Secondary Mathematics
EDITION 1

Grade 8

Family Guides

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

If you have further product questions or to report an error, please email openeducationresources@tea.texas.gov.

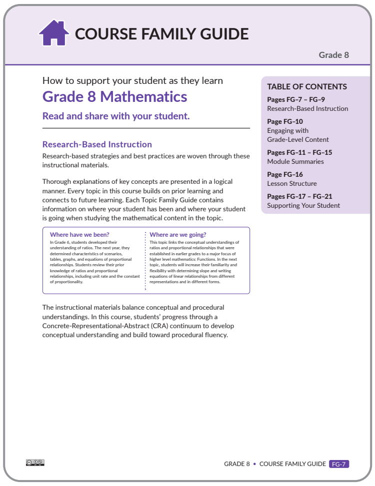
Dear Family,

We recognize that learning outside of the classroom is crucial to your student's success at school. This letter serves as an introduction to the resources designed to assist you as you talk to your student about what they are learning. Resources available to you include:

- Course Family Guide
- Topic Family Guides
- Topic Summaries
- Math Glossary

Course Family Guide

Following this letter, there is the Course Family Guide that will walk you through the research-based instructional approach, how the course is structured, how to bust math myths, using Talking Points from the Topic Family Guide, and using the TEKS mathematical process standards to initiate discussions.



COURSE FAMILY GUIDE

Grade 8

How to support your student as they learn

Grade 8 Mathematics

Read and share with your student.

Research-Based Instruction

Research-based strategies and best practices are woven through these instructional materials.

Thorough explanations of key concepts are presented in a logical manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide contains information on where your student has been and where your student is going when studying the mathematical content in the topic.

Where have we been?

In Grade 6, students developed their understanding of ratios. The next year, they determined characteristics of scenarios, tables, graphs, and equations of proportional relationships. Students review their prior knowledge of ratios and proportional relationships, including unit rate and the constant of proportionality.

Where are we going?

This topic links the conceptual understandings of ratios and proportional relationships that were established in earlier grades to a major focus of higher level mathematics: Functions. In the next topic, students will increase their familiarity and flexibility with determining slope and writing equations of linear relationships from different representations and in different forms.

The instructional materials balance conceptual and procedural understandings. In this course, students' progress through a Concrete-Representational-Abstract (CRA) continuum to develop conceptual understanding and build toward procedural fluency.

TABLE OF CONTENTS

Pages FG–7 – FG–9 Research-Based Instruction

Page FG–10 Engaging with Grade-Level Content

Pages FG–11 – FG–15 Module Summaries

Page FG–16 Lesson Structure

Pages FG–17 – FG–21 Supporting Your Student

GRADE 8 • COURSE FAMILY GUIDE FG-7

Research and classroom experience guided course development, with the foundation being a scientific understanding of how people learn and a real-world understanding of how to apply that science to mathematics instructional materials. The instructional design elements presented in the Course Family Guide incorporate research-based strategies to develop conceptual understanding and creative problem solvers.

The Course Family Guide provides an overview of the structure of the course. The course consists of both a Learning Together component and a Learning Individually component. The teacher facilitates a collaborative learning experience during the Learning Together Days and uses data to target specific skills on the Learning Individually Days.

Next, the Course Family Guide includes Module Overviews of each module in the course, which include a detailed summary of what your student will be learning in each topic within the module. Below the topic summaries are facts and information that connect the concepts to the real world. Read and discuss the information below the topic summaries with your student and continue to come back to these pages as your student moves from one topic to the next within each module.

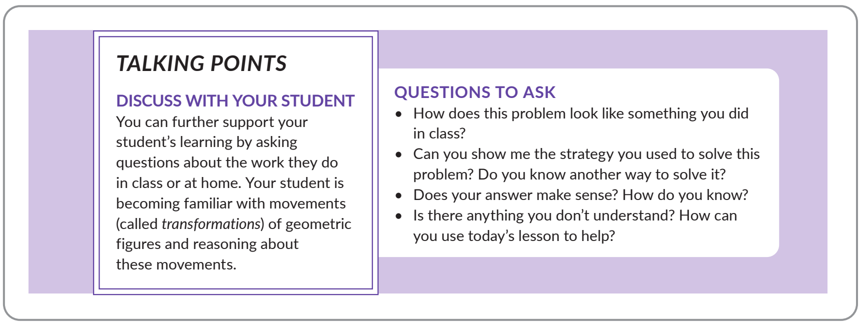
The Course Family Guide also highlights the lesson structure. Each lesson is structured the same way and includes four parts: Objectives & Essential Question, Getting Started, Activities, and the Talk the Talk.

Topic Family Guide

Each course is organized into modules. Each module consists of topics with corresponding Topic Family Guides. These guides all have the same structure. This consistency will allow you and your student to understand how to reference the content of each topic.

The Topic Family Guide begins with an overview of the content in the topic. This introduction includes a brief explanation of what your student will learn in the topic, the prior knowledge they will use to help them understand this topic, and a connection to future learning.

The next section of the Topic Family Guide is the Talking Points section. The Talking Points section provides questions you can ask as your student works through the math of the topic.



TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is becoming familiar with movements (called *transformations*) of geometric figures and reasoning about these movements.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Next, the Topic Family Guide lists all the new key terms of the topic and details some of the math strategies students will learn in the topic. Finally, each Topic Family Guide contains a Math Myth. Busting these Math Myths helps to build confidence and explain how math is accessible to everyone.

MYTH

"Asking questions means you don't understand."

It is universally true that for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. However, there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more on a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, everyone should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

Topic Summary

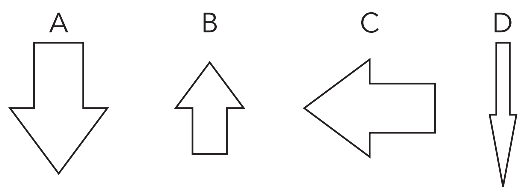
A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all new key terms of the topic and provides a summary of each lesson. Each lesson summary defines new key terms and reviews key concepts, strategies, and/or Worked Examples. The Topic Summary provides an opportunity for you and your student to discuss the key concepts from each lesson, review the examples, and do the math together.

LESSON 1 **Introduction to Congruent Figures**

Figures that have the same size and shape are **congruent figures**. When two figures are congruent, all corresponding sides and all corresponding angles have the same measures. **Corresponding sides** are sides that have the same relative position in geometric figures. **Corresponding angles** are angles that have the same relative position in geometric figures.

When two figures are congruent, you can obtain one figure by a combination of sliding, flipping, and spinning the figure until it lies on the other figure.

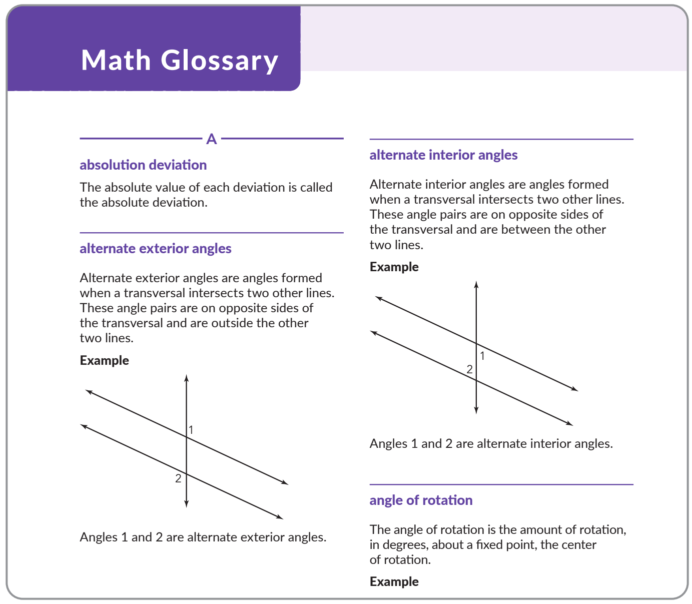
For example, Figure A is congruent to Figure C, but it is not congruent to Figure B or Figure D.



Evidence of the TEKS mathematical process standards are present in the Topic Summaries. Each lesson within the topic highlights one or more of the TEKS mathematical process standards. These processes will help your student develop effective communication and collaboration skills that are essential for becoming a successful learner. Discuss with your student the “I can” statements associated with each of the TEKS mathematical process standards to help them develop their mathematical learning and understanding. The "I can" statements for each of the TEKS mathematical process standards are included in the Course Family Guide. With your help, your student can develop the habits of a productive mathematical thinker.

Math Glossary

The Math Glossary for each course is a tool for your student to utilize and reference during their learning. Along with the definition of a term, the glossary provides examples to help further their understanding.



Math Glossary

A

absolution deviation

The absolute value of each deviation is called the absolute deviation.

alternate exterior angles

Alternate exterior angles are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are outside the other two lines.

Example

Angles 1 and 2 are alternate exterior angles.

alternate interior angles

Alternate interior angles are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are between the other two lines.

Example

Angles 1 and 2 are alternate interior angles.

angle of rotation

The angle of rotation is the amount of rotation, in degrees, about a fixed point, the center of rotation.

Example

We all have the same goal for your student: to become a successful problem solver and use mathematics efficiently and effectively in daily life. Encourage them to use the mathematics they already know when seeing new concepts and communicate their thinking while providing a critical ear to the thinking of others.

Thank you for supporting your student.

COURSE FAMILY GUIDE

Grade 8

How to support your student as they learn

Grade 8 Mathematics

Read and share with your student.

TABLE OF CONTENTS

Research-Based Instruction

Research-based strategies and best practices are woven through these instructional materials.

Thorough explanations of key concepts are presented in a logical manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide contains information on where your student has been and where your student is going when studying the mathematical content in the topic.

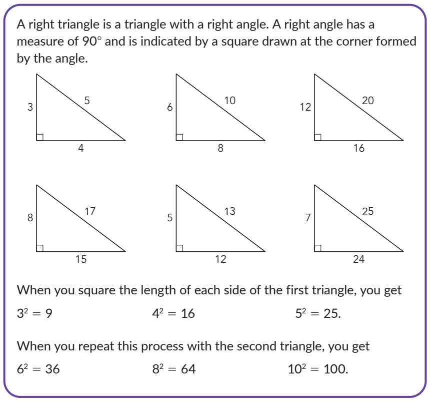
Where have we been?

In Grade 6, students developed their understanding of ratios. The next year, they determined characteristics of scenarios, tables, graphs, and equations of proportional relationships. Students review their prior knowledge of ratios and proportional relationships, including unit rate and the constant of proportionality.

Where are we going?

This topic links the conceptual understandings of ratios and proportional relationships that were established in earlier grades to a major focus of higher level mathematics: Functions. In the next topic, students will increase their familiarity and flexibility with determining slope and writing equations of linear relationships from different representations and in different forms.

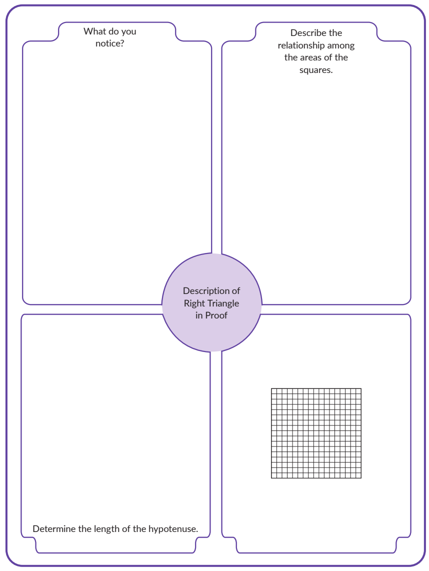
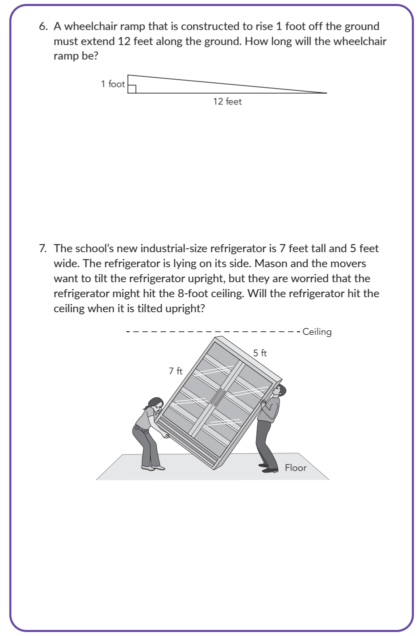
The instructional materials balance conceptual and procedural understandings. In this course, students' progress through a Concrete-Representational-Abstract (CRA) continuum to develop conceptual understanding and build toward procedural fluency.

<table>
<tr><th>Concrete</th><th>Representational</th><th>Abstract</th></tr>
<tr>
<td>Students square the side lengths of right triangles and recognize that the sum of the squares of the two shorter sides equals the square of the longest side.

A right triangle is a triangle with a right angle. A right angle has a measure of 90° and is indicated by a square drawn at the corner formed by the angle.

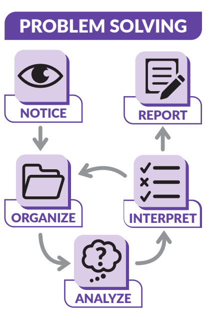
When you square the length of each side of the first triangle, you get

$3^2 = 9$ $4^2 = 16$ $5^2 = 25$.

When you repeat this process with the second triangle, you get

$6^2 = 36$ $8^2 = 64$ $10^2 = 100$.</td>
<td>Students use cutouts and guided directions to explain proofs of the Pythagorean Theorem.

</td>
<td>Students apply the Pythagorean Theorem to solve for the hypotenuse or leg length of a right triangle in mathematical and real-world problems.

6. A wheelchair ramp that is constructed to rise 1 foot off the ground must extend 12 feet along the ground. How long will the wheelchair ramp be?

7. The school's new industrial-size refrigerator is 7 feet tall and 5 feet wide. The refrigerator is lying on its side. Mason and the movers want to tilt the refrigerator upright, but they are worried that the refrigerator might hit the 8-foot ceiling. Will the refrigerator hit the ceiling when it is tilted upright?
</td>
</tr>
</table>



Support is provided to students as they persevere in problem solving. These instructional materials include a problem-solving model, which includes questions your student can ask when productively engaging in real-world and mathematical problems. Prompts will encourage your student to use the problem-solving model throughout the course.

These instructional materials include features that support learners. Worked Examples throughout the product provide explicit instruction and provide a model your student can continually reference.

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think, about the connection between steps.

WORKED EXAMPLE

You can use the slope and y-intercept to determine the equation of a linear relationship.

- First, determine the slope and the *y*-intercept. — $m = 3$; *y*-intercept: (0, 1)
- Next, substitute the slope and *y*-intercept into the slope-intercept form of a line, $y = mx + b$. — $y = mx + b$; $y = 3x + 1$

The equation is $y = 3x + 1$.

Thumbs Up, Thumbs Down, and Who's Correct questions address your student's common misconceptions and provide opportunities for peer work analysis.

Who's Correct

When you see a Who's Correct icon:

- Take your time to read through the correct solution.
- Think about the connection between steps.
- Determine if correct or incorrect.

Ask Yourself

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

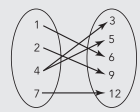
Isabella

The equation $y^2 = x$ represents a function.

x	y
4	2
9	3
25	5

Ethan

My mapping represents a function.



Who's Correct

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or incorrect.

Ask Yourself

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

8. Nakota says that the mean absolute deviation for the points each play scores represents the number of points you should expect each player to score during a game. Is Nakota correct? Explain your reasoning.

Targeted Skills Practice provides support to your student as they work to gain proficiency of the course material. Spaced Practice provides a spaced retrieval of key concepts to your student. Extension opportunities provide challenges to accelerate your student's learning.

Skills Practice

TOPIC 1 From Proportions to Linear Relationships

Name ______________________ Date ____________

I. Representations of Proportional Relationships

Topic Practice

A. At a large company, the total number of employees varies directly with the number of employees that drive to work. The company determines that two out of every three of its total employees drive to work. The other employees walk or use public transportation. Use the information to determine each number of employees.

1. The number of total employees if there are 150 employees that drive to work
2. The number of total employees if there are 85 employees that do not drive to work
3. The number of employees that do not drive to work if there are 240 total employees
4. The number of employees that drive to work if there are 570 total employees

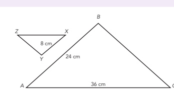
TOPIC 1 From Proportions to Linear Relationships

Extension

Consider the relationship between the side length of a square and the area of the square. Does this represent a proportional relationship? Use a table of values, equation, and graph to justify your answer.

Spaced Practice

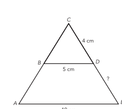
1. In the diagram, $\triangle ABC$, $\triangle XYZ$. State the corresponding sides and angles.



2. In the diagram, $\overline{BD} \parallel \overline{AE}$.

   a. Explain why $\triangle BDC \sim \triangle AEC$.

   b. Determine the length of $\overline{DE}$.



Each lesson features one or more ELPS (English Language Proficiency Standards) and provides the teacher with implementation strategies incorporating best practices for supporting language acquisition. In addition, students are provided with cognates for New Key Terms in the Topic Summaries and Topic Family Guides.

NEW KEY TERMS

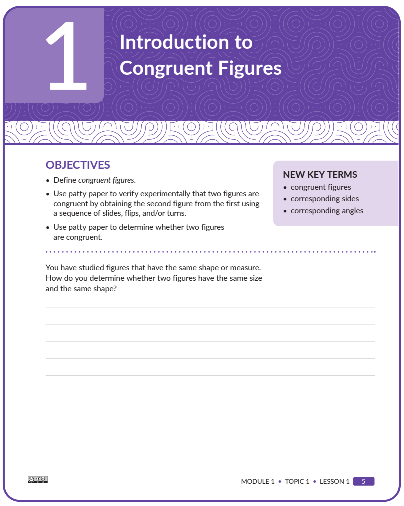
- congruent figures [figuras congruentes]
- corresponding sides
- corresponding angles [ángulos correspondientes]
- plane [plano]
- transformation [transformación]
- rigid motion [movimiento rígido/directo/propio]
- pre-image [preimagen]
- image [imagen]
- translation [traslación]
- reflection [reflexión]
- line of reflection [línea de reflexión]
- rotation [rotación]
- center of rotation [centro de rotación]
- angle of rotation [ángulo de rotación]
- congruent line segments [segmentos de línea congruentes]
- congruent angles [ángulos congruentes]

Refer to the Math Glossary for definitions of the New Key Terms.

Engaging with Grade-Level Content

Your student will engage with grade-level content in multiple ways with the support of the teacher.

Learning Together	Learning Individually
The teacher facilitates active learning of lessons so that students feel confident in sharing ideas, listening to each other, and learning together. Students become creators of their mathematical knowledge.	Skills Practice provides students the opportunity to engage in additional skill building that aligns to each Learning Together lesson. The Learning Individually Days target discrete skills that may require additional practice to achieve proficiency.



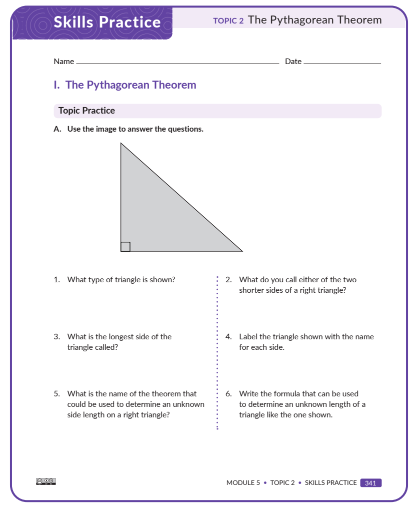
1 Introduction to Congruent Figures

OBJECTIVES
- Define *congruent figures*.
- Use patty paper to verify experimentally that two figures are congruent by obtaining the second figure from the first using a sequence of slides, flips, and/or turns.
- Use patty paper to determine whether two figures are congruent.

NEW KEY TERMS
- congruent figures
- corresponding sides
- corresponding angles

You have studied figures that have the same shape or measure. How do you determine whether two figures have the same size and the same shape?

MODULE 1 • TOPIC 1 • LESSON 1 5



Skills Practice TOPIC 2 The Pythagorean Theorem

Name ____ Date ____

I. The Pythagorean Theorem

Topic Practice

A. Use the image to answer the questions.

1. What type of triangle is shown?
2. What do you call either of the two shorter sides of a right triangle?
3. What is the longest side of the triangle called?
4. Label the triangle shown with the name for each side.
5. What is the name of the theorem that could be used to determine an unknown side length on a right triangle?
6. Write the formula that can be used to determine an unknown length of a triangle like the one shown.

MODULE 5 • TOPIC 2 • SKILLS PRACTICE 341

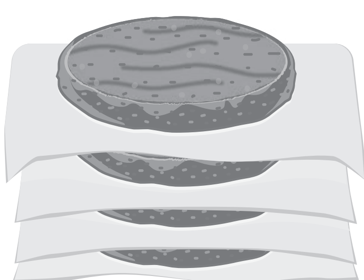
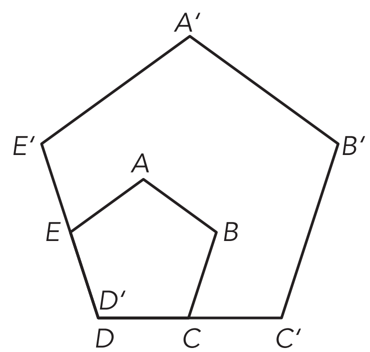
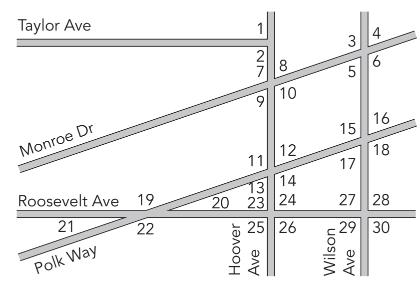
At the end of each topic, your student will take an assessment aligned to the standards covered in the topic. This assessment consists of multiple-choice, multi-select, and open-ended questions designed for your student to demonstrate learning. Each assessment also includes a scoring guide for teachers to ensure consistent scoring. The scoring guide includes ways to support or challenge your student based on their responses to the questions on the assessment. The purpose of the assessment is for the teacher and student to reflect on the learning. Teachers will use your student's assessment results to target individual skills your student needs for proficiency or to accelerate and challenge your student.

Response to Student Performance		
TEKS*	Question(s)	Recommendations
8.10A	2, 6	To support students: • Review Congruence in Motion. • Use Skills Practice Sets I.A, III.A, IV.A, and V.A for additional practice. • Review Lesson 2 Assignment Practice Question 2.
8.10B	5	To support students: • Review Verifying Congruence Using Rigid Motions. • Use Skills Practice Set VI.A, VI.B, and VI.C for additional practice. • Review Lesson 2 Assignment Practice Question 2.
8.10C	1, 3, 4, 7–10	To support students: • Review Verifying Congruence Using Rigid Motions. • Use Skills Practice Sets III.B, IV.B, V.B, and VI.C for additional practice. • Review Lesson 2 Assignment Practice Question 2. To challenge students: • Extend student knowledge with the Skills Practice Extension.
NOTE: Both teachers and administrators should refer to the Assessment Guidance and Analysis section of the Course and Implementation Guide for additional support in analyzing and responding to student data.		

*Bold TEKS = Readiness Standard

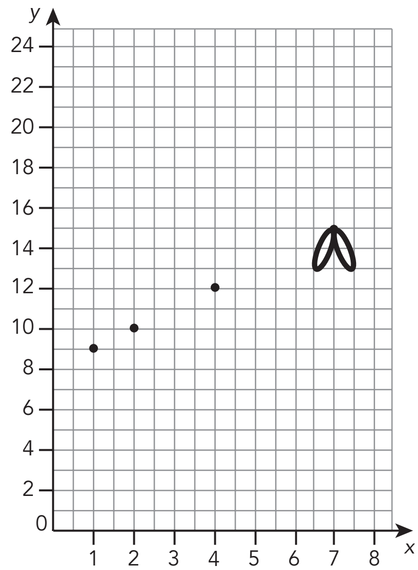
MODULE 1 Transforming Geometric Objects

In this module, your student will develop their understanding of congruence and similarity. There are three topics in this module: *Rigid Motion Transformations*, *Similarity*, and *Line and Angle Relationships*. Your student will use what they already know about geometric objects in this module.

TOPIC 1 Rigid Motion Transformations	TOPIC 2 Similarity	TOPIC 3 Line and Angle Relationships
Your student will use patty paper and the coordinate plane to study the creation of congruent figures with translations, reflections, and rotations.	Your student will study dilations and similarity.	Your student will use their knowledge of transformations, congruence, and similarity to understand the Triangle Sum Theorem, the Exterior Angle Theorem, relationships between angles formed when a transversal cuts parallel lines, and the Angle-Angle Similarity Theorem.
Did you know?  Patty paper separates patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool. You can write on it, trace with it, and see creases when you fold it.	Did you know?  A **dilation** is a transformation that produces a figure that is the same shape as the original figure but not necessarily the same size.	What in the world?  Many city streets are parallel to each other. When another street or multiple streets cross through the parallel roads, special angle relationships are formed. We see these angles at the intersection of the streets. Visualize a 3- or 4-way stop.

MODULE 2 Developing Function Foundations

In this module, your student will deepen their understanding of proportional relationships, lines, and linear equations. There are two topics in this module: *From Proportions to Linear Relationships* and *Linear Relationships*. Your student will use what they already know about ratio and proportional relationships in this module.

TOPIC 1 From Proportions to Linear Relationships	TOPIC 2 Linear Relationships
Your student will build on their knowledge of ratio and proportional relationships to develop connections between proportional relationships, lines, and linear equations.	Your student will develop fluency with analyzing linear relationships, writing equations of lines, and graphing lines. They will define *function* and explore their various representations.
Did you know? Linear equations can be seen on the Rhind Papyrus from Egypt, which was written almost 3700 years ago. 	Did you know? René Descartes had the idea for his graphing method after watching a fly on his ceiling and thinking about how he would tell someone its location. 

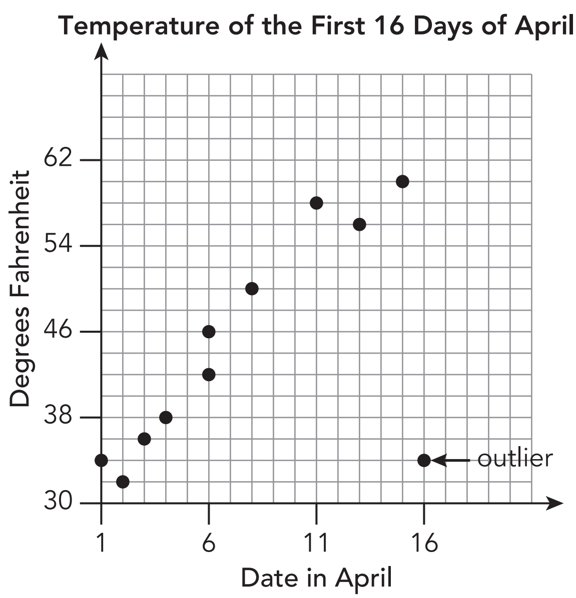
MODULE 3 Data Data Everywhere

In this module, your student will deepen their understanding of analyzing data. There are two topics in this module: *Patterns in Bivariate Data* and *Variability and Sampling*. Your student will use what they already know about statistics.

<table>
<tr><th>TOPIC 1
Patterns in Bivariate Data</th><th>TOPIC 2
Variability and Sampling</th></tr>
<tr><td>Your student will review the statistical process and investigate associations in bivariate data.</td><td>Your student will compare data sets using measures of variability, interquartile range, and mean absolute deviation.</td></tr>
<tr><td>

Did you know?

An **outlier** for bivariate data is a point that varies greatly from the overall pattern of the data.

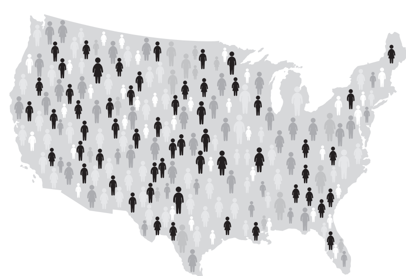


Can you think of an example of something that does not fit with a pattern in the real world?

</td><td>

What in the world?

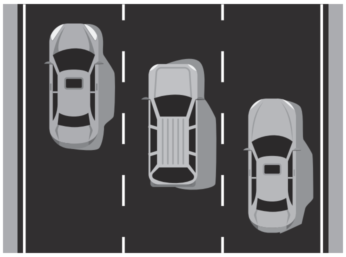
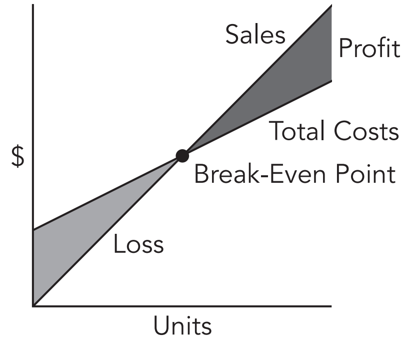
Every 10 years, the U.S. government takes a **census** of the population. They send people to every household to ask questions and count the number of people living in your area. They use this information to decide how much money they will use to build roads, schools, and other important things for the community.



</td></tr>
</table>

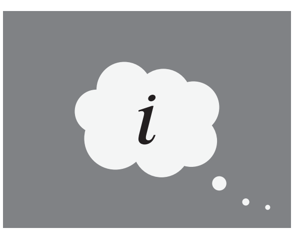
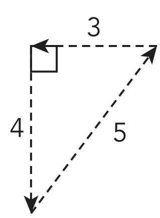
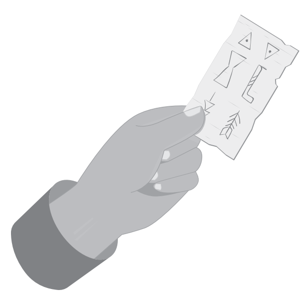
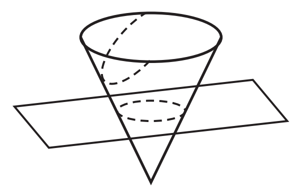
MODULE 4 Modeling Linear Equations

In this module, your student will deepen their understanding of linear equations. There are two topics in this module: *Solving Linear Equations and Inequalities* and *Systems of Linear Equations*. Your student will use what they already know about solving and graphing equations in this module.

TOPIC 1 Solving Linear Equations and Inequalities	TOPIC 2 Systems of Linear Equations
Your student will solve more kinds of one-variable linear equations.	Your student will analyze and solve pairs of linear equations by graphing.
Try it! Linear equations are useful in calculating average speed and distance over time. If you are driving 60 miles per hour for 2 hours, how many miles would you travel? You can substitute the number 2 for the variable t in the linear equation $d = 60t$ to solve for the answer.  [*Answer:* $d = 60(2) = 120$ *miles*]	Did you know? Systems of linear equations can help when starting your own business. They can be used to model supply and demand and to calculate when you will start making a profit by determining the break-even point. 

MODULE 5 Applying Powers

In this module, your student will deepen their understanding of numbers involving powers, including scientific notation and square roots. There are four topics in this module: *Real Numbers*, *The Pythagorean Theorem*, *Financial Literacy: Your Financial Future*, and *Volume of Curved Figures*. Your student will use what they already know about the real number system in this module.

TOPIC 1 Real Numbers	TOPIC 2 The Pythagorean Theorem	TOPIC 3 Financial Literacy: Your Financial Future	TOPIC 4 Volume of Curved Figures
Your student will build on their knowledge of number systems to include the set of irrational numbers.	Your student will explore the Pythagorean Theorem and its converse.	Your student will work with multiple representations of simple and compound interest scenarios.	Your student will solve problems involving surface area of prisms and cylinders, as well as volume of cylinders, cones, and spheres.
Did you know? We use the term *real numbers* because in future math courses, you will learn about *imaginary numbers*. 	Try it! The Pythagorean Theorem can be used to find the distance between two points. If you walk 3 feet to the west and 4 feet to the south, your distance from your original location is 5 feet because $3^2 + 4^2 = 5^2$. 	Interesting . . . Humans have used compound interest for over 4000 years, and the interest rate of 20% used during those times is about the same as credit cards today. 	Did you know? By slicing a cone, you can make several shapes that are studied in algebra. 

Lesson Structure

Each lesson in the course is laid out in the same way to develop deep understanding. Read through the parts of the lesson to learn more about your student's learning in their math classroom.

Objectives & Essential Question

Each lesson begins with objectives, listed to help students understand the objectives. Also included is an essential statement connecting students' learning with a question to ponder. The question is asked again at the end of each lesson to see how much your student understands.

Getting Started

The Getting Started engages your student in the learning. In the Getting Started, your student uses what they already know about the world, what they've already learned, and their intuition to get them thinking mathematically and prepare them for what's to come in the lesson.

Activities

In the Activities, students develop their mathematical knowledge and build a deep understanding of the math. These activities provide your student with the opportunity to communicate and work with others in their math classroom.

When your student is working through these activities, keep in mind:

- It's not just about answer-getting. Doing the math and talking about it is important.
- Making mistakes is an important part of learning, so take risks.
- There is often more than one way to solve a problem.

Talk the Talk

The Talk the Talk gives your student an opportunity to reflect on the main ideas of the lesson and demonstrate their learning.

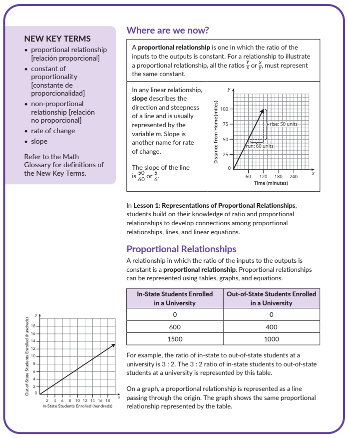
Lesson Assignment

The lesson assignment provides your students with practice to develop fluency and build proficiency. The lesson assignment also includes a section to help prepare students for the next lesson.

Key Concepts of the Lesson

At the end of each topic, the Topic Summary provides a summary of each lesson in the topic. Encourage your student to use these as a tool to review and retrieve the key concepts of a lesson.

Supporting Your Student



The Topic Family Guide

The Topic Family Guide provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides an example of a math model or strategy your student is learning in the topic, busting of a math myth, talking points to discuss and/or questions to ask your student, and all the new key terms your student will learn. You and your student can also use the Math Glossary to check terminology and definitions. Encourage your student to reference the new key terms in the Topic Family Guide and/or Math Glossary when completing math tasks.

Learning outside of the classroom is crucial for your student's success. While we don't expect you to be a math teacher, the Topic Family Guide can assist you as you talk to your student about the mathematical content of the course. The hope is that both you and your student will read and benefit from the guides.

TALKING POINTS

Discuss With Your Student

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to think about linear relationships as objects that can be analyzed, graphed, and compared.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Math Myths

Math myths can lead students and adults to believe that math is too difficult for them, math is an unattainable skill, or that there is only one right way to do mathematics. The Math Myths section in the Topic Family Guides busts these myths and provides research-based explanations of why math is accessible to all students (and adults).

Examples of these myths include:
Myth: Just give me the rule. If I know the rule, then I understand the math.

Memorize the following rule: *All quars are elos*. Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms*. How about now? Can you remember that? Of course you can, because now, it makes sense. Learning does not take place in a vacuum. It must be connected to what you already know. Otherwise, arbitrary rules will be forgotten.

MYTH

"Asking questions means you don't understand."

It is universally true that for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. However, there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more on a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, everyone should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

Supporting Your Student

Myth: There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. When one road is backed up, you can always take a different route. When you know only one route, you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying, "Well, that's one way to do it. Is there another way? What are the pros and cons?" That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

TEKS Mathematical Process Standards

Each module will focus on TEKS mathematical process standards that will help your student become a mathematical thinker. The TEKS mathematical process standards are listed below. Discuss with your student the "I can" statements below the standards to help them develop their mathematical learning and understanding. With your help, your student can become a productive mathematical thinker.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it when necessary.
- ask useful questions to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology, as appropriate; and techniques including mental math, estimation, and number sense, as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions when trying to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:
- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:
- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Reflecting on Learning and Progress

You can support your student by encouraging your student to reflect on the learning process. The instructional resources include a student Topic Self-Reflection for each topic. Encourage your student to accurately and frequently reflect on learning and progress throughout each topic. Talk about the specific concepts in the Topic Self-Reflection with your student and celebrate the progress from the beginning to the end of the Topic. Remind your student to refer to the Topic Self-Reflection on Learning Individual Days after targeting specific skills and concepts. You can have your student explain concepts from the self-reflection using the topic summaries or lesson assignments to demonstrate understanding. In addition, encourage your student to reflect after taking an assessment. An Assessment Reflection is available to your student to assist with this process. Encourage your student to consider what went well and how to prepare for the next assessment. Ask your student how you can support them when preparing for the next assessment.

TOPIC 1 SELF-REFLECTION

Name: ____________________

Solving Linear Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Solving Linear Equations* topic by:

TOPIC 1: *Solving Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
using properties of equality to write equivalent expressions.			
writing one-variable equations or inequalities with variables on both sides of the equal sign or inequality symbol that represent real-world problem situations.			
writing a corresponding real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign or inequality symbol.			
solving one-variable equations with variables on both sides of the equal sign.			
giving examples of linear equations in one variable that have one solution, no solution, or infinitely many solutions.			
solving one-variable inequalities with variables on both sides of the inequality symbol.			

continued on the next page

Thanks!

Enjoy the fun mathematical adventure that is ahead for you and your student! Remember the supports available to you and thank you for supporting your student's learning.

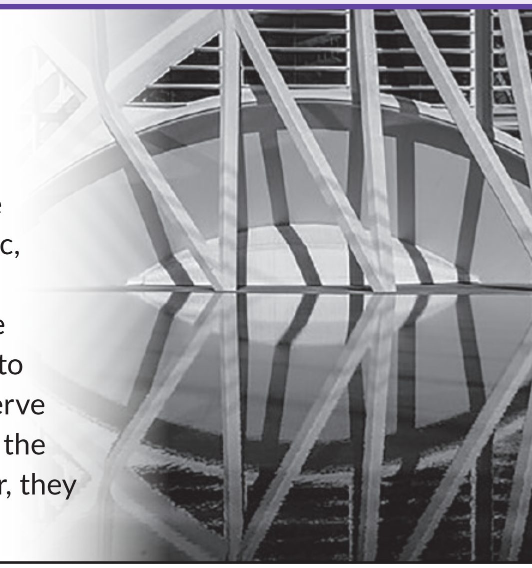
MODULE
1

Transforming Geometric Objects

Family Guide

MODULE 1 Transforming Geometric Objects Grade 8

TOPIC 1 Rigid Motion Transformations



In this topic, students use patty paper (thin, transparent paper) and the coordinate plane to investigate congruent figures. Throughout the topic, students are expected to make and investigate conjectures and justify true results about transformations. They learn that transformations are mappings of a plane and all the points of a figure on a plane according to a common action or operation. They also learn that rigid motions preserve the size and shape of a figure but reflections change the orientation of the vertices of a figure. Note: If students do not have access to patty paper, they can use parchment paper, tracing paper, or even white paper.

Where have we been?

Students review using patty paper to compare figures in a coordinate plane. They review how to compare side lengths and angle measures and how to locate the midpoint of a segment using patty paper. They sort figures according to shape and then according to size and shape. They use patty paper and informal transformation language to verify their sorts.

Where are we going?

This topic begins the study of congruence and sets the stage for similarity. In future courses, students will continue to formalize their knowledge of congruent triangles and use congruence to prove a wide variety of geometric theorems and justify constructions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is becoming familiar with movements (called *transformations*) of geometric figures and reasoning about these movements.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

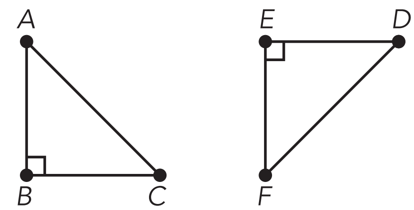
NEW KEY TERMS

- congruent figures [figuras congruentes]
- corresponding sides
- corresponding angles [ángulos correspondientes]
- plane [plano]
- transformation [transformación]
- rigid motion [movimiento rígido/directo/propio]
- pre-image [preimagen]
- image [imagen]
- translation [traslación]
- reflection [reflexión]
- line of reflection [línea de reflexión]
- rotation [rotación]
- center of rotation [centro de rotación]
- angle of rotation [ángulo de rotación]
- congruent line segments [segmentos de línea congruentes]
- congruent angles [ángulos congruentes]

Refer to the Math Glossary for definitions of the New Key Terms.

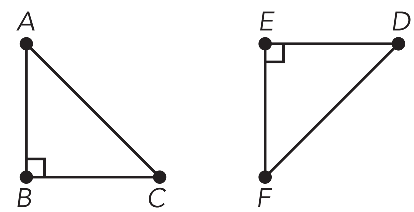
Where are we now?

Corresponding angles are angles that have the same relative positions in geometric figures.



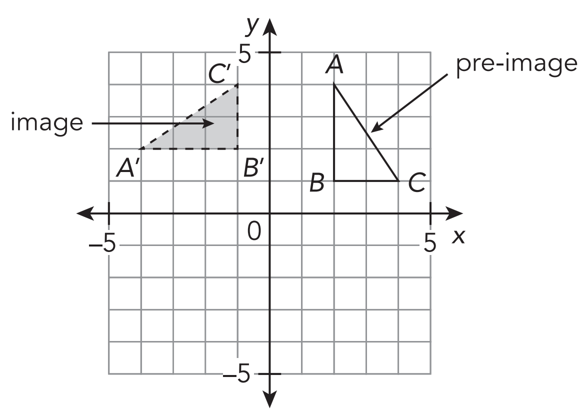
Angle *B* and angle *E* are corresponding angles.

Corresponding sides are sides that have the same relative positions in geometric figures.



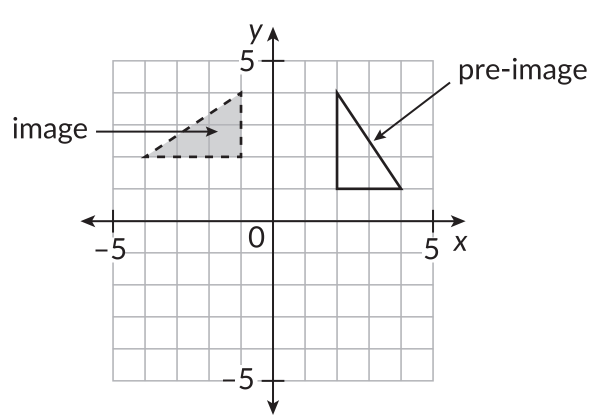
Sides *AB* and *DE* are corresponding sides.

The new figure created from a **transformation** is the **image**. The original figure is the **pre-image**.



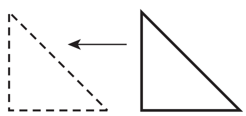
The **center of rotation** is the point around which you rotate a figure. The center of rotation can be a point on the figure, inside the figure, or outside the figure.

The image is a rotation of the pre-image 90° counterclockwise about the center of rotation, which is the origin (0, 0).

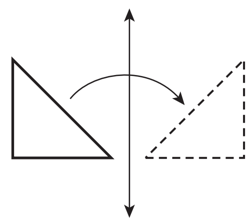


In **Lesson 2: Introduction to Rigid Motions**, students will use everyday language, like *slide*, *flip*, and *turn*, to describe how to map, or move, one figure onto another. In Lessons 3 through 5, students use the mathematical vocabulary of **rigid motion** transformations—translations, reflections, and rotations—and describe how a single rigid motion makes the same change between **congruent figures**. Students also learn that rigid motions preserve, or keep, the size and shape of a figure but reflections change the orientation, or position/direction, of a figure's vertices.

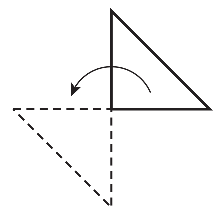
Transformations



translation



reflection



rotation

A **translation** is a rigid motion transformation that slides each point of a figure the same distance and direction along a line.

A **reflection** is a rigid motion transformation that flips a figure across a **line of reflection**.

A **rotation** is a rigid motion transformation that turns a figure on a **plane** about a fixed point.

MYTH

"I don't have the math gene."

Let's be clear about something. There isn't ***a*** gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to our ability to reason mathematically. Some believe mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without any formal instruction. They can use number sense and pattern recognition the same way.

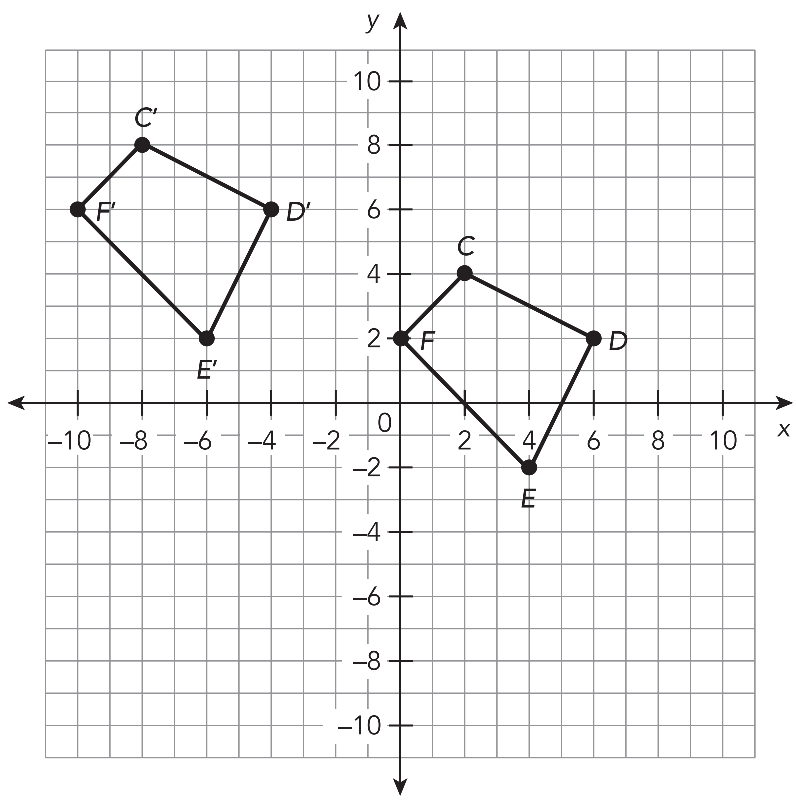
To further nurture your student's mathematical growth, attend to the learning environment. You can think of it as providing a nutritious mathematical diet that includes: discussing math in the real world, offering the right kind of encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and giving them space for plenty of practice.

#mathmythbusted

In **Lesson 3: Translations of Figures on the Coordinate Plane**, students solve problems in order to demonstrate, using translations, that two figures are congruent.

Verifying Congruence Using Translations

A translation "slides" a geometric figure in some direction. Translations can be used to verify, or check, that two figures are congruent. For example, Quadrilateral *CDEF* can be translated up 4 units and left 10 units. This will show that it is congruent to Quadrilateral *C′D′E′F′*.



Family Guide

MODULE 1 Transforming Geometric Objects Grade 8

TOPIC 2 Similarity

In this topic, students investigate dilations. They make connections between scale factors and dilation factors by examining Worked Examples of Euclidean dilations. They then define *similar figures*. Throughout the topic, students relate dilations to scale factors and scaling up and down. Next, students use dilations to map from a figure to a similar figure. Finally, students summarize the relationships between transformations and congruent and similar figures.

Where have we been?

In Grade 6, students reasoned extensively with ratios. They learned to scale ratios up and down to solve real-world and mathematical problems. In Grade 7, students connect the concepts of proportionality they learned in Grade 6 to scale drawings. This topic connects Grade 7 scale drawings with similarity. Students first review content about scale factors from Grade 7 and determine that after an enlargement or reduction, the ratios of corresponding side lengths are equal and the corresponding angles have the same measure. In topic 1, students studied rigid motion transformations. In this topic, students expand their understanding to dilations and use dilations to verify that figures are similar.

Where are we going?

The properties of similar figures are useful for solving real-world problems about scale factors. Students will use similar triangles to explain properties of the slope of a line. They will form similar right triangles on a line and use similarity and proportionality to explain why the slope of the line is the same between any two points on a non-vertical line in a coordinate plane.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to think about mathematical similarity and scaling.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

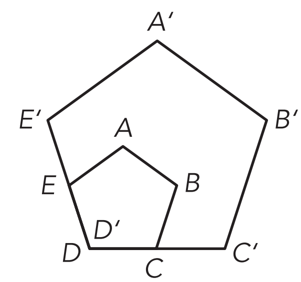
NEW KEY TERMS

- dilation [dilatación]
- center of dilation [centro de dilatación]
- scale factor [factor de escala]
- enlargement
- reduction [reducción]
- similar [similar/ semejante]

Refer to the Math Glossary for definitions of the New Key Terms.

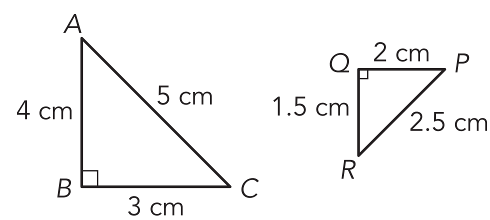
Where are we now?

In a **dilation**, the **scale factor** is the ratio of the distance of the new figure from the **center of dilation** to the distance of the original figure from the center of dilation.



Pentagon *ABCDE* has been dilated by a scale factor of 2 to create pentagon *A′B′C′D′E′*.

When two figures are **similar**, the ratios of their corresponding side lengths are equal.



Triangle *ABC* is similar to $\triangle PQR$.

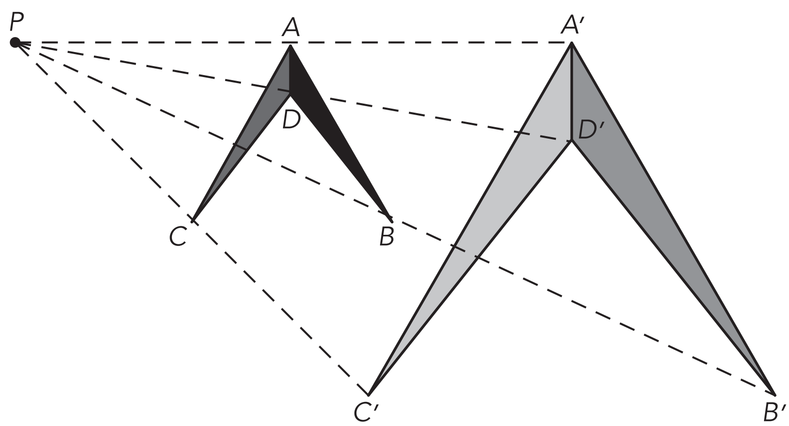
$\overline{AB} : \overline{PQ}$	$\overline{BC} : \overline{QR}$	$\overline{AC} : \overline{RP}$
4 cm : 2 cm	3 cm : 1.5 cm	5 cm : 2.5 cm
2 cm : 1 cm	2 cm : 1 cm	2 cm : 1 cm

In **Lesson 1: Dilations of Figures**, students will study dilations and similar figures.

Dilating Figures with a Scale Factor Greater Than 1

Students will relate dilations to scale factors and scaling up and down.

This image shows the **enlargement** of a logo using point *P* as the center of dilation.

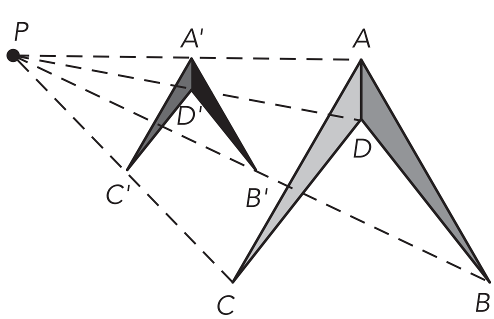


You can express the scale factor as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

When the scale factor is greater than 1, the new figure is an enlargement.

Dilating Figures with a Scale Factor Less Than 1 and Greater Than 0

The image shows a **reduction** of the original logo using point *P* as the center of dilation.



You can express the scale factor as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

When the scale factor is less than 1 and greater than 0, the new figure is a reduction.

MYTH

"If I can get the right answer, then I should not have to explain why."

Sometimes you get the right answer for the wrong reasons. Suppose a student is asked, "What is 4 divided by 2?" and she confidently answers "2!" If she does not explain any further, then it might be assumed that she understands how to divide whole numbers. But what if she used the following rule to solve that problem? "Subtract 2 from 4 one time." Even though she gave the right answer, she has an incomplete understanding of division.

However, if she is asked to explain her reasoning by drawing a picture, creating a model, or giving a different example, the teacher has a chance to remediate her flawed understanding. If teachers aren't exposed to their students' reasoning for both right and wrong answers, then they won't know about or be able to address misconceptions. This is important because mathematics is cumulative in the sense that new lessons build upon previous understandings.

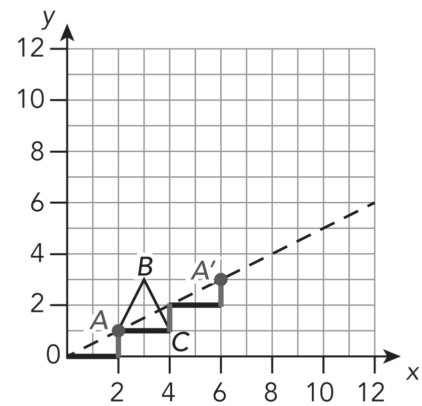
You should ask your student to explain their thinking, when possible, even if you don't know whether the explanation is correct. When students (and adults!) explain something to someone else, it helps them learn. Just the process of trying to explain is helpful.

#mathmythbusted

In **Lesson 2: Dilating Figures on the Coordinate Plane,** students build dilations on the coordinate plane as repeated geometric translations using the origin as the center of dilation.

Scaling Up on the Coordinate Plane

A sequence of repeated horizontal and/or vertical translations also moves a point along a line. You can use this fact to dilate figures.



For example, to dilate $\triangle ABC$ by a scale factor of 3 using the origin (0, 0) as the center of dilation, start by dilating point A, located at (2, 1).

Point A is 2 units right and 1 unit up from the origin.

To dilate point A by a scale factor of 3, translate point A by three repeated sequences: **2 units right** and **1 unit up**.

The dashed line helps you see that point A' is a dilation of point A by a factor of 3.

TOPIC 3 Line and Angle Relationships

In this topic, students use their knowledge of transformations, congruence, and similarity to informally establish the Triangle Sum theorem, the Exterior Angle theorem and the relationships formed between angles when parallel lines are cut by a transversal, and the Angle-Angle Similarity theorem for similarity of triangles. Students determine and informally prove the relationships between the special angle pairs formed when parallel lines are cut by a transversal and use these relationships to solve mathematical problems, including writing and solving equations.

Where have we been?

Students use knowledge from Grade 7 about supplementary angles and rigid motion transformations when proving theorems in this topic and when exploring the angle relationships formed when parallel lines are cut by a transversal.

Where are we going?

Throughout this topic, students are expected to follow lines of logic to reach conclusions, which is a foundation for formal proof in future courses. The geometric results established in this topic via informal arguments will be formally proven in future courses, but students' experiences in this topic provide them with opportunities to build intuition and justify results.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to think about similar triangles as well as different line and angle theorems from geometry.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

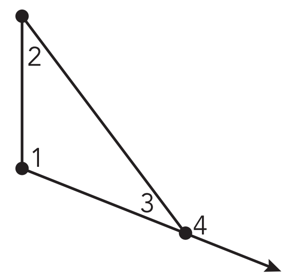
NEW KEY TERMS

- Triangle Sum theorem [teorema de la suma del triángulo]
- exterior angle of a polygon [ángulo exterior de un polígono]
- remote interior angles of a triangle [ángulos interiores remotos (no adyacentes) de un triángulo]
- Exterior Angle theorem [teorema del Ángulo Exterior]
- transversal [transversal]
- alternate interior angles [ángulos interiores alternos]
- alternate exterior angles [ángulos exteriores alternos]
- same-side interior angles
- same-side exterior angles
- Angle-Angle (AA) Similarity theorem [teorema de Similitud (Semejanza) Ángulo-Ángulo (AA)]

Refer to the Math Glossary for definitions of the New Key Terms.

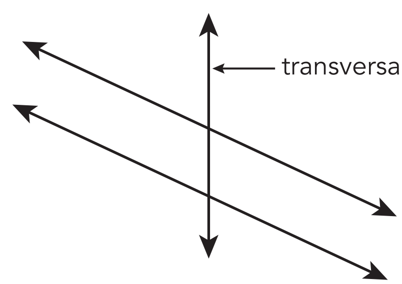
Where are we now?

The **remote interior angles** of a triangle are the two angles that are non adjacent to the specified exterior angle.

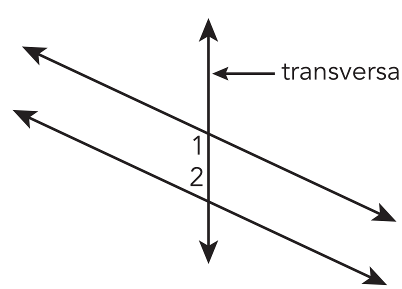


Angles 1 and 2 are remote interior angles of a triangle with respect to $\angle 4$.

A **transversal** is a line that intersects two or more lines at distinct points.



Same-side interior angles form when a transversal intersects two other lines. These angle pairs are on the same side of the transversal and are between the other two lines.



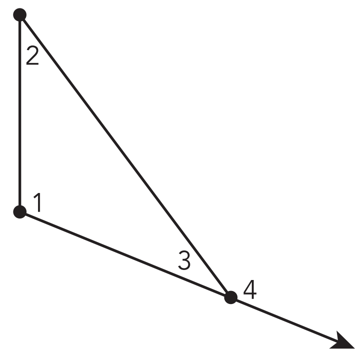
Angles 1 and 2 are same-side interior angles.

In **Lesson 1: Exploring Angle Theorems**, students identify exterior angles and remote interior angles of triangles and explore the relationship between among angles to establish the Exterior Angle theorem.

Exterior Angle Theorem

An **exterior angle of a polygon** is an angle between a side of a polygon and the extension of its adjacent side. You can extend a ray from one side of the polygon to form an exterior angle.

In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are interior angles of the triangle and $\angle 4$ is an exterior angle of the triangle.



If $\angle 1$ and $\angle 2$ are remote interior angles and $\angle 4$ is an exterior angle, then:

$m\angle 1 + m\angle 2 = m\angle 4$.

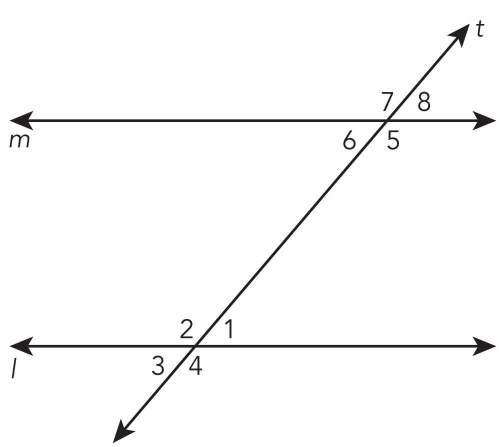
There are different angle relationships found in triangles and between intersecting lines. One relationship between the 3 interior angles of a triangle is represented by the equation $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

Another is between the 2 remote interior angles and their corresponding exterior angle as represented by the equation $m\angle 1 + m\angle 2 = m\angle 4$.

In **Lesson 2: Exploring the Angles Formed by Lines Intersected by a Transversal**, students explore the angles formed when two lines are intersected by a transversal.

Angle Relationships

A transversal is a line that intersects, or crosses, two or more lines. When the two lines intersected by a transversal are parallel, special relationships between the angle measurements form. In this diagram, two parallel lines, *m* and *l*, are intersected by a transversal, *t*.



![MYTH]

MYTH

"Asking questions means you don't understand."

It is universally true that for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. However, there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

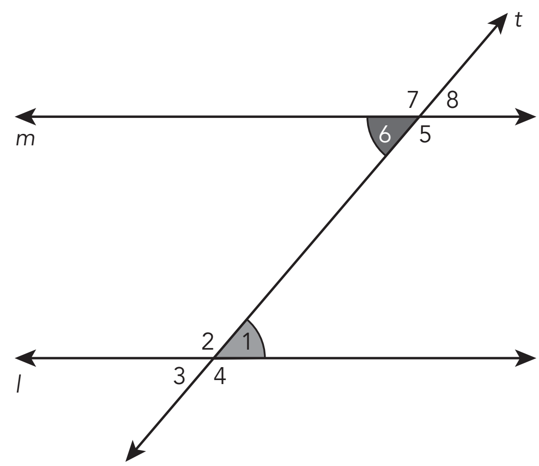
Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more on a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, everyone should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

Special Angle Pairs

Corresponding angles have the same relative positions in geometric figures. An example of corresponding angles are ∠2 and ∠7.

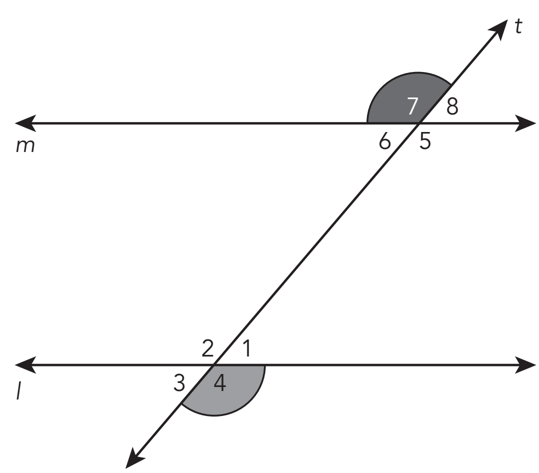
Alternate interior angles



Alternate interior angles are on opposite sides of the transversal and are between the two other lines.

An example of alternate interior angles are ∠1 and ∠6.

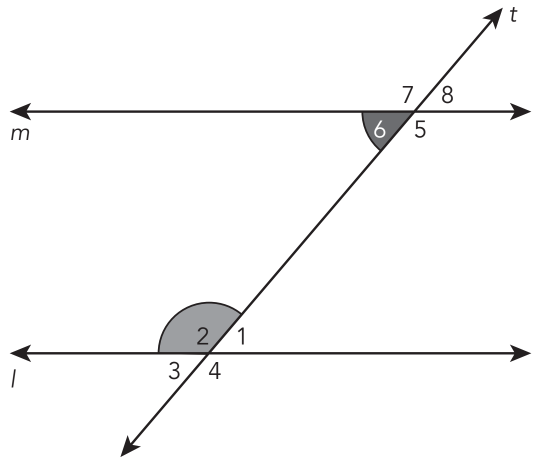
Alternate exterior angles



Alternate exterior angles are on opposite sides of the transversal and are outside the other two lines.

An example of alternate exterior angles are ∠4 and ∠7.

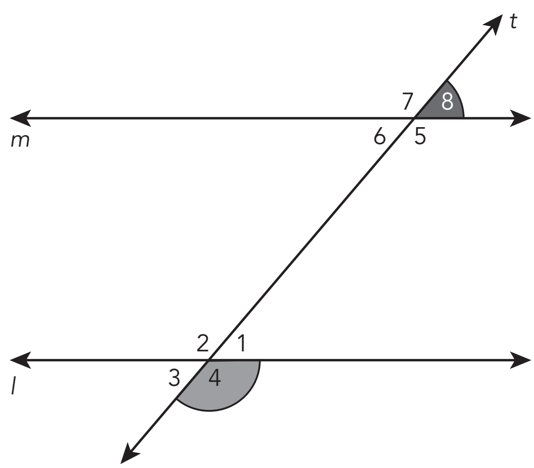
Same-side interior angles



Same-side interior angles are on the same side of the transversal and are between the other two lines.

An example of same-side interior angles are ∠2 and ∠6.

Same-side exterior angles



Same-side exterior angles are on the same side of the transversal and are outside the other two lines.

An example of same-side exterior angles are ∠4 and ∠8.

MODULE 2

Developing Function Foundations

TOPIC 1 From Proportions to Linear Relationships

In this topic, students build on their knowledge of ratio and proportional relationships to develop connections between proportional relationships, lines, and linear equations. Students compare proportional relationships represented in different ways to ensure a firm understanding of the meaning of proportionality. Students then use similar triangles to explain why the slope of a line is always the same between any two points on the line.

Where have we been?

In Grade 6, students developed their understanding of ratios. The next year, they determined characteristics of scenarios, tables, graphs, and equations of proportional relationships. Students review their prior knowledge of ratios and proportional relationships, including unit rate and the constant of proportionality.

Where are we going?

This topic links the conceptual understandings of ratios and proportional relationships that were established in earlier grades to a major focus of higher level mathematics: Functions. In the next topic, students will increase their familiarity and flexibility with determining slope and writing equations of linear relationships from different representations and in different forms.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking them to take a step back and think about a different strategy when they are stuck.

QUESTIONS TO ASK

- What strategy are you using?
- What is another way to solve the problem?
- Can you draw a model?
- Can you come back to this problem after doing some other problems?

NEW KEY TERMS

- proportional relationship [relación proporcional]
- constant of proportionality [constante de proporcionalidad]
- non-proportional relationship [relación no proporcional]
- rate of change
- slope

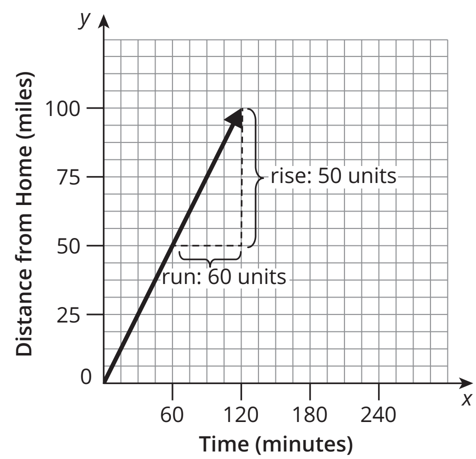
Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **proportional relationship** is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$, must represent the same constant.

In any linear relationship, **slope** describes the direction and steepness of a line and is usually represented by the variable *m*. Slope is another name for rate of change.

The slope of the line is $\frac{50}{60}$ or $\frac{5}{6}$.



In **Lesson 1: Representations of Proportional Relationships**, students build on their knowledge of ratio and proportional relationships to develop connections among proportional relationships, lines, and linear equations.

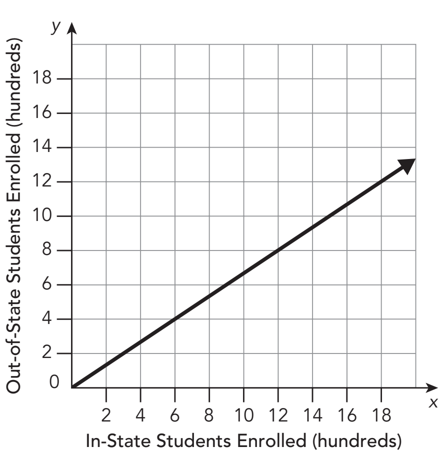
Proportional Relationships

A relationship in which the ratio of the inputs to the outputs is constant is a **proportional relationship**. Proportional relationships can be represented using tables, graphs, and equations.

In-State Students Enrolled in a University	Out-of-State Students Enrolled in a University
0	0
600	400
1500	1000

For example, the ratio of in-state to out-of-state students at a university is 3 : 2. The 3 : 2 ratio of in-state students to out-of-state students at a university is represented by this table.

On a graph, a proportional relationship is represented as a line passing through the origin. The graph shows the same proportional relationship represented by the table.



The equation for a proportional relationship is written in the form $y = kx$, where x represents an input value, y represents an output value, and k represents some constant that is not equal to 0. The constant k is called the **constant of proportionality**. The 3 : 2 ratio of in-state students to out-of-state students at a university is represented by the equation $y = \frac{2}{3}x$. The constant of proportionality is $\frac{2}{3}$.

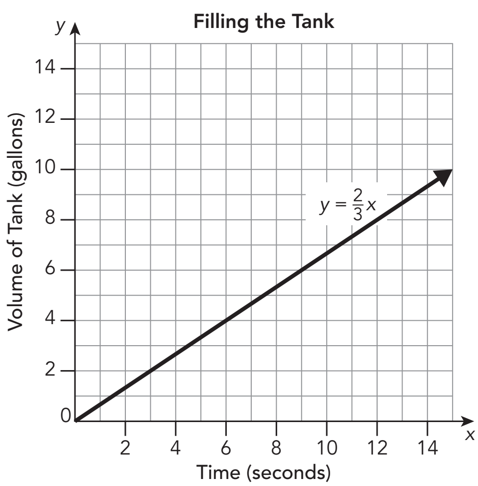
In **Lesson 2: Using Similar Triangles to Describe the Steepness of a Line**, students connect unit rate, constant of proportionality, and scale factor with the concept of rate of change.

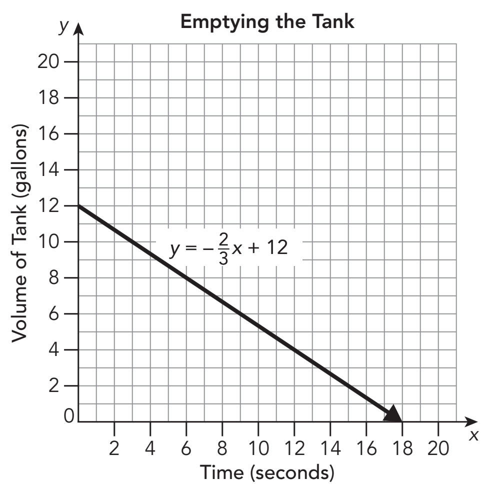
Rate of Change

The **rate of change** for a situation is the amount that the dependent quantity changes compared with the amount that the independent quantity changes.

The sign of the slope indicates the direction of a line. If the slope of a line is positive, then the graph will increase from left to right. If the slope of a line is negative, then the graph will decrease from left to right.

For example, consider the two graphs. The first represents a tank being filled at $\frac{2}{3}$ gallon per second. The second represents the tank being emptied at $\frac{2}{3}$ gallon per second, starting at 12 gallons.





The slope of the line representing the tank being filled is $\frac{2}{3}$. The slope of the line representing the tank being emptied is $-\frac{2}{3}$.

MYTH

Myth: There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. If one road is backed up, then you can always take a different route. If you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: *Well, that's one way to do it. Is there another way? What are the pros and cons?* That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work, or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

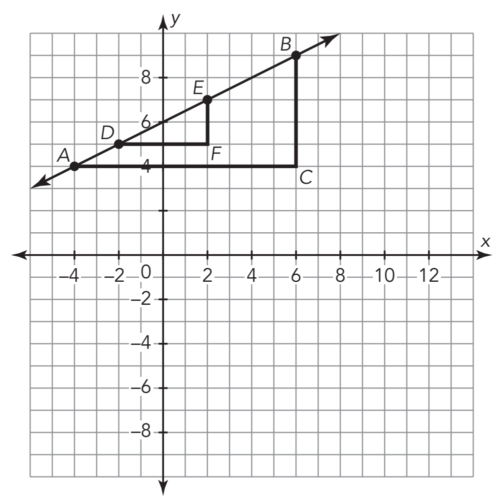
#mathmythbusted

In **Lesson 3: Exploring Slopes Using Similar Triangles**, students use similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line.

Slopes of Similar Triangles

The properties of similar triangles can be used to explain why the slope, m, is the same between any two points on a non-vertical line on the coordinate plane.

For example, points A, B, D, and E along the graphed line can be used to create two right triangles on the coordinate plane.

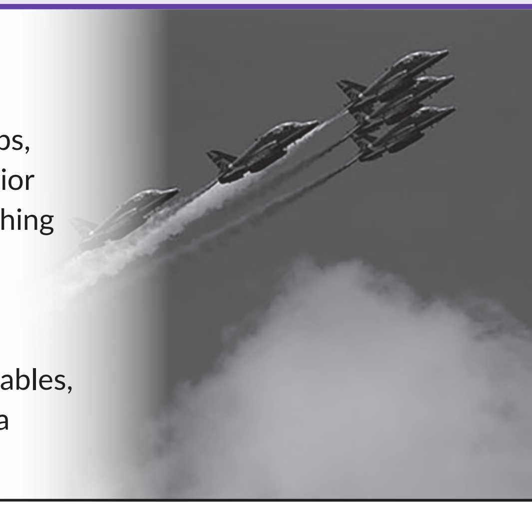


In both triangles, the ratio of the vertical distance to the horizontal distance is $\frac{1}{2}$. The slope of the line is the same between points A and B and between points D and E.

Family Guide

MODULE 2 Developing Function Foundations Grade 8

TOPIC 2 Linear Relationships



In this topic, students develop fluency with analyzing linear relationships, writing equations of lines, and graphing lines. They use intuition and prior knowledge about writing equations, creating tables of values, and graphing equations to compare two linear relationships. Students determine the y-intercept of linear relationships from tables, two points, graphs, and contexts. They graph lines presented in slope-intercept form. Students explore functions in terms of mappings, sets of ordered pairs, graphs, tables, verbal descriptions, and equations. They learn the formal definition of a function and analyze functions and relations.

Where have we been?

Students come into this topic with an understanding of slope as a unit rate of change and as a ratio of vertical change to horizontal change. They have experiences in representing proportional relationships with tables, graphs, and equations.

Where are we going?

This topic provides the foundation for students' algebraic fluency with determining and using equations of linear relationships. Beyond this course, students should understand that different forms of an equation can shed light on a problem situation, reveal characteristics of the function or equation, and make graphing more efficient.

TALKING POINTS

Discuss With Your Student

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to think about linear relationships as objects that can be analyzed, graphed, and compared.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- first differences [primeras diferencias]
- *y*-intercept [intersección con el eje *y*]
- slope-intercept form
- mapping [mapeo o aplicación]
- set
- relation [relación]
- input
- output
- function [función]
- scatterplot
- vertical line test [Prueba de la línea vertical]
- linear function [función lineal]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

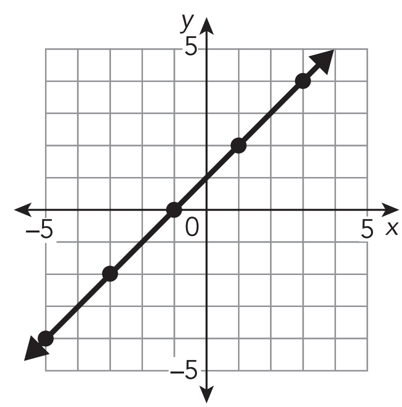
First differences are the values determined by subtracting consecutive *y*-values in a table when the *x*-values are consecutive integers. When the first differences are equal, the points represented by the ordered pairs in the table will form a straight line.

x	*y*	First differences
1	25	
		9
2	34	
		11
3	45	

The first differences are 9 and 11, so the points represented by these ordered pairs will not form a straight line.

The ***y*-intercept** is the *y*-coordinate of the point where a graph crosses the *y*-axis. The *y*-intercept can be written in the form (0, *b*).

The *y*-intercept of the graph is (0, 1).



In **Lesson 3: Linear Relationships in Context**, students develop a deeper understanding of independent and dependent variables.

Independent and Dependent Variables

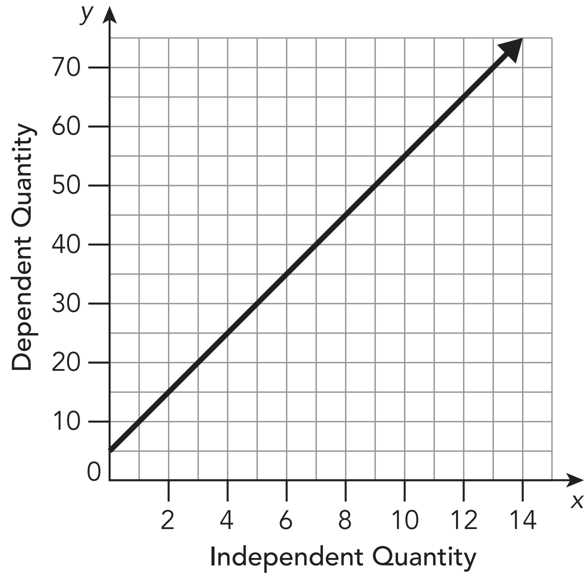
When representing quantities in a table, the **independent** quantity is represented in the left column and the **dependent** quantity is represented in the right column.

When graphing a relationship, the horizontal axis represents the independent quantity and the vertical axis represents the dependent quantity. You should include a label on each axis.

When writing an equation in the slope-intercept form $y = mx + b$, the *x*-value is the independent variable, and the *y*-value is the dependent variable. It is important to define the variables you choose.

For example, the table and graph shown represent the equation $y = 5x + 5$.

Independent Quantity	Dependent Quantity
0	5
1	10
2	15
3	20



In **Lesson 4: Slope-Intercept Form of a Line**, students formalize the concepts of *y*-intercept and the slope-intercept form of a line.

Solving for the *y*-intercept

You can use the slope formula, formula and slope-intercept form of a line to determine the y-intercept for the graph of a linear relationship from a table of values.

For example, you can determine the *y*-intercept of a linear equation from the table of values shown.

x	*y*
2	7
3	10
4	13

First, determine the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 7}{3 - 2} = \frac{3}{1} = 3$$

Next, choose any point from the table: (4, 13).

Now, substitute what you know into the slope-intercept equation of a line, $y = mx + b$, and solve for the *y*-coordinate.

$y = mx + b$

$13 = 3(4) + b$

$13 = 12 + b$

$1 = b$

The *y*-intercept is (0, 1).

MYTH

Myth: Students only use 10% of their brains.

Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of many science fiction movies. These movies ask the audience: *Imagine what you could accomplish if you could use 100% of your brain*!

While this isn't Hollywood, the good news is that you do use 100% of your brain. As you look around the room, your *visual cortex* is busy assembling images, your *motor cortex* is busy moving your neck, and all of the *associative areas* recognize the objects that you see. Meanwhile, the *corpus callosum*, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts . . . like mathematics!

#mathmythbusted

In **Lesson 5: Defining Functional Relationships**, students analyze mappings, sets of ordered pairs, tables, graphs, and equations and determine whether these relations are functions.

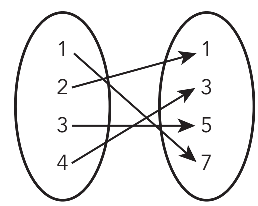
Mapping

The graph of a relationship has meaning because it shows how the dependent quantity changes as the independent quantity changes.

You can use a mapping to show ordered pairs. A **mapping** represents two sets of objects or items. Arrows connect the items to represent a relationship between them. When you write the ordered pairs for a mapping, you are writing a set of ordered pairs.

A **set** is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common. A **relation** is any set of ordered pairs or the mapping between a set of **inputs** and a set of **outputs**.

In the mapping shown, the set of ordered pairs is {(1, 7), (2, 1), (3, 5), (4, 3)}. The set of **inputs** is {1, 2, 3, 4}, and the set of **outputs** is {1, 3, 5, 7}.

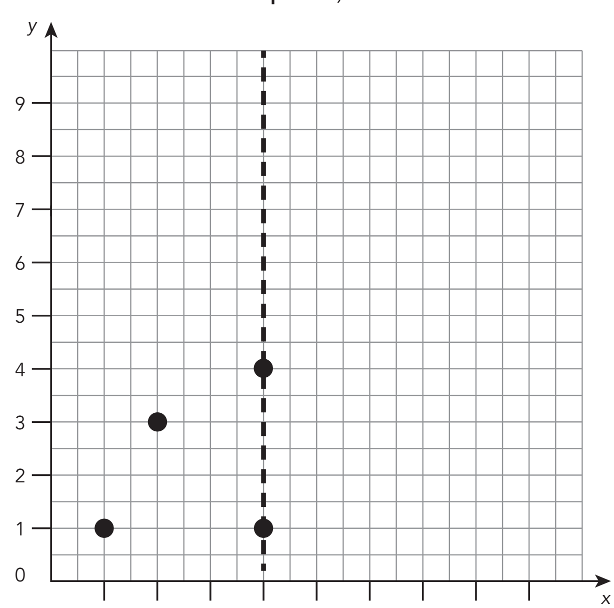


This mapping represents a **function** because each input is mapped to only one output.

Scatterplots

A relation can also be represented as a graph. A **scatterplot** is a graph of a collection of ordered pairs. The **vertical line test** is a method used to determine whether a relation is a function. To apply the vertical line test, think about all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

In this scatterplot, the relation is not a function. The input value 4 can be mapped to two different outputs, 1 and 4. Those two outputs are shown as intersections to the vertical line drawn at $x = 4$.



MODULE
3

Data Data Everywhere

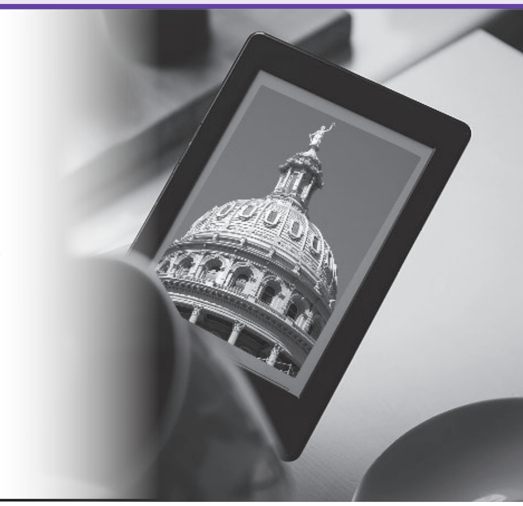
Family Guide

MODULE 3 Data Data Everywhere

Grade 8

TOPIC 1 Patterns in Bivariate Data

In this topic, students review the statistical process and investigate associations in bivariate data, both quantitative and categorical. On scatterplots, students informally fit trend lines and use those lines to make and judge the reasonableness of predictions about the data. They will determine the equations of those lines, also called lines of best fit, and interpret the slopes and y-intercepts of the lines.



Where have we been?

In previous topics, students have interpreted points on coordinate planes in the context of a scenario. In this topic, they identify specific points on scatterplots and informally explain the patterns they notice. Students then use their intuition and new vocabulary to describe patterns in provided scatterplots.

Where are we going?

In this topic, students informally fit trend lines, or lines of best fit, to scatterplots in order to make predictions. They interpret the slope and y-intercept of the trend line in context. In future courses, they will use technology to formally fit lines and other functions to data using regression models.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can support your student's learning by resisting the urge, as long as possible, to solve a problem your student is working on. Students will learn the algebraic shortcuts that you may know about, but only once they have experience in mathematical reasoning. This may seem to take too long at first. But when you practice asking good questions instead of helping your student arrive at the answer, they will learn to rely on their own knowledge, reasoning, patience, and endurance when struggling with math.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

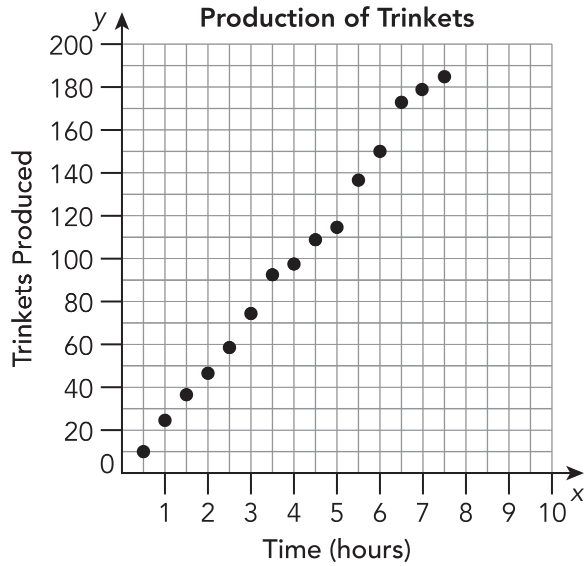
NEW KEY TERMS

- bivariate data [datos bivariados]
- explanatory variable [variable explicativa]
- response variable [variable de respuesta]
- association [asociación]
- linear association [asociación lineal]
- positive association [asociación positiva]
- negative association [asociación negativa]
- outlier
- trend line
- model [modelo]
- interpolating [interpolación]
- extrapolating [extrapolación]

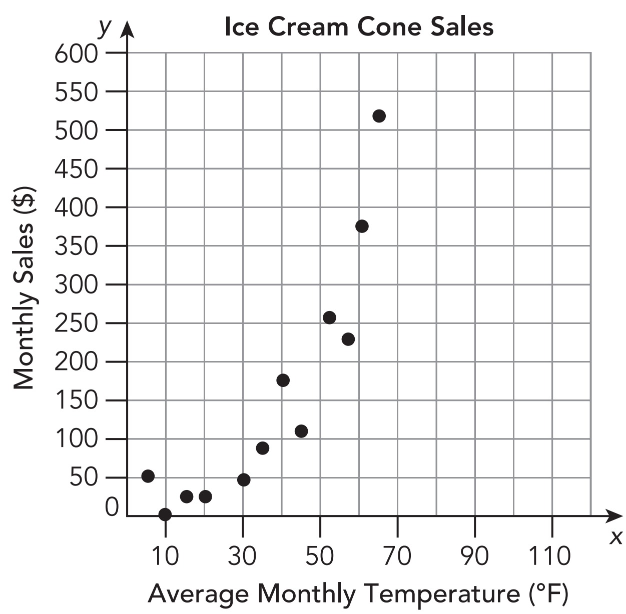
Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **linear association** occurs when the points on the scatterplot are arranged in such a way that, as you look at the graph from left to right, you can imagine a line going through the scatterplot with most of the points being close to the line.

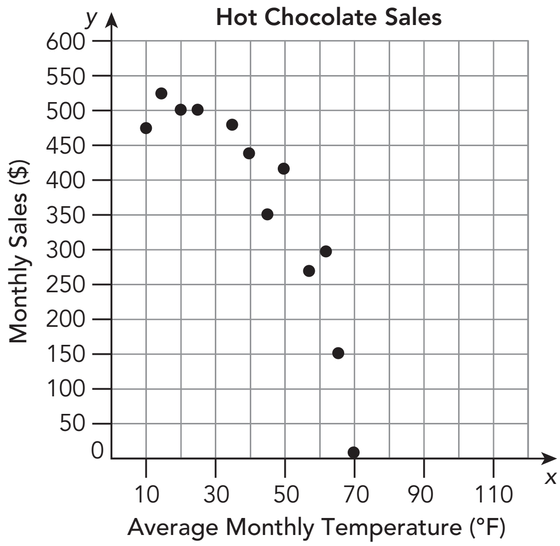


The two variables have a **positive association** if, as the explanatory variable increases, the response variable also increases.



There is a positive association between the average monthly temperature and the amount, in dollars, of monthly ice cream cone sales.

If the response variable decreases as the explanatory variable increases, then the two variables have a **negative association**.



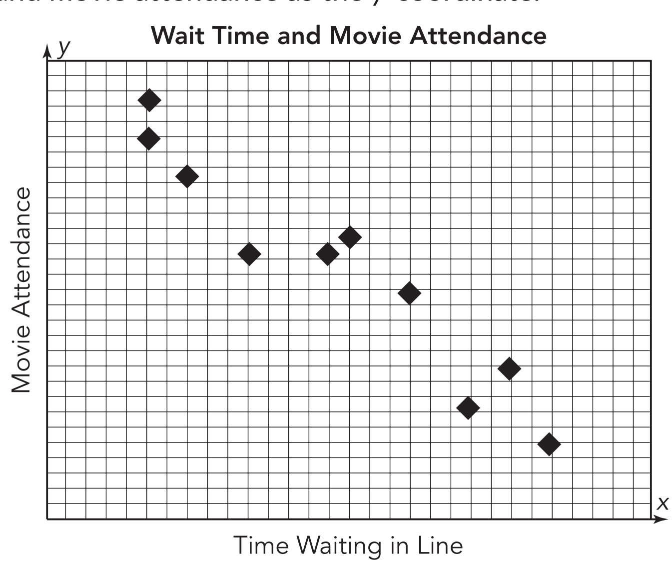
There is a negative association between the average monthly temperature and the amount, in dollars, of monthly hot chocolate sales.

In **Lesson 1: Analyzing Patterns in Scatterplots,** students construct and analyze scatterplots of bivariate data to explore patterns in data.

Bivariate Data

When you collect information about two separate traits for the same person, thing, or event, you have collected **bivariate data**. A **scatterplot** is a graph of a set of ordered pairs. The points in a scatterplot are not connected, but they allow you to investigate patterns in bivariate data by comparing the two variables.

For example, the scatterplot shown represents time waiting in line as the *x*-coordinate and movie attendance as the *y*-coordinate.



MYTH

"I learn best when the instruction matches my learning style."

If asked, most people will tell you they have a learning style—the expressed preference in learning by seeing images, hearing speech, seeing words, or being able to physically interact with the material. Some people even believe that it is the teacher's job to present the information in accordance with that preference.

However, it turns out that the best scientific evidence available does not support learning styles. In other words, when an auditory learner receives instruction about content through a visual model, they do just as well as an auditory learner who receives spoken information. Students may have a preference for visuals or writing or sound, but sticking to their preference doesn't help them learn any better. Far more important is ensuring the student is engaged in an interactive learning activity and the new information connects to the student's prior knowledge.

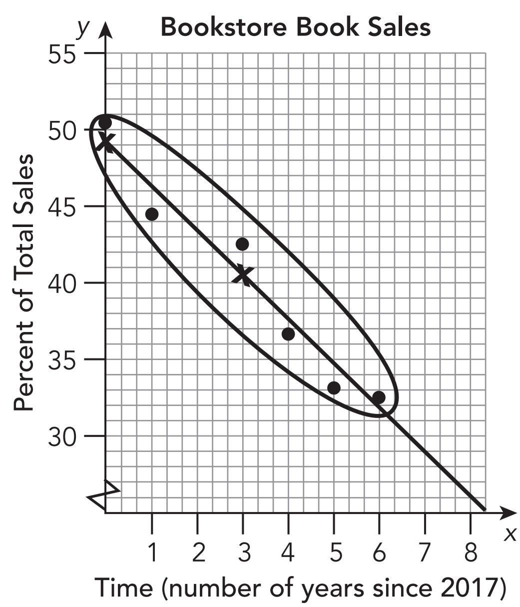
#mathmythbusted

When you look for a relationship in bivariate data, often you are interested in the question: does one variable cause a change in the other variable? In this case, one variable, the **explanatory variable**, is the independent variable. The **response variable** is the dependent variable because this is the variable that responds to what changes in the explanatory variable. In the scatterplot above, time waiting in line is the explanatory variable and movie attendance is the dependent variable.

In **Lesson 2: Drawing Trend Lines,** students construct a trend line and use it to make predictions about bivariate data.

Trend Line

Although a straight line will not pass through all of the points in a scatterplot, you can use a line to fit the data as closely as possible. This kind of line is called a trend line. A **trend line** is a line that is as close to as many points as possible but does not have to go through all of the points. When you use a trend line, the line and its equation are often called a **model** of the data. Constructing a trend line is helpful to predict values not shown on the plot.



To construct a trend line, first plot all of the data.

Next, draw an oval around all of the data.

Then draw a line that divides the enclosed area of the data in half.

The idea is that you want to identify a line that has an equal number of points on either side.

Extrapolating and Interpolating

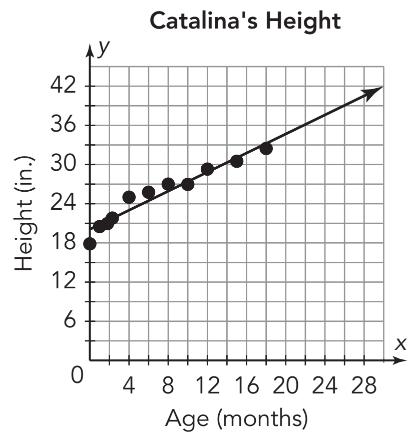
If you are predicting values that are inside the plotted values of a scatterplot, you are **interpolating**. If you are predicting values that are outside the plotted values, you are **extrapolating**. For example, predicting the percent of book sales from bookstores in 2006 using the trend line is an example of interpolation, while predicting the percent of book sales from bookstores in 2012 using the same line would be an example of extrapolation.

In **Lesson 3: Analyzing Trend Lines** and **Lesson 4: Comparing Slopes and Intercepts of Data from Experiments**, students write equations for trend lines and use the equations to make predictions about the data.

Writing and Using Equations for Trend Lines

The equation $y = 0.73x + 20.08$ represents the trend line on the graph. You can use this equation to estimate the child's height at 9 months.

$$y = 0.73(9) + 20.08$$
$$y = 6.57 + 20.08$$
$$y = 26.65 \text{ inches}$$



Family Guide

MODULE 3 Data Data Everywhere Grade 8

TOPIC 2 Variability And Sampling

In this topic, students continue developing their understanding of the statistical process by focusing on the second component of the process: data collection. They learn about samples, populations, censuses, parameters, and statistics. Students display data and compare the difference of the measures of center for two populations to their measures of variability. Students are introduced to mean absolute deviation as a measure of variability. They calculate and use the mean absolute deviation to describe how data are spread out around the mean of the data set. Then, students draw conclusions about two populations using random samples.

Where have we been?

In previous courses, students learned about and used aspects of the statistical problem-solving process: formulating questions, collecting data, analyzing data, and interpreting the results. They also used numerical data displays, including both measures of center and measures of variability.

Where are we going?

In future courses, students will learn about specific types of random sampling and the inherent bias in sampling techniques. They will continue analyzing and comparing random samples from populations and comparing their measures of center and variability.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can support your student's learning by approaching problems slowly. Students may observe a classmate learning things very quickly, and they can easily come to believe that mathematics is about getting the right answer as quickly as possible. When this doesn't happen for them, future encounters with math can raise anxiety, making problem solving more difficult and reinforcing a student's view of themselves as "not good at math." Slowing down is not the ultimate cure for math difficulties. But, it's a good first step for students who are struggling. You can reinforce the view that learning with understanding takes time, and that slow, deliberate work is the rule, not the exception.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- deviation [desviación]
- absolute deviation [desviación absoluta]
- mean absolute deviation (MAD) [desviación media absoluta (DMA)]
- survey
- data [datos]
- population [población]
- census [censo]
- sample
- parameter [parámetro]
- statistic [estadística]
- random sample

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

When data are gathered from a population, the characteristic used to describe the population is called a **parameter**.
When data are gathered from a sample, the characteristic used to describe the sample is called a **statistic**.

In **Lesson 1: Mean Absolute Deviation**, students are introduced to the concept of deviation and mean absolute deviation.

Deviation

One measure of variation that describes the spread of data values is **deviation**. The deviation of a data value indicates how far that data value is from the mean, or average. To calculate deviation, subtract the mean from the data value:

deviation = data value − mean

For example, the mean of the data set 15, 12, 13, 10, 9, and 13 is 12.

The table describes each data point's deviation from the mean.

Data Point	15	12	13	10	9	13
Deviation from the Mean	3	0	1	−2	−3	1

Absolute Deviation

In order to get an idea of the spread of the data values, take the absolute value of each deviation and determine the mean of those absolute values. The absolute value of each deviation is called the **absolute deviation**. The **mean absolute deviation (MAD)** is the mean of the absolute deviations.

For example, the mean absolute deviation of the data shown in the table is

$$\frac{|3| + |0| + |1| + |-2| + |-3| + |1|}{6} = \frac{10}{6}.$$

So, the MAD is about 1.67.

In **Lesson 2: Collecting Random Samples**, students review the statistical process and deepen their understanding of the second component of the process: data collection.

The Statistical Process

There are four parts of the statistical process:

- Create a statistical question.
- Collect data.
- Analyze the data with numbers and graphs.
- Interpret the results.

One data collection strategy you can use is a **survey**. A survey is a method of collecting information about a group of people. It involves asking a question or a set of questions to those people. When information is collected, it is called **data**.

The **population** is the whole set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects. A **census** is the data collected from every member of a population.

In most cases, it is not possible or logical to collect data from the whole population. When data are collected from a part of the population, the data are called a **sample**.

When information is collected from a sample in order to describe something about the population, it is important that such a sample be as representative of the population as possible. A **random sample** is a sample that is selected from the population in a way that every member of the population has the same chance of being selected.

MYTH

Faster = smarter.

In most cases, speed has nothing to do with how smart you are. Why is that? Because it largely depends on how familiar you are with a topic. For example, a bike mechanic can look at a bike for about 8 seconds and tell you details about the bike that you probably didn't even notice (e.g., the front tire is on backwards). Is that person smart? Sure! Suppose, instead, you show the same bike mechanic a car. Will they be able to recall the same amount of detail as for the bike? No!

It's easy to confuse speed with understanding. Speed is associated with the memorization of facts. Understanding, on the other hand, is a methodical, time-consuming process. Understanding is the result of asking lots of questions and seeing connections between different ideas. Many mathematicians who won the Fields Medal (i.e., the Nobel prize for mathematics) describe themselves as extremely slow thinkers. That's because mathematical thinking requires understanding over memorization.

#mathmythbusted

In **Lesson 3: Sample Populations,** students are introduced to the concept and purpose of a census.

When dealing with a large amount of data, it is often impractical to analyze all the data points. A sample is a subset of a larger data set which can be used to determine characteristics of a whole group.

Random Sample

Suppose a state has 67 counties, all with varied populations. Generate a random sample. Then, you can use the sample to determine average statistics for the entire state.

County	Population	Absolute Deviation from the Mean
Adams	91,292	120,152.875
Butler	174,083	37,361.875
Dauphin	251,798	40,353.125
Huntingdon	45,586	165,858.875
Luzerne	319,250	107,805.125
Montgomery	750,097	538,652.125
Potter	18,080	193,364.875
Tioga	41,373	170,071.875

Mean of sample population: 211,444.875
Median of sample population: 132,687.5
Mean absolute deviation of sample population: 171,702.5938

Mean of entire population: 181,212.1618
Median of entire population: 90,366
Mean absolute deviation of entire population: 162,548.439

The mean, median, and mean absolute deviation of the sample population are all slightly larger than the mean, median, and mean absolute deviation of the entire population. However, the mean and mean absolute deviation are quite close. This means the random sample is probably an accurate representation of the entire population.

MODULE
4

Modeling Linear Equations

TOPIC 1 Solving Linear Equations and Inequalities

In this topic, students expand their ability to solve one-variable linear equations by solving equations with variables on both sides of the equal sign. Students review previously learned strategies for solving equations and learn new strategies to make solving equations more efficient. Students develop an understanding of the conditions that lead to equations that have one solution, no solution, or infinitely many solutions. Students then model given situations with one-variable inequalities, including those with variables on both sides of the inequality symbol. They write possible scenarios that could be represented by a given inequality with variables on both sides of the inequality symbol.

Where have we been?

In previous courses, students have solved equations of the forms $px + q = r$ and $p(x + q) = r$, where p, q, and r are rational numbers. They have also used properties, including the distributive property, to factor and expand algebraic expressions. In this topic, students review the strategies learned in previous courses and learn additional strategies to make solving equations simpler and more efficient.

Where are we going?

This topic provides a bridge from solving one-variable equations with variables on one side of the equal sign to solving systems of linear equations algebraically in future courses. Solving equations with no solution or infinitely many solutions also prepares students to solve, algebraically, systems of linear equations with no solution or infinitely many solutions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Working with algebraic expressions, equations, and inequalities can be tricky! You can support your student's learning by allowing them to grapple with the mathematics before quickly arriving at a solution. You can ask questions to help your student develop a deeper understanding of algebraic concepts.

Questions to Ask

- Do you think this equation/inequality will have one solution, no solution, or infinitely many solutions?
- Can you simplify the expressions in this equation/inequality before solving to make the math easier?
- Does your solution make sense? How can you verify your solution?

NEW KEY TERMS

- one solution [una solución]
- no solution [sin solución]
- infinitely many solutions [soluciones infinitas]
- solution set of an inequality

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

Students already know that equations can have one solution. The equation, $3x + 1 = 7$, has one solution. The only value of x that makes the equation true is 2. An equation may have **no solution**. The equation $x = x + 2$, for example, has no solution. No value of x can make the equation true.
An equation may have an infinite number of solutions. The equation $x(1 + 1) = 2x$, for example, has **infinitely many solutions**. An infinite number of values for x makes the equation true.

In **Lesson 1: Equations with Variables on Both Sides**, students solve more complex equations that include variables on both sides of the equal sign.

Solving Linear Equations

An equation with variables on both sides of the equal sign can be solved by moving all the variable terms to one side of the equation and all the constants to the other side of the equation.

You can use properties of equality to rewrite equations and increase your efficiency with solving equations.

Properties of Equality	For all numbers a, b, and c, . . .
addition property of equality	If $a = b$, then $a + c = b + c$.
subtraction property of equality	If $a = b$, then $a - c = b - c$.
multiplication property of equality	If $a = b$, then $ac = bc$.
division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Ethan

$$5x + 3 = 2x + 5$$
$$-5x \qquad -5x$$
$$3 = -3x + 5$$
$$-5 \qquad -5$$
$$\frac{-2}{-3} = \frac{-3x}{-3}$$
$$\frac{2}{3} = x$$
$$x = \frac{2}{3}$$

Samuel

$$5x + 3 = 2x + 5$$
$$-2x \qquad -2x$$
$$3x + 3 = 5$$
$$-3 \quad -3$$
$$3x = 2$$
$$x = \frac{2}{3}$$

In **Lesson 2: Analyzing and Solving Linear Equations**, students interpret solutions and determine when equations have one solution, no solution, or infinitely many solutions.

Solutions to Linear Equations

A linear equation can have **one solution**, **no solution**, or **infinitely many solutions**. When the solution to the equation is a true statement with one value equal to the variable, there is only one solution.

For example, the equation $3(6x - 4) = 2(9x + 5)$ has no solution.

Isabella

$$3(6x - 4) = 2(9x + 5)$$
$$18x - 12 = 18x + 10$$
$$18x - 18x - 12 = 18x - 18x + 10$$
$$-12 \neq 10$$

When the solution to the equation is a true statement for any value of the variable, such as $x = x$, the equation has infinitely many solutions.

Isabella

$$6x + 14 = 2(3x + 7)$$
$$6x + 14 = 6x + 14$$
$$6x + -6x + 14 = 6x + -6x + 14$$
$$14 = 14$$

MYTH

Just give me the rule. If I know the rule, then I understand the math.

Memorize the following rule: *All quars are elos*. Will you remember that rule tomorrow? Nope. Why not? It has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms*. How about now? Can you remember that? Of course you can, because now, it makes sense.

Learning does not take place in a vacuum. It *must be* connected to what you already know. Otherwise, arbitrary rules will be forgotten.

#mathmythbusted

In **Lesson 3: Solving Linear Inequalities**, students solve inequalities with variables on both sides of the inequality symbol.

Solving Linear Inequalities

Not all situations can be represented using equations. Some scenarios require the use of inequalities.

Harper and Diego each solved the inequality $8x + 2 \geq 5x - 10$.

Harper

$$8x + 2 \geq 5x - 10$$
$$8x - 5x + 2 \geq 5x - 5x - 10$$
$$3x + 2 \geq -10$$
$$3x + 2 - 2 \geq -10 - 2$$
$$3x \geq -12$$
$$\frac{3x}{3} \geq \frac{-12}{3}$$
$$x \geq -4$$

Diego

$$8x + 2 \geq 5x - 10$$
$$8x - 8x + 2 \geq 5x - 8x - 10$$
$$2 \geq -3x - 10$$
$$2 + 10 \geq -3x - 10 + 10$$
$$12 \geq -3x$$
$$\frac{12}{-2} \leq \frac{-3x}{-3}$$
$$-4 \leq x$$
$$x \geq -4$$

Family Guide

MODULE 4 Modeling Linear Equations Grade 8

TOPIC 2 Systems of Linear Equations

In this topic, students analyze pairs of linear equations. They represent real-world scenarios by writing and graphing systems of equations in slope-intercept form ($y = mx + b$). Students also determine the solution for both equations using graphs and tables, then verify the solution algebraically.

Where have we been?

In this topic, students utilize what they have learned throughout this course and in previous courses about linear relationships, tables, graphs, and equations. Students also build on their knowledge of proportionality to solve problems and investigate solutions to multiple linear equations.

Where are we going?

Students' experiences in this topic provide the foundation for a more rigorous and abstract study of systems of equations in future courses. In future courses, students will solve systems that include equations that are not linear and use algebraic and graphical techniques to solve systems of inequalities.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning about systems of equations and solving systems by inspection and graphing.

Questions to Ask

- Do the equations in the system have similar structures?
- Do they have the same slope or the same y-intercept?
- What is the point of intersection for the graph of both equations in the system?
- Does your solution make sense in the context of the problem?

NEW KEY TERMS

- point of intersection [punto de intersección]
- break-even point
- system of linear equations [sistema de ecuaciones lineales]
- solution of a linear system [solución de un sistema lineal]
- consistent system [sistema consistente]
- inconsistent system [sistema inconsistente]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

When two or more linear equations define a relationship between quantities, they form a **system of linear equations**.

The **solution of a linear system** is an ordered pair (*x*, *y*) that is a solution to both equations in the system. Graphically, the solution is the point of intersection. Consider the system of linear equations.

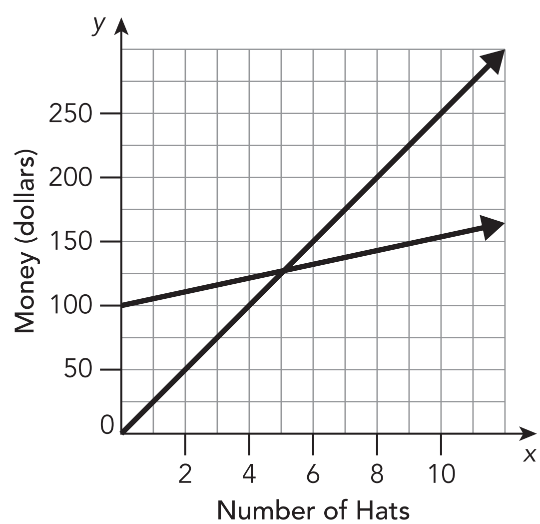
$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

The solution to this system of equations is (1, 6).

In **Lesson 1: Point of Intersection of Linear Graphs**, students graph systems of linear equations and interpret the meaning of the solution in the context of the situation.

Point of Intersection

The **point of intersection** is the point at which two lines cross on a coordinate plane. The point where two linear graphs intersect, or cross, represents the solution to both of the equations. When one line represents the cost of an item and the other line represents the income from selling the item, the point of intersection is called the **break-even point**.



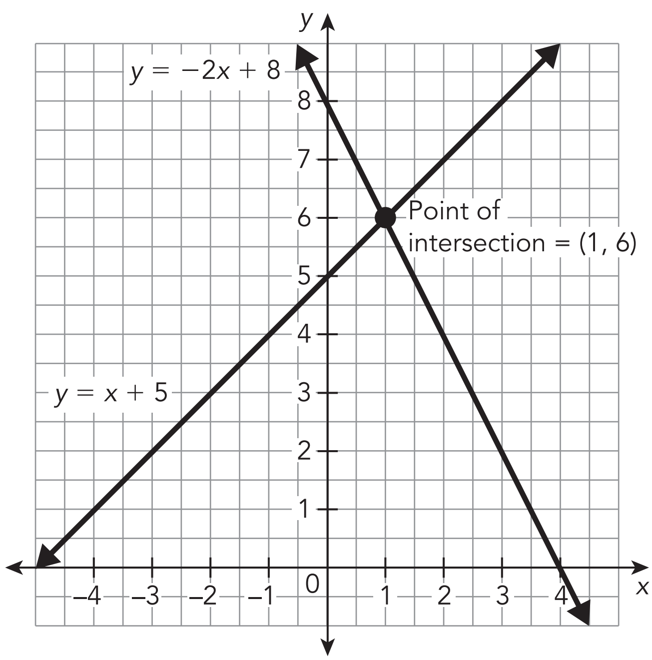
In **Lesson 2: Systems of Linear Equations**, students write and analyze systems of linear equations. They explore systems of linear equations with one solution, no solution, and infinitely many solutions.

Systems of Linear Equations

When two or more linear equations define a relationship between quantities, they form a system of linear equations. The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. Consider this system of linear equations.

$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

Graphically, the solution is the point of intersection, or the point at which two or more lines cross.



You can verify the solution algebraically by substituting the coordinates of the point of intersection into the equations in the system.

$y = x + 5$	$y = -2x + 8$
$6 = 1 + 5$	$6 = -2(1) + 8$
$6 = 6$ ✓	$6 = -2 + 8$
	$6 = 6$ ✓

MYTH

Memory is like an audio or video recording.

Let's play a game. Memorize the following list of words: strawberry, grape, watermelon, banana, orange, peach, cherry, blueberry, raspberry. Got it? Good. Some believe that the brain stores memories in pristine form; memories last for a long time and do not change—like a recording. Without looking back at the original list, was apple on it?

If you answered "yes," then go back and look at the list. You'll see that apple does not appear, even though it seems like it should. In other words, memory is an active, reconstructive process that takes additional information, like the category of words (e.g., fruit), and makes assumptions about the stored information.

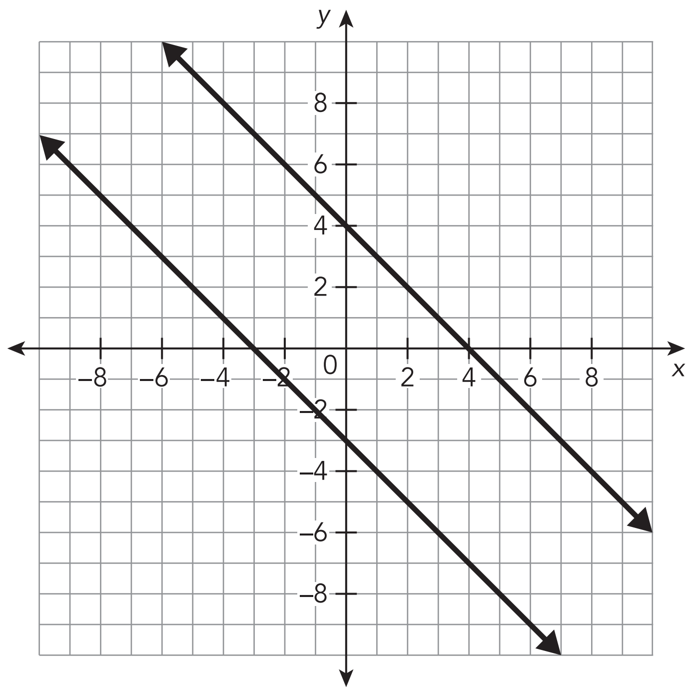
This simple demonstration suggests memory is not like a recording. Instead, it is influenced by prior knowledge and decays over time. Therefore, students need to see and engage with the same information multiple times to minimize forgetting (and distortions).

#mathmythbusted

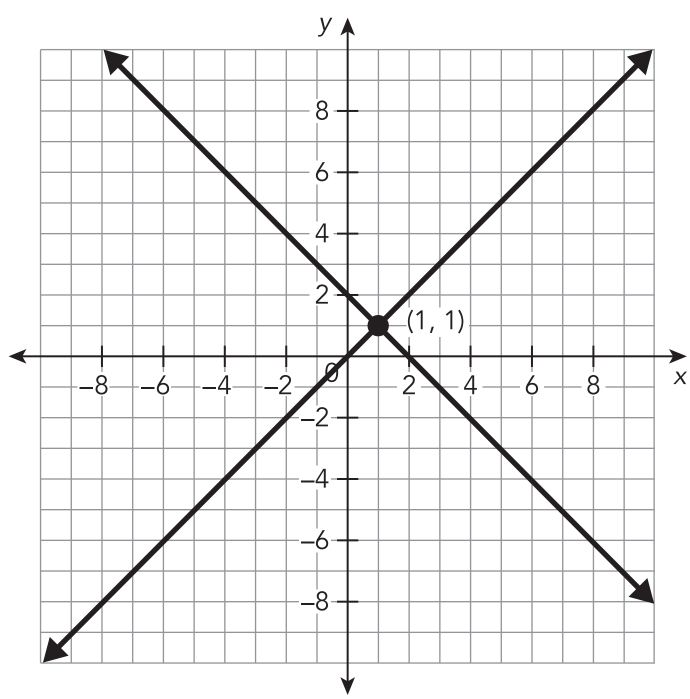
Consistent or Inconsistent

A system of equations may have one solution, infinitely many solutions, or no solution. A system that has one or infinitely many solutions is called a **consistent system**. A system that has no solution is called an **inconsistent system.**

In an inconsistent system, the graphs of the lines never cross each other because they have the same slope and different y-intercepts.



In a consistent system, the graphs of the lines intersect. When the slopes of two lines are different, they will cross at some point.



GRADE 8

MODULE 5

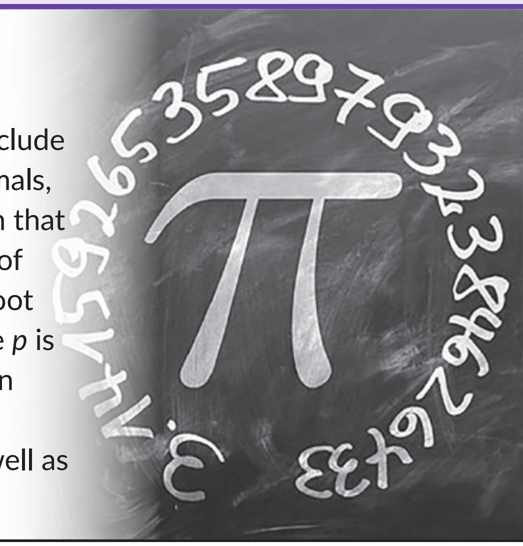
Applying Powers

Family Guide

MODULE 5 Applying Powers

Grade 8

TOPIC 1 Real Numbers



In this topic, students expand their knowledge of number systems to include the set of irrational numbers. Students review writing fractions as decimals, and they learn to write repeating decimals in fractional form. They learn that numbers that are not rational are called *irrational* and the decimal form of irrational numbers does not terminate or repeat. Students use square root symbols to express the solutions to equations of the form $x^2 = p$, where p is a positive rational number. Students are introduced to scientific notation as a means to express very large or very small numbers. They learn to convert between standard decimal notation and scientific notation as well as compare numbers expressed in scientific notation.

Where have we been?

Students review the number sets that they are already familiar with, and they learn the properties of each set. They learn the formal definition of rational numbers before moving on to learn about irrational numbers. Students also review additive identity, additive inverse, multiplicative identity, and multiplicative inverse.

Where are we going?

This topic prepares students to solve problems with non-perfect squares in the next topic. Students begin to understand that mathematics is not arbitrary. Every new number system that students learn results from the need for a number that is not in the currently known number systems. In future math courses, students will regularly operate with irrational numbers. Understanding roots is important when students learn about the imaginary numbers that, along with the set of real numbers, comprise the complex number system.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by resisting the urge, as long as possible, to get to the answer in a problem that your student is working on. Students are encountering irrational numbers formally for the first time in this topic. They will need time and space to struggle with all the implications of working with this expanded number system. Practice asking good questions when your student is stuck.

QUESTIONS TO ASK

- Can you use estimation to help you think about this problem?
- Do you notice any patterns?
- Does your solution seem reasonable?

NEW KEY TERMS

- natural numbers [números naturales]
- whole numbers
- integers [enteros]
- rational numbers [números racionales]
- irrational numbers [números irracionales]
- terminating decimal [decimal terminal]
- repeating decimal [decimal repetitivo/ periódico]
- bar notation [notación de barra]
- square root
- radical [radical]
- radicand [radicando]
- perfect square
- real numbers [números reales]
- scientific notation [notación científica]
- mantissa [mantisa]
- characteristic [característica]
- order of magnitude [orden de magnitud]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **repeating decimal** is a decimal in which a digit, or a group of digits, repeats infinitely. Repeating decimals are rational numbers.

$$\frac{1}{9} = 0.111...$$

$$\frac{7}{12} = 0.58333...$$

$$\frac{22}{7} = 3.142857142857...$$

A **terminating decimal** has a finite number of digits, meaning that after a finite number of decimal places, all following decimal places have a value of 0. Terminating decimals are rational numbers.

$$\frac{9}{10} = 0.9$$

$$\frac{15}{8} = 1.875$$

$$\frac{193}{16} = 12.0625$$

In **Lesson 2: Rational and Irrational Numbers**, students differentiate between rational and *irrational* numbers.

Rational and Irrational Numbers

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are both integers and b is not equal to 0. A rational number can be written as either a *terminating* or *repeating* decimal.

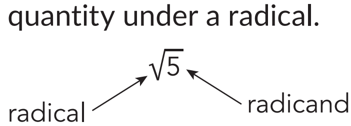
All other decimals are irrational numbers because these decimals cannot be written as fractions in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0.

In **Lesson 3: The Real Numbers**, students study the square roots of numbers that are not perfect squares. Then, they identify the relationships between various sets of numbers within the set of real numbers.

Square Root

A **square root** is one of two equal factors of a given number. Every positive number has two square roots: a positive square root and a negative square root. The positive square root is called the *principal square root*.

The symbol $\sqrt{}$ is called a **radical**. The **radicand** is the quantity under a radical.



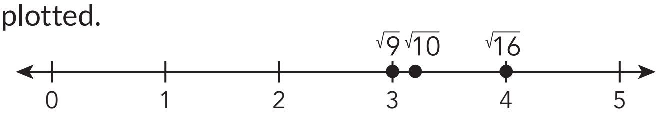
Estimating Square Roots

You can estimate the square root of a number that is not a **perfect square**. Begin by determining the two perfect squares closest to the radicand so that one perfect square is less than the radicand and one perfect square is greater than the radicand. Then, locate the expression on a number line and use approximation to estimate the value.

For example, to estimate $\sqrt{10}$ to the nearest tenth, identify the closest perfect square less than 10 and the closest perfect square greater than 10.

$$\sqrt{9} < \sqrt{10} < \sqrt{16}$$
$$3 < \sqrt{10} < 4$$

This means that the estimate of $\sqrt{10}$ is between 3 and 4. Locate each square root on a number line. The approximate location of $\sqrt{10}$ is closer to 3 than it is to 4 when plotted.



$$\sqrt{10} \approx 3.2$$

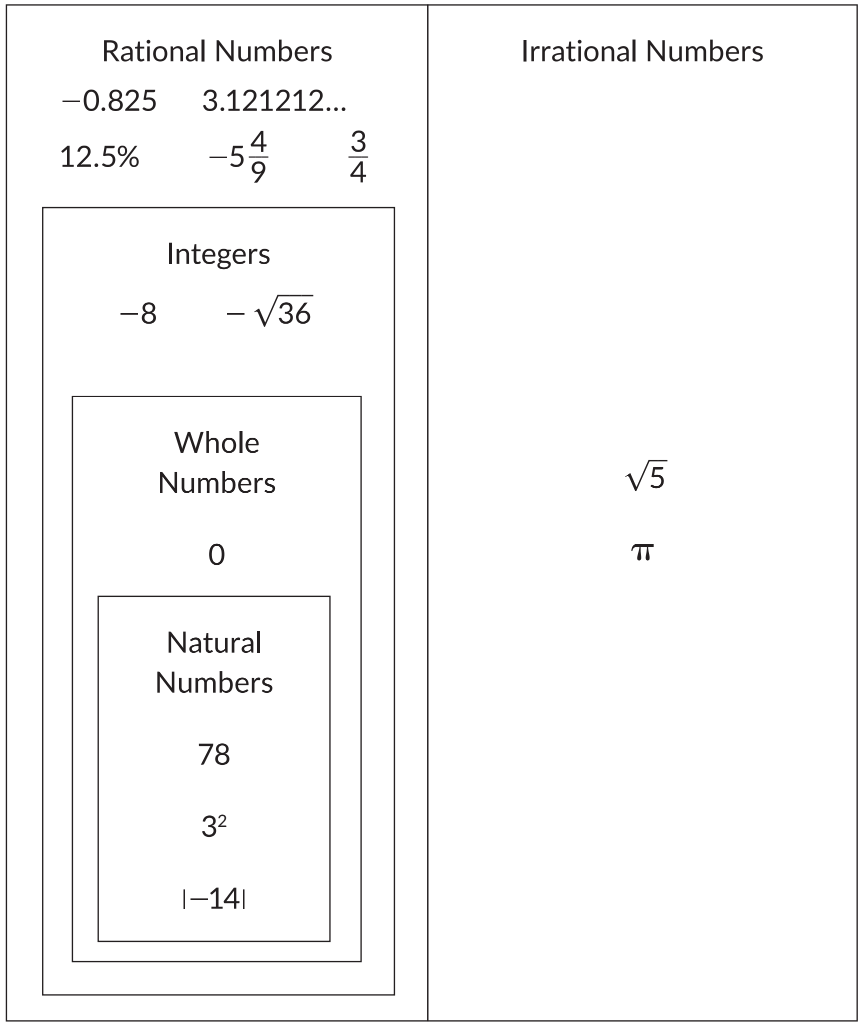
You can check your estimate by calculating the squares of values between 3 and 4.

$$(3.1)(3.1) = 9.61$$
$$(3.2)(3.2) = 10.24$$
$$(3.3)(3.3) = 10.89$$

Real Numbers

Students combine the set of rational numbers and the set of irrational numbers to produce the set of **real numbers**. You can use a Venn diagram to represent how the sets within the set of real numbers are related.

Real Numbers



In Lesson 4: Scientific Notation, students are introduced to scientific notation.

Scientific Notation

Scientific notation is a notation used to express a very large or very small number as the product of two numbers:

- A number that is greater than or equal to 1 and less than 10, and
- A power of 10.

In general terms, $a \times 10^n$ is a number written in scientific notation, where a is greater than or equal to 1 and less than 10, and n is any integer. The number a is called the **mantissa**, and n is called the **characteristic**.

For example, you can write the number 16,000,000,000 in scientific notation.

$$16{,}000{,}000{,}000 = 1.6 \times 10^{10}$$

You can also write 0.00065 in scientific notation.

$$0.00065 = 6.5 \times 10^{-4}$$

MYTH

Cramming for an exam is just as good as spaced practice for long-term retention.

Everyone has been there. You have a big test tomorrow, but you've been so busy that you haven't had time to study. So, you had to learn it all in one night. You may have gotten a decent grade on the test. However, did you remember the material a week, month, or year later?

The honest answer is, "probably not." That's because long-term memory is designed to retain useful information. How does your brain know if a memory is "useful" or not? One way is the frequency with which you encounter a piece of information. If you only see something once (like during cramming), then your brain doesn't deem those memories as important. However, if you sporadically come across the same information over time, then it's probably important. To optimize retention, encourage your student to periodically study the same information over expanding intervals of time.

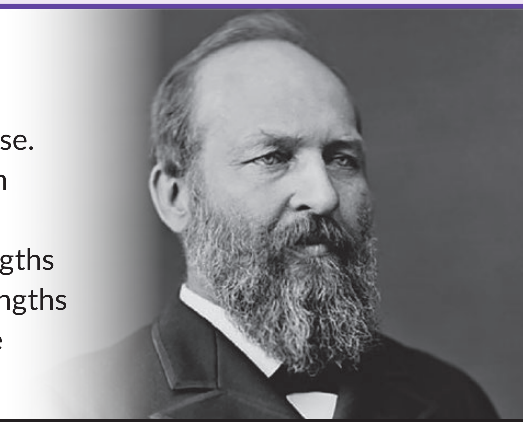
#mathmythbusted

Family Guide

MODULE 5 Applying Powers

Grade 8

TOPIC 2 The Pythagorean Theorem



In this topic, students explore the Pythagorean Theorem and its converse. They learn that knowing two side-lengths of a right triangle allows them to determine the third side length, therefore forming a unique triangle. Students practice applying the theorem to determine unknown side lengths in right triangles and apply the converse of the theorem: if three side lengths are given, determine if the triangle is a right triangle. Students apply the Pythagorean Theorem to real-world and mathematical problems.

Where have we been?

Students learned about right angles and right triangles in previous grades. They evaluated numerical expressions with whole-number exponents in previous courses, and they have continued to use these skills in subsequent courses.

Where are we going?

In Geometry, students will use right triangles and similarity to define ratios of sides, the trigonometric ratios. These new ratios, along with the Pythagorean Theorem, will be used to solve application problems. Students will use the Pythagorean Theorem in the study of analytic geometry when they use coordinates to prove geometric theorems algebraically, including derivation of the distance formula.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning about the Pythagorean Theorem.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Can you draw and label a diagram to model the problem?
- Does your answer seem reasonable for this problem situation?

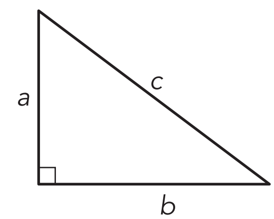
NEW KEY TERMS

- hypotenuse [hipotenusa]
- legs
- Pythagorean Theorem [Teorema de Pitágoras]
- proof
- diagonal of a square
- converse
- converse of the Pythagorean Theorem
- Pythagorean triple [triple Pitagórico]
- diagonal [diagonal]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

In the right triangle shown, the lengths of the sides are a, b, and c. The side opposite the right angle is called the **hypotenuse**. The other two sides are called **legs** of the right triangle. In the figure, the sides with lengths a and b are the legs, and the side with length c is the hypotenuse.



In **Lesson 1: The Pythagorean Theorem,** students conjecture about the lengths of the sides of a right triangle. They then use the Pythagorean Theorem to solve for the length of unknown sides of right triangles set in a variety of contexts.

Pythagorean Theorem

The special relationship that exists among the squares of the lengths of the sides of a right triangle is known as the **Pythagorean Theorem**. The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse: $a^2 + b^2 = c^2$.

The Pythagorean Theorem can be used to determine unknown side lengths in a right triangle if you know two of the other side lengths. For example, suppose you want to determine the length of the hypotenuse of the right triangle with leg lengths of 2 and 4 units.

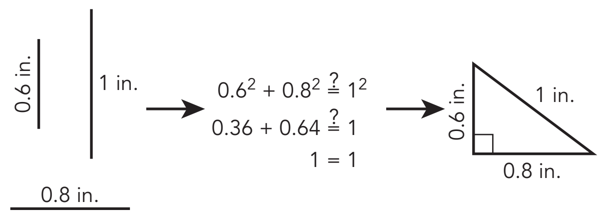
$$c^2 = 2^2 + 4^2$$
$$c^2 = 4 + 16 = 20$$
$$c = \sqrt{20} \approx 4.5$$

The length of the hypotenuse is approximately 4.5 units.

In **Lesson 2: The Converse of the Pythagorean Theorem**, students continue to explore right triangle relationships.

Converse of the Pythagorean Theorem

The **Converse of the Pythagorean Theorem** states that if the sum of the squares of the two shorter sides of a triangle equals the square of the longest side, then the triangle is a right triangle.



Pythagorean Triples

Any set of three positive integers *a*, *b*, and *c* that satisfies the equation $a^2 + b^2 = c^2$ is a **Pythagorean triple**.

3, 4, and 5 is a Pythagorean triple: $3^2 + 4^2 = 5^2$.

In **Lesson 3: Distances in a Coordinate System,** students apply the Pythagorean Theorem to determine the distance between any two given points on the coordinate plane.

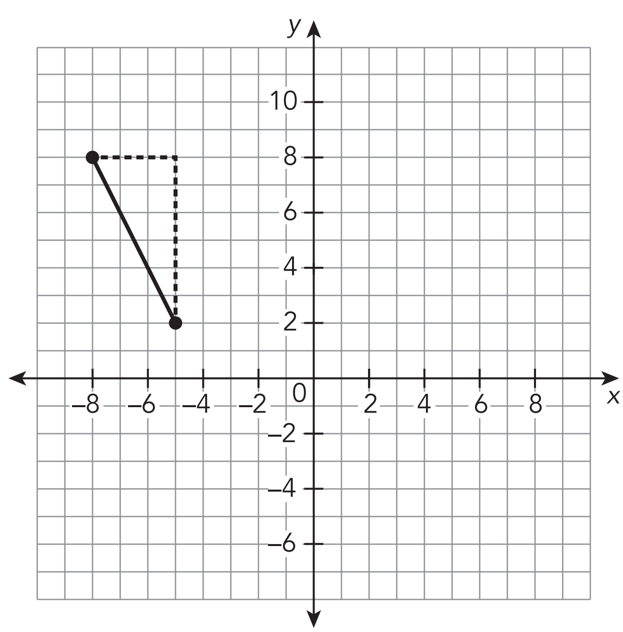
Distance Between Points

You can calculate the distance between two points on the coordinate plane by drawing a right triangle. When you think about this line segment as the hypotenuse of the right triangle, you can use the Pythagorean Theorem.

For example, you can calculate the distance between the points (−5, 2) and (−8, 8) by first determining the length of each leg. One leg measures 3 units, and the other leg measures 6 units.

$$3^2 + 6^2 = c^2$$
$$9 + 36 = c^2$$
$$45 = c^2$$
$$c = \sqrt{45}$$
$$c \approx 6.7$$

The distance between the points is approximately 6.7 units. The distance between two points on a coordinate plane is always a positive number.



MYTH

I'm not smart.

The word "smart" is tricky because it means different things to different people. For example, would you say a baby is "smart"? On the one hand, a baby is helpless and doesn't know anything. But, on the other hand, a baby is exceptionally smart because they are constantly learning new things every day.

This example is meant to demonstrate that "smart" can have two meanings. It can mean "the knowledge that you have," or it can mean, "the capacity to learn from experience." When someone says they are "not smart," are they saying they do not have lots of knowledge, or are they saying they lack the capacity to learn? If it's the first definition, then none of us are smart until we acquire that information. If it's the second definition, then we know that is completely untrue because everyone has the capacity to grow as a result of new experiences.

So, if your student doesn't think that they are smart, encourage them to be patient. They have the capacity to learn new facts and skills. It might not be easy, and it will take some time and effort. But, the brain is automatically wired to learn. Smart should not refer only to how much knowledge you currently have.

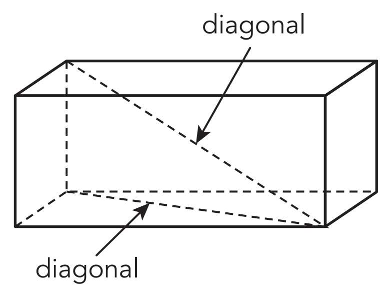
#mathmythbusted

In **Lesson 4: Side Lengths in Two and Three Dimensions,** students apply the Pythagorean Theorem to determine the lengths of the diagonals of various three-dimensional figures.

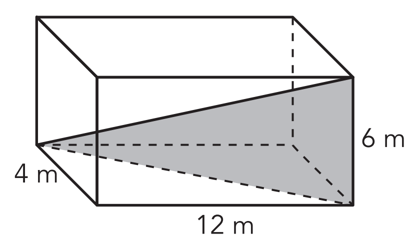
Diagonals

You can use the Pythagorean Theorem to determine the length of a **diagonal** in a two- or three-dimensional figure.

In a three-dimensional figure, a diagonal is a line segment connecting any two non-adjacent vertices. You can use the width and length of the base of the prism to determine the measure of the diagonal of the base.



The diagonal on the base of the prism is also one of the legs of a triangle with an inner diagonal as the hypotenuse. The height of the prism is the length of the other leg.



Family Guide

MODULE 5 Applying Powers

Grade 8

TOPIC 3 Financial Literacy: Your Financial Future

This topic begins with students calculating and comparing simple and compound interest earnings to understand how money invested will grow over time. Students learn how to make financially responsible decisions based on analyzing the terms of loans and investments, which include the amount loaned or invested, the interest rate, and the length of time. Technology, such as online calculators, are used to determine the amount of time it takes to pay off a credit card, the interest amount, and the total debt payment when given the principal, interest rate, and monthly payment. Students also use this tool to determine the amount of monthly student loan payments, the total cost of the student loan, and the salary necessary to pay off the loan. Students learn that there are various options available to help fund their post-secondary tuition.

Where have we been?

Students had to distinguish between debit cards and credit cards, and they learned why it is important to have a good credit history. They were introduced to simple and compound interest, which they continue to study in this topic.

Where are we going?

The culminating lesson of this topic sums up the importance of financial literacy for students' futures. It sends the following three-pronged message: (1) every student can benefit from some form of post-secondary education; (2) post-secondary tuition costs vary widely, and there is an option that makes sense and is affordable for every student; and (3) all students can find a way to manage the cost of post-secondary education.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Your student is learning about financial literacy, including options for a 4 year university, 2 year college, technical school, or obtaining licenses or certifications. While this may seem far away, it is never too early to begin preparing for the future. Talk with your student about the importance of working hard in school and making wise financial decisions.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- terms of an investment [términos de una inversión]
- simple interest [interés simple]
- compound interest [interés compuesto]
- deferment
- online calculator [calculadora en línea]
- cash advance

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

The **terms of an investment** include the type of loan, amount of money invested, and the length of the investment.
A **deferment** is a period of time, usually up to two years, in which people delay paying the principal and interest on their loan.

In **Lesson 1: Simple and Compound Interest,** students develop a conceptual understanding of growth functions. They compare linear and exponential growth through a pay scenario and then compare simple interest and compound interest.

Simple and Compound Interest

Simple interest accounts pay the same amount over each time period, while **compound interest** accounts calculate the interest earned according to the balance after each year. This means that simple interest accounts grow steadily over time because they increase at a constant rate. Compound interest accounts grow more rapidly because a percentage of the principal and interest is added to the balance each year.

Interest Formulas

The formula to determine simple interest is $I = Prt$, where I is interest, P is principal (amount invested), r is the interest rate, and t is time (in years). The formula for compound interest is $A = P(1 + r)^t$, where A represents the final balance, P represents the original principal amount invested, r represents the annual interest rate, and t represents the time in years.

For example, Lucas's family opens two savings accounts for him to start saving for college. They deposit $200 in each account. Both accounts have an interest rate of 2.5%. However, one is a simple interest account and the other is a compound interest account. He keeps the money in the accounts for 18 years.

Simple Interest Account	Compound Interest Account
$I = Prt$	$A = P(1 + r)^t$
$I = (200)(0.025)(18)$	$A = 200(1 + 0.025)^{18}$
$I = 90$	$A = 311.93$
$200 + 90 = 290$	

$$311.93 - 290.00 = 21.93$$

After 18 years, the compound interest account earns $21.93 more than the simple interest account.

In **Lesson 2: Terms of a Loan,** students learn how to make financially responsible decisions based upon analyzing the terms of loans.

Loans

Students review the terms of a loan, which include the amount of the loan, the interest rate on the loan, the time of the loan, and the monthly payments on the loan. A person's credit history can affect the loan they are able to receive. A good credit score can help you get the best possible interest rate available. But interest rates can also vary, depending on the lender and the length of the loan. When applying for a loan, a person should try to get the best terms they can. Shopping around for the best loan terms could save you thousands of dollars.

In **Lesson 3: Online Calculators,** students explore various credit card options and the implications of each one.

Credit Cards

Students learn that the mathematics behind credit card debt is more complicated than just determining compound interest. If the loan is not paid off in full at the end of the month, many factors play a role in determining the details of different payment options. For this reason, technology, such as online calculators, can be valuable tools for comparing credit card payment options.

MYTH

Once I understand something, it has been learned.

Learning is tricky for three reasons. First, even when we learn something, we don't always recognize when that knowledge is useful. For example, you know there are four quarters in a dollar. But if someone asks you, "What is 75 times 2?" you might not immediately recognize that is the same thing as having six quarters.

Second, when you learn something new, it's not as if the old way of thinking goes away. For example, some children think of north as straight ahead. But, have you ever been following directions on your phone and made a wrong turn, only to catch yourself and think, "I know better than that!"?

The final reason that learning is tricky is that it is balanced by a different mental process: forgetting. Even when you learn something (e.g., your phone number), when you stop using it (e.g., when you move), it becomes extremely hard to remember.

There should always be an asterisk next to the word when we say we learned* something.

#mathmythbusted

In **Lesson 4: Financing Your Education,** students explore options to finance post-secondary education.

Paying for College

It is never too early to begin planning for college. There are many things that can be done while in high school that could lead to success after high school.

In Texas, the Recommended High School Program, one of four state-approved graduation programs, offers courses that students should take to best prepare them for post-secondary education. Students can begin earning college credits before even graduating high school.

When planning to pay for college, there are a number of ways to get help.

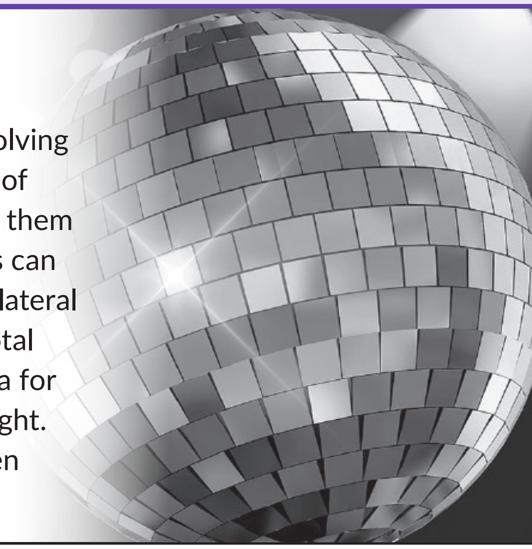
- The Free Application for Federal Student Aid. (FASFA) is a free application that could make students eligible for grants, loans, and work-study funds each year.
- The Texas College Savings Plan also called the *Texas 529 Savings Plan*, is a savings plan that offers tax-free growth, as well as tax-free withdrawals, for things like books, transportation, room and board, and other miscellaneous expenses related to education.
- The Texas Tuition Promise Fund® is a fund that allows you to lock in current undergraduate tuition rates by purchasing "units" to be spent at Texas public colleges and universities, excluding medical and dental institutions.
- Education IRAs, now more formally known as Coverdell Savings Accounts, are a type of investment that allows Texas residents to withdraw money from their retirement accounts, without penalty, to pay for college. Families may deposit up to $2000 per child into this type of account.

Family Guide

MODULE 5 Applying Powers

Grade 8

TOPIC 4 Volume of Curved Figures



In this topic, students solve real-world and mathematical problems involving lateral and total surface area of prisms and cylinders as well as volume of cylinders, cones, and spheres. Students explore cylinders and compare them with right prisms to determine that the volume formula for right prisms can also be applied to cylinders. Students use their prior knowledge about lateral and total surface area of rectangular prisms to determine lateral and total surface area of cylinders. Next, students determine the volume formula for a cone by connecting it to a cylinder that has a congruent base and height. Students learn and apply the volume formula for a sphere and they then solve problems with composite figures.

Where have we been?

Throughout elementary school, students learned about 3-D figures. In Grades 6 through 8, they have learned to calculate the lateral surface area, total surface area, and volume of pyramids and prisms. Students have also learned to calculate area and circumference of circles.

Where are we going?

This topic opens the door for students to engage in geometric design and model real-world situations. As students study polynomial functions in Algebra II, volumes of three-dimensional figures are useful for developing an understanding of graphical characteristics of cubic functions.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning about lateral surface area, total surface area, and volume of 3-D figures.

QUESTIONS TO ASK

- What do you know about this geometric solid? What are you trying to figure out?
- What is the relationship between this geometric solid and other geometric figures that you have learned about?
- Does your solution seem reasonable in this situation?

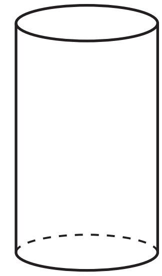
NEW KEY TERMS

- cylinder [cilindro]
- right cylinder [cilindro recto]
- radius of a cylinder [radio de un cilindro]
- height of a cylinder
- cone [cono]
- height of a cone
- sphere [esfera]
- center of a sphere [centro de una esfera]
- radius of a sphere [radio de una esfera]
- diameter of a sphere [diámetro de una esfera]
- great circle

Refer to the Math Glossary for definitions of the New Key Terms.

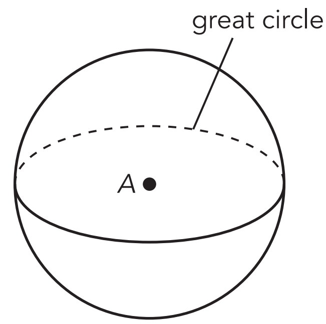
Where are we now?

A **right cylinder** is a cylinder in which the bases are aligned one directly above the other.



A **great circle** is the circumference of the sphere at the sphere's widest part.

Point *A* is the center of the sphere. It is also the center of the great circle.

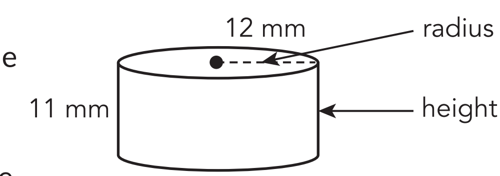


In **Lesson 1: Volume, Lateral and Total Surface Area of a Cylinder,** students develop and apply the formulas for volume, lateral surface area, and total surface area of a cylinder.

Cylinders

A cylinder is a three-dimensional object with two parallel, congruent circular bases. A right cylinder is a cylinder in which the bases are circles that are aligned one directly above the other.

The radius of a cylinder is the distance from the center of the base to any point on the edge of the base. The length of the radius of a cylinder is the same on both bases.

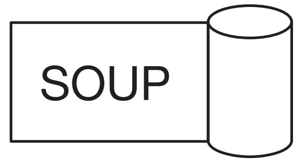


The height of a cylinder is the length of a line segment drawn from one base to the other base, perpendicular to both bases.

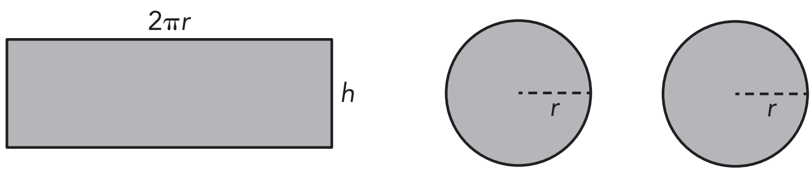
The volume of any cylinder can be calculated by multiplying the area of the circular base by the height of the cylinder. The formula for the area of a circle is $A = \pi r^2$, so the formula for the volume of a cylinder is $V = Bh$, or $V = \pi r^2 h$.

Surface Area of a Cylinder

The total surface area is the sum of the areas that form the surface of a three-dimensional figure. The label of a can covers the surface of the can that does not include the two bases that are circles. You call this *lateral surface area*. If you cut the label from a can, you can see that the label is a rectangle.



The width of the rectangle is the height of the can. Because the label wraps around the can, the length of the rectangle is the circumference of the can. The total surface area of the can is the area of the rectangle plus the area of the two circular bases.



The radius of one of the circular bases is 2 inches. The height of the soup can is 5 inches.

Lateral surface area: $S = 2\pi rh = 2\pi(2)(5) \approx 62.8$ square in.

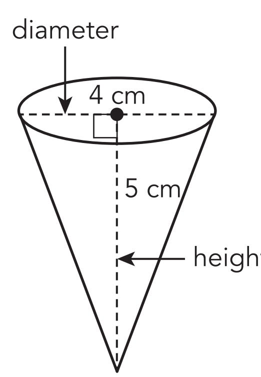
Total surface area: $S = 2\pi rh + 2\pi r^2 = 2\pi(2)(5) + 2\pi(2)^2 \approx 87.92$ square in.

In **Lesson 2: Volume of a Cone,** students learn how to calculate the volume of a cone and solve problems involving cones.

Cones

A **cone** is a three-dimensional object with a circular or oval base and one vertex. The height of a cone is the length of a line segment drawn from the vertex to the base of the cone. In a right cone, this line segment is perpendicular to the base.

The volume of a cone is one-third the volume of a cylinder with the same base and height as the cone. Therefore, the formula for the volume of a cone is $V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$.



$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}\pi\left(\frac{4}{2}\right)^2(5)$$

$$\approx 20.94 \text{ cubic centimeters}$$

The volume of the cone is approximately 20.94 cubic centimeters.

MYTH

Some students are "right-brain" learners, while other students are "left-brain" learners.

As you probably know, the brain is divided into two hemispheres: left and right. Some categorize people by their preferred or dominant mode of thinking. "Right-brain" thinkers are considered to be more intuitive, creative, and imaginative. "Left-brain" thinkers are more logical, verbal, and mathematical.

The brain can also be broken down into lobes. The *occipital lobe* can be found in the back of the brain, and it is responsible for processing visual information. The *temporal lobes*, which sit above your ears, process language and sensory information. The *parietal lobe*, which is a band across the top of your head, controls movement. Finally, the *frontal lobe* is where planning and learning occurs. Another way to think about the brain is from the back to the front, where information goes from highly concrete to abstract.

Why don't we claim that some people are "back of the brain," thinkers who are highly concrete, whereas others are "frontal" thinkers, who are more abstract? The reason is that the brain is a highly interconnected organ. Each lobe hands off information to be processed by other lobes, and they are constantly talking to each other. All of us are *whole-brain* thinkers!

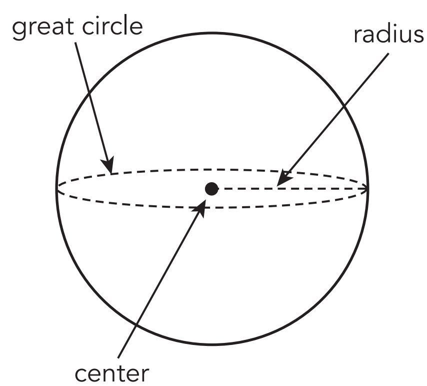
#mathmythbusted

In **Lesson 3: Volume of a Sphere,** students learn how to calculate the volume of a sphere and solve problems involving spheres.

Spheres

A sphere is the set of all points in three dimensions that are the same distance from a given point called the *center of a sphere*. Like a circle, a sphere has radii and diameters. The length of a diameter is twice the length of a radius.

The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r represents the length of the radius of the sphere.



For example, calculate the volume of a sphere with a radius length of 4.5 inches.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(4.5)^3$

$V \approx 381.7$ cubic inches

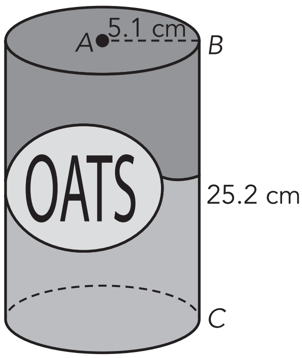
The volume of a sphere with a radius length of 4.5 inches is approximately 381.7 cubic inches.

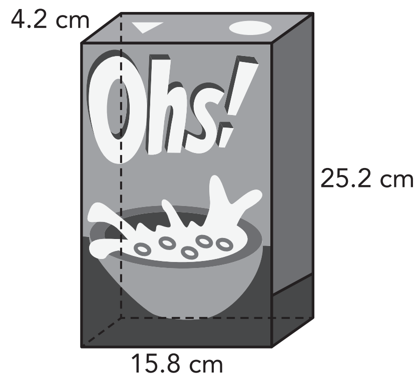
In **Lesson 4: Volume and Surface Area Problems with Prisms, Cylinders, Cones, and Spheres,** students review all of the formulas they have learned up to this point and use them to solve real-world and mathematical problems.

Real-World Problems

You can use the formulas for the lateral and total surface area of prisms and cylinders, or for the volume of cones, cylinders, and spheres, to solve real-world problems.

For example, you can compare the total surface area of a cylinder and a rectangular prism.





Total Surface Area of Cylinder

$S = 2\pi rh + 2\pi r^2$

$S = 2\pi(5.1)(25.2) + 2\pi(5.1)^2$

$S \approx 970.45 \text{ cm}^2$

Total Surface Area of Rectangular Prism

$S = Ph + 2B$

$S = (2(15.8)(25.2) + 2(4.2))(25.2) + 2(15.8)(4.2)$

$S \approx 1140.72 \text{ cm}^2$

The total surface area of the rectangular prism is greater than the total surface area of the cylinder.

ISBN: 178-9-69120-768-9

Printed in the USA

Images

www.pixabay.com