OER Secondary Mathematics
EDITION 1

Grade 8

Volume 1

STUDENT EDITION

Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

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OER Secondary Mathematics

EDITION 1

Grade 8

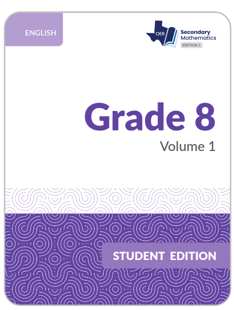
Course Guide

Welcome to the Course Guide for Secondary Mathematics, Grade 8

The instructional materials help you learn math in different ways. There are two types of resources: Learning Together and Learning Individually. These resources provide various learning experiences to develop your understanding of mathematics.

Learning Together

On **Learning Together** days, you spend time engaging in active learning to build mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the materials for each lesson.



STUDENT EDITION

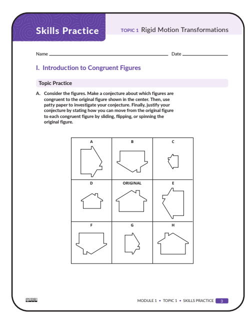
I am a record of your thinking, reasoning, and problem solving.

My lessons allow you to build new knowledge from prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On **Learning Individually** days, Skills Practice offers opportunities to engage with skills, concepts, and applications that you learn in each lesson. It also provides opportunities for interleaved practice, which encourages you to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning. This resource will help you build proficiency in specific skills based on your individual academic needs as indicated by monitoring your progress throughout the course.

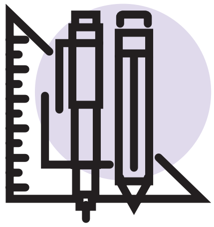


SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide problem sets for additional practice, enrichment, and extension.

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure you thoroughly understand the mathematical concepts. The instructional materials guide you to connect related ideas holistically, supporting the integration of your evolving mathematical understanding and developing proficiency with mathematical processes.



Intentional Mathematics Design

Mathematical Coherence: The path through the mathematics develops logically, building understanding by linking ideas within and across grades so you can learn concepts more deeply and apply what you've learned to more complex problems.

TEKS Mathematical Process Standards: The instructional materials support your development of the TEKS mathematical process standards. They encourage you to experiment, think creatively, and test various strategies. These mathematical processes empower you to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem-solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture your interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for you to build new understanding on the firm foundation of what you already know. You move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem-Solving: Solving problems is an essential life skill that you need to develop. The problem-solving model provides a structure to support you as you analyze and solve problems. It is a strategy you can continue to use as you solve problems in everyday life.

1

Introduction to Congruent Figures

1 OBJECTIVES

- Define *congruent figures*.
- Use patty paper to verify experimentally that two figures are congruent by obtaining the second figure from the first using a sequence of slides, flips, and/or turns.
- Use patty paper to determine whether two figures are congruent.

3 NEW KEY TERMS

- congruent figures
- corresponding sides
- corresponding angles

2 You have studied figures that have the same shape or measure. How do you determine whether two figures have the same size and the same shape?

LESSON STRUCTURE

1 Objectives

Objectives are stated for each lesson to help you take ownership of the objectives.

2 Essential Question

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3 New Key Terms

The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

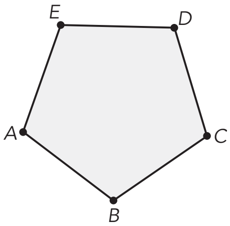
4 Getting Started

It's Transparent!

Patty paper is great paper to investigate geometric properties. You can write on it, trace with it, and see creases when you fold it.

Let's use patty paper to investigate the figure shown.

1. List everything you know about the shape.



2. Use patty paper to compare the sizes of the sides and angles in the figure.

   a. What do you notice about the side lengths?

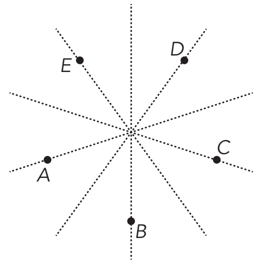
   b. What do you notice about the angle measures?

   c. What can you say about the figure based on this investigation?

Patty paper was originally created for separating patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool.

Trace the polygon onto a sheet of patty paper.

3. Use five folds of your patty paper to determine the center of each side of the shape. What do you notice about where the folds intersect?



5 ACTIVITY 1.1 Analyzing Size and Shape

Cut out each of the figures provided at the end of the lesson.

1. Sort the figures into at least two categories. Provide a rationale for your classification. List your categories and the letters of the figures that belong in each category.

2. List the figures that are the same shape as Figure A. How do you know they are the same shape?

Figures with the same shape but not necessarily the same size are *similar figures*, which you will study in later lessons.

3. List the figures that are both the same shape and the same size as Figure A. How do you know they are the same shape and same size?

Corresponding sides are sides that have the same relative position in geometric figures.

Figures that have the same size and shape are **congruent figures**. When two figures are congruent, all *corresponding sides* and all *corresponding angles* have the same measure.

4. List the figures that are congruent to Figure C.

Corresponding angles are angles that have the same relative position in geometric figures.

5 Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about getting the answer. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Talk the Talk 6

The Core of Congruent Figures

Recall that when two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

Consider these figures.

1. How can you slide, flip, or spin the figure on the left to obtain the figure on the right?

2. Use patty paper to determine the corresponding sides and corresponding angles of the congruent figures.

Lesson 1 Assignment

7 Write

Explain what a conjecture is and how it is used in math.

Remember 8

If two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

9 Practice

1. Determine which figures are congruent to Figure *A*. Follow the steps given as you investigate each shape.
   - Make a conjecture about which figures are congruent to Figure *A*.
   - Use patty paper to investigate your conjecture.
   - Justify your conjecture by stating how you can move from Figure *A* to each congruent figure by sliding, flipping, or spinning Figure *A*.

Figure *A*

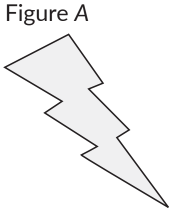


Figure *B*

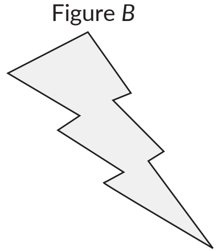


Figure *C*

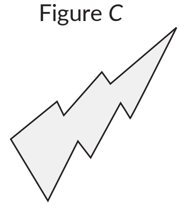


Figure *D*

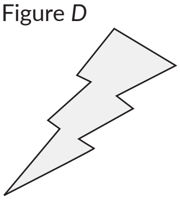


Figure *E*

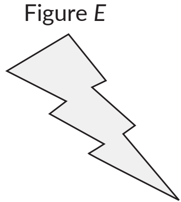


Figure *F*

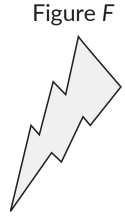


Figure	Your Conjecture	Congruent to Figure *A*?	How Do You Move Figure *A* onto the Congruent Figure?
B			
C			
D			
E			
F			

ASSIGNMENT

7 Write

Reflect on your work and clarify your thinking.

8 Remember

Take note of the key concepts from the lesson.

9 Practice

Use the concepts learned in the lesson to solve problems.

ASSIGNMENT

(10) **Prepare**

Get ready for the next lesson.

Lesson 1 Assignment

(10)

Prepare

Draw all lines of symmetry for each letter.

1. A

2. B

3. H

4. X

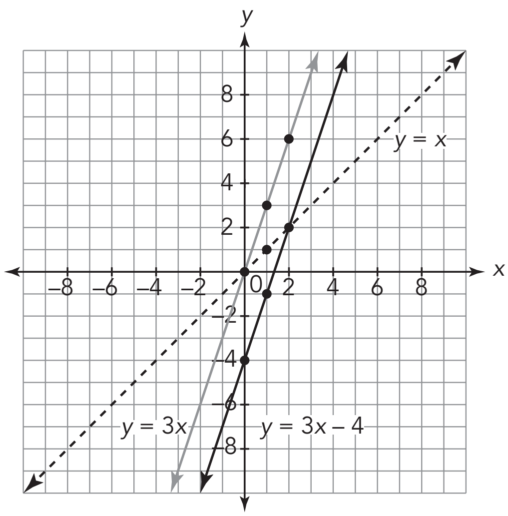
WORKED EXAMPLE

Graph $y = 3x - 4$ using transformations of the basic linear equation $y = x$.

First, graph the basic equation, $y = x$, and consider at least 2 sets of ordered pairs on the line, for example (0, 0), (1, 1), and (2, 2).

Then, dilate the y-values by 3.

Finally, translate all y-values down 4 units.



Ask Yourself . . .

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

WORKED EXAMPLE

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

THUMBS UP

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

THUMBS DOWN

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

WHO'S CORRECT?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine correct whether the work is correct or incorrect.

The Pythagorean Theorem can be used to determine unknown side lengths in a right triangle. Liam and Victoria are using the theorem to determine the length of the hypotenuse, *c*, with leg lengths of 2 and 4. Examine their work.

Victoria

$c^2 = 2^2 + 4^2$
$c^2 = 4 + 16 = 20$
$c = \sqrt{20} \approx 4.5$

The length of the hypotenuse is approximately 4.5 units.

Ask Yourself . . .

- Why is this method correct?
- Have I used this method before?

Liam

$c^2 = 2^2 + 4^2$
$c^2 = 6^2$
$c = 6$

The length of the hypotenuse is 6 units.

Ask Yourself . . .

- Where is the error?
- Why is it an error?
- How can I correct it?

d. Luna claims Ms. Park must begin with the number 1 when assigning numbers to students. Jorge says she can start with any number as long as she assigns every student a different number. Who is correct? Explain your reasoning.

Ask Yourself . . .

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I Can” expectations listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I CAN:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology, as appropriate; and techniques including mental math, estimation, and number sense, as appropriate, to solve problems.

I CAN:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language, as appropriate.

I CAN:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Create and use representations to organize, record, and communicate mathematical ideas.

I CAN:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Analyze mathematical relationships to connect and communicate mathematical ideas.

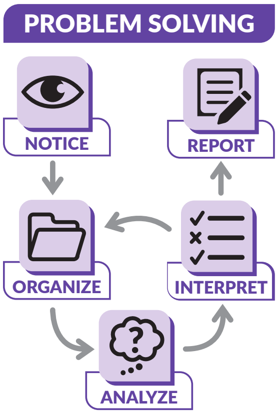
I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.



The Problem-Solving Model

Productive mathematical thinkers are problem-solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

Understanding the Problem-Solving Model

NOTICE

Notice | Wonder

Understand the situation by asking these questions.

- What do I know?
- What do I need to determine?
- What important information is given that I will need to determine a solution?
- What information is given that I do NOT need?
- Is there enough information given to solve the problem?

ORGANIZE

Organize | Mathematize

Devise a plan for your mathematical approach. Ask yourself these questions.

- What is a similar problem to this that I have solved before?
- What strategies may help to solve this problem using the given information?
- How can I represent this problem using a picture, diagram, symbols, graph, or some other visual representation? Which representations make sense for this problem?

PREDICT

Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I show my math work using representations?
- Did I explain my mathematical solution in terms of the problem situation, when applicable?
- Did I describe how I arrived at my solution?
- Did I communicate my strategy and solution clearly using precise mathematical language as necessary?
- Can I make any predictions based on my work?

INTERPRET

Test | Interpret

Look back at your work and ask these questions.

- Does the solution answer the original question/problem?
- Does the reasoning and the solution make sense?
- How could I have used a different strategy to solve this problem? Would it have changed the outcome?

REPORT

Report

As you share your mathematical reasoning with others, ask these questions.

- Did I share my solution with others?
- Do others understand the mathematics I communicated?

The Problem-Solving Model Graphic Organizer

NOTICE **Understand the Problem**	ORGANIZE **Devise a Plan**

PREDICT **Carry Out the Plan**

INTERPRET **Look Back**

REPORT **Report**

Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition

Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Explain Your Reasoning

Definition

Give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

What is Productive Struggle?

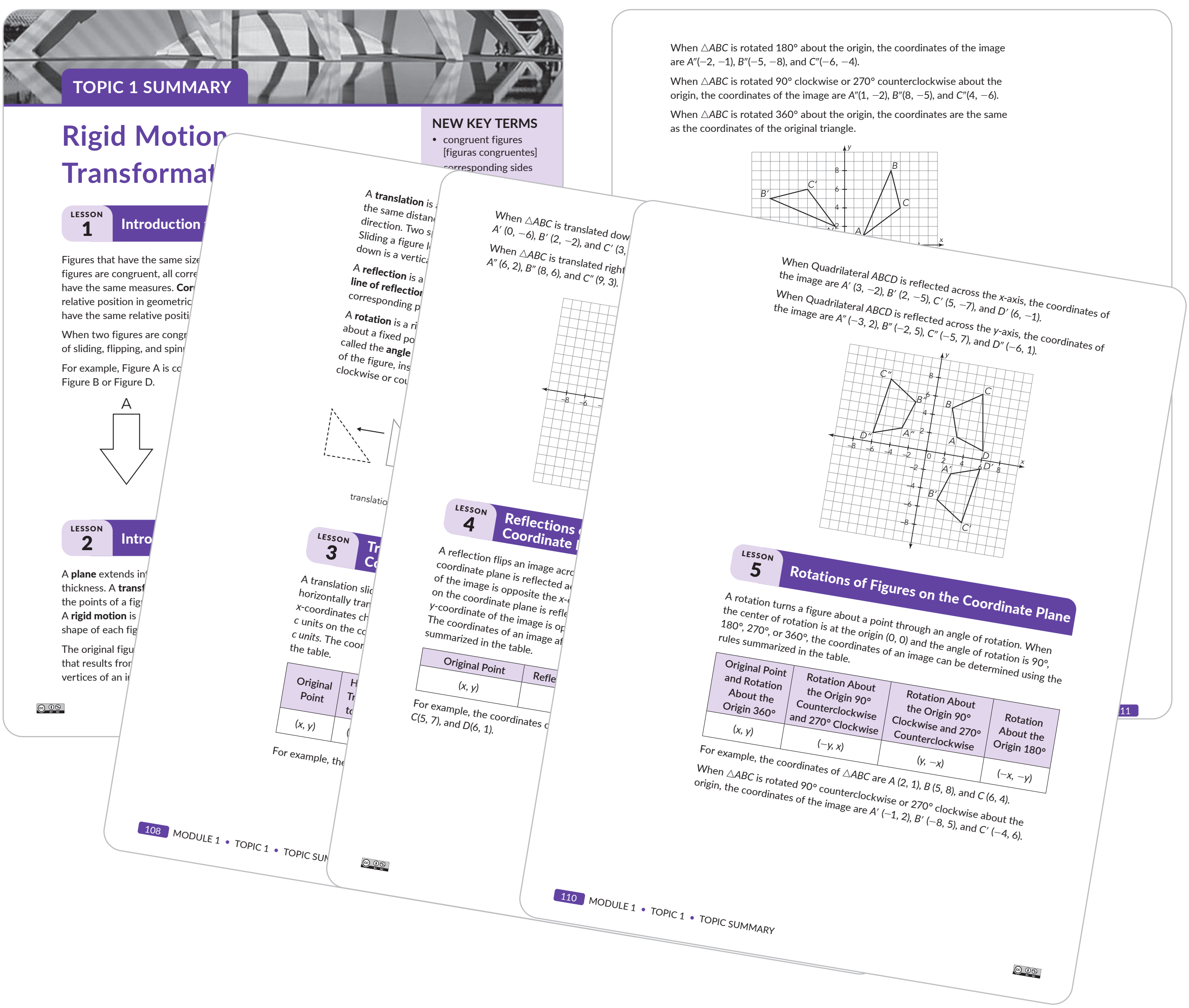
Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process productive struggle. *Productive struggle* is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

Topic Summary

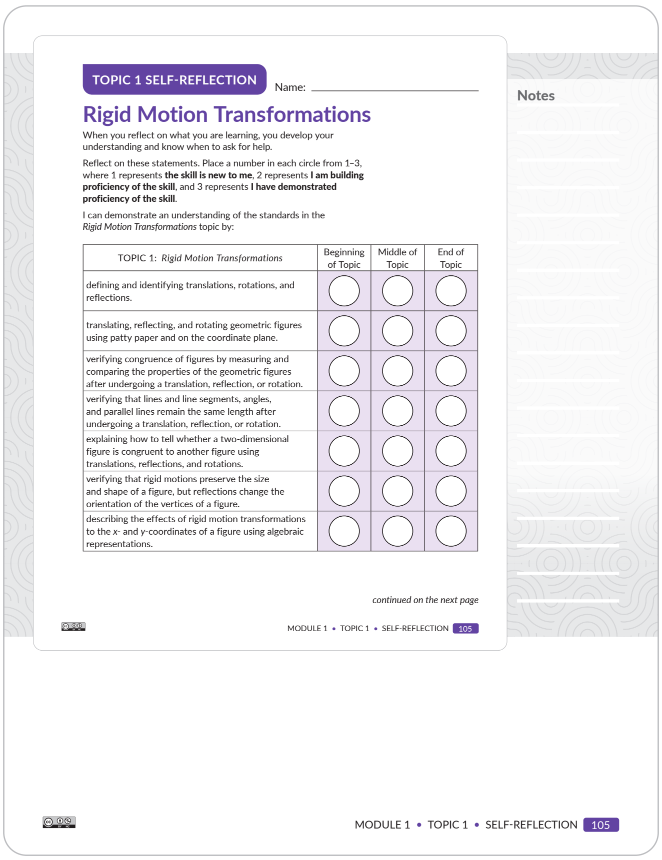
Each topic includes a Topic Summary. The Topic Summary contains a list of all new key terms addressed in the topic and a summary of each lesson, including worked examples and new key term definitions. Use the Topic Summary to review each lesson's major concepts and strategies as you complete assignments and/or share your learning outside of class.



Topic Self-Reflection

The Topic Self-Reflection, provided at the end of each topic, empowers you to develop confidence in your mathematical understanding and monitor your own learning processes. Taking the time for self-reflection helps you identify your strengths and where you want to focus your efforts to improve.

Use the Topic Self-Reflection throughout the topic to monitor your progress toward the mathematical goals for the topic.



TOPIC 1 SELF-REFLECTION Name: ____________

Rigid Motion Transformations

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

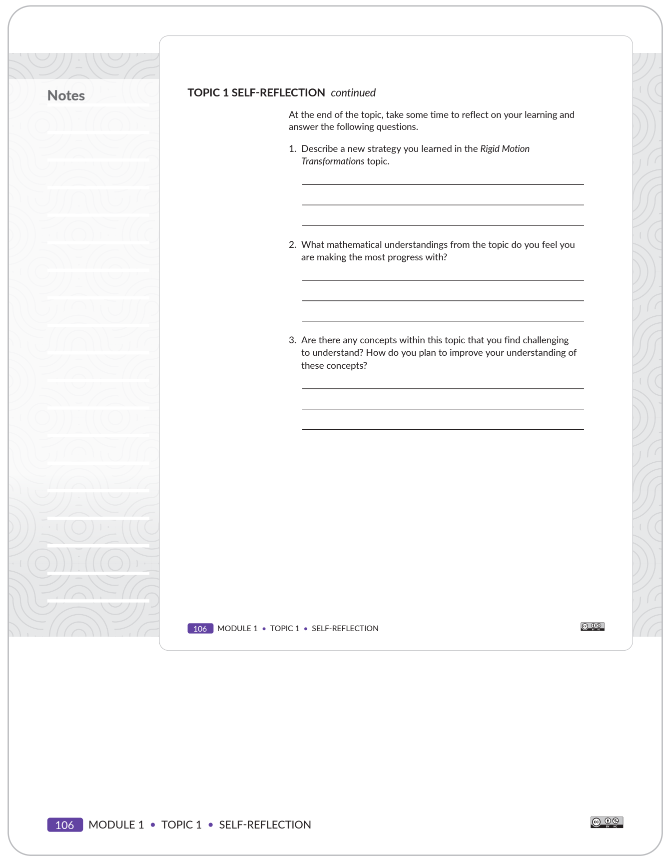
I can demonstrate an understanding of the standards in the *Rigid Motion Transformations* topic by:

TOPIC 1: *Rigid Motion Transformations*	Beginning of Topic	Middle of Topic	End of Topic
defining and identifying translations, rotations, and reflections.	◯	◯	◯
translating, reflecting, and rotating geometric figures using patty paper and on the coordinate plane.	◯	◯	◯
verifying congruence of figures by measuring and comparing the properties of the geometric figures after undergoing a translation, reflection, or rotation.	◯	◯	◯
verifying that lines and line segments, angles, and parallel lines remain the same length after undergoing a translation, reflection, or rotation.	◯	◯	◯
explaining how to tell whether a two-dimensional figure is congruent to another figure using translations, reflections, and rotations.	◯	◯	◯
verifying that rigid motions preserve the size and shape of a figure, but reflections change the orientation of the vertices of a figure.	◯	◯	◯
describing the effects of rigid motion transformations to the x- and y-coordinates of a figure using algebraic representations.	◯	◯	◯

continued on the next page

Notes

MODULE 1 • TOPIC 1 • SELF-REFLECTION 105



Notes

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Rigid Motion Transformations* topic.

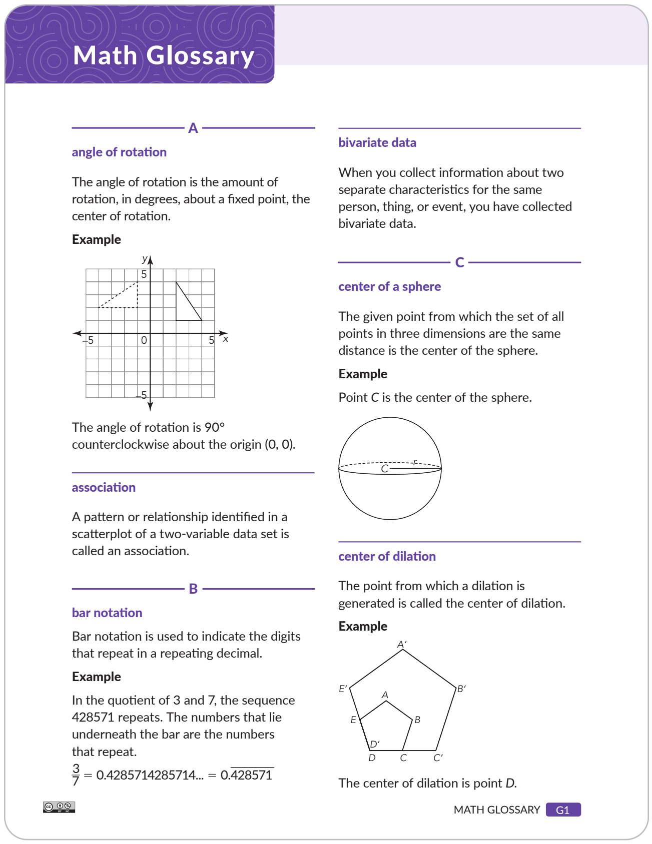
2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

106 MODULE 1 • TOPIC 1 • SELF-REFLECTION

Math Glossary

A course-specific math glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.



Math Glossary

A

angle of rotation

The angle of rotation is the amount of rotation, in degrees, about a fixed point, the center of rotation.

Example

The angle of rotation is 90° counterclockwise about the origin (0, 0).

association

A pattern or relationship identified in a scatterplot of a two-variable data set is called an association.

B

bar notation

Bar notation is used to indicate the digits that repeat in a repeating decimal.

Example

In the quotient of 3 and 7, the sequence 428571 repeats. The numbers that lie underneath the bar are the numbers that repeat.

$\frac{3}{7} = 0.4285714285714... = 0.\overline{428571}$

bivariate data

When you collect information about two separate characteristics for the same person, thing, or event, you have collected bivariate data.

C

center of a sphere

The given point from which the set of all points in three dimensions are the same distance is the center of the sphere.

Example

Point *C* is the center of the sphere.

center of dilation

The point from which a dilation is generated is called the center of dilation.

Example

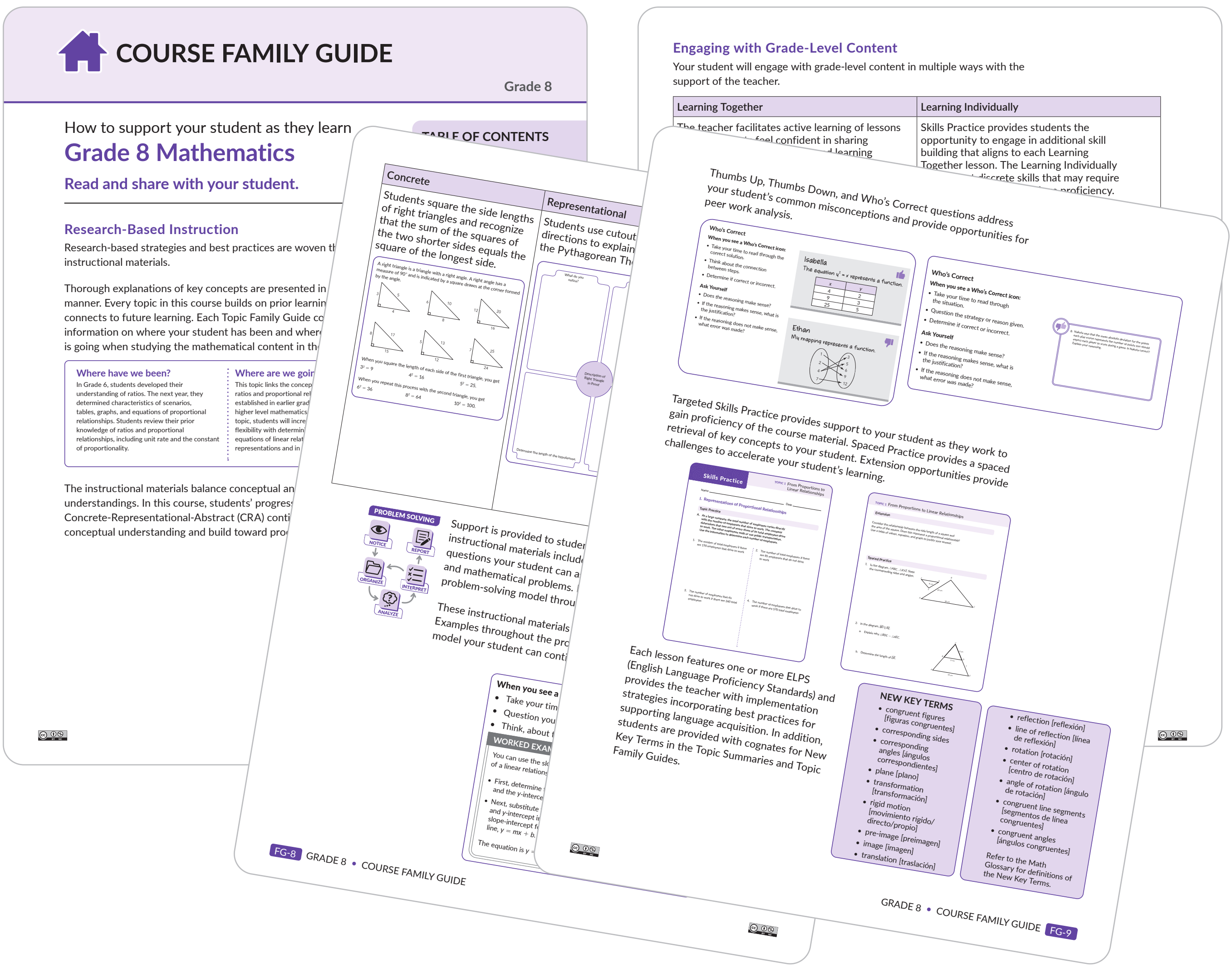
The center of dilation is point *D*.

MATH GLOSSARY G1

Course Family Guide

The Course Family Guide provides you and your family an overview of the course design. The guide details the resources available to support your learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

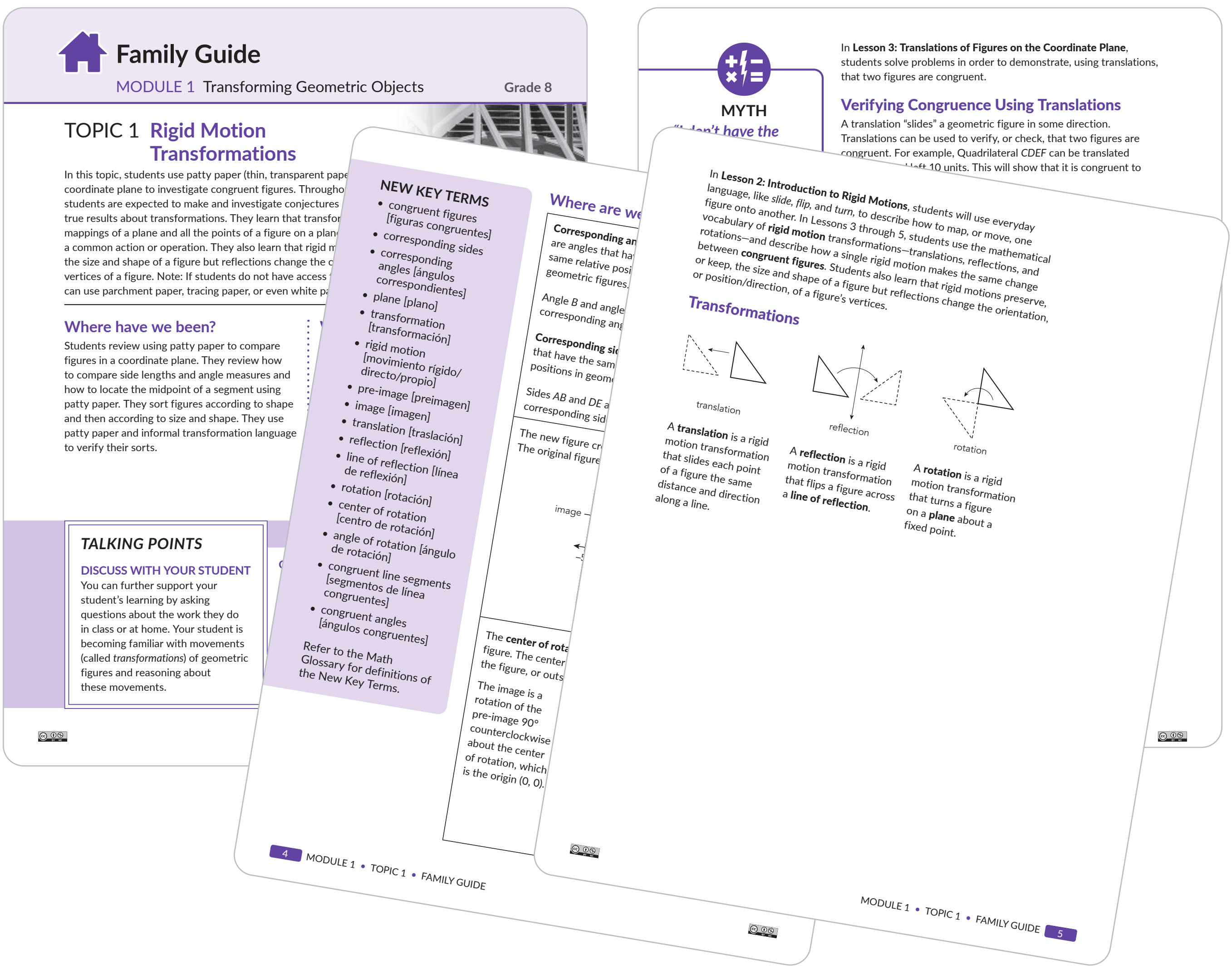
The purpose of the Course Family Guide is to bridge your learning in the classroom to your learning at home. The goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together.



Topic Family Guide

Each topic contains a Topic Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Topic Family Guide are an example of a math model or strategy taught in the topic, definitions of a few key terms, busting of a math myth, and questions family members can ask you to support your learning.

Learning outside of the classroom is crucial to student success at school. The Topic Family Guide serves to assist families in talking to students about the learning that is happening in the classroom.



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MODULE
1

Transforming Geometric Objects

GRADE 8

TOPIC 1

You can see geometric transformations everywhere. What reflections, rotations, and translations can you see in this picture?

Rigid Motion Transformations

INTRODUCTION LESSON

Introduction to the Problem-Solving Model and Learning Resources

OBJECTIVES

- Establish a community of learners.
- Discover learning resources available for this course.
- Apply the problem-solving model to a real-life situation.

In previous math classes, you have analyzed patterns and relationships, learned about numbers and operations in base ten and fractions, measurement and data, and geometry.

What resources are available in this course to help you extend your mathematical thinking?

Getting Started

You Already Know a Lot

Each lesson in this book begins with a Getting Started that gives you the opportunity to use what you know about the world and what you have learned in previous math classes. You know a lot from a variety of learning experiences.

Think back to how you learn something new.

1. List three different skills that you recently learned. Then, describe why you wanted to learn that skill and the strategies that you used.

New Skill	Motivation to Learn the New Skill	Strategies I Used to Learn This Skill

Ask Yourself . . .
How do your strategies change based on what you are learning and what you already know about it?

One learning strategy is to talk with your peers. In this course, you will work with your classmates to solve problems, discuss strategies, and learn together.

Compare and discuss your list with a classmate.

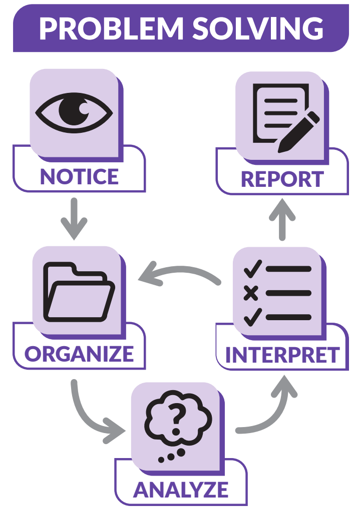
Think About . . .
Listening well, cooperating with others, and appreciating different perspectives are essential life skills.

2. Which strategies do you have in common? Which strategies does your classmate have that you did not think of on your own?

Be prepared to share your list of learning strategies with the class.

ACTIVITY 1.1

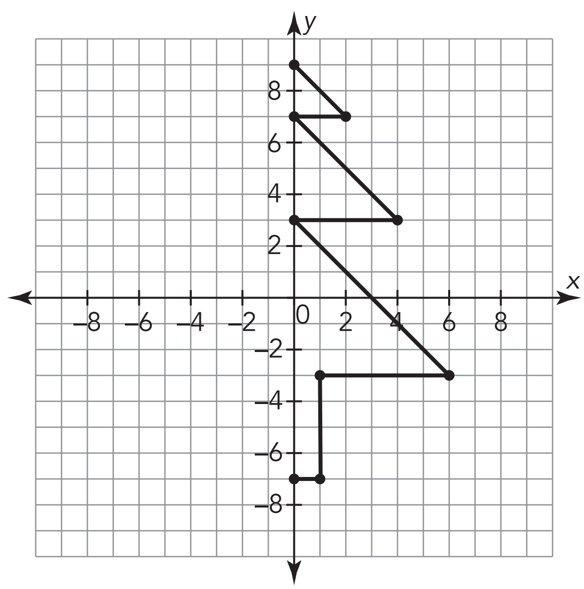
Learning Resources



In this course, you will learn new math concepts by exploring and investigating ideas, reading, writing, and talking to your classmates. You will even learn by making mistakes with concepts you haven't mastered yet.

Let's practice exploring and investigating. You do not need to answer the question yet. You will solve the question as you work through the problem-solving model.

Olivia is drawing a pine tree on a coordinate grid. She plots the points for the first half of her drawing. Determine where Olivia should plot the remaining points so the tree is reflected across the *y*-axis and symmetric. **Describe** your process.



The Academic Glossary is your guide as you engage with the kind of thinking you do as you are learning the content.

1. Locate the word **describe** in the Academic Glossary in the Course Guide. What questions should you ask yourself as you describe your process?

The Academic Glossary provides definitions of terms you will see throughout the course as you think, reason, and communicate your ideas. The Math Glossary provides the definitions of new key terms in each lesson.

2. What is a related word or phrase for the word **describe**?

The problem-solving model provides a structure to help you become a better problem-solver.

Notice and Wonder

The first step in modeling a situation mathematically is to understand the problem, gather information, notice patterns, and formulate mathematical questions about what you notice.

Read through the *Questions to Ask* yourself for the first step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

3. What do you notice about Olivia's drawing?

4. Why do you think the first step of the problem-solving model is important? How will it help you when you solve problems?

Organize and Mathematize

The second step in the problem-solving model is to devise a plan. When devising a plan, you will organize your information and begin to represent it using mathematical notation.

Read through the *Questions to Ask* yourself for the second step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

5. Describe your strategy for determining where Olivia should plot the additional points.

6. Why do you think the second step of the problem-solving model process is important? How can you use the *Questions to Ask* yourself to help develop a strategy to solve the problem?

Predict and Analyze

The third step in the modeling process involves carrying out your plan. As you carry out the plan you will complete operations, analyze mathematical results, make predictions, and extend patterns.

Read through the *Questions to Ask* yourself for the third step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

7. Use your strategy to determine where Olivia should plot the additional points to complete her drawing.

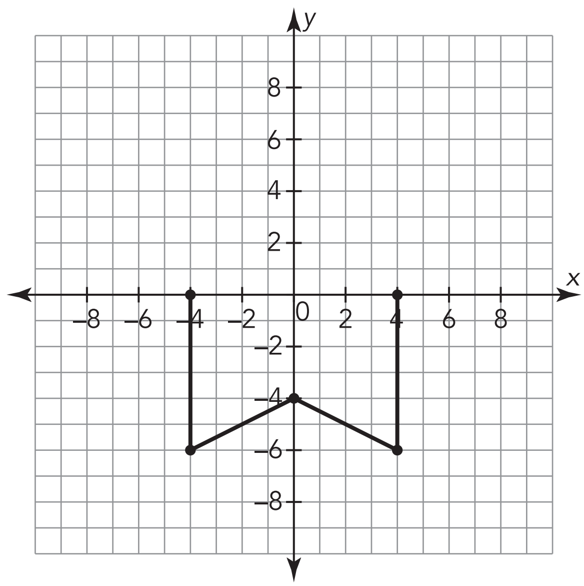
8. How can the *Questions to Ask* for the third step help you communicate your mathematical thinking?

Test and Interpret

The fourth step in the modeling process is to look back, interpret your results, and test your mathematical predictions in the real world. When your predictions are incorrect, or your results are not reasonable, you can revisit your mathematical work and make adjustments—or start all over!

Read through the *Questions to Ask* yourself for the fourth step of the problem-solving model. Use the *Problem-Solving Model Graphic Organizer* to answer the questions and show your thinking.

9. Olivia decides to draw a different symmetric figure that is reflected across the *x*-axis.



Can you apply the same reasoning to determine where she should plot additional points to complete her new picture? Explain your reasoning.

10. How do the questions for the fourth step of the problem-solving model help you evaluate the reasonableness of your solution?

Report

The final step in the modeling process is to share your results.

11. How does listening to others help you learn mathematics?

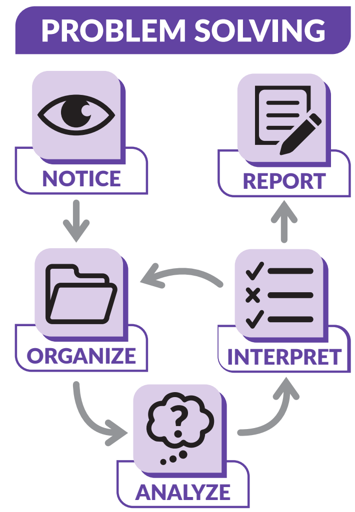
Locate the TEKS mathematical process standards in the Course Guide.

12. Which TEKS mathematical process standard(s) did you use to determine where Olivia should plot the additional points?

You will see this symbol throughout the course to remind you to use the problem-solving model.

Talk the Talk

The Problem-Solving Model



In this lesson, you used a problem-solving model to solve a real-world problem. The basic steps of the problem-solving model are summarized in the diagram.

Summarize what is involved in each phase of this problem-solving model.

Notice and Wonder

___

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Organize and Mathematize

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Predict and Analyze

___

___

Test and Interpret

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Report

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1 Introduction to Congruent Figures

OBJECTIVES

- Define *congruent figures*.
- Use patty paper to verify experimentally that two figures are congruent by obtaining the second figure from the first using a sequence of slides, flips, and/or turns.
- Use patty paper to determine whether two figures are congruent.

NEW KEY TERMS

- congruent figures
- corresponding sides
- corresponding angles

You have studied figures that have the same shape or measure. How do you determine whether two figures have the same size and the same shape?

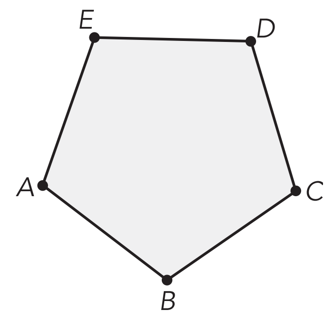
Getting Started

It's Transparent!

Patty paper is great paper to investigate geometric properties. You can write on it, trace with it, and see creases when you fold it.

Let's use patty paper to investigate the figure shown.

1. List everything you know about the shape.



2. Use patty paper to compare the sizes of the sides and angles in the figure.

   a. What do you notice about the side lengths?

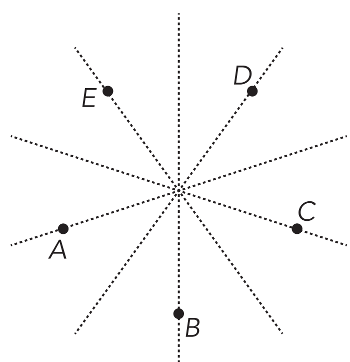
   b. What do you notice about the angle measures?

   c. What can you say about the figure based on this investigation?

Patty paper was originally created for separating patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool.

Trace the polygon onto a sheet of patty paper.

3. Use five folds of your patty paper to determine the center of each side of the shape. What do you notice about where the folds intersect?



ACTIVITY 1.1

Analyzing Size and Shape

Cut out each of the figures provided at the end of the lesson.

1. Sort the figures into at least two categories. Provide a rationale for your classification. List your categories and the letters of the figures that belong in each category.

2. List the figures that are the same shape as Figure A. How do you know they are the same shape?

Figures with the same shape but not necessarily the same size are *similar figures*, which you will study in later lessons.

3. List the figures that are both the same shape and the same size as Figure A. How do you know they are the same shape and same size?

Corresponding sides are sides that have the same relative position in geometric figures.

Figures that have the same size and shape are **congruent figures**. When two figures are congruent, all *corresponding sides* and all *corresponding angles* have the same measure.

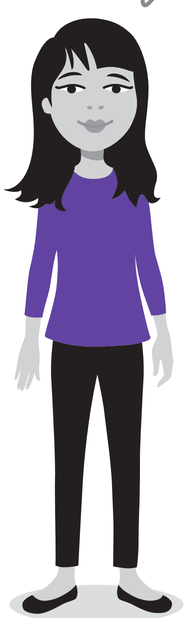
4. List the figures that are congruent to Figure C.

Corresponding angles are angles that have the same relative position in geometric figures.

ACTIVITY 1.2

Congruent or Not?

A *conjecture* is a hypothesis or educated guess that is based on what you know but hasn't been proven yet. Verifying your conjectures and making new ones, if necessary, is an important part of learning in mathematics.



Throughout the study of geometry, as you reason about relationships, study how figures change under specific conditions, and generalize patterns, you will engage in the geometric process of

- making a conjecture about what you think is true,
- investigating to confirm or refute your conjecture, and
- justifying the geometric idea.

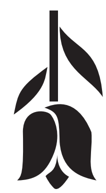
In many cases, you will need to make and investigate conjectures a few times before reaching a true result that can be justified.

Let's use this process to investigate congruent figures.

When two figures are congruent, you can slide, flip, and spin one figure until it lies on the other figure.

1. Consider the flowers shown following the table. For each flower, make a conjecture about which are congruent to the original flower, which is shaded in the center. Then, use patty paper to investigate your conjecture. Finally, justify your conjecture by stating how you can move from the shaded flower to each congruent flower by sliding, flipping, or spinning the original flower.

Flower	Congruent to Original Flower	How Do You Move the Original Flower onto the Congruent Flower?
A		
B		
C		
D		
E		
F		
G		
H		

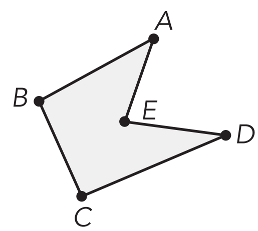
A 	B 	C 
D 	ORIGINAL 	E 
F 	G 	H 

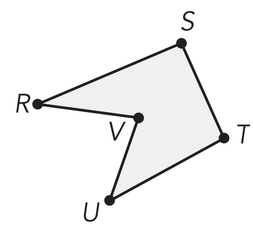
Talk the Talk

The Core of Congruent Figures

Recall that when two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

Consider these figures.

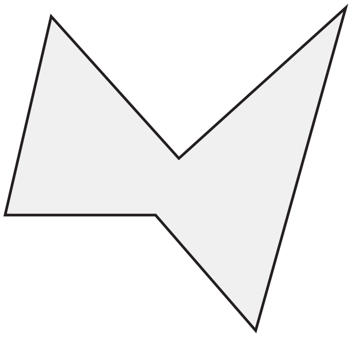




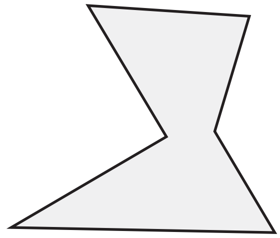
1. How can you slide, flip, or spin the figure on the left to obtain the figure on the right?

2. Use patty paper to determine the corresponding sides and corresponding angles of the congruent figures.

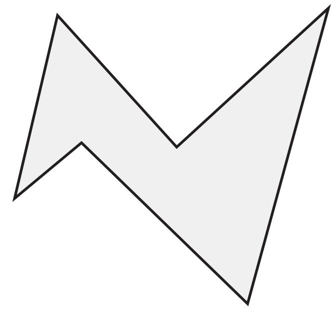
A



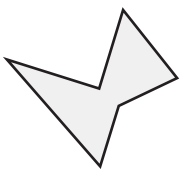
B



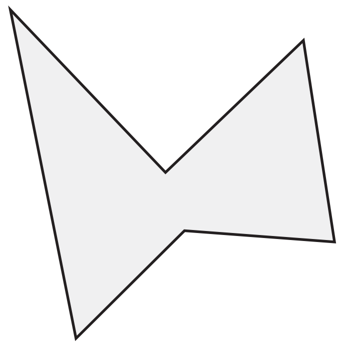
C



D



E



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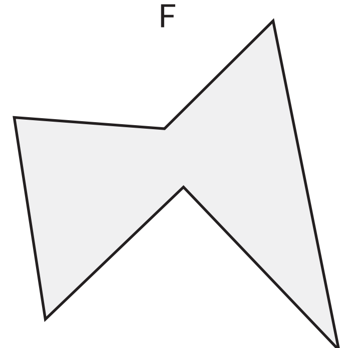

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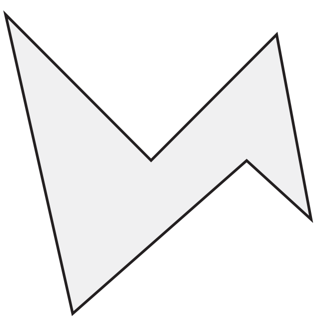
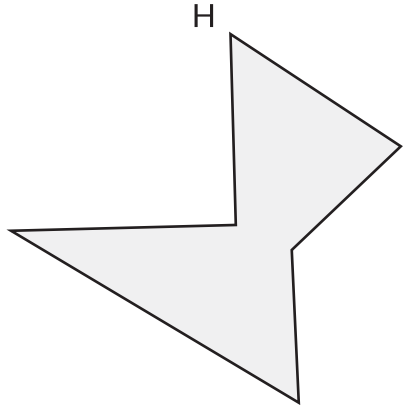


H



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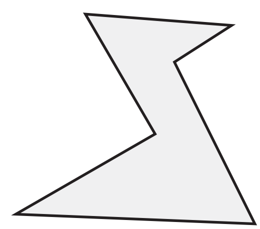
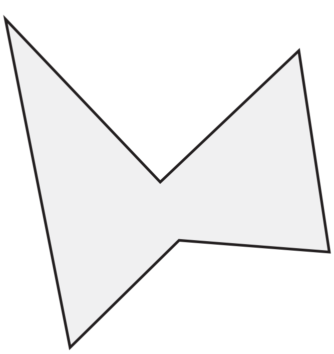
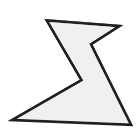


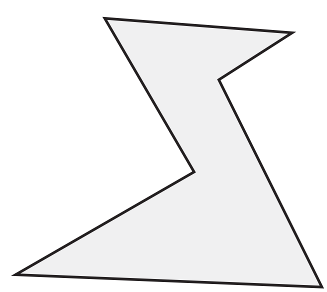
J



K



L



Why is this page blank?

So you can cut out the figures on the other side

Lesson 1 Assignment

Write

Explain what a conjecture is and how it is used in math.

Remember

If two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

Practice

1. Determine which figures are congruent to Figure *A*. Follow the steps given as you investigate each shape.
   - Make a conjecture about which figures are congruent to Figure *A*.
   - Use patty paper to investigate your conjecture.
   - Justify your conjecture by stating how you can move from Figure *A* to each congruent figure by sliding, flipping, or spinning Figure *A*.

Figure *A*

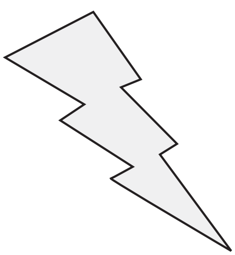


Figure *B*

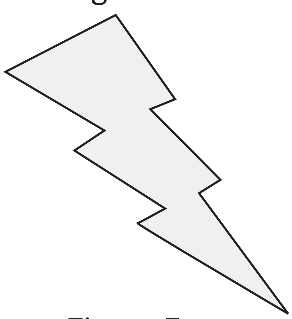


Figure *C*

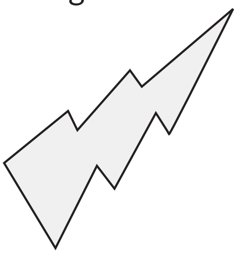


Figure *D*

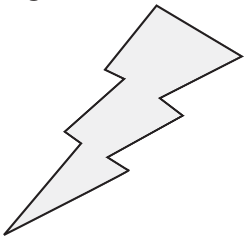


Figure *E*

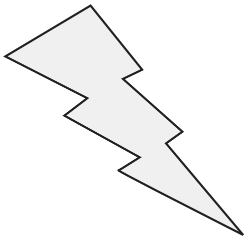


Figure *F*

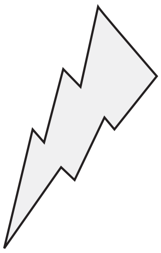


Figure	Your Conjecture	Congruent to Figure *A*?	How Do You Move Figure *A* onto the Congruent Figure?
B			
C			
D			
E			
F			

Lesson 1 Assignment

Prepare

Draw all lines of symmetry for each letter.

1. A

2. B

3. H

4. X

2 Introduction to Rigid Motions

OBJECTIVES

- Model transformations of a plane.
- Translate geometric objects on the plane.
- Reflect geometric objects on the plane.
- Rotate geometric objects on the plane.
- Describe a single rigid motion that maps a figure onto a congruent figure.

NEW KEY TERMS

- plane
- transformation
- rigid motion
- pre-image
- image
- translation
- reflection
- line of reflection
- rotation
- center of rotation
- angle of rotation

When you investigated shapes with patty paper, you used slides, flips, and spins to determine whether shapes are congruent.

What are the names and properties of the actions used to carry a figure onto a congruent figure?

Getting Started

Design Competition

A community track club is holding a 5K to raise money for new uniforms. They want to create a logo for the race that includes the running man icon. However, they want the logo to include at least four copies of the running man.

1. Trace the running man onto a sheet of patty paper. Create a logo for the track team on another sheet of patty paper that includes the original running man and three copies—one example each of sliding, flipping, and spinning the picture of the running man.

Are all the copies of the icon turned the same way?

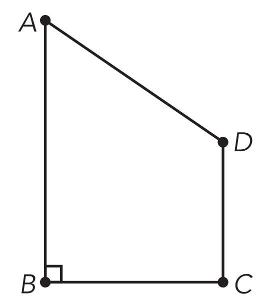
2. What do you know about the copies of the running man compared with the original picture of the running man?

Each sheet of patty paper represents a model of a geometric *plane*. A **plane** extends infinitely in all directions in two dimensions and has no thickness.

ACTIVITY 2.1 Translations on the Plane

In this module, you will explore different ways to transform, or change, planes and figures on planes. A **transformation** is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure. Each of the actions you used to make the running man logo—slide, flip, spin—is a rigid motion transformation.

You are going to start by exploring translations on the plane using the trapezoid shown. Trapezoid *ABCD* has angles *A*, *B*, *C*, and *D*, and sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$.



1. What else do you know about Trapezoid *ABCD*?

2. Use the Translations Mat at the end of the lesson for this exploration.

   a. Use a straightedge to trace the trapezoid on the shiny side of a sheet of patty paper.

   b. Slide the patty paper containing the trapezoid to align $\overline{AB}$ with one of the segments $\overline{A'B'}$.

   c. Record the location of the *image* of Trapezoid *ABCD* on the mat. This image is called Trapezoid *A'B'C'D'*.

Once you have traced the trapezoid on one side, turn the patty paper over and, using a pencil, copy the lines on the back side as well. This will help you to transfer the translated trapezoid back onto the Translations Mat.

$\overline{AB}$ is read, "line segment *AB*."
A' is read, "*A* prime."

The original trapezoid on the mat is called the **pre-image**.

The traced trapezoid is the **image**. It is the new figure that results from the transformation.

3. Examine your pre-image and image.

   a. Which angle in Trapezoid *ABCD* maps to each angle of Trapezoid *A'B'C'D'*? Label the vertices on your drawing of the image of Trapezoid *ABCD*.

   b. Which side of Trapezoid *ABCD* maps to each side of Trapezoid *A'B'C'D'*?

   c. What do you notice about the measures of the corresponding angles in the pre-image and the image?

   d. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

   e. What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?

   f. Is the image congruent to the pre-image? Explain your reasoning.

This type of movement of a plane containing a figure is called a *translation*. A **translation** is a rigid motion transformation that "slides" each point of a figure the same distance and direction. Let's verify this definition.

A figure can be translated in any direction. Two special translations are vertical and horizontal translations. Sliding a figure only left or right is a horizontal translation, and sliding it only up or down is a vertical translation.

4. On the mat, draw segments to connect corresponding vertices of the pre-image and image.

   a. Use a ruler to measure each segment. What do you notice?

   b. Compare your translations and measures with your classmates' translations and measures. What do you notice?

5. Consider the translation you created, as well as your classmates' translations.

   a. What changes about a figure after a translation?

   b. What stays the same about a figure after a translation?

   c. What information do you need to perform a translation?

ACTIVITY 2.2

Reflections on the Plane

The first transformation you explored was a translation. Now, let's see what happens when you flip, or reflect, the trapezoid. Trace Trapezoid *ABCD* onto a sheet of patty paper. Imagine tracing the trapezoid on one side of the patty paper, folding the patty paper in half, and tracing the trapezoid on the other half of the patty paper.

1. Make a conjecture about how the image and pre-image will be alike and different.

To verify or refine your conjecture, let's explore a reflection using patty paper and the Reflections Mat located at the end of the lesson. Trace the trapezoid from the previous activity on the lower left corner of a new piece of patty paper.

2. Align the trapezoid on the patty paper with the trapezoid on the Reflections Mat. Fold the patty paper along ℓ_1. Trace the trapezoid on the other side of the crease and transfer it onto the Reflections Mat. Label the vertices of the image Trapezoid *A'B'C'D'*.

3. Compare the pre-image and image that you created.

   a. What do you notice about the measures of the corresponding angles in the pre-image and the image?

   b. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

c. What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?

d. Is the image congruent to the pre-image? Explain your reasoning.

e. Draw segments connecting corresponding vertices of the pre-image and image. Measure the lengths of these segments and the distance from each vertex to the fold. What do you notice?

Notice that the segments you drew are perpendicular to the crease of the patty paper. Why do you think this is true?

4. Repeat the reflection investigation using Trapezoid *ABCD* and folding along ℓ_2. Record your observations.

How is a reflection in geometry like your reflection in a mirror?

5. Repeat the reflection investigation using Trapezoid *ABCD* and folding along ℓ_3. Record your observations.

Are the vertices of the image in the same relative order as the vertices of the pre-image?

This type of movement of a plane containing a figure is called a *reflection*. A **reflection** is a rigid motion transformation that "flips" a figure across a *line of reflection*. A **line of reflection** is a line that acts as a mirror so that corresponding points are the same distance from the line.

6. Consider the reflections you created.

   a. What changes about a figure after a reflection?

   b. What stays the same about a figure after a reflection?

   c. What information do you need to perform a reflection?

ACTIVITY
2.3 Rotations on the Plane

You have now investigated translating and reflecting a trapezoid on the plane. Let's see what happens when you spin, or rotate, the trapezoid. You are going to use the Rotations Mat found at the end of the lesson for this investigation.

Trace Trapezoid *ABCD* onto the center of a sheet of patty paper. Imagine spinning the patty paper so that the trapezoid is no longer aligned with the trapezoid on the mat.

1. Make a conjecture about how the image and pre-image will be alike and different.

Let's investigate with patty paper to verify or refine your conjecture.

2. Align your trapezoid with the trapezoid on the Rotations Mat.

   Put your pencil on point O_1 and spin the patty paper 90° in a clockwise direction.

   Then, copy the rotated trapezoid onto the Rotations Mat and label the vertices.

3. Compare the pre-image and image created by the rotation.

   a. What do you notice about the measures of the corresponding angles in the pre-image and the image?

b. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

c. What do you notice about the relationship of *A'B'* to *C'D'*? How does this relate to the corresponding sides of the pre-image?

d. Is the image congruent to the pre-image? Explain your reasoning.

4. Draw two segments: one to connect point O_2 to *A* and another to connect point O_2 to *A'*.

a. Measure the lengths of these segments. What do you notice?

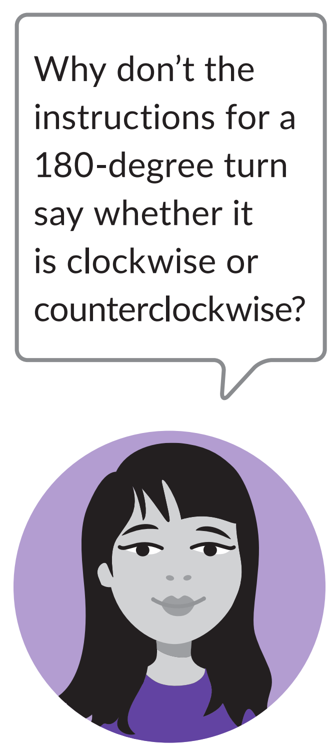
b. Measure the angle formed by the segments. What do you notice?

5. Repeat the process from the previous question with *B* and *B*'.
What do you notice about the segment lengths and angle measures?

6. What do you think is true about the segments connecting *C* and *C*' and the segments connecting *D* and *D*'?

7. Repeat the rotations investigation using Trapezoid *ABCD* and spinning the patty paper 90° in a counterclockwise direction around O_2. Record your observations.

8. Repeat the rotations investigation using Trapezoid *ABCD* and spinning the patty paper 180° around O_3. Record your observations.



Rotations can be clockwise or counterclockwise.

This type of movement of a plane containing a figure is called a *rotation*. A **rotation** is a rigid motion transformation that turns a figure on a plane about a fixed point, called the **center of rotation**, through a given angle, called the **angle of rotation**. The center of rotation can be a point outside the figure, inside the figure, or on the figure.

9. Consider the rotations you created.

   a. Describe the centers of rotation used for each investigation.

   b. How do you identify the angle of rotation, including the direction, in your patty paper rotations?

   c. What changes about a figure after a rotation?

   d. What stays the same about a figure after a rotation?

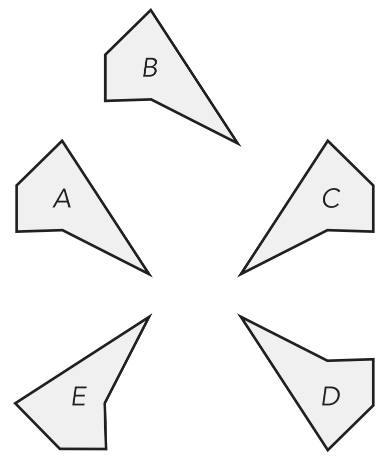
   e. What information do you need to perform a rotation?

Talk the Talk

Congruence in Motion

1. Describe a transformation that maps one figure onto the other. Be as specific as possible.

   a. Figure *A* onto Figure *B*

   b. Figure *A* onto Figure *C*

   c. Figure *A* onto Figure *E*

   d. Figure *C* onto Figure *D*

2. Explain what you know about the images that result from translating, reflecting, and rotating the same pre-image. How are the images related to each other and to the pre-image?

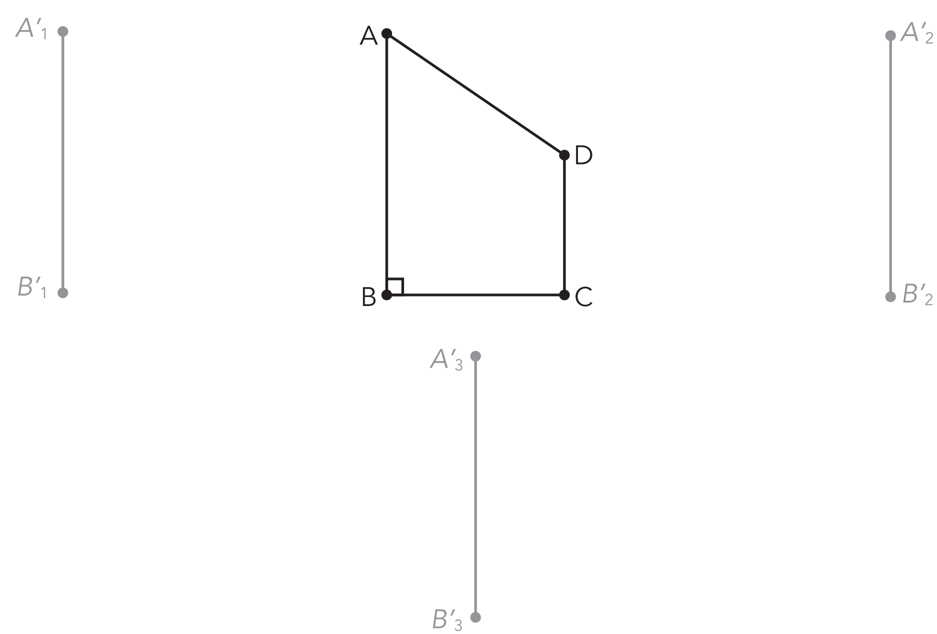
3. If Figure *A* is congruent to Figure *C* and Figure *C* is congruent to Figure *D*, answer each question.

   a. What is true about the relationship between Figures *A* and *D*?

   b. How could you use multiple transformations to map Figure *A* onto Figure *D*?

   c. How could you use a single transformation to map Figure *A* onto Figure *D*?

Translations Mat



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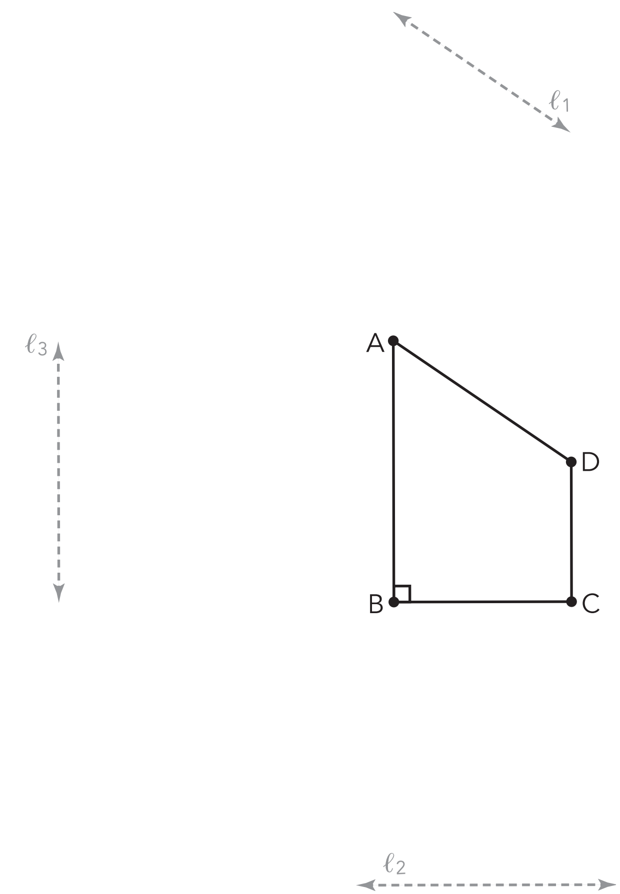
So you can tear out the page and explore the mat on the other side

Reflections Mat



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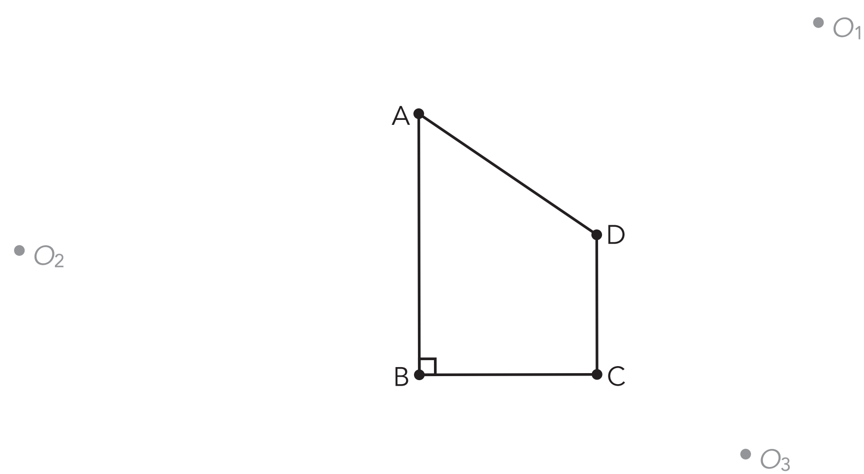
Rotations Mat



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So you can tear out the page and explore the mat on the other side

Lesson 2 Assignment

Write

Explain each term or set of terms in your own words.

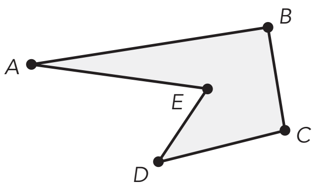
1. Transformation
2. Pre-image and image
3. Translation
4. Reflection and line of reflection
5. Rotation, angle of rotation, and center of rotation

Remember

Rigid motions are transformations that preserve the size and shape of figures. Translations and rotations also preserve the orientation of a figure. The relative order of the vertices is the same in the pre-image and the image of a translation and of a rotation.

Practice

1. Complete each rigid motion transformation of the provided figure. In each case, label the vertices of the image and label your transformation to demonstrate at least one property of the transformation.



   a. Translate the figure in a horizontal direction.

   b. Translate the figure in a vertical direction.

Lesson 2 Assignment

c. Translate the figure in a diagonal direction.

d. Reflect the figure across a vertical line of reflection.

e. Reflect the figure across a horizontal line of reflection.

f. Reflect the figure across a diagonal line of reflection.

Lesson 2 Assignment

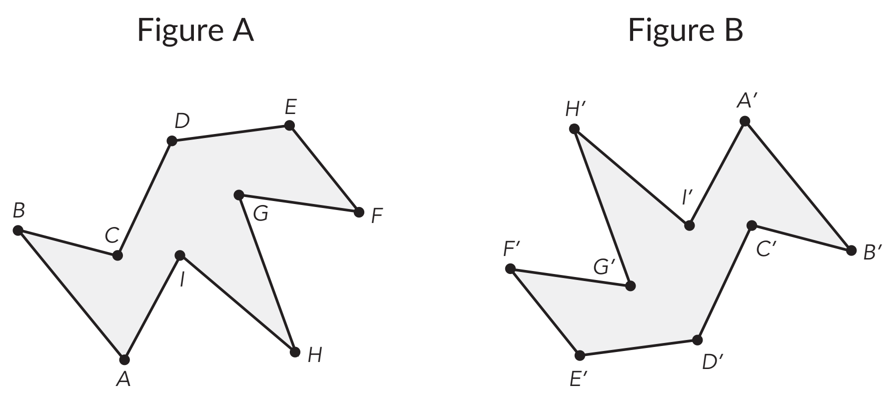
g. Rotate the figure 90° clockwise. Be sure to label the center of rotation.

h. Rotate the figure 90° counterclockwise. Be sure to label the center of rotation.

i. Rotate the figure 180°. Be sure to label the center of rotation.

Lesson 2 Assignment

2. Figure *B* is the image of Figure *A*.

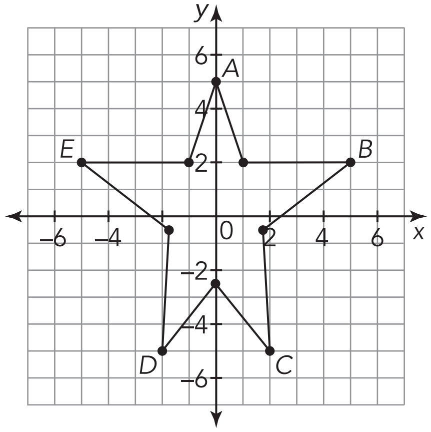


a. What is the relationship between the figures?

b. Explain how Figure *A* was transformed to create Figure *B*.

Prepare

1. Identify the ordered pairs associated with each of the five labeled points of the star.



3 Translations of Figures on the Coordinate Plane

OBJECTIVES

- Translate geometric figures on the coordinate plane.
- Identify and describe the effect of geometric translations on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations.

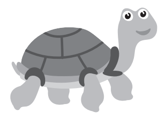
You have learned to model transformations, such as translations, rotations, and reflections.

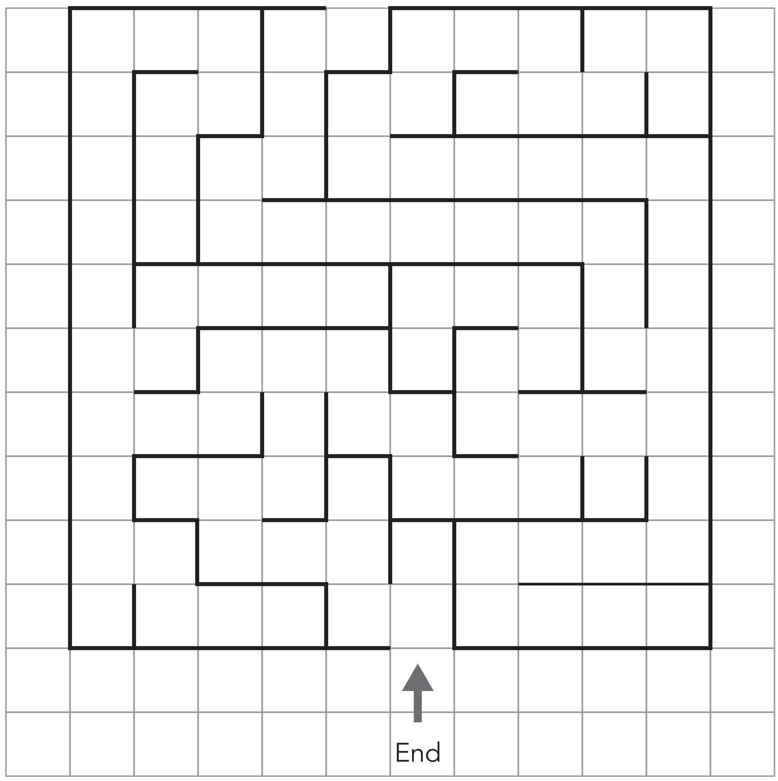
How can you model and describe translations on the coordinate plane?

Getting Started

Stopping for Directions

Consider the maze shown.





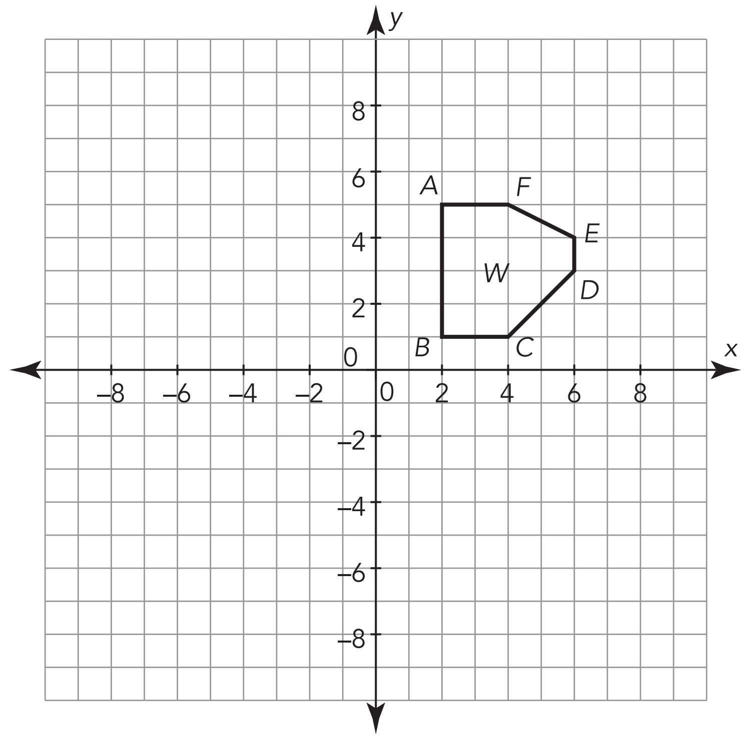
1. Navigate this maze to help the turtle move to the end. Justify your solution by writing the steps you used to solve the maze.

2. How would your steps change if the turtle started at the end and had to make its way to the start of the maze?

ACTIVITY 3.1

Modeling Translations on the Coordinate Plane

You know that translations are transformations that "slide" each point of a figure the same distance and the same direction. Each point moves in a line. You can describe translations more precisely by using coordinates.



1. Place patty paper on the coordinate plane, trace Figure *W*, and copy the labels for the vertices on the patty paper.

   a. Translate the figure down 6 units. Then, draw the translated figure on the coordinate plane and label it as Figure *W*′.

   b. Identify the pre-image and the image.

   c. Did translating Figure *W* vertically change the size or shape of the figure? Justify your answer.

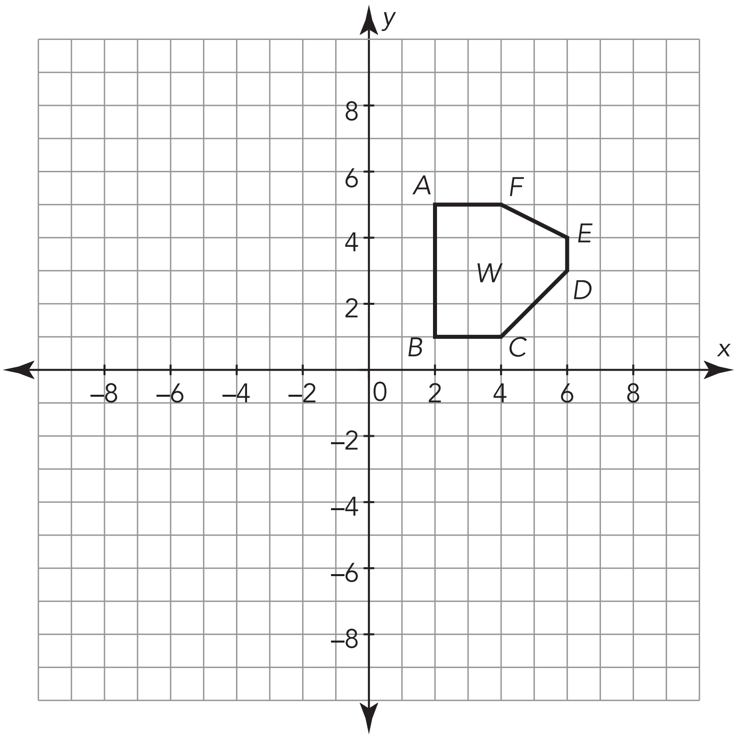
   d. Complete the table with the coordinates of Figure *W*′.

   e. Compare the coordinates of Figure *W*′ with the coordinates of Figure *W*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

Coordinates of *W*	Coordinates of *W*′
A(2, 5)	
B(2, 1)	
C(4, 1)	
D(6, 3)	
E(6, 4)	
F(4, 5)	

Now, let's investigate translating Figure *W* horizontally.

2. Place patty paper on the coordinate plane, trace Figure *W*, and write and copy the labels for the vertices.



a. Translate the figure left 5 units.

b. Draw the translated figure on the coordinate plane with a different color, and label it as Figure *W*″. Then identify the pre-image and the image.

c. Did translating Figure *W* horizontally change the size or shape of the figure? Justify your answer.

Coordinates of *W*	Coordinates of *W*″
A(2, 5)	
B(2, 1)	
C(4, 1)	
D(6, 3)	
E(6, 4)	
F(4, 5)	

d. Complete the table with the coordinates of Figure *W*″.

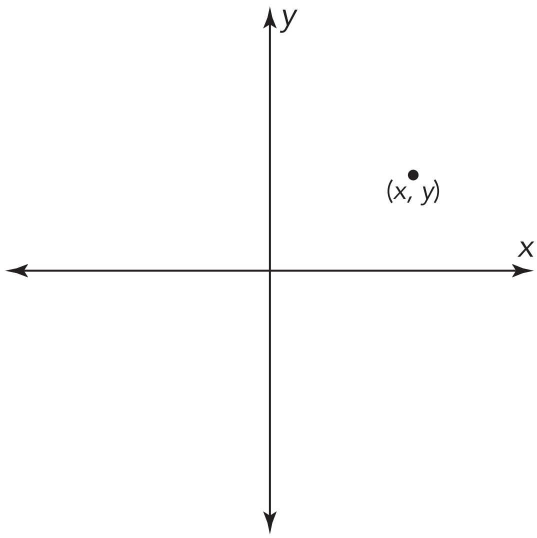
e. Compare the coordinates of Figure *W*″ with the coordinates of Figure *W*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

3. Make a conjecture about how a vertical or horizontal translation affects the coordinates of any point (x, y).

ACTIVITY 3.2

Translating Any Points on the Coordinate Plane

Consider the point (x, y) located anywhere in the first quadrant on the coordinate plane.



How do these coordinates compare with your conjecture in the previous activity?

1. Consider each translation of the point (x, y) according to the descriptions in the table shown. Record the coordinates of the translated points in terms of x and y.

Translation	Coordinates of Translated Point
3 units to the left	
3 units down	
3 units to the right	
3 units up	

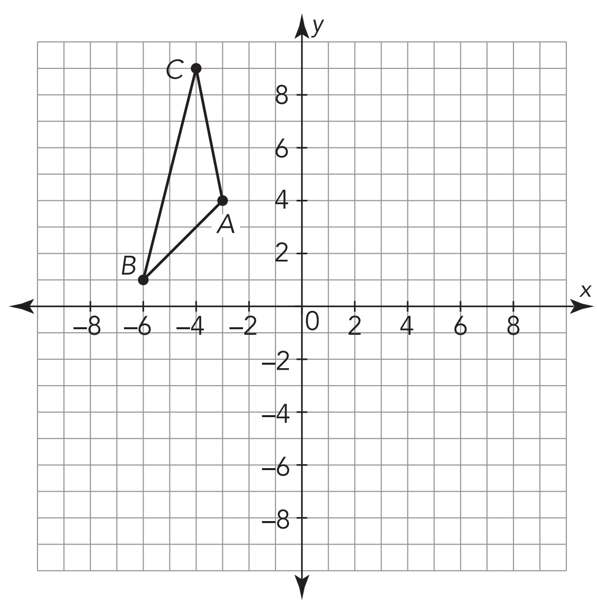
2. Describe a translation in terms of x and y that would move any point (x, y) in Quadrant I into each quadrant.

a. Quadrant II

b. Quadrant III

c. Quadrant IV

Let's consider △*ABC* shown on the coordinate plane.



3. Use the table to record the coordinates of the vertices of each translated triangle.

a. Translate △*ABC* 5 units to the right to form △*A′B′C′*. List the coordinates of points *A′*, *B′*, and *X′*. Then, graph △*A′B′C′*.

b. Translate △*ABC* 8 units down to form △*A″B″C″*. List the coordinates of points *A″*, *B″*, and *C″*. Then graph △*A″B″C″*.

Original Triangle	Triangle Translated 5 Units to the Right	Triangle Translated 8 Units Down
$\triangle ABC$	$\triangle A'B'C'$	$\triangle A''B''C''$
$A(-3, 4)$		
$B(-6, 1)$		
$C(-4, 9)$		

Let's consider translations of a different triangle without graphing.

4. The vertices of $\triangle DEF$ are $D(-7, 10)$, $E(-5, 5)$, and $F(-8, 1)$.

   a. If $\triangle DEF$ is translated to the right 12 units, what are the coordinates of the vertices of the image? Name the triangle.

   b. How did you determine the coordinates of the image without graphing the triangle? Write the algebraic rule to represent the translation.

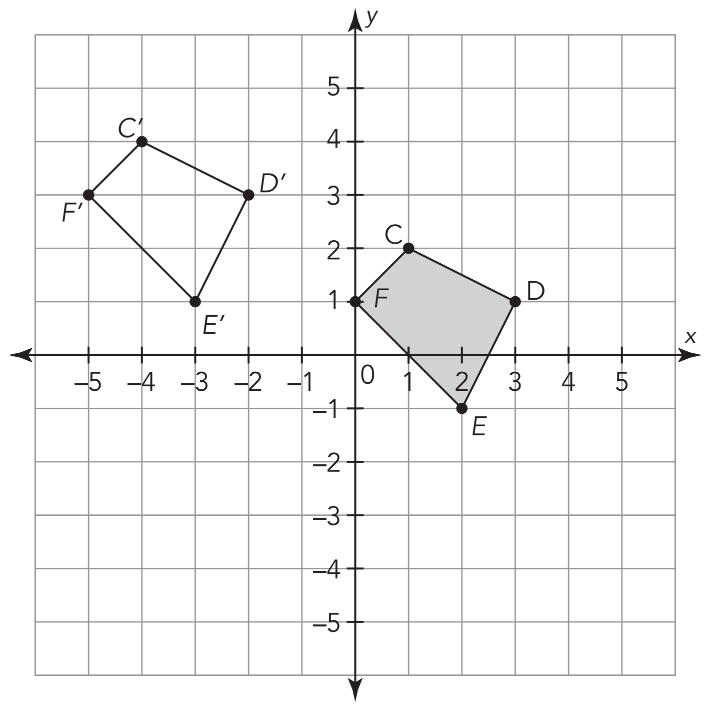
   c. If $\triangle DEF$ is translated up 9 units, what are the coordinates of the vertices of the image? Name the triangle.

   d. How did you determine the coordinates of the image without graphing the triangle? Write the algebraic rule to represent the translation.

ACTIVITY
3.3
Verifying Congruence Using Translations

One way to verify that two figures are congruent is to show that translations move all of the points of one figure to all the points of the other figure.

Consider the two quadrilaterals shown on the coordinate plane.



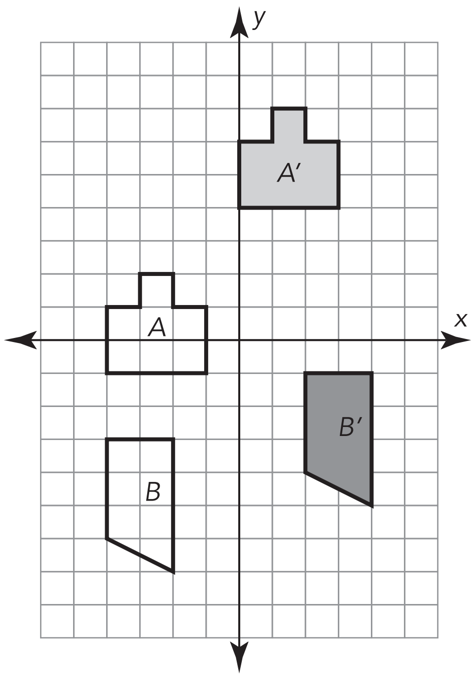
1. Complete the table with the coordinates of each figure and the translation from each vertex in Quadrilateral *CDEF* to the corresponding vertex in Quadrilateral *C′D′E′F′*.

Coordinates of Quadrilateral *CDEF*	Coordinates of Quadrilateral *C′D′E′F′*	Translations

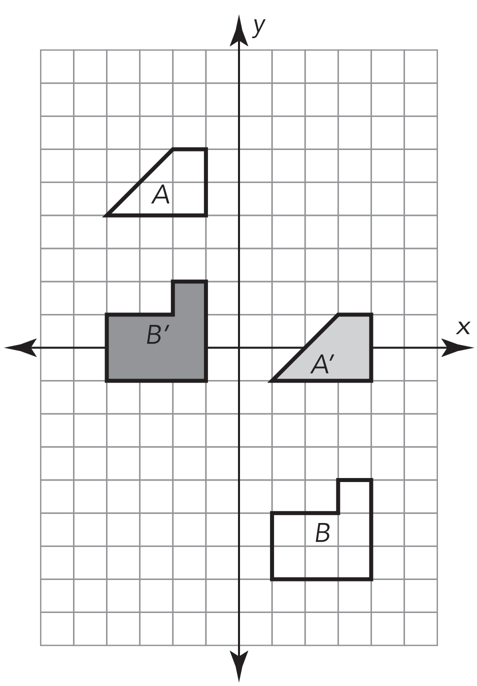
2. Is Quadrilateral *CDEF* congruent to Quadrilateral *C′D′E′F′*?
   Explain how you know.

3. Describe translations that can be used to show that Figures *A* and *A′* are congruent. Show your work and explain your reasoning.

   a.

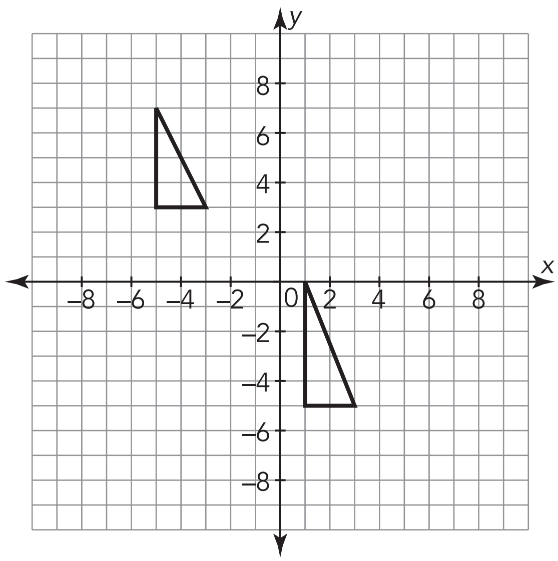


   b.

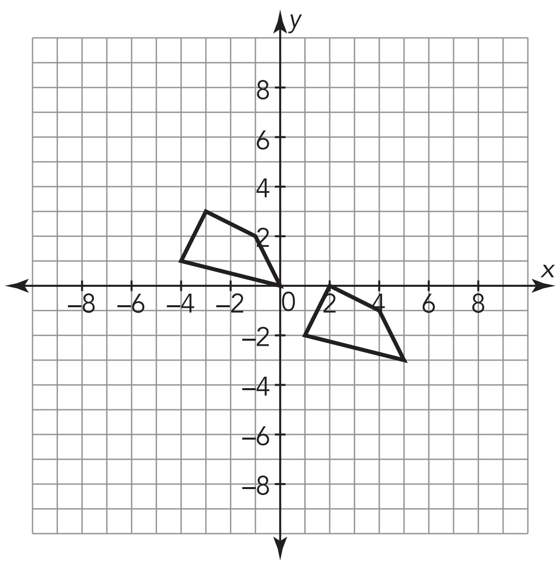


4. For each example, decide whether the figures given are congruent or not congruent using translations. Show your work and explain your reasoning.

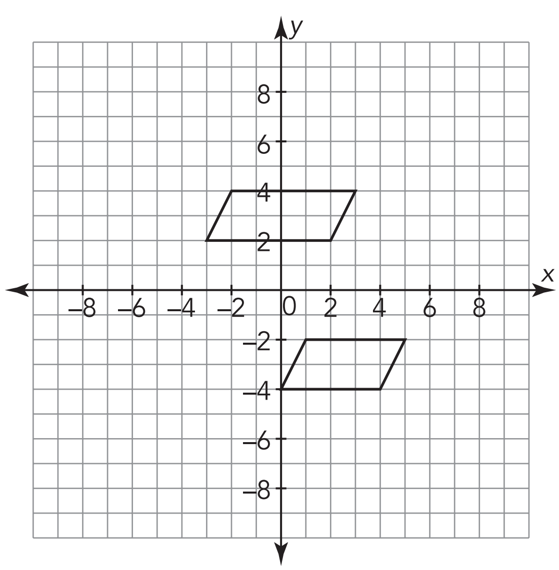
a.



b.



c.



Talk the Talk

Left and Right, Up and Down

1. Suppose the point (x, y) is translated horizontally c units.

   a. How do you know if the point is translated left or right?

   b. Write the coordinates of the image of the point.

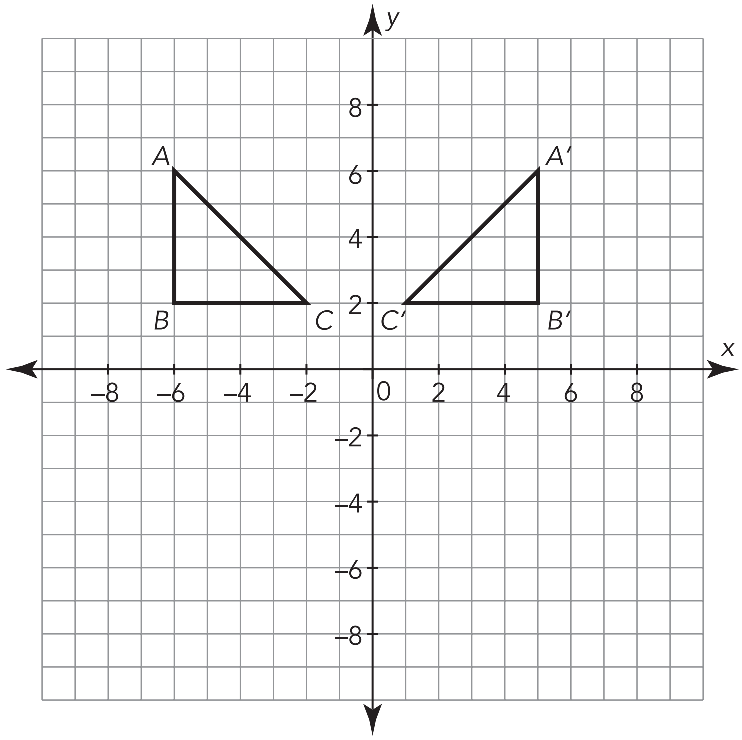
2. Suppose the point (x, y) is translated vertically d units.

   a. How do you know if the point is translated up or down?

   b. Write the coordinates of the image of the point.

3. Suppose a point is translated repeatedly up 2 units and right 1 unit. Does the point remain on a straight line as it is translated? Draw an example to explain your answer.

4. Can you verify that these two figures are congruent using only translations? Explain why or why not.



Lesson 3 Assignment

Write

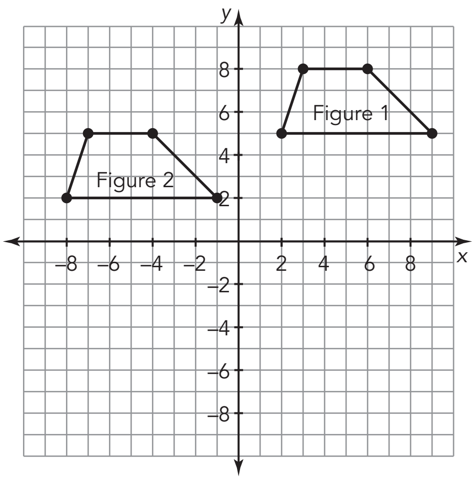
In your own words, explain how horizontal and vertical translations each affect the coordinates of the points of a figure.

Remember

A translation "slides" a figure along a line. A translation is a rigid motion that preserves the size and shape of figures.

Practice

1. Use the figures shown to complete parts (a) through (d).



a. Describe the sequence of translations used to move Figure 1 onto Figure 2.

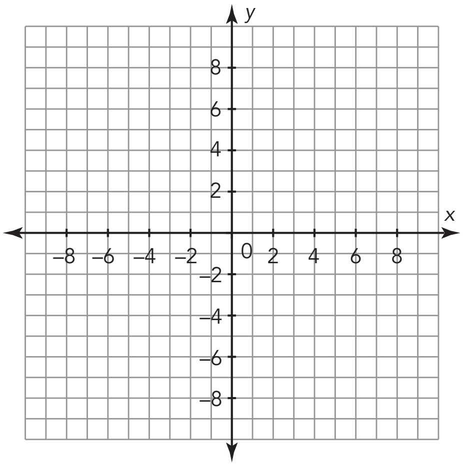
b. Determine the coordinates of the image of Figure 1 if it is translated 1 unit horizontally and −8 units vertically.

c. Explain how you determined the coordinates in part (b).

d. Verify your answer to part (b) by graphing the image. Label it *Figure 3*.

Lesson 3 Assignment

2. Use a coordinate plane to complete parts (a) through (d).



a. Plot the given points and connect them with straight lines in the order in which they are given. Connect the last point to the first point to complete the figure. Label the figure *A*.
(−3, −6), (−3, −3), (0, 0), (3, −3), (3, −6), (0, −3)

b. Translate the figure in part (a) −3 units vertically. Label the image *B*.

c. Translate the figure in part (a) 6 units vertically and 3 units horizontally. Label the image *C*.

d. Translate the figure in part (a) −3 units horizontally and 6 units vertically. Label the image *D*.

Prepare

Determine each product.

1. $-1 \cdot 6$

2. $-\frac{3}{5}(-1)$

3. $-1 \cdot 4.33$

4. $4h(-1)$

4 Reflections of Figures on the Coordinate Plane

OBJECTIVES

- Reflect geometric figures on the coordinate plane.
- Identify and describe the effect of geometric reflections on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations and reflections.

You have learned to model transformations, such as translations, rotations, and reflections.

How can you model and describe reflections on the coordinate plane?

Getting Started

Ambulance

The image shows the front of a typical ambulance.

1. Why does the word *ambulance* appear like this on the front?

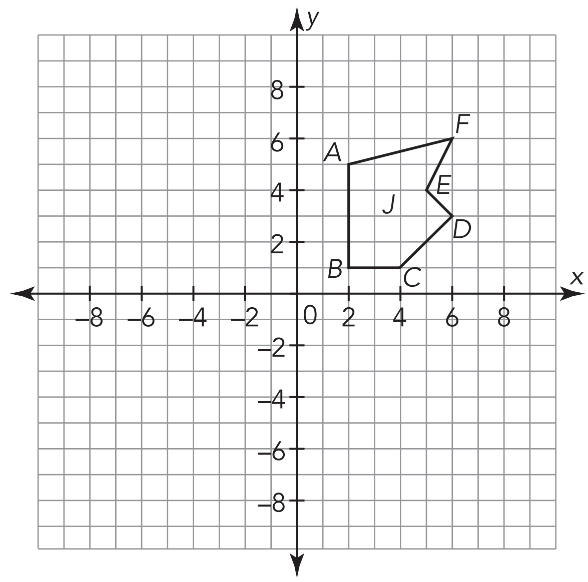
2. Suppose you are going to replace the word *ambulance* with your name. Write your name as it appears on the front of the vehicle. How can you check that it is written correctly?

ACTIVITY 4.1

Modeling Reflections on the Coordinate Plane

In this activity, you will reflect pre-images across the *x*-axis and *y*-axis and explore how the reflection affects the coordinates.

1. Place patty paper on the coordinate plane, trace Figure *J*, and copy the labels for the vertices on the patty paper.



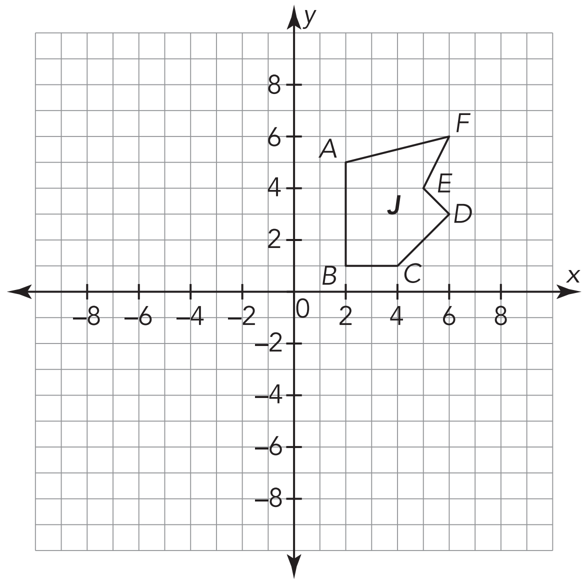
   a. Reflect Figure *J* across the *x*-axis. Then, complete the table with the coordinates of the reflected figure.

Coordinates of *J*	Coordinates of *J'*
A(2, 5)	
B(2, 1)	
C(4, 1)	
D(6, 3)	
E(5, 4)	
F(6, 6)	

   b. Compare the coordinates of Figure *J'* with the coordinates of Figure *J*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

2. Consider Figure *J* again.

   a. Reflect Figure *J* across the *y*-axis. Then, complete the table with the coordinates of the reflected figure.

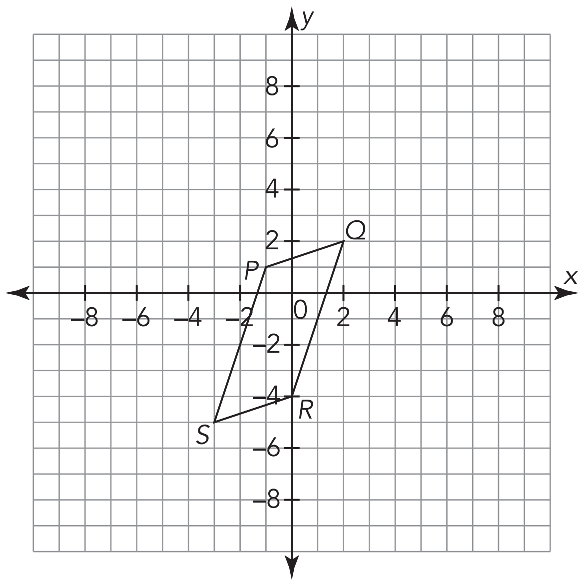


Coordinates of *J*	Coordinates of *J*"
A(2, 5)	
B(2, 1)	
C(4, 1)	
D(6, 3)	
E(5, 4)	
F(6, 6)	

   b. Compare the coordinates of Figure *J*" with the coordinates of Figure *J*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

Let's consider a new figure situated differently on the coordinate plane.

3. Reflect Quadrilateral *PQRS* across the *x*-axis.



Make a conjecture, investigate, and then use the results to verify or justify your conjecture.

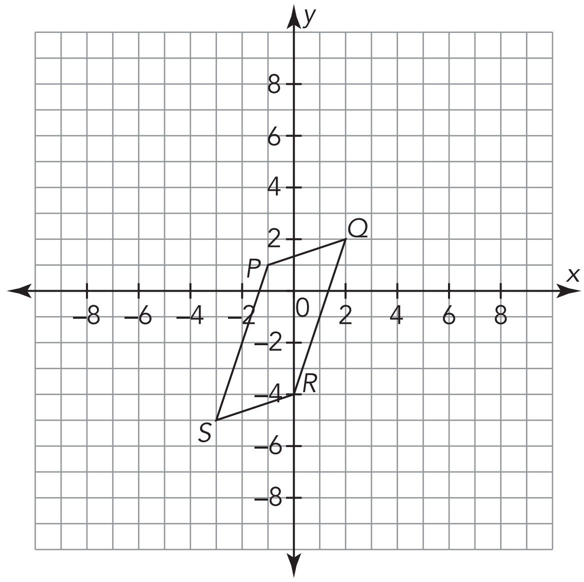
a. Make a conjecture about the ordered pairs for the reflection of the quadrilateral across the *x*-axis.

b. Use patty paper to test your conjecture. Complete the table with the coordinates of the reflection.

Coordinates of Quadrilateral *PQRS*	Coordinates of Quadrilateral *P'Q'R'S'*
P(−1, 1)	
Q(2, 2)	
R(0, −4)	
S(−3, −5)	

c. Compare the coordinates of Quadrilateral *P'Q'R'S'* with the coordinates of Quadrilateral *PQRS*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

4. Reflect Quadrilateral *PQRS* across the *y*-axis.



a. Make a conjecture about the ordered pairs for the reflection of the quadrilateral across the *y*-axis.

b. Use patty paper to test your conjecture. Complete the table with the coordinates of the reflection.

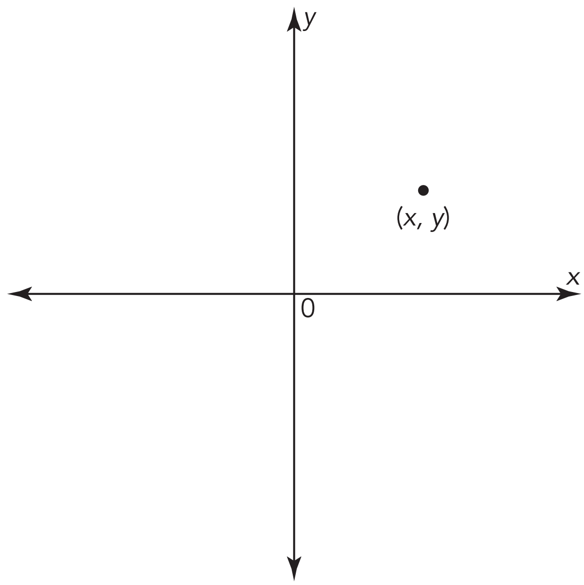
Coordinates of Quadrilateral *PQRS*	Coordinates of Quadrilateral *P"Q"R"S"*
P(−1, 1)	
Q(2, 2)	
R(0, −4)	
S(−3, −5)	

c. Compare the coordinates of Quadrilateral *P"Q"R"S"* with the coordinates of Quadrilateral *PQRS*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

ACTIVITY **4.2**

Reflecting Any Points on the Coordinate Plane

Consider the point (*x*, *y*) located anywhere in the first quadrant.



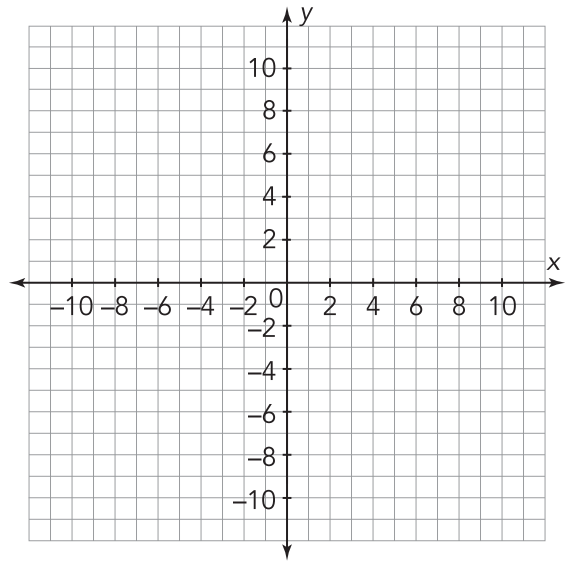
1. Use the table to record the coordinates of each point.

   a. Reflect and graph the point (*x*, *y*) across the *x*-axis on the coordinate plane. What are the new coordinates of the reflected point in terms of *x* and *y*?

   b. Reflect and graph the point (*x*, *y*) across the *y*-axis on the coordinate plane. What are the new coordinates of the reflected point in terms of *x* and *y*?

Original Point	Reflection Across the *x*-axis	Reflection Across the *y*-axis
(*x*, *y*)		

2. Graph △*ABC* by plotting the points *A*(3, 4), *B*(6, 1), and *C*(4, 9).



3. Use the table to record the coordinates of the vertices of each triangle.

   a. Reflect △*ABC* across the *x*-axis to form △*A'B'C'*. Graph the triangle and then list the coordinates of the reflected triangle.

   b. Reflect △*ABC* across the *y*-axis to form △*A''B''C''*. Graph the triangle and then list the coordinates of the reflected triangle.

Original Triangle	Triangle Reflected Across the *x*-axis	Triangle Reflected Across the *y*-axis
△*ABC*	△*A'B'C'*	△*A''B''C''*
A(3, 4)		
B(6, 1)		
C(4, 9)		

Let's consider reflections of a different triangle without graphing.

4. The vertices of $\triangle DEF$ are $D(-7, 10)$, $E(-5, 5)$, and $F(-1, -8)$.

   a. If $\triangle DEF$ is reflected across the x-axis, what are the coordinates of the vertices of the image? Name the triangle.

   b. How did you determine the coordinates of the image without graphing the triangle? Write the algebraic rule to represent the reflection.

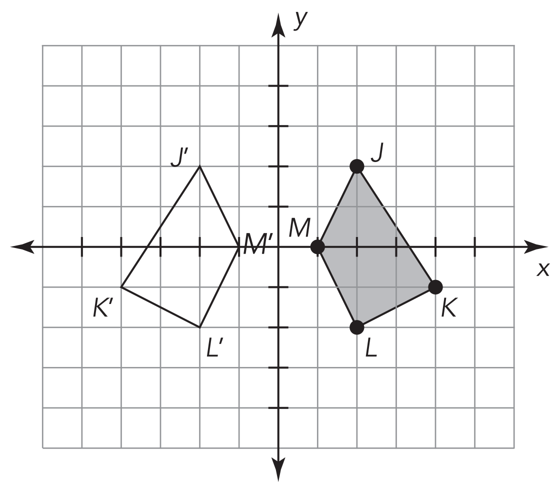
   c. If $\triangle DEF$ is reflected across the y-axis, what are the coordinates of the vertices of the image? Name the triangle.

   d. How did you determine the coordinates of the image without graphing the triangle? Write the algebraic rule to represent the reflection.

ACTIVITY
4.3 Verifying Congruence Using Reflections and Translations

Just as with translations, one way to verify that two figures are congruent is to show that a reflection moves all the points of one figure onto all the points of the other figure.

1. Consider the two figures shown.



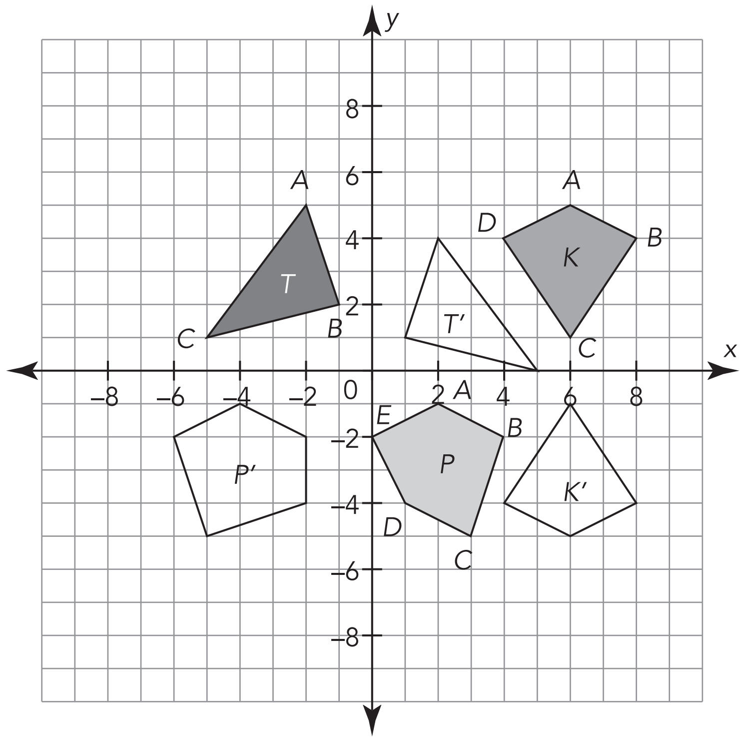
a. Complete the table with the corresponding coordinates of each figure.

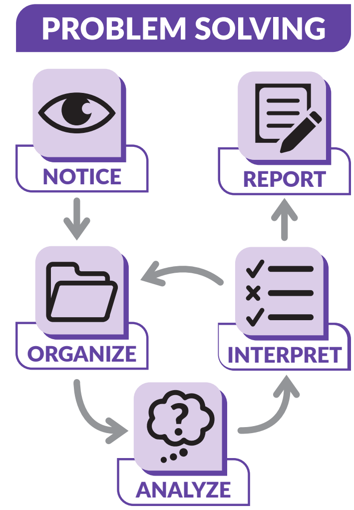
Coordinates of Quadrilateral *JKLM*	Coordinates of Quadrilateral *J'K'L'M'*

Remember, a rigid motion is a transformation that preserves the size and shape of the figure.

b. Is Quadrilateral *JKLM* congruent to Quadrilateral *J'K'L'M'*? Describe the rigid motion to verify your conclusion.

2. Study the figures shown on the coordinate plane.





Use the problem-solving model whenever you see this icon.

Determine whether each pair of figures are congruent. Then, describe the sequence of rigid motions to verify your conclusion.

a. Is Figure *K* congruent to Figure *K*′?

b. Is Figure *P* congruent to Figure *P*′?

c. Is Figure *T* congruent to Figure *T*′?

Talk the Talk

Reflecting on Reflections

1. Describe how the ordered pair (x, y) of any figure changes when the figure is reflected across the x-axis.

2. Describe how the ordered pair (x, y) of any figure changes when the figure is reflected across the y-axis.

Lesson 4 Assignment

Write

In your own words, explain how reflections across the *x*-axis and across the *y*-axis each affect the coordinates of the points of a figure.

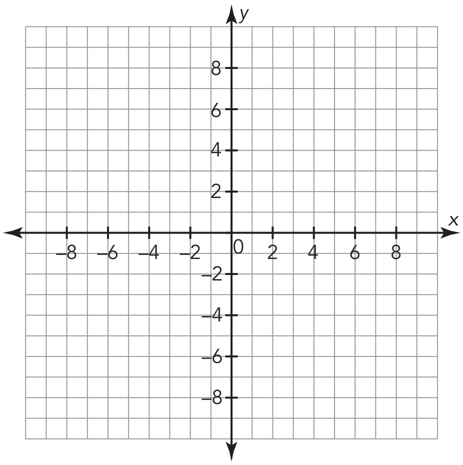
Remember

A reflection "flips" a figure across a line of reflection. A reflection is a rigid motion that preserves the size and shape of figures.

Practice

1. Use a coordinate plane to complete parts (a) through (d).

   a. Plot the points (0, 0), (−7, 5), (−7, 8), (−4, 8) and connect them with straight lines in the order in which they are given. Connect the last point to the first point to complete the figure and label the figure as 1.



   b. List the ordered pairs of Quadrilateral 1 if it is reflected across the *y*-axis. Explain how you can determine the ordered pairs of the reflection without graphing it. Plot the reflection described and label the figure as 2.

   c. List the ordered pairs of Quadrilateral 2 if it is reflected over the *x*-axis. Explain how you can determine the ordered pairs of the reflection without graphing it. Plot the reflection described and label the figure as 3.

Lesson 4 Assignment

d. List the ordered pairs of Quadrilateral 1 if it is reflected over the *x*-axis. Explain how you can determine the ordered pairs of the reflection without graphing it. Plot the reflection described, and label the figure as 4.

2. Write a general statement about how to determine the ordered pairs of the vertices of a figure if it is reflected across the *x*-axis.

3. Write a general statement about how to determine the ordered pairs of the vertices of a figure if it is reflected across the *y*-axis.

Lesson 4 Assignment

Prepare

1. Redraw each given figure as described.

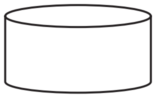
   a. So that it is turned 180°

   Before: After:

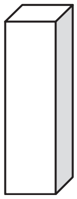
   b. So that it is turned 90° counterclockwise

   Before: After:

   c. So that it is turned 90° clockwise

   Before: After:

5 Rotations of Figures on the Coordinate Plane

OBJECTIVES

- Rotate geometric figures on the coordinate plane 90°, 180°, 270°, and 360°.
- Identify and describe the effect of geometric rotations of 90°, 180°, 270°, and 360° on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations, reflections, and rotations.

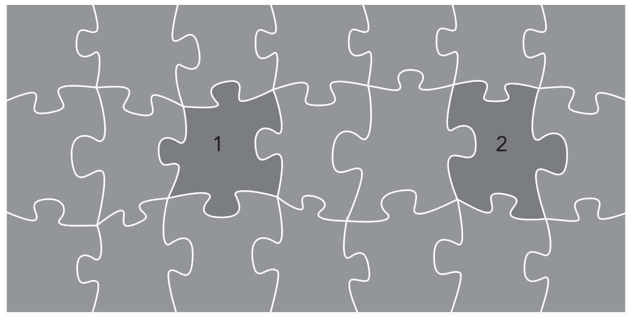
You have learned to model rigid motions, such as translations, rotations, and reflections.

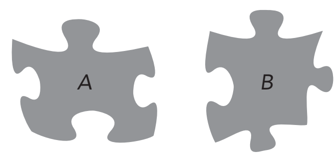
How can you model and describe rotations on the coordinate plane?

Getting Started

Jigsaw Transformations

There are just two pieces left to complete this jigsaw puzzle.





1. Which puzzle piece fills the missing spot at 1? Describe the translations, the reflections, and the rotations needed to move the piece into the spot.

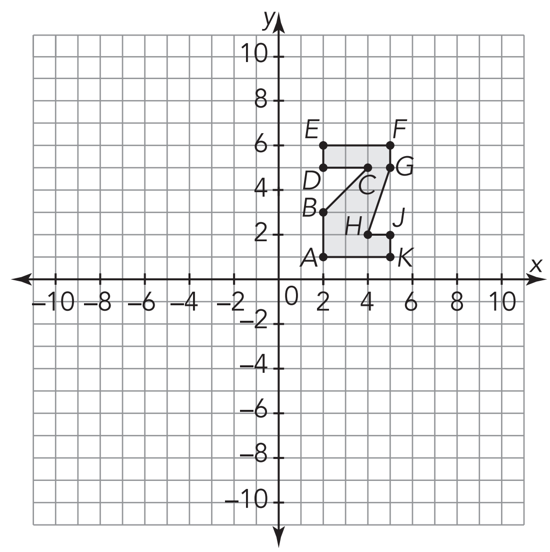
2. Which puzzle piece fills the missing spot at 2? Describe the translations, the reflections, and the rotations needed to move the piece into the spot.

ACTIVITY 5.1

Modeling Rotations on the Coordinate Plane

In this activity, you will investigate rotating pre-images to understand how the rotation affects the coordinates of the image.

1. Rotate the figure 180° about the origin.



   a. Place patty paper on the coordinate plane, trace the figure, and copy the labels for the vertices on the patty paper.

   b. Mark the origin, (0, 0), as the center of rotation. Trace a ray from the origin on the *x*-axis. This ray will track the angle of rotation.

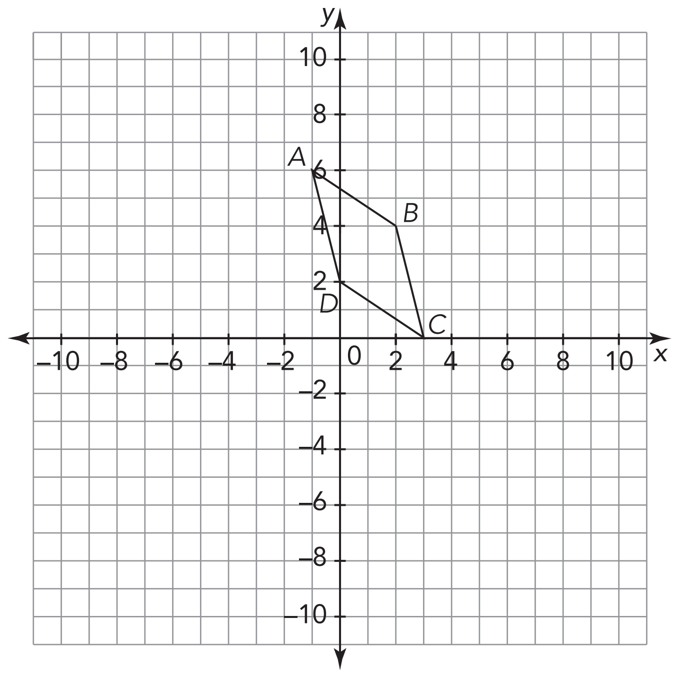
   c. Rotate the figure 180° about the center of rotation. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane. Finally, complete the table with the coordinates of the rotated figure.

   d. Compare the coordinates of the rotated figure with the coordinates of the original figure. How are the values of the coordinates the same? How are they different?

Coordinates of Pre-Image	Coordinates of Image
A(2, 1)	
B(2, 3)	
C(4, 5)	
D(2, 5)	
E(2, 6)	
F(5, 6)	
G(5, 5)	
H(4, 2)	
J(5, 2)	
K(5, 1)	

Now, let's investigate rotating a figure 90° about the origin.

2. Consider the parallelogram shown on the coordinate plane.



a. Place patty paper on the coordinate plane, trace the parallelogram, and then copy the labels for the vertices.

b. Rotate the figure 90° counterclockwise about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.

c. Complete the table with the coordinates of the pre-image and the image.

Coordinates of Pre-Image	Coordinates of Image

d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.

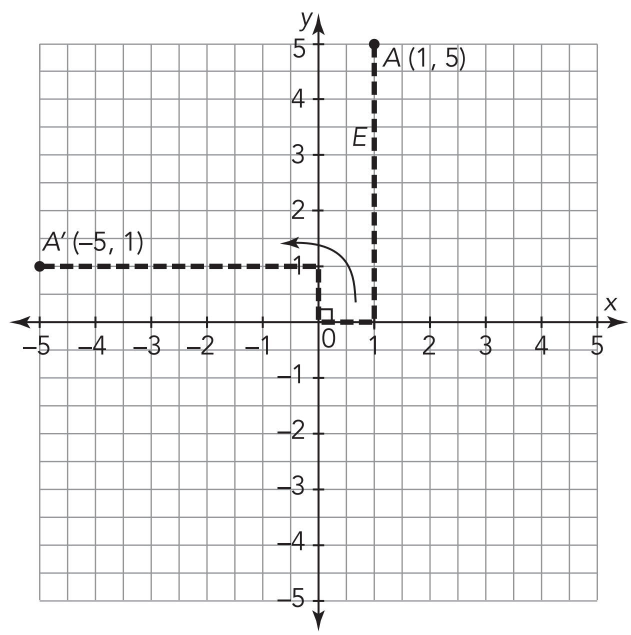
3. Make conjectures about how a counterclockwise 90° rotation and a 180° rotation affect the coordinates of any point (*x*, *y*).

You can use steps to help you rotate geometric objects on the coordinate plane.

WORKED EXAMPLE

Let's rotate a point 90° counterclockwise about the origin.

Step 1: Draw a "hook" from the origin to point *A* using the coordinates and horizontal and vertical line segments as shown.



Step 2: Rotate the "hook" 90° counterclockwise as shown.

Point *A*′ is located at (−5, 1). Point *A* has been rotated 90° counterclockwise about the origin.

4. What do you notice about the coordinates of the rotated point? How does this compare with your conjecture?

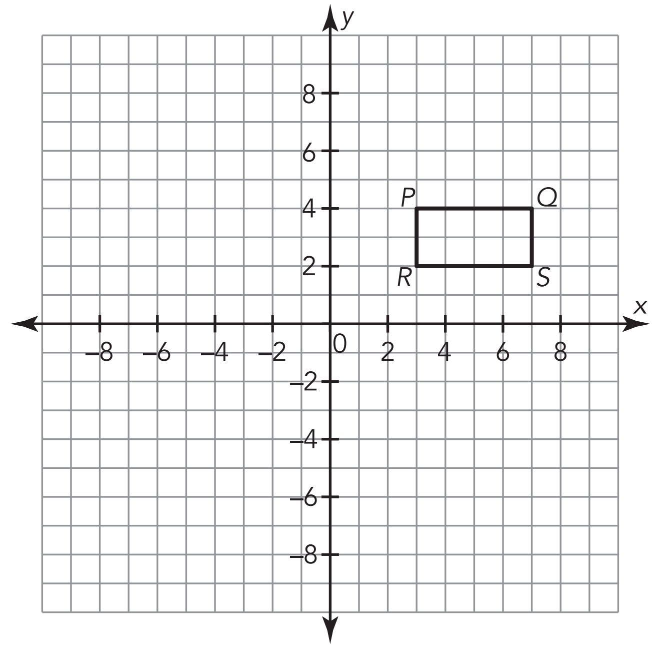
Now, let's investigate rotating figures more than 180° about the origin.

5. Consider the parallelogram shown on the coordinate plane.

   a. Place patty paper on the coordinate plane, trace the parallelogram, and then copy the labels for the vertices.

   b. Rotate the figure 270° clockwise about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.

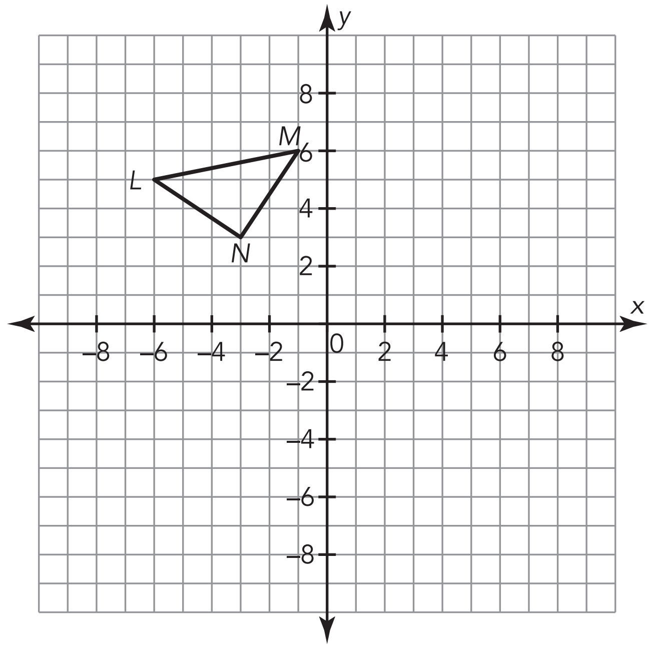
   c. Complete the table with the coordinates of the pre-image and the image.



Coordinates of Pre-Image	Coordinates of Image

   d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.

6. Consider the triangle shown on the coordinate plane.



a. Place patty paper on the coordinate plane, trace the triangle, and then copy the labels for the vertices.

b. Rotate the figure 360° about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.

c. Complete the table with the coordinates of the pre-image and the image.

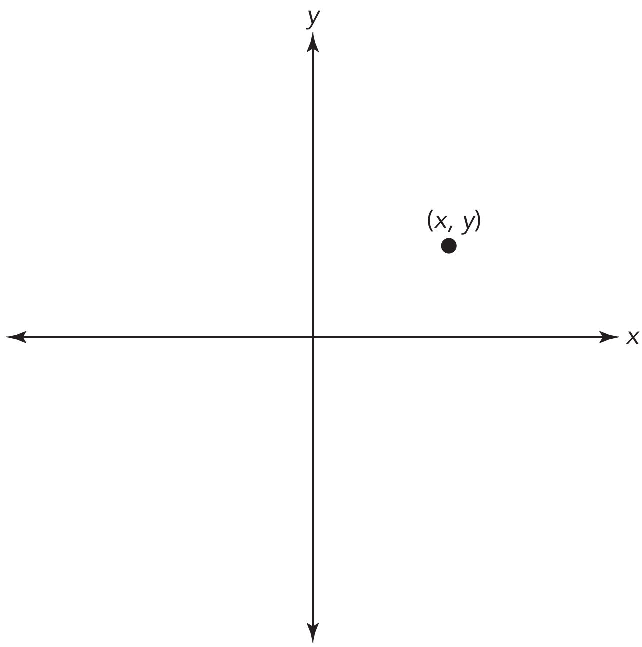
Coordinates of Pre-Image	Coordinates of Image

d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.

ACTIVITY 5.2

Rotating Any Points on the Coordinate Plane

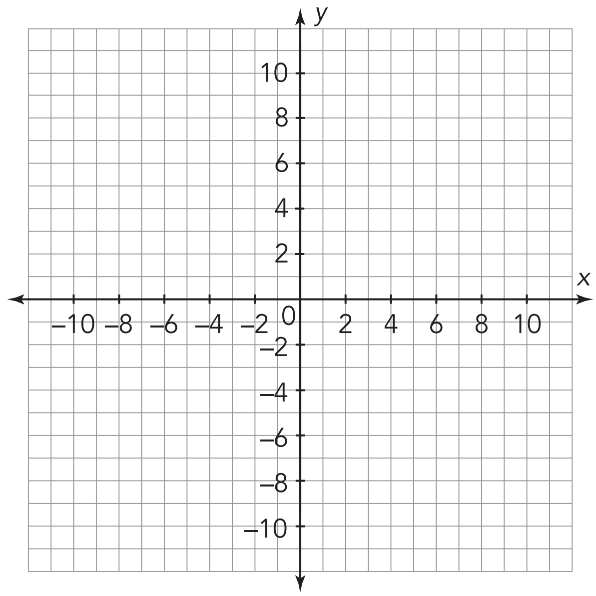
Consider the point (x, y) located anywhere in the first quadrant.



1. Use the origin, (0, 0), as the point of rotation. Rotate the point (x, y) as described in the table and plot and label the new point. Then, record the coordinates of each rotated point in terms of x and y.

Original Point	Rotation About the Origin 90° Counterclockwise	Rotation About the Origin 90° Clockwise	Rotation About the Origin 180°
(x, y)			

2. Graph $\triangle ABC$ by plotting the points $A(3, 4)$, $B(6, 1)$, and $C(4, 9)$.



Use the origin, (0, 0), as the point of rotation. Rotate $\triangle ABC$ as described in the table, graph and label the new triangle. Then, record the coordinates of the vertices of each triangle in the table.

Original Triangle	Rotation About the Origin 90° Counterclockwise	Rotation About the Origin 90° Clockwise	Rotation About the Origin 180°	Rotation About the Origin 270° Clockwise	Rotation About the Origin 360°
$\triangle ABC$	$\triangle A'B'C'$	$\triangle A''B''C''$	$\triangle A'''B'''C'''$	$\triangle A''''B''''C''''$	$\triangle A'''''B'''''C'''''$
$A(3, 4)$					
$B(6, 1)$					
$C(4, 9)$					

3. Consider your table.

   a. What do you notice about the coordinates of the triangle that has been rotated 270° clockwise about the origin? What conjecture can you make about these two triangles?

   b. What do you notice about the coordinates of the triangle that has been rotated 360° about the origin? What conjecture can you make about these two triangles?

Let's consider rotations of a different triangle without graphing.

4. The vertices of $\triangle DEF$ are $D(-7, 10)$, $E(-5, 5)$, and $F(-1, -8)$.

   a. If $\triangle DEF$ is rotated 90° counterclockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

   b. How did you determine the coordinates of the image without graphing the triangle? Write the algebraic rule to represent the rotation.

   c. If $\triangle DEF$ is rotated 90° clockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

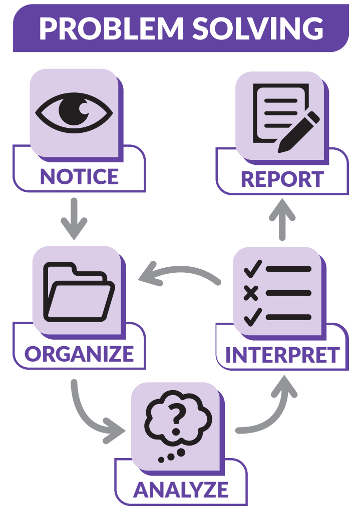
d. How did you determine the coordinates of the image without graphing the triangle? Write the algebraic rule to represent the rotation.

e. If $\triangle DEF$ is rotated 180° about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

f. How did you determine the coordinates of the image without graphing the triangle? Write the algebraic rule to represent the rotation.

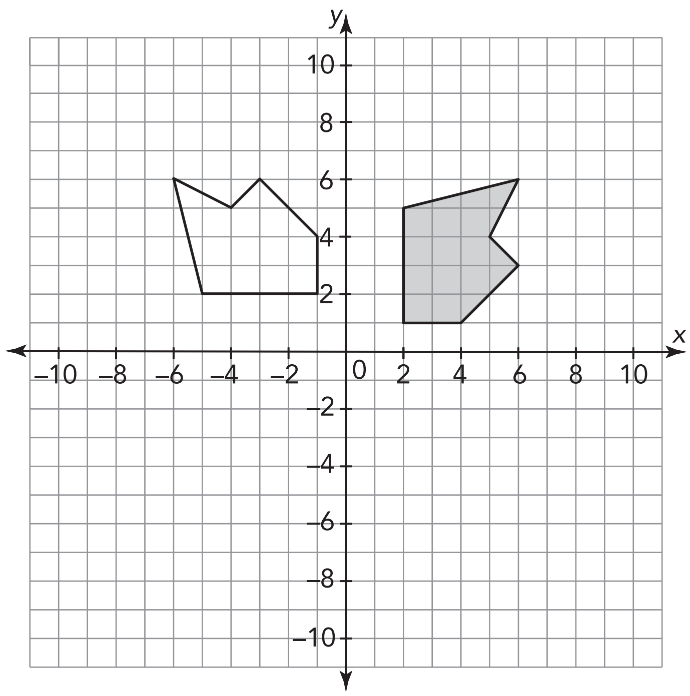
ACTIVITY 5.3

Verifying Congruence Using Rigid Motions



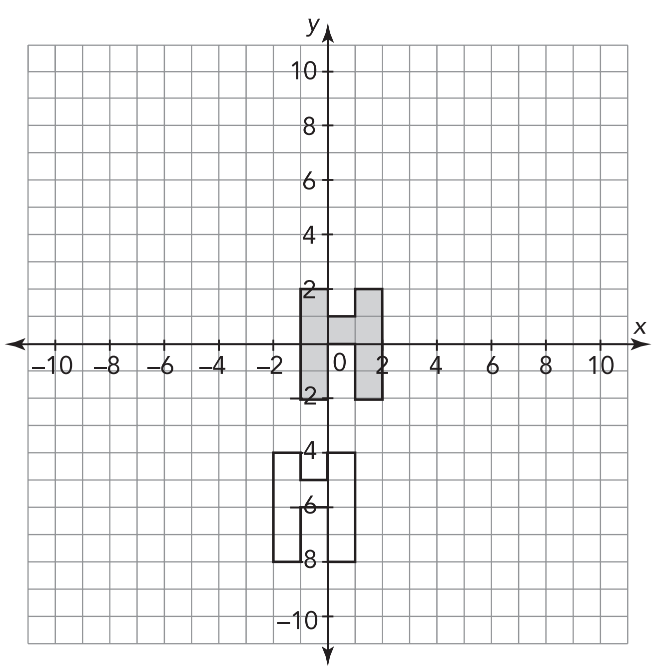
Describe a rigid motion you can use to verify that the shaded pre-image is congruent to the image.

1.

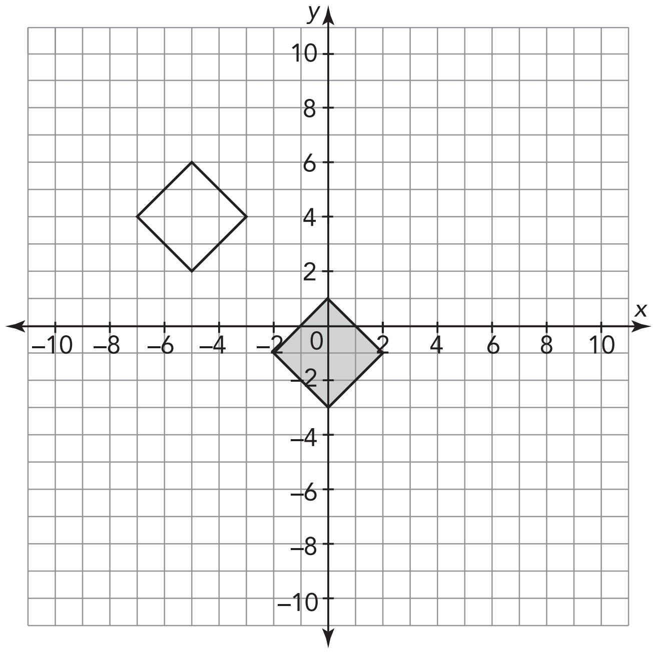


Ask Yourself . . .
Can you restate the problem in your own words?

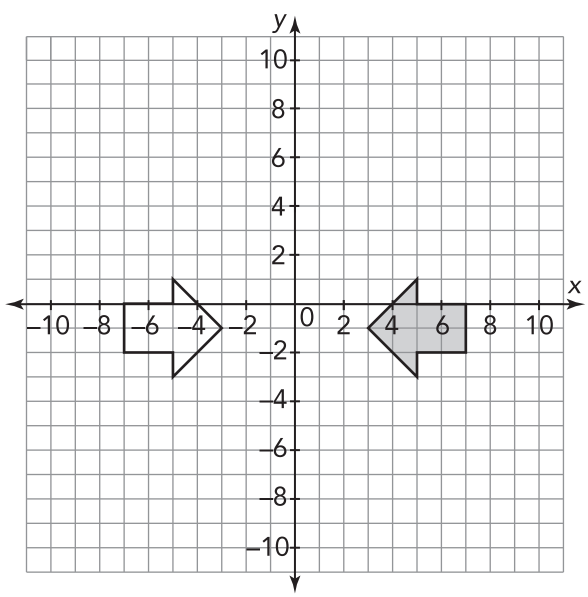
2.



3.



4.



Talk the Talk

Just the Coordinates

Using what you know about rigid motions, determine the transformation used to map the pre-image on to the image. Write the algebraic rule to represent the transformation.

1. Triangle *QRS* has coordinates $Q(1, -1)$, $R(3, -2)$, and $S(2, -3)$. Triangle $Q'R'S'$ has coordinates $Q'(1, 1)$, $R'(2, 3)$, and $S'(3, 2)$.

2. Rectangle *MNPQ* has coordinates $M(3, -2)$, $N(5, -2)$, $P(5, -6)$, and $Q(3, -6)$. Rectangle $M'N'P'Q'$ has coordinates $M'(-3, 2)$, $N'(-5, 2)$, $P'(-5, 6)$, and $Q'(-3, 6)$.

Lesson 5 Assignment

Write

In your own words, explain how each rotation about the origin affects the coordinate points of a figure.

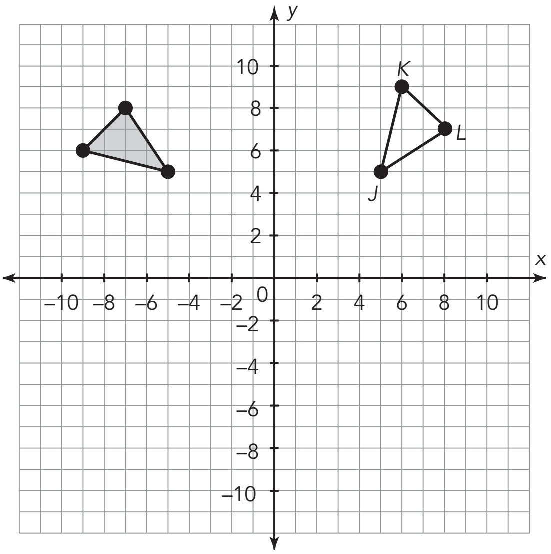
1. A counterclockwise rotation of 90°
2. A clockwise rotation of 90°
3. A rotation of 180°

Remember

A rotation "turns" a figure about a point.
A rotation is a rigid motion that preserves the size and shape of figures.

Practice

1. Use $\triangle JKL$ and the coordinate plane to answer each question.



   a. List the coordinates of each vertex of $\triangle JKL$.

   b. Describe the rotation that you can use to move $\triangle JKL$ onto the shaded area on the coordinate plane. Use the origin as the point of rotation.

   c. Determine what the coordinates of the vertices of the rotated $\triangle J'K'L'$ will be if you perform the rotation you described in your answer to part (b). Explain how you determined your answers.

   d. Verify your answers by graphing $\triangle J'K'L'$ on the coordinate plane.

Lesson 5 Assignment

2. Write an algebraic rule that represents the given transformation. Determine the coordinates of the new image.

   a. Triangle *ABC* with coordinates *A*(3, 4), *B*(7, 7), and *C*(8, 1) is translated 6 units left and 7 units down.

   b. Triangle *DEF* with coordinates *D*(−2, 2), *E*(1, 5), and *F*(4, −1) is rotated 90° counterclockwise about the origin.

   c. Triangle *GHJ* with coordinates *G*(2, −9), *H*(3, 8), and *J*(1, 6) is reflected across the *x*-axis.

   d. Triangle *KLM* with coordinates *K*(−4, 2), *L*(−8, 7), and *M*(3, −3) is translated 4 units right and 9 units up.

   e. Triangle *NPQ* with coordinates *N*(12, −3), *P*(1, 2), and *Q*(9, 0) is rotated 180° about the origin.

Prepare

Determine the distance between each pair of points.

1. (2, 3) and (−5, 3)
2. (−1, −4) and (−1, 8)
3. (6, −2.5) and (6, 5)
4. (−8.2, 5.6) and (−4.3, 5.6)

6 Congruence and Rigid Motions

OBJECTIVES

- Use coordinates to identify rigid motion transformations.
- Write congruence statements.
- Determine a rigid motion that maps a figure onto a congruent figure.
- Generalize the effects of rigid motion transformations on the coordinates of two-dimensional figures.

NEW KEY TERMS

- congruent line segments
- congruent angles

You have explored rigid motion transformations.

How can you use rigid motion transformations to verify the congruence of figures?

Getting Started

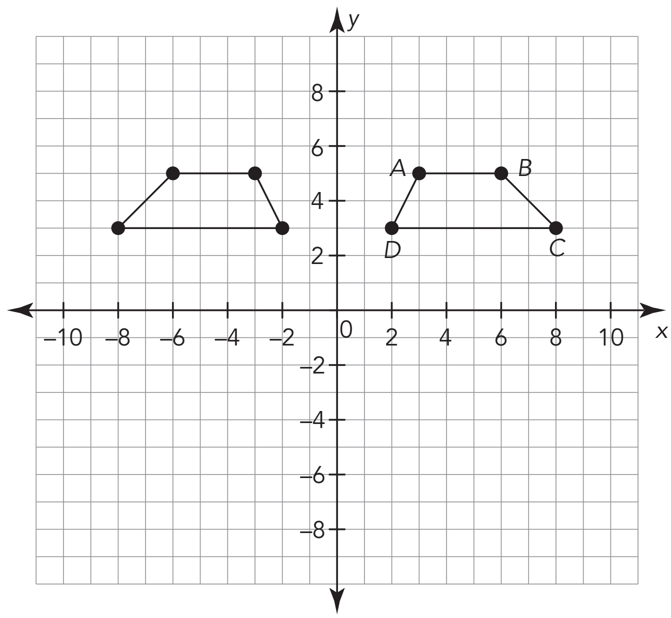
Going Backwards

Use your knowledge of rigid motions and their effects on the coordinates of two-dimensional figures to answer each question.

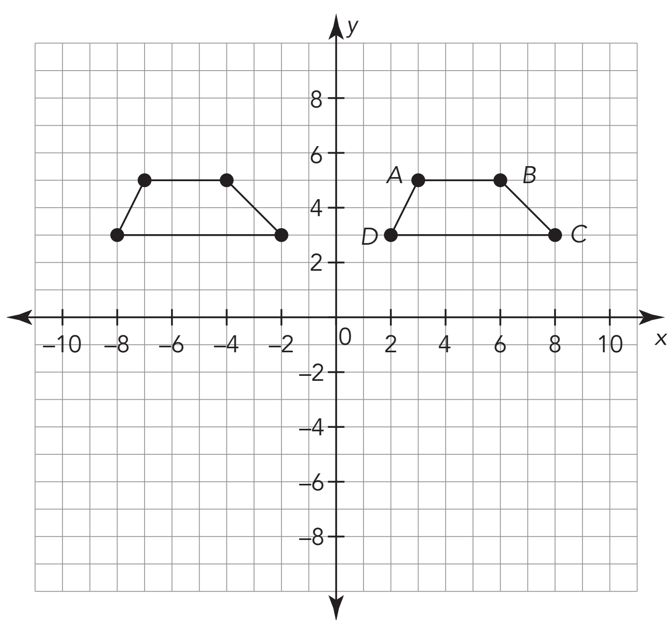
The line of reflection will be an axis, and the center of rotation will be the origin.

1. The pre-image and image of three different single transformations are given. Determine the transformation that maps the pre-image, the labeled figure, to the image. Label the vertices of the image.

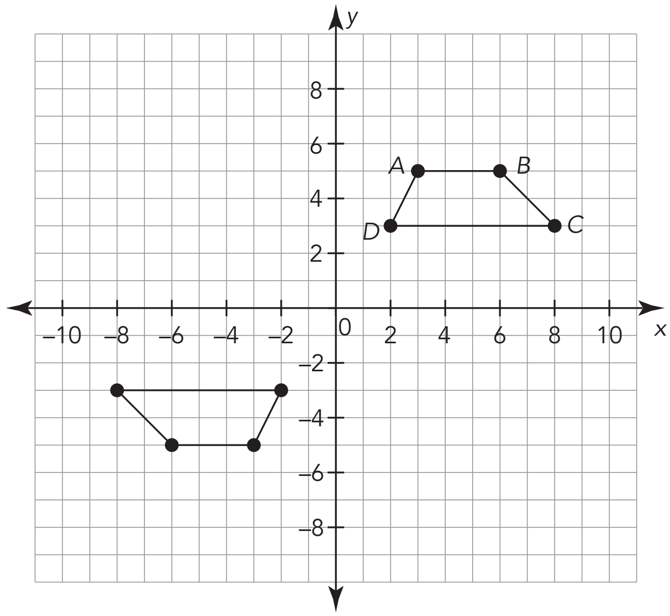
a.



b.

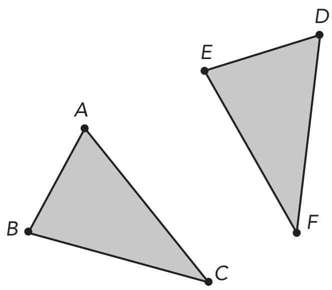


c.



2. Compare the order of the vertices, starting from *A*′, in each image with the order of the vertices, starting from *A*, in the pre-image.

ACTIVITY 6.1 Congruence Statements



You have determined that when a figure is translated, rotated, or reflected, the resulting image is the same size and the same shape as the original figure; therefore, the image and the pre-image are congruent figures.

1. How was $\triangle ABC$ transformed to create $\triangle DEF$?

Congruent line segments are line segments that have the same length.

Because $\triangle DEF$ was created using a rigid motion transformation of $\triangle ABC$, the triangles are congruent. Therefore, all corresponding sides and all corresponding angles have the same measure. In congruent figures, the corresponding sides are *congruent line segments*.

Think About . . .
Congruent figures as a mapping of one figure onto the other. When naming congruent segments, write the vertices in a way that shows the mapping.

WORKED EXAMPLE

If the length of line segment *AB* is equal to the length of line segment *DE*, the relationship can be expressed using symbols. For example,

- $AB = DE$ is read "the distance between *A* and *B* is equal to the distance between *D* and *E*"

If the sides of two different triangles are equal in length, for example, the length of side *AB* in $\triangle ABC$ is equal to the length of side *DE* in Triangle *DEF*, these sides are said to be congruent. This relationship can be expressed using symbols.

- $\overline{AB} \cong \overline{DE}$ is read "line segment *AB* is congruent to line segment *DE*."

Review the definition of **represent** in the Academic Glossary.

2. Write congruence statements for the other two sets of corresponding sides of the triangles.

Likewise, if corresponding angles have the same measure, they are *congruent angles*. **Congruent angles** are angles that are equal in measure.

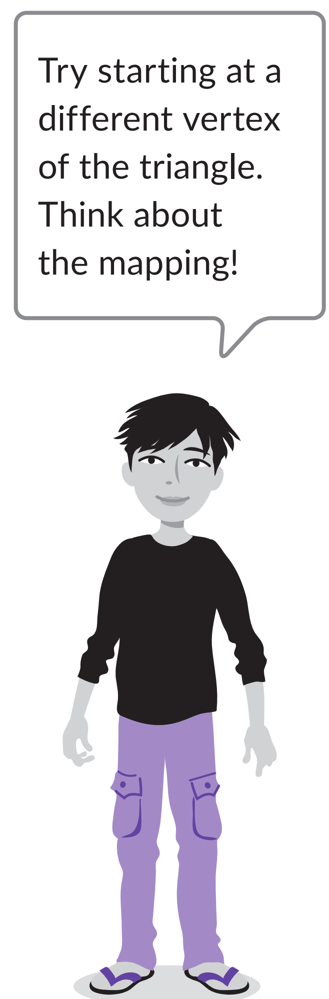
WORKED EXAMPLE

If the measure of angle *A* is equal to the measure of angle *D*, the relationship can be expressed using symbols.

- $m\angle A = m\angle D$ is read "the measure of angle *A* is equal to the measure of angle *D*."

If the angles of two different triangles are equal in measure, for example, the measure of angle *A* in $\triangle ABC$ is equal to the measure of angle *D* in $\triangle DEF$, these angles are said to be congruent. This relationship can be expressed using symbols.

- $\angle A \cong \angle D$ is read "angle *A* is congruent to angle *D*."



3. Write congruence statements for the other two sets of corresponding angles of the triangles.

You can write a single congruence statement about the triangles that shows the correspondence between the two figures. For the triangles in this activity, $\triangle ABC \cong \triangle DEF$.

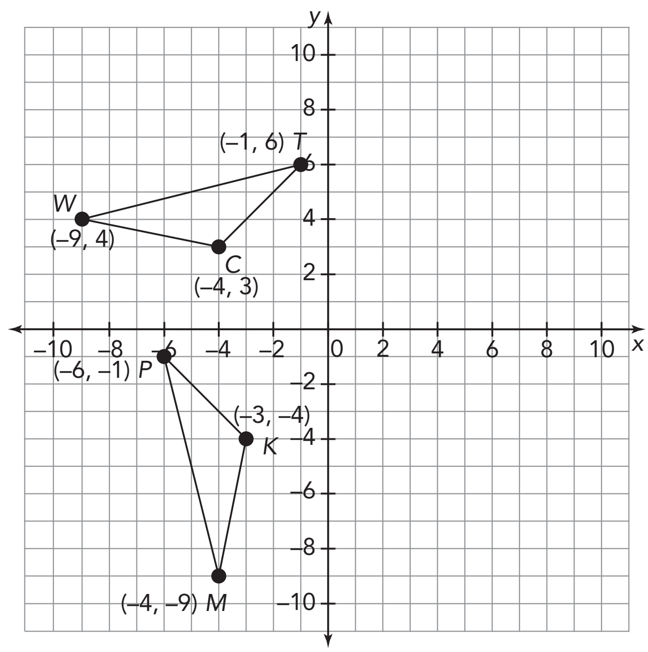
4. Write two additional correct congruence statements for these triangles.

ACTIVITY 6.2

Using Rigid Motions to Verify Congruence

When two figures are congruent, you can determine a rigid motion that maps one figure onto the other.

1. Analyze the two triangles shown.



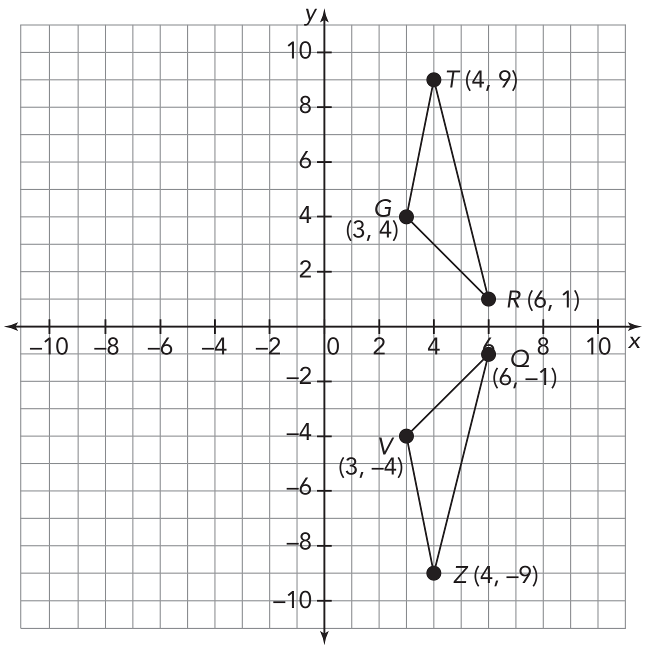
a. Is △*TWC* congruent to △*PMK*? Explain your reasoning.

b. What do you know about the angle measures and side lengths of the two triangles?

c. What do you know about the area of the two triangles?

d. What do you know about the perimeter of the two triangles?

2. Analyze the two triangles shown.



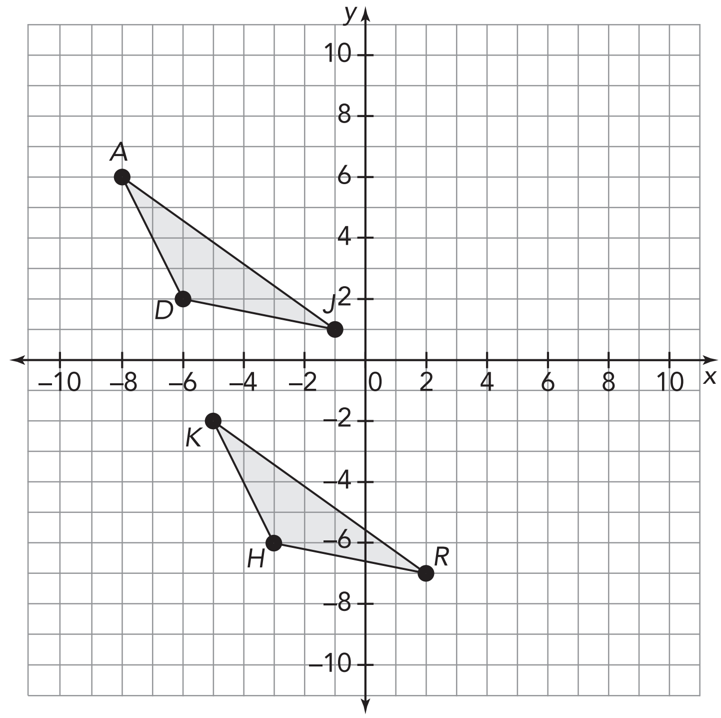
a. Is △*TRG* congruent to △*ZQV*? Explain your reasoning.

b. What do you know about the angle measures and side lengths of the two triangles?

c. What do you know about the area of the two triangles?

d. What do you know about the perimeter of the two triangles?

3. Analyze the two triangles.



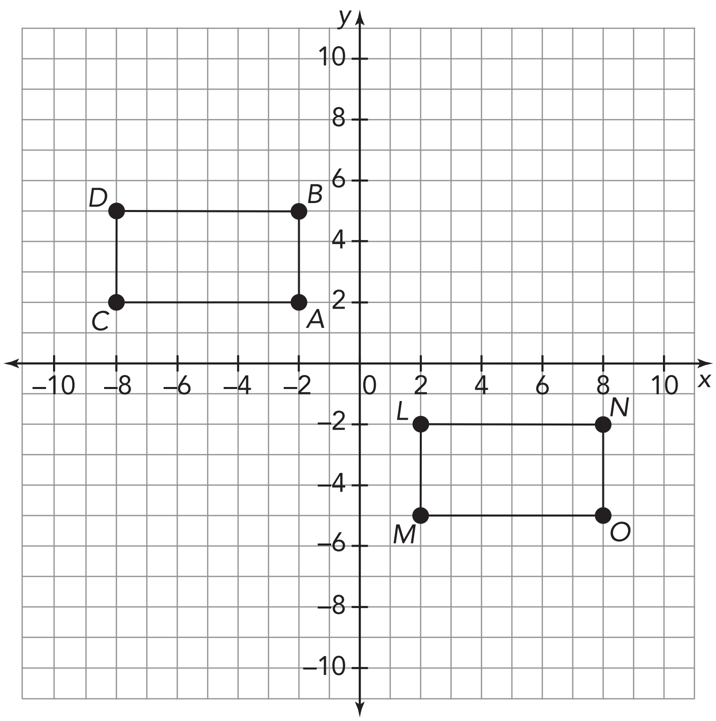
a. Is $\triangle ADJ$ congruent to $\triangle KHR$? Explain your reasoning.

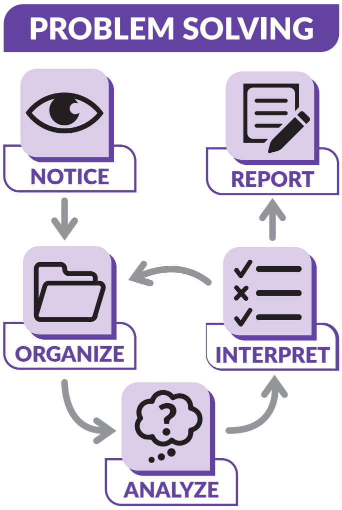
b. What do you know about the angle measures and side lengths of the two triangles?

c. What do you know about the area of the two triangles?

d. What do you know about the perimeter of the two triangles?

4. Maria and Luis are analyzing the rectangles shown. Maria says the rectangles are congruent because she can rotate Rectangle *ABCD* 180° to map onto Rectangle *LMNO*. Luis says the rectangles are congruent because he can reflect Rectangle *ABCD* across the *x*-axis and then translate the resulting figure 4 units to the right to map onto Rectangle *LMNO*. Who's correct? Explain your reasoning.





ACTIVITY 6.3 Transformations with Coordinates

For the triangles in this activity, $\triangle PQR \cong \triangle JME \cong \triangle DLG$.

1. Suppose the vertices of $\triangle PQR$ are $P(4, 3)$, $Q(-2, 2)$, and $R(0, 0)$. Describe the translation used to form each triangle. Explain your reasoning using the algebraic rule.

   a. $J(0, 3)$, $M(-6, 2)$, and $E(-4, 0)$

   b. $D(4, 5.5)$, $L(-2, 4.5)$, and $G(0, 2.5)$

2. Suppose the vertices of $\triangle PQR$ are $P(1, 3)$, $Q(6, 5)$, and $R(8, 1)$. Describe the rotation used to form each triangle. Explain your reasoning using the algebraic rule.

   a. $J(-3, 1)$, $M(-5, 6)$, and $E(-1, 8)$

   b. $D(-1, -3)$, $L(-6, -5)$, and $G(-8, -1)$

3. Suppose the vertices of $\triangle PQR$ are $P(12, 4)$, $Q(14, 1)$, and $R(20, 9)$. Describe the reflection used to form each triangle. Explain your reasoning using the algebraic rule.

   a. $J(-12, 4)$, $M(-14, 1)$, and $E(-20, 9)$

   b. $D(12, -4)$, $L(14, -1)$, and $G(20, -9)$

4. Suppose the vertices of $\triangle PQR$ are $P(3, 2)$, $Q(7, 3)$, and $R(1, 7)$.

   a. Describe a sequence of a translation and reflection to form $\triangle$JME with coordinates $J(8, -2)$, $M(12, -3)$, and $E(6, -7)$.

   b. Describe a sequence of a translation and a rotation to form $\triangle DLG$ with coordinates $D(2, -6)$, $L(3, -10)$, and $G(7, -4)$.

5. Are the images that result from a translation, rotation, or reflection always, sometimes, or never congruent to the original figure?

Talk the Talk

Transformation Match-Up

Remember . . .
A positive rational number is a positive number that is written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Suppose a point (x, y) undergoes a rigid motion transformation. The possible new coordinates of the point are shown. Assume c is a positive rational number.

$(y, -x)$	$(x, y - c)$	$(x, -y)$
$(x + c, y)$	$(x - c, y)$	$(-y, x)$
$(-x, -y)$	$(-x, y)$	$(x, y + c)$

1. Record each set of new coordinates in the appropriate section of the table and then write a verbal description of the transformation. Be as specific as possible.

Translations	
Coordinates	Description

Reflections	
Coordinates	Description

Rotations	
Coordinates	Description

2. Do the rigid motion transformations of translations, reflections, and rotations preserve congruency?

Lesson 6 Assignment

Write

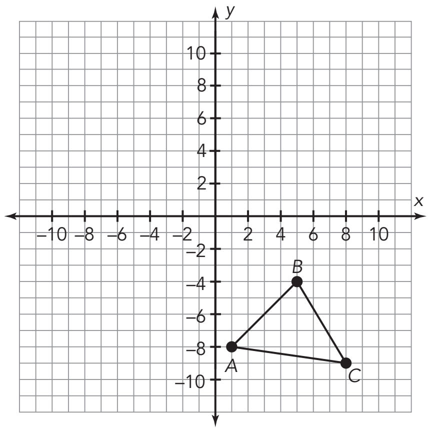
Draw and label a pair of congruent triangles. Since the triangles are congruent, what else do you know about the triangles?

Remember

Rigid motions produce congruent figures. There is often more than one sequence of transformations that can be used to verify that two figures are congruent.

Practice

1. Triangle *ABC* has coordinates *A*(1, −8), *B*(5, −4), and *C*(8, −9).



   a. Describe a transformation that can be performed on △*ABC* that will result in a triangle in the first quadrant.

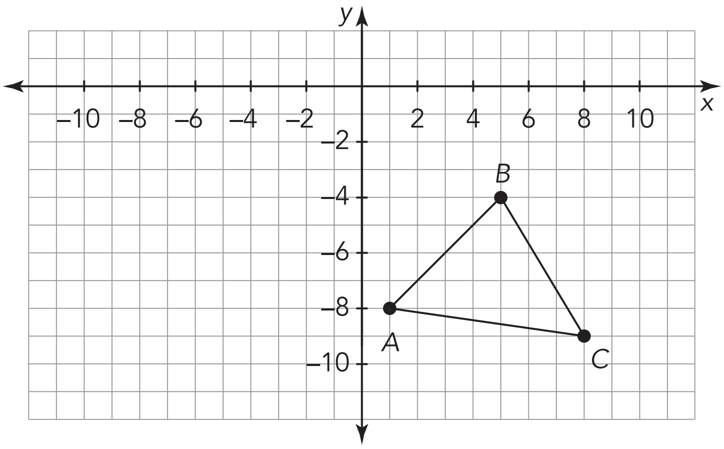
   b. Perform the transformation and label the image △*DEF*.

   c. List the coordinates for the vertices for △*DEF*.

   d. Use an algebraic rule to verify the location of the coordinates of the image.

Lesson 6 Assignment

2. Triangle ABC has coordinates $A(1, -8)$, $B(5, -4)$, and $C(8, -9)$.



a. Describe a transformation that can be performed on $\triangle ABC$ that will result in a triangle in the third quadrant.

b. Perform the transformation and label the image $\triangle DEF$.

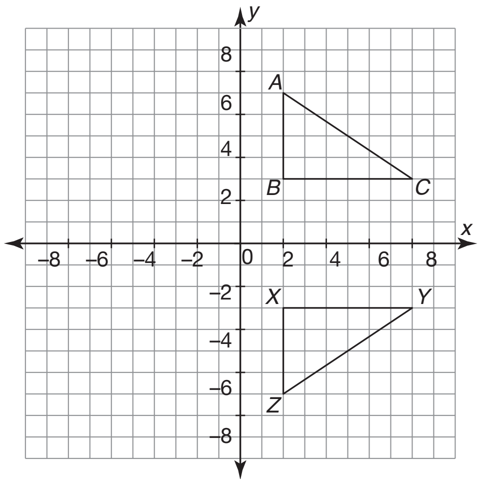
c. List the coordinates for the vertices for $\triangle DEF$.

d. Use an algebraic rule to verify the location of the coordinates of the image.

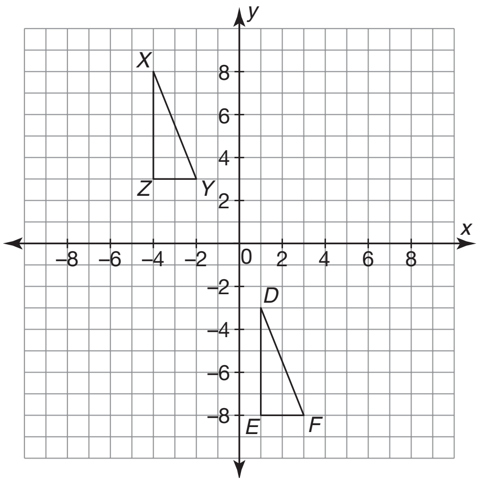
Lesson 6 Assignment

3. Identify the transformation used to create △*XYZ* in each.

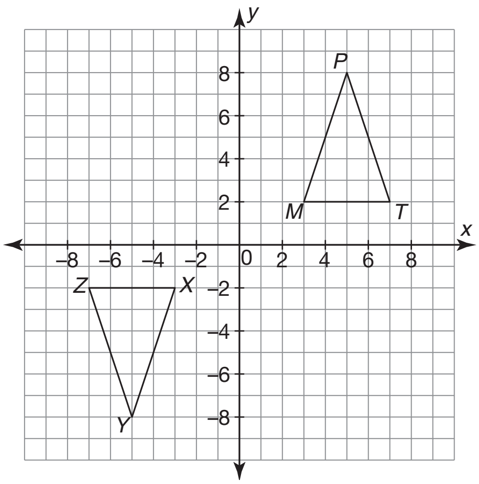
a.



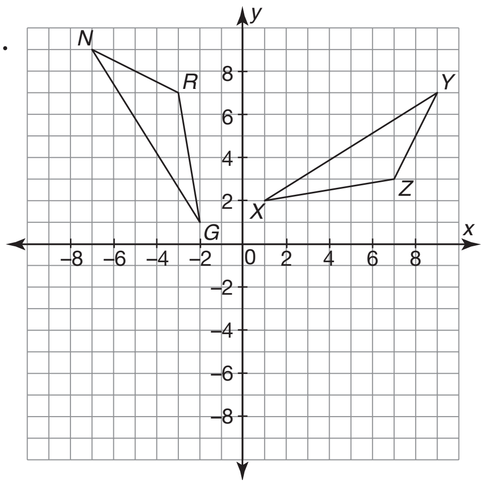
b.



c.



d.



Lesson 6 Assignment

4. Write an algebraic rule that represents the given transformation. Determine the coordinates of the new image.

   a. Triangle *ABC* with coordinates *A*(−8, 1), *B*(−4, 6), and *C*(0, 3) maps onto △*XYZ* with coordinates *X*(−1, −8), *Y*(−6, −4), and *Z*(−3, 0).

   b. Triangle *PRG* with coordinates *P*(2, 8), *R*(−7, 5), and *G*(2, 5) maps onto △*YOB* with coordinates *Y*(−2, 8), *O*(7, 5), and *B*(−2, 5).

   c. Triangle *JCE* with coordinates *J*(−6, 0), *C*(−4, −2), and *E*(0, 2) maps onto △*RAN* with coordinates *R*(−6, −3), *A*(−4, −5), and *N*(0, −1).

Prepare

A billboard advertises a watch. The face of the watch is 2 meters wide on the billboard. The face of the actual watch is 2 centimeters wide. What scale factor was used to create the billboard?

Name: ____________________

Rigid Motion Transformations

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Rigid Motion Transformations* topic by:

TOPIC 1: *Rigid Motion Transformations*	Beginning of Topic	Middle of Topic	End of Topic
defining and identifying translations, rotations, and reflections.	◯	◯	◯
translating, reflecting, and rotating geometric figures using patty paper and on the coordinate plane.	◯	◯	◯
verifying congruence of figures by measuring and comparing the properties of the geometric figures after undergoing a translation, reflection, or rotation.	◯	◯	◯
verifying that lines and line segments, angles, and parallel lines remain the same length after undergoing a translation, reflection, or rotation.	◯	◯	◯
explaining how to tell whether a two-dimensional figure is congruent to another figure using translations, reflections, and rotations.	◯	◯	◯
verifying that rigid motions preserve the size and shape of a figure, but reflections change the orientation of the vertices of a figure.	◯	◯	◯
describing the effects of rigid motion transformations to the *x*- and *y*-coordinates of a figure using algebraic representations.	◯	◯	◯

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Rigid Motion Transformations* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

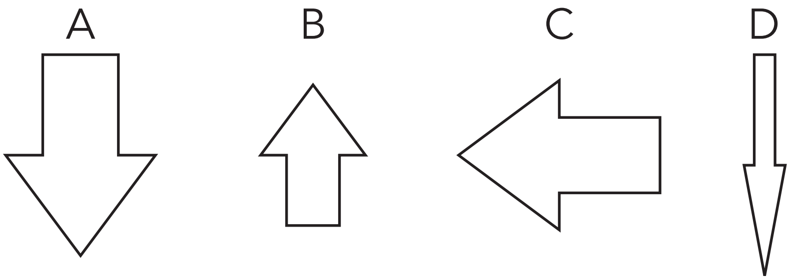
Rigid Motion Transformations Summary

LESSON 1 Introduction to Congruent Figures

Figures that have the same size and shape are **congruent figures**. When two figures are congruent, all corresponding sides and all corresponding angles have the same measures. **Corresponding sides** are sides that have the same relative position in geometric figures. **Corresponding angles** are angles that have the same relative position in geometric figures.

When two figures are congruent, you can obtain one figure by a combination of sliding, flipping, and spinning the figure until it lies on the other figure.

For example, Figure A is congruent to Figure C, but it is not congruent to Figure B or Figure D.



LESSON 2 Introduction to Rigid Motions

A **plane** extends infinitely in all directions in two dimensions and has no thickness. A **transformation** is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of each figure.

The original figure on the plane is called the **pre-image**, and the new figure that results from a transformation is called the **image**. The labels for the vertices of an image use the symbol (′), which is read as "prime."

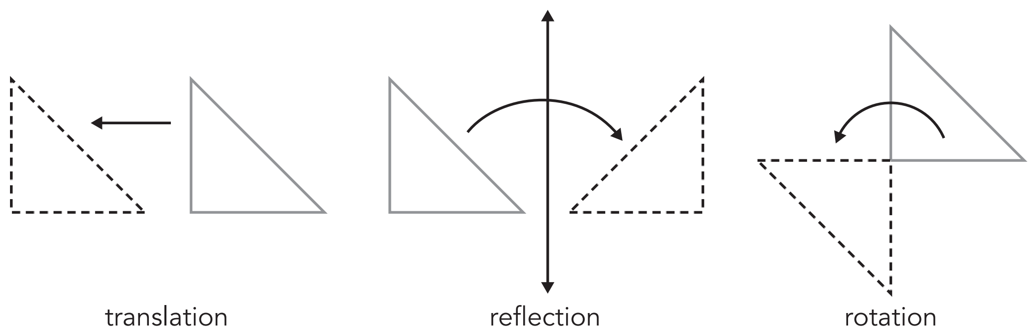
NEW KEY TERMS

- congruent figures [figuras congruentes]
- corresponding sides
- corresponding angles [ángulos correspondientes]
- plane [plano]
- transformation [transformación]
- rigid motion [movimiento rígido/directo/propio]
- pre-image [preimagen]
- image [imagen]
- translation [traslación]
- reflection [reflexión]
- line of reflection [línea de reflexión]
- rotation [rotación]
- center of rotation [centro de rotación]
- angle of rotation [ángulo de rotación]
- congruent line segments[segmentos de línea congruentes]
- congruent angles [ángulos congruentes]

A **translation** is a rigid motion transformation that slides each point of a figure the same distance and direction along a line. A figure can be translated in any direction. Two special translations are vertical and horizontal translations. Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation.

A **reflection** is a rigid motion transformation that flips a figure across a **line of reflection**. A line of reflection is a line that acts as a mirror so that corresponding points are the same distance from the line.

A **rotation** is a rigid motion transformation that turns a figure on a plane about a fixed point, called the **center of rotation**, through a given angle, called the **angle of rotation**. The center of rotation can be a point outside of the figure, inside of the figure, or on the figure itself. Rotation can be clockwise or counterclockwise.



LESSON 3 Translations of Figures on the Coordinate Plane

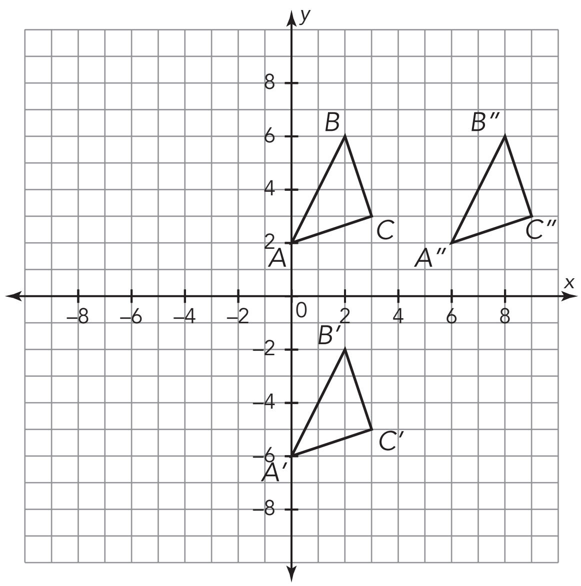
A translation slides an image on the coordinate plane. When an image is horizontally translated c units on the coordinate plane, the value of the x-coordinates changes by c units. When an image is vertically translated c units on the coordinate plane, the value of the y-coordinate changes by c *units*. The coordinates of an image after a translation are summarized in the table.

Original Point	Horizontal Translation to the Left	Horizontal Translation to the Right	Vertical Translation Up	Vertical Translation Down
(x, y)	$(x - c, y)$	$(x + c, y)$	$(x, y + c)$	$(x, y - c)$

For example, the coordinates of $\triangle ABC$ are $A(0, 2)$, $B(2, 6)$, and $C(3, 3)$.

When $\triangle ABC$ is translated down 8 units, the coordinates of the image are $A'(0, -6)$, $B'(2, -2)$, and $C'(3, -5)$.

When $\triangle ABC$ is translated right 6 units, the coordinates of the image are $A''(6, 2)$, $B''(8, 6)$, and $C''(9, 3)$.



LESSON 4 Reflections of Figures on the Coordinate Plane

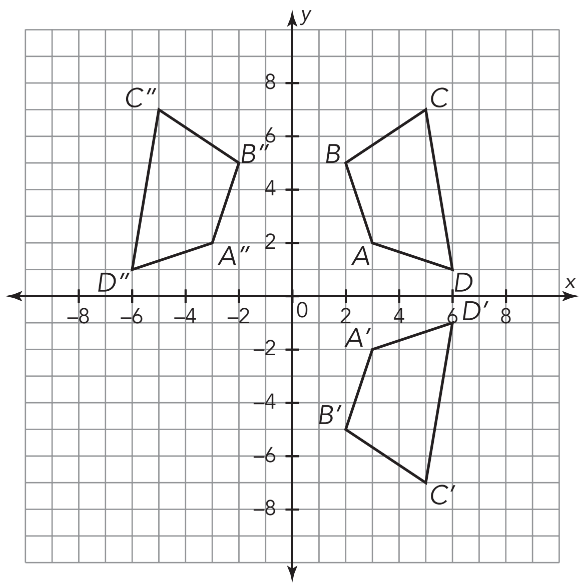
A reflection flips an image across a line of reflection. When an image on the coordinate plane is reflected across the *y*-axis, the value of the *x*-coordinate of the image is opposite the *x*-coordinate of the pre-image. When an image on the coordinate plane is reflected across the *x*-axis, the value of the *y*-coordinate of the image is opposite the *y*-coordinate of the pre-image. The coordinates of an image after a reflection on the coordinate plane are summarized in the table.

Original Point	Reflection Over *x*-Axis	Reflection Over *y*-Axis
(x, y)	$(x, -y)$	$(-x, y)$

For example, the coordinates of Quadrilateral *ABCD* are $A(3, 2)$, $B(2, 5)$, $C(5, 7)$, and $D(6, 1)$.

When Quadrilateral *ABCD* is reflected across the *x*-axis, the coordinates of the image are *A*′(3, −2), *B*′(2, −5), *C*′(5, −7), and *D*′(6, −1).

When Quadrilateral *ABCD* is reflected across the *y*-axis, the coordinates of the image are *A*″(−3, 2), *B*″(−2, 5), *C*″(−5, 7), and *D*″(−6, 1).



LESSON 5

Rotations of Figures on the Coordinate Plane

A rotation turns a figure about a point through an angle of rotation. When the center of rotation is at the origin (0, 0) and the angle of rotation is 90°, 180°, 270°, or 360°, the coordinates of an image can be determined using the rules summarized in the table.

Original Point and Rotation About the Origin 360°	Rotation About the Origin 90° Counterclockwise and 270° Clockwise	Rotation About the Origin 90° Clockwise and 270° Counterclockwise	Rotation About the Origin 180°
(x, y)	$(-y, x)$	$(y, -x)$	$(-x, -y)$

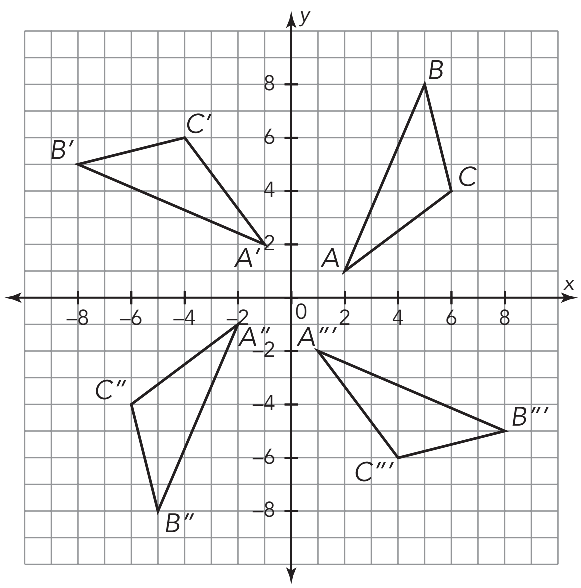
For example, the coordinates of △*ABC* are *A*(2, 1), *B*(5, 8), and *C*(6, 4).

When △*ABC* is rotated 90° counterclockwise or 270° clockwise about the origin, the coordinates of the image are *A*′(−1, 2), *B*′(−8, 5), and *C*′(−4, 6).

When $\triangle ABC$ is rotated 180° about the origin, the coordinates of the image are $A''(-2, -1)$, $B''(-5, -8)$, and $C''(-6, -4)$.

When $\triangle ABC$ is rotated 90° clockwise or 270° counterclockwise about the origin, the coordinates of the image are $A''(1, -2)$, $B''(8, -5)$, and $C''(4, -6)$.

When $\triangle ABC$ is rotated 360° about the origin, the coordinates are the same as the coordinates of the original triangle.



LESSON 6 Congruence and Rigid Motions

In congruent figures, the corresponding sides are congruent line segments. **Congruent line segments** are line segments that have the same length. Likewise, when corresponding angles have the same measure, they are congruent angles. **Congruent angles** are angles that are equal in measure.

For example, when the sides of two different figures are equal in length, such that the length of side *AB* in $\triangle ABC$ is equal to the length of side *DE* in $\triangle DEF$, these sides are said to be congruent.

$\overline{AB} \cong \overline{DE}$ is read "line segment *AB* is congruent to line segment *DE*."

Likewise, when the angles of two different figures are equal in measure, such that the measure of angle *A* in $\triangle ABC$ is equal to the measure of angle *D* in $\triangle DEF$, these angles are said to be congruent.

$\angle A \cong \angle D$ is read "angle *A* is congruent to angle *D*."

There is often more than one sequence of transformations that can be used to verify that two figures are congruent.

GRADE 8

TOPIC 2

Another type of transformation scales a figure up or down in size. The original figure and the new figure are similar to each other.

Similarity

1 Dilations of Figures

OBJECTIVES

- Dilate figures given a center of dilation and scale factor such that the resulting dilation is an enlargement or a reduction of the original figure.
- Identify the scale factor used in a dilation of a figure.
- Determine whether a two-dimensional figure is similar to another by obtaining one from the other using a sequence of dilations.
- Describe a sequence of dilations that demonstrates that two figures are similar.

NEW KEY TERMS

- dilation
- center of dilation
- scale factor
- enlargement
- reduction
- similar

You have learned about geometric transformations that preserve the size and shape of figures. You also know how to use scale factors to produce scale drawings.

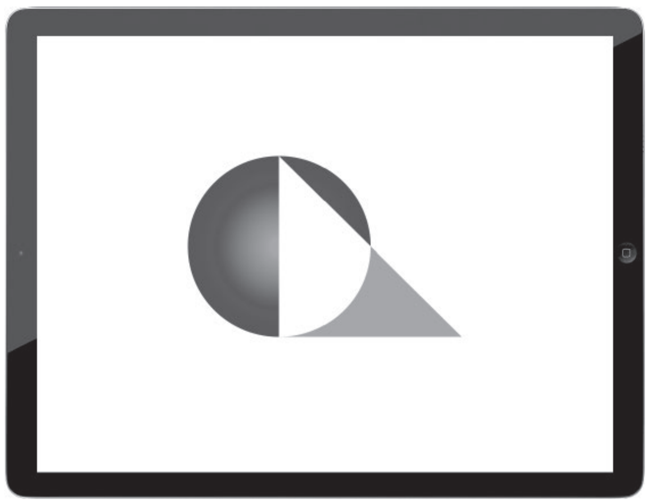
Is there another type of geometric transformation that can change the size of a figure?

Getting Started

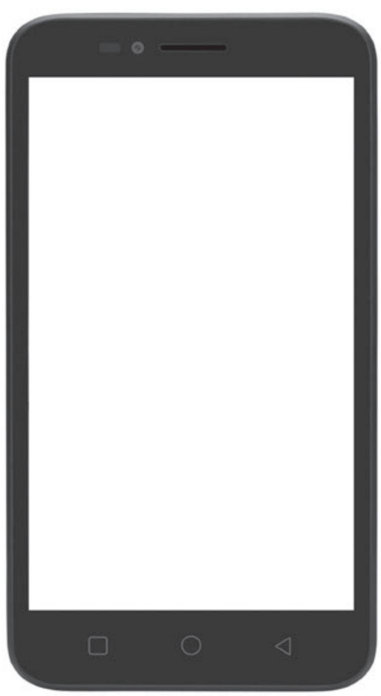
Scale Drawing by Doing

Recall that a scale drawing is a representation of a real object or place that is in proportion to the real object or place it represents. The ratios of corresponding side lengths between the drawing and the object are all the same.

Consider the logo shown on the tablet screen.



1. When the logo on the tablet screen appears on the smartphone screen, it will be reduced by a scale factor of $\frac{1}{2}$. Sketch the logo on the smartphone screen and explain your process.



2. When the logo on the tablet screen appears on the desktop screen, it will be enlarged by a scale factor of 2. Sketch the logo on the desktop screen and explain your process.

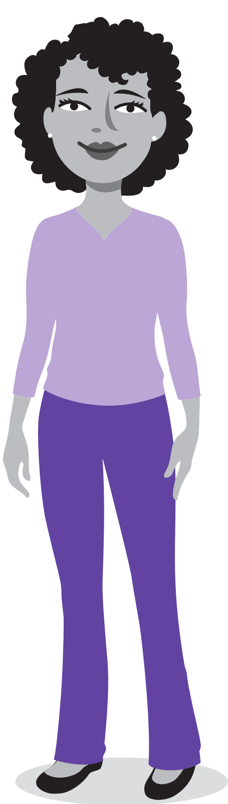
ACTIVITY 1.1

Dilating Figures with a Scale Factor Greater Than 1

Dilations are transformations that produce figures that are the same shape as the original figure but not necessarily the same size. Each point on the original figure is moved along a straight line, and the straight line is drawn from a fixed point known as the **center of dilation**. The distance each point moves is determined by the *scale factor* used.

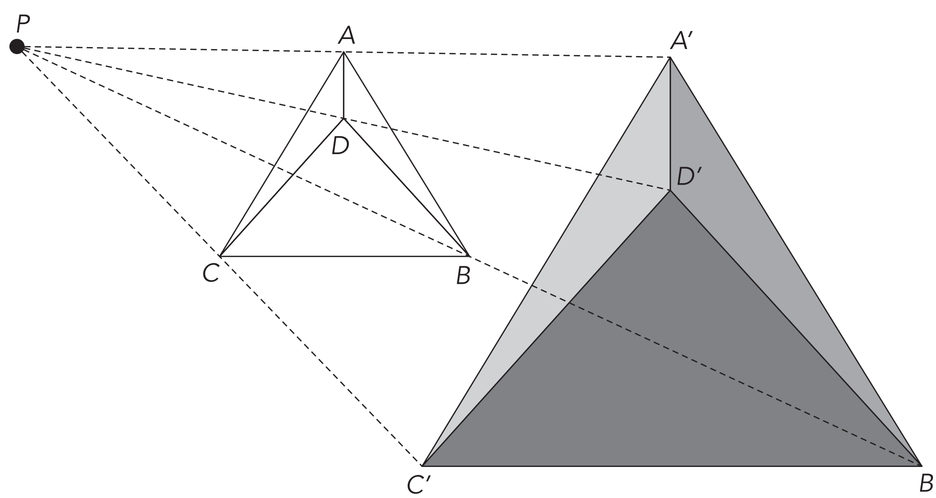
The **scale factor** is the ratio of the distance of the new figure from the center of dilation to the distance of the original figure from the center of dilation. When the scale factor is greater than 1, the new figure is called an **enlargement**.

The image of a dilation can also be called a *scale drawing*.



WORKED EXAMPLE

This image of a logo was dilated to produce an enlargement using point *P* as the center of dilation.



The scale factor can be expressed as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

1. In the Worked Example, the scale factor is represented by 4 equivalent ratios. What distances are represented by each part of those ratios? Is the scale factor between 0 and 1, equal to 1, or greater than 1? Explain your reasoning.

2. Measure the segment lengths of the original logo in millimeters.

   $AB =$ ______________ $AC =$ ______________

   $BC =$ ______________ $AD =$ ______________

The notation $\overline{AB}$ means "segment AB." The notation AB means "the length of segment AB."

3. Measure the segment lengths of the new logo in millimeters.

   $A'B' =$ ______________ $A'C' =$ ______________

   $B'C' =$ ______________ $A'D' =$ ______________

To indicate the measure of the segment, you can write AB.

4. Measure each line segment in millimeters.

   $A'P =$ ______________ $AP =$ ______________

   $B'P =$ ______________ $BP =$ ______________

   $C'P =$ ______________ $CP =$ ______________

   $D'P =$ ______________ $DP =$ ______________

5. Determine each ratio.

$\frac{A'P}{AP} =$ ______________ $\frac{B'P}{BP} =$ ______________

$\frac{C'P}{CP} =$ ______________ $\frac{D'P}{DP} =$ ______________

$\frac{B'C'}{BC} =$ ______________ $\frac{A'B'}{AB} =$ ______________

$\frac{A'D'}{AD} =$ ______________ $\frac{A'C'}{AC} =$ ______________

6. How do you think the angle measures of the new logo will compare with those of the old logo? Make a conjecture. Then, test your conjecture by measuring various angles in the original and new logos. Describe your conclusion.

7. Compare the original logo and the new logo. What do you notice?

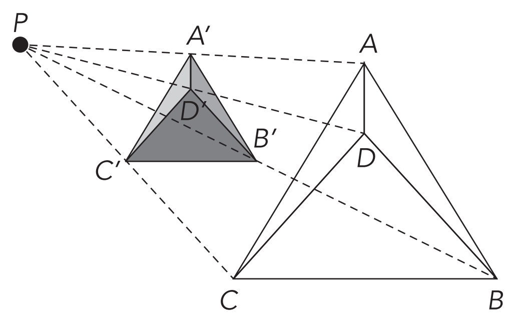
ACTIVITY 1.2

Dilating Figures with a Scale Factor Between 0 and 1

When the scale factor is between 0 and 1, the new figure is called a **reduction**.

WORKED EXAMPLE

The original logo was dilated to produce a reduction using point *P* as the center of dilation.



The scale factor can be expressed as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

The size of the logo and its distance from point *P* are the same as the Worked Example showing an enlargement of the logo.

1. In the Worked Example, the scale factor is represented by 4 equivalent ratios. What distances are represented by each part of those ratios? Is the scale factor between 0 and 1, equal to 1, or greater than 1? Explain your reasoning.

2. Measure the segment lengths of the new logo in millimeters.

   $A'B' =$ ______________ $A'C' =$ ______________

   $B'C' =$ ______________ $A'D' =$ ______________

3. Measure each line segment in millimeters.

$A'P' = $ ______________ $B'P' = $ ______________

$C'P' = $ ______________ $D'P' = $ ______________

4. Determine each ratio.

$\frac{A'P}{AP} = $ ______________ $\frac{B'P'}{BP} = $ ______________

$\frac{C'P}{CP} = $ ______________ $\frac{D'P}{DP} = $ ______________

$\frac{B'C'}{BC} = $ ______________ $\frac{A'B'}{AB} = $ ______________

$\frac{A'D'}{AD} = $ ______________ $\frac{A'C'}{AC} = $ ______________

5. How do you think the angle measures of the new logo will compare with those of the old logo? Make a conjecture. Then, test your conjecture by measuring various angles in the original and new logos. Describe your conclusion.

6. Compare the original logo and the new logo. What do you notice?

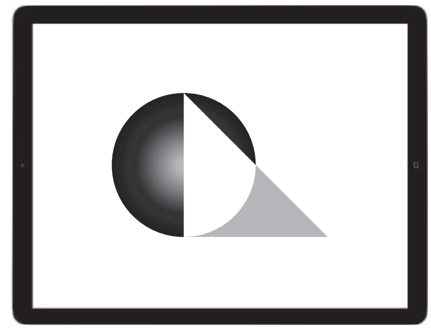
7. What do you think would occur when the scale factor is 1? Explain your reasoning.

ACTIVITY 1.3

Creating and Verifying Similar Figures

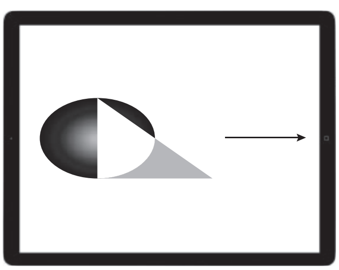
When working with images on a computer, the size of the images can be changed by dragging a corner or side of the image. How you drag the images determines whether or not the scale of the image is maintained.

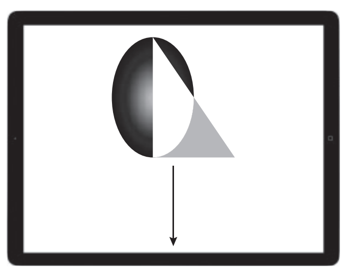
Destiny needs to adjust the original logo to use on different web pages. She plays around with the image to determine how she can adjust the logo and still maintain the same scale.

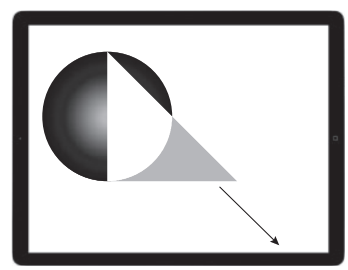


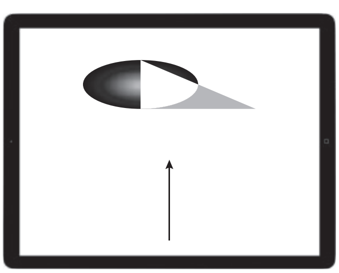
Each image contains an arrow that indicates how Destiny adjusts the logo and the resulting logo.

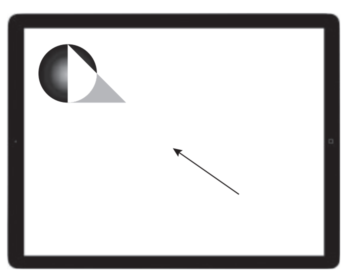
1. Which of the adjusted logos do you think are dilations of the original? Which are not? Explain your thinking.

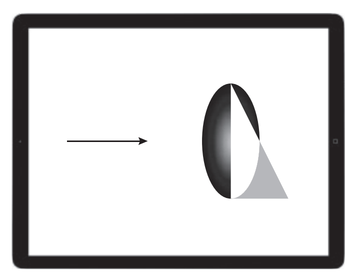
a. 

b. 

c. 

d. 

e. 

f. 

When you dilate a figure, you create a *similar* figure. When two figures are **similar**, the ratios of their corresponding side lengths are equal. This means that you can create a similar figure by multiplying or dividing all of the side lengths of a figure by the same scale factor (except 0). You can multiply or divide by 1 to create a similar figure too. In that case, the similar figures are congruent figures. Corresponding angles in similar figures are congruent.

Many word processing and graphics software programs allow users to change the sizes of images.

WORKED EXAMPLE

Consider the images shown. The height of the original image is 2.66 inches, and the width is 3.48 inches. The original image is then dilated to create a reduction.

2. Are the two images similar? Explain how you know.

3. What is the scale factor used to reduce the image? Describe two different ways you can determine the scale factor.

4. How can you tell that a height of 2.66 in. and a width of 3.48 in. are the original dimensions of the image?

5. Consider each set of new dimensions or scale percents that show adjustments to this original image. Describe how the image changed and whether the new image is similar to the original. Show your work and explain your reasoning.

Height
- (●) Absolute: 2"
- ( ) Relative

Width
- (●) Absolute: 3"
- ( ) Relative

a. Scale

Height: 225 % Width: 225 %

b. Scale

Height: 90 % Width: 110 %

c. Height
- (●) Absolute: 1.5"
- ( ) Relative

Width
- (●) Absolute: 2.25"
- ( ) Relative

d. Height
- (●) Absolute: 2"
- ( ) Relative

Width
- (●) Absolute: 2"
- ( ) Relative

6. Explain why Javier's reasoning is not correct. Draw examples to illustrate your explanation.

> **Javier**
>
> I can dilate a rectangular figure by adding the same value to its length and width.

Talk the Talk

It's a Cloud

1. Dilate the figure shown using scale factors of $\frac{4}{3}$ and $\frac{3}{4}$ and point Q as the center of dilation.

2. Describe the relationship between the corresponding sides in an original figure and the new figure resulting from a dilation.

3. Describe the relationship between the corresponding angles in an original figure and the new figure resulting from a dilation.

4. Determine if each statement is true or false. If a statement is false, include a counterexample. Explain your reasoning.

   a. True False All similar figures are also congruent figures.

   b. True False All congruent figures are also similar figures.

A *counterexample* is a single example that shows that a statement is not true.

Lesson 1 Assignment

Write

In your own words, describe all of the ways you can tell whether two figures are *similar*. Use examples to illustrate your description.

Remember

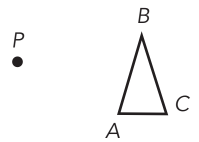
Dilations are transformations that produce figures that are the same shape as the original figure but not the same size. Each point on the original figure is moved along a straight line, and the straight line is drawn from a fixed point known as the *center of dilation*. The distance each point moves is determined by the scale factor used.

The *scale factor* is the ratio of the distance of the new figure from the center of dilation to the distance of the original figure from the center of dilation.

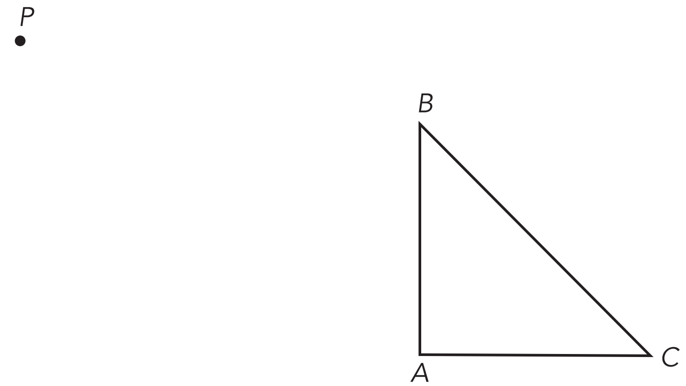
Practice

1. Dilate each triangle with P as the center of dilation and the given scale factor.

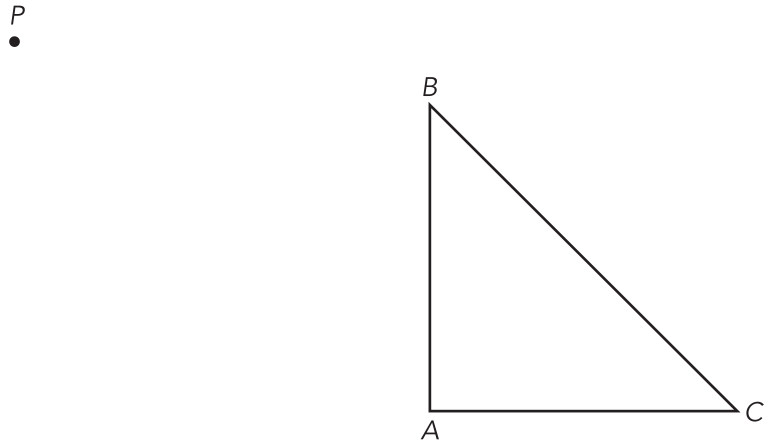
   a. Scale factor of 3



   b. Scale factor of $\frac{1}{3}$

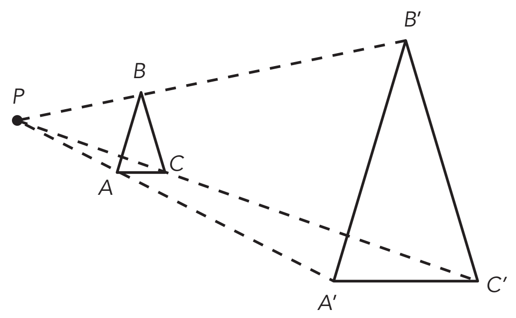


c. Scale factor of $\frac{1}{4}$

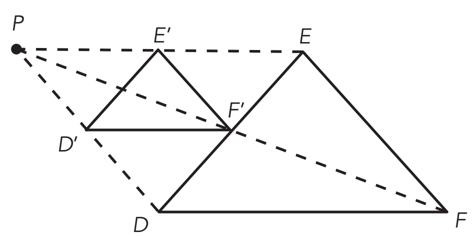


2. The triangles in each pair are similar. Identify the congruent corresponding angles and the corresponding proportional side lengths.

a. Triangle ABC is similar to $\triangle A'B'C'$.



b. Triangle DEF is similar to $\triangle D'E'F'$.



Lesson 1 Assignment

3. Gabriela has a photo of a lasagna dish she made, which she wants to post to various websites. The original image has a width of 300 pixels and a height of 450 pixels. Consider each set of new dimensions or scale percents that show adjustments to this original image. Describe how the image changed and whether the new image is similar to the original. Show your work and explain your reasoning.

   a. New image: 360 pixels width, 540 pixels height

   b. New image: 35% width, 35% height

   c. New image: 150 pixels width, 150 pixels height

Lesson 1 Assignment

Prepare

Scale up or scale down to determine the value of the variable in each equivalent ratio.

1. $3 : 1 = 25.5 : z$

2. $2 : 5 = a : 30$

3. $1 : 4 = x : 80$

4. $9.9 : 10 = 99 : p$

2

Dilating Figures on the Coordinate Plane

OBJECTIVES

- Dilate figures on a coordinate plane.
- Understand the dilation of a figure on the coordinate plane as a scaling up or scaling down of the coordinates of the figure.
- Describe how a dilation of a figure on a coordinate plane affects the coordinates of the figure.

You have used transformations called *dilations* to create similar figures.

How can you use coordinates to determine whether two figures are similar?

Getting Started

Ask Yourself . . .
How can you organize and record your mathematical ideas?

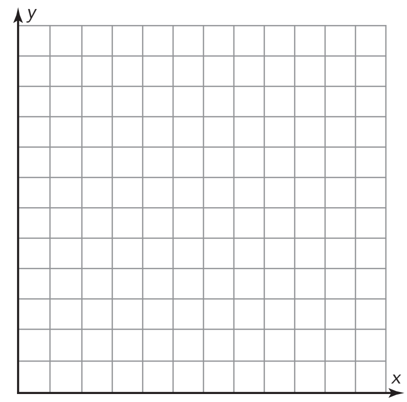
The Escalator or the Stairs

Fernando rides up an escalator. The escalator starts at (0, 0) and drops Fernando off at (12, 8). Hannah starts at the same point but takes the stairs.

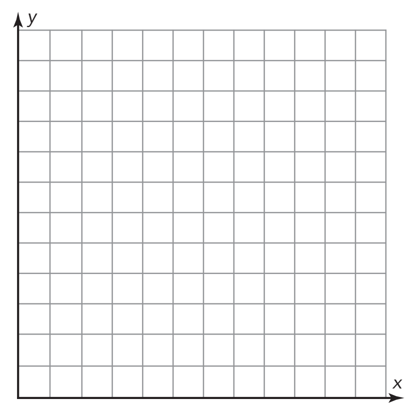
1. Use the coordinate planes given to represent the movement of Fernando and Hannah.

   a. Draw a line to show Fernando's path on the escalator.

   b. Draw steps starting at the origin that will take Hannah to the same location as Fernando. Make all of the steps the same.

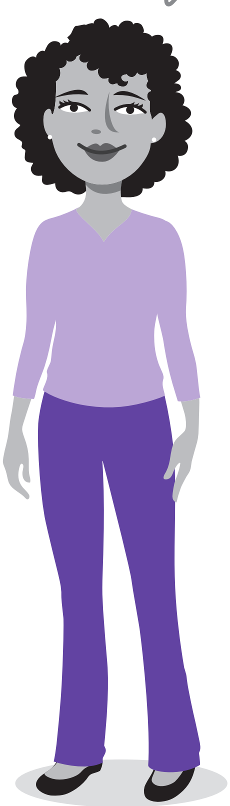


Escalator



Stairs

Think about equivalent ratios, scaling up, and scaling down.



2. How is taking the stairs similar to riding the escalator? How is it different? Explain your reasoning.

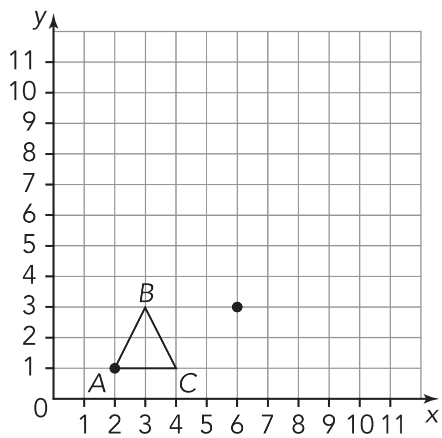
3. Compare the steps that you designed for Hannah with your classmates' steps. How are these steps similar to your steps?

ACTIVITY 2.1

Scaling Up and Down on the Coordinate Plane

You know that a translation moves a point along a line. A sequence of repeated horizontal and/or vertical translations also moves a point along a line. You can use this fact to dilate figures.

WORKED EXAMPLE



Dilate △*ABC* by a scale factor of 3 using the origin as the center of dilation.

Let's start by dilating point *A*, which is located at (2, 1).

Point *A* is 2 units right and 1 unit up from the origin.

To dilate point *A* by a scale factor of 3, translate point *A* by three repeated sequences: 2 units right and 1 unit up.

1. Describe the repeated translations you can use to scale point *B* and point *C*. Then, plot point *B'* and point *C'* on the coordinate plane in the Worked Example.

   a. Point *B* to point *B'*

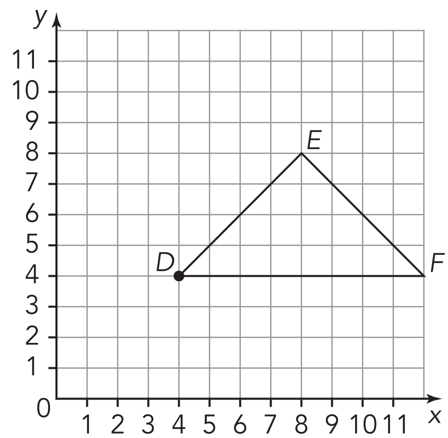
   b. Point *C* to point *C'*

2. Draw △*A'B'C'* on the coordinate plane in the example. Is △*ABC* similar to △*A'B'C'*? Explain your reasoning.

WORKED EXAMPLE

Dilate $\triangle DEF$ by a scale factor of $\frac{1}{4}$ using the origin as the center of dilation.

Let's start by dilating point *D*, located at (4, 4).

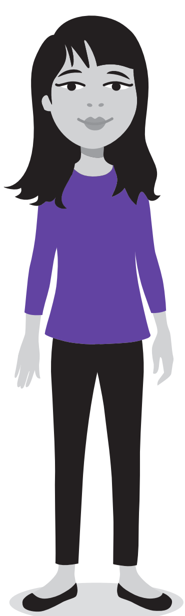


Point *D* is translated from the origin 4 units right and 4 units up (4, 4). This is the same as four translations of 1 unit right and 1 unit up.

Therefore, scaling point *D* to (1, 1) represents a dilation by a scale factor of $\frac{1}{4}$.

3. Determine the coordinates of points *E'* and *F'*. Explain how you determined your answers. Then, draw $\triangle D'E'F'$ on the coordinate plane in the example.

How do the side lengths and angles of the triangles compare?

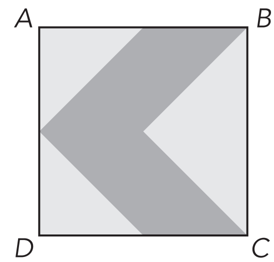


4. Is $\triangle DEF$ similar to $\triangle D'E'F'$? Explain your reasoning.

5. How does dilating a figure, using the origin as the center of dilation, affect the coordinates of the original figure? Make a conjecture using the examples in this activity.

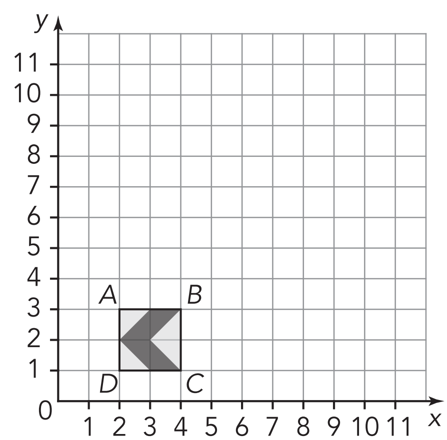
ACTIVITY 2.2

Using the Origin as the Center of Dilation



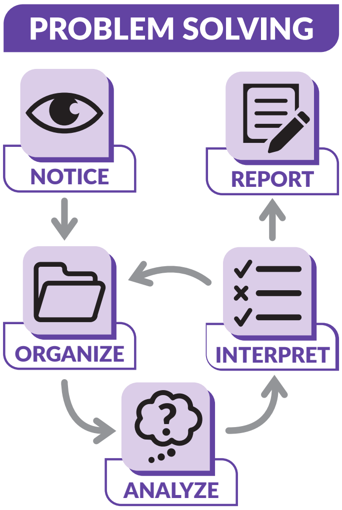
Road signs maintain a constant scale regardless of whether they are on the road or in the drivers' manual. This sign indicates that the road is bending to the left.

1. Dilate the figure on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of 3 to form a new figure.



2. List the ordered pairs for the original figure and for the new figure. How are the values in the ordered pairs affected by the dilation?

3. Compare and contrast the corresponding angles and corresponding side lengths of the new figure and the original figure.



4. Determine the perimeter and area of the original figure and the new figure.

	Perimeter	Area
Original figure		
New figure		

   a. How is the perimeter affected by the dilation?

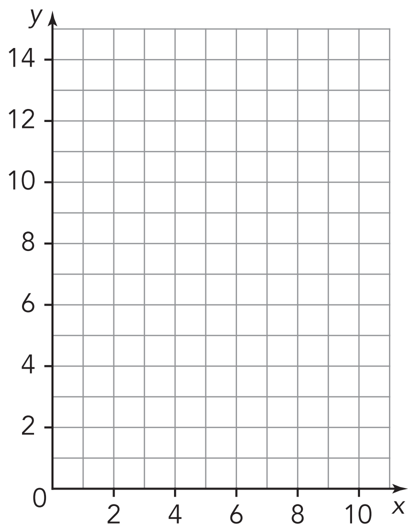
   b. How is the area affected by the dilation?

5. A road sign is represented by the coordinates $A(2, 1)$, $B(2, 12)$, $C(6, 12)$, and $D(6, 1)$. Suppose you were to dilate the figure by a scale factor of $\frac{1}{2}$ using the origin as the center of dilation.

   a. Predict how the perimeter of the figure will be affected by the dilation.

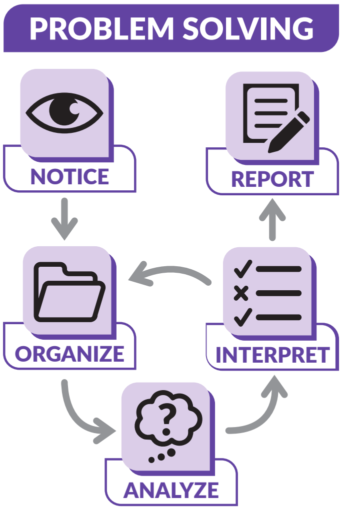
   b. Predict how the area of the figure will be affected by the dilation.

c. Test your prediction by graphing the original figure and the new figure on the coordinate plane.

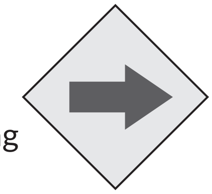


d. Describe your conclusion.

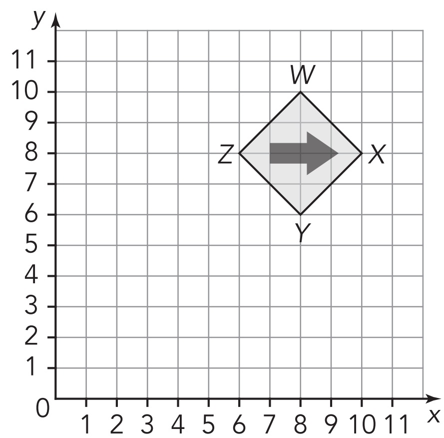
6. How does dilating a figure, using the origin as the center of dilation, affect the perimeter and area of the new figure? Make a conjecture using the examples in this activity.



Let's consider a different road sign. This sign indicates that the road proceeds to the right.



7. Dilate the figure on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of $\frac{1}{2}$ to form a new figure.



8. List the ordered pairs for the original figure and for the new figure. How are the values in the ordered pairs affected by the dilation?

9. Compare and contrast the corresponding angles and corresponding side lengths of the original figure and the new figure.

10. Suppose a point is located at (x, y). Write the ordered pair of the point after a dilation by each given scale factor.

a. $\frac{1}{2}$

b. 5

c. $\frac{4}{3}$

Talk the Talk

Location, Location, Location

Answer each question to summarize what you know about dilating figures on the coordinate plane. Use your answers to plan a presentation for your classmates that demonstrates what you learned in this lesson.

1. What strategies can you use to determine whether two figures are similar when they are:

   a. located on a coordinate plane?

   b. not located on a coordinate plane?

2. A polygon is graphed on a coordinate plane with (x, y) representing the location of a certain point on the polygon. The polygon is transformed using the rule $(x, y) \rightarrow (ax, ay)$.

   a. What will be the impact on the original figure if a is greater than 1?

   b. What will be the impact on the original figure if a is between 0 and 1?

Lesson 2 Assignment

Write

In your own words, explain how to dilate a figure on the coordinate plane using repeated translations. Use examples with scale factors between 0 and 1 and greater than 1 to illustrate your explanation.

Remember

If the dilation of a figure is centered at the origin, you can multiply the coordinates of the points of the original figure by the scale factor to determine the coordinates of the new figure.

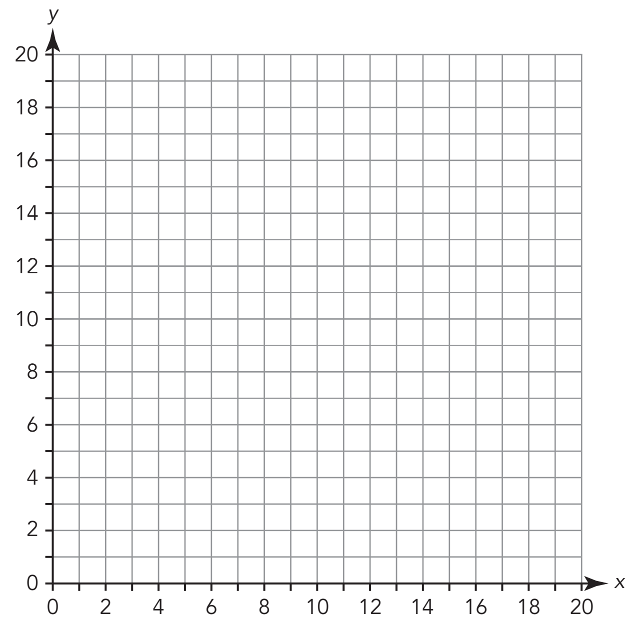
Practice

1. Graph △*XYZ* with the coordinates *X*(3, 18), *Y*(18, 18), and *Z*(18, 9).

    a. Reduce △*XYZ* on the coordinate plane using the origin as the center of dilation and a scale factor of $\frac{1}{3}$ to form △*X*′*Y*′*Z*′.

    b. What are the coordinates of points *X*′, *Y*′, and *Z*′?

    c. What is the area of the pre-image and the image?

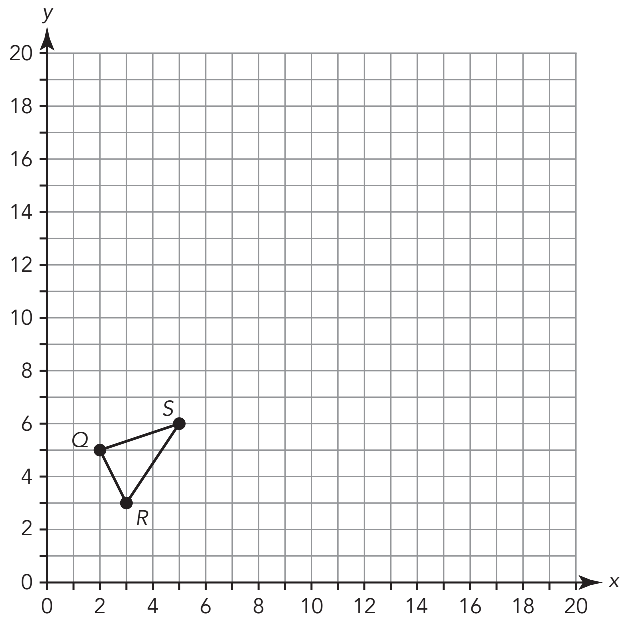


    d. What is the relationship between the two areas?

    e. If the perimeter of the pre-image is 41.49 units, what is the perimeter of the image? Explain your reasoning.

Lesson 2 Assignment

2. Dilate $\triangle QRS$ on the coordinate plane using the origin (0, 0) as the center of dilation and a scale factor of 3 to form $\triangle Q'R'S'$. Label the coordinates of points Q', R', and S'.



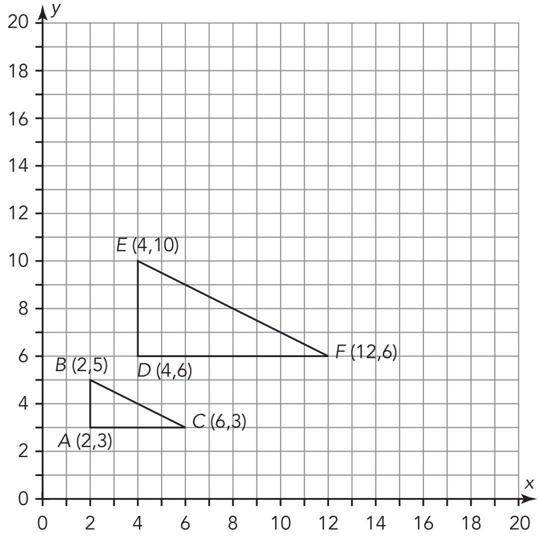
3. Triangle ABC is graphed on a coordinate plane with vertices at $A(-7, 5)$, $B(4, 6)$, and $C(5, 8)$. Triangle ABC is dilated by a scale factor of w with the origin as the center of dilation to create $\triangle A'B'C'$. What are the coordinates of the vertices of $\triangle A'B'C'$?

4. Quadrilateral $QRST$ is graphed on a coordinate plane with vertices as $Q(2, 14)$, $R(10, 18)$, $S(12, -5)$, and $T(4, -1)$. Quadrilateral $QRST$ is dilated by a scale factor of $\frac{1}{u}$ with the origin as the center of dilation to create Quadrilateral $Q'R'S'T'$. What are the coordinates of the vertices of Quadrilateral $Q'R'S'T'$?

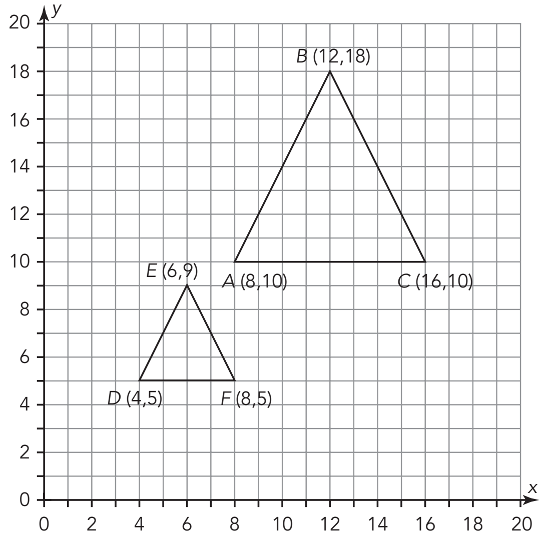
Lesson 2 Assignment

5. Verify that each pair of triangles is similar.

a.



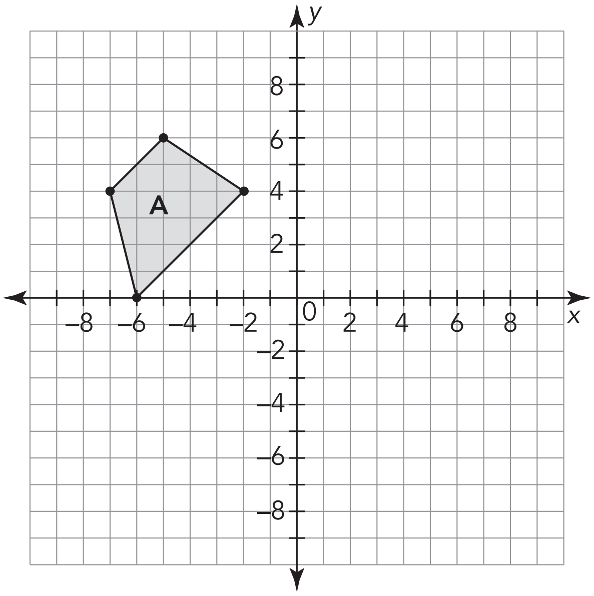
b.



Lesson 2 Assignment

Prepare

1. Use the origin as the center of dilation and dilate *A* on the coordinate plane by a scale factor of $\frac{1}{2}$.



3 Mapping Similar Figures Using Dilations

OBJECTIVES

- Write the ratio of corresponding sides of similar figures.
- Describe a single dilation that maps a two-dimensional figure onto a similar figure.

You have used sequences of translations, reflections, and rotations to verify that two images are congruent.

How can you use dilations to determine whether two images are similar and/or congruent?

Getting Started

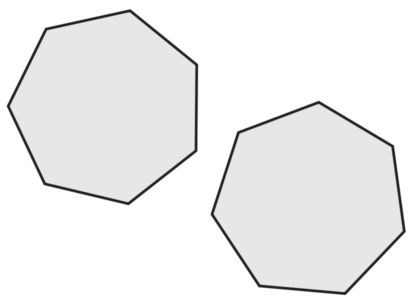
Same Figure or Same Shape?

When two figures are similar, the same scale factor can be applied to all side lengths to map one figure to the other.

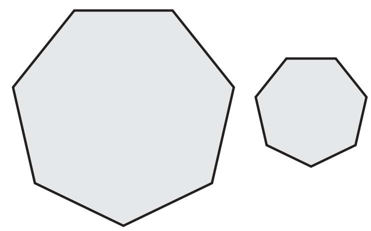
We often say that dilations preserve shape and that rigid motions preserve both size and shape. As a result, it is common to state that similar figures have the same shape and congruent figures have the same size and shape. However, what does it mean for two figures to have the same shape in this context? Are all rectangles similar? Are all triangles similar?

Use the definition of similar figures to determine which figures are similar.

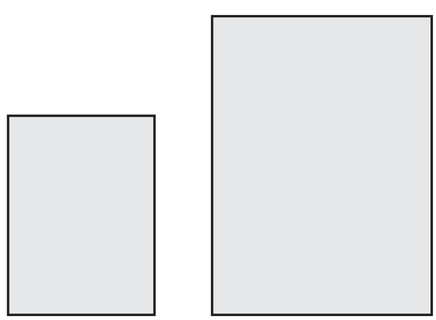
1.



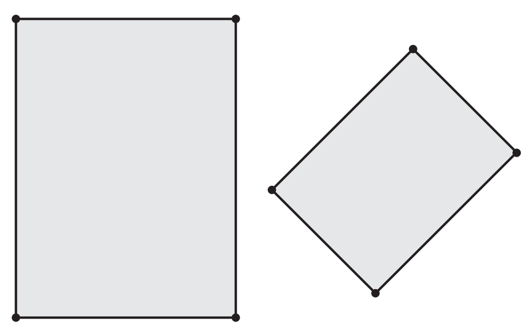
2.



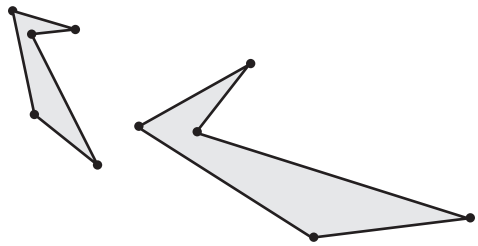
3.



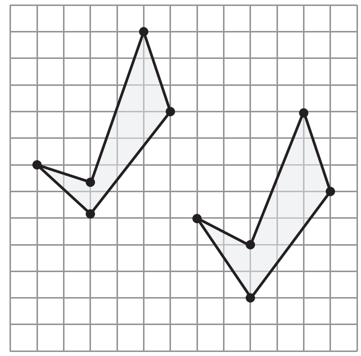
4.



5.



6.

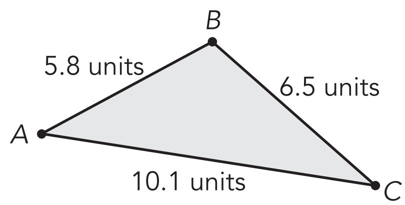


ACTIVITY
3.1

Corresponding Sides of Similar Figures

In this activity, you will use you what you know about dilations to describe the relationship between corresponding sides of similar figures.

1. Triangle *XYZ* is similar to △*ABC* after undergoing a reflection and a dilation.



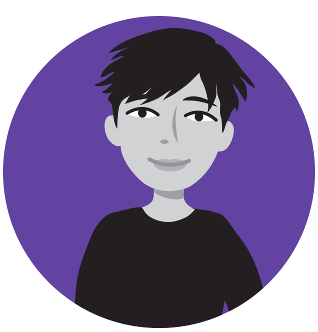
   a. List the corresponding sides and angles for △*ABC* and △*XYZ*.

   b. When the pre-image is △*ABC* and the image is △*XYZ* is the scale factor greater than 1, between 0 and 1, or equal to 1. Explain your reasoning.

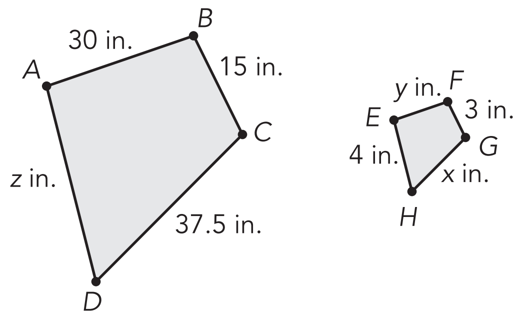
2. What do you know about the ratios of corresponding sides of a similar figure?

3. Why can you describe the ratio of the corresponding sides of a shape and its dilation as a proportional relationship?

A **proportional relationship** is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$ must represent the same constant.

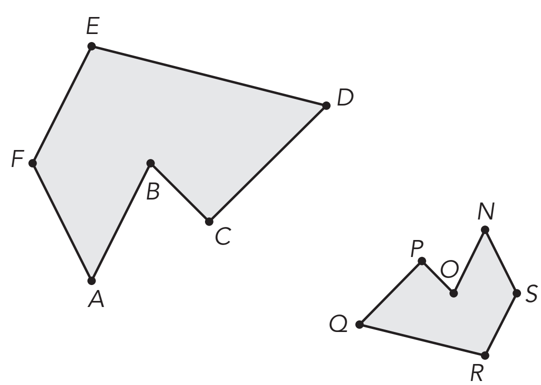


4. Quadrilateral *ABCD* is dilated to produce similar Quadrilateral *EFGH*.



Write 3 ratios equivalent to $\frac{30}{y}$.

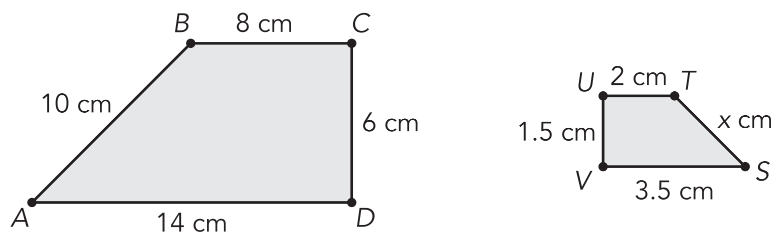
5. Figure *ABCDEF* is dilated and rotated to produce similar Figure *NOPQRS*.



Write 5 ratios that are equivalent to $\frac{AF}{NS}$.

6. Quadrilateral *ABCD* is reflected and dilated to create similar Quadrilateral *STUV*.

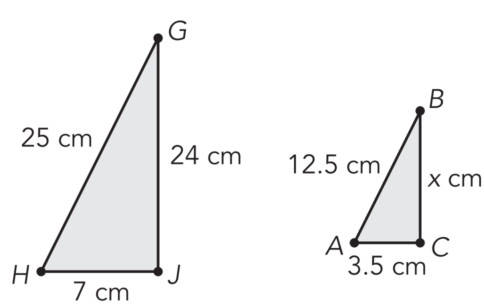
Alexander says he can use the proportion $\frac{8}{2} = \frac{10}{x}$ to determine the value of x. Madison says she can use the proportion $\frac{x}{10} = \frac{3.5}{14}$ to determine the value of x. Who is correct?



7. Determine the value of x in Quadrilateral *STUV*.

8. Triangle *HGJ* is dilated to produce similar $\triangle BAC$.

a. Write a proportion you can use to determine the value of x.



b. Determine the value of x.

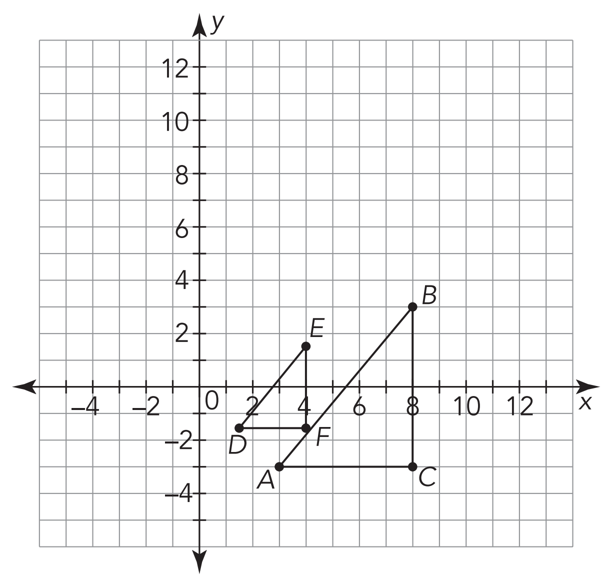
ACTIVITY 3.2

Proving Similarity Through Dilations

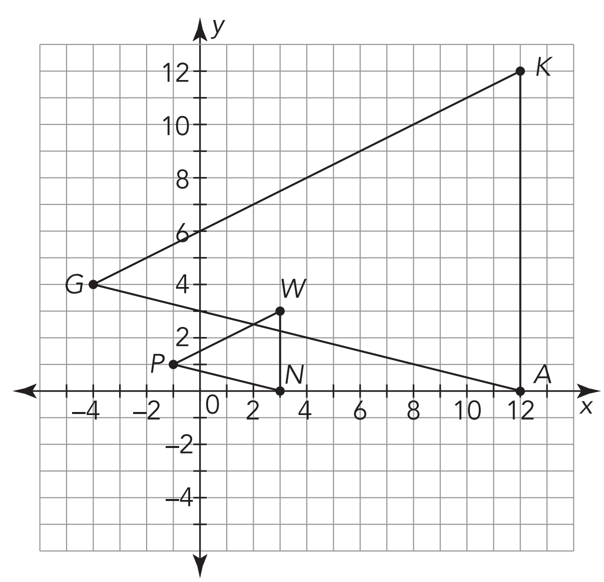
In this activity, you will use what you know about dilations to determine whether figures are similar.

1. Determine whether the figures are similar. When they are similar, state the scale factor, the center of dilation, and the rule that maps a point (x, y) on Figure 1 to Figure 2. When they are not, explain why the figures are not similar.

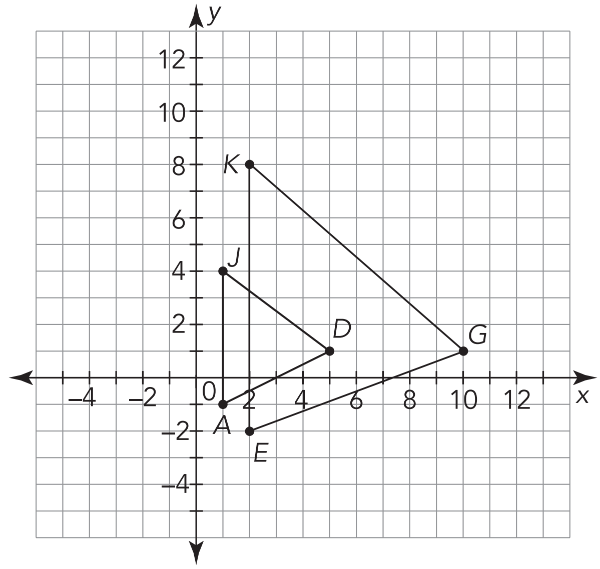
   a. Figure 1: $\triangle ABC$
      Figure 2: $\triangle DEF$



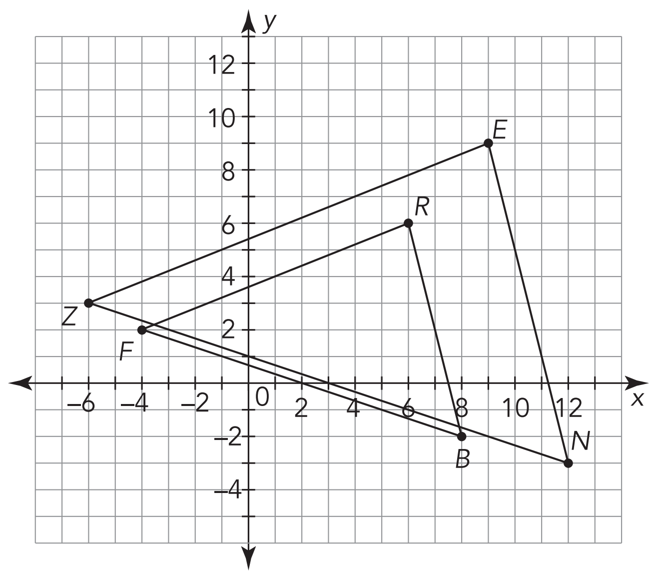
   b. Figure 1: $\triangle PWN$
      Figure 2: $\triangle GKA$



c. Figure 1: △*JDA*
Figure 2: △*KGE*



d. Figure 1: △*ZEN*
Figure 2: △*FRB*



2. Triangle *ABC* is dilated using the origin as the center of dilation to create △*A*′*B*′*C*′. Determine the scale factor and write the rule applied to △*ABC* to create △A′B′C′.
A(−2, −1)   *B*(−2, −2)   *C*(1, 1)
A′(−5, −2.5)   *B*′(−5, −5)   *C*′(2.5, 2.5)

3. Triangle DEF is dilated using the origin as the center of dilation to create $\triangle D'E'F'$. Determine the scale factor and write the rule applied to $\triangle DEF$ to create $\triangle D'E'F'$.
   $D(-6, 4)$, $E(-4, -2.5)$, $F(-6, -3)$
   $D'(-3, 2)$, $E'(-2, -1.25)$, $F'(-3, -1.5)$

4. Triangle JKL is dilated using the origin as the center of dilation to create $\triangle J'K'L'$. Determine the scale factor and write the rule applied to $\triangle JKL$ to create $\triangle J'K'L'$.
   $J(-7, 4)$, $K(7, 2)$, $L(1, -2)$
   $J'(-7, 4)$, $K'(7, 2)$, $L'(1, -2)$

5. Ana says $\triangle JKL$ and $\triangle J'K'L'$ are similar. Jacob says $\triangle JKL$ and $\triangle J'K'L'$ are congruent. Who is correct? Explain your reasoning.

Talk the Talk

Summing Up Similar Figures

Determine if each statement is *always*, *sometimes*, or *never true*. Provide a justification for each answer.

1. Triangle *ABC* is dilated four times with different scale factors. The four images are congruent.

2. Triangle *HIP* is dilated by a scale factor of 8, followed by a scale factor of $\frac{1}{8}$. The final image is congruent to $\triangle HIP$.

Ask Yourself . . .
What observations can you make?

3. Dilations are used to create congruent figures.

4. Dilations are used to create similar figures.

Lesson 3 Assignment

Write

Explain how to use transformations to determine if figures are congruent or similar.

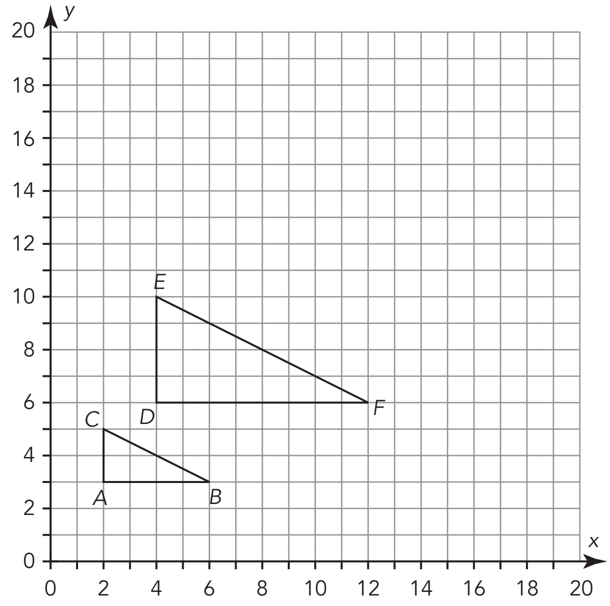
Remember

Images created from the same pre-image are always similar figures.

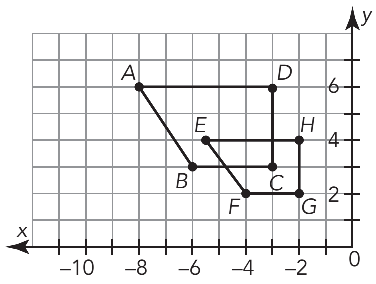
Practice

1. Verify that the two figures are similar by describing a dilation that maps one figure onto the other. Be sure to include the scale factor.

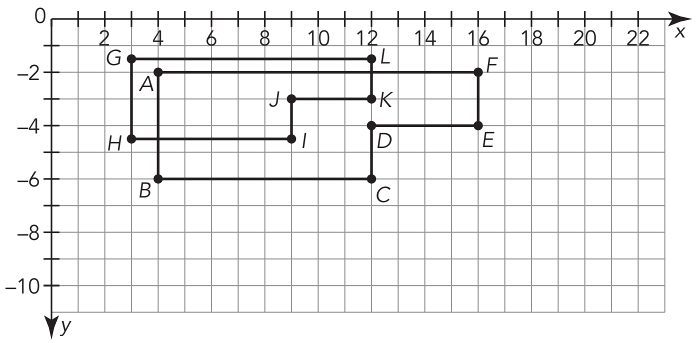
a.



b.



Lesson 3 Assignment

c. 

2. Each triangle is dilated using the origin as the center of dilation to create a new triangle. Determine the scale factor and write the rule applied to create the new triangle.

   a. $C(-3, 4)$ $A(5,1)$ $T(-2, -5)$
   $C'(-6, 8)$ $A'(10, 2)$ $T'(-4, -10)$

   b. $X(9,6)$ $Y(3,9)$ $Z(6,3)$
   $X'(3,2)$ $Y'(1,3)$ $Z'(2,1)$

Prepare

Solve each equation for *x*.

1. $x + 105 = 180$

2. $2x + 65 = 180$

3. $45 + 4x - 15 = 180$

4. $90 + 2x = 180$

TOPIC 2 SELF-REFLECTION

Name: ____________________

Similarity

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents the **skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Similarity* topic by:

TOPIC 2: *Similarity*	Beginning of Topic	Middle of Topic	End of Topic
defining and identifying dilations and similar figures.	○	○	○
identifying the scale factor of a dilation.	○	○	○
dilating geometric figures on the coordinate plane with the origin as the center of the dilation.	○	○	○
describing the changes to the *x*- and *y*-coordinates of a figure after a dilation, including the use of an algebraic representation.	○	○	○
identifying corresponding sides and corresponding angles of similar figures.	○	○	○
generalizing that ratios of corresponding sides of similar figures are proportional and their corresponding angles are congruent.	○	○	○
using prime notation to describe an image after a dilation.	○	○	○
explaining how transformation can be used to prove that two figures are similar.	○	○	○

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Similarity* topic.

    ____

    ____

    ____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

    ____

    ____

    ____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

    ____

    ____

    ____

TOPIC 2 SUMMARY

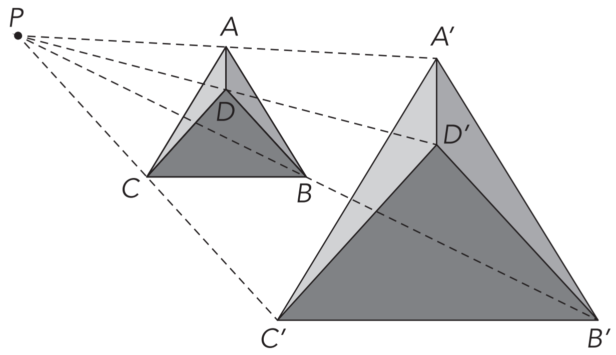
Similarity Summary

LESSON 1 Dilations of Figures

A **dilation** is a transformation that produces figures that are the same shape as the original figure, but not necessarily the same size. Each point on the original figure is moved along a straight line, and the straight line is drawn from a fixed point known as the **center of dilation**. The distance each point moves is determined by the scale factor used. The **scale factor** is the ratio of the *distance of the new figure from the center of dilation* to the *distance of the original figure from the center of dilation*.

When the scale factor is greater than 1, the new figure is called an **enlargement**.

This image of a logo was dilated to produce an enlargement using point *P* as the center of dilation.



The scale factor can be expressed as

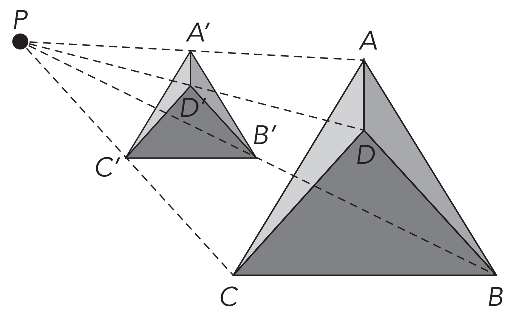
$$\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}.$$

NEW KEY TERMS

- dilation [dilatación]
- center of dilation [centro de dilatación]
- scale factor [factor de escala]
- enlargement
- reduction [reducción]
- similar [similar/semejante]

When the scale factor is between 0 and 1, the new figure is called a **reduction**.

For example, the original logo was dilated to produce a reduction using point *P* as the center of dilation.



The scale factor can be expressed as

$$\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}.$$

When the scale factor is equal to 1, the figure stays the same size.

When you dilate a figure, you create a similar figure. When two figures are **similar**, the ratios of their corresponding side lengths are equal. This means that you can create a similar figure by multiplying or dividing all of the side lengths of a figure by the same scale factor (except 0). You can multiply or divide by 1 to create a similar figure, too. In that case, the similar figures are congruent figures. Corresponding angles in similar figures are congruent.

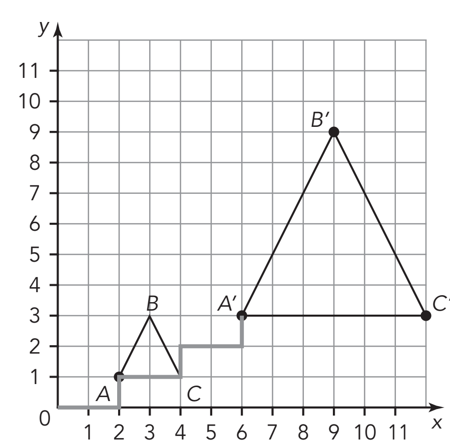
LESSON 2 Dilating Figures on the Coordinate Plane

When the dilation of a figure is centered at the origin, you can multiply the coordinates of the points of the original figure by the scale factor to determine the coordinates of the new figure. For scale factor k, the algebraic representation of the dilation is $(x, y) \rightarrow (kx, ky)$.

For example, to dilate $\triangle ABC$ by a scale factor of 3 using the origin as the center of dilation, repeatedly translate point *A* at (2, 1) by multiplying each of the point's coordinates by 3.

$$A'(2 \cdot 3, 1 \cdot 3) \rightarrow A'(6, 3)$$

Repeat for points *B* and *C*.

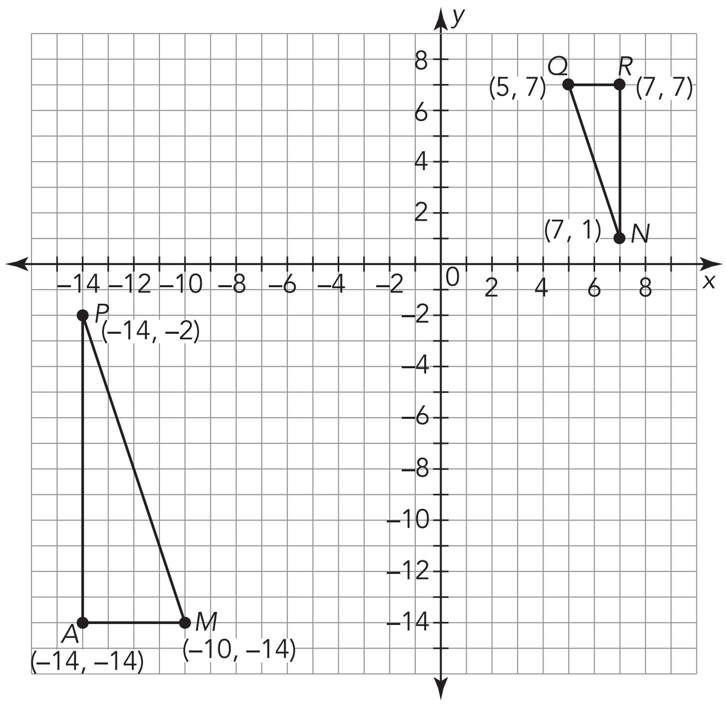


LESSON 3

Mapping Similar Figures Using Dilations

When two figures are similar, the same scale factor can be applied to all side lengths to map one figure onto the other. You can compare the ratios of corresponding side lengths of figures to determine similarity. When the ratio, or scale factor, is the same for all corresponding sides, then the figures are similar.

For example, $\triangle MAP$ is similar to $\triangle QRN$. The ratio of corresponding sides is equal to 2, or $\frac{1}{2}$.



GRADE 8

TOPIC 3

Crisscrossing roads and interstate highways are common sights in big cities. Interchanges like this cost hundreds of millions of dollars and require millions of worker-hours to complete.

Line and Angle Relationships

1 Exploring Angle Theorems

OBJECTIVES

- Establish the Triangle Sum theorem.
- Explore the relationship between the interior angle measures and the side lengths of a triangle.
- Identify the remote interior angles of a triangle.
- Identify the exterior angles of a triangle.
- Use informal arguments to establish facts about exterior angles of triangles.
- Explore the relationship between the exterior angle measures and two remote interior angles of a triangle.
- Prove the Exterior Angle theorem.

NEW KEY TERMS

- Triangle Sum theorem
- exterior angle of a polygon
- remote interior angles of a triangle
- Exterior Angle theorem

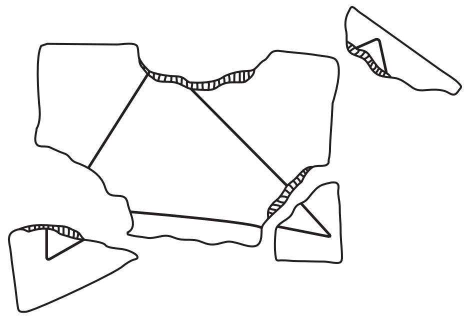
You already know a lot about triangles. In previous grades, you classified triangles by side lengths and angle measures.

What special relationships exist among the angles of a triangle?

Getting Started

Rip 'Em Up

Draw any triangle on a piece of patty paper. Tear off the triangle's three angles. Arrange the angles so that they are adjacent angles.



1. What do you notice about these angles? Write a conjecture about the sum of the three angles in a triangle.

2. Compare your angles and your conjecture with your classmates'. What do you notice?

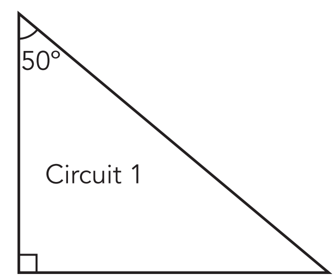
ACTIVITY
1.1 Analyzing Angles and Sides

In the previous activity, what you noticed about the relationship between the three angles in a triangle is called the *Triangle Sum theorem*. The **Triangle Sum theorem** states that the sum of the measures of the interior angles of a triangle is 180°.

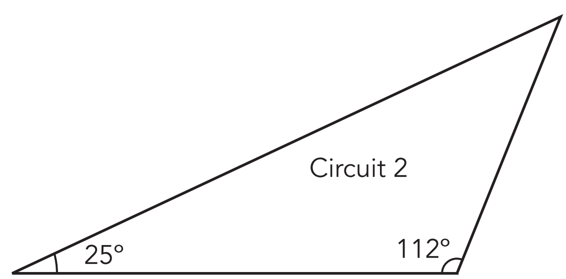
Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Michael is organizing a bike race called the *Tri-Cities Criterium*. Criteriums consist of several laps around a closed circuit. Based on the city map provided to him, Michael designs three different triangular circuits and presents scale drawings of them to the Tri-Cities Cycling Association for consideration.

1. Classify each circuit according to the type of triangle created.



2. Use the Triangle Sum theorem to determine the measure of the third angle in each triangular circuit. Label the triangles with the unknown angle measures.

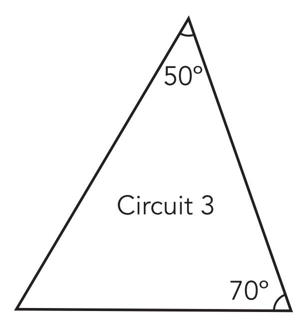


3. Measure the length of each side of each triangular circuit. Label the side lengths in the diagram.

The sharper the angles on a race course, the more difficult the course is for cyclists to navigate.

4. Perform the following tasks for each circuit.

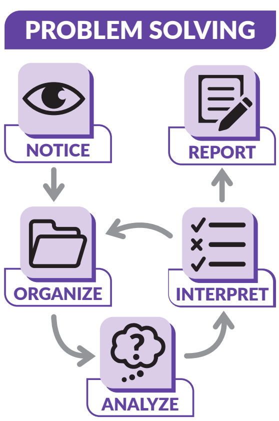
   a. List the angle measures from least to greatest.



   b. List the side lengths from shortest to longest.

c. Describe what you notice about the location of the angle with the least measure and the location of the shortest side.

d. Describe what you notice about the location of the angle with the greatest measure and the location of the longest side.



5. Olivia, the president of the Tri-Cities Cycling Association, presents a fourth circuit for consideration. The measures of two of the interior angles of the triangle are 57° and 61°. Determine the measure of the third angle and then describe the location of each side with respect to the measures of the opposite interior angles without drawing or measuring any part of the triangle.

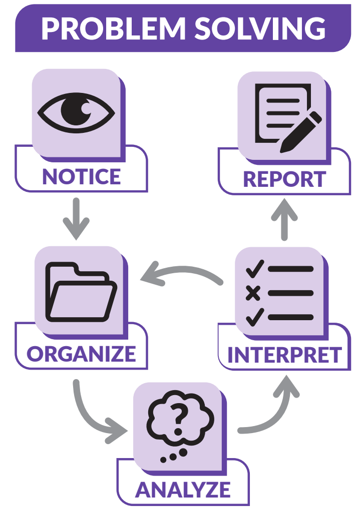
   a. Measure of the third angle

   b. Longest side of the triangle

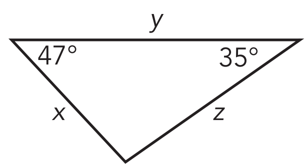
   c. Shortest side of the triangle

Do your answers change depending on the circuit? Which circuit would you select for the race?

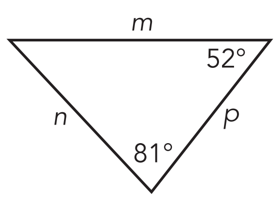
6. List the side lengths from shortest to longest for each diagram.



a.

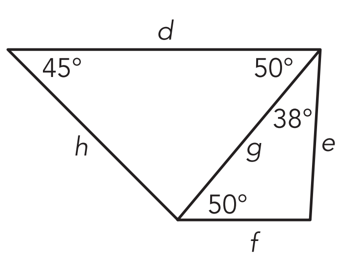


b.



If two angles of a triangle have equal measures, what does that mean about the relationship between the sides opposite the angles?

c.

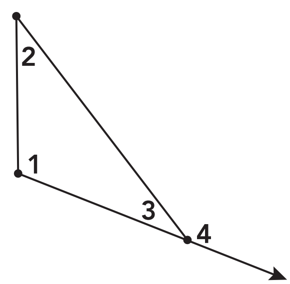


ACTIVITY 1.2

Exterior Angle Theorem

You now know about the relationships among the angles inside a triangle, the *interior angles of a triangle*, but are there special relationships between interior and *exterior angles of a triangle?*

An **exterior angle of a polygon** is an angle between a side of a polygon and the extension of its adjacent side. It is formed by extending a ray from one side of the polygon.



In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are the interior angles of the triangle and $\angle 4$ is an exterior angle of the triangle.

1. Make a conjecture about the measure of the exterior angle in relation to the measures of the other angles in the diagram.

2. Let's investigate the relationships among measures of the angles in the diagram.

   a. What does $m\angle 1 + m\angle 2 + m\angle 3$ equal? Explain your reasoning.

   b. What does $m\angle 3 + m\angle 4$ equal? Explain your reasoning.

   c. State a relationship between the measures of $\angle 1$, $\angle 2$, and $\angle 4$. Explain your reasoning.

3. In a triangle, for each exterior angle there are two "remote" interior angles.

   a. Why would $\angle 1$ and $\angle 2$ be referred to as "remote" interior angles with respect to the exterior angle, $\angle 4$?

   b. Extend another side of the triangle and label the exterior angle $\angle 5$. Then, name the two remote interior angles with respect to $\angle 5$.

The **remote interior angles of a triangle** are the two angles that are non-adjacent to the specified exterior angle.

4. Rewrite $m\angle 4 = m\angle 1 + m\angle 2$ using the terms *sum*, *remote interior angles of a triangle*, and *exterior angle of a triangle*.

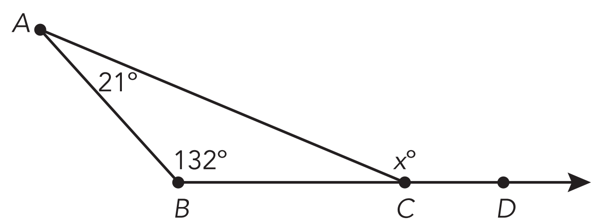
5. The original diagram was drawn as an obtuse triangle with one exterior angle. If the triangle had been drawn as an acute or right triangle, would this have changed the relationship between the measure of the exterior angle and the sum of the measures of the two remote interior angles? Explain your reasoning.

Was your conjecture from Question 1 correct? If so, you have proven an important theorem in the study of geometry!

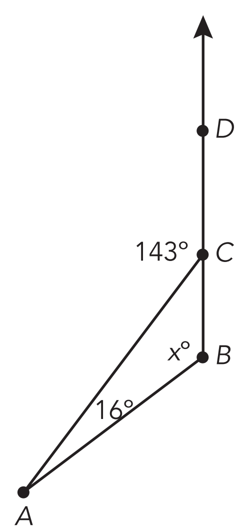
The **Exterior Angle theorem** states that the measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

6. Use the Exterior Angle theorem to determine each unknown angle measure.

a.



b.



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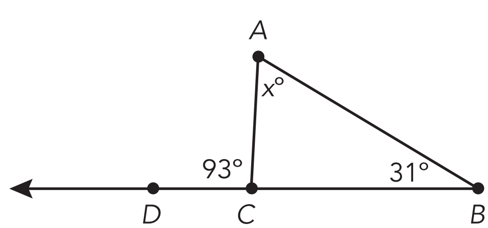
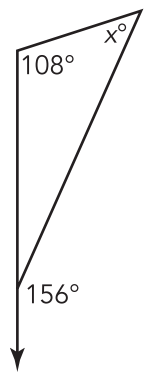
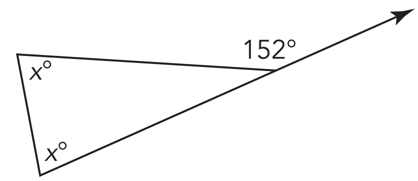


7. Write and solve an equation to determine the value of x in each diagram.

a.



b.



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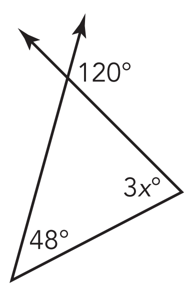

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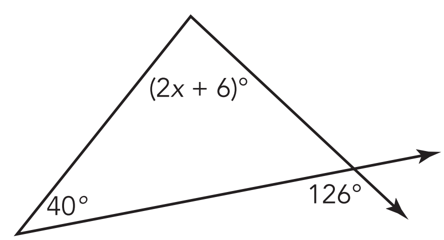
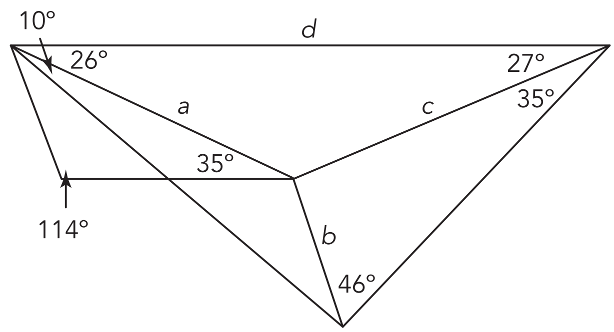


Talk the Talk

So Many Angles!

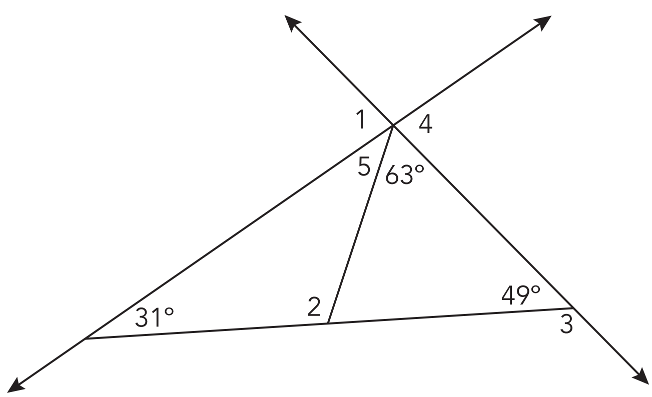
1. Consider the diagram shown.



a. Determine the measures of the eight unknown angle measures inside the figure.

b. List the labeled side lengths in order from least to greatest.

2. Determine the unknown angle measures in the figure.



Lesson 1 Assignment

Write

Write the term that best completes each statement.

1. The ___________ states that the sum of the measures of the interior angles of a triangle is 180°.
2. The ___________ states that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.
3. The ___________ are the two angles that are non-adjacent to the specified exterior angle.
4. A(n) ___________ is formed by extending a side of a polygon.

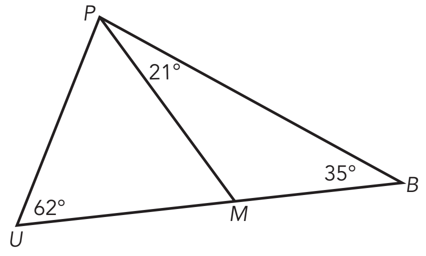
Remember

The sum of the measures of the interior angles of a triangle is 180°.

The measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

Practice

1. Use the figure shown to answer each question.



   a. Explain how you can use the Exterior Angle theorem to calculate the measure of $\angle PMU$.

   b. Calculate the measure of $\angle PMU$.

Lesson 1 Assignment

c. Explain how you can use the Triangle Sum theorem to calculate the measure of $\angle UPM$.

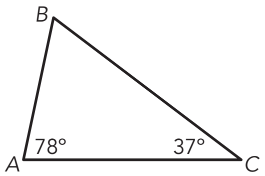
d. Calculate the measure of $\angle UPM$.

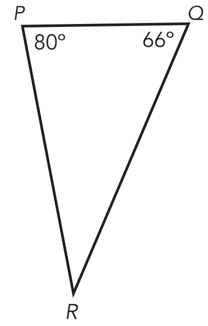
e. List the sides of $\triangle PMB$ in order from shortest to longest. Explain how you determined your answer.

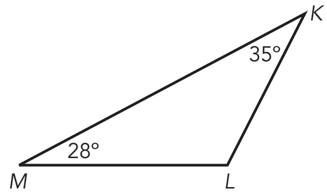
f. List the sides of $\triangle PUB$ in order from shortest to longest. Explain how you determined your answer.

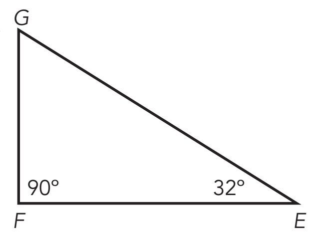
Lesson 1 Assignment

2. Determine the measure of the unknown angle in each triangle.

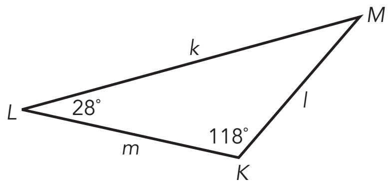
a. 

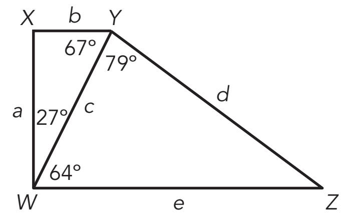
b. 

c. 

d. 

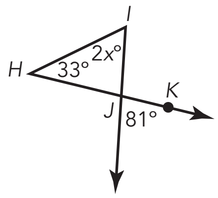
3. List the side lengths from shortest to longest for each diagram.

a. 

b. 

4. Determine the value of x in each diagram.

a.



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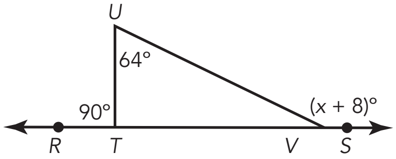

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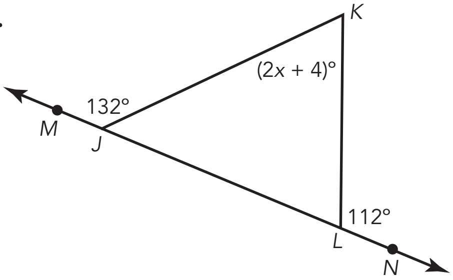

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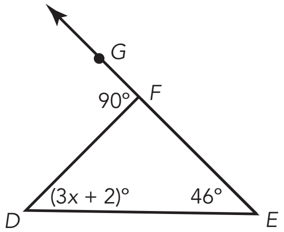
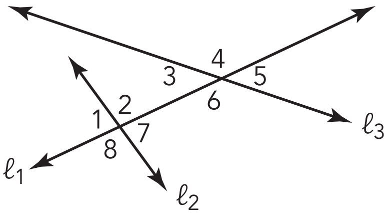


Prepare

Use the numbered angles in the diagram to answer each question.



1. Which angles form vertical angles?

2. Which angles are congruent?

2 Exploring the Angles Formed by Lines Intersected by a Transversal

OBJECTIVES

- Explore the angles determined by two lines that are intersected by a transversal.
- Use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal.
- Identify corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles.
- Determine the measure of alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles.

NEW KEY TERMS

- transversal
- alternate interior angles
- alternate exterior angles
- same-side interior angles
- same-side exterior angles

The intersection of two lines forms special angle pair relationships.

What special angle pair relationships exist when two lines are intersected by a third line?

Getting Started

A *postulate* is a statement that is accepted to be true without proof.

Euclid's Fifth Postulate

Euclid is known as the father of geometry, and he stated five postulates upon which every other geometric relationship can be based. The fifth postulate is known as the *Parallel postulate*. Consider one of the equivalent forms of this postulate:

"Given any straight line and a point not on the line, there exists one and only one straight line that passes through the point and never intersects the line."

1. Draw a picture that shows your interpretation of this statement of the postulate.

2. Why do you think this postulate is called the Parallel postulate?

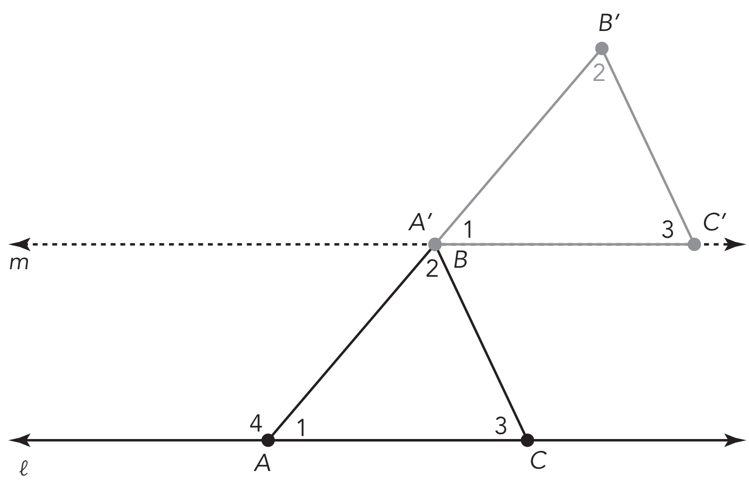
A common definition of parallel lines is co-planar lines that are always equidistant, or the same distance apart.

3. Explain what is meant by this definition and demonstrate it on your diagram.

ACTIVITY 2.1 Creating New Angles from Triangles

In the previous lesson, you determined measures of interior and exterior angles of triangles.

Consider the diagram shown. Lines *m* and ℓ are parallel. This is notated as $m \parallel \ell$.



1. Explain the relationships between the numbered angles in the diagram.

2. Trace the diagram onto two sheets of patty paper and extend $\overline{AB}$ to create a line that contains the side of the triangle. Align the triangles on your patty paper and translate the bottom triangle along $\overline{AB}$ until $\overline{AC}$ lies on line *m*. Trace your translated triangle on the top sheet of patty paper. Label the translated triangle $A'B'C'$.

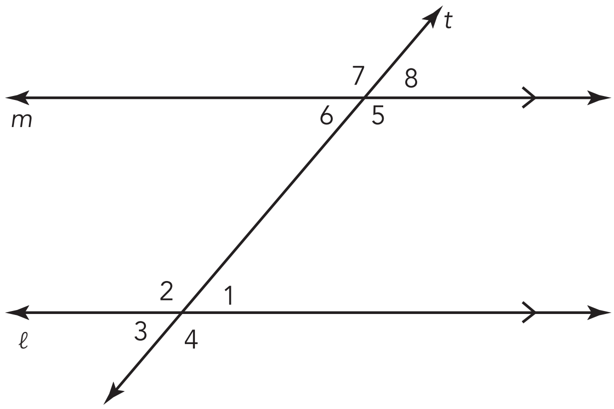
3. Angle 1 in $\triangle A'B'C'$ is a translation of angle 1 in $\triangle ABC$. How are the measures of these angles related to each other? Explain your reasoning.

4. Extend $\overline{CB}$ to create a line. Use what you know about special angle pairs to label all six angles at point B as congruent to $\angle 1$, $\angle 2$, or $\angle 3$. Explain your reasoning. Sketch your patty paper drawing.

ACTIVITY 2.2

Angles Formed by Three Lines

Consider your diagram from the previous activity. If you remove $\overline{BC}$ and the line containing $\overline{BC}$, your diagram might look similar to the diagram shown.



Arrowheads on lines in diagrams indicate parallel lines. Lines or segments with the same number of arrowheads are parallel.

In this diagram, the two parallel lines, *m* and ℓ, are intersected by a *transversal*, *t*. A **transversal** is a line that intersects two or more lines.

The transversal, *t*, in this diagram corresponds to the line that contained side *AB* in your patty paper diagram.

Recall that corresponding angles are angles that have the same relative positions in geometric figures. In the previous activity, when you translated $\triangle ABC$ to create $\triangle A'B'C'$ you created three sets of corresponding angles. You can also refer to corresponding angles in relation to lines intersected by a transversal.

1. Use the diagram to name all pairs of corresponding angles.

2. Analyze each angle pair: $\angle 1$ with $\angle 6$ and $\angle 2$ with $\angle 5$.

   a. Are the angles between (on the *interior* of) lines *m* and ℓ, or are they outside (on the *exterior* of) lines *m* and ℓ?

   b. Are the angles on the same side of the transversal, or are they on opposite (*alternating*) sides of the transversal?

There is a special relationship between angles like $\angle 1$ and $\angle 6$ or $\angle 2$ and $\angle 5$. **Alternate interior angles** are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are between the two other lines.

Alternate exterior angles are also formed when a transversal intersects two lines. These angle pairs are on opposite sides of the transversal and are outside the other two lines.

3. Use your diagram to name all pairs of alternate exterior angles.

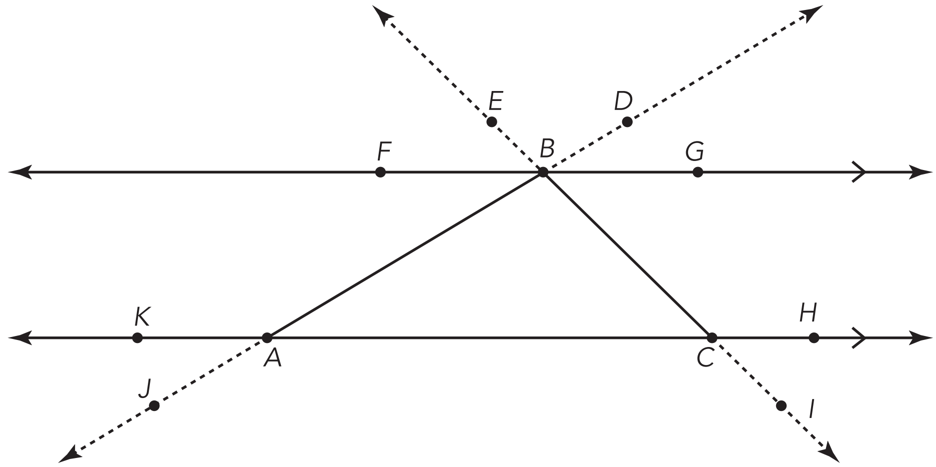
Two additional angle pairs are *same-side interior angles* and *same-side exterior angles.*

4. Use the names to write a definition for each type of angle pair. Identify all pairs of each type of angle pair from the diagram.

   a. Same-side interior angles

   b. Same-side exterior angles

5. In the diagram from the previous activity, each time you extended a side of the triangle, you created a transversal. Identify the angle pairs described by each statement.



a. Corresponding angles if $\overleftrightarrow{BC}$ is the transversal

b. Alternate interior angles if $\overleftrightarrow{BC}$ is the transversal

c. Alternate exterior angles if $\overleftrightarrow{AB}$ is the transversal

Same-side interior angles are on the same side of the transversal and are between the other two lines.

d. Same-side interior angles if $\overleftrightarrow{AB}$ is the transversal

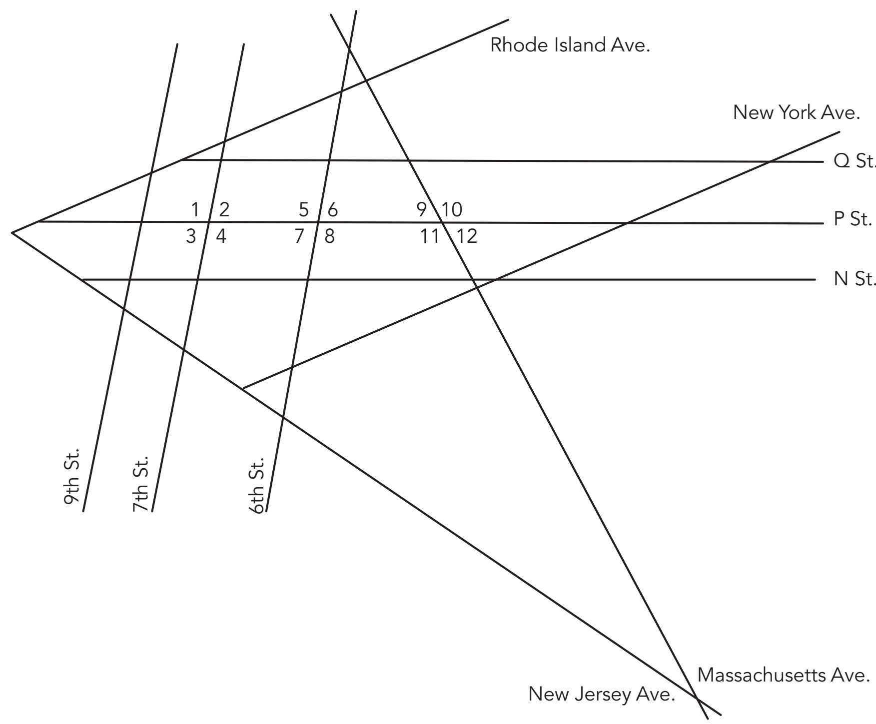
Same-side exterior angles are on the same side of the transversal and are outside the other two lines.

e. Same-side exterior angles if $\overleftrightarrow{AB}$ is the transversal

ACTIVITY 2.3 Analyzing Special Angle Pairs

> **Ask Yourself . . .**
> How can you use special angle pairs in everyday life?

Consider a map of Washington, D.C., shown. Assume that all line segments that appear to be parallel are parallel.



1. Consider only P St., N St., Massachusetts Ave., and 6th St. Which of these streets, if any, are transversals? Explain your reasoning.

Let's explore the relationships between the angles formed from lines cut by transversals.

2. Use a protractor to measure all 12 angles labeled on the diagram.

3. Consider only 6th St., 7th St., and P St.

    a. Which of these streets, if any, are transversals?
    Explain your reasoning.

    b. What is the relationship between 6th St. and 7th St.?

    c. Name the pairs of alternate interior angles. What do you notice about their angle measures?

    d. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

    e. Name the pairs of corresponding angles. What do you notice about their angle measures?

    f. Name the pairs of same-side interior angles. What do you notice about their angle measures?

    g. Name the pairs of same-side exterior angles. What do you notice about their angle measures?

4. Consider only 6th St., Massachusetts Ave., and P St.

   a. Which of these streets, if any, are transversals?

   b. What is the relationship between 6th St. and Massachusetts Ave.?

   c. Name the pairs of alternate interior angles. What do you notice about their angle measures?

   d. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

   e. Name the pairs of corresponding angles. What do you notice about their angle measures?

   f. Name the pairs of same-side interior angles. What do you notice about their angle measures?

   g. Name the pairs of same-side exterior angles. What do you notice about their angle measures?

ACTIVITY 2.4

Line Relationships and Angle Pairs

In the previous activity, you explored angle pairs formed by a transversal intersecting two non-parallel lines and a transversal intersecting two parallel lines.

1. Make a conjecture about the types of lines cut by a transversal and the measures of the special angle pairs.

Refer back to the measurements of the labeled angles on the diagram of Washington, D.C.

2. What do you notice about the measures of each pair of alternate interior angles when the lines are

   a. non-parallel?

   b. parallel?

3. What do you notice about the measures of each pair of alternate exterior angles when the lines are

   a. non-parallel?

   b. parallel?

4. What do you notice about the measures of each pair of corresponding angles when the lines are

   a. non-parallel?

   b. parallel?

5. What do you notice about the measures of the same-side interior angles when the lines are

   a. non-parallel?

   b. parallel?

6. What do you notice about the measures of the same-side exterior angles when the lines are

   a. non-parallel?

   b. parallel?

7. Summarize your conclusions in the table by writing the relationships of the measures of the angles. The relationships are either congruent or not congruent, supplementary or not supplementary.

Angles	Two Parallel Lines Intersected by a Transversal	Two Non-Parallel Lines Intersected by a Transversal
Alternate Interior Angles		
Alternate Exterior Angles		
Corresponding Angles		
Same-Side Interior Angles		
Same-Side Exterior Angles		

8. Use transformations to explain how to map the angle pairs that are congruent.

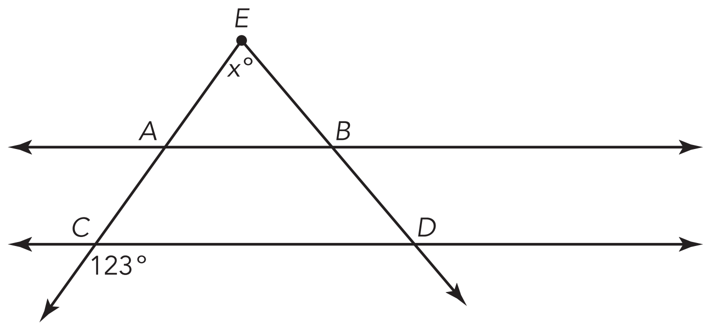
9. Use transformations to explain why certain angle pairs are supplementary.

ACTIVITY 2.5 Solving for Unknown Angle Measures

Use what you know about angle pairs to answer each question.

1. Daniela and Javier were working together to solve the problem shown.

   Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ Solve for x. Show all your work.

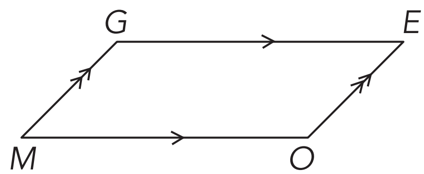


   a. Daniela concluded that $x = 66°$. How did Daniela get her answer?

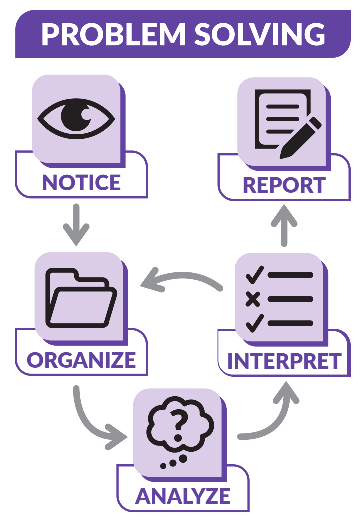
   b. Javier does not agree with Daniela's answer. He thinks there is not enough information to solve the problem. How could Javier alter the figure to show why he disagrees with Daniela's answer?

   c. Who is correct?

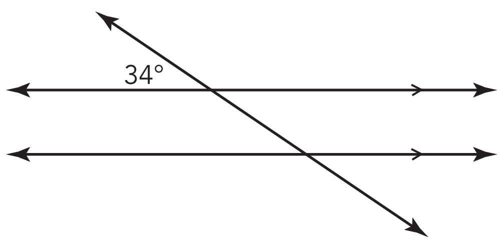
2. Opposite sides of the figure shown are parallel. Suppose that the measure of Angle *M* is equal to 30°. Solve for the measures of angles *G*, *E*, and *O*. Explain your reasoning.



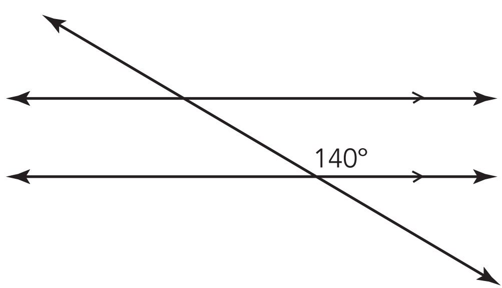
3. Determine the measure of each unknown angle.



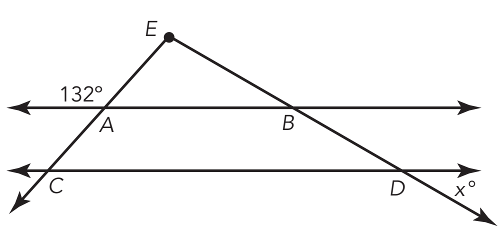
a.

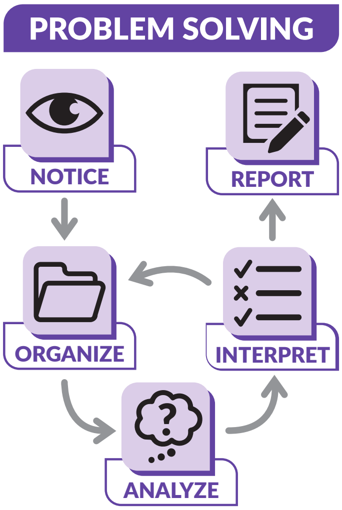


b.

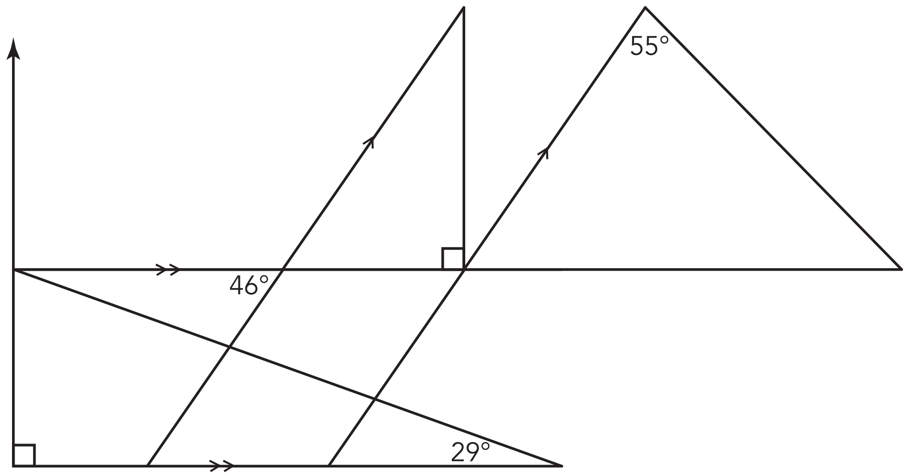


4. In this figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{EC} \perp \overleftrightarrow{ED}$. Solve for *x*. Show all your work.





5. Determine the measure of each angle in this figure.



Talk the Talk

What's So Special?

In this lesson, you explored the special angle relationships formed when a transversal intersects a pair of lines.

1. If two lines are intersected by a transversal, when are

   a. alternate interior angles congruent?

   b. alternate exterior angles congruent?

   c. vertical angles congruent?

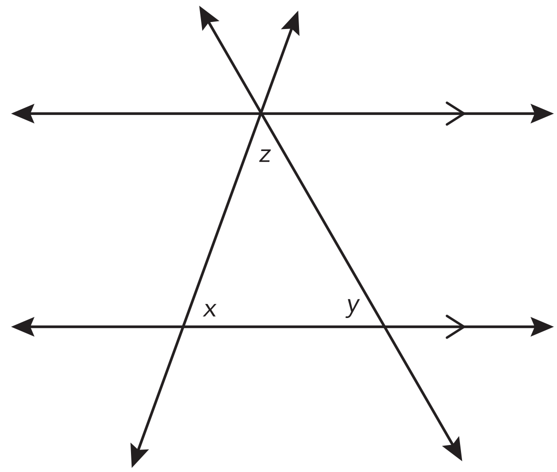
   d. corresponding angles congruent?

   e. same-side interior angles supplementary?

   f. same-side exterior angles supplementary?

   g. linear pairs of angles supplementary?

2. Hannah says that she can use what she learned about parallel lines cut by a transversal to show that the measures of the angles of a triangle sum to 180°. She drew the figure shown.



Explain what Hannah discovered.

Lesson 2 Assignment

Write

Write the term that best completes each sentence.

1. ____________________________ are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on opposite sides of the transversal and are outside the other two lines.

2. A ____________________________ is a line that intersects two or more lines.

3. ____________________________ are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on the same side of the transversal and are outside the other two lines.

4. ____________________________ are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on opposite sides of the transversal and are between the other two lines.

5. ____________________________ are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on the same side of the transversal and are between the other two lines.

Remember

When two parallel lines are intersected by a transversal:

- corresponding angles are congruent.
- alternate interior angles are congruent.
- alternate exterior angles are congruent.
- same-side interior angles are supplementary.
- same-side exterior angles are supplementary.

Lesson 2 Assignment

Practice

The figure shows part of a map of Chicago, Illinois.

1. Use the numbered angles to identify a pair that illustrates each relationship.

   a. Name a pair of alternate interior angles.

   b. Name a pair of alternate exterior angles.

   c. Name a pair of corresponding angles.

   d. Name a pair of same-side interior angles.

   e. Name a pair of same-side exterior angles.

Lesson 2 Assignment

2. Look at the intersection of W. Waveland Ave. and N. Sheffield Ave. Notice the northwest corner is labeled $\angle 1$. Label the other angles of this intersection in clockwise order angles 2, 3, and 4. Next, label the angles created by the intersection of W. Addison St. and N. Sheffield Ave. Angles 14, 15, 16, and 17 clockwise, starting at the northwest corner.

   a. Determine the type of angle pair for $\angle 1$ and $\angle 14$.

   b. Determine the type of angle pair for $\angle 3$ and $\angle 15$.

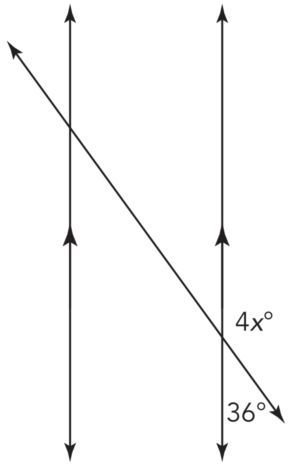
   c. Determine the type of angle pair for $\angle 1$ and $\angle 16$.

   d. Determine the type of angle pair for $\angle 1$ and $\angle 17$.

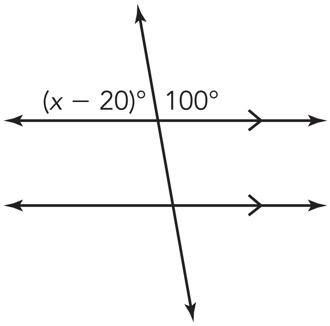
   e. Determine the type of angle pair for $\angle 3$ and $\angle 14$.

3. Determine the measure of all the angles in each diagram.

   a.

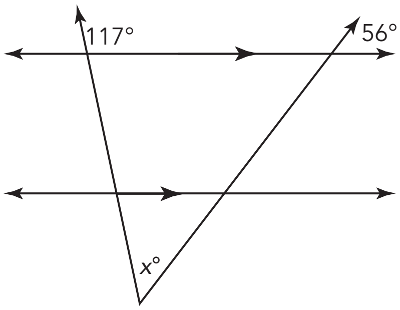


   b.



Lesson 2 Assignment

4. Solve for x. Show all your work.



Prepare

Suppose $\triangle BHX$ is similar to $\triangle KRC$.

1. List the corresponding angles.

2. Write the ratios to identify the proportional side lengths.

3 Exploring the Angle-Angle Similarity Theorem

OBJECTIVES

- Develop the minimum criteria to show that two triangles are similar.
- Use informal arguments to establish facts about the angle-angle criterion for similarity of triangles.
- Use the Angle-Angle Similarity theorem to identify similar triangles.

NEW KEY TERM

- Angle-Angle (AA) Similarity theorem

You have determined that when two triangles are similar, the corresponding angles are congruent and the corresponding sides are proportional.

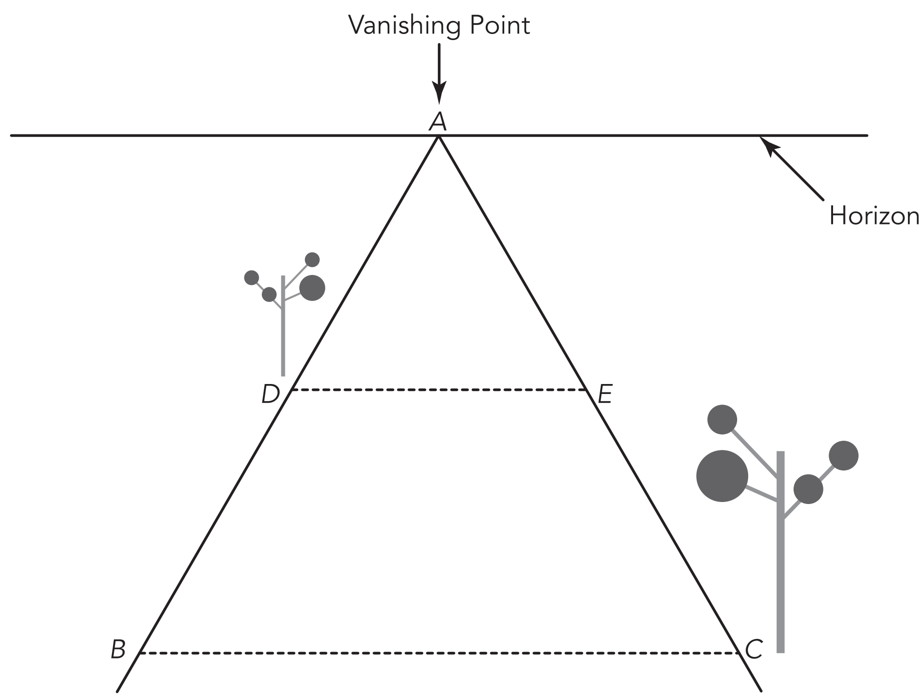
How can you show that two triangles are similar without measuring all the angles and side lengths?

Getting Started

Vanishing Point

Ask Yourself . . .
Did you justify your mathematical reasoning?

Graphic artists use knowledge about similarity to create realistic perspective drawings. Choose where the horizon should be and a vanishing point—a point where all the parallel lines in the drawing should appear to meet—and you too can create a perspective drawing.



The symbol ~ means "is similar to."

1. Suppose the vanishing point is point A and that $\overline{DE} \parallel \overline{BC}$. How could you demonstrate that $\triangle ABC \sim \triangle ADE$?

2. Draw a horizontal line in the path to create another similar triangle. Then, sketch a tree at that line using the appropriate scale factor.

ACTIVITY 3.1

Exploring Angle-Angle Similarity Theorem

You have determined that when two triangles are similar, the corresponding angles are congruent and the corresponding sides are proportional. To show that two triangles are similar, do you need to show that all of the corresponding sides are proportional and all of the corresponding angles are congruent?

Let's explore an efficient method to determine if two triangles are similar.

1. If the measures of two angles of a triangle are known, is that enough information to draw a similar triangle? Let's explore this possibility.

   a. Use a straightedge to draw $\triangle ABC$ in the space provided.

   b. Use a protractor to measure $\angle A$ and $\angle B$ of $\triangle ABC$ and record the measurements.

   $m\angle A =$ ______________ $m\angle B =$ ______________

   c. Use the Triangle Sum theorem to determine $m\angle C$.

d. Draw a second triangle, $\triangle DEF$, in the space provided using the angle measurements from part (b).

e. Based on your knowledge, what other information is needed to determine if the two triangles are similar, and how can you acquire that information?

f. Determine the measurements to get the additional information needed and decide if the two triangles are similar.

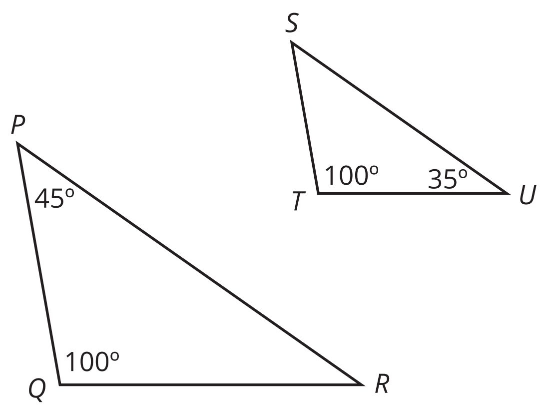
You have just shown that given the measures of two pairs of congruent corresponding angles of two triangles, it is possible to determine that two triangles are similar. In the study of geometry, this is expressed as a theorem.

The **Angle-Angle (AA) Similarity theorem** states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

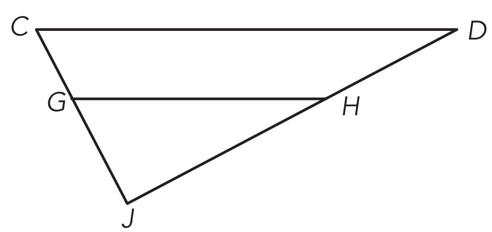
ACTIVITY 3.2 Using the Angle-Angle Similarity Theorem

Identify the triangles that are similar by the AA Similarity theorem. Explain how you know that the triangles are similar.

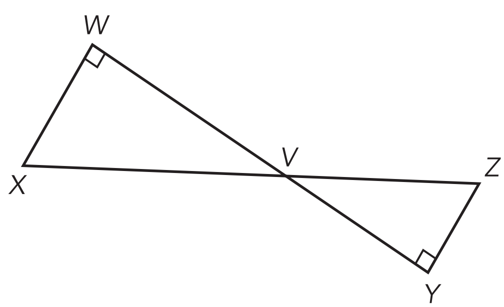
1.



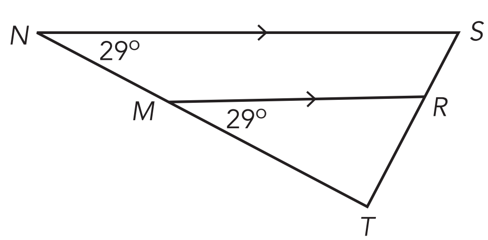
2. $\overline{CD} \parallel \overline{GH}$



3.



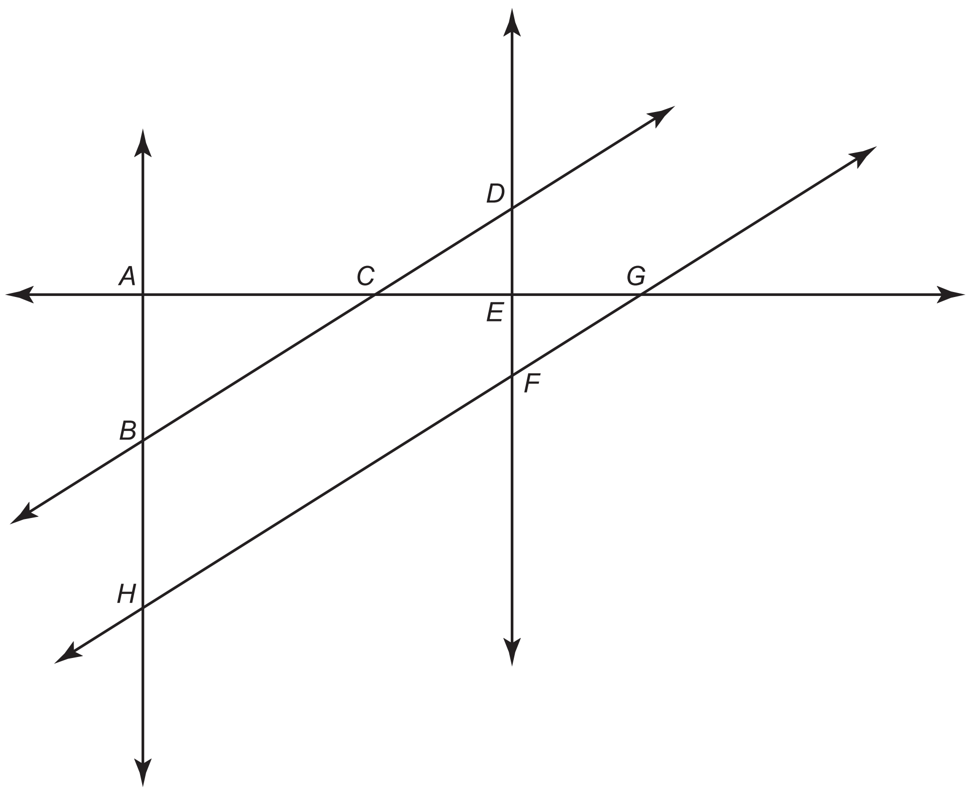
4. $\overline{NS} \parallel \overline{MR}$



ACTIVITY 3.3 Reasoning with the Angle-Angle Similarity Theorem

Use what you have learned about triangle similarity to answer each question.

Given: $\overleftrightarrow{BD} \parallel \overleftrightarrow{HG}$, $\overleftrightarrow{AH} \parallel \overleftrightarrow{DF}$, $\overleftrightarrow{AH} \perp \overleftrightarrow{AG}$, $\overleftrightarrow{DF} \perp \overleftrightarrow{AG}$



1. Is $\triangle ABC \sim \triangle AHG$? Explain your reasoning.

2. Is $\triangle ABC \sim \triangle EDC$? Explain your reasoning.

3. Is $\triangle EDC \sim \triangle EFG$? Explain your reasoning.

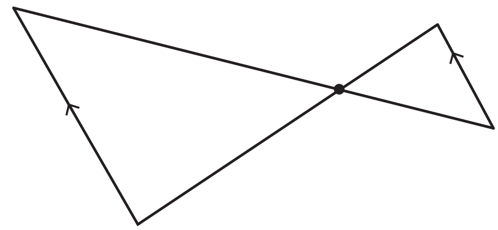
4. Is $\triangle ABC \sim \triangle EFG$? Explain your reasoning.

5. Is $\triangle AHG \sim \triangle EFG$? Explain your reasoning.

Talk the Talk

Bow-Tie Triangles

You can draw special triangles known as *bow-tie triangles*. First, draw a pair of parallel line segments. Then, connect the pairs of endpoints with line segments so that the line segments intersect, like this:



1. Are bow-tie triangles always similar? Show your work and explain your reasoning. Then, compare your work with your classmates' work.

Lesson 3 Assignment

Write

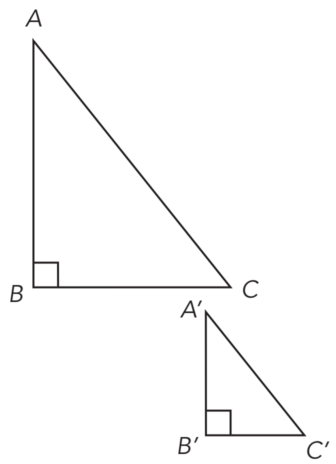
In your own words, explain the Angle-Angle (AA) Similarity theorem.

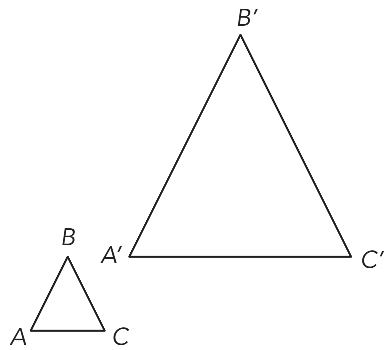
Remember

You can use dilations and other transformations, line and angle relationships, measurements, and/or the Angle-Angle Similarity theorem to demonstrate that two triangles are similar.

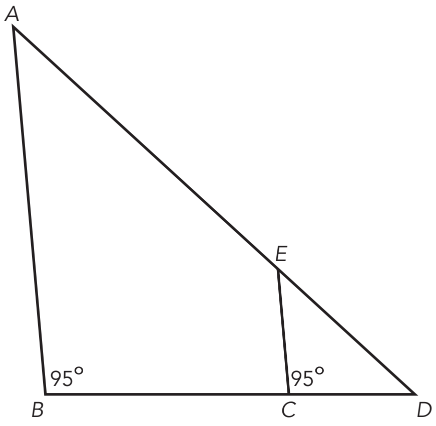
Practice

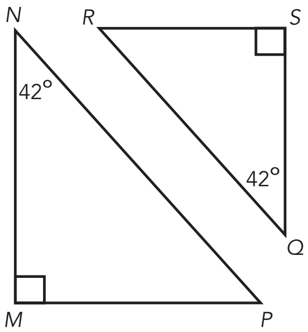
Use the AA Similarity theorem and a protractor or patty paper, if necessary, to demonstrate how the triangles in each pair are similar. Show your work.

1. 

2. 

Lesson 3 Assignment

3. 

4. 

Prepare

Determine each equivalent ratio.

1. $\frac{7}{16} = \frac{x}{48}$

2. $\frac{t}{90} = \frac{5}{9}$

3. $\frac{10}{p} = 1$

4. $250 = \frac{1000}{q}$

Name: ______________________________

Line and Angle Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Line and Angle Relationships* topic by:

TOPIC 3: *Line and Angle Relationships*	Beginning of Topic	Middle of Topic	End of Topic
informally proving that the sum of the interior angles of a triangle is 180°.	◯	◯	◯
informally proving that the measure of an exterior angle of a triangle is equal to the sum of its two remote interior angles.	◯	◯	◯
defining and identifying transversals.	◯	◯	◯
identifying the special angle pairs formed when parallel lines are intersected by a transversal.	◯	◯	◯
informally proving the relationships and measurements of the angles created when two parallel lines are intersected by a transversal.	◯	◯	◯
using transversals to explain why two sets of congruent corresponding angles are sufficient for justifying the fact that two triangles are similar.	◯	◯	◯
using the Angle-Angle Similarity theorem to prove similarity among triangles.	◯	◯	◯

continued on the next page

TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Line and Angle Relationships* topic.

___

___

___

2. What mathematical understandings from the topic do you feel you are making the most progress with?

___

___

___

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

___

___

___

TOPIC 3 SUMMARY

Line and Angle Relationships Summary

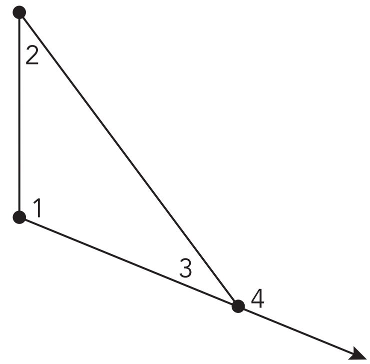
NEW KEY TERMS

- Triangle Sum theorem [teorema de la suma del triángulo]
- exterior angle of a polygon [ángulo exterior de un polígono]
- remote interior angles of a triangle [ángulos interiores remotos (no adyacentes) de un triángulo]
- Exterior Angle theorem [teorema delÁngulo Exterior]
- transversal [transversal]
- alternate interior angles[ángulos interioresalternos]
- alternate exterior angles[ángulos exteriores alternos]
- same-side interior angles
- same-side exterior angles
- Angle-Angle (AA) Similarity theorem [teorema de Similitud (Semejanza) Ángulo-Ángulo (AA)]

LESSON 1 Exploring Angle Theorems

The **Triangle Sum theorem** states that the sum of the measures of the interior angles of a triangle is 180°. The longest side of a triangle is opposite the interior angle with the greatest measure, and the shortest side is opposite the interior angle with the least measure.

An **exterior angle of a polygon** is an angle between a side of a polygon and the extension of its adjacent side. It is formed by extending a ray from one side of the polygon. For example, in the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are the interior angles of the triangle and $\angle 4$ is an exterior angle of the triangle. $\angle 1$ and $\angle 2$ are remote interior angles. The **remote interior angles of a triangle** are the two angles that are non adjacent to the specified exterior angle.

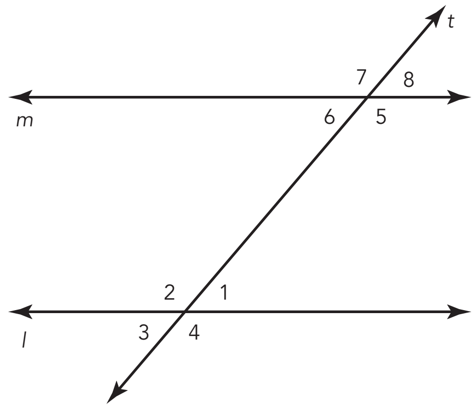


The **Exterior Angle theorem** states that the measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle. In the diagram shown, $m\angle 1 + m\angle 2 = m\angle 4$.

LESSON 2

Exploring the Angles Formed by Lines Intersected by a Transversal

A **transversal** is a line that intersects two or more lines. In this diagram, two parallel lines, *m* and *l*, are intersected by a transversal, *t*.



Corresponding angles have the same relative positions in geometric figures. An example of corresponding angles are ∠2 and ∠7.

Alternate interior angles are on opposite sides of the transversal and are between the two other lines. An example of alternate interior angles are ∠1 and ∠6.

Alternate exterior angles are on opposite sides of the transversal and are outside the other two lines. An example of alternate exterior angles are ∠4 and ∠7.

Same-side interior angles are on the same side of the transversal and are between the other two lines. An example of same-side interior angles are ∠2 and ∠6.

Same-side exterior angles are on the same side of the transversal and are outside the other two lines. An example of same-side exterior angles are ∠4 and ∠8.

When two parallel lines are intersected by a transversal,

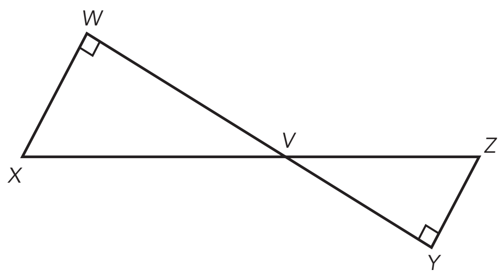
- Corresponding angles are congruent;
- Alternate interior angles are congruent;
- Alternate exterior angles are congruent;
- Same-side interior angles are supplementary;
- Same-side exterior angles are supplementary.

LESSON 3

Exploring the Angle-Angle Similarity Theorem

The **Angle-Angle (AA) Similarity theorem** states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

For example, in the figure shown, $\triangle XWV$ is similar to $\triangle ZYV$ by the AA Similarity theorem. Because $\angle XWV$ and $\angle ZYV$ are right angles, they are congruent to each other. Because $\angle WVX$ and $\angle YVZ$ are vertical angles, they are congruent to each other. Thus, $\triangle XWV$ is similar to $\triangle ZYV$.



You can use dilations and other transformations, line and angle relationships, measurements, and/or the Angle-Angle Similarity theorem to demonstrate that two triangles are similar.

GRADE 8

MODULE
2

Developing Function Foundations

GRADE 8

TOPIC 1

Where might you see the sign shown? What can you say about the triangle on the sign? What do you think 8% represents?

From Proportions to Linear Relationships

1 Representations of Proportional Relationships

OBJECTIVES

- Represent proportional relationships with tables, lines, and linear equations.
- Compare graphs of proportional relationships.
- Compare two different proportional relationships represented in multiple ways.

NEW KEY TERMS

- proportional relationship
- non-proportional relationship
- constant of proportionality

You have studied proportional relationships in previous courses.

How can you represent and compare proportional relationships using graphs, tables, and equations?

Getting Started

Ratio of In-State to Out-of-State Students

When planning for your post-secondary education, it is important to think about whether you will attend a college in your home state or another state. Students who attend school in their home state are known as in-state students, while students from a different state are known as out-of-state students. In-state students typically pay less in tuition than out-of-state students. One study focused on the enrollment of in-state students at a university. The study found that three out of every five students enrolled were in-state students.

Use the findings of the study to write each ratio.

Remember . . .
there are different ways to write a ratio. You can write a ratio as a fraction, with a colon, or using the word *to*.

1. The number of enrolled in-state students to the total number of students

2. The number of enrolled out-of-state students to the total number of students

3. The number of enrolled in-state students to the number of enrolled out-of-state students

4. The number of enrolled out-of-state students to the number of enrolled in-state students

ACTIVITY 1.1

Representing Proportional Relationships

Use the findings of the enrollment study to make predictions.

1. Determine the number of enrolled in-state students for each given total number of enrolled students. Explain your reasoning.

    a. 15 total students

    b. 250 total students

    c. 4000 total students

2. Compare the total number of enrolled students to the number of enrolled out-of-state students.

    a. Complete the table.

Total Students Enrolled in a University	Out-of-State Students Enrolled in a University
0	
250	
6000	
	6000

    b. Explain how you calculated each value.

3. Determine the number of in-state students if 800 enrolled students are out-of-state. Show all work and explain your reasoning.

4. Choose the correct equation to match each description. Then, compare the equations.

$y = \frac{2}{5}x$	$y = 2x + 3$	$y = \frac{2}{3}x$
$y = \frac{3}{2}x$	$y = \frac{3}{5}x$	
$y = \frac{5}{2}x$	$y = 2x + 5$	$y = 3x + 2$

a. The number of in-state students enrolled, *y*, for *x* total number of students enrolled

b. The number of out-of-state students enrolled, *y*, for *x* total number of students enrolled

c. The number of in-state students enrolled, *y*, for *x* out-of-state students enrolled

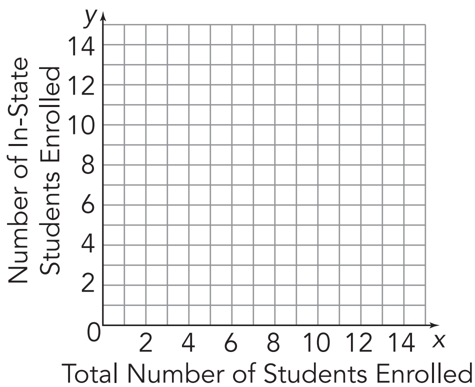
d. The number of out-of-state students enrolled, *y*, for *x* in-state students enrolled

Identify patterns in the language of the questions. How does this help you create the equations?

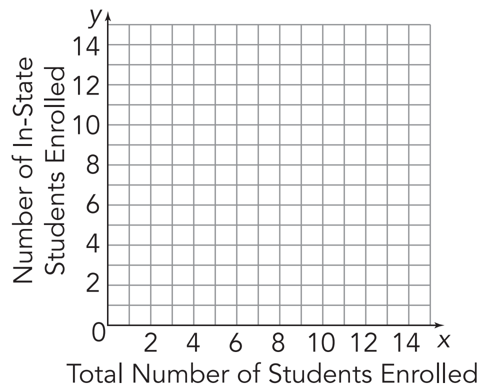
e. Describe the similarities and differences in each of the correct equations.

5. Create graphs that display the ratios. Then, compare the graphs.

a. The total number of in-state students enrolled, y, with respect to the total number of students enrolled, x



b. The total number of out-of-state students enrolled, y, with respect to the total number of students enrolled, x



c. Describe the similarities and differences between the two graphs.

In this lesson, you are studying relationships that are proportional. A **proportional relationship** is a relationship in which the ratio of the inputs to the outputs is constant. For example, the ratio of in-state to out-of-state students at a university is 3 : 2. Proportional relationships are always written in the form $y = kx$, where x represents an input value, y represents an output value, and k represents some constant that is not equal to 0. The constant k is called the **constant of proportionality**.

Remember, a proportional relationship is also referred to as a direct variation.

6. Identify the constant of proportionality for each relationship in Question 4.

7. Identify the constant of proportionality, or rate of change, for each graph in Question 5. Then, explain how to determine k from a graph.

8. Determine the constant of proportionality for each proportional relationship. Assume that y represents all of the outputs and x represents all of the inputs.

   a. $18x = 9y$

   b. $\frac{y}{3} = 4x$

   c. $\frac{3}{4}x = y$

   d. $\frac{5}{3}y = 5x$

When a relationship has a constant rate of change but is not proportional, it is called a **non-proportional relationship**. For example, Alejandro has $200 in his savings account and continues to save an additional $25 each week. This relationship has a constant rate of change, $25 per week. However, since Alejandro already has $200 in his account, the relationship is a *non-proportional relationship*. The total amount of money in Alejandro's account is represented by the equation $y = 25x + 200$.

9. Consider the equations given in Question 4.

   a. Which of the equations represent proportional relationships?

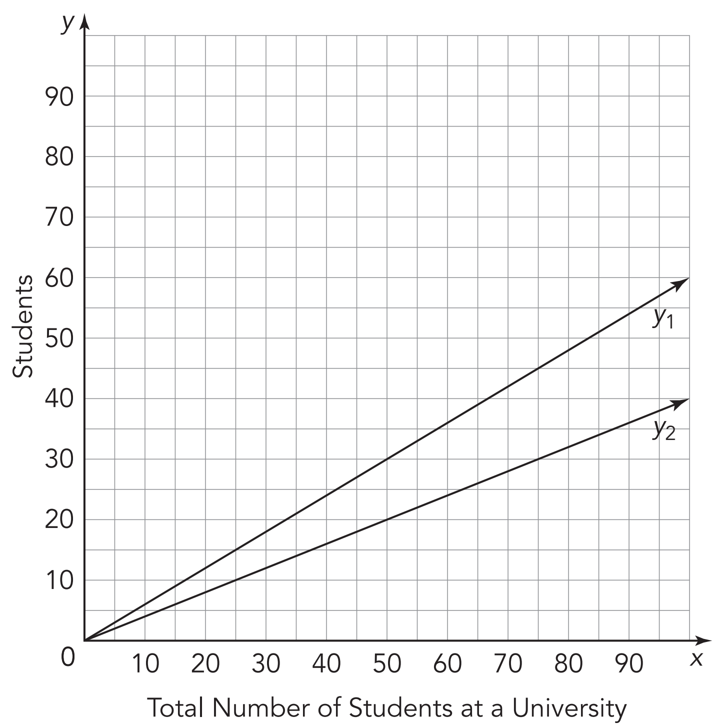
   b. Which of the equations represent non-proportional relationships?

ACTIVITY 1.2

Comparing Ratios and Graphs

Graphs provide a variety of information about relationships between quantities.

1. Examine the lines graphed on the coordinate plane. What can you determine about the relationships between the quantities by inspecting the graph?



The lines y_1 and y_2 each represent a proportional relationship. One line represents the proportional relationship between the number of in-state students enrolled and the total number of students. The other line represents the proportional relationship between the number of out-of-state students enrolled and the total number of students.

2. Determine which line represents each relationship. Explain your reasoning.

   a. The number of in-state students enrolled in a university

   b. The number of out-of-state students enrolled in a university

In a linear relationship, any change in an independent variable will produce a corresponding change in the dependent variable.

The ratio of the number of students who enjoy music to the total number of students is slightly more than the ratio of in-state students to the total number of students.

3. Draw a line on the coordinate plane that might represent the ratio of the number of students who enjoy music to the total number of students. Label this line y_3. Explain your reasoning.

The ratio of students who work full-time to total students is less than the ratio of out-of-state students to total students.

4. Draw a line on the coordinate plane that might represent the ratio of the number of students who work full-time to the total number of students. Label this line y_4. Explain your reasoning.

5. Of the lines on the coordinate plane, which is the steepest? How does this relate to the ratios?

ACTIVITY 1.3

Comparing Speeds

A proportional relationship can also be referred to as a *direct variation*. A situation represents a **direct variation** if the ratio between two quantities is constant for every point. If two quantities vary directly, the points on a graph form a straight line, and the line passes through the origin.

Natalia attends college in another state. During summer break, she drives home from college to visit her family and friends.

1. Natalia decides to keep track of the time it takes her to drive home from school. She records her distance after various numbers of hours. Her data is shown in the table.

Natalia's Drive Home

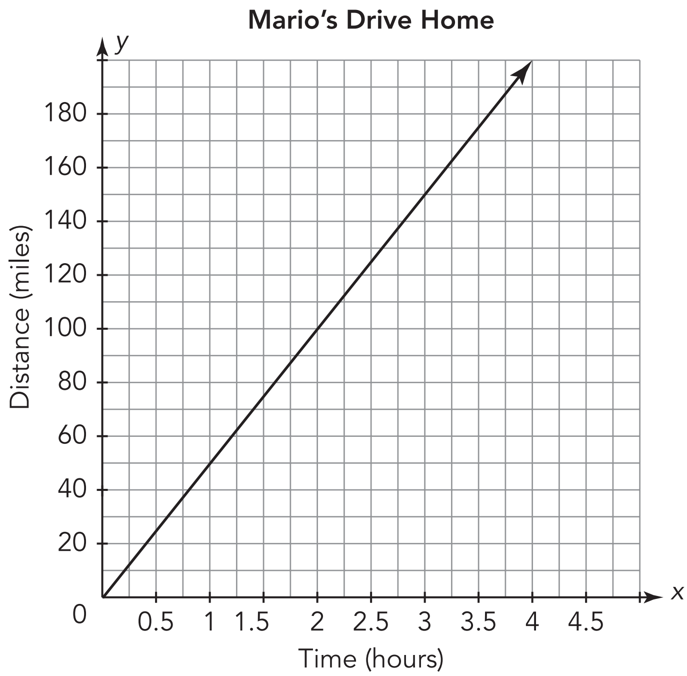
Time (hours)	Distance (miles)
3	180
2	120
1.5	90
2.5	150

a. Does this table represent a direct variation? Explain your reasoning.

b. Write a ratio for distance to time.

c. Write the unit rate for distance per 1 hour.

Unit rate is a comparison of two quantities in which the denominator has a value of one unit.



One of Natalia's high school classmates, Mario, attends college with Natalia. He also drives home during the summer break but takes a different route.

2. Analyze the graph of his trip.

   a. Does the graph represent a direct variation? Explain your reasoning.

   b. Who drives faster—Natalia or Mario? Explain your reasoning.

A third friend, Kaya offers to drive Natalia and Mario home for spring break, so that they can share the cost of gas. When asked how fast she drives, Kaya reported that the distance traveled, y, varies directly with the time, x, and can be expressed as $y = 57x$.

3. Suppose you were to graph Kaya's drive. Explain how the graph would represent a direct variation.

4. The distance traveled varies directly with the time spent driving. Suppose each friend drives for 15 minutes.

   a. Who drives the farthest? Explain your reasoning.

   b. Rank the friends in order from the shortest distance traveled to the longest.

ACTIVITY 1.4 Comparing Depth of Color

Students in a sculpting class at a university are working in teams to create modeling clay. The students learned that they can make different types of clay by changing the ratio of flour to water. Their recipes are shown in the table.

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
Flour	2.5 cups	3 cups	7.5 cups	4 cups	12 cups	3.75 cups	5 cups
Water	1 cup	2 cups	3 cups	2 cups	8 cups	1.5 cups	2 cups

1. How many different recipes for clay did the students create? Show all work and explain your reasoning.

The art professor would like all of the projects to include the same shade of orange. The students have learned that orange paint is created by mixing red and yellow paints. Three groups presented suggestions for the shade of orange to be used for the art projects.

<table>
<tr><th>Andrew's Group</th><th>Alejandro's Group</th><th>Trung's Group</th></tr>
<tr>
<td>$y = \frac{4}{5}x$, where the amount of yellow paint, y, varies directly with the amount of red paint, x</td>
<td>
<table>
<tr><th>Red Paint (parts)</th><th>Yellow Paint (parts)</th></tr>
<tr><td>6</td><td>1.5</td></tr>
<tr><td>8</td><td>2</td></tr>
<tr><td>12</td><td>3</td></tr>
<tr><td>15</td><td>3.75</td></tr>
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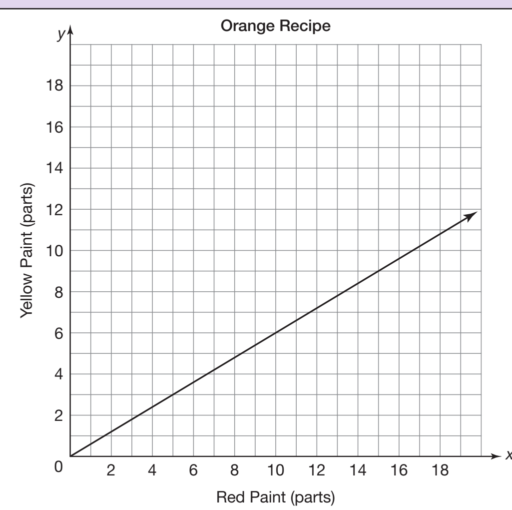


</td>
</tr>
</table>

2. Explain how you know that each group's proposal represents a direct variation.

   a. Andrew's Group

   b. Alejandro's Group

   c. Trung's Group

The greater the ratio of yellow to red paint used, the lighter the shade of orange paint.

3. Rate the group's proposals from lightest orange to deepest orange. Explain your reasoning.

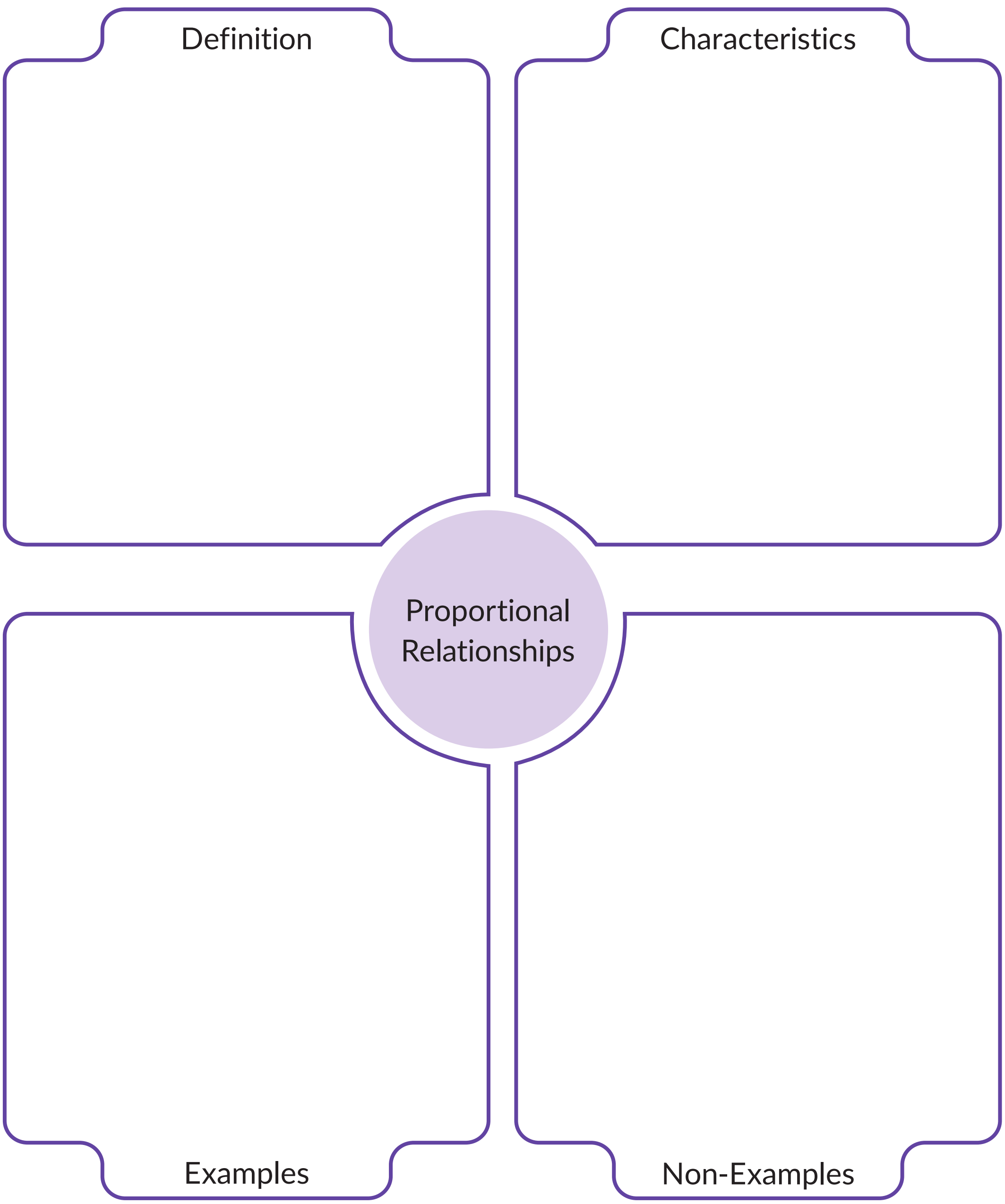
4. Write a direct variation equation, where the amount of yellow paint, y, varies directly with the amount of red paint, x, that would create a shade of orange that is between the two deepest shades. Explain your reasoning.

Talk the Talk

Proportional Relationships

All of the relationships in this lesson are examples of proportional relationships.

1. Complete the graphic organizer to summarize proportional relationships. Include characteristics, examples, and non-examples using tables, equations, and graphs.



Lesson 1 Assignment

Write

Explain how to compare proportional relationships represented in different forms.

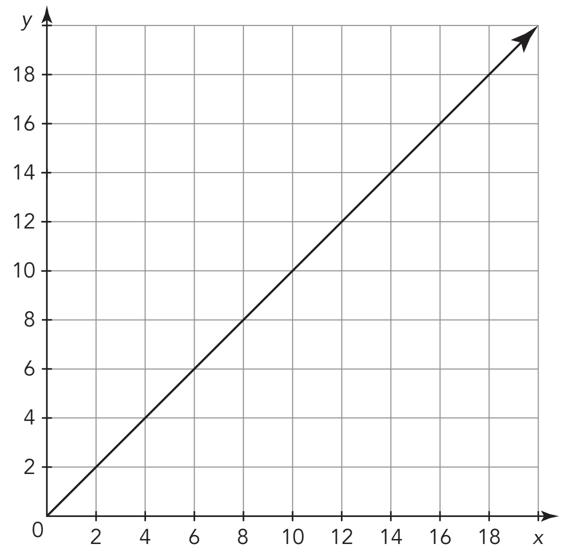
Remember

Proportional relationships, also known as direct variations, can be represented using tables, graphs, and equations. In a table, all the ratios of corresponding *x*- and *y*-values must be constant. On a graph, a proportional relationship is represented as a linear graph passing through the origin. The equation for a proportional relationship, or a direct variation, is written in the form $y = kx$, where k is the constant of proportionality.

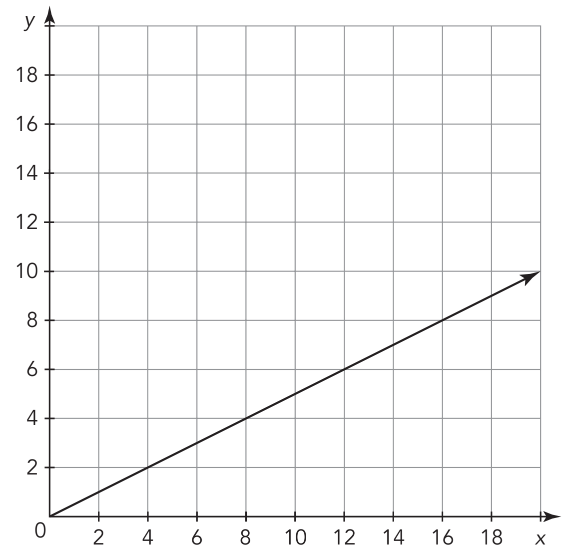
Practice

1. Determine the constant of proportionality represented in each graph.

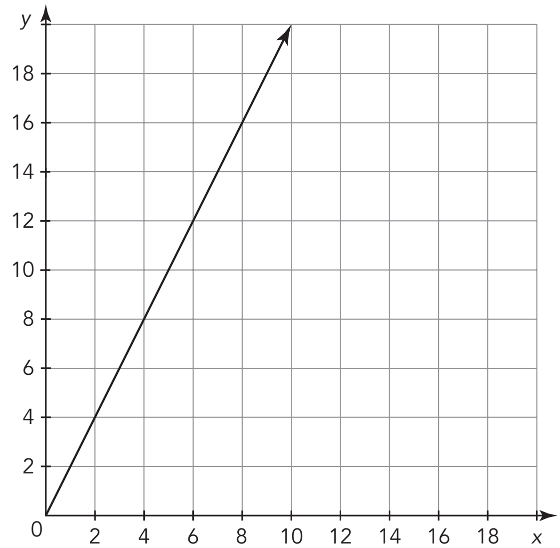
a.



b.



Lesson 1 Assignment

c. 

2. Determine the constant of proportionality for each proportional relationship. Assume that *y* represents all of the outputs and *x* represents all of the inputs.

a. $2x = 10y$

b. $\left(\frac{3}{5}\right)y = 8x$

c. $\frac{y}{10} = 10x$

d. $\left(\frac{1}{2}\right)x = y$

Lesson 1 Assignment

3. Samantha collects coins from different periods in American History. Her favorite coin series are the ones celebrating American women (e.g. Bessie Coleman, Anna May Wong, and Jovita Idar) and the series celebrating National (e.g. Westward Journey nickels) and Texas (e.g. San Antonio Missions quarter) historical moments. She is reorganizing her collection into coins from her American Women collection and coins from other collections. After sorting the coins, she comes to the conclusion that six out of every ten of the coins in her collection come from the American women series.

   a. Write a ratio for the number of American women coins to the total number of coins, the number of non-American women coins to the total number of coins, and the number of American women coins to the number of non-American women coins.

   b. Samantha has 230 coins in her collection. Determine the number of American women and non-American women coins that she has in her collection.

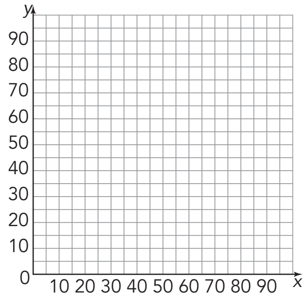
   c. Samantha adds to her collection while keeping the same ratio of coins and now has 180 American women coins. Determine the number of non-American women coins and the total number of coins in her collection.

Lesson 1 Assignment

d. Write an equation to determine the number of American Women coins, W, if Samantha has t total coins. Show your work and identify the constant of proportionality.

e. Write an equation to determine the number of non-American women coins, N, if Samantha has t total coins. Show your work and identify the constant of proportionality.

f. Graph your equations from parts (d) and (e) on a coordinate plane. Label the axes of the graph.



4. Eduardo also collects coins. He organizes his collection into coins from the series celebrating National and Texas historical moments and coins from other collections. After sorting the coins, he comes to the conclusion that three out of every eight of the coins in his collection come from the series celebrating National and Texas historical moments.

   a. Write a ratio for the number of historical moments coins to the total number of coins, the number of non-historical moments coins to the total number of coins, and the number of historical moment coins to the number of non-historical moments coins.

Lesson 1 Assignment

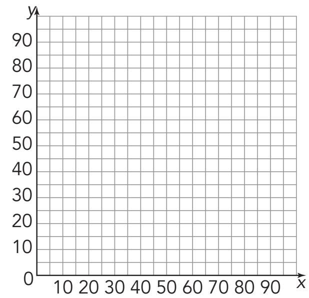
b. Eduardo has 240 coins in his collection. Determine the number of historical moment and non-historical moment coins that he has in his collection.

c. Eduardo adds to his collection while keeping the same ratio of coins and now has 120 historical moment coins. Determine the number of non-historical moment coins and the total number of coins in his collection.

d. Write an equation to determine the number of historical moment coins, H, if Eduardo has t total coins. Show your work and identify the constant of proportionality.

e. Write an equation to determine the number of non-historical moments coins, N, if Eduardo has t total coins. Show your work and identify the constant of proportionality.

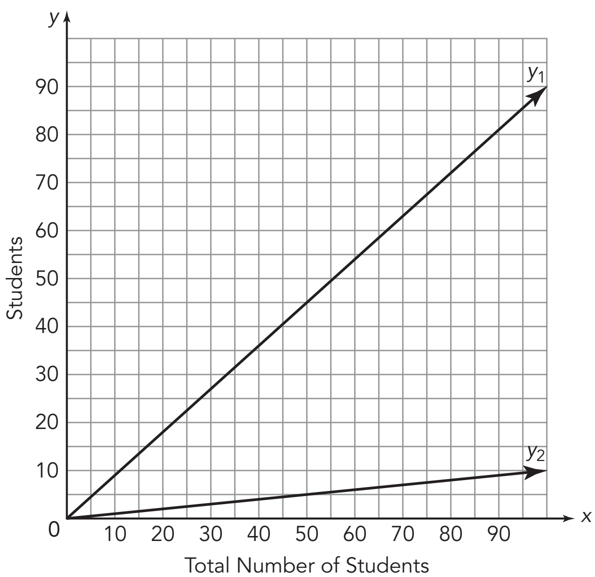
f. Graph your equations from parts (d) and (e) on a coordinate plane. Label the axes of the graph.



Lesson 1 Assignment

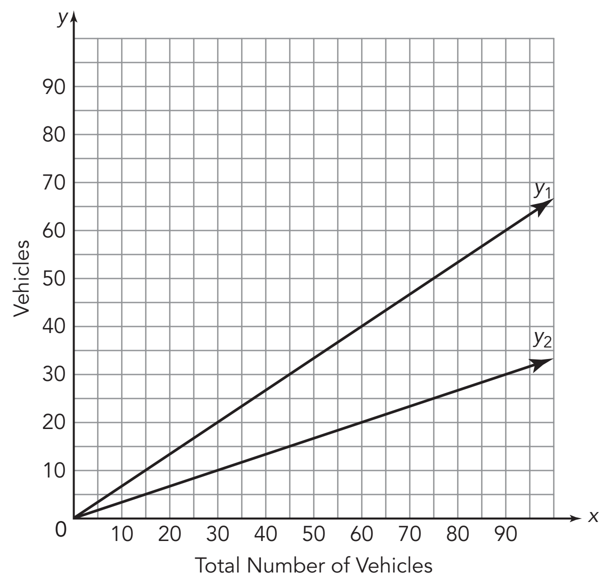
5. Analyze each scenario and graph.

a. A voice instructor notices that only one out of every ten of her students can sing soprano.



Identify the direct variation represented by each line as it relates to the scenario. Explain your reasoning.

b. A store owner notices that in his parking lot, two out of every six vehicles are trucks.



Write an equation for each graph that has a constant of proportionality between those represented on each graph. Explain what relationship is represented by your equations.

Lesson 1 Assignment

Prepare

Identify the coefficient(s) and constant(s) in each equation.

1. $64x + 24$

2. $36 - 8z$

3. $-3a^2 + 18a$

4. $42mn + 27m - 1$

2 Using Similar Triangles to Describe the Steepness of a Line

OBJECTIVES

- Analyze the rate of change between any two points on a line.
- Use similar triangles to explore the steepness of a line.
- Derive the equations $y = mx$ and $y = mx + b$, representing linear relationships.
- Graph proportional relationships, interpreting the unit rate as the slope of the graph.

NEW KEY TERMS

- rate of change
- slope

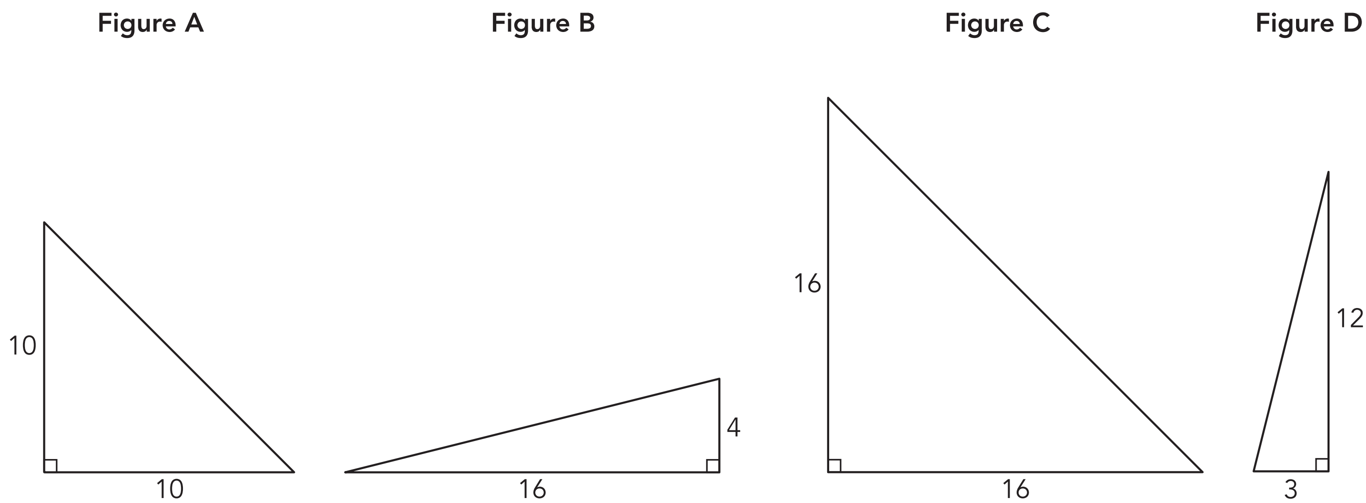
You have learned about rates, unit rates, and the constant of proportionality.

How can you connect all of those concepts to describe the steepness of a line?

Getting Started

Let It Steep

Examine each triangle shown.



1. For each triangle, write a ratio that represents the relationship between the height and the base of each triangle.

2. Write each ratio as a unit rate.

3. How can you use these rates to compare the steepness of the triangles?

ACTIVITY 2.1 Constant of Proportionality as Rate of Change

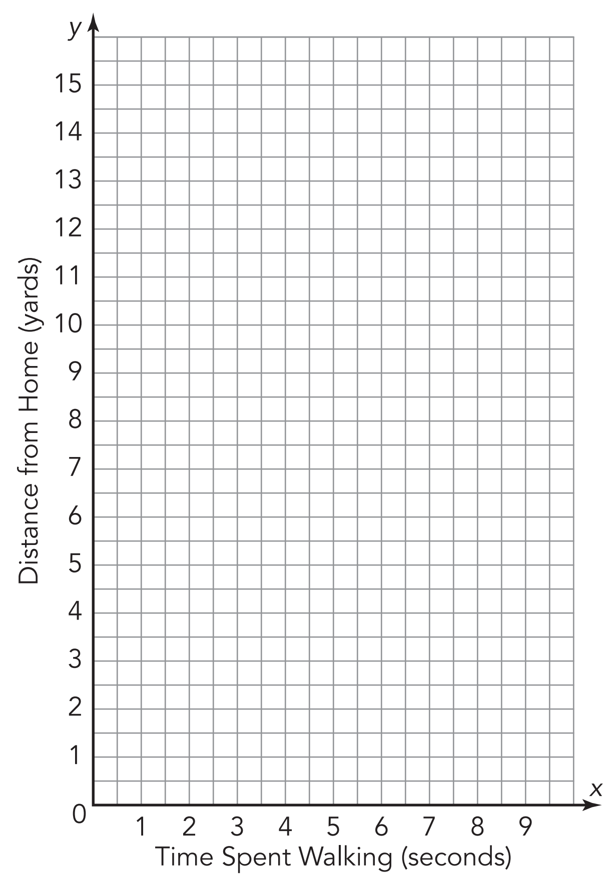
On Monday, Eduardo and Paola walked from their home up a hill to get to the bus stop. They walked 4 yards every 3 seconds.

1. Write an equation to represent the distance, d, Eduardo and Paola walked over time, t.

2. Does this situation represent a proportional relationship? If so, identify the constant of proportionality.

3. Complete the table. Then, graph the points. Finally, draw a line to represent the relationship between the time Eduardo and Paola walked and their distance from home.

Time Spent Walking (seconds)	Distance from Home (yards)
	0
1	
3	
	8
7.5	
9	



4. What is the unit rate? Explain what the unit rate means in terms of this situation.

5. Chris's teacher asked the class to explain why the graph goes up as you move from left to right. Explain why Chris's reasoning is incorrect. Then, explain why the graph goes up as you move from left to right.

> Chris
>
> This graph goes up from left to right because Eduardo and Paola were walking up a hill.

The **rate of change** for a situation describes the amount that the dependent variable changes in comparison to the amount that the independent variable changes.

6. Consider the Eduardo and Paola situation.

   a. Identify the independent and dependent variables. Explain your reasoning.

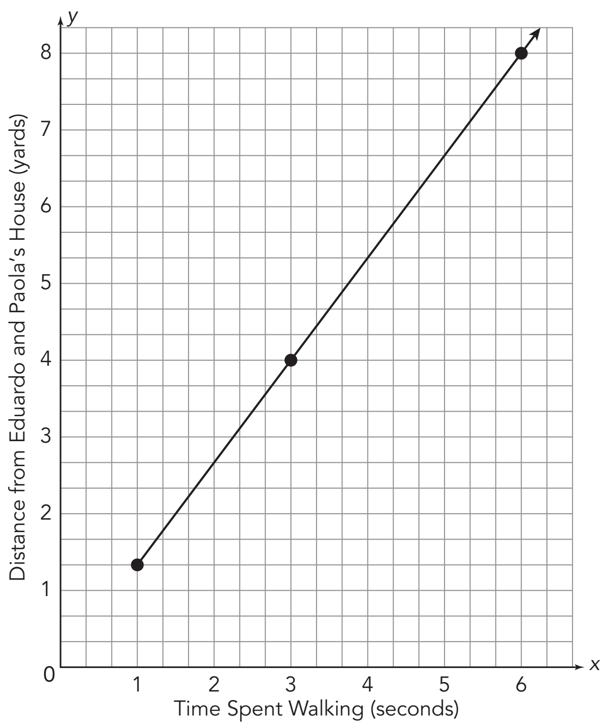
   b. Identify the rate of change.

7. Consider the rate of change, the constant of proportionality, and the unit rate for this situation. What do you notice?

8. How would the rate of change and the graph of the relationship change if Eduardo and Paola walked faster? How would they change if Eduardo and Paola walked more slowly?

ACTIVITY 2.2

Slope of a Line



The graph shown represents the relationship between the time Eduardo and Paola walk and the distance they walk from their home.

Let's analyze three different moments in time during Eduardo and Paola's walk to the bus stop.

$$t = 1 \quad t = 3 \quad t = 6$$

The graph shows a right triangle drawn to represent $t = 1$.

1. Trace the triangle on a piece of patty paper. Label the horizontal and vertical sides of the right triangle with their respective lengths.

2. Draw right triangles to model $t = 3$ and $t = 6$ on the coordinate plane. Then, trace each triangle on a separate piece of patty paper. Label the horizontal and vertical sides of the right triangle with their respective lengths.

3. Determine the steepness of each triangle by writing a ratio of the vertical side length to the horizontal side length. How do these ratios compare?

4. What is the relationship among the three right triangles? Justify your reasoning.

5. Identify and label the triangle that represents the unit rate. Explain how you know.

6. Slide the unit rate triangle along the graph of the line. What do you notice?

7. Slide the other two triangles along the graph of the line. What do you notice?

Keep your patty paper drawings. You will need those in the next lesson.

In the last two activities, you investigated a relationship using a rate of change of $\frac{4}{3}$ to represent Eduardo and Paola walking 4 yards away from their home in 3 seconds, or as a unit rate of $\frac{4}{3}$ yards per second. Because this situation is a proportional relationship, the rate of change can specifically be called the *constant of proportionality*, represented by the variable *k*.

In this activity, you created three similar triangles each using two points from the line to explore the steepness of the line. By sliding the similar triangles along the line, you noticed the steepness of the line remained constant between any two points on the line. In any linear relationship, **slope** describes the direction and steepness of a line and is usually represented by the variable *m*. Slope is another name for rate of change. It represents the ratio of the change in vertical distance to the change in horizontal distance between any two points on the line. The slope of a line is constant between any two points on the line.

You wrote the equation $d = \frac{4}{3}t$ to represent the distance, *d*, Eduardo and Paola walked from home with respect to time, *t*. Let's generalize this linear relationship.

8. Let *y* represent the dependent variable, *x* represent the independent variable, and *m* represent the slope of the line.

   a. Write a general equation to relate these quantities.

   b. How is this equation similar to the equation for the constant of proportionality?

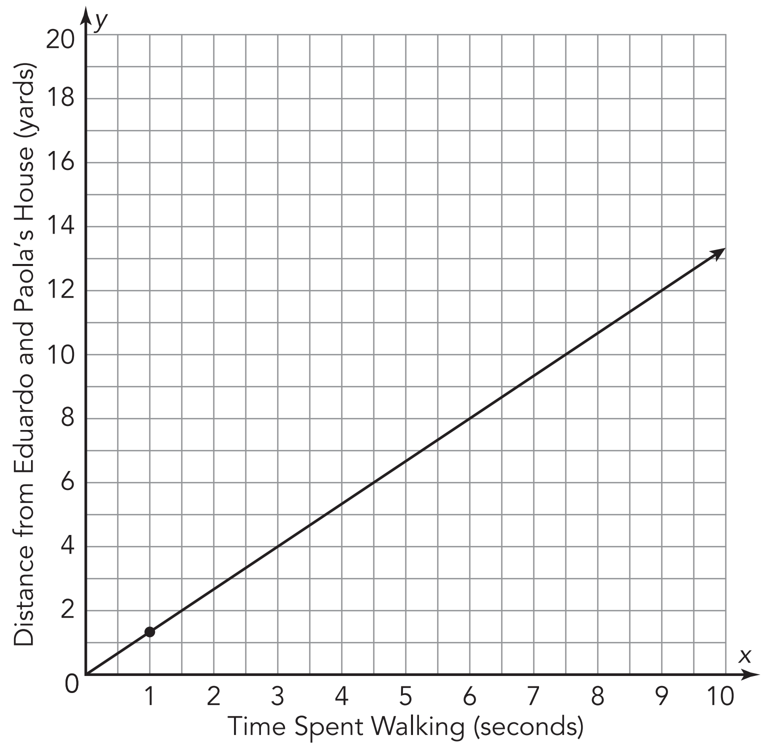
ACTIVITY

2.3

Equation for a Line Not Through the Origin

Eduardo and Paola's Aunt Elena lives 10 yards from their home closer to the bus stop. After spending Monday night at Aunt Elena's house, they leave for the bus stop from there Tuesday morning. They walk at the same rate from either house, 4 yards every 3 seconds.

The graph shows the line $y = \frac{4}{3}x$, which represents the relationship between the time Eduardo and Paola walk and their distance from their house.



1. Compare the two situations.

   a. How do the slopes compare?

   b. How do the starting points compare?

2. Let's graph the line to represent their walk to the bus stop from Aunt Elena's house.

   a. On a piece of patty paper, trace the line $y = \frac{4}{3}x$ that represents Eduardo and Paola's walk to the bus stop from their house. Be sure to include the triangle representing the unit rate in your trace.

   b. Translate this line to represent their walk from Aunt Elena's house and then transfer this line onto the graph.

3. Analyze the translated line.

a. Does your new line represent a proportional or non-proportional relationship? Explain how you know.

b. How does this translation affect the coordinates of the line? Complete the table to show how the translation affects the coordinates of your new line.

Time Spent Walking (seconds)	Distance from Eduardo and Paola's House on Monday (yards)	Distance from Eduardo and Paola's House on Tuesday (yards)
x	y_1	y_2
0	0	
1	$\frac{4}{3}$	
3	4	
6	8	
7.5	10	
9	12	

c. How does this translation affect the unit rate?

d. Write an equation to represent the translated line. Let y_2 represent the distance from Eduardo and Paola's house and let x represent their time spent walking. Explain how this line is the same and different from the line $y_1 = \frac{4}{3}x$.

You have written a general equation, $y = mx$, to relate the independent and dependent variables and the slope in a proportional linear relationship. How does this general equation change when the line is translated vertically by b units?

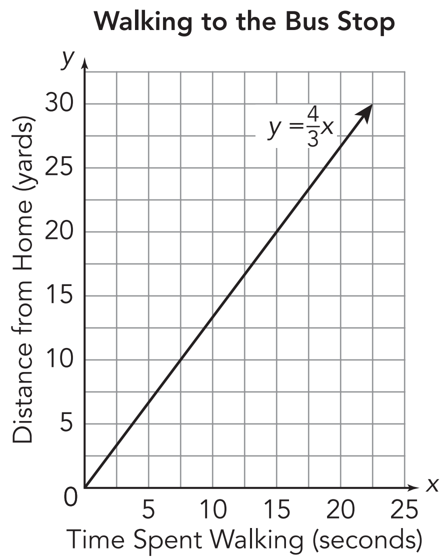
4. Write a general equation to represent the relationship $y = mx$ after it is vertically translated b units.

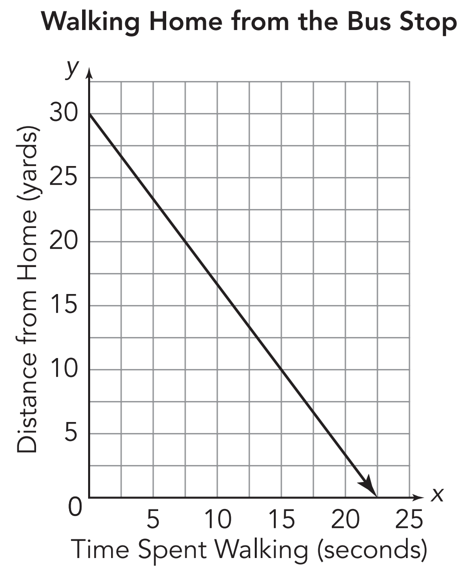
ACTIVITY
2.4

A Negative Unit Rate

Eduardo and Paola are walking back home from the bus stop which is 30 yards from their house. They walk at the same rate, 4 yards every 3 seconds.

Consider the two graphs shown.





1. Analyze the graph of Eduardo and Paola walking home from the bus stop.

   a. Does this situation represent a proportional or non-proportional relationship? Explain your reasoning.

   b. Is the slope of the line positive or negative? Explain how you know.

2. Compare and contrast the rate of change, or slope, of each line.

   a. Use patty paper to trace and create any right triangle that represents the rate of change, or slope, from the *Walking to the Bus Stop* graph.

   b. Place your patty paper on the *Walking Home from the Bus Stop* graph. How can you transform the right triangle you drew from the Walking to the Bus Stop graph to the Walking Home from the Bus Stop graph?

   c. Slide the right triangle along the line of the *Walking Home from the Bus Stop* graph. What do you notice?

   d. What is the slope of the line in the *Walking Home from the Bus Stop* graph? Explain your reasoning.

Remember . . .
the slope of a line represents steepness and direction.

3. Write an equation to represent Eduardo and Paola's walk home from the bus stop. Let *y* represent the distance from home and *x* represent the time spent walking.

4. How does the equation you wrote to represent Eduardo and Paola's walk home from the bus stop compare to the equation that represents their walk to the bus stop?

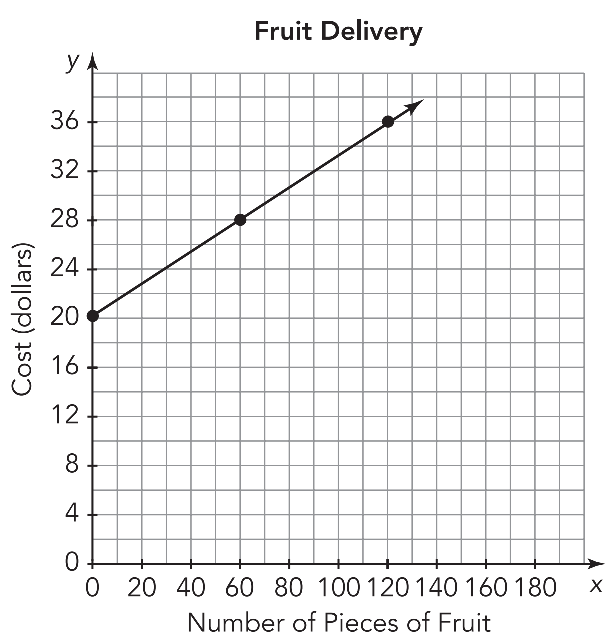
ACTIVITY 2.5 Describing Linear Equations

You have discovered that the equation $y = mx$ represents a proportional relationship. The equation represents every point (x, y) on the graph of a line with slope m that passes through the origin (0, 0).

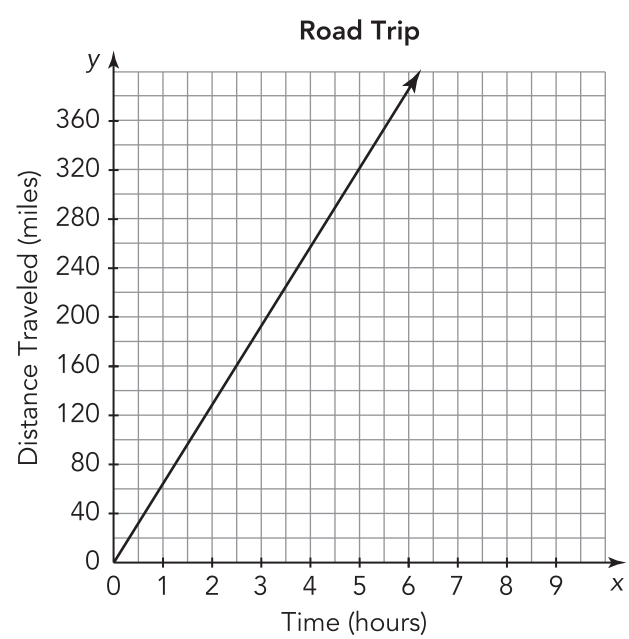
An equation of the form $y = mx + b$, where b is not equal to zero, represents a non-proportional relationship. This equation represents every point (x, y) on the graph of a line with slope m that passes through the point $(0, b)$.

1. Consider each graph shown.
   - Determine whether the graph represents a proportional or non-proportional relationship.
   - Write an equation in the form $y = mx$ or $y = mx + b$ to represent the relationship between the independent and dependent quantities.

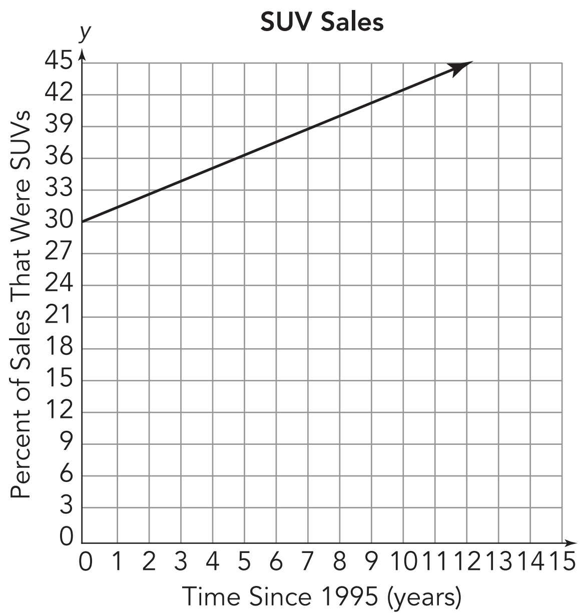
a.



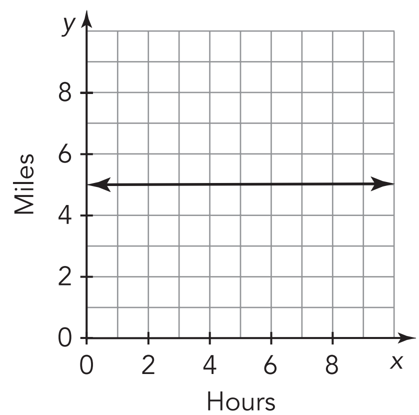
b.



c.



2. Determine the slope of this graph and write an equation to represent it. Describe a situation that could be modeled by this graph.



3. Complete the table of values to represent the linear relationship specified. Then, write an equation to represent the relationship.

a. Proportional relationship

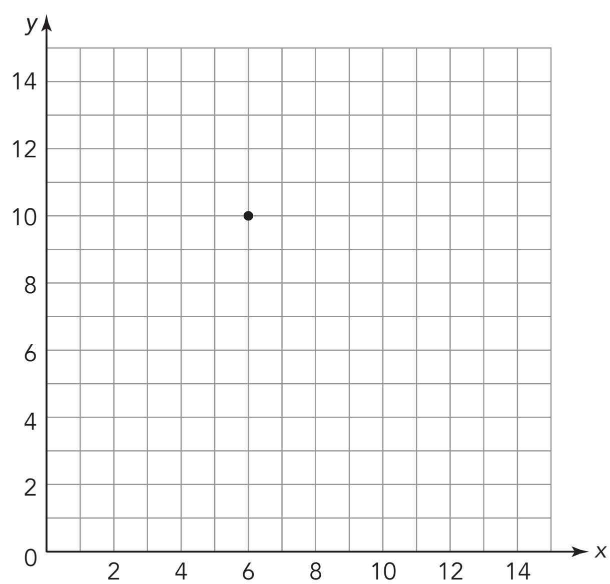
x	***y***
0	
1	
2	12
3	
4	

b. Non-proportional relationship

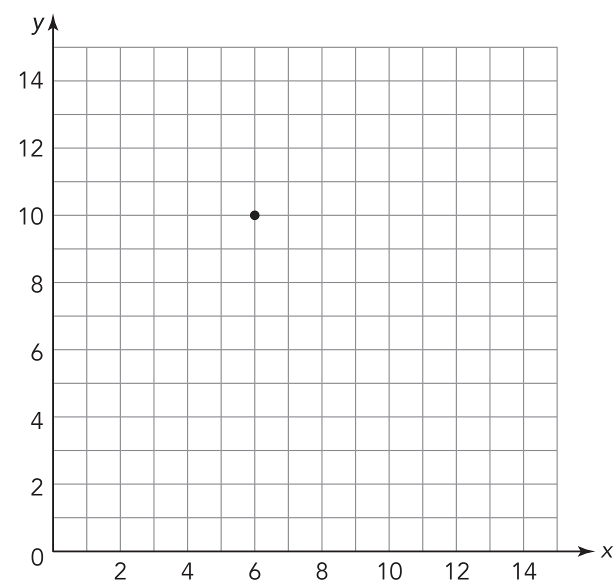
x	*y*
0	
1	
2	12
3	
4	

4. Draw a line through the point and label the graph to represent the linear relationship specified. Then, write an equation.

a. Proportional relationship



b. Non-proportional relationship



Talk the Talk

A Web of Connections

In this lesson, you learned that the steepness of a line can be described by its slope, which is a concept that is connected to many other concepts you have learned previously. Review the definitions from Activity 2.2 and the equations from Activity 2.5 to complete Question 1.

1. Complete the graphic organizer to describe how steepness is related to slope, rate of change, unit rate, and the constant of proportionality. Include definitions, graphs, and equations. Be sure to address both proportional and non-proportional relationships.

Slope

Rate of Change

Steepness of a Line

Unit Rate

Constant of Proportionality

Lesson 2 Assignment

Write

Explain the relationships between the slope of a line and the similar right triangles formed along the line. Use examples to illustrate your explanation.

Remember

- Slope is another name for the rate of change of a linear relationship graphed as a line.
- The equation for a proportional linear relationship is $y = mx$, where m is the slope. The equation represents all of the points (x, y) on the line.
- An equation for a non-proportional linear relationship is $y = mx + b$, where m is the slope and b is the y-coordinate of the point where the graph crosses the y-axis. The equation represents all of the points (x, y) on the line.

Practice

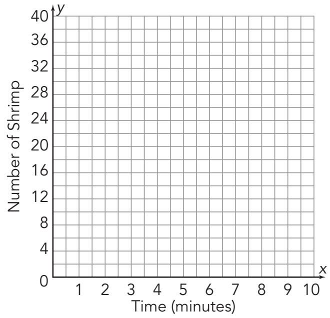
1. Jorge is cleaning shrimp. He cleans 4 shrimp every minute. Use time in minutes as the independent quantity and the number of shrimp as the dependent quantity.

   a. Is the relationship proportional or non-proportional? Explain how you can determine this using a graph and an equation.

   b. Identify the unit rate of this relationship. Explain what the unit rate means in terms of the situation.

   c. Write an equation that determines the number of shrimp cleaned given any time.

   d. Create a graph of the relationship.

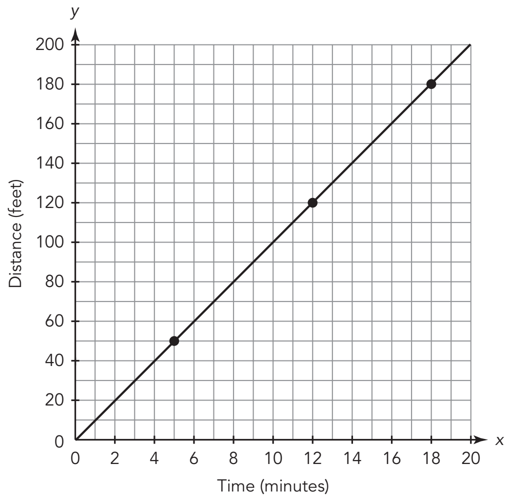


Lesson 2 Assignment

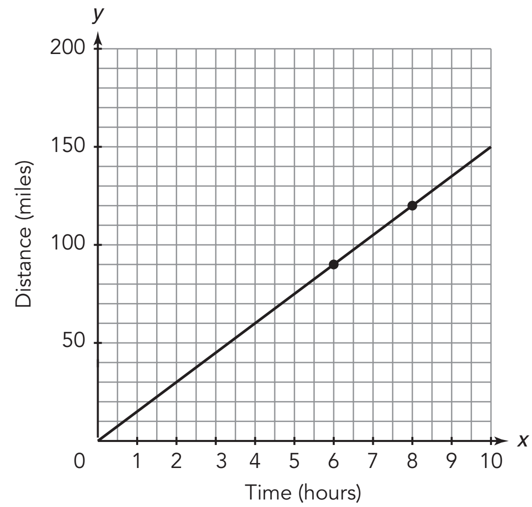
2. Consider each graph shown.

- Determine whether the graph represents a proportional or non-proportional relationship.
- Write an equation in the form $y = mx$ or $y = mx + b$ to represent the relationship between the independent and dependent quantities.

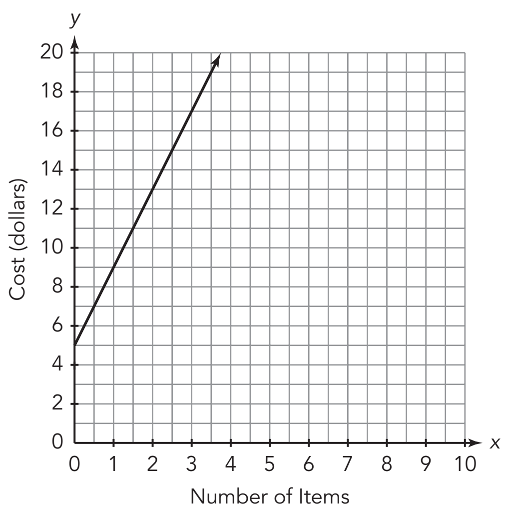
a.



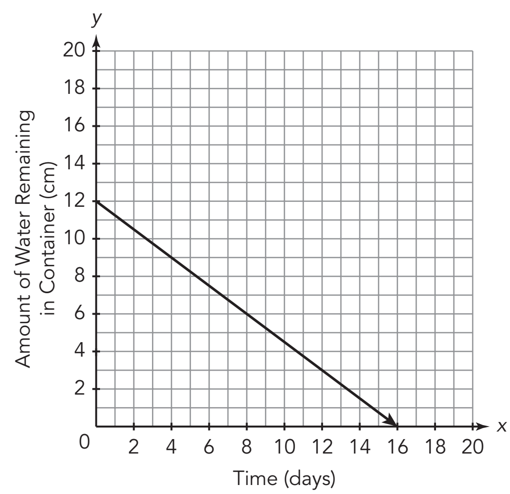
b.



c.



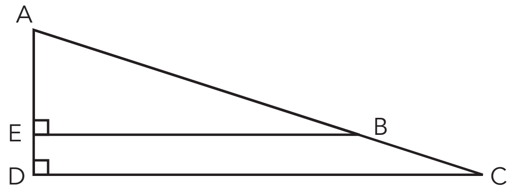
d.

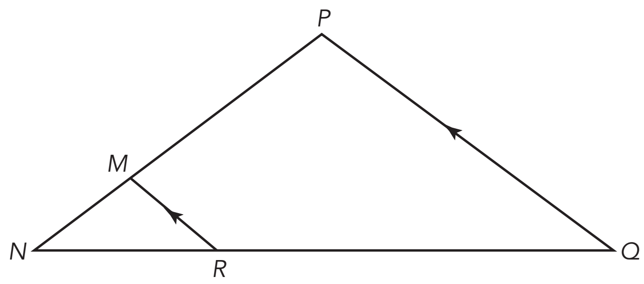


Lesson 2 Assignment

Prepare

For each diagram, describe how you can show that the triangles are similar.

1. 

2. 

3 Exploring Slopes Using Similar Triangles

OBJECTIVES

- Use similar triangles to show that the slope is the same between any two distinct points on a non-vertical line in a coordinate plane.
- Use right triangles to identify the slope of a line from a graph.

You have used similar triangles to describe the steepness of a line. How can you use similar triangles to explain why the slope is the same between any two distinct points on a non-vertical line?

Getting Started

Steep Grade

Consider the three street signs shown.

Discuss each question with your partner.

1. Where might you see each of the signs?

2. What do you know about the triangles on the signs?

3. For the signs that include numbers, what do you think those numbers represent?

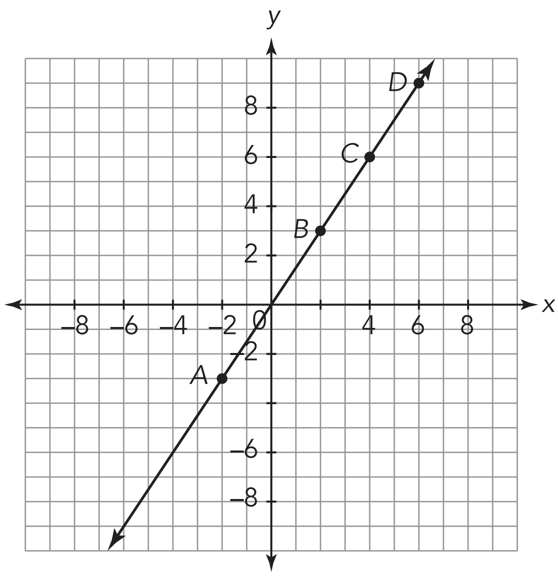
ACTIVITY
3.1 Triangles and the Equation $y = mx$

In the previous lesson, you used patty paper to analyze the slope of the line $y = \frac{4}{3}x$ using similar triangles formed at $x = 1$, $x = 3$, and $x = 6$.

Now, let's investigate if the slope of a line is always the same between any two points on a line.

Remember . . .
slope describes the direction and steepness of a line.

Consider the graph of $y = \frac{3}{2}x$.



1. Is the slope of the line positive or negative? Explain your reasoning.

2. Examine the slope between points *A* and *B*.

   a. Create a right triangle using points *A* and *B* and trace it onto patty paper.

   b. Label the triangle with the vertical and horizontal distances.

   c. Label the patty paper with the slope of the line between points *A* and *B*.

3. Does the orientation of the right triangle matter? Place your patty paper on the graph, use point A as the center of rotation, and rotate your triangle 180°.

   a. Compare and contrast these two triangles. How are they the same? How are they different?

   b. Does the new triangle give you the same slope? Explain your reasoning.

4. Create right triangles using points B and C and then B and D.

   a. Label the horizontal and vertical distances.

   b. Label the patty paper with the slope of the line.

5. Compare the triangles created on the line. How can you verify that all of the triangles are similar?

6. What is the slope of the line?

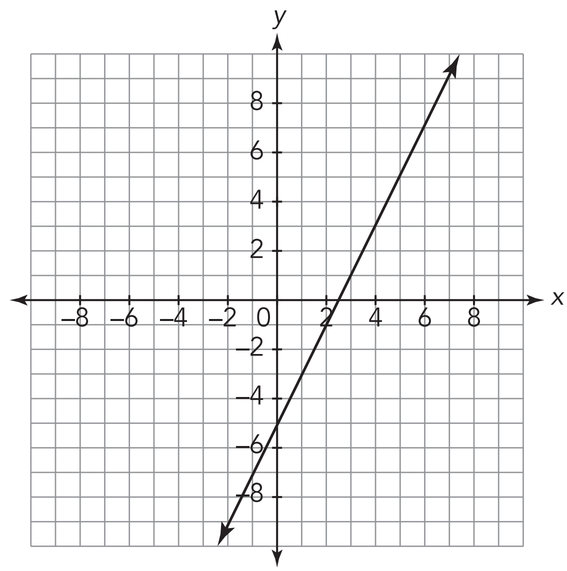
7. Jaylen claims that all right triangles formed on a given line are similar. Is Jaylen correct? Explain your reasoning.

ACTIVITY

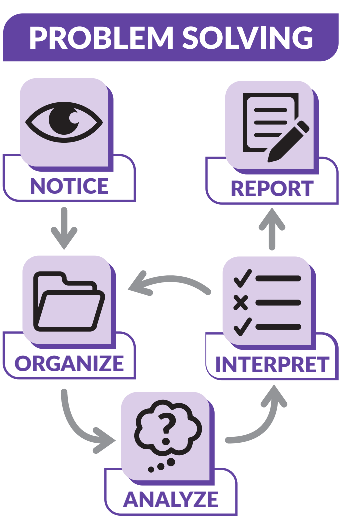
3.2

Triangles, Slope, and the Equation $y = mx + b$

Consider the graph shown.



1. Is the slope of the line positive or negative? Explain your reasoning.



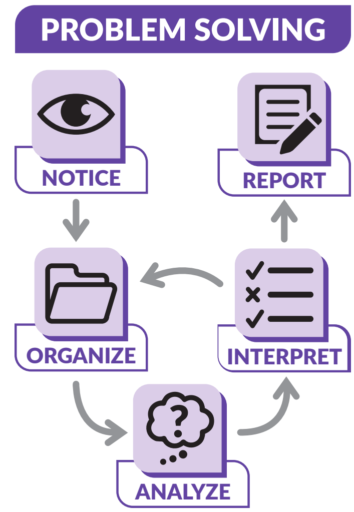
2. Create at least three similar triangles using points on the line.

   a. Use any method to justify that these triangles are similar.

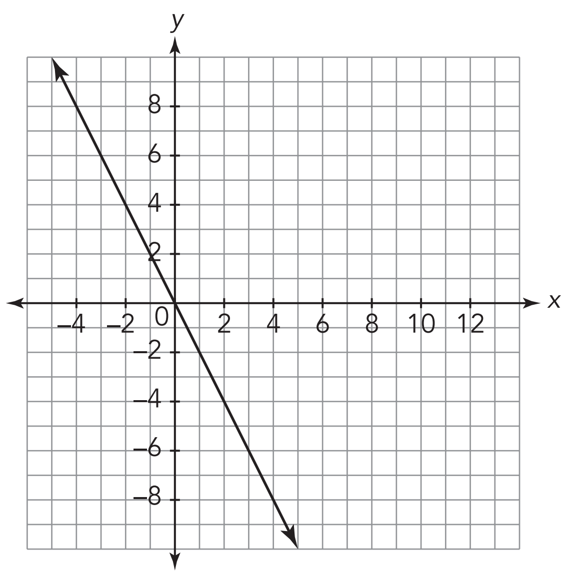
   b. Determine the slope of the line.

3. How many similar triangles can be formed on the graph of a line? How do you know?

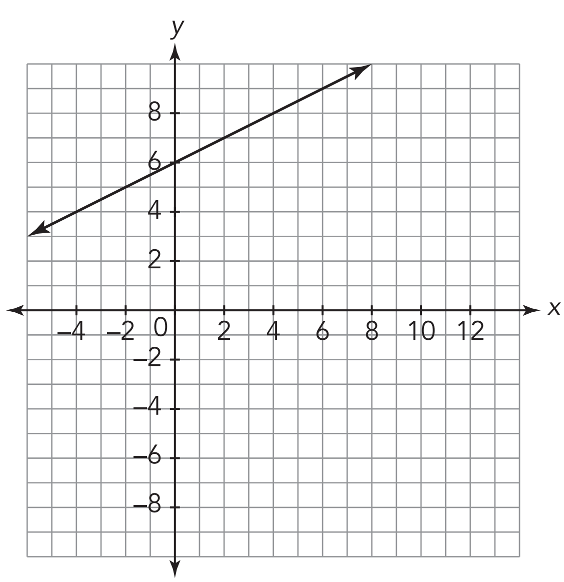
4. Consider each graph shown. Determine the slope of each line and then use similar triangles to justify that the slope is the same between any two points.



a.



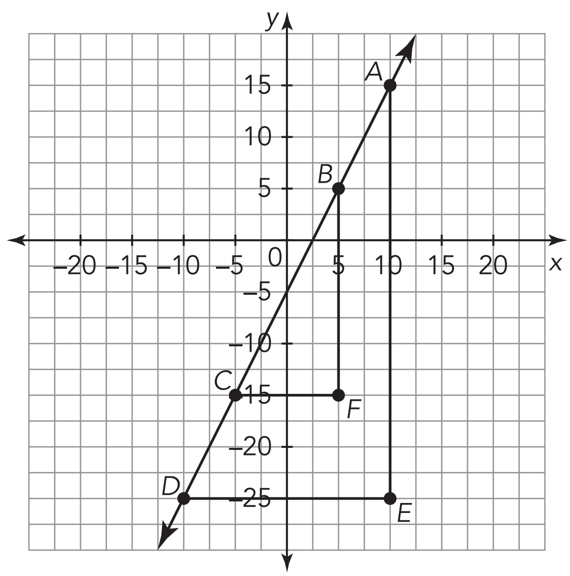
b.



Talk the Talk

Connecting Similar Triangles and Slope of a Line

Elena was absent for the lesson on the connection between similar triangles and the slope of a line. Write an explanation of what you learned in this lesson. Be sure to include how you can use a graph to determine the slope of a non-vertical line, and how you can use similar triangles to show that the slope is the same between any two points on the line.



Lesson 3 Assignment

Write

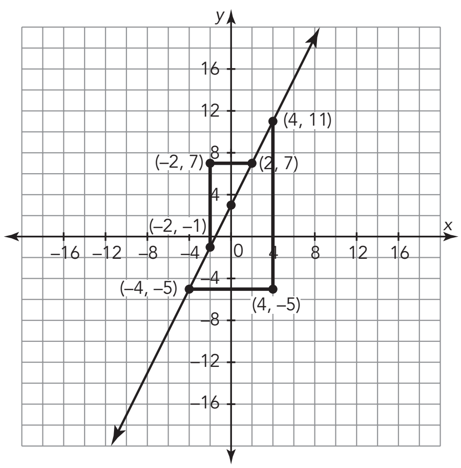
Explain why the slope between any two points on a line is always the same.

Remember

The properties of similar triangles can be used to explain why the slope, m, is the same between any two distinct points on a non-vertical line in the coordinate plane.

Practice

1. Consider the graph of the equation $y = 2x + 3$.

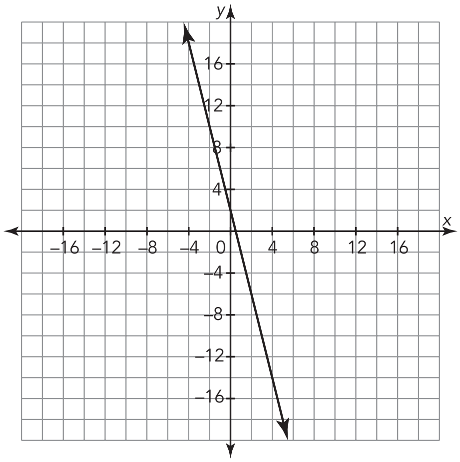


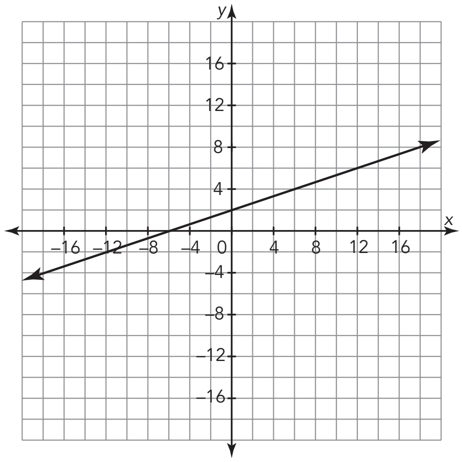
   a. The points on the line were used to create triangles. Describe the relationship between the two triangles.

   b. How can transformations be used to verify the relationship between the triangles?

   c. Use the similar triangles to determine the slope between any two points on the line.

2. Consider each graph shown. Determine the slope of each line and then use similar triangles to justify that the slope is the same between any two points.





Lesson 3 Assignment

Prepare

Identify whether the equation represents a proportional or non-proportional relationship. Then, state whether the graph of the line will increase or decrease from left to right.

1. $y = -2x - 9$

2. $y = 2x - 3$

3. $y = \frac{2}{3}x$

4 Transformations of Lines

OBJECTIVES

- Translate linear graphs horizontally and vertically.
- Use transformations to graph linear relationships.
- Determine the slopes of parallel lines.
- Identify parallel lines.
- Explore transformations of parallel lines.

You have learned how translations, reflections, or dilations of a pre-image affect the coordinates of an image.

How can you use what you know to transform the graphs and equations of linear relationships?

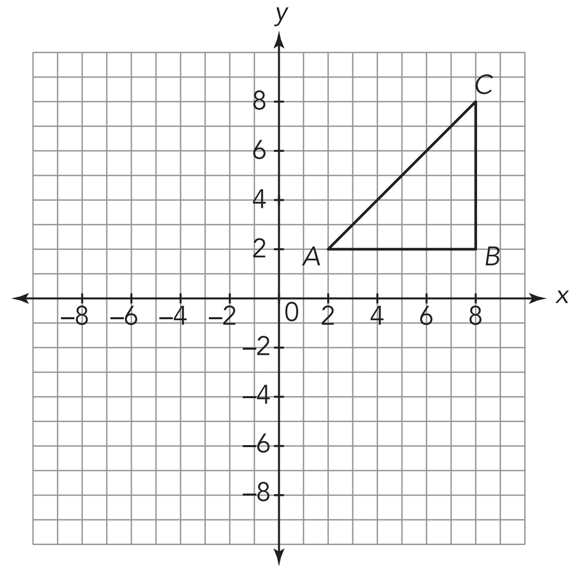
Getting Started

Transformation Station

Consider △ABC with coordinates *A*(2, 2), *B*(8, 2), and *C*(8, 8) shown on the coordinate plane.

Ask Yourself . . .
What patterns do you notice?

1. Suppose the triangle is translated in a single direction. In general, how does this affect the coordinates of the figure?



(*x*, *y*)	4 Units Up	4 Units Down	4 Units Left	4 Units Right
New Coordinates				

2. Suppose the triangle is reflected across an axis. How does this affect the coordinates of the figure?

(*x*, *y*)	*x*-Axis	*y*-Axis
New Coordinates		

3. Suppose the triangle is rotated through an angle with the origin as the center of rotation. How does this affect the coordinates of the figure?

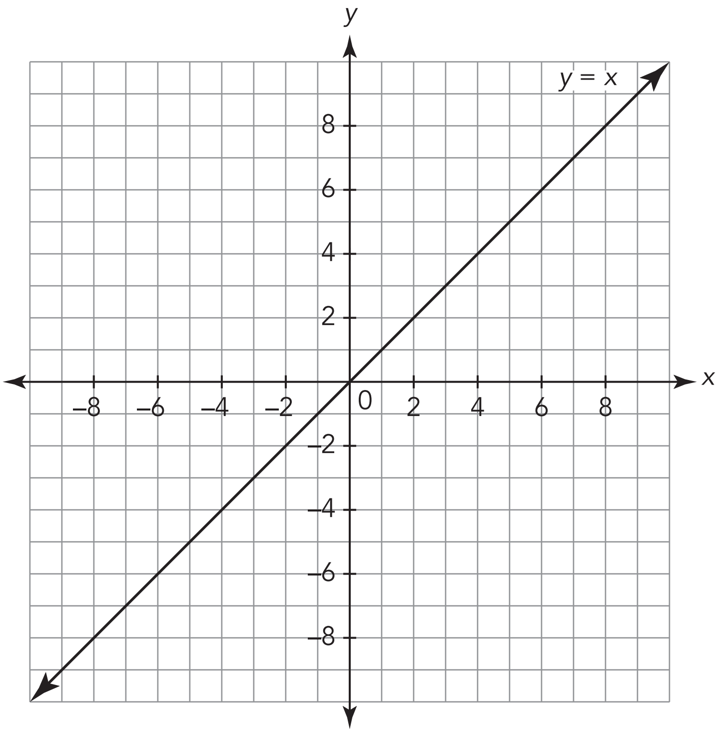
(*x*, *y*)	90° Counterclockwise	180°	270° Counterclockwise
New Coordinates			

4. Suppose the triangle is dilated by a factor of *m* with a center of dilation at the origin. How does this affect the coordinates of the figure?

(*x*, *y*)	Dilation
New Coordinates	

5. How do you think translations, reflections, rotations, and dilations affect lines?

ACTIVITY
4.1
Translating Lines



In this activity, you will investigate how the equation of a line changes as you translate the line up and down the y-axis.

Consider the graph of the basic linear equation $y = x$, which is of the form $y = mx$. The line represents a proportional relationship with a rate of change, or slope, of 1. The graph of $y = x$ has a y-intercept of (0, 0).

1. Trace the axes and the line $y = x$ on a sheet of patty paper.

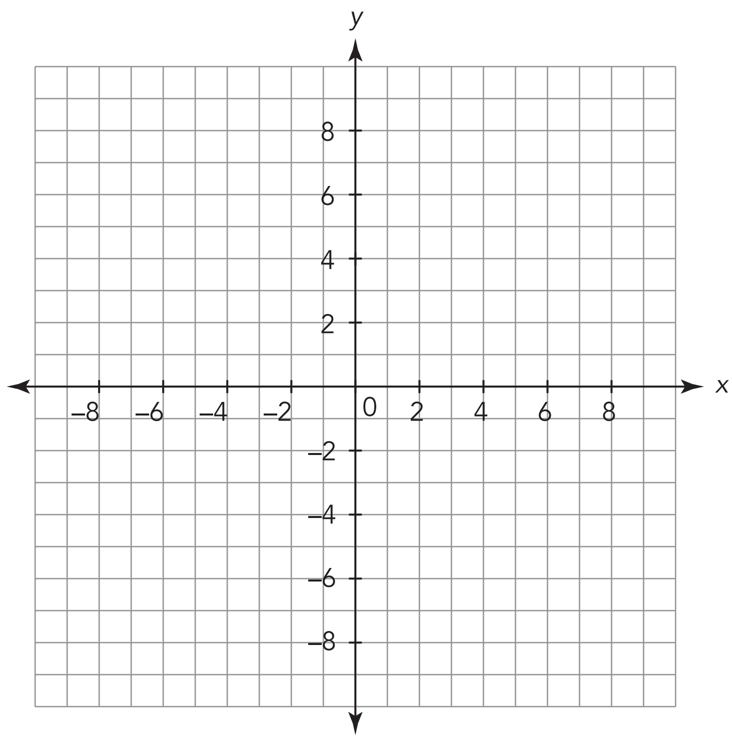
2. Keep the y-axis on your patty paper on top of the corresponding y-axis of the coordinate plane. Slide the line $y = x$ up and down the y-axis.

   a. How does the slope of the line change as you move it up and down the y-axis?

   b. How do the coordinates of the line change as you move it up and down the y-axis?

3. Translate the line $y = x$ up 4 units.

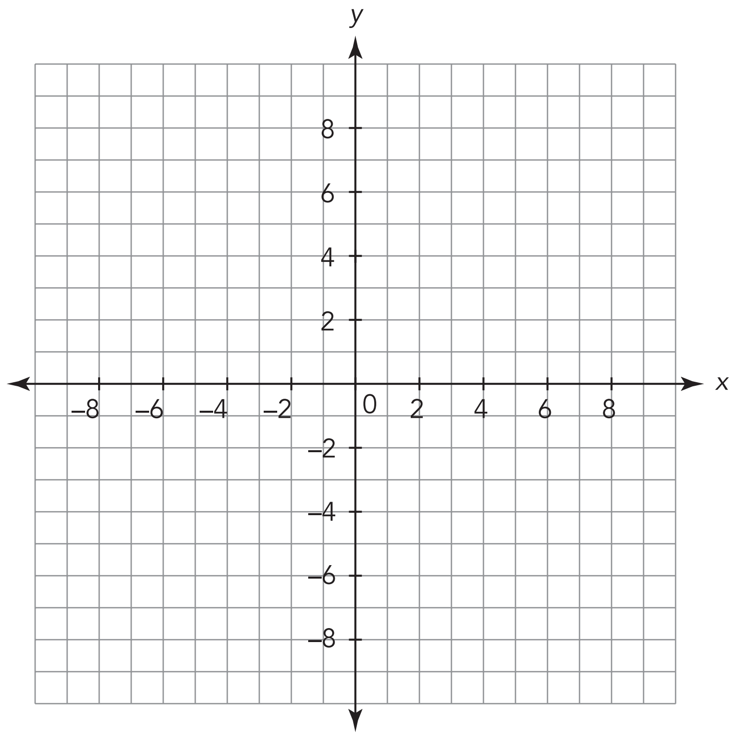
   a. Graph and label the line with its equation.



   b. Compare the equation of $y = x$ to the equation of its translation up 4 units. What do you notice?

4. Translate the line $y = x$ down 4 units.

   a. Graph and label the line with its equation.

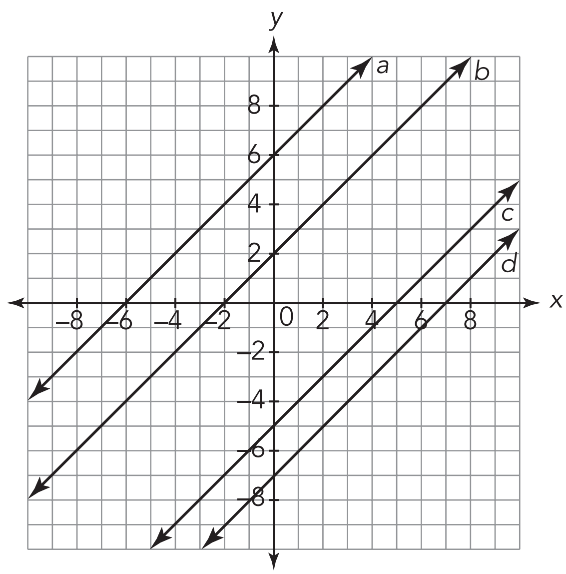


   b. Compare the graph and equation of $y = x$ to the graph and equation of its translation down 4 units. What do you notice?

5. For any x-value, how does the y-value change when you translate $y = x$ up or down?

6. Are the translated lines proportional or non-proportional relationships? Explain your reasoning.

7. The lines on the graph are translations of the line represented by $y = x$.



a. Describe each translation in terms of a translation up or down. Then, write the equation.

b. Identify the slope of each line.

The lines drawn on the coordinate plane in Question 7 represent parallel lines. Remember that parallel lines are lines that lie in the same plane and do not intersect no matter how far they extend. Parallel lines are always equidistant.

8. Analyze the graph of each line and its corresponding equation.

   a. How can you verify that the lines graphed are equidistant?

   b. How can you tell by looking at the set of equations that the lines are parallel?

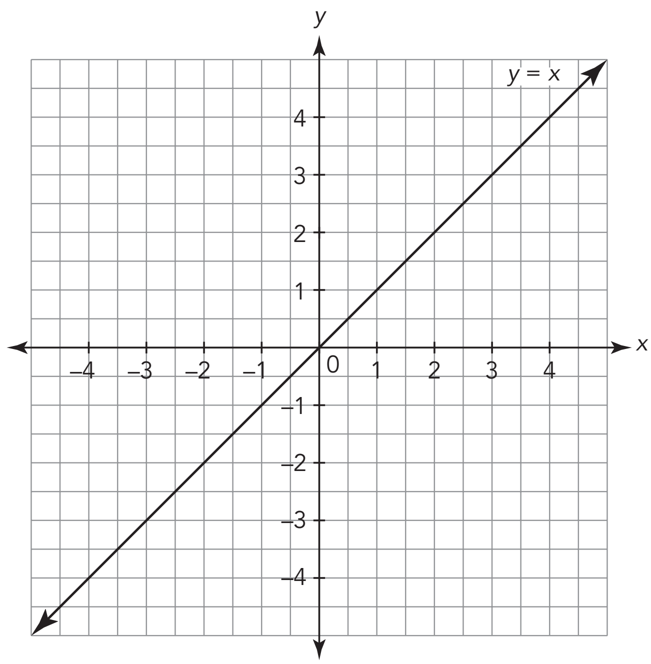
9. Based on your investigation, complete the sentence:

   The line $y = x + b$ is a ____________________ of the line $y = x$ that maps the point (0, 0) onto the point __________ and maps the point (1, 1) onto the point (1, __________).

ACTIVITY 4.2

Dilating Linear Equations

The graph of the basic linear equation $y = x$ is shown on the coordinate plane.



Let's investigate how the line $y = x$ changes when the rate of change, or slope, changes.

1. Use a thin piece of pasta to explore how the characteristics of the lines change as you dilate the line $y = x$ to create the lines with equation $y = 2x$ and $y = \frac{1}{2}x$. Then, complete the table based on your investigation.

x	$y = x$	$y = 2x$	$y = \frac{1}{2}x$	$y = mx$
−2				
0				
1				
2				
4				

2. Based on your investigation, complete the sentence:
The line $y = mx$ is a ______________ of the line $y = x$ that maps the point (0, 0) onto the point ______________ and maps the point (1, 1) onto the point (1, ______________).

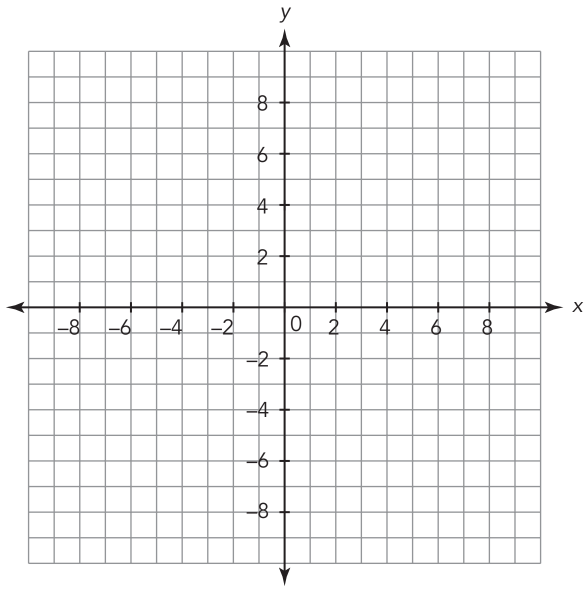
3. Consider the equation $y = \frac{3}{4}x$. Use transformations to complete the table of values. Explain your strategy.

x	$y = x$	$y = \frac{3}{4}x$
−2		
−1		
0		
1		
2		

4. The equation $y = -x$ is a transformation of $y = x$.

   a. How are the equations similar? How are they different?

b. Graph both equations to determine the transformation.



c. Based on your investigation, complete the sentence:

The line $y = -x$ is a ______________ of the line $y = x$ that maps the point (0, 0) onto the point ______________ and maps the point (1, 1) onto the point (1, _______).

ACTIVITY 4.3 Using Transformations to Graph Lines

You have explored how the basic linear equation $y = x$ is translated to create the equation $y = x + b$ or dilated to create the equation $y = mx$. In this activity, you will combine both dilations and translations to graph equations of the form $y = mx + b$.

1. Consider the set of equations.

   $y = 2x$

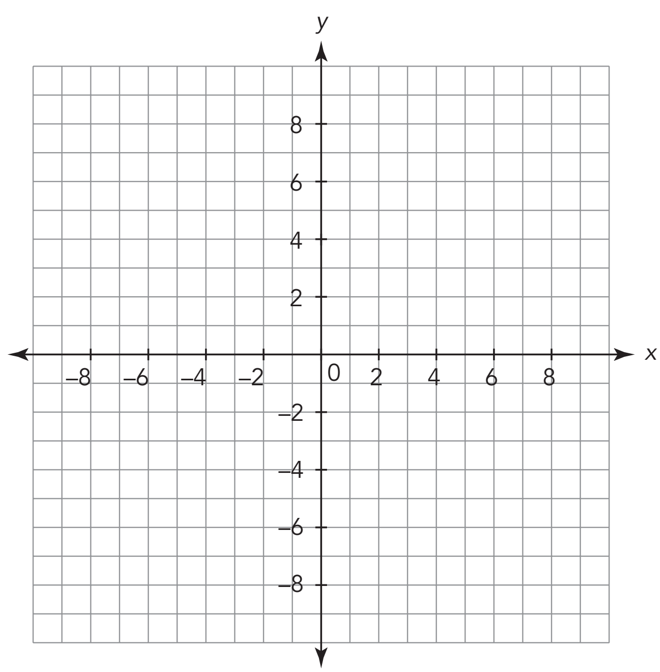
   $y = 2x + 3$

   $y = 2x - 5$

   $y = 2x + 5$

   a. What do all of the equations have in common?

   b. Use transformations to graph each equation on the coordinate plane.



   c. Describe the relationship among the lines.

2. Consider the set of equations.

$y = -3x$

$y = -3x - 2$

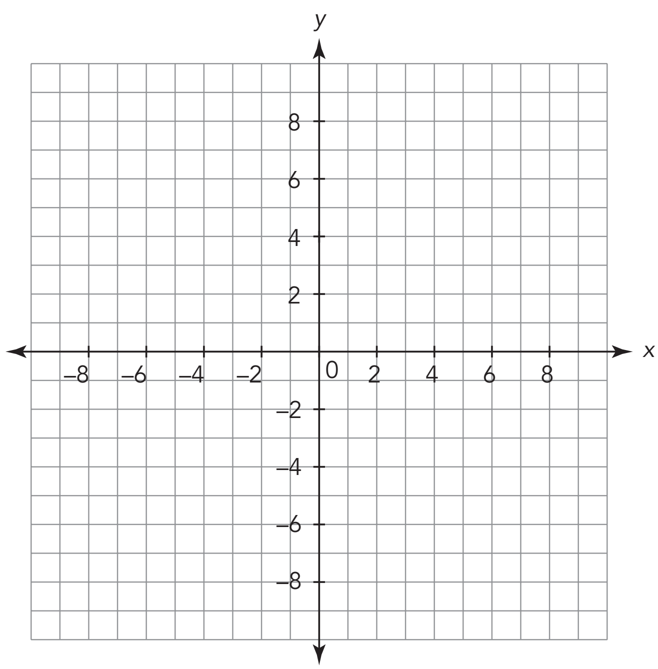
$y = -3x + 5$

$y = -3x - 8$

Conventionally, $y = -x$ is considered a reflection of $y = x$ across the x-axis.

a. What do all of the equations have in common?

b. Use transformations to graph each equation on the coordinate plane.



c. Describe the relationship among the lines.

d. Describe and use a strategy for verifying the relationship among the lines.

3. Consider these equations.

$y = \frac{1}{2}x$

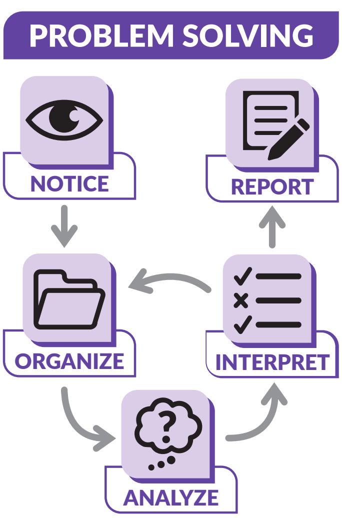
$y = \frac{1}{2}x + 6$

$y = \frac{1}{2}x - 3$

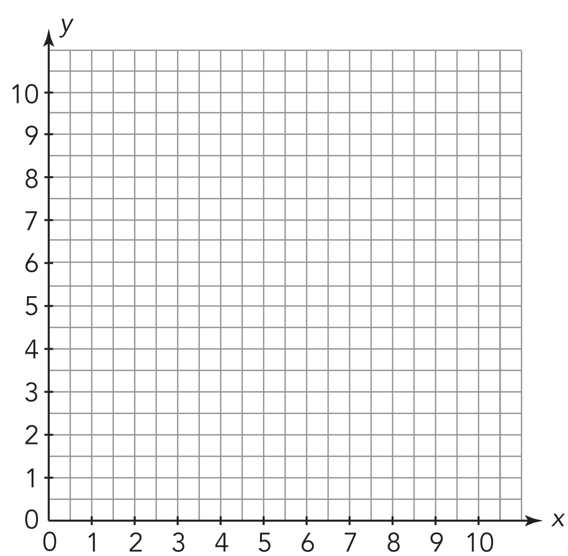
$y = \frac{1}{2}x - 2$

a. Without graphing, describe the graphical relationship among the lines.

b. Explain how you determined the relationship.



4. Determine if the quadrilateral formed by joining the points $A(3, 1)$, $B(8, 1)$, $C(10, 5)$, and $D(5, 5)$ in alphabetical order is a parallelogram.



Now that you understand linear equations in terms of transformations, you can use transformations to graph lines.

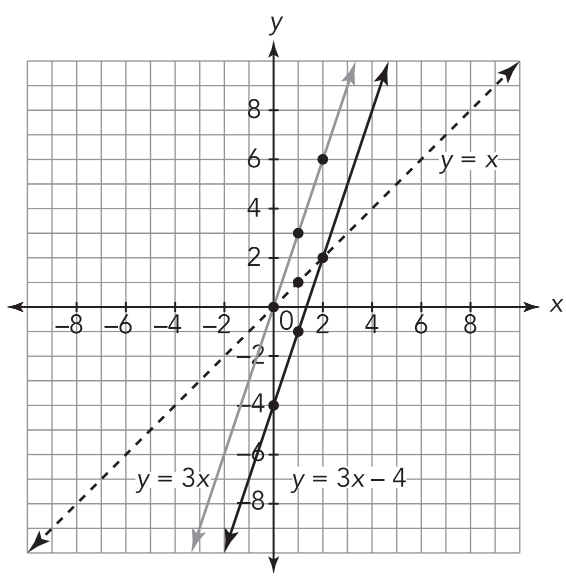
WORKED EXAMPLE

Graph $y = 3x - 4$ using transformations of the basic linear equation $y = x$.

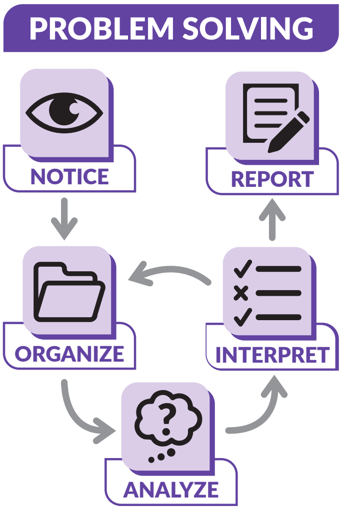
First, graph the basic equation, $y = x$, and consider at least 2 sets of ordered pairs on the line, for example (0, 0), (1, 1), and (2, 2).

Then, dilate the y-values by 3.

Finally, translate all y-values down 4 units.

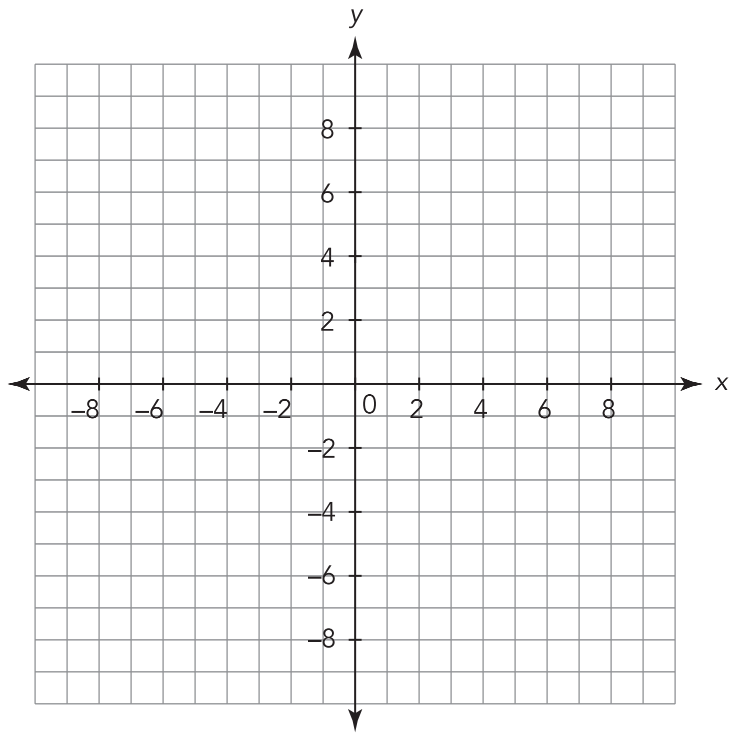


Could you have translated the line $y = x$ down 4 units first and then dilated the y-values by 3?

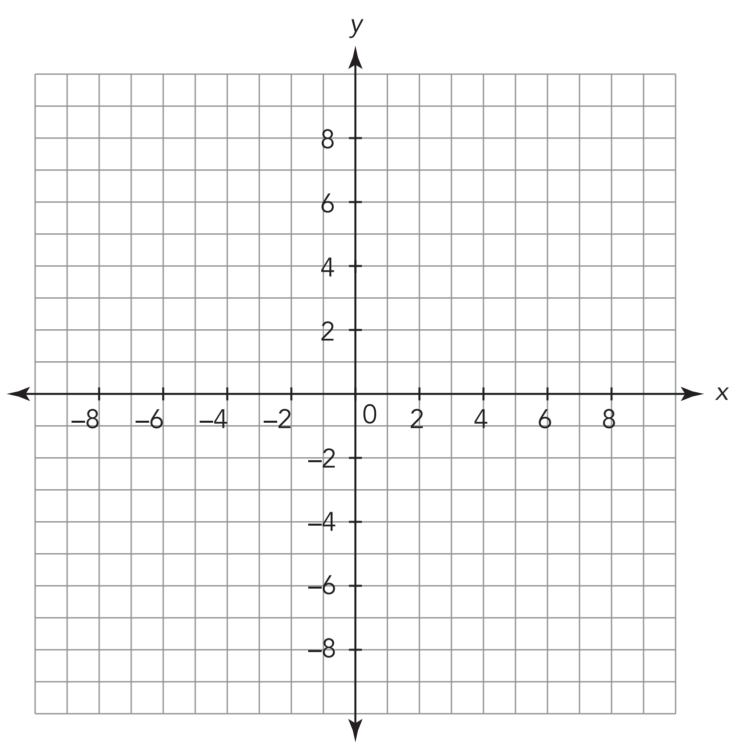


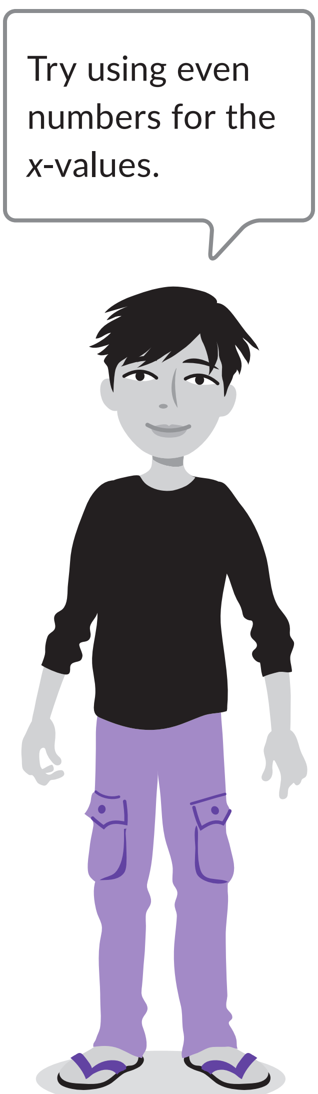
5. Graph each equation using transformations. Specify which transformations you use.

a. $y = \frac{1}{2}x + 5$



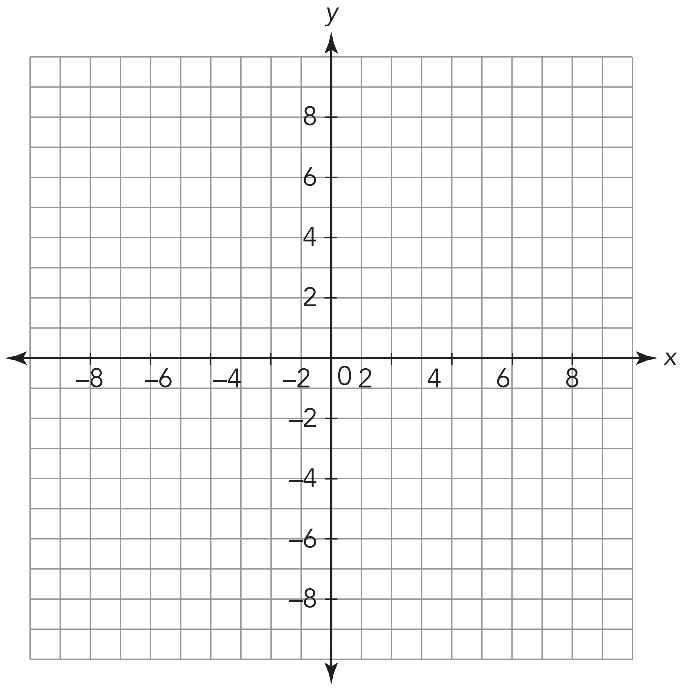
b. $y = \frac{3}{2}x - 3$





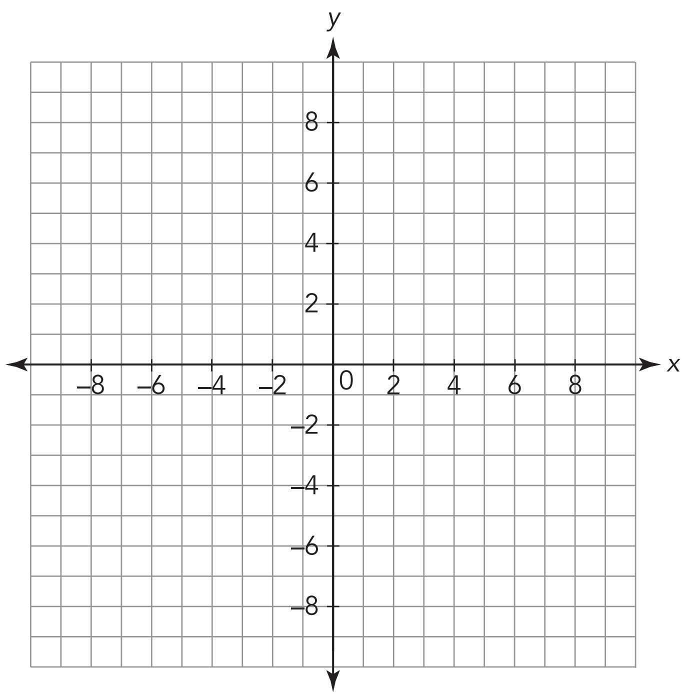
6. Create a table of values, a graph, and an equation to represent the description.

a. The value of y is 5 more than $3x$.



x	y

b. The value of y is 1 less than the opposite of x.



x	y

Talk the Talk

Are They Parallel?

1. Which transformations of linear graphs result in parallel lines? Explain each response.

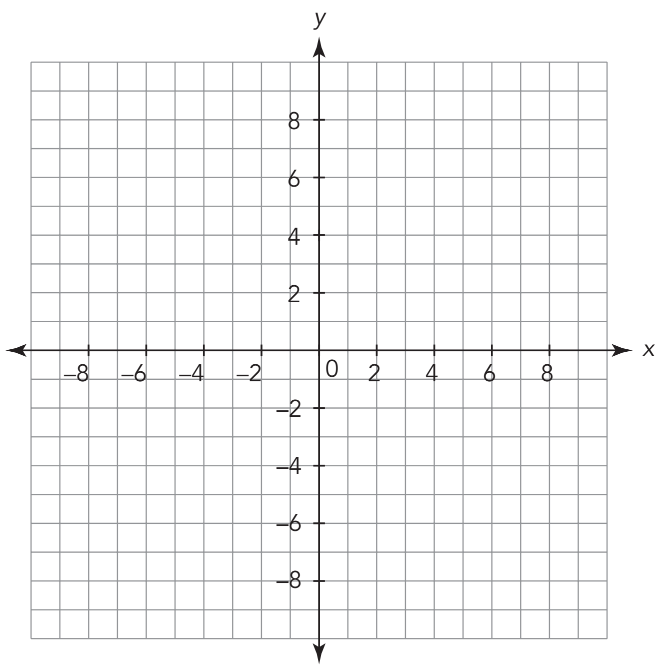
   a. dilation by a non-zero factor other than 1

   b. translation up or down

   c. reflection across an axis

   d. rotation 90° counterclockwise

2. Create and graph four linear equations that represent lines with the same slope. Label each line with its corresponding equation.



Lesson 4 Assignment

Write

Explain how to use transformations of the basic equation $y = x$ to graph the equation $y = mx + b$.

Remember

Translations, reflections, and rotations map parallel lines and line segments to corresponding parallel lines and line segments.

Practice

1. Write an equation for each linear relationship after transforming $y = x$.

   a. Dilation by a factor of $\frac{5}{6}$

   b. Dilation by a factor of 8

   c. Reflection across the x-axis

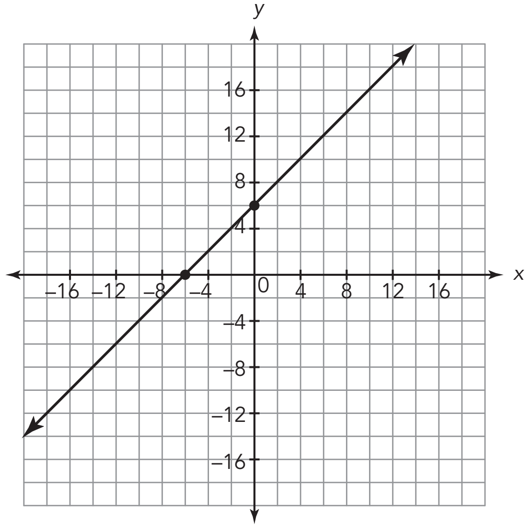
   d. Translation down 6 units

   e. Dilation by a factor of 2, then a translation up 3 units

   f. Reflection across the x-axis, dilation by a factor of 3, and then a translation down 9 units

Lesson 4 Assignment

2. Use the graph of the linear relationship shown to complete each task.



   a. Write the equation of the line.

   b. Write the equation of the line after a translation down 8 units. Graph the line.

   c. Write the equation of the line after a translation up 8 units. Graph the line.

Prepare

Determine the value of y in each equation for the given value of x.

1. $y = -2x + 4, x = 3.5$

2. $y = \frac{1}{2}x + 11, x = -1$

3. $x + y = 1, x = 0$

4. $2x - y = 5, x = 4$

TOPIC 1 SELF-REFLECTION

Name: ______________________________

From Proportions to Linear Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *From Proportions to Linear Relationships* topic by:

TOPIC 1: *From Proportions to Linear Relationships*	Beginning of Topic	Middle of Topic	End of Topic
graphing proportional relationships, interpreting the unit rate as the slope of the graph.	◯	◯	◯
comparing graphs of proportional relationships using the unit rate of change.	◯	◯	◯
comparing proportional relationships using multiple representations.	◯	◯	◯
using a table, an equation, or a graph to determine the unit rate of a proportional relationship.	◯	◯	◯
identifying the rate of change and unit rate of change between two quantities in a linear relationship.	◯	◯	◯
describing the slope of a line as a rate of change of the dependent quantity with respect to the independent quantity.	◯	◯	◯
deriving the equations $y = mx$, for a line through the origin, and $y = mx + b$, for a line intercepting the vertical axis at b.	◯	◯	◯

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *From Proportions to Linear Relationships*	Beginning of Topic	Middle of Topic	End of Topic
using similar triangles to explain why the slope, *m*, is the same between two points on a non-vertical line in a coordinate plane.	◯	◯	◯
distinguishing between proportional and non-proportional situations using tables, graphs, and equations.	◯	◯	◯
vertically translating linear relationships.	◯	◯	◯
using transformations to graph linear functions.	◯	◯	◯
determining the slopes of parallel lines.	◯	◯	◯

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *From Proportions to Linear Relationships* topic.

   ____

   ____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

   ____

   ____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

   ____

   ____

TOPIC 1 SUMMARY

From Proportions to Linear Relationships Summary

NEW KEY TERMS

- proportional relationship [relación proporcional]
- constant of proportionality [constante de proporcionalidad]
- non-proportional relationship [relación no proporcional]
- rate of change
- slope

LESSON 1 Representations of Proportional Relationships

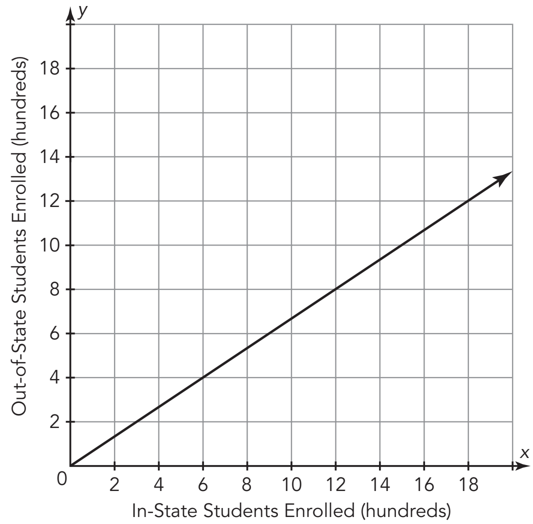
A **proportional relationship** is one in which the ratio of the inputs to the outputs is constant. For example, the ratio of in-state to out-of-state students at a university is 3 : 2.

Proportional relationships can be represented using tables, graphs, and equations.

In-State Students Enrolled in a University	Out-of-State Students Enrolled in a University
0	0
600	400
1500	1000

In a table, the values in a proportional relationship increase or decrease at a constant rate beginning or ending at (0, 0). The 3 : 2 ratio of in-state students to out-of-state students at a university is represented by this table.

On a graph, a proportional relationship is represented as a linear graph passing through the origin. The given graph shows the same proportional relationship as represented by the table.



The equation for a proportional relationship is written in the form $y = kx$, where x represents an input value, y represents an output value, and k represents some constant that is not equal to 0. The constant k is called the **constant of proportionality**. The 3 : 2 ratio of in-state students to out-of-state students at a university is represented by the equation $y = \frac{2}{3}x$. The constant of proportionality is $\frac{2}{3}$.

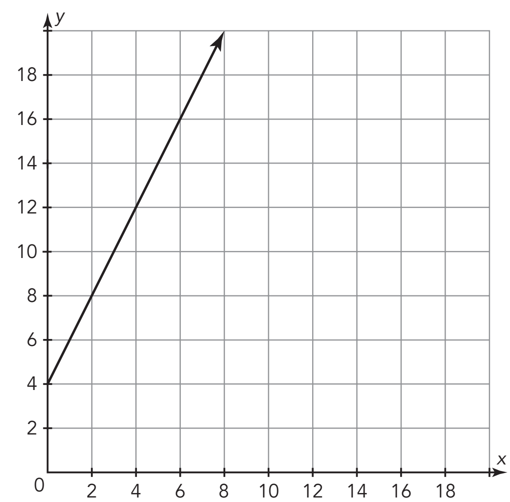
LESSON 2

Using Similar Triangles to Describe the Steepness of a Line

The **rate of change** for a situation is the amount that the dependent quantity changes compared with the amount that the independent quantity changes.

In any linear relationship, **slope** describes the direction and steepness of a line and is usually represented by the variable *m*. Slope is another name for the rate of change of a linear relationship graphed as a line. The slope of the line is constant between any two points on the line. The sign of the slope indicates the direction of a line. When the slope of a line is positive, then the graph will increase from left to right. When the slope of a line is negative, then the graph will decrease from left to right.

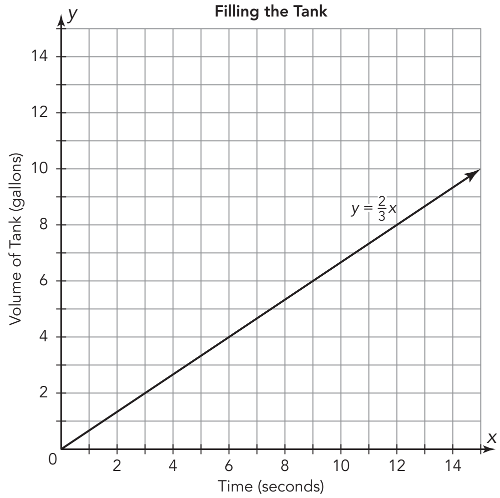
The equation $y = mx$ represents a proportional relationship. The equation represents every point (x, y) on the graph of a line with slope *m* that passes through the origin (0, 0).

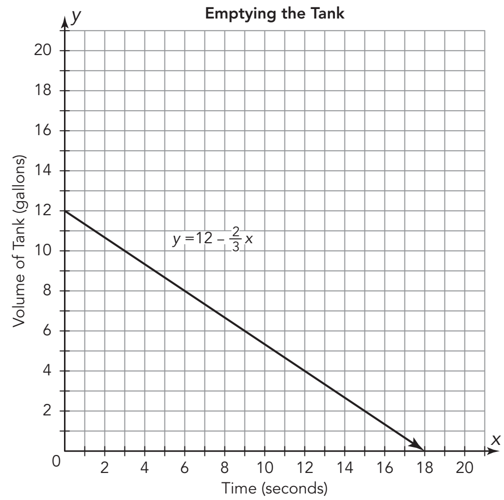


An equation in the form $y = mx + b$, where *b* is not equal to 0, represents a **non-proportional relationship**. This equation represents every point (x, y) on the graph of a line with slope *m* that passes through the point $(0, b)$. For example, the graph shown represents a non-proportional relationship where $m = 2$ and $b = 4$.

A line with a negative slope goes in the opposite direction. It decreases from left to right.

For example, consider the two graphs. The first represents a tank being filled at $\frac{2}{3}$ gallon per second. The second represents the tank being emptied at $\frac{2}{3}$ gallon per second, starting at 12 gallons.

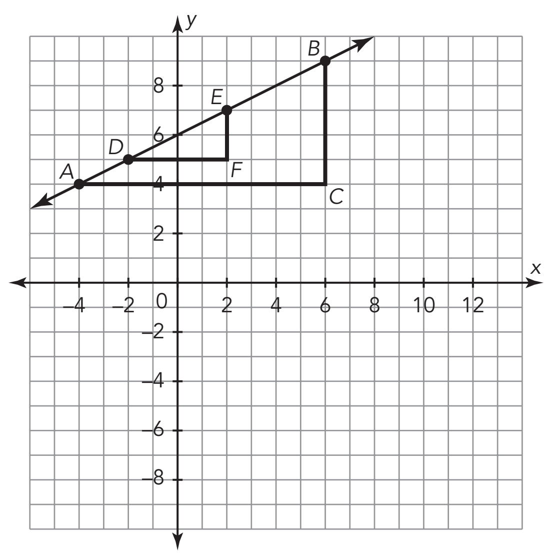




The slope of the line representing the tank being filled is $\frac{2}{3}$. You can draw a triangle to represent the slope of this line and then horizontally reflect it onto the line representing the tank being emptied. This shows that the slope of this line is $-\frac{2}{3}$.

LESSON 3 Exploring Slopes Using Similar Triangles

The properties of similar triangles can be used to explain why the slope, *m*, is the same between any two distinct points on a non- vertical line on the coordinate plane.



For example, Points *A*, *B*, *D*, and *E* along the graphed line can be used to create two right triangles on the coordinate plane.

Because $\angle BAC$ and $\angle EDF$ are corresponding angles on parallel lines cut by a transversal, you know that $\angle BAC \cong \angle EDF$. Likewise, because $\angle ABC$ and $\angle DEF$ are corresponding angles on parallel lines cut by a transversal, you know that $\angle ABC \cong \angle DEF$. Therefore, by the Angle-Angle (AA) Similarity theorem, $\triangle ABC$ is similar to $\triangle DEF$.

In both triangles, the ratio of the vertical distance to the horizontal distance is $\frac{1}{2}$. The slope of the line is the same between points *A* and *B* and between points *D* and *E*.

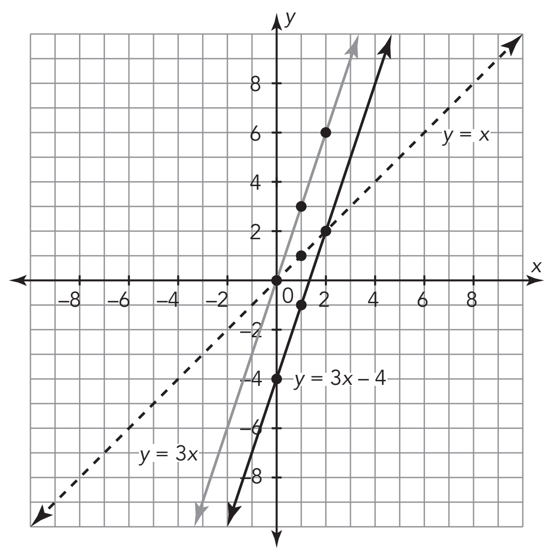
LESSON 4 Transformations of Lines

Translations, reflections, and rotations map parallel lines and line segments to corresponding parallel lines and line segments.

The line $y = x + b$ is a translation of the line $y = x$ that maps the point (0, 0) onto the point (0, b) and maps the point (1, 1) onto the point (1, $1 + b$).

The line $y = mx$ is a dilation of the line $y = x$, which maps the point (0, 0) onto the point (0, 0) and maps the point (1, 1) onto the point (1, m).

The line $y = -x$ is a reflection of the line $y = x$, which maps the point (0, 0) onto the point (0, 0) and maps the point (1, 1) onto the point (1, −1).



For example, you can graph $y = 3x - 4$ using transformations of the basic linear equation $y = x$.

First, graph the basic equation $y = x$, and consider at least two sets of ordered pairs on the line, for example, (0, 0), (1, 1), and (2, 2).

Then, dilate the y-values by 3. This is the graph of the equation $y = 3x$.

Finally, translate all y-values down 4 units.

GRADE 8

TOPIC 2

The elevation of these planes increases over time. The paths formed by these relationships are straight lines.

Linear Relationships

1 Using Tables, Graphs, and Equations

OBJECTIVES

- Construct a table of (x, y) values and a graph to model a linear relationship between two quantities.
- Use different representations to model a problem situation.
- Analyze the characteristics of different linear representations.
- Compare linear representations using tables, graphs, and equations.

You have analyzed linear relationships by considering points on the line and rate of change.

How can you compare two linear relationships in a problem situation?

Getting Started

Cost Analysis

You work at a local custom T-shirt shop over the summer. One of your responsibilities is to calculate the total cost of customers' orders. The shop charges $8 per shirt plus a one-time charge of $15 to set up a T-shirt design.

1. Describe the problem situation and your responsibility in your own words.

2. Is the relationship between the number of shirts ordered and the total cost of an order proportional or non-proportional? Explain how you know.

ACTIVITY
1.1 Modeling a Linear Relationship

Let’s analyze various customer orders with the local shop.

1. What is the total cost of an order for:

    a. 3 shirts?

    b. 10 shirts?

    c. 100 shirts?

    d. Explain how you calculated each total cost.

If the order doubles, does the total cost double?

2. How many shirts can a customer buy if they have:

    a. $50 to spend?

    b. $60 to spend?

    c. $220 to spend?

    d. Explain how you calculated the number of shirts that the customer can buy.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

Variable quantities are quantities that change, and *constant quantities* are quantities that don't change.

3. Identify the variable quantities and constant quantities in this problem situation. Include each quantity's units.

4. Identify the independent and dependent variables in the situation. Explain your reasoning.

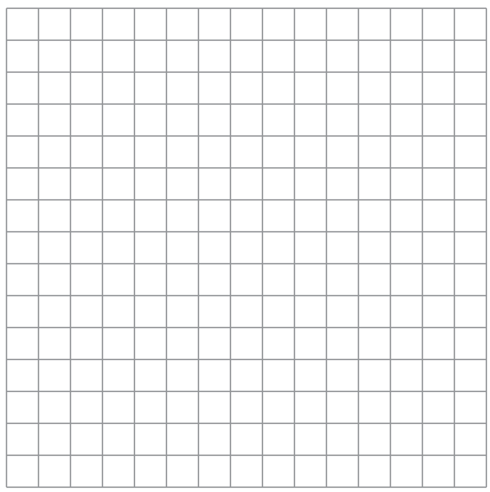
5. Complete the table of values for the local shop.

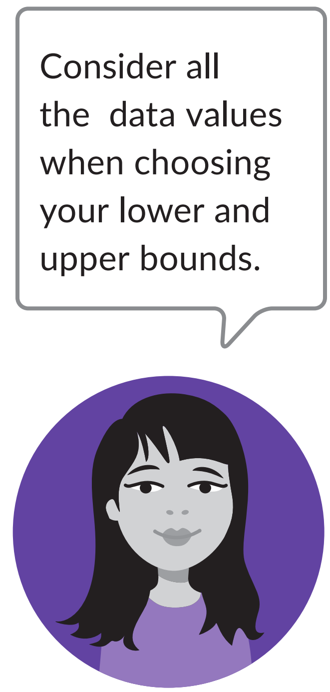
Number of Shirts Ordered	Total Cost (dollars)

6. Create a graph of the data from your table on the grid. First, choose your bounds and intervals by completing the table. Remember to label your graph clearly and provide a title.

Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Shirts			
Total Cost (dollars)			

Remember, you can draw a line through your points to model the relationship. You then need to decide whether or not all points on your line make sense in terms of the problem situation.





7. Define the variables and write an algebraic equation for this problem situation.

ACTIVITY 1.2 An Online Store

In your own words, describe this problem situation and how it will affect the business at the local shop.

Previously, you explored a job at the local shop. One of the local shop's competitors, an online store, advertises that it makes custom T-shirts for $5.50 each with a one-time set-up fee of $49.95 and free shipping. Your boss brings you the advertisement from the online store and asks you to figure out how the competition might affect business.

1. Determine the total customer cost of an order for:

   a. 3 shirts.

   b. 10 shirts.

   c. 50 shirts.

   d. 100 shirts.

2. Determine the number of shirts that a customer can purchase from the online store for:

   a. $50.

   b. $60.

   c. $220.

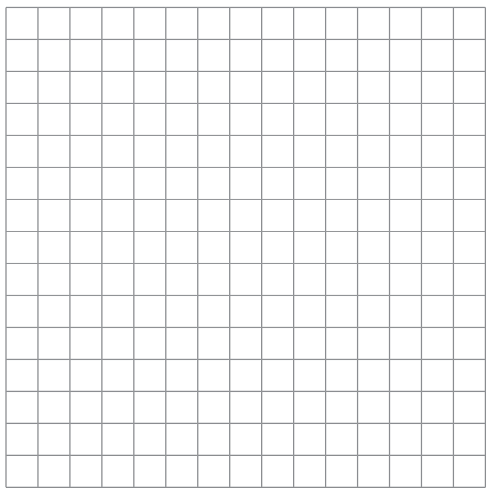
What is your initial prediction? Is the online store a strong competitor for the local shop?

3. Complete the table of values for the online store.

Number of Shirts Ordered	Total Cost (dollars)

4. Create a graph of the data from the table on the grid shown. First, choose your bounds and intervals by completing the table shown. Remember to label your graph clearly and provide a title.

Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Shirts			
Total Cost (dollars)			

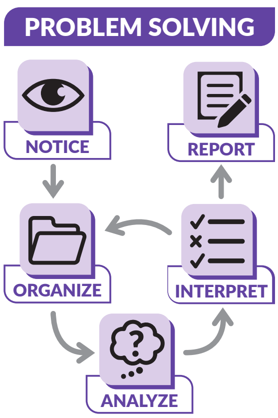


5. Define the variables and write an algebraic equation for this problem situation.

ACTIVITY 1.3

Comparing Linear Relationships

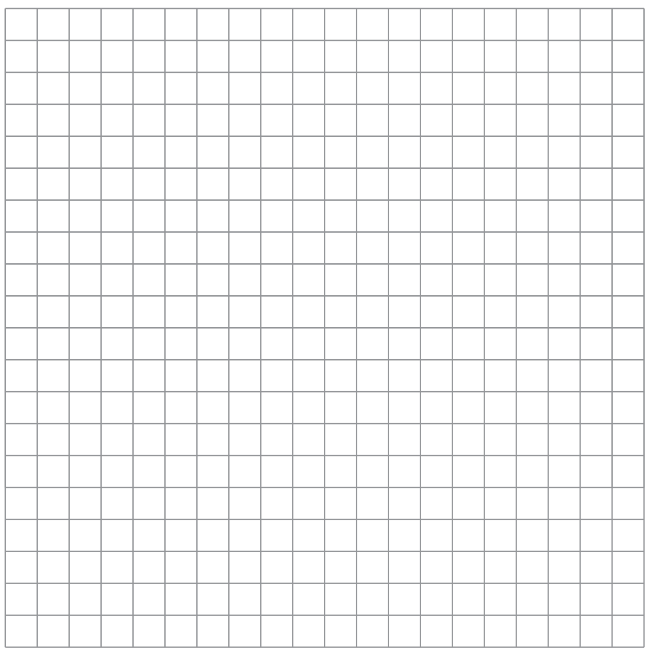
You have explored the costs of ordering T-shirts from two companies, the local shop and an online store. Your boss has asked you to determine which company has the better price for T-shirts in different situations.

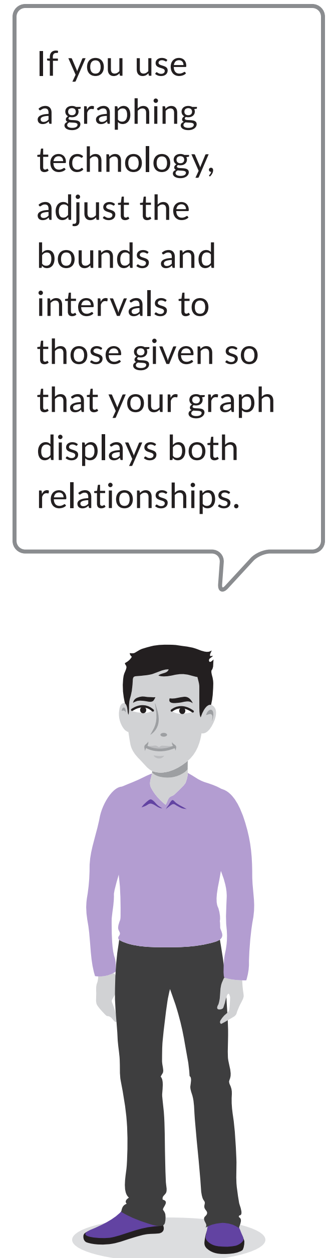


1. Compare the two businesses for orders of 5 or fewer shirts, 18 shirts, and 80 shirts. Is the local shop or the online store the better buy for each? What would each company charge? Describe how you calculated the values.

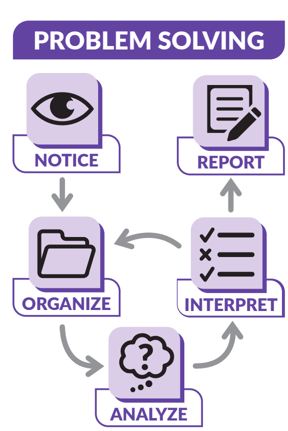
2. Create graphs for the total cost for the local shop and the online store on the grid shown. Use the bounds and intervals for the grid in the table shown. Label each graph and provide a title.

Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Shirts	0	100	5
Total Cost (dollars)	0	1000	50





3. Estimate the number of shirts for which the total cost is the same. Explain how you determined the number of shirts.



Talk the Talk

Business Report Presentation

Consider the graphs for the local shop and the online store. Notice that the graphs intersect at about (14, 127). This point of intersection indicates where the total cost for each business is the same. Therefore, when the local shop sells 14 shirts, the total cost is $127. When the online store sells 14 shirts, the total cost is $126.95, which is about the same as $127.

1. Prepare a presentation for your boss that compares the costs of ordering from each business.
   - Include a statement describing when it's better to buy from the local shop than the online store.
   - Include a statement listing the cost per shirt and set-up fee for each business.
   - Try to answer your boss's question: "Will the online store's prices affect the business at the local shop?"

Lesson 1 Assignment

Write

Describe how tables, graphs, and equations are related. Then, describe the advantages of each representation.

Remember

In mathematics, when representing quantities in a table, it is important to include a row to identify the quantities and units of measure. Typically, the independent quantity is represented in the left column and the dependent quantity is represented in the right column.

When graphing a relationship, the convention is to represent the independent quantity on the horizontal axis of a graph and the dependent quantity on the vertical axis. You should include labels on each axis.

When writing an equation in the form of $y = mx + b$, the x-value represents the independent variable, and the y-value represents the dependent variable. It is important to define the variables you choose.

Practice

1. A local shipping company bases its charges on the weight of the items being shipped. In addition to charging $0.40 per pound, the company also charges a one-time fee of $10 to set up a customer's account.

   a. How much does the shipping company charge a new customer to ship a package that weighs 20 pounds?

   b. How much does the shipping company charge a new customer to ship a package that weighs 50 pounds?

   c. Estimate the weight of a package if the shipping company charges a new customer $45 to ship the package.

   d. Write an equation for the problem situation.

Lesson 1 Assignment

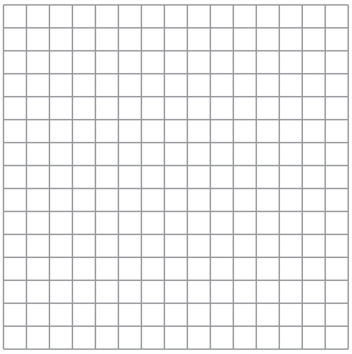
2. Twin brothers, Mario and Matthew, are looking for week-long winter break jobs. They are both offered jobs at grocery stores. Mario is offered a job at one grocery store $10 per hour. Matthew is offered a job at another grocery store making $8 an hour, plus a one-time hiring bonus of $100. Each twin believes that he has been offered the better job.

    a. Complete the table of values for the given number of hours worked.

Time Worked (hours)	Mario's Earnings (dollars)	Matthew's Earnings (dollars)
0		
20		
40		
60		

    b. Create a graph of the data from the table from part (e). First, choose your bounds and intervals. Remember to label your graph clearly and provide a title.

Variable Quantity	Lower Bound	Upper Bound	Interval
Time Worked			
Earnings			

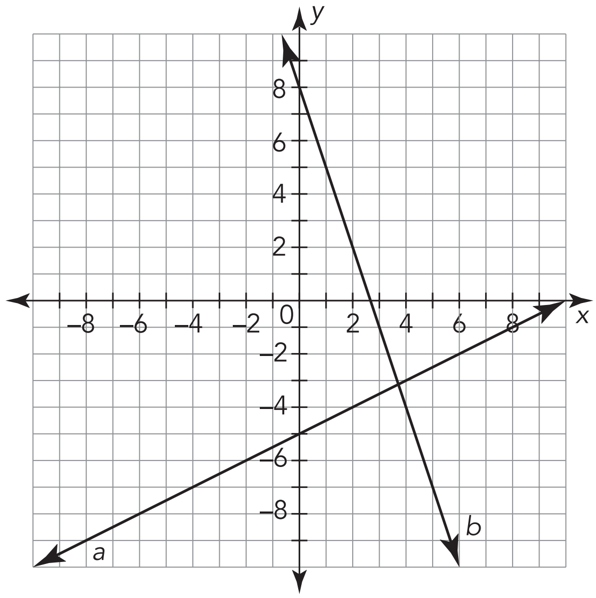


Lesson 1 Assignment

c. Whose job is better, Mario's or Matthew's? Explain your reasoning.

Prepare

1. Use similar right triangles to determine the slope of each line.



2 Linear Relationships in Tables

OBJECTIVES

- Determine the rate of change of a linear relationship by reading (x, y) values from a table.
- Develop a formula to calculate the slope of a line given a table of values.
- Use the slope formula to calculate the rate of change from a table of values or two coordinate pairs.
- Determine whether a table of values represents a linear proportional or linear non-proportional relationship.

NEW KEY TERM

- first differences

You have used graphs to analyze and compare linear relationships. You have used similar right triangles to determine slopes of lines graphed on a coordinate plane.

How can you calculate the slope of a linear relationship given a table of values without creating a graph?

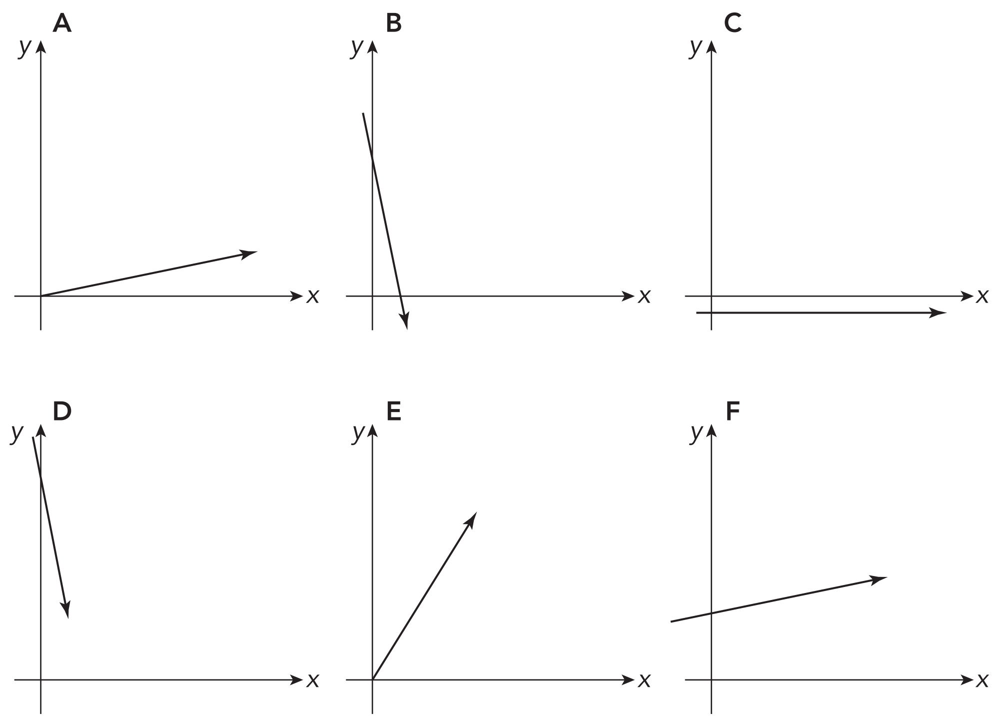
Getting Started

Slope Matching

Remember, the rate of change, or slope, of a line represents a ratio of the change in the dependent quantity to the change in the independent quantity.

You have used slope to describe the steepness and direction of a line. Consider each graph shown.

1. Identify the graph(s) whose line may have the given slope. Then, describe your strategy for matching the graphs to the given slopes.



a. $\frac{1}{4}$

b. 0

c. $\frac{5}{4}$

d. -3

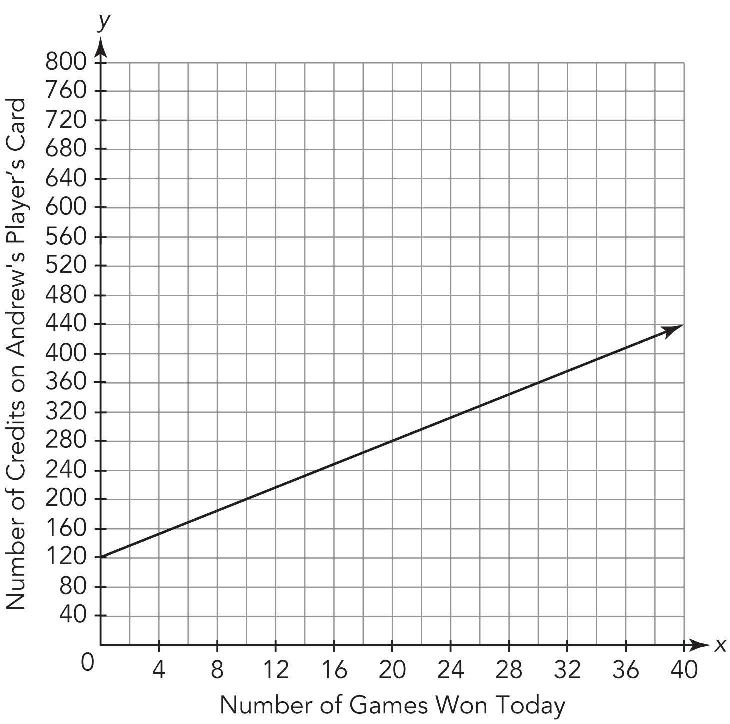
2. How did you use the graphs to estimate their slopes?

ACTIVITY 2.1

Analyzing a Linear Relationship from a Table

Andrew has a player's card for the arcade at the mall. His player's card keeps track of the number of credits he earns as he wins games. Each winning game earns the same number of credits, and those credits can be redeemed for various prizes. Andrew has been saving his credits to collect a prize worth 500 credits.

The table and graph show the number of credits Andrew had on his game card at various times today when he checked his balance at the arcade.



Number of Games Andrew Won Today	Number of Credits on Andrew's Player's Card
0	120
12	216
18	264
25	320
40	440

1. Is this relationship proportional or non-proportional? Explain how you know.

2. Explain the meaning of the ordered pair (0, 120) listed in the table.

3. Use the graph to determine the slope of the line. Then, explain the meaning of the slope in terms of this problem situation.

4. Analyze Kaya's reasoning. Explain why her reasoning is incorrect.

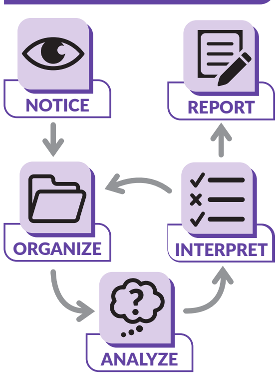
Kaya

$$\frac{440 \text{ credits}}{40 \text{ games won}} = \frac{11 \text{ credits}}{1 \text{ game won}}$$

The slope is 11.

5. Before Andrew started winning games today, how many games had he won for which he had saved the credits on his player's card? Show your work.

PROBLEM SOLVING



6. After Andrew wins 40 games today, how many more games does he need to win to collect a prize worth 500 credits? Show your work and explain your reasoning.

7. Summarize what you know about this scenario based on your analysis. Be sure to include each item listed.

- The initial values of the independent and dependent variables in the context of the problem
- A sentence explaining the rate of change in terms of the context of the problem
- The final values of the independent and dependent variables in the context of the problem

Ask Yourself . . .
What precise mathematical language do you need to communicate your mathematical reasoning?

ACTIVITY 2.2

Calculating Rate of Change from a Table

So far, you have determined the rate of change from a graph using similar triangles and writing a ratio of the vertical distance to the horizontal distance. However, you can also determine the rate of change, or slope, from a table.

1. Complete the steps to determine the slope from a table.

Number of Games Andrew Won Today	0	12	18	25	40
Number of Credits on Andrew's Player's Card	120	216	264	320	440

a. Choose any two values of the independent variable. Calculate their difference.

b. Calculate the difference between the corresponding values of the dependent variable. It is important that the order of values you used for determining the difference of the independent variables be followed for the dependent variables.

c. Write a rate to compare the change in the dependent variable to the change in the independent variable.

d. Rewrite the rate as a unit rate.

2. Examine each example. Follow the arrows to calculate the slope. Was the slope calculated correctly in each case? Explain your reasoning.

Example 1

Number of Games Andrew Won Today	Number of Credits on Andrew's Player's Card
0	120
12	216
18	264
25	320
40	440

Example 2

Number of Games Andrew Won Today	Number of Credits on Andrew's Player's Card
0	120
12	216
18	264
25	320
40	440

Example 3

Number of Games Andrew Won Today	Number of Credits on Andrew's Player's Card
0	120
12	216
18	264
25	320
40	440

There is a formal mathematical process that can be used to calculate the slope of a linear relationship from a table of values with at least two coordinate pairs.

The slope can be calculated using two ordered pairs and the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where the first point is (x_1, y_1) and the second point is (x_2, y_2).

WORKED EXAMPLE

You can calculate the slope of a linear relationship from a table of values. Consider the table showing the number of credits Andrew had on his game card at various times at the arcade.

Number of Games Andrew Won Today	Number of Credits on Andrew's Player's Card
0	120
12	216
18	264
25	320
40	440

Step 1: From the table of values, use (12, 216) as the first point and (25, 320) as the second point.

Step 2: Label the points with the variables.

$$\begin{array}{cc} (12,\ 216) & (25,\ 320) \\ \downarrow\ \ \downarrow & \downarrow\ \ \downarrow \\ (x_1,\ y_1) & (x_2,\ y_2) \end{array}$$

Step 3: Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{320 - 216}{25 - 12}$$

$$= \frac{104}{13}$$

$$= 8$$

The slope is $\frac{8 \text{ credits}}{1 \text{ game}}$ or 8 credits per game.

Does it make a difference which points you choose?

3. Repeat the process to calculate the slope using two different values from the table. Show your work.

4. How is using the slope formula given a table related to using similar triangles given a graph?

ACTIVITY 2.3

Practice with Linear Relationships in Tables

You can now use the slope formula to calculate the slope of a line given a table of values.

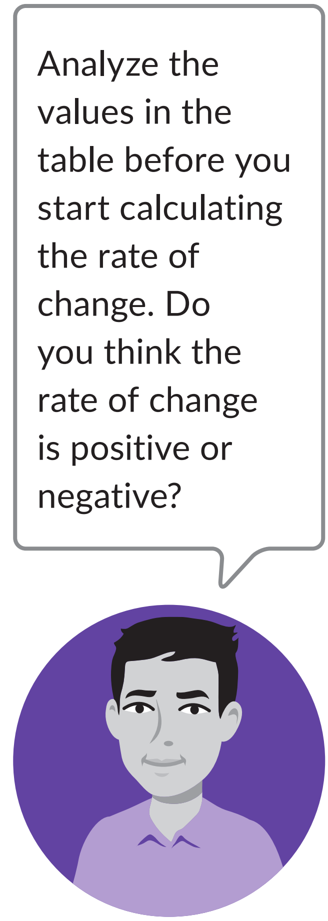
1. Calculate the slope of each linear relationship using the formula. Show all your work.

a.

Number of Carnival Ride Tickets	Cost (dollars)
4	9
8	12
16	18
32	30

b.

x	y
−1	13
0	−2
4	−62
10	−152



c.

Days Passed	7	8	9	10
Vitamins Remaining in Bottle	25	23	21	19

d.

x	7	18	29	40
y	9	9	9	9

2. Which relationships in Question 1 are proportional relationships? Explain your reasoning.

3. Complete each sentence to describe how you can tell whether the slope of a line is positive or negative by analyzing given points.

   a. If the slope of a line is positive, then as the value of x increases the value of y ________________.

   b. If the slope of a line is negative, then as the value of x increases the value of y ________________.

4. Consider the relationship represented in each table shown.

$x = 1$	
x	y
1	−5
1	10
1	15
1	30

$y = 2$	
x	y
5	2
6	2
7	2
8	2

   a. Sketch a graph of each relationship. What do you notice?

   b. Calculate the slope of each line.

ACTIVITY
2.4

Determining Whether a Relationship Is Linear

You previously used similar right triangles to show that if you are given a line on a graph, then the slope is the same between any two points on that line. The converse is also true. If the slope between every ordered pair in a table of values is constant, then the ordered pairs will form a straight line.

So, in order to determine if a table of values represents a linear relationship, show that the slope is the same between every set of ordered pairs.

A conditional statement uses the words "if" and "then" to show assumptions and conclusions. For example, if today is Monday, then tomorrow is Tuesday. A converse statement switches the order. For example, if tomorrow is Tuesday, then today is Monday. For any conditional statement, the converse may or may not be true.

1. Calculate the slope between the given ordered pairs to determine if they form a straight line. Show your work.

x	y
4	13
9	28
11	34
16	47

a. (4, 13) and (9, 28)

b. (9, 28) and (11, 34)

c. (11, 34) and (16, 47)

d. Does the table of values represent a linear relationship? Explain your reasoning.

2. Determine whether the ordered pairs listed in each table will form a straight line when plotted. Show your work. Explain your reasoning.

a.

x	y
2	7
6	13
8	16
20	34

b.

x	y
1	33
2	40
3	47
4	54
5	61

When the values for the independent variable in a table are consecutive integers, you can examine only the column with the dependent variable and calculate the differences between consecutive values. If the differences are the same each time, then you know that the rate of change is the same each time. The relationship is a linear relationship.

Consecutive means *one right after the other*, such as 12, 13, and 14.

WORKED EXAMPLE

The differences have been calculated for the table shown.

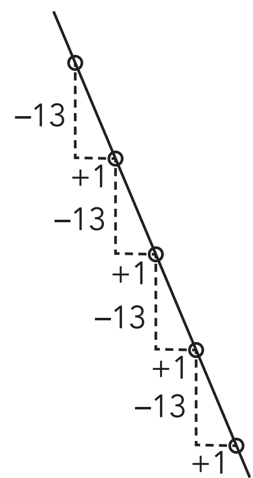
x	y
1	99
2	86
3	73
4	60
5	47

$86 - 99 = -13$

$73 - 86 = -13$

$60 - 73 = -13$

$47 - 60 = -13$



The differences between consecutive values for the dependent variable are the same each time. Therefore, the rate of change is the same each time as well. The ordered pairs in this table will form a straight line when plotted.

In this process, you are calculating *first differences*. **First differences** are the values determined by subtracting consecutive *y*-values in a table when the *x*-values are consecutive integers. The first differences in a linear relationship are constant.

3. Use first differences to determine whether the ordered pairs in each table represent a linear relationship. Show your work and explain your reasoning.

a.

x	y
1	25
2	34
3	45
4	52
5	61

b.

x	y
1	12
2	8
3	4
4	0
5	−4

c.

x	y
1	1
2	4
3	9
4	16
5	25

d.

x	y
1	15
2	18
3	21
4	24
5	27

Talk the Talk

Walk the Walk

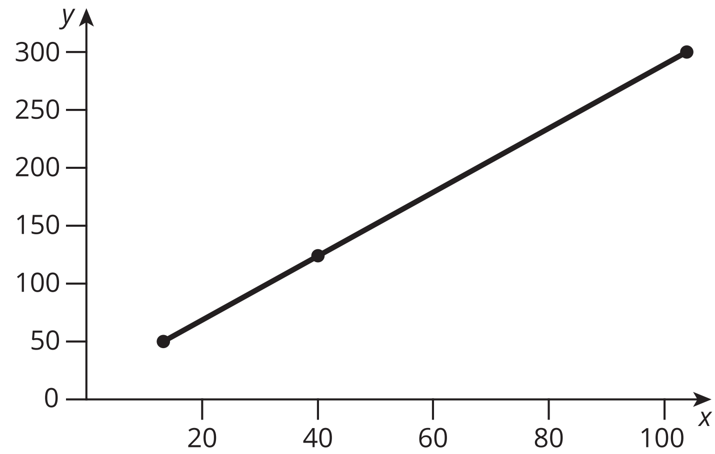
The table shows the distance Samantha walked compared to the number of steps she took.

Number of Steps	Distance Walked (ft)
16	50
40	120
110	300

1. Calculate the slope between each set of ordered pairs. Show your work.

2. Is the graph of the relationship linear? What does this mean in terms of the problem situation?

3. The ordered pairs from the table are represented on the given graph. Show how to use the graph to verify the slope you calculated from the table.



4. How is calculating the slope from a table similar to calculating the slope of a linear relationship from a graph?

Lesson 2 Assignment

Write

Define the term *first differences* in your own words.

Remember

You can use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to determine the rate of change between two points represented in a table of values. If the rate is constant, this formula gives the rate of change for the relationship, or slope. The slope of a horizontal line is 0. The slope of a vertical line is undefined.

Practice

1. Each table represents a linear relationship. Which table(s) represent a slope of 2?

Table 1

x	y
0	32
3	26
5	22
9	14

Table 2

x	y
1	3
2	5
3	7
4	9

Table 3

x	y
0	8
3	14
7	22
9	26

2. Calculate the rate of change between the points listed in each table. Determine if the table represents a proportional relationship.

a.

x	y
2	14
5	35
7	49
10	70

b.

x	y
−10	50
−2	10
4	−20
14	−70

c.

x	y
−1	−24
2	48
4	90
8	192

Lesson 2 Assignment

d.

x	y
−6	12
−3	6
3	−6
6	−10

e.

x	y
2	13.5
5	33.75
10	67.5
15	101.25

f.

x	y
−4	−38
−1	−9.5
2	19
3	27

Prepare

The lunch special at the pizza shop is two slices of pizza for $5.00.

1. Express the cost of the pizza as a unit rate.

2. Create a table to represent this context.

Number of Slices of Pizza					
Cost (dollars)					

3. Write an equation to represent this situation. Define your variables.

3 Linear Relationships in Contexts

OBJECTIVES

- Determine the slope from a context.
- Connect the rate of change represented in a context to the rate of change in other representations.
- Interpret the rate of change of a linear relationship in terms of the situation it models.
- Generate the values of two coordinate pairs from information given in contexts.
- Determine the independent and dependent quantities from contexts.

You have analyzed linear relationships in graphs and tables.

How can you determine rates of change from word problems that do not describe a rate of change?

Getting Started

Dependent on Your Point of View

Identify the dependent quantity and the independent quantity in each problem situation.

1. Trung is purchasing canned vegetables at his local grocery store to donate to the local food pantry. Each can costs $0.59.

2. The amount of electricity used by a light changes as the knob on the dimmer switch is turned.

3. Paola is selling cookies to raise money for her local troop. For each box of cookies she sells, the troop receives $2.00.

4. How would each problem situation change if you switched the independent and dependent quantities? Would each problem still make sense?

ACTIVITY 3.1

Choosing Independent and Dependent Quantities

You can choose different independent and dependent quantities to model the same information, depending on what you want to know. Once you have determined the independent and dependent quantities, you need just two points to determine the slope, or unit rate.

Eduardo took a road trip with his family to visit Big Bend National Park in Texas. Some information about their trip is shown in the table.

Total Miles	Total Cost for Gas ($)	Total Gallons
2600	200	80

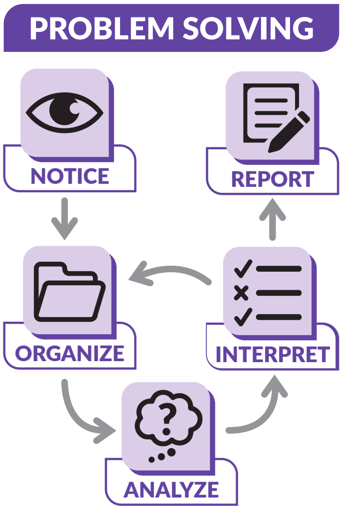
1. After they arrived, Eduardo was curious about how many miles per gallon their car got on the trip.

   a. Given this question, what are the independent and dependent quantities?

   b. Write the ordered pairs of two points you can use to answer the question. Explain what each of your ordered pairs means in terms of the situation.

   c. Determine the rate. Explain what this means in terms of the situation.

   d. How many miles per gallon did their car get on the road trip?



2. The family wants to know about how many gallons of gas on average they used for each mile of the trip.

   a. Given this question, what are the independent and dependent quantities?

   b. Write the ordered pairs of two points you can use to answer the question. Explain what each of your ordered pairs means in terms of the situation.

   c. Determine the rate. Explain what this means in terms of the problem situation.

   d. What was the family's average gallons per mile for the trip? Round to the nearest hundredth.

3. If the family had flown, they would have traveled 2100 miles and spent $3250 for tickets alone. Compare the costs per mile for flying and driving. Determine the independent and dependent quantities and rates for each relationship. Show your work.

ACTIVITY 3.2

Determining Slope from Context

For each context, complete each task.

- Identify the independent and dependent quantities.
- Write the ordered pairs of two points you can use to answer the question. Explain what each of your ordered pairs means in terms of the situation.
- Then, determine the rate described.

Ask Yourself . . .
How can you use slope in everyday life?

1. A pizzeria charges $9.00 for a large pizza. Toppings cost extra, depending on the size of the pizza ordered. Trung ordered a large pizza with three toppings that cost a total of $12.60. What is the cost per number of toppings for a large pizza?

2. After 10 shifts, a maintenance crew paves 1.25 miles of road. After 45 shifts, the crew paves 5.625 miles of road. At what rate is the maintenance crew paving the road in miles per shift?

3. Mariana bakes 15 dozen breakfast rolls in 3 hours. After 8 hours, she has baked 40 dozen breakfast rolls. At what rate does Mariana bake breakfast rolls each hour?

4. Ashley's dog has been put on a diet by his veterinarian. He weighs 149 pounds after 8 weeks on his diet. By Week 13, he weighs 134 pounds. What is his average weight loss per week?

Solve each problem.

5. Lizzie works after school to finish assembling the 82 favors needed for the school dance. When she starts at 3:15 p.m., she counts the 67 favors already assembled. She works until 4:30 p.m. to finish the job.

   a. How many favors can Lizzie assemble each minute?

   b. How many minutes does it take Lizzie to assemble one favor?

   c. Which rate is more meaningful in this situation? Explain your reasoning.

6. Jorge rented a moving van to travel across the country. The odometer registered 34,567 miles after he drove for 4 hours. After 7 hours of driving, the odometer read 34,741 miles. What was Jorge's driving rate in miles per hour?

7. Tiara uses a gift card to buy iced tea for herself and her friends. On Monday, after she and a friend ordered, the balance on Tiara's gift card was $14.85. On Tuesday, after she and three friends ordered, she had a balance of $3.97 on her gift card. Determine the cost for one glass of iced tea.

Talk the Talk

And Stamps and Tickets

A context may or may not provide a rate of change. When a problem situation does not provide the rate of change, you can see whether it is possible to generate two coordinate pairs from the information to determine the rate of change.

1. A book of 20 postage stamps costs $8.80. What is the cost of one postage stamp?

2. Ticket sales for a local concert totaled $101,244 yesterday. After the ticket window closed today, the cashiers counted 968 tickets sold with a two-day total of $143,836. What is the cost of one concert ticket?

Number of Tickets Sold Today	Total Amount of Sales (dollars)

3. List two similarities between Questions 1 and 2.

4. List two differences between Questions 1 and 2.

Lesson 3 Assignment

Write

Describe how to use the independent and dependent quantities in a word problem to determine the rate of change, or slope.

Remember

Two ordered pairs are needed to determine a unit rate given a real-world problem situation.

Practice

1. Elena makes jewelry to sell at a craft fair. On Monday, she makes 12 bracelets. On Tuesday, she works an additional 2.5 hours and has a total of 22 bracelets. Determine the time it takes Elena to make one bracelet.

2. The organizers of a festival expect 10,000 people to attend over four days. At the end of the festival, the organizers say that they have exceeded their expected attendance by 2000 people. Determine the average number of people that attended the festival per day.

3. Alyssa bakes croissants for a community center bake sale. She already has a number of batches baked from the day before. After 2.5 hours, Alyssa has a total of 8 batches. After 4 hours, she has a total of 11 batches. Determine the number of batches Trung baked per hour.

Lesson 3 Assignment

4. Jamal sells his photographs at an art festival. The festival is open for 6 hours each day for 3 days. At the conclusion of the festival, Nelson has sold 54 photographs. Determine the average number of photographs Jamal sold per hour.

5. Chris and Kaya purchase admission to a carnival plus a number of game tickets. Chris purchases 10 game tickets and spends $19.50. Kaya purchases 15 game tickets and spends $23.25. Determine the cost per game ticket.

6. Josh plans a hiking trip. The trail he would like to follow is 7.5 miles long. He plans to start his hike at 10:00 a.m. He hopes to reach the end of the trail at 3:00 p.m. Determine the average number of miles per hour that Josh plans to hike.

Lesson 3 Assignment

Prepare

Solve each equation for *y*.

1. $4 = \frac{y - 5}{3}$

2. $\frac{1}{2} = \frac{y + 3}{7}$

3. $-\frac{3}{4} = \frac{y - 17}{25}$

4. $-\frac{9}{5} = \frac{y + 31}{-8}$

4 Slope-Intercept Form of a Line

OBJECTIVES

- Write the *y*-intercept as an ordered pair.
- Determine the *y*-intercept of a linear equation from a context, a table, a graph, or an equation.
- Explain the meaning of the *y*-intercept, or initial value, when given the context of a linear equation.
- Write equations of lines in slope-intercept form.
- Analyze linear relationships using slopes and initial values.

NEW KEY TERMS

- *y*-intercept
- slope-intercept form

You have learned how to calculate the slope of a line given a graph, table, or context.

How can you determine the initial value in a linear relationship from a table, an equation, or a graph?

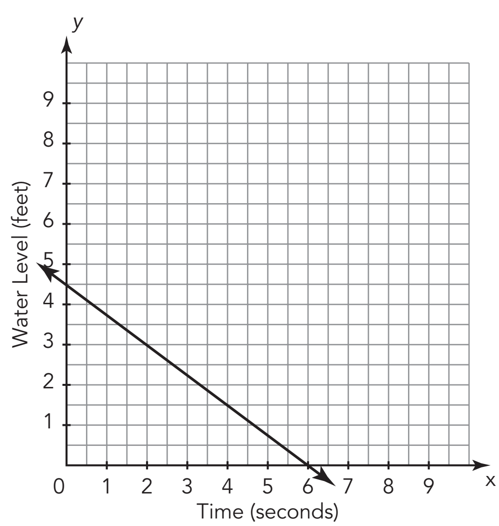
Getting Started

Introducing the *y*-Intercept!

The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept. The **y-intercept** is the y-coordinate of the point where a graph crosses the y-axis represented by the ordered pair $(0, b)$.

For each graph, determine the y-intercept, write it as an ordered pair, and explain its meaning.

1.



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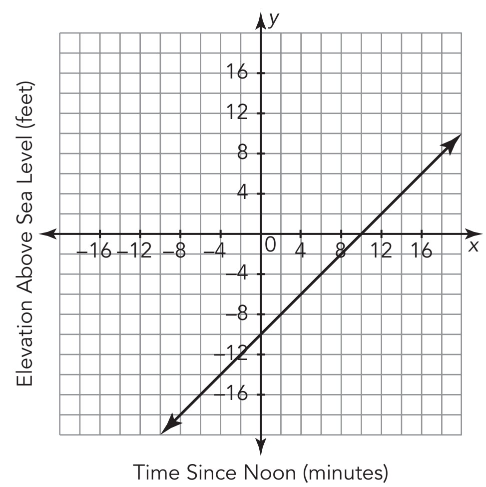
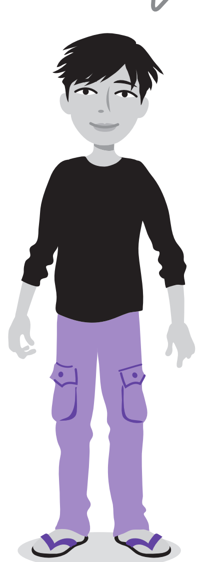
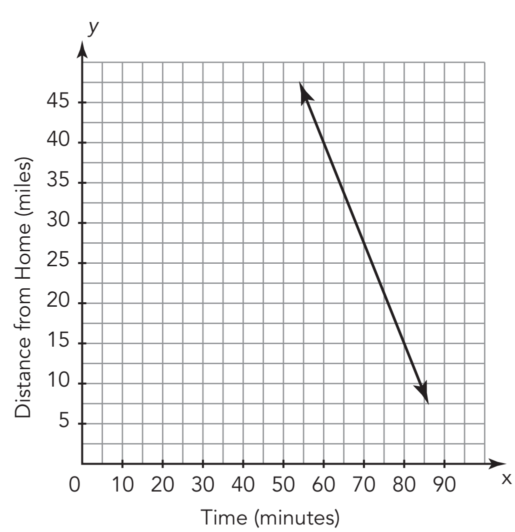


How can you use the slope to think about where each graph would cross the y-axis?



3.



ACTIVITY 4.1

Determining the y-Intercept

Just as you can determine the slope of a linear equation from a table of values or a problem situation, you can also determine the *y*-intercept. Let's start with what you already know: the slope formula and slope-intercept form of a line.

The table of values represents a linear relationship between the variables *x* and *y*.

x	*y*
2	7
3	10
4	13

Review the definition of **represent** in the Academic Glossary.

WORKED EXAMPLE

You can use the slope formula and slope-intercept form of a line to determine the *y*-intercept (0, *b*) for the graph of a linear relationship.

- First, determine the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 7}{3 - 2} = \frac{3}{1} = 3$$

- Next, choose any point from the table.

(4, 13)

- Now, substitute what you know into the slope intercept equation of a line, $y = mx + b$: $m = 3$, and (4, 13).

$$y = mx + b$$
$$13 = 3(4) + b$$
$$13 = 12 + b$$

- Finally, solve for the value of *b*.

$$1 = b$$

The *y*-intercept is (0, 1).

1. How would the Worked Example change if different points were chosen to calculate the slope? Explain your reasoning.

2. Use a different point from the table to calculate the *y*-intercept. Do you get the same *y*-intercept?

Each table represents a linear relationship. Determine the y-intercept using the slope formula and the slope-intercept form of a line. Write the y-intercept in coordinate form.

3.

x	y
200	14
225	16
250	18
275	20
300	22

4.

x	y
16	90
19	91
22	92
25	93
28	94

5. A fitness center charges an initial membership fee to join the center plus an additional fee per month to remain a member. The table shows the total cost, y, of being a fitness center member for x months.

Number of Months	1	3	6	10
Total Cost	140	220	340	500

Each context represents a linear relationship. Determine the y-intercept using the slope formula and slope-intercept form of a line. Write the y-intercept in coordinate form. Explain what the y-intercept represents in each problem situation.

6. Paola spent $18 to purchase a ride-all-day pass for the amusement park and to play 8 games. After playing a total of 20 games, she realized she'd spent $24.

7. Mario saved money he received as gifts and put it toward buying a bike. When he added one week's allowance to his savings, he had $125. After 4 weeks of saving his allowance, he had $161 toward the cost of his bike.

ACTIVITY 4.2

Writing Equations in Slope-Intercept Form

Now that you know how to determine the slope and y-intercept for a linear relationship from a table, graph, or context, you can use this information to write the equation of a line.

Let's use the slope and the y-intercept to determine the equation of the linear relationship represented in the table.

x	y
0	1
2	7
3	10
4	13

WORKED EXAMPLE

You can use the slope and y-intercept to determine the equation of a linear relationship.

- First, determine the slope and the y-intercept.
- Next, substitute the slope and y-intercept into the slope-intercept form of a line, $y = mx + b$.

$m = 3$

y-intercept: (0, 1)

$y = mx + b$

$y = 3x + 1$

The equation is $y = 3x + 1$.

1. Determine the slope, y-intercept, and the slope-intercept form of the linear equation for the relationship represented in the table.

x	y
100	10
105	6
110	2
115	−2
120	−6

2. Write the equation for each linear relationship in slope-intercept form.

By convention, the slope-intercept form is written as $y = mx + b$, but $y = b + mx$ is also correct.

a. $m = -\frac{5}{3}$
   y-intercept: (0, 8)

b. Slope: 6.2
   y-intercept: (0, -2.5)

c. The line containing points (6, 19) and (0, -35)

d. Jorge regularly checks the balance on his bus pass. Friday afternoon, his balance was $26.25. Monday morning, his balance was $1.50.

3. Consider the equations that you wrote in Question 2.

a. Write an equation that represents a line with the same y-intercept as part (a) but a less steep slope.

b. Write an equation that represents a line with the same y-intercept as part (b) but a steeper slope.

ACTIVITY 4.3

Analyzing Linear Relationships

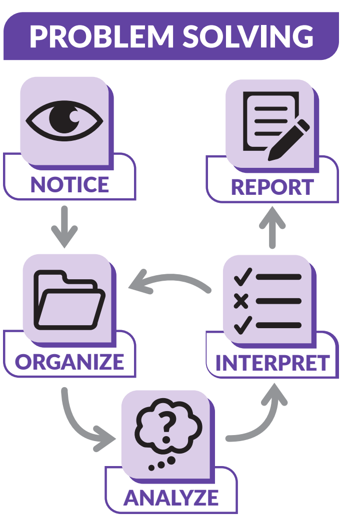
Each year, your class sponsors a go-kart derby to raise money for a local food bank. Jaylen, a member of your class, has claimed the first-place trophy each year for the last four years. Everyone in the class is determined to capture the trophy this year.

Today is Derby Day! You and each member of your group are derby drivers competing against Jaylen and Natalia. Who is going to win? Your teacher will distribute Derby Day cards to your group. These cards contain the information your group needs to determine the winner.

Explain the rules to a partner at your table to make sure that everyone understands them.

Rules:

- The members of your group must work cooperatively to answer all the questions on the cards.
- Each member of your group will be assigned Driver *A*, *B*, *C*, or *D*.
- When you get your driver card, do not show your card to your group members. You may only communicate the information contained on the card.
- Natalia's and Jaylen's cards will be shared with the entire group.
- Be sure everyone in your group discusses the entire problem and its solution.



1. Use the graph paper located at the end of the lesson and your clue cards to help you determine the outcome of the derby.

2. Use the table to organize the information from your graphs and to write equations for the drivers in slope-intercept form.

Driver	Slope	*y*-Intercept	Equation
A:			
B:			
C:			
D:			
Natalia			
Jaylen			

3. What was the speed of the driver who won the race? Explain your reasoning.

Review the definition of **explain your reasoning** in the Academic Glossary.

4. In what order did the drivers finish the derby? List their names or letters and the time it took them to finish.

5. After eight seconds, which driver had traveled the shortest distance from the starting line? Who had traveled the longest distance? Explain your reasoning.

6. Locate and label a point when one driver passed another driver. Describe this point and explain your reasoning.

7. Is there a point when three drivers are tied? If so, describe the point.

8. Suppose the derby was only 20 meters long. Would the order of the winners change? List their names or letters and the time it would take them to finish.

9. After 16 seconds, how far had each driver traveled from the starting line? (Assume the drivers keep traveling at the same rate past the finish line.)

10. How long would the derby have to be for Driver C to win?

Talk the Talk

More or Less

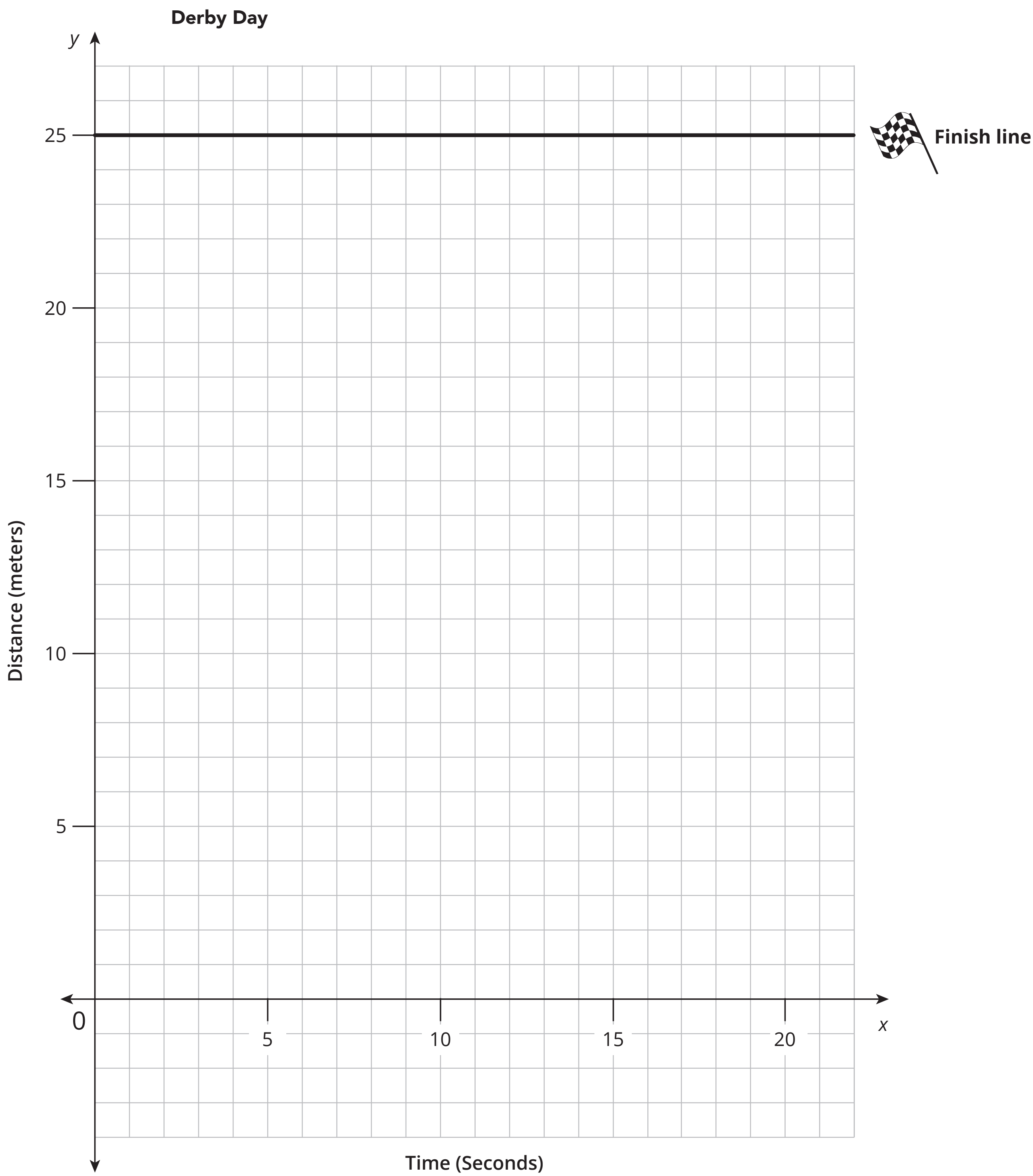
Write an equation in slope-intercept form for a line with each of the given characteristics.

1. The line is decreasing from left to right and has a positive y-intercept.

2. The line is decreasing from left to right. The line is steeper than the line represented by the equation $y = -3x + 8$.

3. The line is increasing from left to right. The line is less steep than the line represented by the equation $y = 7x - 85$.

4. Create a context that represents a linear relationship, with (0, 22) as its y-intercept and a positive slope. Then, write the equation of the line in slope-intercept form.


Derby Day
y
25
20
15
10
5
0
5
10
15
20
x
Distance (meters)
Time (Seconds)
Finish line

Why is this page blank?

So you can tear it out and use the graph on the other side

Lesson 4 Assignment

Write

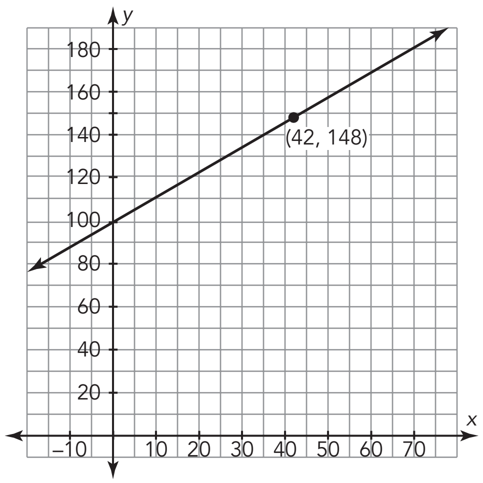
Explain how you can determine the initial value of a linear relationship, the *y*-intercept, when given two points.

Remember

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the *y*-intercept of the line.

Practice

1. Examine the linear graph. Determine the *y*-intercept and write the *y*-intercept in coordinate form. Then, write the equation of the line in slope-intercept form.



2. The table represents a linear relation. Use the table to identify the *y*-intercept. Write the *y*-intercept in coordinate form. Then, write the equation in slope-intercept form.

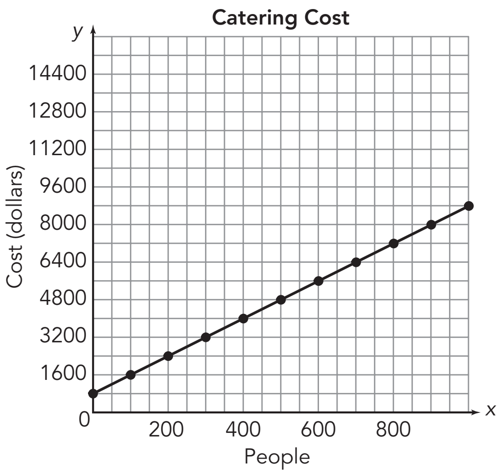
x	*y*
20	144
24	172
28	200
32	228
36	256

Lesson 4 Assignment

3. Each context represents a linear relation. Read each and determine the y-intercept. Write the y-intercept in coordinate form. Explain what the y-intercept represents in the problem situation. Then, write the equation in slope-intercept form.
   a. The water level of a river is 34 feet, and it is receding at a rate of 0.5 foot per day.

   b. Elena worked at a golf course during the summer after eighth grade. After working for two weeks, she added her earnings to the gifts she got for graduation and found she had \$570. After four more weeks of work, she had a total of \$870.

4. Define the variables and write a linear equation in slope-intercept form for each problem situation. Explain the meaning of the y-intercept.
   a. A catering company charges a fixed fee and an additional charge per person.



   b. A line has a constant rate of change of $\frac{3}{7}$ and passes through the point $(0, -8)$.

Lesson 4 Assignment

c. A group bike tour costs $75 plus $12 per bike rental.

d. A salesperson receives a base salary and a percentage of the total sales for the year.

Total Sales (dollars)	Total Income (dollars)
25,000	41,250
30,000	41,500
35,000	41,750
40,000	42,000

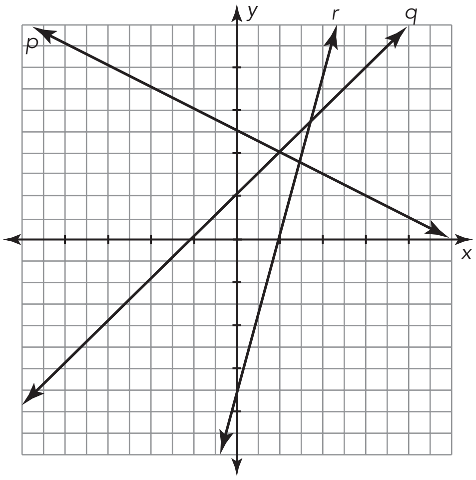
Lesson 4 Assignment

5. The graph shows three lines. The equations of the lines are as follows.

$p: 2y = -x + 10$

$q: y = x + 2$

$r: 7x - 2y = 14$



a. Determine the slope of each line.

b. Write the lines in order from least steep to most steep.

c. Write the equation of a line that is steeper than line *r*.

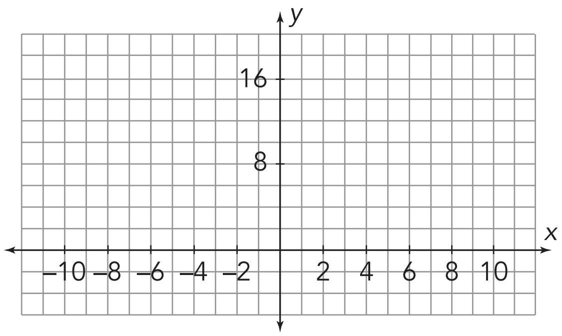
d. Write the equation of a line with a negative slope that is steeper than line *p*.

e. Write the equation of a line with a positive slope that is less steep than line *q*.

f. Write a possible context for each of the lines.

Lesson 4 Assignment

6. Draw a linear graph that decreases and has a *y*-intercept of (0, 4). Write the equation in slope-intercept form.



7. Create a table that represents a linear relation with four values, a *y*-intercept of (0, 6), and a slope of 3.

x	y

5 Defining Functional Relationships

OBJECTIVES

- Describe a functional relationship in terms of a rule which assigns to each input exactly one output.
- Determine whether a relation (represented as a mapping, set of ordered pairs, table, sequence, graph, equation, or context) is a function.

NEW KEY TERMS

- mapping
- set
- relation
- input
- output
- function
- scatterplot
- vertical line test
- linear function

Over the last few years in math class, you have investigated different types of relationships between variable quantities: additive, multiplicative, proportional, and non-proportional.

What are functional relationships?

Getting Started

What's My Rule?

Rules can be used to generate sequences of numbers. They can also be used to generate (x, y) ordered pairs.

Review the definition of **describe** in the Academic Glossary.

1. Write an equation to describe the relationship between each independent variable x and the dependent variable y. Explain your reasoning.

a.

x	y
−6	−12
−3	0
0	12
3	24

b.

x	y
1	−2
5	−10
−1	2
−10	20

c.

x	y
0	2
4	4
5	4.5
20	12

2. Create your own table and have a partner determine the equation you used to build it.

x	y

ACTIVITY 5.1 Functions as Mappings from One Set to Another

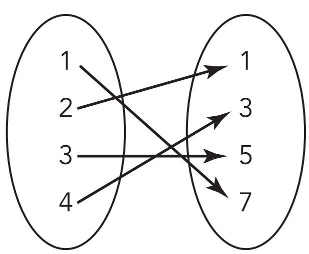
As you learned previously, ordered pairs consist of an *x*-coordinate and a *y*-coordinate. You also learned that a series of ordered pairs on a coordinate plane can represent a pattern. You can also use a *mapping* to show ordered pairs. A **mapping** represents two sets of objects or items. Arrows connect the items to represent a relationship between them.

When you write the ordered pairs for a mapping, you are writing a *set* of ordered pairs. A **set** is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

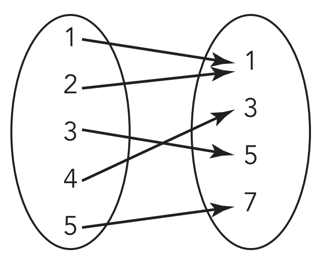
Use braces, { }, to denote a set.

1. Write the set of ordered pairs that represent a relationship in each mapping.

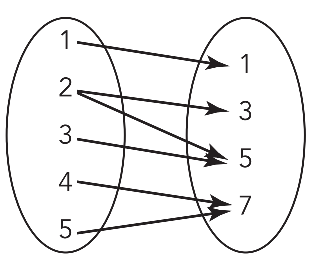
a.



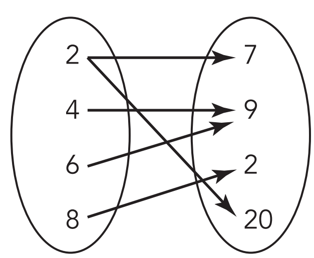
b.



c.



d.



2. Create a mapping from the set of ordered pairs.

   a. {(5, 8), (11, 9), (6, 8), (8, 5)}

   b. {(3, 4), (9, 8), (3, 7), (4, 20)}

3. Write the set of ordered pairs to represent each table.

a.

Input	Output
−10	−20
−5	−10
0	0
5	10
10	20

b.

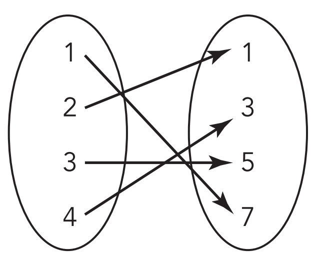
x	y
20	−10
10	−5
0	0
10	5
20	10

The mappings and ordered pairs shown in Questions 1 through 3 form *relations*. A **relation** is any set of ordered pairs or the mapping between a set of *inputs* and a set of *outputs*. The first coordinate of an ordered pair in a relation is the **input**, and the second coordinate is the **output**. A **function** maps each input to one and only one output. In other words, a function has no input with more than one output.

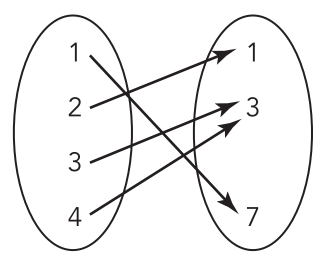
Notice the use of set notation when writing the input values and output values.

WORKED EXAMPLE

In each mapping shown, the input values are {1, 2, 3, 4}.



The output values are {1, 3, 5, 7}.

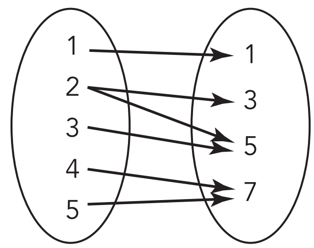


The output values are {1, 3, 7}.

Each mapping represents a function because no input value is mapped to more than one output.

WORKED EXAMPLE

In the mapping shown, the input values are {1, 2, 3, 4, 5} and the output values are {1, 3, 5, 7}.



This mapping does not represent a function.

4. State why the relation in the Worked Example shown is not a function.

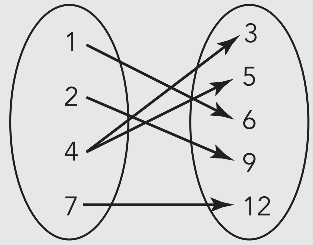
5. State the input values and output values for each relation in Questions 2 and 3. Then, determine which relations represent functions. If the relation is not a function, explain why not.

Think about the mappings as ordered pairs.

6. Review and analyze Alejandro's work. Explain why Alejandro's mapping is not an example of a function.

Alejandro

My mapping represents a function.



ACTIVITY 5.2

Functions as Mapping Inputs to Outputs

You have determined if sets of ordered pairs represent functions. In this activity, you will examine different situations and determine whether they represent functional relationships.

Read each context and decide whether it fits the definition of a function. Explain your reasoning.

1. *Input:* Lizzie writes a thank-you note to her best friend.
   Output: Her best friend receives the thank-you note in the mail.

2. *Input:* A football game is being telecast.
   Output: It appears on televisions in millions of homes.

3. *Input:* There are four puppies in a litter.
   Output: One puppy was adopted by the Smiths, another by the Jacksons, and the remaining two by the Fullers.

4. *Input:* The basketball team has numbered uniforms.
   Output: Each player wears a uniform with her assigned number.

5. *Input:* Beverly Hills, California, has the zip code 90210.
   Output: As of the 2021 Census, there were 32,903 people living in Beverly Hills.

6. *Input:* A sneak preview of a new movie is being shown in a local theater.
   Output: 65 people are in the audience.

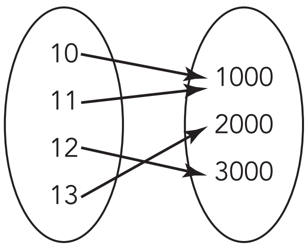
7. *Input:* Tiara works at a fast food restaurant on weekdays and a card store on weekends.
   Output: Tiara's job on any one day.

8. *Input:* Alyssa sends a text message to everyone in her contact list on her cell phone.
   Output: There are 41 friends and family on Alyssa's contact list.

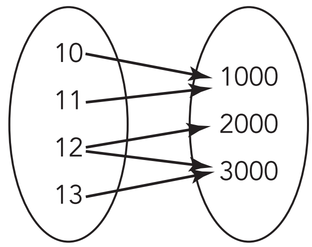
ACTIVITY
5.3
Determining Whether a Relation Is a Function

Analyze the relations in each pair. Determine which relations are functions and which are not functions. Explain how you know.

1. Mapping *A*



Mapping *B*



2. Table *A*

Input	Output
−2	4
−1	1
0	0
1	1
2	4

Table *B*

x	*y*
2	−4
1	−1
0	0
1	1
2	4

3. Set *A*
{(2, 3), (2, 4), (2, 5), (2, 6), (2, 7)}

Set *B*
{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1)}

4. Scenario *A*
Input:
The morning announcements are read over the school intercom system during homeroom period.

Output:
All students report to homeroom at the start of the school day to listen to the announcements.

Scenario *B*
Input:
Each student goes through the cafeteria line.

Output:
Each student selects a lunch option from the menu.

ACTIVITY 5.4

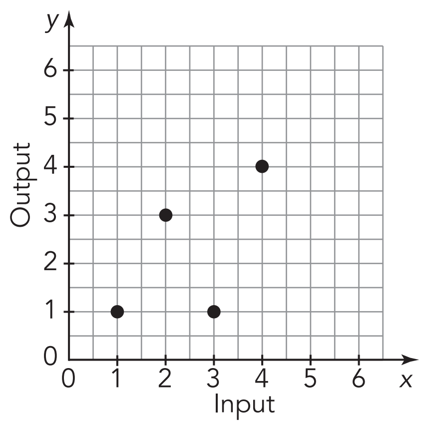
Functions as Graphs

A **scatterplot** is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

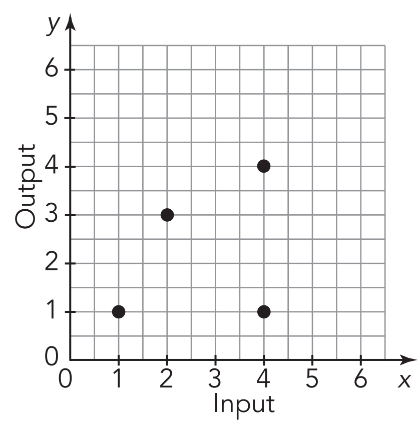
A relation can be represented as a graph.

1. Determine if each scatterplot represents a function. Explain your reasoning.

a.



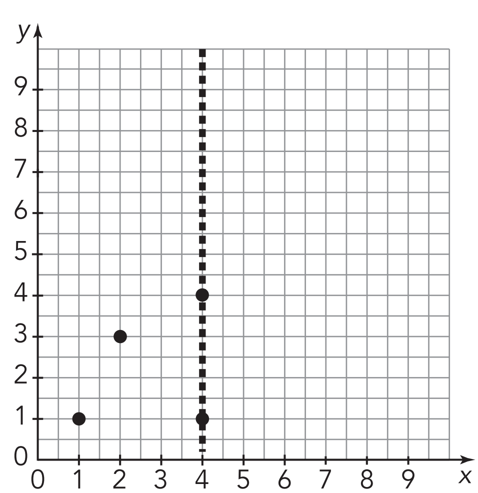
b.



The **vertical line test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all of the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

WORKED EXAMPLE

Consider the scatterplot shown.

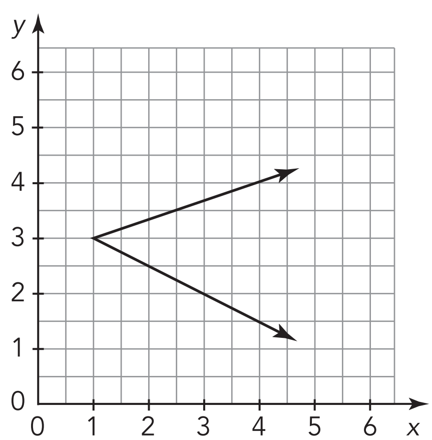


In this scatterplot, the relation is not a function. The input value 4 can be mapped to two different outputs, 1 and 4. Those two outputs are shown as intersections to the vertical line drawn at $x = 4$.

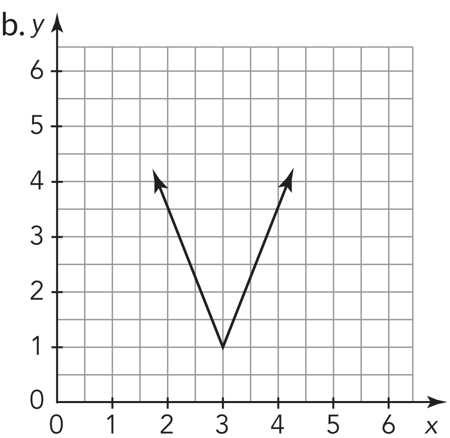
2. Use the definition of function to explain why the vertical line test works.

3. Use the vertical line test to determine if each graph represents a function. Explain your reasoning

a.



b.



4. Cut out the 12 cards at the end of this lesson. Sort the graphs into two groups: functions and non-functions. Use the letter of each graph to record your findings.

Functions	Non-functions

A function whose graph is a straight line is a **linear function**. A function whose graph is not a straight line is a **non-linear function**.

5. Sort the graphs that are functions into two groups: linear functions and non-linear functions. Use the letter of each graph to record your findings.

Linear Functions	Non-Linear Functions

ACTIVITY **5.5**

Functions as Equations

So far, you have determined whether a mapping, context, or a graph represents a function. You can also determine whether an equation is a function.

WORKED EXAMPLE

The given equation can be used to convert yards to feet. Let x represent the number of yards, and let y represent the number of feet.

$$y = 3x$$

To test whether this equation is a function, first, substitute values for x into the equation and then determine if any x-value can be mapped to more than one y-value. If each x-value has exactly one y-value, then it is a function. Otherwise, it is not a function.

x	$y = 3x$
1	3
3	9
4	12
8	24

In this case, every x-value can be mapped to only one y-value. Each x-value is multiplied by 3. Some examples of ordered pairs are (2, 6), (10, 30), and (5, 15). Therefore, this equation is a function.

It is not possible to test every possible input value in order to determine whether or not the equation represents a function. You can graph any equation to see the pattern and use the vertical line test to determine whether it represents a function.

1. Determine whether each equation is a function. List three ordered pairs that are solutions to each. Explain your reasoning.

   a. $y = 5x + 3$

   b. $y = x^2$

   c. $y = |x|$

   d. $x^2 + y^2 = 1$

   e. $y = 4$

   f. $x = 2$

If you do not recognize the graph of the equation, use a graphing calculator to see the pattern.

2. Explain what is wrong with Matthew's reasoning.

Matthew

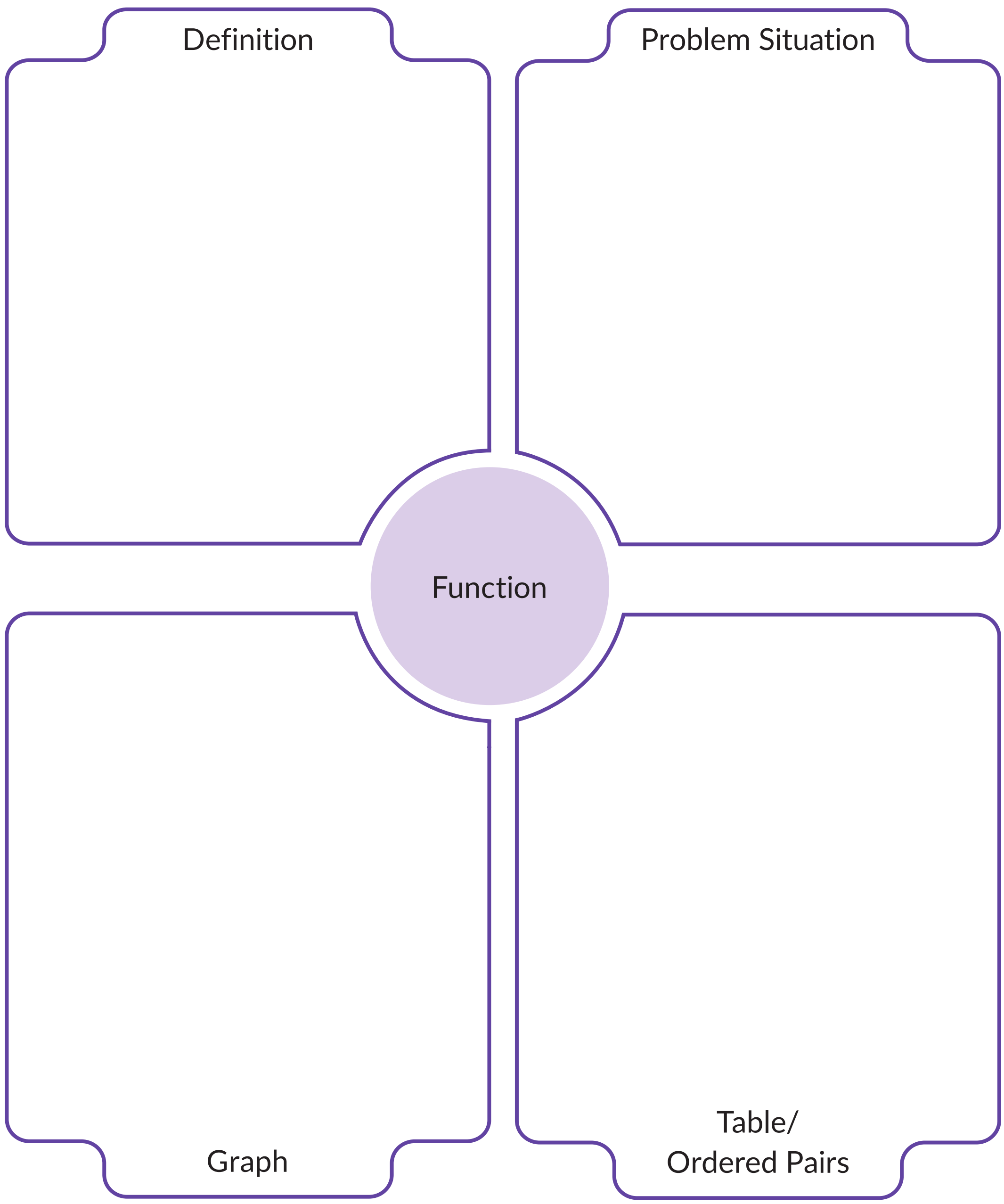
The equation $y^2 = x$ represents a function.

x	y
4	2
9	3
25	5

Talk the Talk

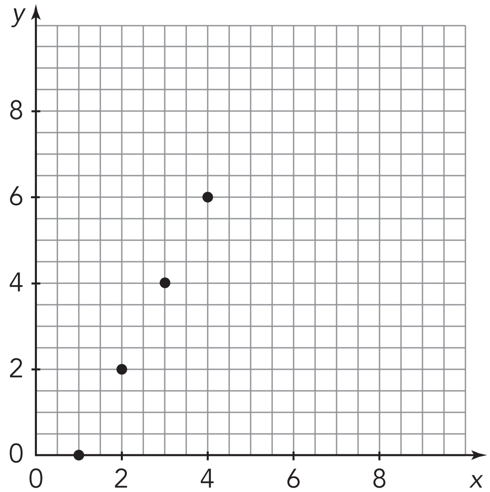
Function Organizer

1. Complete the graphic organizer for the concept of function. Write a definition for function in your own words. Then, create a problem situation that can be represented using a function. Finally, create a table of ordered pairs and sketch a graph to represent the function.

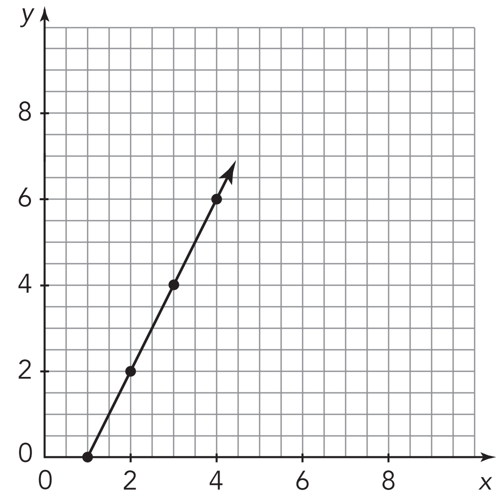


Graph Cutouts

a.



b.



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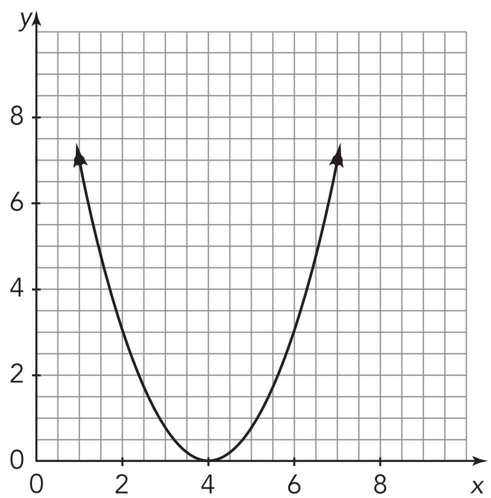

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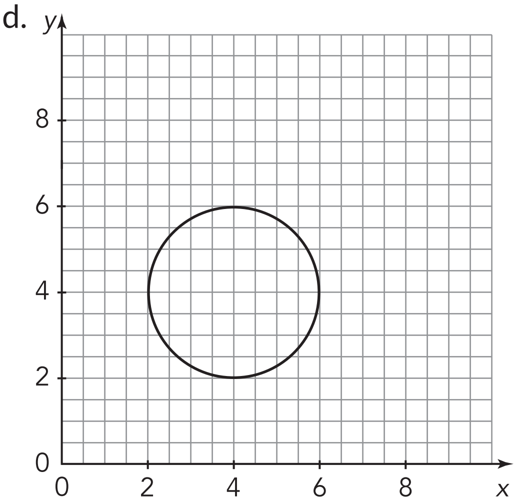

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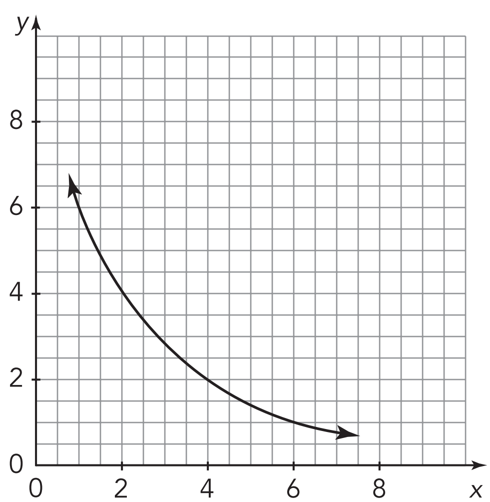

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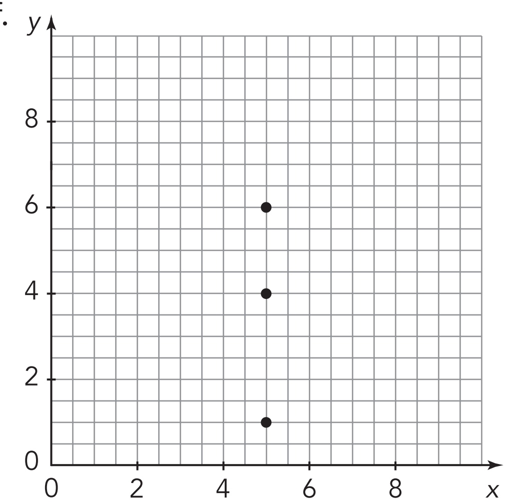

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So you can cut out the graphs on the other side

Graph Cutouts

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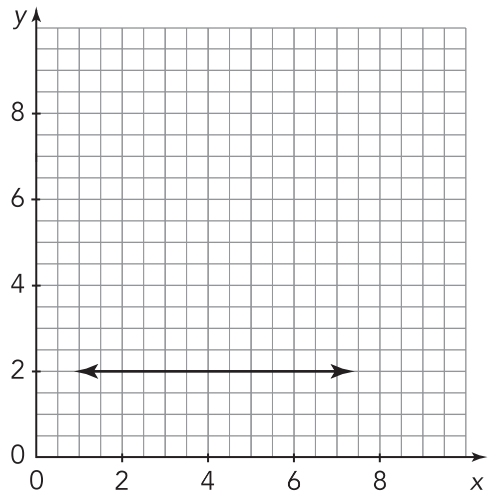

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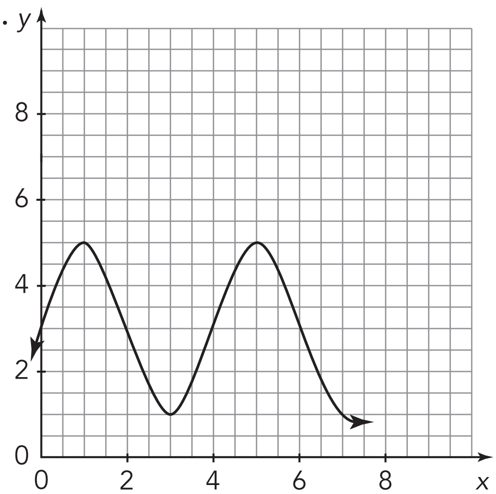

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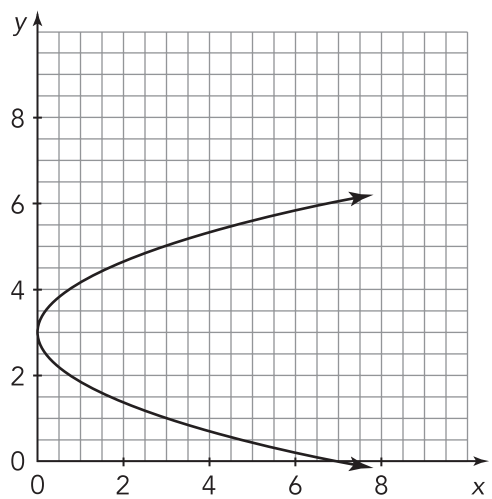

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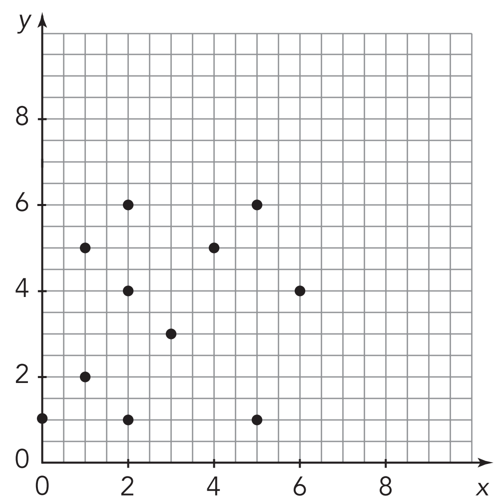

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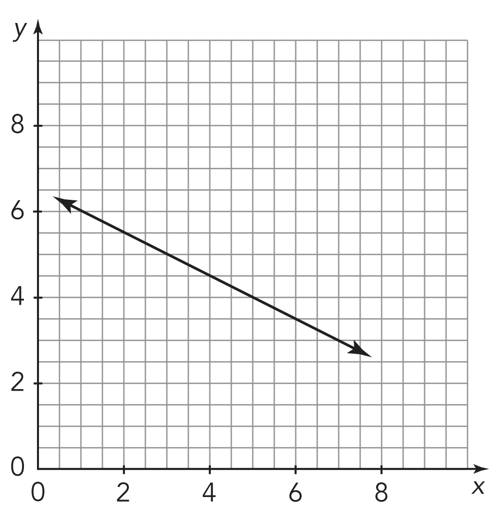

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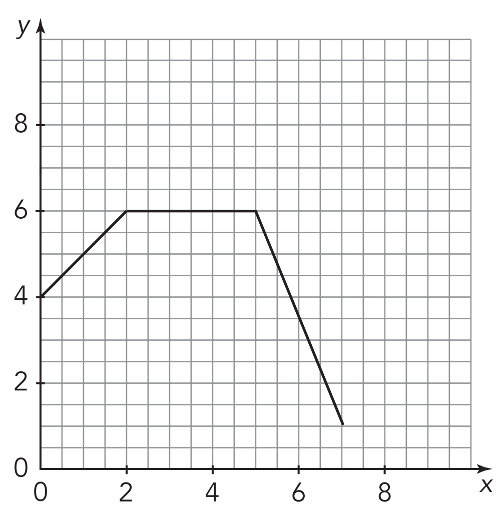

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So you can cut out the graphs on the other side

Lesson 5 Assignment

Write

Write the term from the box that best completes each sentence.

scatterplot	output	relation	input	vertical line test
mapping	set	input values	output values	function

1. A(n) ________________ is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.

2. The first coordinate of an ordered pair in a relation is the ________________.

3. The second coordinate of an ordered pair is the ________________.

4. A(n) ________________ maps each input to one and only one output.

5. A(n) ________________ is a graph of a collection of ordered pairs.

6. The ________________ is a visual method of determining whether a relation represented as a graph is a function by visualizing whether any vertical lines would intersect the graph of the relation at more than one point.

7. A(n) ________________ shows objects in two sets connected together to represent a relationship between the two sets.

8. A(n) ________________ is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

9. The ________________ of a function are the set of all inputs of the function.

10. The ________________ of a function are the set of all outputs of the function.

Lesson 5 Assignment

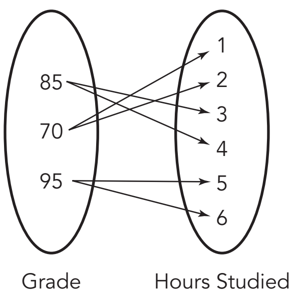
Remember

A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.

A relation is a function when each input value maps to one and only one output value.

Practice

1. A history teacher asks six of her students the number of hours that they studied for a recent test. The diagram shown maps the grades that they received on the test to the number of hours that they studied.

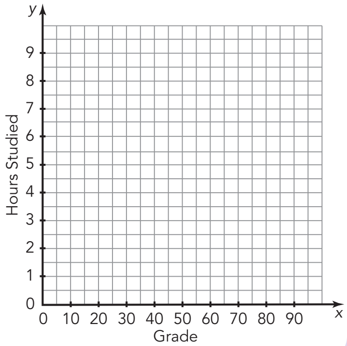


   a. Is the relation a function? If the relation is not a function, explain why not.

   b. Write the set of ordered pairs to represent the mapping.

   c. What does the first value in each ordered pair in part (b) represent? What does the second value in each ordered pair represent?

   d. Create a scatterplot. Does the graph agree with your conclusion from part (a)? Explain your reasoning.



Lesson 5 Assignment

2. The science teacher created the set of ordered pairs {(100, 6), (90, 5), (80, 3), (70, 1), (90, 4), (80, 2)} to represent six students' grades on the midterm to the number of hours that they had studied. Create a mapping from this set of ordered pairs.

   a. Is the relation a function? If the relation is not a function, explain why not.

   b. List all the inputs of the relation.

   c. List all the outputs of the relation.

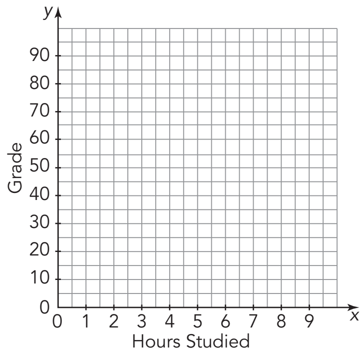
   d. Instead of mapping grades to hours studied, the teacher decides to create a new diagram. This diagram maps hours studied to grades. Show the mapping that would result.

Lesson 5 Assignment

e. Write the set of ordered pairs to represent the mapping in part (d).

f. Is the relation in part (d) a function? If the relation is not a function, explain why not.

g. Create a scatterplot. Does the graph agree with your conclusion from part (f)? Explain your reasoning.

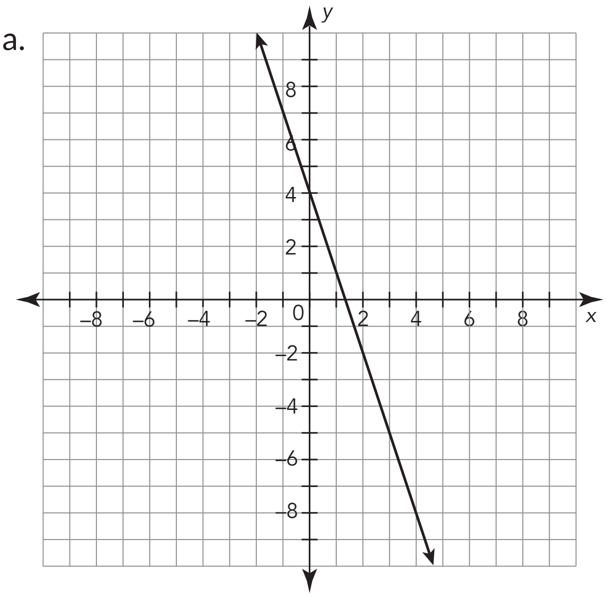


Lesson 5 Assignment

3. At the end of the year, a principal decides to create the given mapping.
   Input: The 82 total students in the history class
   Ouput: The final grades they received for the class
   Does this mapping fit the definition of a function? Explain your reasoning.

4. Use the vertical line test to determine if each graph represents a function. Explain your reasoning.

a.



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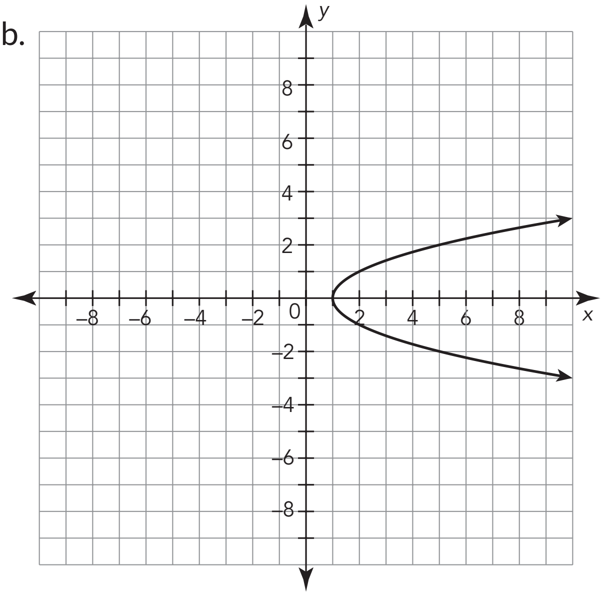


Lesson 5 Assignment

Prepare

Describe the relationship, if there is one, between the number of hours spent in a bookstore and the amount of money spent.

Hours In the Bookstore	Amount of Money Spent
0.5	60
1	85
1.5	10
2	35
2.5	20
3	0
3.5	100
4	25

TOPIC 2 SELF-REFLECTION

Name: ______________________

Linear Relationships

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Linear Relationships* topic by:

TOPIC 2: *Linear Relationships*	Beginning of Topic	Middle of Topic	End of Topic
comparing representations of linear relationships using tables, graphs, and equations.	◯	◯	◯
creating a linear equation to model a linear relationship between two quantities.	◯	◯	◯
interpreting the slope and y-intercept of linear relationships in terms of the context and relate them to a graph or a table of values.	◯	◯	◯
defining slope as the rate of change and a ratio of the vertical change to the horizontal change between any two points.	◯	◯	◯
defining the y-intercept as the initial value, where $x = 0$.	◯	◯	◯
determining the slope and y-intercept of a linear relationship from a table of values, two coordinate pairs, a context, a graph, or an equation.	◯	◯	◯
writing equations of lines in slope-intercept form, $y = mx + b$.	◯	◯	◯
graphing lines from an equation written in slope-intercept form.	◯	◯	◯

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

TOPIC 2: *Linear Relationships*	Beginning of Topic	Middle of Topic	End of Topic
identifying functions using sets of ordered pairs, tables, mappings, verbal descriptions, graphs, and equations.	○	○	○
generating a set of ordered pairs from a function and graphing the function.	○	○	○
comparing the characteristics of linear and non-linear functions using various representations.	○	○	○
comparing and contrasting two functions represented in different ways.	○	○	○
providing examples of non-linear functions using multiple representations (tables, graphs, and equations).	○	○	○
explaining that the equation $y = mx + b$ is the equation of a function whose graph is a straight line, where m is the slope and b is the y-intercept.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Linear Relationships* topic.

    ____

    ____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

    ____

    ____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

    ____

    ____

TOPIC 2 SUMMARY

Linear Relationships Summary

NEW KEY TERMS

- first differences [primeras diferencias]
- *y*-intercept [intersección con el eje *y*]
- slope-intercept form
- mapping [mapeo o aplicación]
- set
- relation [relación]
- input
- output
- function [función]
- scatterplot
- vertical line test [prueba de la línea vertical]
- linear function [función lineal]

LESSON 1 Using Tables, Graphs, and Equations

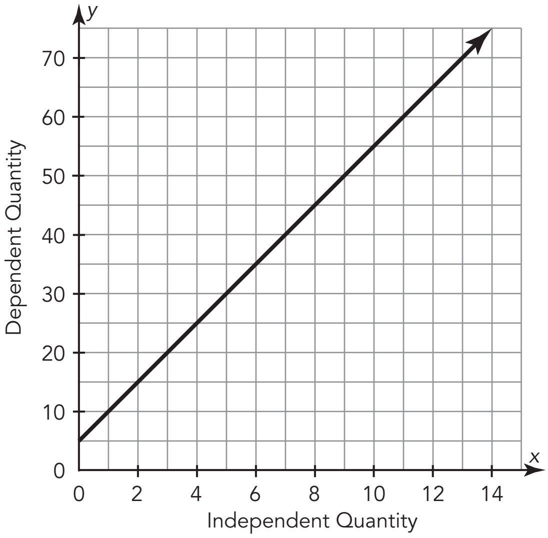
In mathematics, when representing quantities in a table, it is important to include a row to identify the quantities and units of measure. Typically, the independent quantity is represented in the left column and the dependent quantity is represented in the right column.

Independent Quantity	Dependent Quantity
0	5
1	10
2	15
3	20

When graphing a relationship, the convention is to represent the independent quantity on the horizontal axis of a graph and the dependent quantity on the vertical axis. You should include labels on each axis.

When writing an equation in the form $y = mx + b$, the *x*-value is the independent variable and the *y*-value is the dependent variable. It is important to define the variables you choose.

For example, the table and graph shown represent the equation $y = 5x + 5$.



LESSON 2 Linear Relationships in Tables

You can use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, to determine the rate of change between two points represented in a table of values. If the rate is constant, this formula gives the rate of change for the relationship, or slope.

Independent Quantity	Dependent Quantity
0	5
1	10
2	15
3	20

For example, the table shows a linear relationship with a slope of 5.

$$m = \frac{20 - 10}{3 - 1} = \frac{10}{2} = 5$$

When the values for the independent variable in a table are consecutive integers, you can examine only the column with the dependent variable and calculate the differences between consecutive values. In this process, you are calculating *first differences*. **First differences** are the values determined by subtracting consecutive y-values in a table when the x-values are consecutive integers. The first differences in a linear relationship are constant.

LESSON 3 Linear Relationships in Context

You can choose different independent and dependent quantities to model the same information, depending on what you want to know. Once you have determined the independent and dependent quantities, you need just two points to determine the slope, or unit rate.

Total Miles	Total Cost for Gas ($)	Total Gallons
2600	200	80

For example, using the information in this table, you can model the number of miles per gallon or the number of gallons per mile.

Number of miles per gallon $= \frac{2600}{80} = 32.5$ miles per gallon

Number of gallons per mile $= \frac{80}{2600} \approx 0.03$ gallon per mile

LESSON 4

Slope-Intercept Form of a Line

The **y-intercept** is the *y*-coordinate of the point where a graph crosses the *y*-axis. It is the value of the dependent quantity when the independent quantity is 0. The *y*-intercept can be written as the ordered pair $(0, y)$.

When you know the slope and the *y*-intercept, you can use this information to write a linear equation in *slope-intercept form*. The **slope-intercept form** of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the *y*-intercept.

x	y
2	7
3	10
4	13

- First, identify the slope. $m = 3$
- Next, choose any point from the table. $(4, 13)$
- Now, substitute what you know into the slope-intercept form of an equation. $y = mx + b$

  $13 = 3(4) + b$
- Finally, solve for the value of the *y*-coordinate. $13 = 12 + b$

  $1 = b$

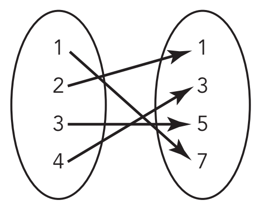
The *y*-intercept is $(0, 1)$.

LESSON 5

Defining Functional Relationships

You can use a *mapping* to show ordered pairs. A **mapping** represents two sets of objects or items. Arrows connect the items to represent a relationship between them. When you write the ordered pairs for a mapping, you are writing a set of ordered pairs. A **set** is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common. A **relation** is any set of ordered pairs or the mapping between a set of inputs and a set of outputs. The first coordinate of an ordered pair in a relation is the **input**, and the second coordinate is the **output**. A **function** maps each input to one and only one output.

For example, in the mapping shown, the set of ordered pairs is {(1, 7), (2, 1), (3, 5), (4, 3)}. The input values are {1, 2, 3, 4}, and the output values are {1, 3, 5, 7}.



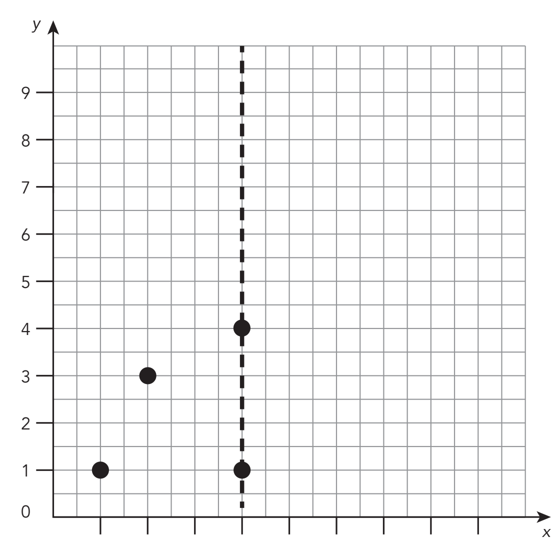
This mapping represents a function because each input value is mapped to only one output.

A relation can be represented as a graph. A **scatterplot** is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

The **vertical line test** is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all of the vertical lines that could be drawn on the graph of a relation. When any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

In this scatterplot, the relation is not a function. The input value 4 can be mapped to two different outputs, 1 and 4. Those two outputs are shown as intersections to the vertical line drawn at $x = 4$.

A **linear function** is a function in which its graph is a straight line.



Math Glossary

A

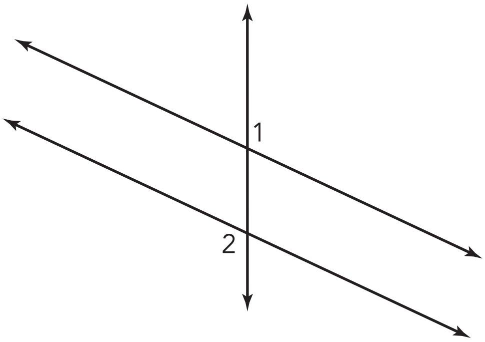
absolution deviation

The absolute value of each deviation is called the absolute deviation.

alternate exterior angles

Alternate exterior angles are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are outside the other two lines.

Example

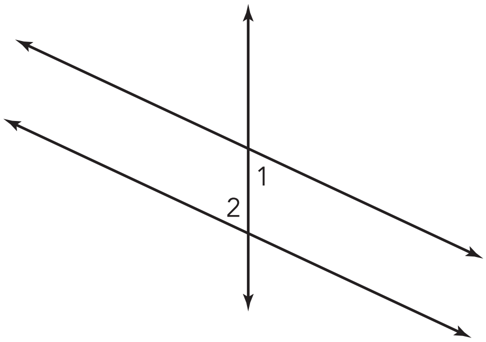


Angles 1 and 2 are alternate exterior angles.

alternate interior angles

Alternate interior angles are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are between the other two lines.

Example

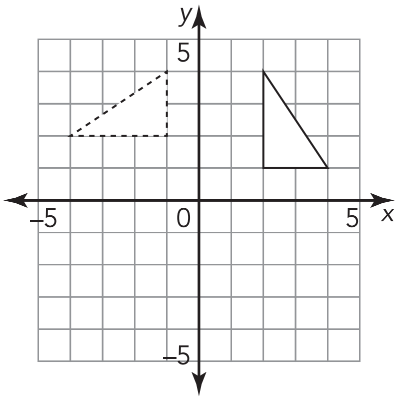


Angles 1 and 2 are alternate interior angles.

angle of rotation

The angle of rotation is the amount of rotation, in degrees, about a fixed point, the center of rotation.

Example



The angle of rotation is 90° counterclockwise about the origin (0, 0).

Angle-Angle Similarity theorem

The Angle-Angle Similarity theorem states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

association

A pattern or relationship identified in a scatter plot of a two-variable data set is called an association.

B

bar notation

Bar notation is used to indicate the digits that repeat in a repeating decimal.

Example

In the quotient of 3 and 7, the sequence 428571 repeats. The numbers that lie underneath the bar are the numbers that repeat.

$\frac{3}{7} = 0.4285714285714... = 0.\overline{428571}$

base

The base of a power is the factor that is multiplied repeatedly in the power.

Examples

$2^3 = 2 \times 2 \times 2 = 8$ (base: 2)

$8^0 = 1$ (base: 8)

bivariate data

When you collect information about two separate characteristics for the same person, thing, or event, you have collected bivariate data.

break-even point

When one line represents the cost of an item and the other line represents the income from selling the item, the point of intersection is called the break-even point.

C

cash advance

A cash advance is a service provided by credit card companies that allows their customers to take out money directly from a bank or ATM.

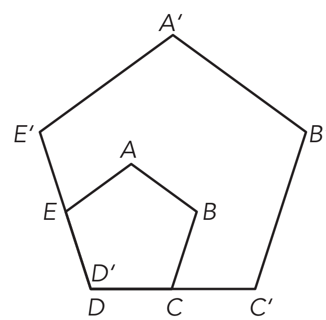
census

A census is the data collected from every member of a population.

center of dilation

The point from which a dilation is generated is called the center of dilation.

Example

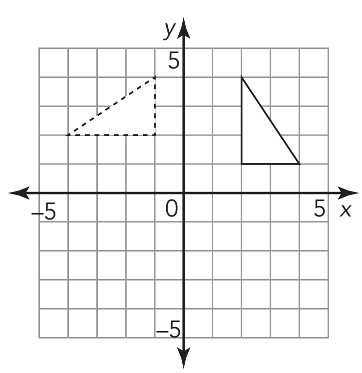


The center of dilation is point *D*.

center of rotation

The center of rotation is the point around which a figure is rotated. The center of rotation can be a point on the figure, inside the figure, or outside the figure.

Example



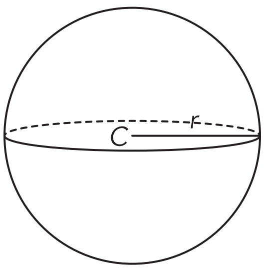
The figure has been rotated 90° counterclockwise about the center of rotation, which is the origin (0, 0).

center of a sphere

The given point from which the set of all points in three dimensions are the same distance is the center of the sphere.

Example

Point *C* is the center of the sphere.



characteristic

In the expression $a \times 10^n$, the variable n is called the characteristic.

Example

$6.1 \times 10^5 = 610{,}000$

↑

characteristic

closed

A set of numbers is said to be closed under an operation if the result of the operation on two numbers in the set is a defined value also in the set.

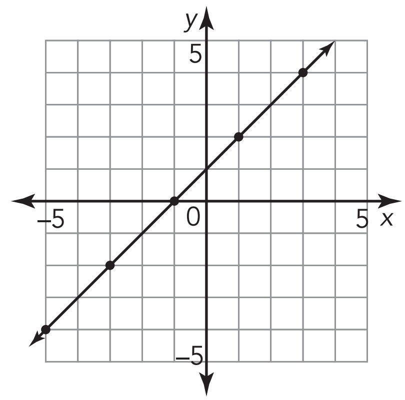
Example

The set of integers is closed under the operation of addition because for every two integers a and b, the sum $a + b$ is also an integer.

collinear points

Collinear points are points that lie in the same straight line.

Example



All the points on the graph are collinear points.

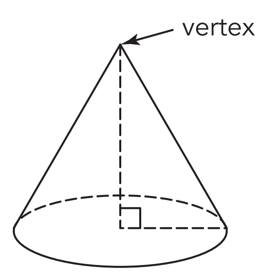
compound interest

Compound interest is a percentage that is paid on the principal and interest after each time period.

cone

A cone is a three-dimensional object with a circular or oval base and one vertex.

Example



congruent angles

Congruent angles are angles that are equal in measure.

congruent figures

Figures that have the same size and shape are congruent figures. If two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

congruent line segments

Congruent line segments are line segments that have the same length.

consistent system

Systems that have one or an infinite number of solutions are called consistent systems.

constant function

When the y-value of a function does not change, or remains constant, the function is called a constant function.

constant of proportionality

In a proportional relationship, the ratio of all y-values to their corresponding x-values is constant. This specific ratio, $\frac{y}{x}$, is called the constant of proportionality. Generally, the variable k is used to represent the constant of proportionality.

converse

The converse of a theorem is created when the if-then parts of that theorem are exchanged.

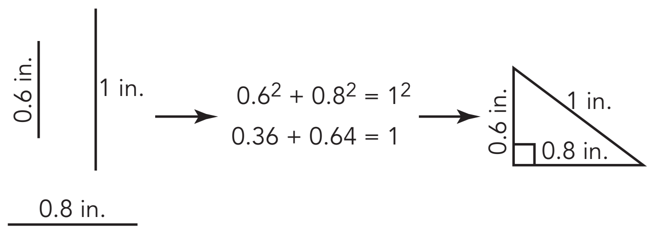
Example

Triangle Inequality theorem:	converse of Triangle Inequality theorem:
If a polygon is a triangle, then the sum of any two of its side lengths is always greater than the length of the third side.	If you have three side lengths, and the sum of any two of the side lengths is greater than the third side length, then the side lengths can form a triangle.

Converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem states that if the sum of the squares of the two shorter sides of a triangle equals the square of the longest side, then the triangle is a right triangle.

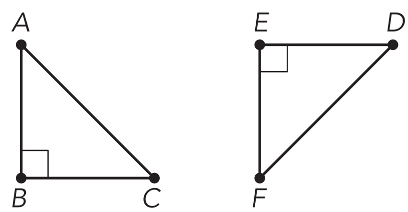
Example



corresponding angles

Corresponding angles are angles that have the same relative positions in geometric figures.

Example

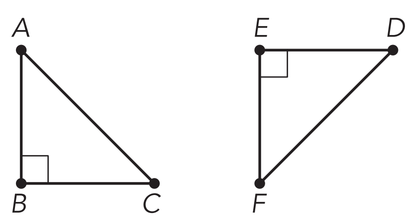


Angle B and Angle E are corresponding angles.

corresponding sides

Corresponding sides are sides that have the same relative positions in geometric figures.

Example



Sides AB and DE are corresponding sides.

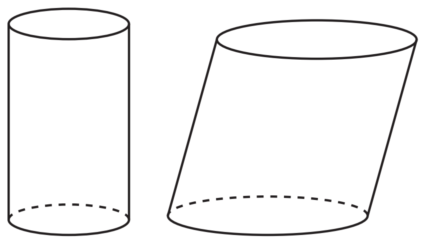
cubic function

A cubic function is a function that can be written in the form $y = ax^3 + bx^2 + cx + d$, where each coefficient or constant a, b, c, and d is a real number and a is not equal to 0.

cylinder

A cylinder is a three-dimensional object with two parallel, congruent circular bases.

Examples



D

data

When information is collected, the facts or numbers gathered are called data.

decreasing function

When the value of a dependent variable decreases as the independent variable increases, the function is called a decreasing function.

deferment

A deferment is a period of time, usually up to two years, in which students delay paying the principal and interest on their loan.

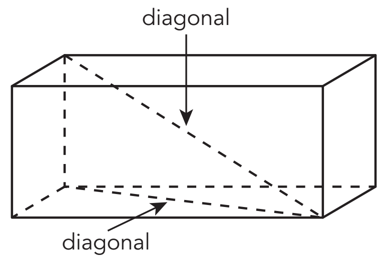
deviation

The deviation of a data value indicates how far that data value is from the mean.

diagonal

In a three-dimensional figure, a diagonal is a line segment connecting any two non-adjacent vertices.

Example



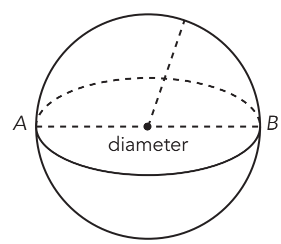
diagonal of a square

A diagonal of a square is a line segment connecting opposite vertices of the square.

diameter of the sphere

A segment drawn between two points on the sphere that passes through the center of the sphere is a diameter of the sphere.

Example

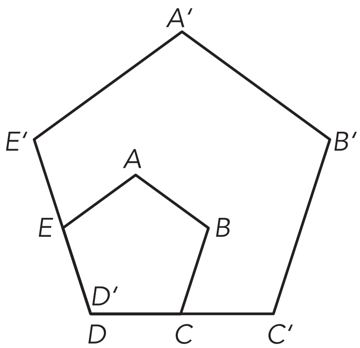


The diameter of the sphere is labeled.

dilation

A dilation is a transformation that produces a figure that is the same shape as the original figure, but not necessarily the same size.

Example

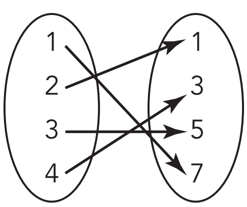


Pentagon *A'B'C'D'E'* is a dilation of Pentagon *ABCDE*.

domain

The domain of a function is the set of all inputs of the function.

Example



The domain in the mapping shown is {1, 2, 3, 4}.

E

ellipsis

An ellipsis is a set of three periods which stands for "and so on."

Example

3, 9, 27, 81, ...

↑

ellipsis

enlargement

When the scale factor is greater than 1, the image is called an enlargement.

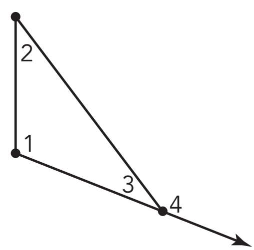
explanatory variable

The independent variable can also be called the explanatory variable.

exterior angle of a polygon

An exterior angle of a polygon is an angle between a side of a polygon and the extension of its adjacent side.

Example

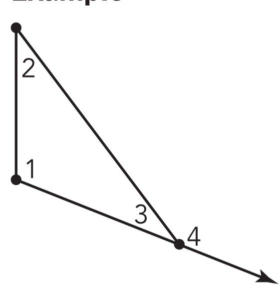


Angle 4 is an exterior angle of a polygon.

Exterior Angle theorem

The Exterior Angle theorem states that the measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

Example



According to the Exterior Angle theorem, $m\angle 4 = m\angle 1 + m\angle 2$.

extrapolating

Extrapolating is predicting values that fall outside the plotted values on a scatter plot.

F

first differences

First differences are the values determined by subtracting consecutive *y*-values in a table when the *x*-values are consecutive integers. When the first differences are equal, the points represented by the ordered pairs in the table will form a straight line.

Example

x	y
1	25
2	34
3	45

9

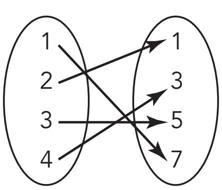
11

The first differences are 9 and 11, so the points represented by these ordered pairs will not form a straight line.

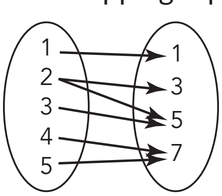
function

A function maps each input to one and only one output.

Example



This mapping represents a function.



This mapping does NOT represent a function.

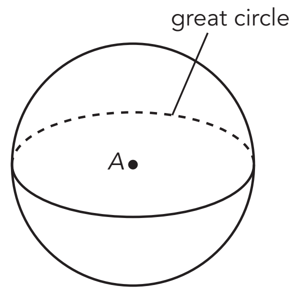
G

great circle

A great circle is the circumference of the sphere at the sphere's widest part.

Example

Point *A* is the center of the sphere. It is also the center of the great circle.

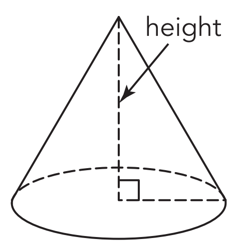


H

height of a cone

The height of a cone is the length of a line segment drawn from the vertex to the base of the cone. In a right cone, this line segment is perpendicular to the base.

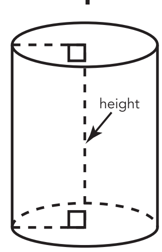
Example



height of a cylinder

The height of a cylinder is the length of a line segment drawn from one base to the other base, perpendicular to both bases.

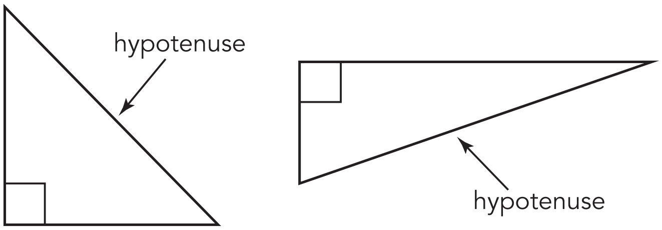
Example



hypotenuse

The side opposite the right angle in a right triangle is called the hypotenuse.

Examples

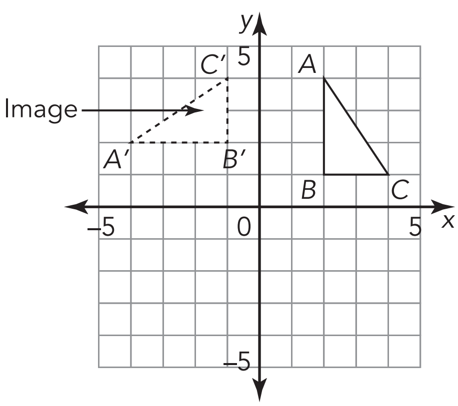


I

image

The new figure created from a transformation is called the image.

Example



inconsistent system

Systems that have no solution are called inconsistent systems.

increasing function

When both values of a function increase together, the function is called an increasing function.

infinitely many solutions

When the solution to the equation is a true statement for any value of the variable, the equation has infinitely many solutions.

Example

The equation $x = x$ has infinitely many solutions.

input

The first coordinate of an ordered pair in a relation is the input.

integers

Integers are the set of whole numbers and their additive inverses.

Example

The set of integers can be represented as {... −3, −2, −1, 0, 1, 2, 3, ...}

interpolating

Interpolating is predicting values that fall within the plotted values on a scatter plot.

irrational numbers

Numbers that cannot be written as fractions in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0, are irrational numbers.

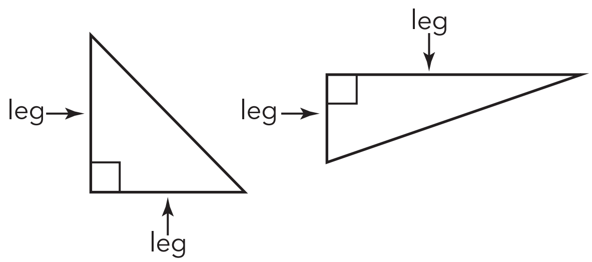
Examples

The numbers $\sqrt{2}$, 0.313113111..., and π are irrational numbers

leg

A leg of a right triangle is either of the two shorter sides. Together, the two legs form the right angle of a right triangle.

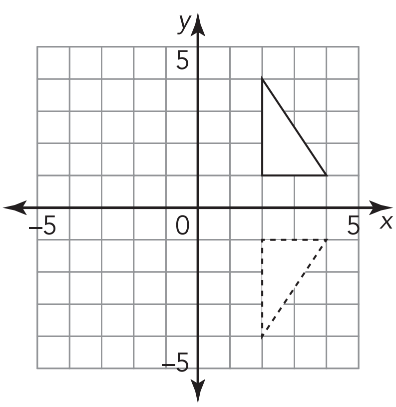
Examples



line of reflection

A line of reflection is a line that acts as a mirror so that corresponding points are the same distance from the line.

Example

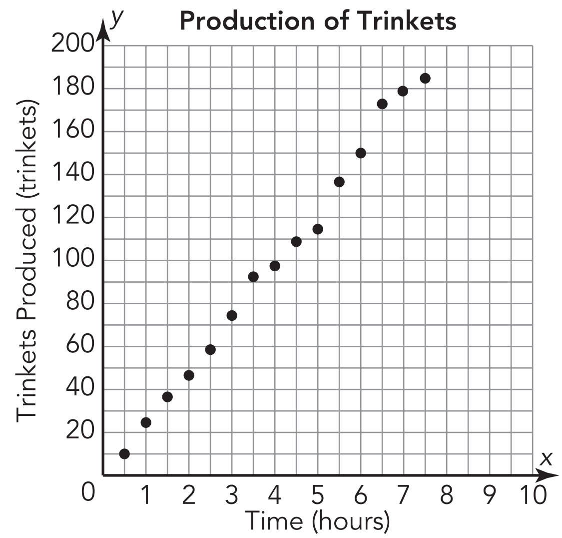


The *x*-axis is the line of reflection.

linear association

A linear association occurs when the points on the scatterplot seem to form a line.

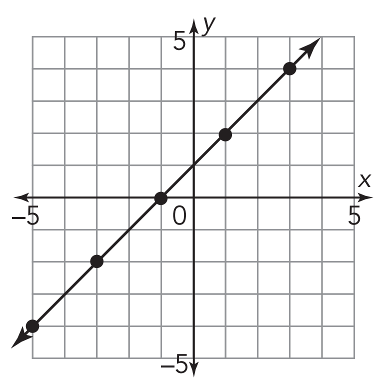
Example



linear function

A function whose graph is a straight line is a linear function.

Example



The function $f(x) = x + 1$ is a linear function.

M

mantissa

In the expression $a \times 10^n$, the variable a is called the mantissa. In scientific notation, the mantissa is greater than or equal to 1 and less than 10.

Example

$6.1 \times 10^5 = 610{,}000$

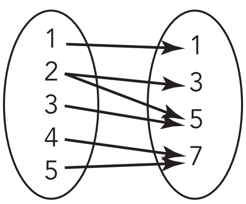
↑

mantissa

mapping

A mapping represents two sets of objects or items. Arrows connect the items to represent a relationship between them.

Example



mean absolute deviation

The mean absolute deviation (MAD) is the mean of the absolute deviations.

model

When you use a trend line, the line and its equation are often referred to as a model of the data. (See *trend line*.)

N

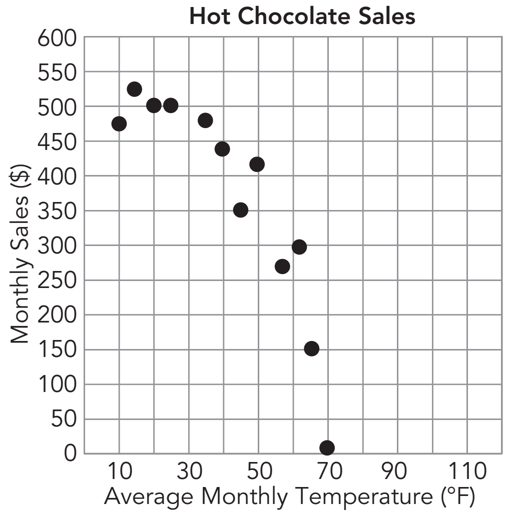
natural numbers

Natural numbers consist of the numbers that you use to count objects: {1, 2, 3, . . .}.

negative association

If the response variable decreases as the explanatory variable increases, then the two variables have a negative association.

Example



There is a negative association between average monthly temperature and hot chocolate sales.

non-proportional relationship

An equation in the form $y = mx + b$, where b is not equal to 0, represents a non-proportional relationship.

no solution

When the solution to the equation is a false statement, the equation has no solution.

Example

The equation $x + 0 = x + 1$ has no solution.

O

online calculator

An online calculator is an Internet-based application that quickly performs calculations for the user.

order of magnitude

The order of magnitude is an estimate of size expressed as a power of ten.

Example

The Earth's mass has an order of magnitude of about 10^{24} kilograms.

one solution

When the solution to the equation is a true statement with one value equal to the variable, there is only one solution.

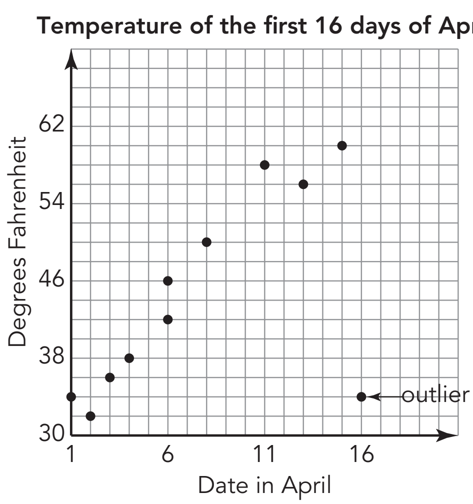
Example

The equation $x + 2 = 8$ has only one solution: $x = 6$.

outlier

An outlier for bivariate data is a point that varies greatly from the overall pattern of the data.

Example



output

The second coordinate of an ordered pair in a relation is the output.

P

parameter

When data are gathered from a population, the characteristic used to describe the population is called a parameter.

perfect square

A perfect square is the square of an integer.

Example

9 is a perfect square: $3^2 = 9$

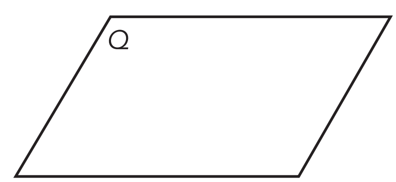
25 is a perfect square: $5^2 = 25$

plane

A plane is a flat surface. It has infinite length and width, but no depth. A plane extends infinitely in all directions in two dimensions. Planes are determined by three points, but are usually named using one uppercase letter.

Example

Plane *Q* is shown.



point of intersection

The point of intersection is the point at which two lines cross on a coordinate plane. In a system of linear equations, a point of intersection indicates a solution to both equations.

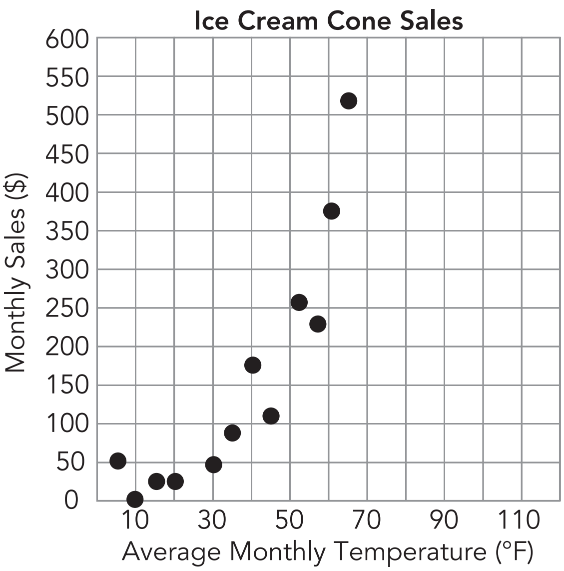
population

The population is the entire set of items from which data can be selected.

positive association

The two variables have a positive association if, as the explanatory variable increases, the response variable also increases.

Example

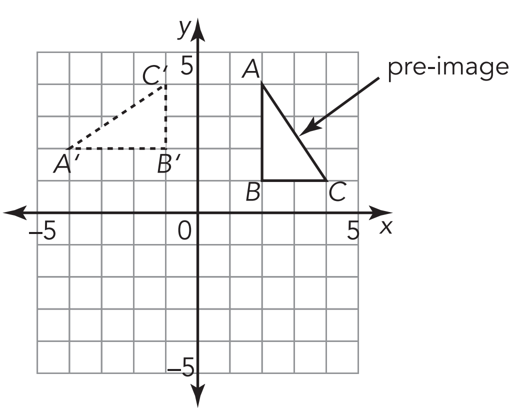


There is a positive association between the average monthly temperature and ice cream cone sales.

pre-image

The original figure in a transformation is called the pre-image.

Example



proof

A proof is a line of reasoning used to validate a theorem.

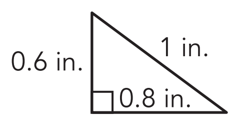
proportional relationship

A proportional relationship is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$, must represent the same constant.

Pythagorean Theorem

The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If a and b are the lengths of the legs, and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Example



$0.6^2 + 0.8^2 = 1^2$
$0.36 + 0.64 = 1$

Pythagorean triple

Any set of three positive integers a, b, and c that satisfies the equation $a^2 + b^2 = c^2$ is a Pythagorean triple.

Example

3, 4, and 5 is a Pythagorean triple: $3^2 + 4^2 = 5^2$

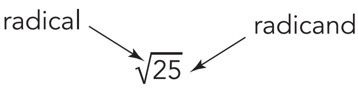
R

radical

The symbol $\sqrt{}$ is a radical.

Example

The expression shown is read as "the square root of 25" or as "radical 25"

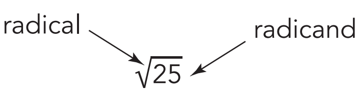


radicand

The quantity under the radical symbol is the radicand.

Example

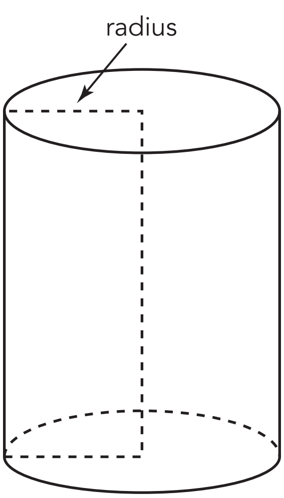
$\sqrt{25}$ The expression shown is read as "the square root of 25" or as "radical 25"



radius of a cylinder

The radius of a cylinder is the distance from the center of the base to any point on the edge of the base.

Example

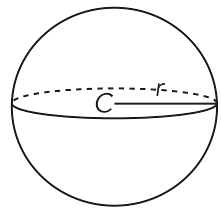


radius of the sphere

A segment drawn from the center of a sphere to a point on the sphere is called a radius of the sphere.

Example

Point C is the center of the sphere, and r is the radius of the sphere.



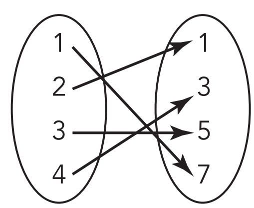
random sample

A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

range

The range of a function is the set of all outputs of the function.

Example



The range in the mapping shown is {1, 3, 5, 7}.

rate of change

The rate of change for a situation describes the amount that the dependent variable changes compared with the amount that the independent variable changes.

rational numbers

Rational numbers are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples

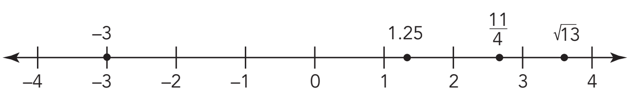
$-4, \frac{1}{2}, \frac{2}{3}$, 0.67, and $\frac{22}{7}$ are examples of rational numbers.

real numbers

Combining the set of rational numbers and the set of irrational numbers produces the set of real numbers. Real numbers can be represented on the real number line.

Examples

The numbers -3, 1.25, $\frac{11}{4}$ and $\sqrt{13}$ shown are real numbers.



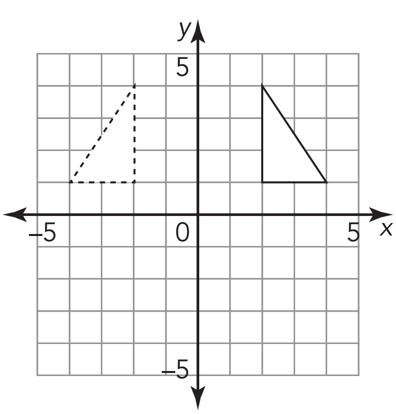
reduction

When the scale factor is less than 1, the image is called a reduction.

reflection

A reflection is a rigid motion transformation that “flips” a figure across a line of reflection.

Example



The figure has been reflected across the y-axis.

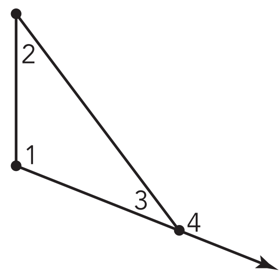
relation

A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.

remote interior angles of a triangle

The remote interior angles of a triangle are the two angles that are non-adjacent to the specified exterior angle.

Example



Angles 1 and 2 are remote interior angles of a triangle.

repeating decimal

A repeating decimal is a decimal in which a digit, or a group of digits, repeat(s) infinitely. Repeating decimals are rational numbers.

Examples

$\frac{1}{9} = 0.111...$ $\frac{7}{12} = 0.58333...$

$\frac{22}{7} = 3.142857142857...$

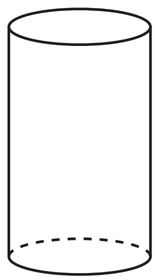
response variable

The dependent variable can also be called the response variable, because this is the variable that responds to what occurs to the explanatory variable.

right cylinder

A right cylinder is a cylinder in which the bases are aligned one directly above the other.

Example



rigid motion

A rigid motion is a special type of transformation that preserves the size and shape of the figure.

Examples

Translations, reflections, and rotations are examples of rigid motion transformations.

rotation

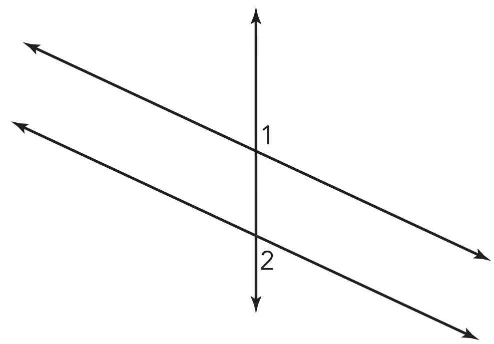
A rotation is a rigid motion transformation that turns a figure on a plane about a fixed point, called the center of rotation, through a given angle, called the angle of rotation.

S

same-side exterior angles

Same-side interior angles are formed when a transversal intersects two other lines. These angle pairs are on the same side of the transversal and are outside the other two lines.

Example

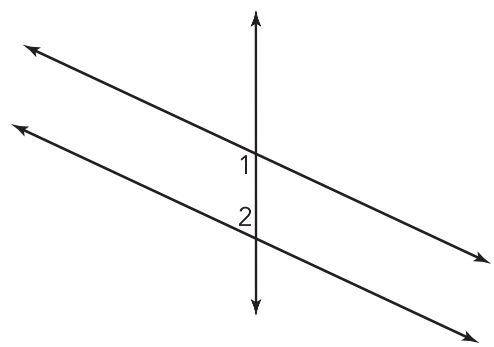


Angles 1 and 2 are same-side exterior angles.

same-side interior angles

Same-side interior angles are formed when a transversal intersects two other lines. These angle pairs are on the same side of the transversal and are between the other two lines.

Example



Angles 1 and 2 are same-side interior angles.

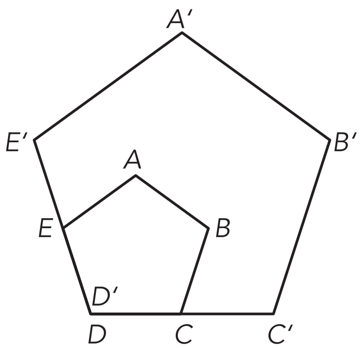
sample

When data are collected from a part of the population, the data are called a sample.

scale factor

In a dilation, the scale factor is the ratio of the distance of the new figure from the center of dilation to the distance of the original figure from the center of dilation.

Example

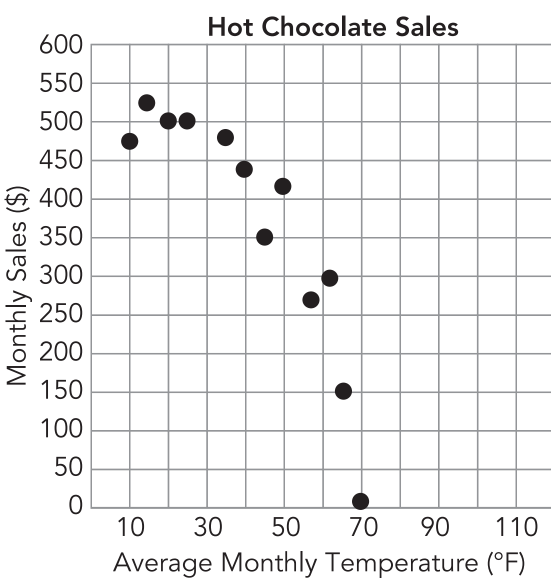


Pentagon *ABCDE* has been dilated by a scale factor of 2 to create Pentagon *A'B'C'D'E'*.

scatterplot

A scatterplot is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

Example



scientific notation

Scientific notation is a notation used to express a very large or a very small number as the product of a number greater than or equal to 1 and less than 10 and a power of 10.

Example

The number 1,345,000,000 is written in scientific notation as 1.345×10^9.

sequence

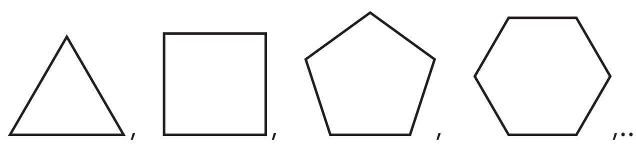
A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.

Examples

Sequence A:

2, 4, 6, 8, 10, 12, . . .

Sequence B:



set

A set is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

Examples

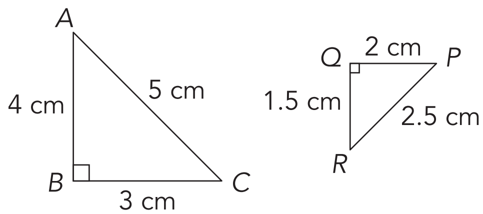
The set of counting numbers is {1, 2, 3, 4, ...}

The set of even numbers is {2, 4, 6, 8, ...}

similar

When two figures are similar, the ratios of their corresponding side lengths are equal.

Example



Triangle *ABC* is similar to Triangle *PQR*.

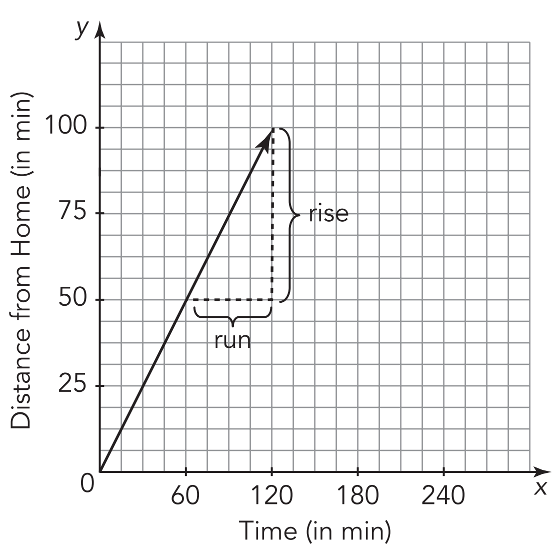
simple interest

Simple interest is a percentage that is paid only on the original principal.

slope

In any linear relationship, slope describes the direction and steepness of a line and is usually represented by the variable *m*. Slope is another name for rate of change. (See *rate of change*.)

Example



The slope of the line is $\frac{50}{60}$, or $\frac{5}{6}$.

slope-intercept form

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and $(0, b)$ is the y-intercept.

solution of a linear system

The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system. Graphically, the solution is the point of intersection.

Example

$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

The solution to this system of equations is (1, 6).

solution set of an inequality

The solution set of an inequality is the set of all points that make an inequality true.

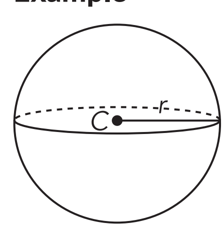
square root

A square root is one of two equal factors of a given number.

sphere

A sphere is the set of all points in three dimensions that are the same distance from a given point called the center of the sphere.

Example



statistic

When data are gathered from a sample, the characteristic used to describe the sample is called a statistic.

survey

A survey is a method of collecting information about a certain group of people.

system of linear equations

When two or more linear equations define a relationship between quantities they form a system of linear equations.

Example

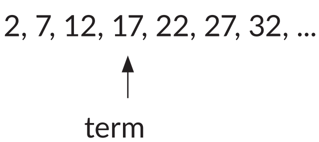
$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

T

term

A term in a sequence is an individual number, figure, or letter in the sequence.

Example



terminating decimal

A terminating decimal has a finite number of digits, meaning that after a finite number of decimal places, all following decimal places have a value of 0. Terminating decimals are rational numbers.

Examples

$\frac{9}{10} = 0.9$ $\frac{15}{8} = 1.875$ $\frac{193}{16} = 12.0625$

terms of an investment

The terms of an investment include the type of loan, amount of money invested, and the length of the investment.

transformation

A transformation is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation.

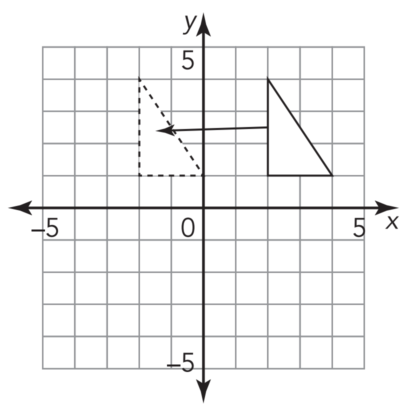
Examples

Translations, reflections, rotations, and dilations are examples of transformations.

translation

A translation is a rigid motion transformation that "slides" each point of a figure the same distance and direction.

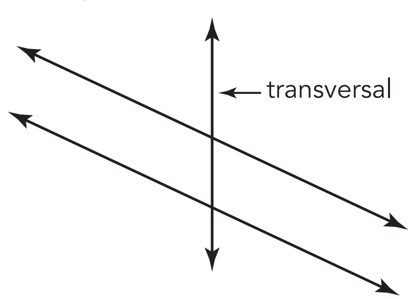
Example



transversal

A transversal is a line that intersects two or more lines at distinct points.

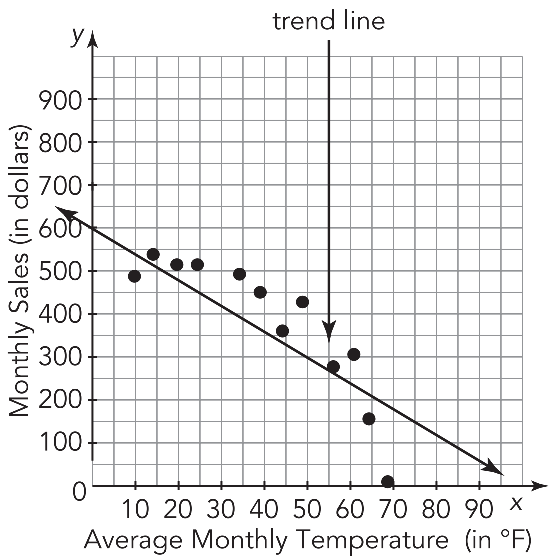
Example



trend line

A trend line is a line that is as close to as many points as possible but doesn't have to go through all of the points.

Example



Triangle Sum theorem

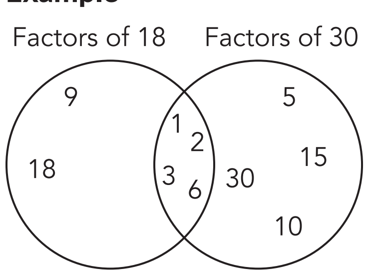
The Triangle Sum theorem states that the sum of the measures of the interior angles of a triangle is 180°.

V

Venn diagram

A Venn diagram uses circles to show how elements among sets of numbers or objects are related.

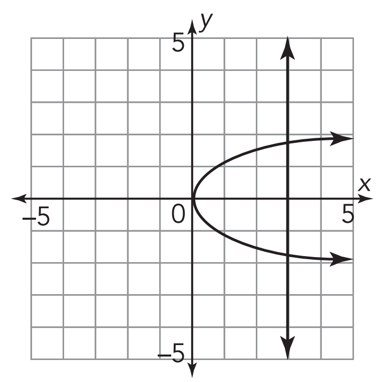
Example



vertical line test

The vertical line test is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

Example



The line drawn at $x = 3$ crosses two points on the graph, so the relation is not a function.

W

whole numbers

Whole numbers are made up of the set of natural numbers and the number 0, the additive identity.

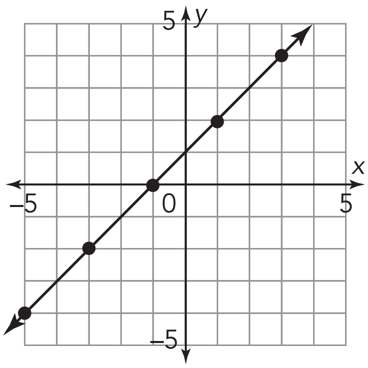
Example

The set of whole numbers can be represented as {0, 1, 2, 3, 4, 5, ...}.

y-intercept

The *y*-intercept is the *y*-coordinate of the point where a graph crosses the *y*-axis. The *y*-intercept can be written in the form (0, *y*).

Example



The *y*-intercept of the graph is (0, 1).

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