses often buy products from a wholesaler or distributor for one amount and add to that amount to determine the price they use to sell the product to their customers. This is called a markup.

means

In a proportion that is written $a : b = c : d$, the two values in the middle, b and c, are the means.

Example

7 books : 14 days = 3 books : 6 days

means

N

net worth

Your net worth is basically a calculation of the value of everything that you own minus the amount of money that you owe.

non-uniform probability model

A non-uniform probability model occurs when all the probabilities in a probability model are not equal to each other.

Example

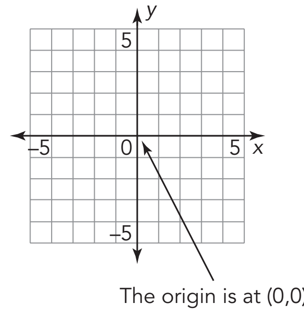
Outcome	Red	Green	Blue
Probability	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{3}{8}$

O

origin

The origin is a point on a graph with the ordered pair (0, 0).

Example



outcome

An outcome is the result of a single trial of a probability experiment.

Example

The numbers on the faces of a six-sided number cube are the outcomes that can occur when rolling a six-sided number cube.

P

parameter

When data are gathered from a population, the characteristic used to describe the population is called a parameter.

Example

If you wanted to determine the average height of the students at your school, and you measured every student at the school, the characteristic "average height" would be a parameter.

percent decrease

A percent decrease occurs when the new amount is less than the original amount. It is a ratio of the amount of decrease to the original amount.

Example

The price of a $12 shirt has decreased to $8.

$\frac{12 - 8}{12} = \frac{4}{12} = 0.3 = 33.3\%$

The percent decrease is 33.3%

percent equation

A percent equation can be written in the form percent $\times$ whole $=$ part, where the percent is often written as a decimal.

Example

40% of 25 = 10

(0.40) (25) = 10

↑ Percent ↑ Whole ↑ Part

percent error (estimation)

Calculating percent error is one way to compare an estimated value to an actual value. To compute percent error, determine the difference between the estimated and actual values and then divide by the actual value.

Example

An airline estimates that they will need an airplane that sits 320 passengers for a flight. An actual 300 tickets were booked for the flight.

Percent Error $= \frac{300 - 320}{300} = \frac{-20}{300} \approx -6.7\%$

percent error (probability)

In probability, the percent error describes how far off the experimental probability is from the theoretical probability as a percent ratio.

Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. You draw the blue marble 20 times out of 50 trials.

The experimental probability, P_E(blue), is $\frac{20}{50}$ or $\frac{2}{5}$. The theoretical probability, P_T(blue), is $\frac{1}{4}$.

The percent error is $\frac{\frac{2}{5}-\frac{1}{4}}{\frac{1}{4}} = \frac{\frac{3}{20}}{\frac{1}{4}} = \frac{3}{5}$

$= 0.6 = 60\%$

percent increase

A percent increase occurs when the new amount is greater than the original amount. It is a ratio of the amount of increase to the original amount.

Example

The price of a $12 shirt has increased to $13.20.

$\frac{13.20 - 12}{12} = \frac{1.20}{12} = 0.1 = 10\%$

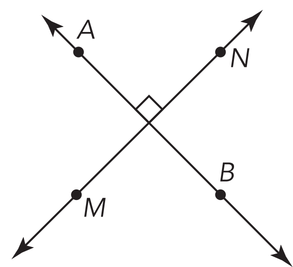
The percent increase is 10%.

perpendicular

Two lines, line segments, or rays are perpendicular if they intersect to form 90° angles. The symbol for perpendicular is $\perp$.

Example

Line *AB* is perpendicular to line *MN*



personal budget

A personal budget is an estimate of the amount of money that a person or family will need for specific financial items. It generally includes current expenses as well as savings for anticipated future expenses.

principal

A principal is the term for an original amount of money on which interest is calculated.

pi

The number pi (π) is the ratio of the circumference of a circle to its diameter. That is $\pi = \frac{C}{d}$, where *C* is the circumference of the circle, and *d* is the diameter of the circle.

population

A population is an entire set of items from which data are collected.

Example

If you wanted to determine the average height of the students at your school, the number of students at the school would be the population.

probability

Probability is the measure of the likelihood that an event will occur. It is a way of assigning a numerical value to the chance that an event will occur by dividing the number of times an event can occur by the number of possible outcomes.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the probability of rolling a 5, or $P(5)$, is $\frac{1}{6}$.

probability model

A probability model is a list of each possible outcome along with its probability, often shown in a table.

Example

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

This is a probability model for rolling a six-sided number cube with the numbers 1 through 6 on each face.

proportion

A proportion is an equation that states that two ratios are equal.

Example

$\frac{1}{2} = \frac{4.5}{9}$

proportional relationship

A proportional relationship is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$ must represent the same constant.

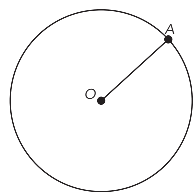
R

radius

The radius of a circle is a line segment formed by connecting a point on the circle and the center of the circle.

Example

In the circle, *O* is the center and segment *OA* is the radius.



random number table

A random number table is a table that displays random digits. These tables can contain hundreds of digits.

Example

Line 7	54621	62117	55516	40467

random sample

A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

Example

If you wanted to determine the average height of the students at your school, you could choose just a certain number of students randomly and measure their heights. This group of students would be a random sample.

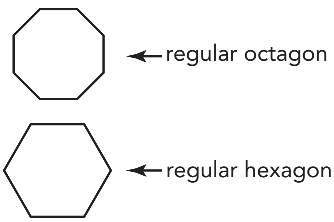
rebate

A rebate is a refund of part of the amount paid for an item.

regular polygon

A regular polygon is a polygon with all sides congruent and all angles congruent.

Examples



S

sale

A sale is an event at which products are sold at reduced prices.

sales tax

Sales tax is a percentage of the selling price of a good or service which is added to the price.

Example

You want to purchase an item for $8.00 in a state where the sales tax is 6.25%, therefore you will pay 8×0.0625 or $0.50 in sales tax. You will pay a total of $8.50 for the item.

sample

A sample is a selection from a population.

Example

If you wanted to determine the average height of the students in your school, you could choose a certain number of students and measure their heights. The heights of the students in this group would be your sample.

sample space

A list of all possible outcomes of an experiment is called a sample space.

Example

When rolling a six-sided number cube that has one number, from 1 through 6, on each face, the sample space is {1, 2, 3, 4, 5, 6}.

scale

A scale is a ratio that compares two measures.

Example

1 cm : 4 cm

scale drawing

A scale drawing is a representation of a real object or place that is in proportion to the real object or place it represents.

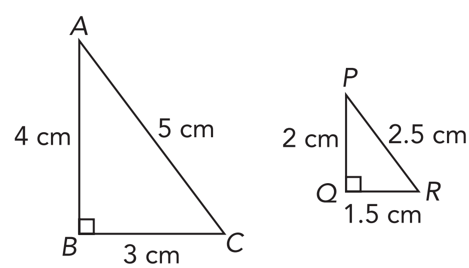
Examples

A map or a blueprint is an example of a scale drawing.

scale factor

When you multiply a measure by a scale to produce a reduced or enlarged measure, the scale is called a scale factor.

Example

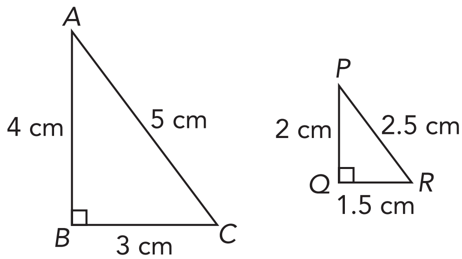


The scale factor from Triangle *ABC* to Triangle *PQR* is $\frac{1}{2}$.

similar figures

Figures that are proportional in size, or that have proportional dimensions, are called similar figures.

Example



Triangle *ABC* and Triangle *PQR* are similar figures.

simple event

A simple event is an event consisting of one outcome.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, rolling a 5 is a simple event.

simple interest

Simple interest is a type of interest that is a fixed percent of the principal. Simple interest is paid over a specific period of time—either twice a year or once a year, for example. The formula for simple interest is $I = P \cdot r \cdot t$, where I represents the interest earned, P represents the amount of the principal, r represents the interest rate, and t represents the time that the money earns interest.

Example

Kim deposits $300 into a savings account at a simple interest rate of 5% per year. The formula can be used to calculate the simple interest Kim will have earned at the end of 3 years.

Interest = Principal · rate · time

Interest = (300)(0.05)(3)

= $45

simulation

A simulation is an experiment that models a real-life situation.

solve a proportion

To solve a proportion means to determine all the values of the variables that make the proportion true.

stacked bar graph

A stacked bar graph is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole.

statistic

When data are gathered from a sample, the characteristic used to describe the sample is called a statistic.

Example

If you wanted to determine the average height of the students in your school, and you chose just a certain number of students randomly and measured their heights, the characteristic "average height" would be called a statistic.

straight angle

A straight angle is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

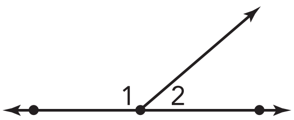
Example

Angle *CAB* is a straight angle.

supplementary angles

Two angles are supplementary angles if the sum of their angle measures is equal to 180°.

Example



Angles 1 and 2 are supplementary angles.

surface area

The **surface area** of a prism or pyramid is the total area of all its two-dimensional faces.

survey

A survey is one method of collecting data in which people are asked one or more questions.

Example

A restaurant may ask its customers to complete a survey with the following question:

On a scale of 1–10, with 1 meaning "poor" and 10 meaning "excellent," how would you rate the food you ate?

T

theoretical probability

The theoretical probability of an event is the ratio of the number of desired outcomes to the total possible outcomes.

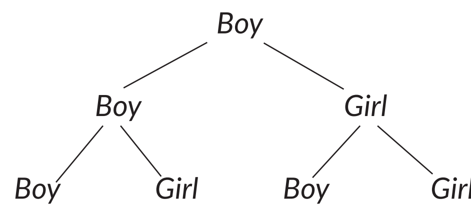
Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. The theoretical probability of drawing a blue marble, P_T(blue), is $\frac{1}{4}$.

tree diagram

A tree diagram illustrates the possible outcomes of a given situation. It has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

Example



two-step equation

A two-step equation requires that two inverse operations be performed to isolate the variable.

U

uniform probability model

A uniform probability model occurs when all the probabilities in a probability model are equally likely to occur.

Example

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

unit rate

A unit rate is a comparison of two different measurements in which the numerator or denominator has a value of one unit.

Example

The speed 60 miles in 2 hours can be written as a unit rate:

$\frac{60 \text{ mi}}{2 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}}$.

The unit rate is 30 miles per hour.

unit rate of change

The unit rate of change describes the amount the dependent variable changes for every unit the independent variable changes.

V

variable

A variable is a letter or symbol that is used to represent a number.

Examples

$3x = 81$ $\frac{4}{p}$ z^2

variables

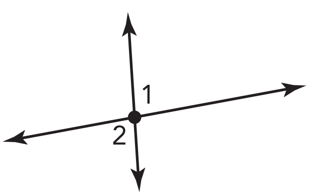
variable expenses

Variable expenses are expenses that can be different from month to month.

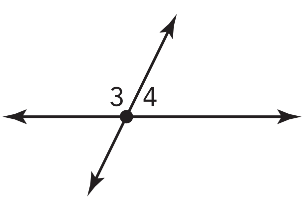
vertical angles

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

Examples



Angles 1 and 2 are vertical angles.



Angles 3 and 4 are NOT vertical angles.

Z

zero pair

A positive counter and a negative counter together make a zero pair since the total value of the pair is zero.

Example

⊕ + ⊖ = 0

ISBN: 189-7-09673-476-3

Printed in the USA

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