Secondary Mathematics

EDITION 1

Grade 7

Volume 2

TEACHER EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

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OER Secondary Mathematics

EDITION 1

Grade 7

Course and Implementation Guide

GRADE 7

Welcome to the Course and Implementation Guide for Secondary Mathematics, Grade 7

The Instructional Approach FM-6

Instructional Design FM-7

Connecting Learning Experiences FM-11

Course-Level Documents FM-13

Connecting Content and Practice FM-16

Research-Based Strategies FM-22

Facilitating Student Learning FM-26

Preparing to Facilitate Learning FM-54

Course Table of Contents FM-59

The Instructional Approach

These instructional materials follow the path that has been proven most effective by research and classroom experience. The instructional approach and design are informed by books like *Adding It Up: Helping Children Learn Mathematics* (National Research Council, 2001), *How People Learn: Brain, Mind, Experience, and School: Expanded Edition* (National Research Council, 2000), *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics, 2014).

The instructional approach utilized is a culmination of the collective knowledge of researchers, instructional designers, cognitive learning scientists, and master practitioners. The approach is based on a scientific understanding of how people learn as well as an understanding of how to apply the science to the classroom. At its core, the instructional approach is based on three key components:

ENGAGE

Getting Started

- Building Off Intuition
- Accessing Prior Mathematical Knowledge
- Establishing a Scenario

DEVELOP

Activities

- Problem Solving
- Classification
- Investigation
- Explicit Instruction
  - Worked Example
  - Peer Work Analysis

DEMONSTRATE

Talk the Talk

- Graphic Organizer
- Presentation
- Generalization
- Writing Task
- Procedure
- Application

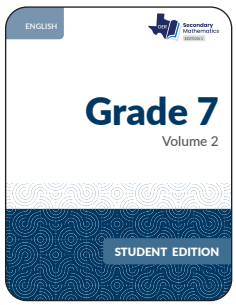
The Getting Started for each lesson activates student thinking by tapping into prior knowledge and real-world experiences. The Activities in each lesson vary in complexity and build a deep understanding of the mathematics. The Talk the Talk for each lesson is an opportunity to engage students in reflection and assess their learning.

Instructional Design

To provide both a toolkit of instructional strategies and opportunities for targeted individualized instruction, the course consists of both a Learning Together and Learning Individually component. These two components work together to engage students with the various learning experiences they need to understand the mathematics in this course.

Learning Together

On **Learning Together** days, you spend time facilitating active learning so that students build their mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the student-facing materials for each lesson.



STUDENT EDITION

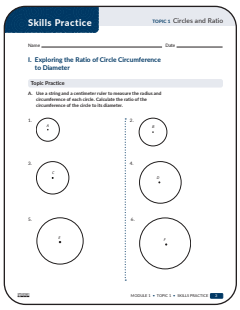
I am a record of student thinking, reasoning, and problem solving.

My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

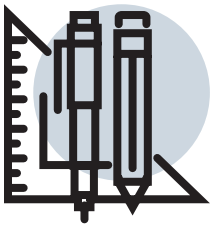
On **Learning Individually** days, you spend time on targeted instruction to meet the needs of each student. Skills Practice offers students the opportunity to engage with problems aligned to each lesson's essential ideas. It also provides opportunities for interleaved practice, which encourages students to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning.



SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide additional problem sets for teachers to assign as needed for differentiated instruction, enrichment, and extension.



The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure students thoroughly understand the mathematical concepts. The instructional materials guide students to connect related ideas holistically, supporting the integration of their evolving mathematical understanding and developing proficiency with mathematical processes.

Intentional Mathematics Design

Mathematical Coherence: The arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

TEKS Mathematical Process Standards: The instructional materials embody the TEKS mathematical process standards as they encourage experimentation, creativity, and various solution strategies. These mathematical processes empower students to persevere when presented with complex real-world problems.

Multiple Representations: The instructional resources connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem-solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work (Chi et. al, 2008). The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture student interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for students to build their new understanding on the firm foundation of what they already know. Students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem Solving: Solving problems is an essential life skill that students need to develop. The problem-solving model provides a structure to support students as they analyze and solve problems. It is a strategy they can continue to use as they solve problems in everyday life.

Seeing Connections: Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. Activities focus on relevant real-world situations to demonstrate the usefulness of mathematics.

Exploring Structure: Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships.

Reflecting and Communicating: Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. During collaborative activities and peer work analysis, students given and receive feedback, which leads to verifying, clarifying, and/or improving the strategy. Using the Topic Self-Reflection and Assessment Reflection tools students reflect on their own learning. This allows students to communicate where they need support, adjust strategies, and set learning goals.

Connecting Learning Experiences

The instructional development aids students in the effective transition from an intuitive understanding of the world to the abstract language of mathematics.

Once students have ample opportunities to build understanding, procedural problems and exercises support the development of computational fluency.

A thoughtful progression from the use of manipulatives and visual aids to representations and drawings that bridge to more abstract understanding benefits all students.

By linking these learning experiences, students can make meaningful connections. These progressions occur across topics, across lessons, and within lessons.

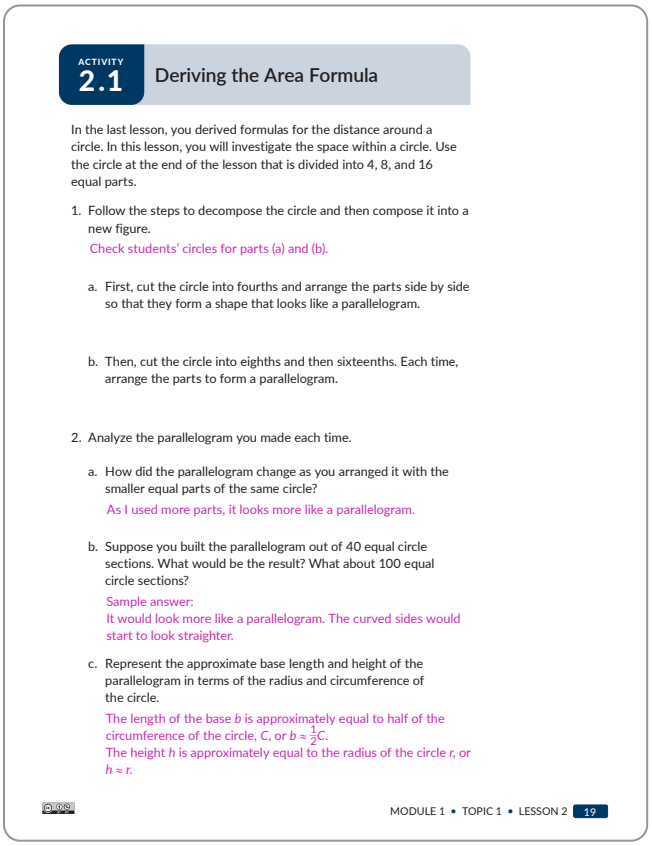
One example of this sequence occurs in the topic *Circles and Ratio*. Students use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas (**TEKS 7.8C**). They then determine the circumference and area of circles (**TEKS 7.9B**).

Think About . . .

Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks.

Building Understanding

In the first activity of Lesson 2, *Area of Circles*, students deconstruct a circle and reconstruct it into a rectangle.

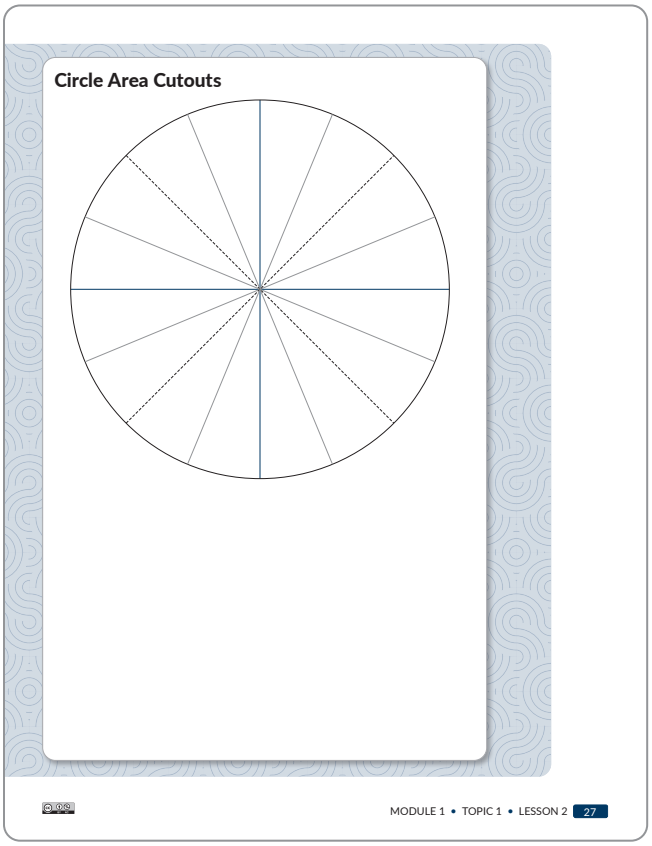


ACTIVITY 2.1 Deriving the Area Formula

In the last lesson, you derived formulas for the distance around a circle. In this lesson, you will investigate the space within a circle. Use the circle at the end of the lesson that is divided into 4, 8, and 16 equal parts.

1. Follow the steps to decompose the circle and then compose it into a new figure.
   Check students' circles for parts (a) and (b).
   a. First, cut the circle into fourths and arrange the parts side by side so that they form a shape that looks like a parallelogram.
   b. Then, cut the circle into eighths and then sixteenths. Each time, arrange the parts to form a parallelogram.
2. Analyze the parallelogram you made each time.
   a. How did the parallelogram change as you arranged it with the smaller equal parts of the same circle?
      As I used more parts, it looks more like a parallelogram.
   b. Suppose you built the parallelogram out of 40 equal circle sections. What would be the result? What about 100 equal circle sections?
      Sample answer:
      It would look more like a parallelogram. The curved sides would start to look straighter.
   c. Represent the approximate base length and height of the parallelogram in terms of the radius and circumference of the circle.
      The length of the base b is approximately equal to half of the circumference of the circle, C, or $b \approx \frac{1}{2}C$.
      The height h is approximately equal to the radius of the circle r, or $h \approx r$.

MODULE 1 • TOPIC 1 • LESSON 2 19



Circle Area Cutouts

MODULE 1 • TOPIC 1 • LESSON 2 27

Representing the Concept

As Activity 1 continues, students analyze each shape they form and discover that the shape appears to approach that of a rectangle as the equal parts get smaller. Students write the formula for the area of a rectangle and substitute the radius for the height and half of the circumference for the base to develop the formula for the area of a circle.

2. Analyze the parallelogram you made each time.

   a. How did the parallelogram change as you arranged it with the smaller equal parts of the same circle?

   As I used more parts, it looks more like a parallelogram.

   b. Suppose you built the parallelogram out of 40 equal circle sections. What would be the result? What about 100 equal circle sections?

   Sample answer:
   It would look more like a parallelogram. The curved sides would start to look straighter.

   c. Represent the approximate base length and height of the parallelogram in terms of the radius and circumference of the circle.

   The length of the base b is approximately equal to half of the circumference of the circle, C, or $b \approx \frac{1}{2}C$.
   The height h is approximately equal to the radius of the circle r, or $h \approx r$.

Moving to Symbols

Finally, students use the newly derived formula to calculate the area of circles given different measurements for a radius.

3. Use different representations for π to calculate the area of a circle.

   a. Calculate the area of each circle with the given radius. Round your answers to the nearest ten-thousandths, when necessary.

   See table below.

Value for π	$r = 6$ units	$r = 1.5$ units	$r = \frac{1}{2}$ unit
π	36π	2.25π	$\frac{1}{4}\pi$
Use the π key on a calculator	$\approx$ 113.0973 sq units	$\approx$ 7.0686 sq units	$\approx$ 0.7854 sq units
Use 3.14 for π	$\approx$ 113.04 sq units	$\approx$ 7.065 sq units	$\approx$ 0.785 sq units
Use $\frac{22}{7}$ for π	$\approx$ 113.1429 sq units	$\approx$ 7.0714 sq units	$\frac{11}{14}$ or $\approx$ 0.7857 sq units

   b. Compare your area calculations for each circle. How do the different values of π affect your calculations?

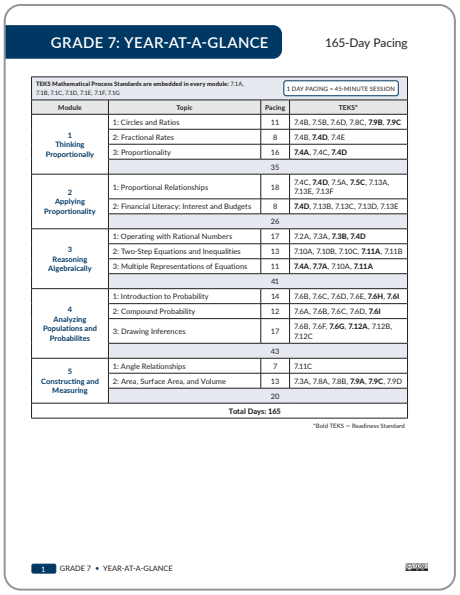
   The calculations that leave π in the answer represent the exact area. The calculations using the π key on the calculator are the best area approximations. The calculations using 3.14 for π are slightly less than those approximations using the p key. The calculations using $\frac{22}{7}$ for π are slightly greater than those approximations using the π key.

Look for similar sequences throughout the textbook.

Course-Level Documents

Year-at-a-Glance

The Year-at-a-Glance highlights the sequence of topics and the number of instructional days allocated for the course.



GRADE 7: YEAR-AT-A-GLANCE

165-Day Pacing

TEKS Mathematical Process Standards are embedded in every module: 7.1A, 7.1B, 7.1C, 7.1D, 7.1E, 7.1F, 7.1G

1 DAY PACING = 45-MINUTE SESSION

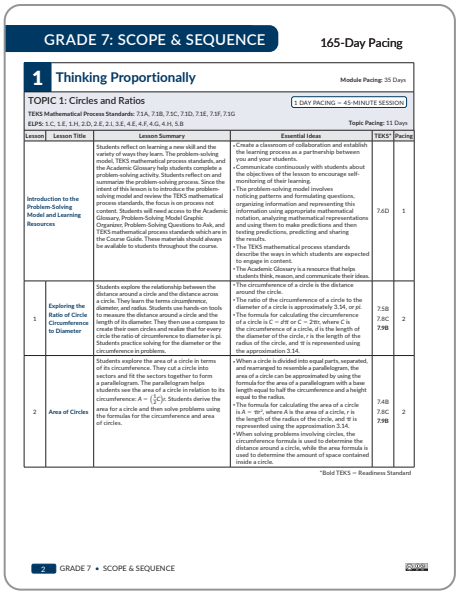
Module	Topic	Pacing	TEKS*
1 Thinking Proportionally	1: Circles and Ratios	11	7.4B, 7.5B, 7.6D, 7.8C, **7.9B**, **7.9C**
	2: Fractional Rates	8	7.4B, **7.4D**, 7.4E
	3: Proportionality	16	**7.4A**, 7.4C, **7.4D**
		35	
2 Applying Proportionality	1: Proportional Relationships	18	7.4C, **7.4D**, 7.5A, **7.5C**, 7.13A, 7.13E, 7.13F
	2: Financial Literacy: Interest and Budgets	8	**7.4D**, 7.13B, 7.13C, 7.13D, 7.13E
		26	
3 Reasoning Algebraically	1: Operating with Rational Numbers	17	7.2A, 7.3A, **7.3B**, **7.4D**
	2: Two-Step Equations and Inequalities	13	7.10A, 7.10B, 7.10C, **7.11A**, 7.11B
	3: Multiple Representations of Equations	11	**7.4A**, **7.7A**, 7.10A, **7.11A**
		41	
4 Analyzing Populations and Probabilities	1: Introduction to Probability	14	7.6B, 7.6C, 7.6D, 7.6E, **7.6H**, **7.6I**
	2: Compound Probability	12	7.6A, 7.6B, 7.6C, 7.6D, **7.6I**
	3: Drawing Inferences	17	7.6B, 7.6F, **7.6G**, **7.12A**, 7.12B, 7.12C
		43	
5 Constructing and Measuring	1: Angle Relationships	7	7.11C
	2: Area, Surface Area, and Volume	13	7.3A, 7.8A, 7.8B, **7.9A**, **7.9C**, 7.9D
		20	
	Total Days: 165		

*Bold TEKS = Readiness Standard

1 GRADE 7 • YEAR-AT-A-GLANCE

Scope and Sequence

The Scope and Sequence provides lesson highlights, pacing suggestions, and essential ideas for each lesson.



GRADE 7: SCOPE & SEQUENCE

165-Day Pacing

1 Thinking Proportionally

Module Pacing: 35 Days

TOPIC 1: Circles and Ratios

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1A, 7.1B, 7.1C, 7.1D, 7.1E, 7.1F, 7.1G

ELPS: 1.C, 1.E, 1.H, 2.D, 2.E, 2.I, 3.E, 4.E, 4.F, 4.G, 4.H, 5.B

Topic Pacing: 11 Days

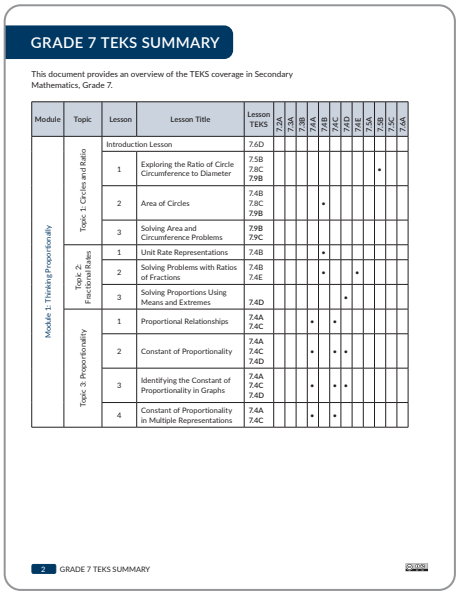
Lesson	Lesson Title	Lesson Summary	Essential Ideas	TEKS*	Pacing
	Introduction to the Problem-Solving Model and Learning Resources	Students reflect on learning a new skill and the variety of ways they learn. The problem-solving model, TEKS mathematical process standards, and the Academic Glossary help students complete a problem-solving activity. Students reflect on and summarize the problem-solving process. Since the intent of this lesson is to introduce the problem-solving model and review the TEKS mathematical process standards, the focus is on process not content. Students will need access to the Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Questions to Ask, and TEKS mathematical process standards which are in the Course Guide. These materials should always be available to students throughout the course.	• Create a classroom of collaboration and establish the learning process as a partnership between you and your students. • Communicate continuously with students about the objectives of the lesson to encourage self-monitoring of their learning. • The problem-solving model involves noticing patterns and formulating questions, organizing information and representing this information using appropriate mathematical notation, analyzing mathematical representations and using them to make predictions and then testing predictions, predicting and sharing the results. • The TEKS mathematical process standards describe the ways in which students are expected to engage in content. • The *Academic Glossary* is a resource that helps students think, reason, and communicate their ideas.	7.6D	1
1	Exploring the Ratio of Circle Circumference to Diameter	Students explore the relationship between the distance around a circle and the distance across a circle. They learn the terms *circumference*, *diameter*, and *radius*. Students use hands-on tools to measure the distance around a circle and the length of its diameter. They then use a compass to create their own circles and realize that for every circle the ratio of circumference to diameter is pi. Students practice solving for the diameter or the circumference in problems.	• The circumference of a circle is the distance around the circle. • The ratio of the circumference of a circle to the diameter of a circle is approximately 3.14, or pi. • The formula for calculating the circumference of a circle is $C = \pi d$ or $C = 2\pi r$, where C is the circumference of a circle, d is the length of the diameter of the circle, r is the length of the radius of the circle, and π is represented using the approximation 3.14.	7.5B 7.8C **7.9B**	2
2	Area of Circles	Students explore the area of a circle in terms of its circumference. They cut a circle into sectors and fit the sectors together to form a parallelogram. The parallelogram helps students see the area of a circle in relation to its circumference: $A = (\frac{1}{2}c)r$. Students derive the area for a circle and then solve problems using the formulas for the circumference and area of circles.	• When a circle is divided into equal parts, separated, and rearranged to resemble a parallelogram, the area of a circle can be approximated by using the formula for the area of a parallelogram with a base length equal to half the circumference and a height equal to the radius. • The formula for calculating the area of a circle is $A = \pi r^2$, where A is the area of a circle, r is the length of the radius of the circle, and π is represented using the approximation 3.14. • When solving problems involving circles, the circumference formula is used to determine the distance around a circle, while the area formula is used to determine the amount of space contained inside a circle.	7.4B 7.8C **7.9B**	2

*Bold TEKS = Readiness Standard

2 GRADE 7 • SCOPE & SEQUENCE

TEKS Summary

The TEKS Summary visually represents where each standard is addressed within the course.



GRADE 7 TEKS SUMMARY

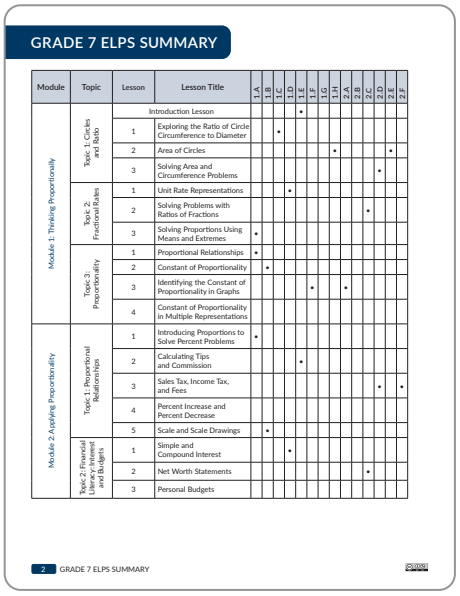
This document provides an overview of the TEKS coverage in Secondary Mathematics, Grade 7.

Module	Topic	Lesson	Lesson Title	Lesson TEKS	7.2A	7.3A	7.3B	7.4A	7.4B	7.4C	7.4D	7.4E	7.5A	7.5B	7.5C	7.7A
Module 1: Thinking Proportionally	Topic 1: Circles and Ratio	Introduction Lesson		7.6D												
		1	Exploring the Ratio of Circle Circumference to Diameter	7.5B 7.8C 7.9B										•		
		2	Area of Circles	7.4B 7.8C 7.9B					•							
		3	Solving Area and Circumference Problems	7.9B 7.9C												
	Topic 2: Fractional Rates	1	Unit Rate Representations	7.4B					•							
		2	Solving Problems with Ratios of Fractions	7.4B 7.4E					•			•				
		3	Solving Proportions Using Means and Extremes	7.4D							•					
	Topic 3: Proportionality	1	Proportional Relationships	7.4A 7.4C					•		•					
		2	Constant of Proportionality	7.4A 7.4C 7.4D					•		•	•				
		3	Identifying the Constant of Proportionality in Graphs	7.4A 7.4C 7.4D					•		•	•				
		4	Constant of Proportionality in Multiple Representations	7.4A 7.4C					•		•					

2 GRADE 7 TEKS SUMMARY

ELPS Summary

The ELPS Summary visually represents where each standard is addressed within the course.



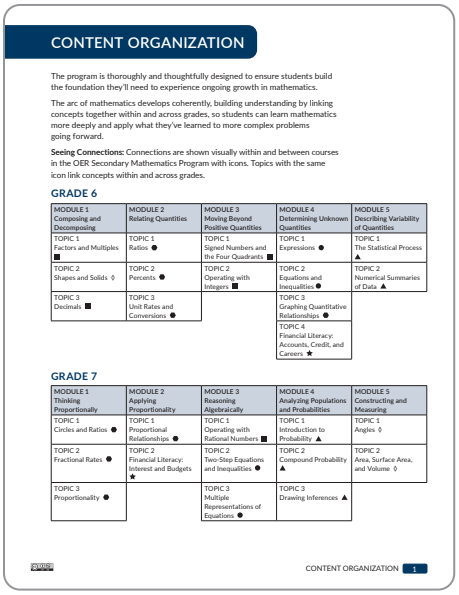
GRADE 7 ELPS SUMMARY

Module	Topic	Lesson	Lesson Title	1.A	1.B	1.C	1.D	1.E	1.F	1.G	1.H	2.B	2.C	2.D	2.E	2.I
Module 1: Thinking Proportionally	Topic 1: Circles and Ratio	Introduction Lesson						•								
		1	Exploring the Ratio of Circle Circumference to Diameter			•										
		2	Area of Circles							•					•	
		3	Solving Area and Circumference Problems											•		
	Topic 2: Fractional Rates	1	Unit Rate Representations				•									
		2	Solving Problems with Ratios of Fractions										•			
		3	Solving Proportions Using Means and Extremes	•												
	Topic 3: Proportionality	1	Proportional Relationships	•												
		2	Constant of Proportionality		•											
		3	Identifying the Constant of Proportionality in Graphs						•		•					
		4	Constant of Proportionality in Multiple Representations													
Module 2: Applying Proportionality	Topic 1: Proportional Relationships	1	Introducing Proportions to Solve Percent Problems	•												
		2	Calculating Tips and Commission					•								
		3	Sales Tax, Income Tax, and Fees											•		•
		4	Percent Increase and Percent Decrease													
		5	Scale and Scale Drawings	•												
	Topic 2: Financial Literacy: Interest and Budgets	1	Simple and Compound Interest				•									
		2	Net Worth Statements										•			
		3	Personal Budgets													

2 GRADE 7 ELPS SUMMARY

Content Organization

The Content Organization document provides insights into how the content is organized. It also identifies content connections within and across different courses.



CONTENT ORGANIZATION

The program is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

The arc of mathematics develops coherently, building understanding by linking concepts together within and across grades, so students can learn mathematics more deeply and apply what they've learned to more complex problems going forward.

Seeing Connections: Connections are shown visually within and between courses in the OER Secondary Mathematics Program with icons. Topics with the same icon link concepts within and across grades.

GRADE 6

MODULE 1 Composing and Decomposing	MODULE 2 Relating Quantities	MODULE 3 Moving Beyond Positive Quantities	MODULE 4 Determining Unknown Quantities	MODULE 5 Describing Variability of Quantities
TOPIC 1 Factors and Multiples ■	TOPIC 1 Ratios ●	TOPIC 1 Signed Numbers and the Four Quadrants ■	TOPIC 1 Expressions ●	TOPIC 1 The Statistical Process ▲
TOPIC 2 Shapes and Solids ◊	TOPIC 2 Percents ●	TOPIC 2 Operating with Integers ■	TOPIC 2 Equations and Inequalities ●	TOPIC 2 Numerical Summaries of Data ▲
TOPIC 3 Decimals ■	TOPIC 3 Unit Rates and Conversions ◆		TOPIC 3 Graphing Quantitative Relationships ●	
			TOPIC 4 Financial Literacy: Accounts, Credit, and Careers ★	

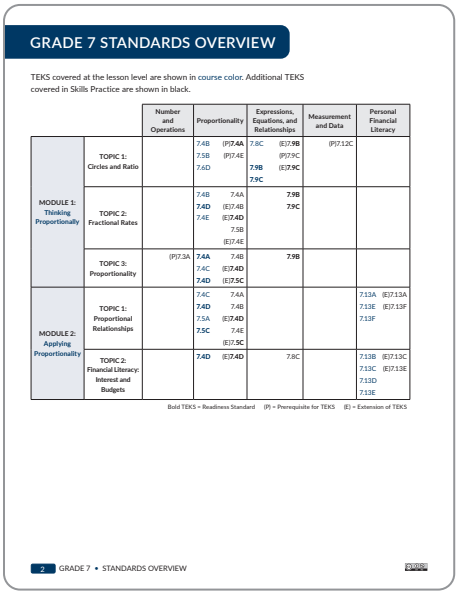
GRADE 7

MODULE 1 Thinking Proportionally	MODULE 2 Applying Proportionality	MODULE 3 Reasoning Algebraically	MODULE 4 Analyzing Populations and Probabilities	MODULE 5 Constructing and Measuring
TOPIC 1 Circles and Ratios ●	TOPIC 1 Proportional Relationships ●	TOPIC 1 Operating with Rational Numbers ■	TOPIC 1 Introduction to Probability ▲	TOPIC 1 Angles ◊
TOPIC 2 Fractional Rates ●	TOPIC 2 Financial Literacy: Interest and Budgets ★	TOPIC 2 Two-Step Equations and Inequalities ●	TOPIC 2 Compound Probability ▲	TOPIC 2 Area, Surface Area, and Volume ◊
TOPIC 3 Proportionality ●		TOPIC 3 Multiple Representations of Equations ●	TOPIC 3 Drawing Inferences ▲	

2.0 | CONTENT ORGANIZATION 1

Standards Overview

The Standards Overview lists the standards addressed in each topic. The Standards Overview shows the lesson and skills practice standards.



GRADE 7 STANDARDS OVERVIEW

TEKS covered at the lesson level are shown in course color. Additional TEKS covered in Skills Practice are shown in black.

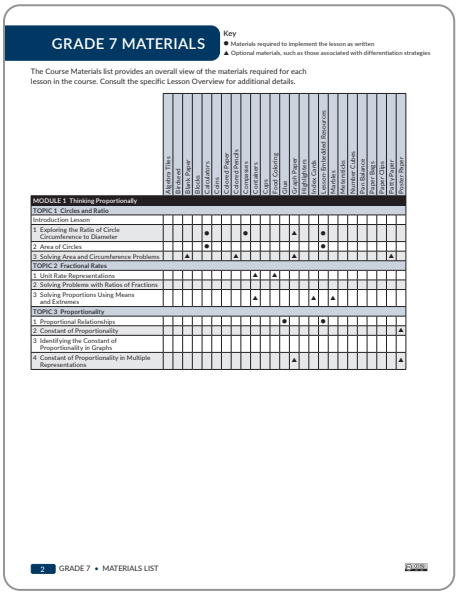
		Number and Operations	Proportionality	Expressions, Equations, and Relationships	Measurement and Data	Personal Financial Literacy
MODULE 1: Thinking Proportionally	TOPIC 1: Circles and Ratio		7.4B (P)7.4A 7.5B (P)7.4E 7.6D	7.8C (E)7.9B (P)7.9C 7.9B (E)7.9C 7.9C	(P)7.12C	
	TOPIC 2: Fractional Rates		7.4B 7.4A 7.4D (E)7.4B 7.4E (E)7.4D 7.5B (E)7.4E	7.9B 7.9C		
	TOPIC 3: Proportionality	(P)7.3A	7.4A 7.4B 7.4C (E)7.4D 7.4D (E)7.5C	7.9B		
MODULE 2: Applying Proportionality	TOPIC 1: Proportional Relationships		7.4C 7.4A 7.4D 7.4B 7.5A (E)7.4D 7.5C 7.4E (E)7.5C			7.13A (E)7.13A 7.13E (E)7.13F 7.13F
	TOPIC 2: Financial Literacy: Interest and Budgets		7.4D (E)7.4D	7.8C		7.13B (E)7.13C 7.13C (E)7.13E 7.13D 7.13E

Bold TEKS = Readiness Standard (P) = Prerequisite for TEKS (E) = Extension of TEKS

2 GRADE 7 • STANDARDS OVERVIEW 2.0

Materials List

The Materials List highlights the materials needed for the course.



GRADE 7 MATERIALS

Key
● Materials required to implement the lesson as written
▲ Optional materials, such as those associated with differentiation strategies

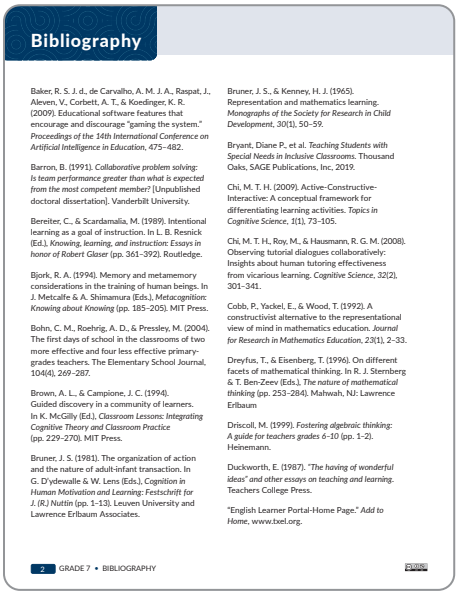
The Course Materials list provides an overall view of the materials required for each lesson in the course. Consult the specific Lesson Overview for additional details.

	Algebra Tiles	Blackline Masters	Blank Paper	Blocks	Calculators	Cards	Colored Pencils	Compasses	Construction Paper	Cups	Exit Ticket	Food Coloring	Glue	Graph Paper	Highlighters	Index Cards	Manipulatives	Markers	Measuring Tape	Number Cubes	Pan Balance	Patty Paper	Pencils	Rulers	Scissors	Spinners	Sticky Notes	Straws	Tape	Tracing Paper	Transparencies	Other Embedded Resources	Poster Paper
MODULE 1 Thinking Proportionally																																	
TOPIC 1 Circles and Ratio																																	
Introduction Lesson																																	
1 Exploring the Ratio of Circle Circumference to Diameter				●				●					▲		●																		
2 Area of Circles				●											●																		
3 Solving Area and Circumference Problems		▲					▲						▲																				
TOPIC 2 Fractional Rates																																	
1 Unit Rate Representations										▲		▲																					
2 Solving Problems with Ratios of Fractions																																	
3 Solving Proportions Using Means and Extremes										▲						●	▲																
TOPIC 3 Proportionality																																	
1 Proportional Relationships																																	
2 Constant of Proportionality																																	
3 Identifying the Constant of Proportionality in Graphs																																	
4 Constant of Proportionality in Multiple Representations													▲																				

2 GRADE 7 • MATERIALS LIST 2.0

Bibliography

The Bibliography contains a list of peer-reviewed articles and research used to influence the content of the instructional materials.



Bibliography

Baker, R. S. J. d., de Carvalho, A. M. J. A., Raspat, J., Aleven, V., Corbett, A. T., & Koedinger, K. R. (2009). Educational software features that encourage and discourage "gaming the system." *Proceedings of the 14th International Conference on Artificial Intelligence in Education*, 475–482.

Barron, B. (1991). *Collaborative problem solving: Is team performance greater than what is expected from the most competent member?* [Unpublished doctoral dissertation]. Vanderbilt University.

Bereiter, C., & Scardamalia, M. (1989). Intentional learning as a goal of instruction. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 361–392). Routledge.

Bjork, R. A. (1994). Memory and metamemory considerations in the training of human beings. In J. Metcalfe & A. Shimamura (Eds.), *Metacognition: Knowing about Knowing* (pp. 185–205). MIT Press.

Bohn, C. M., Roehrig, A. D., & Pressley, M. (2004). The first days of school in the classrooms of two more effective and four less effective primary-grades teachers. The Elementary School Journal, 104(4), 269–287.

Brown, A. L., & Campione, J. C. (1994). Guided discovery in a community of learners. In K. McGilly (Ed.), *Classroom Lessons: Integrating Cognitive Theory and Classroom Practice* (pp. 229–270). MIT Press.

Bruner, J. S. (1981). The organization of action and the nature of adult-infant transaction. In G. D'ydewalle & W. Lens (Eds.), *Cognition in Human Motivation and Learning: Festschrift for J. (R.) Nuttin* (pp. 1–13). Leuven University and Lawrence Erlbaum Associates.

Bruner, J. S., & Kenney, H. J. (1965). Representation and mathematics learning. *Monographs of the Society for Research in Child Development*, 30(1), 50–59.

Bryant, Diane P., et al. *Teaching Students with Special Needs in Inclusive Classrooms.* Thousand Oaks, SAGE Publications, Inc, 2019.

Chi, M. T. H. (2009). Active-Constructive-Interactive: A conceptual framework for differentiating learning activities. *Topics in Cognitive Science*, 1(1), 73–105.

Chi, M. T. H., Roy, M., & Hausmann, R. G. M. (2008). Observing tutorial dialogues collaboratively: Insights about human tutoring effectiveness from vicarious learning. *Cognitive Science*, 32(2), 301–341.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, 23(1), 2–33.

Dreyfus, T., & Eisenberg, T. (1996). On different facets of mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 253–284). Mahwah, NJ: Lawrence Erlbaum

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers grades 6–10* (pp. 1–2). Heinemann.

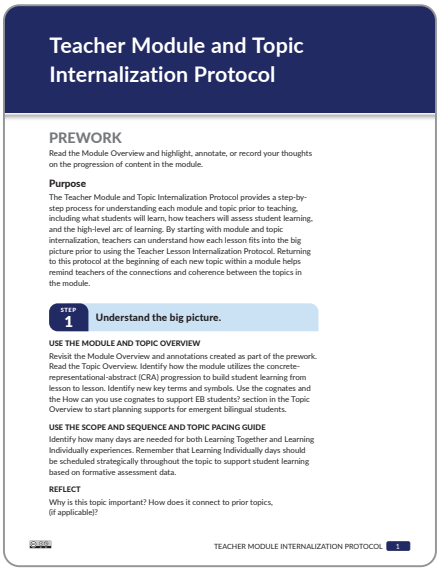
Duckworth, E. (1987). *"The having of wonderful ideas" and other essays on teaching and learning.* Teachers College Press.

"English Learner Portal-Home Page." *Add to Home*, www.txel.org.

2 GRADE 7 • BIBLIOGRAPHY 2.0

Module and Topic Internalization Protocol

The Module and Topic Internalization Protocol supports understanding of what students will learn, how students will be assessed, and the high-level arc of learning over the course of the module and topic. There is a Teacher and Coach version of the Module and Topic Internalization Protocol.



Teacher Module and Topic Internalization Protocol

PREWORK

Read the Module Overview and highlight, annotate, or record your thoughts on the progression of content in the module.

Purpose

The Teacher Module and Topic Internalization Protocol provides a step-by-step process for understanding each module and topic prior to teaching, including what students will learn, how teachers will assess student learning, and the high-level arc of learning. By starting with module and topic internalization, teachers can understand how each lesson fits into the big picture prior to using the Teacher Lesson Internalization Protocol. Returning to this protocol at the beginning of each new topic within a module helps remind teachers of the connections and coherence between the topics in the module.

STEP 1 Understand the big picture.

USE THE MODULE AND TOPIC OVERVIEW

Revisit the Module Overview and annotations created as part of the prework. Read the Topic Overview. Identify how the module utilizes the concrete-representational-abstract (CRA) progression to build student learning from lesson to lesson. Identify new key terms and symbols. Use the cognates and the How can you use cognates to support EB students? section in the Topic Overview to start planning supports for emergent bilingual students.

USE THE SCOPE AND SEQUENCE AND TOPIC PACING GUIDE

Identify how many days are needed for both Learning Together and Learning Individually experiences. Remember that Learning Individually days should be scheduled strategically throughout the topic to support student learning based on formative assessment data.

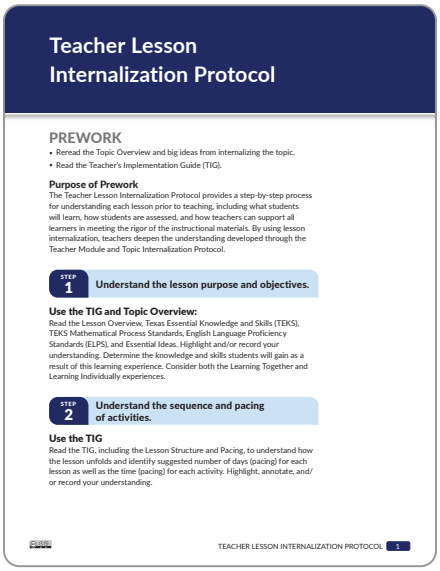
REFLECT

Why is this topic important? How does it connect to prior topics, (if applicable)?

TEACHER MODULE INTERNALIZATION PROTOCOL 1

Lesson Internalization Protocol

The Lesson Internalization Protocol focuses on what students will learn in a specific lesson, how students will be assessed, and helps determine how to make decisions about teaching the lesson to support all students' success. There is a Teacher and Coach version of the Lesson Internalization Protocol.



Teacher Lesson Internalization Protocol

PREWORK

- Reread the Topic Overview and big ideas from internalizing the topic.
- Read the Teacher's Implementation Guide (TIG).

Purpose of Prework

The Teacher Lesson Internalization Protocol provides a step-by-step process for understanding each lesson prior to teaching, including what students will learn, how students are assessed, and how teachers can support all learners in meeting the rigor of the instructional materials. By using lesson internalization, teachers deepen the understanding developed through the Teacher Module and Topic Internalization Protocol.

STEP 1 Understand the lesson purpose and objectives.

Use the TIG and Topic Overview:

Read the Lesson Overview, Texas Essential Knowledge and Skills (TEKS), TEKS Mathematical Process Standards, English Language Proficiency Standards (ELPS), and Essential Ideas. Highlight and/or record your understanding. Determine the knowledge and skills students will gain as a result of this learning experience. Consider both the Learning Together and Learning Individually experiences.

STEP 2 Understand the sequence and pacing of activities.

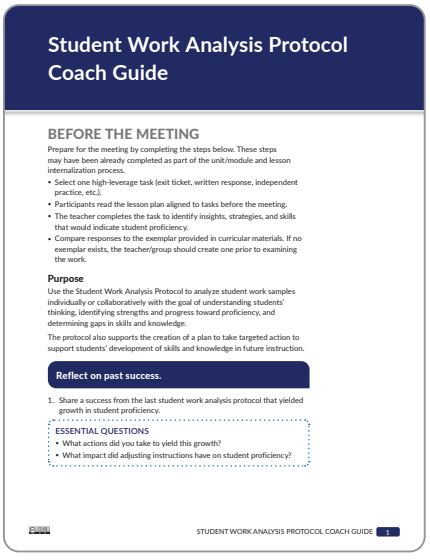
Use the TIG

Read the TIG, including the Lesson Structure and Pacing, to understand how the lesson unfolds and identify suggested number of days (pacing) for each lesson as well as the time (pacing) for each activity. Highlight, annotate, and/or record your understanding.

TEACHER LESSON INTERNALIZATION PROTOCOL 1

Student Work Analysis Protocol

The Student Work Analysis Protocol is a tool for analyzing student work samples individually or collaboratively to understand students' thinking, identify strengths and progress toward mastery, and determine gaps in skills. There is a Teacher and Coach version of the Student Work Analysis Protocol.



Student Work Analysis Protocol Coach Guide

BEFORE THE MEETING

Prepare for the meeting by completing the steps below. These steps may have been already completed as part of the unit/module and lesson internalization process.

- Select one high-leverage task (exit ticket, written response, independent practice, etc.).
- Participants read the lesson plan aligned to tasks before the meeting.
- The teacher completes the task to identify insights, strategies, and skills that would indicate student proficiency.
- Compare responses to the exemplar provided in curricular materials. If no exemplar exists, the teacher/group should create one prior to examining the work.

Purpose

Use the Student Work Analysis Protocol to analyze student work samples individually or collaboratively with the goal of understanding students' thinking, identifying strengths and progress toward proficiency, and determining gaps in skills and knowledge.

The protocol also supports the creation of a plan to take targeted action to support students' development of skills and knowledge in future instruction.

Reflect on past success.

1. Share a success from the last student work analysis protocol that yielded growth in student proficiency.

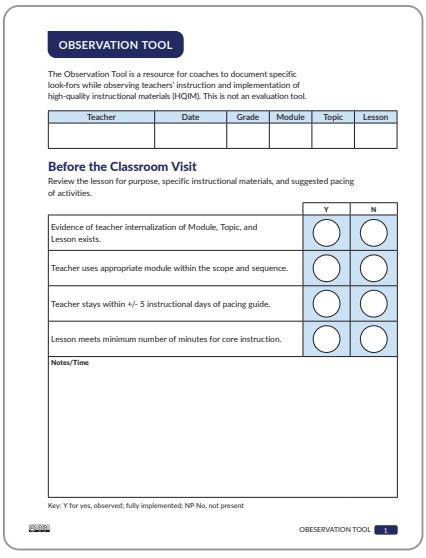
ESSENTIAL QUESTIONS

- What actions did you take to yield this growth?
- What impact did adjusting instructions have on student proficiency?

STUDENT WORK ANALYSIS PROTOCOL COACH GUIDE 1

Observation Tool

The Observation Tool is a resource to document specific look-fors while observing instruction and implementation of High Quality Instructional Materials (HQIM). You can use these document to reflect on your instruction.



OBSERVATION TOOL

The Observation Tool is a resource for coaches to document specific look-fors while observing teachers' instruction and implementation of high-quality instructional materials (HQIM). This is not an evaluation tool.

Teacher	Date	Grade	Module	Topic	Lesson

Before the Classroom Visit

Review the lesson for purpose, specific instructional materials, and suggested pacing of activities.

	Y	N
Evidence of teacher internalization of Module, Topic, and Lesson exists.	◯	◯
Teacher uses appropriate module within the scope and sequence.	◯	◯
Teacher stays within +/- 5 instructional days of pacing guide.	◯	◯
Lesson meets minimum number of minutes for core instruction.	◯	◯

Notes/Time

Key: Y for yes, observed; fully implemented; NP No, not present

OBSERVATION TOOL 1

Connecting Content and Practice

LESSON STRUCTURE
Each lesson of the course has the same structure. This consistency allows both you and your students to internalize the lesson progression. Key features of each lesson are noted.

1 Objectives
Objectives are stated for each lesson to help you take ownership of the objectives.

2 Essential Question
Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3 New Key Terms
The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

1 Exploring the Ratio of Circle Circumference to Diameter

1 OBJECTIVES

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Understand the formula for the circumference of a circle and use the formula to solve problems.

3 NEW KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

2

You have learned about ratios.

How can you use ratios to analyze the properties of geometric figures, such as circles?

Engage

Establishing Mathematical Goals to Focus Learning

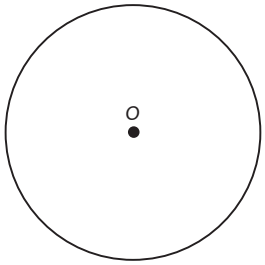
Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the objectives of the lesson to encourage self-monitoring of their learning.

4 Getting Started

Across and Around

Consider the circle with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.



1. Analyze the distance around the circle.

   a. Use a string and a centimeter ruler to determine the distance around the circle.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

2. Draw a line from a point on the circle to the center of the circle, point *O*.

   a. Measure your line using your centimeter ruler.

   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning. Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

5 Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

ACTIVITY 5 1.1

Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a bit harder. Can you define a circle without using the word *round*? Investigating how a circle is formed will help you mathematically define a circle.

1. Follow the given steps to investigate how a circle is formed.

Step 1: In the space provided, draw a point and label the point *A*.

Step 2: Use a centimeter ruler to locate and draw a second point that is exactly 5 cm from point *A*. Label this point *B*.

Step 3: Locate a third point that is exactly 5 cm from point *A*. Label this point *C*.

Step 4: Repeat this process until you have drawn at least ten distinct points that are each exactly 5 cm from point *A*.

2. How many other points could be located exactly 5 cm from point *A*? How would you describe this collection of points in relation to point *A*?

3. Define the term *circle* without using the word *round*.

Develop

Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

Talk the Talk 6

Twice

Use what you have learned to compare circles by their characteristics.

1. Using your compass, draw each circle.

   a. Radius length of 3 centimeters

   b. Diameter length of 3 centimeters

2. Describe the similarities and differences between your two circles.

3. Describe the relationship between the circumferences of the two circles.

4. Describe the circumference-to-diameter ratio of all circles.

6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.

Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Demonstrate

Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the objectives. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

ASSIGNMENT
An intentionally designed Assignment follows each lesson. The assignment is an additional component that is useful for Tier 1 instruction.

7 Write
Reflect on your work and clarify your thinking.

8 Remember
Take note of the key concepts from the lesson.

9 Practice
Use the concepts learned in the lesson to solve problems.

There is one assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the assignment students can complete based on that day's progress.

Use the assignment as homework throughout or after a lesson, as additional practice before students leave class, or as a review tool before assessments.

Lesson 1 Assignment

7 Write

Define each term in your own words.

1. Circle
2. Radius
3. Diameter
4. Pi

Remember 8

The circumference of a circle is the distance around the circle. The formulas to determine the circumference of a circle are $C = \pi d$, or $C = 2\pi r$, where d represents the diameter, r represents the radius, and π is a constant value equal to approximately 3.14 or $\frac{22}{7}$.

The constant pi (π) represents the ratio of the circumference of a circle to its diameter.

9 Practice

Answer each question. Use 3.14 for π. Round your answer to the nearest tenth, when necessary.

1. Although he's only in middle school, Mason loves to drive go-carts! His favorite place to drive go-carts has 3 circular tracks. Track 1 has a radius of 60 feet. Track 2 has a radius of 85 feet. Track 3 has a radius of 110 feet.

   a. Compute the circumference of Track 1.

   b. Compute the circumference of Track 2.

   c. Compute the circumference of Track 3.

   d. The go-cart place is considering building a new track. They have a circular space with a diameter of 150 feet. Compute the circumference of the circular space.

Lesson 1 Assignment

2. Mason wants to build a circular go-cart track in his backyard. Use 3.14 for π.

   a. Suppose he wants the track to have a circumference of 157 feet. What does the radius of the track need to be?

   b. Suppose he wants the track to have a circumference of 314 feet. What does the radius of the track need to be?

   c. Suppose he wants the track to have a circumference of 471 feet. What does the diameter of the track need to be?

10 Prepare

Determine a unit rate for each situation.

1. $38.40 for 16 gallons of gas

2. 15 miles jogged in 3.75 hours

3. $26.99 for 15 pounds

ASSIGNMENT

10 Prepare

Get ready for the next lesson.

The **Prepare** section provides spaced retrieval of concepts related to previous learning and fluency skills important for the course.

Note

Look at each lesson and assignment in the context of the over-arching story of the mathematics for the course. One lesson or activity may not fully cover a particular TEKS. However, in these cases, the lesson or activity is a building block to another lesson or topic where the TEKS is fully covered.

Internal researchers collaborated with various independent research organizations, tirelessly working to understand more about how people learn and how to best facilitate learning. This information was supplemented with feedback and data from educators, students, and the community to continuously evaluate and elevate the instructional approach and delivery.

The embedded strategies, tools, and guidance provided in these instructional resources are informed by books like *Adding it Up*, *How People Learn*, and *Principles to Actions*.

Worked Examples

- Worked Examples provide a means for students to view each step taken to solve the example problem.
- The questions that follow serve as a model for self-questioning and self-explanations. They represent and mimic an internal dialog about the mathematics and the strategies.
- This approach doesn't allow students to skip over the example without interacting with it, thinking about it, and responding to the questions.
- This approach will help students recognize the value in identifying steps, organizing their thinking, and codifying processes for problem solving.

WORKED EXAMPLE

Isabella's first client of the day spent $150 to have her hair dyed and cut and gave Isabella a $30 tip.

Use a Proportion

$$\frac{t}{100} = \frac{30}{150}$$

$$t = \frac{(30)(100)}{150}$$

$$t = 20$$

Use a Percent Equation

$$(t)(150) = 30$$

$$150t = 30$$

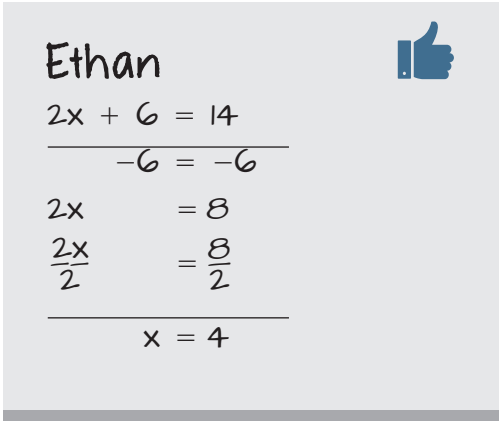
$$\frac{150t}{150} = \frac{30}{150}$$

$$t = \frac{30}{150}$$

$$t = 0.2$$

Thumbs Up/Thumbs Down

- Thumbs Up problems allow students the opportunity to analyze viable methods and problem-solving strategies. We present questions to help students think more in-depth about the various strategies and analyze correct responses.
- Research shows that only providing positive examples does not eliminate some of the common misconceptions students may have. Negative examples provide a way to directly address common misconceptions.
- Thumbs Down problems, showing incorrect responses, allow students to identify and explain errors and make corrections.



Who's Correct?

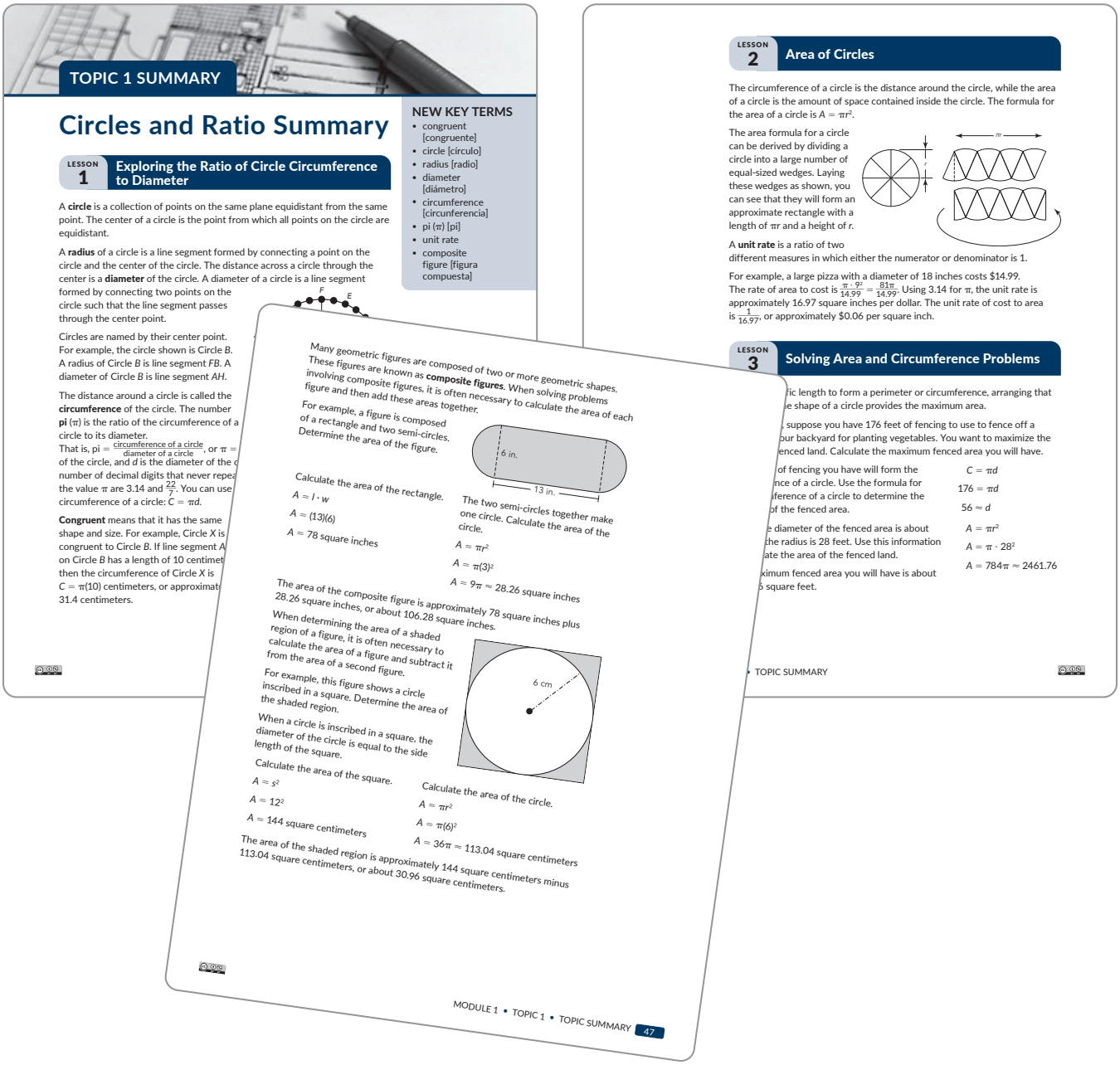
Who's Correct? problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean and whether each of the solutions makes sense.

5. Harper says that the product being less than 10 and the product being more than 10 are complementary events. Mia disagrees. Who is correct? Explain your reasoning.

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all new key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or Worked Examples.

Consider using the Topic Summary to help students review content when they were absent or are preparing for a test. The Topic Summary can also provide insight into the key ideas and examples from each lesson.



TOPIC 1 SUMMARY

Circles and Ratio Summary

NEW KEY TERMS
- congruent [congruente]
- circle [círculo]
- radius [radio]
- diameter [diámetro]
- circumference [circunferencia]
- pi (π) [pi]
- unit rate
- composite figure [figura compuesta]

LESSON 1 Exploring the Ratio of Circle Circumference to Diameter

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant.

A **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is a **diameter** of the circle. A diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.

Circles are named by their center point. For example, the circle shown is Circle *B*. A radius of Circle *B* is line segment *FB*. A diameter of Circle *B* is line segment *AH*.

The distance around a circle is called the **circumference** of the circle. The number **pi** (π) is the ratio of the circumference of a circle to its diameter.
That is, pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi =$ of the circle, and d is the diameter of the c number of decimal digits that never repea the value π are 3.14 and $\frac{22}{7}$. You can use circumference of a circle: $C = \pi d$.

Congruent means that it has the same shape and size. For example, Circle *X* is congruent to Circle *B*. If line segment *A* on Circle *B* has a length of 10 centimet then the circumference of Circle *X* is $C = \pi(10)$ centimeters, or approximat 31.4 centimeters.

LESSON 2 Area of Circles

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. The formula for the area of a circle is $A = \pi r^2$.

The area formula for a circle can be derived by dividing a circle into a large number of equal-sized wedges. Laying these wedges as shown, you can see that they will form an approximate rectangle with a length of πr and a height of r.

A **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

For example, a large pizza with a diameter of 18 inches costs $14.99. The rate of area to cost is $\frac{\pi \cdot 9^2}{14.99} = \frac{81\pi}{14.99}$. Using 3.14 for π, the unit rate is approximately 16.97 square inches per dollar. The unit rate of cost to area is $\frac{1}{16.97}$, or approximately $0.06 per square inch.

LESSON 3 Solving Area and Circumference Problems

ic length to form a perimeter or circumference, arranging that e shape of a circle provides the maximum area.

suppose you have 176 feet of fencing to use to fence off a our backyard for planting vegetables. You want to maximize the enced land. Calculate the maximum fenced area you will have.

of fencing you have will form the nce of a circle. Use the formula for ference of a circle to determine the of the fenced area.

$C = \pi d$
$176 = \pi d$
$56 \approx d$

e diameter of the fenced area is about the radius is 28 feet. Use this information ate the area of the fenced land.

$A = \pi r^2$
$A = \pi \cdot 28^2$
$A = 784\pi \approx 2461.76$

ximum fenced area you will have is about square feet.

TOPIC SUMMARY

Many geometric figures are composed of two or more geometric shapes. These figures are known as **composite figures**. When solving problems involving composite figures, it is often necessary to calculate the area of each figure and then add these areas together.

For example, a figure is composed of a rectangle and two semi-circles. Determine the area of the figure.

Calculate the area of the rectangle.

$A = l \cdot w$
$A = (13)(6)$
$A = 78$ square inches

The two semi-circles together make one circle. Calculate the area of the circle.

$A = \pi r^2$
$A = \pi(3)^2$
$A = 9\pi \approx 28.26$ square inches

The area of the composite figure is approximately 78 square inches plus 28.26 square inches, or about 106.28 square inches.

When determining the area of a shaded region of a figure, it is often necessary to calculate the area of a figure and subtract it from the area of a second figure.

For example, this figure shows a circle inscribed in a square. Determine the area of the shaded region.

When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.

Calculate the area of the square.

$A = s^2$
$A = 12^2$
$A = 144$ square centimeters

Calculate the area of the circle.

$A = \pi r^2$
$A = \pi(6)^2$
$A = 36\pi \approx 113.04$ square centimeters

The area of the shaded region is approximately 144 square centimeters minus 113.04 square centimeters, or about 30.96 square centimeters.

MODULE 1 • TOPIC 1 • TOPIC SUMMARY 47

Topic Self-Reflection

The Topic Self-Reflection allows students to reflect on their understanding of the concepts and skills they learn in the topic. The Topic Self-Reflection is designed for students to reflect on their understanding of concepts at the beginning, middle, and end of the topic, so they can track and monitor their own progress and growth.

You can use this information to guide conversations with students about their strengths, areas for improvement, and strategies for growth. Additionally, the Topic Self-Reflection can highlight areas for additional practice or review.

TOPIC 1 SELF-REFLECTION Name: _____

Circles and Ratios

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents I **am building proficiency of the skill**, and 3 represents I **have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Circles and Ratios* topic by:

TOPIC 1: *Circles and Ratios*	Beginning of Topic	Middle of Topic	End of Topic
describing pi (π) as the ratio of the circumference to the diameter of a circle.	○	○	○
using models to derive and explain the relationship between circumference and area of a circle.	○	○	○
justifying the formulas for area and circumference of a circle and how they relate to pi (π).	○	○	○
applying the circumference and area formulas to solve mathematical and real-world problems.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Circles and Ratios* topic.

_____

_____

_____

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

_____

_____

_____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

_____

_____

_____

Teacher's Edition

The Teacher's Edition is designed to fully support a wide range of teachers implementing our materials: from new teachers to experienced teachers.

One goal in developing the Teacher's Edition was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

What makes this Teacher's Edition useful?

Effective Lesson Design

Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing

Each course is designed to be taught in a 165-day school year. Pacing suggestions are provided for each lesson. Each day in the Pacing Guide is an equivalent to about a 45-minute instructional session.

Instructional Supports

Guiding questions are provided for teachers to use as they're circulating the room as well as differentiation strategies, common student misconceptions, and student look-fors.

Clearly Defined Mathematics

The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The Teacher's Edition is critical to understanding how the mathematics that students encounter should be realized in the classroom. The Teacher's Edition describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students.

Module and Topic Overviews

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics and how topics build to achieve understanding throughout the course.

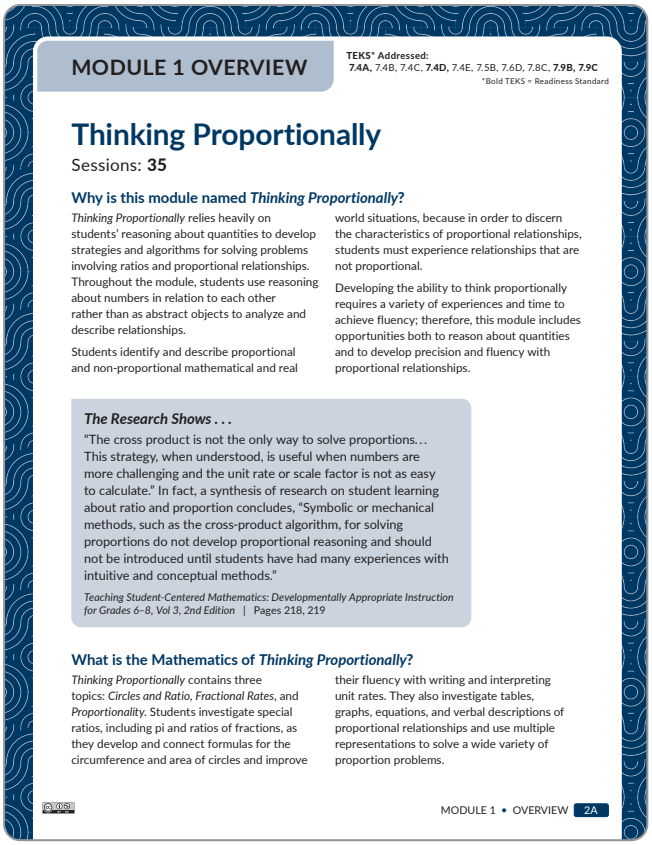
Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, and the connections to future learning.

Topic Overview

The Topic Overview describes how the topic is organized from the entry point for students to how students will demonstrate understanding. The Topic Overview explains why the mathematics of the topic is important and how the activities within the topic promote expertise in the TEKS mathematical process standards. A materials list, visual representations or strategies used, and suggested pacing information is also provided.

> "Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)"
>
> *Drake & Sherin*, 2009 | Page 325



MODULE 1 OVERVIEW

TEKS* Addressed:
7.4A, 7.4B, 7.4C, **7.4D,** 7.4E, 7.5B, 7.6D, 7.8C, **7.9B, 7.9C**
*Bold TEKS = Readiness Standard

Thinking Proportionally

Sessions: **35**

Why is this module named *Thinking Proportionally*?

Thinking Proportionally relies heavily on students' reasoning about quantities to develop strategies and algorithms for solving problems involving ratios and proportional relationships. Throughout the module, students use reasoning about numbers in relation to each other rather than as abstract objects to analyze and describe relationships.

Students identify and describe proportional and non-proportional mathematical and real world situations, because in order to discern the characteristics of proportional relationships, students must experience relationships that are not proportional.

Developing the ability to think proportionally requires a variety of experiences and time to achieve fluency; therefore, this module includes opportunities both to reason about quantities and to develop precision and fluency with proportional relationships.

The Research Shows . . .

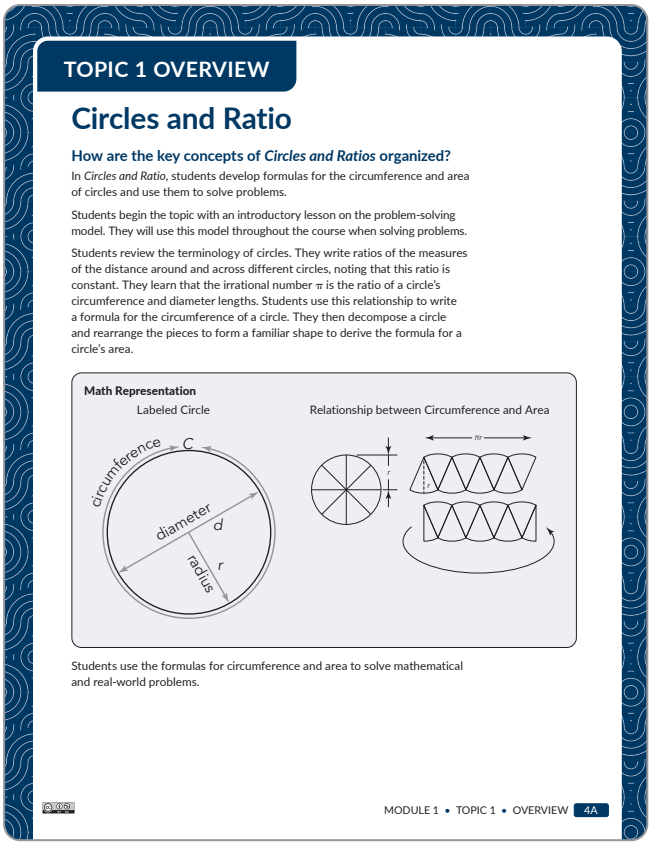
"The cross product is not the only way to solve proportions… This strategy, when understood, is useful when numbers are more challenging and the unit rate or scale factor is not as easy to calculate." In fact, a synthesis of research on student learning about ratio and proportion concludes, "Symbolic or mechanical methods, such as the cross-product algorithm, for solving proportions do not develop proportional reasoning and should not be introduced until students have had many experiences with intuitive and conceptual methods."

Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 6–8, Vol 3, 2nd Edition | Pages 218, 219

What is the Mathematics of *Thinking Proportionally*?

Thinking Proportionally contains three topics: *Circles and Ratio*, *Fractional Rates*, and *Proportionality*. Students investigate special ratios, including pi and ratios of fractions, as they develop and connect formulas for the circumference and area of circles and improve their fluency with writing and interpreting unit rates. They also investigate tables, graphs, equations, and verbal descriptions of proportional relationships and use multiple representations to solve a wide variety of proportion problems.

MODULE 1 • OVERVIEW 2A



TOPIC 1 OVERVIEW

Circles and Ratio

How are the key concepts of *Circles and Ratios* organized?

In *Circles and Ratio*, students develop formulas for the circumference and area of circles and use them to solve problems.

Students begin the topic with an introductory lesson on the problem-solving model. They will use this model throughout the course when solving problems.

Students review the terminology of circles. They write ratios of the measures of the distance around and across different circles, noting that this ratio is constant. They learn that the irrational number π is the ratio of a circle's circumference and diameter lengths. Students use this relationship to write a formula for the circumference of a circle. They then decompose a circle and rearrange the pieces to form a familiar shape to derive the formula for a circle's area.

Math Representation

Labeled Circle

Relationship between Circumference and Area

Students use the formulas for circumference and area to solve mathematical and real-world problems.

MODULE 1 • TOPIC 1 • OVERVIEW 4A

Facilitation Notes Support your planning process. Point-of-use strategies are your resource for planning, guiding, and facilitating student learning.

3 Determining Experimental Probability of Simple Events

1 MATERIALS

Coins
Paper or Plastic Cups
Paper Clips

2 LESSON OVERVIEW

Students flip a coin multiple times to determine the probabilities of heads and tails based on the results of the experiment. The terms *theoretical probability* and *experimental probability* are defined in this lesson. Students conduct trials of a game to estimate probabilities of the three outcomes. They also conduct trials of a spinner game to calculate experimental probabilities using data. Students use those experimental probabilities to predict the number of outcomes for a given number of trials. They then compare the experimental probabilities to the theoretical probabilities using *percent error*.

3 GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1E create and use representations to organize, record, and communicate mathematical ideas.

7.1F analyze mathematical relationships to connect and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.
The student is expected to:

7.6C make predictions and determine solutions using experimental data for simple and compound events.

7.6D make predictions and determine solutions using theoretical probability for simple and compound events.

7.6H solve problems using qualitative and quantitative predictions and comparisons from simple experiments.

7.6I determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

4 ELPS

(1) Learning Strategies
The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(4) Reading
The student is expected to:

(B) recognize directionality of English reading such as left to right and top to bottom.

(C) develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials.

5 ESSENTIAL IDEAS

- *Experimental probability* is the ratio of the number of times an event occurs to the total number of trials performed.
- *Theoretical probability* is the ratio of the number of desired outcomes to the total possible outcomes.
- Percent error is one way to measure the difference between experimental and theoretical probabilities.

1 Materials

Materials required for the lesson are identified.

2 Lesson Overview

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

3 TEKS Addressed

The highlighted TEKS mathematical process standards and TEKS content standards for each lesson are listed. A visual display accompanies the content standards to show coverage of the TEKS throughout the course. When the circle is completely shaded in course color, that is the last lesson associated with that particular TEKS. A star inside the circle indicates a readiness standard.

4 ELPS Addressed

Highlighted English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

5 Essential Ideas

These statements are derived from the standards and state the concepts students will develop.

LESSON STRUCTURE AND PACING: 2 DAYS (6)

DAY 1

(7)

ENGAGE

Getting Started: Flip It! 10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students flip a coin 30 times and determine the probabilities of heads and tails based on the results of the experiment. They then combine their results to see how the probability changes with more trials included. This activity is designed to engage students in thinking about experimental probability from an intuitive perspective.

DEVELOP

Activity 3.1: Determining Experimental Probability 10–15 minutes

INVESTIGATION

Students play a game. Working with a partner, they will toss a cup 25 times and record each trial using tally marks. The terms *experimental probability* and *theoretical probability* are introduced and students calculate the experimental probability of each event using their data. They conclude that the game is not a fair game, and it is not possible to determine the actual probability of each event due to the shape of the cup.

Activity 3.2: Predicting Theoretical Probability Using Experimental Probability 15–20 minutes

INVESTIGATION

A spinner is divided unequally into three sectors. Students list the sample space and predict the probability of each outcome. They conclude that a protractor could be used to determine the actual probabilities. Using a paper clip as the pointer and working with a partner, students conduct 50 trials and record the data in a table using tally marks. From this information, they calculate the experimental probabilities. Students revisit their predictions and compare them to their experimental probability.

DAY 2

Activity 3.3: Comparing Probabilities Using Proportional Reasoning 20 minutes

INVESTIGATION, WORKED EXAMPLE

Students use given probabilities of specified events occurring in conjunction with proportional reasoning to make predictions. Students follow a Worked Example to learn how to compare theoretical and experimental probabilities using *percent error*—a ratio of the difference between the theoretical and experimental probabilities to the theoretical probability.

Activity 3.4: Practice with Probability and Percent Error 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students revisit the coin flipping scenario from the Getting Started. This time, students observe how the difference between the experimental and theoretical probabilities is decreased as the number of trials is increased. The percent error decreases as a result.

CC BY NC

(6) Pacing

Lessons often span more than one 45-minute instructional session. Suggested pacing is provided for each lesson so that the entire course can be completed in a 165-Day instructional calendar.

(7) Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity is included.

CC BY NC

8 Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, common misconceptions, differentiation strategies, student look-fors, and an activity summary.

9 Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the objectives.

10 Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners. For example, to extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

8 Getting Started

ENGAGE

Flip It!

9 Facilitation Notes

In this activity, students flip a coin 30 times and determine the probabilities of heads and tails based on the results of the experiment. They then combine their results to see how the probability changes with more trials included. This activity is designed to engage students in thinking about experimental probability from an intuitive perspective.

Provide a coin to each pair of students for this experiment. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses with the class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• According to your experiment results, which outcome has a greater probability of occurring? • Were your results what you expected? Why or why not? • Why might someone say the results are 50-50? • What is the probability of flipping tails after 60 trials? 90 trials? • How did your results change when you included all the class data?
Gathering	• How did you determine the actual probability?

10 DIFFERENTIATION STRATEGIES

Access for All

- To combine results systematically, have students write their results in a table on the board. For example:

Student Pairs	*P*(heads)	Combined *P*(heads)	*P*(tails)	Combined *P*(tails)
Partners A & B	$\frac{12}{30}$	$\frac{12}{30} = 0.400$	$\frac{18}{30}$	$\frac{18}{30} = 0.600$
Partners C & D	$\frac{16}{30}$	$\frac{28}{60} = 0.467$	$\frac{14}{30}$	$\frac{32}{60} = 0.533$
Partners E & F	$\frac{17}{30}$	$\frac{45}{90} = 0.500$	$\frac{13}{30}$	$\frac{45}{90} = 0.500$

- Rewrite the probability as a decimal for easier comparison.

Students should recognize:

- The sum of the probabilities of complementary events equals 1.
- The probability for both heads and tails approaches 0.50 as the number of trials increases.

Summary

Experimental probabilities approach theoretical probabilities as the number of trials increase.

STAMP THE LEARNING

ACTIVITY 3.1 Determining Experimental Probability

DEVELOP

Facilitation Notes

In this activity, students play a game. Working with a partner, they toss a cup 25 times and record each trial using tally marks. The terms *experimental probability* and *theoretical probability* are introduced, and students calculate the experimental probability of each event using their data. They conclude that the game is not a fair game, and it is not possible to determine the actual probability of each event due to the shape of the cup.

Ask a student to read the introduction aloud. Discuss the directions to the game and complete Questions 1 through 4 as a class.

11 QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the sum of your probabilities? • How do your probabilities compare to those of your classmates? • How would you describe the experiment related to the game? • What are the possible outcomes of the game? • Who is more likely to win the game, Player 1 or Player 2?
Seeing structure	• Why aren't *the cup landing upright* and *upside-down* complementary events?

11 **Questions to Support Discourse**

As you facilitate the lesson, use the Questions to Support Discourse to assess students' sense-making and reasoning, to gauge what they know, and to generate evidence of student learning.

Ask a student to read the definition. Analyze the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do P_E and P_T represent?
Probing	• Explain the steps to solve this problem using decimals.
Seeing structure	• Does the order of the subtraction matter? What would a negative difference represent? • Explain the calculations to rewrite the complex expression on the last line as 14.3%.

COMMON MISCONCEPTION

Percent error does not mean that something is wrong, it just means that the experimental results are not exactly what the theoretical probability expected them to be.

Note that students may not have encountered the approximate symbol (≈) before. Discuss its meaning and cases where it is used.

12 **Have students work with a partner or in a group to complete Questions 2 through 5. Share responses as a class.**

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Is the experimental result higher or lower than the theoretical probability? How can you tell? • How might using the class experimental data rather than your individual data affect the percent error? Explain your thinking. • How do you interpret negative percent error?

Summary

The percent error is a ratio of the difference between the theoretical and experimental probabilities to the theoretical probability.

STAMP THE LEARNING

ACTIVITY 3.4 Practice with Probability and Percent Error

Facilitation Notes

In this activity, students experiment with flipping a coin again. Attention is paid to the difference between the experimental and theoretical probabilities as the number of trials increase. The percent error decreases as a result.

12 Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Research supports that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

COMMON MISCONCEPTION

Students may incorrectly think the complement of *P*(odd number) is *P*(even number). The complement of *P*(odd number) must contain all the remaining outcomes in the sample space; therefore, it is most efficient to state the complement as *P*(not odd) to include the even number, 2, and the letters, *X* and *Z*.

DIFFERENTIATION STRATEGY

Access for All

Have students interact with Catalina's and Linh's work. For each student, have them write the sample space, the number of times the desired outcome occurs and then the probability. Discuss why Catalina's work says *P*(not 5 or less) and Linh's work says *P*(5 or less), but they got the same final result.

Summary

Complementary events are events that together contain all of the outcomes in a sample space and, therefore, add up to one complete whole. The sum of the probabilities of any two complementary events is 1.

STAMP THE LEARNING

ACTIVITY 1.3 Simple Probability and Spinners

Facilitation Notes

In this activity, students use spinners to describe events, list possible outcomes, and calculate probabilities. They also predict the probabilities of specified events occurring.

Have students work with a partner or in a group to complete Questions 1 through 6. Emphasize to students that if the spinner stops on a line, rather than in a section, it must be spun again until it stops within a section. Share responses as a class.

13 **AS STUDENTS WORK, LOOK FOR**

Students who misinterpret the probability of a number to only include numbers in the sample space or the probability of a letter to only include letters in the sample space. For example, *P*(odd number) is $\frac{3}{6}$ or $\frac{1}{2}$, not $\frac{3}{4}$. The sample space should include all six items on the spinner, regardless of whether the probability of a number or letter is being determined.

14 **COMMON MISCONCEPTION**

Use this activity to disprove the misconception that complements are always opposites. For example, in this context, the complement of *P*(odd number) is *P*(not odd number); *P*(even number) is incorrect because it does not contain all the remaining outcomes in the sample space.

13 As Student Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

14 Common Misconceptions

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

15 White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

16 Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

Summary

Tables can be used to create probability models and those models can be used to answer related questions.

STAMP THE LEARNING

DEMONSTRATE

15

Talk the Talk

COMPARE AND CONTRAST

Facilitation Notes

In this activity, students answer questions related to uniform probability models and non-uniform probability models.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials Needed: Poster Paper

To extend the lesson, have groups of students create posters with examples (including a scenario and probability model) that demonstrate a uniform probability model or a non-uniform probability model. Have students present their posters to the class. Discuss Questions 1 and 2 throughout the presentations.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What two features are necessary in a probability model?
Gathering	• What are the advantages of not writing probabilities in lowest terms? • What is a uniform probability model? • What is a non-uniform probability model? • When is the sum of all the probabilities of the outcomes of an event not equal to 1?

Have students read and answer the Essential Question on the lesson opener page.

16

Summary

A uniform probability model occurs when all of the probabilities in the model are equally likely to occur, and a non-uniform probability model occurs when all the probabilities in the model are not equally likely to occur.

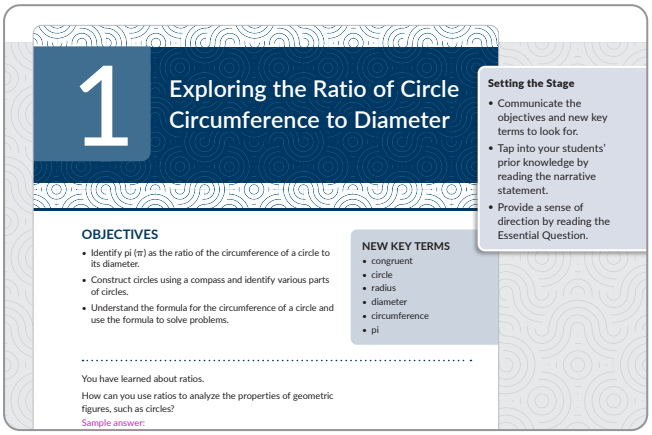
STAMP THE LEARNING

Embedded Teacher Supports

Within the lesson, additional teacher embedded notes, when applicable, are provided at point of use within a lesson.

Setting the Stage

Each Lesson Overview provides guidance to help students anticipate how the new information will connect to prior learning.



1

Exploring the Ratio of Circle Circumference to Diameter

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior knowledge by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Understand the formula for the circumference of a circle and use the formula to solve problems.

NEW KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

You have learned about ratios.

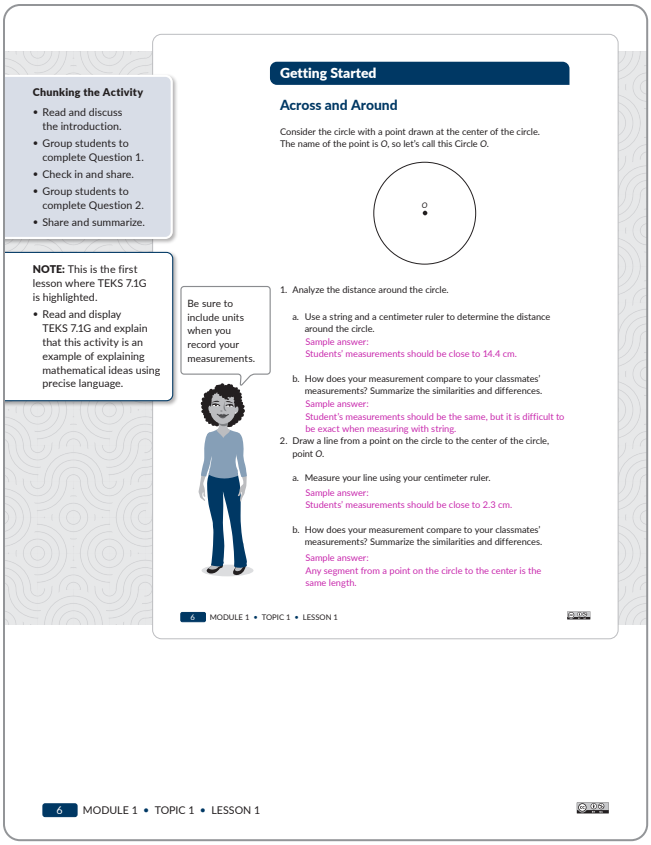
How can you use ratios to analyze the properties of geometric figures, such as circles?

Sample answer:

Chunking the Activity This section provides suggestions for separating the activity into smaller meaningful parts or chunks to help students process the information. This is the same activity chunking provided within the facilitation notes.

TEKS Mathematical Process Standards Notes

Each note references a particular TEKS mathematical process standard. The first instance of a TEKS mathematical process standard is highlighted in a lesson and encourages you to introduce the standard to your students. After the first time a process standard is highlighted, additional notes help you assess whether students are demonstrating proficiency with the process standards.



Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Question 1.
- Check in and share.
- Group students to complete Question 2.
- Share and summarize.

NOTE: This is the first lesson where TEKS 7.1G is highlighted.

- Read and display TEKS 7.1G and explain that this activity is an example of explaining mathematical ideas using precise language.

Getting Started

Across and Around

Consider the circle with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.

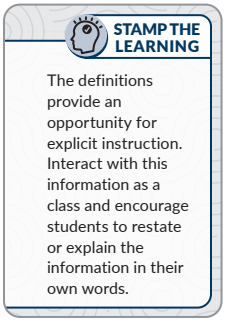
O

Be sure to include units when you record your measurements.

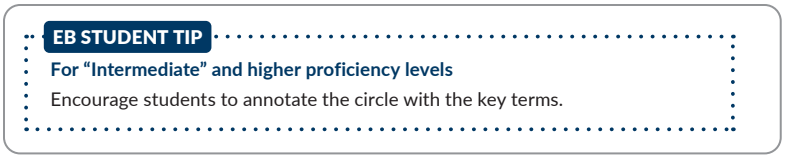
1. Analyze the distance around the circle.
   a. Use a string and a centimeter ruler to determine the distance around the circle.
   Sample answer:
   Students' measurements should be close to 14.4 cm.
   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.
   Sample answer:
   Student's measurements should be the same, but it is difficult to be exact when measuring with string.
2. Draw a line from a point on the circle to the center of the circle, point *O*.
   a. Measure your line using your centimeter ruler.
   Sample answer:
   Students' measurements should be close to 2.3 cm.
   b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.
   Sample answer:
   Any segment from a point on the circle to the center is the same length.

6 MODULE 1 • TOPIC 1 • LESSON 1

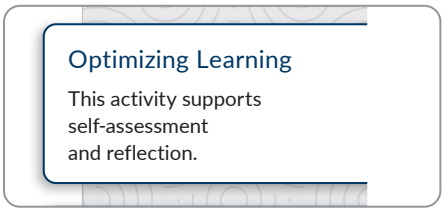
Stamp the Learning The Stamp the Learning icon provides prompts and guided instructions to support you in communicating, explaining, and modeling the concepts directly and explicitly.



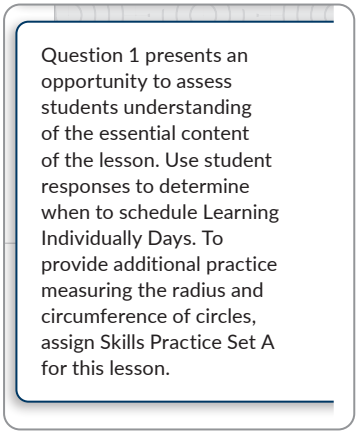
EB Student Tip EB Student Tips provide instructional strategies to support students with varying levels of English language proficiency.



Optimizing Learning These notes indicate opportunities for purposeful learning. These strategies provide access to the course content for all learners.



Skills Practice Alignment Notes These notes indicate the section(s) of Skills Practice that align to activities within the lessons.



Modeling Moment These notes provide instructional guidance surrounding when and how to utilize the Problem-Solving Model Graphic Organizer.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Refer students to the problem-solving model in the Course Guide.
- Have students read the *Questions to Ask* aloud for each step.
- Work through Question 1 together as a class to complete the graphic organizer.
- Have students work in pairs and use the model to complete Question 2.

Self-Monitoring Strategies Some lessons include self-monitoring strategy notes at point of use.

Providing students with purposeful guidance as they develop self-monitoring skills is critical to their growth as holistic learners. Throughout the text, there are callouts placed strategically where there are opportunities for students to practice self-monitoring. Support students by acknowledging those demonstrating the actions outlined by the callout, and encouraging other students to make an intentional attempt at implementing the strategies. Over time, these actions will become habits for students that they can apply throughout all their academic courses and their lives more broadly. As the Self-Monitoring Strategies indicate, look for students:

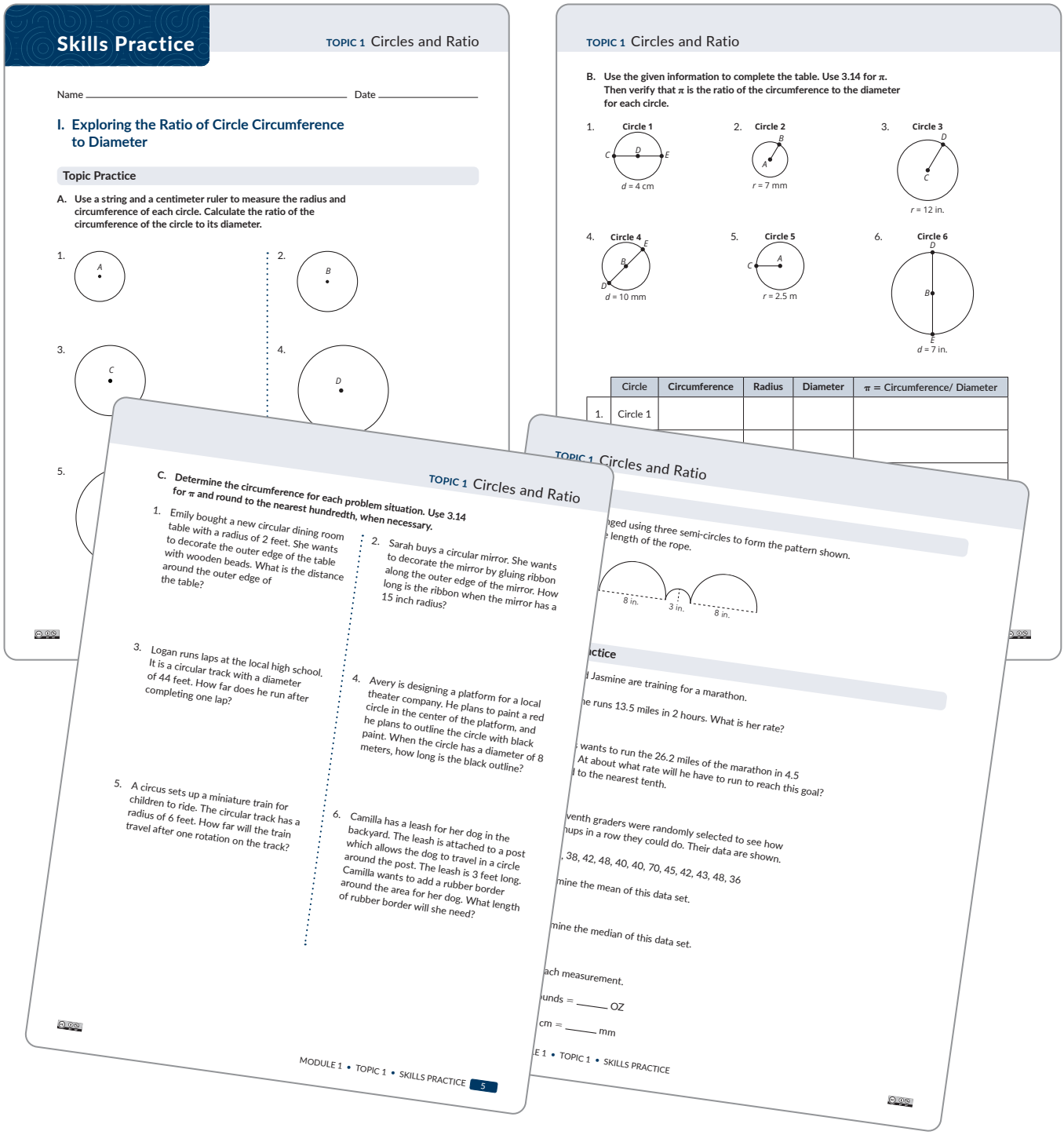
- Making a reasoned judgment.
- Using self-motivation and self-discipline to persevere in problem-solving.
- Engaging in productive struggle as they use their reasoning to solve problems.
- Recognizing the importance of developing critical thinking skills.
- Showing a sense of confidence and optimism as they approach the problem.
- Accurately perceiving their strengths and limitations.
- Using organizational strategies while solving problems.
- Monitoring their understanding by referring back to concrete examples.
- Demonstrating self-awareness by perseverance through problem-solving.
- Reviewing and reflecting on their goals.

SELF-MONITORING STRATEGIES

Look for students using self-motivation and self-discipline to persevere in problem solving. Refer to the Course and Implementation Guide for further details on these look-fors.

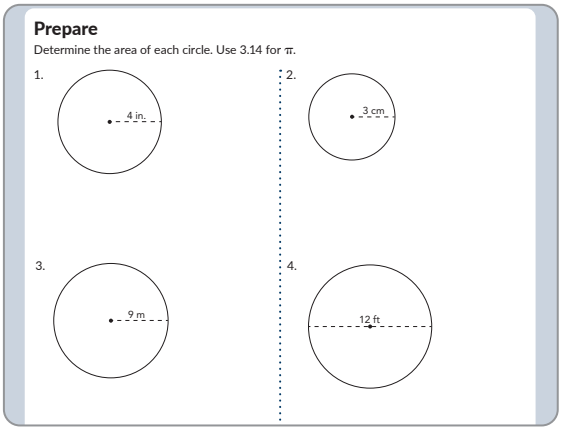
Skills Practice

Each topic includes a set of Skills Practice problems that focus on skills and concepts students learn in the topic. Use Skills Practice during Learning Individually days to support students in developing skill proficiency. You can use formative and summative assessment data to determine when to schedule Learning Individually days. Skills Practice provides opportunity for targeted instruction as you determine which problem sets to assign to individual students.



Formative Assessments

To prepare for the upcoming lesson, use the Prepare section of the assignment from the previous lesson as a diagnostic tool you can use to determine whether your students are ready for new learning. When time is available, you can use this as a warm-up to assess students' prior knowledge.

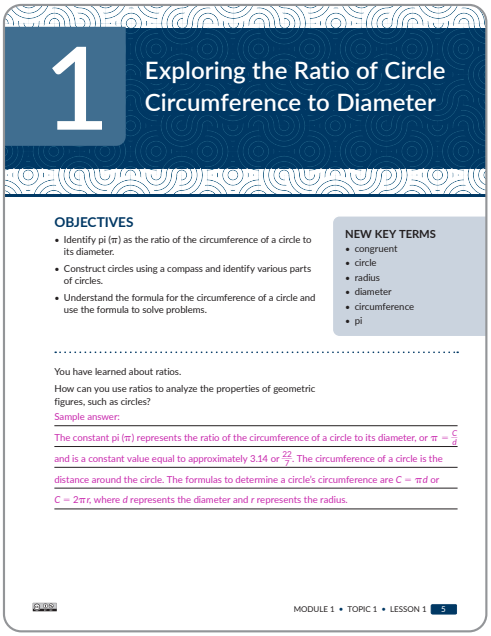


Prepare

Determine the area of each circle. Use 3.14 for π.

1. 4 in.
2. 3 cm
3. 9 m
4. 12 ft

To set the stage for new learning, Lesson Overviews provide Objectives and New Key Terms. Most importantly, you will see a statement that connects to prior knowledge and an Essential Questions that anticipates new learning. At the end of the lesson, students return to and answer the Essential Question to demonstrate their learning.



1 Exploring the Ratio of Circle Circumference to Diameter

OBJECTIVES

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Understand the formula for the circumference of a circle and use the formula to solve problems.

NEW KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

You have learned about ratios.

How can you use ratios to analyze the properties of geometric figures, such as circles?

Sample answer:

The constant pi (π) represents the ratio of the circumference of a circle to its diameter, or $\pi = \frac{C}{d}$ and is a constant value equal to approximately 3.14 or $\frac{22}{7}$. The circumference of a circle is the distance around the circle. The formulas to determine a circle's circumference are $C = \pi d$ or $C = 2\pi r$, where d represents the diameter and r represents the radius.

MODULE 1 • TOPIC 1 • LESSON 1 5

As you facilitate lessons, use the Questions to Support Discourse to assess students' sense-making and reasoning, to gauge what they know, and to generate evidence of student learning.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why isn't the height of the parallelogram one of its sides? • What notation lets you know which segment is the height?
Seeing structure	• Did you change the lengths of the base and height when you rearranged the parallelogram to create a rectangle? How can you tell? • Did you change the amount of space inside the parallelogram when you rearranged it? Explain your thinking. • Why do both the parallelogram and the rectangle have the same area formula?

As you facilitate the learning, use the stamp the learning icon to identify opportunities to provide explicit instruction on key concepts within the lesson. The stamp the learning icon highlights these opportunities. To assess students' understanding of these key concepts, have them restate the information in their own words.

STAMP THE LEARNING

The formulas provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

As students self-reflect during the Talk the Talk, you can interpret how well they have demonstrated the learning outcomes and prepare for what's next.

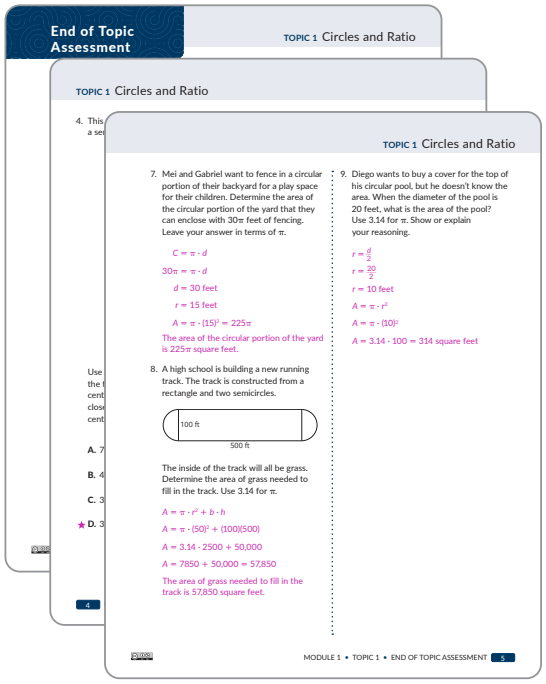
Talk the Talk

Go With the Flow

1. Residents of a community are trying to determine which configuration would allow more water to flow through the pipe(s), one pipe with a radius of 8 cm or two pipes that each have a radius of 4 cm. Which configuration allows the most water to flow through the pipe(s), and what is the difference between the two configurations? Show your work and explain your reasoning.

   Sample answer:

   The 8 cm pipe will allow more water to flow through the pipe. The area of the 8 cm pipe is 200.96 sq cm, and the area of the two 4 cm pipes is 100.48 sq cm. The 8 cm pipe is double the

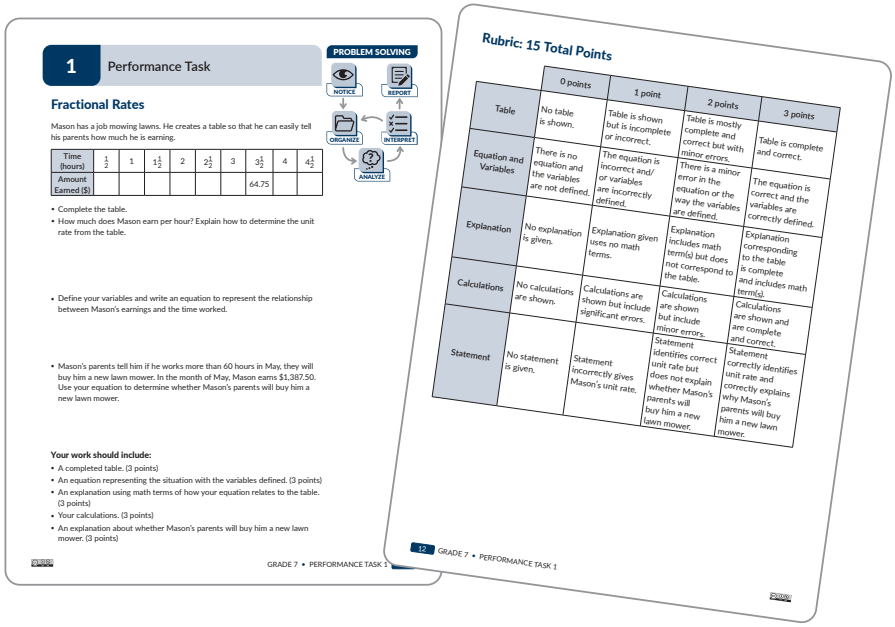


Summative Assessments

End of Topic Assessments are provided to measure student performance on a clearly denoted set of standards. There are three problem types students will encounter on the assessment: multiple-choice, multiselect, and open-response questions. These questions are thoughtfully designed to prepare students for digitally enhanced standardized tests.

The answer keys provide teachers with sample answers for open-response questions as well as for the multiple-choice and multiselect questions when applicable.

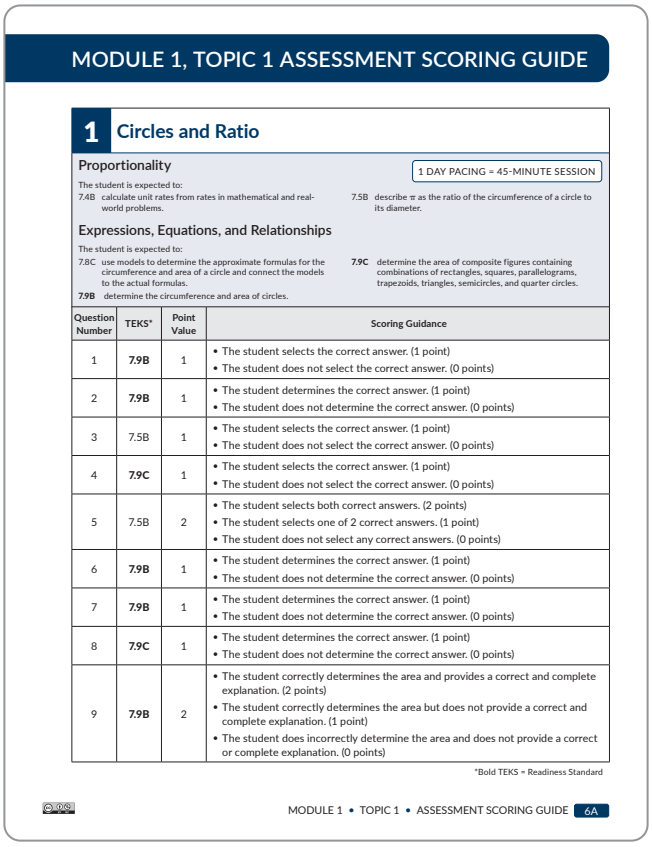
Question & Test Interoperability (QTI) files are provided for the implementation of digital assessments aligned to each End of Topic Assessment. There are many problem types that students will encounter on digital assessments: multiple-choice, multiselect, text entry/equation editor, graphing, inline choice, hot spot, drag and drop, and match table grid. It is recommended to administer either the print or the digital assessment method of the End of Topic Assessment. Students should not take both assessment types at the end of a topic



There are a set of optional Performance Tasks students can complete after certain modules/topics. These tasks cover selected priority TEKS content from the course. You can use the Performance Task as either a formative or summative assessment. These tasks include a rubric that you can utilize to assess individual or class depth of understanding as aligned to the TEKS.

Assessment Guidance and Analysis

An Assessment Scoring Guide is provided for each End of Topic Assessment. The scoring guide highlights the TEKS aligned to each question on the assessment. It identifies possible point values for each question, along with a rubric for how to score each question. Each Assessment Scoring Guide also provides recommendations for how to respond to students' performance on the assessment.



MODULE 1, TOPIC 1 ASSESSMENT SCORING GUIDE

1 Circles and Ratio

Proportionality

1 DAY PACING = 45-MINUTE SESSION

The student is expected to:

7.4B calculate unit rates from rates in mathematical and real-world problems.

7.5B describe π as the ratio of the circumference of a circle to its diameter.

Expressions, Equations, and Relationships

The student is expected to:

7.8C use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas.

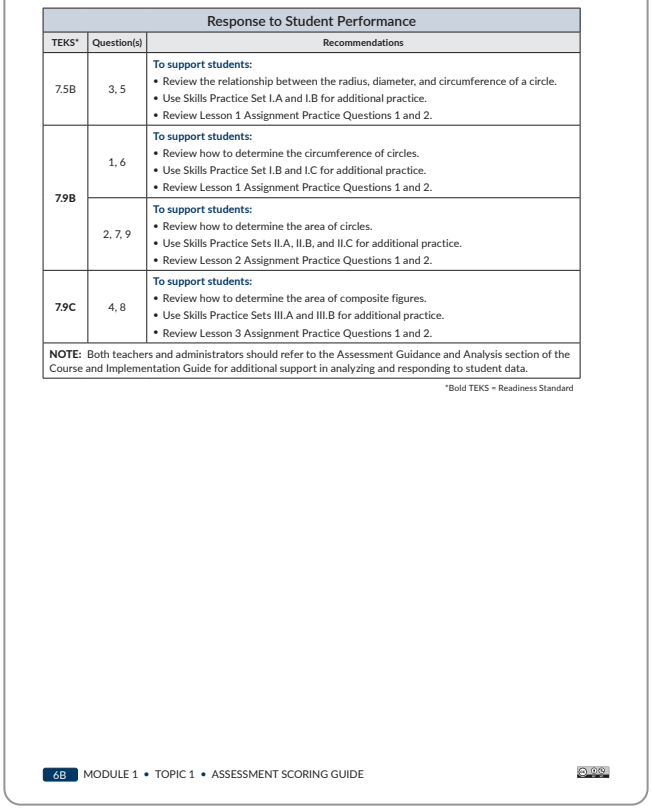
7.9B determine the circumference and area of circles.

7.9C determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles.

Question Number	TEKS*	Point Value	Scoring Guidance
1	**7.9B**	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
2	**7.9B**	1	• The student determines the correct answer. (1 point) • The student does not determine the correct answer. (0 points)
3	7.5B	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
4	**7.9C**	1	• The student selects the correct answer. (1 point) • The student does not select the correct answer. (0 points)
5	7.5B	2	• The student selects both correct answers. (2 points) • The student selects one of 2 correct answers. (1 point) • The student does not select any correct answers. (0 points)
6	**7.9B**	1	• The student determines the correct answer. (1 point) • The student does not determine the correct answer. (0 points)
7	**7.9B**	1	• The student determines the correct answer. (1 point) • The student does not determine the correct answer. (0 points)
8	**7.9C**	1	• The student determines the correct answer. (1 point) • The student does not determine the correct answer. (0 points)
9	**7.9B**	2	• The student correctly determines the area and provides a correct and complete explanation. (2 points) • The student correctly determines the area but does not provide a correct and complete explanation. (1 point) • The student does incorrectly determine the area and does not provide a correct or complete explanation. (0 points)

*Bold TEKS = Readiness Standard

MODULE 1 • TOPIC 1 • ASSESSMENT SCORING GUIDE 6A



Response to Student Performance		
TEKS*	**Question(s)**	**Recommendations**
7.5B	3, 5	**To support students:** • Review the relationship between the radius, diameter, and circumference of a circle. • Use Skills Practice Set I.A and I.B for additional practice. • Review Lesson 1 Assignment Practice Questions 1 and 2.
7.9B	1, 6	**To support students:** • Review how to determine the circumference of circles. • Use Skills Practice Set I.B and I.C for additional practice. • Review Lesson 1 Assignment Practice Questions 1 and 2.
	2, 7, 9	**To support students:** • Review how to determine the area of circles. • Use Skills Practice Sets II.A, II.B, and II.C for additional practice. • Review Lesson 2 Assignment Practice Questions 1 and 2.
7.9C	4, 8	**To support students:** • Review how to determine the area of composite figures. • Use Skills Practice Sets III.A and III.B for additional practice. • Review Lesson 3 Assignment Practice Questions 1 and 2.
NOTE: Both teachers and administrators should refer to the Assessment Guidance and Analysis section of the Course and Implementation Guide for additional support in analyzing and responding to student data.		

*Bold TEKS = Readiness Standard

6B MODULE 1 • TOPIC 1 • ASSESSMENT SCORING GUIDE

Assessments are designed to cover the focus TEKS of the topic at the depth and rigor of the standard. When a TEKS addressed in a topic is not assessed on a topic assessment it will be addressed to its full depth in another topic. The only TEKS that are not assessed in the course are the unassessed standards in the course.

The Assessment Scoring Guide's Response to Student Performance section is still applicable if digital assessments are utilized.

Promoting Self-Reflection

The Crew

Characters are embedded throughout the course to remind students to stop and think to promote productive reflection. The characters are used in a variety of ways: they may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

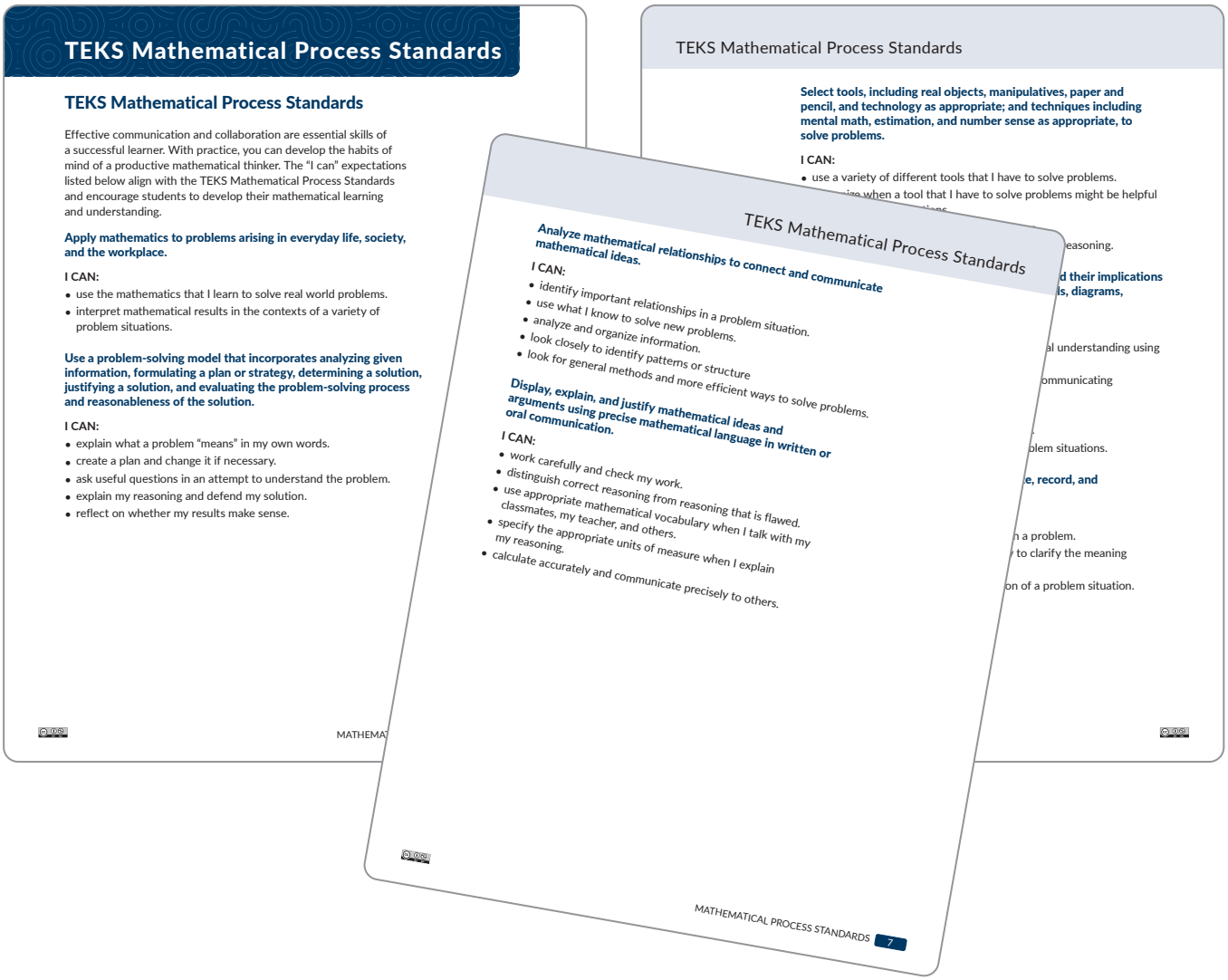
4 THE CREW

TEKS Mathematical Process Standards

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

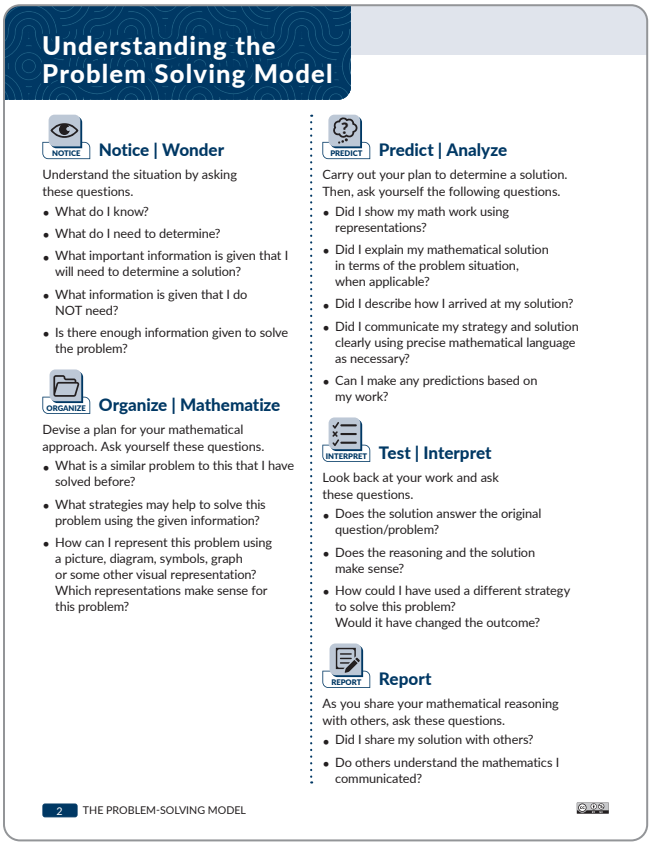
When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

Consider having students create "I Can" statements to promote their self-reflection.



Understanding the Problem-Solving Model

Productive mathematical thinkers are problem-solvers. These instructional materials include a problem-solving model to help students develop proficiency with the TEKS mathematical process standards and to make sense of the problems they must solve. As students engage with the problem-solving model, have them use the provided questions to guide their thinking. As students collaborate, suggest they use the provided questions to spark discussion. When appropriate, provide students with the Problem-Solving Model Graphic Organizer to complete as they solve problems.



Understanding the Problem Solving Model

Notice | Wonder

Understand the situation by asking these questions.

- What do I know?
- What do I need to determine?
- What important information is given that I will need to determine a solution?
- What information is given that I do NOT need?
- Is there enough information given to solve the problem?

Organize | Mathematize

Devise a plan for your mathematical approach. Ask yourself these questions.

- What is a similar problem to this that I have solved before?
- What strategies may help to solve this problem using the given information?
- How can I represent this problem using a picture, diagram, symbols, graph or some other visual representation? Which representations make sense for this problem?

Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I show my math work using representations?
- Did I explain my mathematical solution in terms of the problem situation, when applicable?
- Did I describe how I arrived at my solution?
- Did I communicate my strategy and solution clearly using precise mathematical language as necessary?
- Can I make any predictions based on my work?

Test | Interpret

Look back at your work and ask these questions.

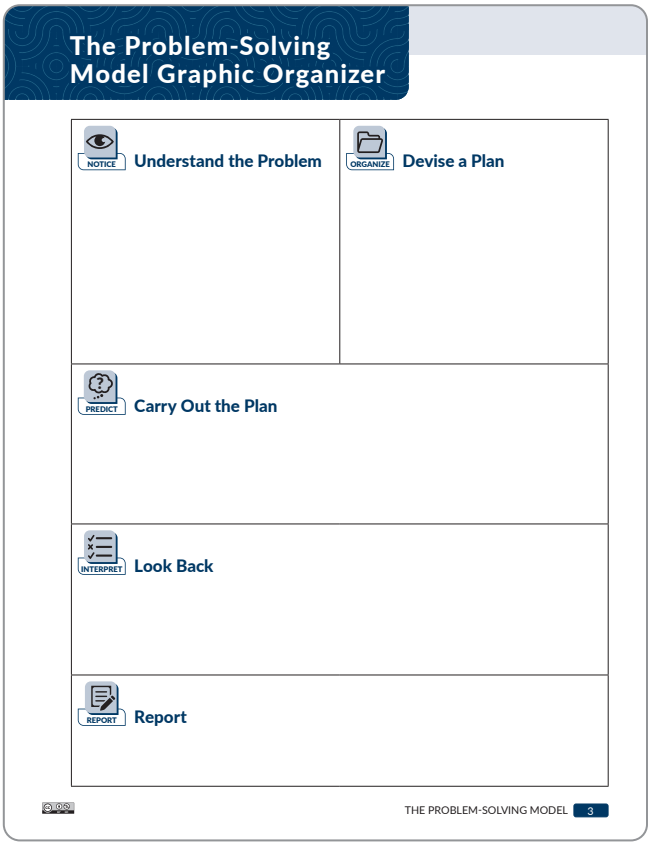
- Does the solution answer the original question/problem?
- Does the reasoning and the solution make sense?
- How could I have used a different strategy to solve this problem? Would it have changed the outcome?

Report

As you share your mathematical reasoning with others, ask these questions.

- Did I share my solution with others?
- Do others understand the mathematics I communicated?

2 THE PROBLEM-SOLVING MODEL



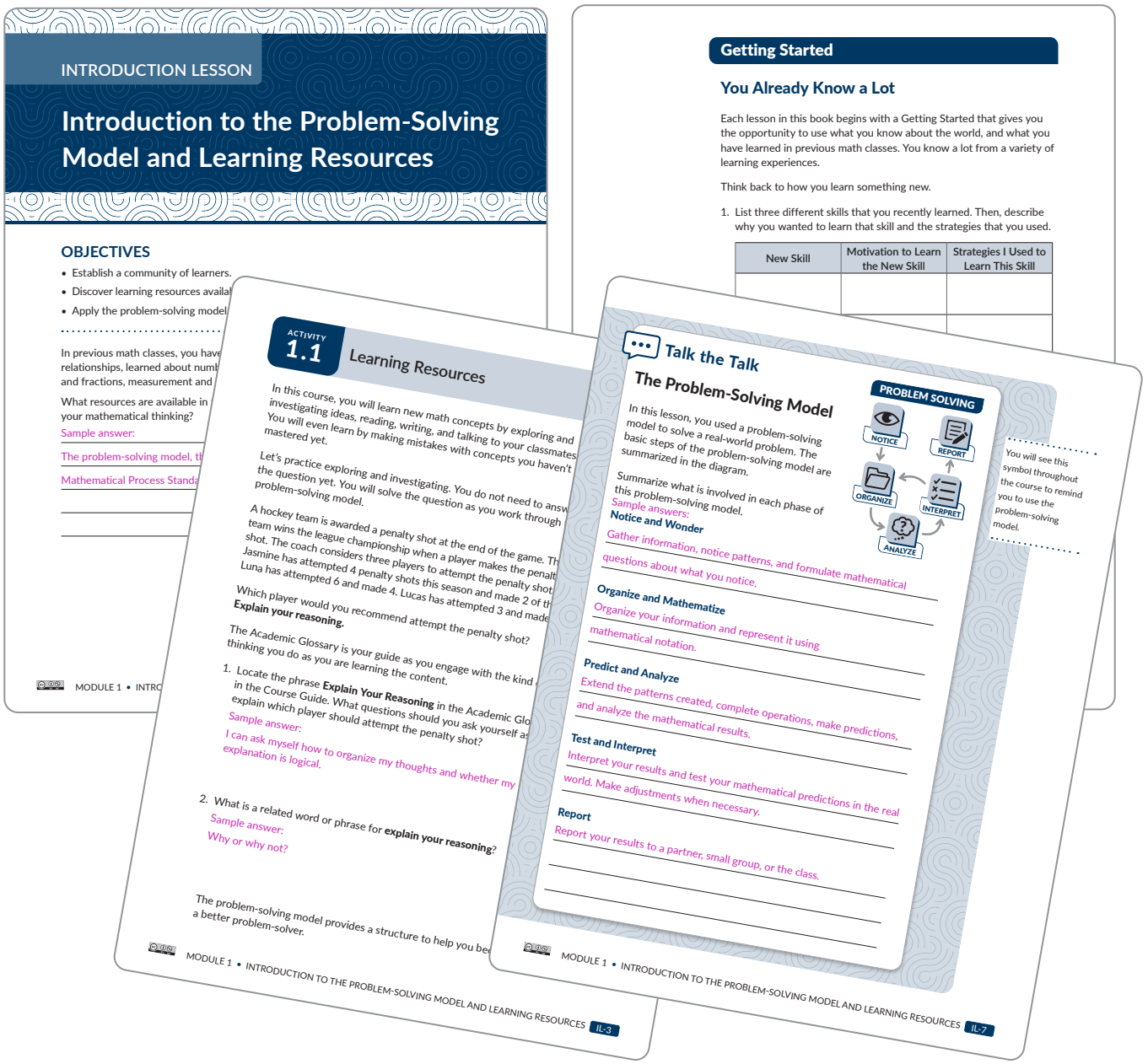
The Problem-Solving Model Graphic Organizer

Understand the Problem	Devise a Plan
Carry Out the Plan	
Look Back	
Report	

THE PROBLEM-SOLVING MODEL 3

Introduction Lesson

The introduction lesson is designed to establish a community of learners. This lesson introduces students to the learning process and the learning resources they will utilize throughout the course. The lesson introduces or reviews the TEKS mathematical process standards, the Academic Glossary, the Math Glossary, and the Problem-Solving Model Graphic Organizer. Students will access these tools throughout the entire course. The Problem-Solving Model Graphic Organizer helps students organize and communicate their mathematics. The introduction lesson empowers students with tools to persevere during challenging mathematical tasks.



Academic Glossary

Language Expectations

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

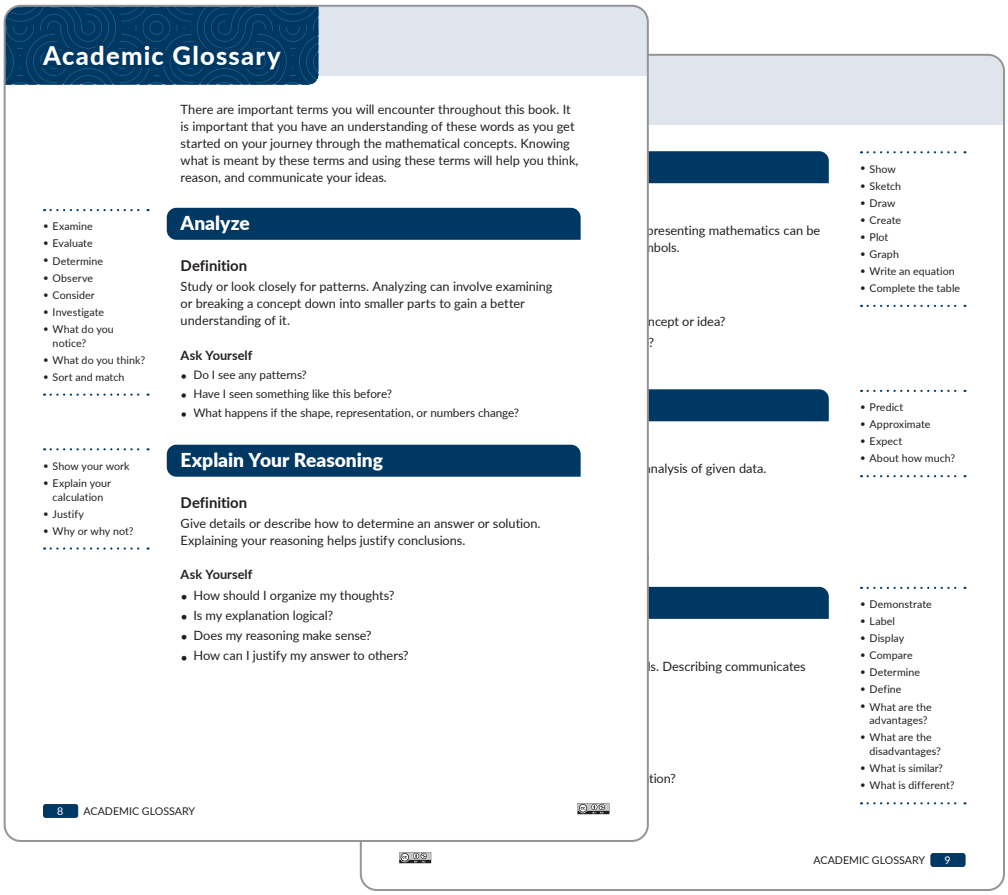
Be explicit about your expectations of language use and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

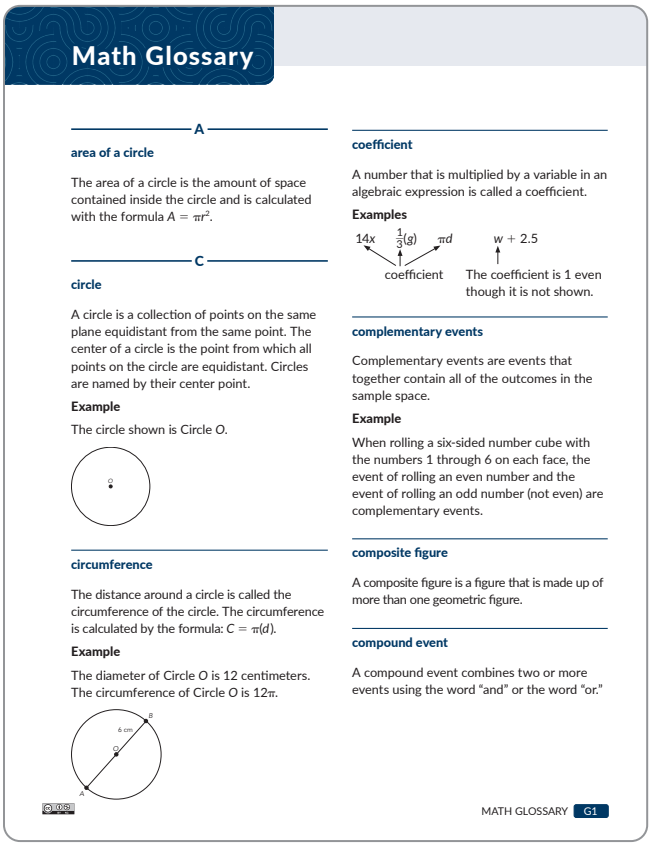
Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different from quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.



Math Glossary

A course-specific math glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.



Math Glossary

A

area of a circle

The area of a circle is the amount of space contained inside the circle and is calculated with the formula $A = \pi r^2$.

C

circle

A circle is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.

Example

The circle shown is Circle *O*.

circumference

The distance around a circle is called the circumference of the circle. The circumference is calculated by the formula: $C = \pi(d)$.

Example

The diameter of Circle *O* is 12 centimeters. The circumference of Circle *O* is 12π.

coefficient

A number that is multiplied by a variable in an algebraic expression is called a coefficient.

Examples

$14x$ $\frac{1}{3}(g)$ πd $w + 2.5$

coefficient

The coefficient is 1 even though it is not shown.

complementary events

Complementary events are events that together contain all of the outcomes in the sample space.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the event of rolling an even number and the event of rolling an odd number (not even) are complementary events.

composite figure

A composite figure is a figure that is made up of more than one geometric figure.

compound event

A compound event combines two or more events using the word "and" or the word "or."

MATH GLOSSARY G1

A Collaborative Classroom

Early research on teaching and learning shows that what happens in the classroom in the first three days determines the environment for the entire year. An effective implementation is most likely to occur in the collaborative classroom, a classroom where knowledge is shared.

The instructional materials are grounded in the belief that students develop understanding and skills by actively participating in their environment. Furthermore, effective communication and collaboration are essential skills for the successful learner. Through dialogue and discussion of different strategies and perspectives, students become knowledgeable independent learners.

Defining A Collaborative Classroom

A collaborative classroom is an environment where teachers and students share knowledge and authority. In a collaborative classroom, teachers are facilitators, and students are active participants. All students, not segregated by ability level, linguistic proficiency, interest, or achievement, benefit from the environment created in the collaborative classroom.

Teachers in the collaborative classroom combine their extensive knowledge about teaching and learning, content, and skills with their students' informal and formal knowledge, strategies, and individual experiences. The collaborative classroom differs from the traditional classroom, where the teacher is seen as an information giver.

Characteristics Of The Collaborative Classroom

The collaborative classroom is identified by discussion, with in-depth, accountable talk and two-way interactions, whether among members of the whole class or small groups. It is a well-structured environment that values questioning and dialogue and sets appropriate parameters for active learning. Given the importance of dialogue, the teacher needs to understand the linguistic proficiencies of your emergent bilingual students - to foster a safe environment for students to speak freely. Careful planning by the teacher ensures students can work together to attain individual and collective goals and develop learning strategies.

In the collaborative classroom, students are encouraged to take responsibility for their learning through monitoring and reflective self-evaluation. The collaborative classroom is where teachers spend more time in true academic interactions, guiding students to search for information and share what they know. As facilitators, teachers support each student's needs by providing appropriate hints, probing questions, feedback, linguistic support, and help clarifying, thinking, or using a particular strategy.

Learning In The Collaborative Classroom

Critical to teaching and learning in the collaborative environment is the ability to define the teacher's and students' responsibilities. For effective collaboration and teamwork, teachers and students must agree to specific responsibilities that support the learning process. The table reflects the parallel responsibilities of teachers and students.

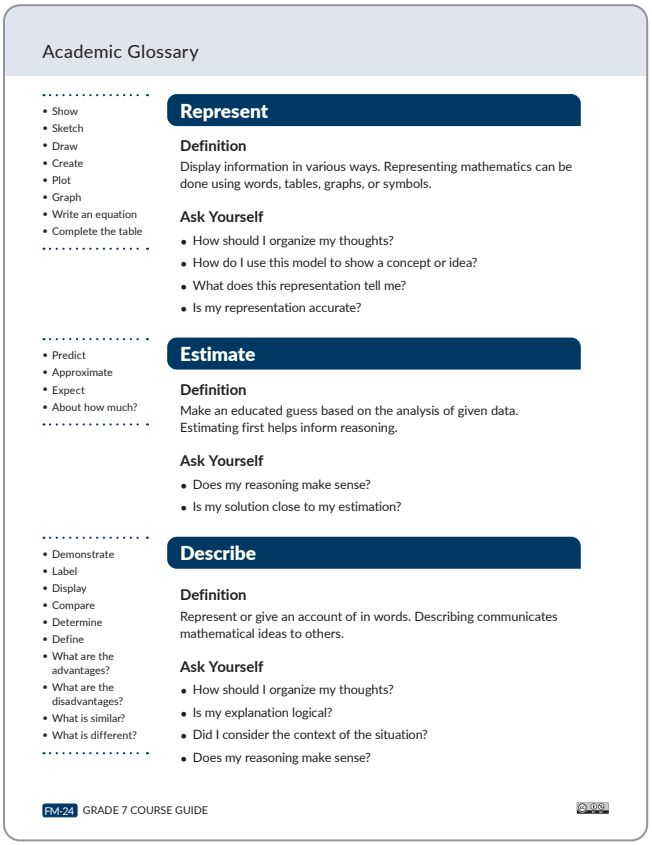
Teacher Responsibilities	Student Responsibilities
Monitor student behavior.	Develop the skills to work cooperatively.
Provide assistance when needed.	Learn to talk and discuss problems with each other to accomplish the group goal.
Answer questions only when they are group questions.	Ask for help only after each group member has considered the problem and the group has a question for the teacher.
Interrupt the process to reinforce cooperative skills or to provide direct instruction to all students.	Believe that all members of the group work together toward a common goal. Understand all members share the success or failure of the group.
Provide closure for the lesson.	Reflect on the work of the group.
Evaluate the group process by discussing the actions of the group members.	Appreciate that working together is a process and encourage group members to interact with and relate to the rest of the group members.
Help students to become individually accountable for learning and reinforce this understanding regularly.	Realize that each member must contribute as much as possible to the group goal. Understand that the success of the group depends on each member's individual work and that students are accountable for their own learning.

Facilitating Productive Struggle

As students learn mathematics, it is critical that they develop characteristics, such as creative thinking, resilience, and belief in their own ability as problem solvers. *Productive struggle* is a process that supports the development of those characteristics. Engaging in productive struggle motivates students to exert effort, develop persistence, and employ effective problem-solving strategies, leading to deeper learning and improved academic performance. Research also suggests that experiencing productive struggle can increase students' self-efficacy, resilience, and perseverance through problem solving, ultimately preparing them to tackle future challenges with confidence and adaptability.

Consider teaching practices when creating opportunities for students to engage in productive struggle.

- Select non-routine tasks that allow all students to engage in productive struggle.
- Provide time for students to try different strategies and ask questions.
- Support students as they develop resilience by having them share their thought process instead of their solutions or conclusions.



Academic Glossary

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

FM-24 GRADE 7 COURSE GUIDE

Supporting All Learners

The course materials are designed to support all learners. The course supports provide adaptations to meet the needs of emergent bilingual students, students of special populations, and gifted and talented students.

Supporting Students of Special Populations

This course is specifically designed to engage and support all learners. Supports included within this course provide ways to adapt and differentiate instruction.

To support students of special populations, differentiate instruction by:

- using embedded Differentiation Strategies.
- providing options for answering questions, such as drawing a picture, or answering aloud.
- shortening assignments on a one-by-one basis.
- using the Math Glossary as a pre-teaching tool.
- utilizing the Skills Practice to target specific skills and complete prerequisite or space practice sets as needed.
- using the Prepare section of the lesson assignment to preview the material in the upcoming lesson.
- using alternative grouping strategies.
- having students interact with Worked Examples by highlighting key steps or information.

DIFFERENTIATION STRATEGY

Just in Time Support

To scaffold support, have students first draw a circle with a radius of 1 unit and a second circle with a radius of 2 units and then calculate and compare the areas. Next, have them draw a third circle with a radius of 4 units. Ask them to calculate the area and compare it to the area of the circle with a radius of 2 units.

Supporting Gifted and Talented Students

To support gifted and talented students or any student who is showing proficiency in a standard and is ready for a challenge and/or extension differentiate instruction by:

- using embedded Differentiation Strategies labeled as Challenge Opportunities.
- utilizing the Extension section of the Skills Practice.
- scaffolding up the academic glossary by encouraging students to apply the terminology across disciplines and in real-world applications.
- using alternative grouping strategies.

Optimizing Learning

This differentiation strategy highlights patterns, critical features, big ideas, and relationships.

DIFFERENTIATION STRATEGIES

Access for All

- Have students write the word *example* above the Thumbs Up strategy as an indicator that it is a correct solution to reference later. As students read through the strategy and think about the connections between steps, suggest they ask themselves:
  - Why is this method correct?
  - Have I used this method before?

Challenge Opportunities

- Ask students to determine the cost per slice of pizza and the amount in slices per dollar. Then, ask students whether they could use the slices-per-dollar or dollars-per-slice ratios to determine the best buy. This strategy gives the same correct answer to this problem; however, this strategy may not always work because it doesn't consider the area of each pizza slice.
- Have students reprice the pizzas so that they all have the same unit rate.

Emergent Bilingual Students

Emergent bilingual students often face multiple challenges in the mathematics classroom beyond language development skills, including peer-to-peer understanding and building solid conceptual proficiency. The instructional materials seek to support emergent bilingual students as they develop skills in both mathematics and language.

The Topic Overview includes cognates for new key terms, when applicable. It also includes guidance on how to use cognates to support emergent bilingual students.

How can you use cognates to support EB students?

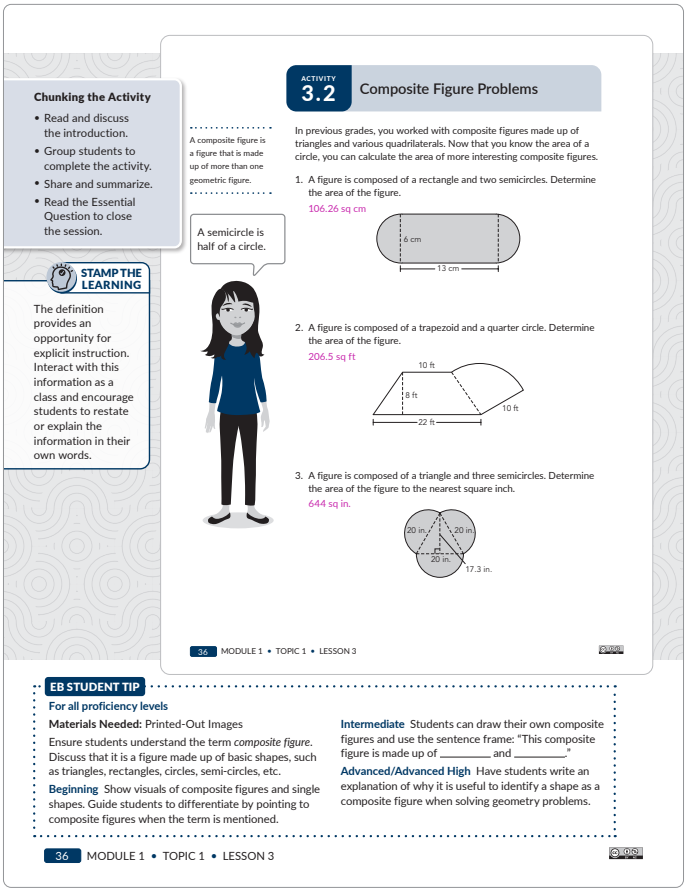
Cognates are provided for new key terms when applicable. Encourage students to keep a bilingual math journal, recording reflections and background knowledge on new topics, in either written or verbal format, with added visuals for clarity. Incorporate journal excerpts into a shared word wall or digital bilingual glossary, with a focus on highlighting cognates.

Throughout instruction, EB Student Tips are placed for teachers at point-of-use on the mini-lesson page in the Teacher's Edition. They provide additional scaffolds to support this population.

These tips:

- Inform teachers of potential learning barriers specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.



Chunking the Activity
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Read the Essential Question to close the session.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 3.2 Composite Figure Problems

A composite figure is a figure that is made up of more than one geometric figure.

A semicircle is half of a circle.

In previous grades, you worked with composite figures made up of triangles and various quadrilaterals. Now that you know the area of a circle, you can calculate the area of more interesting composite figures.

1. A figure is composed of a rectangle and two semicircles. Determine the area of the figure.
106.26 sq cm

2. A figure is composed of a trapezoid and a quarter circle. Determine the area of the figure.
206.5 sq ft

3. A figure is composed of a triangle and three semicircles. Determine the area of the figure to the nearest square inch.
644 sq in.

36 MODULE 1 • TOPIC 1 • LESSON 3

EB STUDENT TIP

For all proficiency levels

Materials Needed: Printed-Out Images

Ensure students understand the term *composite figure*. Discuss that it is a figure made up of basic shapes, such as triangles, rectangles, circles, semi-circles, etc.

Beginning Show visuals of composite figures and single shapes. Guide students to differentiate by pointing to composite figures when the term is mentioned.

Intermediate Students can draw their own composite figures and use the sentence frame: "This composite figure is made up of _____ and _____."

Advanced/Advanced High Have students write an explanation of why it is useful to identify a shape as a composite figure when solving geometry problems.

36 MODULE 1 • TOPIC 1 • LESSON 3

Preparing to Facilitate Learning

The classroom teachers make the material come alive for students, transforming the way math is taught. Implementation requires integrating Learning Together and Learning Individually. It is important to balance the two and set a structure that students can follow.

- Use the structure of the lesson to guide teaching on Learning Together days.
  - Begin by establishing mathematical objectives to focus and activate student learning.
  - Align teaching to learning by facilitating collaborative discourse and providing time for students to engage in productive struggle during learning activities.
  - Have students reflect and demonstrate their learning and provide feedback.
- Set clear expectations for the structure of the Learning Individually day to maximize class time. Set up the classroom environment for stations or small groups, providing space for the teacher or student leaders to circulate around the room to support students.

Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Develop and understanding of how each lesson fits into the big picture by using the Teacher Module and Topic Internalization Protocol.
- Understand the lesson prior to teaching by using the Teacher Lesson Internalization Protocol.
- Prepare team-building activities to intentionally create a student-centered environment.

Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of working individually and why practice is important.

- Read through Skills Practice for the topic.
- Read through skills practice alignment notes at point of use in the topic lessons.
- Determine how you will interpret student's strengths, weaknesses and/or gaps, and common misconceptions from the identified activities within lessons that correspond to Skills Practice problem sets.

Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of new key terms.

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
    - Their written work is a record of their learning and is to be used as a reference for any assignments or assessments you give.
    - They will be doing the thinking, talking, and writing in your classroom.
    - They will be working and sharing their strategies and reasoning with their peers.
    - Mistakes and struggles are normal and necessary.

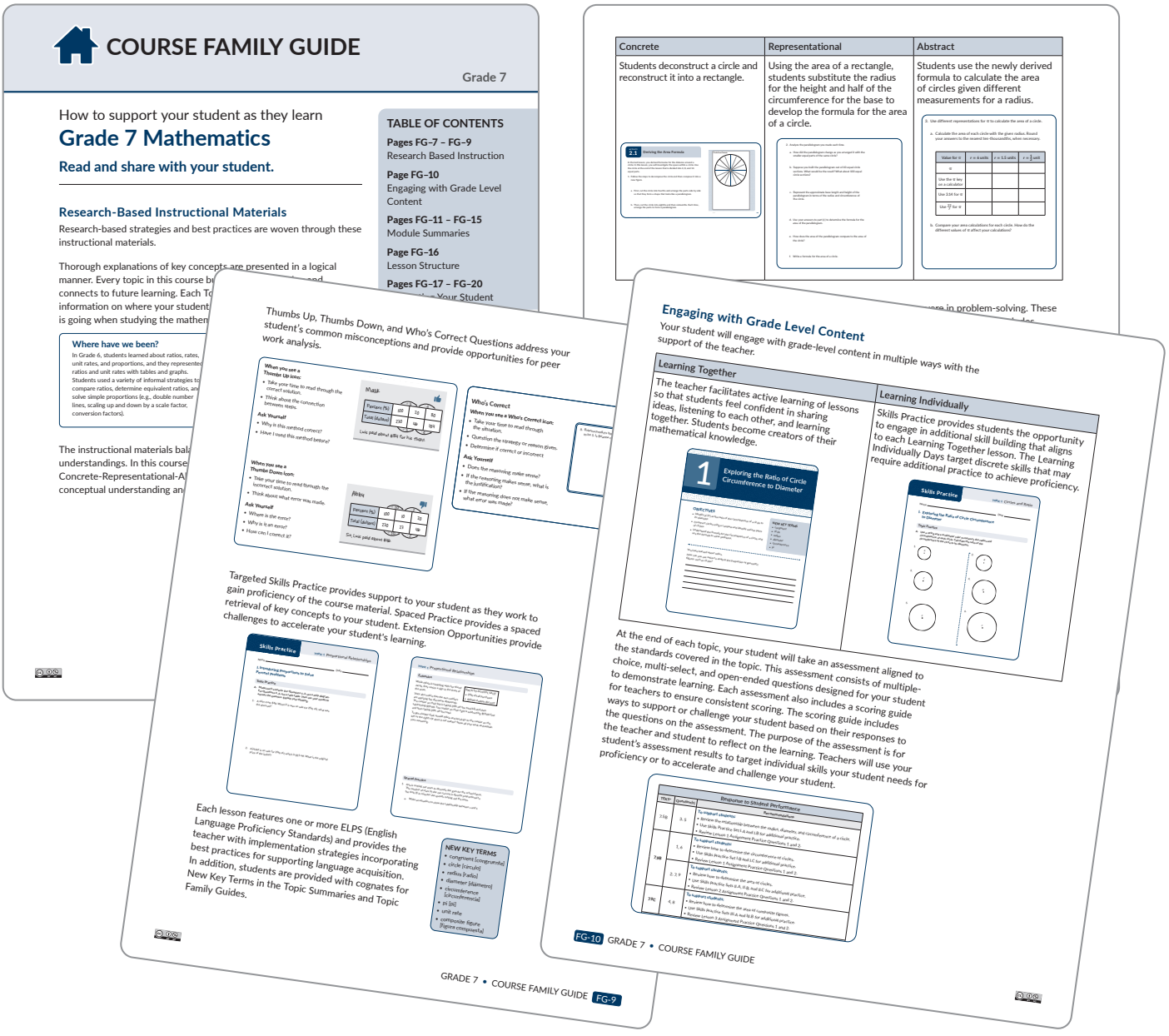
Prepare the Support

- Send home the Course Family Letter and the Course Family Guide on the first day.
- Encourage families and caregivers to read the Student Course Guide.
- Ensure that families receive the Topic Family Guide at the start of each topic.
- Consider a Family Math Night some time within the first few weeks of the school year.

Course Family Guide

The Course Family Guide, available in both English and Spanish, provides families with strategies they can use to support their students. The guide explains the research-based instructional approach of the course. It details the course structure as well as an overview of the content in each module of the course. The guide details the resources available to support learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

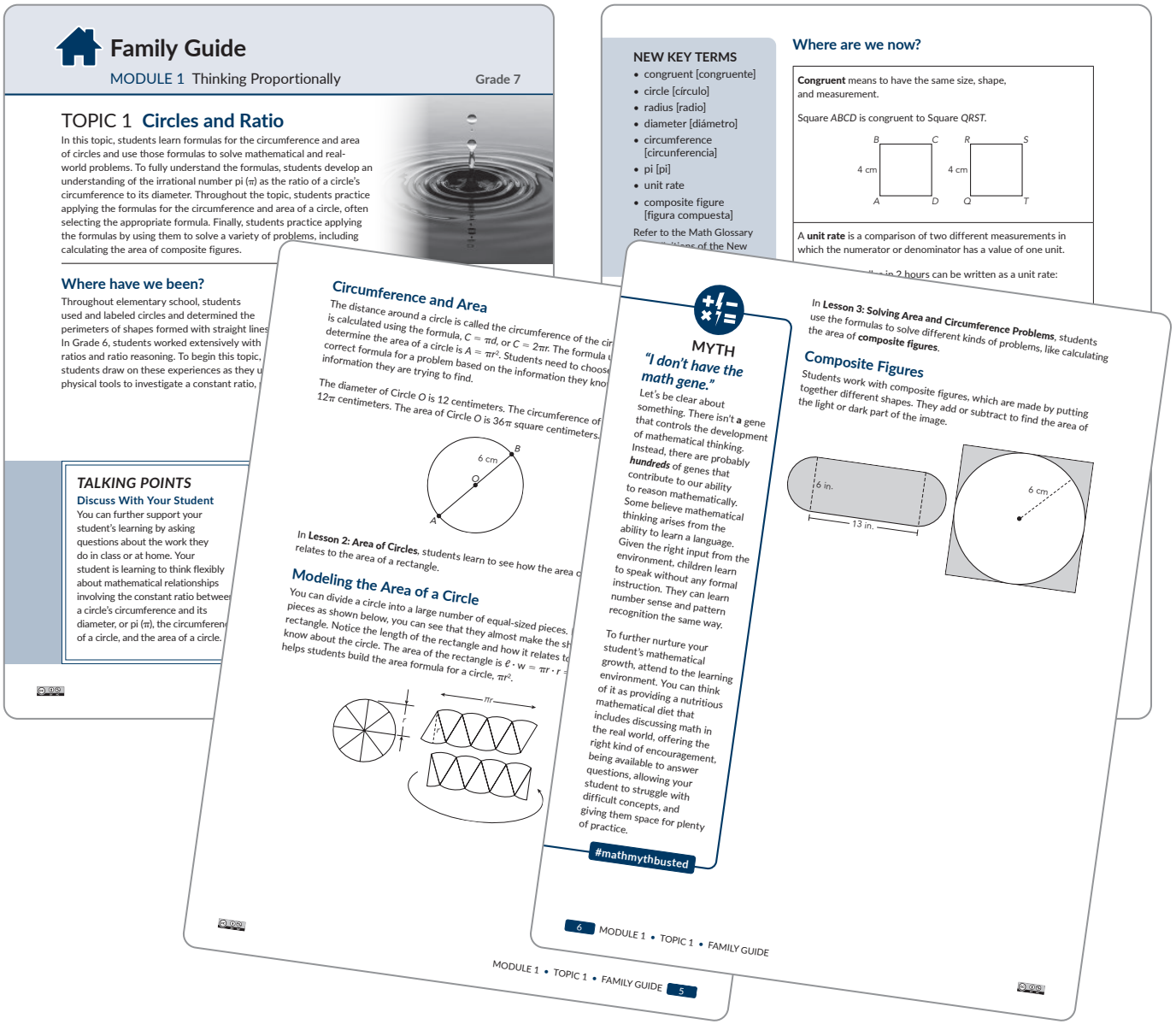
The purpose of the Course Family Guide is to bridge student learning in the classroom to student learning at home. The goal is to empower families to understand the concepts and skills learned in the classroom so that families can review, discuss, and solidify the understanding of these key concepts together.



Topic Family Guides

Each topic contains a Topic Family Guide, which is available in both English and Spanish. The Topic Family Guide provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how they will use that knowledge in future learning. It provides families an example of a math model or strategy their student is learning in the topic, busting of a math myth, questions to ask their student to support their learning, and the new key terms their student will learn.

Learning outside of the classroom is crucial to students' success at school. While families are not expected to be math teachers, the Topic Family Guides are designed to assist families as they talk to their students about what they are learning. The hope is the students and their families will read and benefit from the guides.



Technology Use

Technology is a useful tool in the classroom. Even though students are expected to complete calculations without a calculator in Grade 7, specific lessons call for a calculator to aid in the students in their computation, so they can focus on the conceptual understanding of the mathematics. In addition, there are benefits to occasionally using graphing technology to aid students in their mathematical concept development as they explore patterns. There are also places where additional online tools or resources are suggested in the materials for a lesson to enhance student learning. When using any website or online tool in the classroom, it is important to preview and vet the tool prior to classroom use.

Course Table of Contents

MODULE 1 Thinking Proportionally

TOPIC 1 Circles and Ratio 3

Introduction to the Problem-Solving Model and Learning Resources

LESSON 1 Exploring the Ratio of Circle Circumference to Diameter

LESSON 2 Area of Circles

LESSON 3 Solving Area and Circumference Problems

TOPIC 2 Fractional Rates. 49

LESSON 1 Unit Rate Representations

LESSON 2 Solving Problems with Ratios of Fractions

LESSON 3 Solving Proportions Using Means and Extremes

TOPIC 3 Proportionality 95

LESSON 1 Proportional Relationships

LESSON 2 Constant of Proportionality

LESSON 3 Identifying the Constant of Proportionality in Graphs

LESSON 4 Constant of Proportionality in Multiple Representations

MODULE 2 Applying Proportionality

TOPIC 1 Proportional Relationships 175

LESSON 1 Introducing Proportions to Solve Percent Problems

LESSON 2 Calculating Tips, Commission, and Simple Interest

LESSON 3 Sales Tax, Income Tax, and Fees

LESSON 4 Percent Increase and Percent Decrease

LESSON 5 Scale and Scale Drawings

TOPIC 2 Financial Literacy: Interest and Budgets 273

LESSON 1 Simple and Compound Interest

LESSON 2 Net Worth Statements

LESSON 3 Personal Budgets

MODULE 3 Reasoning Algebraically

TOPIC 1 Operating with Rational Numbers 325

LESSON 1 Adding and Subtracting Rational Numbers

LESSON 2 Quotients of Integers

LESSON 3 Simplifying Expressions to Solve Problems

LESSON 4 Using Number Properties to Interpret Expressions with Signed Numbers

LESSON 5 Evaluating Algebraic Expressions

LESSON 6 Rewriting Expressions Using the Distributive Property

TOPIC 2 Two-Step Equations and Inequalities 411

LESSON 1 Modeling Equations as Equal Expressions

LESSON 2 Solving Equations Using Algebra Tiles

LESSON 3 Solving Equations on a Double Number Line

LESSON 4 Using Inverse Operations to Solve Equations

LESSON 5 Using Inverse Operations to Solve Inequalities

TOPIC 3 Multiple Representations of Equations **513**

LESSON 1 Representing Equations with Tables and Graphs

LESSON 2 Building Inequalities and Equations to Solve Problems

LESSON 3 Using Multiple Representations to Solve Problems

MODULE 4 Analyzing Populations and Probabilities

TOPIC 1 Introduction to Probability **585**

LESSON 1 Defining and Representing Probability

LESSON 2 Probability Models

LESSON 3 Determining Experimental Probability of Simple Events

LESSON 4 Simulating Simple Experiments

TOPIC 2 Compound Probability **673**

LESSON 1 Using Arrays to Organize Outcomes

LESSON 2 Using Tree Diagrams

LESSON 3 Determining Compound Probability

LESSON 4 Simulating Probability of Compound Events

TOPIC 3 Drawing Inferences **757**

LESSON 1 Collecting Random Samples

LESSON 2 Using Random Samples to Draw Inferences

LESSON 3 Bar Graphs

LESSON 4 Comparing Two Populations

LESSON 5 Using Random Samples from Two Populations to Draw Conclusions

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MODULE 5 Constructing and Measuring

TOPIC 1 Angle Relationships 891

LESSON 1 Solving Equations Using the Triangle Sum Theorem

LESSON 2 Relationships Between 90° and 180° Angles

LESSON 3 Special Angle Relationships

TOPIC 2 Area, Surface Area, and Volume 947

LESSON 1 Composite Figures

LESSON 2 Total Surface Area of Prisms and Pyramids

LESSON 3 Volume of Prisms and Pyramids

Math Glossary G1

GRADE 7

MODULE 4

Analyzing Populations and Probabilities

TOPIC 1 Introduction to Probability **585**

TOPIC 2 Compound Probability **673**

TOPIC 3 Drawing Inferences **757**

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MODULE 4 OVERVIEW

TEKS Addressed:
7.6A, 7.6B, 7.6C, 7.6D, 7.6E, 7.6F, **7.6G, 7.6H, 7.6I, 7.12A**, 7.12B, 7.12C

*Bold TEKS = Readiness Standard

Analyzing Populations and Probabilities

Sessions: **43**

Why is this module named *Analyzing Populations and Probabilities?*

Analyzing Populations and Probabilities formally introduces and explores probabilities and uses numeric and graphical displays students learned in previous courses to compare populations. Students learn to analyze the representativeness of experimental probabilities and statistics drawn from samples of populations, focusing on the validity and usefulness of appropriately generated data.

The Research Shows . . .

"Probability is an important part of any mathematical education. It is a part of mathematics that enriches the subject as a whole by its intersection with other uses of mathematics. Probability is an essential tool in applied mathematics and mathematical modeling. It is also an essential tool in statistics."

GAISE Report | 8

What is the mathematics of *Analyzing Populations and Probabilities?*

Analyzing Populations and Probabilities contains three topics: *Introduction to Probability*, *Compound Probability*, and *Drawing Inferences*. Students learn the basics of probability and use experimental and theoretical probability to make predictions. They construct probability models and design simulations. They generate random samples to determine statistics from populations and use them to make conclusions.

1 DAY PACING = 45-MINUTE SESSION

14 SESSIONS
13 LEARNING • 1 ASSESSMENT

TOPIC 1 *Analyzing Populations and Probabilities*

Learning Together: 9 Sessions

TEKS: 7.6B, 7.6C, 7.6D, 7.6E, **7.6H, 7.6I**

Students conduct probability experiments and determine theoretical and experimental probabilities of events.

- Students use probability models to organize the probabilities of outcomes in a sample space.
- Students use proportional reasoning to predict expected frequencies of favorable outcomes in larger samples.
- Students calculate the percent error between theoretical and experimental probabilities.
- Students use various tools to simulate the results of experiments.

Learning Individually: 4 Sessions

Targeted Skills Practice for *Analyzing Populations and Probabilities*

- Students estimate probabilities of events.
- Students determine theoretical and experimental probabilities of events.
- Students construct uniform and non-uniform probability models.
- Students describe a simulation to model a situation and then describe one trial.
- Students conduct experiments, record results, and determine experimental probabilities.

12 SESSIONS
11 LEARNING • 1 ASSESSMENT

TOPIC 2 *Compound Probability*

Learning Together: 8 Sessions

TEKS: 7.6A, 7.6B, 7.6C, 7.6D, **7.6I**

Students use arrays, lists, and tree diagrams to organize the possible outcomes of an experiment.

- Students create probability models, calculate experimental and theoretical probabilities, and use proportional reasoning to determine percent error and make predictions of expected numbers of outcomes.
- Students learn about compound events that use the conjunctions *and* and *or*.
- Students design and conduct simulations for compound probability problems.

Learning Individually: 3 Sessions

Targeted Skills Practice for *Compound Probability*

- Students use arrays to construct sample spaces.
- Students use tree diagrams to construct probability models.
- Students determine compound probabilities.
- Students use simulations to answer questions.

Learning Together: 11 Sessions

TEKS: 7.6B, 7.6F, **7.6G, 7.12A**, 7.12B, 7.12C

Students use random samples to collect representative data from a specified population.

- Students use the results of the sample and proportional reasoning to estimate population parameters.
- Students use data displays and measures of center and variation to compare populations.
- Students use random samples to draw inferences about populations and to compare two populations.

Learning Individually: 5 Session

Targeted Skills Practice for *Drawing Inferences*

- Students determine whether data collected in a survey represent a census or a sample and whether each survey result is a parameter or a statistic.
- Students use random samples to calculate measures.
- Students compare the measures of center and spread in problem situations.
- Students analyze bar graphs, double bar graphs, stacked bar graphs, circle graphs, stem-and-leaf plots, and box plots.

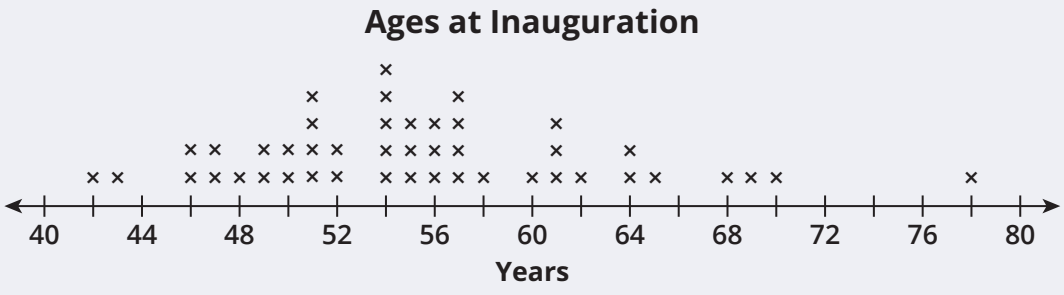
How is *Analyzing Populations and Probabilities* connected to prior learning?

This module builds on students' experiences with proportional reasoning, including percent error. They have developed formal strategies for solving proportions and calculating percent error. Students use these skills to make predictions and compare the percent error between experimental and theoretical probabilities and between statistics and parameters.

In previous courses, students calculated the mean, the median, and the interquartile range of data sets. They also displayed numeric data using dot plots, stem-and-leaf plots, histograms, and box plots.

Math Representation

Catalina collected data on the ages of the 46 presidents of the United States at their first inauguration. She created data displays of the presidents' ages at inauguration.



When will students use knowledge from *Analyzing Populations and Probabilities* in future learning?

Students will develop a more in-depth and more algorithmic approach to probability in high school, including compound and conditional probabilities.

Math Representation

Conditional probability is the probability of event B, given that event A has already occurred: $P(B \mid A)$. You can represent the conditional probability, $P(B \mid A)$ for independent events as $\frac{\text{desired outcomes}}{\text{total outcomes}}$.

The probability of event B, given that A has already occurred is the probability of A and B divided by the probability of A.

$$P(B \mid A) = \frac{\text{desired outcomes}}{\text{total outcomes}}$$

$$= \frac{A \text{ and } B}{A}$$

$$= \frac{A \text{ and } B}{A} \cdot \frac{\frac{1}{\text{total}}}{\frac{1}{\text{total}}}$$

$$= \frac{\frac{A \text{ and } B}{\text{total}}}{\frac{A}{\text{total}}}$$

$$= \frac{P(A \text{ and } B)}{P(A)}$$

Students will also engage in varied types of data collection and need to understand the role of randomization and simulation in making valid inferences.

4 Analyzing Populations and Probabilities

MODULE 1 Assessment Summary

Topic	Topic Title	Name	Administered	TEKS*
1	**Introduction to Probability**	End of Topic Assessment	After Topic 1	7.6B 7.6C 7.6D 7.6E **7.6H** **7.6I**
2	**Compound Probability**	End of Topic Assessment	After Topic 2	7.6A 7.6B 7.6C 7.6D **7.6I**
3	**Drawing Inferences**	End of Topic Assessment	After Topic 3	**7.6G** 7.12A 7.12B 7.12C

*Bold TEKS = Readiness Standard

GRADE 7

TOPIC 1

Behind two of the doors are delicious cakes. Behind one of the doors is a delicious piece of chocolate. If you can only pick a door once, what are your chances of eating a cake versus snacking on a piece of chocolate?

Introduction to Probability

LESSON 1 Defining and Representing Probability **587**

LESSON 2 Probability Models **611**

LESSON 3 Determining Experimental Probability of Simple Events . . . **625**

LESSON 4 Simulating Simple Experiments **647**

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TOPIC 1 OVERVIEW

Introduction to Probability

How are the key concepts of *Introduction to Probability* organized?

In *Introduction to Probability*, students conduct simple experiments and determine theoretical and experimental probabilities of simple events. They use familiar objects, such as number cubes, marbles in a bag, and spinners, to learn the terminology of probability, including *outcome*, *experiment*, *sample space*, *event*, *simple event*, *probability*, *complementary events*, and *equally likely*.

Students use probability models to organize the probabilities of outcomes in a sample space differentiating uniform and non-uniform probability models. They create models from experiments, but they also use models to calculate probabilities.

Math Representation

The number line shown represents the probabilities, from 0 to 1, of any event occurring.

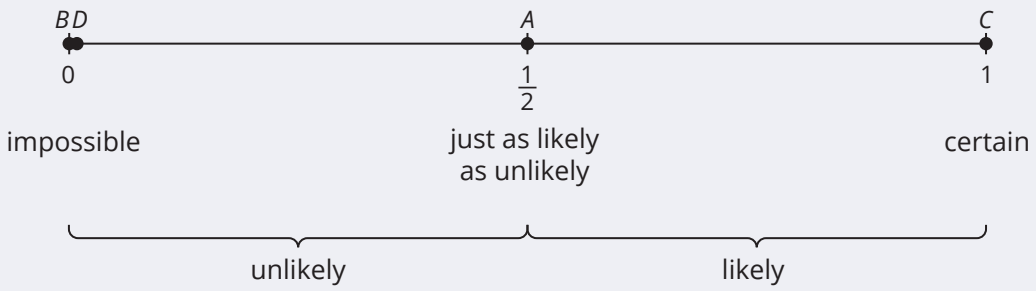
You can plot the estimated probability of events occurring on the number line.

A. A card selected from a standard 52-card deck is red.

B. Your neighbors will get a pet dinosaur.

C. You will have a test in one class this month.

D. A seventh-grader is more than 6 feet tall.



Students conduct probability experiments and use proportional reasoning to predict expected frequencies of favorable outcomes in larger samples and to calculate the percent error between theoretical and experimental probabilities. For real-world probability situations that require a large number of trials, students use simulation techniques, including random number tables, to simulate the results of experiments. Through simulation, students learn that as the number of trials increases, experimental probabilities approach theoretical probabilities.

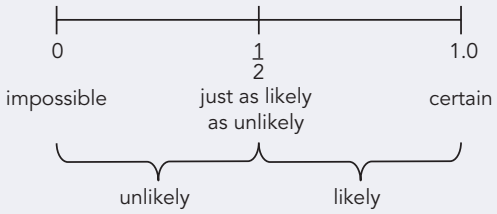
What is the entry point for students?

Although Grade 7 provides students' formal introduction to probability, students have encountered probability situations throughout their lives. The topic opens with asking students to interpret the meaning of a meteorologist's forecast. Students use their intuition of the meaning of "chance of rain" and rewrite the percent as a fraction. They perform an experiment to simulate the likelihood of guessing correct answers on a multiple choice test. They are also familiar with the tools they use to learn the basics of probability: six-sided number cubes, coins, and spinners.

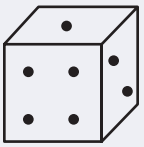
Math Representation

Probability Representations and Tools Used

- Number Lines



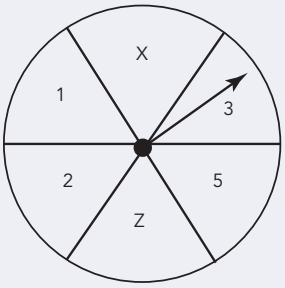
- Six-Sided Number Cubes



- Probability Models

Outcome	2	3	4	5	6	7
Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

- Spinners



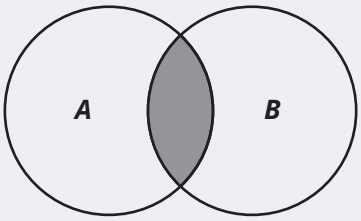
Why is *Introduction to Probability* important?

Students have already started exploring the role of probability in mathematics and statistics. Like measures of variability, probabilities can be used to make conclusions and predictions about situations. In an upcoming topic, students will use probability and ideas of randomness to explore sampling and drawing inferences about data, which is the start of the formal study of statistical inference.

The ideas developed in this topic will be used in future topics, as students build on the basic ideas of experimental and theoretical probabilities and simulation techniques to analyze and explore compound events.

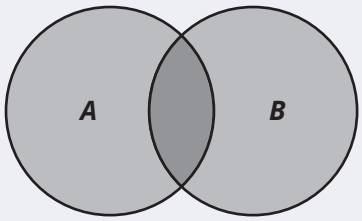
Math Representation

Determining the probability of a compound event with the word *and* is different from a compound event with *or*.



A and *B*

When determining a compound event with *and*, you determine the probability that both events occur.



A or *B*

When determining a compound event with *or*, you determine the probability that one, or the other, or both outcomes occur.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic when they can:

- Identify the sample space of an experiment.
- Recognize and explain that the probability of a chance event is a number between 0 and 1 that expresses how likely an event is to occur.
- Determine the probabilities of a simple event and its complement and describe the relationship between the two.
- Calculate experimental probability as the number of times an outcome occurs divided by the total number of times the experiment is completed.
- Calculate theoretical probability as the number of favorable outcomes in the sample space divided by the total number of outcomes in the sample space.
- Make predictions and determine solutions using experimental and theoretical probability for simple events.
- Determine experimental and theoretical probabilities related to simple events using data and sample spaces.

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NEW KEY TERMS

- outcome
- experiment [experimento]
- sample space
- event [evento]
- simple event [evento simple]
- probability [probabilidad]
- complementary events [eventos complementarios]
- equally likely
- probability model [modelo probabilístico]
- uniform probability model [modelo probabilístico uniforme]
- non-uniform probability model [modelo probabilístico no uniforme]
- theoretical probability [probabilidad teórica]
- experimental probability [probabilidad experimental]
- percent error [error porcentual]
- simulation [simulación]
- random number table

- Use probability models to determine the probability of events.
- Design and use a simulation to determine experimental probabilities of simple events.

How do the activities in *Introduction to Probability* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

Novice users of probability and statistics may believe that the uncertainty and variability inherent in probability and statistics contradict the need for precision. However, precision is necessary for sampling and simulation methods; students must select appropriate tools or assignments of numbers or values in simulations (7.1C). Precision is also important as students use probabilities and proportional reasoning to make predictions and calculate percent error (7.1G). Students are expected to reason about probability concepts and use probabilities to model real-world situations while attending to precision in language, methods, and computation (7.1A).

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Strategically grouping students with varying language proficiencies and factoring in group members' shared languages helps foster a collaborative learning environment. Peer discourse enables students to explain concepts to each other in both languages and build off of each other's language, enhancing understanding and language skills simultaneously.

MODULE 4, TOPIC 1 PACING GUIDE

165-Day Pacing

4 Analyzing Populations and Probabilities

TOPIC 1: Introduction to Probability

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1A, 7.1B, 7.1C, 7.1D, 7.1E, 7.1F, 7.1G

ELPS: 1.E, 2.C, 2.G, 3.D, 3.F, 3.G, 4.B, 4.C, 4.K, 5.B

Topic Pacing: 14 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	**Defining and Representing Probability**	Students conduct an experiment that involves rolling a one six-sided number cube. The terms *outcome*, *experiment*, *sample space*, *event*, *simple event*, *probability*, *complementary events*, and *equally likely* are defined. Students calculate probabilities rolling number cubes, using spinners, and drawing marbles from a bag. **Materials Needed:** None	7.6E **7.6H** **7.6I**	3
2	**Probability Models**	The terms *probability model*, *uniform probability model*, and *non-uniform probability model* are defined in this lesson. Students will develop a probability model for an experiment and use it to determine probabilities of events. They will construct and interpret uniform and non-uniform probability models. **Materials Needed:** None	**7.6I**	2
3	**Determining Experimental Probability of Simple Events**	Students flip a coin multiple times to determine the probabilities of heads and tails based on the results of the experiment. The terms *theoretical probability* and *experimental probability* are defined in this lesson. Students conduct trials in an experiment to estimate probabilities of the three outcomes. They also conduct trials of a spinner game to calculate experimental probabilities using data. Students use those experimental probabilities to predict the number of outcomes for a given number of trials. They then compare the experimental probabilities to the theoretical probabilities using *percent error*. **Materials Needed:** Coins, Paper or Plastic Cups, Paper Clips	7.6C 7.6D **7.6H** **7.6I**	2
4	**Simulating Simple Experiments**	The term *simulation* is defined in this lesson. A coin toss serves as a simulation to determine the experimental probability of the percent of female chickens hatched. Students note that as the number of trials increases, the experimental probability approaches the theoretical probability. Other situations used in this lesson are a five-question multiple-choice test, a ten-question true or false test, a number cube game, and a card game. Students describe simulation models that fit each situation. **Materials Needed:** Coins, Paper Clips, Number Cubes, Technology to Simulate a Spinner, Note Cards, Problem-Solving Graphic Organizer	7.6B 7.6C 7.6D	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				4

*Bold TEKS = Readiness Standard

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MODULE 4, TOPIC 1 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.6E, **7.6H, 7.6I** **LESSON 1** Defining and Representing Probability **GETTING STARTED** **ACTIVITY 1**	**LESSON 1** continued **ACTIVITY 2** **ACTIVITY 3**	**LESSON 1** continued **ACTIVITY 4** **ACTIVITY 5** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **7.6I** **LESSON 2** Probability Models **GETTING STARTED** **ACTIVITY 1**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 2 continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: 7.6C, 7.6D, **7.6H, 7.6I** **LESSON 3** Determining Experimental Probability of Simple Events **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 11	**Day 12**	**Day 13**	**Day 14**	
TEKS: 7.6B, 7.6C, 7.6D **LESSON 4** Simulating Simple Experiments **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 4** continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**	

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are four Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets can be assigned to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Defining and Representing Probability

MATERIALS

None

LESSON OVERVIEW

Students conduct an experiment that involves rolling one six-sided number cube. The terms *outcome*, *experiment*, *sample space*, *event*, *simple event*, *probability*, *complementary events*, and *equally likely* are defined. Students calculate probabilities rolling number cubes, using spinners, and drawing marbles from a bag.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1E create and use representations to organize, record, and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6E find the probabilities of a simple event and its complement and describe the relationship between the two.

7.6H solve problems using qualitative and quantitative predictions and comparisons from simple experiments.

7.6I determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

ELPS

(2) Listening

The student is expected to:

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

(4) Reading

The student is expected to:

(C) develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials.

(K) demonstrate English comprehension and expand reading skills by employing analytical skills such as evaluating written information and performing critical analyses commensurate with content area and grade-level needs.

ESSENTIAL IDEAS

- An *experiment* is a situation involving chance that leads to results or outcomes.
- An *outcome* is the result of a single trial of an experiment.
- A *sample space* is the list of all possible outcomes of an experiment.
- An *event* is one or a group of possible outcomes for a given situation.
- A *simple event* is an event consisting of one outcome.
- *Probability* is a measure of the likelihood that an event will occur.
- The probability of an event can be determined using the formula:

$$\text{probability} = \frac{\text{number of times an event can occur}}{\text{number of possible outcomes}}$$

- When the probability of an event is equal to 0, there is no chance that the event will occur.
- When the probability of an event is equal to 1, there is certainty that the event will occur.
- Complementary events are events that consist of the desired outcomes and the remaining events that consist of all the undesired outcomes.
- The sum of the probabilities of any two complementary events is 1.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: What Are the Chances? 10–15 minutes

INVESTIGATION, BUILD OFF INTUITION

Students analyze two different statements that include probabilities. They write the meaning of the statement and then convert the probability from a percent to a fraction or a fraction to a percent.

DEVELOP

Activity 1.1: Calculating Simple Probabilities 25–30 minutes

INVESTIGATION, WORKED EXAMPLE

The terms *experiment*, *outcome*, *sample space*, *event*, *simple event*, and *probability* are defined. Students create a list of all possible numbers that can be shown on the top face of a six-sided number cube. They then list the sample space for different events and determine the probability of various outcomes.

DAY 2

Activity 1.2: Complementary Events 15 minutes

INVESTIGATION, PEER WORK ANALYSIS

The term *complementary events* is defined and notation is given. Students explore complementary events for rolling a number cube. They draw conclusions about the sum of the probabilities of any two complementary events.

Activity 1.3: Simple Probability and Spinners 25–30 minutes

MATHEMATICAL PROBLEM SOLVING, PEER WORK ANALYSIS

In the first problem situation, a spinner is divided into six sectors of equal size. Each section is labeled using a single number or a letter. Students describe different events, list possible outcomes, and calculate probabilities of the specified events occurring. In the second problem situation, a spinner is divided into two sectors of unequal size. Both sections are labeled using a single letter. Students predict the probabilities of specified events occurring.

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DAY 3

Activity 1.4: Probabilities from 0 to 1 10–15 minutes

REAL-WORLD PROBLEM SOLVING, MATHEMATICAL PROBLEM SOLVING

Through a series of questions, students conclude the certainty of an event occurring is associated with 1, whereas the impossibility of an event occurring is associated with 0. All other probabilities range between 0 and 1. If events are equally likely to occur, their probability is $\frac{1}{2}$. Students complete a chart using these conclusions, expressing the probabilities as fractions, decimals, and percents. Finally, they estimate the probabilities of various events as fractions, decimals, and percents.

Activity 1.5: A Simple Probability Experiment 20 minutes

REAL-WORLD PROBLEM SOLVING

In this experiment, there are two blue, three green, and five yellow marbles in the bag. Students list the sample space of the experiment and calculate the probabilities of each event to conclude that probabilities of the outcomes are different. Students conclude that the sum of all the probabilities in one experiment is equal to 1.

DEMONSTRATE

Talk the Talk: Absolutely/Never 10 minutes

GENERALIZATION

Students explain why the sum of the probabilities for all outcomes of an experiment cannot be greater than 1. They describe an event that has a probability of 0 and an event that has a probability of 1. Students answer questions reviewing that the sum of the probabilities of complementary events is 1.

Getting Started

ENGAGE

What are the Chances?

Facilitation Notes

In this activity, students explain and rewrite given statements involving probabilities. They convert the probability from a percent to a fraction or a fraction to a percent.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Is it more likely to rain or not rain today? How do you know? • What do both statements in Question 2 have in common?
Reflecting and justifying	• How do you rewrite any fraction as a percent?

COMMON MISCONCEPTION

There may be confusion over the meaning of the phrase "40 percent chance of rain today." This means that there is a 40 percent chance that rain will occur at any given point in the area.

The purpose of this activity is to introduce students to examples of probability in everyday life. The details of interpreting weather forecast language is not the priority.

Summary

Probabilities can be expressed using decimals, percents, and fractions.

STAMP THE LEARNING

ACTIVITY 1.1

Calculating Simple Probabilities

DEVELOP

Facilitation Notes

In this activity, terms associated with probability are defined and students answer questions related to a six-sided number cube.

Ask a student to read the information and definitions aloud. Then, complete Questions 1 and 2 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Does every number have the same chance of appearing on the top face when you roll a number cube? Explain.
Probing	• What are other examples of an *experiment*? Describe a possible sample space. • What is the difference between an *outcome* and a *sample space*?

Ask a student to read the definitions following Question 2 aloud. Discuss the definitions and Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is another example of a simple event when rolling a number cube? An event?
Gathering	• What is the event? • What is meant by *possible outcomes in the event*?

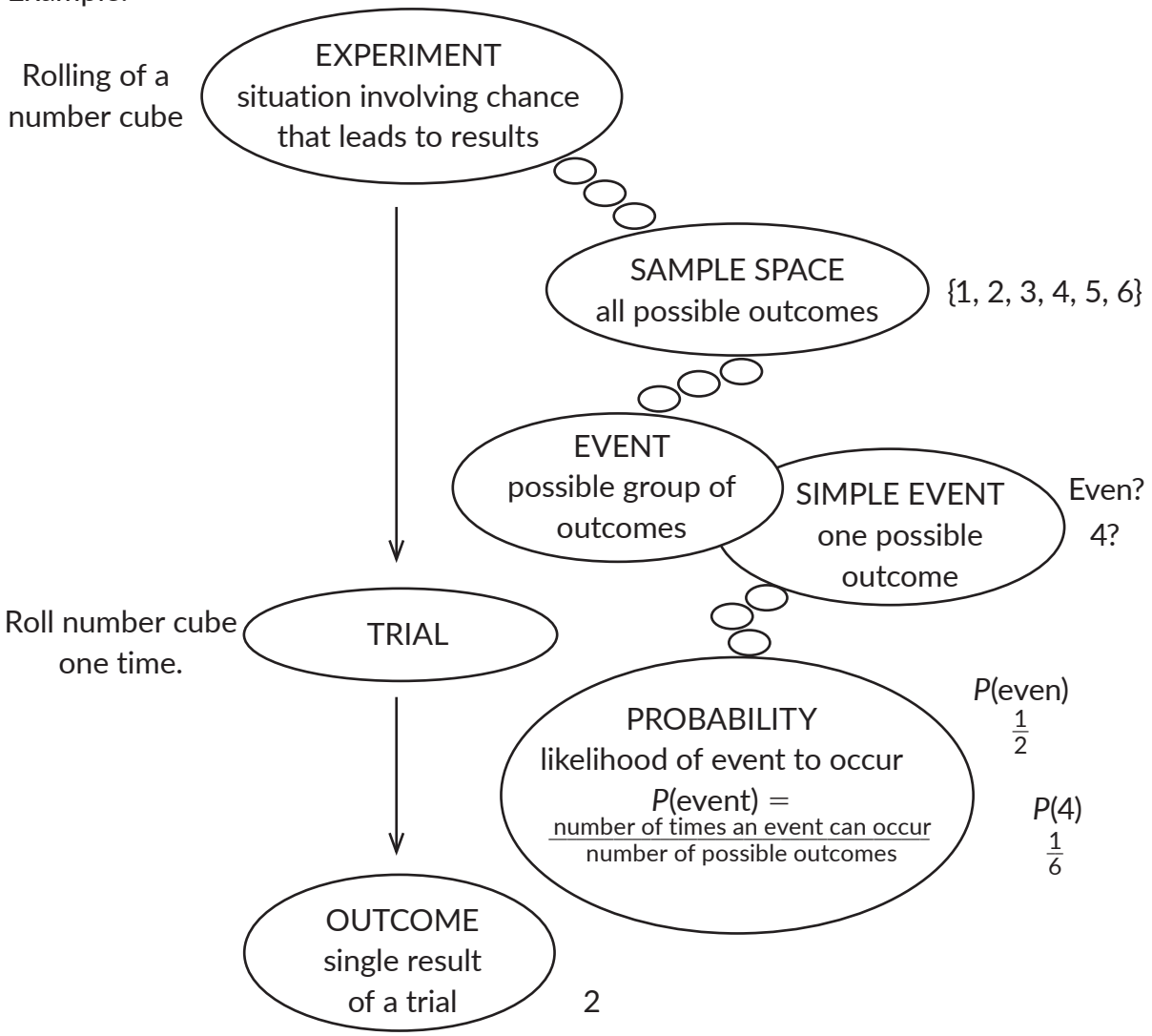
DIFFERENTIATION STRATEGY

Access for All

Have students create a graphic organizer to make sense of the vocabulary in this activity.

- First, have students create a list in chronological order of what is visible: experiment, trial, and outcome.
- Next, provide examples.
- Then, have students add the thinking bubbles for sample space, event, simple event, and probability.
- Lastly, have students add the examples.

Example:



Have students work with a partner or in a group to complete Questions 3 through 8. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Is rolling an even number a simple event? Why or why not? • What are the two quantities in your ratio? What do they represent?

COMMON MISCONCEPTIONS

- Students sometimes confuse the outcome and the number of ways the outcome can occur. For example, when rolling a number cube, $P(4) = \frac{1}{6}$, not $\frac{4}{6}$.

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- Students sometimes do not understand the meaning of the probability when read as a fraction. To help students connect the fractional answer with the context, have them read a fraction such as $\frac{1}{6}$ as "one out of six" rather than "one-sixth."

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Calculating the probability is measuring the likelihood that an event will occur. It assigns a numeric value to the chance that an event will occur.

ACTIVITY 1.2

Complementary Events

Facilitation Notes

In this activity, students will explore complementary events for rolling a number cube. They draw conclusions about the sum of the probabilities of any two complementary events.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the definition and information. Discuss as a class.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Can you think of additional complementary events for the number cube experiment?
Seeing structure	• Suppose the probability of an event is x, what is the probability of its complementary event?
Probing	• How did Catalina get a 1 for the numerator of the probability ratio? • Why did Linh subtract the probability from 1?
Seeing structure	• Suppose you know the probability of an event, what are two ways you can calculate the probability of the complementary event?

COMMON MISCONCEPTION

Students may incorrectly think the complement of *P*(odd number) is *P*(even number). The complement of *P*(odd number) must contain all the remaining outcomes in the sample space; therefore, it is most efficient to state the complement as *P*(not odd) to include the even number, 2, and the letters, *X* and *Z*.

DIFFERENTIATION STRATEGY

Access for All

Have students interact with Catalina's and Linh's work. For each student, have them write the sample space, the number of times the desired outcome occurs, and then the probability. Discuss why Catalina's work says *P*(not 5 or less) and Linh's work says *P*(5 or less), but they got the same final result.

Summary

STAMP THE LEARNING

Complementary events are events that together contain all of the outcomes in a sample space and, therefore, add up to one complete whole. The sum of the probabilities of any two complementary events is 1.

ACTIVITY 1.3 Simple Probability and Spinners

Facilitation Notes

In this activity, students use spinners to describe events, list possible outcomes, and calculate probabilities. They also predict the probabilities of specified events occurring.

Have students work with a partner or in a group to complete Questions 1 through 6. Emphasize to students that if the spinner stops on a line, rather than in a section, it must be spun again until it stops within a section. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

Students who misinterpret the probability of a number to only include numbers in the sample space or the probability of a letter to only include letters in the sample space. For example, *P*(odd number) is $\frac{3}{6}$ or $\frac{1}{2}$, not $\frac{3}{4}$. The sample space should include all 6 items on the spinner, regardless of whether the probability of a number or letter is being determined.

COMMON MISCONCEPTION

Use this activity to disprove the misconception that complements are always opposites. For example, in this context, the complement of *P*(odd number) is *P*(not odd number); *P*(even number) is incorrect because it does not contain all the remaining outcomes in the sample space.

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QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which characteristic of the spinner tells you the number of possible outcomes in the experiment? The sample space? • How does the probability of the spinner stopping on a number compare to the probability of the spinner stopping on a letter? • What is another way to calculate *P*(not a letter)? • Why isn't *P*(even number) the complement of *P*(odd number)? • Why isn't *P*(vowel) the complement of *P*(consonant)?
Seeing structure	• What is the probability of an event that is impossible? • What is the probability of an event that is certain to happen?

Have students work with a partner or in a group to complete Questions 7 and 8. Share responses as a class.

QUESTION TO SUPPORT DISCOURSE

Probing	• How did you get numeric answers for these probabilities?

COMMON MISCONCEPTION

Although students can see visually that the *P*(*A*) is greater than *P*(*B*), they may not realize that to calculate the probability correctly, the entire sample space must be divided into pieces of the same size. This was intuitive for the number cubes and was not an issue in the first spinner problem.

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Spinners can be used to compute simple probability.

ACTIVITY 1.4

Probabilities from 0 to 1

Facilitation Notes

In this activity, students answer questions about the certainty of an event occurring and the impossibility of an event occurring. A chart is completed expressing these probabilities in fractions, decimals, and percents.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

- Students may struggle with the fact that 100 percent = 1. Remind them that percent means "per 100." To rewrite a decimal as a percent, you multiply by 100.
- Connect the meaning of the probability $\frac{1}{2}$, 0.5, and 50% with the common phrase "fifty-fifty."

COMMON MISCONCEPTION

Students sometimes struggle with the fact that 100% = 1. To make sense of this equality, use a quiz score as an example, where there are 20 questions with each question worth 1 point. A score of $\frac{15}{20}$ or "15 out of 20" demonstrates the portion of quiz questions which were answered correctly. A score of $\frac{20}{20}$ equals 1, which means that the whole quiz was answered correctly, or 100% of the quiz questions were answered correctly.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which fractions represent probabilities of events that are more likely to occur? Decimal? Percents? • When an event is certain to happen, how do the numerator and denominator of the probability ratio relate? • Which fractions represent probabilities of events that are less likely to occur? Decimal? Percents? • When it is impossible for an event to happen, what is the numerator of the probability ratio? • What is another example of an event with a probability of 0 percent? 50 percent? 100 percent?
Seeing structure	• What is a possible probability of an event that is more likely to occur than not occur? Explain your thinking. • When an event has a probability of $\frac{1}{4}$, is it more or less likely to happen? Explain your thinking.

Summary

STAMP THE LEARNING

The certainty of an event occurring is associated with 1, whereas the impossibility of an event occurring is associated with 0. All other probabilities range between 0 and 1. If events are equally likely to occur, their probability is $\frac{1}{2}$.

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ACTIVITY 1.5

A Simple Probability Experiment

Facilitation Notes

In this activity, students list the sample space of an experiment and calculate the probabilities of each event.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

Students may be in the habit of expressing fractions in lowest terms. However, when calculating the sum of probabilities, it is more efficient to keep the individual fractions in their original state so that they have a common denominator. Use Question 4 to make this explicit to students. For $P(B)$, keep the fraction written as $\frac{2}{10}$ rather than expressing it as $\frac{1}{5}$, so that it can be combined easily with $\frac{3}{10}$ and $\frac{5}{10}$.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why isn't the sample space {blue, green, yellow}? • What is $P(B) + P(Y)$? • What is $P(\text{not } B)$?
Gathering	• How could you change the marble bag so that the outcomes are equally likely?

Have students work with a partner or in a group to complete Questions 7 through 11. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Is each outcome equally likely? • How many students did James survey? How do you know?
Reflecting and justifying	• What is another way to calculate the probability of not purple?

STAMP THE LEARNING

Summary

The sum of all the probabilities in one experiment is equal to 1.

DEMONSTRATE

Talk the Talk

ABSOLUTELY/NEVER

Facilitation Notes

In this activity, students answer questions about probability.

Have students complete Questions 1 through 5 independently. Then, share the responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• How can you use this fact in your answer to Question 1 to check your work when solving probability problems? • What is the sum of two complementary events? Why?
Seeing Structure	• What do events with a probability of 1 have in common? • What do events with a probability of 0 have in common?

Have students read and answer the Essential Question on the lesson opener page.

Summary

The sum of the probabilities for all outcomes of any experiment will never be greater than 1.

STAMP THE LEARNING

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1 Defining and Representing Probability

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Differentiate between an outcome and an event for an experiment.
- List the sample space for an experiment.
- Determine the probability for an event and the complement of an event.
- Identify the probability of an event as a number between 0 and 1, which can be expressed as a fraction, decimal, or percent.
- Determine that the sum of the probabilities of the outcomes of an experiment is always 1.

NEW KEY TERMS

- outcome
- experiment
- sample space
- event
- simple event
- probability
- complementary events
- equally likely

You have used ratios to represent relationships between two quantities.

How can you use ratios to represent the likelihood of an event?

Sample answer:

Probability is a measure of the likelihood that an event will occur. To calculate the probability of an event, or *P*(event), determine the ratio of the number of ways the event can occur to the total number of outcomes.

MODULE 4 • TOPIC 1 • LESSON 1 587

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.

Getting Started

What Are the Chances?

Consider each statement.

1. The local weatherman broadcasted that there is a 40% chance of rain today.

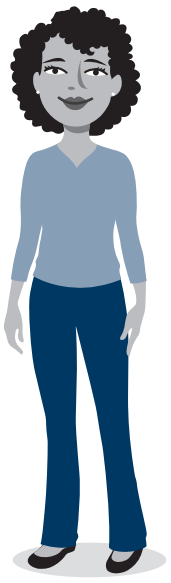
   a. In your own words, explain what the statement means.

   It means that it is more likely that it will not rain today. There is a 60% chance it will not rain today.

   b. Rewrite Question 1 using an equivalent fraction in place of the percent. Explain what it means.

   The local weatherman broadcasted that there is a $\frac{2}{5}$ chance of rain today. It means that there is a 2 out of 5 chance it will rain today and a 3 out of 5 chance it will not rain today.

Can a meteorologist predict the future? A meteorologist uses lots of science and math to provide you with the probability of what may happen with the weather.



2. For a multiple-choice question with four answer choices, the likelihood of guessing the correct answer is $\frac{1}{4}$.

   a. In your own words, explain what the statement means.

   It means that there is a 1 out of 4 chance that a person will guess the correct answer and a 3 out of 4 chance that a person will guess an incorrect answer.

   b. Rewrite Question 2 using an equivalent percent in place of the fraction. Explain what it means.

   For a multiple-choice question with four answer choices, the likelihood of guessing the correct answer is 25%. It means that there is a 25% chance that a person will guess the correct answer and a 75% chance a person will guess an incorrect answer.

588 MODULE 4 • TOPIC 1 • LESSON 1

ACTIVITY 1.1

Calculating Simple Probabilities

A six-sided number cube has one number, from 1 through 6, on each face. Number cubes are often used when playing board games.

Remember. . .
A cube is a three-dimensional figure with 6 sides called faces.

1. Create a list of all the possible numbers that can appear on the top face if you roll a six-sided number cube.

   The numbers 1, 2, 3, 4, 5, and 6 can appear on the top face.

The numbers on the faces of a six-sided number cube are the *outcomes* that can occur when rolling a six-sided number cube. An **outcome** is the result of a single trial of a probability *experiment*. An **experiment** is a situation involving chance that leads to results, or outcomes. A list of all possible outcomes of an experiment is called a **sample space**.

A sample space is typically enclosed in braces, { }, with commas between the outcomes.

2. List the sample space for the experiment of rolling a six-sided number cube.

   The sample space is {1, 2, 3, 4, 5, 6}.

An **event** is one possible outcome or a group of possible outcomes for a given situation. A **simple event** is an event consisting of one outcome. For example, in the number cube experiment, an event could be rolling an even number. However, rolling a 5 is a simple event.

An event is a subset of the sample space.

Probability is a measure of the likelihood that an event will occur. It is a way of assigning a numerical value to the chance that an event will occur. The probability of an event is often written as *P*(event). For example, in the number cube experiment, the probability of rolling a 5 could be written as *P*(5). The probability of rolling an even number could be written as *P*(even).

There is a formula to determine the probability of an event.

$$\text{probability} = \frac{\text{number of times an event can occur}}{\text{number of possible outcomes}}$$

Chunking the Activity

- Read and discuss the information and definitions.
- Complete Questions 1 and 2 as a class.
- Read and discuss the definitions and Worked Example.
- Group students to complete Questions 3-8.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Optimizing Learning

This activity clarifies vocabulary and symbols.

MODULE 4 • TOPIC 1 • LESSON 1 589

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

WORKED EXAMPLE

To determine the probability of rolling an odd number, or *P*(odd), follow these steps.

Step 1: First, list all the possible outcomes in the event.
The possible odd numbers that can be rolled are 1, 3, and 5.

Step 2: Add the number of outcomes.
There are 3 possible outcomes of rolling an odd number.

Step 3: Use the equation to determine the probability of rolling an odd number.

$$P(\text{odd}) = \frac{\text{number of times an odd can occur}}{\text{number of possible outcomes}} = \frac{3 \text{ possible odd numbers}}{6 \text{ possible numbers}}$$

The probability of rolling an odd number is $\frac{3}{6}$ or $\frac{1}{2}$.

So, to determine *P*(4) consider how many times a 4 can occur when you roll a six-sided number cube.

3. What is the probability of rolling a 4, or *P*(4)?
Explain your reasoning.

   P(4) is equal to $\frac{1}{6}$. There are 6 possible outcomes, and the event of rolling a 4 is 1 out of the 6 outcomes.

4. What is the probability of rolling a 6, or *P*(6)?
Explain your reasoning.

   P(6) is equal to $\frac{1}{6}$. There are 6 possible outcomes, and the event of rolling a 6 is 1 out of the 6 outcomes.

5. Determine the probability of rolling an even number.

   a. Which outcome or outcomes make up the event of rolling an even number?

      The outcomes 2, 4, and 6 make up the event of rolling an even number.

   b. Calculate the probability of rolling an even number.

      $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

590 MODULE 4 • TOPIC 1 • LESSON 1

6. Determine the probability of rolling a number that is *not* even.

   a. Which outcome or outcomes make up the event of rolling a number that is *not* even?

      The outcomes of 1, 3, and 5 make up the event of rolling a number that is not even.

   b. Calculate the probability of rolling a number that is *not* even.

      $P(\text{not even}) = \frac{3}{6} = \frac{1}{2}$

7. Determine the probability of rolling a number greater than 4.

   a. Which outcome or outcomes make up the event of rolling a number greater than 4?

      The outcomes of 5 and 6 make up the event of rolling a number greater than 4.

   b. Calculate the probability of rolling a number greater than 4.

      $P(\text{greater than 4}) = \frac{2}{6} = \frac{1}{3}$

8. Determine the probability of rolling a number that is *not* greater than 4.

   a. Which outcome or outcomes make up the event of rolling a number that is *not* greater than 4?

      The outcomes of 1, 2, 3, and 4 make up the event of rolling a number that is not greater than 4.

   b. Calculate the probability of rolling a number that is *not* greater than 4.

      $P(\text{number not greater than 4}) = \frac{4}{6} = \frac{2}{3}$

Activity 1.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining the probability of simple events, assign Skills Practice Set A for this lesson.

MODULE 4 • TOPIC 1 • LESSON 1 591

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the definition and information.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 1.2

Complementary Events

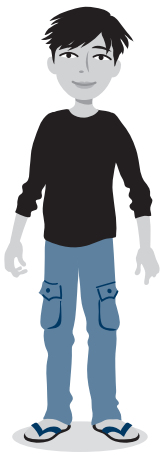
In the previous activity, you calculated the probability of *complementary events*. **Complementary events** are events that together contain all of the outcomes in the sample space.

One way to notate the complement of an event A is *not A*. For instance, suppose that *P*(even) represents the probability of rolling an even number on a number cube. Then, *P*(not even) represents the probability of rolling a number that is *not* even on a number cube.

1. Consider the events from Activity 1.1, Questions 5 and 6—"rolling an even number" and "rolling a number that is not even." What do you notice about the sum of the probabilities of these two complementary events?

   The sum of the probabilities of these two complementary events is 1.

Complements are events that add up to one COMPLETE whole. Compliments are what you say when you are being kind.



2. Consider the events from Activity 1.1, Questions 7 and 8—"rolling a number greater than 4" and "rolling a number that is not greater than 4." What do you notice about the sum of the probabilities of these two complementary events?

   The sum of the probabilities of these two complementary events is 1.

EB STUDENT TIP

For "Beginning" and "Intermediate" proficiency levels

Discuss the meanings of the homonyms *complimentary* and *complementary*. In Module 5, students will learn the term *complementary angles*.

592 MODULE 4 • TOPIC 1 • LESSON 1

3. What is the sum of the probabilities of *any* two complementary events? Explain why your answer makes sense.

The sum of the probabilities of any two complementary events is 1. This makes sense because together, complementary events include the entire set of possible outcomes from an experiment.

4. The probability of rolling a 5 or less on a number cube is $P(5 \text{ or less}) = \frac{5}{6}$. Catalina and Linh calculated $P(\text{not 5 or less})$. Their work is shown.

Catalina

$$P(\text{not 5 or less}) = \frac{\text{number of possible outcomes}}{\text{total number of outcomes}} = \frac{1}{6}$$

Linh

$$P(\text{not 5 or less}) = 1 - P(5 \text{ or less}) = 1 - \frac{5}{6} = \frac{1}{6}$$

Explain the difference between Catalina's strategy and Linh's strategy.

Catalina is correct because she used the correct formula for determining the probability of an event. Linh is correct because the sum of the probabilities of an event and its complement is 1.

Activity 1.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with complementary events, assign Skills Practice Set B for this lesson.

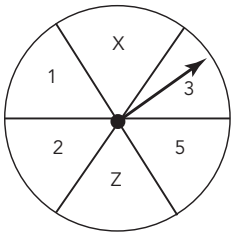
MODULE 4 • TOPIC 1 • LESSON 1 593

Chunking the Activity

- Read and discuss the situation.
- Group students to complete 1–6.
- Check in and share.
- Group students to complete 7 and 8.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 1.3

Simple Probability and Spinners



Consider the spinner shown. All sections of the spinner are the same size. An experiment consists of spinning the spinner one time.

1. How many possible outcomes are there in the experiment? How did you determine your answer?

   There are 6 possible outcomes. I counted the number of sections that were in the spinner which was 6. Therefore, there are 6 possible outcomes.

2. List the sample space for the experiment.

   {*X*, *Z*, 1, 2, 3, 5}

When the probabilities of all the outcomes of an experiment are equal, the outcomes are called **equally likely**.

3. Are the possible outcomes of the experiment equally likely. Explain your reasoning.

   Yes. All of the sections of the spinner are the same size and each section displays a unique character. The probability for each outcome on the spinner is $\frac{1}{6}$.

4. Determine the probability that the spinner lands on a letter.

   a. Describe the event and the possible outcomes of the event.

      The event is the spinner landing on a letter. The possible outcomes in this event are *X* and *Z*.

   b. Calculate *P*(letter).

      $P(\text{letter}) = \frac{2}{6} = \frac{1}{3}$

594 MODULE 4 • TOPIC 1 • LESSON 1

c. Describe the complement of this event and the possible outcomes of the complement.

The complement of this event is the spinner not landing on a letter. The possible outcomes of the complement of this event are 1, 2, 3, and 5.

d. Calculate *P*(not a letter).

$P(\text{not a letter}) = 1 - \frac{1}{3}$

$= \frac{2}{3}$

e. Are these complementary events equally likely? Explain your reasoning.

No. The probability of each event is not the same.

5. Determine the probability that the spinner lands on an odd number.

a. Describe the event and the possible outcomes of the event.

The event is the spinner landing on an odd number. The possible outcomes in this event are 1, 3, and 5.

b. Calculate *P*(odd number).

$P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$

c. Describe the complement of this event and the possible outcomes of the complement.

The complement of this event is the spinner not landing on an odd number. The possible outcomes of the complement of this event are 2, *X*, and *Z*.

d. Calculate *P*(not an odd number).

$P(\text{not an odd number}) = 1 - \frac{1}{2} = \frac{1}{2}$

MODULE 4 • TOPIC 1 • LESSON 1 595

e. Are these complimentary events equally likely? Explain your reasoning.

Yes. The probability of each event is $\frac{1}{2}$.

6. Determine the probability that the spinner lands on a vowel.

a. Describe the event and the possible outcomes of the event.

The event is the spinner landing on a vowel. There are no outcomes in this event.

b. Calculate *P*(vowel).

P(vowel) = 0

c. Describe the complement of this event and the possible outcomes of the complement.

The complement of this event is the spinner not landing on a vowel. The possible outcomes of the complement of this event are 1, 2, 3, 5, *X*, and *Z*.

d. Calculate *P*(not a vowel).

P(not a vowel) = 1 − 0 = 1

e. Are these complementary events equally likely?

No. The probability of each event is not the same.

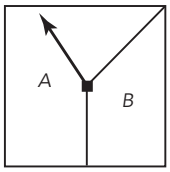
EB STUDENT TIP

For "Beginning" and "Intermediate" proficiency levels

Some students may not be familiar with the word *vowel* or which letters are vowels in English. To make this question accessible for all students, show the English alphabet and underline the vowels (*a*, *e*, *i*, *o*, and *u*).

596 MODULE 4 • TOPIC 1 • LESSON 1

The Spinning Square Game is a game that consists of spinning the square spinner. If a player takes a spin and the spinner lands on *B*, the player wins a prize. If the spinner lands on *A*, the player does not receive a prize.



7. Predict each probability.

   a. $P(A) =$

   The probability is about $\frac{5}{8}$.

   b. $P(B) =$

   The probability is about $\frac{3}{8}$.

8. Brianna predicts the probability that the spinner will land on *A* to be 5. Is Brianna correct? Explain your reasoning.

   Brianna is not correct.
   Sample explanation:
   Brianna may have made an estimate of the probability of the spinner landing on *A*. However, probability is a fraction of the number of possible outcomes over the total number of possible outcomes. Brianna must use two numbers or a fractional form to describe the probability that an event will occur.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1G:

- Do students defend their mathematical reasoning?
- Do students use precise mathematical language when communicating?

MODULE 4 • TOPIC 1 • LESSON 1 597

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Group students to complete the activity.
- Share and summarize.

ACTIVITY 1.4

Probabilities from 0 to 1

1. What is the greatest possible probability in any experiment? Explain your reasoning.

   The greatest possible probability is 1.
   Sample explanation:
   Since probability $= \frac{\text{number of times an event can occur}}{\text{number of possible outcomes}}$ when an event always occurs, then it also makes up all of the outcomes. The numerator and the denominator are the same, and the fraction would be equivalent to $\frac{1}{1}$, or 1.

2. What is the least possible probability in any experiment? Explain your reasoning.

   The least possible probability is 0.
   Sample explanation:
   If an event never occurs, then $\frac{\text{number of times an event can occur}}{\text{number of possible outcomes}} = \frac{0}{\text{number of possible outcomes}} = 0$.

3. What is the probability of an event that is just as likely to occur as not occur? Explain your reasoning.

   The probability of an event that is just as likely to occur as not occur is $\frac{1}{2}$.
   Sample explanation:
   There would be 2 possible outcomes: the event occurs, and the event does not occur.

Probabilities can be expressed as fractions, decimals, or percents.

The probability of an event occurring is a number between 0 and 1. If the event is certain to happen, then the probability is 1. If an event is impossible to happen, then the probability is 0. If an event is just as likely to happen as not happen, then the probability is 0.5, or $\frac{1}{2}$.

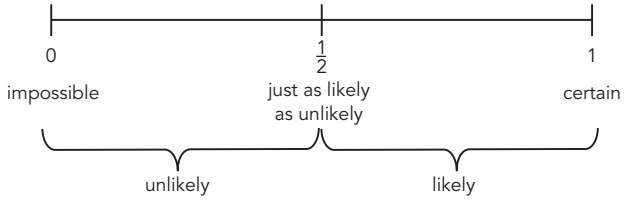
598 MODULE 4 • TOPIC 1 • LESSON 1

Complete the chart representing the different probabilities.

4.

	Fraction	Decimal	Percent
P(certain event)	$\frac{1}{1}$	1.0	100%
P(event that is just as likely as unlikely to occur)	$\frac{1}{2}$	0.5	50%
P(impossible event)	0	0.0	0%

The number line shown represents the probabilities, from 0 to 1, of any event occurring.



5. Estimate the probability of each event occurring. Then, place the letter corresponding to the estimated probability of the event on the number line.

	Fraction	Decimal	Percent
a. A card selected from a standard 52-card deck is red.	$\frac{1}{2}$	0.5	50%
b. Your neighbors will get a pet dinosaur.	0	0	0%
c. You will have a test in one of your classes this month.	$\frac{9}{10}$	0.9	90%
d. A seventh-grader is more than 6 feet tall.	$\frac{1}{100}$	0.01	1%

Activity 1.4 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with probabilities from 0 to 1, assign Skills Practice Set C for this lesson.

MODULE 4 • TOPIC 1 • LESSON 1 599

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Student Look-Fors

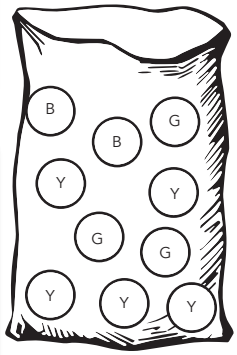
Whether students are demonstrating proficiencies related to TEKS 7.1D:

- Do students represent their mathematical reasoning with appropriate representations?
- Can students communicate how a chosen representation demonstrates their reasoning?
- Can students identify the advantages and disadvantages of different representations?

ACTIVITY 1.5

A Simple Probability Experiment

Suppose there are 2 blue, 3 green, and 5 yellow marbles in a bag. One marble will be drawn from the bag.



1. List the sample space for the experiment.

   S = {blue, blue, green, green, green, yellow, yellow, yellow, yellow, yellow}

2. Calculate each probability.

   a. $P(B) = \frac{2}{10} = 0.2$

   b. $P(G) = \frac{3}{10} = 0.3$

   c. $P(Y) = \frac{5}{10} = 0.5$

3. Are the outcomes in the marble experiment equally likely? Explain your reasoning.

   The probabilities are not equally likely. The probabilities are all different because the number of each colored marble is different.

4. Determine the sum of all the probabilities.

   $P(B) + P(G) + P(Y) =$

   $\frac{2}{10} + \frac{3}{10} + \frac{5}{10} = \frac{10}{10} = 1$

5. Determine the sum of the probabilities for all the outcomes of the first spinner in Activity 1.3.

   $P(1) + P(2) + P(3) + P(5) + P(X) + P(Z) =$

   $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + = \frac{6}{6} = 1$

6. Do you think the sum of the probabilities for all outcomes of any experiment will always be 1? Explain your reasoning.

   I think the sum of the probabilities for the outcomes of any experiment will always be 1 because 1 is equivalent to 100%, and all of the outcomes together make up 100% of the experiment.

600 MODULE 4 • TOPIC 1 • LESSON 1

James surveys some members of his class at random and asks about their favorite color. Each student chooses 1 color. The results of his survey are in the table below.

Color	Number
Red	4
Orange	9
Yellow	6
Green	3
Blue	10
Purple	4

Given that one of the surveyed students is chosen at random, answer each question.

7. What is the probability the chosen student's favorite color is blue?
   $\frac{10}{36} = \frac{5}{18}$

8. What is the probability the chosen student's favorite color is not purple?
   $\frac{32}{36} = \frac{8}{9}$

9. How many times more likely is it that the chosen student favorite color is yellow than green?
   It is twice as likely for the student's favorite color to be yellow than green.

10. Which colors are equally likely to the be the favorite color of the chosen student?
    Purple and red

MODULE 4 • TOPIC 1 • LESSON 1 601

11. What is the sum of $P(\text{red}) + P(\text{orange}) + P(\text{yellow}) + P(\text{green}) + P(\text{blue}) + P(\text{purple})$? Does the sum match your prediction in Question 6? Explain your reasoning.

$\frac{36}{36} = 1$

Sample answer:

Yes, the sum matches my prediction. The sum is 1 because 1 is equivalent to 100%, and all the outcomes together make up 100% of the experiment.

602 MODULE 4 • TOPIC 1 • LESSON 1

Talk the Talk

Absolutely/Never

1. Can the sum of the probabilities for all outcomes of some experiment ever be greater than 1? Explain your reasoning.

   No. The sum of the probabilities for all outcomes of any experiment will never be greater than 1. Because the denominator of the fraction is the number of all possible outcomes, the numerator can't ever be greater than the denominator.

2. Write an event that has a probability of 1.

   Sample answer:
   The sun will rise in the east and set in the west.

3. Write an event that has a probability of 0.

   Sample answer:
   I can throw a baseball 100 miles.

4. If $P(\text{not event } X) = 1$, what is $P(\text{event } X)$?

   $P(\text{event } X) = 1 - 1 = 0$

5. If $P(\text{not event } Y) = 0$, what is $P(\text{event } Y)$?

   $P(\text{event } Y) = 1 - 0 = 1$

Chunking the Activity

- Have students complete the activity individually.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

MODULE 4 • TOPIC 1 • LESSON 1 603

604 MODULE 4 • TOPIC 1 • LESSON 1

Lesson 1 Assignment

Write

Complete each statement using a term from the word box.

experiment	*probability*	*event*	*equally likely*
outcome	*sample space*	*simple event*	*complementary events*

1. A(n) _____ is one or a group of possible outcomes for a given situation.
2. A list of all possible outcomes of an experiment is called a(n) _____.
3. A(n) _____ is a situation involving chance that leads to results.
4. The measure of the likelihood that an event will occur is its _____.
5. The result of an experiment is a(n) _____.
6. An event consisting of one outcome is a(n) _____.
7. When the probability of all the outcomes of an experiment are equal, then the probabilities are called _____.
8. Two events that together contain all of the outcomes in the same space is/are _____.

Remember

Probability is a measure of the likelihood that an event will occur. To calculate the probability of an event, or *P*(event), determine the ratio of the number of times the event occurs to the total number of outcomes.

Write

1. event
2. sample space
3. experiment
4. probability
5. outcome
6. simple event
7. equally likely
8. complementary events

MODULE 4 • TOPIC 1 • LESSON 1 ASSIGNMENT 605

Lesson 1 Assignment

Practice

1. Ricardo is getting dressed in the dark. He reaches into his sock drawer to get a pair of socks. He knows that his sock drawer contains 6 pairs of socks, and each pair is a different color. Each pair of socks is folded together. The pairs of socks in the drawer are red, brown, green, white, black, and blue.

   a. How many possible outcomes are there in the experiment?

   There are 6 possible outcomes.

   b. What are the possible outcomes of the experiment?

   The possible outcomes are a pair of red, brown, green, white, black, or blue socks.

   c. List the sample space for the experiment.

   The sample space is {red socks, brown socks, green socks, white socks, black socks, blue socks}.

   d. Calculate $P(\text{blue})$.

   $P(\text{blue}) = \frac{1}{6}$

   e. Calculate $P(\text{green})$.

   $P(\text{green}) = \frac{1}{6}$

   f. Calculate $P(\text{not red})$.

   $P(\text{not red}) = \frac{5}{6}$

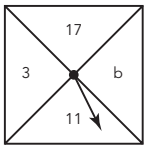
   g. Calculate $P(\text{not purple})$.

   $P(\text{not purple}) = 1$

606 MODULE 4 • TOPIC 1 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

2. Consider the square spinner shown and assume all sections are the same size. An experiment consists of spinning the spinner one time.



a. How many possible outcomes are there in the experiment?

There are 4 possible outcomes.

b. What are the possible outcomes of the experiment?

The possible outcomes are the spinner stopping on a section with a 17, 3, 11, or *b* written on it.

c. List the sample space for the experiment.

The sample space is {17, 3, 11, *b*}.

d. Calculate *P*(*b*).

$P(b) = \frac{1}{4}$

e. Calculate *P*(number).

$P(\text{number}) = \frac{3}{4}$

f. Calculate *P*(not a number greater than 10).

$P(\text{not a number greater than 10}) = \frac{1}{2}$

MODULE 4 • TOPIC 1 • LESSON 1 ASSIGNMENT 607

Lesson 1 Assignment

g. Calculate P(number less than 2).

P(number less than 2) = 0

3. Determine whether each event is certain to occur, just as likely to occur as not to occur, or impossible to occur. Then, write the probability.

a. A coin is flipped and the coin lands heads up. Express the probability as a fraction.

It is just as likely that the coin lands heads up as it is that the coin lands tails up.

$P(\text{heads}) = \frac{1}{2}$

b. Tuesday follows Monday in the week. Express the probability as a percent.

Tuesday always follows Monday in the week. The event is certain to occur.

P(Tuesday following Monday) = 1, or 100%

c. You have only white shirts in your closet. Express the probability of reaching into your closet and choosing a red shirt as a fraction.

Because you only have white shirts in your closet, it is impossible to choose a red shirt.

$P(\text{red}) = 0$

d. A box contains 2 green balls and 2 yellow balls. You reach into the box and grab a yellow ball. Express the probability as a decimal.

It is just as likely that you will grab a green ball as it is that you will grab a yellow ball.

$P(\text{yellow}) = \frac{1}{2}$, or 0.5

608 MODULE 4 • TOPIC 1 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

Prepare

Determine each ratio.

1. Number of heads to number of sides of coin

   $\frac{1}{2}$

2. Number of 6s to number of faces on a six-sided number cube

   $\frac{1}{6}$

3. Number of day names ending in "day" to total number of day names

   $\frac{7}{7}$, or 1

MODULE 4 • TOPIC 1 • LESSON 1 ASSIGNMENT 609

610 MODULE 4 • TOPIC 1 • LESSON 1 ASSIGNMENT

2 Probability Models

MATERIALS

None

LESSON OVERVIEW

The terms *probability model*, *uniform probability model*, and *non-uniform probability model* are defined in this lesson. Students will develop a probability model for an experiment and use it to determine probabilities of events. They will construct and interpret uniform and non-uniform probability models.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6I determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

ELPS

(2) Listening

The student is expected to:

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

(3) Speaking

The student is expected to:

(F) ask and give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments.

(G) express opinions, ideas, and feelings ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics.

ESSENTIAL IDEAS

- A *probability model* is a list of each possible outcome along with its probability. The sum of all the probabilities for the outcomes will always be 1.
- A *uniform probability model* is a model in which all of the probabilities are equally likely to occur.
- A *non-uniform probability model* is a model in which all of the probabilities are not equally likely to occur.

CC BY NC

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Pocket Probabilities 10–15 minutes

CONNECT TO PRIOR KNOWLEDGE

Students use a scenario to complete a table of probabilities to solve a problem. This activity is designed to engage students in thinking about deconstructing the probabilities in a situation so they can represent them using a model.

DEVELOP

Activity 2.1: Using Probability Models 10–15 minutes

REAL-WORLD PROBLEM SOLVING

The situation involves the likelihood of an audience member picking each of five different color cards during a magic trick. Students list the sample space and construct a probability model. They are introduced to uniform and non-uniform probability models.

Activity 2.2: Constructing and Interpreting Probability Models 15 minutes

REAL-WORLD PROBLEM SOLVING

Students construct a probability model for four different problem situations. They then identify which models are uniform probability models and which models are non-uniform probability models.

DAY 2

Activity 2.3: More with Probability Models 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students are given a table listing five letters of the alphabet and the number of students in a homeroom who have first names beginning with each letter. They create a probability model for selecting a student at random whose name begins with each of the letters and use the model to determine specified probabilities.

DEMONSTRATE

Talk the Talk: Compare and Contrast 25–30 minutes

WRITING TASK

Students answer questions related to uniform probability models and non-uniform probability models.

Getting Started

ENGAGE

Pocket Probabilities

Facilitation Notes

In this activity, students use a scenario to complete a table of probabilities. This activity is designed to engage students in thinking about deconstructing the probabilities so they can represent them using a model.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

Suggest that students write fractions in both their original form and lowest terms when entering them in the table. This allows them to check their work and respond to Question 2 more efficiently.

Just in Time Support

Suggest students draw a basic diagram to model the situation. For example:

P P P N N N N D D D Q Q

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the sample space? • Why is it important to list duplicates in the sample space? • Explain how you determined each probability. • How does the table help you to organize your information?
Seeing structure	• Which outcomes are equally likely to occur?
Probing	• Why is it helpful to verify that the sum of the probabilities is 1?

Summary

Probabilities can be represented and organized in a table. The sum of the probabilities is 1.

STAMP THE LEARNING

ACTIVITY 2.1

Using Probability Models

DEVELOP

Facilitation Notes

In this activity, students list the sample space and construct a probability model. They are introduced to uniform and non-uniform probability models.

Ask a student to read the information and definitions aloud. Complete Questions 1 through 3 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What information in the problem helped you determine the sample space?
Probing	• How did you determine each probability?
Seeing structure	• If the sum of the probabilities was less than or greater than 1, what would this tell you? • What are the two features of a probability model? • How are both features of a probability model dependent upon one another?

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Highlighters

Have students highlight the definition of a probability model and number its two features.

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What term can Nakota use to identify outcomes that have the same probability? • How could you rewrite the situation and model to show Nakota why the probabilities are not always equal in a probability model?

Ask a student to read the information and definitions aloud. Then, discuss the information as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why is the table in the Getting Started considered a probability model? • Why is the table in the Getting Started an example of a non-uniform probability model?

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Highlighters

Have students highlight *uniform probability model* and *non-uniform probability model* and the phrases that define each. Discuss how the definitions coincide with the meaning of *uniform*.

Summary

The sum of the probabilities in a probability model is always equal to 1.

STAMP THE LEARNING

ACTIVITY 2.2 Constructing and Interpreting Probability Models

Facilitation Notes

In this activity, students construct probability models and identify them as uniform probability models or non-uniform probability models.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Are the steps any different to construct a uniform and non-uniform probability model? Explain. • Do you prefer to write the probabilities as fractions, decimals, or percents? Why or why not?
Reflecting and justifying	• How can you check that your work is correct?
Probing	• How can you use the form of the probability ratio to compare different outcomes in the situation? • How did you use the probability model to solve Question 2? • Why is it a good practice to construct a probability model before answering probability questions?
Seeing structure	• What is a situation that could apply to this probability model?

AS STUDENTS WORK, LOOK FOR

Students may show confusion between the number of outcomes and the probability of the outcomes or a total disregard of the bottom row of the probability model. For example, thinking $P(3) = \frac{1}{6}$ rather than $\frac{3}{12}$ or $\frac{1}{4}$.

DIFFERENTIATION STRATEGY

Challenge Opportunity

To extend the activity, have students create a situation for the probability model. One possibility could be a spinner with 12 equal-sized sectors.

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COMMON MISCONCEPTION

Students may think that marbles in Question 1 part (b) represent uniform probability since each individual marble is equally likely to be chosen. Facilitate a discussion about the probability of each color marble being chosen to help students differentiate between these ideas.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

A uniform probability model occurs when all of the probabilities in the model are equally likely to occur, and a non-uniform probability model occurs when all the probabilities in the model are not equally likely to occur.

ACTIVITY 2.3

More with Probability Models

Facilitation Notes

In this activity, students use a given table to create a probability model and use the model to determine specified probabilities.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What letter do most of Mr. Flores's students' names begin with? • How did you determine how many students are in Mr. Flores's homeroom?
Seeing structure	• How does your probability model contain the two required features according to its definition? • Is this probability model a uniform probability model or a non-uniform probability model?
Reflecting and justifying	• Did you write each fraction in your probability model in lowest terms? Why or why not?
Probing	• How would you phrase the complement of this probability? What is its value?

DIFFERENTIATION STRATEGY

Access for All

Replace the probability table in Question 2 with one that represents the names of the students in your class.

Summary

Tables can be used to create probability models and those models can be used to answer related questions.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

COMPARE AND CONTRAST

Facilitation Notes

In this activity, students answer questions related to uniform probability models and non-uniform probability models.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Challenge Opportunity

Materials Needed: Poster Paper

To extend the lesson, have groups of students create posters with examples (including a scenario and probability model) that demonstrate a uniform probability model or a non-uniform probability model. Have students present their posters to the class. Discuss Questions 1 and 2 throughout the presentations.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What two features are necessary in a probability model?
Gathering	• What are the advantages of not writing probabilities in lowest terms? • What is a uniform probability model? • What is a non-uniform probability model? • When is the sum of all the probabilities of the outcomes of an event not equal to 1?

Have students read and answer the Essential Question on the lesson opener page.

Summary

A uniform probability model occurs when all of the probabilities in the model are equally likely to occur, and a non-uniform probability model occurs when all the probabilities in the model are not equally likely to occur.

STAMP THE LEARNING

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2 Probability Models

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Develop a probability model for an experiment and use it to determine probabilities of events.
- Develop and interpret a uniform probability model assigning equal probability to all outcomes.
- Construct and interpret a non-uniform probability model.

NEW KEY TERMS

- probability model
- uniform probability model
- non-uniform probability model

You now know about simple probabilities and how to write them.

How can you use models to represent probabilities and solve problems?

Sample answer:

When the probabilities of all the outcomes in a situation appear in a probability model, the sum of the probabilities is 1.

MODULE 4 • TOPIC 1 • LESSON 2 611

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Pocket Probabilities

Alejandra has 3 pennies, 4 nickels, 3 dimes, and 2 quarters in her pocket. She takes one coin out of her pocket and hides it in the palm of her hand. She wants her best friend to guess which coin she is holding.

1. Complete the table. Use a fraction to represent each probability.

Outcome	Penny	Nickel	Dime	Quarter
Probability	$\frac{3}{12} = \frac{1}{4}$	$\frac{4}{12} = \frac{1}{3}$	$\frac{3}{12} = \frac{1}{4}$	$\frac{2}{12} = \frac{1}{6}$

2. Determine the sum of the probabilities.

   The sum of the probabilities is 1.

3. What coin would you advise Alejandra's best friend to guess? Explain your reasoning.

   Sample answer:

   A nickel has the greatest probability of being chosen. There are more nickels in Alejandra's pocket than any other coin.

612 MODULE 4 • TOPIC 1 • LESSON 2

ACTIVITY 2.1

Using Probability Models

Jackson and Nakota are training to be magicians and are working on a card trick. They have 5 cards of different colors—red, blue, yellow, green, and purple. They are discussing the likelihood of an audience member picking each color, assuming that they can't see the card's color.

1. List the sample space for choosing a card.

   $S = \{R, B, Y, G, P\}$

2. What is the probability of selecting each card? Write the probability for each in the table.

Outcome	R	B	Y	G	P
Probability	$P(R) = \frac{1}{5}$, or 0.2	$P(B) = \frac{1}{5}$, or 0.2	$P(Y) = \frac{1}{5}$, or 0.2	$P(G) = \frac{1}{5}$, or 0.2	$P(P) = \frac{1}{5}$, or 0.2

When solving a probability problem, it is helpful to construct a *probability model*. A **probability model** is a list of each possible outcome along with its probability. Probability models are often shown in a table.

A probability model will list all the outcomes. The probability of each outcome will be greater than 0 but less than 1. The sum of all the probabilities for the outcomes will always be 1.

3. Why is the sum of the probabilities in a probability model always 1?

   The sum is always 1 because the probability model lists every possible outcome that can occur, so the probabilities must add to 1 to account for 100% of the outcomes.

Chunking the Activity

- Read and discuss the information and definitions.
- Complete Questions 1–3 as a class.
- Group students to complete Question 4.
- Check in and share.
- Read and discuss the information and definitions.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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MODULE 4 • TOPIC 1 • LESSON 2 613

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

4. Nakota claims, "In any probability model, the probabilities for all outcomes are equal to each other." Jackson disagrees and says, "The sum of all the probabilities is 1 for any probability model, but that does not mean the probabilities of all the outcomes are equal." Who is correct? Explain your reasoning.

   Jackson is correct. The sum of all probabilities for the outcomes will always be 1. In some sample spaces, some outcomes are more likely than other outcomes. Therefore, sometimes some outcomes may have a greater chance of occurring than other outcomes.

Ask Yourself . . .
Did you justify your mathematical reasoning?

A **uniform probability model** occurs when all the probabilities in a probability model are equally likely to occur. Each color card in the magicians' trick had the same probability of being chosen.

When all probabilities in a probability model are not equal to each other, it is called a **non-uniform probability model**. An example would be a weather forecast that states there is a 30 percent chance of rain. This means that there is a 70 percent chance that it will not rain. The sum of these two probabilities is 1, but the outcomes do not have the same probability.

EB STUDENT TIP

For "Beginning" and "Intermediate" proficiency levels

Discuss the Latin root of the word uniform. Something uniform has only "one form" from the Latin word *unus*, meaning one, and *forma*, meaning form. Discuss how this relates to a uniform probability model.

614 MODULE 4 • TOPIC 1 • LESSON 2

ACTIVITY 2.2

Constructing and Interpreting Probability Models

1. Construct a probability model for each situation. Explain how you constructed the model. Then, determine whether or not the probability model is a uniform probability model.

   a. Rolling an 8-sided polyhedron with the numbers 1 through 8:

Outcome	1	2	3	4	5	6	7	8
Probability	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

When rolling an 8-sided polyhedron with the numbers 1 through 8 on it, there is the same chance of landing on each number. This is a uniform probability model since all probabilities are equivalent to each other.

   b. Choosing a marble from a bag of marbles containing 1 green marble, 2 red marbles, and 7 blue marbles:

Outcome	Green	Red	Blue
Probability	0.1	0.2	0.7

There are 10 marbles in the bag, so there is a 1 in 10 chance of choosing green, 2 in 10 chance of choosing red, and 7 in 10 chance of choosing blue. This is a not a uniform probability model since all probabilities are not equivalent to each other.

An eight-sided polyhedron is like a six-sided number cube, but instead it has 8 faces.

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with constructing probability models and identifying whether the model is a uniform probability model or a non-uniform probability model, assign Skills Practice Set A for this lesson.

MODULE 4 • TOPIC 1 • LESSON 2 615

c. Selecting a member of the chess club whose members are Omar, Hailey, Gracie, Ben, Nicky, Xavier, Joey:

Outcome	Omar	Hailey	Gracie	Ben	Nicky	Xavier	Joey
Probability	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

There are 7 people in the club, and each person has the same chance of being chosen. This is a uniform probability model since all probabilities are equivalent to each other.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1C:

- Are students considering which strategies or tools to use?

d. Selecting a member of the chess club with a three-syllable name out of Brianna, Hailey, Gracie, Nia, Nicky, Xavier, Lucia:

Outcome	Three-Syllable Name	Two-Syllable Name
Probability	$\frac{3}{7}$	$\frac{4}{7}$

There are 3 members with three-syllable names and 4 members with two-syllable names in the club, so members with three-syllable names have a 3 in 7 chance of being chosen. A member with a two-syllable name has a 4 in 7 chance of being chosen. This is not a uniform probability model since all probabilities are not equivalent to each other.

616 MODULE 4 • TOPIC 1 • LESSON 2

2. Use the probability model to calculate each probability.

Outcome	2	3	4	5	6	7
Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

a. $P(3) =$ $\frac{1}{4}$

b. $P(8) =$ 0

c. $P(\text{number less than 8}) =$ 1

d. $P(\text{prime number}) =$ $\frac{5}{6}$

e. $P(\text{even number}) =$ $\frac{3}{12}$

Question 2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining probability from a probability model, assign Skills Practice Set B for this lesson.

MODULE 4 • TOPIC 1 • LESSON 2 617

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Activity 2.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice using creating and using a probability model from a table of data, assign Skills Practice Set C for this lesson.

ACTIVITY 2.3

More with Probability Models

When Mr. Flores receives his homeroom list for this year, he cannot believe it—all of the last names in his homeroom start with one of 5 letters! The table shows how many students in Mr. Flores's homeroom have last names beginning with the letters listed.

1. How many students are in Mr. Flores's homeroom?

   There are 30 students in Mr. Flores's homeroom.

Letter	Number of Students
A	7
B	4
M	5
O	2
S	12

2. Create a probability model for selecting a student from Mr. Flores's homeroom.

Outcome	A	B	M	O	S
Probability	$\frac{7}{30}$	$\frac{4}{30}$	$\frac{5}{30}$	$\frac{2}{30}$	$\frac{12}{30}$

618 MODULE 4 • TOPIC 1 • LESSON 2

3. Use the probability model to determine each probability.

a. Choosing a student at random with a last name that begins with a vowel

$\frac{3}{10}$

b. Choosing a student at random with a last name that does not begin with S

$\frac{3}{5}$

c. Choosing a student at random with a last name that begins with *C*.

0

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1A:

- Do students persevere in real-world problem solving?
- Do students recognize they can use probability models outside of the math classroom?

MODULE 4 • TOPIC 1 • LESSON 2 619

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

Optimizing Learning

This activity develops self-assessment and reflection.

Talk the Talk

Compare and Contrast

Your best friend is not in school today and misses this lesson. Help your friend understand what you have learned. Write 1 or 2 paragraphs that explain the answers to these two questions.

1. What is the difference between a uniform probability model and a non-uniform probability model?

   Sample answer:

   A uniform probability model shows that all the outcomes of the event are equally likely to occur. In a non-uniform probability model, not all of the probabilities for the outcomes are equivalent to each other.

2. Will the sum of all the probabilities of the outcomes of an event be equal to 1 for both uniform and non-uniform probability models?

   Sample answer:

   Yes. The sum of the probabilities of all outcomes of an event will always be equal to 1, whether all the probabilities are equally likely or not.

620 MODULE 4 • TOPIC 1 • LESSON 2

Lesson 2 Assignment

Write

Complete each sentence.

1. When all probabilities in a probability model are the same, it is called a(n) _____.
2. When all probabilities in a probability model are not the same, it is called a(n) _____.
3. A(n) _____ is a list of each possible outcome along with its probability.

Remember

When the probabilities of all the outcomes in a situation are represented in a probability model, the sum of the probabilities is 1.

Practice

1. Use the probability model to determine each probability.

Outcome	1	2	3	4	5	6	7	8
Probability	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{1}{25}$	$\frac{5}{25}$	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{3}{25}$	$\frac{4}{25}$

a. $P(4)$

$P(4) = \frac{5}{25} = \frac{1}{5}$

b. $P(7)$

$P(7) = \frac{3}{25}$

c. $P(\text{less than } 7)$

$P(\text{less than } 7)$

$= P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$

$= \frac{2}{25} + \frac{4}{25} + \frac{1}{25} + \frac{5}{25} + \frac{2}{25} + \frac{4}{25}$

$= \frac{18}{25}$

Write

Sample answer:

1. Uniform probability model
2. Non-uniform probability model
3. Probability model

MODULE 4 • TOPIC 1 • LESSON 2 ASSIGNMENT 621

Lesson 2 Assignment

d. P(greater than 8)

P(greater than 8) $= 0$

e. P(odd number)

P(odd number) $= P(1) + P(3) + P(5) + P(7)$

$= \frac{2}{25} + \frac{1}{25} + \frac{2}{25} + \frac{3}{25}$

$= \frac{8}{25}$

f. P(less than 10)

P(less than 10) $= 1$

g. Is this a uniform probability model? Explain your answer.

No. All probabilities are not equal.

Explanations will vary.

2. Use the probability model to determine each probability.

Outcome	A	B	C	D	E	F
Probability	$\frac{2}{20}$	$\frac{5}{20}$	$\frac{2}{20}$	$\frac{7}{20}$	$\frac{1}{20}$	$\frac{3}{20}$

a. P(B)

$P(B) = \frac{5}{20} = \frac{1}{4}$

b. P(F)

$P(F) = \frac{3}{20}$

622 MODULE 4 • TOPIC 1 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

c. $P(\text{not C})$

$$P(\text{not C}) = P(\text{A}) + P(\text{B}) + P(\text{D}) + P(\text{E}) + P(\text{F})$$
$$= \frac{2}{20} + \frac{5}{20} + \frac{7}{20} + \frac{1}{20} + \frac{3}{20}$$
$$= \frac{18}{20} = \frac{9}{10}$$

d. $P(\text{consonant})$

$$P(\text{consonant}) = P(\text{B}) + P(\text{C}) + P(\text{D}) + P(\text{F})$$
$$= \frac{5}{20} + \frac{2}{20} + \frac{7}{20} + \frac{3}{20}$$
$$= \frac{17}{20}$$

e. $P(\text{not A})$

$$P(\text{not A}) = P(\text{B}) + P(\text{C}) + P(\text{D}) + P(\text{E}) + P(\text{F})$$
$$= \frac{5}{20} + \frac{2}{20} + \frac{7}{20} + \frac{1}{20} + \frac{3}{20}$$
$$= \frac{18}{20} = \frac{9}{10}$$

f. $P(\text{vowel})$

$$P(\text{vowel}) = P(\text{A}) + P(\text{E})$$
$$= \frac{2}{20} + \frac{1}{20}$$
$$= \frac{3}{20}$$

g. Is this a uniform probability model? Explain your answer.

No. All probabilities are not equal.

Explanations will vary.

MODULE 4 • TOPIC 1 • LESSON 2 ASSIGNMENT 623

Lesson 2 Assignment

Prepare

Write each fraction as a percent.

1. $\frac{3}{10}$

   30%

2. $\frac{1}{4}$

   25%

3. $\frac{9}{20}$

   45%

4. $\frac{2}{5}$

   40%

624 MODULE 4 • TOPIC 1 • LESSON 2 ASSIGNMENT

3 Determining Experimental Probability of Simple Events

MATERIALS

Coins
Paper or Plastic Cups
Paper Clips

LESSON OVERVIEW

Students flip a coin multiple times to determine the probabilities of heads and tails based on the results of the experiment. The terms *theoretical probability* and *experimental probability* are defined in this lesson. Students conduct trials of a game to estimate probabilities of the three outcomes. They also conduct trials of a spinner game to calculate experimental probabilities using data. Students use those experimental probabilities to predict the number of outcomes for a given number of trials. They then compare the experimental probabilities to the theoretical probabilities using *percent error.*

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1E create and use representations to organize, record, and communicate mathematical ideas.

7.1F analyze mathematical relationships to connect and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.
The student is expected to:

7.6C make predictions and determine solutions using experimental data for simple and compound events.

7.6D make predictions and determine solutions using theoretical probability for simple and compound events.

7.6H solve problems using qualitative and quantitative predictions and comparisons from simple experiments.

7.6I determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

ELPS

(1) Learning Strategies
The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(4) Reading
The student is expected to:

(B) recognize directionality of English reading such as left to right and top to bottom.

(C) develop basic sight vocabulary, derive meaning of environmental print, and comprehend English vocabulary and language structures used routinely in written classroom materials.

ESSENTIAL IDEAS

- *Experimental probability* is the ratio of the number of times an event occurs to the total number of trials performed.
- *Theoretical probability* is the ratio of the number of desired outcomes to the total possible outcomes.
- Percent error is one way to measure the difference between experimental and theoretical probabilities.

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LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Flip It! 10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students flip a coin 30 times and determine the probabilities of heads and tails based on the results of the experiment. They then combine their results to see how the probability changes with more trials included. This activity is designed to engage students in thinking about experimental probability from an intuitive perspective.

DEVELOP

Activity 3.1: Determining Experimental Probability 10–15 minutes

INVESTIGATION

Students play a game. Working with a partner, they will toss a cup 25 times and record each trial using tally marks. The terms *experimental probability* and *theoretical probability* are introduced and students calculate the experimental probability of each event using their data. They conclude that the game is not a fair game, and it is not possible to determine the actual probability of each event due to the shape of the cup.

Activity 3.2: Predicting Theoretical Probability Using Experimental Probability 15–20 minutes

INVESTIGATION

A spinner is divided unequally into three sectors. Students list the sample space and predict the probability of each outcome. They conclude that a protractor could be used to determine the actual probabilities. Using a paper clip as the pointer and working with a partner, students conduct 50 trials and record the data in a table using tally marks. From this information, they calculate the experimental probabilities. Students revisit their predictions and compare them to their experimental probability.

DAY 2

Activity 3.3: Comparing Probabilities Using Proportional Reasoning 20 minutes

INVESTIGATION, WORKED EXAMPLE

Students use given probabilities of specified events occurring in conjunction with proportional reasoning to make predictions. Students follow a Worked Example to learn how to compare theoretical and experimental probabilities using *percent error*—a ratio of the difference between the theoretical and experimental probabilities to the theoretical probability.

Activity 3.4: Practice with Probability and Percent Error 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students revisit the coin flipping scenario from the Getting Started. This time, students observe how the difference between the experimental and theoretical probabilities is decreased as the number of trials is increased. The percent error decreases as a result.

DEMONSTRATE

Talk the Talk: Theoretical or Experimental? 5–10 minutes

EXIT TICKET APPLICATION

Students explain the difference between theoretical probability and experimental probability. Four situations are described, and students determine if each probability can be calculated experimentally, theoretically, or both.

CC BY NC

Getting Started

ENGAGE

Flip It!

Facilitation Notes

In this activity, students flip a coin 30 times and determine the probabilities of heads and tails based on the results of the experiment. They then combine their results to see how the probability changes with more trials included. This activity is designed to engage students in thinking about experimental probability from an intuitive perspective.

Provide a coin to each pair of students for this experiment. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses with the class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• According to your experiment results, which outcome has a greater probability of occurring? • Were your results what you expected? Why or why not? • Why might someone say the results are 50-50? • What is the probability of flipping tails after 60 trials? 90 trials? • How did your results change when you included all the class data?
Gathering	• How did you determine the actual probability?

DIFFERENTIATION STRATEGIES

Access for All

- To combine results systematically, have students write their results in a table on the board. For example:

Student Pairs	*P*(heads)	Combined *P*(heads)	*P*(tails)	Combined *P*(tails)
Partners A & B	$\frac{12}{30}$	$\frac{12}{30} = 0.400$	$\frac{18}{30}$	$\frac{18}{30} = 0.600$
Partners C & D	$\frac{16}{30}$	$\frac{28}{60} = 0.467$	$\frac{14}{30}$	$\frac{32}{60} = 0.533$
Partners E & F	$\frac{17}{30}$	$\frac{45}{90} = 0.500$	$\frac{13}{30}$	$\frac{45}{90} = 0.500$

- Rewrite the probability as a decimal for easier comparison.

Students should recognize:

- The sum of the probabilities of complementary events equals 1.
- The probability for both heads and tails approaches 0.50 as the number of trials increases

Summary

Experimental probabilities approach theoretical probabilities as the number of trials increase.

STAMP THE LEARNING

ACTIVITY 3.1 Determining Experimental Probability

DEVELOP

Facilitation Notes

In this activity, students play a game. Working with a partner, they toss a cup 25 times and record each trial using tally marks. The terms *experimental probability* and *theoretical probability* are introduced, and students calculate the experimental probability of each event using their data. They conclude that the game is not a fair game, and it is not possible to determine the actual probability of each event due to the shape of the cup.

Ask a student to read the introduction aloud. Discuss the directions to the game and complete Questions 1 through 4 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the sum of your probabilities? • How do your probabilities compare to those of your classmates? • How would you describe the experiment related to the game? • What are the possible outcomes of the game? • Who is more likely to win the game, Player 1 or Player 2?
Seeing structure	• Why aren't *the cup landing upright* and *the cup landing upside-down* complementary events?

Provide plastic or paper cups to each pair or group of students to complete the experiment. Have students work with a partner or in a group to complete Questions 5 through 8. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Are you surprised by the results? Explain your thinking.
Seeing structure	• How do you know when you are calculating the theoretical probability? • How are the two ratios different? The same? • How do you obtain the values to substitute into each ratio?
Gathering	• What makes a game a fair game?
Probing	• Did you consider the cup landing on its side in your decision in Question 7? Why or why not?
Seeing structure	• Is Question 8 asking about experimental or theoretical probability? How can you tell?

DIFFERENTIATION STRATEGY

Access for All

Compare and contrast the definitions of theoretical probability and experimental probability. In the margin, have students write: theoretical probability $= \frac{\text{number of desired outcomes}}{\text{number of total possible outcomes}}$. Discuss how the formulas are related and how the values to substitute into the formula are obtained.

STAMP THE LEARNING

Summary

There are cases where theoretical probability cannot be calculated due to the nature of a situation. Experimental probability $= \frac{\text{number of times the event occurs}}{\text{total number of trials performed}}$ and may be determined in cases where it is not possible to calculate theoretical probability.

ACTIVITY 3.2 Predicting Theoretical Probability Using Experimental Probability

Facilitation Notes

In this activity, a spinner is divided unequally into three sectors. Students list the sample space and predict the probability of each outcome. They conclude that a protractor could be used to determine the actual probabilities. From this information, they calculate the experimental probabilities. Students revisit their predictions and compare them to their experimental probability.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How is this problem different from other spinner problems you have encountered?
Probing	• What criteria did you use to make your decision in Question 3? • What strategy did you use to estimate the probability? • If you spin the spinner 100 times, about how many times would the spinner land of each symbol? • How does verifying that the sum of the probabilities was not 1 help you analyze Miguel's mistake? • Why does it make sense to compare the angle measures? • Could you compare the areas for each figure on the spinner? Why or why not?

DIFFERENTIATION STRATEGIES

Access for All

To assist in comparing the spinner sections, have students extend the radius between the fish and the soccer ball to create a diameter of the circle. This process helps students to see that the fish segment is larger.

Challenge Opportunity

Materials Needed: Protractors

Have students use a protractor to calculate the theoretical probabilities by measuring the central angles. Compare them with their experimental probabilities from Question 9.

Provide each pair or group with a paperclip for the experiment. Have students work with a partner or in a group to complete Questions 7 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How does your data compare to your classmates' data?
Seeing structure	• Do you think you would get a more accurate probability by combining the class data? Explain your thinking.
Probing	• Did you compare your responses as fractions, decimals, or percents? Why did you choose that strategy?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

The sample space alone cannot be used to determine theoretical probabilities when the outcomes are not equally likely.

ACTIVITY 3.3 Comparing Probabilities Using Proportional Reasoning

Facilitation Notes

In this activity, students use given the probabilities of specified events occurring in conjunction with proportional reasoning to make predictions. Students compare theoretical and experimental probabilities using percent error.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction to the activity aloud. Discuss the Worked Example as a class.

DIFFERENTIATION STRATEGIES

Access for All

To ground students in the Worked Example and remainder of the activity, post the spinner from Activity 2 on the board.

Just in Time Support

Provide intermediate steps between the lines in Step 2.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• Why does the value 60 correspond to the value 20 in the proportion? • What strategy was used to solve the proportion? • What is another way to solve this proportion?

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you solve each proportion? • What does your result represent? • How can you check your work?

Ask a student to read the definition. Analyze the Worked Example as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do P_E and P_T represent?
Probing	• Explain the steps to solve this problem using decimals.
Seeing structure	• Does the order of the subtraction matter? What would a negative difference represent? • Explain the calculations to rewrite the complex expression on the last line as 14.3%.

COMMON MISCONCEPTION

Percent error does not mean that something is wrong, it just means that the experimental results are not exactly what the theoretical probability expected them to be.

Note that students may not have encountered the approximate symbol (≈) before. Discuss its meaning and cases where it is used.

Have students work with a partner or in a group to complete Questions 2 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Is the experimental result higher or lower than the theoretical probability? How can you tell? • How might using the class experimental data rather than your individual data affect the percent error? Explain your thinking. • How do you interpret negative percent error?

Summary

The percent error is a ratio of the difference between the theoretical and experimental probabilities to the theoretical probability.

STAMP THE LEARNING

ACTIVITY 3.4

Practice with Probability and Percent Error

Facilitation Notes

In this activity, students experiment with flipping a coin again. Attention is paid to the difference between the experimental and theoretical probabilities as the number of trials increase. The percent error decreases as a result.

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Provide a coin to each pair of students for this experiment. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you represent the ratios in your calculations? Explain your steps.
Seeing structure	• How does the sign of the percent error help you make sense of the problem situation? • Did you expect a change in the percent error? Why or why not? • How does an increase in the number of trials affect the experimental probability? • What happens to the percent error as the experimental probability gets closer to the theoretical probability?

DIFFERENTIATION STRATEGY

Access for All

If you implemented the differentiation strategy for Question 2 of the Getting Started, revisit the data table to investigate the percent error without flipping coins again.

STAMP THE LEARNING

Summary

As the number of trials increase, experimental probability approaches theoretical probability and the percent error decreases.

DEMONSTRATE

Talk the Talk

THEORETICAL OR EXPERIMENTAL?

Facilitation Notes

In this activity, students explain the difference between theoretical probability and experimental probability. Four situations are described, and students determine if each probability can be calculated experimentally, theoretically, or both.

You may want to have a four-sided polyhedron available for demonstration purposes for Question 2, part (b).

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How can you tell whether you are calculating the experimental and theoretical probability? • What is the ratio for experimental probability? Theoretical probability?
Reflecting and justifying	• Why can't you always calculate the theoretical probability for a situation? • Why can't you always calculate the experimental probability for a situation?

Have students read and answer the Essential Question on the lesson opener page.

Summary

Theoretical probability is determined mathematically, and experimental probability is determined by performing an experiment.

STAMP THE LEARNING

3 Determining Experimental Probability of Simple Events

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Predict the experimental probability of a chance event by collecting data in a probability experiment.
- Develop a probability model using frequencies in data generated from a probability experiment.
- Use proportional reasoning to determine the theoretical probability of random events.
- Calculate the difference between a theoretical prediction and the experimental results as a percent error ratio.

NEW KEY TERMS

- theoretical probability
- experimental probability
- percent error

You can create probability models of simple events.

Do these models always reflect what actually happens when you toss a coin, roll a number cube, or spin a spinner? How are the probability models related to these actions in the real world?

Sample answer:

Probability models reflect what happens when you perform an experiment many times. As the number of trials increase, experimental probability approaches theoretical probability.

MODULE 4 • TOPIC 1 • LESSON 3 625

Chunking the Activity

- Group students to complete the activity.
- Provide a coin to each group of students.
- Share and summarize.

Getting Started

Flip It!

1. With a partner, flip a coin 30 times, record the results, and then calculate the probability of each event.

Outcome	Tally	Total	Probability
Heads			
Tails			

Answers will vary.

For this probability experiment, the probability is the number of times the outcome occurred over the total number of flips.

2. Combine the results of your experiment with those of your classmates. Then, calculate the probability of each event.

Answers will vary.

3. What is the actual probability of flipping heads?

The actual probability of flipping heads is $\frac{1}{2}$.

4. How does the actual probability compare to the calculated probability in Question 2?

Answers will vary.

626 MODULE 4 • TOPIC 1 • LESSON 3

ACTIVITY 3.1

Determining Experimental Probability

Two friends are designing a game.

The game is played between two players. To play the game, a paper or plastic cup is needed. To start the game, the cup is tossed in the air.

- When the cup lands on its bottom, Player 1 wins a point.
- When the cup lands on its top, Player 2 wins a point.
- When the cup lands on its side, neither player receives a point.

1. Predict the probability for each position in which the cup can land.

   Answers will vary.

2. List the sample space for the game.

   S = {top, bottom, side}

3. Can you use the sample space to determine the probability that the cup lands on its top, bottom, or side? Explain why or why not.

   No. You cannot use the sample space to determine the probabilities because the outcomes are not equally likely.

4. Do you think all the outcomes are equally likely? Explain your reasoning.

   No. The cup will land on its side more often than on its top or bottom.

Chunking the Activity

- Read and discuss the introduction and the directions to the game.
- Complete Questions 1–4 as a class.
- Group students to complete Questions 5–8.
- Provide paper or plastic cups to each group.
- Share and summarize.

MODULE 4 • TOPIC 1 • LESSON 3 627

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

5. Play the game 25 times with a partner. Decide who will be Player 1 and who will be Player 2.

Sample data provided.

Result	Tally Marks	Total
Player 1 (lands on bottom)	|	1
Player 2 (lands on top)	𝍸 | |	6
Side	𝍸 𝍸 𝍸 |||	18

a. Record your results in the table using tally marks. Then, write your and your opponent's total score and write the number of times the cup landed on its side.

See sample data in table.

b. Summarize your results.

Sample answer based on sample data.
The cup landed on its side most of the time, only landed on the bottom once, and on its top 6 times.

EB STUDENT TIP

For all proficiency levels

Materials Needed: Six-Sided Number Cube

To help students understand the distinction between theoretical and experimental probability, focus on the root words theory and experiment. Explain that a theory is an idea about how something works, and an experiment can help test the theory in real life.

Beginning: Use a six-sided number cube and explain that the theoretical probability of rolling any specific number is 1 out of 6. Model saying "theory" when pointing to the theoretical probability, and "experiment" after rolling the die. Have students also practice using these words or responding to prompts with these words as appropriate.

(continued on next page)

The **theoretical probability** of an event is the ratio of the number of desired outcomes to the total number of possible outcomes. This is the kind of probability you have been working with in previous lessons.

Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

$$\text{Experimental Probability} = \frac{\text{number of times the event occurs}}{\text{total number of trials performed}}$$

6. What is the experimental probability of the cup landing:

   Sample answers based on sample data.

   a. On its bottom?

      $\frac{1}{25}$

   b. On its top?

      $\frac{6}{25}$

   c. On its side?

      $\frac{18}{25}$

7. Do you think this is a fair game to play? Why or why not?

   Sample answer:

   This is not a fair game to play. The cup will almost always land on its side.

8. Is it possible to determine the exact probability of the cup landing on its top, bottom, or side? Explain your reasoning.

   No. It is not possible. When tossing the cup, even though there are three possible outcomes, the way the cup is shaped makes it impossible to determine exactly what the chances are that it will land on its top, bottom, or side.

Question 6 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining experimental probability from a table of tally marks, assign Skills Practice Set A for this lesson.

EB STUDENT TIP *(continued)*

Intermediate: Provide this sentence frame: "In theory, the chance of _____ happening is _____, so we call this the theoretical probability. I can determine the experimental probability by _____."

Advanced/Advanced High: Ask students to think of how the word theory and theoretical are used differently in mathematics than some other disciplines and to explain why they think comparing theoretical and experimental probabilities might be important in real-life contexts.

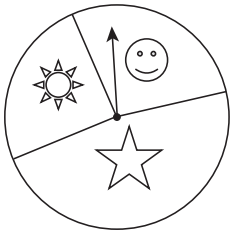
Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–6.
- Check in and share.
- Group students to complete Questions 7–9.
- Provide a paperclip to each group.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 3.2

Predicting Theoretical Probability Using Experimental Probability

Examine the spinner shown.



1. List the sample space.

   S = { }

2. Can you use the sample space to determine the probabilities of the spinner landing on each symbol? Explain why or why not.

   No. I cannot use the sample space to determine the probabilities because the outcomes are not equally likely.

3. What section of the spinner represents the greatest probability? What section of the spinner represents the least probability?

   The star has the greatest chance for the spinner to land. The sun has the least chance for the spinner to land.

630 MODULE 4 • TOPIC 1 • LESSON 3

4. Predict the probability of the spinner landing on each symbol.

   a. $P($ $) =$

   Answers will vary; approximately 30%.

   b. $P($ $) =$

   Answers will vary; approximately 25%.

   c. $P($ $) =$

   Answers will vary; approximately 45%.

5. Miguel and Juliana make the following predictions for the spinner landing on each symbol. Explain why each student is incorrect.

Miguel

$P($ $) = \frac{1}{4}$

$P($ $) = \frac{1}{4}$

$P($ $) = \frac{2}{5}$

Juliana

$P($ $) = \frac{1}{3}$

$P($ $) = \frac{1}{3}$

$P($ $) = \frac{1}{3}$

Miguel's predictions are incorrect because the sum of the probabilities of the three outcomes is not equal to 1. Juliana's predictions are incorrect because the outcomes are not equally likely.

MODULE 4 • TOPIC 1 • LESSON 3 631

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1E:

- Do students create representations to present and organize their mathematical reasoning?

6. Is there a way to determine the exact probabilities of landing on each of the shapes? Explain your reasoning.

A protractor could be used to measure each angle on the spinner. Then, take each angle and divide it by 360 to determine the fraction of the circle made up by each angle in each section of the spinner.

Let's determine the experimental probability of the spinner landing on each of the symbols. Use a paper clip as the arrow part of the spinner. Place a pencil point through the paper clip, and then on the center of the circle. Working with a partner, one person will spin the spinner and the other person will record the result of each spin.

7. Spin the spinner 50 times and record the data using tally marks. Then, complete the table.

Shape	Tally	Total	Probability

Answers will vary.

632 MODULE 4 • TOPIC 1 • LESSON 3

8. Calculate the experimental probabilities using your data.

   a. $P($ $) =$

   b. $P($ $) =$

   c. $P($ $) =$

   Answers for all parts will vary.

Activity 3.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with predicting probabilities on spinners with unequal sections, assign Skills Practice Set B for this lesson.

9. Compare the experimental probabilities with your predictions from Question 4. What do you notice? Why did this happen?

   The probabilities and the predictions of probabilities are probably different. The experimental probability is determined from doing an experiment. I do not always get the same results when doing the experiment.

MODULE 4 • TOPIC 1 • LESSON 3 633

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction and the Worked Example.
- Group students to complete Question 1.
- Check in and share.
- Read and discuss the definition and the Worked Example.
- Group students to complete Questions 2–5.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 3.3

Comparing Probabilities Using Proportional Reasoning

If you know the probability of an event, then you can use proportional reasoning to predict the number of times that event will occur throughout an experiment.

WORKED EXAMPLE

The probability that a spinner will land on is $\frac{7}{20}$.

If you spin the spinner 60 times, you can predict the number of times the spinner will land on the section.

Step 1: Set up a proportion.

$$\frac{\text{number of times an event will occur}}{\text{number of possible outcomes}} \quad \frac{7}{20} = \frac{x}{60}$$

Step 2: Solve the proportion.

$$\frac{7}{20} = \frac{x}{60}$$

$$7(60) = 20x$$

$$\frac{420}{20} = \frac{20x}{20}$$

$$21 = x$$

If you spin the spinner 60 times, you can expect it to land on the 21 times.

634 MODULE 4 • TOPIC 1 • LESSON 3

1. Suppose these are the probabilities for the symbols on the spinner.

$P(\text{☺}) = \frac{7}{20}$ $P(\text{✹}) = \frac{2}{5}$ $P(\text{☆}) = \frac{1}{4}$

a. If you spin the spinner 40 times, predict the number of times the spinner would land on each symbol.

☺ ✹ ☆

14 times, 16 times, 10 times

b. If you spin the spinner 100 times, predict the number of times you would land on each symbol.

☺ ✹ ☆

35 times, 40 times, 25 times

c. How did you predict the number of times the spinner would land on each symbol for the given number of times the spinner would be spun?

I can use proportional reasoning to predict the number of times the spinner would land on each symbol. I will up a proportion using the probabilities for each symbol as one ratio, and then set up a ratio with the given number of spins in the denominator. I will then solve for the numerator.

d. Do you think you would land on the symbol exactly the number of times you calculated in parts (a) and (b) if you spin the spinner? Why or why not?

The actual number of times the spinner would land on a symbol is random. The prediction gives a close estimate of about how many times you could expect the spinner to land on the symbol.

Activity 3.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining probability using proportional reasoning, assign Skills Practice Set C for this lesson.

MODULE 4 • TOPIC 1 • LESSON 3 635

STAMP THE LEARNING

The definition and Worked Example provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

To compare the theoretical and experimental probabilities, you can use a measure called *percent error*. In probability, the **percent error** describes how far off the experimental probability is from the theoretical probability as a percent ratio.

WORKED EXAMPLE

Suppose you spun the 😊 20 times out of 50 trials. The experimental probability, P_E (😊), is $\frac{20}{50}$ or $\frac{2}{5}$. The theoretical probability, P_T (😊), is $\frac{7}{20}$.

The percent error can be calculated using this formula:

$$\frac{P_E - P_T}{P_T} \cdot 100 = \frac{\frac{2}{5} - \frac{7}{20}}{\frac{7}{20}} \cdot 100$$

$$= \frac{\frac{8}{20} - \frac{7}{20}}{\frac{7}{20}} \cdot 100$$

$$= \frac{\frac{1}{20}}{\frac{7}{20}} \cdot 100 \approx 14.3\%$$

2. Write a sentence to interpret the result in the Worked Example.

   The experimental probability is approximately 14.3% greater than the theoretical probability in this example.

3. What would it mean to have a 0% error? Use an example with the formula to explain your reasoning.

   A 0% error would mean that the theoretical and experimental probabilities are the same.

636 MODULE 4 • TOPIC 1 • LESSON 3

4. What would it mean to have a negative percent error? Use an example with the formula to explain your reasoning.

   A negative percent error would mean that the experimental probability is less than the theoretical probability.

5. Use the spinner results from the previous activity to calculate the percent errors for $P($ $)$ and $P($ $)$ and describe their meaning.

   Answers will vary.

MODULE 4 • TOPIC 1 • LESSON 3 637

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Provide a coin to each group.
- Share and summarize.

Activity 3.4 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with probability and percent error, assign Skills Practice Set D for this lesson.

ACTIVITY 3.4

Practice with Probability and Percent Error

As the number of trials increases, the experimental probability tends to get closer and closer to the theoretical probability. What do you think will happen to the percent error as the number of trials increases?

1. Flip a coin 10 times. Record each measure.

Outcome	Tally
Heads	
Tails	

a. Experimental probability of flipping heads

Answers will vary.

b. Theoretical probability of flipping heads

$\frac{1}{2}$

c. Percent error for P(heads)

Answers will vary.

d. Interpret the percent error in the context of your outcomes.

Sample answer:
A positive percent error means that the student landed on heads more than the theoretical probability of $\frac{1}{2}$. A negative percent error means that the student landed on heads less than the theoretical probability.

638 MODULE 4 • TOPIC 1 • LESSON 3

2. Combine your results with a partner. Describe how the experimental probability, theoretical probability, and percent error changed.

   Check students' work. The theoretical probability remains $\frac{1}{2}$. The percent error is likely less.

3. Combine your results with the class. Describe how the experimental probability, theoretical probability, and percent error changed.

   Check students' work. The theoretical probability remains $\frac{1}{2}$. The percent error is likely less.

4. What seems to happen to the percent error as the number of trials increases? Why do you think this happens?

   The percent error decreases because the experimental probability gets closer and closer to the theoretical probability as the number of trials increases.

MODULE 4 • TOPIC 1 • LESSON 3 639

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1F:

- Can students identify and explain mathematical relationships?

Talk the Talk

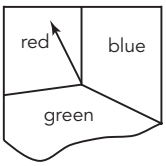
Theoretical or Experimental?

1. Explain the difference between experimental and theoretical probability.

   Experimental probability is determined by doing an experiment. Theoretical probability is determined mathematically.

2. Determine whether each probability of the scenario can be calculated experimentally, theoretically, or both. Explain your reasoning.

   a. The probability of the spinner landing on red.

   Both.

   Sample explanation:

   I could do an experiment and record the number of times the spinner landed on red. I could also calculate the theoretical probability by using a protractor to measure each of the angles in the spinner. Then, I can take each angle and divide by 360 to give the fraction of the spinner section made up by each angle.

   b. *P*(sum of 4) when a four-sided number polyhedron with numbers 1 through 4 is rolled twice.

   Both.

   Sample explanation:

   I could do an experiment and record the sums, or I could determine all the possible sums for two rolls.

A four-sided polyhedron is like a six-sided number cube. It's just that it has four faces instead of six! What do you think a four-sided polyhedron looks like?

640 MODULE 4 • TOPIC 1 • LESSON 3

c. Probability a particular medicine will cure a disease.

Experimental.

There is no way to mathematically calculate the exact number of people for whom the medicine will cure their disease.

d. Probability a car will last for more than 100,000 miles.

Theoretical.

It would not make sense to build a number of cars and drive them all for at least 100,000 miles.

MODULE 4 • TOPIC 1 • LESSON 3 641

642 MODULE 4 • TOPIC 1 • LESSON 3

Lesson 3 Assignment

Write

Define each term in your own words.

1. *experimental probability*
2. *theoretical probability*

Remember

The *percent error* describes how far off the experimental probability is from the theoretical probability.

The percent error is the ratio $\frac{P_E - P_T}{P_T} \cdot 100$.

- If $P_E > P_T$, then the percent error is positive.
- If $P_E < P_T$, then the percent error is negative.
- If $P_E = P_T$, then the percent error is 0.

Practice

1. Suppose the probabilities for the letters on a spinner are known to be:
   $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{3}$ $P(C) = \frac{5}{12}$

Predict the number of times you would land on each letter if you were to spin the spinner the number of times given.

a. You spin the spinner 12 times.

$P(A)$:	$P(B)$:	$P(C)$:
A: 3 times	*B*: 4 times	*C*: 5 times

b. You spin the spinner 96 times.

$P(A)$:	$P(B)$:	$P(C)$:
A: 24 times	*B*: 32 times	*C*: 40 times

c. You spin the spinner 6000 times.

$P(A)$:	$P(B)$:	$P(C)$:
A: 1500 times	*B*: 2000 times	*C*: 2500 times

Write

Sample answers:

1. Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

2. Theoretical probability is the ratio of the number of desired outcomes to the total number of outcomes.

MODULE 4 • TOPIC 1 • LESSON 3 ASSIGNMENT 643

Lesson 3 Assignment

2. A six-sided number cube was rolled 30 times. Use the results listed in the table to answer each question.

Outcome	Tally
1	𝍸 III
2	II
3	𝍸 III
4	IIII
5	III
6	𝍸

   a. What is the theoretical probability of rolling a number less than 4?

   $\frac{1}{2}$

   b. What is the experimental probability of rolling a number less than 4?

   $\frac{3}{5}$

   c. What is the percent error for *P*(less than 4)? Interpret your results.

   20%. In the experiment, the number cube landed on a number less than 4 20% more frequently than the theoretical probability predicted.

   d. What is the theoretical probability of rolling the cube once with the outcome being a 2?

   $\frac{1}{6}$

644 MODULE 4 • TOPIC 1 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

e. What is the experimental probability of rolling a 2?

$\frac{1}{15}$

f. What is the percent error for $P(2)$? Interpret your results.

−60%; In the experiment, the number cube landed on 2 60% less frequently than the theoretical probability predicted.

g. How might you adjust the experiment to decrease the percent error?

Increasing the number of trials should decrease the percent error.

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MODULE 4 • TOPIC 1 • LESSON 3 ASSIGNMENT 645

Lesson 3 Assignment

Prepare

Solve each proportion.

1. $\frac{3}{x} = \frac{1}{24}$

   $x = 72$

2. $\frac{4}{9} = \frac{10}{y}$

   $y = 22.5$

3. $\frac{3}{5} = \frac{p}{212}$

   $p = 127.2$

646 MODULE 4 • TOPIC 1 • LESSON 3 ASSIGNMENT

4 Simulating Simple Experiments

LESSON OVERVIEW

The term *simulation* is defined in this lesson. A coin toss serves as a simulation to determine the experimental probability of the percent of female chickens hatched. Students note that as the number of trials increases, the experimental probability approaches the theoretical probability. Other situations used in this lesson are a five-question multiple-choice test, a ten-question true-or-false test, a number cube game, and a card game. Students describe simulation models that fit each situation.

MATERIALS

- Coins
- Paper Clips
- Number Cube
- Technology to Simulate a Spinner
- Note Cards
- Problem-Solving Model Graphic Organizer

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

7.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6B select and use different simulations to represent simple and compound events with and without technology.

7.6C make predictions and determine solutions using experimental data for simple and compound events.

7.6D make predictions and determine solutions using theoretical probability for simple and compound events.

ELPS

(2) Listening

The student is expected to:

(G) understand the general meaning, main points, and important details of spoken language ranging from situations in which topics, language, and contexts are familiar to unfamiliar.

(3) Speaking

The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

(5) Writing

The student is expected to:

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary.

ESSENTIAL IDEAS

- A *simulation* is an experiment that models a real-world situation. When conducting a simulation, you must choose a model that has the same probability as the event.
- A *trial* is a repetition of an experiment. Each time the experiment is repeated, it is called a trial.
- The experimental probability of an event approaches the theoretical probability as the number of trials increases, which occurs with a large number of trials.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Designing Sims 5–10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students design spinners to model probabilities of real-world situations, such as a spinner divided into halves to model the probabilities of guessing correctly on a true-or-false question. This activity is designed to engage students in thinking about modeling an experimental probability situation.

DEVELOP

Activity 4.1: Simulating Chicken Hatchings 20 minutes

INVESTIGATION

Students create a model to simulate the percent of female chickens hatched. They list the sample space to describe this event. Students use a coin toss as the model and conduct a trial consisting of two tosses. Using the classes' cumulative data, students then create a dot plot. Next, they increase the number of tosses to 8 and create a new dot plot. After conducting 50 more trials, students notice that the experimental probability approaches the theoretical probability.

Activity 4.2: Guessing on Tests 15 minutes

INVESTIGATION

A history test consists of five questions, and each question is multiple choice with four possible answers. A spinner is divided equally into four sections. Three of the sections represent an incorrect answer and one section represents a correct answer. Students determine the probability of guessing the correct answer to one question. One trial involves spinning the spinner five times, one spin for each question on the test. Using a paperclip and the spinner, students will conduct 50 trials of the simulation, record the results in a table, and create a dot plot. They then conclude that if they guess the answer to all five questions, they would expect to get a little more than one question correct.

DAY 2

Activity 4.3: Simulation Using Random Numbers 20–25 minutes

INVESTIGATION REAL-WORLD PROBLEM SOLVING

A history test consists of five questions, and each question is multiple-choice with five possible answers. Students use a random number table to simulate trials for guessing the correct answer. They use the table to simulate 25 trials, record the results in a table, create a dot plot using the data, and calculate experimental probabilities related to guessing the correct answer.

DEMONSTRATE

Talk the Talk: Mind Readers 20 minutes

INVESTIGATION EXIT TICKET APPLICATION

Students review the lesson by describing a simulation to model each of three situations. They then describe one trial for each situation. Finally, students conduct the simulations and answer related questions.

Getting Started

ENGAGE

Designing Sims

Facilitation Notes

In this activity, students design different spinners to model probabilities of real-world situations. This activity is designed to engage students in thinking about modeling an experimental probability situation.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why is it necessary that the situation states that the responses to the test are guesses?
Probing	• How did you decide how many equal sections to place on the spinner? • Can you have 10 sections on the spinner? How should you label the spinner so that it correctly generates the experimental probability? • Why is it important that the sections are all the same size? • What is the probability of guessing correctly? Incorrectly? How does your spinner correspond to those probabilities? • What is another possible spinner to simulate this context?
Seeing structure	• How does the number of possible outcomes compare to the number of sections on the spinner?

COMMON MISCONCEPTION

Students may partition the spinner into the correct number of sections but not make the sections equal sized. Facilitate a discussion about the importance of the sections being equal size to insure equally likely outcomes.

Summary

Spinners can be used to model real-world situations and calculate experimental probabilities.

STAMP THE LEARNING

CC BY NC

ACTIVITY 4.1

Simulating Chicken Hatchings

DEVELOP

Facilitation Notes

In this activity, students create a coin toss model to simulate a scenario and use the class' cumulative data to create different dot plots.

Ask a student to read the definition aloud and complete Questions 1 through 4 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why does your model work for this sample space? • How could you model this situation with a coin? A six-sided number cube?

Provide each pair or group of students with a coin for the experiment. Have students work with a partner or in a group to complete Question 5, Question 6, parts (a) and (b), and Question 7, parts (a) and (b). Then, complete part (c) for Questions 6 and 7 as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

To help students create the dot plot, provide equivalent fractions $\left(\frac{0}{8}, \frac{2}{8}, \frac{4}{8}, \frac{6}{8}, \frac{8}{8}\right)$ under the labeled values.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What does tossing a coin 2 times represent in Question 5? • What does the result HH mean? HT? TH? TT? • What does tossing a coin 8 times represent in Question 7?
Probing	• How did you use your response to Question 6 part (a) to answer part (b)? • What does a response of 1 in Question 6 part (b) mean? Which coin tosses resulted in that experimental probability? • How did you determine the probability based on your coin tosses? • What does a probability above $\frac{1}{2}$ mean?
Seeing structure	• Does the dot plot look like you expected it to look? Why or why not? • Which dot plot more accurately reflects the theoretical probability? Why do you think that is the case?

Have students work with a partner or in a group to complete Question 8. Complete Questions 9 and 10 as a class. Have a student read the summary at the end of the activity.

DIFFERENTIATION STRATEGY

Just in Time Support

To help students create the dot plot, provide decimal equivalents under the labeled values.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• According to your results, what percent of chickens hatched are female?
Seeing structure	• Consider two towns, one with 20 chickens hatched this year and the other with 500 chickens hatched this year. Which town would come closer to having 50% male and 50% female chickens?

Summary

Simulations are experiments that model a real-world situation. As the number of trials in an experiment increases, the experimental probability approaches the theoretical probability.

ACTIVITY 4.2 Guessing on Tests

Facilitation Notes

In this activity, a spinner is used to model the possible outcomes of guessing on a multiple-choice test question. Using a paperclip, students use the spinner to conduct experiments. The data is used to determine probabilities of guessing correct answers to test questions and create a dot plot. Students conclude that if they guess the answer to all five questions, they would expect to get a little more than one question correct.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How is this situation the same and different from the one in the previous activity? • Why is the spinner divided into four sectors? • How are the 5 questions on the test represented in the simulation? • If there are 5 questions on the test, why isn't the spinner divided into 5 sectors? • If you were guessing the answer to all 5 questions on the test, how many questions would you expect to get correct?
Gathering	• What is an example result for one trial?
Seeing structure	• Are you more likely to get all the questions correct or incorrect on one trial of the test? Explain your thinking.

Provide each student with a paperclip for the experiment. Have students work with a partner or in a group to complete Questions 5 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• To conduct 50 trials, how many times will you have to spin the spinner? • How does your dot plot compare to your classmates' dot plots? • Did anyone in the class have a trial with all the questions incorrect? If so, what is the ratio of number of trials with a score of 0 to the total number of class trials? • Did anyone in the class have a trial with all the questions correct? If so, what is the ratio of number of trials with a score of 5 to the total number of class trials?
Reflecting and justifying	• How did your results compare to your estimate in Question 1?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Spinners are used to model probability scenarios. As the number of trials in an experiment increases, the experimental probability approaches the theoretical probability.

ACTIVITY 4.3

Simulation Using Random Numbers

Facilitation Notes

In this activity, students use a random number table to simulate trials for guessing the correct answer on a multiple-choice test. They record the results in a table, create a dot plot using the data, and calculate experimental probabilities related to guessing the correct answer.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction and definition aloud. Analyze the Worked Example as a class.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how to interpret the numbers you generated. • What number will you enter for Trial 1 in the table? • What is the advantage of using a random number generator? • How does your dot plot compare to the dot plot for a multiple-choice test with 4 choices? Is that what you expected? Explain your thinking. • How did you calculate the experimental probability for each number of questions? • How many questions are you most likely to get correct by guessing? • Did the results show that you would score about the same, better, or worse on the 5-choice multiple-choice test?

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Technology

Provide directions to use technology to generate random numbers. Consider graphing technology, random digit apps and websites, or voice-enabled devices. When using any website or online tool in the classroom, it is important to preview and vet the tool prior to classroom use.

Summary

A random number table is a table that displays random digits and can be used to carry out a simulation.

STAMP THE LEARNING

CC BY NC

DEMONSTRATE

Talk the Talk

MIND READERS

Facilitation Notes

In this activity, students describe a simulation to model different situations and describe one trial. They also conduct the simulations and answer related questions.

Provide students supplies such as a coin, a number cube, and note cards for the experiments. Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How did you choose which simulation to model each situation? • Explain how you defined the outcomes in your model to represent the outcomes of the real-life situation.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Simulations such as coin flips, number cube rolls, and note cards can be used to model probability experiments and determine experimental probability data.

4 Simulating Simple Experiments

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Conduct trials using a simulation to determine probability.
- Conduct a large number of trials and observe the long-run relative frequency of outcomes to demonstrate that experimental probability approaches theoretical probability.

NEW KEY TERMS

- simulation
- random number table

Conducting probability experiments are helpful when you are rolling a number cube, flipping a coin, spinning a spinner, etc.

What about more complicated experiments? How can you use models to run simulations of experiments when it's not possible to do the experiments in real life?

Sample answer:

I can use a model that has the same probability of the event to conduct a simulation and generate experimental probabilities when it is not possible to do the experiment in real life.

MODULE 4 • TOPIC 1 • LESSON 4 647

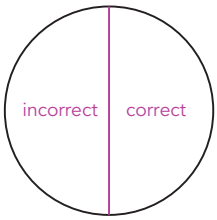
Chunking the Activity

- Group students to complete the activity.
- Share and summarize.

Getting Started

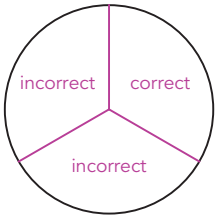
Designing Sims

1. Design a spinner that could be used to generate the experimental probability of guessing the correct answer on a true or false test question. Explain your design.



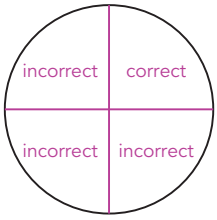
The probability of guessing the correct answer is $\frac{1}{2}$. Students may use *true* and *false* as labels.

2. Design a spinner that could be used to generate the experimental probability of guessing the correct answer on a multiple-choice test where each question has three possible answers. Explain your design.



The probability of guessing the correct answer is $\frac{1}{3}$. Students may use A, B, and C as labels.

3. Design a spinner that could be used to generate the experimental probability of guessing the correct answer on a multiple-choice test where each question has four possible answers. Explain your design.



The probability of guessing the correct answer is $\frac{1}{4}$. Students may use A, B, C, and D as labels.

648 MODULE 4 • TOPIC 1 • LESSON 4

ACTIVITY 4.1

Simulating Chicken Hatchings

What percent of chickens hatching are female? One way to answer the question is to perform a *simulation*. A **simulation** is an experiment that models a real-world situation. When conducting a simulation, you must choose a model that has the same probability of the event.

1. What model might be appropriate for creating a simulation of this event?

   Sample answer:

   Tossing a coin

2. Describe the sample space for this situation.

   S = {female, male}

3. What is the event you are trying to determine?

   The event is the percent of chickens hatching that are female.

4. Suppose the probability of a chicken hatching a female chicken is $\frac{1}{2}$. What percent of all chickens would you expect to be female?

   I would expect that the number of female chickens to be 50% of the total hatchings.

Let's use the toss of a coin as the model. Heads will represent the hatching of a female, and tails will represent the hatching of a male.

5. If you toss a coin twice, list all the possible outcomes from this simulation.

   (H, H), (H, T), (T, H), and (T, T)

Chunking the Activity

- Read and discuss the definition.
- Complete Questions 1–4 as a class.
- Group students to complete Question 5 and parts (a) and (b) of Questions 6 and 7.
- Provide each group with a coin.
- Complete part (c) of Questions 6 and 7 as a class.
- Group students to complete Question 8.
- Check in and share.
- Complete Questions 9 and 10 as a class.
- Read and discuss the summary.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

6. Toss your coin 2 times.

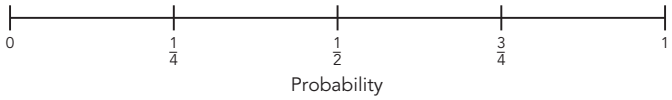
   a. How many of the coin tosses resulted in heads?

      Answers will vary.

   b. According to your simulation, what is the experimental probability of a female chicken being hatched?

      Answers will vary.

   c. Share your results with your classmates. Then, create a dot plot of all the experimental probabilities. Did everyone end up with the same results as the theoretical probability of a female chicken being hatched?



7. Toss your coin 8 times.

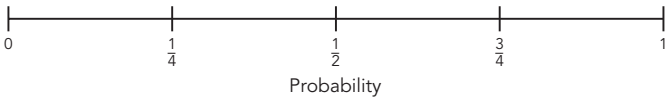
   a. How many of the coin tosses resulted in heads?

      Answers will vary.

   b. According to your simulation, what is the experimental probability of a female chicken being hatched?

      Answers will vary.

   c. Share your results with your classmates. Then, create a dot plot of all the experimental probabilities. How do these results compare to the results from Question 6?



Optimizing Learning

This activity guides information processing and visualization.

650 MODULE 4 • TOPIC 1 • LESSON 4

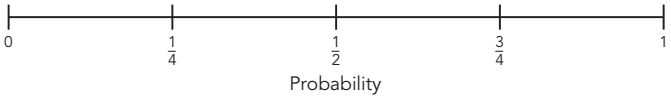
8. Conduct 50 trials of the simulation. Record the results in the table using tally marks.

Result	Tally	Total	Percent
Heads			
Tails			

Answers will vary.

9. Share your results with your classmates. Then, create a dot plot of all the probabilities. What do you notice?

Answers will vary.



10. What can you conclude about the experimental probability of an event and its theoretical probability as the number of trials of the simulation increases?

As the number of trials of the simulation increase, the experimental probability gets closer to the theoretical probability.

Using simulations to generate experimental probabilities is very useful in estimating the probability of an event for which the theoretical probability is difficult, or impossible, to calculate. As you have investigated, the experimental probability of an event approaches the theoretical probability when the number of trials increases.

Question 8 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with conducting experiments to determine probability, assign Skills Practice Set A for this lesson.

MODULE 4 • TOPIC 1 • LESSON 4 651

Chunking the Activity

- Read and discuss the situation.
- Group students to complete Questions 1–4.
- Check in and share.
- Provide each student with a paperclip.
- Group students to complete Questions 5–9.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

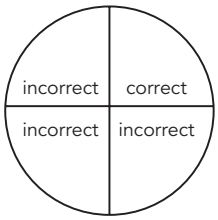
ACTIVITY 4.2

Guessing on Tests

Mr. Garcia, your history teacher, is giving a five-question multiple-choice test. Each question has 4 possible answers. How many questions can you expect to get correct simply by guessing?

1. Estimate the number of questions you expect to get correct by guessing.
   Answers will vary.

One model that you could use to simulate this problem situation is a spinner divided into 4 sections that are the same size.



2. According to the spinner shown, what is the theoretical probability of correctly guessing the answer to one question?
   $\frac{1}{4}$

3. Describe one trial of the experiment if you want to simulate guessing on every question of the test.
   One trial will consist of spinning the spinner 5 times.

4. Will one trial provide a good estimate of how many questions you should expect to get correct? Explain your reasoning.
   No. I will need a large number of trials to get a good estimate of the experimental probability.

5. How can you use technology to simulate the guessing on every question of the test?
   I could find a virtual tool that chooses the outcome at random for me. For example, I could use a virtual spinner instead of a real one.

Of course, if you know the answer, you don't have to guess. This is just an experiment, and you should probably study so that you don't have to guess!

652 MODULE 4 • TOPIC 1 • LESSON 4

6. Conduct 50 trials of the simulation using technology. Record your results.

Results will vary.

Trial Number	Number Correct
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	

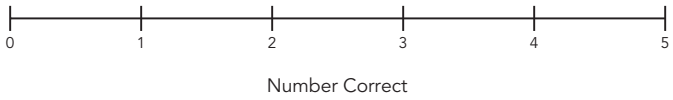
Trial Number	Number Correct
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	
31	
32	
33	
34	

Trial Number	Number Correct
35	
36	
37	
38	
39	
40	
41	
42	
43	
44	
45	
46	
47	
48	
49	
50	

MODULE 4 • TOPIC 1 • LESSON 4 653

7. Graph your results on the dot plot.

Answers will vary.



8. According to your simulation, about how many questions would you expect to get correct on the test by only guessing?

I would expect to get one question correct by guessing.

9. Do you think that guessing on a five-question multiple-choice test will result in a good grade on the test?

No. It appears that guessing will not result in a good grade. One correct out of five questions is 20%, which is a failing grade.

654 MODULE 4 • TOPIC 1 • LESSON 4

ACTIVITY 4.3

Simulation Using Random Numbers

What if each multiple choice question had 5 answer choices instead of 4? How does this change the probability of guessing correct answers?

Instead of creating a new spinner for this simulation, you can design and carry out a simulation for the five-question test guessing experiment using a random *number table*. A **random number table** is a table that displays random digits.

Choose any line in the random number table and let the numbers 00–19 represent a correct guess and 20–99 represent incorrect guesses since each answer choice in a question has a 20% probability of being selected.

The digits of a random number table are often displayed with a space after every 5 digits so that the digits are easier to read. Some random number tables also place a space after every 5 lines for the same reason.

WORKED EXAMPLE

This line of a random number table shows one trial of the simulation. The numbers 12, 64, 56, 20, and 00 are chosen. This corresponds to correct, incorrect, incorrect, incorrect, correct.

Line 4	12645	62000	61555	76404	86210	11808	12841	45147	97438	60022

In this trial, the correct answer was guessed two times.

Use the random number table at the end of the lesson to select random two-digit numbers as a simulation.

1. Conduct one trial of the experiment. List and interpret the results of your first trial.
   Answers will vary.

You can also use technology to generate random numbers.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction, definition, and Worked Example.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The definitions and Worked Example provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

MODULE 4 • TOPIC 1 • LESSON 4 655

2. Conduct a total of 25 trials. Record your results.

Results will vary.

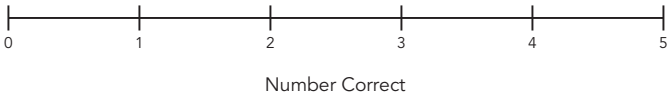
Trial Number	Number Correct
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

Trial Number	Number Correct
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	

656 MODULE 4 • TOPIC 1 • LESSON 4

3. Represent the results from your table on the dot plot shown.

Answers will vary.



4. What is the experimental probability of guessing:

Answers will vary.

a. 0 questions correctly?

b. 1 question correctly?

c. 2 questions correctly?

d. 3 questions correctly?

e. 4 questions correctly?

f. all 5 questions correctly?

5. How do the results from this experiment with 5 answer choices for each question compare to the results from the experiment with 4 answer choices for each question?

Answers will vary.

MODULE 4 • TOPIC 1 • LESSON 4 657

Chunking the Activity

- Read and discuss the directions.
- Provide students supplies such as a coin, a number cube, and note cards for the experiments.
- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Question 2, have students work individually or with a partner to complete the graphic organizer.
- See the Facilitating Productive Struggle section in the Course and Implementation Guide for additional guidance on supporting students through problem-solving activities.

Talk the Talk

PROBLEM SOLVING

Mind Readers

Describe a simulation to model each situation and then describe one trial and the experiment. Conduct the simulation and answer the question.

1. How many questions would you get correct on a ten-question true or false test simply by guessing?

   Simulation without technology:

   Use a coin with heads, representing the correct guess, and tails, representing the incorrect guess.

   Trial:

   One trial would be a single coin toss.

   Experiment:

   The experiment would consist of tossing the coin 10 times and recording the number of heads. This indicates how many questions were guessed correctly.

   Conduct the simulation one time:

   Answers will vary.

2. A board game requires you to roll a 6 on a number cube before you can begin playing. How many times would you expect to roll the cube before you can begin the game?

   Simulation with technology:

   I can use a virtual number roller.

658 MODULE 4 • TOPIC 1 • LESSON 4

Trial:

One trial would be a single roll of a virtual cube.

Experiment:

The experiment would consist of rolling the virtual cube as many times as necessary to roll a 6.

Conduct the simulation 10 times and calculate the mean of your answers:

Answers will vary.

3. James claims he can read your mind. He gives you four cards: a red 1, a blue 2, a blue 3, and a red 4. You draw one of the cards and look at it without showing James.

   a. If you ask James to guess the color, what percent of the time could he guess correctly?

      Simulation without technology:

      Select one of the cards as the correct guess. Use four slips of paper or note cards. Label the slips of paper the same as the cards.

      Trial:

      One trial would be to choose one of the cards at random.

      Experiment:

      The experiment would consist of choosing one of the cards at random and recording whether it was the card you selected as the correct guess.

      Conduct the simulation 10 times and count the number of times your card is selected:

      Answers will vary.

This activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice describing simulations and trials, assign Skills Practice Set B for this lesson.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1B:

- Are students referring to the problem-solving model?
- Do students evaluate the reasonableness of their solution?
- Do students adapt their plan as needed?

MODULE 4 • TOPIC 1 • LESSON 4 659

b. If you ask James to guess the numbers, what percent of the time should he guess correctly?

Simulation without technology:

Select one of the cards as the correct guess. Use four slips of paper or note cards. Label the slips of paper the same as the cards.

Trial:

One trial would be to choose one of the cards at random.

Experiment:

The experiment would consist of choosing one of the cards at random and recording whether or not it was the card you selected as the correct guess.

Conduct the simulation 10 times and count the number of times your card is selected:

Answers will vary.

660 MODULE 4 • TOPIC 1 • LESSON 4

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61404	43516	17020
Line 2	17264	57327	38224	29301	18164	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86214	11808	12840	55147	97438	60222	12645	62090
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19107	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	11415	36457
Line 8	18975	95428	33226	55901	31605	43816	22259	00317	46999	98571
Line 9	59138	39542	71168	57609	91510	27904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

MODULE 4 • TOPIC 1 • LESSON 4 661

662 MODULE 4 • TOPIC 1 • LESSON 4

Lesson 4 Assignment

Write

1. Each time you repeat an experiment, it is called a(n) _____.
2. A(n) _____ is an experiment that models a real-world situation.

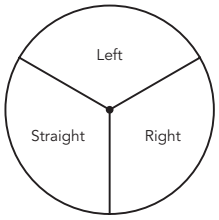
Remember

As the number of trials increases, the experimental probability gets closer and closer to the theoretical probability.

Practice

Conduct each experiment as described and record your results in a table. Use your results to determine the experimental probability.
Answers for all questions will vary.

1. At the first intersection of a corn maze, a person can go left, right, or straight. Use the spinner to model the person choosing the direction they will go. Use a paper clip as the arrow part of the spinner. Place a pencil point through the paper clip and then on the center of the circle. Perform 30 trials of the experiment. Record the results in a table using tally marks.



   What is your experimental probability that the person turns right?

2. A theater audience is made up of half adults and half children. One person is chosen at random to volunteer on stage. Toss a coin to model the person being chosen from the audience. Perform 40 trials of the experiment. Record the results in a table using tally marks.

   What is your experimental probability that the volunteer is a child?

3. Two-thirds of the fish in a lake are trout. A fisherman catches 1 fish. Roll a number cube to model the fisherman catching the fish. Perform 25 trials of the experiment. Record the results in a table using tally marks.

   What is your experimental probability that the fisherman catches a fish that is not a trout?

Write

1. trial
2. simulation

MODULE 4 • TOPIC 1 • LESSON 4 ASSIGNMENT 663

Lesson 4 Assignment

4. A drawer contains 10 white socks and 10 brown socks. The socks are mixed up. Catalina chooses 1 sock without looking. Use a number cube to model Catalina choosing the sock. Perform 30 trials of the experiment. Record the results in a table using tally marks.

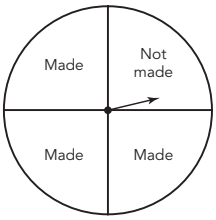
   What is your experimental probability that Catalina chooses a brown sock?

5. A multiple-choice quiz has 4 questions. Each question has 3 possible answers. You guess the answer to each question. Use 3 slips of paper, one labeled correct, one labeled incorrect, and another labeled incorrect, to model guessing the answer to one question. Perform 10 trials of the experiment, where each trial consists of pulling a slip of paper from a bag without looking 4 times. Be sure to return the paper you chose back into the bag before choosing again. Record the results in a table.

   What is your experimental probability that you get at least 2 questions correct?

6. A basketball player makes a foul shot 75% of the time. He is given the chance to make 2 foul shots. Use the spinner to model the player attempting a foul shot. Perform 20 trials of the experiment, where each trial consists of spinning the spinner 2 times. Record the results in a table.

   What is your experimental probability that the player makes both foul shots?



664 MODULE 4 • TOPIC 1 • LESSON 4 ASSIGNMENT

Lesson 4 Assignment

Prepare

Solve each proportion for the unknown value.

1. $\frac{x}{150} = \frac{1}{12}$

   12.5

2. $\frac{12}{x} = \frac{1}{36}$

   432

3. $\frac{1}{8} = \frac{x}{100}$

   12.5

4. $\frac{x}{16} = \frac{1}{4}$

   4

MODULE 4 • TOPIC 1 • LESSON 4 ASSIGNMENT 665

666 MODULE 4 • TOPIC 1 • LESSON 4 ASSIGNMENT

TOPIC 1 SELF-REFLECTION

Name: _____

Introduction to Probability

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Introduction to Probability* topic by:

TOPIC 1: *Introduction to Probability*	Beginning of Topic	Middle of Topic	End of Topic
identifying the sample space of an experiment.	○	○	○
recognizing and explaining that the probability of a chance event is a number between 0 and 1 that expresses how likely an event is to occur.	○	○	○
determining the probabilities of a simple event and its complement and describing the relationship between the two.	○	○	○
calculating experimental probability as the number of times an outcome occurs divided by the total number of times the experiment is completed.	○	○	○
calculating theoretical probability as the number of favorable outcomes in the sample space divided by the total number of outcomes in the sample space.	○	○	○
making predictions and determining solutions using experimental and theoretical probability for simple events.	○	○	○
determining experimental and theoretical probabilities related to simple events using data and sample spaces.	○	○	○

continued on the next page

Notes

MODULE 4 • TOPIC 1 • SELF-REFLECTION 667

Notes

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Introduction to Probability*	Beginning of Topic	Middle of Topic	End of Topic
using probability models to determine the probability of events.	○	○	○
designing and using a simulation to determine experimental probabilities of simple events.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Introduction to Probability* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

668 MODULE 4 • TOPIC 1 • SELF-REFLECTION

TOPIC 1 SUMMARY

Introduction to Probability Summary

LESSON 1 Analyzing Populations and Probabilities

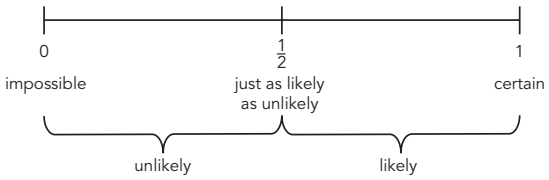
A six-sided number cube that has one number, from 1 through 6, on each face is rolled.

The numbers on the faces of the cube are the *outcomes* that can occur. An **outcome** is the result of a single trial of a probability *experiment*. An **experiment** is a situation involving chance that leads to results, or outcomes. The experiment is rolling the number cube. A list of all possible outcomes of an experiment is called a **sample space**. For example, the sample space for rolling the six-sided number cube is {1, 2, 3, 4, 5, 6}. An **event** is one possible outcome or a group of possible outcomes for a given situation. It is a subset of the sample space. A **simple event** is an event consisting of one outcome. In the number cube experiment, an event could be rolling an even number, while rolling a 5 is a simple event.

Probability is a measure of the likelihood that an event will occur. The probability of an event is often written as *P*(event). In the number cube experiment, the probability of rolling a 5 could be written as *P*(5), and the probability of rolling an even number could be written as *P*(even).

The probability of an event occurring is a number between 0 and 1. If the event is certain to happen, then the probability is 1. If an event is impossible, then the probability is 0. If an event is just as likely to happen as not happen, then the probability is 0.5, or $\frac{1}{2}$.

The number line shown represents the probabilities, from 0 to 1, of any event occurring.



NEW KEY TERMS

- outcome
- experiment [experimento]
- sample space
- event [evento]
- simple event [evento simple]
- probability [probabilidad]
- complementary events [eventos complementarios]
- equally likely
- probability model [modelo probabilístico]
- uniform probability model [modelo probabilístico uniforme]
- non-uniform probability model [modelo probabilístico no uniforme]
- theoretical probability [probabilidad teórica]
- experimental probability [probabilidad experimental]
- percent error [error porcentual]
- simulation [simulación]
- random number table

Notes

MODULE 4 • TOPIC 1 • TOPIC SUMMARY 669

Notes

Probability is the ratio of the number of times an event can occur to the number of possible outcomes.

$$\text{probability} = \frac{\text{number of times an event can occur}}{\text{number of possible outcomes}}$$

To determine the probability of rolling an odd number on a six-sided number cube, or $P(\text{odd})$, follow these steps:

- First, list all the possible outcomes. The possible odd numbers that can be rolled are 1, 3, and 5.
- Next, add the number of outcomes. There are 3 possible outcomes of rolling an odd number.
- Then, use the equation to determine the probability of rolling an odd number.

$$P(\text{odd}) = \frac{3 \text{ possible odd numbers}}{6 \text{ possible numbers}}$$

The probability of rolling an odd number is $\frac{3}{6}$, or $\frac{1}{2}$.

Complementary events are events that together contain all of the outcomes in the sample space. If $P(\text{even})$ represents the probability of rolling an even number, then $P(\text{not even})$ is the complementary event.

When the probabilities of all the outcomes of an experiment are equal, then the outcomes are called **equally likely**.

LESSON 2 Probability Models

When solving a probability problem, it is helpful to construct a *probability model*. A **probability model** is a list of each possible outcome along with its probability. Probability models are often shown in a table.

For example, the probability model for rolling a six-sided number cube is shown.

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

A probability model lists all the outcomes. The probability of each outcome will be greater than 0 but less than 1. The sum of all the probabilities for the outcomes will always be 1.

A **uniform probability model** occurs when all the probabilities in a probability model are equally likely to occur. When all the probabilities in a probability model are not equal to each other, it is called a **non-uniform probability model**.

670 MODULE 4 • TOPIC 1 • TOPIC SUMMARY

LESSON 3 Determining Experimental Probability of Simple Events

Notes

The **theoretical probability** of an event is the ratio of the number of desired outcomes to the total possible outcomes. **Experimental probability** is the ratio of the number of times an event occurs to the total number of trials performed.

$$\text{experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of trials performed}}$$

When you know the probability of an event, you can use proportional reasoning to predict the number of times the event will occur throughout an experiment.

For example, the probability that a spinner will land on blue is $\frac{2}{3}$. When you spin the spinner 60 times, you can set up and solve a proportion to predict the number of times the spinner will land on the blue section.

$$\frac{2}{3} = \frac{x}{60}$$
$$2(60) = 3x$$
$$\frac{120}{3} = \frac{3x}{3}$$
$$40 = x$$

When you spin the spinner 60 times, you can expect it to land on blue 40 times.

To compare theoretical and experimental probabilities, you can use a measure called *percent error*. In probability, the **percent error** describes how far off the experimental probability is from the theoretical probability as a percent ratio.

For example, suppose you spin blue 30 times out of 40 trials. The experimental probability, P_E(blue), is $\frac{30}{40}$ or $\frac{3}{4}$. The theoretical probability, P_T(blue), is $\frac{2}{3}$.

The percent error can be calculated using this formula:

$$\frac{P_E - P_T}{P_T} \cdot 100 = \frac{\frac{3}{4} - \frac{2}{3}}{\frac{2}{3}} \cdot 100$$
$$= \frac{\frac{9}{12} - \frac{8}{12}}{\frac{8}{12}} \cdot 100$$
$$= \frac{\frac{1}{12}}{\frac{8}{12}} \cdot 100 = 12.5\%$$

MODULE 4 • TOPIC 1 • TOPIC SUMMARY 671

Notes

LESSON 4

Simulating Simple Experiments

A **simulation** is an experiment that models a real-world situation. When conducting a simulation, you must choose a model that has the same probability of the event. Using simulations to generate experimental probabilities is very useful in estimating the probability of an event for which the theoretical probability is difficult, or impossible, to calculate.

You can design and carry out a simulation for an experiment using technology or a *random number table*. A **random number table** is a table that displays random digits. You assign a range of numbers to each outcome that models the same probability of an event and then choose any line from the table to perform a trial.

For example, in a five-question multiple-choice test, each question has five possible answer choices. How many questions can you expect to get correct simply by guessing?

Each answer choice has a 20% chance of being selected, but only $\frac{1}{5}$ of the guesses are correct, while the others are incorrect. Let the numbers 00–19 represent correct guesses and 20–99 represent incorrect guesses.

This line of a random number table shows one trial of the simulation.

Line 4	12645	62000	61555	76404	86210	11808	12841	45147	97438	60022

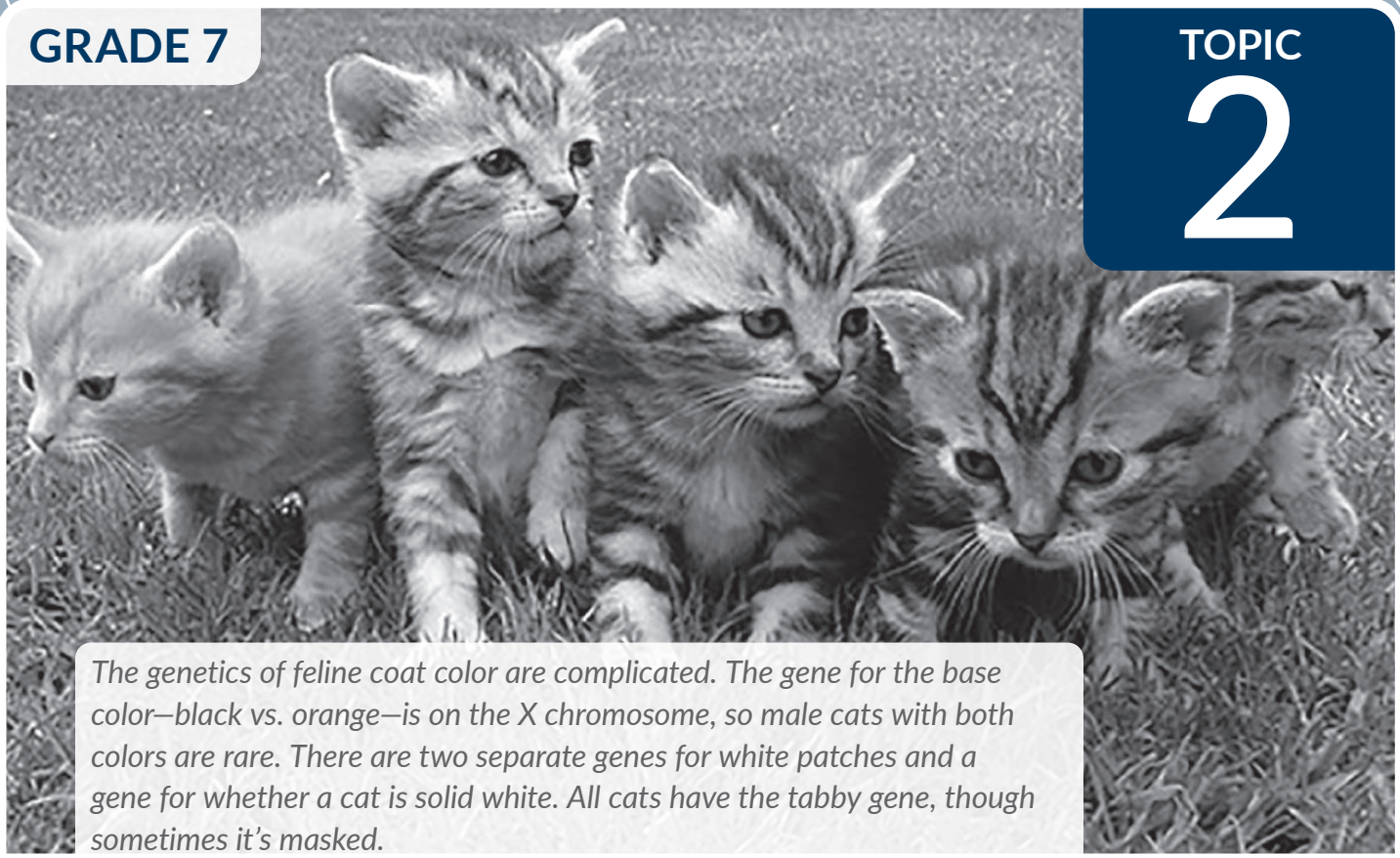
The numbers 12, 64, 56, 20, and 00 are chosen. This corresponds to correct, incorrect, incorrect, incorrect, correct. In this trial, the correct answer was guessed 2 times out of 5.

You need to run multiple trials of this simulation to predict the number of questions you can expect to get correct simply by guessing.

672 MODULE 4 • TOPIC 1 • TOPIC SUMMARY

GRADE 7

TOPIC 2



The genetics of feline coat color are complicated. The gene for the base color—black vs. orange—is on the X chromosome, so male cats with both colors are rare. There are two separate genes for white patches and a gene for whether a cat is solid white. All cats have the tabby gene, though sometimes it's masked.

Compound Probability

LESSON 1 Using Arrays to Organize Outcomes **675**

LESSON 2 Using Tree Diagrams **695**

LESSON 3 Determining Compound Probability **711**

LESSON 4 Simulating Probability of Compound Events **729**

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TOPIC 2 OVERVIEW

Compound Probability

How is *Compound Probability* organized?

In *Compound Probability*, students build on their understanding of probability concepts from the previous topic. They use arrays and lists to organize the possible outcomes of an experiment that includes two simple events. Students continue to calculate experimental and theoretical probabilities of events and use proportional reasoning to determine percent error and make predictions of expected numbers of outcomes. In this topic, students are introduced to and use tree diagrams as a method for listing outcomes in a sample space when there are more than two simple events. They use their tree diagrams to create probability models and determine specified probabilities.

Math Representation

This array shows the sample space—all the possible outcomes—for rolling two 6-sided number cubes and calculating the product of the numbers shown.

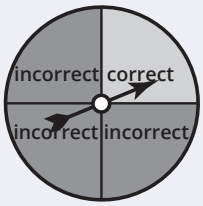
		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

The probability of the product being a 6 is $\frac{4}{36}$, or $\frac{1}{9}$.

Students learn about compound events that use the conjunctions *and* and *or*. They list outcomes that are contained in an event, distinguishing between *and* and *or* situations. Students engage in multiple activities to build understanding of compound events and to help them distinguish between the types of events.

Math Representation

Mr. Garcia gives a multiple-choice test with five questions on it. Each question has 4 possible answers. How many questions can you expect to get correct by merely guessing?



One model that you could use to simulate this problem situation is a spinner divided into 4 sections that are the same size.

What is the entry point for students?

Students have used arrays to represent relationships among numbers. Here, they apply their knowledge of arrays to represent outcomes from conducting two simple events simultaneously. Students then use arrays and organized lists to list sample spaces and calculate probabilities for compound events. Throughout *Compound Probability*, they reinforce and deepen their understanding of probability concepts learned in the previous topic—simple events, experimental versus theoretical, making predictions, and simulation.

Why is *Compound Probability* important?

Compound Probability provides students with the opportunity to build intuition about compound events. Probability and randomness are central in the study of statistical inference, and compound probability is an essential building block for the next topic. Students will continue to deepen their understandings of probability concepts as they explore random sampling and draw inferences about a population. In future courses, students will engage with probability from a more formal and formula-driven perspective as they learn about mutually exclusive events and conditional probability.

Math Representation

Suppose you have 6 cards: 3 have a yellow dot, 1 has a purple dot, and 2 have a blue dot. You select two cards, at random, one at a time.

You can write an organized list to represent the sample space for this situation.

Y_1Y_2	Y_2Y_1	Y_3Y_1	PY_1	B_1Y_1	B_2Y_1
Y_1Y_3	Y_2Y_3	Y_3Y_2	PY_2	B_1Y_2	B_2Y_2
Y_1P	Y_2P	Y_3P	PY_3	B_1Y_3	B_2Y_3
Y_1B_1	Y_2B_1	Y_3B_1	PB_1	B_1P	B_2P
Y_1B_2	Y_2B_2	Y_3B_2	PB_2	B_1B_2	B_2B_1

Key: Y = Yellow
P = Purple
B = Blue

The circle shows the outcomes that match the first event, and the rectangles show the outcomes that match the second event. You can use this to determine the probability of choosing a yellow card first and a blue card second.

$P(Y \text{ and } B) = \frac{6}{30} = \frac{1}{5}$

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic when they can:

- Choose the appropriate method, such as an organized list, a table, or a tree diagram, to represent sample spaces and create probability models for compound events.
- Represent the outcomes in the sample space that make up a compound event.
- Determine theoretical and experimental probabilities for compound events using data and sample spaces.
- Make predictions and determine solutions using experimental and theoretical probabilities for compound events.
- Design and use a simulation to predict the probability of a compound event.

NEW KEY TERMS

- tree diagram
- compound event

How do the activities in *Compound Probability* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

The focus of the mathematical process standards in this topic is on using appropriate tools and modeling real-world situations with mathematics (7.1C). This topic builds on the initial understanding of probability, probability models, and simulation developed in the prior topic (7.1F). Students use a variety of tools (e.g., organized lists, arrays, and simulations) to create sample spaces and to determine probabilities of compound (7.1E).

MODULE 4, TOPIC 2 PACING GUIDE

165-Day Pacing

4 Analyzing Populations and Probabilities

TOPIC 2: Compound Probability

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1A, 7.1B, 7.1C, 7.1E, 7.1F, 7.1G

ELPS: 1.A, 1.E, 1.F, 2.C, 2.E, 2.G, 3.D, 4.G, 5.B

Topic Pacing: 12 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	**Using Arrays to Organize Outcomes**	This lesson provides the foundation for probability of compound events. Students use arrays and lists to determine sample spaces and then to calculate probabilities. First, they conduct an experiment tossing two coins and predict the theoretical probability of each outcome based on their experiment. Next, students conduct trials using a six-sided number cube, record the data in a table, and determine the experimental probabilities of each possible sum of the numbers on the cubes. They use an array to organize the outcomes, determine the sample space, and then determine the theoretical probabilities. These probabilities are used to calculate probabilities of compound events (but not stated as such) and make predictions using proportional reasoning. Finally, students practice these skills with a 4-section spinner and the Getting Started activity. **Materials Needed:** Coins (one per student), Six-Sided Number Cubes (one per student), Problem-Solving Model Graphic Organizer	7.6A 7.6C 7.6D **7.6I**	2
2	**Using Tree Diagrams**	Students use experimental data to create a probability model and then construct a second probability model using theoretical probabilities for comparison purposes. Tree diagrams are introduced as another method to illustrate all the possible outcomes in a sample space. They then analyze a given tree diagram modeling the same situation and create a third probability model. In the second activity, a five-sided spinner and a tree diagram are used to generate all possible outcomes to create a probability model and answer related questions. To demonstrate their understanding, students create a tree diagram for all possible outcomes of correctly guessing the answers to a 3-question true or false test. They then use the tree diagram to create a probability model and use the model to determine specified probabilities. **Materials Needed:** None	7.6A **7.6I**	2

*Bold TEKS = Readiness Standard

MODULE 4, TOPIC 2 PACING GUIDE

165-Day Pacing

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	**Determining Compound Probability**	The term *compound event* is defined in this lesson. Within the context of the first situation, students determine the probability for three events by calculating the sum of the probabilities of each event. In the second situation, students calculate and compare the probability of a compound event with the word *and* to the probability of a compound event with the word *or*. They distinguish between the two compound events; they state that the compound event associated with the word *and* means simple events both (or all) occur, while the compound event associated with the word *or* means any combination of one or more simple events occurring. When calculating the probability of a compound event, students learn not to count repeated outcomes if the same outcome appears in more than one simple event. **Materials Needed:** None	7.6A 7.6D	2
4	**Simulating Probability of Compound Events**	Students simulate the probability of compound events. They use given rations of past performance to predict future outcomes through simulations. Students simulate the number of free throws a player would make before missing a shot, the number of people donating blood before a person with Type B blood participates, and the number of extra-point kick attempts before missing one. **Materials Needed:** Six-sided Number Cubes, Other Simulation Tools, as desired, Problem-Solving Model Graphic Organizer	7.6B 7.6C 7.6D	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				3

*Bold TEKS = Readiness Standard

MODULE 4, TOPIC 2 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.6A, 7.6C, 7.6D, **7.6I** **LESSON 1** Using Arrays to Organize Outcomes **GETTING STARTED** **ACTIVITY 1**	**LESSON 1** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	TEKS: 7.6A, 7.6I **LESSON 2** Using Tree Diagrams **GETTING STARTED** **ACTIVITY 1**	**LESSON 2** continued **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
TEKS: 7.6A, 7.6D **LESSON 3** Determining Compound Probability **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: 7.6B, 7.6C, 7.6D **LESSON 4** Simulating Probability of Compound Events **GETTING STARTED** **ACTIVITY 1**	**LESSON 4** continued **ACTIVITY 2** **TALK THE TALK**
Day 11	**Day 12**			
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**			

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are three Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice set of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Using Arrays to Organize Outcomes

MATERIALS

Coins (one per student)
Six-Sided Number Cubes (one per student)
Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

This lesson provides the foundation for probability of compound events. Students use arrays and lists to determine sample spaces and calculate probabilities. First, they conduct an experiment in which they toss two coins and predict the theoretical probability of each outcome based on their experiment. Next, students conduct trials using a six-sided number cube, record the data in a table, and determine the experimental probabilities of each possible sum of the numbers on the cubes. They use an array to organize the outcomes, determine the sample space, and then determine the theoretical probabilities. These probabilities are used to calculate probabilities of compound events (but are not stated as such) and make predictions using proportional reasoning. Finally, students practice these skills with a four-section spinner and the Getting Started.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

7.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

7.1E create and use representations to organize, record, and communicate mathematical ideas.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.
The student is expected to:

7.6A represent sample spaces for simple and compound events using lists and tree diagrams.

ELPS

(1) Learning Strategies
The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(3) Speaking
The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

(5) Writing
The student is expected to:

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary.

GRADE 7 TEKS *(TEKS continued from previous page)*

7.6C make predictions and determine solutions using experimental data for simple and compound events.

7.6D make predictions and determine solutions using theoretical probability for simple and compound events.

7.6I determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

ESSENTIAL IDEAS

- *Experimental probability* is the ratio of the number of times an event occurs to the total number of trials performed.
- *Theoretical probability* is the mathematical calculation that an event will occur in theory (long-run relative frequency).
- Experimental probability can be used to predict theoretical probability.
- Arrays and lists are useful for organizing outcomes and determining the sample space of an experiment.
- Proportional reasoning is used to make predictions about the expected number of times an outcome will occur based on the probability of the outcome.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Tossing Coins 20 minutes

CONNECT TO PRIOR KNOWLEDGE

Students consider an experiment of tossing two coins. They create an array to determine the outcomes and use the array to write the sample space. Students conduct the experiment and calculate their experimental probabilities. They then combine their results with their classmates and calculate the class experimental probabilities. Students use this information to predict the actual probabilities of each outcome.

DEVELOP

Activity 1.1: Experimental Probability of Sums 20–25 minutes

INVESTIGATION

Students generate a list of all of the outcomes when rolling two 6-sided number cubes. They list the sample space for a game in which they calculate the sum of rolling two 6-sided number cubes. Students conduct the experiment and predict who might win this game. Results of the trials are recorded in a table, and the experimental probabilities are calculated. Students create a plan for determining the theoretical probabilities in this experiment.

DAY 2

Activity 1.2: Theoretical Probability of Sums 15–20 minutes

REAL-WORLD PROBLEM SOLVING

Students continue analyzing the number cube game. They create an array to display the possible sums and calculate the theoretical probabilities of each sum. From these calculations, students conclude that the game is fair, as both players are equally likely to win. Theoretical probability is compared to experimental probability, and the probabilities are used to determine various compound probabilities and expected values.

Activity 1.3: Probabilities of Products 10–15 minutes

REAL-WORLD PROBLEM SOLVING

Students practice using arrays to organize outcomes and determine theoretical probabilities. A spinner is divided equally into four sections with a different number written in each section. The spinner is spun twice, and the product is calculated and recorded. Students determine the sample space and use the array to calculate the probability of each product. The probabilities are used to calculate compound probabilities and expected values.

DEMONSTRATE

Talk the Talk: Tossing More Coins 10 minutes

EXIT TICKET APPLICATION

Students return to the experiment from the Getting Started and create an array for the number of heads when tossing two coins. They list the sample space, determine the theoretical probabilities for different numbers of heads, and answer questions using the probabilities.

CC BY NC

Getting Started

ENGAGE

Tossing Coins

Facilitation Notes

In this activity, students perform an experiment by tossing two coins. They create an array and write the sample space. Students calculate experimental probabilities and use this information to predict the actual probabilities of outcomes.

Have students work with a partner to complete Questions 1 through 5. Each partner should be assigned a number: 1 or 2. Student 1 landing on heads and Student 2 landing on tails would be marked in a different cell than Student 2 landing on heads and Student 1 landing on tails. Share results as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How is this experiment different from others you have performed so far? • Explain the organization of the array and how it helps you identify all the possible outcomes. • What is the difference between the two outcomes with one heads and one tails? • Is this a uniform probability model? Why or why not?
Probing	• Why are your probabilities different from the class's probabilities?

COMMON MISCONCEPTION

Students may think the experimental probably must match the theoretical probably. Facilitate a discussion with students about the difference between theoretical and experimental probability.

STAMP THE LEARNING

Summary

As the number of trials of an experiment increase, the probabilities of a specific event occurring approach theoretical probability.

ACTIVITY 1.1

Experimental Probability of Sums

DEVELOP

Facilitation Notes

In this activity, an experiment uses two 6-sided number cubes. Students generate a list of all of the outcomes, list the sample space, and calculate the sum of rolling two 6-sided number cubes. Data are used to make predictions, results of the trials are recorded in a table, and the experimental probabilities are calculated.

Have students work with a partner to complete Questions 1 through 8. For Question 5, consider compiling the class data, similar to the previous activity. Discuss results as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Provide students with an empty array to list the outcomes.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How did you organize your work to be sure you listed all of the outcomes? • Why should you consider (2, 1) and (1, 2) as two different outcomes? • Why is the sample space not the same as the list of possible outcomes? • Why isn't 1 included in your sample space? • Which sum is most likely to occur? In how many ways can it occur? • Why is the probability called *experimental probability*? • Why can you consider your table a probability model?
Probing	• Why are the outcomes not equally likely when you have an equal chance of rolling a 1 through 6 on each cube? • How can you use your list of outcomes and sample space in your calculations? • How can you verify that you didn't make mistakes in creating your probability model?
Reflecting and justifying	• Does this game have a uniform or non-uniform probability model? Explain your reasoning. • Explain how you used your data to make this decision.

		Number cube 2					
		1	2	3	4	5	6
Number cube 1	1						
	2				(2, 4)		
	3						
	4						
	5						
	6						

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Simulations can be performed to calculate the experimental probability of sums.

STAMP THE LEARNING

CC BY NC

ACTIVITY 1.2

Theoretical Probability of Sums

Facilitation Notes

In this activity, students continue analyzing the number cube game from the previous activity. They create an array and calculate the theoretical probabilities of each sum. From these calculations, students conclude that the game is fair, as both players are equally likely to win. The probabilities are then used to determine various compound probabilities and expected values.

In probability, it is often most convenient to write probabilities in fractional form with a common denominator, which represents the number of possible outcomes. Students can more easily compute additional probabilities this way. Refrain from requiring students to write the fractions in lowest terms.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What are two different ways you could get 9 as the response? • What patterns do you notice in the array? • How is this array different from your list of possible outcomes in the previous activity?
Reflecting and justifying	• How can you check your work in the probability model?

Have students work with a partner or in a group to complete Questions 6 through 10. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why is the probability called *theoretical probability*?
Probing	• Explain how you used the probability model to calculate *P*(odd sum). • How did you know the other probability without doing any calculations? • Explain your steps to calculate the percent error. • What does the $\frac{1}{2}$ in the denominator represent in this situation?

Have students work with a partner or in a group to complete Questions 11 through 13. For Question 11, students can choose how to calculate the probabilities. Do not focus on rules, but rather focus on using the array and probabilities together. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How could you set up a proportion to make your prediction? • What is another way to solve this problem? • What is the probability of getting a sum of 4? • Is Catalina's response greater or less than the correct response? Explain your thinking.

Summary

Proportional reasoning can be used to predict how often an event may occur for any number of trials.

STAMP THE LEARNING

ACTIVITY 1.3

Probabilities of Products

Facilitation Notes

In this activity, arrays are used to organize outcomes and determine theoretical probabilities. A game is described that uses a spinner, and students calculate theoretical probabilities. These probabilities are used to calculate compound probabilities and expected values.

Have students work with a partner or in a group to complete Questions 1 through 8. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Provide students with an empty array to list the outcomes.

		Spinner 2			
		1	2	3	6
Spinner 1	1				
	2				
	3		(3, 2)		
	6				

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How is this situation the same as the previous one? How is it different?
Probing	• How did you organize your work to be sure you listed all of the outcomes? • Explain why there are 16 outcomes. • What is the purpose of creating this array? • Why are the outcomes not equally likely when you have an equal chance of spinning a 1, 2, 3, or 6? • Explain how you used the array to calculate each probability. • How can you verify that you didn't make mistakes in creating your probability model? • Explain how you used the probability model to calculate *P*(even product). • What are two different ways to calculate *P*(odd product)? • Which strategy did you use to make your prediction?
Reflecting and justifying	• Which product is most likely to occur? In how many ways can it occur? • Does this game have a uniform or non-uniform probability model? Explain your reasoning.

STAMP THE LEARNING

Summary

An array can be used to list a sample space, which can be used to calculate theoretical probability. Theoretical probability is probability that is calculated using mathematics.

DEMONSTRATE

Talk the Talk

TOSSING MORE COINS

Facilitation Notes

In this activity, students use the scenario from the first activity to create an array, list the sample space, determine the theoretical probabilities, and answer questions using the probabilities.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How is this situation different from the Getting Started situation?
Probing	• Explain the meaning of the entries in your array. • What information did you use to calculate the probabilities? • What does the probability $\frac{2}{4}$ represent? • How did you use your probability model to calculate this probability?

Have students read and answer the Essential Question on the lesson opener page.

Summary

An array can be used to list a sample space that can be used to calculate theoretical probability. Theoretical probability is probability that is calculated using mathematics.

STAMP THE LEARNING

CC BY NC

1 Using Arrays to Organize Outcomes

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Conduct trials of an experiment.
- Predict the theoretical probability of an event using the results from the trials of an experiment.
- Represent and organize sample spaces for compound events using arrays, lists, and tables.
- Identify the outcomes in the sample space that compose a compound event.
- Calculate the experimental and theoretical probabilities of an experiment.
- Calculate probabilities of multiple outcomes using theoretical probabilities.
- Use proportional reasoning to predict the probability of random events.

You have calculated experimental and theoretical probabilities of simple events.

How can you organize the outcomes of multiple events to determine theoretical probabilities?

Sample answer:

To organize the outcomes in a number array, list the outcomes for one trial along one side and the outcomes for the other trial along the other side of an array. Combine the results in the intersections of each row and column.

MODULE 4 • TOPIC 2 • LESSON 1 675

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Tossing Coins

Suppose you and a partner each flip a coin at the same time. What are the different outcomes that could occur?

1. Use an array to list the possible outcomes of tossing two coins into the air at the same time.

		Student 1	
		Heads	Tails
Student 2	Heads	Heads, Heads	Heads, Tails
	Tails	Tails, Heads	Tails, Tails

2. Use your array to write the sample space for tossing two coins at the same time.

   S = {(Heads, Heads), (Heads, Tails), (Tails, Tails), (Tails, Heads)}

3. Conduct this experiment for 30 trials and record the results.

   a. Calculate the experimental probability of each outcome in the table.

      Answers will vary.

Outcome Student 1	Outcome Student 2	Tally	Total	Probability	Class Probability
Heads	Heads				
Heads	Tails				
Tails	Heads				
Tails	Tails				

676 MODULE 4 • TOPIC 2 • LESSON 1

b. Combine the results of your experiment with those of your classmates. Then, calculate the experimental probability of each outcome and record the class probabilities in the table.

Answers will vary.

4. Which probability is more accurate: the probabilities from the experiment with your partner or the class probabilities? Explain your reasoning.

Sample answer:

The class probabilities are more accurate because they are derived from the results of many more trials than the results my partner and I got.

5. Use your table to predict the actual probabilities of each outcome. Create a probability model table to display your predictions.

Sample answer:

Outcome	Heads, Heads	Heads, Tails	Tails, Tails	Tails, Heads
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Questions 1–5 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice using arrays to organize outcomes to answer questions, assign Skills Practice Set A for this lesson.

MODULE 4 • TOPIC 2 • LESSON 1 677

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 1.1

Experimental Probability of Sums

How can you organize your list to make sure you account for all the outcomes?

James and Ricardo are playing a game using two 6-sided number cubes. The number cubes are rolled, and the sum of the 2 numbers shown is calculated. If the sum is even, James wins a point. If the sum is odd, Ricardo wins a point.

1. Make a list of all possible outcomes when rolling two 6-sided number cubes.

   (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
   (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
   (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
   (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
   (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
   (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

2. Use your list to create the sample space for the possible sums in the game.

   S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

3. Do you think each of the outcomes in the sample space is equally likely? Explain your reasoning.

   Sample answer:

   In listing all the possible results of each number occurring when 2 number cubes are rolled, I can see that some outcomes occur more often than others. Therefore, it appears that not all the sums are equally likely to result from rolling the two number cubes.

4. Predict who has a better chance of winning this game. Explain your reasoning.

   Answers will vary, but some students may predict that James will win the game because there are more even sums than odd sums in the sample space. Therefore, James has a better chance.

SELF-MONITORING STRATEGY

Look for students using self-motivation and self-discipline to persevere in problem solving.

Refer to the Course and Implementation Guide for further details on these look-fors.

678 MODULE 4 • TOPIC 2 • LESSON 1

5. Play the game 25 times with a partner.

   a. List each possible outcome in the first column. Tally your results in the second column.

   Answers will vary.

   b. Calculate the experimental probability for each outcome. Express your probability as a fraction and as a percent. Record your probabilities in the third column of the table.

   Answers will vary.

Outcome	Tally	Probability
2		$\frac{0}{25} = 0\%$
3		$\frac{1}{25} = 4\%$
4		$\frac{1}{25} = 4\%$
5		$\frac{3}{25} = 12\%$
6		$\frac{5}{25} = 20\%$
7		$\frac{3}{25} = 12\%$
8		$\frac{2}{25} = 8\%$
9		$\frac{5}{25} = 20\%$
10		$\frac{1}{25} = 4\%$
11		$\frac{2}{25} = 8\%$
12		$\frac{2}{25} = 8\%$

6. According to your experiment, who would win the game between James and Ricardo? Explain your reasoning.

   Sample answer:

   Based on the data in Question 5, the $P(\text{even}) = \frac{11}{25} = 44\%$ and $P(\text{odd}) = \frac{14}{25} = 56\%$, so Ricardo would win.

7. Does the outcome of the game match your prediction of the winner? Explain your reasoning.

   Sample answer:

   Ricardo won the game, but I predicted that James would win the game. I thought that James had a better chance to win the game because there are more even sums than odd sums.

8. Suppose you wanted to calculate the theoretical probability of each sum. Explain how you might proceed.

   I can determine the number of possible sums from rolling two 6-sided number cubes and then determine the number of times I can roll a particular sum.

Make a plan, but do not carry it out. You will calculate the theoretical probabilities of this experiment in the next activity.

MODULE 4 • TOPIC 2 • LESSON 1 679

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete Questions 1–5.
- Check in and share.
- Group students to complete Questions 6–10.
- Check in and share.
- Group students to complete Questions 11–13.
- Share and summarize.

ACTIVITY 1.2

Theoretical Probability of Sums

Recall the game James and Ricardo are playing with two 6-sided number cubes. Now, you will calculate the theoretical probability of each sum.

By playing the game in the experiment, you can see that this game has a non-uniform probability model. However, calculating the theoretical probabilities of each outcome is difficult without knowing them all. One method to determine the probabilities of the outcomes is to make a list of all the possible outcomes.

1. How many different outcomes are there when rolling two 6-sided number cubes?

   There are 36 different outcomes.

When there are a large number of possible outcomes, an array like the one you used before is useful in organizing the outcomes. The entries in the array should be related to the experiment.

2. The array shown has two numbers filled in: 2 and 7.

		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	2					
	2						
	3						
	4			7			
	5						
	6						

680 MODULE 4 • TOPIC 2 • LESSON 1

a. What does the 2 represent in the array?

Two is the sum of 1 showing on the first number cube and 1 showing on the second number cube.

b. What does the 7 represent in the array?

Seven is the sum of 3 showing on the first number cube and 4 showing on the second number cube.

3. Complete the array.

		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

4. How many different outcomes are in the number array?

There are 36 outcomes.

5. Does it appear that the list of all the outcomes when rolling 2 number cubes has the same number of outcomes as the number array?

Yes. The number array and the list result in the same number of outcomes: 36 outcomes.

Questions 1–13 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice using arrays to determine theoretical probability, assign Skills Practice Set B for this lesson.

Ask Yourself . . .
How can you organize and record your mathematical ideas?

MODULE 4 • TOPIC 2 • LESSON 1 681

6. Using the array, determine the number of times each sum appears and the theoretical probability, in fraction form, of each sum. Record your answers in the table.

Outcome	Number of Times	Probability
2	1	$\frac{1}{36}$
3	2	$\frac{2}{36}$
4	3	$\frac{3}{36}$
5	4	$\frac{4}{36}$
6	5	$\frac{5}{36}$
7	6	$\frac{6}{36}$
8	5	$\frac{5}{36}$
9	4	$\frac{4}{36}$
10	3	$\frac{3}{36}$
11	2	$\frac{2}{36}$
12	1	$\frac{1}{36}$

7. Is there an equally likely chance for each outcome to occur from rolling 2 number cubes according to the theoretical probabilities?

   No. Some outcomes have a greater theoretical chance of occurring than others.

8. According to the theoretical probabilities, who should win the game? Explain your reasoning.

   The game should be a tie. The $P(\text{odd sum}) = \frac{18}{36} = \frac{1}{2}$, so $P(\text{even sum}) = \frac{18}{36} = \frac{1}{2}$.

682 MODULE 4 • TOPIC 2 • LESSON 1

9. Did the experimental probability of your experiment in the previous activity match the theoretical probability of the game? If not, why do you think the results of the experimental and theoretical probabilities were different?

Sample answer:

No. The experimental probability of my experiment and the theoretical probability did not match. When someone does an experiment, each outcome varies, so outcomes probably will not occur exactly the number of times the theoretical probability predicts it should.

10. Calculate the percent error between the experimental and theoretical probabilities of Ricardo winning the game.

Sample answer:

Based on the results in Activity 1.1, the percent error is 12%.

11. Calculate each probability when rolling 2 number cubes and summing the resulting numbers. Explain your calculations.

a. P(prime number)

$\frac{15}{36}$

b. P(greater than 7)

$\frac{15}{36}$

c. P(1)

0

12. If the number cubes are tossed 180 times, how many times do you predict the following sums would occur?

a. 1

0 times

b. 4

15 times

c. 9

20 times

d. 10

15 times

e. 12

5 times

f. prime number

75 times

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- See the Facilitating Productive Struggle section in the Course and Implementation Guide. for additional guidance on supporting students through problem-solving activities.

13. Catalina claims that the probability of getting a sum of 4 when rolling two number cubes should be 1 out of 11 since there are 11 possible outcomes. Is Catalina correct? If not, how could you convince Catalina that her thinking is incorrect?

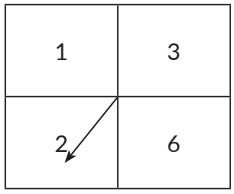
Catalina claim is not correct. Sample explanation: I would explain to Catalina that even though there are 11 outcomes, not all of the outcomes have the same chance of occurring. I would show her that a sum of 4 can occur multiple ways, such as rolling a 2 and a 2, rolling a 1 and a 3, and rolling a 3 and a 1. Therefore, there is more than a 1 out of 11 chance of rolling a 4.

684 MODULE 4 • TOPIC 2 • LESSON 1

ACTIVITY
1.3 Probabilities of Products

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.



The square spinner shown is spun twice, and the results of the two spins are multiplied together to produce a product that is recorded.

1. Determine all the outcomes for obtaining the products using a list.

   (1, 1), (1, 2), (1, 3), (1, 6), (2, 1), (2, 2), (2, 3), (2, 6), (3, 1), (3, 2), (3, 3), (3, 6), (6, 1), (6, 2), (6, 3), (6, 6)

2. Complete the array to determine the possible products.

		Spin 1			
		1	2	3	6
Spin 2	1	1	2	3	6
	2	2	4	6	12
	3	3	6	9	18
	6	6	12	18	36

3. List the sample space.

   S = {1, 2, 3, 4, 6, 9, 12, 18, 36}

4. Are all outcomes equally likely? Explain your calculations.

   All outcomes are not equally likely. There is only one way to get some of the products, and there is more than one way to get other products.

MODULE 4 • TOPIC 2 • LESSON 1 685

5. Calculate the probability of each product. Record your answers in the table.

Outcome	Probability
1	$\frac{1}{16}$
2	$\frac{2}{16}$
3	$\frac{2}{16}$
4	$\frac{1}{16}$
6	$\frac{4}{16}$
9	$\frac{1}{16}$
12	$\frac{2}{16}$
18	$\frac{2}{16}$
36	$\frac{1}{16}$

6. Did you calculate the experimental or theoretical probability? Explain how you know.

   I calculated the theoretical probability because I did the mathematical calculation, not an experiment to determine the probabilities.

Remember, a perfect square is the product of a factor multiplied by itself!

7. Calculate each probability shown for the experiment with the square spinner.

   a. *P*(even product)

   $\frac{12}{16} = \frac{3}{4}$

   b. *P*(odd product)

   $\frac{4}{16} = \frac{1}{4}$

   c. *P*(spin results in a multiple of 3)

   $\frac{12}{16} = \frac{3}{4}$

686 MODULE 4 • TOPIC 2 • LESSON 1

d. *P*(perfect square)

$\frac{4}{16} = \frac{1}{4}$

e. *P*(less than 50)

1

Ask Yourself . . .
Did you make a plan to solve the problem?

8. If this experiment is conducted 200 times, how many times do you predict you would get each product?

a. 2

25 times

b. 3

25 times

c. 6

50 times

d. 36

12 or 13 times

e. An even product

150 times

f. An odd product

50 times

Question 8 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice predicting outcomes using theoretical probability, assign Skills Practice Set C for this lesson.

MODULE 4 • TOPIC 2 • LESSON 1 687

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Optimizing Learning

This activity provides options that supports students to highlight patterns, critical features, big ideas, and relationships.

Talk the Talk

Tossing More Coins

Think back to the experiment you conducted in the *Tossing Coins* activity. You listed the sample space and made an array of outcomes for tossing two coins.

Suppose you are now interested in the number of heads when you toss two coins.

1. Create an array to display the sample space.

		First Coin	
		Heads	Tails
Second Coin	Heads	2	1
	Tails	1	0

2. List the sample space for the number of heads when you toss two coins.

   $S = \{0, 1, 2\}$

3. Using the array, determine the theoretical probability of each outcome. Record your answers in the table.

Outcome	Probability
0	$\frac{1}{4}$
1	$\frac{2}{4}$
2	$\frac{1}{4}$

688 MODULE 4 • TOPIC 2 • LESSON 1

4. Determine the probability of tossing at least 1 head.

$\frac{3}{4}$

5. If you were to conduct 100 trials of the experiment, how many times would you expect to toss 2 heads?

25 times

MODULE 4 • TOPIC 2 • LESSON 1 689

Lesson 1 Assignment

Write

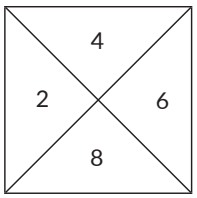
Describe the advantages of using an array to determine the sample space for an experiment.

Remember

To organize the outcomes in a number array, list the outcomes for one trial along one side and the outcomes for the other trial along the other side of an array. Combine the results in the intersections of each row and column.

Practice

1. Linh received a dart board for his birthday. The rule book says that two darts are to be thrown and that an individual's score is the sum of the two numbers.



   a. Complete an array to determine all the outcomes for obtaining the sums.

	Dart 1				
Dart 2	+	2	4	6	8
	2	4	6	8	10
	4	6	8	10	12
	6	8	10	12	14
	8	10	12	14	16

   b. List the sample space for the sum of the numbers.

   S = {4, 6, 8, 10, 12, 14, 16}

   c. Are all outcomes equally likely? Explain your reasoning.

   No. There is only one way to get some of the sums, and there is more than one way to get some of the other sums.

Write

Sample answer:

An array helps organize outcomes.

Lesson 1 Assignment

d. How many outcomes are in the array?

There 16 outcomes.

e. Use the array to help create a probability model listing the theoretical probabilities for each sum.

Outcome	Probability
4	$\frac{1}{16}$
6	$\frac{2}{16}$
8	$\frac{3}{16}$
10	$\frac{4}{16}$
12	$\frac{3}{16}$
14	$\frac{2}{16}$
16	$\frac{1}{16}$

f. Calculate each probability of an even sum, a sum greater than 8, and an odd sum.

$P(\text{sum even}) = 1$;
$P(\text{sum greater than } 8) = \frac{5}{8}$;
$P(\text{odd sum}) = 0$

g. If two darts are thrown 80 times, how many times do you predict each of the following sums would occur? 8? 10? 14?

The sum of 8 would occur 15 times. The sum of 10 would occur 20 times. The sum of 14 would occur 10 times.

692 MODULE 4 • TOPIC 2 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

2. Jackson writes the numbers 1, 2, and 3 on papers and puts them in a bag. He chooses one paper without looking, writes the number down, returns it to the bag, and chooses another paper without looking. He calculates the absolute value of the difference between the two numbers.

   a. Complete an array to determine all the outcomes for obtaining the differences.

		First Pick		
		1	2	3
Second Pick	1	0	1	2
	2	1	0	1
	3	2	1	0

   b. List the sample space for the difference of the numbers.

   S = {0, 1, 2}

   c. Are all outcomes equally likely? Explain your reasoning.

   All outcomes are not equally likely.
   Each difference occurs a different number of times.

   d. How many outcomes are in the array?

   There are 9 outcomes.

MODULE 4 • TOPIC 2 • LESSON 1 ASSIGNMENT 693

Lesson 1 Assignment

e. Use the array to help create a probability model listing the theoretical probabilities for each difference.

Outcome	0	1	2
Probability	$\frac{3}{9} = \frac{1}{3}$	$\frac{4}{9}$	$\frac{2}{9}$

f. If Jackson repeats this experiment 100 times, how many times do you predict he would get a difference of 2?

I would expect Jackson to get a difference of 2 around 22 times.

Prepare

Lauren's treat bag has 5 chocolate bars, 7 peanut butter cups, and 8 sour gummies. If she selects 1 piece of candy at random, determine each probability.

1. *P*(chocolate bar)

   $\frac{1}{4}$

2. *P*(peanut butter cups)

   $\frac{7}{20}$

3. *P*(sour gummy)

   $\frac{2}{5}$

4. *P*(not sour gummy)

   $\frac{3}{5}$

694 MODULE 4 • TOPIC 2 • LESSON 1 ASSIGNMENT

2 Using Tree Diagrams

MATERIALS

None

LESSON OVERVIEW

Students use experimental data to create a probability model and then construct a second probability model using theoretical probabilities for comparison purposes. Tree diagrams are introduced as another method to illustrate all the possible outcomes in a sample space. Students then analyze a given tree diagram modeling the same situation and create a third probability model. In the second activity, a five-number spinner and a tree diagram are used to generate all possible outcomes to create a probability model and answer related questions. To demonstrate understanding, students create a tree diagram for all possible outcomes of correctly guessing the answers to a three-question true-or-false test. They then use the tree diagram to create a probability model and use the model to determine specified probabilities.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6A represent sample spaces for simple and compound events using lists and tree diagrams.

7.6I determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces.

ELPS

(1) Learning Strategies

The student is expected to:

(F) use accessible language and learn new and essential language in the process.

(2) Listening

The student is expected to:

(E) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

ESSENTIAL IDEAS

- Another method to determine the theoretical probability of an event is to construct a tree diagram.
- A *tree diagram* is a tree-shaped diagram that illustrates the possible outcomes of a given situation.
- A tree diagram shows how each possible outcome of an event affects the probabilities of the other events.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Three Puppies, Three Females 15–20 minutes

ESTABLISH A SITUATION

Students simulate the event of a litter of 3 puppies being all females using coin tosses. They construct a probability model using the results from the simulation.

DEVELOP

Activity 2.1: Introduction to Tree Diagrams 20–25 minutes

PEER WORK ANALYSIS, WORKED EXAMPLE

Student construct a probability model to demonstrate the theoretical probability of the simulation conducted in the Getting Started. Tree diagrams are then introduced as another method for determining theoretical probability. Students analyze a tree diagram of the same situation to create a probability model that matches the probability model created from theoretical probabilities.

DAY 2

Activity 2.2: Using Tree Diagrams to Determine Probabilities 25–30 minutes

REAL-WORLD PROBLEM SOLVING, PEER WORK ANALYSIS

Students consider the possible results from spinning a five-number spinner twice and computing the product of the two numbers. They construct a tree diagram to determine all the possible outcomes and list the product of each outcome. Using the tree diagram, students construct a probability model. Using the probability model, they calculate the probability of several events and answer related questions about complementary events.

DEMONSTRATE

Talk the Talk: True-False Trees 10–15 minutes

EXIT TICKET APPLICATION

Students create a tree diagram for all possible outcomes of correctly guessing the answers to a three-question true-or-false test. They then use the tree diagram to create a probability model, and they use the model to determine specified probabilities.

Getting Started

ENGAGE

Three Puppies, Three Females

Facilitation Notes

In this activity, coin tosses are used to simulate the birth of three females in a litter of three puppies. A probability model is constructed using the results from the simulation.

Ask a student to read the information. Complete Question 1 as a class and discuss the different ways to simulate this event.

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Coins

There are many ways to simulate this event. You can allow each group to choose their own method or decide as a class to all use the same method. One way to simulate the event of a litter of puppies being comprised of three females is to use 3 coin flips; let heads represent a female, and let tails represent a male.

Have students work with a partner or in a group to complete Questions 2 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What other animals have more than one offspring at birth? • Why do you think it is appropriate to use $\frac{1}{2}$ as the theoretical probability?
Probing	• Which model did you select for your simulation? Explain how your model has the same probability of the event. • How will you represent the birth of a female puppy using your model? • How could you simulate this situation with a number cube? A spinner? • How did you use your simulation results to create the probability model? • How does your probability model compare to your classmates' probability models? Why do you think that is the case? • What is the probability that the litter would contain 2 males? How can you tell?

Seeing structure	• Why are the probabilities lower for 0 and 3 females? • Why do you think the probabilities are relatively close for 1 female and 2 females?

Summary

A simulation, such as a coin toss, can be used as a probability model. This model can be used to calculate experimental probabilities.

ACTIVITY 2.1 Introduction to Tree Diagrams

DEVELOP

Facilitation Notes

Student construct a probability model to demonstrate the theoretical probability of the simulation conducted in the Getting Started. Tree diagrams are introduced as another method for determining theoretical probability. Students use the tree diagram to create a probability model that matches the probability model created from theoretical probabilities.

Ask a student to read the introduction aloud. Then, discuss the information as a class. Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What should Nakota compare to write his ratio? • How did you organize your work to be sure you listed all of the probabilities?
Gathering	• What is the difference between the MFF outcomes and the FFM outcome? • How does this theoretical probability model compare to your experimental probability model in the Getting Started?
Seeing structure	• How could you scale up those probabilities to compare them to the experimental probabilities?

CC BY NC

Ask a student to read the information and definition aloud. Discuss the Worked Example as a class. Have students work with a partner or in a group to complete Questions 6 through 12. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why do you think it's called a *tree diagram*? • How do you know the number of events represented in a tree diagram? • What is the advantage of using a tree diagram? • When do you think it is inefficient to use a tree diagram? • Why don't you want to use an array to generate the list of outcomes for this situation?
Seeing structure	• Explain how to determine the possible outcomes from the branches of the tree diagram.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

A tree diagram is a visual representation used to organize outcomes that involve more than one event.

ACTIVITY 2.2

Using Tree Diagrams to Determine Probabilities

Facilitation Notes

In this activity, students consider a five-number spinner and the possible results if it is spun twice and they calculate the product. Students create a tree diagram to determine all the possible outcomes. Using the tree diagram, they construct a probability model. Using the probability model, they calculate the probability of several events and answer related questions about complementary events.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the sample space for this experiment?
Probing	• Why is the denominator 25 for your probabilities? • Why does a product of 4 have the highest probability? • How did you use the results in the probability model to answer this question? • What is another way to solve Question 3 part (d)?

COMMON MISCONCEPTION

Students may want to rewrite the probabilities as fractions in simplest form. Encourage them to keep the denominator of the fraction as the total number of outcomes.

Have students work with a partner or in a group to complete Questions 4 through 7. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are *complementary events*?
Probing	• How could you rewrite the probabilities so that they are complementary? • What is another example of complementary events for this situation?

Summary

A tree diagram allows you to see all the possible outcomes of an event and calculate probability. The sum of the probabilities of two complementary events is equal to one.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

TRUE-FALSE TREES

Facilitation Notes

In this activity, students create a tree diagram for all possible outcomes of correctly guessing the answers to a three-question true-or-false test. They then use the tree diagram to create a probability model and use the model to determine specified probabilities.

CC BY NC

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

Give students a three-question true-false quiz to make sense of the situation.

- The quiz should consist merely of guessing true or false.
- Have students check their quiz and score their work.
- If time permits, collect the data and calculate the experimental probability to compare to Question 2.

Just in Time Support

Students may be uncertain how to transfer the tree diagram correct-incorrect results to the outcome row of the probability model. Remind them to list the number of correct answers at the end of each tree branch, similar to what they did in Activity 2. Those values are the possible outcomes or sample space.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Select one branch of your tree diagram and explain what it represents. • How do you determine the sample space from your tree diagram? • How can you check that your probability model is correct?
Seeing structure	• What is the probability of getting 1 or 2 questions wrong? How can you tell?
Reflecting and justifying	• How did you use your tree diagram to identify the desired outcomes for each probability?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Tree diagrams can be used to create a probability model. A probability model can be used to determine theoretical probabilities.

2 Using Tree Diagrams

Setting the Stage

- Communicate the objectives and new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Develop a probability model and use it to determine probabilities.
- Construct a tree diagram to determine the theoretical probability of an event.
- Construct and interpret a non-uniform probability model.

NEW KEY TERM

- tree diagram

Organized lists and arrays are two strategies for analyzing experiments that have a number of different outcomes.

How can you use tree diagrams to display outcomes and their probability of occuring?

Sample answer:

I can use a tree diagram to organize outcomes that involve more than one event. It allows me to see all the possible outcomes and calculate theoretical probability.

MODULE 4 • TOPIC 2 • LESSON 2 695

Chunking the Activity

- Read and discuss the information.
- Complete Question 1 as a class.
- Group students to complete Questions 2–5.
- Share and summarize.

Getting Started

Three Puppies, Three Females

What is the probability that if a dog has a litter of 3 puppies, those 3 puppies are females? Let's say that the theoretical probability of a female being born is equal to the theoretical probability of a male being born, which is $\frac{1}{2}$.

Let's simulate the event of a litter of 3 puppies being 3 females.

1. Choose an appropriate model to simulate the probability of a litter of 3 puppies being all females. Explain how you will represent females and males in your model.

   Sample answer:

   I can use 3 coin flips to simulate the event of a litter of puppies being 3 females. Let heads represent a female, and let tails represent a male.

2. Conduct 25 trials of the simulation. Record the results in the table shown.

   Sample answer:

Trial	Results
1	HTH
2	TTH
3	TTT
4	HHT
5	TTT
6	THT
7	TTH
8	HTH
9	HTH

Trial	Results
10	TTH
11	THH
12	HTT
13	THT
14	HHH
15	HTT
16	HTT
17	TTH
18	THT

Trial	Results
19	HTH
20	HTT
21	HTH
22	HHT
23	TTT
24	HHT
25	THH

EB STUDENT TIP

For "Beginning" and "Intermediate" proficiency levels

A *litter* of puppies refers to the number of puppies born at the same time by the same female dog.

696 MODULE 4 • TOPIC 2 • LESSON 2

3. List all possible outcomes for the number of females among a litter of 3 puppies.

   {0, 1, 2, 3}

4. Use the results from your simulation to construct a probability model.

Outcome	0 females	1 female	2 females	3 females
Probability	$\frac{3}{25}$	$\frac{10}{25}$	$\frac{11}{25}$	$\frac{1}{25}$

5. What is the experimental probability that a litter of 3 puppies is all females according to your probability model?

   Sample answer:

   According to the model, the probability is $\frac{1}{25}$.

MODULE 4 • TOPIC 2 • LESSON 2 697

Chunking the Activity

- Read and discuss the directions.
- Group students to complete Questions 1–5.
- Check in and share.
- Read and discuss the information and definition.
- Read and discuss the Worked Example.
- Group students to complete Questions 6–12.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Questions 1–12 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice completing probability models to determine theoretical probabilities, assign Skills Practice Set A for this lesson.

ACTIVITY 2.1

Introduction to Tree Diagrams

In the previous simulation, your probability model was based on experimental probabilities. In some cases, this is the only method of constructing a probability model. For example, in an earlier lesson, you determined the experimental probabilities of a cup landing on its top, bottom, or side when it was tossed. It would be difficult or impossible to determine the theoretical probabilities for the cup toss. However, it is possible to determine the theoretical probability for a litter of 3 puppies being 3 females.

Ask Yourself . . .
Did you justify your mathematical reasoning?

One method to calculate the theoretical probability for a litter of puppies being 3 females is to list all of the possible outcomes for a litter of 3 puppies. You can then determine how many of those outcomes include 3 females.

1. Nakota says, "I think that the probability of a litter of puppies being 3 females is 1 out of 3 because there is only one outcome that has all 3 puppies being female. There are only two other outcomes."

   Joey says, "I don't think that's correct. I think the probability is much lower since there are many combinations of males and females in a litter of three puppies."

   Who's correct? Explain your reasoning.

   Joey is correct.

   Nakota is not correct. There are more than 3 total outcomes. For example, there could be 2 females and 1 male, but the first and last puppies could be females and the middle puppy a male, or the first 2 puppies could be females and the last puppy a male.

698 MODULE 4 • TOPIC 2 • LESSON 2

2. List all of the possible outcomes for having 3 puppies in a litter, using *F* to represent females and *M* to represent males.

   (FFF), (FFM), (FMF), (MFF), (FMM), (MFM), (MMF), (MMM)

3. What does the outcome MFF represent?

   MFF represents the litter being a male as the first puppy, a female as the second puppy, and a female as the third puppy.

4. Complete the probability model using all possible outcomes.

Outcome	0 females	1 female	2 females	3 females
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

5. What is the theoretical probability that a litter of 3 puppies is comprised of 3 females?

   $\frac{1}{8}$

MODULE 4 • TOPIC 2 • LESSON 2 699

STAMP THE LEARNING

The definition and Worked Example provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Optimizing Learning

This activity offers alternatives for visual information.

A tree diagram has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

Another method of determining the theoretical probability of an event is to construct a *tree diagram*. A **tree diagram** illustrates the possible outcomes of a given situation. Tree diagrams can be constructed vertically or horizontally.

You can construct a tree diagram to show all the possible outcomes for a litter of 3 puppies.

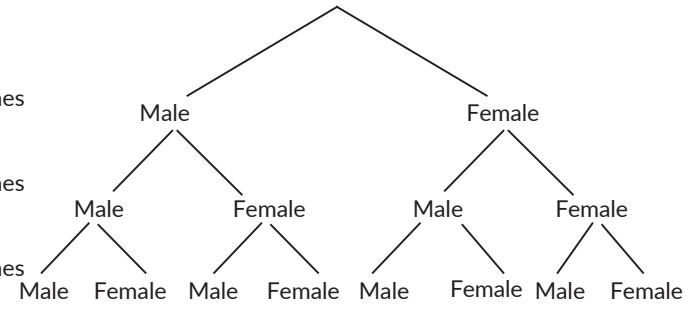
WORKED EXAMPLE

What are the possible outcomes for a litter of 3 puppies?

List the possible outcomes for the 1st puppy.

List the possible outcomes for the 2nd puppy.

List the possible outcomes for the 3rd puppy.



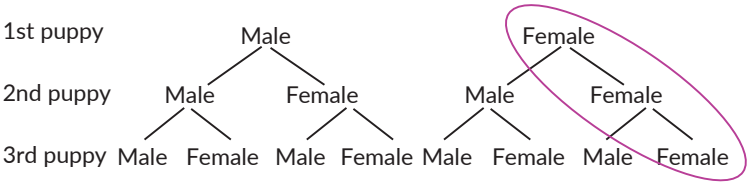
6. How would this tree diagram change if you were trying to determine the possible outcomes for a litter of 4 puppies?

   The tree diagram would need to be extended to include a fourth row.

7. How does the tree diagram in the Worked Example compare to the list you made in Question 2?

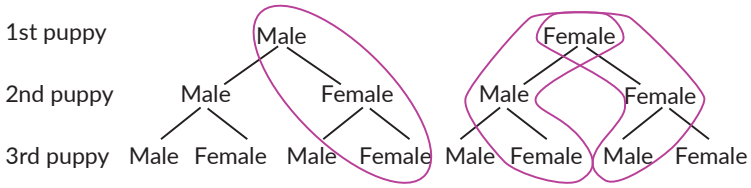
   Both show the same possible outcomes.

8. Circle the outcome(s) of a litter of three puppies that are all females on the tree diagrams shown.

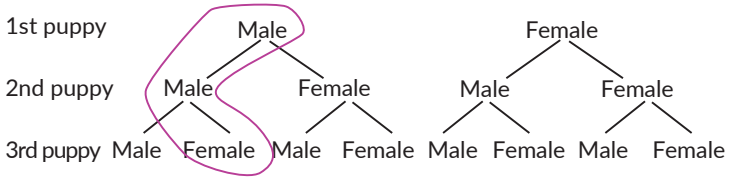


700 MODULE 4 • TOPIC 2 • LESSON 2

9. Circle the outcome(s) of a litter of three puppies in which two of the puppies are females in the tree diagrams shown.



10. Circle the outcome MMF in the tree diagrams shown.



11. Complete the probability model shown with the information from the tree diagrams.

Outcome	0 females	1 female	2 females	3 females
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

12. Is there a difference in the theoretical probability of each outcome between the list of outcomes you wrote and the tree diagrams you analyzed?

No. The probability of each outcome is the same whether I create a list of all outcomes or use a tree diagram.

MODULE 4 • TOPIC 2 • LESSON 2 701

Chunking the Activity

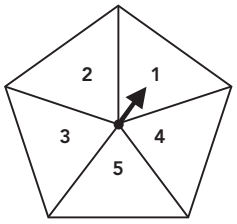
- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Questions 4–7.
- Share and summarize.

Questions 1–7 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice using tree diagrams to determine probabilities, assign Skills Practice Set B for this lesson.

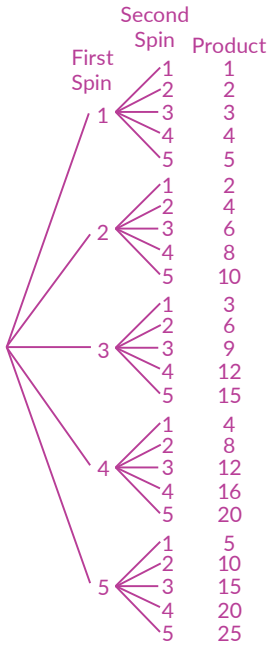
ACTIVITY 2.2

Using Tree Diagrams to Determine Probabilities

The five-number spinner is spun twice and a product is calculated.



1. Construct a tree diagram to determine all the possible outcomes. Then, list the product at the end of each branch of the tree.



Product	Probability
1	$\frac{1}{25}$
2	$\frac{2}{25}$
3	$\frac{2}{25}$
4	$\frac{3}{25}$
5	$\frac{2}{25}$
6	$\frac{2}{25}$
8	$\frac{2}{25}$
9	$\frac{1}{25}$
10	$\frac{2}{25}$
12	$\frac{2}{25}$
15	$\frac{2}{25}$
16	$\frac{1}{25}$
20	$\frac{2}{25}$
25	$\frac{1}{25}$

2. Construct a probability model for spinning the spinner twice and recording the product.

702 MODULE 4 • TOPIC 2 • LESSON 2

3. Use the probability model you created to calculate the probability for each event shown.

   a. $P(10)$
   $\frac{2}{25}$

   b. $P(\text{less than } 10)$
   $\frac{15}{25}$

   c. $P(\text{multiple of } 5)$
   $\frac{9}{25}$

   d. $P(\text{not a multiple of } 5)$
   $\frac{16}{25}$

4. Which events from Question 3 represent complementary events? Explain your reasoning.

   The events of the product being a multiple of 5 and the product not being a multiple of 5 represent complementary events. Complementary events are events that consist of a desired outcome and the remaining events that consist of all the undesired outcomes. Together, they include every possible outcome in the sample space.

5. Alejandra says that the product being less than 10 and the product being more than 10 are complementary events. Hailey disagrees. Who is correct? Explain your reasoning.

   Alejandra is not correct. The product being less than 10 and the product being more than 10 do not include the entire sample space—the product being equivalent to 10 is not included with either event.

6. What event is complementary to the event that the product is an even number? Determine the probability of both events.

   The product being an odd number would be complementary to the event that the product is an even number. $P(\text{product is an even number})$ is $\frac{16}{25}$, and $P(\text{product is an odd number})$ is $\frac{9}{25}$.

MODULE 4 • TOPIC 2 • LESSON 2 703

7. What is the sum of the probabilities of two complementary events? Explain why your answer makes sense.

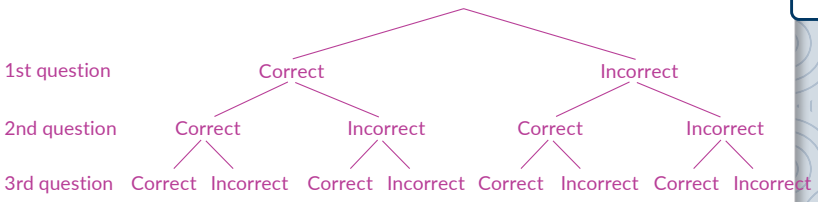
The sum of the probabilities of two complementary events is 1. This makes sense because complementary events together include the entire set of outcomes for an experiment.

704 MODULE 4 • TOPIC 2 • LESSON 2

Talk the Talk

True-False Trees

1. Construct a tree diagram to determine all the possible outcomes for correctly guessing the answers to a three-question true-or-false quiz.



2. Construct a probability model for the tree diagram you completed in Question 1.

Outcome	Probability
3 correct	$\frac{1}{8}$
2 correct	$\frac{3}{8}$
1 correct	$\frac{3}{8}$
0 correct	$\frac{1}{8}$

3. Calculate the probabilities:

   a. $P(3 \text{ questions correct}) = \frac{1}{8}$

   b. $P(1 \text{ or } 2 \text{ questions correct}) = \frac{6}{8}$

   c. $P(0 \text{ questions correct}) = \frac{1}{8}$

Ask Yourself . . .
How can you use tree diagrams in everyday life?

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

MODULE 4 • TOPIC 2 • LESSON 2 705

706 MODULE 4 • TOPIC 2 • LESSON 2

Lesson 2 Assignment

Write

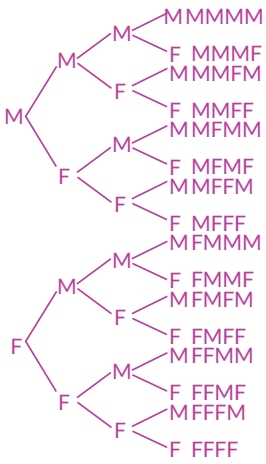
How are tree diagrams useful when constructing probability models?

Remember

A *tree diagram* illustrates the possible outcomes of a given situation.

Practice

1. Brianna's dog gives birth to a litter of 4 puppies. The birth order of the puppies is F, F, M, F. Create a tree diagram to list all the possible birth orders of a litter of 4 puppies. Then, determine the probability of the puppies' birth order.



The probability of the puppies' birth order is $\frac{1}{16}$.

Write

Sample answer:

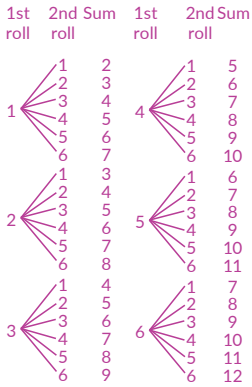
Tree diagrams are useful because I can use the outcomes to determine the theoretical probability of an event.

MODULE 4 • TOPIC 2 • LESSON 2 ASSIGNMENT 707

Lesson 2 Assignment

2. Gracie is learning probability in middle school while her little brother, Nicky, is learning arithmetic in first grade. Gracie uses a six-sided number cube to help Nicky learn how to add one-digit numbers. She rolls two cubes, numbered 1 through 6, and Nicky adds up the two numbers on the faces.

   a. Construct a tree diagram to determine all the possible outcomes. List the sum at the end of each branch of the tree.



   b. Construct a probability model for rolling two 6-sided number cubes and determining the sum of the faces.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

   c. What is the probability that the sum is 7?

   $\frac{1}{6}$

   d. What is the probability that the sum is 11?

   $\frac{1}{18}$

708 MODULE 4 • TOPIC 2 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

e. Calculate the probability that the sum is an even number.

$\frac{1}{2}$

f. Calculate the probability that the sum is greater than 5.

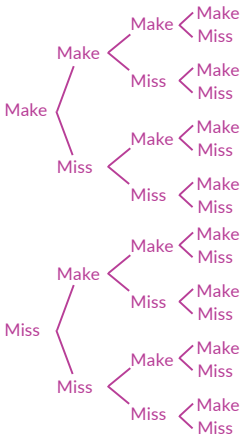
$\frac{13}{18}$

g. What event would be complementary to the event that the sum is greater than 5? Explain your reasoning.

The sum being less than or equal to 5 would be the complementary event.

3. When Gracie and Nicky finish their math homework, they go outside to shoot some hoops. On average, Gracie makes one-half of all of the shots she takes.

a. She shoots the basketball 4 times. Construct a tree diagram for all possible outcomes of the 4 shots.



MODULE 4 • TOPIC 2 • LESSON 2 ASSIGNMENT 709

Lesson 2 Assignment

b. Construct the probability model.

Outcome	Makes All 4 Shots	Makes 3 Shots	Makes 2 Shots	Makes 1 Shots	Makes 0 Shots
Probability	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

c. What is the probability she makes all 4 shots?

$\frac{1}{16}$

d. Calculate the probability she makes 3 or more shots.

$\frac{5}{16}$

e. Calculate the probability she makes 2 or more shots.

$\frac{11}{16}$

Prepare

Miguel has 6 crayons. The crayons are blue, red, green, purple, brown, and yellow. He chooses one crayon at random.

1. What is the probability that the crayon Miguel chooses will be red or purple?

   $P(\text{red or purple}) = \frac{1}{3}$

2. What is the complementary event?

   The complementary event is that Miguel will choose a blue, green, brown, or yellow crayon.

3. What is the probability of the complementary event?

   $P(\text{blue, green, brown, or yellow}) = \frac{2}{3}$

710 MODULE 4 • TOPIC 2 • LESSON 2 ASSIGNMENT

3 Determining Compound Probability

MATERIALS

None

LESSON OVERVIEW

The term *compound event* is defined in this lesson. Within the context of the first situation, students determine the probability for three events by calculating the sum of the probabilities of each event. In the second situation, students calculate and compare the probability of a compound event with the word *and* to the probability of a compound event with the word *or*. They distinguish between the two compound events; they state that the compound event associated with the word *and* means simple events both (or all) occur, while the compound event associated with the word *or* means any combination of one or more simple events occurring. When calculating the probability of a compound event, students learn not to count repeated outcomes if the same outcome appears in more than one simple event.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1F analyze mathematical relationships to connect and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6A represent sample spaces for simple and compound events using lists and tree diagrams.

7.6D make predictions and determine solutions using theoretical probability for simple and compound events.

ELPS

(1) Learning Strategies

The student is expected to:

(A) use prior knowledge and experiences to understand meanings in English.

(2) Listening

The student is expected to:

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

ESSENTIAL IDEAS

- A compound event combines two or more events, using the word *and* or the word *or*.
- The probability of a compound event with the word *and* is the probability of two or more events occurring at the same time.
- The probability of a compound event with the word *or* is the probability of one or more of the named simple events occurring.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: And Or What?! 10–15 minutes

ESTABLISH A SITUATION

Students will group themselves according to a number of different categories by sitting or standing. Through this activity, they learn the difference between the terms *or* and *and* as they prepare to calculate compound probabilities.

DEVELOP

Activity 3.1: Compound Events 25–30 minutes

REAL-WORLD PROBLEM SOLVING, PEER WORK ANALYSIS

A pet shelter has five different kinds of pets available for adoption. Students create a non-uniform probability model for adopting a pet and use the model to answer related questions. Examples of student work for calculating a compound event with the word *or* are provided, and questions related to the different solutions are posed. The term *compound event* is defined, and the word *or* is distinguished from the word *and* with regard to one or two events. A compound event that includes three events is given, and students determine the probability of the event occurring.

DAY 2

Activity 3.2: Tree Diagrams and Compound Events 25–30 minutes

REAL-WORLD PROBLEM SOLVING

Students use a tree diagram to list outcomes of a compound event. They draw cards from two piles: the first pile contains three cards, and the second pile contains four cards. They use a tree diagram or an organized list to answer questions related to the problem situation.

DEMONSTRATE

Talk the Talk: Evens and Odds 10–15 minutes

EXIT TICKET APPLICATION

A six-sided number cube and a four-number spinner are used in a game. Rules for winning and losing the game are based on compound probabilities. Students list possible outcomes for playing the game. They use the set of possible outcomes to answer questions related to the game and determine that the game is fair.

CC BY NC

Getting Started

ENGAGE

And Or What?!

Facilitation Notes

In this activity, students answer questions by either standing up or remaining seated. The purpose of this activity is for students to learn the difference between the terms *or* and *and* as they prepare to calculate compound probabilities.

Have the students stand to answer "yes" or remain seated to answer "no" to Questions 1 through 3. After each question, they should record the number of students in the class, including themselves if appropriate, who are standing.

COMMON MISCONCEPTION

Students may misinterpret the statement cat or dog owners and think it pertains to students who own only cats or only dogs. Clarify that this statement includes owners of cats, dogs, or both.

DIFFERENTIATION STRATEGIES

Access for All

When completing parts (a) and (b), if no student owns both a cat and dog, include yourself as an owner of both a dog and cat. These data will create a richer discussion and help to compare the situations in Questions 1 and 2. Students will encounter a similar overlap situation in Activity 2.

Challenge Opportunity

Have students investigate the relationship among responses to part (a) through part (d) in both questions. Then, have them consider why this relationship occurs.

Conclusion: part (a) response + part (b) response = part (c) response + part (d) response.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• In Question 1, how does your answer to part (c) compare to your answer to parts (a) and (b)? Why is that the case? • When will the number of cat or dog owners equal the sum of the cat owners and the dog owners? • Why is the number of cat or dog owners less than the sum of the cat owners and the dog owners? • Which questions resulted in no students standing? Why do you suppose this happened?

Seeing structure	• In Question 2, how does your answer to part (c) compare to your answers to parts (a) and (b)? Why will that always be the case? • Why is the response to part (d) always 0?

Have students work with a partner or in a group to answer Question 4. Share responses as a class. Ask a student to read aloud the margin note and the information following the question. Discuss as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• How are the structures of the situations in Questions 1 and 2 different? • Explain the difference between *or* and *and* statements.

Summary

The probability of *A* or *B* occurring is different from the probability of *A* and *B* occurring.

STAMP THE LEARNING

ACTIVITY 3.1

Compound Events

DEVELOP

Facilitation Notes

In this activity, a pet shelter has five different kinds of pets available for adoption. Students create a non-uniform probability model for adopting a pet and use the model to answer related questions. Examples of student work for calculating a compound event with the word *or* are provided, and questions related to the different solutions are posed. The term *compound event* is defined, and the word *or* is distinguished from the word *and* with regard to one or two events. A compound event that includes three events is given, and students determine the probability of the event occurring.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain how you determined each probability. • What is the probability the next pet adopted is not a dog? How did you determine your answer?
Seeing structure	• What is another way to solve Question 5 part (c)?

CC BY NC

Have students work with a partner or in a group to analyze the peer work analysis and complete Question 6. Share responses as a class.

QUESTION TO SUPPORT DISCOURSE

Seeing structure	• Did any of the students solve the problem the same way you did?

Ask a student to read the information and definition following Question 6 aloud. Discuss the information as a class. Have students work with a partner or in a group to complete Question 7. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you know whether to use *and* or *or* in your statement? • What method did you use to solve this problem? • What is another way to solve this problem?

COMMON MISCONCEPTION

Students may have difficulty understanding the difference between "and" and "or". Keep referring to the situations from the getting started to help students make sense of the difference between these words.

Summary

A compound event combines two or more events, using the word *and* or the word *or*. A compound event with the word *and* means that you are determining the probability that both events occur. A compound event with the word *or* means that you are determining the probability that one or the other or both events occur.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

ACTIVITY 3.2 Tree Diagrams and Compound Events

Facilitation Notes

In this activity, two piles of numbered cards are used in the scenario. Students use a tree diagram or an organized list to list outcomes of a compound event. The diagram or list is then used to answer questions related to the problem situation.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• If you created an organized list, explain your process. • For those who created a tree diagram, how many branches did you include? What do those branches represent? • Why should you count (2, 3) and (3, 2) as different outcomes? • How did you know whether to use *and* or *or* in your statement? • How did you determine the probability? What is another way? • What outcomes result in a sum of 5? • Why didn't you list (4, 1) as an outcome?
Gathering	• Does selecting one card from each pile describe one or two events? • If the same outcome occurs in more than one event, what do you do? • How many outcomes occur more than once? • How can you tell when an outcome occurs more than once? • What does it mean when an outcome occurs more than once?
Probing	• How is Question 7 different from the previous ones in this activity? • Explain your solution path to solve this probability question with *and*.
Seeing structure	• What is meant by *unique outcomes*? • Why are there more outcomes for Question 8 than Question 7? • What are things to look for when solving probability questions with *or*? • What do you notice about the outcomes for the and statement in Question 7 part (b) and the outcomes listed in both events for Question 8 part (c)? Do you think this will happen all the time? Why or why not?

Summary

Two strategies for determining the outcomes and probabilities of a compound event include using organized lists or tree diagrams, which can be used to make predictions about future events.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

EVENS AND ODDS

Facilitation Notes

In this activity, a number cube and a spinner are used in the scenario. Rules for winning and losing a game are based on compound probabilities. Students list the possible outcomes for playing the game, use that list to answer related questions, and determine whether or not the game is fair.

Have students complete Questions 1 through 5 individually. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Suggest students draw a diagram and create an organized list or construct a tree diagram before placing the outcomes in the appropriate row in the table.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What strategy did you use to list the possible outcomes? • Is this a fair game? Why or why not? • What outcomes applied to each event? • How did you calculate each compound probability?
Seeing structure	• How could you change the rules of the game so that you are more likely to win?

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Games often include compound events. Organized lists, tree diagrams, and probability models are all tools to help determine possible outcomes and probabilities as well as evaluating whether or not a game is fair.

3 Determining Compound Probability

Setting the Stage

- Communicate the objectives and new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Calculate probabilities of compound events using tables.
- Represent the sample space for compound events and calculate probabilities of compound events using tree diagrams.

NEW KEY TERM

- compound event

You know how to calculate the probabilities of simple events. You have used organized lists, arrays, and tree diagrams to calculate the probabilities of more complicated events.

How do you define a probability event that combines multiple simple events? How can you calculate the probability of these events?

Sample answer:

A compound event combines two or more events using the word *and* or the word *or*. I can use the same methods used to calculate the probabilities of simple events to calculate the probabilities of compound events.

MODULE 4 • TOPIC 2 • LESSON 3 711

Chunking the Activity

- Read and discuss the introduction.
- Complete Question 1–3 as a class.
- Group students to complete Question 4.
- Read and discuss the margin note and information.
- Share and summarize.
- Read and discuss the margin note and information.

Optimizing Learning

This activity fosters collaboration and community.

Getting Started

And Or What?!

What's the difference between the words *and* and *or*? You've been using them your whole life, so you can imagine that you are pretty sure of the difference. However, these words have a very specific meaning in mathematics, and it is important to understand the difference.

For each situation, identify how many of your classmates are in each category. Answers will vary.

1. Cat owners vs. dog owners
   a. Cat owners
   b. Dog owners
   c. Cat or dog owners
   d. Cat and dog owners
2. Even birth dates vs. odd birth dates
   a. Even birthdays
   b. Odd birthdays
   c. Even or odd birthdays
   d. Even and odd birthdays
3. Athletes vs. musicians
   a. Athletes
   b. Musicians
   c. Athletes or musicians
   d. Athletes and musicians

712 MODULE 4 • TOPIC 2 • LESSON 3

What is the difference between *and* and *or*?

4. If you know the answers to parts (a) and (b) from Questions 1–3, can you determine the answers to the other two situations? Explain why or why not.

   a. Part (c)?

      Knowing the answers to parts (a) and (b) will tell me the answer to part (c). Anyone who stood up for part (a) or part (b) will stand up for part (c).

   b. Part (d)?

      For Questions 1 and 3, I cannot determine part (d) based on parts (a) and (b).

      For Question 2, no one can be standing for part (d) because a birth date cannot be both even and odd.

You can calculate the probability of multiple events, say events *A* and *B*. Sometimes, you will want to know the probability of *A* or *B* occurring. Other times, you will want to know the probability of *A* and *B* occurring.

The word *or* is used to identify a union—the set of items that are in either set or both. The word *and* is used to identify an intersection—the set of items that are in both sets.

STAMP THE LEARNING

The margin note provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

MODULE 4 • TOPIC 2 • LESSON 3 713

Chunking the Activity

- Read and discuss the situation.
- Group students to complete Questions 1–5.
- Check in and share.
- Group students to analyze the peer work analysis and complete Question 6.
- Check in and share.
- Read and discuss the information and definition.
- Group students to complete Question 7.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 3.1

Compound Events

A pet shelter has the following animals available for adoption. You will complete a probability model and use it to determine the probability that the next pet chosen at random is a dog or a cat.

Pets	Number Available
Cat	4
Dog	7
Snake	1
Rabbit	3
Bird	5

1. How many pets are available for adoption?
   There are 20 pets available for adoption.

2. Complete the probability model for adopting a pet from a pet shelter, at random.

Outcome	Cat	Dog	Snake	Rabbit	Bird
Probability	$\frac{4}{20}$	$\frac{7}{20}$	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{5}{20}$

3. Is the model a uniform or a non-uniform probability model? Explain your reasoning.
   This model is a non-uniform probability model because the possible outcomes are not equally likely.

4. What is the probability the next pet adopted is:

   a. a dog?
   $P(\text{dog}) = \frac{7}{20}$

   b. a snake?
   $P(\text{snake}) = \frac{1}{20}$

   c. a cat?
   $P(\text{cat}) = \frac{4}{20}$

714 MODULE 4 • TOPIC 2 • LESSON 3

5. Determine the probability that the next pet adopted is a dog or a cat.

   a. How many of the pets are dogs or cats?

      11

   b. How many total number of pets are there?

      20

   c. What is the probability the next pet adopted at random is a dog or a cat?

      $P(\text{dog or cat}) = \frac{11}{20}$

Examples of students' methods to solve Question 5 are shown.

Nia

I added the number of cats and dogs and got a total of 11. 4 + 7 = 11
That's 11 pets out of 20.
$\frac{\text{desired}}{\text{total}} = \frac{11}{20}$

Xavier

Don't want 9 out of 20. So, what's left is 11 out of 20.

Ben

CATS $\frac{4}{20}$

DOGS $+\frac{7}{20}$

$\frac{11}{20}$

Antonio

$\frac{2}{5}$
Cats and dogs
That's 2 animals listed out of 5

Omar

Probability of a cat $\frac{4}{20}$ or $\frac{1}{5}$
Probability of a dog $\frac{7}{20}$
$\frac{1}{5} + \frac{7}{20} = \frac{8}{20}$

MODULE 4 • TOPIC 2 • LESSON 3 715

Review the definition of **analyze** in the Academic Glossary.

6. Analyze each student's responses to the question.

   a. Describe the strategy Xavier used.

   Xavier used the idea of complementary events. He counted all the pets that did not include cats and dogs. He calculated a total of 9 pets. Then, he realized that out of 20 pets, the 11 remaining had to be cats or dogs. Then, he made a fraction: 11 cats and dogs out of 20 pets.

   b. Describe the strategy Ben used.

   Ben used the theoretical probability formula twice (once for cats and once for dogs) and then added his results together.

   c. What is the difference between Nia's work and Ben's work?

   Nia first added the number of cats and dogs available for adoption. Then, she wrote a ratio using the theoretical probability formula. Ben first calculated the probability for a cat to be adopted next, and then the probability for a dog to be adopted next. Finally, he added the two fractions together.

   d. Explain to both Antonio and Omar why their methods are incorrect for determining the probability that the next pet adopted will be a cat or dog.

   Antonio needs to count the number of each type of pet, not the types of pets. Omar needs to add the fractions using common denominators.

EB STUDENT TIP

For all proficiency levels

The beginning of the word compound is com- which means with or together and is related to the Spanish word con. When we describe a compound event, we relate two events to each other using the conjunction and or the conjunction or.

Beginning: Demonstrate compound events, using the correct conjunction to narrate them, while contrasting these with single events as you draw differently colored items from a bag. Have students say "single" or "compound" or point to the correct phrase to show their understanding.

(continued on next page)

A **compound event** combines two or more events, using the word *and* or the word *or*.

Determining the probability of a compound event with the word *and* is different from a compound event with *or*.

The difference is that a compound event with the word *and* means that you are determining the probability that both events occur.

When determining a compound event with *or*, you are determining the probability that one, the other, or both outcomes occur.

7. What is the probability that the next pet adopted is a four-legged animal?

   a. What events make up "the next pet adopted being a four-legged animal"?

      The events that make up the next pet adopted being a four-legged animal are adopting a cat, a dog, or a rabbit.

   b. For how many events are you determining the probability? How do you know?

      I am determining the probability for 3 events. I know that I am determining the probability for 3 events because the next adoption could be a dog, or a cat, or a rabbit.

   c. Rewrite Question 7 using the events you wrote in part (a).

      What is the probability the next pet adopted is a cat, or a dog, or a rabbit?

How are compound events in math like compound sentences in grammar?

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP *(continued)*

Intermediate: Offer students this sentence frame: "Drawing a _____ from a bag is a single event, but drawing a _____ first and then _____ second is a compound event because _____. Drawing either _____ or _____ is also a compound event because _____."

Advanced/Advanced High: Ask students to brainstorm everyday examples of compound events, including some with more than two coinciding events. Challenge students to calculate which compound event would be more or less likely and explain their reasoning using precise mathematical language and visuals such as a tree diagram.

d. Determine the probability that the next pet adopted is a four-legged animal. Show how you determined your answer.

$P(\text{next pet adopted is four-legged}) = P(\text{next pet adopted is a cat, or a dog, or a rabbit}) = P(\text{next pet adopted is a cat}) + P(\text{next pet adopted is a dog}) + P(\text{next pet adopted is a rabbit})$

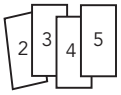
$= \frac{4}{20} + \frac{7}{20} + \frac{3}{20} = \frac{14}{20}$ or $\frac{7}{10}$

718 MODULE 4 • TOPIC 2 • LESSON 3

ACTIVITY 3.2

Tree Diagrams and Compound Events

The cards shown were placed facedown in two piles so that they can be chosen at random.



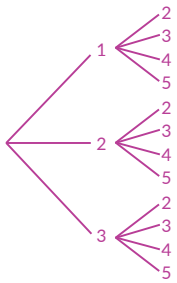
1. If one card is chosen at random from each pile, determine the probability of choosing a matching pair at random.

   a. What are the possible outcomes for choosing one card from each pile at random? Make sure you show your work by either creating an organized list or constructing a tree diagram.

   Organized List

   (1, 2), (1,3), (1,4), (1,5),
   (2, 2), (2,3), (2,4), (2,5),
   (3, 2), (3,3), (3,4), (3,5)

   Tree Diagram



   b. How many possible outcomes are there?

   There are 12 possible outcomes for this event.

   c. What events make up choosing a matching pair at random?

   The events that make up choosing a matching pair at random are choosing a pair of 2s or a pair of 3s.

   d. Rewrite Question 1 using the events you wrote in part (c).

   What is the probability of choosing a pair of 2s or a pair of 3s?

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Activity 3.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice listing and using sample spaces determine compound probabilities with no outcomes in common, assign Skills Practice Set A and/or B for this lesson.

MODULE 4 • TOPIC 2 • LESSON 3 719

e. Determine the probability of choosing a pair at random.

$P(\text{pair}) = \frac{2}{12} = \frac{1}{6}$

2. Determine the probability of choosing two cards at random that have a sum of 5. Show your work.

$P(\text{sum of 5}) = P(1, 4) + P(2, 3) + P(3, 2) = \frac{3}{12} = \frac{1}{4}$

3. The class is asked to determine the probability of choosing 2 odd cards at random.. Lucy says, "The probability of drawing 2 odd cards is 4 out of 7 because there are 7 cards and 4 of them are odd." Do you agree with Lucy's statement? If not, explain to Lucy why her reasoning is not correct.

Sample answer:

No. I do not agree with Lucy. Lucy is thinking of only one event with 7 possible outcomes. Lucy was not thinking about how the cards are in 2 separate piles.

4. Write a problem using the number cards for which Lucy's answer would be correct.

There are 7 number cards as shown. What is the probability of selecting an odd number card at random?

5. What outcomes make up the event of "choosing 2 odd cards" given one card is chosen from each pile?

The outcomes are (1, 3), (1, 5), (3, 3), (3, 5).

6. Determine the probability choosing 2 odd cards at random.

$P(\text{2 odd cards}) = \frac{4}{12} = \frac{1}{3}$

720 MODULE 4 • TOPIC 2 • LESSON 3

7. Determine the probability of selecting one card from each pile where the first card is a 2 and the second card is odd.

   a. List the event(s) for determining the probability.

   There are 2 events that must occur simultaneously. The first card must be a 2, and the second card must be odd.

   b. List the outcome(s) for the event(s).

   The outcomes for the compound event are (2, 3) and (2, 5).

   c. Determine the probability of selecting one card from each pile where the first card is a 2 and the second card is odd.

   $P(\text{1st card is 2 and 2nd card is odd}) = \frac{2}{12} = \frac{1}{6}$

Remember to carefully read the event in the scenario. There is a big difference between determining a compound event with *and*, and a compound event with *or*.

8. Determine the probability that the first card is a 2 or the second card is odd.

   a. List the event(s) for determining the probability.

   There are 2 events. The first event is the first card being a 2. The second event is the second card being odd. One or both of these must occur to be included.

   b. Determine the outcome(s) for the event(s).

   The outcomes for the first event are (2, 2), (2, 3), (2, 4), and (2, 5). The outcomes for the second event are (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), and (3, 5).

   c. How many outcomes are listed? Are any of the outcomes listed in both events?

   There are 10 outcomes listed. The outcomes (2, 3) and (2, 5) are listed in both events.

When there are two or more events for which you are determining the probability, an outcome might occur for both events. When this occurs and you are determining the number of outcomes, count the repeated outcome only once.

STAMP THE LEARNING

The margin note provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

MODULE 4 • TOPIC 2 • LESSON 3 721

d. How many unique outcomes are there for the two events?

There are 8 unique outcomes for the two events.

e. Determine the probability that the first card is a 2 or the second card is odd.

$P(\text{1st card is a 2 or 2nd card is odd}) = \frac{8}{12} = \frac{2}{3}$

f. Why would you not count all of the outcomes when calculating the probability?

Sample answer:

I would not count all 10 outcomes because 2 of the outcomes would be counted twice.

9. Explain the difference between the events in Questions 7 and 8.

In Question 7, for a pair to be counted, it must have a 2 for the first card and an odd number for the second card. In Question 8, the pair can either have a 2 for the first card, or it can have an odd number for the second card (or both a 2 for the first card and an odd number as the second).

722 MODULE 4 • TOPIC 2 • LESSON 3

Talk the Talk

Evens and Odds

You and a friend are playing a game in which you take turns rolling a six-sided number cube and spinning a spinner with 4 congruent sections numbered 1–4.

You and your friend take turns.

- If your friend rolls an odd number and spins an even number, he or she wins.
- If you roll an even number and spin an odd number, you win.

1. List the possible outcomes for playing the game.

   (1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4)

2. Explain why the events of your friend winning or of you winning are compound events.

   Sample answer:

   The event of my friend winning is a compound event because it consists of the outcome of 2 events that must occur: rolling an odd number and spinning an even number. The event of me winning is also a compound event because it consists of the outcome of 2 events that must occur: me rolling an even number and spinning an odd number.

3. List the outcomes for the event of your friend winning the game and for the event of you winning the game. Then, calculate each probability and record in the table.

	Outcomes of the Event	Probability
Your Friend Wins	(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)	$P(F) = \frac{6}{24} = \frac{1}{4}$
You Win	(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)	$P(Y) = \frac{6}{24} = \frac{1}{4}$

Chunking the Activity

- Read and discuss the situation.
- Have students complete the activity individually.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

MODULE 4 • TOPIC 2 • LESSON 3 723

4. Do you and your friend have equally likely chances of winning the game? Explain your reasoning.

   Yes. Both my friend and I have equally likely chances of winning because the probability of the events that result in a win for me or my friend are equivalent to each other.

5. List the outcomes for and determine the probability of each statement.

   a. *P*(even number on the number cube and even number on the spinner)

      The outcomes are (2, 2), (2, 4), (4, 2), (4, 4), (6, 2), and (6, 4).
      $P(E \text{ and } E) = \frac{6}{24} = \frac{1}{4}$

   b. *P*(even number on the number cube or even number on the spinner)

      Outcomes for even number on the number cube:

      (2, 1), (2, 2), (2, 3), (2, 4), (4, 1), (4, 2), (4, 3), (4, 4), (6, 1), (6, 2), (6, 3), (6, 4)

      Outcomes for even number on the spinner:

      (1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4), (4, 2), (4, 4), (5, 2), (5, 4), (6, 2), (6, 4)

      The following outcomes are listed in both events: (2, 2), (2, 4), (4, 2), (4, 4), (6, 2), and (6, 4). I will only count these outcomes once when determining the probability of rolling an even number on the number cube or spinning an even number on the spinner. $P(E \text{ or } E) = \frac{18}{24} = \frac{3}{4}$

724 MODULE 4 • TOPIC 2 • LESSON 3

Lesson 3 Assignment

Write

Explain how to determine the probability of a compound event. Be sure to include both types of compound events.

Remember

A *compound event* combines two or more events using the word *and* or the word *or*.

Practice

1. Lucia is selecting colored tiles out of a bag to use for an art project. The table shows the number of tiles of each color that are in the bag. Lucia selects one tile at random from the bag.

Color	Number of Tiles
Blue	10
Yellow	12
Pink	6
Green	3
Purple	9

   a. How many tiles are in her bag?

   There are 40 tiles in the bag.

   b. Complete the probability model for selecting tiles from the bag.

Outcome	Blue	Yellow	Pink	Green	Purple
Probability	$\frac{10}{40}$	$\frac{12}{40}$	$\frac{6}{40}$	$\frac{3}{40}$	$\frac{9}{40}$

   c. What is the probability that Lucia selects a green or purple tile?

   The probability that Lucia selects a green or purple tile is $\frac{12}{40}$, or $\frac{3}{10}$.

   d. What is the probability that Lucia selects a pink, green, or purple tile?

   The probability that Lucia selects a pink, green, or purple tile is $\frac{18}{40}$, or $\frac{9}{20}$.

   e. What is the probability that Lucia selects a pink and purple tile in one draw?

   Because there are no pink and purple tiles, the probability is 0.

Write

Answers will vary. Students should explain that in an *or* compound event, they sum the number of outcomes for each part of the compound event, making sure not to count an outcome twice, and divide by the total number of outcomes. In an *and* compound event, students only count the number of outcomes in which both components of the event occur and divide by the total number of outcomes.

MODULE 4 • TOPIC 2 • LESSON 3 ASSIGNMENT 725

Lesson 3 Assignment

2. Once Lucia finishes placing the tiles in her art project, she needs to determine the color of the grout that goes in between the tiles and the color of the frame around the project. She flips a coin to decide if she is going to use blue or yellow grout. She assigns heads to blue grout and tails to yellow grout. She puts a yellow, green, blue, and purple tile in a bag and pulls one out to determine the frame color.

   a. Determine the possible outcomes for flipping a coin once and selecting one tile out of the bag at random.

      (blue, yellow), (blue, green), (blue, blue), (blue, purple), (yellow, yellow), (yellow, green), (yellow, blue), (yellow, purple)

   b. How many possible outcomes are there?

      There are 8 possible outcomes.

   c. What events make up the outcome of having the same color for grout and the frame at random?

      The events that make up selecting the same color at random for grout and the frame are selecting blue grout with a blue frame or selecting yellow grout with a yellow frame.

   d. Determine the probability of the outcome where the same color for grout and the frame are chosen at random.

      $P(\text{same color}) = \frac{2}{8} = \frac{1}{4}$. The probability of selecting the same color for the grout and frame is $\frac{1}{4}$.

726 MODULE 4 • TOPIC 2 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

e. What events make up selecting blue for the grout or the frame?

The events are selecting blue for the grout or selecting blue for the frame.

f. Determine the probability of selecting blue for the grout or the frame.

$P(\text{blue grout or frame}) = \frac{5}{8}$. The probability of Lucia selecting blue for the grout or the frame is $\frac{5}{8}$.

Prepare

1. Explain how to use a six-sided number cube to simulate whether someone prefers gymnastics, soccer, or baseball, given that these outcomes are equally likely.

   Sample answer:

   Assign 1–2 to gymnastics, 3–4 to soccer, and 5–6 to baseball. Roll the cube 25 times and calculate the experimental probability of each outcome.

2. Explain how to use a random number table to simulate whether an evenly divided spinner lands on red, blue, green, or yellow.

   Sample answer:

   Assign 00–24 to red, 25–49 to blue, 50–74 to green, and 75–99 to yellow. Select 25 two-digit numbers and record their corresponding colors. Calculate the experimental probability of each outcome.

MODULE 4 • TOPIC 2 • LESSON 3 ASSIGNMENT 727

728 MODULE 4 • TOPIC 2 • LESSON 3 ASSIGNMENT

4 Simulating Probability of Compound Events

LESSON OVERVIEW

Students simulate the probability of compound events. They use given ratios of past performance to predict future outcomes through simulation. Students simulate the number of free throws a player would make before missing a shot, the number of people donating blood before a person with Type B blood participates, and the number of extra-point kick attempts before missing one.

MATERIALS

Six-Sided Number Cubes
Other Simulation Tools, as desired
Problem-Solving Model Graphic Organizer

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

7.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.
The student is expected to:

7.6B select and use different simulations to represent simple and compound events with and without technology.

7.6C make predictions and determine solutions using experimental data for simple and compound events.

7.6D make predictions and determine solutions using theoretical probability for simple and compound events.

ELPS

(1) Learning Strategies
The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(2) Listening
The student is expected to:

(G) understand the general meaning, main points, and important details of spoken language ranging from situations in which topics, language, and contexts are familiar to unfamiliar.

(4) Reading
The student is expected to:

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

ESSENTIAL IDEAS

- Simulations are used to estimate compound probabilities.
- A greater number of trials of a simulation should show that the experimental probability of an event approaches the same value as the theoretical probability of that event.
- Depending on the question posed, one trial of a simulation may consist of a fixed or variable number of observations.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Shoot Out 10 minutes

ESTABLISH A SITUATION

Students predict the number of free throws they would expect James's a 5 out of 6 free throw shooter, to make. They also predict the likelihood of James making his next shots. Students use their prior knowledge and intuition to answer the questions.

DEVELOP

Activity 4.1: Simulating Free Throws 30–35 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

Students design a model for simulating the player from the Getting Started shooting free throws. They then conduct 10 trials and use the results to determine the experimental probability of the player making all 3 free throws. Students also design and conduct a simulation to model the number of times the player would make free throws before missing a shot.

DAY 2

Activity 4.2: Using Random Numbers to Simulate Compound Probability
25–30 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

Students are given data involving the percents of the population having group A, group B, group O, and group AB blood. They will determine various probabilities of a person or persons having a specified blood group. Using a random number table, students model the problem, run simulations consisting of 20 trials, and use the data to answer related questions. The results should be close to the theoretical probabilities.

DEMONSTRATE

Talk the Talk: It's Good! 10–15 minutes

EXIT TICKET APPLICATION

Students design and conduct two simulations based on a kicker who makes 7 out of 10 extra point attempts. In the first simulation, students determine the probability that the kicker will make 3 out of the next 4 attempts. In the second simulation, students determine the number of extra point attempts the kicker makes before he misses. In each case, students conduct 20 trials of the simulation.

CC BY NC

Getting Started

ENGAGE

Shoot Out

Facilitation Notes

In this activity, students predict the number of free throws they would expect James, a 5 out of 6 free throw shooter, to make. They also predict the likelihood of James making his next shots. Students use their prior knowledge and intuition to answer the questions.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• On what did you base your decision? • What mathematics did you use to come to this conclusion? • Explain how you can use a proportion to solve Question 2. • Out of 5 free throws, how many is James likely to make? • Is it impossible for James to make 5 free throws in a row? Why or why not? • How did you come to your conclusion in Question 5? • Do you think James's previous 5 free throws affect his performance on his next free throw? Explain your thinking.
Gathering	• How many free throws does Question 3 imply?
Seeing structure	• Could you use the same strategy you used to solve Question 2 to solve Question 3? Explain your thinking.

STAMP THE LEARNING

Summary

Predictions for probabilities can be made using given statistics.

ACTIVITY 4.1

Simulating Free Throws

DEVELOP

Facilitation Notes

In this activity, students design a model for simulating the scenario in the previous activity. The model is used to conduct trials, and the data is used to calculate experimental probabilities.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What requirements must your simulation tool meet? • Why is a six-sided number cube a good choice? • What is another simulation tool you could use to model this situation? • How many numbers do you need to signify a success? How do you know? • Does it make a difference which numbers you assign for success and failure? Why or why not? • Should one trial consist of 1 or 3 free throws? Explain your thinking.
Seeing structure	• What is the purpose of calculating the mean of the class data? • Do you think you'd get the same result if you compiled the class data and then calculated a single probability? Why or why not? • How do the trials in the simulation in Question 4 vary from those in Question 3? • Why do you think the answer in Question 4 is close to 6?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Answers may vary using different simulation tools, but the probabilities should still be close.

STAMP THE LEARNING

ACTIVITY 4.2 Using Random Numbers to Simulate Compound Probability

Facilitation Notes

In this activity, students are given a table of data written as percents and use it to determine probabilities. Using a random number table, students model the problem, run simulations, and use the data to answer related questions.

To begin the Day 2 session, have a student read the Essential Question aloud.

DIFFERENTIATION STRATEGY

Access for All

You could have students complete 10 trials only and compile the class data for more accurate results.

CC BY NC

Have students work with a partner or in a group to complete Question 1 through Question 3, part (c). Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why is Question 2 considered a compound probability question? • How would you solve any compound probability question with *or* for this context? • What is a compound probability question with *and* for this context? What is its answer? Why?
Probing	• How did you determine the ranges of numbers for the four different blood types?

Have students work with a partner or in a group to complete Question 3, parts (d), (e), and (f), and Question 4. Share responses as a class. Note that the answer for Question 4 part (d) provides the theoretical probability. This is beyond the scope of what students are required to do in Grade 7. The answer is provided for teachers as a reference to determine how close students get to the theoretical probability using their simulation data.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Describe how you interpreted the results of one trial in Question 3.
Gathering	• How do the trials in the simulation in Question 4 vary from those in Question 3?
Seeing structure	• Describe a problem you previously solved with a structure similar to the one in Question 4.

STAMP THE LEARNING

Summary

A random number table can be a tool used in a simulation to determine compound probabilities.

DEMONSTRATE

Talk the Talk

IT'S GOOD!

Facilitation Notes

In this activity, students design and conduct two simulations to answer related questions.

Have students work with a partner or in a group to complete Question 1. Share responses as a class. Note that the answer for Question 1 part (a) provides the theoretical probability. This is a reference point for teachers to determine the accuracy of student simulation data.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is another way to design the simulation? • Which values in your simulation represent success? • Describe one trial of your simulation.
Seeing structure	• Describe a problem you previously solved with a structure similar to this one.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Simulations can use tools such as random number tables to run trials and calculate probabilities.

4 Simulating Probability of Compound Events

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Use simulations to estimate compound probabilities.
- Design and use a simulation to generate frequencies for compound events.
- Determine probabilities of compound events using simulation.

Calculating the theoretical probability of many compound events requires advanced rules and formulas.

How can you simulate real-world compound events using tools you already have?

Sample answer:

I can use a coin, a number cube, a random number generator, or slips of paper in a bag to model real-world compound events.

MODULE 4 • TOPIC 2 • LESSON 4 729

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

Getting Started

Shoot Out

James is the star player on his basketball team. So far this season, he has made 5 out of 6 free throws.

Use your knowledge of probability to answer each question.

1. Out of the next 6 free throws, how many would you expect him to make?

   I would expect him to make 5 out of the next 6 free throws.

2. Out of the next 10 free throws, how many would you expect him to make?

   I would expect him to make 8 of the next 10 free throws.

3. Predict the likelihood that James will make his next free throw.

   He has an 83% chance of making his next free throw.

4. Predict the likelihood that he will make his next 5 free throws.

   Sample answer:

   Even though he has an 83% chance of making each shot, it would not be likely for him to make the next five in a row.

5. Suppose James made 5 free throws in a row. Predict what will happen on his next free throw.

   Sample answer:

   James will miss the next free throw because he has made 5. Each free throw has an 83% chance of being made, so James still has an 83% chance of making the next shot.

730 MODULE 4 • TOPIC 2 • LESSON 4

ACTIVITY 4.1

Simulating Free Throws

James is also the team's top 3-point shooter. Suppose James is fouled while attempting a 3-point shot; he then attempts 3 free throws.

1. What might be a good simulation tool for James attempting the free throws?

   Sample answer:

   Because James makes 5 out of 6 free throws, a standard six-sided number cube would be a good model for simulating James attempting free throws.

2. Explain what constitutes a success, or James making a free throw, and what constitutes a failure, or James missing a free throw, using your tool.

   Answers will vary based on the simulation tool selected.

   For a number cube, the numbers 1–5 could indicate a success and 6 can indicate a failure, or a missed free throw attempt.

3. James's coach wants to know the probability that James will make all 3 shots. Use a simulation to determine the probability.

   a. Describe one trial of the simulation.

      Answers will vary based on the simulation tool selected.

      For a number cube, roll one cube three times (or 3 cubes one time) and record how many times the numbers 1–5 landed on top. That number is the number of free throws made out of 3 free throw attempts.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Activity 4.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice simulating probability of compound events, assign Skills Practice Set A for this lesson.

Optimizing Learning

This activity optimizes individual choice and autonomy.

MODULE 4 • TOPIC 2 • LESSON 4 731

How is calculating the mean of the probabilities similar to compiling all of the class data and then computing the probability?

b. Conduct 10 trials of the simulation and record your results in the table. Answers will vary.

Trial Number	Number of Free Throws Made
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

c. Count the number of times that your simulation resulted in James making all 3 free throws.

Answers will vary.

d. According to your simulation, what is the probability that James makes all 3 free throws?

Answers will vary.

732 MODULE 4 • TOPIC 2 • LESSON 4

e. Calculate the mean of the probabilities from the simulations conducted by your classmates. What does this tell you?

Answers will vary, but the probability should be close to 0.58 or 58%.

James has been on a "hot streak" lately!

4. Design and conduct a simulation to model the number of times James would shoot before missing a shot. That is, on what shot number would you expect James to miss?

   a. Describe one trial of the simulation.

   Answers will vary based on the simulation tool selected.

   For a number cube, roll one cube until a 6 appears.

   b. Conduct 10 trials of the simulation and record the results in the table. Answers will vary.

Trial Number	Number of Shots For First Miss
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

MODULE 4 • TOPIC 2 • LESSON 4 733

c. Calculate the mean number of shots it takes for James to miss a shot.

Answers will vary but should be near 6.

d. Compile your data with your classmates' data. Calculate the mean number of shots it takes for James to miss a shot.

Answers will vary but should be near 6.

e. How was this simulation different from the first simulation?

In the first simulation, I rolled exactly 3 times.
In this simulation, I rolled until I met the criteria.

734 MODULE 4 • TOPIC 2 • LESSON 4

ACTIVITY 4.2

Using Random Numbers to Simulate Compound Probability

Overall, the percent of people in the U.S. having each blood group is given in the table. The percents have been rounded to the nearest whole number percent.

Blood Groups	A	B	O	AB
Percent of Population	42%	10%	44%	4%

Suppose the Red Cross is having a blood drive at the community center.

1. What is the probability that the next person who enters the community center to donate blood has Group A blood?

   The probability of the next person to donate blood having Group A blood is 42%.

2. What is the probability that the next person who enters the community center to donate blood has Group A or Group O blood?

   The probability of the next person to donate blood having Group A or Group O blood is 86%.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete Question 1 through Question 3 part (c).
- Check in and share.
- Group students to complete Question 3 part (d) through Question 4.
- Share and summarize.

MODULE 4 • TOPIC 2 • LESSON 4 735

Activity 4.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice using a random number table to simulate compound probability, assign Skills Practice Set B for this lesson.

3. Determine the probability that out of the next 5 people to donate blood, at least 1 person has Group AB blood.

   a. Describe different strategies with and without technology you could use to simulate the blood groups of people who give blood.

   Answers will vary, but students may suggest a random number table or generator, 100 slips of paper in a bag, 50 slips of paper, using an online number generator, etc.

A random number table is provided at the end of the lesson.

   b. Because of the values of the percents, use a random number table or technology for this simulation. How could you assign numbers to people to account for the different blood groups?

   I could assign the numbers 00 through 41 to people with Group A blood. I could then assign the numbers 42 through 51 to people with Group B blood. Next, I could assign the numbers 52 through 95 to people with Group O blood. Finally, I could assign 96 through 99 to people with Group AB blood.

   c. Describe one trial of the simulation.

   One trial could consist of choosing five 2-digit numbers from the random digit table and counting how many of the 2-digit numbers represent Group AB.

   d. Conduct 20 trials of the simulation and record your results in the table. Answers will vary.

Trial Number	Number of AB Blood Donors
1	
2	
3	
4	
5	
6	

736 MODULE 4 • TOPIC 2 • LESSON 4

Trial Number	Number of AB Blood Donors
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

e. Out of the 20 trials, how many had at least one number that represented an AB blood donor?

Answers will vary.

f. According to your simulation, what is the probability that out of the next 5 people to donate blood, at least one of them has Group AB blood?

Answers will vary.

The theoretical probability is ≈ 0.185, or 18.5%.

MODULE 4 • TOPIC 2 • LESSON 4 737

4. How many people would be expected to donate blood before a person with Group B blood would donate blood?

a. Describe one trial of the simulation.

One trial would consist of choosing 2-digit random numbers until a number representing Group B is chosen.

b. Conduct 20 trials of the simulation and record your results in the table. Answers will vary.

Trial Number	Number of Donors Until a Group B
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

738 MODULE 4 • TOPIC 2 • LESSON 4

Trial Number	Number of Donors Until a Group B
13	
14	
15	
16	
17	
18	
19	
20	

c. Calculate the average for your 20 trials.

Answers will vary.

d. About how many people would be expected to donate blood before a person with Group B blood enters?

Answers will vary, but the theoretical number is 10.

MODULE 4 • TOPIC 2 • LESSON 4 739

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Talk the Talk

It's Good!

The kicker of the high school football team makes 7 out of every 10 extra point attempts.

1. Design and conduct a simulation with or without technology for each situation. Conduct 20 trials of your simulation and prepare to present your results to your classmates.

   Answers will vary, but students' answers for proper assignment of simulation values to a success and a failure. For a random number table, 0–6 can be assigned to a success and 7–9 can be assigned to a failure, or miss.

   a. Determine the probability that the kicker will successfully make 3 out of the next 4 extra points he attempts.

   Answers will vary.
   The theoretical probability of the kicker making 3 of the next 4 extra point attempts is 0.4116 = 41.16%.

   b. Determine the number of extra point attempts it takes for his first miss.

   Answers will vary.
   Theoretically, his first miss would come around attempt 3 or 4.

740 MODULE 4 • TOPIC 2 • LESSON 4

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61404	43516	17020
Line 2	17264	57327	38224	29301	18164	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86214	11808	12840	55147	97438	60222	12645	62090
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19107	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	11415	36457
Line 8	18975	95428	33226	55901	31605	43816	22259	00317	46999	98571
Line 9	59138	39542	71168	57609	91510	27904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

MODULE 4 • TOPIC 2 • LESSON 4 741

742 MODULE 4 • TOPIC 2 • LESSON 4

Lesson 4 Assignment

Write

Explain the difference between designing and conducting a simulation that asks for the probability of a specific number of successes in a given number of observations and a simulation that asks for the number of observations until the first success.

Remember

Many events involve very advanced rules for probability. In most cases, a simulation can be used to model the event.

Practice

1. In 1900, half of the babies born in America were born with blue eyes. What is the probability that 3 out of 4 babies born had blue eyes?

   a. What might be a good model for simulating the probability of a baby being born with blue eyes in 1900?

   Sample answer:

   Flipping a coin would be a good model to determine eye color.

   b. Describe how you would assign outcomes and then describe one trial of the simulation.

   Sample answer:

   I could assign heads to represent a baby being born with blue eyes and tails to represent a baby being born with a different eye color. One trial would consist of flipping 4 coins and recording how many of the coins landed heads up.

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Write

Sample answer:

When designing a simulation that asks for the probability of a specific number of successes in a given number of observations, say 5, each trial consists of 5 observations and determining the probability of success based on multiple trials of exactly 5 observations. When designing a simulation that asks for the number of observations until the first success, each trial could consist of a different number of observations, depending on when the first success occurred.

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MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT 743

Lesson 4 Assignment

c. Conduct 20 trials of the simulation and record your results in a table. Sample answers:

Trial Number	Outcome	Number of Heads from the 4 Coins
1	H, H, T, H	3
2	H, T, T, H	2
3	H, H, H, T	3
4	H, T, T, T	1
5	T, H, T, T	1
6	H, T, H, T	2
7	H, T, T, T	1
8	H, H, H, T	3
9	H, T, H, T	2
10	T, T, H, H	2

Trial Number	Outcome	Number of Heads from the 4 Coins
11	T, H, H, T	2
12	T, T, H, T	1
13	T, H, H, H	3
14	T, T, T, T	0
15	H, T, T, H	2
16	H, H, T, T	2
17	T, T, H, T	1
18	H, T, H, H	3
19	T, T, T, T	0
20	H, T, T, T	1

d. According to your simulation, what is the probability that 3 out of 4 babies born have blue eyes?

Sample answer:

$P(\text{3 out of 4 babies have blue eyes}) = \frac{5}{20} = 0.25$.
(Note that the theoretical probability is $= 0.25$.)

744 MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT

Lesson 4 Assignment

2. By the start of the 21st century, only 1 in 6 babies in America was born with blue eyes.

   What is the probability that at least 1 out of 2 babies has blue eyes?

   a. What might be a good model for simulating the probability of a baby being born with blue eyes in 2001?

      Sample answer:

      Rolling a six-sided number cube would be a good model.

   b. Describe how you would assign outcomes and then describe one trial of the simulation.

      Sample answer:

      I could assign number 1 to represent the baby being born with blue eyes and the rest of the numbers to represent babies born with eyes of another color. One trial would consist of rolling a number cube 2 times and recording how many times the cube showed number 1.

   c. Conduct 20 trials of the simulation and record your results in a table. Sample answers:

Trial Number	Outcome	Number of Times the Cube Showed Number 1
1	2, 5	0
2	3, 6	0
3	1, 4	1
4	5, 3	0
5	6, 5	0

MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT 745

Lesson 4 Assignment

Trial Number	Outcome	Number of Times the Cube Showed Number 1
6	4, 4	0
7	3, 1	1
8	2, 6	0
9	4, 3	0
10	1, 2	1
11	5, 2	0
12	1, 3	1
13	5, 1	1
14	2, 3	0
15	4, 2	0
16	4, 6	0
17	5, 6	0
18	6, 1	1
19	5, 3	0
20	2, 4	0

746 MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT

Lesson 4 Assignment

d. According to your simulation, what is the probability that at least 1 out of 2 babies born in 2001 has blue eyes?

Sample answer:

$P(\text{at least 1 out of 2 babies have blue eyes}) = \frac{6}{20} = 0.3$.
(Note that the theoretical probability is ≈ 0.31.)

3. The preferences of customers who rent movies online are given in the table. Design and conduct a simulation for each question. Be sure to describe how you would assign outcomes and what makes up one trial. Conduct 10 trials for each.

Movie Type	Comedy	Drama	Science Fiction	Documentary
Percent of Customers	31%	42%	22%	5%

a. Determine the probability that out of the next 5 customers to rent a movie, at least 1 rents a science fiction movie.

Trial Number	Numbers	Number of Customers Who Rent Science Fiction
1	65, 28, 59, 71, 12	0
2	13, 85, 30, 10, 94	2
3	11, 58, 38, 16, 56	0
4	10, 04, 43, 51, 61	0
5	20, 17, 26, 45, 27	0
6	38, 22, 93, 01, 18	1
7	16, 43, 81, 09, 97	1

MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT 747

Lesson 4 Assignment

Trial Number	Numbers	Number of Customers Who Rent Science Fiction
8	66, 56, 92, 98, 56	1
9	95, 50, 95, 63, 97	0
10	54, 31, 19, 99, 25	0

Sample answer:

Use a random number table. Assign the numbers 00 to 30 to the people who rent a Comedy movie. Assign the numbers 31 to 72 to the people who rent a Drama movie. Assign the numbers 73 to 94 to the people who rent a Science Fiction movie. Assign the numbers 95 to 99 to the people who rent a Documentary. One trial consists of selecting 5 sets of two-digit numbers. Each time the number is 73 to 94, that is a success.

Out of the 10 trials, 4 trials had at least 1 customer rent a science fiction movie.

$P(\text{at least 1 customer rents science fiction}) = \frac{4}{10}$
$= 0.40$. (Note: The theoretical probability is ≈ 0.71.)

b. Determine the number of customers you would expect to rent movies until someone rents a science fiction movie.

Trial Number	Numbers	Number of Customers Until One Rents Science Fiction
1	93	1
2	27, 27, 12, 5, 72, 1, 42, 30, 97, 2, 83	11
3	20, 98, 72, 30, 24, 94	6
4	92	1
5	31, 8, 99, 19, 39, 11, 24, 85	8

748 MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT

Lesson 4 Assignment

Trial Number	Numbers	Number of Customers Until One Rents Science Fiction
6	1, 1, 69, 70, 88	5
7	11, 45, 98, 69, 54, 63, 92	7
8	62, 88	2
9	81	1
10	61, 80	2

Sample answer:

Use a random number table. Assign the numbers 00 to 30 to the people who rent a Comedy movie. Assign the numbers 31 to 72 to the people who rent a Drama movie. Assign the numbers 73 to 94 to the people who rent a Science Fiction movie. Assign the numbers 95 to 99 to the people who rent a Documentary. One trial consists of selecting two-digit numbers until I get a number from 73 to 94.

Based on the simulation, the average number of customers until someone rents a science fiction movie is 4.4 customers. (Note that the theoretical number is $\approx$ 4.55 customers.)

MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT 749

Lesson 4 Assignment

Prepare

A light bulb company tested 24 of the bulbs it just produced and found that 3 of them were defective. Use proportions to predict how many light bulbs would be defective in shipments of each size.

1. 100 light bulbs

   12.5; About 12 or 13 light bulbs would be defective.

2. 400 light bulbs

   50 light bulbs would be defective.

3. 750 light bulbs

   93.75; About 93 or 94 light bulbs would be defective.

750 MODULE 4 • TOPIC 2 • LESSON 4 ASSIGNMENT

TOPIC 2 SELF-REFLECTION

Name: _____

Compond Probability

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represent **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Compound Probability* topic by:

TOPIC 2: *Compound Probability*	Beginning of Topic	Middle of Topic	End of Topic
choosing the appropriate method, such as an organized list, a table, or a tree diagram, to represent sample spaces and create probability models for compound events.	○	○	○
representing the outcomes in the sample space that make up a compound event.	○	○	○
determining theoretical and experimental probabilities for compound events using data and sample spaces.	○	○	○
making predictions and determining solutions using experimental and theoretical probabilities for compound events.	○	○	○
designing and using a simulation to predict the probability of a compound event.	○	○	○

Notes

continued on the next page

MODULE 4 • TOPIC 2 • SELF-REFLECTION 751

Notes

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Compound Probability* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

752 MODULE 4 • TOPIC 2 • SELF-REFLECTION

TOPIC 2 SUMMARY

Notes

Compound Probability Summary

NEW KEY TERMS
- tree diagram
- compound event

LESSON 1 Using Arrays to Organize Outcomes

To organize the outcomes for two events in a number array, list the outcomes for one event along one side and the outcomes for the other event along the other side. Combine the results in the intersections of each row and column.

For example, this array shows the sample space—all the possible outcomes—for rolling two 6-sided number cubes and calculating the product of the numbers shown.

		Number Cube 1					
		1	2	3	4	5	6
Number Cube 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

The probability of the product being 6 is $\frac{4}{36}$, or $\frac{1}{9}$.

MODULE 4 • TOPIC 2 • TOPIC SUMMARY 753

Notes

LESSON 2

Using Tree Diagrams

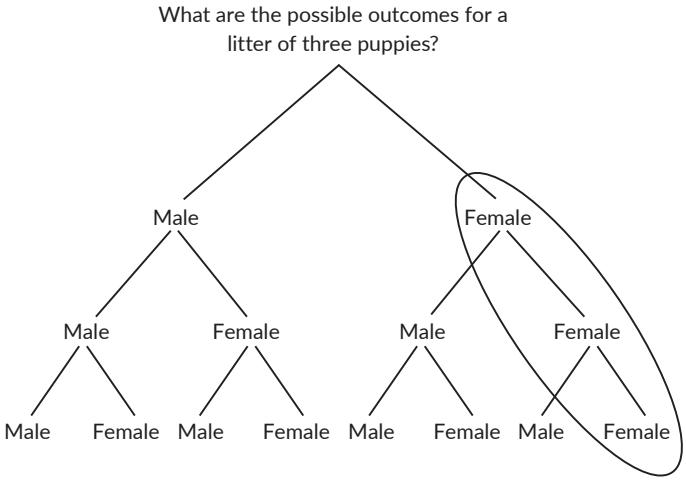
One method of determining the theoretical probability of an event is to construct a *tree diagram*. A **tree diagram** illustrates the possible outcomes of a given situation. Tree diagrams can be constructed vertically or horizontally. A tree diagram has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

For example, you can construct a tree diagram to show all the possible outcomes for a litter of three puppies.

List the possible outcomes of the 1st puppy.

List the possible outcomes of the 2nd puppy.

List the possible outcomes of the 3rd puppy.



You can use a tree diagram to determine the probability of an event. In the tree diagram above, there are 8 possible outcomes for a litter of three puppies. The probability for a litter having three female puppies is $\frac{1}{8}$.

754 MODULE 4 • TOPIC 2 • TOPIC SUMMARY

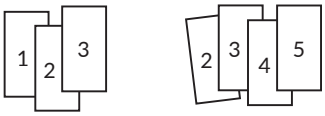
LESSON 3

Determining Compound Probability

A **compound event** combines two or more events using the word *and* or the word *or*. Determining the probability of a compound event with the word *and* is different from determining the probability of a compound event with the word *or*.

The difference is that a compound event with the word *and* means that you are determining the probability that both events occur.

For example, the cards shown were placed facedown in two piles so that they could be chosen at random.



The possible outcomes for choosing one card at random from each pile are (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 2), (3, 3), (3, 4), and (3, 5).

Determine the probability that the first card is a 1 and the second card is even.

The possible desired outcomes are (1, 2) and (1, 4). The probability that the first card is a 1 and the second card is even is $\frac{2}{12}$, or $\frac{1}{6}$.

When determining the probability of a compound event with the word *or*, you are determining the probability that one or the other or both outcomes occur.

For example, using the same experiment above, determine the probability that the first card is a 1 or the second card is even.

The possible desired outcomes are (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 4), (3, 2), and (3, 4).

The probability that the first card is a 1 or the second card is even is $\frac{8}{12}$, or $\frac{2}{3}$.

Notes

MODULE 4 • TOPIC 2 • TOPIC SUMMARY 755

Notes

LESSON 4

Simulating Probability of Compound Events

Many events involve very advanced rules for probability. In most cases, a simulation can be used to model the event. A random number table or technology can be used to simulate compound probability in many of these events.

For example, the preferences of customers who rent movies online are provided in the table below. You can design and conduct a simulation to predict the probability that out of the next 10 customers to rent a movie, at least 3 rent a comedy.

Movie Type	Comedy	Drama	Science Fiction	Documentary
Percent of Customers	31%	42%	22%	5%

You could use a random number table for this simulation. First, assign each movie type a range of two-digit numbers that correspond to the percent of customers who prefer that type.

- Comedy: 00–30
- Drama: 31–72
- Science Fiction: 73–94
- Documentary: 95–99

Then, run trials of the experiment. Each trial consists of selecting 10 two-digit numbers from the random number table. The experimental probability of at least 3 of the next 10 customers renting a comedy is the number of trials that contained at least 3 the numbers from 00 to 30 divided by the total number of trials.

756 MODULE 4 • TOPIC 2 • TOPIC SUMMARY

GRADE 7

TOPIC 3

Choosing a sample randomly from a population is a good way to select items that are representative of the entire population.

Drawing Inferences

LESSON 1 Collecting Random Samples 759

LESSON 2 Using Random Samples to Draw Inferences 783

LESSON 3 Bar Graphs 807

LESSON 4 Comparing Two Populations 835

LESSON 5 Using Random Samples from Two Populations to Draw Conclusions 851

TOPIC 3 OVERVIEW

Drawing Inferences

How are the key concepts of *Drawing Inferences* organized?

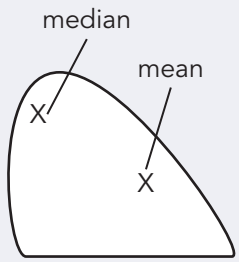
In *Drawing Inferences*, students continue developing their understanding of the statistical process. They learn about samples, populations, censuses, parameters, and statistics. Students then discuss the importance of representative samples, including random samples, for the purpose of making generalizations about the populations represented by the samples. Students collect random and non-random samples, using hands-on tools and simulation strategies, and they then use proportional reasoning to estimate the value of the parameter of interest. Students conclude that statistics obtained from samples are more likely to represent the population's parameter when the sample is random.

Next, students analyze categorical data presented in circle graphs, bar graphs, double bar graphs, and stacked bar graphs, which allow for part-to-part and part-to-whole comparisons. Students continue to create data displays, such as dot plots, stem-and-leaf plots, and box plots to compare two populations. They calculate the measures of center and measures of variation for two data sets, including the five-number summary and interquartile range (IQR), for comparison purposes.

They compare the calculated measures of center and measures of variation to interpret the differences among the two populations. They also display the data to compare the differences in these measures visually. Students then use random samples to make inferences and draw conclusions about two

Math Representation

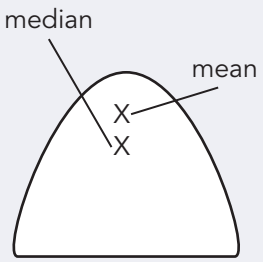
There are three common distributions of data: skewed left, skewed right, and symmetric. The distribution of data can help you determine whether the mean or median is a better measure of center.



skewed right

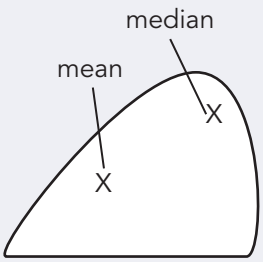
The mean of a data set is greater than the median when the data are skewed to the right.

The median is the better measure of center because the median is not affected by very large data values.



symmetric

The mean and median are equal when the data are symmetric.



skewed left

The mean of a data set is less than the median when the data are skewed to the left.

The median is the better measure of center because the median is not affected by very small data values.

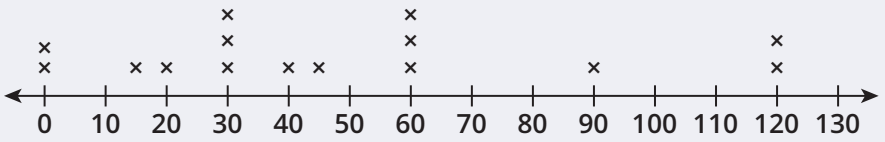
populations. The characteristics of the two populations are analyzed using graphical displays, such as stem-and-leaf plots, box plots, and histograms, as students compare their shapes, centers, and spreads.

What is the entry point for students?

Students have learned and used aspects of the statistical problem-solving process: formulating questions, collecting data, analyzing data, and interpreting the results. They formulated statistical questions, collected survey data, and analyzed data using numerical data displays, measures of center (mean, median, mode), and measures of variation (range and interquartile range).

Math Representation

The principal wants to understand how much time the students at a middle school spend exercising each weekday. She gives a survey to 15 anonymous students and displays her data in a dot plot.



The median of 40 minutes indicates that half the students spend less than 40 minutes exercising each weekday, and half the students spend more than 40 minutes exercising each weekday.

The mean of 48 minutes tells you the average number of minutes the students spend exercising each weekday.

Previously, students were introduced to the idea of comparing multiple data sets using various graphical displays. In *Drawing Inferences*, students extend this work to calculate measures for multiple samples or populations, and they use those measures to compare data sets.

Students used random number tables and began working with simulations in the previous topic. As students move through this topic, they will first engage with the statistical process and simulation tools, eventually combining this knowledge with their prior knowledge of displaying data and calculating measures of center and measures of variation, as they draw informal inferences about populations.

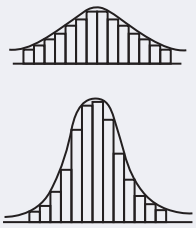
Why is *Drawing Inferences* important?

In future courses, students will continue developing strategies that allow them to engage in higher levels of statistical sophistication in terms of data collection, analysis, and interpretation. They will learn about specific types of random sampling and the inherent bias in sampling techniques. Students continue analyzing random samples from populations, comparing their measures of center and measures of variation, until they realize the need for a standard method to compare variation. This realization will lead to use of the standard normal distribution for calculating the likelihood of a specific outcome, which leads students to significance testing, margin of error, and confidence intervals.

Math Representation

A **normal curve** is a bell-shaped curve that is symmetric about the mean of the data. A normal curve models a theoretical data set that is said to have a normal distribution.

Normal Distributions



Not Normal Distributions

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic when they can:

- Apply knowledge of probability and proportional relationships to analyze data and make inferences about a population.
- Apply knowledge of the statistical process to analyze data and make inferences about a population.
- Use tools to generate a random sample that represents the population.
- Create and interpret data displays, such as bar graphs, dot plots, and circle graphs, including part-to-part and part-to-whole comparisons.
- Generate random samples for two populations to identify their similarities and differences.

NEW KEY TERMS

- survey
- data [datos]
- population [población]
- census [censo]
- sample
- parameter [parámetro]
- statistic [estadística]
- random sample

- Compare and draw informal inferences about two populations based on their measures of center and measures of variation (mean, median, range, interquartile range).
- Compare two numerical data distributions on a graph, such as a comparative dot plot, stem-and-leaf plot, histogram, or box plot, by visually comparing the shapes, centers, and spreads of the data displays.

How do the activities in *Drawing Inferences* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

This topic is designed to help students reason statistically and use appropriate tools to generate random samples and compare populations (7.1C). Students are expected to describe data displays, compute measures of center and measures of variation, compare data sets, and reason about representative and random samples (7.1D). Throughout the topic, students use real-world data as they engage in the statistical process (7.1A).

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Introduce a "Cognate Art Gallery" where students create visual representations of cognates. This could involve drawing, digital art, or any other artistic medium. Displaying their art around the classroom will serve as continued visual reminders of these connections.

MODULE 4, TOPIC 3 PACING GUIDE

165-Day Pacing

4 Analyzing Populations and Probabilities

TOPIC 3: Drawing Inferences

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1A, 7.1B, 7.1C, 7.1D, 7.1E, 7.1G

ELPS: 1.D, 1.E, 1.G, 2.D, 2.H, 3.A, 3.D, 3.E, 3.G, 4.A, 4.G, 4.J, 5.B, 5.G

Topic Pacing: 17 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Collecting Random Samples	Students review the statistical process and deepen their understanding of the second component of the process: data collection. They are introduced to new terms related to data collection. Students then read various problem situations and differentiate between *census* and *sample* and *parameter* and *statistic*. Students learn that a sample is smaller than the population, and it represents characteristics of the population. They encounter methods for selecting samples from a population and determine if methods inadvertently misrepresent the population. Students use two tools to generate random numbers: randomly selecting numbers from a bag and using provided random number tables. **Materials Needed:** Paper Bags, Scissors, Blank Paper	7.6F 7.12B	2
2	Using Random Samples to Draw Inferences	Students use statistical information gathered from a sample to determine a parameter for a population. They complete this process two times with one scenario. The first time students may select the sample using various methods; however, the second time they follow a specific strategy to select a random sample. In each case, students use proportional reasoning to estimate the parameter. They compute percent error and conclude that statistics obtained from samples are more likely to represent the parameter of the population if the sample is randomly chosen. They then analyze data from 100 samples and predict the parameter from the data. Finally, students are provided with a scenario and must design and carry out a sampling plan to estimate the parameter. **Materials Needed:** Problem-Solving Model Graphic Organizer	7.6B 7.6F 7.12B	3
3	Bar Graphs	In this lesson, students analyze categorical data presented in bar graphs. Students analyze three types of bar graphs–single bar graphs (with horizontal or vertical bars), double bar graphs, and stacked bar graphs. Students then answer questions about data provided and create their own graphs from data sets. **Materials Needed:** None	**7.6G** 7.12C	2

*Bold TEKS = Readiness Standard

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MODULE 4, TOPIC 3 PACING GUIDE

165-Day Pacing

Lesson	Lesson Title	Highlights	TEKS*	Pacing
4	**Comparing Two Populations**	Within the context of a situation, students calculate the measures of center and measures of variability for two different populations. They compare the difference of the measures of center for the two populations to their measures of variation. Students construct dot plots and determine a five-number summary for a data set for comparison purposes. A stem-and-leaf plot is used to display data in one situation. **Materials Needed:** None	**7.6G** **7.12A**	2
5	**Using Random Samples from Two Populations to Draw Conclusions**	Students use random samples to draw conclusions about two populations. The characteristics of the two populations are analyzed using graphical displays in the form of stem-and-leaf plots and box plots. In the first situation, students are given a table of values containing data for two populations. In the second situation, students are given two histograms containing data for two distinct populations. Students create graphical displays to answer questions related to each problem situation. Questions focus on means, medians, ranges, mean absolute deviation, and interquartile ranges. **Materials Needed:** Problem-Solving Model Graphic Organizer	7.6F **7.12A** 7.12C	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				5

*Bold TEKS = Readiness Standard

MODULE 4, TOPIC 3 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.6F, 7.12B **LESSON 1** Collecting Random Samples **GETTING STARTED** **ACTIVITY 1**	**LESSON 1** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: 7.6B, 7.6F, 7.12B **LESSON 2** Using Random Samples to Draw Inferences **GETTING STARTED** **ACTIVITY 1**	**LESSON 2** continued **ACTIVITY 2** **ACTIVITY 3**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 2 continued **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: **7.6G**, 7.12C **LESSON 3** Bar Graphs **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
TEKS; **7.6G, 7.12A** **LESSON 4** Comparing Two Populations **GETTING STARTED** **ACTIVITY 1**	**LESSON 4** continued **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: 7.6F, **7.12A**, 7.12C **LESSON 5** Using Random Samples from Two Populations to Draw Conclusions **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 5** continued **ACTIVITY 3** **TALK THE TALK**
Day 16	**Day 17**			
LEARNING INDIVIDUALLY **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**			

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are five Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets can be assigned to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Collecting Random Samples

MATERIALS

Paper Bags
Scissors
Blank Paper

LESSON OVERVIEW

Students review the statistical process and deepen their understanding of the second component of the process: data collection. They are introduced to new terms related to data collection. Students then read various problem situations and differentiate between *census*, *sample*, *parameter*, and *statistic*. Students learn that a sample is smaller than the population, and that it represents characteristics of the population. They encounter methods for selecting samples from a population and determine if methods inadvertently misrepresent the population. Students use two tools to generate random numbers: randomly selecting numbers from a bag and using provided random number tables.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6F use data from a random sample to make inferences about a population.

ELPS

(1) Learning Strategies

The student is expected to:

(D) speak using learning strategies such as requesting assistance, employing non-verbal cues, and using synonyms and circumlocution (conveying ideas by defining or describing when exact English words are not known).

(3) Speaking

The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

(E) share information in cooperative learning interactions.

(TEKS continued on next page)

GRADE 7 TEKS *(continued)*

Measurement and Data

(12) The student applies mathematical process standards to use statistical representations to analyze data.

The student is expected to:

7.12B use data from a random sample to make inferences about a population.

ESSENTIAL IDEAS

- A *survey* is a method of collecting information from a population or sample of a population.
- A *population* is the entire set of items from which data can be selected.
- A *census* is the collection of data from every member of a population.
- The characteristic used to describe the population is called a *parameter*.
- A *statistic* describes the sample from a population and can be used to make a prediction about a parameter.
- A *random sample* is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.
- A sample generated randomly is more likely to be representative of the population than one that is not generated randomly.
- Random number tables are used to generate random numbers when the population size is large.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Reviewing the Statistical Process 10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students review the four components of the statistical process. They formulate statistical questions about their class and propose a data collection method. Students review the definition of *survey*. They complete a short survey to use in the lesson.

DEVELOP

Activity 1.1: Representative Samples 30–35 minutes

REAL-WORLD PROBLEM SOLVING

Students are introduced to the terms *population*, *census*, *parameter*, *sample*, and *statistic*. They differentiate among these terms in various contexts, including the survey information they completed at the start of this lesson. Students then begin with a context, suggest several methods of selecting a sample, and discuss how some of the samples could inaccurately represent the population. They discuss the concept of a random sample, along with examples and non-examples of selecting a random sample.

DAY 2

Activity 1.2: Selecting a Random Sample 20–25 minutes

INVESTIGATION, PEER WORK ANALYSIS

Students begin with a context and analyze several methods of selecting a sample, discuss how some of the samples could inaccurately represent the population. They physically select a random sample of five random students. Students compare and contrast their samples and results to those of their classmates.

Activity 1.3: Using Random Number Tables to Select a Sample 10 minutes

INVESTIGATION, REAL-WORLD PROBLEM SOLVING

Students draw a random sample from a population of 450. They revisit the idea of using a random number table to randomly select three-digit numbers. Students then use the table to select a sample of 20 three-digit numbers.

DEMONSTRATE

Talk the Talk: Lunching with Ms. Brown 10 minutes

EXIT TICKET APPLICATION

Students begin with a context and identify the population and sample. They determine whether a sampling method is representative of the population. Then, they recommend a method for assigning random numbers in order to use the random number table to generate the random sample.

CC BY NC

ENGAGE

Reviewing the Statistical Process

Facilitation Notes

In this activity, students summarize the four components of the statistical process and complete a short survey for use in subsequent lessons.

Note that sample data is provided at the end of the lesson if you prefer not to use class data.

Emphasize that the term *survey* can be used as a noun or a verb. Remind students that statistical questions anticipate variability. As students work, listen for anyone struggling with the broad definitions presented in this lesson. Many of the newly defined terms have mainstream definitions and mathematical definitions.

Ask a student to read the introduction aloud. Discuss as a class. Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What makes a statistical question different from other questions? • What are different ways to collect data? • What are different ways to analyze data using a graph? Using numbers? • Explain how the last step of the statistical process relates to the first step. • Explain why your question is a statistical question. • Does a statistical question ever ask for an opinion? • Why do you think your method of collection is better than others?

Have students complete Question 3 (the survey) independently. Publicly collect the data from Question 3 of the survey. This data will be used in the next activity.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What surveys do you hear about in the news?
Probing	• Why would someone want to survey 7th-grade students about this information?
Gathering	• Do all survey questions ask an opinion?
Reflecting and justifying	• Does the class's data collected from the survey have the results you expected? Why or why not?

Summary

There are four components of a statistical process: formulating a statistical question, collecting appropriate data, analyzing the data graphically and numerically, and interpreting the results of the analysis.

ACTIVITY 1.1

Representative Samples

DEVELOP

Facilitation Notes

In this activity, students are introduced to the terms *population*, *census*, *parameter*, *sample*, and *statistic*. They differentiate among these terms in various contexts, including the survey information they completed at the start of this lesson. Students then begin with a context, suggest several methods of selecting a sample, and discuss how some of the samples could inaccurately represent the population. Students discuss the concept of a random sample, along with examples and non-examples of selecting a random sample.

DIFFERENTIATION STRATEGY

Access for All

Ask students to create concept maps connecting the terms *population*, *census*, *parameter*, *sample*, and *statistic*. Display these maps for reference throughout this topic.

Ask a student to read the definitions aloud. Discuss as a class. Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How is the mathematical definition of *population* different from its everyday use?
Probing	• Does a census always refer to a count of people? What other information can you collect?

CC BY NC

Ask a student to read the information and definitions following Question 3 aloud. Discuss the definitions as a class. Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How can you tell from the terms used whether data relates to a population or a sample?
Probing	• What strategy did you use to answer Question 5 part (d)? What is another way? • How could you use this information to determine a parameter for the population?

Have students work with a partner or in a group to complete Question 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you compute the percent of students in your class that carry a cell phone? • Why is the percent of students carrying a cell phone considered a parameter and not a statistic?

Have students work with a partner or in a group to complete Questions 7 and 8. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What characteristics might the students in Question 7 share that makes them not representative of the entire population? • How do you know when a sample is a good sample? • What is another way to get an appropriate sample?

Ask a student to read the information following Question 8 aloud. Discuss the definition as a class.

QUESTION TO SUPPORT DISCOURSE

Gathering	• Why is it important to collect a random sample?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

A sample of data should have characteristics that are similar to the entire population. Parameters are used to describe populations. Statistics are used to describe samples and make estimates about parameters.

STAMP THE LEARNING

ACTIVITY 1.2

Selecting a Random Sample

Facilitation Notes

In this activity, students analyze methods for selecting a sample and discuss how some of the samples could inaccurately represent the population.

To begin the Day 2 session, have a student read the Essential Question aloud.

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Technology

If you have access to spreadsheets, graphing technology, or random number apps, students can use those tools instead of physically drawing numbers. The instructions for each tool differ, so refer to user's manuals or online help for how to generate random samples with technology. When using any website or online tool in the classroom, it is important to preview and vet the tool prior to classroom use.

Note that if the graphing technology have never been used before to generate random numbers, students may actually all get the same set of random numbers. This is because the random numbers are all initially seeded by the same number. To guarantee this does not happen (prior to the activity), have students follow the steps to seed the graphing technology with different values.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Once Ms. Flores randomly selects students, what data is she going to collect from them? Why?
Probing	• Explain why Ms. Flores's method generates a random sample. • What does James not understand about the meaning of a random sample?

COMMON MISCONCEPTION

Students may have overgeneralized "the number 7 is most likely to occur" from calculating the sum of two number cubes in the previous topic.

CC BY NC

Explain that Ms. Flores's method created a random sample, so all numbers have the same chance of being selected. Use the sum of two number cubes as an example of non-randomness because in this case, the number 7 does have the greatest likelihood of occurring.

You will need a bag and 30 small pieces of paper to complete this part of the activity. Ask a student to read Question 4 aloud and complete as a class.

QUESTION TO SUPPORT DISCOURSE

Probing	• Is it possible for the same number to appear more than once? How is it related to the concept of randomness?

Have students work with a partner or in a group to complete Questions 5 and 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Suppose Ms. Flores selects five consecutive numbers. Is the sample still random?
Seeing structure	• What determines whether a sample is random: the method of selection or the results?

STAMP THE LEARNING

Summary

Methods for sample selection should be random and representative of the entire population.

ACTIVITY 1.3

Using Random Number Tables to Select a Sample

Facilitation Notes

In this activity, students use a random number table to select three-digit numbers and answer related questions.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you suppose the numbers in the random number table were generated? • Could the random number table be used to generate a four or five digit number? How?

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Highlighters

When using a random number table, have students alternately put a line segment above or below sets of three numbers for a few lines at a time. Then, have students identify the successes by highlighting or circling them on the table.

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• Which method do you prefer: the random number table, or just writing numbers on slips of paper, placing them in a box, shaking the box, and choosing slips of paper with numbers on them? Why?
Probing	• How do you know which digits to eliminate when choosing Ms. Brown's sample? • How do you know which digits to select when choosing Ms. Brown's sample? • Which digits were eliminated? Why? • Why would you ignore the second time the same random number is selected? • Is it necessary to randomly choose a line to select the sample? Why or why not?

Summary

A random number table can be used to generate numbers for a sampling.

DEMONSTRATE

Talk the Talk

LUNCHING WITH MS. BROWN

Facilitation Notes

In this activity, students identify the population and sample from a real-world problem situation. They recommend a method for assigning random numbers.

Have students work with a partner to answer Questions 1 through 4. Have students share responses with another pair before sharing responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What are some problems you may encounter when using a sample of the population? • What are some reasons a sample might not be representative of the population? • How can you tell when a sample is or is not representative of the population? • What method should the principal use to select 10 random students?
Reflecting and justifying	• Which method would be easiest for the principal to use? Why? • Is one method fairer than another? Explain.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

A random number table is a tool used to generate a random sample.

1 Collecting Random Samples

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Differentiate between a population and a sample.
- Differentiate between a parameter and a statistic.
- Differentiate between a random sample and a non-random sample.
- Identify the benefits of random sampling of a population, including supporting valid statistical inferences and generalizations.
- Use several methods to select a random sample.

NEW KEY TERMS

- survey
- data
- population
- census
- sample
- parameter
- statistic
- random sample

The statistical process is a structure for answering questions about real-world phenomena.

How can you make sure that the data you collect accurately answer your statistical questions?

Sample answer:

You should collect data through a random sample to get a more accurate representation of the characteristics of a population.

MODULE 4 • TOPIC 3 • LESSON 1 759

Getting Started

Reviewing the Statistical Process

There are four components of the statistical process:

- Formulating a statistical question.
- Collecting appropriate data.
- Analyzing the data graphically and numerically.
- Interpreting the results of the analysis.

1. Summarize each of the four components. You may want to use examples to support your answers.

   Sample answers:

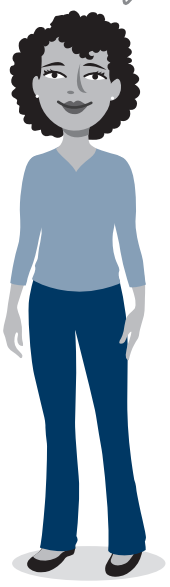
   - Formulating a statistical question: Posing questions that anticipate variability, or variation in data. For example, "How tall am I?" is not a statistical question, but "How tall are seventh-graders?" is a statistical question.
   - Collecting appropriate data: Based on the question, conduct a survey or experiment or collect observational data.
   - Analyzing the data graphically and numerically: Once the data are collected, create a plot (e.g., dot plot, stem plot, box plot, histogram) to analyze the distribution, and based on the shape of the distribution and the context, calculate the summary statistics (e.g., mean, median, mode, mean absolute deviation, interquartile range).
   - Interpreting the results of the analysis: Write a conclusion to answer the statistical question based on the analysis.

How could you describe the students in your classroom? How do the students in your classroom compare to other groups of students in your school or to other seventh-graders in the United States?

2. Formulate a statistical question about your classmates. How might you collect the information to answer your question?

   Answers will vary.

In this lesson, you will explore the second component of the statistical process, data collection. You will learn strategies for generating samples.



Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1 and 2.
- Share and summarize.
- Have students complete Question 3 individually and collect the data.

Optimizing Learning

This activity fosters collaboration and community.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

760 MODULE 4 • TOPIC 3 • LESSON 1

One data collection strategy you can use is a *survey*. A **survey** is a method of collecting information about a certain group of people. It involves asking a question or a set of questions to those people. When information is collected, the facts or numbers gathered are called **data**.

3. Answer each question in the survey shown. You will use the results in the next activity.

   Answers will vary.

   a. What is your approximate height? _____

   b. Do you have a cell phone? Yes _____ No _____

   c. About how many text messages do you send each day? _____

MODULE 4 • TOPIC 3 • LESSON 1 761

Chunking the Activity

- Read and discuss the definitions.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the information and definitions.
- Group students to complete Questions 4 and 5.
- Check in and share.
- Group students to complete Question 6.
- Check in and share.
- Group students to complete Questions 7 and 8.
- Check in and share.
- Read and discuss the definition.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 1.1

Representative Samples

Some examples of populations include:
- every person in the United States
- every person in your math class
- every person in your school
- all the apples in your supermarket
- all the apples in the world

The **population** is the entire set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects.

A **census** is the data collected from every member of a population.

1. Use your survey to answer each question.

   a. Besides you, who else took the math class survey?
   The other students in my math class took the 7th-grade math survey.

   b. What is the population in your class survey?
   The population is all my classmates in my math class.

   c. Are the data collected in the class survey a census? Explain your reasoning.
   The data collected are a census because every classmate in my math class took the survey.

According to the 2020 census, approximately 331,450,000 people live in the United States!

Ever since 1790, the United States has taken a census every 10 years to collect data about population and state resources. The original purposes of the census were to decide the number of representatives a state could send to the U.S. House of Representatives and to determine the federal tax burden.

2. Describe the population for the United States census.
   The population is every person living in the United States.

EB STUDENT TIP

For all proficiency levels

- Have students create a concept map relating the terms *population*, *census*, *parameter*, *sample*, and *statistic*.
- Clarify the difference between the everyday meaning and the mathematical meaning of *population*. In mathematics, a population does not necessarily imply people.
- Use the everyday meaning of *sample* to make sense of its meaning. Relate this to a sample of food that shares the dish's characteristics but is smaller in size.

762 MODULE 4 • TOPIC 3 • LESSON 1

3. Why do you think this collection of data is called "the census"?

   It is called a "census" because information is collected for every person living in the United States.

In most cases, it is not possible or logical to collect data from every member of the population. When data are collected from a part of the population, the data are called a **sample**. A sample should be representative of the population. In other words, it should have characteristics that are similar to the entire population.

When data are gathered from a population, the characteristic used to describe the population is called a **parameter**.

When data are gathered from a sample, the characteristic used to describe the sample is called a **statistic**. A statistic is used to make an estimate about the parameter.

At a particular middle school, 29% of students had perfect attendance last year. This is a parameter because you can count every single student at the school.

Nationally, 16% of all teachers miss 3 or fewer days per year. This characteristic is a statistic because the population is too big to survey every single teacher.

4. After the 2000 census, the United States Census Bureau reported that 7.4% of Georgia residents were between the ages of 10 and 14. Was a parameter or a statistic reported? Explain your reasoning.

   A parameter was reported because every person in Georgia was counted for the census.

5. A recent survey of 1000 teenagers from across the United States shows that 4 out of 5 carry a cell phone with them.

   a. What is the population in the survey?

      The population is teenagers from across the United States.

   b. Were the data collected in the survey a census? Why or why not?

      The data collected were not a census because only 1000 teenagers were surveyed.

Question 4 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice differentiating between parameters and statistics, assign Skills Practice Set A for this lesson.

Question 5 part (b) presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice differentiating between a census and a sample, assign Skills Practice Set B for this lesson.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

MODULE 4 • TOPIC 3 • LESSON 1 763

c. Does the given statement represent a parameter or a statistic? Explain how you determined your answer.

The "4 out of 5" is a statistic used to describe the portion of the 1000 teenagers surveyed who carry a cell phone with them.

d. Of those 1000 teenagers surveyed, how many carry a cell phone? How many do not carry a cell phone?

According to the survey, about 800 of the 1000 teenagers surveyed carry a cell phone with them, and 200 teenagers do not carry a cell phone.

6. Use your math class survey, or the data from a sample class survey provided at the end of this lesson, to answer each question. Use a complete sentence to justify each answer.

a. How many students in the class have a cell phone?

The answers provided are from the sample data at the end of the lesson. Your students' responses will vary. Eighteen of the 20 students carry a cell phone.

b. What percent of the students in the class have a cell phone?

Eighteen out of 20 means 90 percent of the students carry a cell phone.

c. Does the percent of students in the class that have a cell phone represent a parameter or a statistic? Explain how you determined your answer.

The percent of students in my class who carry a cell phone is a parameter because it describes the population of all students in my class. I am interested in only the responses of my classmates, not the responses of all seventh-graders in the United States.

764 MODULE 4 • TOPIC 3 • LESSON 1

7. Suppose you only want to survey a sample of the class about whether they have a cell phone. Discuss whether or not these samples would provide an accurate representation of all students in a class. Use complete sentences to justify your answers.

   a. the selection of all of the 12-year-olds for the sample

      This sample does not represent all students because the students that are not 12 years old are not represented in the sample.

   b. the selection of the students in the first seat of every row

      This sample might not represent all students because it is possible that the students in the first seat of every row have a characteristic that makes them different from the remainder of the students.

   c. the selection of every fourth student alphabetically

      This sample would more than likely represent all students in the class.

   d. the selection of the first 10 students to enter the classroom

      This sample might not represent all students in the classroom, because the first 10 students to enter the classroom may have characteristics that all the students do not have. They may have been in a class that not all students take, such as band or chorus. The last students in the class might be those with cell phones if they are allowed to use their phones between classes.

MODULE 4 • TOPIC 3 • LESSON 1 765

8. Suppose you wanted to determine the number of students who have a cell phone across the entire seventh grade.

   a. What is the population?

      The population is all of the students in the seventh grade at my school.

   b. Suggest and justify a method of surveying students in the seventh grade to obtain a representative sample.

      Answers will vary, but students should justify why they believe their strategy would provide a representative sample.

When information is collected from a sample to describe a characteristic about the population, it is important that such a sample be as representative of the population as possible. A **random sample** is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

766 MODULE 4 • TOPIC 3 • LESSON 1

ACTIVITY 1.2

Selecting a Random Sample

Ms. Flores is purchasing standing desks for her classroom. She wants to set up the desks to minimize the amount of desk height adjusting by the students and decides to use the mean height of students in her math class as a guide. Rather than using the heights of all students in her class, she decides to collect a random sample of students in her class.

1. What is the population for this problem?

   The population is all the students in Ms. Flores's class.

2. Ms. Flores received two suggestions to randomly sample her class. Decide if each strategy represents a random sample. If not, explain why not.

   a. Ms. Flores chooses the 12-year-olds in the class.

      This is not a random sample because the height of the students that are not 12 years old is not considered.

   b. Ms. Flores chooses all of the students wearing white sneakers.

      This is not a random sample. There may be a school rule that does not allow students to wear sneakers except during gym. Or, there may be a basketball game that day, and all members of the team are required to wear white sneakers.

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the situation.
- Group students to complete Questions 1–3.
- Check in and share.
- Complete Question 4 as a class.
- Group students to complete Questions 5 and 6.
- Share and summarize.

Question 2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with identifying random samples, assign Skills Practice Set C for this lesson.

MODULE 4 • TOPIC 3 • LESSON 1 767

Review the definitions of **select** and **estimate** in the Academic Glossary.

3. Ms. Flores decides to select a random sample of five students in her class and then calculate the mean height. She assigns each student in her class a different number. Then, she randomly selects 5 numbers.

   a. Explain why Ms. Flores's method of taking a sample is a random sample.

   The method provides a random sample because every student in the classroom has the same chance of being selected.

   b. Do you think randomly selecting five students will accurately represent the population of her class? If not, do you think she should pick more or fewer students?

   I think that randomly selecting five students will accurately represent the population of the class. If the population was larger, she might want to select additional students.

c. James hopes Ms. Flores will assign him the number 7 because it will have a better chance of being selected for the sample. Do you agree or disagree with James? Explain your reasoning.

James is incorrect. The number 7 will have the same chance of being chosen as any other number.

b. Catalina claims Ms. Flores must begin with the number 1 when assigning numbers to students. Ricardo says she can start with any number as long as she assigns every student a different number. Who is correct? Explain your reasoning.

Ricardo is correct. When assigning numbers, Ms. Flores can begin with any number she chooses as long as every student receives a number, and no students are assigned the same number.

768 MODULE 4 • TOPIC 3 • LESSON 1

One way to select the students is to write the numbers (or student names) on equal-sized pieces of paper, put the papers in a bag, draw out a piece of paper, and record the result. To create a true random sample, the papers should be returned to the bag after each draw.

Help Ms. Flores randomly select five students from her class.

4. With your partner, create a bag with 30 numbers from which to select your sample.

   a. Draw five numbers and record your results.

      Sample answer:
      4, 2, 22, 1, 13

   b. Compare your sample with the samples of your classmates. What do you notice?

      Sample answer:
      I noticed that no two lists of five numbers are the same. None of the numbers were less than 1 or greater than 30.

5. Suppose Ms. Flores starts with the number 15 when she assigns each of the 30 students in her class a number. How can you change your selection process to accommodate the list beginning at 15?

   I would write the numbers 15–44 on slips of paper rather than the numbers 1–30.

6. Suppose that the five numbers selected from your bag resulted in five 12-year-olds.

   a. Is the sample still a random sample? Explain your reasoning.

      The situation would still be a random sample because every student in the class had the same chance of being chosen.

   b. How is this outcome different from choosing all 12-year-olds to represent the sample in Question 2?

      The difference is that Ms. Flores purposely chose only the 12-year-olds for her sample in Question 2, part (a). In that case, anyone who is not 12 years old had no chance of being chosen. In this outcome, the students were chosen randomly so that every student had the same chance of being chosen. If five 12-year-olds are chosen, it is truly by random chance.

Optimizing Learning

This activity supports planning and strategy development.

MODULE 4 • TOPIC 3 • LESSON 1 769

Chunking the Activity

- Read and discuss the situation.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Questions 4 and 5.
- Share and summarize.

ACTIVITY 1.3

Using Random Number Tables to Select a Sample

The standing desks improved student motivation, attendance, and achievement so much in Ms. Flores's class that the principal, Ms. Brown, has decided to order standing work desks for every seventh-grade class in the school.

The school has 450 seventh-grade students, and Ms. Brown would like to take a random sample of 20 seventh-graders, determine their heights, and use their mean height for the initial set-up of the standing desks. However, Ms. Brown does not want to write the 450 names on slips of paper.

Technology, such as spreadsheets, graphing technology, and random number generator applications, can be used to generate random numbers.

There are other ways to select a random sample. One way to select a random sample is to use a random number table like you used previously to simulate events.

You can use a random number table to choose a number that has any number of digits in it. For example, if you are choosing 6 three-digit random numbers and begin with Line 7, the first 6 three-digit random numbers would be: 242, 166, 344, 421, 283, and 070.

Random Number Table										
Line 6	62490	99215	84987	28759	19177	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	10415	36457
Line 8	16975	95428	33226	55903	31605	43817	22250	03918	46999	98501
Line 9	59138	39542	71168	57609	91510	77904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825

770 MODULE 4 • TOPIC 3 • LESSON 1

1. What number does "070" represent when choosing a three-digit random number? Why are the zeros in the number included? Explain your reasoning.

   If selecting a three-digit random number, 070 represents the number 70. The zeros hold a place value: the leftmost "0" means "0 hundreds" and the rightmost "0" means "0 ones."

2. If selecting a three-digit random number, how would the number 5 be displayed in the table?

   The number 5 would be displayed as 005.

3. Begin on Line 10 and select 5 three-digit random numbers.

   The following numbers would be chosen: 294, 785, 965, 250, 414.

4. Explain how to assign numbers to the 450 seventh-grade students so that Ms. Brown can take a random sample.

   Sample answer:
   If I begin at the number "1," I would use 3 digits to represent each number: 001 to 450. Any numbers over 450 would be ignored in my selection process.

5. Use Line 6 as a starting place to generate a random sample of 20 students.

   a. What is the first number that appears?

      The first number to be chosen would be 624.

Question 5 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with random number tables, assign Skills Practice Set D for this lesson.

MODULE 4 • TOPIC 3 • LESSON 1 771

b. What do you think Ms. Brown should do with that number?

Since no student was assigned the number 624, Ms. Brown should ignore the number.

c. Continuing on Line 6, what are the 20 three-digit numbers to be used to select Ms. Brown's sample?

The total list of 20 numbers is 909, 921, 584, 987, 287, 591, 917, 714, 733, 245, 502, 806, 768, 894, 384, 902, 421, 663, 444, 212

In this lesson, you have engaged with the first two phases of the statistical process: formulating questions and collecting data. In the next lessons, you will analyze and interpret findings.

d. What should you do if a three-digit number appears twice in the random number table?

I should choose the first occurrence of the number, but I should ignore any subsequent occurrences of the number.

e. Will choosing a line number affect whether Ms. Brown's sample is random?

The line number does not affect whether or not the sample is random, since all the lines of the table contain random digits.

f. Will choosing a line number affect who will be chosen for the sample?

The line number does affect who will be chosen, but the people chosen are still chosen randomly.

772 MODULE 4 • TOPIC 3 • LESSON 1

Talk the Talk

Lunching with Ms. Brown

Ms. Brown wishes to randomly select 10 students for a lunch meeting to discuss ways to improve school spirit. There are 1500 students in the school.

1. What is the population for this problem?
   The population is the 1500 students at the school.

2. What is the sample for this problem?
   The sample is the 10 students Ms. Brown selects.

3. Ms. Brown selects three to four student council members from each grade to participate. Does this sample represent all of the students in the school? Explain your answer.
   Sample answer:
   This sample would not represent the entire school. The students on student council may already be involved in school events and think there is plenty of school spirit.

4. Ms. Flores recommended that Ms. Brown use a random number table to select her sample of 10 students. How would you recommend Ms. Brown assign numbers and select her random sample?
   Sample answer:
   Ms. Brown could list all students alphabetically and then assign each a number from 0001 to 1500. She could use the random number table to select 10 four-digit numbers. She would ignore all numbers greater than 1500.

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Have groups share responses with another group.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

MODULE 4 • TOPIC 3 • LESSON 1 773

Results from a Sample Class Survey			
Student	1. What is your approximate height?	2. Do you carry a cell phone with you?	3. About how many text messages do you send in one day?
1-Catalina	60 in.	Yes	75
2-James	68 in.	Yes	5
3-Ricardo	63 in.	No	0
4-Linh	65 in.	Yes	20
5-Jackson	62 in.	Yes	50
6-Lauren	68 in.	Yes	100
7-Nakota	56 in.	Yes	60
8-Alejandra	70 in.	No	0
9-Hailey	69 in.	Yes	50
10-Joey	66 in.	Yes	100
11-Brianna	61 in.	Yes	0
12-Gracie	68 in.	Yes	60
13-Nicky	64 in.	Yes	50
14-Nia	66 in.	Yes	40
15-Miguel	60 in.	Yes	90

774 MODULE 4 • TOPIC 3 • LESSON 1

Results from a Sample Class Survey			
Student	1. What is your approximate height?	2. Do you carry a cell phone with you?	3. About how many text messages do you send in one day?
16-Lucia	67 in.	Yes	10
17-Xavier	64 in.	Yes	0
18-Ben	65 in.	Yes	25
19-Juliana	63 in.	Yes	30
20-Omar	58 in.	Yes	80

MODULE 4 • TOPIC 3 • LESSON 1 775

776 MODULE 4 • TOPIC 3 • LESSON 1

Lesson 1 Assignment

Write

Match each definition to the corresponding term.

1. the facts or numbers gathered by a survey
2. the characteristic used to describe a sample
3. the collection of data from every member of a population
4. a method of collecting information about a certain group of people by asking a question or set of questions
5. a sample that is selected from the population in such a way that every member of the population has the same chance of being selected
6. the characteristic used to describe a population
7. the entire set of items from which data can be selected
8. the data collected from part of a population

a. census

b. data

c. parameter

d. population

e. sample

f. statistic

g. survey

h. random sample

Remember

Statistics obtained from data collected through a random sample are more likely to be representative of the population than those statistics obtained from data collected through non-random samples.

Write

1. b
2. f
3. a
4. g
5. h
6. c
7. d
8. e

MODULE 4 • TOPIC 3 • LESSON 1 ASSIGNMENT 777

Lesson 1 Assignment

Practice

1. Explain which sampling method is more representative of the population.

   a. Lauren and Linh live in Springfield, RI, and are interested in the average number of skateboarders who use their town's skate park in one week. Lauren recorded the number of skateboarders who used the park each week in June, July, and August. Linh recorded the number of skateboarders who used the park the second week of each of the 12 months.

   Lauren's sample will be more representative. More skateboarders will use the park in June than in January.

   b. Nakota and Alejandra want to determine the most popular lunch choice in the school cafeteria among seventh-graders. They decide to collect data from a sample of seventh-graders at school. Nakota surveys 20 seventh-graders that are in line at the cafeteria. Alejandra surveys 20 seventh-graders whose student ID numbers end in 9.

   Alejandra's sample is more representative. Her sampling strategy represents a random sampling of seventh-graders. Nakota's sampling strategy only represents students who ate school lunch on a particular day.

2. The coach of the soccer team is asked to select 5 students to represent the team in the Homecoming Parade. The coach decides to randomly select the 5 students out of the 38 members of the team.

   a. What is the population for this problem?

   The population is the members of the soccer team.

778 MODULE 4 • TOPIC 3 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

b. What is the sample for this problem?

The sample is the 5 students selected to participate in the parade.

c. Suggest a method for selecting the random sample of 5 students.

Sample answer:

The coach could use the players' uniform numbers as their assigned random number.

3. Consider the population of integers from 8 to 48.

a. Select a sample of 6 numbers. Is this a random sample? Explain your reasoning.

Answers will vary.

If students placed the numbers in a bag or used a random number generator, this was a random sample.

b. How can you assign random numbers to select a sample using a random number table?

Assign the numbers 08—48 to the numbers 8—48. I will ignore two-digit numbers less than 08 and greater than 48.

c. Use the random number table to choose 6 numbers from this population.

Answers will vary.

MODULE 4 • TOPIC 3 • LESSON 1 ASSIGNMENT 779

Lesson 1 Assignment

d. Use a different line of the random number table to choose 6 numbers from this population.

Answers will vary.

e. Compare the results from each sample. Do the results surprise you? Explain.

Sample answer:

None of the samples were the same. This is what I expected, because each number has an equal chance of being selected in each sample.

4. The manager of the local mall wants to know the mean age of the people who shop at the mall and the stores in which they typically shop. Jackson has been put in charge of collecting data for the mall. He decides to interview 100 people one Saturday because it is the mall's busiest shopping day.

a. What is the population for this situation?

The population is all of the people who shop at the mall.

b. What is the sample?

The sample is the 100 shoppers that Jackson interviews.

780 MODULE 4 • TOPIC 3 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

c. When Jackson calculates the mean age of the people who shop at the mall, will he be calculating a parameter or a statistic? Explain your reasoning.

Jackson will be calculating a statistic because he is gathering his data from a sample of 100 shoppers. He is not gathering his data from the entire population.

d. Describe three different ways Jackson can take a sample. Describe how any of these three possible samples may cause the results of Jackson's survey to inaccurately reflect the average age of shoppers at the mall.

Sample answer:
Jackson can interview: (1) the first 100 shoppers who come into the mall. This could result in bias because the early morning shoppers are typically older people; (2) 100 shoppers who are shopping at a trendy clothing store. This could result in bias because shoppers at these stores tend to be in their teens and early twenties; (3) 100 shoppers at the toy store. This could result in bias because these shoppers tend to be parents with children.

e. Jackson decides to use a random number table to choose the next 10 people to interview. Explain how to choose 10 two-digit numbers between 1 and 80 from a random number table.

Because you are choosing numbers between 1 and 80, you need to use two digits to represent each number. Use 01 for 1, 02 for 2, etc.

MODULE 4 • TOPIC 3 • LESSON 1 ASSIGNMENT 781

Lesson 1 Assignment

f. Record the numbers of the people who would be interviewed. Be sure to specify which line you used to generate your list.

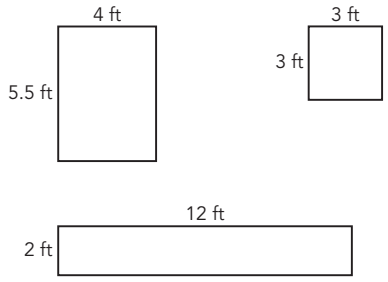
Answers will vary but should include 10 numbers between 01 and 80 and specify which line number was used as a starting point.

g. Suppose Jackson uses a random number table to generate his sample, resulting in Jackson interviewing 10 people who all go into the gaming store. Does this mean the sample is not random? Explain.

This sample would still be random because every person who shops at the mall that day has the same chance of being chosen.

Prepare

Mr. Lopez has three bulletin boards in his classroom. What is the average amount of space per bulletin board?



The average amount of space per bulletin board is approximately 18.3 square feet.

782 MODULE 4 • TOPIC 3 • LESSON 1 ASSIGNMENT

2 Using Random Samples to Draw Inferences

MATERIALS

Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

Students use statistical information gathered from a sample to determine a parameter for a population. They complete this process two times with one scenario. The first time, students may select the sample using various methods; however, the second time, they follow a specific strategy to select a random sample. In each case, students use proportional reasoning to estimate the parameter. They compute percent error and conclude that statistics obtained from samples are more likely to represent the parameter of the population when the sample is randomly chosen. They then analyze data from 100 samples and predict the parameter from the data. Finally, students are provided with a scenario and must design and carry out a sampling plan to estimate the parameter.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

7.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

(TEKS continued on next page)

ELPS

(1) Learning Strategies

The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(2) Listening

The student is expected to:

(H) understand implicit ideas and information in increasingly complex spoken language commensurate with grade-level learning expectations.

(5) Writing

The student is expected to:

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary

CC BY NC

GRADE 7 TEKS *(continued)*

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6B select and use different simulations to represent simple and compound events with and without technology.

7.6F use data from a random sample to make inferences about a population.

Measurement and Data

(12) The student applies mathematical process standards to use statistical representations to analyze data.

The student is expected to:

7.12B use data from a random sample to make inferences about a population.

ESSENTIAL IDEAS

- Statistics obtained from samples are more likely to represent the parameter of the population when the sample is randomly chosen.
- Statistics are used to estimate parameters.
- Proportional reasoning can be used with statistics to estimate parameters.
- Percent error can be used as a measure of the variation between a statistic and a parameter.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Selecting Squares 10–15 minutes

ESTABLISH A SITUATION

In this scenario, an Art Club is planning to tile a designated area of square sections. Students suggest a method to select a sample of squares to determine the number of tiles needed for the larger area. Students reference this scenario throughout the lesson.

DEVELOP

Activity 2.1: Using Multiple Samples to Make Predictions 30 minutes

INVESTIGATION, WORKED EXAMPLE

Students take samples of the squares in the Art Club scenario to predict the area of all the squares on a floor plan. They select a sample, calculate the area of their sample, and create a dot plot of the class data. Students then use their sample and proportional reasoning to predict the total area of the squares.

DAY 2

Activity 2.2: Using Random Samples to Make Predictions 20–25 minutes

INVESTIGATION

This activity parallels the previous activity, but this time, students take random samples of the squares in the Art Club scenario. They select a random sample, calculate the area of their sample, and create a dot plot of the class data. They then use their sample and proportional reasoning to predict the total area of the squares. Students compare these results to the results of the previous activity. Finally, they are given the actual total area and calculate the percent error for the statistics calculated in both this and the previous activity.

Activity 2.3: Using Samples to Justify Predictions 15–20 minutes

REAL-WORLD PROBLEM SOLVING

Students use data from 100 samples of the same population to make inferences about the population. They create and interpret a dot plot of the results of the samples. Students then use the results and the true value of the population parameter to further analyze the scenario.

DAY 3

DEMONSTRATE

Talk the Talk: Pumpkin Patch 40–45 minutes

EXIT TICKET PROCEDURE

Students use a diagram of a pumpkin and gourd field to estimate the number of pumpkins in the field. They are expected to use random sampling and proportional reasoning to complete this task.

CC BY NC

Getting Started

ENGAGE

Selecting Squares

Facilitation Notes

In this activity, students are presented with a scenario and suggest a method to sample the population. This situation will be used in subsequent activities.

Ask a student to read the introduction aloud. Discuss the context and complete Question 1 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain the situation in your own words. • Does it appear that there are more large or small squares in the diagram? • Do you think more of the total colored area is from the small squares or the large squares? Explain your thinking.

Summary

Sampling can be used to make predictions in a challenging problem.

ACTIVITY 2.1

Using Multiple Samples to Make Predictions

DEVELOP

Facilitation Notes

In this activity, students use the Art Club scenario to select a sample, calculate the area of the sample, and create a dot plot using class data. They answer related questions and make predictions using proportional reasoning.

Students may question why they are using a sample for this problem when the actual area can be calculated. This problem was developed so students could use a sample and compare it to the actual area for learning purposes. Once students are convinced of the usefulness of this process, it can be duplicated on situations when using the entire population is impossible or inconvenient to calculate precisely.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTION TO SUPPORT DISCOURSE

Probing	• Do you think Hailey's method uses random selection? Why or why not?

Ask a student to read the Worked Example aloud and discuss as a class. Have students work with a partner or in a group to complete Questions 3 and 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you select which squares represented the sample?
Seeing structure	• Why were all the areas perfect square numbers?

Ask a student to read the information after Question 4 aloud and complete Question 5 as a class. Note that students will need to record their areas on a class dot plot or in a central location. The purpose of the dot plot is to show the data visually. Discuss that the mean may be a decimal number and show students how to represent that number on the dot plot.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is the purpose of using the dot plot to display your data? • Why didn't all groups get the same total area? • Was your sample randomly selected?
Reflecting and justifying	• Who from the class selected a random sample? Who did not? Explain.

COMMON MISCONCEPTION

Students may think that closing their eyes and randomly pointing at a square is a way of randomly selecting a sample. This is not the case; the larger squares have a greater chance of being selected because their areas are larger. This misconception is addressed in the next activity.

Have students work with a partner or in a group to complete Questions 6 through 8. Share responses as a class. Note that for Questions 6 and 7, students should use their personal estimates, not the class estimate calculated in Question 5, part (c).

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What does the number in the numerator of the ratio represent with respect to the problem situation? • What does the number in the denominator of the ratio represent with respect to the problem situation? • What does the variable represent in the proportion? • What is the unit of measurement in this situation? • Where do the majority of the estimated total area lie on the dot plot?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Common distributions of data, such as skewed left, skewed right, and symmetric, can help determine whether the mean or median is a better measure of center. Proportional reasoning can be used to estimate and make predictions.

ACTIVITY 2.2 Using Random Samples to Make Predictions

Facilitation Notes

This activity parallels the previous activity, but this time, students select a random sample, calculate the area of their sample, and create a dot plot of the class data. They then use their sample and proportional reasoning to make predictions and compare to those from the previous activity. Actual data is used to calculate the percent error for the statistics previously stated.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction aloud. Complete Questions 1 through 3 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Explain why all squares have an equal chance of being selected using a random number generator.
Seeing structure	• How could Brianna change her method so it can be a random sample?

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How did you set up your random number generator to select five random numbers? • How did you set up your ratio to estimate the total area of the color tiles?

Ask a student to read Question 6 aloud and complete as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you use the terms *peaks*, *clusters*, or *gaps* to describe the dot plot and assist with your estimation? • Why do you think the dot plot has that distribution?

Have students work with a partner or in a group to complete Question 7. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How did you set up your ratio to estimate the total area of the color tiles?

Ask a student to read Question 8 aloud and complete as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you use the terms *peaks*, *clusters*, or *gaps* to describe the dot plot and assist with your estimation? • Why do you think the dot plot has that distribution?!

Have students work with a partner or in a group to complete Question 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Is your percent error positive or negative? What does this indicate about your estimate?
Seeing structure	• Why would a random sample more likely have a smaller percent error than other types of samples?

Summary

Proportional reasoning can be used to make predictions, and actual data can be used to calculate percent error.

STAMP THE LEARNING

ACTIVITY 2.3

Using Samples to Justify Predictions

Facilitation Notes

In this activity, students are presented with a scenario and use data from a table to make inferences about the population. Then, they create and interpret a dot plot. They use the results and the true value of the population parameter to answer related questions.

Ask a student to read the introduction aloud. Discuss as a class. Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Choose a row of the table and explain what it means. • How would you describe the distribution of the data?
Probing	• Did you use the table or dot plot to make your decision? Explain your strategy. • How did you determine how many students will win the store credit? • Explain how you calculated the percent error.

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

Random samples and proportional reasoning can be used to make predictions for an entire population.

DEMONSTRATE

Talk the Talk

PUMPKIN PATCH

Facilitation Notes

In this activity, students are presented with a scenario that includes a diagram. They use the diagram for estimation purposes. Students devise a method for random sampling and use proportional reasoning to solve the problem situation.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete this task. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Provide additional scaffolding by limiting the size of the pumpkin patch that they estimate.

Alternatively, you can assist them in selecting a random sample:

- Assign each row a random number and randomly select one of the rows.
- Assign each column a random number and randomly select one of the columns.
- Divide the pumpkin patch into 10 or 20 regions and randomly select one of them.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How do your results compare to those of your classmates?
Probing	• How did you include sampling in your data collection? • Explain how you used proportions in your data analysis.
Seeing structure	• How is this problem related to the other problems you solved?
Reflecting and justifying	• How confident are you in your estimation? • Explain how your strategies are best practices when estimating a parameter of a population.

Have students read and answer the Essential Question on the lesson opener page.

Summary

Sampling, in conjunction with proportional reasoning, can be used to estimate and predict a population's parameter.

STAMP THE LEARNING

2 Using Random Samples to Draw Inferences

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Investigate how results from a random sample are more reliable for representing the population than results from a sample that is not random.
- Use data from a random sample and proportional reasoning to draw inferences about a population parameter.
- Calculate the percent error between population parameters and sample statistics.

You know how to generate random samples.

How can you use random samples to make inferences about population parameters?

Sample answer:

When you estimate a parameter that is a count rather than a percent, you can use proportional reasoning to scale up from the ratio of the number of observations in your sample to the statistic.

MODULE 4 • TOPIC 3 • LESSON 2 783

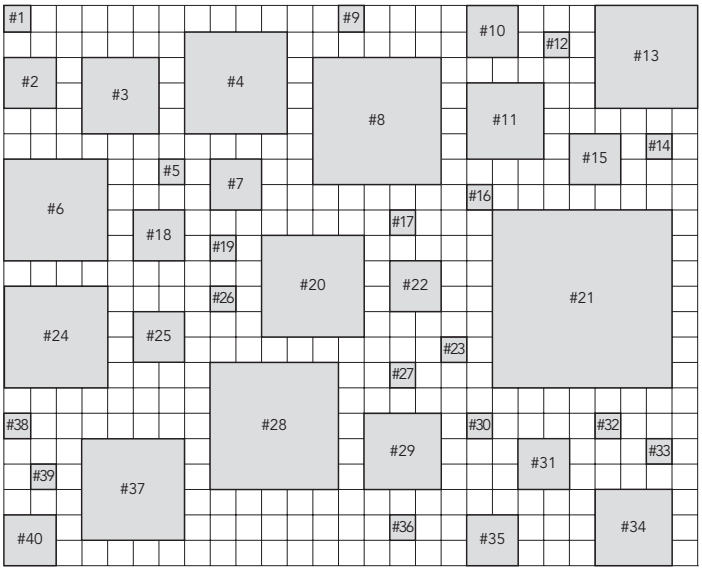
Chunking the Activity

- Read and discuss the introduction.
- Complete the activity as a class.
- Share and summarize.

Getting Started

Selecting Squares

The Art Club created a design on the floor of the art room. Each of the 40 numbered squares on the floor will have colored tiles. The club needs to calculate how many colored tiles they must buy. Each small grid square represents a square that is one foot long and one foot wide.



784 MODULE 4 • TOPIC 3 • LESSON 2

The Art Club needs to complete the art room floor plan project by tomorrow! Because they are short of time, they decide they do not have enough time to measure all 40 squares.

1. Suggest a method for the Art Club to use to sample the 40 squares. What would they do once they collect their sample?

   Sample answer:
   The Art Club could write down the numbers 1 through 40 on equal-sized pieces of paper, put the papers in a bag, draw out a piece of paper, and record the result. They could repeat this process a given number of times to randomly select a sample of squares. Then, they could determine the area of each square that is included in the sample. They could write a ratio of the number of squares included in the sample to the total area of the sample. Finally, they could write a proportion to scale up this ratio and estimate the area of all 40 squares on the Art Club floor.

MODULE 4 • TOPIC 3 • LESSON 2 785

Chunking the Activity

- Read and discuss the introduction.
- Group students to answer Questions 1 and 2.
- Check in and share.
- Read and discuss the Worked Example.
- Group students to complete Questions 3 and 4.
- Check in and share.
- Read and discuss the information.
- Complete Question 5 as a class.
- Group students to complete Questions 6–8.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 2.1

Using Multiple Samples to Make Predictions

I wonder why Hailey picked those squares?

Hailey says, "Since we don't have a lot of time, why don't we just select 5 squares and calculate their total area? That should be good enough for us to estimate the total area of all the squares that will need colored tiles."

1. Do you think Hailey's idea could work to estimate the total area for all the squares that need colored tiles? Explain your reasoning.

   Sample answer:
   Yes. Hailey's idea could work if they collect a random sample. Then, they need to scale up to the area of 40 squares.

2. What is the population for this problem?

   The population is the 40 numbered squares on the floor plan.

WORKED EXAMPLE

As you learned, you can select a sample to estimate parameters of a population. In this problem situation, the Art Club is going to set up a ratio using the sample of squares they select to the total area of those sample squares.

They will use the ratio:
number of squares in the sample : total area of the sample squares.

Hailey decides to select the following squares: 1, 15, 21, 37, and 40.

Total area

#1: $1 \cdot 1 = 1$ square foot

#15: $2 \cdot 2 = 4$ square feet

#21: $7 \cdot 7 = 49$ square feet

#37: $4 \cdot 4 = 16$ square feet

#40: $2 \cdot 2 = 4$ square feet

Ratio

number of squares in the sample	:	total area of the sample squares
5 squares	:	74 square feet

So, the total area of these 5 squares is 74 square feet.

786 MODULE 4 • TOPIC 3 • LESSON 2

3. Select 5 numbered squares that you think best represent the 40 squares that need colored tiles. Record the numbers of the squares you selected.

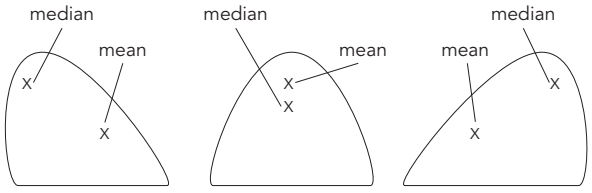
   Answers will vary, because students can pick any 5 squares they want.

4. Calculate the total area of the 5 numbered squares you selected.

   The areas of all 40 squares are provided in the margin for your reference, not for the students.

   Sample answer:
   The total area of the 5 squares I selected is 100 square feet.

Now that you have collected your data, you need to analyze the data. Remember, there are three common distributions of data: skewed left, skewed right, and symmetric. The distribution of data can help you determine whether the mean or median is a better measure of center. Examine the diagrams shown.



Skewed Right

The mean of a data set is greater than the median when the data are skewed to the right.

The median is the best measure of center because the median is not affected by very large data values.

Symmetric

The mean and median are equal when the data are symmetric.

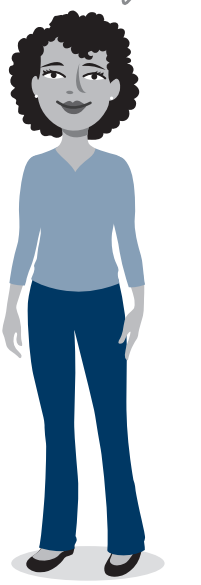
Skewed Left

The mean of a data set is less than the median when the data are skewed to the left.

The median is the best measure of center because the median is not affected by very small data values.

The median is not affected by very large or very small data values, but the mean is affected by these large and small values.

You will use these descriptions throughout this lesson.



STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

#1: $1 \cdot 1 = 1$ square foot
#2: $2 \cdot 2 = 4$ square feet
#3: $3 \cdot 3 = 9$ square feet
#4: $4 \cdot 4 = 16$ square feet
#5: $1 \cdot 1 = 1$ square foot
#6: $4 \cdot 4 = 16$ square feet
#7: $2 \cdot 2 = 4$ square feet
#8: $5 \cdot 5 = 25$ square feet
#9: $1 \cdot 1 = 1$ square foot
#10: $2 \cdot 2 = 4$ square feet
#11: $3 \cdot 3 = 9$ square feet
#12: $1 \cdot 1 = 1$ square foot
#13: $4 \cdot 4 = 16$ square feet
#14: $1 \cdot 1 = 1$ square foot
#15: $2 \cdot 2 = 4$ square feet
#16: $1 \cdot 1 = 1$ square foot
#17: $1 \cdot 1 = 1$ square foot
#18: $2 \cdot 2 = 4$ square feet
#19: $1 \cdot 1 = 1$ square foot
#20: $4 \cdot 4 = 16$ square feet
#21: $7 \cdot 7 = 49$ square feet

MODULE 4 • TOPIC 3 • LESSON 2 787

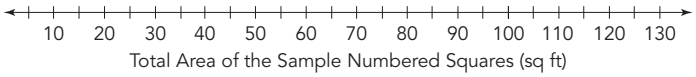
Optimizing Learning

This activity fosters collaboration and community.

5. Compare the total area of your sample to the total areas of your classmates' samples.

   a. Record the total area you calculated for your sample on the dot plot shown. Then, record the total areas your classmates calculated on the same dot plot.

   Answers will vary.



   b. Describe the distribution of the dot plot.

   Answers will vary.

   c. Estimate the total area for a sample of 5 squares using the data values in the dot plot.

   Answers will vary but should refer to where the majority of the sums lie on the dot plot.

You can set up a ratio of the sample of 5 squares to the total area of those 5 sample squares, as Hailey did, and you can then set up a proportion to estimate the total area of those 40 squares in the Art Club's floor plan design. In doing so, you are scaling up from your sample to the population of the squares.

6. Write a ratio of the number of squares in your sample to the total area of the squares in your sample.

   Sample answer:
   5 squares to 100 square feet.

#22: $2 \cdot 2 = 4$ square feet
#23: $1 \cdot 1 = 1$ square foot
#24: $4 \cdot 4 = 16$ square feet
#25: $2 \cdot 2 = 4$ square feet
#26: $1 \cdot 1 = 1$ square foot
#27: $1 \cdot 1 = 1$ square foot
#28: $5 \cdot 5 = 25$ square feet
#29: $3 \cdot 3 = 9$ square feet
#30: $1 \cdot 1 = 1$ square foot
#31: $2 \cdot 2 = 4$ square feet
#32: $1 \cdot 1 = 1$ square foot
#33: $1 \cdot 1 = 1$ square foot
#34: $3 \cdot 3 = 9$ square feet
#35: $2 \cdot 2 = 4$ square feet
#36: $1 \cdot 1 = 1$ square foot
#37: $4 \cdot 4 = 16$ square feet
#38: $1 \cdot 1 = 1$ square foot
#39: $1 \cdot 1 = 1$ square foot
#40: $2 \cdot 2 = 4$ square feet

7. Estimate the total area of all 40 squares on the floor plan using proportional reasoning.

Sample answer:

$$\frac{\text{number of squares in the sample}}{\text{total area in the sample}} = \frac{\text{total number of squares}}{\text{total area of all squares}}$$

$$\frac{5}{100} = \frac{40}{x}$$

$$5x = 4000$$

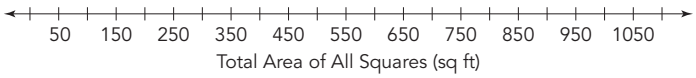
$$x = 800 \text{ sq ft}$$

The estimated total area of all 40 squares on the floor plan is 800 square feet.

8. Compare the estimated total area of the 40 squares on the floor plan with your classmates' estimated total areas.

   a. Record the estimated total area of the 40 squares on the floor plan on the dot plot shown. Then, record your classmates' estimates of the total area of the 40 squares.

   Answers will vary.



   b. Look at the distribution of the dot plot shown in section a, above. Then, look at the distribution of the dot plot shown in Question 5. What are some similarities or some differences you see between this dot plot and the one pictured on Question 5? What inferences can be made about the means and medians of the data sets based on these dot plots?.

   Answers will vary.

   c. Estimate the total area of the squares in the floor plan using data values in the dot plot.

   Answers will vary but should refer to where the majority of the estimated total areas lie on the dot plot.

Activity 2.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with describing distributions, assign Skills Practice Set A for this lesson.

MODULE 4 • TOPIC 3 • LESSON 2 789

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Complete Questions 1–3 as a class.
- Group students to complete Questions 4 and 5.
- Check in and share.
- Read and complete Question 6 as a class.
- Group students to complete Question 7.
- Check in and share.
- Read and complete Question 8 as a class.
- Group students to complete Question 9.
- Share and summarize.

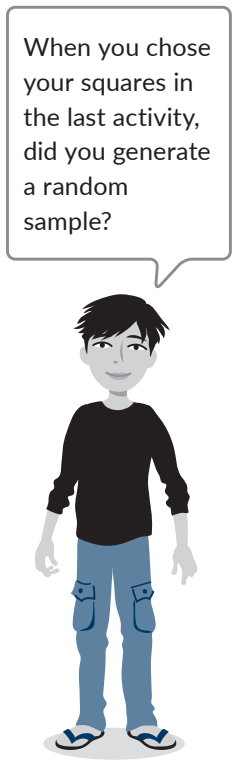
ACTIVITY 2.2

Using Random Samples to Make Predictions

Samples chosen by looking at the squares and trying to pick certain squares will probably contain many more of the larger squares in the floor plan. Most of the squares actually have small areas (17 of the 40 squares have an area of 1 square foot, and 10 of the 40 squares have an area of 4 square feet); therefore, you need to use another method to randomly choose squares.

1. How might you randomly choose 5 squares for your sample?

   Sample answer:
   I can use a random number table, use a random number generator, or place numbers on equal-sized slips of paper and randomly choose 5 slips of paper.



2. Brianna has a suggestion on how to randomly select the numbered squares. She says, "I can cut out the squares from the floor plan and then I can put these squares in a bag. That will help me randomly select squares." Will Brianna's method result in a random sample? Explain your reasoning or suggest a way to modify her strategy.

   Brianna's method will not result in a random sample. The squares with larger dimensions will have a better chance of being chosen because of their size. The larger squares are more easily selected in a bag than the smaller squares.

3. You have used random number tables to generate a random sample. You can also use technology, in the form of a random number generator, to select a random sample. How can you use a random number generator to choose 5 numbered squares for your sample?

   I can set the random number generator to have a minimum, or lower limit, of 1 and a maximum, or upper limit, of 40. Then, I can click "enter" or "generate" to obtain a random number. I can repeat this process 4 more times to get a total of 5 random numbers.

790 MODULE 4 • TOPIC 3 • LESSON 2

4. Use a random number generator to choose 5 numbered squares using two-digit numbers ranging between 1 and 40. Record the square numbers.

   Answers will vary, but each number should be within the range of 1 through 40.

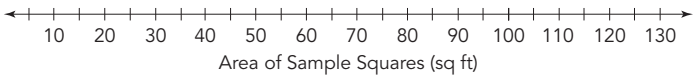
5. Calculate the total area of the 5 numbered squares you selected.

   Answers will vary.

6. Compare the total area of your sample to the total areas your classmates calculated from their random samples.

   a. Record the total areas your classmates calculated and the total area you calculated on the dot plot shown.

      Answers will vary.



   b. How do the values plotted on this dot plot compare to the values plotted in Activity 2.1, Question 5? Compare the shapes and the centers of the data values for both dot plots.

      Sample answer:
      The data values are probably lower than those in the dot plot in Activity 2.1, Question 5.

Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

How does using a random number generator compare to using a random number table?

MODULE 4 • TOPIC 3 • LESSON 2 791

7. Using proportional reasoning, estimate the total area of all 40 squares on the floor plan using the area you calculated from the random sample.

Sample answer:
With a total area of 19 square feet for the 5 randomly chosen squares:

$$\frac{\text{number of squares in the sample}}{\text{total area in the sample}} = \frac{\text{total number of squares}}{\text{total area of all squares}}$$

$$\frac{5}{19} = \frac{40}{x}$$

$$5x = 760$$

$$x = 152$$

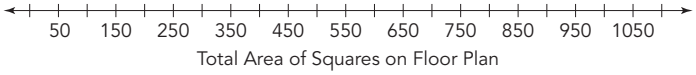
The estimated total area of all 40 squares is 152 square feet.

Activity 2.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice answering questions using data and a random number generator, assign Skills Practice Set B for this lesson.

8. Compare your estimated total area from the random sample for all 40 squares with your classmates' total area estimates.

a. Record your estimated total area of the 40 squares on the dot plot shown. Then, record your classmates' estimates of the total area of the 40 squares on the dot plot.

Answers will vary.



Total Area of Squares on Floor Plan

b. Estimate the total area of the squares in the floor plan using data values in the dot plot.

Answers will vary but should refer to where the majority of the estimated total areas lie on the dot plot.

c. How do the values plotted on this dot plot compare to the values plotted in Activity 2.1, Question 8? Compare the distributions and the centers of the data values for both dot plots.

Sample answer:
The data values in this dot plot are also probably lower than the dot plot data values in Activity 2.1, Question 8.

792 MODULE 4 • TOPIC 3 • LESSON 2

9. The actual total area of the 40 numbered squares is 288 square feet.

   a. Is 288 a parameter or a statistic? Explain your reasoning.

   The total area of 288 square feet is a parameter because it describes the population of all the numbered squares.

Percent error is the absolute value of the ratio of the difference between the statistic and parameter to the parameter.

$$\left|\frac{\text{Statistic - Parameter}}{\text{Parameter}}\right|$$

   b. Locate 288 on each dot plot you created in the previous activity and this activity. What do you notice?

   Sample answer:
   Most samples gave a total area greater than the total area of all the numbered squares. It appears that the randomly selected samples gave a more accurate estimated total area for all 40 squares than the 5 squares my classmates and I selected.

   c. Calculate the percent error for the parameter and your statistics from this activity and the previous activity for the total sum of the areas of the squares.

   Sample answer:
   For my statistic of 800 square feet from Activity 2.1, the percent error is 177.8%. For my statistic of 152 square feet, the percent error is 47.2%.

   d. Based on your percent error, which sample is more accurate? Is this what you expected? Explain your reasoning.

   Sample answer:
   The statistics from the random sample had a smaller percent error. This is what I expected because random samples are better representations of populations than other types of samples.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words. Have students reflect on where they see *percent error* used in real-world situations.

MODULE 4 • TOPIC 3 • LESSON 2 793

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 2.3

Using Samples to Justify Predictions

Ask Yourself . . .
Can you restate the problem in your own words?

The student council holds regular fundraisers to raise money for community service projects. To raise money for Back-to-School Backpacks for the local homeless shelter, they hold a Gumball Guessing Competition. They place differently colored gumballs in a large, clear gumball machine. Students pay $1.00 to predict the percent of red gumballs in the machine. Any students who predict within 5% of the actual percent win a $5.00 credit at the school store and a share of the gumballs.

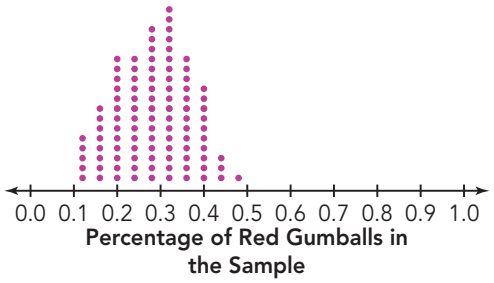
To make their predictions, students take a sample of 25 gumballs (and then return the gumballs to the machine), and they use the percent of red gumballs in the sample to make their guess. The results from the first 100 students' samples are provided in the table.

Percent of Red Gumballs in the Sample	Number of Samples
12%	5
16%	8
20%	13
24%	13
28%	16
32%	18
36%	13
40%	10
44%	3
48%	1

1. What do you notice about the data in the table?

Sample answer:
I notice that 20%, 24%, and 36% red gumballs are equally likely. It is twice as likely to have 40% red gumballs than 12% red gumballs. It's more likely to have 32% red gumballs than 48% red gumballs.

2. Create a dot plot of the results. Be sure to label your dot plot.



794 MODULE 4 • TOPIC 3 • LESSON 2

3. Use the results to predict the likely percent of gumballs that are red. Explain your reasoning.

   Sample answer:
   Because the most frequent sample percentage was 32%, I think about 32% of the gumballs are red.

4. How many of the students obtained a sample that was less than 25% red gumballs?

   39 of the 100 samples had less than 25% red gumballs.

5. The gumball machine holds 10,000 gumballs, and there are 2936 red gumballs in the machine.

   a. How many students will split the gumballs? How many gumballs will each student receive?

      The percentages within 5 points of 29.36% are 28% and 32%, so 34 students will share the gumballs. Each will receive 294 or 295 gumballs.

   b. Is it reasonable that none of the estimates were equal to the actual percent of red gumballs? Explain your reasoning.

      Yes. Because all of the predictions are equal to the quotient of a whole number of gumballs from 0 to 25 total gumballs, the only percentages that could be predicted were multiples of 4.

Activity 2.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with using data to draw inferences, assign Skills Practice Set C for this lesson.

MODULE 4 • TOPIC 3 • LESSON 2 795

c. Suppose a disgruntled student argued that there must be at least 40% red gumballs. Use the analysis to explain why this is unlikely.

Sample answer:
Out of the 100 samples, only 14 were at least 40% red. In contrast, 60% of the samples were from 24% to 36%, so it is more likely that the true percentage was in that range.

d. The principal did not take a random sample to create his estimate. Instead, he based his estimate on a visual inspection of the gumball machine. His guess was 35%. Calculate the percent error of the principal's guess from the true percent of red gumballs.

The percent error is 19.2%.

796 MODULE 4 • TOPIC 3 • LESSON 2

Talk the Talk

Pumpkin Patch

Right before pumpkin picking season, you are hired to work at a Pumpkin Patch. Your first task is to determine the number of pumpkins available for picking. In addition to growing pumpkins in the pick-your-own field, the Pumpkin Patch also grows gourds.

The diagram on the next page shows the field that contains the pumpkins and the gourds. The stars represent the gourds. Notice that there are also gaps in the field.

You and your manager agree that it would take too long to count all the pumpkins in the field.

1. Design and carry out a method to estimate the total number of pumpkins in the field without counting all the shapes. Then, prepare a presentation for your classmates that includes an explanation of your method, your results, and a justification of your estimate.

   Answers will vary, but students should describe an appropriate sampling method that takes into account the different proportions of pumpkins and gourds. They should divide the field into equally-sized areas and sample from the different areas. Students should use proportional reasoning in their solutions.

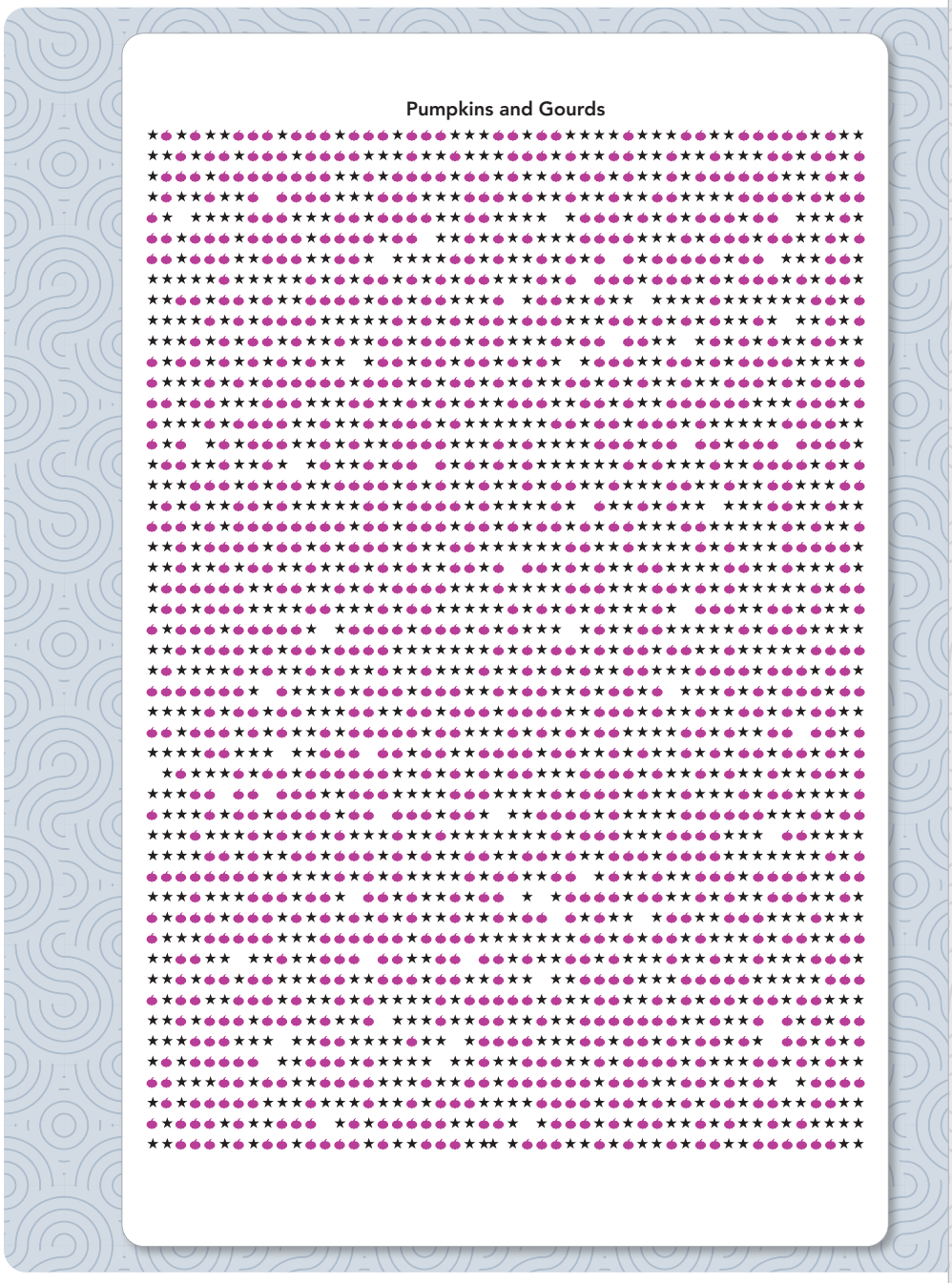
Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.
- See the Facilitating Productive Struggle section in the Course and Implementation Guide for additional guidance on supporting students through problem-solving activities.

MODULE 4 • TOPIC 3 • LESSON 2 797



798 MODULE 4 • TOPIC 3 • LESSON 2

Lesson 2 Assignment

Write

Explain the purpose and process of taking a sample when you are interested in a characteristic of a population.

Remember

When you are estimating a parameter that is a count, rather than a percentage, you can use proportional reasoning to scale up from the ratio of the number of observations in your sample to the statistic.

Practice

The table at the end of this assignment shows the names and ages at inauguration of 46 Presidents of the United States.

1. You want to determine the mean age of the U.S. Presidents at their inaugurations. Instead of calculating the mean using the 46 Presidents' ages, you will take a sample.

   a. What is the population for this situation?

   The population for this situation is all U.S. Presidents.

   b. Select 10 presidents whose ages best represent the mean age of a U.S. President at inauguration.

   Sample answer:
   George Washington, Thomas Jefferson, John Quincy Adams, Rutherford B. Hayes, Grover Cleveland (2nd inauguration), Woodrow Wilson, Herbert Hoover, Lyndon B. Johnson, Richard Nixon, and George W. Bush.

   c. Record the ages of these presidents.

   Sample answer:
   57, 57, 57, 54, 55, 56, 54, 55, 56, 54

Write

The purpose of taking a sample is to use it to learn about a characteristic of a population because it is often difficult to directly calculate the corresponding population parameter. Statistics obtained from random samples can be used as an estimate of the population parameter.

MODULE 4 • TOPIC 3 • LESSON 2 ASSIGNMENT 799

Lesson 2 Assignment

d. Explain why you chose these Presidents.

Sample answer:
When I looked over the ages in the list of presidents, I noticed that many of them were in their mid-fifties. So, I chose 10 presidents whose age at their inauguration was in the mid-fifties.

e. Is this a random sample? Explain your reasoning.

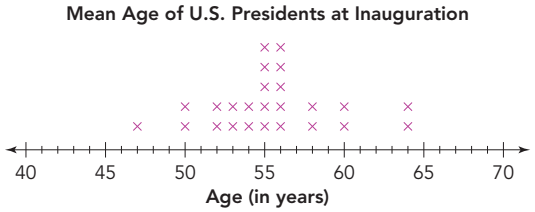
This is not a random sample because all of the presidents did not have an equal chance of being selected.

f. Calculate the mean age of the presidents you selected. Round to the nearest year.

Sample answer:
The mean age at inauguration of the presidents I selected is about 56 years.

g. Record the mean age you calculated and the mean age your classmates calculated on a dot plot.

Sample dot plot:



800 MODULE 4 • TOPIC 3 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

h. Describe the distribution of the dot plot in part (g).

Sample answer:
The data is symmetric.

2. You decide to use another method to choose presidents.

a. Randomly select 10 presidents. Record the ages of these presidents.

Sample answer:
Bill Clinton, James Buchanan, Warren G. Harding, William Howard Taft, Grover Cleveland (1st inauguration), Franklin Pierce, Harry S. Truman, Andrew Jackson, James K. Polk, and John F. Kennedy.
46, 65, 55, 51, 47, 48, 60, 61, 49, 43

b. Is this a random sample? Explain your reasoning.

This is a random sample because all of the presidents have an equal chance of being selected.

c. Calculate the mean age of the presidents you selected. Round to the nearest year.

Sample answer:
The mean age at inauguration of the presidents I selected is about 53 years.

d. Is the mean age of the 10 presidents you selected a statistic or parameter? Explain your reasoning.

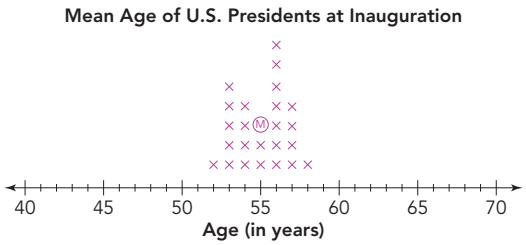
The mean age is a statistic because it describes the mean of a sample, not a population.

MODULE 4 • TOPIC 3 • LESSON 2 ASSIGNMENT 801

Lesson 2 Assignment

e. Record the mean age you calculated and the mean age your classmates calculated on a dot plot.

Sample dot plot:



f. Describe the distribution of the dot plot in part (e).

Sample answer:
The dot plot in part (e) is symmetric.

g. Based on the distribution of the data in your dot plot, what is the relationship between the median and the mean?

Sample answer:
The median and the mean are approximately equal.

h. Calculate the actual mean age at inauguration of the 46 Presidents. Round to the nearest year. Plot this age with an M on the dot plot in part (e).

The actual mean age at inauguration of the 46 presidents is about 55.

i. Calculate the percent error between the statistic from your random sample and the true mean age.

Sample answer:
The percent error between my statistic of 53 and the parameter 55 is about 3.6%.

3. How would the dot plot change if you had purposefully selected 10 presidents who were older in age as your sample?

Sample answer:
The dot plot would still likely be symmetrical, just centered around a greater median.

802 MODULE 4 • TOPIC 3 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

Prepare

Determine each measure given the data provided. Show your work.

47, 35, 37, 32, 39, 38, 34, 36, 35

1. Mean

   37

2. Median

   36

3. Mode

   35

4. Range

   15

MODULE 4 • TOPIC 3 • LESSON 2 ASSIGNMENT 803

Lesson 2 Assignment

Presidents of the United States			
President	Age at Inauguration	President	Age at Inauguration
George Washington	57	Grover Cleveland	55
John Adams	61	William McKinley	54
Thomas Jefferson	57	Theordore Roosevelt	42
James Madison	57	William Howard Taft	51
James Monroe	58	Woodrow Wilson	56
John Quincy Adams	57	Warren G. Harding	55
Andrew Jackson	61	Calvin Coolidge	51
Martin Van Buren	54	Herbert Hoover	54
William Henry Harrison	68	Franklin D. Roosevelt	51
John Tyler	51	Harry S. Truman	60
James K. Polk	49	Dwight D. Eisenhower	62
Zachary Taylor	64	John F. Kennedy	43
Millard Fillmore	50	Lyndon B. Johnson	55
Franklin Pierce	48	Richard Nixon	56
James Buchanan	65	Gerald Ford	61

804 MODULE 4 • TOPIC 3 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

Presidents of the United States			
President	Age at Inauguration	President	Age at Inauguration
Abraham Lincoln	52	Jimmy Carter	52
Andrew Johnson	56	Ronald Reagan	69
Ulysses S. Grant	46	George H.W. Bush	64
Rutherford B. Hayes	54	Bill Clinton	46
James A. Garfield	49	George W. Bush	54
Chester A. Arthur	51	Barack Obama	47
Grover Cleveland	47	Donald Trump	70
Benjamin Harrison	55	Joe Biden	78

MODULE 4 • TOPIC 3 • LESSON 2 ASSIGNMENT 805

3 Bar Graphs

MATERIALS

None

LESSON OVERVIEW

In this lesson, students analyze categorical data presented in bar graphs. Students analyze three types of bar graphs: single bar graphs (with horizontal or vertical bars), double bar graphs, and stacked bar graphs. Students then answer questions about data provided and create their own graphs from data sets.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1E create and use representations to organize, record, and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6G solve problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents.

Measurement and Data

(12) The student applies mathematical process standards to use statistical representations to analyze data.

The student is expected to:

7.12C compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations.

ELPS

(1) Learning Strategies

The student is expected to:

(G) demonstrate an increasing ability to distinguish between formal and informal English and an increasing knowledge of when to use each one commensurate with grade-level learning expectations.

(3) Speaking

The student is expected to:

(G) express opinions, ideas, and feelings ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics.

(4) Reading

The student is expected to:

(A) learn relationships between sounds and letters of the English language and decode (sound out) words using a combination of skills such as recognizing sound-letter relationships and identifying cognates, affixes, roots and base words.

ESSENTIAL IDEAS

- Bar graphs display data using vertical or horizontal bars. The height or length of each bar indicates its value. A scale must be provided to read bar graphs.
- Double bar graphs and stacked bar graphs may be used when each category contains two different groups of data. A key must be provided to tell the two different groups apart, and a scale must be provided to read the values of the data.
- The side-by-side bars in a double bar graph make it easy to compare how much larger, or how many times bigger, one value is than the other.
- Stacked bars in a stacked bar graph make it easy to compare parts to the whole within a category.
- Circle graphs can also be used to compare parts to the whole.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: The Birthday Paradox 20 minutes

ESTABLISH A SITUATION

Students are introduced to the probability phenomenon known as the Birthday Paradox. They survey everyone in the classroom and record the month and day of their birthday. Students create a vertical data display to show the months in which everyone was born and a horizontal data display to show the day of the month on which everyone was born.

DEVELOP

Activity 3.1: Bar Graphs 25 minutes

REAL-WORLD PROBLEM SOLVING

Students begin with two different contexts and two bar graphs (one vertical and one horizontal). They use a scale to determine the values of the bars. Students also create a bar graph to represent data from a table.

DAY 2

Activity 3.2: Double Bar Graphs 20 minutes

REAL-WORLD PROBLEM SOLVING

Students begin with a review of double bar graphs. Double bar graphs are useful when comparing two groups within a category and require a key to differentiate each group. Students use a key to interpret information provided in a double bar graph and then create a double bar graph from a table of values. They end the activity by comparing double bar graphs with circle graphs.

Activity 3.3: Stacked Bar Graphs 15 minutes

REAL-WORLD PROBLEM SOLVING

Students begin with a review of stacked bar graphs. Stacked bar graphs, like double bar graphs, contain two different groups of data within each category. Stacked bar graphs require a scale and a key to differentiate between the groups of data, and they are useful when comparing parts to the whole within a category. Students analyze and interpret information provided in a stacked bar graph and create a stacked bar graph from a table of values. Students end the activity by analyzing circle graphs.

DEMONSTRATE

Talk the Talk: Runners and Jumpers and Jerseys, Oh My! 10 minutes

EXIT TICKET APPLICATION

Students are given a stacked bar graph displaying the results of the selections of two different groups of athletes. They use that data to create a double bar graph. Students answer questions that require them to analyze the data and then they state which of the two data displays provides the information in a way that best allows them to answer the question.

Getting Started

ENGAGE

The Birthday Paradox

Facilitation Notes

In this activity, students are introduced to the probability phenomenon known as the Birthday Paradox. They survey everyone in the classroom and record the month and day of their birthday. Students create a vertical data display to show the months in which everyone was born and a horizontal data display to show the day of the month on which everyone was born.

Ask a student to read the introduction aloud and discuss as a class. Have students complete Questions 1 through 3 with a partner or in a group. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What type of data is being collected?
Probing	• How could you display the data you've collected?
Reflecting and justifying	• Could you display the data in a way that did not use the axes provided? Explain.

Complete and discuss Question 4 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• According to the Birthday Paradox, how many people in your class should have the same birthday? • If you were to survey the entire seventh grade, how many people should have the same birthday?

STAMP THE LEARNING

Summary

Data can be displayed in a variety of different ways, some of which are vertical, and some of which are horizontal.

ACTIVITY 3.1 Bar Graphs

DEVELOP

Facilitation Notes

In this activity, students begin with two different contexts and two bar graphs (one vertical and one horizontal). They use a scale to determine the values of the bars. Students also create a bar graph to represent data from a table.

Ask a student to read the introduction regarding bar graphs aloud. Have students work with a partner or in groups to complete Question 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How could you change the situations in Questions 1 and 2 so that bar graphs would not be appropriate to represent the data?
Seeing structure	• Do you think the bars on a bar graph should all be the same width? Why or why not?
Reflecting and justifying	• Do you prefer to read bar graphs that are horizontal or vertical? Explain.

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine what scale to use for this situation? • What are some differences between the table and the bar graph?
Reflecting and justifying	• What are some advantages and disadvantages of using a bar graph, rather than a table, to display data?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Bar graphs display categorical data using horizontal or vertical bars. The height or length of the bars indicates its value for a specific category.

ACTIVITY 3.2

Double Bar Graphs

Facilitation Notes

In this activity, students begin with a review of double bar graphs. Double bar graphs are useful when comparing two different groups within a category and require a key to differentiate each group. Students use a key and interpret information provided in a double bar graph. They create a double bar graph from a table of values. Students end the activity by comparing double bar graphs with circle graphs.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction regarding the context and the double bar graphs before Question 1. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Why is it important to have spaces between the categories? • What is the scale on the graph?
Probing	• How many kilograms of peaches were sold Saturday and Sunday combined? How did you determine your answer?
Seeing structure	• Estimate which day saw more sales of fruit. How did you determine your answer without calculating?

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What information is easier to see on the double bar graph than in the table of values? • What information is lost when viewing a double bar graph rather than a table of values?

COMMON MISCONCEPTION

The magnitude of the numbers may make it challenging for students to determine what scale to use when creating their graph. When students start at a value other than zero, encourage the use of a "broken graph symbol" on the *y*-axis to signify that a portion of the bars are missing. Calculations involving differences will still make sense when viewing the top portions of the bars. However, when students are asked to make comparisons, such as "twice as big," the bars would not show the proportional relationship, since they weren't included in their entirety.

Have students complete Question 4 independently. Discuss responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How is a circle graph different from a double bar graph?
Seeing structure	• What information is lost when viewing a circle graph rather than a double bar graph? • When a section in one circle graph is about the same size as a section in another circle graph, does that mean the data values also are the same? • What information do you need to know to compare sections in two or more circle graphs? • What data might be best represented using a circle graph?

Summary

Double bar graphs are used for categorical data when each category contains two different groups of data. The bars may be vertical or horizontal, and a key explains the colors or patterns for each group. The two bars representing the same category are side by side, and space is used to separate the categories.

ACTIVITY 3.3

Stacked Bar Graphs

Facilitation Notes

In this activity, students begin with a review of stacked bar graphs. Stacked bar graphs, like double bar graphs, contain two different groups of data within each category. Stacked bar graphs require a scale and a key to differentiate between the groups of data. Stacked bar graphs are useful when comparing parts to the whole within a category. Students analyze and interpret information provided in a stacked bar graph and then create a stacked bar graph from a table of values. Students end the activity by analyzing circle graphs.

Ask a student to read the introduction regarding the context and the stacked bar graph aloud. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How is a stacked bar graph different from a double bar graph? • How is a stacked bar graph similar to a double bar graph?

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Ask a student to read the information in Question 2 aloud. Have students work with a partner or in a group to complete Questions 2 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What information is more visible in the stacked bar graph than in the table of values? • What information cannot be determined when viewing a stacked bar graph rather than a table of values?
Reflecting and justifying	• Describe the advantages and disadvantages of using a table of values, a stacked bar graph, and a circle graph to represent data.

Summary

A stacked bar graph is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole. Each bar represents a total for the whole category but still shows how many data pieces make up each group within the entire category.

DEMONSTRATE

Talk the Talk

RUNNERS AND JUMPERS AND JERSEYS, OH MY!

Facilitation Notes

In this activity, students are given a stacked bar graph displaying the results of the jersey color selections of two different groups of athletes. They use that data to create a double bar graph. Students answer questions that require them to analyze the data, and they then state which of the two data displays provides the information in a way that best allows them to answer the question.

Ask a student to read the introduction aloud. Discuss the graph as a class. Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How may total athletes are being considered in the data?
Probing	• What other data displays could have been used to show the data?
Seeing structure	• What ratios are easier to determine by looking at a stacked bar graph? Which are easier to determine by looking at a double bar graph? • What is a set of data from this same scenario that could not be displayed using a stacked bar graph or a double bar graph?

Have students read and answer the Essential Question on the lesson opener page.

Summary

The same data can be displayed in a variety of ways, including different forms of bar graphs. Different forms of bar graphs make various aspects of the data easier to see.

STAMP THE LEARNING

CC BY NC

3 Bar Graphs

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Organize data into single bar graphs, double bar graphs, stacked bar graphs, and circle graphs.
- Analyze data and interpret results of data analysis from single bar graphs, double bar graphs, stacked bar graphs, and cirlce graphs.

You have learned about various data displays including dot plots, histograms, and stem-and-leaf plots.

How can bar graphs help you organize data? What information can you interpret from their different forms?

Sample answer:

Bar graphs display the frequency of categorical data using horizontal and vertical bars.

MODULE 4 • TOPIC 3 • LESSON 3 807

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–3.
- Check in and share.
- Complete Question 4 as a class.
- Share and summarize.

Getting Started

The Birthday Paradox

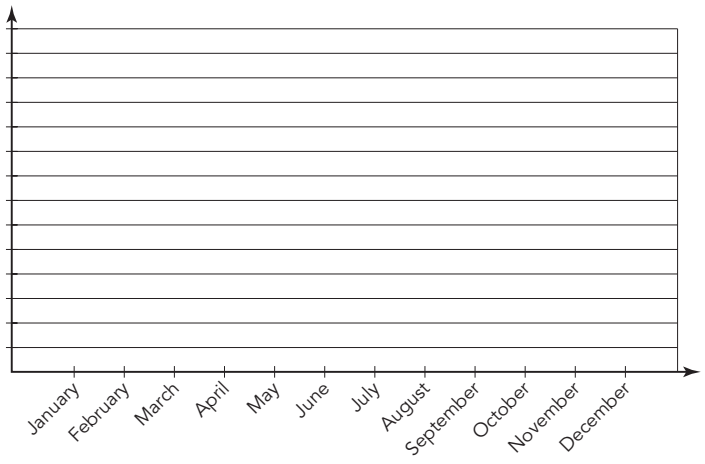
In a room of just 23 people, there is a 50-50 chance of at least two people having the same birthday. If there are 75 people, there is a 99.9% chance of at least two people sharing the same birthday. This set of facts is known as the Birthday Paradox.

1. Survey everyone in your classroom, including your teacher, and record the month and day of their birthday.

   Answers will vary.

2. Use the *x*- and *y*-axes below to create a data display to show the months in which everyone in your classroom was born.

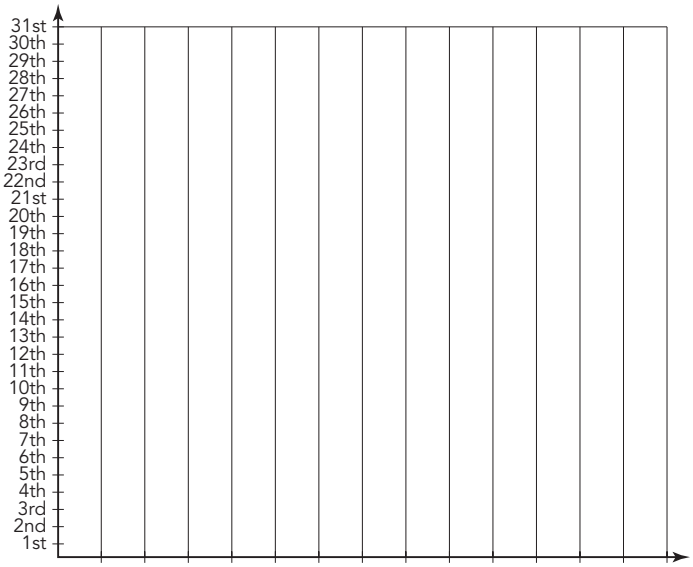
   Answers will vary.



808 MODULE 4 • TOPIC 3 • LESSON 3

3. Use the *x*- and *y*-axes below to create a data display to show the day of the month on which everyone in your classroom was born.

Answers will vary.



4. Does the number of people in your classroom fit the criteria of the Birthday Paradox? Does anyone in your classroom have the same birthday as someone else?

Answers will vary.

MODULE 4 • TOPIC 3 • LESSON 3 809

Chunking the Activity

- Read and discuss the introduction and the double bar graphs.
- Group students to complete Questions 1–3.
- Check in and share.
- Group students to complete Question 4.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

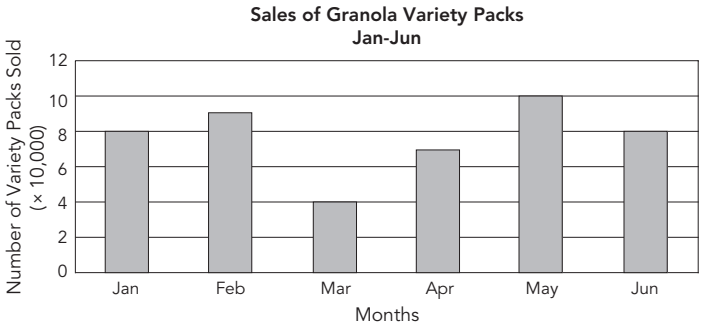
The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 3.1

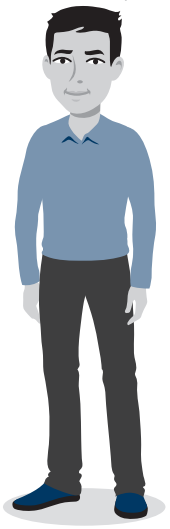
Bar Graphs

Some graphs are used to display data that consists of different categories. *Bar graphs* display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

1. The bar graph shows the number of granola variety packs sold in the first six months of the year. The bars are vertical.



Notice that all of the bars are the same width. Why do you think this is?



a. What does each bar represent?

Each bar represents the number of granola variety packs sold for the month.

b. Which month has the most sales of granola variety packs? How many were sold?

May had the most sales of granola bar variety packs. There were 100,000 granola variety packs sold in May; (10 · 10,000).

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

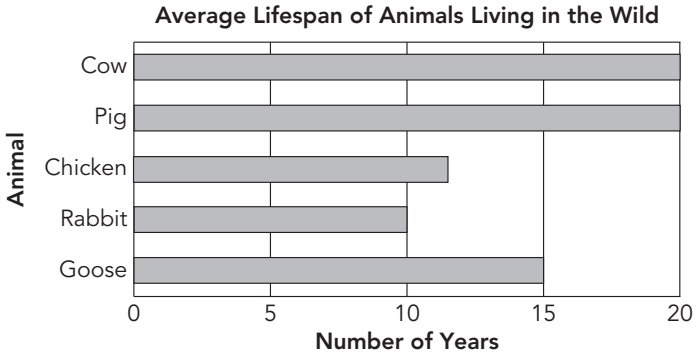
Explain and point out the parts of the bar graph, such as the title, scale, *x*-axis, *y*-axis, and labels. Provide additional clarification of the abbreviations used for the months in a year. Ask students to identify the parts of the bar graph in Questions 1 and 2.

810 MODULE 4 • TOPIC 3 • LESSON 3

c. How many granola variety packs were sold in the first half of the year (January through June) according to the data in the bar graph?

There were a total of 460,000 variety packs sold between January and June; [(8 + 9 + 4 + 7 + 10 + 8) · 10,000].

2. The graph titled "Average Lifespan of Animals Living in the Wild" is an example of a bar graph where the bars are horizontal.



a. Which animal has the shortest lifespan according to the graph?

According to the graph, rabbits have the shortest lifespan.

b. Which animal has, on average, the longest lifespan according to the graph?

Cows and pigs have the longest life span.

c. How long does the average chicken live? How did you calculate your answer from the graph?

Chickens must live about 11 years, because the bar goes just a little past the 10 line.

d. What interval is used to record the number of years on the bar graph?

The intervals go up by 5 years.

MODULE 4 • TOPIC 3 • LESSON 3 811

e. Why do you think the bar graph includes "average" in its title? Explain your reasoning.

Sample answer:
I think it says "average" because it is impossible to record the lifespan of every animal displayed in the bar graph. The data must have been compiled to create the bar graph.

Ask Yourself . . .
Did you justify your mathematical reasoning?

f. What other name could you use to replace "average" in the title?

Sample answer:
I could change the word "average" to "mean" in the bar graph title, and it would have the same meaning.

3. Compare the "Average Lifespan of Animals Living in the Wild" bar graph with the "Sales of Granola Variety Packs" bar graph.

a. Gracie says that the graphs are similar because in both graphs, the frequency of the data in each interval is represented by the height or length of the bar. Joey says that the graphs are similar because in both graphs, the intervals are the same.

Who is correct? Explain your reasoning.

Gracie is correct. Joey is incorrect because the intervals are different. In the Granola graph, the intervals are in two-unit intervals, while the intervals in the Average Lifespan bar graph are in five-unit intervals.

b. In what other ways are the bar graphs similar? Name at least two similarities in your answer.

Sample answers:
The two bar graphs are similar in the following ways:

All bars within each graph have the same width.

Each graph has equal intervals, which are shown on one of the axes.

812 MODULE 4 • TOPIC 3 • LESSON 3

c. How are the bar graphs different? Name at least two differences in your answer.

The two bar graphs are different in the following ways:

One graph displays sales that took place, while the other shows the mean lifespan of a variety of animals.

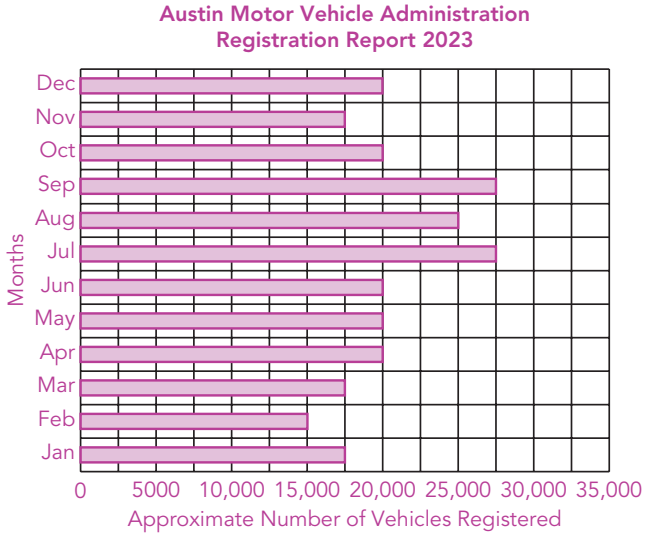
One graph is a horizontal bar graph, and the other is a vertical bar graph.

4. Use the data in the table shown to create a bar graph with vertical or horizontal bars.

Austin Motor Vehicle Administration Registration Report 2023	
Month	Approximate Number of Vehicles Registered
January	17,500
February	15,000
March	17,500
April	20,000
May	20,000
June	20,000
July	27,500
August	25,000
September	27,500
October	20,000
November	17,500
December	20,000

MODULE 4 • TOPIC 3 • LESSON 3 813

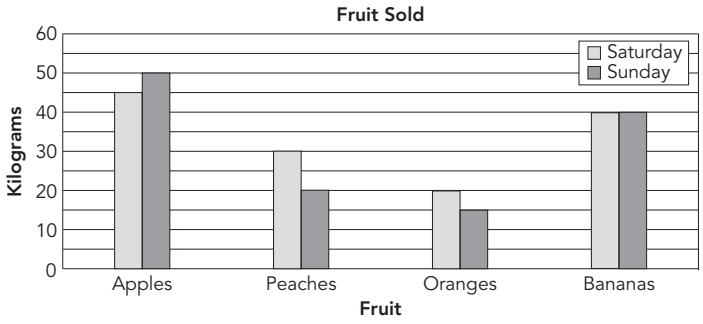
Sample graph:



814 MODULE 4 • TOPIC 3 • LESSON 3

ACTIVITY
3.2 Double Bar Graphs

Miguel owns a fruit stand at a local farmers market that he only opens on Saturdays and Sundays. Miguel recorded the amount of fruit he sold last weekend.



Miguel displayed his data in a *double bar graph*, which is used when each category contains two different groups of data. The bars may be vertical or horizontal. A *key* explains the colors or patterns for each group. The two bars representing the same category are side-by-side. Space is used to separate the categories.

1. Use the double bar graph to answer each question.

   a. What does the key tell you?

   The light gray bar represents the amount of fruit sold on Saturday, and the dark gray bar represents the amount of fruit sold on Sunday.

   b. How many kilograms of apples were sold on Sunday?

   Fifty kilograms of apples were sold on Sunday.

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Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction and double bar graphs.
- Group students to complete Question 1.
- Check in and share.
- Group students to complete Questions 2 and 3.
- Check in and share.
- Have students complete Question 4 individually.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For "Beginning" and "Intermediate" proficiency levels

Use the introduction of the double bar graph to expose students to the meaning of *double*. Provide students with visuals of items or pictures which represent *doubling* in number, size, or value. Ask students to provide examples of *doubling*.

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c. Which fruit showed the greatest difference in sales over the two days?

Peaches showed the greatest difference in sales.

d. What was the difference between Saturday's sales and Sunday's sales of your answer to part (c)?

The difference between the peach sales on Saturday and Sunday was 10 kilograms.

e. Which fruit showed the smallest difference in sales over the two days?

Bananas had the same amount of sales on the two days.

f. What was the difference between Saturday's sales and Sunday's sales of your answer to part (e)?

There was no difference in sales between Saturday and Sunday.

816 MODULE 4 • TOPIC 3 • LESSON 3

2. In many countries, reports are kept for all sorts of statistics. In addition to recording the number of births in the United States, different research companies compile the same information in different ways. The table below shows the number males and females born in the United States between 2000 and 2006. You will create your own double bar graph using the information shown in the table.

U.S. Male and Female Births 2000–2006		
Year	Male	Female
2000	2,076,969	1,981,845
2001	2,057,922	1,968,011
2002	2,057,979	1,963,747
2003	2,093,535	1,996,415
2004	2,150,228	1,961,794
2005	2,118,982	2,019,367
2006	2,184,237	2,081,318

a. What will your key show?

My key will show which colors or patterns will represent female versus male births.

b. Create a name for your double bar graph.

I will name my double bar graph "U.S. Male and Female Births 2000–2006."

MODULE 4 • TOPIC 3 • LESSON 3 817

c. What are your categories?

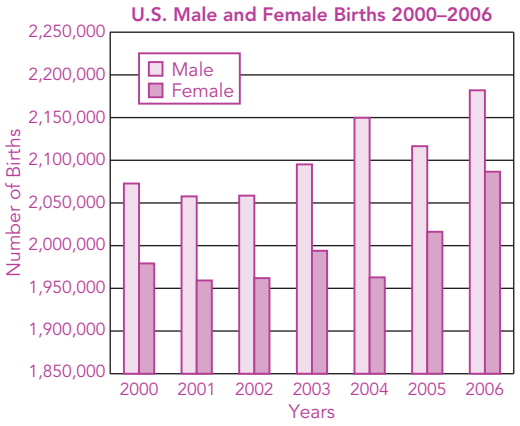
Sample answer:
The categories in my double bar graph are the different years in which the birth data were compiled.

d. How will you number the axes of your graph? State the intervals you will use.

Sample answer:
I will start my graph at 1,850,000 and end at 2,250,000. I will divide it into intervals of 50,000.

e. Create a double bar graph.

Sample graph:



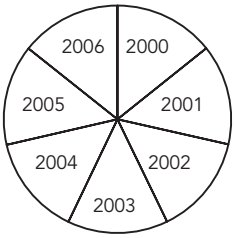
3. When is a double bar graph a better graph choice than a single bar graph?

Sample answer:
A double bar graph is preferred when you want to show frequencies of 2 different groups within one category.

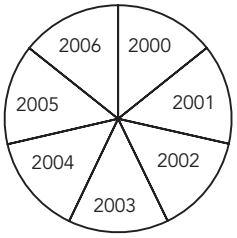
818 MODULE 4 • TOPIC 3 • LESSON 3

4. You can also display the data using *circle graphs* to represent the relationship between each part and the whole.

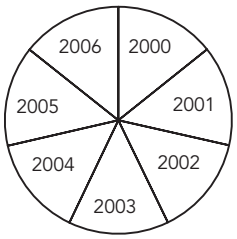
U.S. Male Births 2000–2006



U.S. Female Births 2000–2006



U.S. Male and Female Births 2000–2006



a. By analyzing the U.S. Female Births circle graph, can you determine whether more females were born in 2004 or 2005? Explain.

The size of the 2004 and 2005 categories in the U.S. Female Births circle are about the same size. To determine which year more females were born, you would have to compare the actual numbers.

b. The 2002 category for the U.S. Male Births circle graph is about the same size as the 2002 category for the U.S. Male and Female Births circle graph. Does that mean that in 2002 the actual number of male births is about the same as the actual number of male and female births? Explain.

No. When comparing categories between two circle graphs, it's important to take the total of each circle into consideration. Even though the 2002 category in the U.S. Male Births circle graph is about the same size as the 2002 U.S. Male and Female category, the actual number of male births is about half as many as the actual number of male and female births.

c. Is it more meaningful to display the data using a double bar graph or circle graphs? Explain.

A double bar graph is better suited for an accurate comparison of male births to female births.

MODULE 4 • TOPIC 3 • LESSON 3 819

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Question 1.
- Check in and share.
- Read and discuss the information.
- Group students to complete Questions 2–6.
- Share and summarize.

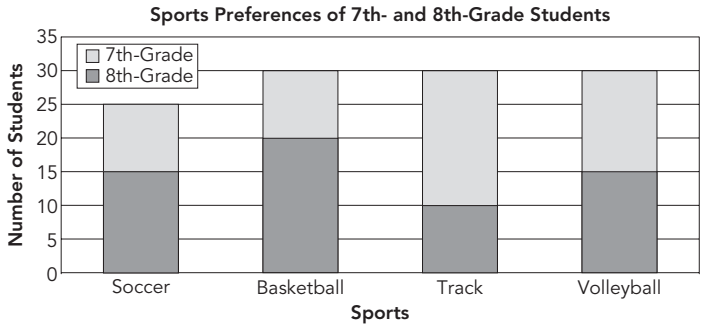
STAMP THE LEARNING

The paragraph provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 3.3

Stacked Bar Graphs

Seventh- and eighth-grade students in an after-school program were asked which sport they preferred. Their answers are recorded in the graph below.



The "Sports Preferences of 7th- and 8th-Grade Students" display is a *stacked bar graph*. A *stacked bar graph* is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole. Each bar represents a total for the whole category but still shows how many data pieces make up each group within the entire category.

1. Use the stacked bar graph to answer each question.

   a. What does the key indicate?

   Sample answer:
   The key tells me that the 7th-grade students frequency is represented by the solid light gray bar, and the 8th-grade students frequency is represented by the solid dark gray bar.

   b. How many total students prefer basketball over the other choices?

   30 students prefer basketball.

820 MODULE 4 • TOPIC 3 • LESSON 3

c. Write a ratio for each group of students that prefers basketball. Then, determine which group of students has the higher preference.

The ratio of 7th-grade students that prefer basketball is 10 : 55, or 2 : 11. The ratio of 8th-grade students that prefer basketball is 20 : 60 or 1 : 3. Eighth-grade students have a higher preference for basketball than seventh-grade students.

d. Which sport is preferred equally by the 7th- and 8th-grade students in the after-school program? How can you tell?

Sample answer:
The same number of 7th- and 8th-grade students prefer volleyball. I can tell this because the bar is split exactly in half.

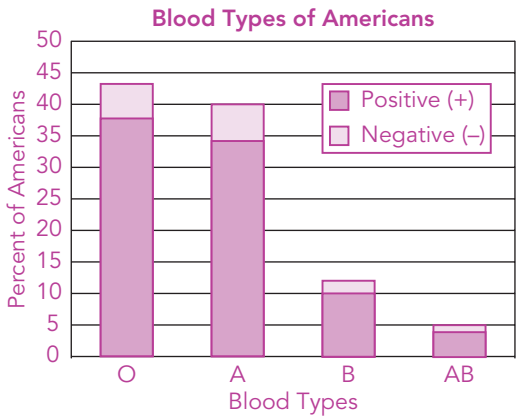
2. Do you know what blood type you have? Do you know anyone in your class who has your same blood type? There are eight different human blood types and four blood groups. (O, A, B, and AB) These groups indicate what types of blood a person can receive should they need a transfusion. Some blood types are more common than others.

Blood Type	Percent of Americans with Blood Type
O+	37
O−	6
A+	34
A−	6
B+	10
B−	2
AB+	4
AB−	1

MODULE 4 • TOPIC 3 • LESSON 3 821

a. Create a stacked bar graph to represent the data listed in the table.

Sample graph:



b. Using the information in your graph, what is the most common blood type found in Americans?

The most common blood type found in Americans is O+.

c. What is the least common blood group, when its positive and negative carriers are combined?

The least common blood group is AB.

d. What percent of Americans have Group B blood with either a positive or negative factor?

According to the graph, about 12% of Americans have Group B blood.

822 MODULE 4 • TOPIC 3 • LESSON 3

3. How is a stacked bar graph the same as other bar graphs?

Stacked bar graphs are the same as other bar graphs because they all use same sized bars to represent the categories. Also, they all clearly show the frequencies for each category.

4. How is a stacked bar graph different from other bar graphs?

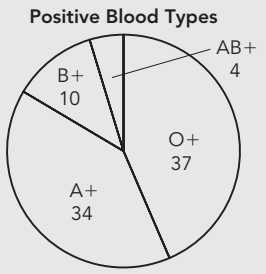
Sample answer:
In a stacked bar graph, I can see the different frequencies that make up the whole category because the different frequencies are stacked on top of each other.

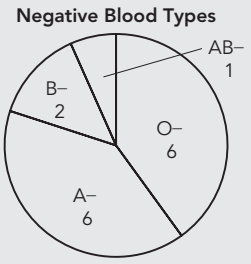
5. When would you use a stacked bar graph?

Sample answer:
I would use a stacked bar graph when I'm trying to show how the frequency of the sub-groups in a category compares to the total frequency.

6. Nicky and Lucia created circle graphs using the blood type data.

Nicky

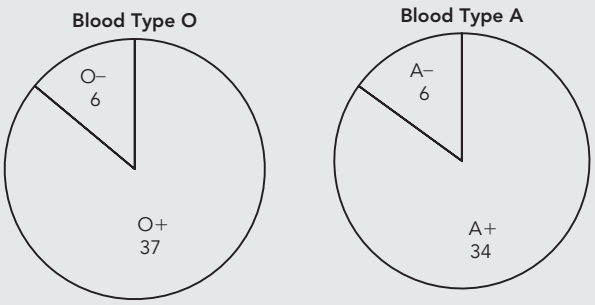


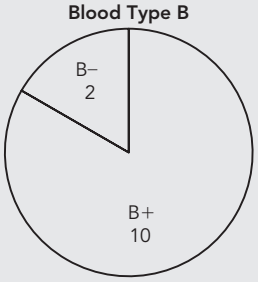


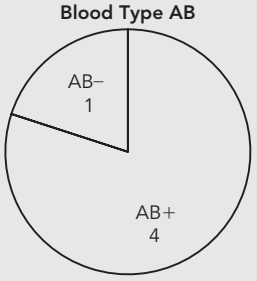
MODULE 4 • TOPIC 3 • LESSON 3 823

Activity 3.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with creating bar and circle graphs, assign Skills Practice Set A for this lesson.

Lucia







Who is correct? Explain your reasoning.

Both Nicky and Lucia are correct.

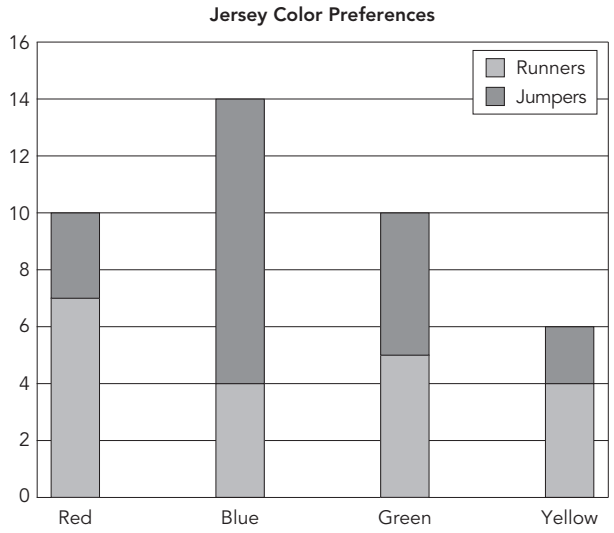
Nicky and Lucia simply organized the information in their circle graphs differently

824 MODULE 4 • TOPIC 3 • LESSON 3

Talk the Talk

Runners and Jumpers and Jerseys, oh My!

A group of 7th-graders selected the color jersey they wanted to wear for the school track and field games. The results of their selections are shown in the stacked bar graph.



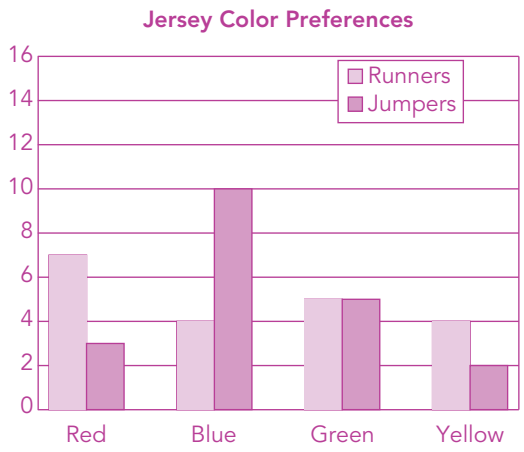
Chunking the Activity

- Read and discuss the introduction and graph.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

MODULE 4 • TOPIC 3 • LESSON 3 825

1. Create a double bar graph that displays the same information as the stacked bar graph.

   Sample graph:



2. Answer each regarding the students' color choice. Then, write which graph, double bar graph or stacked bar graph, displays the information better to answer each question.

   a. How many more jumpers than runners preferred blue?

      Six more jumpers than runners preferred blue. The double bar graph was the better display.

   b. How many more runners than jumpers preferred red?

      Four more runners than jumpers preferred red. The double bar graph was the better display.

826 MODULE 4 • TOPIC 3 • LESSON 3

c. What fraction of students who preferred green were runners?

Half of the students who preferred green were runners. The stacked bar graph was the better display.

d. What fraction of students who preferred yellow were runners?

Two-thirds of the students who preferred yellow were runners. The stacked bar graph was the better display.

e. What type of question is best answered by a double bar graph? Explain.

A question that asks for a comparison as to how much smaller or larger one value is compared to another is best answered by a double bar graph. When the graphs are side-by-side, it makes it easy to see the difference between them.

f. What type of question is best answered by a stacked bar graph? Explain.

A question that asks for a comparison of a portion of responses to all responses of that type is best answered by a stacked graph. A stacked graph makes it easy to see the whole and then it can be broken into portions by using the scale.

g. What ratio represents the number of jumpers that preferred red compared to the number of runners that preferred red? Which type of graph is a more meaningful way to display the information?

The ratio $\frac{3}{7}$ represents the number of jumpers who preferred red compared to the number of runners who preferred red. The double bar graph made this easy to see because the graphs were compared side-by-side.

Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

Talk the Talk presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with analyzing bar and circle graphs, assign Skills Practice Set B for this lesson.

MODULE 4 • TOPIC 3 • LESSON 3 827

828 MODULE 4 • TOPIC 3 • LESSON 3

Lesson 3 Assignment

Write

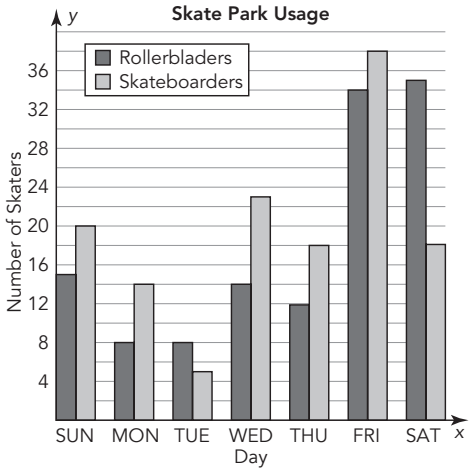
Explain the similarities and differences between bar graphs, double bar graphs, and stacked bar graphs.

Remember

Bar graphs display the frequency of categorical data using horizontal and vertical bars.

Practice

The double bar graph shows the number of skaters who used a skate park in one week. Use the double bar graph to answer the questions.



1. What does the key tell you?

   The dark bars represent the rollerbladers, and the light bars represent the skateboarders.

Write

Sample answer:
Each bar graph may use either horizontal or vertical bars to represent the data. Single bar graphs are used when each category contains only one group of data. Double and stacked bar graphs are used when each category contains two groups of data. The bars on a double bar graph are side-by-side. In a stacked bar graph, one bar is stacked on the other.

MODULE 4 • TOPIC 3 • LESSON 3 ASSIGNMENT 829

Lesson 3 Assignment

2. How many rollerbladers used the park during the week? Explain how you determined your answer.

   $15 + 8 + 9 + 14 + 12 + 34 + 35 = 127$

   I added the number of rollerbladers that used the park each day and got a total of 127 for the week.

3. What is the difference between the number of skateboarders that used the park on Friday and the number of skateboarders that used it on Saturday? Show your work.

   $38 - 17 = 21$

   Twenty-one more skateboarders used the park on Friday than on Saturday.

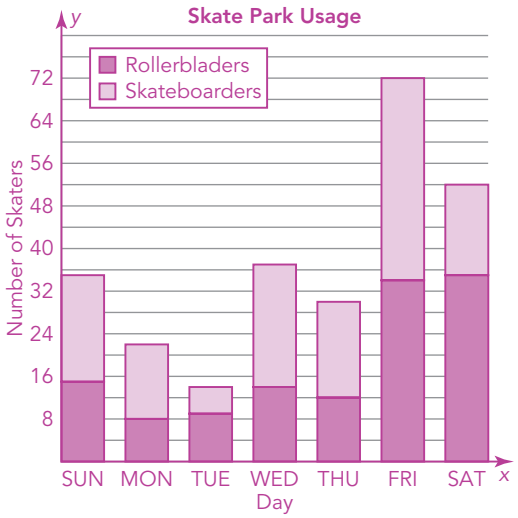
4. On how many days were there more skateboarders using the park than rollerbladers? Explain how you determined your answer.

   On five days, there were more skateboarders using the park than rollerbladers. To determine my answer, I looked at the bar graph and noticed on which days the bar representing the skateboarders was taller than the bar representing the rollerbladers.

830 MODULE 4 • TOPIC 3 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

5. Create a stacked bar graph using the data represented in the double bar graph.



6. How many total skaters used the park on Wednesday?

   Thirty-seven skaters used the park on Wednesday.

7. How many rollerbladers and how many skateboarders used the park on Tuesday?

   Nine rollerbladers and five skateboarders used the park on Tuesday.

MODULE 4 • TOPIC 3 • LESSON 3 ASSIGNMENT 831

Lesson 3 Assignment

8. On which day did the most skaters use the park? Explain how you determined your answer.

   Friday was the day on which the most skaters used the park. The bar for Friday is tallest on the stacked bar graph, so I know it is the day on which the most skaters used the park.

9. The city would like to close the park one day a week for maintenance. Which day would you recommend they close the park based on the data shown in the bar graph? Explain your reasoning.

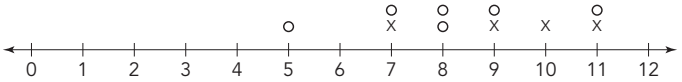
   I would suggest that the city close the park on Tuesday. The bar for Tuesday is shortest on the graph, so it was the day that the least number of skaters used the park.

832 MODULE 4 • TOPIC 3 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

Prepare

The dot plot shows the number of football games 7th- and 8th-grade students attended. The "o" represents 7th-graders' responses, and the "x" represents 8th-graders' responses.



1. Estimate the mean number of games the 7th-grade students attended.

   About 8 games

2. Estimate the mean number of games the 8th-grade students attended.

   About 9 games

3. What observations can you make from your estimations of the data?

   Sample answer:

   Eighth-grade students attend more football games than seventh-grade students.

MODULE 4 • TOPIC 3 • LESSON 3 ASSIGNMENT 833

834 MODULE 4 • TOPIC 3 • LESSON 3 ASSIGNMENT

4 Comparing Two Populations

MATERIALS

None

LESSON OVERVIEW

Within the context of a situation, students calculate the measures of center and measures of variation for two different populations. They compare the difference of the measures of center for the two populations to their measures of variation. Students construct dot plots and determine a five-number summary for a data set for comparison purposes. A stem-and-leaf plot is used to display data in one situation.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1E create and use representations to organize, record, and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6G solve problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to-part comparisons and equivalents.

Measurement and Data

(12) The student applies mathematical process standards to use statistical representations to analyze data.

The student is expected to:

7.12A compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads.

ELPS

(3) Speaking

The student is expected to:

(G) express opinions, ideas, and feelings ranging from communicating single words and short phrases to participating in extended discussions on a variety of social and grade-appropriate academic topics.

(4) Reading

The student is expected to:

(J) demonstrate English comprehension and expand reading skills by employing inferential skills such as predicting, making connections between ideas, drawing inferences and conclusions from text and graphic sources, and finding supporting text evidence commensurate with content area needs.

(5) Writing

The student is expected to:

(G) narrate, describe, and explain with increasing specificity and detail to fulfill content area writing needs as more English is acquired.

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ESSENTIAL IDEAS

- Measures of center, including the mean and median, can be used to compare data sets for two populations.
- Measures of variation, including the spread, distribution, interquartile range (IQR), and five-number summary, can be used to compare data sets for two populations.
- A dot plot or a stem-and-leaf plot can be used to display two sets of data to compare measures of center and measures of variation.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Couch Potatoes 15–20 minutes

CONNECT TO PRIOR KNOWLEDGE

Students analyze and compare data from two data sets by calculating the mean, median, and interquartile range (IQR) for each data set. They then describe what these measures say about the comparison of the two populations.

DEVELOP

Activity 4.1: Comparing Measures of Center and Measures of Variation 20–25 minutes

REAL-WORLD PROBLEM SOLVING, WORKED EXAMPLE

Students analyze and use a table of two data sets. They display both data sets on the same dot plot and use the plot to make observations and comparisons related to the means, distributions, and variations of the data sets. Students then answer questions related to the context of the problem situation by comparing the difference between the two means.

DAY 2

Activity 4.2: Comparing Distributions 25–30 minutes

REAL-WORLD PROBLEM SOLVING

Students analyze a stem-and-leaf plot containing two data sets. They interpret the stem-and-leaf plot and describe the distribution of data for each data set. Students then determine the five-number summary and IQR for each set of data. They compare the measures of center and IQR for both sets and relate the comparisons back to the problem situation.

DEMONSTRATE

Talk the Talk: Summarize 10–15 minutes

WRITING TASK

Students explain how they can use the five-number summary to interpret and compare the data of two populations.

Getting Started

ENGAGE

Couch Potatoes

Facilitation Notes

In this activity, a scenario containing two data sets is given. Students calculate and interpret the mean, median, and interquartile range (IQR) for each data set.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the mean? • How is the mean calculated?
Probing	• What does the mean tell you about this problem situation?
Gathering	• What is the median? • How is the median calculated?
Probing	• What does the median tell you about this problem situation?
Gathering	• What is the IQR? • How is the IQR calculated?
Probing	• What does the IQR tell you about this problem situation?

STAMP THE LEARNING

Summary

Measures of center and measures of variation, such as the mean, median, and interquartile range (IQR) can provide additional information about a population.

ACTIVITY 4.1

Comparing Measures of Center and Measures of Variation

DEVELOP

Facilitation Notes

In this activity, a scenario containing two data sets in a table of values is given. Students create and interpret a single dot plot with respect to the mean, the distribution, and variations for both sets of data.

Ask a student to read the introduction aloud. Discuss the table containing the two data sets as a class.

Have students work with a partner or in a group to complete Questions 1 through 10. Share responses as a class.

DIFFERENTIATION STRATEGIES

Materials Needed: Colored Pencils

Just in Time Support

- Suggest students use colors to differentiate between the two sets of data on the same dot plot or recreate two separate dot plots to see the distributions more clearly.
- Provide a word bank for students.
  DISTRIBUTION: Symmetric, Skewed Left, Skewed Right
  MEASURE OF CENTER: Mean, Median
  MEASURE OF SPREAD OR VARIABILITY: Interquartile Range (IQR)

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What observations can you make from the table of values? • When the data are symmetric, what measures should you use for the center and spread?
Probing	• What is the range for each data set?
Gathering	• Which measures determine consistency?
Probing	• How can you identify consistency from a data display?
Seeing structure	• Are you more confident in a decision when there is or is not an overlap of the data sets? Explain your thinking.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

Data sets displayed in a table of values can be used to show measures of center and measures of variation, such as the mean, median, and interquartile range (IQR), which provide additional information about a population.

ACTIVITY 4.2 Comparing Distributions

Facilitation Notes

In this activity, a scenario with a stem-and-leaf plot containing two data sets is given. Students interpret the stem-and-leaf plot, describe the distribution of data, then determine the five-number summary and IQR. They compare the measures of center and IQR and answer related questions.

To begin the Day 2 session, have a student read the Essential Question aloud.

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Ask a student to read the introduction aloud. Discuss the stem-and-leaf plot containing the two data sets as a class.

Have students work with a partner or in a group to complete Questions 1 through 8. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are the most points scored by the team in a single game in 2022? In 2023?
Probing	• How can you tell from looking at the dot plot whether the team scored more points in 2022 or 2023?
Seeing structure	• How do you know whether the data is symmetric or skewed? • What does the distribution indicate about the mean and median of the data? • What do the different medians tell you about the football team's performance for the two seasons? • What does the same IQR imply about the team? • Is a higher or lower measure of spread preferred in this situation? Explain your thinking.
Probing	• Explain your calculations to make this comparison.

Summary

Stem-and-leaf plots can be used to show measures of center and measures of variation, such as the mean, median, and interquartile range (IQR), which provide additional information about a population.

DEMONSTRATE

Talk the Talk

SUMMARIZE

Facilitation Notes

In this activity, students summarize the key points in the lesson by explaining that it is possible to determine the mean and the variation of data for two populations from a dot plot or stem-and-leaf plot. They explain that they can compare the means, along with the minimums and maximums of the two sets of data, and calculate the IQR to compare the spreads.

Have students work with a partner or in a group to complete the question. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What measure of center and spread should you use when both sets of data are symmetric? Skewed? • How is a larger mean visible in a dot plot? • How is a larger measure of spread visible in a dot plot? • What does a measure of spread tell you about the consistency of the data? • Do you prefer a larger or smaller measure of spread when you have to make a decision based on data? Explain your thinking.

Have students read and answer the Essential Question on the lesson opener page.

Summary

The means, minimums, and maximums of two data sets can be compared, along with the interquartile ranges (IQR), to compare the spreads of the two sets.

STAMP THE LEARNING

CC BY NC

4 Comparing Two Populations

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Calculate the measures of center and measures of variation for two populations.
- Construct comparative dot plots for two populations.
- Compare two populations of data using measures of center and measures of variation.

You have used measures of center and measures of variation to analyze single data sets.

How can you use statistics to compare two different data sets in terms of their measures of center and measures of variation?

Sample answer:

Data for two populations may overlap. Comparing the measures of center and measures of variation for the two populations can help you interpret the differences between the two populations.

MODULE 4 • TOPIC 3 • LESSON 4 835

Chunking the Activity

- Read and discuss the situation and tables.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Couch Potatoes

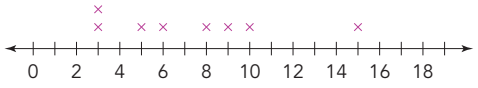
Several teenagers were surveyed to determine the number of hours they spend watching TV during a typical weekend. Another group was surveyed about the number of hours they spend playing outside. Eight surveys were randomly chosen from each group.

Survey Number	Hours Spent Watching TV (Per Weekend)
1	5
2	3
3	10
4	6
5	15
6	9
7	8
8	3

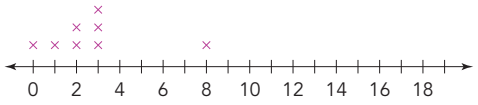
Survey Number	Hours Spent Playing Outside (Per Weekend)
1	1
2	2
3	0
4	3
5	8
6	2
7	3
8	3

1. Create a dot plot for each data set.

Hours Spent Watching TV



Hours Spent Playing Outside



836 MODULE 4 • TOPIC 3 • LESSON 4

2. Calculate the mean, median, and interquartile range (IQR) for each data set.

   Watching TV: mean: 7.375, median: 7, IQR: 5.5

   Playing Outside: mean: 2.75, median: 2.5, IQR: 1.5

The IQR is the difference between the third quartile and the first quartile.

3. What do these measures of center and measures of variation tell you about the data from the surveys?

   Sample answer:
   The first group spent about 4.5 more hours watching TV than the second group spent playing outside over a typical weekend. The variation of the first group was more than the second group.

MODULE 4 • TOPIC 3 • LESSON 4 837

Chunking the Activity

- Read and discuss the introduction and table.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

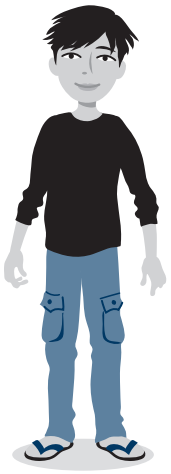
ACTIVITY 4.1

Comparing Measures of Center and Measures of Variation

Nia has just opened a new restaurant, which serves nothing but chocolate milk. She is experimenting with two new flavors— a spicy chocolate milk and a dark chocolate milk.

Nia has asked you to provide a report analyzing customer feedback about the new flavors. You have conducted a survey of 20 random customers, asking each customer to rate a flavor on a scale of 0 to 100.

The 4 steps of the statistical process are:
1. Formulate a statistical question.
2. Collect data.
3. Analyze the data.
4. Interpret the data.

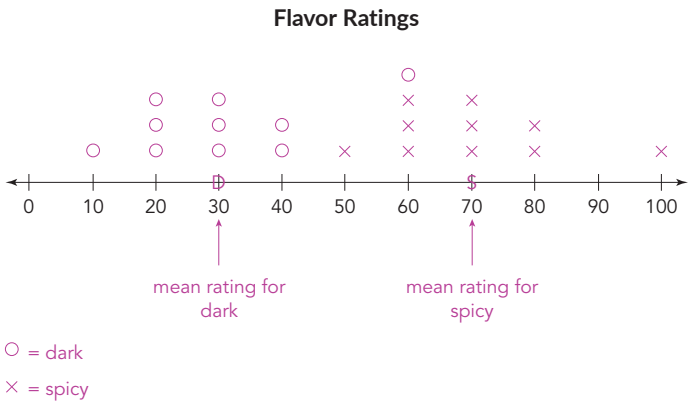


Flavor	Rating	Flavor	Rating
spicy	50	spicy	70
dark	20	dark	30
dark	30	dark	40
spicy	100	spicy	70
spicy	60	dark	20
spicy	80	dark	60
spicy	60	spicy	80
dark	10	dark	20
dark	30	spicy	60
dark	40	spicy	70

838 MODULE 4 • TOPIC 3 • LESSON 4

1. Display the results on a dot plot. Use an "x" to represent the spicy flavor responses and an "o" to represent the dark flavor responses.

Review the definition of **represent** in the Academic Glossary.



2. What observations can you make from your dot plot?

   The ratings for the flavors range from 10 to 100. The average rating is around 50.

3. Describe the distribution of data values for the spicy flavor and for the dark flavor.

   Each flavor's data distribution appears to be symmetric.

MODULE 4 • TOPIC 3 • LESSON 4 839

4. Analyze the data values on your dot plot for spicy and dark.

Remember, you can describe distributions as skewed right, symmetric, or skewed left.

a. Estimate the mean rating for the spicy flavor. Mark the mean on your dot plot with an "s." Explain how you determined your estimate.

The mean rating for the spicy flavor is about 70. It appears that spicy data distribution is almost symmetric, so the mean is probably in the middle of the data points.

b. Estimate the mean rating for the dark flavor. Mark the mean on your dot plot with a "d." Explain how you determined your estimate.

The mean rating for the dark flavor is about 30. It appears that the dark data distribution is almost symmetric, so the mean is probably in the middle of the data points.

5. Calculate the actual mean rating for the spicy flavor.

The mean for the spicy flavor is 70.

6. Calculate the actual mean rating for the dark flavor.

The mean for the dark flavor is 30.

7. What observations can you make about the spread of the two data sets?

Sample answer:

Each data set has the same range and interquartile range.

840 MODULE 4 • TOPIC 3 • LESSON 4

8. Can you report on which flavor has a more consistent rating? Explain your reasoning.

Both flavors are rated equally consistently. The dark flavor rates consistently around 30, and the spicy flavor rates consistently around 70.

WORKED EXAMPLE

Comparing the difference of means with the variation in each data set can be an important way of determining just how different two data sets are.

Consider these data sets.

5, 3, 4, 5, 10	5, 3, 100, 5, 10
Mean = 5.4	Mean = 24.6

The difference in their means is 19.2. Depending on what you are measuring, that can be a big difference.

9. For the spicy flavor and dark flavor data, compare the difference in the means. What observations can you make?

The difference between the mean ratings of the flavors is 40. This means that the two flavors get very distinct ratings— the spicy flavor consistently outperforms the dark flavor.

10. What recommendation would you give to Nia about the two new flavors?

Sample answer:

I would recommend that Nia choose to sell the spicy chocolate milk over the dark chocolate milk since the ratings for the dark chocolate milk were consistently lower with not much overlap between the data sets.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Question 10 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice comparing measures of center and measures of variation, assign Skills Practice Set A for this lesson.

MODULE 4 • TOPIC 3 • LESSON 4 841

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction and stem-and-leaf-plot.
- Group students to complete the activity.
- Share and summarize.

ACTIVITY 4.2

Comparing Distributions

In 2022, a middle school's football team won only 2 games. The school decided to give the coach another chance at improving the team the next year. In 2023, the team won 7 games. Did scoring points have something to do with the team improving their record? The stem-and-leaf plot shows the number of points scored in each game by the football team in 2022 and 2023.

Middle School Football Team

Points scored 2022						Points scored 2023			
		7	3	0	0				
7	4	4	2	0	1	4	7	7	
			9	8	2	4	7	8	8
				5	3	5	8		
					4	2	5		

Key = 0|1|4 means 10 and 14

1. In which year did the football team score more points?
   The team scored more points in 2023.

2. Describe the distribution of the stem-and-leaf plots for each year.
   Both plots seem to be skewed right.

EB STUDENT TIP

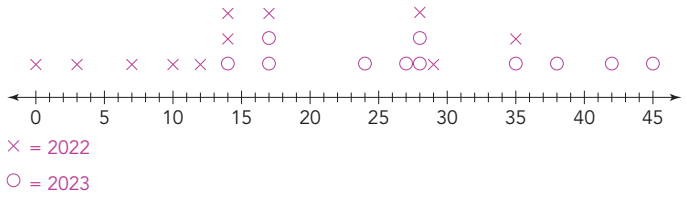
For "Intermediate" and higher proficiency levels

Refer students to the mathematical key used for the stem-and-leaf plot. Prompt students to represent additional single-digit values, as noted in the key. Model and explain the notation used for 2-digit values, such as 23. To check for understanding, ask students to provide other examples.

842 MODULE 4 • TOPIC 3 • LESSON 4

3. Create a combined dot plot to represent the data. Use the letters "x" and "o" to represent data from the different years.

Points Scored by the Football Team, 2022 and 2023



a. How does the shape of the stem-and-leaf plot distribution compare with the shape of the dot plot distributions?

Sample answer:
If I rotate the stem-and-leaf plot 90° counterclockwise, the shape of the 2023 data matches the dot plot of the 2023 data. If I return the stem-and-leaf plot to its original position and rotate it 90° clockwise, the shape of the 2022 data is a reflection of the dot plot of the 2022 data.

4. Determine the five-number summary and IQR for each data set. Then, complete the table.

	2022	2023
Minimum	0	14
Q1	7	17
Median	14	28
Q3	28	38
Maximum	35	45
IQR	$28 - 7 = 21$	$38 - 17 = 21$

Optimizing Learning

This activity offers ways of customizing the display of information.

MODULE 4 • TOPIC 3 • LESSON 4 843

Question 8 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice comparing distributions, assign Skills Practice Set B for this lesson.

5. To calculate the five-number summary and IQR, did you use the data from the stem-and-leaf plot or the dot plots? Explain.

   Sample answer:

   I used the data from the stem-and-leaf plot because it was easier for me to see which scores went with which year.

6. Compare the median and the IQR for the two data sets.

   The median for the number of points scored in 2023 is 14 points greater than the median for 2022. Both years had an IQR of 21 points.

7. How does the difference in the medians compare to the IQR?

   The difference in the medians is two-thirds of the IQR.

The final step in the statistical process is to interpret the data.

8. Do you think that scoring more points may have been one reason the football team improved its record?

   Yes. It does appear that scoring more points could have been a reason for the football team improving its record.

844 MODULE 4 • TOPIC 3 • LESSON 4

Talk the Talk

Summarize

Write 1–2 paragraphs to summarize this lesson. Be sure to answer the question below in your response.

How can you use the five-number summary to interpret and compare the data of two populations?

Sample answer:

I can determine the mean and spread of data for two populations on a dot plot. I have to determine the values from the dot plot before calculating the mean and the five-number summary. I can compare those means, along with the minimums and maximums of the two sets of data, and calculate the IQR to compare the spreads.

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

MODULE 4 • TOPIC 3 • LESSON 4 845

846 MODULE 4 • TOPIC 3 • LESSON 4

Lesson 4 Assignment

Write

How can dot plots and stem-and-leaf plots help you to compare two sets of data?

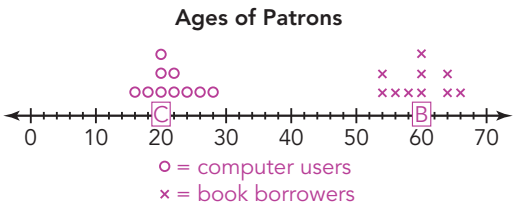
Remember

Data for two populations may overlap. Comparing the measures of center and measures of variation for the two populations can help you interpret the differences between the two populations.

Practice

The head librarian at the local public library is investigating the current trends in technology and the effects of computers and electronic books on the loaning of books. She thinks that the users at the library on the computers are generally younger than the people who actually check out books. She asks the ages of a sample of both computer users and book borrowers. The results are shown in the table.

1. Display the results on a dot plot. Use an "o" to represent the computer users' ages and an "x" to represent the book borrowers' ages from the information in the table.



Patron		Age
Computer user	Book borrower	
C		27
C		16
	B	57
C		20
	B	55
	B	60
C		22
	B	59
	B	63
C		20
C		24
	B	63
	B	60
C		22
C		20
C		17
	B	66
	B	60
	B	55
C		25

Write

Sample answer:
I can determine the mean and spread of data for two data sets on a dot plot or a stem-and-leaf plot. I have to determine the values from the data display before calculating the mean and the five-number summary. I can then calculate the IQR to compare the spreads.

MODULE 4 • TOPIC 3 • LESSON 4 ASSIGNMENT 847

Lesson 4 Assignment

2. Display the results using a stem-and-leaf plot. Be sure to include a key.

Computer users		Book borrowers
7 6	1	
7 5 4 2 2 0 0 0	2	
	3	
	4	
	5	5 5 7 9
	6	0 0 0 3 3 6
Key = 0 |1| means 10 |5| 5 means 55		

3. Describe the distribution of data values for the computer users and the book borrowers.

   Both group's distributions appear to be symmetric.

4. Calculate the mean age of the computer users and the mean age of the book borrowers.

   Mean of computer users: 21.3; mean of book borrowers: 59.8

848 MODULE 4 • TOPIC 3 • LESSON 4 ASSIGNMENT

Lesson 4 Assignment

5. Determine the five-number summary for the computer users and the book borrowers.

   Computer user: Min = 16, Q1 = 20, Med = 21, Q3 = 24, Max = 27
   Book borrower: Min = 55, Q1 = 57, Med = 60, Q3 = 63, Max = 66

6. Calculate, interpret, and compare the IQR for both the computer users and the book borrowers.

   Computer user: IQR = 4

   Book borrower: IQR = 6

7. What can you say about these two populations?

   Sample answer:

   There is not an overlap between the data sets, so I can see that the book borrowers are older than the computer users on average.

MODULE 4 • TOPIC 3 • LESSON 4 ASSIGNMENT 849

Lesson 4 Assignment

Prepare

Determine the five-number summary, mean, and IQR of the data set represented in the stem-and-leaf plot.

0	1 2 4
1	0 1
2	3
3	1 2 3 5
4	
5	5 6

Key: 3|1 = 31

Minimum: 1

Q1: 7

Median: 27

Q3: 34

Maximum: 56

Mean: approximately 24.42

IQR: 27

850 MODULE 4 • TOPIC 3 • LESSON 4 ASSIGNMENT

5 Using Random Samples from Two Populations to Draw Conclusions

MATERIALS

Problem-Solving Model
Graphic Organizer

LESSON OVERVIEW

Students use random samples to draw conclusions about two populations. The characteristics of the two populations are analyzed using graphical displays in the forms of stem-and-leaf plots and box plots. In the first situation, students are given a table of values containing data for two populations. In the second situation, students are given two histograms containing data for two distinct populations. Students create graphical displays to answer questions related to each problem situation. Questions focus on means, medians, ranges, and interquartile ranges (IQR).

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1E create and use representations to organize, record, and communicate mathematical ideas.

Proportionality

(6) The student applies mathematical process standards to use probability and statistics to describe or solve problems involving proportional relationships.

The student is expected to:

7.6F use data from a random sample to make inferences about a population.

Measurement and Data

(12) The student applies mathematical process standards to use statistical representations to analyze data.

The student is expected to:

7.12A compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads.

7.12C compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations.

ELPS

(2) Listening

The student is expected to:

(D) monitor understanding of spoken language during classroom instruction and interactions and seek clarification as needed.

(3) Speaking

The student is expected to:

(A) practice producing sounds of newly acquired vocabulary such as long and short vowels, silent letters, and consonant clusters to pronounce English words in a manner that is increasingly comprehensible.

(4) Reading

The student is expected to:

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

ESSENTIAL IDEAS

- Measures of center for samples from two populations are compared.
- Graphical displays, such as stem-and-leaf plots and box plots, are used to determine the characteristics of two populations.

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LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Downloading Podcasts 5 minutes

ESTABLISH A SITUATION

Students preview the next activity by analyzing a scenario describing two populations. They determine the populations described and then outline a plan for determining random samples from each of the populations.

DEVELOP

Activity 5.1: Simulating Random Samples 20 minutes

REAL-WORLD PROBLEM SOLVING, PEER WORK ANALYSIS

Students use provided tables of data to do a random sampling of two populations. The samples are used to create dot plots. Students describe the distribution of data, estimate, and then calculate the mean for each dot plot. They conclude similar results from both sets of samples with respect to the problem situation.

Activity 5.2: Comparing Random Samples 20 minutes

REAL-WORLD PROBLEM SOLVING, PEER WORK ANALYSIS

Students utilize a table of data containing two populations. From this information, they determine a method for analyzing and comparing data. Students then examine the mean and median for both populations. Next, they construct a stem-and-leaf plot containing the two populations and draw conclusions from their graph. Students determine and interpret the five-number summaries and the interquartile ranges to make additional observations. Finally, they construct a box plot containing both populations and answer questions related to the problem situation.

DAY 2

Activity 5.3: Analyzing Displays of Data from Random Samples 15–20 minutes

REAL-WORLD PROBLEM SOLVING, PEER WORK ANALYSIS

Students are given two histograms that represent the minutes per view for different customers on two sections of a website. They use the histograms to answer questions about variations, means, medians, and distribution of data related to the problem situation. Students analyze unnamed box plots. They analyze two box plots to determine which histogram displays the matching data. This graphical display of the populations is also used to answer questions related to the problem situation.

DEMONSTRATE

Talk the Talk: Into Each Life Some Rain Must Fall 20–25 minutes

EXIT TICKET APPLICATION

Students demonstrate what they have learned in the lesson by comparing two people's interpretations about the weather with data. They can choose any method they wish to determine whether the interpretations given are valid.

Getting Started

ENGAGE

Downloading Podcasts

Facilitation Notes

In this activity, students are presented with a scenario about a local radio station. They identify two different populations and outline a plan for determining random samples from each of the populations.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What problem are you trying to solve? • Which question is a statistical question? How can you tell? • Why might the radio station want to separate the responses of the radio listeners and podcast listeners?

Summary

Real-world problem situations can be solved using random sampling techniques.

STAMP THE LEARNING

ACTIVITY 5.1 Simulating Random Samples

DEVELOP

Facilitation Notes

In this activity, students use tables of data to conduct a random sampling of two populations, create dot plots, and describe the distribution of data for information from a local radio station. For comparison purposes, they estimate and calculate the mean for each dot plot.

Emphasize that the random sampling of data multiple times usually results in similar conclusions.

Have students work with a partner or in a group to complete Questions 1 through 8. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Allow students to create two separate dot plots to see the distributions more clearly.

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QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you select your random sample?
Gathering	• What does the horizontal axis of your dot plot represent?
Seeing structure	• How do you know which measure of variation to use to summarize the spread? • How do these results help you compare the data for the two groups? • Does an overlap of data sets make it easier or more difficult to make a conclusion? • How did having a larger sample affect your results?

Combine the class data to complete Question 9 and then have students complete Question 10. Share responses as a class.

STAMP THE LEARNING

Summary

Random sampling of data multiple times usually results in similar conclusions.

ACTIVITY 5.2

Comparing Random Samples

Facilitation Notes

In this activity, students are presented with a table of data containing the monthly average temperatures in two cities. They analyze the data by determining the mean and median for both populations, constructing a stem-and-leaf plot, and constructing a box plot. Students then determine and interpret the five-number summaries and IQRs and answer related questions.

Ask a student to read the introduction aloud. Discuss the table as a class.

Have students work with a partner or in a group to complete Questions 1 through 8. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Complete the stem-and-leaf plot for Ashland as a class. Then, have students repeat the process for Belsano with a partner.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What observations can you make from the table? • What other mathematical information might help Juliana make her decision?
Probing	• Explain how you calculated the five-number summary. • What observations can you make from this table?
Gathering	• What information can you determine from the box plot that wasn't as evident in either table? • Which part of the graph represents the IQR? • What percent of data appears in the box portion? • Which measures of spread does a box plot display?
Reflecting and justifying	• Which city would Juliana choose if she based her decision on the median only? IQR only? Range only?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Graphical displays, such as stem-and-leaf plots and box plots are helpful when determining characteristics of a population, such as variations in spread, mean, median, and interquartile range.

ACTIVITY 5.3 Analyzing Displays of Data from Random Samples

Facilitation Notes

In this activity, students are presented with two histograms that show data of customer visits to a website. They use the histograms to answer questions about variations in data, means, medians, and distribution of data. Students analyze two box plots to determine which histogram displays the matching data and answer a related question.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction aloud. Discuss the information on the histograms as a class.

Have students work with a partner or in a group to complete Questions 1 through 8. Share responses as a class.

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DIFFERENTIATION STRATEGY

Just in Time Support

Have students refer to Lesson 4 in this topic for graphs showing the relationship between the mean and median based on the data distribution.

COMMON MISCONCEPTION

Students often think that the longer sections of a box plot include more data. Remind them that 25 percent of the data lies in each section, so an extended section means that the 25 percent of the data it contains is more spread out.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What does the first bin in the Articles histogram represent? • What does the tallest bin in the Photo Gallery histogram represent? • Describe the type of customer represented in the last bin of the Articles histogram. • According to the data, which section of the website is more popular with customers? Explain your thinking.
Probing	• How is the range evident in the histograms?
Gathering	• Do the viewing patterns reflect what you would expect for articles and photos? Explain your thinking.
Seeing structure	• How can you use the graph rather than calculations to make these conclusions? • Which characteristics of the histograms and box plots did you consider to make your decisions?
Gathering	• What percent of the data lies in each section of a box plot?
Seeing structure	• How can you use the fact that 25 percent of the data lies in each section to explain how the box plot shows variation?

Summary

Graphical displays, such as histograms and box plots, are helpful when determining characteristics of a population, such as variations in spread, mean, median, and interquartile range.

DEMONSTRATE

Talk the Talk

INTO EACH LIFE SOME RAIN MUST FALL

Facilitation Notes

In this activity, students are presented with a table of data showing the average monthly rainfall for two cities. They review two people's interpretations of the data and use any method to determine the validity of their statements.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Did you display the data in a graph? If so, what information were you able to obtain from the graph to determine who is correct? • Did you calculate any measures of center and spread? If so, how did your results help you validate the given statements? • What do the measures of center and spread tell you about each city's rainfall?

Have students read and answer the Essential Question on the lesson opener page.

Summary

Data sets shown using a table of values can be used to create graphical displays. Graphical displays can be used to describe characteristics of a population, such as variations in spread, mean, median, and interquartile range.

STAMP THE LEARNING

5 Using Random Samples from Two Populations to Draw Conclusions

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Compare the measures of variation for random samples from two populations.
- Express the difference between the centers of two data distributions as a multiple of a measure of variation.
- Use measures of variation to draw conclusions about two populations.

You have learned about measures of variation and have calculated the variations of different data sets.

How can you compare data sets in terms of their variation to solve problems?

Sample answer:

You can use the means and mean absolute deviations to compare two populations with approximately symmetric data sets. You can use the medians and the interquartile range to compare two populations with skewed data sets.

MODULE 4 • TOPIC 3 • LESSON 5 851

Chunking the Activity

- Read and discuss the scenario.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Downloading Podcasts

Square Roots is a radio show that airs 10 times a week on local radio station WMTH.

WMTH is trying to raise its commercial airtime rates during *Square Roots*. The station claims that while this music show is listened to by hundreds of middle school students via the radio, there are actually a greater number of middle school students who listen to the show by regularly downloading the podcast. Advertisers disagree with WMTH's claim. Advertisers want the station to verify its claim that there are more students listening to downloaded podcasts than listening to live radio. To do so, WMTH and the advertisers choose the local middle school to collect data. They send out a survey and ask the following two questions:

- Do you listen to *Square Roots* on the radio or download the podcast?
- How often do you listen to *Square Roots* per week?

All 389 students at the local middle school who listen to *Square Roots* responded to the survey.

1. What are the two populations for the *Square Roots* survey WMTH is conducting?

   The two populations are regular radio listeners and podcast listeners.

WMTH decides to select a random sample for each population.

2. There are 180 regular radio listeners and 209 podcast listeners at the local middle school. Describe how WMTH and the advertisers can randomly select students for their sample.

   Sample answer:
   WMTH can hand out tickets, each with a three-digit number starting at 001, for each population. They can then throw the tickets into two separate baskets and randomly select the tickets for each population.

852 MODULE 4 • TOPIC 3 • LESSON 5

ACTIVITY
5.1 Simulating Random Samples

Let's use a random number table to simulate random samples from the data. The random number table and data are at the end of the lesson.

1. Use the random number table and the list of radio and podcast listeners at the local middle school at the end of the lesson to help WMTH randomly select a sample.

   a. Randomly select 10 radio listeners. Record each student's last name. Then, use the list to record the number of times each student listened to *Square Roots* during the week.

   Answers will vary.

   b. Randomly select 10 podcast listeners. Record each student's last name. Then, use the list to record the number of podcasts each student downloaded in one week.

   Answers will vary.

When you are assigning each student a number, each number should have the maximum number of digits in the largest number of a population. Therefore, if there are 300 people in a population, each number assigned should have three digits.

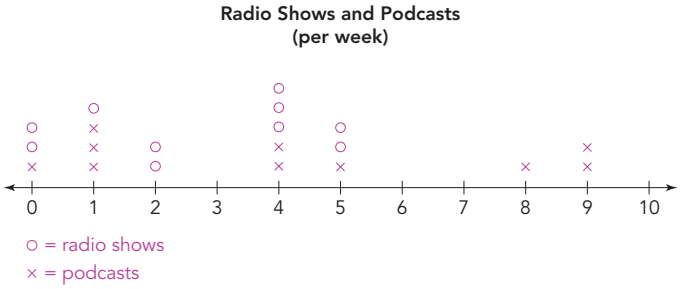
Chunking the Activity

- Read and discuss the introduction.
- Group students to complete Questions 1–8.
- Check in and share.
- Complete Question 9 as a class.
- Group students to complete Question 10.
- Share and summarize.

MODULE 4 • TOPIC 3 • LESSON 5 853

2. Construct a combined dot plot for the two groups. What conclusions can you draw?

Sample answer:



Activity 5.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice interpreting data in dot plots, assign Skills Practice Set A for this lesson.

Review the definition of **describe** in the Academic Glossary.

3. Describe the distribution for each graph. Describe any clusters or gaps in the data values in each graph.

Sample answer:

The graph for the radio shows listened to seems to be symmetric. There seems to be a gap in the data around 3. The graph for the podcasts downloaded doesn't seem to be symmetric or skewed to the left or right. There are gaps in the data values between 1 and 4 and between 5 and 8.

4. Estimate the mean for each dot plot. Explain how you determined your estimate.

Answers will vary.

854 MODULE 4 • TOPIC 3 • LESSON 5

5. Calculate the mean number of radio shows listened to in a week and the mean number of podcasts downloaded in a week.

Sample answer:
Radio shows: 2.7; Podcasts: 4.2

6. Compare the two samples. Are more shows listened to on the radio, or are more podcasts downloaded?

Sample answer:
It appears that podcasts are downloaded more often during the week.

7. Compare the measures of center and measures of variation.

Sample answer:
The difference in means is 1.5. There is likely a lot of overlap in the two data sets.

8. Xavier says,"If we had started on a different line number in the random number table, our results would have been the same." Is Xavier correct? Explain your reasoning.

Xavier is not correct. The means might not have been the same since different students would be chosen.

EB STUDENT TIP

For all proficiency levels

Support students in using the language skewed right, skewed left, symmetric, clusters, and data values while discussing dot plots.

Beginning: Connect the language of dot plots to students manipulating everyday objects or manipulatives to make their own real-life "dot plot" displayed flat on their desks. For example, when you say, "Make a cluster in your data values," students can accordingly arrange the objects to show this on their "dot plot." Model with some examples before prompting students to try it on their own.

(continued on next page)

The third step in the statistical process is to interpret the data.

9. Combine your data with the data from other classmates. Calculate measures of center and measures of variation for the two combined random samples and interpret your results.

Check students' work.

10. Determine the difference of means for the two samples and describe this difference as a multiple of the measure of variation.

Check students' work.

EB STUDENT TIP *(continued)*

Intermediate: Use sentence frames to support students in describing the distribution of dot plots, such as "The dot plot is (skewed left / skewed right / symmetric) because _____," and "There is a cluster in the data values around _____, which indicates _____."

Advanced/Advanced High: Have students critique the use of dot plots in representing data, comparing them with other types of graphs like histograms or box plots. For example, using precise mathematical vocabulary, students can write a paragraph on a given dataset, comparing the advantages and limitations of dot plots in data visualization.

ACTIVITY 5.2

Comparing Random Samples

Juliana graduated from college and now has a choice of two jobs. One of the jobs is in Ashland, and the other job is in Belsano. Since Juliana enjoys mild weather and average temperatures in the 60s (°F), she decides to compare the monthly average temperatures of the two cities. She gathered the following sample of average monthly temperatures for a previous year for the two cities, as shown in the table.

Month	Ashland Average Monthly Temperatures (°F)	Belsano Average Monthly Temperatures (°F)
January	56	48
February	58	55
March	60	59
April	61	62
May	65	66
June	70	69
July	75	78
August	82	88
September	73	82
October	68	69
November	60	59
December	56	49

Chunking the Activity

- Read and discuss the introduction and table.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

MODULE 4 • TOPIC 3 • LESSON 5 857

1. What are some ways Juliana could analyze the data to determine which city is warmer?

   Sample answer:
   She could create a dot plot displaying the temperatures in each city using different symbols. She could calculate the measures of center and measures of variation of the data for each city.

2. Juliana decides to calculate the mean and median temperature for each city to determine which city is warmer overall. Calculate the mean and median temperature for both cities. What do you notice?

   The mean temperature for the entire year for both cities is $65.\overline{3}$°F. The median temperature for Ashland is 63°F, and the median temperature for Belsano is 64°F.

3. Ben says that since Ashland and Belsano have very similar mean and median temperatures, Juliana could choose either city to live in because they both have mild temperatures. Do you agree or disagree? Explain your reasoning.

   Sample answer:
   I agree with Ben. Since the mean and median temperatures for both cities are in the 60s, Juliana could choose either city to live in to meet her requirement.

4. Construct a back-to-back stem-and-leaf plot for the average monthly temperature for each city.

Ashland						Belsano				
					4	8	9			
		8	6	6	5	5	9	9		
8	5	1	0	0	6	2	6	9	9	
		5	3	0	7	8				
				2	8	2	8			

Key: 0|6|2 = 60°F and 62°F

858 MODULE 4 • TOPIC 3 • LESSON 5

5. What conclusions can you draw from the plot?

The temperatures in Belsano are more spread out. Belsano has average temperatures in the high 40s in December and January. The temperatures in Ashland have less variability and are more consistent. The data values show that Ashland did not have any temperatures in the 40s.

6. Determine and interpret the five-number summary and IQR for each data set. Then, describe some observations from the data of each five-number summary.

	Ashland	Belsano
Minimum:	56	48
Q1:	59	57
Median:	63	64
Q3:	71.5	73.5
Maximum:	82	88
IQR:	12.5	16.5

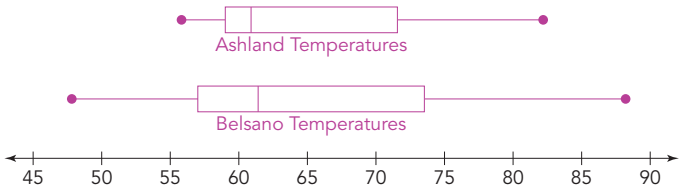
Ashland and Belsano have very similar medians. However, 50% of the sample temperatures for Ashland are between 59°F and 71.5°F, while 50% of the sample temperatures for Belsano are between 57°F and 73.5°F. The temperatures for Ashland go from 56°F to 82°F, for a range of 26 degrees, while the temperatures for Belsano go from 48°F to 88°F, for a range of 40 degrees.

MODULE 4 • TOPIC 3 • LESSON 5 859

Question 7 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice creating box plots, assign Skills Practice Set B for this lesson.

Question 8 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice interpreting data in box plots, assign Skills Practice Set C for this lesson.

7. Construct and label box plots for each data set using the same number line for both. What conclusions can you make?



8. Compare the mean and variation of the samples. If you were Juliana, which city would you choose to live in? Explain your reasoning.

Sample answer:

Belsano has a higher median temperature than Ashland, but the IQR for the temperatures in Ashland is less than the IQR for the temperatures in Belsano. This means that the temperatures in Ashland are more consistently around the median temperature of 61°F, so I would choose to live in Ashland.

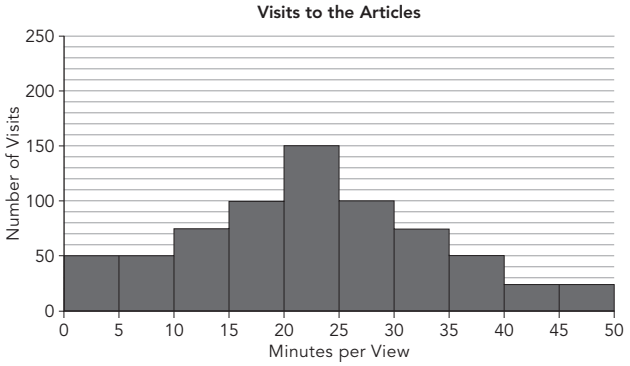
860 MODULE 4 • TOPIC 3 • LESSON 5

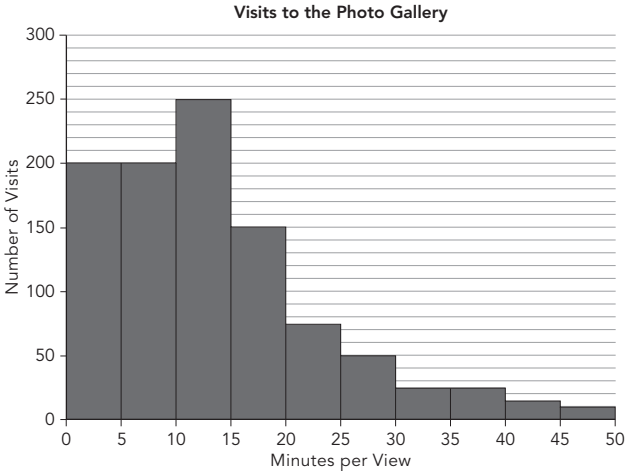
ACTIVITY 5.3

Analyzing Displays of Data from Random Samples

Websites often analyze customer visits to see if there are patterns or trends. Gaining information about patterns helps companies display the information users want.

A sample of customer visits to a news and opinion website is shown in the histograms. The histograms display the number of visits customers made to the Articles and Photo Gallery sections of the website, along with how long each customer spent viewing the content in each section.





Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction and histograms.
- Group students to complete the activity.
- Share and summarize.

MODULE 4 • TOPIC 3 • LESSON 5 861

1. Describe the populations and samples for this problem.

   The populations are all views for the website. The samples are the views for Articles and Photo Gallery.

2. What do the intervals along the *x*-axis and *y*-axis represent for each histogram?

   The intervals along the *x*-axis represent the minutes per view. The intervals along the *y*-axis represent the number of visits.

3. About how many visits were made to the Articles sample? The Photo Gallery sample? Explain how you determined your answer.

   There were about 700 views in the sample for the Articles and about 1000 views in the sample for the Photo Gallery.

   Explanations will vary.

4. Antonio claims that the variation in the length of views of each section in terms of the range is about the same for both sections. Do you agree or disagree?

   Both sections have a similar range—minutes per view are between 0 and 50 minutes.

862 MODULE 4 • TOPIC 3 • LESSON 5

5. Describe the shape of each histogram and explain what this means in terms of the number of minutes per view.

The histogram for the Articles is more symmetric. Most of the minutes per view were closer to the center of the time intervals along the *x*-axis. The histogram for the Photo Gallery is skewed right. That means most of the minutes per view were shorter, and fewer views were longer than 30 minutes.

6. Determine whether the mean or the median is greater for each section. Then, explain why that measure of center is greater in value for each section.

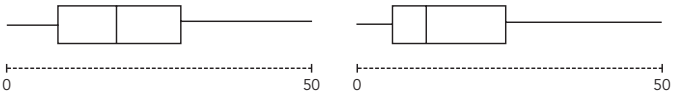
For the Photo Gallery, the mean is greater than the median because the data are skewed right. The median is closer to the majority of the data.

For the Articles, the mean and median are probably close to each other because the graph is symmetric.

So, I cannot tell by looking at the graph whether the mean or median is greater.

MODULE 4 • TOPIC 3 • LESSON 5 863

7. The box plots shown represent the minutes per view for the Photo Gallery and Articles sections. Using the information you know from the histograms for each section, which box plot do you think represents the minutes per view for the Articles, and which represents the minutes per view for the Photo Gallery? Explain your choice.



The box plot on the left is for the Articles; the plot on the right is for the Photo Gallery.

8. In terms of the box plots, which section has more variation in the minutes per view? Explain your reasoning.

Photo Gallery has more variation in the minutes per view. The upper 50% of the data is much more spread out than the lower 50% of the data.

For the Articles, the upper and lower 50% of the data have about the same spread.

Question 8 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice comparing box plots and describing the variation, assign Skills Practice Set D for this lesson.

864 MODULE 4 • TOPIC 3 • LESSON 5

Talk the Talk

Into Each Life Some Rain Must Fall

PROBLEM SOLVING

Omar lives in Seattle, Washington, and he says it seems like it rains all the time. Ricardo lives in Washington, DC, and he says it seems like it doesn't rain very much.

The table contains the average monthly rainfall for both cities over the past 30 years.

1. Use any method you want to determine the validity of Omar's and Ricardo's statements.

Month	Seattle, Washington Average Monthly Rainfall (inches)	Washington, DC. Average Monthly Rainfall (inches)
January	5.24	3.21
February	4.09	2.63
March	3.92	3.60
April	2.75	2.77
May	2.03	3.82
June	1.55	3.13
July	0.93	3.66
August	1.16	3.44
September	1.61	3.79
October	3.24	3.22
November	5.67	3.03
December	6.06	3.05

Answers will vary. Students may compare the median, range, and IQR of the data sets.

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

Optimizing Learning

This activity optimizes individual choice and autonomy.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate their problem-solving strategies as a class.

MODULE 4 • TOPIC 3 • LESSON 5 865

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)
1	Abunto	1	21	D'Ambrosio	0	41	Granger	0
2	Adler	3	22	Datz	4	42	Guca	2
3	Aizawa	3	23	Delecroix	2	43	Haag	8
4	Alescio	4	24	Difiore	6	44	Heese	5
5	Almasy	8	25	Dobrich	7	45	Hilson	1
6	Ansari	6	26	Donoghy	1	46	Holihan	1
7	Aro	7	27	Donaldson	5	47	Hudack	1
8	Aung	2	28	Dreher	2	48	Ianuzzi	3
9	Baehr	7	29	Dubinsky	1	49	Islamov	4
10	Bellmer	1	30	Dytko	8	50	Jacobsen	5
11	Bilski	4	31	Fabry	7	51	Jessell	4
12	Blinn	6	32	Fetcher	1	52	Ji	1
13	Bonetto	3	33	Fontes	5	53	Johnson	2
14	Breznai	1	34	Frick	3	54	Jomisko	1
15	Cabot	3	35	Furmanek	5	55	Jones	6
16	Chacalos	0	36	Gadgil	4	56	Joy	5
17	Cioc	0	37	Gavlak	4	57	Jumba	1
18	Cole	3	38	Gibbs	0	58	Juth	7
19	Creighan	4	39	Gloninger	2	59	Jyoti	6
20	Cuthbert	6	40	Goff	1	60	Kachur	2

866 MODULE 4 • TOPIC 3 • LESSON 5

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)
61	Kanai	0	81	McNary	7	101	Nzuyen	2
62	Keller	2	82	Meadows	0	102	O'Bryon	0
63	Khaing	7	83	Merks	8	103	Obitz	3
64	Kindler	5	84	Mickler	5	104	Oglesby	8
65	Kneiss	5	85	Minniti	4	105	Ono	1
66	Kolc	1	86	Mohr	3	106	Paclawski	0
67	Kuisis	2	87	Mordecki	5	107	Pappis	6
68	Labas	1	88	Mueser	3	108	Peery	3
69	Lasek	8	89	Musati	3	109	Phillips	5
70	Leeds	5	90	Myron	2	110	Potter	3
71	Lin	0	91	Nadzam	4	111	Pribanic	7
72	Litsko	2	92	Nazif	7	112	Pwono	5
73	Lodi	3	93	Newby	0	113	Quinn	2
74	Lookman	2	94	Ng	1	114	Rabel	5
75	Lucini	1	95	Nino	2	115	Rayl	2
76	Lykos	0	96	Northcutt	4	116	Rea	4
77	MacAllister	1	97	Novi	3	117	Reynolds	5
78	Magliocca	6	98	Null	8	118	Rhor	8
79	Marchick	5	99	New	5	119	Rielly	8
80	McGuire	1	100	Nyiri	1	120	Risa	7

MODULE 4 • TOPIC 3 • LESSON 5 867

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)	Student Number	Student Name	Radio Shows Listened To (per week)
121	Robinson	8	141	Stevens	4	161	Volk	5
122	Roethlein	1	142	Tabor	0	162	Vyra	4
123	Romanski	7	143	Tevis	5	163	Wadhwani	5
124	Rouce	7	144	Thomas	0	164	Warnaby	0
125	Rubio	7	145	Thompson	1	165	Weasley	2
126	Rutland	3	146	Thorne	1	166	Weidt	8
127	Rychcik	1	147	Tiani	0	167	Whitelow	0
128	Sabhnani	6	148	Tokay	6	168	Wilson	4
129	Sandroni	2	149	Toomey	6	169	Woller	1
130	Saxon	3	150	Trax	1	170	Woo	0
131	Scalo	4	151	Truong	8	171	Wunderlich	6
132	Schessler	1	152	Tunstall	1	172	Wycoff	4
133	Seeley	0	153	Twiss	6	173	Xander	8
134	Shanahan	3	154	Tyler	0	174	Yahya	0
135	Siejk	1	155	Ueki	0	175	Yezovich	4
136	Skaro	3	156	Uriah	6	176	Youse	0
137	Slonaker	8	157	Vagnelli	6	177	Yuzon	8
138	Sobr	2	158	Van Dine	5	178	Za Khai	3
139	Spatz	4	159	Vella	1	179	Ziff	2
140	Sramac	2	160	Vidnic	0	180	Zuk	5

868 MODULE 4 • TOPIC 3 • LESSON 5

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
1	Aaronson	2	21	Chang	4	41	Frena	1
2	Abati	0	22	Clarke	9	42	Galdi	8
3	Ackerman	4	23	Crnkovich	0	43	Gansberger	3
4	Aderholt	2	24	Dahl	0	44	Gianni	1
5	Akat	7	25	Dax	7	45	Glencer	1
6	Aleck	9	26	Defoe	1	46	Godec	7
7	Alessandro	5	27	Dengler	4	47	Goldstein	0
8	Allen	3	28	Di Minno	4	48	Graef	6
9	Ansil	1	29	Dilla	5	49	Gula	1
10	Archer	5	30	Draus	5	50	Hagen	2
11	Badgett	9	31	Duffy	0	51	Haupt	8
12	Bartle	2	32	Ecoff	3	52	Herc	4
13	Bibby	9	33	Esparra	7	53	Hnat	9
14	Bilich	5	34	Fakiro	7	54	Hodak	3
15	Bloom	3	35	Ferlan	4	55	Hoyt	2
16	Boccio	5	36	Fetherman	2	56	Huang	3
17	Bracht	3	37	Fillipelli	6	57	Iannotta	4
18	Bujak	7	38	Fisher	2	58	Irwin	5
19	Caliari	9	39	Folino	9	59	Jackson	7
20	Cerminara	8	40	Forrester	9	60	Jamil	1

MODULE 4 • TOPIC 3 • LESSON 5 869

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
61	Jessop	1	81	Ling	6	101	Moorey	6
62	Johnson	9	82	Loch	4	102	Mox	7
63	Joos	9	83	Lorenzo	4	103	Mrkali	7
64	Joseph	5	84	Lovejoy	5	104	Mu	0
65	Jubic	3	85	Luba	8	105	Muller	3
66	Juhl	7	86	Lukitsch	4	106	Murphy	2
67	Jung	9	87	Luzzi	8	107	Mwambazi	6
68	Jurgensen	4	88	Lyman	5	108	Myers	4
69	Jyoti	0	89	MacIntyre	8	109	Nangle	3
70	Kaib	5	90	Maddex	5	110	Neilan	7
71	Kapoor	6	91	Marai	2	111	Nicolay	5
72	Kennedy	2	92	Mato	9	112	Niehl	6
73	Kimel	5	93	McCaffrey	0	113	Nix	2
74	Klaas	4	94	McElroy	5	114	Noga	3
75	Ko	9	95	McMillan	3	115	Nowatzki	7
76	Krabb	1	96	Meng	9	116	Nuescheler	5
77	Ladley	9	97	Michelini	5	117	Nye	6
78	Lawson	1	98	Misra	0	118	Nytra	6
79	Lemieux	7	99	Miller	8	119	O'Carrol	6
80	Lewan	6	100	Modecki	7	120	Obedi	7

870 MODULE 4 • TOPIC 3 • LESSON 5

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)
121	Oehrle	8
122	Olds	5
123	Oleary	0
124	Ondrey	1
125	Owusu	9
126	Palamides	9
127	Pappas	0
128	Pecori	3
129	Pennix	4
130	Pendleton	1
131	Phillippi	2
132	Pieton	6
133	Ploeger	2
134	Pressman	4
135	Puzzini	1
136	Qu	4
137	Qutyan	8
138	Raab	5
139	Raeff	0
140	Rav	1
141	Rea	5
142	Renard	7
143	Rex	7
144	Richards	7
145	Ridout	7
146	Rivera	6
147	Roberts	4
148	Rodwich	0
149	Roney	7
150	Ross	6
151	Rothering	0
152	Rua	4
153	Russo	8
154	Ryer	8
155	Sagi	8
156	Sallinger	6
157	Sau	7
158	Sbragia	7
159	Schaier	5
160	Schmit	2
161	Scopaz	4
162	Sebula	4
163	Shah	1
164	Sidor	6
165	Skraly	6
166	Sokolowski	5
167	Speer	6
168	T'Ung	9
169	Tamar	9
170	Tebelius	1
171	Tesla	4
172	Thuma	0
173	Tibi	2
174	Tobkes	9
175	Torelli	8
176	Tozzi	0
177	Traut	6
178	Trax	0
179	Tu	8
180	Tumicki	2

MODULE 4 • TOPIC 3 • LESSON 5 871

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)
181	Tyson	1
182	Uansa	0
183	Ulan	4
184	Urbano	7
185	Uzonyi	7
186	Vaezi	8
187	Vinay	6
188	Vu	8
189	Wallee	0
190	Waldock	4
191	Wallace	2
192	Webb	9
193	Weisenfeld	0
194	Whalen	7
195	Wiley	9
196	Williams	5
197	Williamson	6
198	Witek	9
199	Wojcik	3
200	Woollett	6
201	Wulandana	8
202	Wysor	9
203	Xiao	4
204	Yee	6
205	Yost	1
206	Young	7
207	Yuros	6
208	Zaki	0
209	Zimmerman	1

872 MODULE 4 • TOPIC 3 • LESSON 5

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61004	43516	17020
Line 2	17264	57327	38224	29301	31381	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86210	11808	12841	45147	97438	60022	12645	62000
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19177	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	10415	36457
Line 8	16975	95428	33226	55903	31605	43817	22250	03918	46999	98501
Line 9	59138	39542	71168	57609	91510	77904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

MODULE 4 • TOPIC 3 • LESSON 5 873

874 MODULE 4 • TOPIC 3 • LESSON 5

Lesson 5 Assignment

Write

Explain how to use measures of center and measures of variation to compare two populations.

Remember

You can use the means to compare two populations with approximately symmetric data sets. You can use the medians and the interquartile range to compare two populations with skewed data sets.

Practice

1. Repeat the sampling procedure you used with the data in Activity 5.1. Choose a different line number in the random number table.

   Answers will vary.

   a. Record the results for 20 students.

   b. Construct a combined dot plot for the two groups, using the same scale.

   c. Describe the distribution of the data for each group.

   d. Estimate and calculate the mean for the data in each group.

   e. Compare your new results with your results from Activity 4.1.

Write

Sample answer: Measures of center provide a summary, or typical values, to describe a population. Variation provides information about how much the data varies within the population.

MODULE 4 • TOPIC 3 • LESSON 5 ASSIGNMENT 875

Lesson 5 Assignment

2. Ratings can be used to show people's opinions of hotels and restaurants. One of the things that can help a restaurant get good ratings is the time it takes to be seated (the wait time) at a restaurant without reservations. A steak house and a pizza parlor both claim to have the shortest wait times in town. To check out the claims, Omar a restaurant reviewer, records the time it takes to be seated without reservations. The results of wait times from 8 visits to each restaurant are shown in the table below.

Steak House Wait Times (minutes)	Pizza Parlor Wait Times (minutes)
5	11
13	19
22	14
7	14
20	15
21	20
20	10
12	17

a. Describe the populations and samples for this problem.

The populations are the wait times at the steak house and the pizza parlor. The sample was collected from observations of wait times from 8 visits to each restaurant.

876 MODULE 4 • TOPIC 3 • LESSON 5 ASSIGNMENT

Lesson 5 Assignment

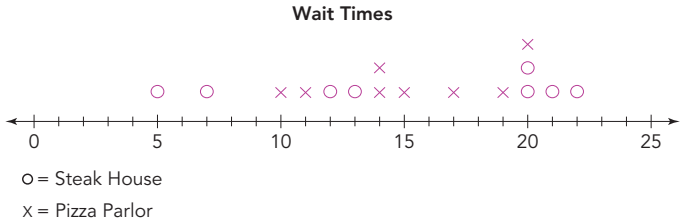
b. Calculate the mean wait times for the two restaurants. Which restaurant seems to have faster service?

Steak house mean: 15 minutes

Pizza parlor mean: 15 minutes

The mean wait time is the same for both restaurants.

c. Complete the dot plot of the times. What do you notice?



d. Determine the median wait time for each restaurant.

Steak house median: 16.5 minutes

Pizza parlor median: 14.5 minutes

e. Explain the difference in median times for the two restaurants.

The median wait time for the pizza parlor is shorter than for the steak house.

f. Which measure of center would you use if:

- you are the steak house and want to claim you have the shortest wait time?

  For the steak house to show the shortest wait time, they should use the mean.

MODULE 4 • TOPIC 3 • LESSON 5 ASSIGNMENT 877

Lesson 5 Assignment

- you are the pizza parlor and want to claim you have the shortest wait time?

  For the pizza parlor to show the shortest wait time, they should use the median.

- you are a customer and want the shortest wait time?

  The median wait time for the pizza parlor shows the shortest wait time.

g. Suppose another restaurant reviewer records wait times at each restaurant several times. Do you think it is possible that the wait times might be different from Omar's wait times? Explain your reasoning.

   Yes. The wait times could change based on the time of day the sample was taken.

h. How could we be more certain which restaurant has the shortest wait time?

   Sample answer:
   You could calculate and interpret the variations in data.

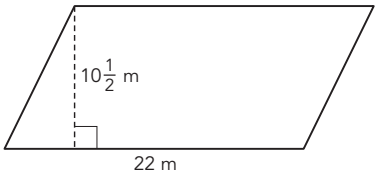
878 MODULE 4 • TOPIC 3 • LESSON 5 ASSIGNMENT

Lesson 5 Assignment

Prepare

Use a formula to determine the area of each figure.

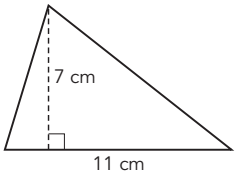
1.



231 square meters

$22 \cdot 10\frac{1}{2} = 231$

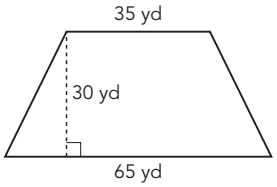
2.



38.5 square centimeters

$\frac{1}{2}(11)(7) = 38.5$

3.



1500 square yards

$\frac{1}{2}(30)(65 + 35) = 1500$

MODULE 4 • TOPIC 3 • LESSON 5 ASSIGNMENT 879

880 MODULE 4 • TOPIC 3 • LESSON 5 ASSIGNMENT

TOPIC 3 SELF-REFLECTION

Name: _____

Drawing Inferences

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Drawing Inferences* topic by:

TOPIC 3: *Drawing Inferences*	Beginning of Topic	Middle of Topic	End of Topic
applying knowledge of probability and proportional relationships to analyze data and make inferences about a population.	○	○	○
applying knowledge of the statistical process to analyze data and make inferences about a population.	○	○	○
using tools to generate a random sample that represents the population.	○	○	○
creating and interpreting data displays, such as bar graphs, dot plots and circle graphs, including part-to-part and part-to-whole comparisons.	○	○	○
generating random samples for two populations to identify their similarities and differences.	○	○	○
comparing and drawing informal inferences about two populations based on their measures of center and measures of variation. (mean, median, range, interquartile range).	○	○	○

continued on the next page

Notes

MODULE 4 • TOPIC 3 • SELF-REFLECTION 881

Notes

TOPIC 3 SELF-REFLECTION *continued*

TOPIC 3: *Drawing Inferences*	Beginning of Topic	Middle of Topic	End of Topic
comparing two numerical data distributions on a graph, such as a comparative dot plot, stem-and-leaf plot, histogram, or box plot, by visually comparing the shapes, centers, and spreads of the data displays.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Drawing Inferences* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

882 MODULE 4 • TOPIC 3 • SELF-REFLECTION

TOPIC 3 SUMMARY

Drawing Inferences Summary

LESSON 1 Collecting Random Samples

There are four components of the statistical process:

- Formulating a statistical question.
- Collecting appropriate data.
- Analyzing the data graphically and numerically.
- Interpreting the results of the analysis.

One data collection strategy you can use is a survey. A **survey** is a method of collecting information about a certain group of people. It involves asking a question or a set of questions to those people. When information is collected, the facts or numbers gathered are called **data**.

The **population** is the entire set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects. A **census** is the data collected from every member of a population.

Often, it is not possible or logical to collect data from every member of the population. When data are collected from a part of the population, the data are called a **sample**.

When data are gathered from a population, the characteristic used to describe the population is called a **parameter**. When data are gathered from a sample, the characteristic used to describe the sample is called a **statistic**. A statistic is used to make an estimate about the parameter.

When information is collected from a sample to describe a characteristic about the population, it is important that such a sample be as representative of the population as possible. A **random sample** is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

NEW KEY TERMS

- survey
- data [datos]
- population [población]
- census [censo]
- sample
- parameter [parámetro]
- statistic [estadistica]
- random sample

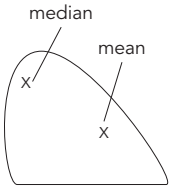
Notes

Notes

LESSON 2 Using Random Samples to Draw Inferences

When you are estimating a parameter that is a count, rather than a percentage, you can use proportional reasoning to scale up from the ratio of the number of observations in your sample to the statistic.

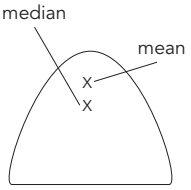
There are three common distributions of data: skewed left, skewed right, and symmetric. The distribution of data can help you determine whether the mean or median is a better measure of center.



Skewed Right

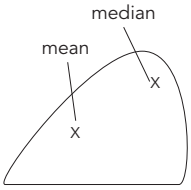
The mean of a data set is greater than the median when the data are skewed to the right.

The median is the better measure of center because the median is not affected by very large data values.



Symmetric

The mean and median are equal when the data are symmetric.



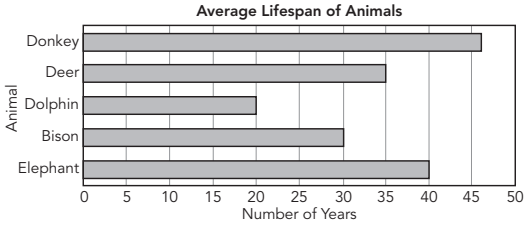
Skewed Left

The mean of a data set is less than the median when the data are skewed to the left.

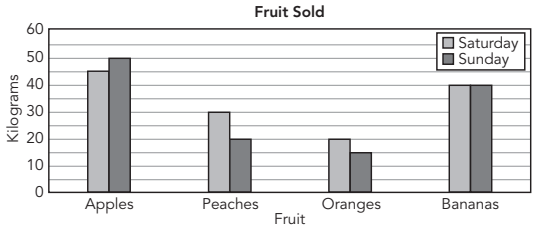
The median is the better measure of center because the median is not affected by very small data values.

LESSON 3 Bar Graphs

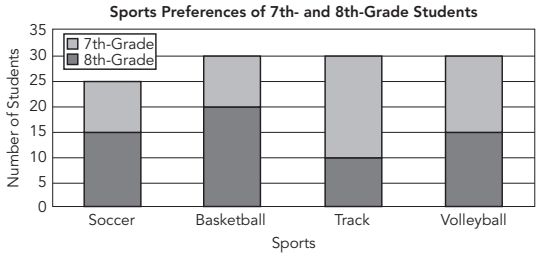
Some graphs are used to display data that consists of different categories. A bar graph displays data using horizontal or vertical bars, so that the height or length of the bars indicates its value for a specific category.



A *double bar graph* can be used when each category contains two different groups of data. The bars may be vertical or horizontal, and a key explains the colors or patterns for each group. The two bars representing the same category are side by side, and space is used to separate the categories.

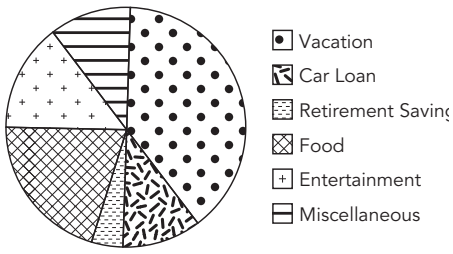


A *stacked bar graph* is a graph that stacks the frequencies of two different groups for a given category on top of one another, so that you can compare the parts to the whole. Each bar represents a total for the whole category but still shows the data for each group within the entire category.



You can also use *circle graphs* to represent the relationship between each part and the whole. For example, this circle graph shows the expenses for families in random cities across the state. As shown in the graph, families tend to spend the greatest amount of money on vacation and the least amount of money on retirement savings.

Average Family Expenses



Notes

Notes

LESSON 4 Comparing Two Populations

Data for two populations may overlap. Comparing the measures of center and measures of variation for the two populations can help you interpret the differences between the two populations.

Comparing the difference of means with the variation in each data set can be an important way of determining just how different two data sets are.

For example, consider these data sets.

5, 3, 4, 5, 10	5, 3, 100, 5, 10
Mean = 5.4	Mean = 24.6

The difference in their means is 19.2. Depending on what you are measuring, this could be a big difference. You can create a comparative dot plot to visually represent and observe the difference in the data sets. Upon investigating the values in each data set, it is apparent that the data sets overlap. The right data set is the same as the left one, with the exception of one number. The data value 100 in the right data set is skewing the data to the right and causing the difference of 19.2 between the means of the two data sets.

LESSON 5 Using Random Samples from Two Populations to Draw Conclusions

You can use the means to compare two populations with approximately symmetric data sets. You can use the medians and the interquartile ranges to compare two populations with skewed data sets.

For example, suppose Antonio wanted to compare the number of text messages sent each day by seventh-grade students to the number of text messages sent each day by eighth-grade students. He randomly selected and surveyed 10 students from each grade. The data sets provided represent the number of text messages each student said they sent per day.

Grade 7: 44, 40, 43, 73, 74, 76, 56, 59, 66, 54

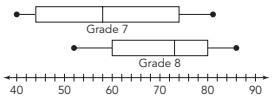
Grade 8: 77, 86, 69, 52, 76, 79, 60, 61, 57, 85

Antonio calculated the mean of each data set. He determined that the mean number of texts sent per day is 58.5 for seventh-graders and 70.2 for eighth-graders.

Antonio created a back-to-back stem-and-leaf plot as well as a box plot, to display the data.

Grade 7		Grade 8
Leaf	Stem	Leaf
4 3 0	4	
9 6 4	5	2 7
6	6	0 1 9
6 4 3	7	6 7 9
	8	5 6

Box Plot



Antonio used his data displays to determine the five-number summary and IQR for each data set.

	Grade 7	Grade 8
Minimum	40	52
Q1:	44	60
Median	57.5	72.5
Q3:	73	79
Maximum	76	86
IQR	29	19

Antonio compared and analyzed the data for the two populations. He used the measures of center to conclude that on average, eighth-graders sent a greater number of texts per day, and he used the measures of variation to conclude that seventh-graders had more variation in the number of texts sent per day.

Notes

Notes

888 MODULE 4 • TOPIC 1 • TOPIC SUMMARY

GRADE 7

MODULE 5

Constructing and Measuring

TOPIC 1 Angle Relationships **891**

TOPIC 2 Area, Surface Area, and Volume **947**

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MODULE 5 OVERVIEW

TEKS Addressed:
7.3A, 7.8A, 7.8B, **7.9A**, **7.9C**, 7.9D, 7.11C
*Bold TEKS = Readiness Standard

Constructing and Measuring

Sessions: **20**

Why is this module named *Constructing and Measuring*?

This module introduces students to informal geometric constructions of angles, using straightedges, protractors, and patty paper to analyze angle relationships. Students build on their knowledge of measures—lengths, areas, surface areas, and volumes—of two- and three-dimensional geometric objects.

Students decompose unfamiliar two-dimensional figures into familiar two-dimensional figures with known area formulas to determine the area of the original composite figure. This knowledge is then applied to working with three-dimensional figures. Students calculate the surface areas of three-dimensional figures by determining the areas of their two-dimensional nets. This builds towards developing formulas for the lateral and total surface areas of rectangular and triangular prisms and pyramids. Then, students develop the formulas for the volumes of rectangular and triangular prisms and pyramids by comparing prisms and pyramids with congruent bases and heights.

The Research Shows . . .

"Spatial visualization—building and manipulating mental representations of two- and three-dimensional objects and perceiving an object from different perspectives—is an important aspect of geometric thinking."

Principles and Standards of School Mathematics | Page 41

What is the mathematics of *Constructing and Measuring*?

Constructing and Measuring contains two topics: *Angle Relationships* and *Area, Surface Area, and Volume*. Students operate with rational numbers. Students build on their prior knowledge of basic geometric ideas and computations of area and volume.

1 DAY PACING = 45-MINUTE SESSION

7 SESSIONS
6 LEARNING • 1 ASSESSMENT

TOPIC 1 *Angle Relationships*

Learning Together: 4 Sessions

TEKS: 7.11C

Students write and solve equations involving the sum of angles in triangles, including isosceles triangles.

- Students explore different pairs of angles including supplementary angles, complementary angles, vertical angles, and adjacent angles.
- Students write and solve equations to determine unknown values in angle pairs.

Learning Individually: 2 Sessions

Targeted Skills Practice for *Angle Relationships*

- Students use the Triangle Sum theorem to determine unknown values.
- Students complete sketches involving angles.
- Students determine unknown angle measures given various types of angle pairs.

13 SESSIONS:
12 LEARNING • 1 ASSESSMENT

TOPIC 2 *Area, Surface Area, and Volume*

Learning Together: 9 Sessions

TEKS: 7.3A, 7.8A, 7.8B, **7.9A**, **7.9C**, 7.9D

Students develop different strategies for decomposing unfamiliar shapes into shapes with known area formulas to determine the area of composite figures.

- Students develop the formulas for lateral and total surface area of rectangular and triangular prisms and pyramids by determining the surface areas of their two-dimensional nets.
- Students investigate the relationship between the volumes of rectangular and triangular prisms and pyramids with congruent bases and heights.
- Students learn that the formulas for the volumes of right rectangular prisms and pyramids can be generalized to any right prisms and pyramids.

Learning Individually: 3 Sessions

Targeted Skills Practice for *Area, Surface Area, and Volume*

- Students calculate the area of composite figures.
- Students sketch nets to represent 3-D figures.
- Students determine the total surface area and lateral surface area of prisms and pyramids.
- Students determine the volume of prisms and pyramids.

How is *Constructing and Measuring* connected to prior learning?

This module builds on students' experiences with two- and three-dimensional shapes. Students have composed two- and three-dimensional shapes and named those shapes, including pentagons, hexagons, and cubes. They have defined and used perimeter, area, volume, and angle measures. They have drawn and identified points, lines, line segments, rays, and angles. Students used decomposition to determine the areas of a variety of two-dimensional shapes. They also determined the volumes of right rectangular prisms.

Math Representation

Suppose that carpeting costs $1.20 per square foot. How much would it cost to carpet every room on this floor except the kitchen?

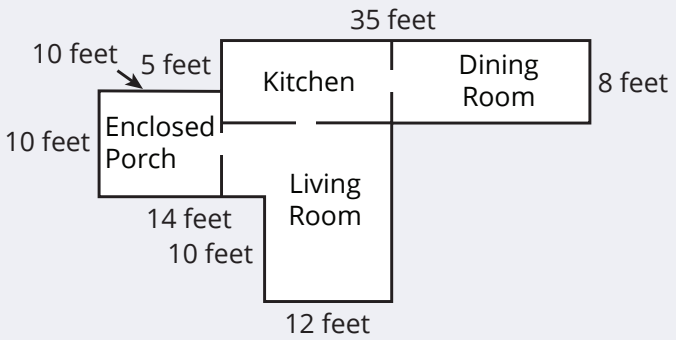
Porch:
10(10) = 100 square feet

Living room:
12(17) + 4(7) = 232 square feet

Dining room:
(19)8 = 152 square feet

Total = 484 square feet
484(1.20) = 580.80

The total cost to carpet every room on this floor except the kitchen is $580.80.



When will students use knowledge from *Constructing and Measuring* in future learning?

Students will continue using construction tools and geometric properties to create and justify constructions in future courses. Students will consider what happens when more than two lines intersect. They will use their knowledge of special angle relationships formed by two intersecting lines to justify relationships. Students will use all of these relationships when they do formal proofs in Geometry. In future courses, students will apply volume formulas to three-dimensional shapes with curved sides.

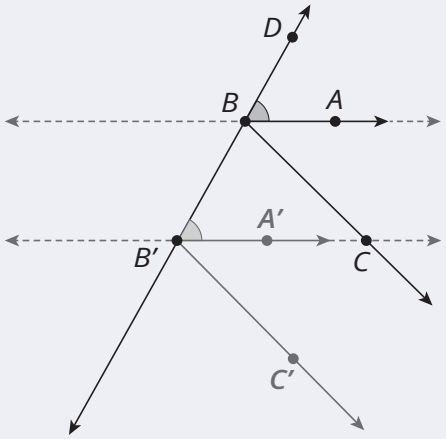
Math Representation

You can prove that corresponding angles are congruent.

Starting with $\angle ABC$ and $\overline{BB'}$, you can translate $\angle ABC$ along $\overline{BB'}$.

Lines AB and $A'B'$ are parallel lines because a translated line is either identical to the original line or parallel to it.

$\angle ABD$ and $\angle A'B'B$ are corresponding angles and are congruent because translations take angles to angles of the same measure.



5 Constructing and Measuring

MODULE 5 Assessment Summary

Topic	Topic Title	Name	Administered	TEKS*
1	**Angle Relationships**	End of Topic Assessment	After Topic 1	7.11C
2	**Area, Surface Area, and Volume**	End of Topic Assessment	After Topic 2	**7.9A** **7.9C** 7.9D

*Bold TEKS = Readiness Standard

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GRADE 7

TOPIC 1

A jib is a triangular sail that sits ahead of the foremast of a sailing ship. Its most crucial function is as an airfoil, increasing performance and overall stability by reducing turbulence on the leeward side of the mainsail.

Angle Relationships

LESSON 1 Solving Equations Using the Triangle Sum Theorem **893**

LESSON 2 Relationships Between 90° and 180° Angles **905**

LESSON 3 Special Angle Relationships **921**

TOPIC 1 OVERVIEW

Angle Relationships

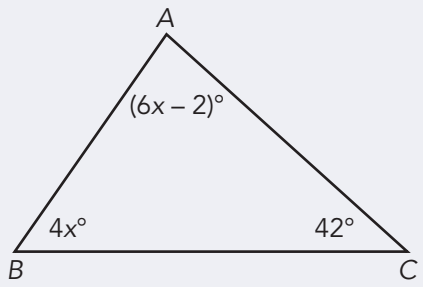
How are the key concepts of *Angle Relationships* organized?

In this topic, students build on their existing knowledge of triangles and angles. They write and solve equations involving the sum of angles in a triangle, including isosceles triangles.

Math Representation

For example, write and solve an equation to determine the value of x. Then, determine each unknown angle measure.

$$(6x - 2) + 4x + 42 = 180$$
$$10x + 40 = 180$$
$$10x = 140$$
$$x = 14$$
$$m\angle B = 4(14) = 56°$$
$$m\angle A = 6(14) - 2 = 82°$$



Next, students explore the relationships between 90° and 180° angles. They use a protractor to explore the relationship between complementary and supplementary angles.

Students explore different pairs of angles, including linear pairs, vertical angles, and adjacent angles. They use patty paper to create angles, and explore relationships between them. Students then write and solve equations involving the sum of angles in angle relationships.

What is the entry point for students?

Students have identified angles as right, obtuse, and acute to classify triangles and to classify two-dimensional figures based on presence or absence of specified angle measures. They have learned the Triangle Sum theorem.

Math Representation

The **Triangle Sum theorem** states that the sum of the measures of the interior angles of a triangle is 180°. The longest side of a triangle is opposite the interior angle with the greatest measure, and the shortest side is opposite the interior angle with the least measure.

The Triangle Sum theorem can be used to determine the measure of the third angle of a triangle when two angle measures of the same triangle are given.

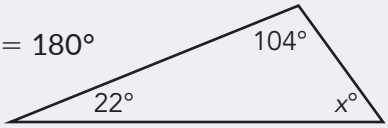
$x + 22 + 104 = 180$

$x + 126 = 180$

$x = 54$

The measure of the third angle in this triangle is 54°.

$22° + 104° + 54° = 180°$



In this topic, students will use prior knowledge of angles and angle relationships as well as writing and solving single variable two-step equations from earlier in this course. Students will use these experiences to solve equations involving geometry concepts as they determine the sum of angles in a triangle and explore angle relationships.

Why is *Angle Relationships* important?

In future courses, students will build from analyzing what happens when two lines intersect (e.g., vertical angles, supplementary angles) to analyzing what happens when more than two lines intersect (e.g., alternate interior angles, same side interior angles). They are expected to build a strong foundation with simple angle pairs to engage with the more complex relationships that result when more than two lines intersect. Students will also be expected to use formal and informal tools to explore and justify these new relationships. The use of patty paper and construction tools prepares students for such explorations because construction experiences provide a more robust understanding of geometric properties and the ability to investigate them.

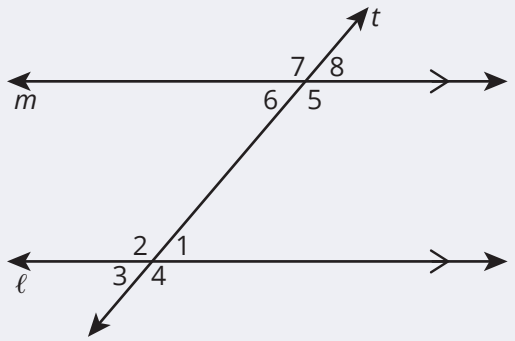
Math Representation

In this diagram, a transversal, t, intersects the two parallel lines, m and ℓ.

The corresponding angles are $\angle 1$ and $\angle 8$, $\angle 4$ and $\angle 5$, $\angle 2$ and $\angle 7$, and $\angle 3$ and $\angle 6$.

The alternate interior angle pairs are $\angle 1$ and $\angle 6$, and $\angle 2$ and $\angle 5$.

The alternate exterior angle pairs are $\angle 3$ and $\angle 8$, and $\angle 4$ and $\angle 7$.



The same-side interior angles are $\angle 1$ and $\angle 5$, and $\angle 2$ and $\angle 6$.

The same-side exterior angles are $\angle 3$ and $\angle 7$, and $\angle 4$ and $\angle 8$.

NEW KEY TERMS

- base angles [ángulos de la base]
- Base Angles theorem [teorema de los ángulos de la base]
- congruent sides
- congruent angles [ángulos congruentes]
- straight angle
- collinear [colineal]
- supplementary angles [ángulos suplementarios]
- complementary angles [ángulos complementarios]
- perpendicular [perpendicular]
- adjacent angles [ángulos adyacentes]
- linear pair [par lineal]
- vertical angles [ángulos verticales]

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic when they can:

- Determine unknown angle measures by writing and solving algebraic equations using the Triangle Sum theorem.
- Recognize the special angle pairs formed when two lines intersect: supplementary angles, complementary angles, vertical angles, adjacent angles, and linear pairs.
- Determine unknown angle measures by writing and solving algebraic equations based on relationships between angles.

How do the activities in *Angle Relationships* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

This topic prompts students to use precision with construction tools and laguage. Students use a protractor, straightedge, and patty paper to build intuition and engage in informal constructions without needing to know the properties required for more formal constructions (7.1C). Students should become comfortable with different tools, so that they can choose strategically and appropriately, according to their purpose, in future courses.

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Integrate technology using multimedia resources from both languages, such as videos or interactive websites. This visual and auditory exposure can solidify the understanding of cognates as students see and hear the practical use of these words in different contexts. When using any website in the classroom, it is important to preview and vet the website prior to classroom use.

MODULE 5, TOPIC 1 PACING GUIDE

165-Day Pacing

5 Constructing and Measuring

TOPIC 1: Angle Relationships

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1C, 7.1D, 7.1E, 7.1F, 7.1G

ELPS: 1.A, 1.C, 1.E, 2.C, 3.D, 3.F, 4.F, 4.G, 5.B

Topic Pacing: 7 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	**Solving Equations Using the Triangle Sum Theorem**	In this lesson, students build on their previous knowledge of the Triangle Sum theorem. They write and solve equations involving the sum of angles in triangles, including isosceles triangles. **Materials Needed:** None	7.11C	1
2	**Relationships Between 90° and 180° Angles**	Students explore the relationships between 90° and 180° angles. They learn the definitions of *complementary angles*, *supplementary angles*, and *perpendicular lines*. Students use a protractor to explore the relationship between complementary and supplementary angles. They then use patty paper to deepen their knowledge of perpendicular lines and the relationship to complementary and supplementary angles. Students then apply what they know about complementary angles to write and solve two different equations in one variable. **Materials Needed:** Protractors, Straightedges, Patty Paper (5 sheets per student)	7.11C	1
3	**Special Angle Relationships**	Students explore the types of angles formed when two lines intersect. They learn the definitions of *adjacent angles*, *linear pairs of angles*, and *vertical angles*. Throughout the lesson, students use patty paper to illustrate the special angle pairs and any special relationships between the measures of angle pairs. **Materials Needed:** Protractors, Straightedges, Patty Paper (5 sheets per student)	7.11C	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				2

*Bold TEKS = Readiness Standard

MODULE 5, TOPIC 1 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.11C	TEKS: 7.11C		TEKS: 7.11C	
LESSON 1 Solving Equations Using the Triangle Sum Theorem **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2** **TALK THE TALK**	**LESSON 2** Relationships Between 90° and 180° Angles **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **SKILLS PRACTICE** *This is a suggested placement. Move based on student data and individual needs.*	**LESSON 3** Special Angle Relationships **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 3** continued **ACTIVITY 3** **TALK THE TALK**

Day 6	Day 7
LEARNING INDIVIDUALLY **SKILLS PRACTICE** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are two Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets can be assigned to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Solving Equations Using the Triangle Sum Theorem

LESSON OVERVIEW

Students build on their previous knowledge of the Triangle Sum theorem. They write and solve equations involving the sum of angles in triangles, including isosceles triangles.

MATERIALS

None

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1F analyze mathematical relationships to connect and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Expressions, Equations, and Relationships

(11) The student applies mathematical process standards to solve one-variable equations and inequalities.

The student is expected to:

7.11C write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.

ELPS

(1) Learning Strategies

The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(3) Speaking

The student is expected to:

(F) ask and give information ranging from using a very limited bank of high-frequency, high-need, concrete vocabulary, including key words and expressions needed for basic communication in academic and social contexts, to using abstract and content-based vocabulary during extended speaking assignments.

(4) Reading

The student is expected to:

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

ESSENTIAL IDEAS

- An equilateral triangle is also equiangular.
- The base angles of an isosceles triangle are congruent.
- Apply the Triangle Sum theorem to determine unknown values or angle measures in a triangle.
- Apply prior knowledge of solving equations to determine unknown values in geometric figures.

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LESSON STRUCTURE AND PACING: 1 DAY

ENGAGE

Getting Started: Triangles, Triangles, Triangles 10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students use what they have previously learned about triangles to answer questions about the angle measures in isosceles, right, equilateral, and obtuse triangles.

DEVELOP

Activity 1.1: Applying the Triangle Sum Theorem 15 minutes

MATHEMATICAL PROBLEM SOLVING

Students apply what they know about the Triangle Sum theorem to write equations with one variable. They solve the equations for the unknown value and determine the measures of the angles in a triangle.

Activity 1.2: Isosceles Triangles 15 minutes

MATHEMATICAL PROBLEM SOLVING

Students use the Base Angles theorem to write equations and determine unknown values and angle measures in isosceles triangles.

DEMONSTRATE

Talk the Talk: Determining Unknowns 5 minutes

EXIT TICKET APPLICATION

Students apply what they know about the Triangle Sum theorem and the Base Angles theorem to write and solve equations for unknown variables. They then make a generalization about how to write and solve an equation for the perimeter of a geometric figure.

Getting Started

ENGAGE

Triangles, Triangles, Triangles

Facilitation Notes

In this activity, students use what they have previously learned about triangles to answer questions about the angle measures in isosceles, right, equilateral, and obtuse triangles.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What must always be true about the angle measures of an isosceles triangle? • What must always be true about the side lengths of an equilateral triangle?
Reflecting and justifying	• When you know the measure of two interior angles of a triangle, how can you determine the measure of the third angle?

DIFFERENTIATION STRATEGY

Just in Time Support

Have students try to draw a triangle with 2 right angles to visualize why it is not possible. Have students explain their reasoning on why this does not work with their group.

Summary

STAMP THE LEARNING

An isosceles triangle is a triangle with two sides and two angles that are congruent. An equilateral triangle is a triangle with three sides and three angles that are congruent.

ACTIVITY 1.1

Applying the Triangle Sum Theorem

DEVELOP

Facilitation Notes

In this activity, students apply what they know about the Triangle Sum theorem to write equations in one variable. They solve the equations for the unknown value, and determine the measures of the angles in a triangle.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What does the Triangle Sum theorem say about the angles in a triangle? • What operation is indicated by the use of the word "sum"?
Probing	• How many unknown angles are in the triangle? • What is the equation that we can use to solve for x? • Is the value of x the answer to the question?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Have students determine the measure of one unknown angle in each triangle.
- Separate students into four different groups, and have each group complete one of the questions. Regroup students into quads so that each student in the group has completed a different question. Have each student explain their question to the rest of the group and how they solved it.

STAMP THE LEARNING

Summary

You can use the Triangle Sum theorem to create an equation with the sum of the three angles in the triangle equal to 180°. This equation sometimes includes unknown values represented by a variable. After solving that equation for the variable, you can substitute that value into the expressions that represent the unknown angles to determine the angles' measure.

ACTIVITY 1.2

Isosceles Triangles

Facilitation Notes

In this activity, students use the Base Angles theorem to write equations and determine unknown values and angle measures in isosceles triangles.

Have a student read the introduction and definitions aloud. Discuss as a class. Have students work with a partner or in a group to complete Questions 1 through 5. Shares responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why do you think the Base Angles theorem only applies to isosceles triangles? • When you know a right triangle is an isosceles triangle, are you able to determine the measure of the other two angles? Which angles would be the base angles?

Summary

The Base Angles theorem states that when two sides of a triangle are congruent, then the angles opposite the congruent sides are also congruent.

STAMP THE LEARNING

Talk the Talk

DETERMINING UNKNOWNS

Facilitation Notes

In this activity, students apply what they know about the Triangle Sum theorem and the Base Angles theorem to write and solve equations for unknown variables. They then make a generalization about how to write and solve an equation for the perimeter of a geometric figure.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• When writing an equation for an isosceles triangle, what will always be true? • How can you apply what you know about determining an unknown angle on a triangle to determining the unknown side length on a rectangle?

Have students read and answer the Essential Question on the lesson opener page.

Summary

You can use the Triangle Sum theorem to create an equation with the sum of the three angles in the triangle equal to 180°. You can use the definition for the perimeter of a geometric figure to create an equation to solve for an unknown dimension.

STAMP THE LEARNING

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1 Solving Equations Using the Triangle Sum Theorem

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Use facts about isosceles triangles to write and solve equations for unknown angles.
- Apply the Triangle Sum theorem to determine unknown values or angle measures in a triangle.
- Apply the formula for the perimeter of a geometric figure to determine unknown values or side lengths.

NEW KEY TERMS

- base angles
- Base Angles theorem
- congruent sides
- congruent angles

You already know a lot about triangles. In previous grades, you classified triangles by side lengths and angle measures.

How can you apply the Triangle Sum theorem to write and solve equations?

Sample answer:

The sum of the measures of the interior angles of a triangle is 180°. You can write an equation to show that the sum of the three angles in a triangle is 180°. Then, you can use the equation to solve for unknown values.

MODULE 5 • TOPIC 1 • LESSON 1 893

Getting Started

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Optimizing Learning

This activity supplies background knowledge.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1G:

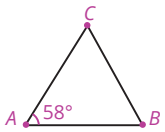
- Do students defend their mathematical reasoning?
- Do students use precise mathematical language when communicating?

Triangles, Triangles, Triangles

You have previously studied triangles. Use what you know about triangles to answer the questions.

Remember:
An isosceles triangle is a triangle with two sides that are the same length. An equilateral triangle is a triangle with three angles that are the same measure.

1. Triangle *ABC* is an isosceles triangle. The measure of angle *B* is 58° and the measure of angle *C* is 64°. Determine the measure of angle *A*.



$m\angle A = 58°$

Triangle *DEF* is an equilateral triangle.

2. What do you notice about the measure of $\angle A$ and the measure of $\angle B$?

   The measure of $\angle A$ equals the measure of $\angle B$.

3. All three angles in $\triangle DEF$ have the same measure. What is the measure of each angle in the triangle? Explain your reasoning.

   $m\angle D = m\angle E = m\angle F = 60°$. The three angles of a triangle sum to 180 degrees so when the three angles are the same each angle must measure 60°.

Remember:
A right triangle has one angle that measures 90°. An obtuse triangle has one obtuse angle.

4. How many right angles can a triangle have? Explain your reasoning.

   A triangle can only have one right angle because the sum of the three angles in a triangle is 180°.

5. Can two angles in a right triangle have the same measure? Explain your reasoning.

   Yes, in a right triangle one angle is 90 degrees. The other two angles could each measure 45 degrees.

EB STUDENT TIP

For all proficiency levels

Support emergent bilingual students by reviewing the terms *isosceles* and *equilateral*. Inform students that one way to remember the meaning of *isosceles* is to focus on the letters "sos" near the beginning of the word. Two of these letters are the same (the letter s), and the o is the "odd one out." Similarly, an isosceles triangle has two sides of the same length and one that is different. The word *equilateral* also clues us into its meaning of three equal side lengths through its initial spelling.

894 MODULE 5 • TOPIC 1 • LESSON 1

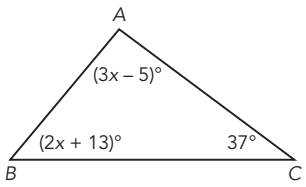
ACTIVITY 1.1

Applying the Triangle Sum Theorem

You have learned about the Triangle Sum theorem, which states that the sum of the measures of the interior angles of a triangle is 180°. You can use that information to write equations and solve for an unknown value.

Write and solve an equation to determine the measure of each unknown angle.

1.



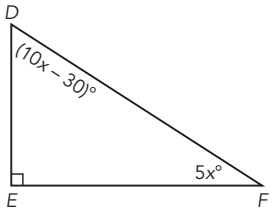
$(3x - 5) + (2x + 13) + 37 = 180$

$x = 27$

$m\angle A = 76°$

$m\angle B = 67°$

2.



$(10x - 30) + 5x + 90 = 180$

$x = 8$

$m\angle D = 50°$

$m\angle F = 40°$

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The theorem provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Activity 1.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice applying the Triangle Sum theorem to determine unknown angle measures, assign Skills Practice Set A for this lesson.

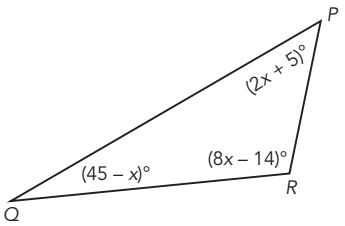
EB STUDENT TIP *(continued)*

Beginning: Use pictures or tangible shapes for each kind of triangle. Have students sort these into two groups and prompt them to say or point to the words isosceles or equilateral as they place a triangle in its respective group.

Intermediate: Provide sentence frames such as "This triangle is isosceles because it has _____ sides that are _____," and "This triangle is equilateral because _____."

Advanced/Advanced High: Encourage students to delve deeper into the properties of isosceles and equilateral triangles by discussing the internal angles and how they relate to the triangle's side lengths. Students can use sentence frames like "In an isosceles/equilateral triangle, the angles are _____ because _____."

3.



$(45 - x) + (2x + 5) + (8x - 14) = 180$

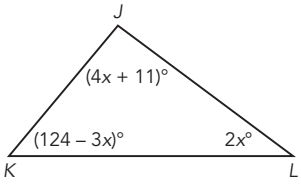
$x = 16$

$m\angle P = 37°$

$m\angle Q = 29°$

$m\angle R = 114°$

4.



$(124 - 3x) + (4x + 11) + 2x = 180$

$x = 15$

$m\angle J = 71°$

$m\angle K = 79°$

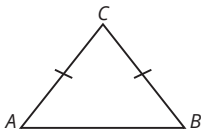
$m\angle L = 30°$

896 MODULE 5 • TOPIC 1 • LESSON 1

ACTIVITY 1.2

Isosceles Triangles

An isosceles triangle is a triangle with two sides that are the same length. The angles opposite these sides are called **base angles**. The **Base Angles theorem** states that the angles opposite the congruent sides of an isosceles triangle are congruent. **Congruent sides** are sides that have the same length. **Congruent angles** are angles that have the same measure.



The two sides of the triangle are each marked with one mark. Since the sides have the same marks, the sides are congruent.

1. Which sides in $\triangle ABC$ have the same length?
   $\overline{AC}$ and $\overline{BC}$

Remember:
$m\angle A$ is a symbol. The notation means the measure of angle A.

2. Which angles in the $\triangle ABC$ have the same measure?
   $\angle A$ and $\angle B$ have the same measure.

3. Write and solve equation to determine the value of x when $m\angle A = (2x + 7)°$ and $m\angle B = 63°$.
   $2x + 7 = 63$
   $x = 28$

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Chunking the Activity

- Read and discuss the introduction and definitions.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Activity 1.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice determining the unknown angle measures of isosceles triangles, assign Skills Practice Set B for this lesson.

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MODULE 5 • TOPIC 1 • LESSON 1 897

4. $\triangle DEF$ is an isosceles triangle. $m\angle D = m\angle E$, $m\angle E = (4x - 11)°$, and $m\angle F = 98°$. Write and solve an equation to determine the value of x.

$(4x - 11) + (4x - 11) + 98 = 180$

$8x + 76 = 180$

$x = 13$

5. a. The two base angles of an isosceles triangle each measure $(5z + 19)°$. The third angle of the triangle measures 122°. Write and solve an equation to determine the value of z.

$(5z + 19) + (5z + 19) + 122 = 180$

$10z + 160 = 180$

$10z = 20$

$z = 2$

b. What is the measure of each base angle in the triangle?

$5(2) + 19$

$= 10 + 19$

$= 29°$

898 MODULE 5 • TOPIC 1 • LESSON 1

Talk the Talk

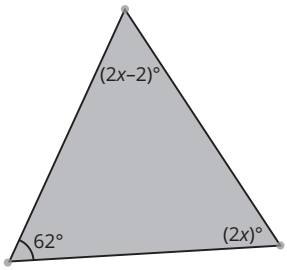
Determining Unknowns

1. An isosceles triangle has base angles that measure 58°. Write an equation to determine the measure of the third angle.

$58 + 58 + w = 180$

$116 + w = 180$

2. The angle measures of a triangle are shown in the diagram.



Write an equation to determine the value of x. Then, solve for x.

$62 + (2x) + (2x - 2) = 180$

$4x + 60 = 180$

$4x = 120$

$x = 30$

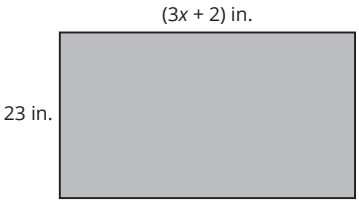
Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

MODULE 5 • TOPIC 1 • LESSON 1 899

Remember:
Perimeter is the distance around a figure.

3. The perimeter of the rectangle shown is 128 inches.



Write an equation to determine the value of x. Then, solve for x.

$23 + (3x + 2) + 23 + (3x + 2) = 128$

$6x + 50 = 128$

$6x = 78$

$x = 13$

900 MODULE 5 • TOPIC 1 • LESSON 1

Lesson 1 Assignment

Write

In your own words, explain the Base Angles theorem.

Remember

The sum of the measures of the interior angles of a triangle is 180°.
You can use what you know about solving equations and the Triangle Sum theorem to determine unknown values for the interior angles of a triangle.

Practice

1. Write and solve an equation to determine the measure of the third angle when the base angle of an isosceles triangle is 52°.

$52 + 52 + x = 180$

$104 + x = 180$

$x = 76$

The third angle of the isosceles triangle is 76°.

2. Write and solve an equation to determine the measure of the base angles of an isosceles triangle when the non-base angle is 88°.

$88 + x + x = 180$

$88 + 2x = 180$

$2x = 92$

$x = 46$

Each base angle of the isosceles triangle is 46°.

Write

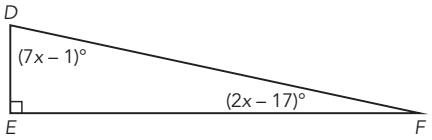
The Base Angles theorem states that the two base angles of an isosceles triangle are congruent. The sides opposite the congruent angles are also congruent.

MODULE 5 • TOPIC 1 • LESSON 1 ASSIGNMENT 901

Lesson 1 Assignment

3. Determine the measure of each unknown angle.

a.



$(7x - 1) + (2x - 17) + 90 = 180$

$9x + 72 = 180$

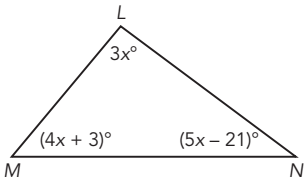
$9x = 108$

$x = 12$

$m\angle D = 83°$

$m\angle F = 7°$

b.



$(4x + 3) + (5x - 21) + 3x = 180$

$12x - 18 = 180$

$12x = 198$

$x = 16.5$

$m\angle L = 49.5°$

$m\angle M = 69°$

$m\angle N = 61.5°$

902 MODULE 5 • TOPIC 1 • LESSON 1 ASSIGNMENT

Lesson 1 Assignment

4. The perimeter of a rectangle is 58 inches. The rectangle has a width of 15 inches and a length of 3.5x. Write an equation and solve for the value of x.

$15 + 15 + 3.5x + 3.5x = 58$

$30 + 7x = 58$

$7x = 28$

$x = 4$

Prepare

1. $90 = 5x + x$

   $x = 15$

2. $x + 3x = 180$

   $x = 45$

3. $(180 - x) + (90 - x) = 210$

   $x = 30$

MODULE 5 • TOPIC 1 • LESSON 1 ASSIGNMENT 903

904 MODULE 5 • TOPIC 1 • LESSON 1 ASSIGNMENT

2 Relationships Between 90° and 180° Angles

LESSON OVERVIEW

Students explore the relationships between 90° and 180° angles. They learn the definitions of *complementary angles*, *supplementary angles*, and *perpendicular lines*. Students use a protractor to explore the relationship between complementary and supplementary angles. They then use patty paper to deepen their knowledge of perpendicular lines and the relationship to complementary and supplementary angles. Students then apply what they know about complementary angles to write and solve two different equations in one variable.

MATERIALS

Protractors

Straightedges

Patty Paper (at most 5 sheets per student)

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1C select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1E create and use representations to organize, record, and communicate mathematical ideas.

Expressions, Equations, and Relationships

(11) The student applies mathematical process standards to solve one-variable equations and inequalities.

The student is expected to:

7.11C write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.

ELPS

(1) Learning Strategies

The student is expected to:

(A) use prior knowledge and experiences to understand meanings in English.

(4) Reading

The student is expected to:

(F) use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language.

(5) Writing

The student is expected to:

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary.

ESSENTIAL IDEAS

- Two angles are supplementary if the sum of their angle measures is equal to 180°.
- Two angles are complementary if the sum of their angle measures is equal to 90°.
- Two lines, line segments, or rays are perpendicular if they intersect to form 90° angles.

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LESSON STRUCTURE AND PACING: 1 DAY

ENGAGE

Getting Started: Splitting Up 10 minutes

CONNECT TO PRIOR KNOWLEDGE

Students use a straightedge to create pairs of angles by splitting straight angles and right angles. They use a protractor to measure the angles. Students should notice that the sum of the angles formed by splitting a straight angle is 180°, and the sum of the angles formed by splitting a right angle is 90°.

DEVELOP

Activity 2.1: Supplements and Complements 15 minutes

INVESTIGATION

Supplementary angles and *complementary angles* are defined. Using a protractor, students draw sets of supplementary and complementary angles. They then use their knowledge of supplementary and complementary angles to write and solve equations to determine the unknown measures of angles.

Activity 2.2: Perpendicular Lines 15 minutes

INVESTIGATION

Students learn the term *perpendicular* and the symbol for perpendicular. They draw perpendicular lines and determine the number of right angles that are formed. Students then use patty paper to create perpendicular lines, complementary angles, and supplementary angles.

DEMONSTRATE

Talk the Talk: More Than One Way 5 minutes

EXIT TICKET APPLICATION

Students apply what they know about complementary angles to write and solve two different equations in one variable.

Getting Started

ENGAGE

Splitting Up

Facilitation Notes

In this activity, students use a straightedge and protractor to investigate some special relationships between angles. Working with straight angles and right angles, students draw rays to split each angle. They notice that the sum of the two angles formed when splitting a straight angle is 180°, and the sum of the two angles formed when splitting a right angle is 90°.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the measure of a straight angle?
Probing	• What is the sum of the measures of the two angles formed? • Do you think the sum is the same for any two angles forming a straight angle? Explain your thinking.
Seeing structure	• Does it matter where you labeled points *A*, *B*, and *C* on your straight angle? Did the order of the points matter?
Gathering	• What is the measure of a right angle?
Probing	• What is the sum of the measures of the two angles formed? • Do you think the sum is the same for any two angles that form a right angle? Explain your thinking.

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Blank Paper

To make connections and distinctions among the new key terms in the lesson, distribute a sheet of paper to each student. Have each student create three columns titled **Terms Related to 180°**, **Terms Related to 90°**, and **Other Terms**.

- When terms appear in the lesson, have students enter key terms and a diagram or written definition into the appropriate column.
- As they add new terms, urge students to make sure their diagrams or definitions are specific enough to differentiate among the terms.

Optimizing Learning

This differentiation strategy provides options that guides information processing and visualization.

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Sample table:

Terms Related to 180°	Terms Related to 90°	Other Terms
Straight angle Supplementary angles	Right angle Complementary angles	

STAMP THE LEARNING

Summary

The sum of the two angles formed when splitting a straight angle is 180°.
The sum of the two angles formed when splitting a right angle is 90°.

ACTIVITY 2.1

Supplements and Complements

DEVELOP

Facilitation Notes

In this activity, *supplementary angles* and *complementary angles* are defined. Using a protractor, students draw sets of supplementary and complementary angles. They then use their knowledge of supplementary and complementary angles to write and solve equations to determine the unknown measures of angles.

Have a student read the definitions aloud and discuss as a class. Have students complete Questions 1 through 6 with a partner or in a group. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• When you draw a pair of supplementary angles such that the two angles share a common side, what kind of angle do you form?
Seeing structure	• Is it possible for a pair of supplementary angles to share a vertex even when they don't share a side? Explain your thinking.
Gathering	• If you add the measure of $\angle KJL$ to the measure of the angle that is supplementary to $\angle KJL$, what is the result? • When a pair of complementary angles share a common side, what kind of angle do you form? • What notation can you use to show that the measure of the larger angle composed by the two smaller angles is 90°? • Did you decompose a 90° angle into two angles? Or did you compose two angles into a 90° angle?
Reflecting and justifying	• Is it possible for a pair of complementary angles to share a vertex even when they don't share a side? Explain your thinking.
Gathering	• What is the sum of the measures of complementary angles?

COMMON MISCONCEPTION

Students often believe that the orientation of angles is fixed. Encourage them to draw their angles in unique orientations.

- Straight angles do not need to be horizontal or vertical.
- Right angles do not need to have horizontal and vertical rays.

Have students work with a partner or in a group to complete Questions 7 and 8. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

Assign each group a different problem to start with. Let students know the problem they start with is what they will share with the class. Allow time for groups to present to the class. Encourage students to ask their classmates questions.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you interpret each verbal statement?
Gathering	• What type of angle is $\angle WXY$? How do you know? • How does knowing the measures of $\angle WXY$ and $\angle WXZ$ help you calculate the measure of $\angle ZXY$?

Summary

You can use algebra to determine the measures of complementary and supplementary angles.

STAMP THE LEARNING

ACTIVITY 2.2 Perpendicular Lines

Facilitation Notes

In this activity, the term *perpendicular* is defined. Students answer questions related to perpendicular lines and use patty paper to illustrate perpendicular lines, complementary angles, and supplementary angles.

Have a student read the definition of *perpendicular* aloud. Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Outline the steps to draw and label $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point E. For example,

Step 1: Draw a line and label it $\overleftrightarrow{AB}$.
Step 2: Draw a line perpendicular to that line.
Step 3: Label the other line $\overleftrightarrow{CD}$.
Step 4: Label your diagram to show the lines are perpendicular.
Step 5: Label the point of intersection E.

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QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you tell the other angles are right angles without measuring them?
Seeing structure	• If *BC* were a ray instead of a line, would that change the number of right angles formed?
Probing	• Why is it important to know that points *A*, *D*, and *B* lie on the same line? • Why can't you say that $\angle DCF$ is a right angle?

Distribute 5 sheets of patty paper to each student, 2 sheets for general experimentation, and 3 sheets for official notes. Ask a student to read the side note and introductory sentence aloud. Complete Question 4 as a class.

Have students work with a partner or in a group to complete Questions 5 through 7. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How did you fold your patty paper so that the angles are not congruent? • How did you fold your patty paper so that the angles are congruent?
Probing	• How do you know the angles are congruent without measuring them?
Seeing structure	• What are the measures of the supplementary angles when they are congruent?

STAMP THE LEARNING

Summary

Perpendicular lines form right angles. You can use patty paper to create right angles and illustrate complementary and supplementary angles.

DEMONSTRATE

Talk the Talk

MORE THAN ONE WAY

Facilitation Notes

In this activity, students apply what they know about complementary angles to write and solve two different equations in one variable.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the sum of the measures of complementary angles?
Seeing structure	• Can you determine the measure of angle J without knowing the value of x? • How does this help to create a different equation that can be used to determine the value of x?

Have students read and answer the Essential Question on the lesson opener page.

Summary

You can use angle relationships to determine unknown angle measurements.

STAMP THE LEARNING

2 Relationships Between 90° and 180° Angles

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Calculate the supplement of an angle.
- Calculate the complement of an angle.
- Use facts about supplementary and complementary angles to write and solve simple equations for unknown angles.

NEW KEY TERMS

- straight angle
- supplementary angles
- complementary angles
- perpendicular
- collinear

You know how to write and solve equations using the Triangle Sum theorem. Now consider other angle relationships.

What types of special relationships exist between 90° and 180° angles?

Sample answer:

Two angles are supplementary angles if the sum of their angle measures is equal to 180°.

Two angles are complementary angles if the sum of their angle measures is equal to 90°.

MODULE 5 • TOPIC 1 • LESSON 2 905

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Getting Started

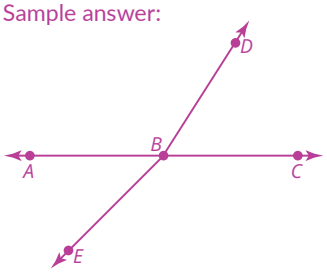
Splitting Up

A **straight angle** is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

Let's use a straightedge and a protractor to investigate some special relationships between angles.

1. Draw a *straight angle*, $\angle ABC$, and label each point.

   Sample answer:



   a. Draw a point D above line AC and use your straightedge to draw a ray, $\overrightarrow{BD}$. Next, use your protractor to measure the angles formed by $\overrightarrow{BD}$ and $\overleftrightarrow{AC}$.

   Sample answer:

   The angles formed by $\overrightarrow{BD}$ and $\overleftrightarrow{AC}$ measure 126° and 54°.

   b. Draw a point E different from point D below line AC and use your straightedge to draw another ray, $\overrightarrow{BE}$. Next, use your protractor to measure the angles formed by $\overrightarrow{BE}$ and $\overleftrightarrow{AC}$.

   Sample answer:

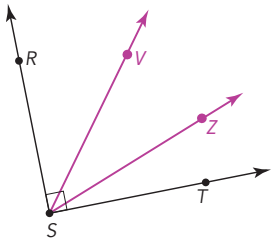
   The angles formed by $\overrightarrow{BE}$ and $\overleftrightarrow{AC}$ measure 38° and 142°.

   c. What do you notice about the measures of the angles formed by a single ray coming off of a line?

   The sum of the two angles formed when splitting a straight angle is 180°.

906 MODULE 5 • TOPIC 1 • LESSON 2

2. Use the right angle shown to answer each question.



a. Draw a point V in the interior of $\angle RST$ and use your straightedge to draw a ray, $\overrightarrow{SV}$. Next, use your protractor to measure the angles formed by $\overrightarrow{SV}$ and $\angle RST$.

Sample answer:

The angles formed by $\overrightarrow{SV}$ and $\angle RST$ measure 47° and 43°.

b. Draw a point Z in the interior of $\angle RST$ and use your straightedge to draw another ray, $\overrightarrow{SZ}$. Next, use your protractor to measure the angles formed by $\overrightarrow{SZ}$ and $\angle RST$.

Sample answer:

The angles formed by $\overrightarrow{SZ}$ and $\angle RST$ measure 15° and 75°.

c. What do you notice about the measures of the angles formed by a single ray that divides a right angle into two angles?

The sum of the two angles formed when splitting a right angle is 90°.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Since an angle is not a closed shape, ensure students understand what is meant by the *interior* of an angle.

Help students recognize the prefix *in-* and connect to the idea of being inside a specific area.

MODULE 5 • TOPIC 1 • LESSON 2 907

Chunking the Activity

- Read and discuss the introduction and definitions.
- Group students to complete Questions 1–6.
- Check in and share.
- Group students to complete Questions 7 and 8.
- Share and summarize.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 2.1

Supplements and Complements

Two angles are **supplementary angles** if the sum of their angle measures is equal to 180°.

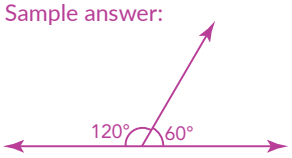
Two angles are **complementary angles** if the sum of their angle measures is equal to 90°.

In the previous activity, you created *supplementary angles* and *complementary angles*.

Let's create sets of supplementary angles.

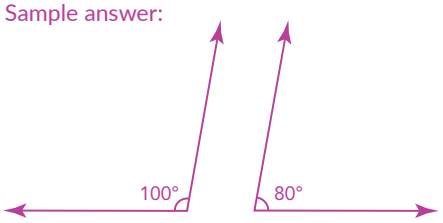
1. Use a protractor to draw a pair of supplementary angles that share a side. What is the measure of each angle?

   Sample answer:

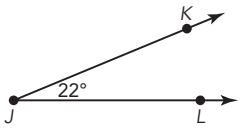


2. Use a protractor to draw a pair of supplementary angles that do not share a side. What is the measure of each angle?

   Sample answer:



3. Calculate the measure of an angle that is supplementary to $\angle KJL$.



An angle that is supplementary to $\angle KJL$ has a measure of 158°.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

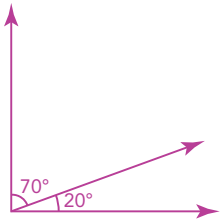
English words, like *complementary*, often have multiple meanings. Students may confuse *complementary angles* with *complementary events* in probability. Clarify the difference between complementary events in probability and complementary angles in geometry.

908 MODULE 5 • TOPIC 1 • LESSON 2

Now, let's create sets of complementary angles.

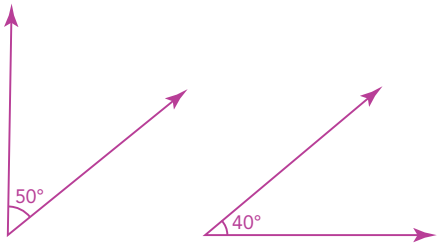
4. Use a protractor to draw a pair of complementary angles that share a side. What is the measure of each angle?

Sample answer:

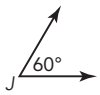


5. Use a protractor to draw a pair of complementary angles that do not share a side. What is the measure of each angle?

Sample answer:



6. Calculate the measure of an angle that is complementary to $\angle J$.



An angle that is complementary to $\angle J$ has a measure of 30°.

How can you check that your measurements are correct?

Congruent angles are angles that have the same measure.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1D:

- Do students represent their mathematical reasoning with appropriate representations?

MODULE 5 • TOPIC 1 • LESSON 2 909

7. Given each statement, write and solve an equation to determine the measure of each angle in the angle pair.

a. Two angles are both congruent and supplementary.

$x + x = 180$

The measure of each angle is 90°.

b. Two angles are both congruent and complementary.

$x + x = 90$

The measure of each angle is 45°.

c. The supplement of an angle is half the measure of the angle itself.

$x + 0.5x = 180.$

The angle measure is 120°, and the supplement of the angle is 60°.

d. The supplement of an angle is 20° more than the measure of the angle itself.

$x + (x + 20) = 180$

The angle measure is 80°, and the supplement of the angle is 100°.

910 MODULE 5 • TOPIC 1 • LESSON 2

e. Angles 1 and 2 are complementary. The measure of Angle 2 is 10° larger than the measure of Angle 1.

$x + (x + 10) = 90$

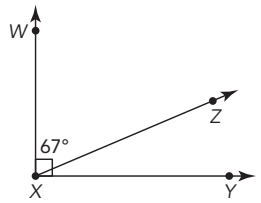
The angle measure is 40°, and the complement of the angle is 50°.

f. Angles 1 and 2 are supplementary. The measure of Angle 1 is three degrees less than twice the measure of Angle 2.

$x + (2x - 3) = 180$

The angle measure is 61°, and the supplement of the angle is 119°.

8. Use the figure to determine the measure of $\angle ZXY$.



$67 + x = 90$

The angle measure is 67°, and the complement of the angle is 23°.

MODULE 5 • TOPIC 1 • LESSON 2 911

Chunking the Activity

- Read and discuss the definition.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the side note and introductory sentence.
- Complete Question 4 as a class.
- Group students to complete Questions 5–7.
- Share and summarize.

STAMP THE LEARNING

The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 2.2

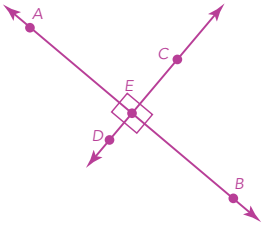
Perpendicular Lines

Two lines, line segments, or rays are **perpendicular** if they intersect to form 90° angles. The symbol for perpendicular is ⊥.

Let's explore angles formed by *perpendicular* lines.

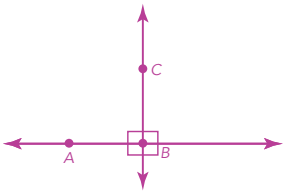
1. Draw and label $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point *E*. How many right angles are formed?

   Four right angles are formed.



2. Draw and label $\overleftrightarrow{BC} \perp \overleftrightarrow{AB}$ at point *B*. How many right angles are formed?

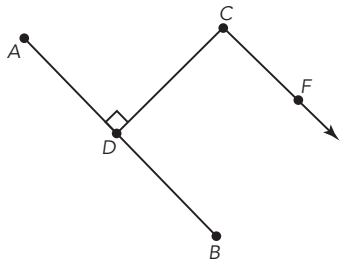
   Four right angles are formed.



Compare your drawings with your partner's drawings. What do you notice?

912 MODULE 5 • TOPIC 1 • LESSON 2

3. Name all angles that you know are right angles in the figure shown. Note: Points *A*, *D*, and *B* lie on the same line segment.



I know that $\angle ADC$ is a right angle because it is labeled as a right angle. Because $\overline{AB}$ is a straight angle, then I also know that $\angle BDC$ is a right angle.

When points lie on the same line or line segment, they are said to be **collinear**.

You can use patty paper to create special angle pairs, just as you can use a protractor and straightedge.

4. Draw a figure on a sheet of patty paper. What do you notice when you write on the paper? What about when you fold the paper?

Sample answer:

When I write on the patty paper, I can see the reflection of the figure I drew on the reverse side of the paper.

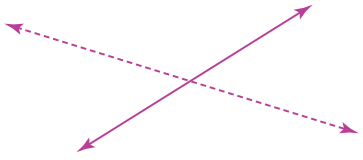
When I fold the patty paper, the fold creates a line through the figure.

Patty paper is traditionally a food service product, used to separate hamburger patties, meats, cheeses, and pastries.

Read each question in its entirety before using your patty paper to illustrate the angles. You should use one sheet of patty paper for each question.

5. Draw a straight angle on your patty paper. Then *fold* your patty paper to create a pair of supplementary angles that are not congruent. Label the patty paper "Supplementary Angles."

Supplementary Angles



Activity 2.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days.

To provide additional practice identifying perpendicular lines, complementary angles, and supplementary angles, assign Skills Practice Set A for this lesson.

To provide additional practice sketching perpendicular lines, complementary angles, and supplementary angles, assign Skills Practice Set B for this lesson.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1C:

- Are students considering which strategies or tools to use?

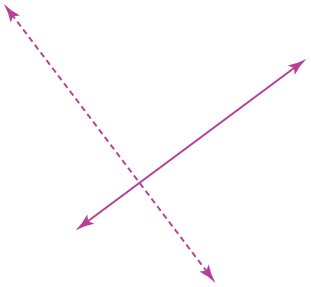
MODULE 5 • TOPIC 1 • LESSON 2 913

6. Draw a straight angle on your patty paper. Then *fold* your patty paper to create a pair of supplementary angles that are congruent. What do you know about the angles? What do you know about the straight angle and the line you created with your fold? Label the patty paper accordingly.

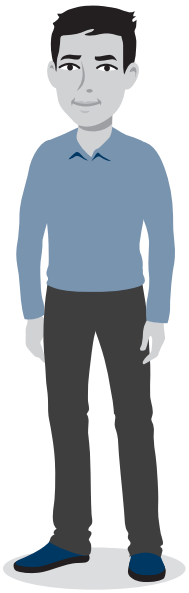
When folded, the two supplementary angles should be the same size. The straight angle and the fold are perpendicular to each other.

Perpendicular Lines

Right Angles

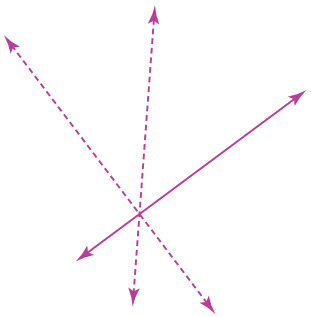


Store your patty paper notes in a safe place. You can use them to study.



7. Draw a right angle on your patty paper. Then *fold* your patty paper to create a pair of complementary angles that are not congruent. Label the patty paper "Complementary Angles."

Complementary Angles



914 MODULE 5 • TOPIC 1 • LESSON 2

Talk the Talk

More Than One Way

Angle Q and angle J are complementary angles. The measure of angle Q is 32°. The measure of angle J is $(3.5x + 2)°$.

1. Use what you know about complementary angles to write and solve an equation to determine the value of x and the measure of angle J.

$32 + (3.5x + 2) = 90$

$3.5x + 34 = 90$

$3.5x = 56$

$x = 16$

$3.5(16) + 2$

$m\angle J = 58°$.

2. Is there another equation that you can write and solve to determine the value of x?

Since angle Q is 32°, I know that $m\angle J = 90 - 32$ or 58°.

$3.5x + 2 = 58$

$3.5x = 56$

$x = 16$

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

The Talk the Talk presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice writing and solving equations involving complementary and supplementary angles, assign Skills Practice Set C for this lesson.

MODULE 5 • TOPIC 1 • LESSON 2 915

916 MODULE 5 • TOPIC 1 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

Write

Draw an example of each term. Provide an explanation when necessary.

1. supplementary angles
2. complementary angles
3. perpendicular lines

Remember

Two angles are *supplementary angles* when the sum of their angle measures is equal to 180°. Two angles are *complementary angles* when the sum of their angle measures is equal to 90°.

Practice

1. Draw a pair of supplementary angles. One of the angles should have a measure of 85°. Label each angle with its measure.

   Check students' drawings. All angles drawn and labeled should measure 95° and 85°.

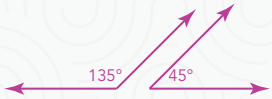
2. Draw a pair of complementary angles. One of the angles should have a measure of 62°. Label each angle with its measure.

   Check students' drawings. All angles drawn and labeled should measure 28° and 62°.

Write

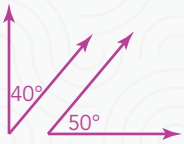
Sample answers:

1. Supplementary angles



$135° + 45° = 180°$

2. Complementary angles



$40° + 50° = 90°$

3. Perpendicular lines

These lines intersect to form four 90° angles.

Lesson 2 Assignment

Write and solve an equation to determine the measure of each unknown angle.

3. Angles C and D are complementary. When $m\angle D$ is 25° is greater than $m\angle C$, what is the measure of each angle?

   $x + (x + 25) = 90$

   $m\angle C = 32.5°$

   $m\angle D = 57.5°$

4. When the supplement of an angle is 30 degrees more than the measure of the angle, what is the measure of the angle?

   $x + (x + 30) = 180$

   The measure of the angle is 105°, and the measure of the supplement of the angle is 75°.

5. When the supplement of an angle is 12 degrees less than twice the measure of the angle, what is the measure of the angle?

   $x + (2x - 12) = 180$

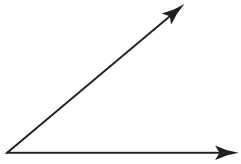
   The measure of the angle is 64°, and the measure of the supplement of the angle is 116°.

918 MODULE 5 • TOPIC 1 • LESSON 2 ASSIGNMENT

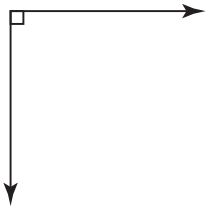
Lesson 2 Assignment

Prepare

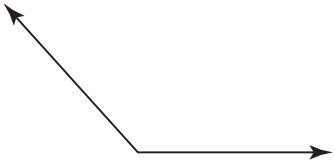
Classify each angle.

1. 

Acute angle

2. 

Right angle

3. 

Obtuse angle

920 MODULE 5 • TOPIC 1 • LESSON 2 ASSIGNMENT

3 Special Angle Relationships

LESSON OVERVIEW

Students explore the types of angles formed when two lines intersect. They learn the definitions of *adjacent angles*, *linear pairs of angles*, and *vertical angles*. Throughout the lesson, students use patty paper to illustrate the special angle pairs and any special relationships between the measures of angle pairs.

MATERIALS

Protractors
Straightedges
Patty Paper (5 sheets per student)

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1E create and use representations to organize, record, and communicate mathematical ideas.

7.1F analyze mathematical relationships to connect and communicate mathematical ideas.

Expressions, Equations, and Relationships

(11) The student applies mathematical process standards to solve one-variable equations and inequalities.
The student is expected to:

7.11C write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.

ELPS

(1) Learning Strategies
The student is expected to:

(C) use strategic learning techniques such as concept mapping, drawing, memorizing, comparing, contrasting, and reviewing to acquire basic and grade-level vocabulary.

(2) Listening
The student is expected to:

(C) learn new language structures, expressions, and basic and academic vocabulary heard during classroom instruction and interactions.

(3) Speaking
The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

ESSENTIAL IDEAS

- Adjacent angles are two angles that share a common vertex and a common side.
- A linear pair of angles is two adjacent angles with non-common sides that form a line. Linear pairs are supplementary.
- Vertical angles are two non-adjacent angles that are formed by two intersecting lines. Vertical angles are congruent.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Name Those Angles! 5 minutes

CONNECT TO PRIOR KNOWLEDGE

Students apply what they know about straight lines and angle relationships. They name all the angles that are less than 180° formed by the intersection of two lines.

DEVELOP

Activity 3.1: Adjacent Angles 15–20 minutes

WORKED EXAMPLE, INVESTIGATION

Students encounter examples of adjacent angles and non-adjacent angles. From these examples, they describe and draw adjacent angles. The formal definition of *adjacent angles* is provided. Students then use patty paper to create adjacent angles with specific conditions.

Activity 3.2: Linear Pairs 15–20 minutes

WORKED EXAMPLE

Students analyze examples and non-examples of linear pairs of angles. From these examples, they describe and draw a linear pair. The formal definition of *linear pair* is provided. Students then answer questions about linear pairs and supplementary angles.

DAY 2

Activity 3.3: Vertical Angles 20–25 minutes

WORKED EXAMPLE, INVESTIGATION

Students analyze examples and non-examples of vertical angles. From these examples, they describe and draw vertical angles. The formal definition of *vertical angles* is provided. Students then use patty paper, protractors, and reasoning about supplementary angles to conclude that vertical angles are congruent, or have the same measure.

DEMONSTRATE

Talk the Talk: Perfect Pairs 15–20 minutes

GENERALIZATION

Students summarize the types of angles that occur when two lines intersect. They use and explain the relationships among the angles. Students then draw a diagram illustrating each of the relationships learned in the lesson.

Getting Started

ENGAGE

Name Those Angles!

Facilitation Notes

In this activity, students apply what they know about straight lines and angle relationships. They name all the angles that are less than 180° formed by the intersection of two lines.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the measure of a straight angle?
Seeing structure	• Where are there 8 different angles present in the diagram? • When naming angles, in what order do you write the points?

Summary

The sum of the two angles formed when splitting a straight angle is 180°.

STAMP THE LEARNING

ACTIVITY 3.1

Adjacent Angles

Facilitation Notes

In this activity, students encounter examples of adjacent angles and non-adjacent angles. From these examples, they describe and draw adjacent angles. The formal definition of *adjacent angles* is then provided. Students then use patty paper to create adjacent angles with specific conditions.

Analyze the Worked Example and definition of *adjacent angles* as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How are the diagrams for ∠1 and ∠2 different from the diagrams for ∠3 and ∠4? • How are the two diagrams for ∠1 and ∠2 the same? How are they different?

Distribute 2 sheets of patty paper to each student. Have students work with a partner or in a group to complete Questions 1 through 8. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Does your drawing match others in your group? Why do you think that is? • What is the definition of the side of an angle? • What do adjacent angles look like when they are supplementary? • What do adjacent angles look like when they are not supplementary?
Gathering	• Which figure in the Worked Example has angles that share a vertex but not a common side? • Why do two rays need to start at the same point to be the same ray?

Summary

Adjacent angles are a pair of angles that share a common vertex and share a common side. Patty paper can be used to illustrate adjacent angles.

ACTIVITY 3.2

Linear Pairs

Facilitation Notes

In this activity, the term *linear pair* is defined. Students draw linear pairs of angles and use patty paper to illustrate linear pairs of angles.

Analyze the Worked Example and definition of *linear pair* as a class.

Distribute 1 sheet of patty paper to each student. Have students work with a partner or in a group to complete Questions 1 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you think you form a linear pair? • Describe the angles formed by a linear pair. • Is every linear pair of angles also a pair of adjacent angles? Explain your thinking. • What is the measure of an angle that forms a straight line? • What has to be true if you wanted to make a linear pair using three rays? • When you draw two lines perpendicular to each other, how many different linear pairs of angles do you form? • Is an equation necessary to solve this problem? Why or why not? • Is there another way to write an equation to solve this problem?
Gathering	• Which angles form a linear pair with $\angle 1$? • How many pairs of angles form linear pairs in this figure? • What does it mean for angles to share a common side? • What does it mean when angles do not share a common side?

To close the Day 1 session, have students reread the Essential Questions and read the activity summaries to the class.

Summary

A linear pair of angles is two adjacent angles with non-common sides that form a line. Patty paper can be used to illustrate a linear pair of angles.

STAMP THE LEARNING

ACTIVITY 3.3 Vertical Angles

Facilitation Notes

In this activity, the term *vertical angles* is defined. Students draw a pair of vertical angles and use patty paper to illustrate pairs of vertical angles. They use patty paper, protractors, and reasoning about supplementary angles to conclude that vertical angles are congruent.

To begin the Day 2 session, have a student read the Essential Question aloud.

Analyze the Worked Example and definition of *vertical angles* as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Students may have difficulty defining vertical angles in their own terms. Ask students to create a list of things that are true about vertical angles and things that are not true about vertical angles.

Distribute 2 sheets of patty paper to each student. Have students work with a partner or in a group to complete Questions 1 through 9. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Are vertical angles also adjacent angles? How do you know? • How are vertical angles formed? • Do you think vertical angles have to share a common vertex? Explain your thinking. • What does it mean for two angles to be supplementary?
Gathering	• How does $\angle 1$ relate to $\angle 3$? • How does $\angle 2$ relate to $\angle 4$? • What type of relationship exists between the labeled angles? • What relationship exists between the labeled angles and the angles without labels?

STAMP THE LEARNING

Summary

Vertical angles are two non-adjacent angles that are formed by two intersecting lines. Patty paper can be used to illustrate a pair of vertical angles.

DEMONSTRATE

Talk the Talk

PERFECT PAIRS

Facilitation Notes

In this activity, students identify different types of angles formed by intersecting lines and draw diagrams that contain different angle relationships.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What does it mean when two angles are complementary?
Reflecting and justifying	• How are supplementary angles different from linear pairs? • Are the angles in a linear pair always adjacent? • Do adjacent angles always form a straight line?
Probing	• What type of angles are formed by intersecting lines? • What does it mean for two angles to form a linear pair? • Which type of angles are formed by perpendicular lines? • How many relationships are present when you draw perpendicular lines?
Gathering	• Which pairs of angles are vertical angles?

Have students read and answer the Essential Question on the lesson opener page.

Summary

You can use intersecting lines to illustrate different angle relationships.

3 Special Angle Relationships

Setting the Stage

- Communicate the objectives and key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Classify adjacent angles, linear pairs, and vertical angles.
- Use facts about adjacent angles, linear pairs, and vertical angles in multi-step problems to write and solve simple equations for unknown angles.

NEW KEY TERMS

- adjacent angles
- linear pair
- vertical angles

You know how to classify individual angles based on their measure.

What types of special relationships exist between pairs of angles?

Sample answer:

Many geometric figures contain a mixture of special angle pairs. Understanding the relationships between special angle pairs will help you understand more complex geometric diagrams.

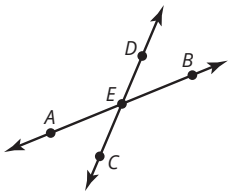
MODULE 5 • TOPIC 1 • LESSON 3 921

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.

Getting Started

Name Those Angles!



1. How many different angles are formed by the intersection of $\overline{AB}$ and $\overline{CD}$?

   There are eight different angles formed by the intersection of two lines.

2. Name all of the angles less than 180° formed by the intersection of $\overline{AB}$ and $\overline{CD}$.

   $\angle AED$, $\angle DEB$, $\angle BEC$, $\angle CEA$

3. Describe the location of all of the angles equal to 180° formed by the intersection of $\overline{AB}$ and $\overline{CD}$.

   Two straight angles are formed by $\overline{CD}$ (one on the left side of $\overline{CD}$ and one on the right side of $\overline{CD}$), and two straight angles are formed by $\overline{AB}$ (one above $\overline{AB}$ and one below $\overline{AB}$).

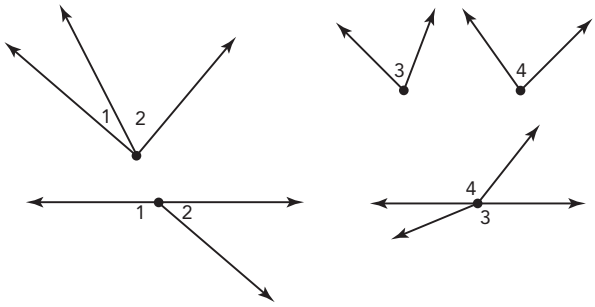
922 MODULE 5 • TOPIC 1 • LESSON 3

ACTIVITY
3.1 Adjacent Angles

In each of the next three activities you will explore special angle pairs.

WORKED EXAMPLE

∠1 and ∠2 are adjacent angles. ∠3 and ∠4 are *not* adjacent angles.



1. Use the figures in the Worked Example to define *adjacent angles* in your own words.

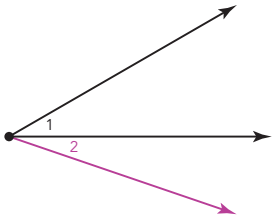
   Sample answer:

   Two angles are adjacent if they have a common vertex and a common side.

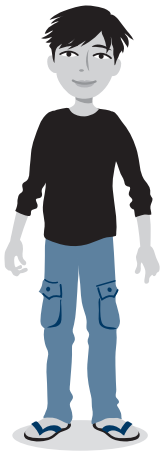
2. Draw and label ∠2 so that it is adjacent to ∠1.

   Check students' drawings.

   Sample drawing:



Can you think of other ways to draw ∠2 so that it is adjacent to ∠1?



Chunking the Activity

- Read and discuss the Worked Example and definition.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

MODULE 5 • TOPIC 1 • LESSON 3 923

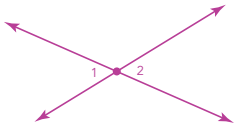
STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Adjacent angles are two angles that share a common vertex and share a common side.

3. Is it possible to draw two angles that share a vertex but do not share a common side? If so, draw an example. Would the angles be adjacent angles? If not, explain your reasoning.

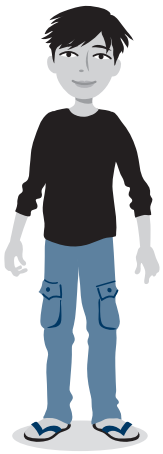
   Check students' drawings. These two angles are not adjacent.



4. Is it possible to draw two angles that share a side but do not share a vertex? If so, draw an example. If not, explain your reasoning.

   It is not possible. The sides of an angle are rays, so if two angles share a side, they also share a ray. The endpoint of the shared ray is the vertex of both angles.

How could you illustrate 2 non-adjacent angles on patty paper?

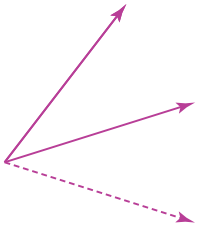


5. Use one sheet of patty paper to create a set of *congruent* adjacent angles. Describe your process. Label the patty paper "Congruent Adjacent Angles."

   Sample answer:

   I used my straightedge to draw an angle. Then I folded the patty paper along one ray of the angle and traced the other ray. When I unfolded the patty paper, I could see the ray of the adjacent angle that was congruent to the first angle.

   Congruent Adjacent Angles



EB STUDENT TIP

For "Intermediate" and higher proficiency levels

The common use of the term *adjacent* refers to two things that touch each other or share a wall. The mathematical definition of *adjacent* refers to both sharing a side and a vertex.

Make sure students have an opportunity to clarify the two definitions as they discuss Question 4.

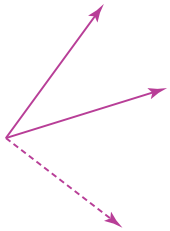
924 MODULE 5 • TOPIC 1 • LESSON 3

6. Use one sheet of patty paper to create a set of *non-congruent* adjacent angles. Describe your process. Label the patty paper "Adjacent Angles."

   Sample answer:

   I used my straightedge to draw an angle. Then I used my straightedge to draw another angle using one of the rays of the original angle as a ray of the new angle.

   Adjacent Angles



7. Are all adjacent angles supplementary? Explain your reasoning.

   No. The measures of adjacent angles do not have any relation to each other.

8. Are all supplementary angles adjacent? Explain your reasoning.

   No. Supplementary angles are not necessarily adjacent.

MODULE 5 • TOPIC 1 • LESSON 3 925

Chunking the Activity

- Read and discuss the Worked Example and definition.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

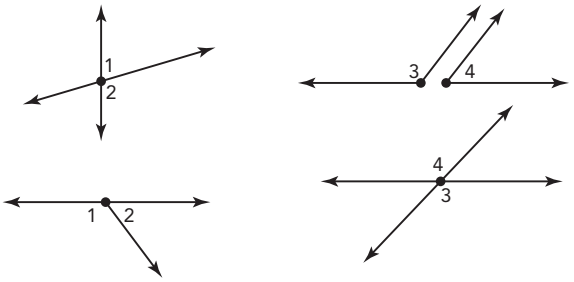
ACTIVITY 3.2

Linear Pairs

Let's explore a different angle relationship.

WORKED EXAMPLE

$\angle 1$ and $\angle 2$ form a linear pair. $\angle 3$ and $\angle 4$ are do *not* form a linear pair.



1. Use the figures in the Worked Example to define a *linear pair of angles* in your own words.

   Sample answer:

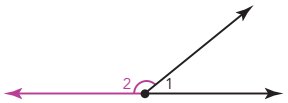
   Two angles are a linear pair if they are adjacent and their non-common sides form a line.

A **linear pair** of angles is formed by two adjacent angles that have noncommon sides that form a line.

2. Draw $\angle 2$ so that it forms a linear pair with $\angle 1$. Use one sheet of patty paper and record your response. Label your patty paper "Linear Pair."

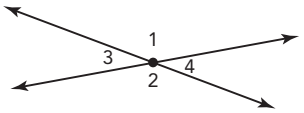
   Check students' drawings.

   Sample answer:



926 MODULE 5 • TOPIC 1 • LESSON 3

3. Name all linear pairs in the figure shown.



The linear pairs are ∠1 and ∠4, ∠1 and ∠3, ∠2 and ∠4, and ∠2 and ∠3.

4. If the angles that form a linear pair are congruent, what can you conclude?

If the angles that form a linear pair are congruent, then the intersecting lines, line segments, or rays forming the linear pair are perpendicular.

5. What is the difference between a linear pair of angles and supplementary angles that share a common side?

There is no difference. A linear pair of angles form a line, and so do a pair of supplementary angles that share a common side.

6. What is the difference between a linear pair of angles and supplementary angles that do not share a common side?

The pair of supplementary angles are not adjacent, but a linear pair must be adjacent.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

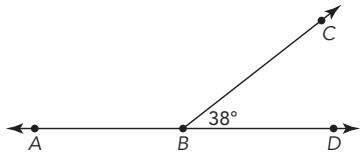
Help students answer Questions 4–6 by providing them with sentence frames or sentence stems.

For example, a sentence frame for Question 4:

If the angles form _____, then you know that _____.

MODULE 5 • TOPIC 1 • LESSON 3 927

7. Angle *ABC* and angle *CBD* form a linear pair. Write and solve an equation to determine the measure of $\angle ABC$.

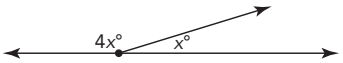


$x + 38 = 180$

$x = 142$

The measure of $\angle ABC$ is 142°.

8. The angles shown are a linear pair of angles. Solve for *x*.



$x + 4x = 180$

$x = 36\ 4(36) = 144$

The measures of the angles are 36° and 144°.

9. Write and complete the sentence on your Linear Pairs patty paper.

If two angles form a linear pair, then the sum of the measures of the linear pair of angles is 180°.

928 MODULE 5 • TOPIC 1 • LESSON 3

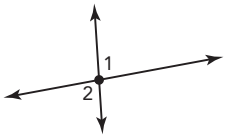
ACTIVITY 3.3

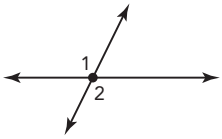
Vertical Angles

Let's explore one more special angle relationship.

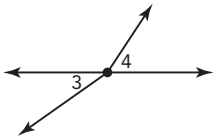
WORKED EXAMPLE

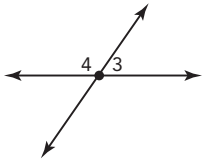
∠1 and ∠2 are vertical angles.





∠3 and ∠4 are *not* vertical angles.



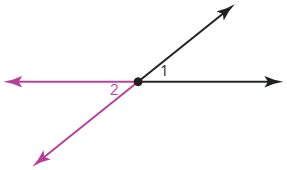


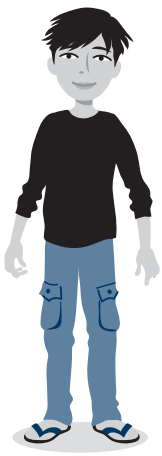
1. Use the figures in the Worked Example to define *vertical angles* in your own words.

   Sample answer:

   Vertical angles are nonadjacent angles formed by two intersecting lines.

2. Draw ∠2 so that it forms a vertical angle pair with ∠1.





Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the Worked Example and definition.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For "Intermediate" and higher proficiency levels

Students typically associate the term *vertical* to an upright orientation. The mathematical meaning of *vertical angles* does not have any connection to the common use of the term.

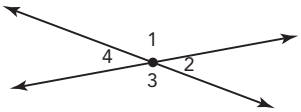
MODULE 5 • TOPIC 1 • LESSON 3 929

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

3. Name all vertical angle pairs in the diagram shown.



∠1 and ∠3 are vertical angles. ∠2 and ∠4 are vertical angles.

4. Trace the figure in Question 3 on two different sheets of patty paper. Be sure you number the angles. Use the patty paper to investigate the measures of vertical angles. What do you notice?

The vertical angles appear to be the same size (congruent).

5. Use your protractor to measure each angle in Question 3. What do you notice?

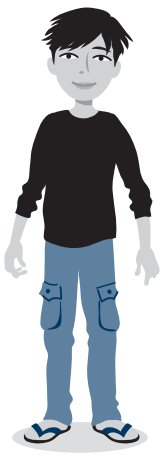
The vertical angles have the same measure.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1F:

- Can students identify and explain mathematical relationships?

How can you rotate the patty paper to investigate vertical angles?



6. Use what you know about supplementary angles and linear pairs to justify your investigations in Questions 4 and 5.

Sample answer:

If angle 1 has a measure of x degrees, then both angles 2 and 4 must have measures of $(180 - x)$ degrees because angles 1 and 2 form a linear pair and angles 1 and 4 form a linear pair. So angles 2 and 4 have the same measure and they are vertical angles. Therefore, vertical angles have the same measure.

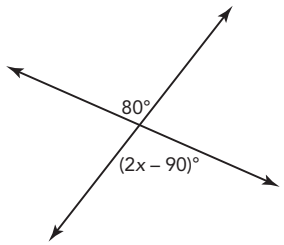
7. Label one sheet of your patty paper "Vertical Angles" and write and complete the sentence.

When two lines intersect to form vertical angles, each pair of vertical angles _____has the same measure_____.

930 MODULE 5 • TOPIC 1 • LESSON 3

8. Write and solve an equation to determine the measures of all four angles in each diagram.

a.

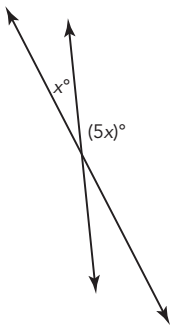


$2x - 90 = 80$

$x = 85$

The angles labeled with expressions are both 80°.
The non-labeled angles are 100° each.

b.



$x + 5x = 180$

$x = 30$

The angle labeled x and its vertical angle are both 30°.
The angle labeled $5x$ and its vertical angle are both 150°.

Activity 3.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days.

To provide additional practice using a diagram to answer questions about special angle relationships, assign Skills Practice Set A for this lesson.

To provide additional practice sketching angles given special angle relationships, assign Skills Practice Set B for this lesson.

To provide additional practice writing and solving equations given special angle relationships, assign Skills Practice Set C for this lesson.

To provide additional practice determining the value of x given special angle relationships, assign Skills Practice Set D for this lesson.

MODULE 5 • TOPIC 1 • LESSON 3 931

Chunking the Activity

- Read and discuss the directions.
- Group students to complete the activity.
- Share and summarize.
- Have students read and answer the Essential Question on the lesson opener page.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1E:

- Do students create representations to present and organize their mathematical reasoning?

Talk the Talk

Perfect Pairs

Answer each question. Draw a figure to justify your response.

1. Two intersecting lines form how many

   a. Pairs of supplementary angles?
   Two intersecting lines create four pairs of supplementary angles.

   b. Pairs of complementary angles?
   Two intersecting lines do not form complementary angles.

   c. Pairs of adjacent angles?
   Two intersecting lines create four pairs of adjacent angles.

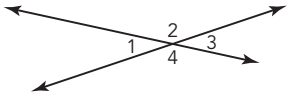
   d. Linear pairs of angles?
   Two intersecting lines create four linear pairs of angles.

   e. Pairs of vertical angles?
   Two intersecting lines create two pairs of vertical angles.

2. Suppose two lines intersect. When you are given the measure of one angle, can you determine the measures of the remaining angles without using a protractor? Explain your reasoning.
   Yes. Sample explanation: When I am given the measure of one angle, then the measure of the angle that forms a vertical pair with the given angle is the same. The two remaining angles each form a linear pair with the given angle. So, by subtracting the given angle measure from 180°, I can determine the measure of each of the two remaining angles.

932 MODULE 5 • TOPIC 1 • LESSON 3

3. When two lines intersect, four different angles are formed as shown.



a. Describe the relationship between vertical angles.

Vertical angles have the same measure.

b. Describe the relationship between adjacent angles.

Adjacent angles are supplementary.

4. Draw and label a diagram that includes at least one of each relationship. Then, identify the angles that satisfy each description.

- Complementary angles
- Supplementary angles
- Perpendicular lines
- Adjacent angles
- Linear pair
- Vertical angles

Check students' drawings and angle notation.

Optimizing Learning

This activity highlights patterns, critical features, big ideas, and relationships.

MODULE 5 • TOPIC 1 • LESSON 3 933

934 MODULE 5 • TOPIC 1 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

Write

Draw an example of each term. Provide an explanation when necessary.

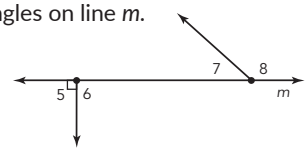
1. Adjacent angles
2. Linear pair
3. Vertical angles

Remember

Many geometric figures contain a mixture of special angle pairs. Understanding the relationships between special angle pairs will help you understand more complex geometric diagrams.

Practice

1. Use the diagram to identify the specified angles on line *m*.



   a. All adjacent angles

   ∠5 and ∠6, ∠7 and ∠8

   b. All linear pairs

   ∠5 and ∠6 form a linear pair, and ∠7 and ∠8 form a linear pair.

   c. All vertical angles

   There are no vertical angles in the diagram.

   d. All right angles

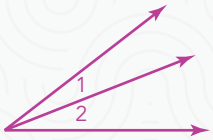
   ∠5 and ∠6 are right angles.

   e. All supplementary angles

   ∠5 and ∠6, ∠7 and ∠8

Write

1. Adjacent angles



∠1 and ∠2 form adjacent angles.

2. Linear pair

∠1 and ∠2 form a linear pair.

3. Vertical angles

∠1 and ∠2 are vertical angles.

MODULE 5 • TOPIC 1 • LESSON 3 ASSIGNMENT 935

Lesson 3 Assignment

2. Draw the angle pairs described and answer each question. Label each angle with its measure.

   a. Draw a linear pair of angles. One of the angles should have a measure of 123°.

   Check students' drawings. All angles drawn and labeled should measure 57° and 123°.

   b. Draw two angles with the same measures as those in part (a), such that they are not a linear pair.

   Check students' drawings. All angles drawn and labeled should measure 57° and 123°.

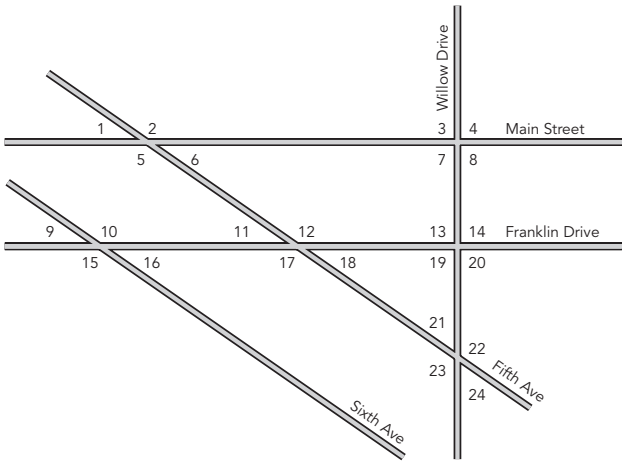
   c. Explain the differences and similarities between linear pairs and supplementary angles.

   The sum of the measures of a linear pair of angles is 180°. The sum of the measures of supplementary angles is also 180°. Linear pairs are always supplementary. However, supplementary angles are not always linear pairs, because they do not have to be adjacent or share a vertex.

936 MODULE 5 • TOPIC 1 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

3. Suppose each street in the map shown represents a line. Provide an example of each angle relationship.



Sample answers:

a. Vertical angles
∠2 and ∠5

b. Supplementary angles
∠9 and ∠10

c. Linear pair
∠11 and ∠12

d. Adjacent angles
∠17 and ∠18

e. Congruent angles
∠12 and ∠17

MODULE 5 • TOPIC 1 • LESSON 3 ASSIGNMENT 937

Lesson 3 Assignment

Write and solve an equation to determine the measure of each unknown angle.

4. When two angles form a linear pair and the measure of the first angle is one-fifth the measure of the second angle, what is the measure of each angle?

$\frac{1}{5}x + x = 180$

$x = 150$

$m\angle 1 = 30°$

$m\angle 2 = 150°$

5. When two angles form a linear pair and the measure of the first angle is three times the measure of the second angle, what is the measure of each angle?

$3x + x = 180$

$x = 45$

$m\angle 1 = 135°$

$m\angle 2 = 45°$

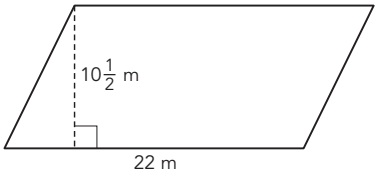
938 MODULE 5 • TOPIC 1 • LESSON 3 ASSIGNMENT

Lesson 3 Assignment

Prepare

Use a formula to determine the area of each figure.

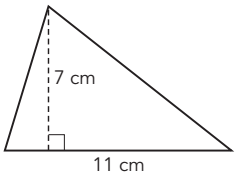
1.



231 square meters

$22 \cdot 10\frac{1}{2} = 231$

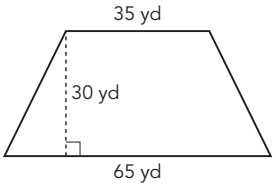
2.



38.5 square centimeters

$\frac{1}{2}(11)(7) = 38.5$

3.



1500 square yards

$\frac{1}{2}(30)(65 + 35) = 1500$

940 MODULE 5 • TOPIC 1 • LESSON 3 ASSIGNMENT

TOPIC 1 SELF-REFLECTION

Name: _____

Angle Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Angle Relationships* topic by:

TOPIC 1: *Angle Relationships*	Beginning of Topic	Middle of Topic	End of Topic
determining unknown angle measures by writing and solving algebraic equations using the Triangle Sum theorem.	○	○	○
recognizing the special angle pairs formed when two lines intersect: supplementary angles, complementary angles, vertical angles, adjacent angles, and linear pairs.	○	○	○
determining unknown angle measures by writing and solving algebraic equations based on relationships between angles.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Angle Relationships* topic.

   Answers will vary.

continued on the next page

Notes

MODULE 5 • TOPIC 1 • SELF-REFLECTION 941

Notes

TOPIC 1 SELF-REFLECTION *continued*

2. What mathematical understandings from the topic do you feel you are making the most progress with?

Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

Answers will vary.

942 MODULE 5 • TOPIC 1 • SELF-REFLECTION

TOPIC 1 SUMMARY

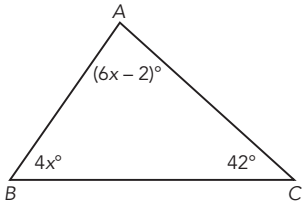
Angle Relationships Summary

LESSON 1 Solving Equations Using the Triangle Sum Theorem

You have learned about the Triangle Sum theorem, which states that the sum of the measures of the interior angles of a triangle is 180°. You can use this information, along with knowledge about angle relationships, to write equations and solve for an unknown value or unknown angle measure.

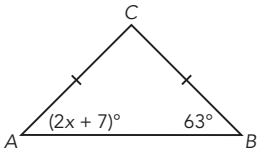
For example, write and solve an equation to determine the value of x. Then, determine each unknown angle measure.

$$(6x - 2) + 4x + 42 = 180$$
$$10x + 40 = 180$$
$$10x = 140$$
$$x = 14$$
$$m\angle B = 4(14) = 56°$$
$$m\angle A = 6(14) - 2 = 82°$$



An isosceles triangle is a triangle with two sides that are the same length. The angles opposite these sides are called **base angles**. The **Base Angles theorem** states that theangles opposite the congruent sides of an isosceles triangle are congruent. **Congruent sides** are sides that have the same length. **Congruent angles** are angles that have the same measure.

For example, △ABC is an isosceles triangle. You can write and solve an equation to determine the value of x. Since ∠A and ∠B are the base angles, their measures are the same.



$$2x + 7 = 63$$
$$2x = 56$$
$$x = 28$$

NEW KEY TERMS

- base angles [ángulos de la base]
- Base Angles theorem [teorema de los ángulos de la base]
- congruent sides
- congruent angles [ángulos congruentes]
- straight angle
- collinear [colineal]
- supplementary angles [ángulos suplementarios]
- complementary angles [ángulos complementarios]
- perpendicular [perpendicular]
- adjacent angles [ángulos adyacentes]
- linear pair [par lineal]
- vertical angles [ángulos verticales]

Notes

MODULE 5 • TOPIC 1 • TOPIC SUMMARY 943

Notes

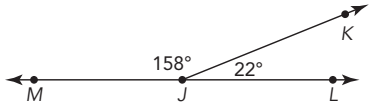
LESSON 2

Relationships Between 90° and 180° Angles

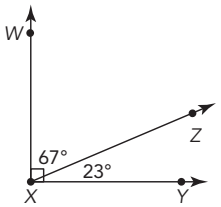
A **straight angle** is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

When points lie on the same line or line segment, they are said to be **collinear**. For example points *M*, *J*, and *L* are collinear.

Two angles are **supplementary angles** if the sum of their angle measures is equal to 180 degrees. For example, Angles *MJK* and *KJL* are supplementary angles.



Two angles are **complementary angles** if the sum of their angle measures is equal to 90 degrees. For example, Angles *WXZ* and *ZXY* are complementary angles.



Two lines, line segments, or rays are **perpendicular** if they intersect to form 90 degree angles. The symbol for perpendicular is ⊥.

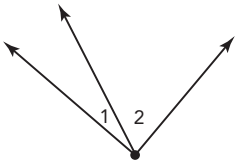
944 MODULE 5 • TOPIC 1 • TOPIC SUMMARY

LESSON 3

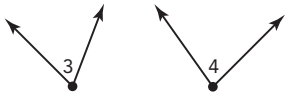
Special Angle Relationships

Adjacent angles are two angles that share a common vertex and a common side.

∠1 and ∠2 are adjacent angles.

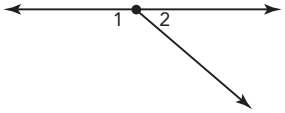


∠3 and ∠4 are *not* adjacent angles.

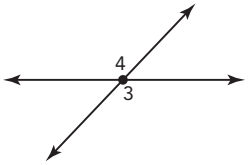


A **linear pair** of angles is formed by two adjacent angles that have noncommon sides that form a line. Linear pairs are supplementary.

∠1 and ∠2 form a linear pair.

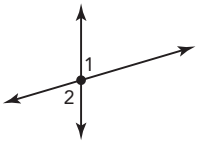


∠3 and ∠4 do *not* form a linear pair.

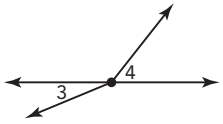


Vertical angles are two nonadjacent angles that are formed by two intersecting lines. Vertical angles are congruent.

∠1 and ∠2 are vertical angles.



∠3 and ∠4 are *not* vertical angles.



Notes

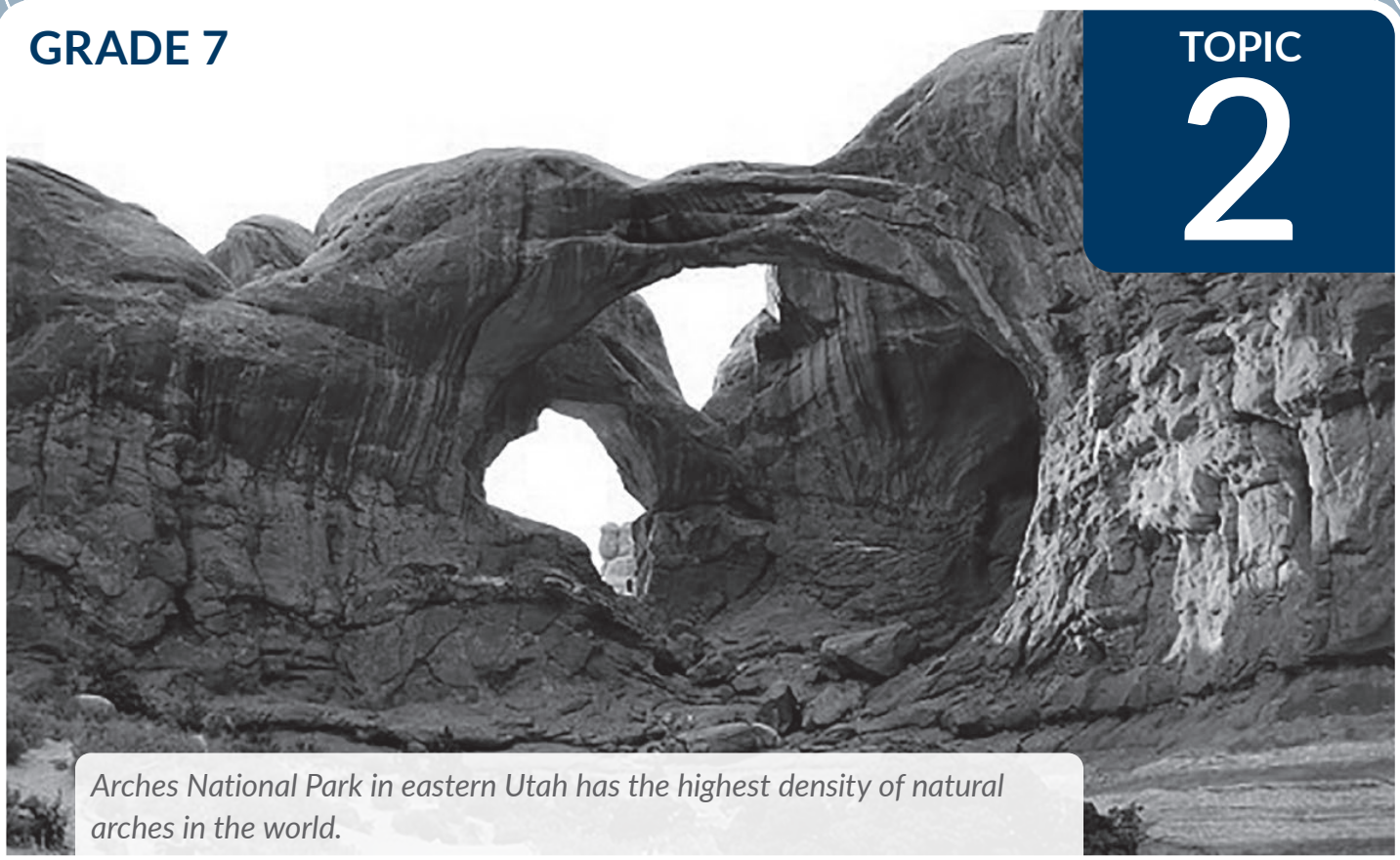
MODULE 5 • TOPIC 1 • TOPIC SUMMARY 945

Notes

946 MODULE 5 • TOPIC 1 • TOPIC SUMMARY

GRADE 7

TOPIC 2



Arches National Park in eastern Utah has the highest density of natural arches in the world.

Area, Surface Area, and Volume

LESSON 1 Composite Figures **949**

LESSON 2 Total Surface Area of Prisms and Pyramids **959**

LESSON 3 Volume of Prisms and Pyramids **981**

LESSON 4 Volume and Surface Area Problems with Prisms and Pyramids **1001**

CC BY NC

TOPIC 2 OVERVIEW

Area, Surface Area, and Volume

How are the key concepts of *Area, Surface Area, and Volume* organized?

In this topic, students build on their existing knowledge of area, volume, and properties of operations, reinforcing procedural fluency with previously learned formulas and number facts. Students use reasoning to develop different strategies for decomposing unfamiliar shapes into shapes with known area formulas to determine the areas of composite figures.

Next, students apply their knowledge of composite figures to determine the surface area of geometric solids. Students use nets, or two-dimensional representations of three-dimensional figures, to determine the area of familiar faces that make up the solid. They physically decompose three-dimensional solids into two-dimensional nets and compose solids from nets.

They use nets of familiar solids (right rectangular prisms and pyramids) to discover that the volume of a pyramid is one-third the volume of a prism with the same base and height. Students practice using the volume formulas while solving real-world problems. They then calculate the total and lateral surface areas of familiar pyramids and prisms.

Math Representation

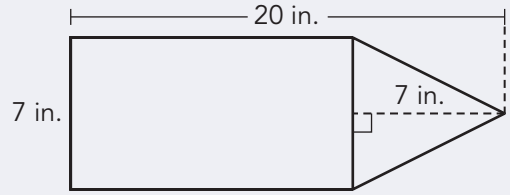
Area is additive. The area of a composite figure can be determined by decomposing it into familiar shapes and then adding the areas of those shapes together.

For example, the composite figure shown is composed of a rectangle and a triangle.

Area of composite figure = Area of Rectangle + Area of Triangle

$$= (7)(13) + \frac{1}{2}(7)(7)$$
$$= 91 + 24\frac{1}{2}$$
$$= 115\frac{1}{2}$$

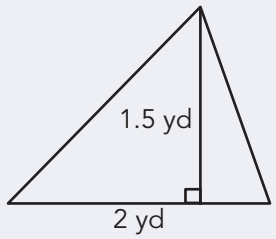
The area of the composite figure is $115\frac{1}{2}$ square inches.



Math Representation

For example, in this triangle, the base, b, is equal to 2 yards, and the height, h, is equal to 1.5 yards.

$$\text{Area of a triangle} = \frac{1}{2}bh$$
$$= \frac{1}{2}(2)(1.5)$$
$$= 1.5 \text{ square yards}$$



What is the entry point for students?

Students have developed and applied area formulas for rectangles, triangles, parallelograms, and trapezoids to solve problems. At the start of this course, students developed the formulas for circumference and area of circles. This development of area formulas emphasized student flexibility in decomposing and composing shapes and in understanding that area is additive.

In this topic, students build upon this knowledge to determine the area of composite figures by decomposing them into shapes with known area formulas. Students determine the area of these familiar shapes and then add their areas together to determine the area of the original composite figure. This concept is then applied to determine the surface areas of prisms and pyramids, as students calculate the areas of two-dimensional nets that represent three-dimensional solids.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic when they can:

- Apply the techniques of composing and/or decomposing to determine the area of composite figures to solve mathematical and real-world problems.
- Determine the areas of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, and triangles.
- Represent three-dimensional figures using nets made up of rectangles and triangles.
- Combine the areas for rectangles and triangles in the net to calculate the total surface area of a three-dimensional figure.
- Solve real-world and mathematical problems involving lateral and surface area using nets for rectangular and triangular prisms and pyramids.
- Model and explain the relationship between the volume of a rectangular prism and a rectangular pyramid having congruent bases and heights and connect to the formulas.
- Explain how the volume of a triangular prism relates to the volume of a triangular pyramid with both congruent bases and heights and connect to their formulas.
- Solve problems involving the volumes of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids.

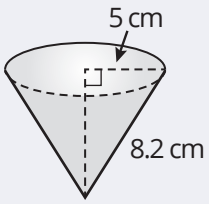
Why is *Area, Surface Area, and Volume* important?

Students will use generalizations of the formulas for the volume of prisms and pyramids in future courses as they continue determining volumes of solids. They will relate the volume of a cone to the volume of a cylinder.

Students will use Cavalieri's Principle to formally prove these familiar formulas in Geometry. They will build on the knowledge established in this topic to solve more complicated volume and surface area problems.

Math Representation

Consider the cone with the given measurements.



To determine the volume of the cone, you need its perpendicular height.

$$5^2 + b^2 = 8.22$$
$$25 + b^2 = 67.24$$
$$b^2 = 42.24$$
$$b \approx 6.5 \text{ cm}$$

$V = \frac{1}{3}\pi(5^2)(6.5) \approx 170.17$ cubic centimeters

How do the activities in *Area, Surface Area, and Volume* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

This topic highlights the necessity of seeing patterns and structure in mathematics, specifically shapes. Students should decompose composite two-dimensional figures into familiar shapes with known area formulas, calculate the area of each smaller familiar shape, and then recompose to determine area for the composite figure. This process transfers to determining surface areas of three-dimensional solids. Students should see

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NEW KEY TERMS

- net
- surface area [área de (la) superficie / área superficial]
- pyramid [pirámide]
- slant height
- lateral surface area [área de la superficie lateral]

three-dimensional solids as solids composed from a net of two-dimensional figures (7.1F). This leads to strategies for calculating the surface areas of prisms and pyramids, which are similar to those used for determining the areas of composite figures.

Using precision with construction tools and language are prominent mathematical process standards in the final lesson of this topic. Students use a protractor, straightedge, and patty paper to build intuition and engage in informal constructions without needing to know the properties required for more formal constructions (7.1C). Students should become comfortable with different tools so they can choose strategically and appropriately, according to their purpose, in future courses.

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Launch a *Cognate Spelling Pattern of the Week* activity where each week, a new spelling pattern is introduced and explored through various activities, including writing sentences, creating visual aids, and discussing examples of them in mathematical contexts. This strategy provides ongoing exposure and reinforces the importance of cognates and recognizing spelling patterns in learning mathematical vocabulary.

MODULE 5, TOPIC 2 PACING GUIDE

165-Day Pacing

5 Constructing and Measuring

TOPIC 2: Area, Surface Area, and Volume

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1A, 7.1B, 7.1D, 7.1F, 7.1G

ELPS: 1.A, 1.C, 1.E, 1.G, 2.I, 3.H, 4.D, 4.F, 4.G, 5.B, 5.C, 5.D

Topic Pacing: 13 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
1	Composite Figures	In this lesson, students calculate the area of complex figures. They compare two methods: decomposing a figure into familiar shapes and composing a figure into a rectangle. Students then solve problems in context, including the area of countries using map scales to approximate areas. They use given dimensions and problem solving to calculate the area of a triangle embedded in a square. **Materials Needed:** Centimeter Rulers, Problem-Solving Model Graphic Organizer	**7.9C**	2
2	Total Surface Area of Prisms and Pyramids	Students apply mathematical and spatial reasoning to determine the surface areas of prisms and pyramids using nets, drawings, and measurements. Students solve a variety of surface area problems and distinguish between volume and surface area measurements. **Materials Needed:** Scissors, Tape, Rulers, Problem-Solving Model Graphic Organizer	7.3A 7.9D	3
3	Volume of Prisms and Pyramids	Students cut out given nets and assemble an open rectangular prism and an open rectangular pyramid. They compare the models and determine that the heights and bases are congruent. Students fill the models with birdseed to discover that the volume of the pyramid is one-third the volume of the prism, and they then write the formula for the volume of each. Students repeat the activity for a triangular prism and a triangular pyramid with congruent heights and bases. They then use the volume formulas to solve problems involving rectangular and triangular prisms and pyramids. **Materials Needed:** Scissors, Tape, Birdseed or Rice, Paper, Rulers	7.8A 7.8B **7.9A**	2

*Bold TEKS = Readiness Standard

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MODULE 5, TOPIC 2 PACING GUIDE

165-Day Pacing

Lesson	Lesson Title	Highlights	TEKS*	Pacing
4	**Volume and Surface Area Problems with Prisms and Pyramids**	Students compare two different pieces of acoustical foam: one that is made up of square pyramids and one that is made up of triangular prisms. Students are introduced to the term *lateral surface area*, and they compare the surface areas and lateral surface area of the foam pieces. They then determine the total amount of foam that covers the top surface of the two pieces of foam. Finally, students calculate the volume, surface area, and lateral surface area of rectangular and triangular prisms and pyramids to solve real-world problems. **Materials Needed:** Rulers, Calculators (optional), Problem-Solving Model Graphic Organizer	**7.9A** 7.9D	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				3

*Bold TEKS = Readiness Standard

MODULE 5, TOPIC 2 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: **7.9C** **LESSON 1** Composite Figures **GETTING STARTED** **ACTIVITY 1**	**LESSON 1** continued **ACTIVITY 2** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: 7.3A, 7.9D **LESSON 2** Total Surface Area of Prisms and Pyramids **GETTING STARTED** **ACTIVITY 1**	**LESSON 2** continued **ACTIVITY 2** **ACTIVITY 3**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 2 continued **ACTIVITY 4** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	TEKS: 7.8A, 7.8B, **7.9A** **LESSON 3** Volume of Prisms and Pyramids **GETTING STARTED** **ACTIVITY 1**	**LESSON 3** continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	TEKS: **7.9A** 7.9D **LESSON 4** Volume and Surface Area Problems with Pyramids and Prisms **GETTING STARTED** **ACTIVITY 1**
Day 11	**Day 12**	**Day 13**		
LESSON 4 continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**END OF TOPIC ASSESSMENT**		

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are three Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice sets of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.

1 Composite Figures

MATERIALS

Centimeter Rulers
Problem-Solving Model Graphic Organizer

LESSON OVERVIEW

In this lesson, students calculate the area of complex figures. They compare two methods: decomposing a figure into familiar shapes and composing a figure into a rectangle. Students then solve problems in context, including the area of countries, using map scales to approximate areas. They use given dimensions and problem solving to calculate the area of a triangle embedded in a square.

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

Expressions, Equations, and Relationships

(9) The student applies mathematical process standards to solve geometric problems.
The student is expected to:

7.9C determine the areas of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles.

ELPS

(1) Learning Strategies
The student is expected to:

(C) demonstrate an increasing ability to distinguish between formal and informal English and an increasing knowledge of when to use each one commensurate with grade-level learning expectations.

(3) Speaking
The student is expected to:

(H) narrate, describe, and explain with increasing specificity and detail as more English is acquired.

(4) Reading:
The students is expected to:

(D) use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text.

ESSENTIAL IDEAS

- The area of a composite figure can be determined by decomposing the figure into rectangles, parallelograms, or triangles and then adding the areas of those figures.
- The area of a composite figure can be determined by composing the figure into a rectangle and then subtracting the area of the shape that is not part of the composite figure.
- When calculating the area of composite figures, additional steps, such as determining dimensions and using a scale, may be necessary.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Compose or Decompose? 15–20 minutes

CONNECT TO PRIOR KNOWLEDGE

Students compare decomposition and composition. They decompose figures by breaking them into familiar shapes, compose figures by making them into rectangles, and compare a problem in context where both methods are used to calculate area.

DEVELOP

Activity 1.1: Area of Composite Figures 20–25 minutes

REAL-WORLD PROBLEM SOLVING

Students calculate the area of complex figures in both mathematical and real-world contexts.

DAY 2

Activity 1.2: Area of Complex Figures 25–30 minutes

REAL-WORLD PROBLEM SOLVING, MATHEMATICAL PROBLEM SOLVING

Students solve more complex area problems involving decomposition and unlabeled dimensions that are more difficult to determine. They are given scales and estimation strategies to approximate the area of irregular shapes.

DEMONSTRATE

Talk the Talk: Use Your Powers of Mathematical Reasoning 10–15 minutes

PRESENTATION

Students calculate the area of a triangle embedded in a square. They use problem solving to measure the dimensions and determine the area.

Getting Started

ENGAGE

Compose or Decompose?

Facilitation Notes

In this activity, students compare decomposition and composition. They decompose figures by breaking them into familiar shapes, compose figures by making them into rectangles, and compare a problem in a context where both methods are used to calculate area.

Ask a student to read the directions aloud. Have students work with a partner or in a group to complete Questions 1 and 2. Discuss responses as a class

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Is there more than one way to decompose the figure? Explain. • To calculate the area of the composite figure, would you add or subtract the areas that you created? Why does addition make sense? • Is there more than one way to compose the figure into a larger familiar figure? Explain. • To calculate the area of the composite figure, would you add or subtract the areas that you created? Why does subtraction make sense?

Ask a student to read the scenario for Question 3 aloud. Have students work with a partner or in a group to complete the question. Discuss responses as a class.

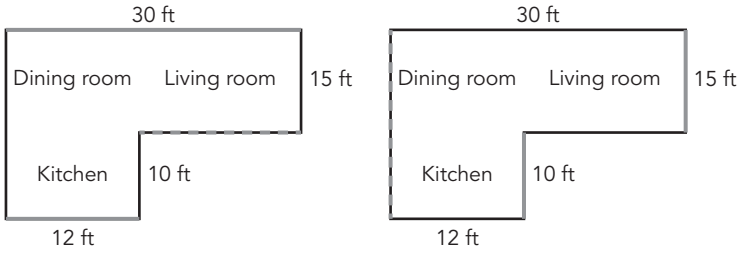
QUESTIONS TO SUPPORT DISCOURSE

Probing	• What is another way Isabella could have created two rectangles? • Do you have enough information to know the dimensions of the dining room? If so, what are they?

DIFFERENTIATION STRATEGY

Just in Time Support

Demonstrate how to use given horizontal measurement to determine an unknown horizontal measurement and then do the same for vertical measurements.

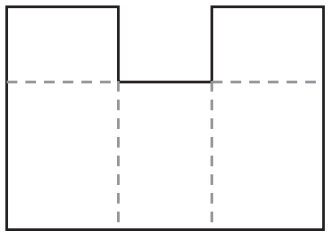


large − small = small
30 − 12 = 18

small + small = large
15 + 10 = 25

COMMON MISCONCEPTION

Students may think it is best to divide the figure into the smallest rectangles or greatest number of rectangles possible, which in this case is five rectangles. Explain that it is more efficient to use the minimum number of rectangles to calculate area.



Summary

STAMP THE LEARNING

When calculating the area of a composite figure, you can decompose the figure into rectangles and triangles and add the areas. You can compose the figure into a rectangle and subtract the area that is not part of the figure.

ACTIVITY 1.1

Area of Composite Figures

DEVELOP

Facilitation Notes

In this activity, students calculate the area of complex figures in both mathematical and real-world contexts.

Ask a student to read the directions aloud. Have students work with a partner or in a group to complete Questions 1 through 3. Discuss responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the dimensions of each room? • Do you have to determine the cost per room to calculate the total cost? If not, what process could you use?
Gathering	• What figures did you use to decompose the back of the house?

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Probing	• Can you assume any line segments are congruent in this diagram? Which ones? Why are they congruent? • How did you determine how much paint is needed? • How did you determine the dimensions of the point of the arrow?

Ask a student to read the directions aloud. Have students work with a partner or in a group to complete Questions 4 and 5. Discuss responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Did you calculate the area of the shaded portion directly or calculate the area of the white portion and subtract it from the rectangle? Explain your process. • Did you use a trapezoid or a rectangle and triangle in your calculations? Explain your process. • Did you calculate the area of the squares or the triangles? Explain your process. • If you used triangles, did you make them all the same size? Explain your process.

DIFFERENTIATION STRATEGY

Just in Time Support

Have students complete Questions 2 and 5 only.

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

STAMP THE LEARNING

Summary

When solving real-world problems involving area, you might have extra steps, such as identifying dimensions and performing calculations, to determine the cost or amount of materials.

ACTIVITY 1.2

Area of Complex Figures

Facilitation Notes

In this activity, Students solve more complex area problems involving decomposition and unlabeled dimensions that are more difficult to determine. They are given scales and estimation strategies to approximate the area of irregular shapes.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction aloud and discuss as a class. Have students answer Questions 1 and 2 with a partner or in a group. Discuss responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What polygons or figures did you create when you decomposed this figure?
Probing	• Which side lengths would you need to calculate the area of this figure based on how you decomposed it?
Reflecting and justifying	• Do you prefer to use trapezoids or just rectangles and triangles? Why?
Gathering	• What units did you use for your final answer?
Probing	• What is another way to decompose this figure? • Did you use parallelograms or two triangles? Explain your reasoning. • How did you determine the lengths not provided?

Have students answer Questions 3 and 4 with a partner or in a group. Discuss responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How many polygons did you use to calculate the area?
Probing	• Explain how you used the scale to determine the dimensions. • Do you think your final answer is an overestimate or an underestimate? Why?
Seeing structure	• Did you use composition or decomposition to determine the area? Why did you choose that method?
Reflecting and justifying	• How close were you to the actual area? • What is another way to solve this problem?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Guide students to create a ruler using the scale. Alternatively, complete Question 3 with them and have them complete Question 4 with a partner.

Challenge Opportunity

- Recommend students research and use Pick's Theorem to determine the area of this irregular figure.

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STAMP THE LEARNING

Summary

Sometimes, it is not obvious where to draw auxiliary lines to decompose a figure into familiar figures; in these cases, use the vertices and sides with given lengths as a guide. In addition, estimation and the use of a scale may be required to approximate the area of an irregular shape.

DEMONSTRATE

Talk the Talk

USE YOUR POWERS OF MATHEMATICAL REASONING

Facilitation Notes

In this activity, students calculate the area of a triangle embedded in a square. They use problem-solving to measure the dimensions and determine the area.

Ask a student to read the introduction aloud and discuss as a class. Have students complete Question 1 in with a partner or in a group. Discuss responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the base and height of the shaded triangle? • Do you need to know the dimensions of the shaded triangle to determine its area? Explain.
Seeing structure	• Did you use composition or decomposition to determine the area of the shaded region? Explain. • How do you know that the three non-shaded triangles are right triangles?
Reflecting and justifying	• Do composition problems always require a subtraction step? Explain your thinking.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Sometimes, problem-solving strategies are limited, based on the information provided. Therefore, it is best to know how to use both composition and decomposition to solve for the areas of complex figures.

1 Composite Figures

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Decompose composite geometric figures into rectangles, parallelograms, and/or triangles to determine their areas.
- Solve real-world problems by composing and decomposing shapes into triangles and rectangles.

You know how to calculate the area of triangles, rectangles, parallelograms, and trapezoids.

How can you use what you know about the areas of these shapes to determine areas of more complex shapes?

Sample answer:

I can determine the area of a composite figure by decomposing it into familiar figures and then adding together the areas of those figures.

EB STUDENT TIP

For all proficiency levels

Remind students that the verb *compose*, the noun *composition*, and the describing word *composite* are all related and that the words *decompose* and *decomposition* mean the opposite of *compose* and *composition* because of the prefix *de-*. These words also have similar Spanish cognates: *(des)componer* for the verbs, *(des)composición* for the nouns, and *compuesto* for *composite*.

Beginning: Model how to decompose a complex shape into simpler shapes like triangles, squares, and parallelograms using physical objects such as tangrams and have students do the same. Ask students if they are composing or decomposing and

(continued on next page)

Chunking the Activity

- Read and discuss the directions.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Read and discuss the scenario.
- Group students to complete Question 3.
- Share and summarize.

STAMP THE LEARNING

The definition provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Getting Started

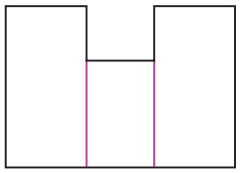
Compose or Decompose?

Remember:
A composite figure is a figure that is made up of more than one geometric figure.

Consider each composite figure shown.

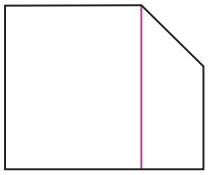
1. Show how you could decompose each figure into two or more familiar figures. Describe the shapes that make up the composite figure.

a.



Sample answer:
I decomposed the figure into three rectangles.

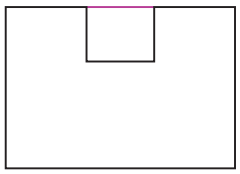
b.



Sample answer:
I decomposed the figure into a rectangle and a trapezoid.

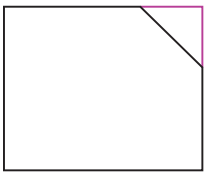
2. Show how you could compose each figure into a larger, familiar figure. Describe the shapes that make up the composite figure.

a.



Sample answer:
I added a smaller rectangle to compose a larger rectangle.

b.



Sample answer:
I added a triangle to compose a larger rectangle.

EB STUDENT TIP *(continued)*

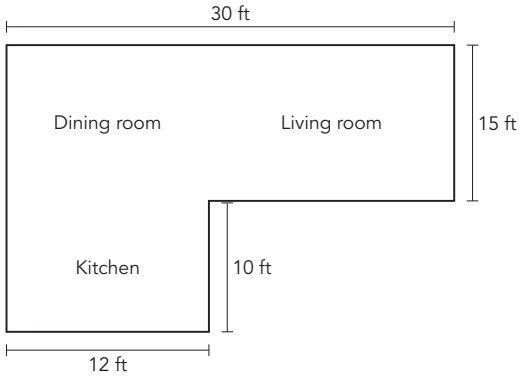
whether figures are composite or not composite. If students are ready to say these words, they can also verbally indicate their answers.

Intermediate: Have students use these sentence frames: "By decomposing the shape into _____ and _____, I can find the _____ by _____ of each part and then _____."

Advanced/Advanced High: Students use precise mathematical language to justify their choice of decomposition and explain how it facilitates determining calculations such as area: "To calculate _____, I decomposed the figure into _____ because _____. Another way to decompose it would be to _____." Students decide how to compose a given figure using the fewest number of different shapes.

950 MODULE 5 • TOPIC 2 • LESSON 1

Consider the blueprint of a floor plan for a kitchen, dining room, and living room combination.



3. Suppose the homeowner wants to replace the floors in all the rooms.

a. Isabella says she can determine the total area of flooring needed by decomposing the blueprint into two rectangles. Ethan says he can determine the total area of flooring needed by composing the blueprint into one large rectangle and then subtracting the unused area. Who is correct?

Both Isabella and Ethan are correct.

b. Show how to calculate the amount of flooring needed using Isabella's method.

$(30)(15) + (12)(10)$

$= 450 + 120$

$= 570$ square feet

c. Show how to calculate the amount of flooring needed using Ethan's method.

$(30)(25) - (10)(18)$

$= 750 - 180$

$= 570$ square feet

MODULE 5 • TOPIC 2 • LESSON 1 951

Chunking the Activity

- Read and discuss the directions.
- Group students to complete Questions 1–3.
- Check in and share.
- Read and discuss the directions.
- Group students to complete Questions 4 and 5.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1A:

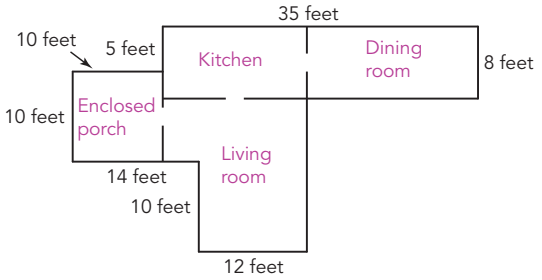
- Do students persevere in real-world problem solving?
- Do students recognize they can use (insert math being used, for example ratios) outside of the math classroom?

ACTIVITY 1.1

Area of Composite Figures

Solve each problem. Show your work.

1. Suppose that carpeting costs $1.20 per square foot. How much would it cost to carpet every room in this house, except the kitchen?



Porch: $10 \cdot 10 = 100$ square feet

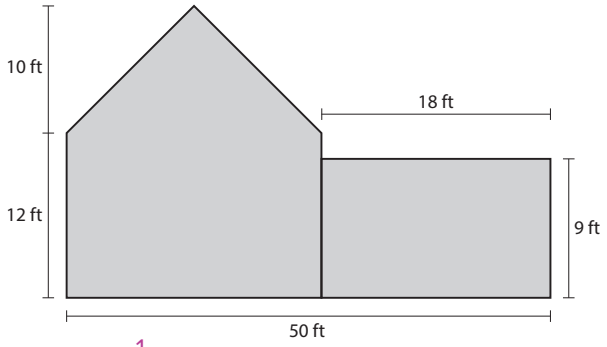
Living room: $12(10 + 10 + 5 - 8) + 4(10 + 5 - 8) = 232$ square feet

Dining room: $(35 + 10 - 14 - 12)8 = 152$ square feet

Total $= 484$ square feet; $484 \cdot \$1.20 = \$580.80$

Be sure that the simpler figures you draw do not have overlapping areas.

2. Suppose a gallon of paint covers about 400 square feet. How much paint would you need to paint the entire back of this house?



Triangular roof: $\frac{1}{2}(50 - 18)(10) = 160$ square feet

Rectangle beneath roof: $12(50 - 18) = 384$ square feet

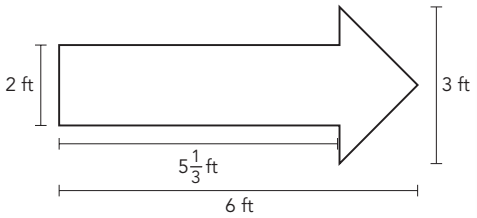
Rectangle without roof: $(18)(9) = 162$ square feet

Total area $= 706$ square feet

$706 \div 400 = 1.765$, so you would need 2 gallons of paint to paint the entire back of the house.

952 MODULE 5 • TOPIC 2 • LESSON 1

3. Samuel is spray painting an arrow on the side of a building to point to the entrance of his store. The can of gold spray paint he wants to use covers up to 12 square feet. Does Samuel have enough spray paint for his arrow?



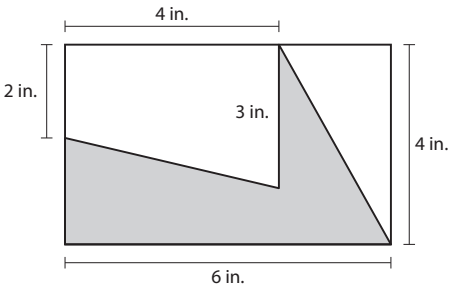
Area of arrow: $11\frac{2}{3}$ sq ft

Yes. Samuel has enough spray paint to paint the arrow.

Activity 1.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining the area of shaded regions, assign Skills Practice Set A for this lesson.

Determine the area of the shaded region in each figure. Show your work.

4. A right triangle and a trapezoid are drawn within a rectangle to create the shaded region in the figure.



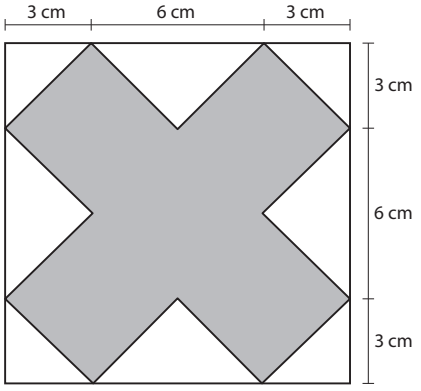
Check students' work.

Area of shaded right triangle: $\frac{1}{2}(2)(4) = 4$ sq in.

Area of shaded trapezoid: $\frac{1}{2}(2 + 1)4 = 6$ sq in.

Area of shaded region: 10 sq in.

5. Eight triangles are drawn within a square to create the shaded region in the figure.



Check students' work.

Area of square: $(12)(12) = 144$ sq cm

Area of 4 smaller triangles:

$4 \cdot \frac{1}{2}(3)(3) = 18$ sq cm

Area of 4 larger triangles:

$4 \cdot \frac{1}{2}(6)(3) = 36$ sq cm

Area of shaded region:

$144 - 18 - 36 = 90$ sq cm

MODULE 5 • TOPIC 2 • LESSON 1 953

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Group students to complete Questions 3 and 4.
- Share and summarize.

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS 7.1B:

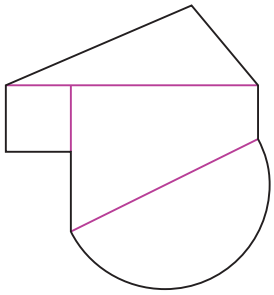
- Are students referring to the problem-solving model?
- Do students evaluate the reasonableness of their solution?
- Do students adapt their plan as needed?

ACTIVITY 1.2

Area of Complex Figures

You can divide more complex figures with oddly-figured regions into smaller, familiar regions to calculate the approximate area.

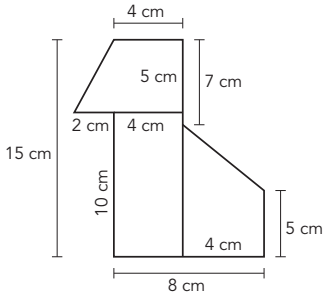
1. Draw lines in the figure to divide the figure into smaller, familiar figures. Then, name the familiar figures that make up the total figure.



Sample answer:
Triangle, rectangle, trapezoid, semi-circle

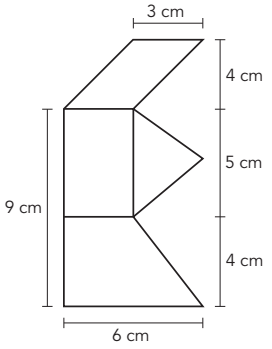
2. Determine the area of each complex figure.

a.



91 square centimeters
Top trapezoid: 25 sq cm
Bottom trapezoid: 26 sq cm
Rectangle: 40 sq cm

b.



52.5 square centimeters
Top parallelogram: 12 sq cm
Middle triangle: 7.5 sq cm
Bottom trapezoid: 18 sq cm
Rectangle: 15 sq cm

954 MODULE 5 • TOPIC 2 • LESSON 1

3. Estimate the area of France.

Answers will vary.

The actual area of France is 248,573 square miles.

4. Estimate the area of Namibia.

Answers will vary.

The actual area of Namibia is 318,772 square miles.

Optimizing Learning

This activity optimizes relevance, value, and authenticity.

Activity 1.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining the area of composite figures, assign Skills Practice Set B for this lesson.

MODULE 5 • TOPIC 2 • LESSON 1 955

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- Have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- See the Facilitating Productive Struggle section in the Course and Implementation Guide for additional guidance on supporting students through problem-solving activities.

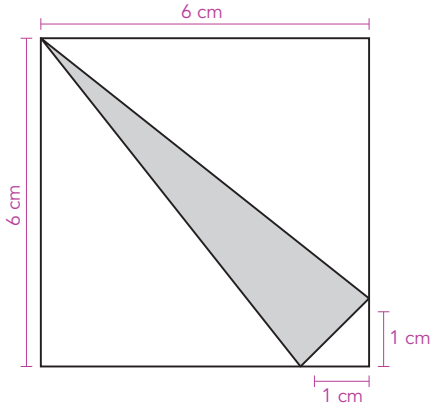
Talk the Talk

PROBLEM SOLVING

Use Your Powers of Mathematical Reasoning

Sometimes, a figure you are already familiar with is divided into smaller figures. You can use what you know about the areas of these figures to determine the area of a specific region.

1. Use a centimeter ruler to determine the measurements required to calculate the area of the shaded triangle inside the square. Then, determine the area. Explain your strategy.



Create a presentation of your solution strategy for the class.

Check students' measurements, as they will vary based on printing dimensions and scale.

Sample answer:

The area of the square is 36 square centimeters. The area of each of the two large triangles (not the target one) is $\frac{1}{2}(6)(5) = 15$ square centimeters, and the area of the smallest triangle (not the target one) is $\frac{1}{2}(1)(1) = 0.5$ square centimeters.

$36 - 15 \cdot 2 - 0.5 = 5.5$ square centimeters.

Optimizing Learning

This activity supports planning and strategy development.

956 MODULE 5 • TOPIC 2 • LESSON 1

Lesson 1 Assignment

Write

Define *composite figure* and draw an example.

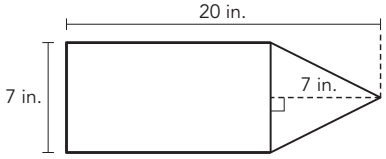
Remember

The area of a composite figure can be determined by decomposing it into familiar shapes and then adding the areas of those shapes together.

Practice

1. Calculate the area of the composite figure.

a.

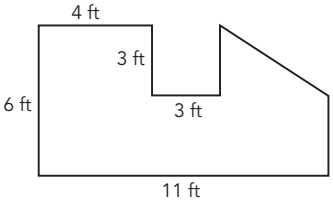


115.5 square inches

Rectangle: 91 sq in.

Triangle: 24.5 sq in.

b.



51 square feet

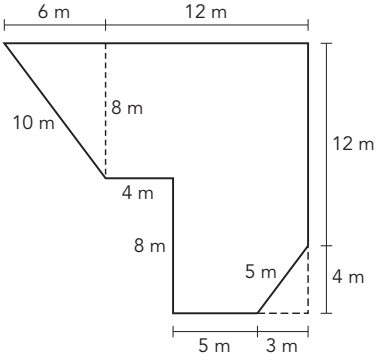
Sample answer:

Left rectangle: 24 sq ft

Middle rectangle: 9 sq ft

Trapezoid: 18 sq ft

2. A city wants to create a garden according to the plan below. Calculate the area of the garden.



178 square meters

Sample answer:

Top triangle: 24 sq m

Top rectangle: 96 sq m

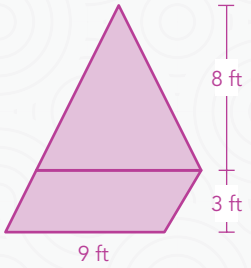
Middle rectangle: 32 sq m

Trapezoid: 26 sq m

Write

A composite figure is a figure that is made up of more than one geometric figure.

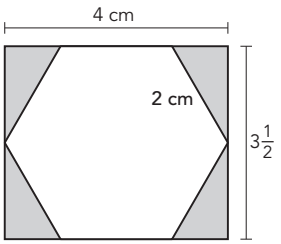
Sample drawing:



MODULE 5 • TOPIC 2 • LESSON 1 ASSIGNMENT 957

Lesson 1 Assignment

3. The figure shown is composed of a rectangle and a hexagon. The length of each side of the hexagon is 2 centimeters. Determine the area of the shaded region.



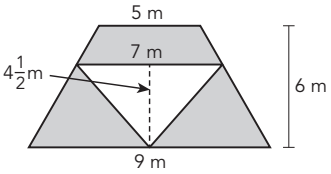
$3\frac{1}{2}$ square centimeters

Rectangle: 14 sq cm

Hexagon: 10.5 sq cm

14 − 10.5 = 3.5

4. The figure shown is composed of a triangle within a trapezoid. Determine the area of the shaded region.



$26\frac{1}{4}$ square meters

Prepare

Solve each equation.

1. $90 = 5x + x$

   $x = 15$

2. $x + 3x = 180$

   $x = 45$

3. $(180 - x) + (90 - x) = 210$

   $x = 30$

958 MODULE 5 • TOPIC 2 • LESSON 1 ASSIGNMENT

2 Total Surface Area of Prisms and Pyramids

LESSON OVERVIEW

Students apply mathematical and spatial reasoning to determine the surface areas of prisms and pyramids using nets, drawings, and measurements. Students solve a variety of surface area problems and distinguish between volume and surface area measurements.

MATERIALS

Scissors
Tape
Rulers
Problem-Solving Model Graphic Organizer

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Numbers and Operations

(3) The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions.
The student is expected to:

7.3A add, subtract, multiply, and divide rational numbers fluently.

Expressions, Equations, and Relationships

(9) The student applies mathematical process standards to solve geometric problems.
The student is expected to:

7.9D solve problems involving the lateral and total surface areas of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net.

ELPS

(1) Learning Strategies
The student is expected to:

(A) use prior knowledge and experiences to understand meanings in English

(4) Reading
The student is expected to:

(F) use visual and contextual support and support from peers and teachers to read grade-appropriate content area text, enhance and confirm understanding, and develop vocabulary, grasp of language structures, and background knowledge needed to comprehend increasingly challenging language.

(5) Writing
The student is expected to:

(C) spell familiar English words with increasing accuracy, and employ English spelling patterns and rules with increasing accuracy as more English is acquired.

ESSENTIAL IDEAS

- A *net* is a two-dimensional representation of a three-dimensional geometric figure.
- The surface area of a three-dimensional figure can be calculated by determining the areas of each face of the figure.

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LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Breaking Down a Cube 15–20 minutes

BUILD OFF INTUITION

Students decide whether a net can be folded into a cube. This activity engages students in visuospatial thinking about the two-dimensional faces of cubes, given what they have learned about rectangular prisms.

DEVELOP

Activity 2.1: Nets of Rectangular Prisms 20–25 minutes

PEER WORK ANALYSIS, MATHEMATICAL PROBLEM SOLVING

Students use the cube they built in the Getting Started to determine total surface area. Students calculate the total surface area from given nets, and they draw nets from given rectangular prisms to calculate the total surface area. Students use decimal and fraction multiplication to determine the surface areas of these figures.

DAY 2

Activity 2.2: Prisms and Pyramids 10–15 minutes

INVESTIGATION

Students complete a table to identify and compare the numbers of faces, edges, and vertices of prisms and pyramids. They identify various prisms and pyramids by their nets.

Activity 2.3: Nets of Other Solids 25–30 minutes

MATHEMATICAL PROBLEM SOLVING

Students cut out and build three-dimensional models of a triangular prism and a triangular pyramid from their nets and determine their surface areas. Students calculate the total surface area from given nets, and they draw nets from given solid figures to calculate the total surface area.

DAY 3

Activity 2.4: Surface Area Problems 25 minutes

REAL-WORLD PROBLEM SOLVING

Students solve real-world problems involving the surface areas of various candle molds.

DEMONSTRATE

Talk the Talk: Nothing but Net 15–20 minutes

EXIT TICKET APPLICATION

Students solve various surface area problems. They draw a net to calculate the total surface area of a prism, and they label a given net to calculate the total surface area of a pyramid.

Getting Started

ENGAGE

Breaking Down a Cube

Facilitation Notes

In this activity, students decide whether a net can be folded into a cube. This activity engages students in visuospatial thinking about the two-dimensional faces of cubes given what they have learned about rectangular prisms.

Ask a student to read the definition aloud and discuss as a class. Have students complete Question 1 to assemble the net of a cube. Allow students to work with a partner or in a group to answer Questions 2 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Are there certain nets that you could immediately tell would not create a cube? How could you tell? • How does the position of the squares help you determine whether it could be folded into a cube? • Would a row of six squares form a cube? Why or why not?

DIFFERENTIATION STRATEGIES

Just in Time Support

- Have students concentrate on one row of nets.

Access for All

- Provide enlargements that students can cut out to verify their responses.

Summary

STAMP THE LEARNING

A net is a two-dimensional representation of a three-dimensional geometric figure. A net can be assembled to create a model of a geometric solid, such as a cube. The number of faces, edges, and vertices are considered when identifying alternate nets for cubes.

ACTIVITY 2.1

Nets of Rectangular Prisms

DEVELOP

Facilitation Notes

In this activity, students use the cube they built in the Getting Started to determine the total surface area. Students calculate the total surface area from given nets, and they draw nets from given rectangular prisms to calculate the total surface area. Students use decimal and fraction multiplication to determine the surface areas of these figures.

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Ask a student to read the properties of a net and the definition of *surface area of a polyhedron* aloud and discuss as a class. Allow students to work with a partner or in a group to answer Questions 1 through 7. Circulate and ask students leading questions when needed. Share responses as a class.

DIFFERENTIATION STRATEGIES

Access for All

- Have students write the word *example* above Diego's strategy as an indicator that it is a correct solution to reference later. As students read through the strategy and think about the connections between steps, suggest they ask themselves:
  - Why is this method correct?
  - Have I used this method before?
- As students read the strategy, recommend they think about the connections between the steps, annotate what was done correctly on each step, and correct the error in the strategy. Support students by suggesting they ask themselves:
  - Where is the error?
  - Why is it an error?
  - How can I correct it?

Just in Time Support

- Provide a template for students to organize their process and use throughout this lesson.

Sketch of Rectangle with Dimensions	Number of Identical Rectangles	Area of One Rectangle	Total Area of All Identical Rectangles
Sum:			

Materials Needed: Colored Pencils

- Suggest students use three different colored pencils to color the three faces shown and label both dimensions for each face. Then, have them use color coding similar to the diagram for Question 5 to draw their net.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is the difference between area and surface area?
Seeing structure	• How can you use the activities where you filled a prism with cubes to help you remember what units to use?

Probing	• Why did you use multiplication rather than addition to calculate the surface area?
Seeing structure	• Will your method work for other rectangular prisms that are not cubes? Explain why or why not.
Probing	• How did you identify both dimensions of each rectangle when only one dimension is labeled on each rectangle?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Nets can be drawn to represent three-dimensional figures, such as right rectangular prisms. The surface area of a polyhedron is the total area of all of its two-dimensional faces.

ACTIVITY 2.2 Prisms and Pyramids

Facilitation Notes

In this activity, students complete a table to identify and compare the numbers of faces, edges, and vertices of prisms and pyramids. Students identify various prisms and pyramids by their nets.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the information and definitions of *pyramid* and *slant height* aloud. Allow students to work with a partner or in a group to answer Questions 1 through 3. Share responses as a class.

COMMON MISCONCEPTION

Students may think all pyramids are square pyramids based on pyramids they have seen in their history books. They will analyze pyramids with bases other than squares in Question 1.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How can you tell whether a figure is a prism or a pyramid? • What's the difference between height and slant height?
Seeing structure	• Do you think the height or slant height is longer? Why? • Do you think that you will use the height or slant height to calculate surface area? Why? • If you know which polygon is the base of a prism, how can you determine the number of faces?
Probing	• Why can't you identify the figure in Question 3, part (b) as a rectangular prism?

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DIFFERENTIATION STRATEGIES

Just in Time Support

Materials Needed: Colored Pencils

- Have students color the faces of the solids to better visualize them before creating their nets. They can shade the bases of the prisms with the same color to see that they are identical and parallel to one another. Then, have them shade in the base of the pyramid to make the difference between prisms and pyramids explicit.

Challenge Opportunity

- Have students investigate the relationship among vertices, edges, and faces of a polyhedron and then check their findings by researching Euler's Formula, $V - E + F = 2$.

STAMP THE LEARNING

Summary

Nets can be drawn to represent prisms and pyramids with rectangular and non-rectangular bases. The characteristics of pyramids and prisms are identified.

ACTIVITY 2.3

Nets of Other Solids

Facilitation Notes

In this activity, students cut out and build three-dimensional models of a triangular prism and a triangular pyramid from their nets and determine their surface areas. They calculate total surface area when given a net. They draw nets to represent solid figures and calculate the total surface area.

Allow students to work with a partner or in a group to cut out and measure the nets of the triangular pyramid and the triangular prism. For the triangular pyramid, ensure that students measure each side length. In order to calculate the total surface area, students also need to measure the height of each triangle. Have them answer Questions 1 through 3 and share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How many different-sized polygons are in your net? • What formula did you use for the area of the triangles? • What units should you use for your final answer?
Probing	• Explain how you calculated the surface area. • Is there another way to draw this net? If so, what is it?
Gathering	• What values did you use for the base and height of the triangle? • What are the dimensions of each rectangle?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Have students draw the nets to scale for Question 3.

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

Nets can be drawn to represent three-dimensional figures, including triangular pyramids and triangular prisms. The surface area of a polyhedron is the total area of all of its two-dimensional faces.

STAMP THE LEARNING

ACTIVITY 2.4 Surface Area Problems

Facilitation Notes

In this activity, students solve real-world problems involving the surface areas of various candle molds.

To begin the Day 3 session, have a student read the Essential Question aloud.

Students identify four solids and calculate the surface area of each in a real-world scenario. Allow students to work with a partner or in a group to answer Questions 1 and 2. Share responses as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Have students complete the problem using one of the candle molds. Then, have them pair with another student who worked on the other mold and share their work.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Identify each solid figure. • How many faces are rectangles? Triangles? • What are the dimensions of each polygon? • Which face is considered the base in the solid?
Probing	• What else might the company take into consideration when pricing the candles?

Summary

A net is a two-dimensional drawing of a three-dimensional geometric figure. A net can be used to determine the total surface area of a 3-D figure, which is the sum of the areas of each of its faces.

STAMP THE LEARNING

CC BY NC

DEMONSTRATE

Talk the Talk

NOTHING BUT NET

Facilitation Notes

In this activity, students solve various surface area problems. They draw and label nets to calculate the total surface area of solid figures and explain how to determine the total surface area of a pyramid.

Students identify four solids and calculate the surface area of each in a real-world scenario. Allow students to work with a partner or in a group to answer Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What patterns do you see in the measurement provided and the dimensions of the rectangles? • Why were your calculations for this pyramid different than those for the other pyramids?
Reflecting and justifying	• What formula did you include in your explanation? • How did your explanation address what units to use in the final answer? • How did you explain what to do when you have more than one polygon with the same dimensions in the net?

DIFFERENTIATION STRATEGY

Challenge Opportunity

Have students investigate how their process compares to calculating 2(area of the base) + height(perimeter of the base).

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

Drawing and labeling a net of a three-dimensional figure can be used to model and solve problems about surface area.

2 Total Surface Area of Prisms and Pyramids

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Represent solid figures using two-dimensional nets made up of rectangles and triangles.
- Use nets of solid figures to determine the surface areas of the figures.
- Solve real-world and mathematical problems involving surface area.
- Fluently multiply and divide multi-digit decimals using standard algorithms.

NEW KEY TERMS

- net
- surface area
- pyramid
- slant height

You know how to determine how many cubic units fill a rectangular prism.

How can you calculate the number of square units it takes to cover the outside of a prism or pyramid?

Sample answer:

The surface area of a polyhedron is the sum of all the areas of the faces of the polyhedron.

MODULE 5 • TOPIC 2 • LESSON 2 959

Chunking the Activity

- Read and discuss the definition.
- Have students complete Question 1 individually.
- Check in and share.
- Group students to complete Questions 2–4.
- Share and summarize.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

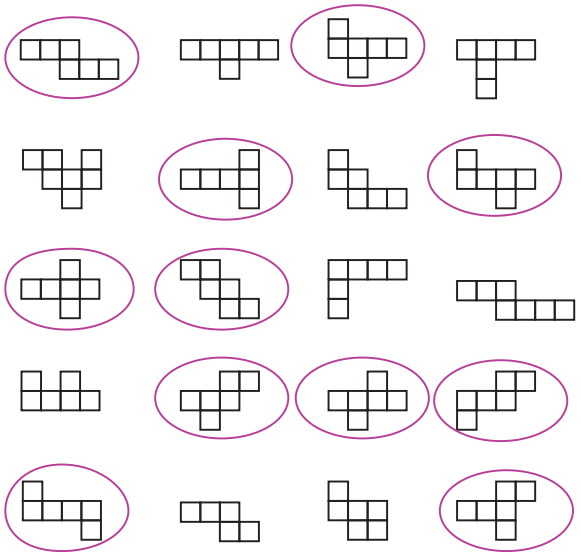
Getting Started

Breaking Down a Cube

A **net** is a two-dimensional representation of a three-dimensional geometric figure. A net is cut out, folded, and taped to create a model of a geometric solid.

1. Cut, fold, and tape the cube net found at the end of the lesson.
   Check students' models.

2. Are there other nets that form a cube? Circle the 11 cutouts that can form a cube.



3. How did you determine which are nets of cubes?
   Answers will vary.

4. What do all of the nets for a cube have in common? Consider the number of faces, edges, and vertices in your explanation.
   Answers will vary.

960 MODULE 5 • TOPIC 2 • LESSON 2

ACTIVITY 2.1

Nets of Rectangular Prisms

A net has all these properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn such that they share common edges.

The **surface area** of a polyhedron is the total area of all its two-dimensional faces.

Consider the cube you created.

1. How is the area of a face of a cube measured? Analyze the two responses and explain why Harper is incorrect in her reasoning.

Harper
This is a 3D figure, which means that its measurements are cubic units.

Diego
Surface area is still measuring area, which is always measured in square units.

Harper is not thinking about the faces of the three-dimensional figure. She may be thinking about the volume of a three-dimensional figure.

Chunking the Activity

- Read and discuss the properties and definition.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP

For all proficiency levels

Support students in distinguishing between the terms *surface area* and *volume* as they relate to three-dimensional figures. Have them relate the concepts to familiar situations. For example, the amount of wrapping paper it takes to wrap a present is an example of surface area, and the amount of orange juice that can fit in a jug is an example of volume. Ask students to generate their own examples.

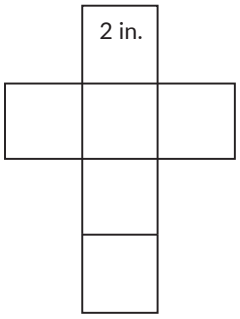
MODULE 5 • TOPIC 2 • LESSON 2 961

2. Describe a strategy that you can use to determine the surface area of a cube.

Sample answer:
I can find the area of one face by squaring the side length and then multiply that area by 6.

3. Use a ruler to determine the surface area of the cube you created.

Check students' measurements, as they will vary based on printing dimensions and scale. The surface are should be close to 6 square inches.

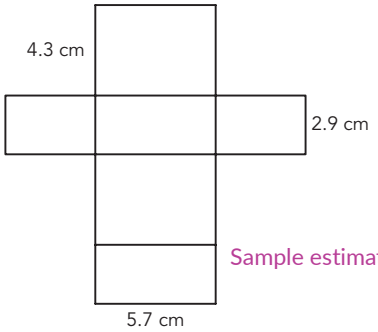


4. Consider the cube net shown. Calculate the surface area.

24 square inches

5. Let's consider a different rectangular prism.

a. Use the net to estimate the surface area of the right rectangular prism.



Sample estimate: 108 square cm

Review the definitions of **consider, describe, explain your calculation,** and related phrases in the Academic Glossary.

b. Calculate the surface area of the right rectangular prism. Explain your calculation.

$2(5.7 \cdot 2.9) + 2(5.7 \cdot 4.3) + 2(4.3 \cdot 2.9) = 107.02$ square cm

Sample explanation:

I can think about the rectangular prism as a box and find the area of each face.

Bottom and top: $2 \cdot 5.7 \cdot 2.9 = 33.06$

Left and right sides: $2 \cdot 4.3 \cdot 2.9 = 24.94$

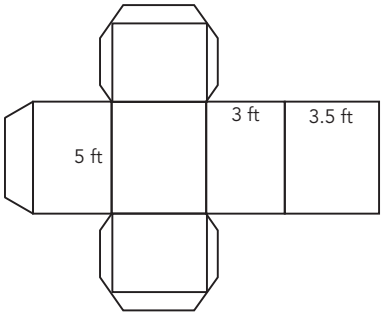
Front and back sides: $2 \cdot 5.7 \cdot 4.3 = 49.02$

Surface area: $33.06 + 24.94 + 49.02 = 107.02$ sq cm

962 MODULE 5 • TOPIC 2 • LESSON 2

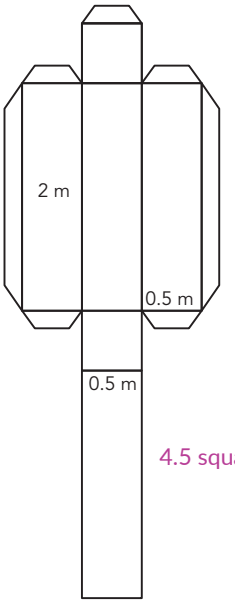
6. Calculate the surface area of the solid figure represented by each net.

a.



86 square feet

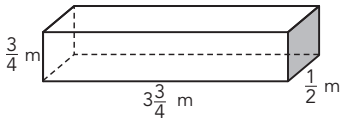
b.

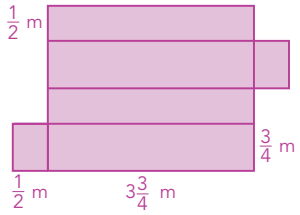


4.5 square meters

7. Draw a net to represent each solid figure. Label each net with measurements and then calculate the surface area of the solid figure.

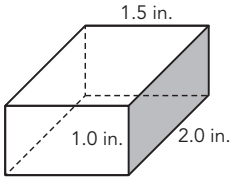
a.

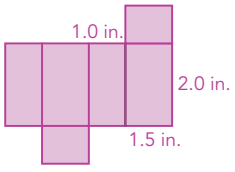




$10\frac{1}{8}$ square meters

b.





13 square inches

Question 7 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with sketching nets, assign Skills Practice Set A for this lesson.

MODULE 5 • TOPIC 2 • LESSON 2 963

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the information and definitions.
- Group students to complete the activity.
- Share and summarize.

STAMP THE LEARNING

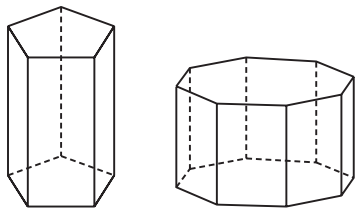
The definitions provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Question 1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with analyzing figures, assign Skills Practice Set B for this lesson.

ACTIVITY 2.2

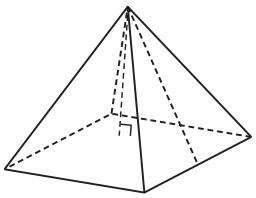
Prisms and Pyramids

The base of a prism does not have to be rectangular. The base of a prism can be a triangle, a pentagon, a hexagon, and so on.



A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The vertex of a pyramid is the point at which all the triangular faces intersect.

A **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint of an edge of the base.



1. Analyze the figures. Then, complete the table using the figures.

Figure *A*

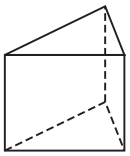


Figure *B*

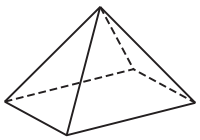


Figure *C*

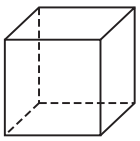


Figure *D*

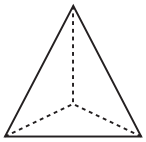


Figure	Is it a Prism or Pyramid?	Shape of Base	Number of Faces	Number of Vertices	Number of Edges
A	Prism	Triangle	5	6	9
B	Pyramid	Rectangle	5	5	8
C	Prism	Rectangle	6	8	12
D	Pyramid	Triangle	4	4	6

EB STUDENT TIP

For all proficiency levels

You can identify prisms and pyramids by their bases. Just add the name of the polygon base as an adjective, for example, a *triangular pyramid* and a *pentagonal prism*.

964 MODULE 5 • TOPIC 2 • LESSON 2

2. Write the names of Figures *A*, *B*, *C*, and *D* from your completed table.

Figure *A*: Triangular Prism

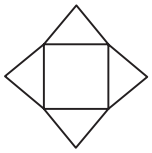
Figure *B*: Rectangular Pyramid

Figure *C*: Rectangular Prism

Figure *D*: Triangular Pyramid

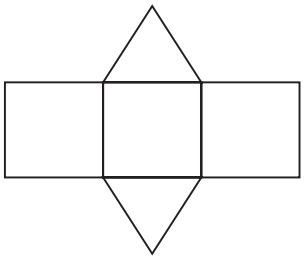
3. Label each net with the name of the solid figure it forms.

a.



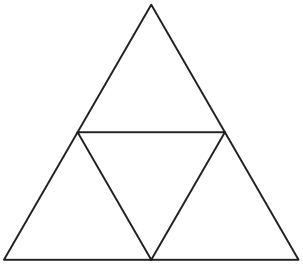
Square pyramid

b.



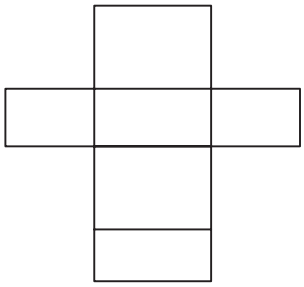
Triangular prism

c.



Triangular pyramid

d.



Rectangular prism

MODULE 5 • TOPIC 2 • LESSON 2 965

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

Question 2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining the surface area based on a net, assign Skills Practice Set C for this lesson.

Optimizing Learning

This activity uses multiple tools for construction and composition.

ACTIVITY 2.3

Nets of Other Solids

1. Locate the nets for the triangular prism and triangular pyramid at the end of the lesson.

   a. Measure the edge lengths of each net with a centimeter ruler. Label the lengths.

   Check students' measurements as they will vary based on printing dimensions and scale.

   b. Calculate the surface area of each solid figure.

   The surface area of the triangular prism is approximately 61.56 square centimeters, and the surface area of the triangular pyramid is approximately 81.9 square centimeters.

   Note: These values are based on the nets in the student edition, when printed at 100% scale on $8\frac{1}{2}$ by 11 in. paper.

   c. Cut out, fold, and tape each net.

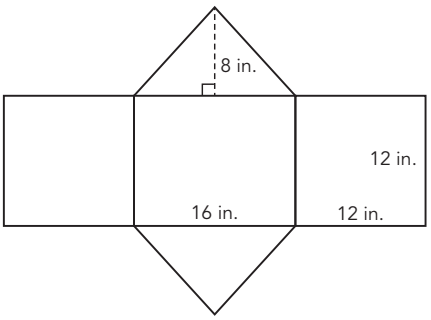
   Check students' models.

   d. Name each solid figure.

   Check that students correctly identify the triangular prism and the triangular pyramid.

2. Calculate the surface area of the solid figure represented by each net.

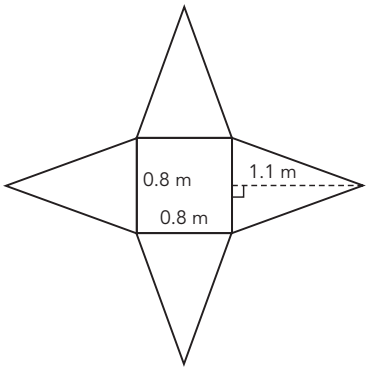
   a.



608 square inches

966 MODULE 5 • TOPIC 2 • LESSON 2

b.

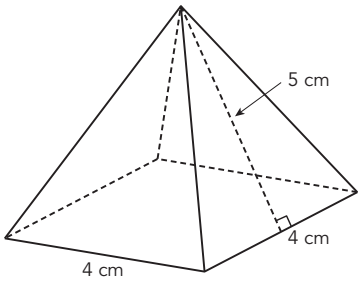


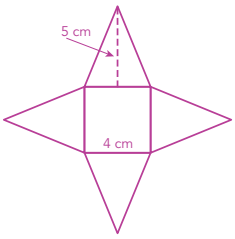
2.4 square meters

3. Draw a net to represent each solid figure. Label each net with measurements and then calculate the surface area of the solid figure.

Review the definition of **represent** in the Academic Glossary.

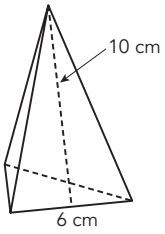
a.

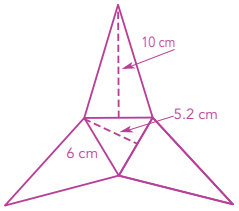




56 square centimeters

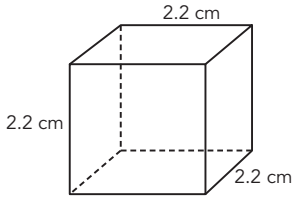
b. The slant heights are all equal. The height of the base is 5.2 cm.

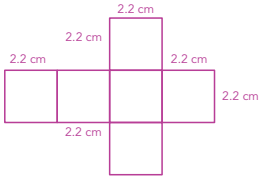




105.6 square centimeters

c.





29.04 square centimeters

Question 3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining the surface area of a figure, assign Skills Practice Set D for this lesson.

MODULE 5 • TOPIC 2 • LESSON 2 967

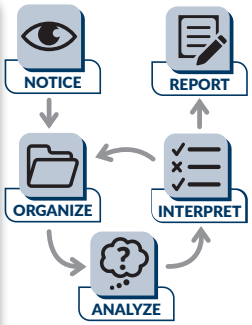
Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Read and discuss the scenario.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Question 2, have students work individually or with a partner to complete the graphic organizer.

PROBLEM SOLVING

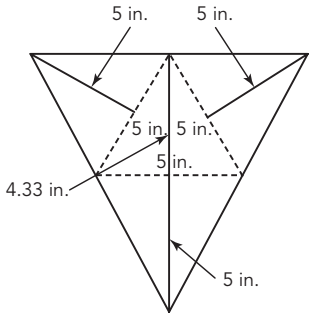


ACTIVITY 2.4

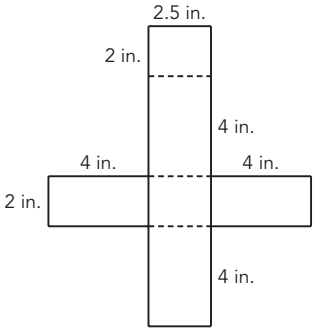
Surface Area Problems

A company produces candles in a variety of shapes. To produce each candle, the company first creates a mold, then pours hot wax into the mold. When the hot wax cools and solidifies, the mold is removed.

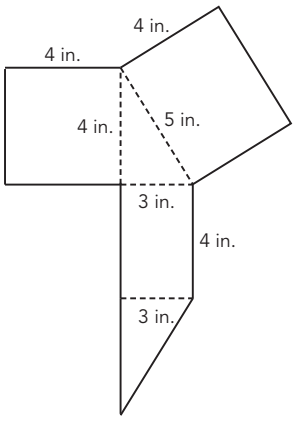
Candle Mold *A*



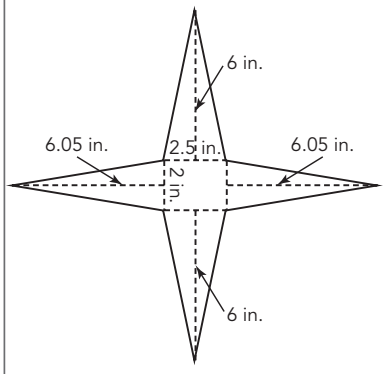
Candle Mold *B*



Candle Mold *C*



Candle Mold *D*



968 MODULE 5 • TOPIC 2 • LESSON 2

1. Classify the shape of each candle, based on the candle mold.

   a. Candle Mold *A*
   Triangular pyramid

   b. Candle Mold *B*
   Rectangular prism

   c. Candle Mold *C*
   Triangular prism

   d. Candle Mold *D*
   Rectangular pyramid

2. Use each candle mold to answer each question.

   a. Calculate the surface area of each candle.
   Candle *A*: 48.325 square inches
   Candle *B*: 46 square inches
   Candle *C*: 54 square inches
   Candle *D*: 32.1 square inches.

   b. How could the company use the surface area of the candles to determine how to price each candle?
   Sample answer:
   They can multiply the surface area of a candle by an approximate cost per square inch. This will give the company an idea how much it costs them to make the candle, then they can set a selling price.

Activity 2.4 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with applying the surface area of a figure, assign Skills Practice Set E for this lesson.

MODULE 5 • TOPIC 2 • LESSON 2 969

Chunking the Activity

- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

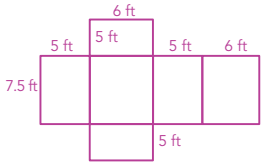
Talk the Talk

Nothing but Net

1. A rectangular prism has a height of 6 feet, a length of 7.5 feet, and a width of 5 feet.

   a. Draw a net of the rectangular prism and label its measurements.

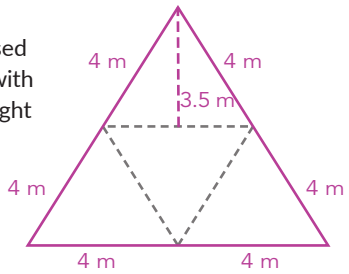
   Sample drawing:



   b. Calculate the surface area of the prism.

   225 square feet

2. Consider the net of the triangular pyramid shown. The net is composed of four equilateral triangles, each with a side length of 4 meters and a height of approximately 3.5 meters.



   a. Label the pyramid with its measurements.

   b. Calculate the surface area of the pyramid.

   28 square meters

Ask Yourself . . .
Did you justify your mathematical reasoning?

3. Explain in your own words how to determine the surface area of a pyramid.

   Sample answer:

   I would make a net of the pyramid and label the dimensions of each side. Then, I would find the area of each face and add them together.

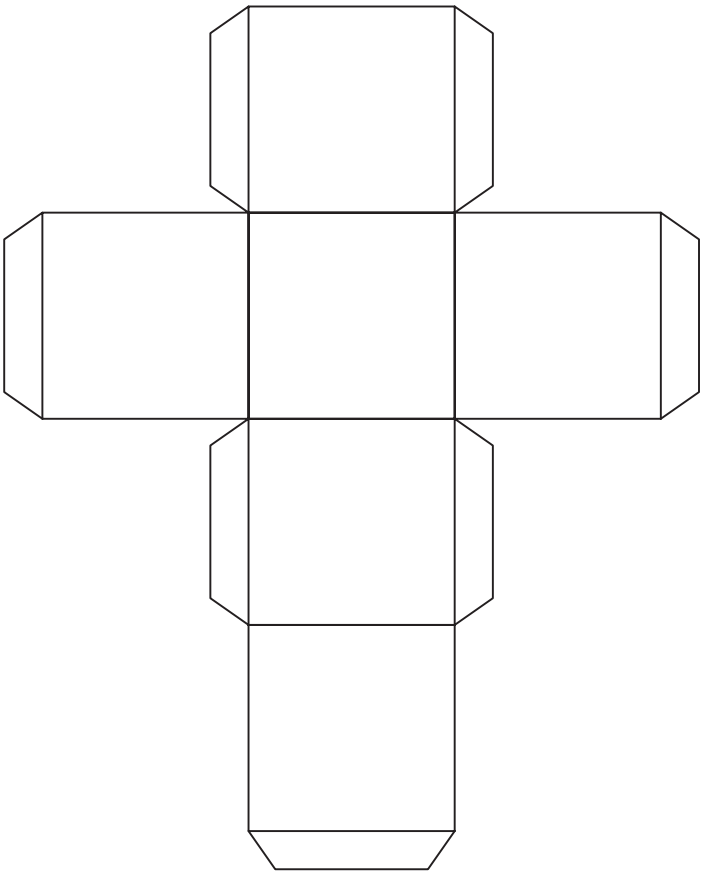
EB STUDENT TIP

For all proficiency levels

The last portion of this section asks students to explain. Have the students engage in a partner interview, where one person begins the explanation and the other asks clarifying questions. When they are satisfied with the explanation, the two students should switch roles. This exercise not only helps to solidify the students' mathematical thinking but also allows emergent bilingual students an opportunity to speak using proper mathematical terms.

970 MODULE 5 • TOPIC 2 • LESSON 2

Cube Net



MODULE 5 • TOPIC 2 • LESSON 2 971

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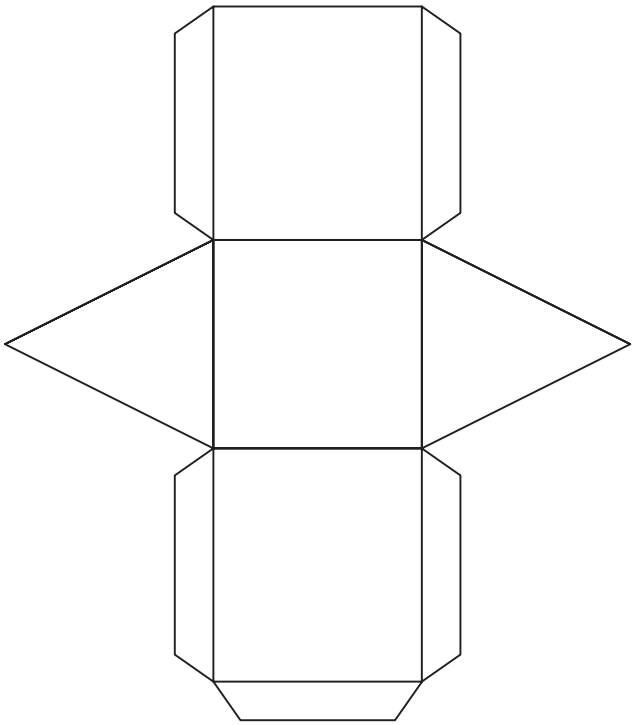
So you can cut out the net on the other side.

972 MODULE 5 • TOPIC 2 • LESSON 2

Triangular Prism Net



MODULE 5 • TOPIC 2 • LESSON 2 973

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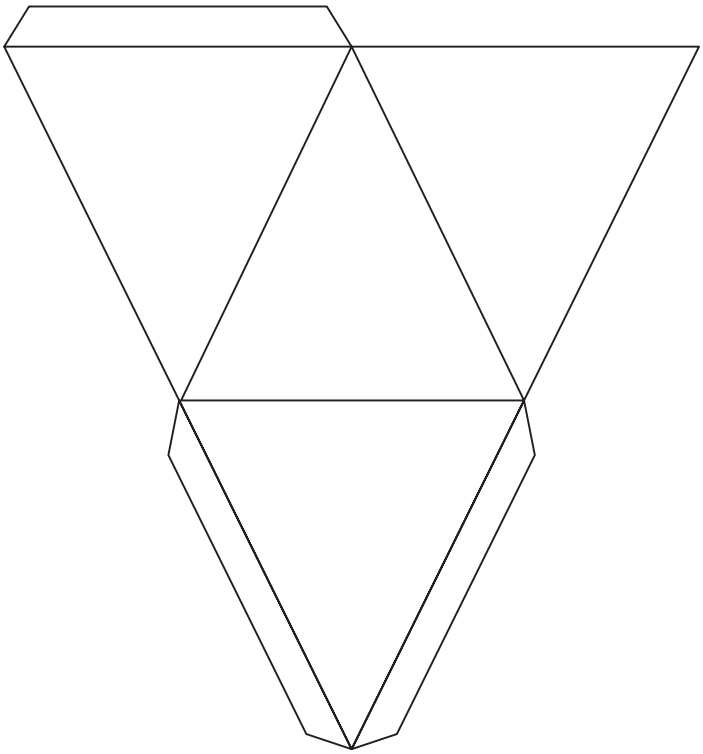
So you can cut out the net on the other side.

974 MODULE 5 • TOPIC 2 • LESSON 2

Triangular Pyramid Net



MODULE 5 • TOPIC 2 • LESSON 2 975

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So you can cut out the net on the other side.

976 MODULE 5 • TOPIC 2 • LESSON 2

Lesson 2 Assignment

Write

Match each definition to its corresponding term.

1. The amount of space occupied by an object
2. A regular polyhedron whose six faces are congruent squares
3. The total area of the two-dimensional surfaces that make up a three-dimensional object
4. The distance across a circle through its center
5. A geometric solid that is made up of polygons
6. The intersection of two faces of a polyhedron
7. A closed figure formed by three or more line segments
8. A two-dimensional representation of a three-dimensional geometric figure
9. A cube that is one unit in length, one unit in width, and one unit in height
10. A bounded three-dimensional geometric figure
11. A portion of a line that includes two points and all the points in between those two points
12. The polygons that make up a polyhedron
13. A location in space
14. The point where the edges of a polyhedron meet

a. *polygon*

b. *polyhedron*

c. *cube*

d. *unit cube*

e. *surface area*

f. *volume*

g. *point*

h. *line segment*

i. *geometric solid*

j. *faces of a polyhedron*

k. *edge of a polyhedron*

l. *vertex of a polyhedron*

m. *net*

n. *diameter*

Remember

The surface area of a polyhedron is the sum of all the areas of the faces of the polyhedron.

Write

1. f. volume
2. c. cube
3. e. surface area
4. n. diameter
5. b. polyhedron
6. k. edge of a polyhedron
7. a. polygon
8. m. net
9. d. unit cube
10. i. geometric solid
11. h. line segment
12. j. faces of a polyhedron
13. g. point
14. l. vertex of a polyhedron

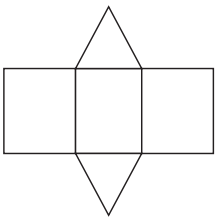
MODULE 5 • TOPIC 2 • LESSON 2 ASSIGNMENT 977

Lesson 2 Assignment

Practice

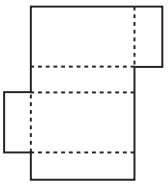
1. Name the solid figure formed by each net.

a.



Triangular prism

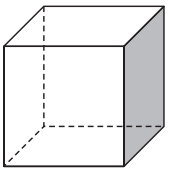
b.



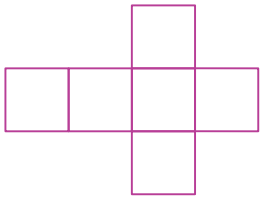
Rectangular prism

2. Draw a net that will form each solid figure.

a.



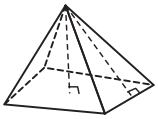
Check students' drawings



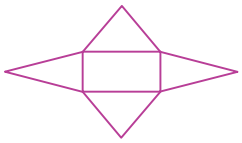
978 MODULE 5 • TOPIC 2 • LESSON 2 ASSIGNMENT

Lesson 2 Assignment

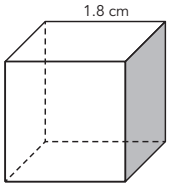
b.



Check students' drawings

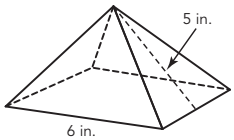


3. Calculate the surface area of the cube.



19.44 square centimeters

4. The pyramid shown has a square base and congruent triangular faces. Calculate the surface area of the pyramid.



96 square inches

5. Estimate and calculate the surface area of a rectangular prism with a length of 9.06 ft, a width of 4.11 ft, and a height of 6.2 ft.

Possible estimate: 228 square feet

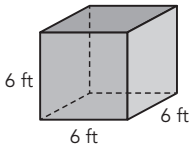
Actual answer: 237.7812 square feet

MODULE 5 • TOPIC 2 • LESSON 2 ASSIGNMENT 979

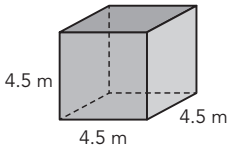
Lesson 2 Assignment

Prepare

Calculate the volume of each cube.

1. 

216 cubic feet

2. 

91.125 cubic meters

3. A cube that has a side length of 150 millimeters

3,375,000 cubic millimeters

980 MODULE 5 • TOPIC 2 • LESSON 2 ASSIGNMENT

3 Volume of Prisms and Pyramids

LESSON OVERVIEW

Students cut out given nets and assemble an open rectangular prism and an open rectangular pyramid. They compare the models and determine that the heights and bases are congruent. Students fill the models with birdseed to discover that the volume of the pyramid is one-third the volume of the prism and then write the formula for the volume of each. Students repeat the activity for a triangular prism and a triangular pyramid with congruent heights and bases. They then use the volume formulas to solve problems involving rectangular and triangular prisms and pyramids.

MATERIALS

Scissors
Tape
Birdseed or Rice
Paper
Rulers

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.
The student is expected to:

7.1B use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.

7.1F analyze mathematical relationships to connect and communicate mathematical ideas.

7.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to develop geometric relationships with volume.
The student is expected to:

7.8A model the relationship between the volume of a rectangular prism and a rectangular pyramid with both congruent bases and heights, and connect this relationship to the formulas.

7.8B explain verbally and symbolically the relationship between the volume of a triangular prism and a triangular pyramid having both congruent bases and heights, and connect that relationship to the formulas.

(TEKS continued on next page)

ELPS

(1) Learning Strategies
The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment

(5) Writing
The student is expected to:

(B) write using newly acquired basic vocabulary and content-based grade-level vocabulary

(D) edit writing for standard grammar and usage, including subject-verb agreement, pronoun agreement, and appropriate verb tenses commensurate with grade-level expectations as more English is acquired.

ESSENTIAL IDEAS

- A pyramid is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The triangular faces are called *lateral faces*.
- A rectangular pyramid is a pyramid that has a rectangle as its base.
- A triangular pyramid is a pyramid that has a triangle as its base.

CC BY NC

GRADE 7 TEKS *(continued)*

(9) The student applies mathematical process standards to solve geometric problems.

The student is expected to:

7.9A solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: You're a Real Cut-Up 15–20 minutes

CONNECT TO PRIOR LEARNING

Students cut out nets and tape together the faces to form part of a rectangular prism and part of a rectangular pyramid. Each of these figures is missing a face so that students can use them in the next activity to investigate the volume of prisms and pyramids. They then compare the heights and bases and predict which figure has the greater volume.

DEVELOP

Activity 3.1: Rectangular Prism Volume vs. Rectangular Pyramid Volume 20–25 minutes

INVESTIGATION

Students explore the volume of a rectangular pyramid and how it relates to the volume of a rectangular prism with the same base and same height. They discover that the volume of the pyramid is one-third the volume of the prism.

DAY 2

Activity 3.2: Triangular Prism Volume vs. Triangular Pyramid Volume 15–20 minutes

INVESTIGATION

Students explore the volume of a triangular pyramid and how it relates to the volume of a triangular prism with the same base and same height. The nets for a triangular prism and a triangular pyramid with the same base and height are provided. Students cut them out, tape them together, and fill them with birdseed to discover that the volume of the pyramid is one-third the volume of the prism.

Activity 3.3: Applying the Volume of Pyramids 20–25 minutes

REAL-WORLD PROBLEM SOLVING, MATHEMATICAL PROBLEM SOLVING

Students use the volume formula that they wrote in the previous activity, $V_{pyramid} = \frac{1}{3}Bh$, where B is the area of the base and h is the height, to solve problems involving triangular and rectangular pyramids.

DEMONSTRATE

Talk the Talk: The Sun and the Moon 5 minutes

EXIT TICKET PROCEDURES

Students apply the volume of a pyramid formula to solve a real-world problem.

CC BY NC

Getting Started

ENGAGE

You're a Real Cut-Up

Facilitation Notes

In this activity, students cut out nets and tape the faces together to form part of a rectangular prism and part of a rectangular pyramid. Each of these figures is missing a face so that students can use them in the next activity to investigate the volume of prisms and pyramids. Students compare the heights and bases and predict which figure has the greater volume.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• Is it a prism or a pyramid? How do you know? • What is the shape of the base?

DIFFERENTIATION STRATEGY

Access for All

Have pairs of students share a set of nets instead of having each student cut out nets.

STAMP THE LEARNING

Summary

The volume of a rectangular prism appears to be larger than the volume of a rectangular pyramid with the same base and height.

ACTIVITY 3.1 Rectangular Prism Volume vs. Rectangular Pyramid Volume

DEVELOP

Facilitation Notes

In this activity, students explore the volume of a rectangular pyramid and how it relates to the volume of a rectangular prism with the same base and height. The nets for a rectangular prism and a rectangular pyramid with the same base and height are provided in the Getting Started. Students will discover that the volume of the pyramid is one-third the volume of the prism.

DIFFERENTIATION STRATEGY

Access for All

Have half of the class complete Activity 3.1 and the other half complete Activity 3.2. Share the results of both activities as a class.

Students need birdseed or rice to complete this activity.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• If the contents of three pyramids fit into one prism, what fraction represents the pyramid's volume compared to the prism's volume?
Reflecting and justifying	• How are the nets you created different from the nets that you were provided? • How do your nets compare to the nets your classmates provided?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

When the areas of the bases are the same and the heights are the same, the volume of a rectangular pyramid is one-third the volume of a rectangular prism.

ACTIVITY 3.2 Triangular Prism Volume vs. Triangular Pyramid Volume

Facilitation Notes

In this activity, students explore the volume of a triangular pyramid and how it relates to the volume of a triangular prism with the same base and height. The nets for a triangular prism and a triangular pyramid with the same base and height are provided. Students cut them out, tape them together, and fill them with birdseed to discover that the volume of the pyramid is one-third the volume of the prism. Note that students need to leave one side of their figure untaped to fill it with birdseed.

To begin the Day 2 session, have a student read the Essential Question aloud.

CC BY NC

Students need birdseed or rice to complete this activity.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How is your comparison related to the number of times you poured from the pyramid into the prism? • How did you ensure that your triangular prism and triangular pyramid have the same height?
Seeing structure	• Do you think this formula is true for any prism and pyramid with the same height, base shape, and area of the base? Explain your thinking.

DIFFERENTIATION STRATEGY

Just in Time Support

For students who have difficulty visualizing the nets of a triangular prism and triangular pyramid, have an extra set of uncut nets that they can reference.

STAMP THE LEARNING

Summary

When the areas of the bases are the same and the heights are the same, the volume of a triangular pyramid is one-third the volume of a triangular prism.

ACTIVITY 3.3

Applying the Volume of Pyramids

Facilitation Notes

In this activity, students use the volume formula that they wrote in the previous activity, $V_{pyramid} = \frac{1}{3}Bh$, where B is the area of the base and h is the height, to solve problems involving triangular and rectangular pyramids.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the area of the base of the Walter Pyramid? • How did you determine the area of the base of the Pyramid Arena?
Gathering	• What unit is used to describe the area of the base?
Seeing structure	• Can you determine which pyramid is larger without calculating the volume? Explain your thinking.

DIFFERENTIATION STRATEGY

Access for All

Materials Needed: Calculators

Consider allowing students to use a calculator to complete this activity. The purpose of the activity to have students experience real-world applications of prisms and pyramids.

Optimizing Learning

The differentiation strategy varies demands and resources to optimize challenge.

Summary

The formula to calculate the volume of a triangular or rectangular pyramid is $V_{pyramid} = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the solid.

STAMP THE LEARNING

DEMONSTRATE

Talk the Talk

THE SUN AND THE MOON

Facilitation Notes

In this activity, students apply the volume of a pyramid formula to solve a real-world problem.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How does the height of each pyramid compare to the number of stories in a building?
Probing	• Are these two pyramids similar to one another? In other words, do they have the same height to length of base ratio? Explain your thinking.

DIFFERENTIATION STRATEGIES

Access for All

Materials Needed: Calculators

- Consider allowing students to use a calculator to complete this activity. The purpose of the activity to have students experience real-world applications of prisms and pyramids.
- Pair students and have each member of the pair focus on calculating the volume of a different pyramid. Have students discuss their strategies to determine the volume.

Have students read and answer the Essential Question on the lesson opener page.

Summary

The volume formula, $V_{pyramid} = \frac{1}{3}Bh$, where B is the area of the base and h is the height, is used to solve problems involving triangular and rectangular pyramids.

STAMP THE LEARNING

3 Volume of Prisms and Pyramids

Setting the Stage

- Communicate the objectives.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Determine the volume of pyramids.
- Determine how prisms.
- Solve mathematical and real-world problems involving volumes of pyramids and objects composed of solid figures.

You have learned about the volume of rectangular prisms.

How does the volume of pyramids compare?

Sample answer:

The volume of a pyramid is one-third the volume of a prism with the same base and height.

EB STUDENT TIP

For all proficiency levels

Encourage emergent bilingual students to use pointing or simple phrases like *taller* or *wider* to describe their observations about the comparison between the height and base length of different pyramids. Provide sentence frames to scaffold precise mathematical language: "The pyramid with a height of _____ and a base length of _____ has a height-to-base length ratio of _____, which means it is _____ (steeper/flatter) than the pyramid with a height of _____ and a base length of _____."

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Getting Started

You're a Real Cut-Up

Cut out Figures *A* and *B* at the end of the lesson and fold along the dashed lines to form solid figures. Tape the sides together so that the sides do not overlap.

1. Name each solid figure.
   Figure *A* is part of a rectangular prism.
   Figure *B* is part of a rectangular pyramid.

2. Compare the bases of the figures. What do you notice?
   The bases are the same size and shape.

3. Compare the heights of the figures. What do you notice?
   The heights are the same.

4. Which figure appears to have a greater volume? Explain your reasoning.
   Sample answer:
   Figure *A* appears to have a greater volume. Both figures have the same base and height, but Figure *B* has slanted sides that meet at a vertex, which appear to give the figure less volume.

5. Use a ruler to measure the dimensions the of the rectangular prism. Then, calculate the volume of the rectangular prism.
   Check students' measurements as they will vary based on printing dimensions and scale.

982 MODULE 5 • TOPIC 2 • LESSON 3

ACTIVITY 3.1

Rectangular Prism Volume vs. Rectangular Pyramid Volume

Use the pyramid you built in the previous activity. Fill the pyramid with birdseed. Then, dump the birdseed from the pyramid into the prism. Repeat this process until the prism is full.

1. How many times did you fill the pyramid?

   3 times

2. Compare the volume of a rectangular pyramid to the volume of a rectangular prism, when the bases are congruent polygons and the heights are of equal measure.

   The volume of a rectangular pyramid is one-third the volume of the rectangular prism.

3. In the Getting Started, you determined the volume of the rectangular prism. Based on you exploration, approximate the volume of the pyramid based on the volume of the prism.

   Check students' answer as they will vary based on printing dimensions and scale. The volume of the pyramid should be approximately $\frac{1}{3}$ the volume of the rectangular prism.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

MODULE 5 • TOPIC 2 • LESSON 3 983

STAMP THE LEARNING

Question 6 provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

4. Write an equation that shows the relationship between the volume of a rectangular prism and the volume of a rectangular pyramid that have congruent bases and heights of equal measure. Let V_{prism} represent the volume of the prism, and let $V_{pyramid}$ represent the volume of the pyramid.

   $\frac{1}{3} \cdot V_{prism} = V_{pyramid}$ or

   $V_{prism} = 3 \cdot V_{pyramid}$

5. Use a separate piece of paper to create your own nets for a rectangular prism and a rectangular pyramid that have congruent bases and heights of equal measure. What do you notice about their volumes?

   Sample answer:

   The volume of the rectangular pyramid is one-third the volume of the rectangular prism.

6. Write formulas for both the volume of a rectangular prism and the volume of a rectangular pyramid. Use V for the volume, B for the area of the base, and h for the height.

   $V_{prism} =$ Bh

   $V_{pyramid} =$ $\frac{1}{3}Bh$

984 MODULE 5 • TOPIC 2 • LESSON 3

ACTIVITY 3.2

Triangular Prism Volume vs. Triangular Pyramid Volume

Cut out Figures *C* and *D* at the end of the lesson and fold along the dashed lines to form a triangular prism and a triangular pyramid. Tape the sides together so that the sides do not overlap. Leave one side of your figure untaped so that you can fill it with birdseed.

1. Fill the pyramid with birdseed. Then, dump the birdseed from the pyramid into the prism. Repeat this process until the prism is full. How many times did you fill the pyramid?

   3 times

2. Compare the volume of a triangular pyramid to the volume of a triangular prism, when the bases are congruent polygons and the heights are of equal measure.

   The volume of a triangular pyramid is one-third the volume of the triangular prism.

3. Write an equation that shows the relationship between the volume of a triangular prism and the volume of a triangular pyramid that have congruent bases and heights of equal measure. Let V_{prism} represent the volume of the prism, and let $V_{pyramid}$ represent the volume of the pyramid.

   $\frac{1}{3} \cdot V_{prism} = V_{pyramid}$ or $V_{prism} = 3 \cdot V_{pyramid}$

Ask Yourself . . .
What observations can you make?

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

MODULE 5 • TOPIC 2 • LESSON 3 985

STAMP THE LEARNING

Question 5 provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

4. Use a separate piece of paper to create your own nets for a triangular prism and triangular pyramid that have congruent bases and heights of equal measure. What do you notice about their volumes?

   Sample answer:

   The volume of the triangular pyramid is one-third the volume of the triangular prism.

5. Write formulas for the volume of a triangular prism and the volume of a triangular pyramid. Use V for the volume, B for the area of the base, and h for the height.

   $V_{prism} = Bh$

   $V_{pyramid} = \frac{1}{3}Bh$

6. Compare your formulas for the volume of a triangular prism and the volume of a triangular pyramid to the formulas for the volume of a rectangular prism and the volume of a rectangular pyramid. What do you notice?

   The formula for the volume of a triangular prism is the same as the formula for the volume of a rectangular prism when written in terms of the area of the base B. The formula for the volume of a triangular pyramid is the same as the formula for the volume of a rectangular pyramid when written in terms of the area of the base B.

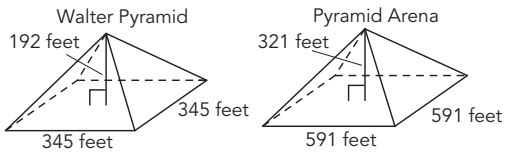
986 MODULE 5 • TOPIC 2 • LESSON 3

ACTIVITY 3.3

Applying the Volume of Pyramids and Prisms

Now, apply what you know about pyramid and prism volume to solve mathematical and real-world problems.

1. Models of the Walter Pyramid and the Pyramid Arena are shown. Calculate the volume of each pyramid. Which is larger?



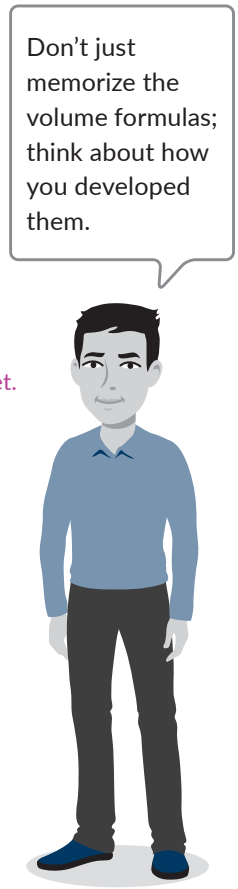
The volume of the Walter Pyramid is 7,617,600 cubic feet. The volume of the Pyramid Arena is 37,373,067 cubic feet. The Pyramid Arena is larger than the Walter Pyramid.

2. A pyramid designed by I.M. Pei, a well-known Chinese-American architect, sits in front of the Louvre art museum in Paris. This square pyramid has a base that has a side length of 115 feet and a height of about 70 feet. Calculate the volume of this pyramid.

The volume of the pyramid designed by I.M. Pei is $308{,}583\frac{1}{3}$ cubic feet.

3. A storage box is the shape of a rectangular prism. The box has a side length of 2.5 feet and a height of 3 feet. What is the volume of the storage box?

The volume of the storage box is 18.75 cubic feet.



Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Questions 2–5, have students work individually or with a partner to complete the graphic organizer.

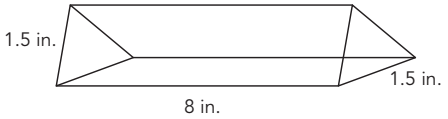
Optimizing Learning

Question 2 optimizes relevance, value, and authenticity.

MODULE 5 • TOPIC 2 • LESSON 3 987

Activity 1.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days.
To provide additional practice with comparing the volumes of pyramids and prisms with the same base and height, assign Skills Practice Set A for this lesson.
To provide additional practice with comparing the volumes of prisms with the same base but different heights, assign Skills Practice Set B for this lesson.
To provide additional practice with determining the volumes of prisms and pyramids, assign Skills Practice Set C for this lesson.

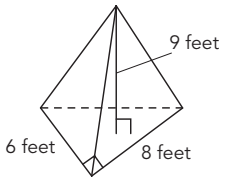
4. A type of candy comes in a box the shape of a triangular prism. Determine the volume of the box when the height is 2 in., the width is 1.5 in., and the length is 8 in.



The volume of the box is 12 cubic inches.

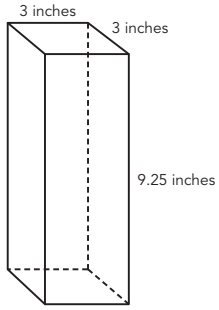
5. Calculate the volume of each prism or pyramid.

a.



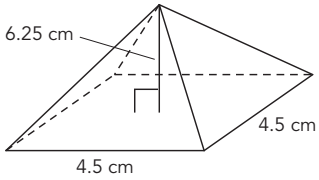
72 cubic feet

b.



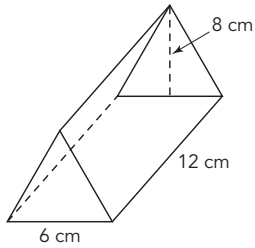
83.25 cubic inches

c.



42.1875 cubic centimeters

d.



288 cubic centimeters

6. The volume of a rectangular pyramid is 520 cubic meters. The area of the base of the pyramid is 156 square meters. Determine the height of the pyramid.

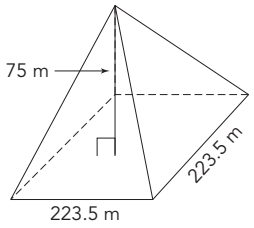
10 meters

988 MODULE 5 • TOPIC 2 • LESSON 3

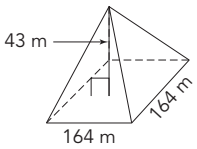
Talk the Talk

The Sun and the Moon

Most people know about Egypt's Great Pyramids at Giza. However, there are a number of pyramids located in the Americas. The Mayan, Aztec, and Inca civilizations all built pyramids to bury their kings. Two of these pyramids are the Pyramid of the Sun and the Pyramid of the Moon, located in Teotihuacán, Mexico.



Pyramid of the Sun



Pyramid of the Moon

1. Determine the volume of each pyramid. Show your work.

   Check students' work.

   Pyramid of the Sun: 1,248,806.25 cubic meters;

   Pyramid of the Moon: 385,509. $\overline{3}$ cubic meters

Ask Yourself . . .
Did you make a plan to solve the problem?

Chunking the Activity

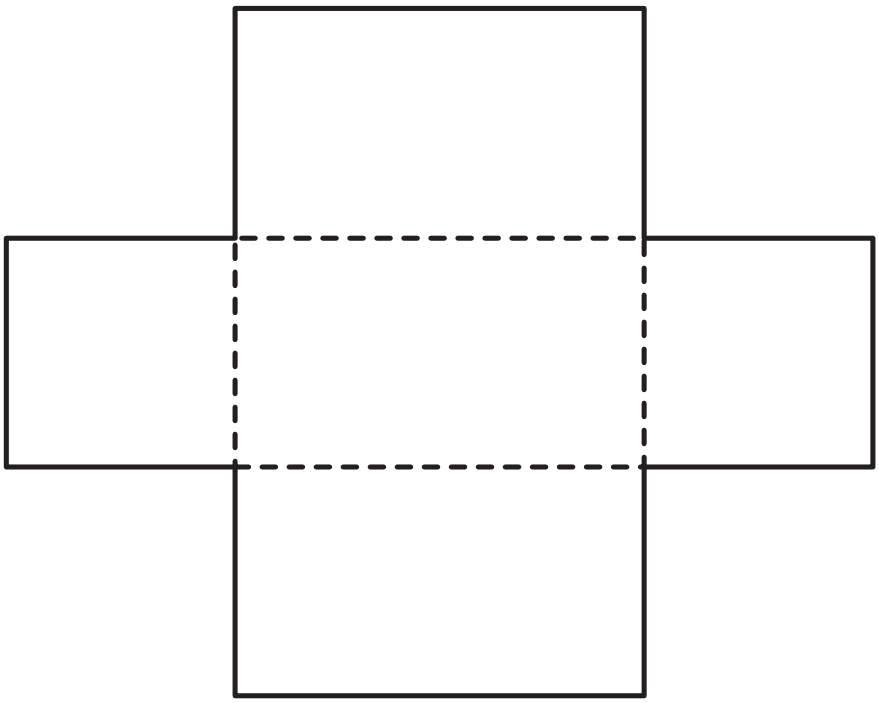
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize
- Have students answer the Essential Question on the lesson opener.

Talk the Talk presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with determining the volume of pyramids, assign Skills Practice Set D for this lesson.

MODULE 5 • TOPIC 2 • LESSON 3 989

990 MODULE 5 • TOPIC 2 • LESSON 3

Figure *A*



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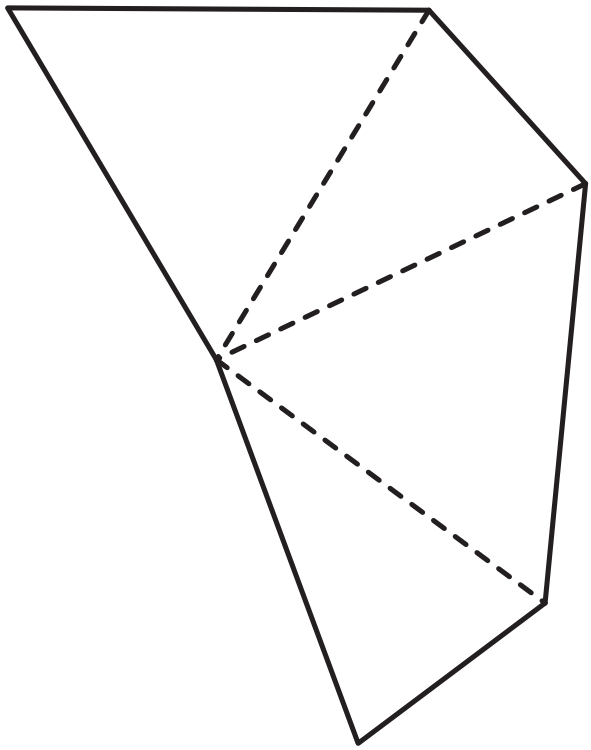
992 MODULE 5 • TOPIC 2 • LESSON 3

Figure *B*



MODULE 5 • TOPIC 2 • LESSON 3 993

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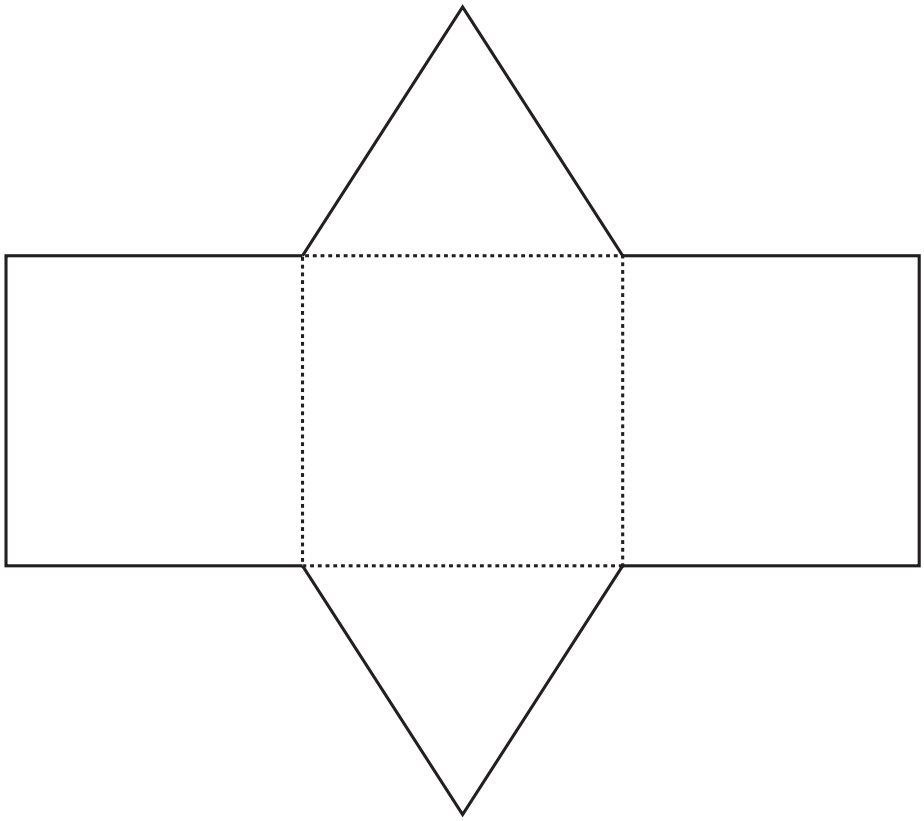
994 MODULE 5 • TOPIC 2 • LESSON 3

Figure C



MODULE 5 • TOPIC 2 • LESSON 3 995

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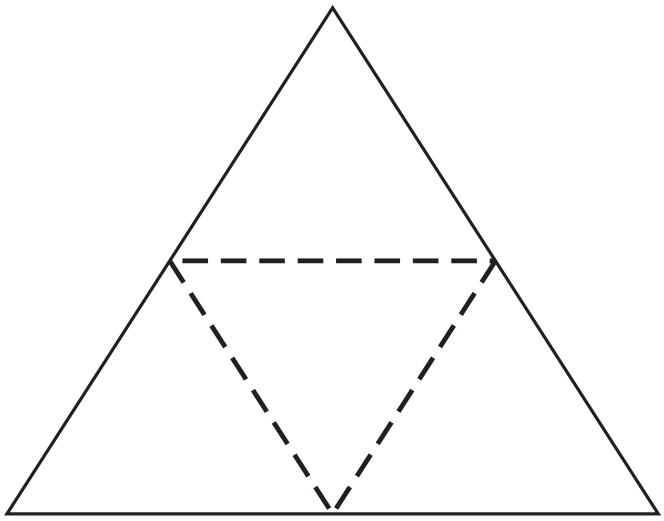
996 MODULE 5 • TOPIC 2 • LESSON 3

Figure *D*



MODULE 5 • TOPIC 2 • LESSON 3 997

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998 MODULE 5 • TOPIC 2 • LESSON 3

Lesson 3 Assignment

Write

Describe the differences between pyramids and prisms. Use examples to help illustrate your description.

Remember

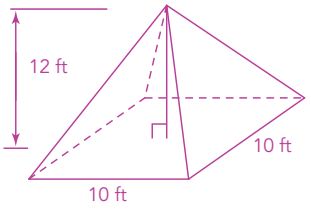
The volume of a pyramid is one-third the volume of a prism with the same base and height.

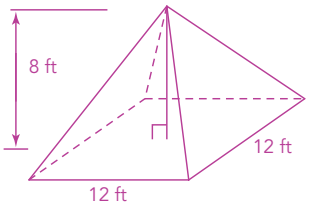
$$V = \frac{1}{3}Bh$$

Practice

Pyramid tents were popular for a time in the 19th century. Although their popularity declined during the 20th century, they have recently begun to regain popularity. The design is ideal for shaping canvas, and it only requires one pole and some stakes to secure it. Joe wants to make a right square pyramid tent and is considering two different sizes. He will either make one with a base that is 10 feet by 10 feet and has a height of 12 feet, or he will make one with a base that is 12 feet by 12 feet and has a height of 8 feet.

1. Sketch the two pyramid designs Joe is considering and label them with the given measurements.





2. How can you determine which pyramid tent will have the most interior space?

   Calculate the volume of each pyramid tent to determine how much interior space each one will have.

Write

Sample answer:

Prisms have two congruent bases and rectangular sides. Pyramids have one base and triangular sides that meet at a vertex. Check students' examples.

MODULE 5 • TOPIC 2 • LESSON 3 ASSIGNMENT 999

Lesson 3 Assignment

3. Calculate the volume of each proposed pyramid tent. Show your work.

   1st pyramid: 400 cubic feet; 2nd pyramid: 384 cubic feet

4. Which tent would you recommend Joe make? Explain your reasoning.

   Sample answer:

   I would recommend making the first pyramid tent because it has more room inside.

Prepare

Calculate the volume of each right rectangular prism or pyramid.

1. Prism: length $= \frac{4}{3}$ ft; width $= 1.8$ ft; height $= 3$ ft

   7.2 cubic feet

2. Prism: area of the base $= 26$ cm^2; height $= 5$ cm

   130 cubic centimeters

3. Pyramid: length $= \frac{4}{3}$ ft; width $= 1.8$ ft; height $= 3$ ft

   2.4 cubic feet

4. Pyramid: area of the base $= 26$ cm^2; height $= 5$ cm

   $4\frac{1}{3}$ cubic centimeters

1000 MODULE 5 • TOPIC 2 • LESSON 3 ASSIGNMENT

4 Volume and Surface Area Problems with Prisms and Pyramids

LESSON OVERVIEW

Students compare two different pieces of acoustical foam: one that is made up of square pyramids and one that is made up of triangular prisms. Students are introduced to the term *lateral surface area*, and they compare the total and lateral surface areas of the foam pieces. They then determine the total amount of foam that covers the top surface of the two foam boards. Finally, students calculate the volume, total surface area, and lateral surface area of rectangular and triangular prisms and pyramids to solve real-world problems.

MATERIALS

Rulers

Calculators (optional)

Problem-Solving Model Graphic Organizer

GRADE 7 TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

7.1A apply mathematics to problems arising in everyday life, society, and the workplace.

7.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

7.1F analyze mathematical relationships to connect and communicate mathematical ideas.

Expressions, Equations, and Relationships

(9) The student applies mathematical process standards to solve geometric problems.

The student is expected to:

7.9A solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids.

7.9D solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net.

ELPS

(1) Learning Strategies

The student is expected to:

(G) demonstrate an increasing ability to distinguish between formal and informal English and an increasing knowledge of when to use each one commensurate with grade-level learning expectations.

(2) Listening

The student is expected to:

(I) demonstrate listening comprehension of increasingly complex spoken English by following directions, retelling or summarizing spoken messages, responding to questions and requests, collaborating with peers, and taking notes commensurate with content and grade-level needs.

(4) Reading

The student is expected to:

(G) demonstrate comprehension of increasingly complex English by participating in shared reading, retelling or summarizing material, responding to questions, and taking notes commensurate with content area and grade-level needs.

CC BY NC

ESSENTIAL IDEAS

- A prism is a polyhedron with two parallel and congruent faces called *bases*. All other faces are parallelograms and are called *lateral faces*.
- A rectangular prism is a prism that has rectangles as its bases.
- A triangular prism is a prism that has triangles as its bases.
- A pyramid is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The triangular faces are called *lateral faces*.
- A rectangular pyramid is a pyramid that has a rectangle as its base.
- A triangular pyramid is a pyramid that has a triangle as its base.

LESSON STRUCTURE AND PACING: 2 DAYS

DAY 1

ENGAGE

Getting Started: Noise-Canceling Geometry 5–10 minutes

BUILD OFF INTUITION

Acoustical foam is placed on walls to control sound. The types of foam used are pyramidal foam and wedge foam. Students compare the top surface of each piece of foam board.

DEVELOP

Activity 4.1: Lateral and Total Surface Area of a Pyramid 35–40 minutes

MATHEMATICAL PROBLEM SOLVING

Students investigate pyramids and nets of pyramids to determine their lateral and total surface areas. They compare parts of surface areas of pyramids and triangular prisms.

DAY 2

Activity 4.2: Surface Area Problems 25 minutes

REAL-WORLD PROBLEM SOLVING, MATHEMATICAL PROBLEM SOLVING

Students solve problems involving prisms and pyramids. They use the context of the problem to decide whether to calculate the volume, total surface area, or lateral surface area.

Activity 4.3: Building a Surface Area Strategy 15 minutes

MATHEMATICAL PROBLEM SOLVING

Students analyze the faces and base of a square pyramid to derive a strategy for calculating the lateral and total surface area of a pyramid. They then use their strategy to calculate the lateral and total surface areas of pyramids.

DEMONSTRATE

Talk the Talk: The Information You Need 5 minutes

GENERALIZATION

Students determine the information they need to determine the volume, lateral surface area, and total surface area of a pyramid.

Getting Started

ENGAGE

Noise-Canceling Geometry

Facilitation Notes

In this activity, students are presented with a real-world problem involving polyhedrons formed using square pyramids and triangular prisms. They describe and compare the top surface areas.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How many polyhedra make up the pyramidal foam board? • How many polyhedra make up the wedge foam board? • Which piece of foam board appears to have a greater surface area? • Which foam board appears to contain a larger number of faces? • How does a larger number of faces affect the total surface area?

STAMP THE LEARNING

Summary

Polyhedrons can be formed using square pyramids and triangular prisms, and they may appear in real-world problem situations.

ACTIVITY 4.1

Lateral and Total Surface Area of a Pyramid

DEVELOP

Facilitation Notes

In this activity, students work with nets of the polyhedrons used in the previous activity to calculate lateral surface area.

Have students work with a partner or in a group to complete Questions 1 through 9. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Have models of the three-dimensional figures available.
- Suggest a format for students to organize their calculations. For example, have them record the areas of individual faces on the net or organize their calculations in a table.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Do you use the height or the slant height of the pyramid when labeling your net? Explain your thinking. • Why don't you use the height of the pyramid when calculating the surface area? • Do you think it is possible to calculate the surface area when you know the slant height but do not know the pyramid's height? • Why do you think a prism doesn't have a slant height? • Is calculating the surface area for a prism different from calculating the surface area of a pyramid? Why or why not?

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

The dimensions of a polyhedron are used to calculate lateral and total surface areas in a real-world problem situation.

ACTIVITY 4.2 Surface Area Problems

Facilitation Notes

In this activity, students solve real-world problems by calculating volume, total surface area, or lateral surface area.

To begin the Day 2 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Confusion between the height of a geometric solid and the slant height of the solid.
- Difficulty visualizing a triangular pyramid.

DIFFERENTIATION STRATEGY

Access for All

Give each group only one of the four scenarios. Allow time for group presentations to the class. Encourage students to ask their classmates questions.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know whether to calculate volume or surface area? • Should you include the area of the base in your calculations? • How is a triangular pyramid different than a square pyramid? • If the square pyramids are sharing their square base, how does this change the strategy for determining the total surface area of the two square pyramids?

STAMP THE LEARNING

Summary

Surface area and volume are used to solve real-world problem situations.

ACTIVITY 4.3

Building a Surface Area Strategy

Facilitation Notes

In this activity, students use a deconstructed net to derive a strategy for calculating the lateral and total surface area of a square pyramid.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Why does the base of the square pyramid look like a rhombus?
Seeing structure	• What does each term of your equation represent from the diagram? • How can b represent both the side/length of the base and the base of the triangular faces.
Gathering	• What is the area of the base of the Walter Pyramid? • What is the unit of measure used to describe the surface area of both the Walter Pyramid and the Pyramid Arena.

STAMP THE LEARNING

Summary

The total surface area of a pyramid can be determined by calculating the sum of the areas of each face of the pyramid. The lateral surface area of a pyramid can be determined by calculating the sum of the areas of each face of the pyramid, not including the base.

DEMONSTRATE

Talk the Talk

THE INFORMATION YOU NEED

Facilitation Notes

In this activity, students answer questions related to solving for the volume and surface area of a pyramid.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What is the relationship between the slant height of the pyramid and the height of the face? • How can you use the base to identify how many faces the pyramid has?

Have students read and answer the Essential Question on the lesson opener page.

Summary

To determine the volume of a pyramid, the area of the base and the height must be known. To determine the surface area of a pyramid, the area of the base, the length of the base of the triangular faces, and the slant height of the pyramid, which is the the height of the triangular faces of the pyramid, must be known.

STAMP THE LEARNING

4 Volume and Surface Area Problems with Prisms and Pyramids

Setting the Stage

- Communicate the objectives and the new key term to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Compare and contrast the lateral and total surface areas of geometric solids.
- Apply volume and surface area concepts to solve real-world and mathematical problems involving surface areas of pyramids and composite solids.

NEW KEY TERM

- lateral surface area

You have determined the surface area of prisms and pyramids.

How can you use the lateral and total surface areas of pyramids or prisms to solve problems?

Sample answer:

The total surface area of a pyramid or prism is the sum of the areas of all of its faces. The lateral surface area of a pyramid is the total surface area of the pyramid, excluding the base. Similarly, the lateral surface are of a prism is the total surface area of the prism excluding the base.

MODULE 5 • TOPIC 2 • LESSON 4 1001

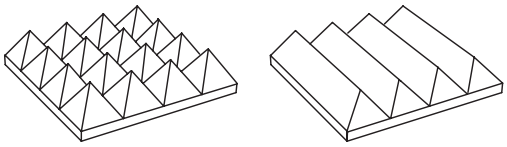
Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Getting Started

Noise-Canceling Geometry

In small recording studios and large auditoriums, acoustical foam is placed on walls and ceilings to control sound and correct sound problems. The size and shape of the foam that is used depends on the sound problem. Two different acoustical foam shapes are shown.



Pyramidal foam

Wedge foam

1. Describe the polyhedrons used in each type of acoustical foam board.

   The pyramidal foam board is formed from square pyramids and a rectangular prism. The wedge foam board is formed from triangular prisms and a rectangular prism.

2. Compare the top surface areas of each piece of foam board.

   The pyramidal foam board appears to have a greater surface area, because the polyhedrons contain a greater number of faces, compared to the polyhedrons that form the top surface of the wedge foam board.

EB STUDENT TIP

For all proficiency levels

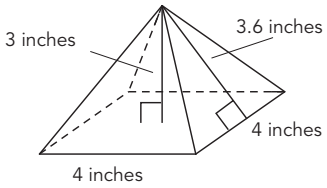
Students may need additional scaffolds with contextual words like *auditorium* and *acoustic foam*. Support students in making connections to contexts or experiences more familiar to them such as a school auditorium or a band room at school. Students may also be unfamiliar with the term *polyhedron*. Use pictures and physical items to help students understand that a polyhedron is a solid figure with multiple faces.

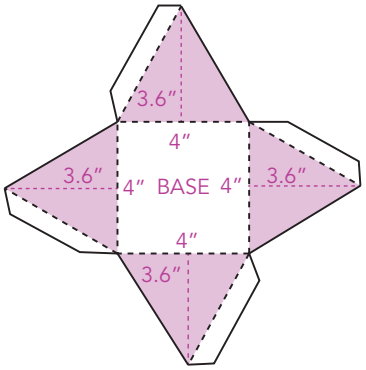
1002 MODULE 5 • TOPIC 2 • LESSON 4

ACTIVITY 4.1

Lateral and Total Surface Area of a Pyramid

Consider the square pyramid and it's net.





All pyramids have a height and a slant height. The slant height is not the actual height of the pyramid. Rather, it is the height of an individual triangle that is a face of the pyramid.

1. Label all known dimensions on the net shown.

2. Calculate the total surface area of the pyramid.
   44.8 square inches

The **lateral surface area** of a prism or pyramid is the sum of the areas of the lateral faces. You can calculate the lateral surface area of a prism or pyramid by determining the total surface area of the figure and then subtracting the area of the base(s).

3. Label the base and shade the lateral faces of the net.
   Check students' nets.

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

STAMP THE LEARNING

The definition provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Optimizing Learning

This activity guides information processing and visualization.

EB STUDENT TIP

For all proficiency levels

Use sentence prompts to guide students in the understanding of *lateral surface area*. Guide students in completing the following statements related to *lateral surface area*.

- I think *lateral surface* area is _____.
- *Lateral surface area* is not _____.
- _____ is an example of *lateral surface area*.

MODULE 5 • TOPIC 2 • LESSON 4 1003

Activity 4.1 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with calculating the lateral surface area and surface area of a polyhedron from its net, assign Skills Practice Set A for this lesson.

4. Compare the lateral surface area of the pyramid to the total surface area of the pyramid.

   The total surface area of the pyramid is the sum of the area of the square base and the areas of the four triangular faces. The lateral surface area of the pyramid is the sum of the areas of the four triangular faces. So, the lateral surface area is less than the total surface area.

5. Calculate the lateral surface area of the pyramid.

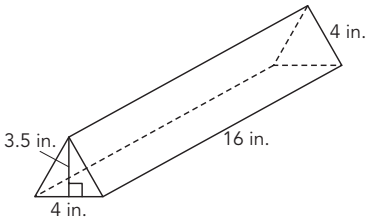
   Lateral Surface Area $= 4(0.5 \cdot 4 \cdot 3.6) = 28.8$

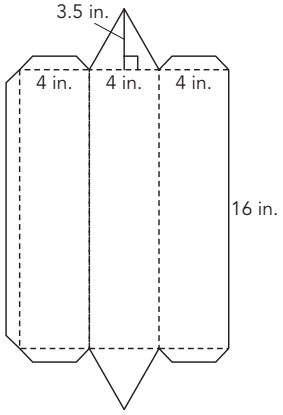
   The lateral surface area of the pyramid is 28.8 square inches.

6. Consider the original piece of pyramidal foam board. Could you use the lateral surface area of a pyramid to determine the surface area of the foam board? Explain your reasoning.

   I could use the lateral surface area of a pyramid when determining the surface area of the foam board. The exposed parts of each pyramid are the lateral sides.

One triangular prism from the wedge foam board is shown. A net of the triangular prism and its dimensions are also shown.





7. Calculate the total surface area of the prism.

   206 square inches

1004 MODULE 5 • TOPIC 2 • LESSON 4

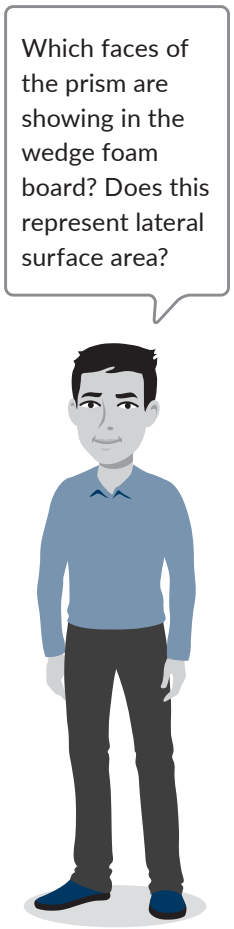
8. Compare the lateral surface area of the prism to the total surface area of the prism.

   The total surface area of the prism is the sum of the areas of the two triangular bases and the areas of the three rectangular faces. The lateral surface area of the prism is the sum of the areas of the three rectangular faces. So, the lateral surface area is less than the total surface area.

9. Calculate the lateral surface area of the prism.

   Lateral Surface Area $= 3(16 \cdot 4) = 192$

   The lateral surface area of the prism is 192 square inches.



10. Consider the original piece of wedge foam board.Could you use the lateral surface area of a triangular prism to determine the surface area of the foam board? Explain your reasoning.

    I could not use the lateral surface area of a triangular prism to determine the surface area. The exposed parts of each triangular prism includes only 2 lateral sides and the 2 bases.

MODULE 5 • TOPIC 2 • LESSON 4 1005

Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Optimizing Learning

This activity fosters collaboration and community.

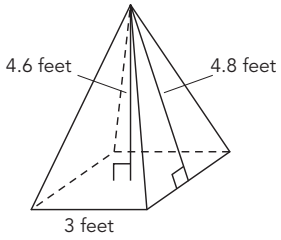
Activity 4.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice calculating the lateral surface area of a polyhedron, assign Skills Practice Set B for this lesson.

ACTIVITY 4.2

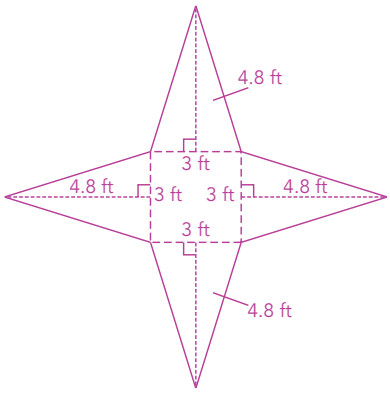
Surface Area Problems

When solving a problem, it may not be obvious whether you should determine the volume or surface area of a solid. So, it is important to think about the context of the problem situation before making a decision about how to answer the question.

1. Consider the square pyramid shown.



a. Draw a net to represent the square pyramid. Label the dimensions.



b. Suppose that the base of the pyramid is made of wood, and the triangular faces are made of plastic covered in blue felt. How much blue felt do you need?

28.8 square feet

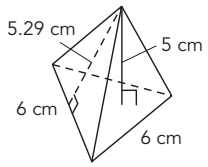
Modeling Moment

- Provide students with the Problem-Solving Model Graphic Organizer.
- For Question 1, have students work individually or with a partner to complete the graphic organizer.
- Have students share and evaluate problem-solving strategies as a class.
- For Questions 2–4, have students work individually or with a partner to complete the graphic organizer.

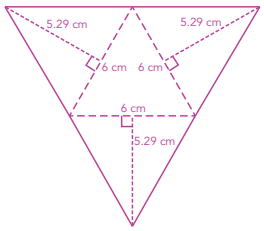
c. Suppose that the pyramid is filled with rice. How much rice is needed?

13.8 cubic feet

2. Consider the triangular pyramid shown. The base is an equilateral triangle. The area of the base is 15.59 square centimeters.



a. Draw a net to represent the triangular pyramid. Label the dimensions.



b. Suppose Sofia fills the triangular pyramid with sand, how much sand will she need?

Approximately 25.98 cubic centimeters

MODULE 5 • TOPIC 2 • LESSON 4 1007

c. Suppose Sofia covers the triangular pyramid with foil, how much foil will she need?

63.2 square centimeters

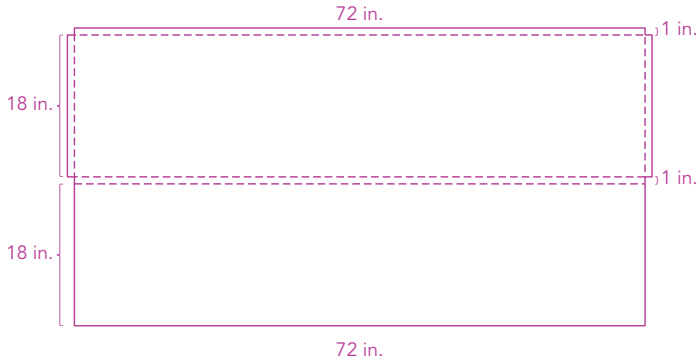
d. Suppose Sofia covers the entire pyramid, except for the base, with foil. How much foil will she need?

47.61 square centimeters

Ask Yourself . . .
How can you use surface area and volume in everyday life?

3. Daniel has a picnic table that has two wooden seats. Each seat is a rectangular prism that is 72 inches long, 18 inches wide, and 1 inch thick.

a. Draw a net that represents one seat from the picnic table. Label the dimensions.



1008 MODULE 5 • TOPIC 2 • LESSON 4

b. Daniel wants to cover the seats with a weather-proofing stain. How much stain will he need?

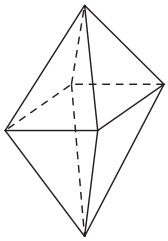
5544 square inches of stain

c. Suppose that Daniel decides he is not going to stain the sides that are 18 inches by 1 inch. How much stain will he need?

5472 square inches of stain

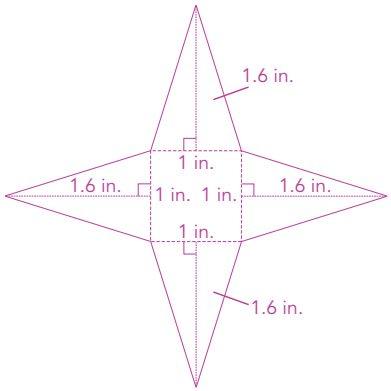
MODULE 5 • TOPIC 2 • LESSON 4 1009

4. Juan is creating an ornament by gluing the bases of two square pyramid blocks together, as shown.



The square base of each pyramid has a side length of 1 inch, the height of each pyramid is 1.5 inches, and the slant height of each pyramid is 1.6 inches.

a. Draw a net to represent one of the square pyramids. Label the dimensions.



b. After gluing the pyramids together, Juan decides to cover the ornament with metallic paint. How much of the ornament will be covered in paint?

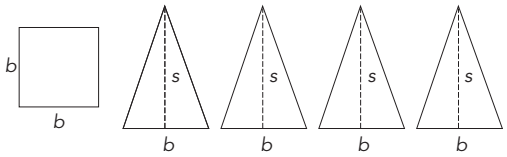
6.4 square inches of the ornament will be covered in metallic paint.

SELF-MONITORING STRATEGY

Look for students using self-motivation and self-discipline to persevere in solving problems. Refer to the Course and Implementation Guide for further details on these look-fors.

ACTIVITY
4.3 Building a Surface Area Strategy

The five faces of a square pyramid are shown.



The variable s represents the height of each triangular face of the pyramid.

1. Write equations to represent the lateral and total surface area of a square pyramid. Explain how the equations describe each value.

   $L = 2bs$

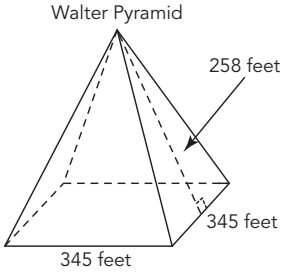
   $SA = 2bs + b^2$

   The lateral surface area of the pyramid is the sum of the areas of the four triangular faces. The area of one of the faces is equal to $\frac{1}{2}bs$. Multiplying this by 4, due to the four faces, you get $4 \cdot \frac{1}{2}bs = 2bs$.

   For total surface area, we need to take the lateral surface area plus the area of the square base. The area of the square base is equal to b^2. So, $SA = 2bs + b^2$.

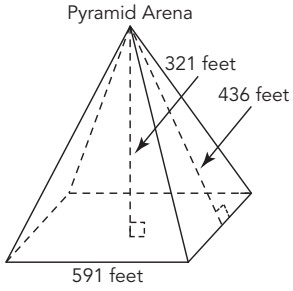
2. Use your equations to calculate the lateral and total surface area of each square pyramid.

   a. Walter Pyramid

   Lateral surface area: 178,020 square feet; total surface area: 297,045 square feet

   b. Pyramid Arena

   Lateral surface area: 515,352 square feet; total surface area: 864,633 square feet

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.

Activity 4.3 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with calculating the total surface area of a polyhedron, assign Skills Practice Set C for this lesson.

MODULE 5 • TOPIC 2 • LESSON 4 1011

Chunking the Activity

- Read and discuss the introduction.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

Activity 4.2 presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with calculating the volume, lateral surface area, or total surface area of a polyhedron, assign Skills Practice Set D for this lesson.

Talk the Talk

The Information You Need

1. What information do you need to determine the volume of a pyramid? Provide an example to explain your reasoning.

   I need to know the area of the base of the pyramid and the height of the pyramid.

   Sample explanation:

   When I know the area of the base of a pyramid is 100 square feet and the height is 30 feet, I can calculate the volume. It does not matter what shape the pyramid has for the base if I know the area of the base.

   $V = \frac{1}{3}(100)(3) = 1000$ cubic feet

2. What information do you need to determine the lateral surface area of a pyramid? What additional information do you need to determine the total surface area? Provide an example to explain your reasoning.

   To determine the lateral surface area of a pyramid, I need to know the length of the base of the triangular faces and the slant height of the pyramid or the height of the triangular faces of the pyramid. To find the total surface area, I also need to know the area of the base.

   Sample explanation:

   The area of the base of a pyramid is 100 square feet. Each triangular face has a base length of 10 feet and a height of 30.4 feet.

   The lateral surface area of the pyramid is $4 \cdot \frac{1}{2}(10)(30.4)$, or 608, square feet.

   The total surface area of the pyramid is 608 + 100, or 708, square feet.

EB STUDENT TIP

For all proficiency levels

Encourage students to start with a picture and talk about the information they need before recording their answers. Allow students to record answers as bullet points or provide sentence frames.

1012 MODULE 5 • TOPIC 2 • LESSON 4

Lesson 4 Assignment

Write

In your own words, explain how to determine the total surface area of any square pyramid. Use an example to illustrate your explanation.

Remember

The *total surface area* of a figure is the sum of the areas of all of its faces, including the base(s). The *lateral surface area* of a figure is the total surface area of the figure, excluding the base(s).

Practice

Calculate the volume, total surface area, and lateral surface area of each solid described.

1. The base of a pyramid is an equilateral triangle with a side length of 12 inches. The height of the base is 10.4 inches. The height of the pyramid is 10 inches, and its slant height is 10.6 inches.

   a. Volume: 208 cubic inches

   b. Total surface area: 253.2 square inches

   c. Lateral surface area: 190.8 square inches

2. A square pyramid has a base length of 1.5 meters, a height of 2.3 meters, and a slant height of 2.4 meters.

   a. Volume: 1.725 cubic meters

   b. Total surface area: 9.45 square meters

   c. Lateral surface area: 7.2 square meters

Write

Sample answer:

The total surface area of a square pyramid whose base lengths are b and whose lateral faces have a height of s is $2bs + b^2$.

MODULE 5 • TOPIC 2 • LESSON 4 ASSIGNMENT 1013

Lesson 4 Assignment

3. The base of a prism is a square with a side length of 2.5 centimeters. The height of the prism is 16 centimeters.

   a. Volume: 100 cubic centimeters

   b. Total surface area: 172.5 square centimeters

   c. Lateral surface area: 160 square centimeters

1014 MODULE 5 • TOPIC 2 • LESSON 4 ASSIGNMENT

TOPIC 2 SELF-REFLECTION

Name: _____

Area, Surface Area, and Volume

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1-3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Area, Surface Area, and Volume* topic by:

TOPIC 2: *Area, Surface Area, and Volume*	Beginning of Topic	Middle of Topic	End of Topic
applying the techniques of composing and/or decomposing to determine the area of composite figures to solve mathematical and real-world problems.	○	○	○
determining the areas of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, and triangles.	○	○	○
representing three-dimensional figures using nets made up of rectangles and triangles.	○	○	○
combining the areas for rectangles and triangles in the net to calculate the total surface area of a three-dimensional figure.	○	○	○
solving real-world and mathematical problems involving total surface area using nets for rectangular and triangular prisms and pyramids.	○	○	○
modeling and explaining the relationship between the volume of a rectangular prism and a rectangular pyramid having congruent bases and heights and connecting to the formulas.	○	○	○

continued on the next page

Notes

MODULE 5 • TOPIC 2 • SELF-REFLECTION 1015

Notes

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Area, Surface Area, and Volume*	Beginning of Topic	Middle of Topic	End of Topic
explaining how the volume of a triangular prism relates to the volume of a triangular pyramid with both congruent bases and heights, and connects to their formulas.	○	○	○
solving problems involving the volumes of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Area, Surface Area, and Volume* topic.

   Answers will vary.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   Answers will vary.

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   Answers will vary.

1016 MODULE 5 • TOPIC 2 • SELF-REFLECTION

TOPIC 2 SUMMARY

Area, Surface Area, and Volume Summary

Notes

NEW KEY TERMS

- net
- surface area [área de (la) superficie / área superficial]
- pyramid [pirámide]
- slant height
- lateral surface area [área de la superficie lateral]

LESSON 1 Composite Figures

A **composite figure** is a figure that is made up of more than one geometric figure.

Area is additive. The area of a composite figure can be determined by decomposing it into familiar shapes and then adding the areas of those shapes together.

For example, the composite figure shown is composed of a rectangle and a triangle.

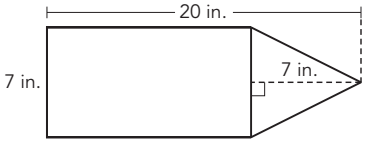
Area of composite figure = Area of Rectangle + Area of Triangle

$$= (7)(13) + \frac{1}{2}(7)(7)$$

$$= 91 + 24\frac{1}{2}$$

$$= 115\frac{1}{2}$$

The area of the composite figure is $115\frac{1}{2}$ square inches.



LESSON 2 Total Surface Area of Prisms and Pyramids

A **net** is a two-dimensional representation of a three-dimensional geometric figure.

A net has the following properties:

- The net is cut out as a single piece.
- All of the faces of the geometric solid are represented in the net.
- The faces of the geometric solid are drawn so that they share common edges.

The **surface area** of a three-dimensional geometric figure is the total area of all of its two-dimensional faces.

MODULE 5 • TOPIC 2 • TOPIC SUMMARY 1017

Notes

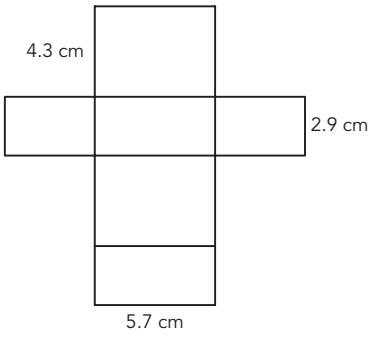
For example, you can use the net to calculate the surface area of the right rectangular prism.

Determine the area of each unique face.

$4.3 \text{ cm} \cdot 5.7 \text{ cm} = 24.51 \text{ cm}^2$

$2.9 \text{ cm} \cdot 5.7 \text{ cm} = 16.53 \text{ cm}^2$

$4.3 \text{ cm} \cdot 2.9 \text{ cm} = 12.47 \text{ cm}^2$

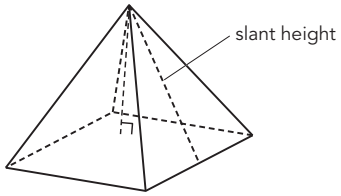


Determine the sum of all faces of the right rectangular prism.

$$\begin{aligned} &2(24.51) + 2(16.53) + 2(12.47) \\ &= 49.02 + 33.06 + 24.94 \\ &= 107.02 \end{aligned}$$

The surface area of the right rectangular prism is 107.02 cm^2.

A **pyramid** is a polyhedron with one base and the same number of triangular faces as there are sides of the base. The vertex of a pyramid is the point at which all the triangular faces intersect. A **slant height** of a pyramid is the distance measured along a triangular face from the vertex of the pyramid to the midpoint, or center, of the base.



LESSON 3 Volume of Prisms and Pyramids

The volume of a pyramid is one-third the volume of a prism with the same base and height, so $V = \frac{1}{3}Bh$.

For example, calculate the volume of the rectangular pyramid shown.

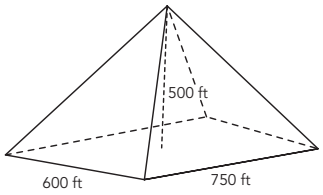
$B = 750 \cdot 600$

$B = 450{,}000$

$V = \frac{1}{3}\text{Bh}$

$V = \frac{1}{3}(450{,}000)(500)$

$V = 75{,}000{,}000$



The volume of the pyramid is 75,000,000 cubic feet.

1018 MODULE 5 • TOPIC 2 • TOPIC SUMMARY

LESSON 4 Volume and Surface Area Problems with Prisms and Pyramids

The total surface area of a pyramid is the sum of the areas of all of its faces. The **lateral surface** area of a pyramid is the total surface area of the pyramid, excluding the base. The dimensions of a polyhedron are used to calculate the lateral surface area in a real-world problem situation.

All pyramids have a height and a slant height. The slant height is not the actual height of the pyramid. Rather, it is the height of an individual triangle that is a face of the pyramid.

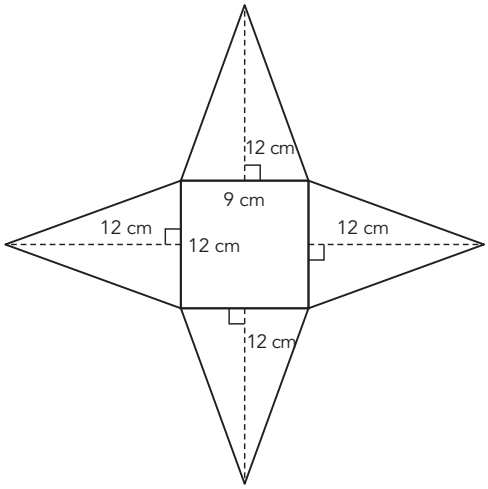
For example, calculate the total and lateral surface areas of the pyramid whose net is shown.

$SA = 4 \cdot \frac{1}{2}(9)(12) + 9^2$ $\quad L = 4 \cdot \frac{1}{2}(9)(12)$

$SA = 4 \cdot 54 + 81$ $\quad L = 4 \cdot 54$

$SA = 216 + 81$ $\quad L = 216$

$SA = 297$



The total surface area of the pyramid is 297 square centimeters. The lateral surface area of the pyramid is 216 square centimeters.

Similarly, the total surface area of a prism is the sum of the area of all of its faces. The *lateral surface* area of a prism is the total surface area of the prism excluding the area of the base.

When solving a problem, it may not be obvious whether you should determine the volume, total surface area, or lateral surface area of a solid. Therefore, it is important to think about the context of the problem situation before making a decision about how to answer the question.

Notes

MODULE 5 • TOPIC 2 • TOPIC SUMMARY 1019

Notes

1020 MODULE 5 • TOPIC 2 • TOPIC SUMMARY

Math Glossary

A

401(k) plan

A 401(k) plan is a retirement investment account set up by an employer. A portion of an employee's pay is invested into the account with the employer often matching a certain amount of it.

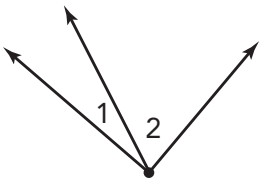
403(b) plan

A 403(b) plan is a retirement plan generally for public school employees or other tax exempt groups.

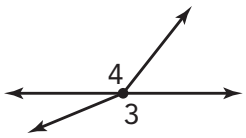
adjacent angles

Adjacent angles are two angles that share a common vertex and share a common side.

Examples



Angles 1 and 2 are adjacent angles.



Angles 3 and 4 are NOT adjacent angles.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $\quad 2a + b$ $\quad xy$ $\quad \frac{4}{p}$ $\quad z^2$

appreciation

Appreciation is an increase in price or value.

asset

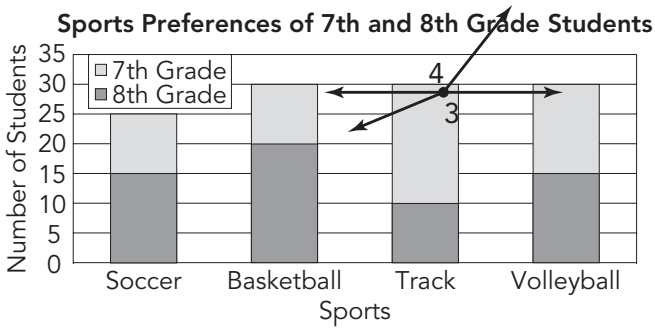
Assets include the value of all accounts, investments, and things that you are own. They are positive and add to your net worth.

B

bar graph

Bar graphs display data using horizontal or vertical bars so that the height or length of the bars indicates its value for a specific category.

Examples

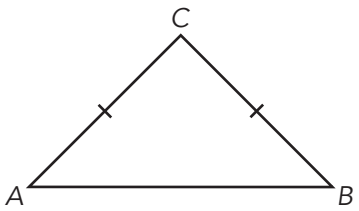


base angles

The angles opposite the two sides that have the same length in an isosceles triangle are called base angles.

Example

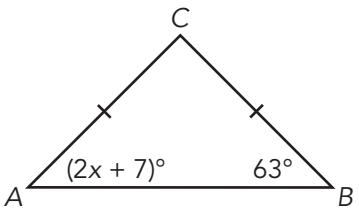
$\angle A$ and $\angle B$ are base angles of $\triangle ABC$



Base Angles theorem

The Base Angles theorem states the angles opposite the congruent sides of an isosceles triangle are congruent.

Example



$\triangle ABC$ is an isosceles triangle. Since, $\angle A$ and $\angle B$ are base angles, their measures are the same.

$2x + 7 = 63$

C

census

A census is the data collected from every member of a population.

Example

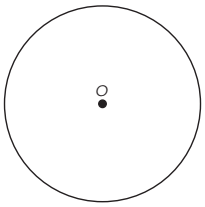
The U.S. Census is taken every 10 years. The U.S. government counts every member of the population every 10 years.

circle

A circle is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.

Example

The circle shown is Circle O.

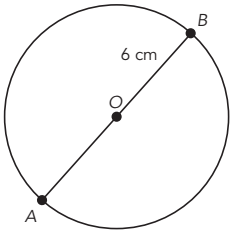


circumference

The distance around a circle is called the circumference of the circle. The circumference is calculated by the formula: $C = \pi(d)$.

Example

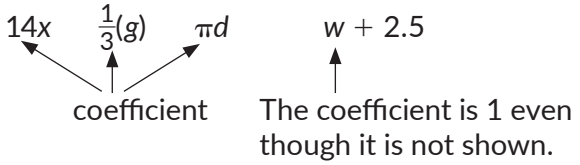
The diameter of Circle O is 12 centimeters. The circumference of Circle O is 12π.



coefficient

A number that is multiplied by a variable in an algebraic expression is called a coefficient.

Examples



collinear

When points lie on the same line or line segment, they are said to be collinear.

Example

Points *C*, *A*, and *B* are collinear.

commission

A commission is an amount of money a salesperson earns after selling a product. Many times, the commission is a certain percent of the product.

Example

5% commission on $350

$0.05 \times 350 = \$17.50 \leftarrow$ commission

common factor

A common factor is a number that is a factor of two or more numbers.

Example

Factors of 60: **1**, **2**, **3**, **4**, 5, **6**, 10, **12**, 15, 20, 30, 60

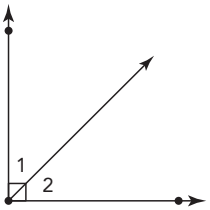
Factors of 24: **1**, **2**, **3**, **4**, **6**, 8, **12**, 24

Common factors of 60 and 24: 1, 2, 3, 4, 6, and 12

complementary angles

Two angles are complementary angles if the sum of their angle measures is equal to 90°.

Example



Angles 1 and 2 are complementary angles.

complementary events

Complementary events are events that together contain all of the outcomes in the sample space.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the event of rolling an even number and the event of rolling an odd number (not even) are complementary events.

complex ratio

A ratio in which one or both of the quantities being compared are written as fractions is a complex ratio.

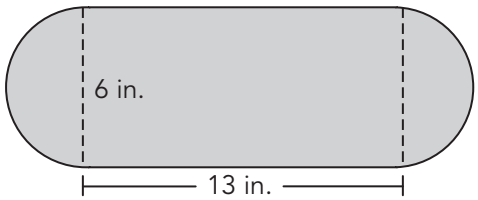
Example

Traveling $\frac{1}{3}$ mile in $\frac{1}{2}$ hour represents a ratio of fractions, or a complex ratio.

composite figure

Composite figures are geometric figures composed of two or more geometric shapes.

Example



The composite figure is composed of a rectangle and two semi-circles.

compound event

A compound event combines two or more events, using the word “and” or the word “or.”

compound interest

Compound interest is a percentage of the principal and the interest that is added to the principal over time.

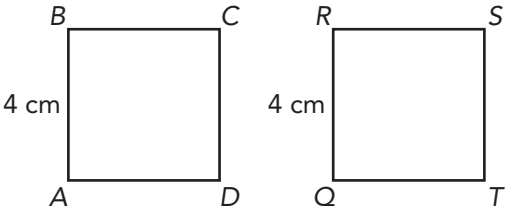
CC BY NC

congruent

Congruent means to have the same size, shape, and measure.

Example

Square *ABCD* is congruent to Square *QRST*.



congruent angles

Congruent angles are angles that have the same measure.

congruent sides

Congruent sides are sides that have the same length.

constant of proportionality

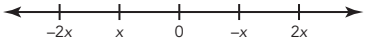
In a proportional relationship, the ratio of all *y*-values to their corresponding *x*-values is constant. This specific ratio, $\frac{y}{x}$, is called the constant of proportionality. Generally, the variable *k* is used to represent the constant of proportionality.

constraint

A constraint is a condition that a solution or problem must satisfy. A constraint can be a restriction set in advance of solving a problem or a limit placed on a solution or graph so the answer makes sense in terms of a real-world scenario.

Example

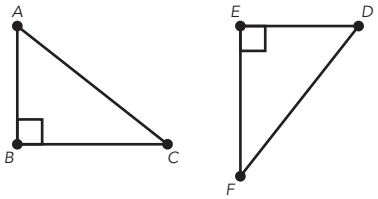
The expressions 0, x, $2x$, $-x$, and $-2x$ are graphed on a number line using the constraint $x < 0$.



corresponding

Corresponding means to have the same relative position in geometric figures, usually referring to sides and angles.

Example



Sides *AB* and *DE* are corresponding sides.

Angle *B* and Angle *E* are corresponding angles.

coupon

A coupon is a detachable part of a ticket or advertisement that entitles the holder to a discount.

D

data

Data are categories, numbers, or observations gathered in response to a statistical question.

Examples

Favorite foods of sixth-graders
Heights of different animals at the zoo

depreciation

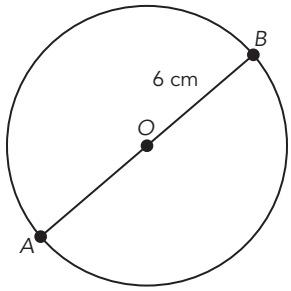
Depreciation is a decrease in price or value.

diameter

The diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.

Example

In Circle O, segment AB is a diameter. The length of diameter AB is two times the length of radius OA. The length of radius OA is 6 centimeters, so the length of diameter AB is 12 centimeters.



direct variation

A situation represents a direct variation if the ratio between the y-value and its corresponding x-value is constant for every point. The quantities are said to vary directly.

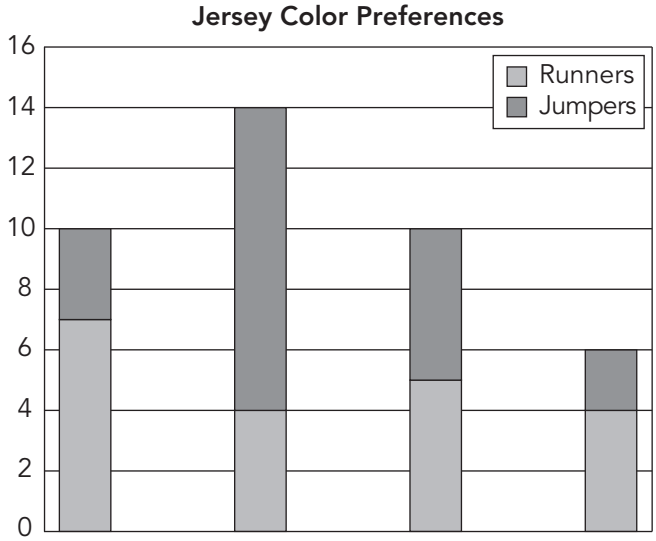
Example

If Melissa earns $8.25 per hour, then the amount she earns is in direct variation with the number of hours she works. The amount $8.25 is the constant of proportionality.

double bar graph

A double graph is used when each category contains two different groups of data.

Example



E

equally likely

When the probabilities of all the outcomes of an experiment are equal, the outcomes are called equally likely.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the probability of rolling each number from 1 through 6 is equally likely.

equation

An equation is a mathematical sentence that uses an equals sign to show that two quantities are the same as one another.

Examples

$$y = 2x + 4$$

$$6 = 3 + 3$$

$$2(8) = 26 - 10$$

$$\frac{1}{4} \cdot 4 = \frac{8}{4} - \frac{4}{4}$$

evaluate an algebraic expression

To evaluate an algebraic expression means to determine the value of the expression for a given value of each variable.

Example

Evaluate the expression $\frac{4x + (2^3 - y)}{p}$ for $x = 2.5$, $y = 8$, and $p = 2$.

- First replace the variables with numbers: $\frac{4(2.5) + (2^3 - 8)}{2}$.
- Then, calculate the value of the expression: $\frac{10 + 0}{2} = \frac{10}{2} = 5$.

event

An event is one possible outcome or a group of possible outcomes for a given situation.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, an event could be rolling an even number.

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experiment

An experiment is a situation involving chance that leads to results, or outcomes.

Example

Rolling a six-sided number cube is an experiment.

experimental probability

Experimental probability is the ratio of the number of times an event occurs to the total number of trials performed.

Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. You draw the blue marble 20 times out of 50 trials. The experimental probability, P_E(blue), is $\frac{20}{25}$ or $\frac{2}{5}$.

extremes

In a proportion that is written $a : b = c : d$, the two values on the outside, a and d, are the extremes.

Example

7 books : 14 days = 3 books : 6 days

extremes

F

Family Budget Estimator

The Family Budget Estimator is a tool that people can use to determine the estimated cost of raising a family in a particular city.

fixed expenses

Fixed expenses are expenses that don't change from month to month.

factor

To factor an expression means to rewrite the expression as a product of factors.

Example

$5(12) + 5(9) = 5(12 + 9)$

G

greatest common factor (GCF)

The greatest common factor, or GCF, is the largest factor two or more numbers have in common.

Example

Factors of 16: **1**, **2**, **4**, 8, 16

Factors of 12: **1**, **2**, 3, **4**, 6, 12

Common factors: 1, 2, 4

Greatest common factor: 4

I

income tax

Income tax is a percentage of a person's or company's earnings that is collected by the government.

Example

If a person earns $90,000 in one year and has to pay an income tax rate of 28%, then that person owes $90{,}000 \times 0.28$ or $25,200 in income tax to the government.

inverse operations

Inverse operations are pairs of operations that reverse the effects of each other.

Examples

Addition and subtraction are inverse operations: $351 + 25 - 25 = 351$.

Multiplication and division are inverse operations: $351 \cdot 25 \div 25 = 351$.

isolate the variable

When you isolate the variable in an equation, you perform an operation, or operations, to get the variable by itself on one side of the equals sign.

Example

In the equation $\frac{a}{b} = \frac{c}{d}$, you can multiply both sides by b to isolate the variable a.

$b \cdot \frac{a}{b} = b \cdot \frac{c}{d} \longrightarrow a = \frac{bc}{d}$

L

lateral surface area

The lateral surface area of a prism or pyramid is the sum of the areas of the lateral faces.

liability

A liability is a financial obligation, or debt, that you must repay. It is negative and take away from your net worth.

like terms

In an algebraic expression, like terms are two or more terms that have the same variable raised to the same power.

Examples

like terms

$4x + 3p + x + 2 = 5x + 3p + 2$

like terms

$24a^2 + 2a - 9a^2 = 13a^2 + 2a$

no like terms

$m + m^2 - x = x^3$

linear expression

A linear expression is any expression in which each term is either a constant or the product of a constant and a single variable raised to the first power.

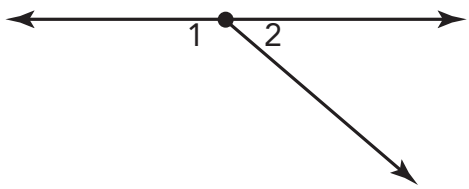
Examples

$\frac{1}{2}x + 2$, $-3 + 12.5x$, $-1 + 3x + \frac{5}{2}x - \frac{4}{3}$

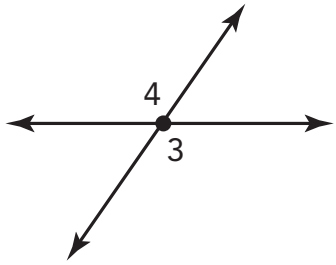
linear pair

A linear pair of angles is formed by two adjacent angles that have noncommon sides that form a line.

Examples



Angles 1 and 2 form a linear pair.



Angles 3 and 4 do NOT form a linear pair.

literal equation

A literal equation is an equation in which the variables represent specific measures.

Examples

$A = \ell w$ $A = \frac{1}{2}bh$ $d = rt$

M

markdown

When businesses sell an item at a lower price than the original price, it is called a markdown.

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markup

To make money, businesses often buy products from a wholesaler or distributor for one amount and add to that amount to determine the price they use to sell the product to their customers. This is called a markup.

means

In a proportion that is written $a : b = c : d$, the two values in the middle, b and c, are the means.

Example

7 books : 14 days = 3 books : 6 days

means

N

net worth

Your net worth is basically a calculation of the value of everything that you own minus the amount of money that you owe.

non-uniform probability model

A non-uniform probability model occurs when all the probabilities in a probability model are not equal to each other.

Example

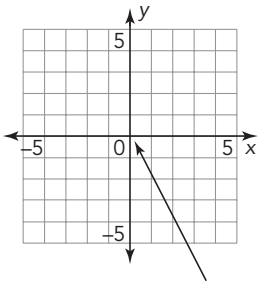
Outcome	Red	Green	Blue
Probability	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{3}{8}$

O

origin

The origin is a point on a graph with the ordered pair (0, 0).

Example



The origin is at (0,0)

outcome

An outcome is the result of a single trial of a probability experiment.

Example

The numbers on the faces of a six-sided number cube are the outcomes that can occur when rolling a six-sided number cube.

P

parameter

When data are gathered from a population, the characteristic used to describe the population is called a parameter.

Example

If you wanted to determine the average height of the students at your school, and you measured every student at the school, the characteristic "average height" would be a parameter.

percent decrease

A percent decrease occurs when the new amount is less than the original amount. It is a ratio of the amount of decrease to the original amount.

Example

The price of a $12 shirt has decreased to $8.

$\frac{12 - 8}{12} = \frac{4}{12} = 0.3 = 33.3\%$

The percent decrease is 33.3%

percent equation

A percent equation can be written in the form percent $\times$ whole $=$ part, where the percent is often written as a decimal.

Example

40% of $25 = 10$

$(0.40)\,(25) = 10$

Percent, Part, Whole

percent error (estimation)

Calculating percent error is one way to compare an estimated value to an actual value. To compute percent error, determine the difference between the estimated and actual values and then divide by the actual value.

Example

An airline estimates that they will need an airplane that sits 320 passengers for a flight. An actual 300 tickets were booked for the flight.

$$\text{Percent Error} = \frac{300 - 320}{300} = \frac{-20}{300} \approx -6.7\%$$

percent error (probability)

In probability, the percent error describes how far off the experimental probability is from the theoretical probability as a percent ratio.

Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. You draw the blue marble 20 times out of 50 trials.

The experimental probability, $P_E(\text{blue})$, is $\frac{20}{50}$ or $\frac{2}{5}$. The theoretical probability, $P_T(\text{blue})$, is $\frac{1}{4}$.

$$\text{The percent error is } \frac{\frac{2}{5} - \frac{1}{4}}{\frac{1}{4}} = \frac{\frac{3}{20}}{\frac{1}{4}} = \frac{3}{5} = 0.6 = 60\%$$

percent increase

A percent increase occurs when the new amount is greater than the original amount. It is a ratio of the amount of increase to the original amount.

Example

The price of a \$12 shirt has increased to \$13.20.

$$\frac{13.20 - 12}{12} = \frac{1.20}{12} = 0.1 = 10\%$$

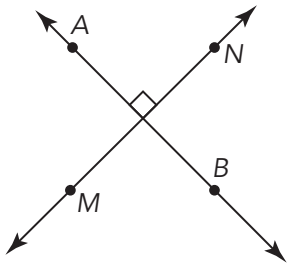
The percent increase is 10%.

perpendicular

Two lines, line segments, or rays are perpendicular if they intersect to form 90° angles. The symbol for perpendicular is $\perp$.

Example

Line AB is perpendicular to line MN



personal budget

A personal budget is an estimate of the amount of money that a person or family will need for specific financial items. It generally includes current expenses as well as savings for anticipated future expenses.

principal

A principal is the term for an original amount of money on which interest is calculated.

pi

The number pi (π) is the ratio of the circumference of a circle to its diameter. That is $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle.

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population

A population is an entire set of items from which data are collected.

Example

If you wanted to determine the average height of the students at your school, the number of students at the school would be the population.

probability

Probability is the measure of the likelihood that an event will occur. It is a way of assigning a numerical value to the chance that an event will occur by dividing the number of times an event can occur by the number of possible outcomes.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, the probability of rolling a 5, or $P(5)$, is $\frac{1}{6}$.

probability model

A probability model is a list of each possible outcome along with its probability, often shown in a table.

Example

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

This is a probability model for rolling a six-sided number cube with the numbers 1 through 6 on each face.

proportion

A proportion is an equation that states that two ratios are equal.

Example

$\frac{1}{2} = \frac{4.5}{9}$

proportional relationship

A proportional relationship is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$ must represent the same constant.

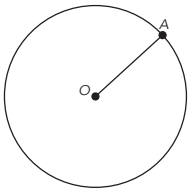
R

radius

The radius of a circle is a line segment formed by connecting a point on the circle and the center of the circle.

Example

In the circle, O is the center and segment OA is the radius.



random number table

A random number table is a table that displays random digits. These tables can contain hundreds of digits.

Example

Line 7	54621	62117	55516	40467

random sample

A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

Example

If you wanted to determine the average height of the students at your school, you could choose just a certain number of students randomly and measure their heights. This group of students would be a random sample.

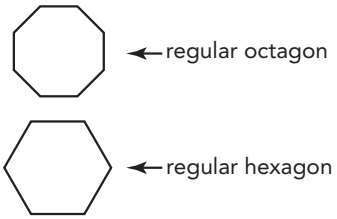
rebate

A rebate is a refund of part of the amount paid for an item.

regular polygon

A regular polygon is a polygon with all sides congruent and all angles congruent.

Examples



S

sale

A sale is an event at which products are sold at reduced prices.

sales tax

Sales tax is a percentage of the selling price of a good or service which is added to the price.

Example

You want to purchase an item for $8.00 in a state where the sales tax is 6.25%, therefore you will pay 8×0.0625 or $0.50 in sales tax. You will pay a total of $8.50 for the item.

sample

A sample is a selection from a population.

Example

If you wanted to determine the average height of the students in your school, you could choose a certain number of students and measure their heights. The heights of the students in this group would be your sample.

sample space

A list of all possible outcomes of an experiment is called a sample space.

Example

When rolling a six-sided number cube that has one number, from 1 through 6, on each face, the sample space is {1, 2, 3, 4, 5, 6}.

scale

A scale is a ratio that compares two measures.

Example

1 cm : 4 cm

scale drawing

A scale drawing is a representation of a real object or place that is in proportion to the real object or place it represents.

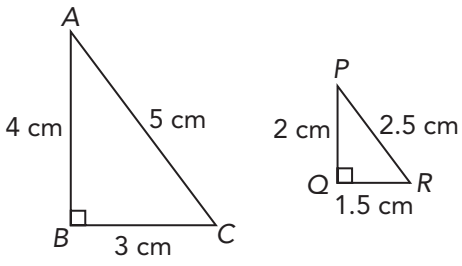
Examples

A map or a blueprint is an example of a scale drawing.

scale factor

When you multiply a measure by a scale to produce a reduced or enlarged measure, the scale is called a scale factor.

Example

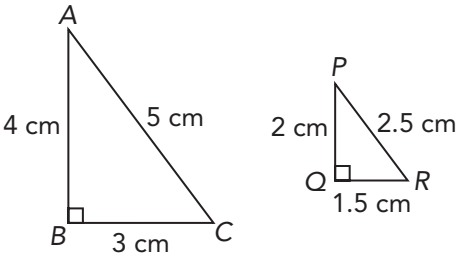


The scale factor from Triangle *ABC* to Triangle *PQR* is $\frac{1}{2}$.

similar figures

Figures that are proportional in size, or that have proportional dimensions, are called similar figures.

Example



Triangle *ABC* and Triangle *PQR* are similar figures.

simple event

A simple event is an event consisting of one outcome.

Example

When rolling a six-sided number cube with the numbers 1 through 6 on each face, rolling a 5 is a simple event.

simple interest

Simple interest is a type of interest that is a fixed percent of the principal. Simple interest is paid over a specific period of time—either twice a year or once a year, for example. The formula for simple interest is $I = P \cdot r \cdot t$, where I represents the interest earned, P represents the amount of the principal, r represents the interest rate, and t represents the time that the money earns interest.

Example

Kim deposits $300 into a savings account at a simple interest rate of 5% per year. The formula can be used to calculate the simple interest Kim will have earned at the end of 3 years.

Interest = Principal · rate · time

Interest = (300)(0.05)(3)

= $45

simulation

A simulation is an experiment that models a real-life situation.

solve a proportion

To solve a proportion means to determine all the values of the variables that make the proportion true.

stacked bar graph

A stacked bar graph is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole.

statistic

When data are gathered from a sample, the characteristic used to describe the sample is called a statistic.

Example

If you wanted to determine the average height of the students in your school, and you chose just a certain number of students randomly and measured their heights, the characteristic "average height" would be called a statistic.

straight angle

A straight angle is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

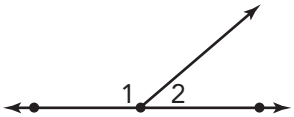
Example

Angle *CAB* is a straight angle.

supplementary angles

Two angles are supplementary angles if the sum of their angle measures is equal to 180°.

Example



Angles 1 and 2 are supplementary angles.

surface area

The **surface area** of a prism or pyramid is the total area of all its two-dimensional faces.

survey

A survey is one method of collecting data in which people are asked one or more questions.

Example

A restaurant may ask its customers to complete a survey with the following question:

On a scale of 1–10, with 1 meaning "poor" and 10 meaning "excellent," how would you rate the food you ate?

T

theoretical probability

The theoretical probability of an event is the ratio of the number of desired outcomes to the total possible outcomes.

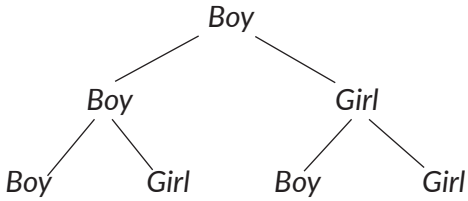
Example

Suppose there is one red, one blue, one green, and one yellow marble in a jar. The theoretical probability of drawing a blue marble, P_T(blue), is $\frac{1}{4}$.

tree diagram

A tree diagram illustrates the possible outcomes of a given situation. It has two main parts: the branches and the ends. An outcome of each event is written at the end of each branch.

Example



two-step equation

A two-step equation requires that two inverse operations be performed to isolate the variable.

U

uniform probability model

A uniform probability model occurs when all the probabilities in a probability model are equally likely to occur.

Example

Outcome	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

unit rate

A unit rate is a comparison of two different measurements in which the numerator or denominator has a value of one unit.

Example

The speed 60 miles in 2 hours can be written as a unit rate:

$\frac{60 \text{ mi}}{2 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}}$.

The unit rate is 30 miles per hour.

unit rate of change

The unit rate of change describes the amount the dependent variable changes for every unit the independent variable changes.

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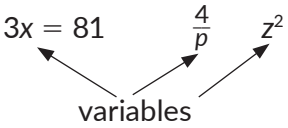
V

variable

A variable is a letter or symbol that is used to represent a number.

Examples



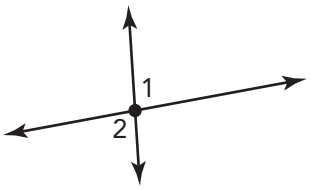
variable expenses

Variable expenses are expenses that can be different from month to month.

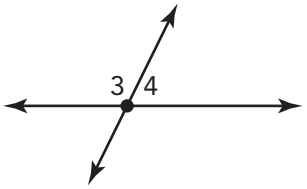
vertical angles

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

Examples



Angles 1 and 2 are vertical angles.



Angles 3 and 4 are NOT vertical angles.

Z

zero pair

A positive counter and a negative counter together make a zero pair since the total value of the pair is zero.

Example

⊕ + ⊖ = 0

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