OER Secondary Mathematics
EDITION 1

Grade 8

Volume 2

STUDENT EDITION

Acknowledgment
Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

OER Secondary Mathematics
EDITION 1

Grade 8

Course Guide

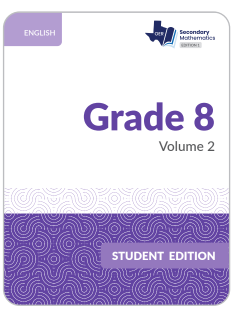
Welcome to the Course Guide for Secondary Mathematics, Grade 8

Instructional Design

The instructional materials help you learn math in different ways. There are two types of resources: Learning Together and Learning Individually. These resources provide various learning experiences to develop your understanding of mathematics.

Learning Together

On **Learning Together** days, you spend time engaging in active learning to build mathematical understanding and confidence in sharing ideas, listening to one another, and learning together. The Student Edition is a consumable resource that contains the materials for each lesson.



STUDENT EDITION

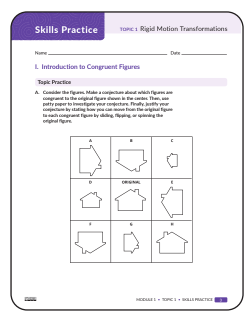
I am a record of your thinking, reasoning, and problem solving.

My lessons allow you to build new knowledge from prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

Learning Individually

On **Learning Individually** days, Skills Practice offers opportunities to engage with skills, concepts, and applications that you learn in each lesson. It also provides opportunities for interleaved practice, which encourages you to flexibly move between individual skills, enhancing connections between concepts to promote long-term learning. This resource will help you build proficiency in specific skills based on your individual academic needs as indicated by monitoring your progress throughout the course.

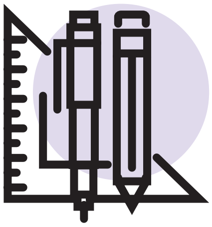


SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student edition.

My purpose is to provide problem sets for additional practice, enrichment, and extension.

The instructional materials intentionally emphasize active learning and sense-making. Deep questions ensure you thoroughly understand the mathematical concepts. The instructional materials guide you to connect related ideas holistically, supporting the integration of your evolving mathematical understanding and developing proficiency with mathematical processes.



Intentional Mathematics Design

Mathematical Coherence: The path through the mathematics develops logically, building understanding by linking ideas within and across grades so you can learn concepts more deeply and apply what you've learned to more complex problems.

TEKS Mathematical Process Standards: The instructional materials support your development of the TEKS mathematical process standards. They encourage you to experiment, think creatively, and test various strategies. These mathematical processes empower you to persevere when presented with complex real-world problems.

Multiple Representations: The instructional materials connect multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: The instructional materials focus on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

What principles guide the design and organization of the instructional materials?

Active Learning: Studies show that learning becomes more robust as instruction moves from entirely passive to fully interactive (Chi, 2009). Classroom activities require active learning as students engage by problem-solving, explaining and justifying reasoning, classifying, investigate, and analyze peer work.

Discourse Through Collaborative Learning: Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities intentionally promote active dialogue centered on structured activities.

Personalized Learning: Research has proven that problems that capture your interests are more likely to be taken seriously (Baker et. al, 2009). In each lesson, learning is linked to prior knowledge and experiences, creating valuable and authentic opportunities for you to build new understanding on the firm foundation of what you already know. You move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures.

Focus on Problem-Solving: Solving problems is an essential life skill that you need to develop. The problem-solving model provides a structure to support you as you analyze and solve problems. It is a strategy you can continue to use as you solve problems in everyday life.

1 Introduction to Congruent Figures

1 OBJECTIVES

- Define *congruent figures*.
- Use patty paper to verify experimentally that two figures are congruent by obtaining the second figure from the first using a sequence of slides, flips, and/or turns.
- Use patty paper to determine whether two figures are congruent.

3 NEW KEY TERMS

- congruent figures
- corresponding sides
- corresponding angles

2 You have studied figures that have the same shape or measure. How do you determine whether two figures have the same size and the same shape?

LESSON STRUCTURE

1 Objectives

Objectives are stated for each lesson to help you take ownership of the objectives.

2 Essential Question

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3 New Key Terms

The new key terms for each lesson are identified to help you connect your everyday and mathematical language.

4 Getting Started

Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

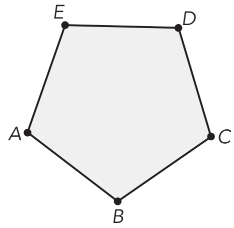
4 Getting Started

It's Transparent!

Patty paper is great paper to investigate geometric properties. You can write on it, trace with it, and see creases when you fold it.

Let's use patty paper to investigate the figure shown.

1. List everything you know about the shape.



2. Use patty paper to compare the sizes of the sides and angles in the figure.

   a. What do you notice about the side lengths?

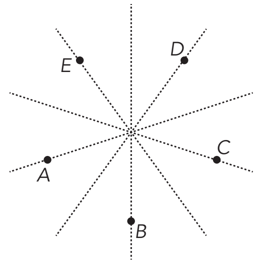
   b. What do you notice about the angle measures?

   c. What can you say about the figure based on this investigation?

Patty paper was originally created for separating patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool.

Trace the polygon onto a sheet of patty paper.

3. Use five folds of your patty paper to determine the center of each side of the shape. What do you notice about where the folds intersect?



5

ACTIVITY 1.1

Analyzing Size and Shape

Cut out each of the figures provided at the end of the lesson.

1. Sort the figures into at least two categories. Provide a rationale for your classification. List your categories and the letters of the figures that belong in each category.

2. List the figures that are the same shape as Figure A. How do you know they are the same shape?

Figures with the same shape but not necessarily the same size are *similar figures*, which you will study in later lessons.

3. List the figures that are both the same shape and the same size as Figure A. How do you know they are the same shape and same size?

Corresponding sides are sides that have the same relative position in geometric figures.

Figures that have the same size and shape are **congruent figures**. When two figures are congruent, all *corresponding sides* and all *corresponding angles* have the same measure.

4. List the figures that are congruent to Figure C.

Corresponding angles are angles that have the same relative position in geometric figures.

5 Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about getting the answer. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

6 Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

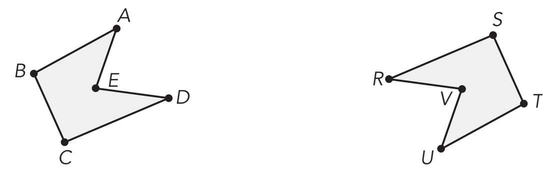
Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Talk the Talk 6

The Core of Congruent Figures

Recall that when two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

Consider these figures.



1. How can you slide, flip, or spin the figure on the left to obtain the figure on the right?

2. Use patty paper to determine the corresponding sides and corresponding angles of the congruent figures.

Lesson 1 Assignment

7 Write

Explain what a conjecture is and how it is used in math.

Remember 8

If two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

9 Practice

1. Determine which figures are congruent to Figure *A*. Follow the steps given as you investigate each shape.
   - Make a conjecture about which figures are congruent to Figure *A*.
   - Use patty paper to investigate your conjecture.
   - Justify your conjecture by stating how you can move from Figure *A* to each congruent figure by sliding, flipping, or spinning Figure *A*.

Figure *A*

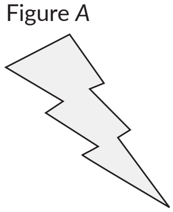


Figure *B*

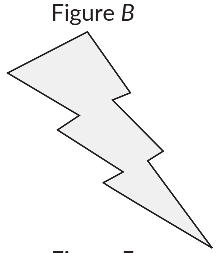


Figure *C*

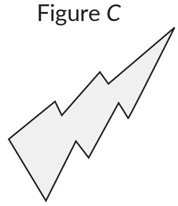


Figure *D*

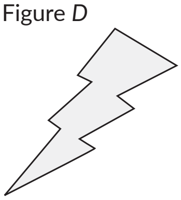


Figure *E*

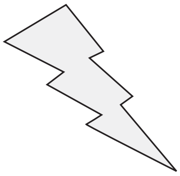


Figure *F*

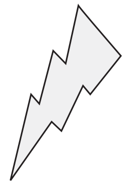


Figure	Your Conjecture	Congruent to Figure *A*?	How Do You Move Figure *A* onto the Congruent Figure?
B			
C			
D			
E			
F			

ASSIGNMENT

7 Write

Reflect on your work and clarify your thinking.

8 Remember

Take note of the key concepts from the lesson.

9 Practice

Use the concepts learned in the lesson to solve problems.

ASSIGNMENT

(10) **Prepare**

Get ready for the next lesson.

Lesson 1 Assignment

(10)

Prepare

Draw all lines of symmetry for each letter.

1. A

2. B

3. H

4. X

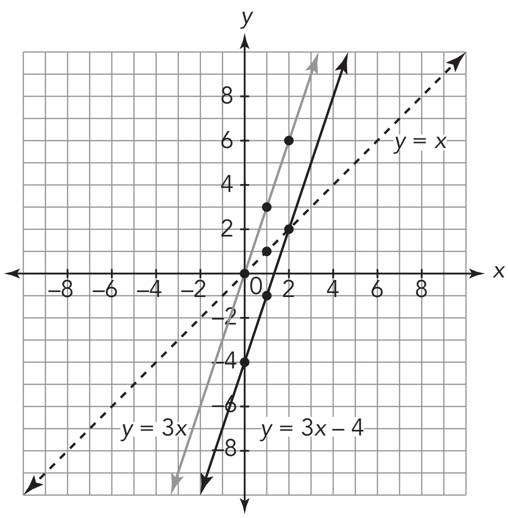
WORKED EXAMPLE

Graph $y = 3x - 4$ using transformations of the basic linear equation $y = x$.

First, graph the basic equation, $y = x$, and consider at least 2 sets of ordered pairs on the line, for example (0, 0), (1, 1), and (2, 2).

Then, dilate the y-values by 3.

Finally, translate all y-values down 4 units.



Ask Yourself . . .

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

WORKED EXAMPLE

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

THUMBS UP

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

THUMBS DOWN

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

WHO'S CORRECT?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine correct whether the work is correct or incorrect.

The Pythagorean Theorem can be used to determine unknown side lengths in a right triangle. Liam and Victoria are using the theorem to determine the length of the hypotenuse, *c*, with leg lengths of 2 and 4. Examine their work.

Victoria

$c^2 = 2^2 + 4^2$
$c^2 = 4 + 16 = 20$
$c = \sqrt{20} \approx 4.5$

The length of the hypotenuse is approximately 4.5 units.

Liam

$c^2 = 2^2 + 4^2$
$c^2 = 6^2$
$c = 6$

The length of the hypotenuse is 6 units.

Ask Yourself . . .

- Why is this method correct?
- Have I used this method before?

Ask Yourself . . .

- Where is the error?
- Why is it an error?
- How can I correct it?

d. Luna claims Ms. Park must begin with the number 1 when assigning numbers to students. Jorge says she can start with any number as long as she assigns every student a different number. Who is correct? Explain your reasoning.

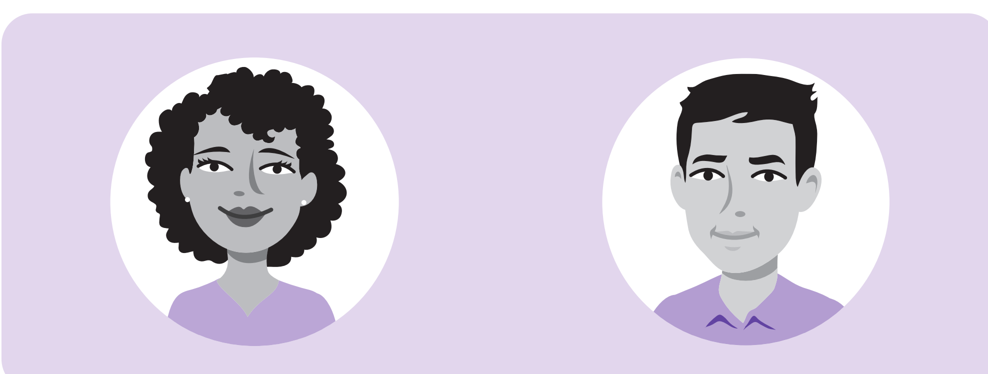
Ask Yourself . . .

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

The Crew

The Crew is here to help you on your journey. Sometimes, they will remind you about things you already learned. Sometimes, they will ask you questions to help you think about different strategies. Sometimes, they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.



TEKS Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I Can" expectations listed below align with the TEKS mathematical process standards and encourage students to develop their mathematical learning and understanding.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I CAN:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I CAN:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology, as appropriate; and techniques including mental math, estimation, and number sense, as appropriate, to solve problems.

I CAN:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language, as appropriate.

I CAN:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Create and use representations to organize, record, and communicate mathematical ideas.

I CAN:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Analyze mathematical relationships to connect and communicate mathematical ideas.

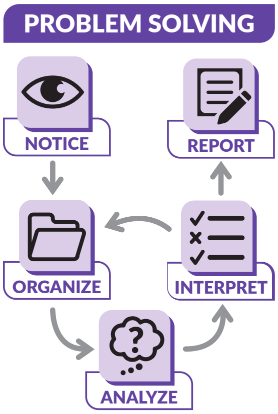
I CAN:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I CAN:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.



The Problem-Solving Model

Productive mathematical thinkers are problem-solvers. These instructional materials include a problem-solving model to help you develop proficiency with the TEKS mathematical process standards. An opportunity to use the problem-solving model is present every time you see the problem-solving model icon.

The problem-solving model represents a strategy you can use to make sense of problems you must solve. The model provides questions you can use to guide your thinking and the graphic organizer provides a place for you to show and organizer your work.

Understanding the Problem-Solving Model

NOTICE Notice | Wonder

Understand the situation by asking these questions.

- What do I know?
- What do I need to determine?
- What important information is given that I will need to determine a solution?
- What information is given that I do NOT need?
- Is there enough information given to solve the problem?

ORGANIZE Organize | Mathematize

Devise a plan for your mathematical approach. Ask yourself these questions.

- What is a similar problem to this that I have solved before?
- What strategies may help to solve this problem using the given information?
- How can I represent this problem using a picture, diagram, symbols, graph, or some other visual representation? Which representations make sense for this problem?

PREDICT Predict | Analyze

Carry out your plan to determine a solution. Then, ask yourself the following questions.

- Did I show my math work using representations?
- Did I explain my mathematical solution in terms of the problem situation, when applicable?
- Did I describe how I arrived at my solution?
- Did I communicate my strategy and solution clearly using precise mathematical language as necessary?
- Can I make any predictions based on my work?

INTERPRET Test | Interpret

Look back at your work and ask these questions.

- Does the solution answer the original question/problem?
- Does the reasoning and the solution make sense?
- How could I have used a different strategy to solve this problem? Would it have changed the outcome?

REPORT Report

As you share your mathematical reasoning with others, ask these questions.

- Did I share my solution with others?
- Do others understand the mathematics I communicated?

The Problem-Solving Model Graphic Organizer

NOTICE **Understand the Problem**	ORGANIZE **Devise a Plan**
PREDICT **Carry Out the Plan**	
INTERPRET **Look Back**	
REPORT **Report**	

Academic Glossary

Knowing what these terms mean and using them will help you demonstrate proficiency with the TEKS mathematical process standards as you think, reason, and communicate your ideas.

Analyze

Definition

Study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Explain Your Reasoning

Definition

Give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

- Show your work
- Explain your calculation
- Justify
- Why or why not?

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Represent

Definition

Display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

- Predict
- Approximate
- Expect
- About how much?

Estimate

Definition

Make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Describe

Definition

Represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

What is Productive Struggle?

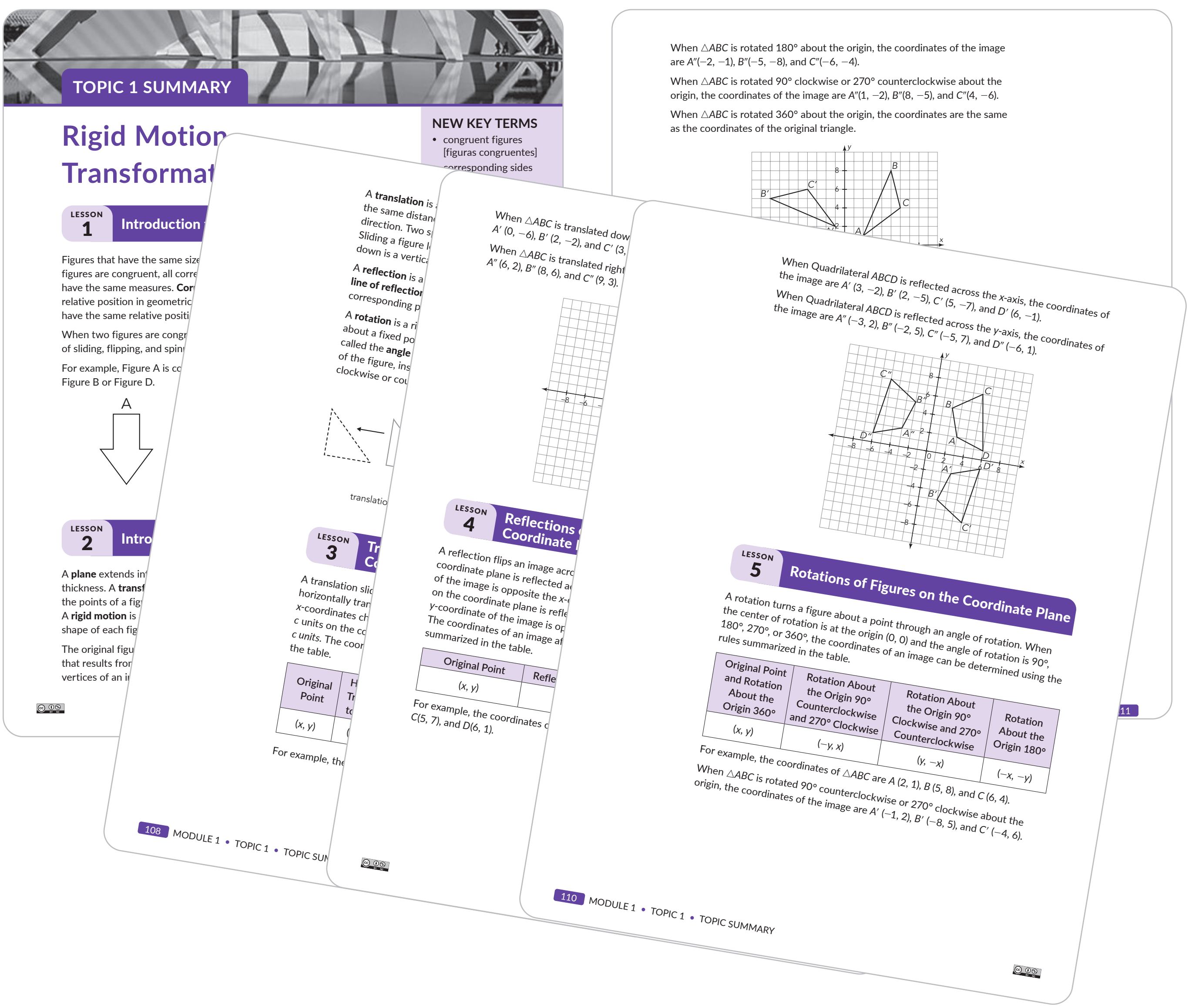
Imagine you are trying to beat a game or solve a puzzle. At some point, you are not sure what to do next. You don't give up; you keep trying repeatedly until you accomplish your goal. You call this process productive struggle. *Productive struggle* is when you choose to persist, think creatively, and try various strategies to accomplish your goal. Productive struggle supports you as you develop characteristics and habits that you will use in everyday life.

Things to do:	Things not to do:
• Persevere. • Think creatively. • Try different strategies. • Look for connections to other questions or ideas. • Ask questions that help you understand the problem. • Help your classmates without telling them the answers.	• Get discouraged. • Stop after trying your first attempt. • Focus on the final answer. • Think you have to make sense of the problem on your own.

This course will challenge you by providing you with opportunities to solve problems that apply the mathematics you know to the real-world. As you work through these problems, your first approach may not work. This is okay, even when your initial strategy does not work, you are learning. In addition, you will discover multiple ways to solve a problem. Looking at various strategies will help you evaluate the problem-solving process and identify an efficient approach. As you persist in problem-solving, you will better understand the math.

Topic Summary

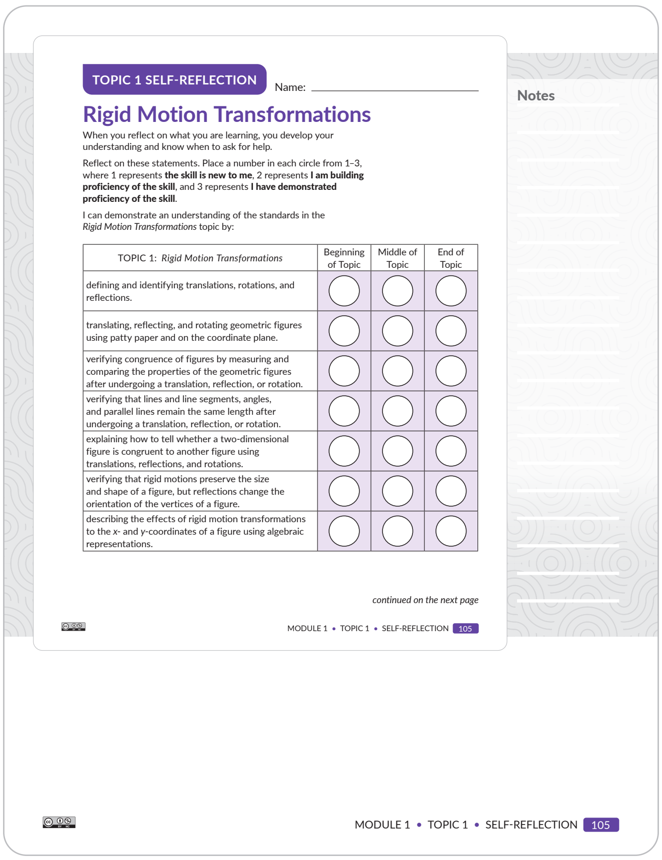
Each topic includes a Topic Summary. The Topic Summary contains a list of all new key terms addressed in the topic and a summary of each lesson, including worked examples and new key term definitions. Use the Topic Summary to review each lesson's major concepts and strategies as you complete assignments and/or share your learning outside of class.



Topic Self-Reflection

The Topic Self-Reflection, provided at the end of each topic, empowers you to develop confidence in your mathematical understanding and monitor your own learning processes. Taking the time for self-reflection helps you identify your strengths and where you want to focus your efforts to improve.

Use the Topic Self-Reflection throughout the topic to monitor your progress toward the mathematical goals for the topic.



TOPIC 1 SELF-REFLECTION Name: ______________

Rigid Motion Transformations

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

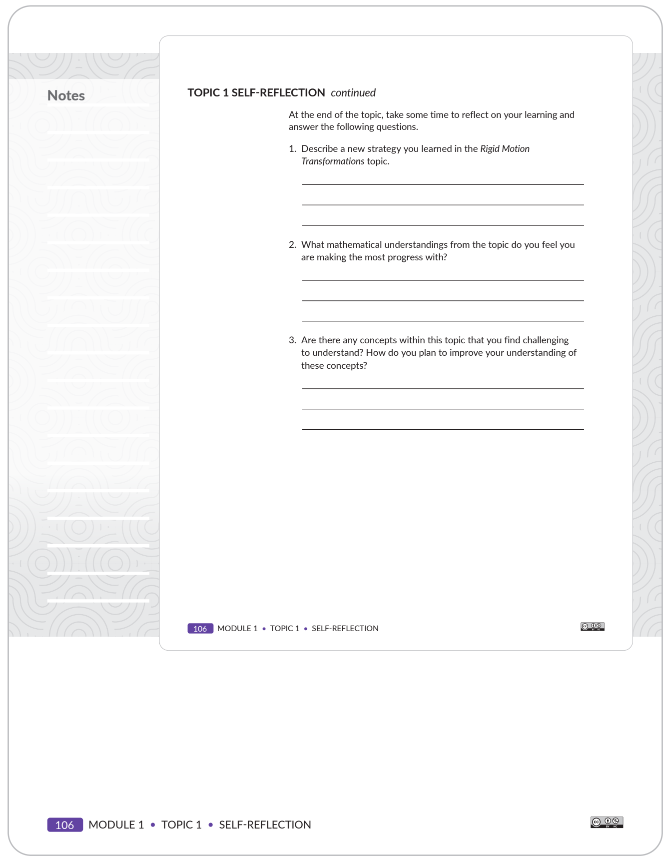
I can demonstrate an understanding of the standards in the *Rigid Motion Transformations* topic by:

TOPIC 1: *Rigid Motion Transformations*	Beginning of Topic	Middle of Topic	End of Topic
defining and identifying translations, rotations, and reflections.	◯	◯	◯
translating, reflecting, and rotating geometric figures using patty paper and on the coordinate plane.	◯	◯	◯
verifying congruence of figures by measuring and comparing the properties of the geometric figures after undergoing a translation, reflection, or rotation.	◯	◯	◯
verifying that lines and line segments, angles, and parallel lines remain the same length after undergoing a translation, reflection, or rotation.	◯	◯	◯
explaining how to tell whether a two-dimensional figure is congruent to another figure using translations, reflections, and rotations.	◯	◯	◯
verifying that rigid motions preserve the size and shape of a figure, but reflections change the orientation of the vertices of a figure.	◯	◯	◯
describing the effects of rigid motion transformations to the *x*- and *y*-coordinates of a figure using algebraic representations.	◯	◯	◯

Notes

continued on the next page

MODULE 1 • TOPIC 1 • SELF-REFLECTION 105



Notes

TOPIC 1 SELF-REFLECTION *continued*

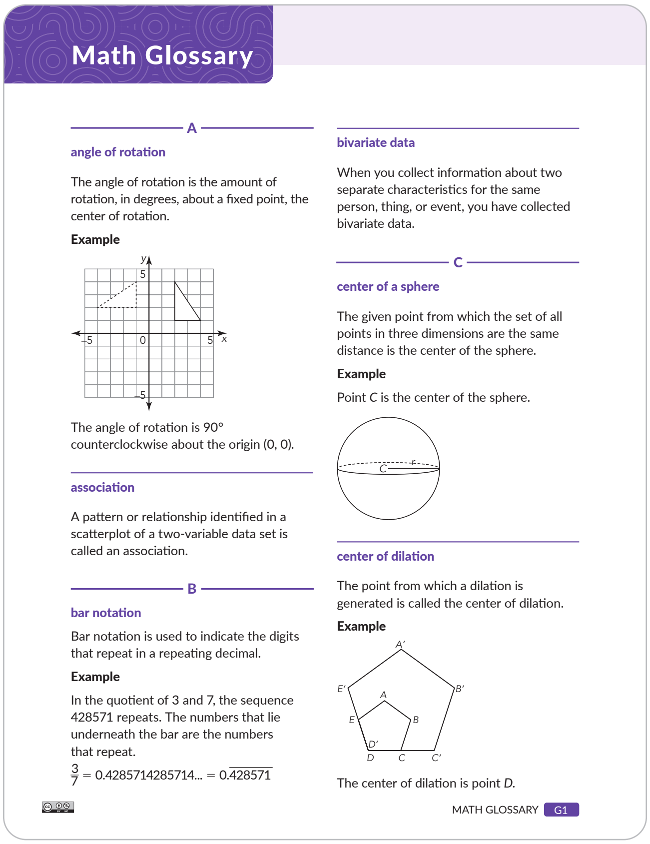
At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Rigid Motion Transformations* topic.
2. What mathematical understandings from the topic do you feel you are making the most progress with?
3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

106 MODULE 1 • TOPIC 1 • SELF-REFLECTION

Math Glossary

A course-specific math glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.



Math Glossary

A

angle of rotation

The angle of rotation is the amount of rotation, in degrees, about a fixed point, the center of rotation.

Example

The angle of rotation is 90° counterclockwise about the origin (0, 0).

association

A pattern or relationship identified in a scatterplot of a two-variable data set is called an association.

B

bar notation

Bar notation is used to indicate the digits that repeat in a repeating decimal.

Example

In the quotient of 3 and 7, the sequence 428571 repeats. The numbers that lie underneath the bar are the numbers that repeat.

$\frac{3}{7} = 0.4285714285714... = 0.\overline{428571}$

bivariate data

When you collect information about two separate characteristics for the same person, thing, or event, you have collected bivariate data.

C

center of a sphere

The given point from which the set of all points in three dimensions are the same distance is the center of the sphere.

Example

Point *C* is the center of the sphere.

center of dilation

The point from which a dilation is generated is called the center of dilation.

Example

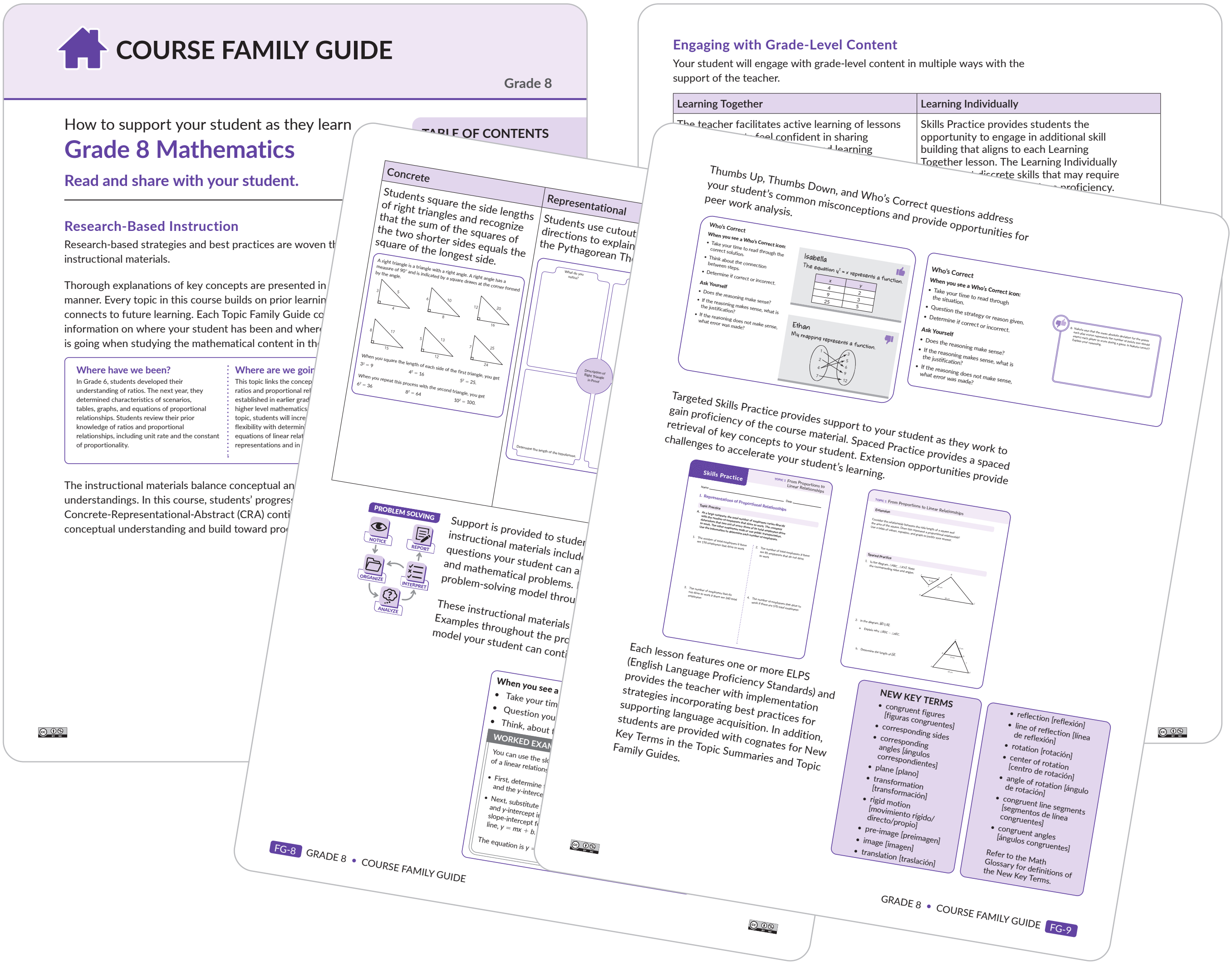
The center of dilation is point *D*.

MATH GLOSSARY G1

Course Family Guide

The Course Family Guide provides you and your family an overview of the course design. The guide details the resources available to support your learning, such as the Math Glossary, the Topic Family Guide, the Topic Self-Reflection, and the Topic Summaries.

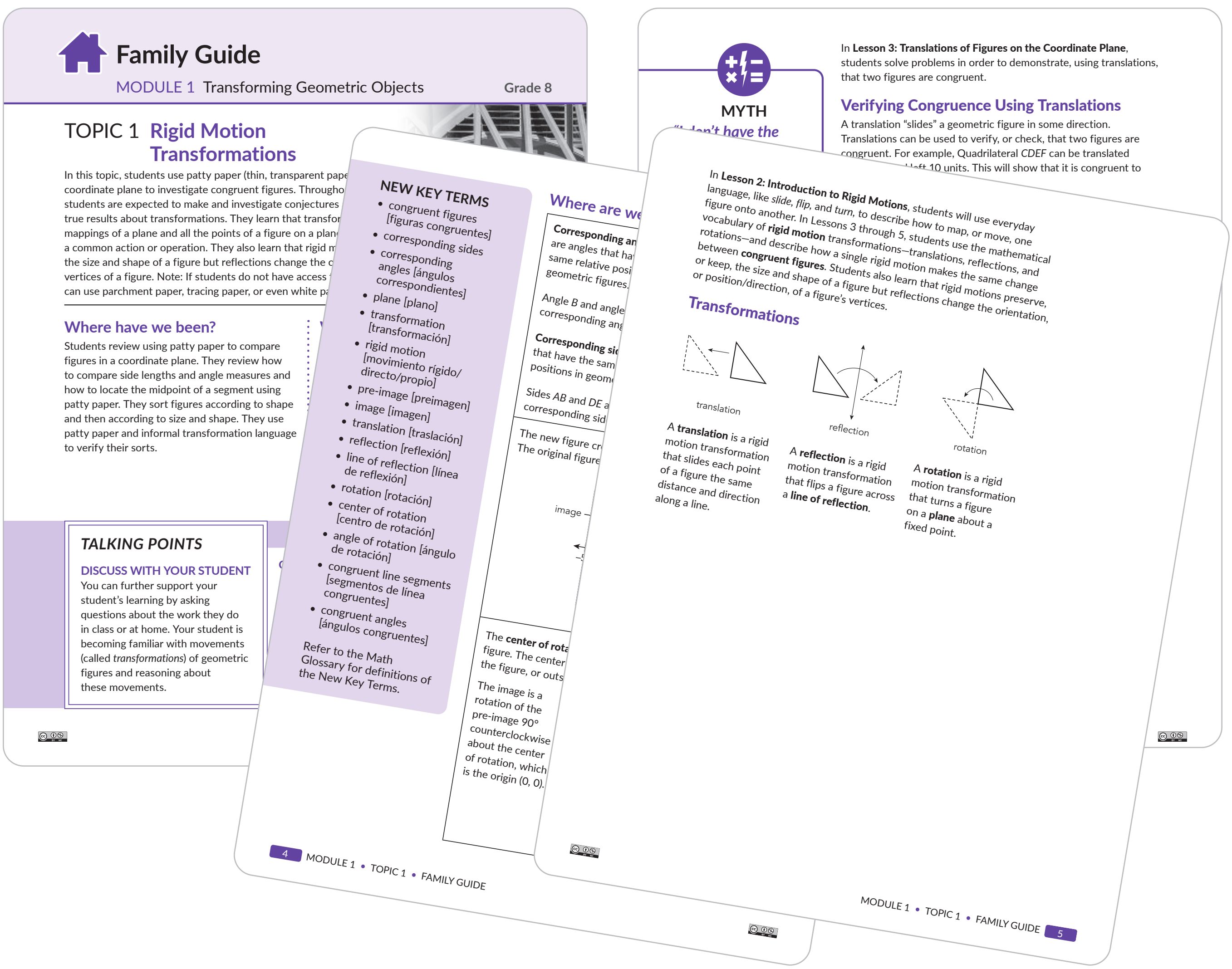
The purpose of the Course Family Guide is to bridge your learning in the classroom to your learning at home. The goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together.



Topic Family Guide

Each topic contains a Topic Family Guide that provides an overview of the math of the topic and answers the questions, “Where have we been?” and “Where are we going?” Additional components of the Topic Family Guide are an example of a math model or strategy taught in the topic, definitions of a few key terms, busting of a math myth, and questions family members can ask you to support your learning.

Learning outside of the classroom is crucial to student success at school. The Topic Family Guide serves to assist families in talking to students about the learning that is happening in the classroom.



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MODULE 3 Data Data Everywhere

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MODULE 5 Applying Powers

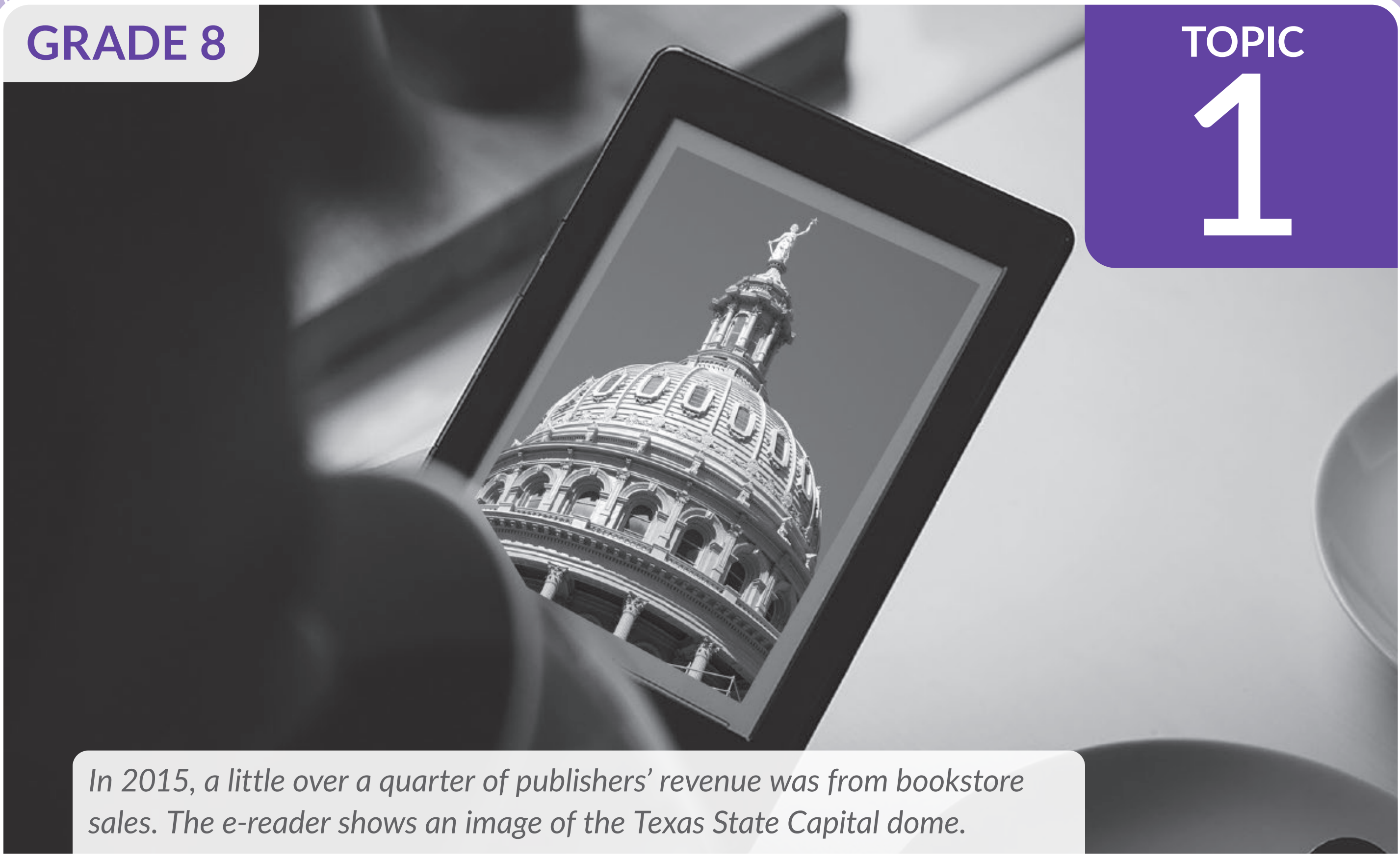
GRADE 8

MODULE
3

Data Data Everywhere

GRADE 8

TOPIC 1



In 2015, a little over a quarter of publishers' revenue was from bookstore sales. The e-reader shows an image of the Texas State Capital dome.

Patterns in Bivariate Data

1 Analyzing Patterns in Scatterplots

OBJECTIVES

- Define bivariate data.
- Collect and record bivariate data.
- Construct and interpret scatterplots for bivariate data to investigate patterns of association.
- Interpret collected data displayed on a scatterplot and in a table.
- Use a scatterplot to determine if there is no relationship or a linear or non-linear relationship between two quantities.
- Identify potential outliers in a scatterplot.

NEW KEY TERMS

- bivariate data
- explanatory variable
- response variable
- association
- linear association
- positive association
- negative association
- outlier

You have analyzed relationships and characteristics of many graphs.

How can you describe patterns of association in a scatterplot of bivariate data?

Getting Started

You Have the Nerve

Your class is going to explore the speed of nerve impulses in the body by performing an experiment that involves a human chain.

- In this experiment, a group of students forms a circle with each person facing out of the circle.
- Another student must be the timekeeper.
- To begin the experiment, the timekeeper says, "Go," and the students carefully, but quickly, pass an empty cup from one student to the student on their right.
- After the last student receives the cup, he or she says, "Stop."
- The amount of time from when the word "Go" is spoken until the word "Stop" is spoken (the amount of time it takes to complete the chain) is recorded by the timekeeper.

In the next activity, you will conduct the experiment using a different number of students in the chain ten times. For each new group of students in the chain, you will conduct three trials and record the average times.

1. Why do you think three trials are needed for each of the different groups of students in the chain?

The Statistical Process

- Formulate a Question
- Collect Data
- Analyze Data
- Interpret the Results

2. Make a prediction about what will happen during the experiment.

ACTIVITY 1.1

Analyzing Scatterplots

It's time to run the Human Chain experiment.

1. Record the data for the experiment in the table shown. Then, calculate the mean time for each row and record the result in the last column of the table. Round your average times to the nearest tenth.

Human Chain Experiment Results

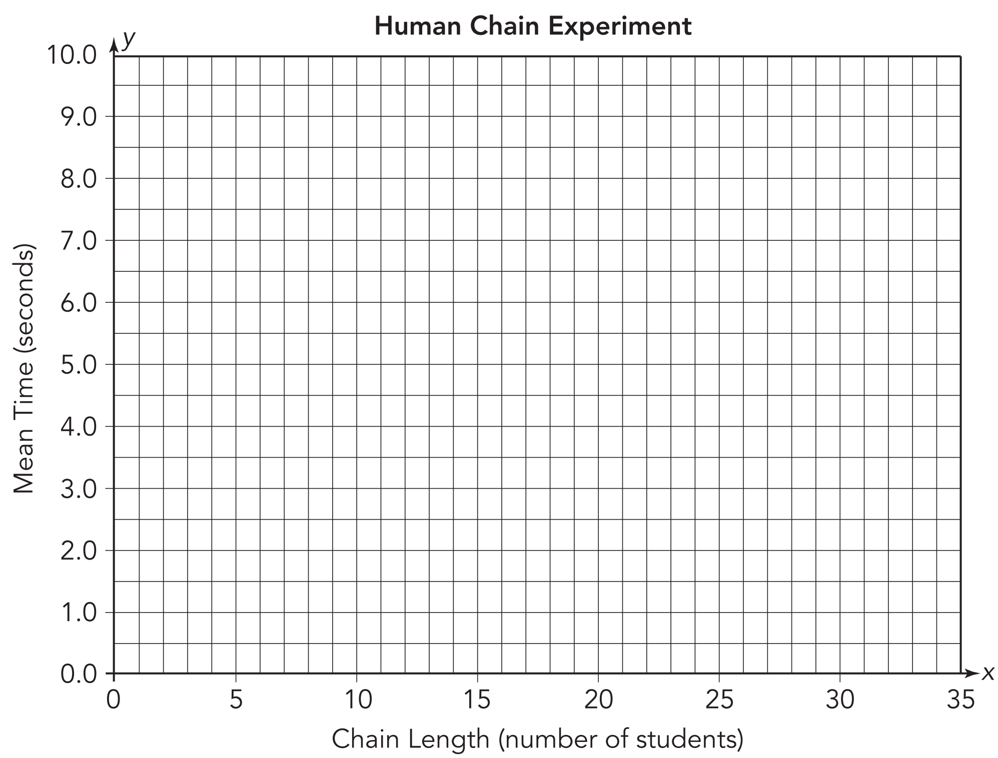
Chain Length (number of students)	Trial 1	Trial 2	Trial 3	Average Time (seconds)

Remember . . .

The *independent variable* is the variable that represents the independent quantity, which is the quantity the *dependent quantity* depends on. The dependent variable is the variable that represents the dependent quantity, which is the quantity that depends on another in a problem situation.

2. Write the ordered pairs from the table with the number of students in the chain as the independent variable and the mean time as the dependent variable.

3. Create a scatterplot of the ordered pairs on the coordinate plane.



4. On the scatterplot, identify the point representing the longest chain. Then, identify the values of the point in the table. Explain how you identified the point and values.

5. On the scatterplot, identify the point representing the least mean time. Then, identify the values of the point in the table. Explain how you identified the point and values.

6. What pattern(s) do you notice about the scatterplot? Compare the scatterplot with your prediction.

ACTIVITY 1.2 Bivariate Data

The School Spirit Club plans to sell sweatpants and sweatshirt sets with the school's logo. The club is determining whether there is a way to package sweatshirt and sweatpants sets so that most of the students can buy a set that will fit.

1. Do you think there might be a relationship between the sweatpant size and the sweatshirt size a person would buy? Why or why not?

When collecting information about a person or thing, the specific characteristic of the information gathered can be called a variable. Previously, you have seen variables in mathematics refer to a letter or symbol to represent a number. In this case, a variable can refer to any characteristic that can change, or vary.

2. Name a variable that can affect a sweatshirt size.

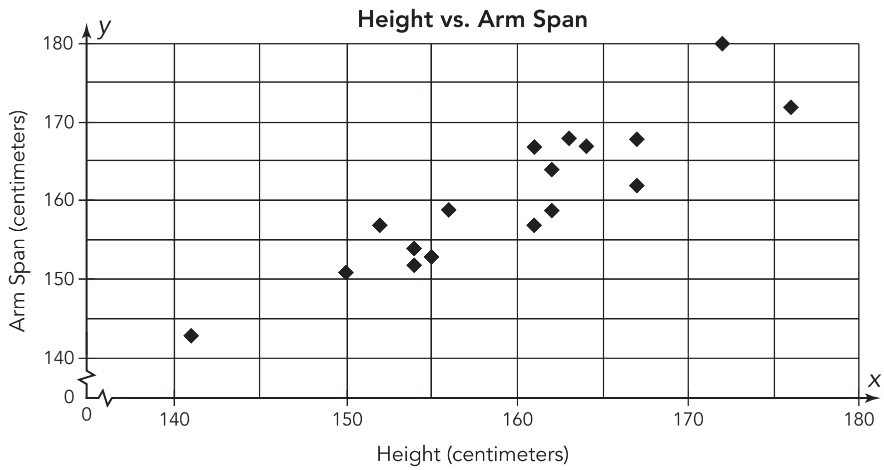
3. Do you think collecting information about one sweatshirt characteristic is enough to determine which shirt sizes should be paired with which pant sizes?

The School Spirit Club decides to collect students' heights and arm spans. They hope that collecting this information can determine if there is a relationship between sweatshirt size and sweatpant size. When you collect information about two separate characteristics for the same person, thing, or event, you have collected **bivariate data**.

4. Why is it important to record each student's height and arm span?

One way to show the relationship between bivariate data is to create a graph that can represent the two variables. The School Spirit Club created a scatterplot using height as the *x*-coordinate and arm span as the *y*-coordinate.

5. What patterns do you notice in the scatterplot?

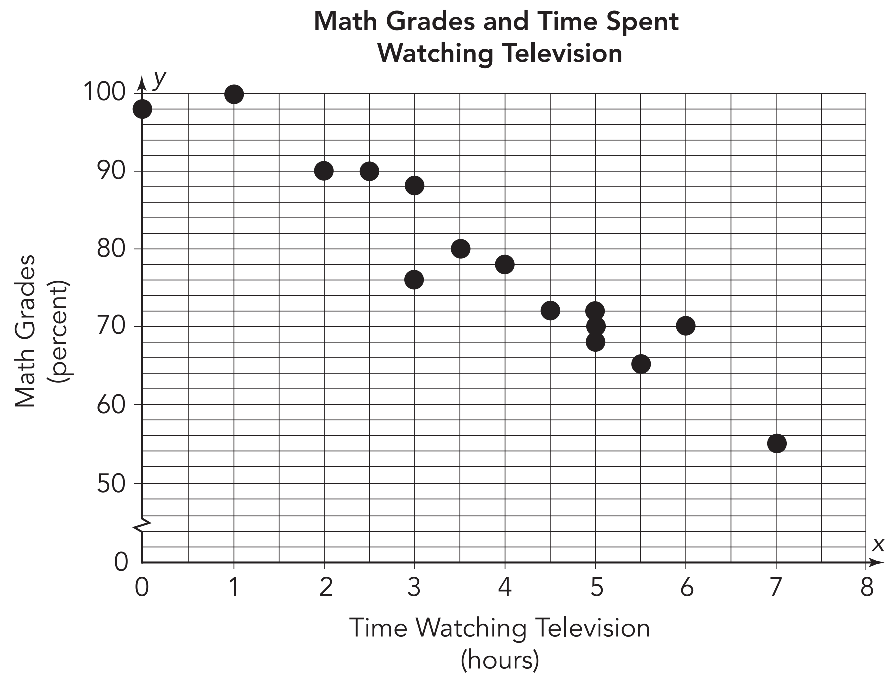


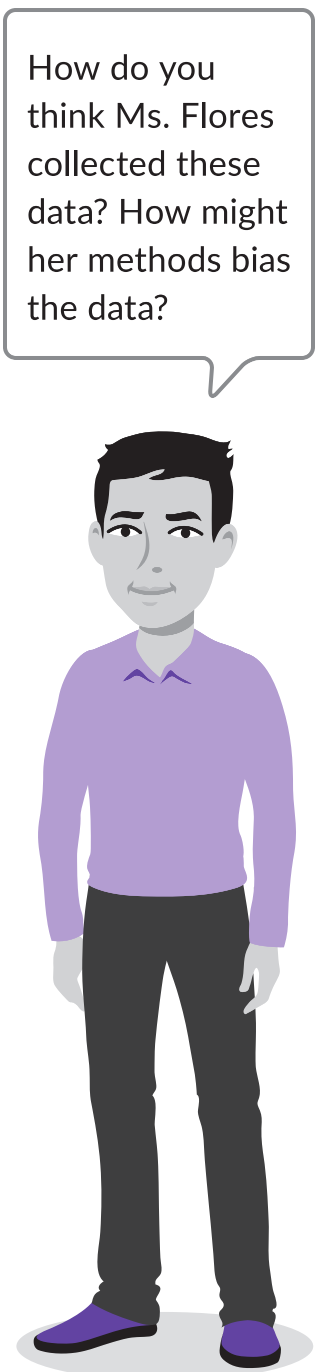
The symbol, -⁄\/-, represents a break in the graph.

6. Now that you have informally analyzed the data represented by a scatterplot, what conclusions can you reach about the relationship between a student's height and arm span?

7. How could your conclusions help the School Spirit Club decide how to package their sets of sweatshirts and sweatpants?

Ms. Flores is trying to determine whether there is a relationship between her students' math grade percentage, and the time spent watching television. She constructed the scatterplot for the number of hours her math students spent watching television per weeknight (Monday through Thursday), and their grade percent in her math class.





8. Circle the point (3.5, 80) on the scatterplot. Explain the meaning of the point.

9. Describe any patterns you see in Ms. Flores's scatterplot.

10. What conclusions can you make about the relationship between her students' math grade percentage and time spent watching television?

ACTIVITY
1.3 Describing Patterns in Scatterplots

When you look for a relationship in bivariate data, often you are interested in determining whether one variable causes a change in the other variable. In this case, one variable, the *explanatory variable*, is designated as the independent variable, and the *response variable*, is designated as the dependent variable.

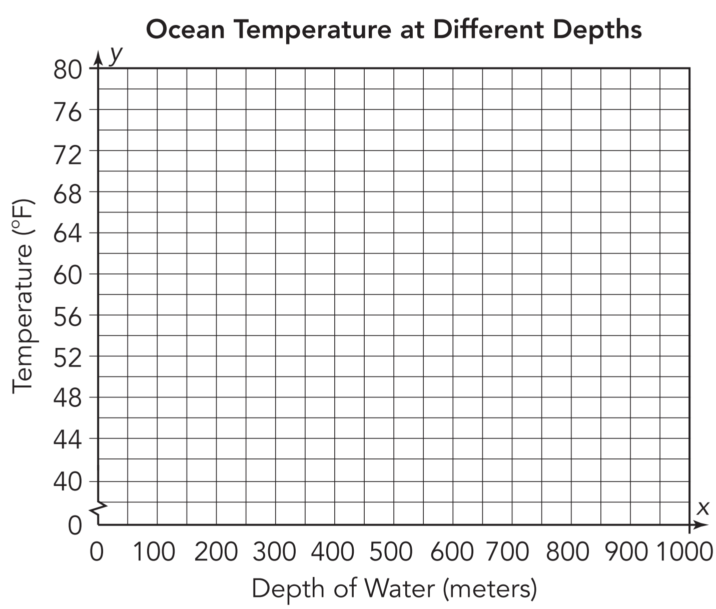
The independent variable can also be called the **explanatory variable**. The dependent variable can also be called the **response variable**, because this is the variable that responds to what occurs to the explanatory variable.

1. James, who is an oceanographer, is measuring the temperature of the ocean at different depths. His results are listed in the table.

Depth (m)	100	200	300	400	500	600	700	800	900
Temperature (°F)	76	73	70	66	61	56	52	48	43

a. Identify the explanatory and response variables in James's data table.

b. Create a scatterplot using the data James gathered for the ocean temperatures at different depths.



Think About . . .
A *cluster* is a group of similar items positioned or occurring closely together. What does this mean in the context of a scatterplot?

c. Explain the meaning of the point (400, 66).

d. What relationship does the scatterplot show between the depth of the ocean water and the temperature of the water?

A **linear association** occurs when the points on the scatterplot seem to form a line. When the data have a distinct pattern that is not linear, that is a *non-linear association*.

As you have experienced, scatterplots can be great tools to identify patterns in bivariate data. Sometimes, these patterns or relationships are called **associations**. One common pattern that exists in data is when the points on a scatterplot form a *linear association*. In that case, the data values are arranged in such a way that, as you look at the graph from left to right, you can imagine a line going through the scatterplot with most of the points being close to the line.

e. Explain how there seems to be a linear association between the depth of the ocean water and the water temperature.

How do positive and negative association relate to positive and negative slope?

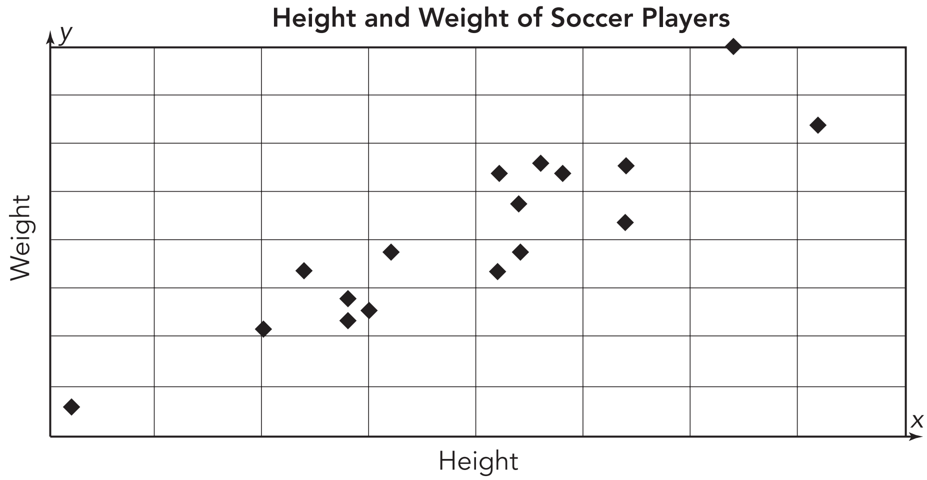
When two variables have a linear association, you can then determine the type of association between the two variables. The two variables have a **positive association** when, as the explanatory variable increases, the response variable also increases. when the response variable decreases as the explanatory variable increases, then the two variables have a **negative association**. Once you identify the pattern for two variables with a linear relationship, you can state the association between the two variables.

When the data do not appear to have any distinct pattern, the scatterplot shows no association between the variables.

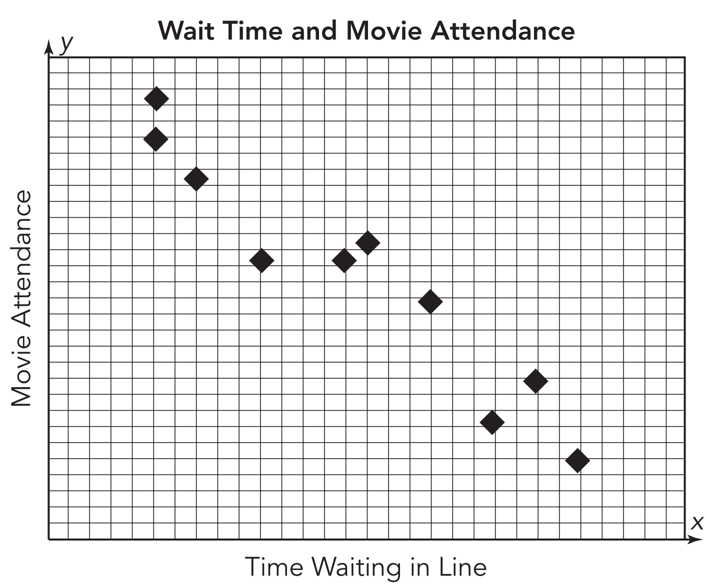
f. Describe the type of association that exists between the depth of the ocean water and the water temperature. State the association in terms of the variables.

2. Analyze each scatterplot shown. Identify the explanatory and response variables. Then, determine whether the scatterplot shows a linear association, nonlinear association, or no association. When there is a linear relationship, determine whether it has a positive association or a negative association, and state the association in terms of the variables.

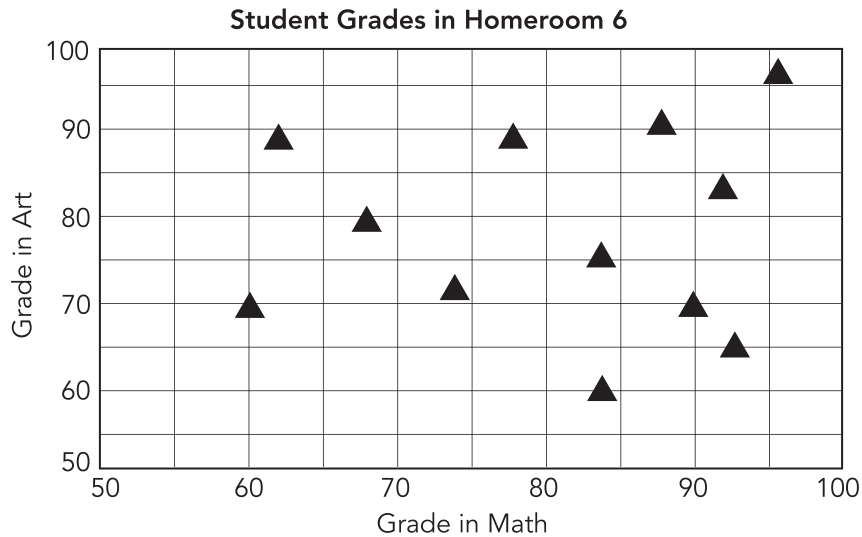
a.



b.

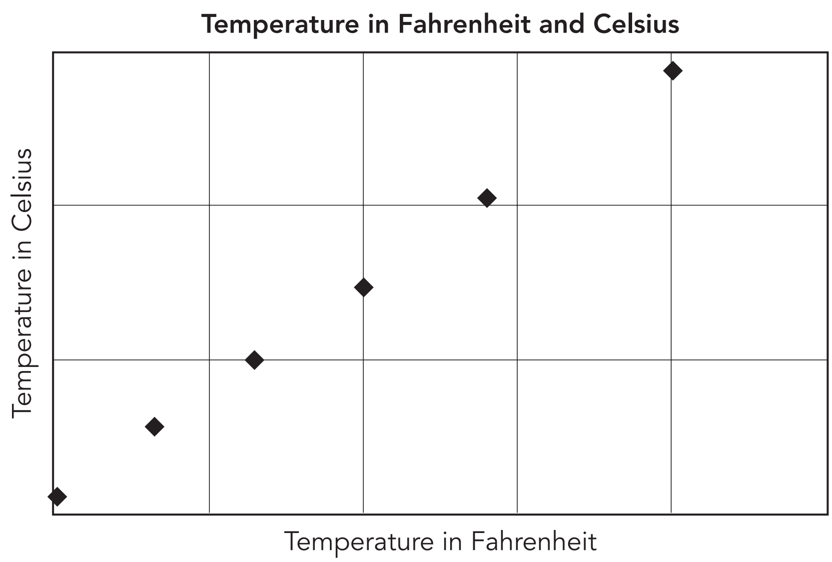


c.



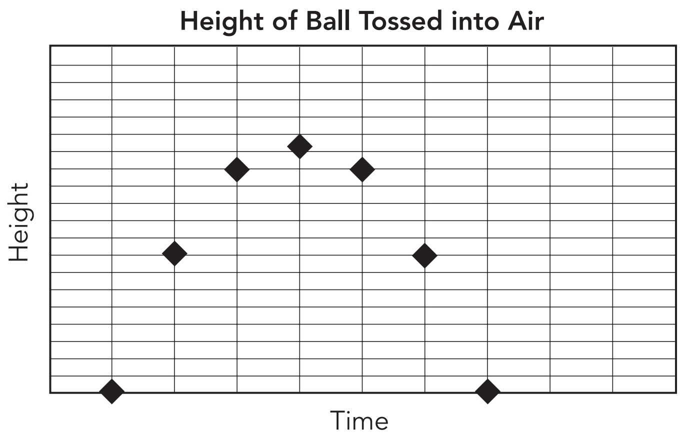
Student Grades in Homeroom 6
100
90
80
70
60
50
Grade in Art
50
60
70
80
90
100
Grade in Math

d.

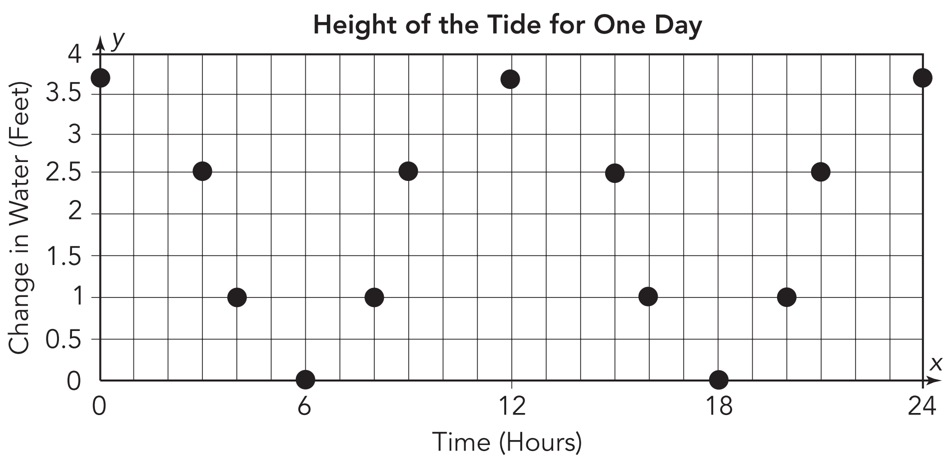


Temperature in Fahrenheit and Celsius
Temperature in Celsius
Temperature in Fahrenheit

e.



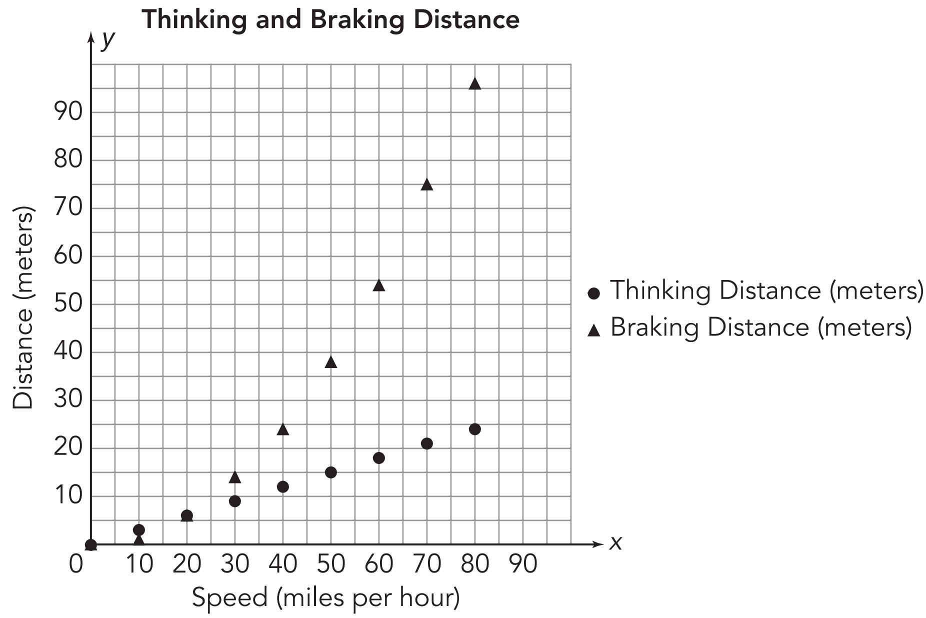
f.



To be a safe driver, you need to understand the factors that affect a car's stopping distance. The stopping distance depends on two factors:

- The thinking distance is the distance traveled in between the driver realizing he needs to brake and actually braking.
- The braking distance is the distance taken to stop once the brakes are applied.

The graph shows how, under normal driving conditions, thinking distance and braking distance depend on the speed of the car.



3. Use the scenario and graph to answer each question.

   a. Identify the explanatory and response variables.

   b. Do you think that there is a linear, non-linear, or no relationship between the speed and the thinking distance? Explain your reasoning.

   c. Do you think that there is a linear, non-linear, or no relationship between the speed and the braking distance? Explain your reasoning.

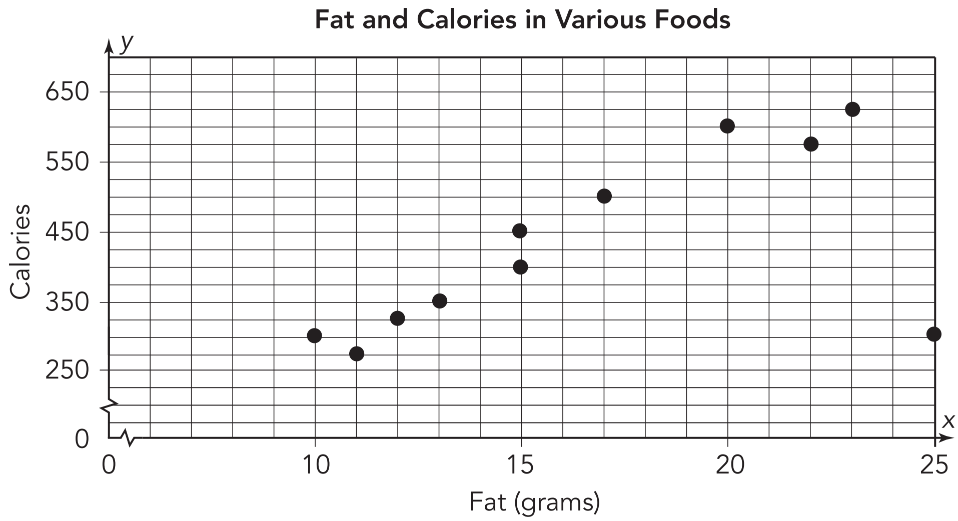
   d. What conclusions can you make from this scatterplot?

ACTIVITY 1.4

Outliers

Another pattern that can occur in a scatterplot is an *outlier*. An **outlier** for bivariate data is a point that varies greatly from the overall pattern of the data.

1. The scatterplot shows the fat and calories in 11 different foods.



Fat (g)	10	11	12	13	15	15	17	20	22	23	25
Calories	300	275	325	350	400	450	500	600	575	625	300

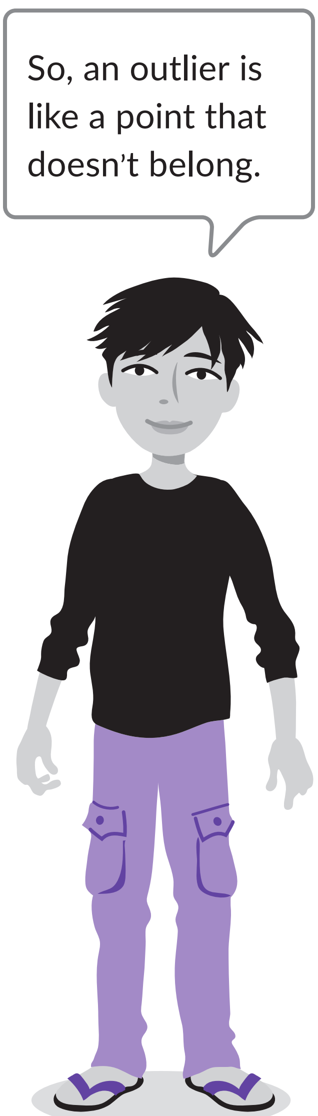
a. Determine the explanatory and response variables in the bivariate data set.

b. Does there appear to be a linear association between the fat and calories of the foods?

c. Do any of the points appear to vary greatly from the other points? When so, circle any outliers in the scatterplot and identify the outlier in the table.

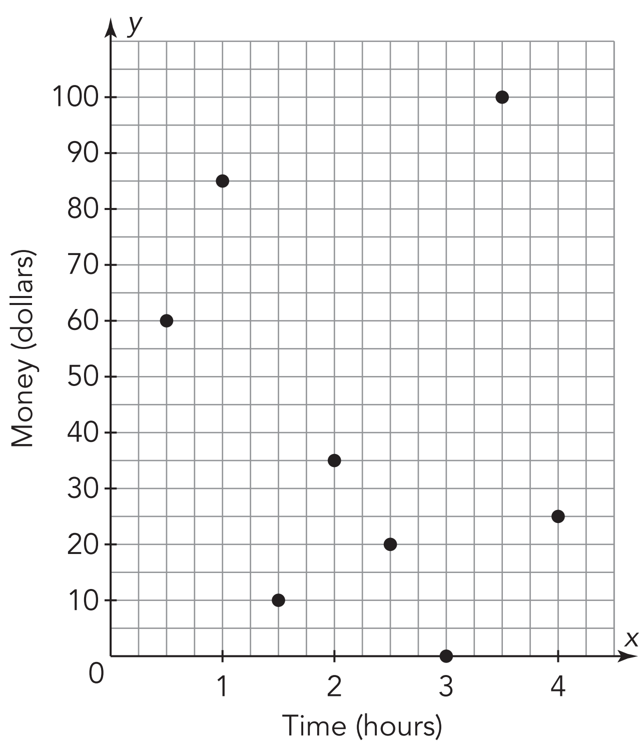
d. Explain why the point (25, 300) is a potential outlier.

e. Examine the values in the table. How can you determine that (25, 300) is a possible outlier?



f. Use your finger to cover up the point (25, 300) and examine the scatterplot. What do you notice?

2. The scatterplot shows the amount of time customers were in a bookstore and the amount of money spent by each customer.



a. Does there appear to be a linear association between the time and money?

b. Use your finger to cover up the point (3.5, 100) and examine the scatterplot. What do you notice?

Talk the Talk

Recognizing the Difference

1. Explain how you can determine whether a scatterplot shows a linear, non-linear, or no association.

2. Explain the difference between a positive association and a negative association of bivariate data.

3. Explain how you can identify an outlier in bivariate data. Do the data need to have a linear association?

Lesson 1 Assignment

Write

Match each term to its corresponding definition.

1. explanatory variable
2. response variable
3. linear association
4. cluster
5. positive association
6. negative association
7. outlier

a. when points on a scatterplot seem to form a line

b. when, as the independent variable increases, the dependent variable also increases

c. the variable whose value is not determined by the other variable

d. a point that varies greatly from the overall pattern of the data

e. when points on a scatterplot are not in a perfect line but are grouped close to an imagined line

f. the variable that changes according to changes in the other variable

g. when the dependent variable decreases as the independent variable increases

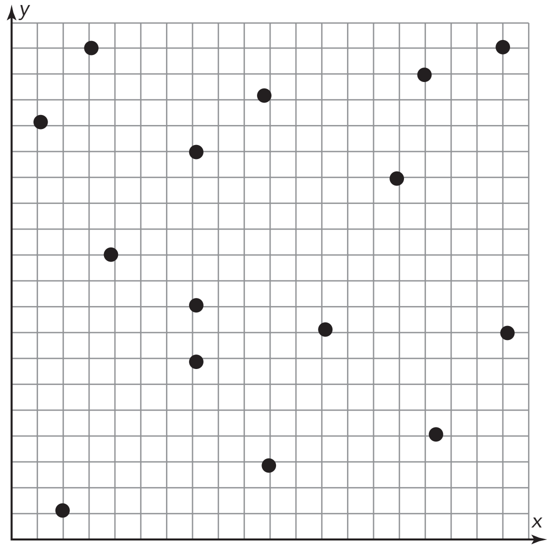
Remember

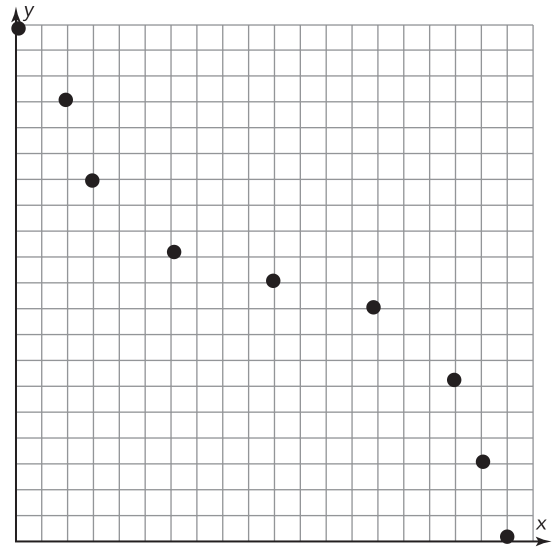
A scatterplot is a graph of a set of ordered pairs. The points in a scatterplot are not connected, but they allow you to investigate patterns in bivariate data. *Bivariate data* are used when collecting information regarding two characteristics for the same person, thing, or event.

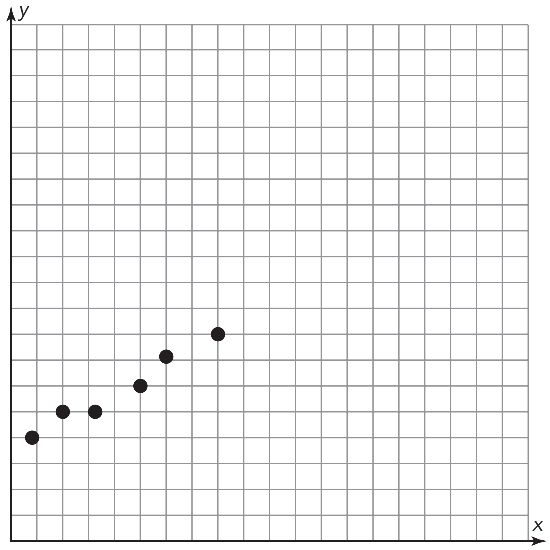
Lesson 1 Assignment

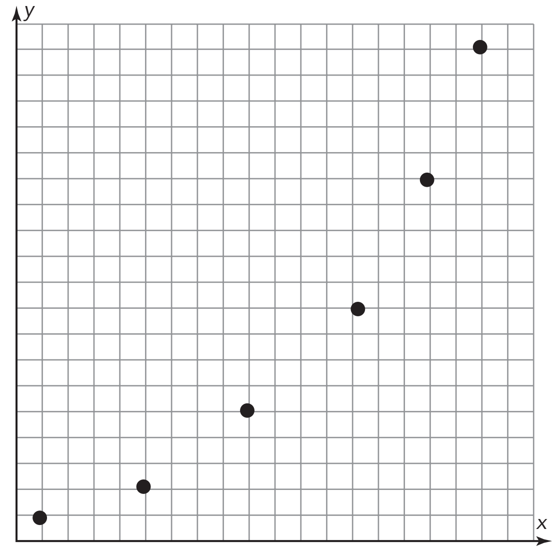
Practice

1. Determine whether each scatterplot represents a linear relationship, a non-linear relationship, or no relationship.

a. 

b. 

c. 

d. 

Lesson 1 Assignment

2. The table shows the relationship between a runner's average speed and the average number of steps the runner takes each second.

Speed (feet per second)	Steps (steps per second)
16.00	3.00
17.00	3.10
17.50	3.20
18.50	3.25
20.00	3.40
21.00	3.50
22.00	3.60
23.00	3.75
24.00	3.80
24.50	3.90

   a. Identify the explanatory and response variables.

   b. Construct a scatterplot using the data. Be sure to label the axes and the graph.

   c. What relationship seems to exist between the runner's average speed and their average steps per second?

   d. Explain the meaning of the point (23.00, 3.75).

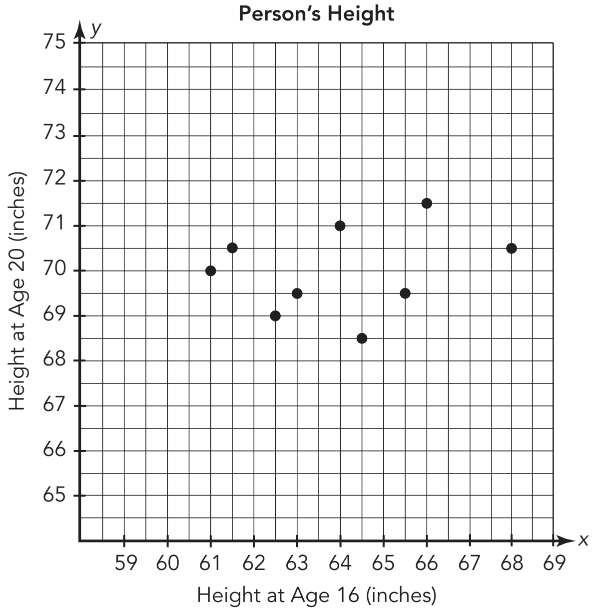
Lesson 1 Assignment

e. Does there appear to be a linear association between the average speed of the runners and their average steps per second?

f. What type of association exists between the runners' average speed and their average steps per second? State the association in terms of the quantities.

g. Do any of the points appear to vary greatly from the other points? Identify the potential outlier. Explain why it is a potential outlier.

3. Mr. Lopez's 12th-grade biology class wondered whether they could predict a person's height at age 20 if they knew the person's height at age 16. The class collected data and displayed it as a scatterplot. Do you think you could predict person's height at age 16 by knowing their height at age 20? Explain your reasoning.

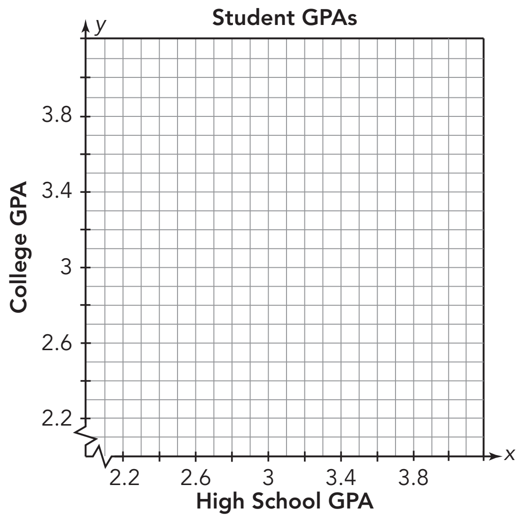


Lesson 1 Assignment

4. Ms. Tanaka is a guidance counselor at a local high school. She is giving a presentation to the freshman class about the importance of studying and getting good grades. She displays data she collected on 12 previous students and their progress in college. The data includes the students' high school GPA and their first year college GPA. The table shows the data she has collected.

   a. Identify the explanatory and response variables.

   b. Graph the data to complete the scatterplot.



   c. Does there appear to be a linear association between the high school GPA and the college GPA? Explain your reasoning.

Student	High School GPA	College GPA
1	2.22	2.35
2	2.50	2.80
3	3.42	3.88
4	3.45	3.40
5	2.45	2.95
6	2.67	3.10
7	3.24	3.55
8	3.80	3.92
9	3.11	3.40
10	3.15	3.50
11	3.25	3.52
12	2.88	2.90

d. Is there a positive or negative association between high school GPA and college GPA?

e. Write the ordered pair for the student with the highest high school GPA. Then explain the meaning of each of the coordinates.

f. Write the ordered pair for the students who have the same college GPA. What was the college GPA for each student, and what was their high school GPA?

Prepare

1. Solve for x.

   a. $50 = 3.5x + 24.2$

   b. $30 = -2.9x + 50.3$

2. Solve for y when $x = 6$.

   a. $y = 3.5x + 24.2$

   b. $y = -2.9x + 50.3$

2 Drawing Trend Lines

OBJECTIVES

- Identify the trend line as a straight line used to model relationships between two quantitative variables.
- Informally fit a straight line to a set of data.
- Write and interpret the equation of a trend line.
- Use a trend line to make predictions.
- Compare trend lines.

NEW KEY TERMS

- model
- trend line
- interpolating
- extrapolating

You have used equations to represent graphs of linear relationships.

How do you create a model for a scatterplot that displays a linear association?

Getting Started

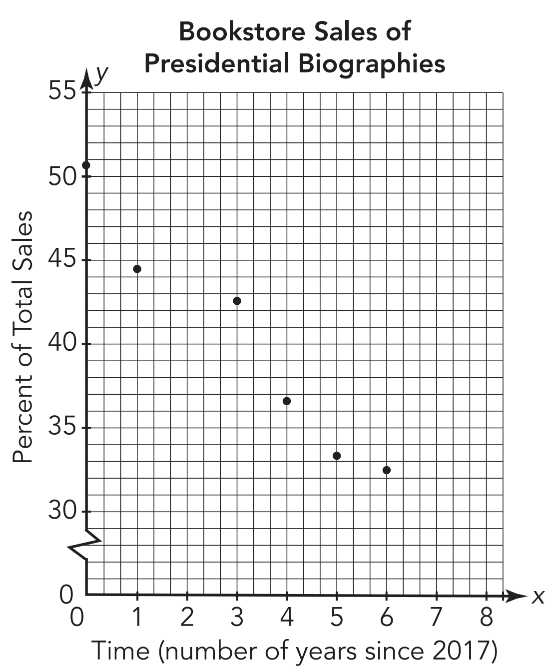
Brick-and-Mortar Book Sales

You can purchase books from many different places: a bookstore, a department store, the Internet, a book club, and many other places. The source for purchasing books changes as the available formats for books change.

Suppose the table and scatterplot show the percent of presidential biographies sales that came from bookstores for the years 2017 through 2023, but the data for 2019 is unknown.

Year	Percent of Total Sales
2017	50.8
2018	44.5
2020	42.5
2021	36.8
2022	33.2
2023	32.5

When you use 0 to indicate a particular year, such as 2017, you should indicate this on your graph with the appropriate axis label. The break in the *y*-axis of the graph symbolizes an interruption in the scale.



1. Describe the relationship between the explanatory and response variables.

2. Do all of the points in the scatterplot lie on the same line? What does this tell you about the percent of total sales as the time changes?

3. Use a piece of thin pasta as a "movable line" to estimate the percent of presidential biographies sales from bookstores for each year. Explain your strategy.

Ask Yourself . . .
Will all of your classmates have the same estimates?

   a. 2019

   b. 2021

   c. 2023

   d. 2024

4. Explain why you should not use this graph to estimate the percent of presidential biographies sales from bookstores in the year 2029.

ACTIVITY 2.1

A Trend Line

A trend line is sometimes formally referred to as a *line of best fit*.

Sometimes, it may seem that there is not a linear relationship between the data points in a scatterplot. However, some of the data points may be close to where a straight line might pass. Although a straight line will not pass through all of the points in your scatterplot, you can use a line to approximate the data as closely as possible. This kind of line is called a *trend line*. A **trend line** is a line that is as close to as many points as possible but doesn't have to go through all of the points.

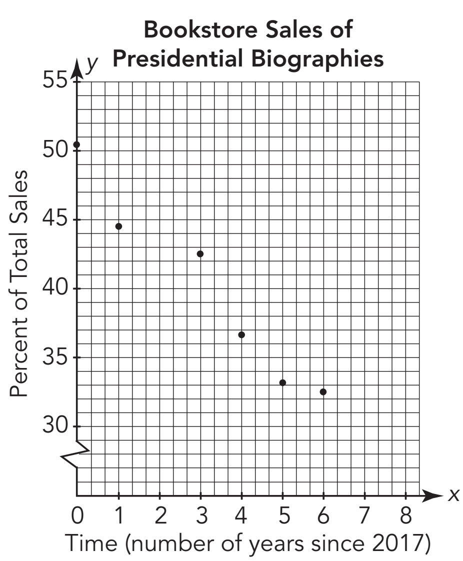
When data are displayed with a scatterplot, constructing a trend line is helpful to predict values that are not displayed on the plot. You want to begin by analyzing the data and asking yourself these questions:

- Does the data look like a line?
- Does the data seem to have a positive or negative association?

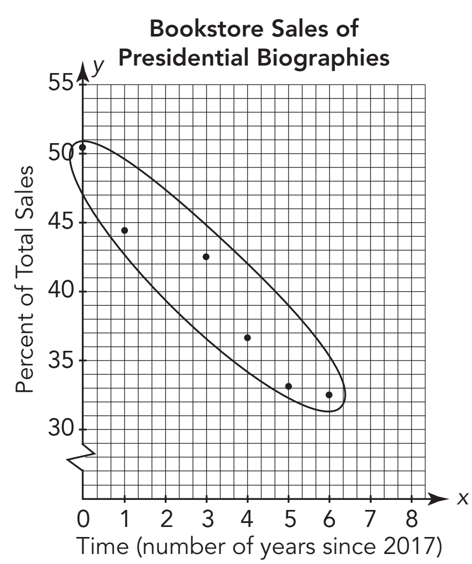
WORKED EXAMPLE

Let's construct a trend line.

Step 1: Begin by plotting all of the data.



Step 2: Draw a shape that encloses all of the data.

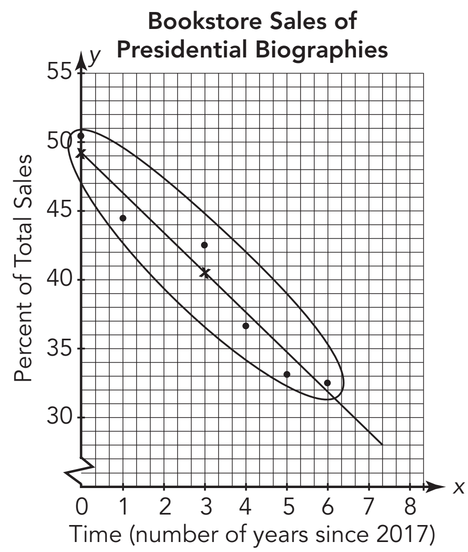


Try to draw a smooth and relatively even shape that represents how the data are clustered.

Step 3: Draw a line that divides the enclosed area of the data in half.

Note that the trend line does not have to go through any of the data values.

Step 4: Determine the equation of your trend line.



- Begin by identifying two points on your trend line. In this example, two points were chosen and marked with an "x." The estimated ordered pairs are (0, 49.2) and (3, 40.5). These points may or may not be data points, but they must be on the trend line.
- Calculate the slope of the line through the two points.

$$m = \frac{49.2 - 40.5}{0 - 3} = \frac{8.7}{-3} = -2.9$$

- Write the equation of the line.

Let x represent the number of years since 2017, and let y represent the percent of all sales.

$$y = -2.9x + 49.2$$

This equation is the algebraic representation of the trend line.

The idea is that you want to identify a line that divides the area in half.

It is possible to choose two different points and estimate those ordered pairs in a slightly different way. Determining the trend line may lead to different equations depending upon the estimated ordered pairs chosen to construct the line. However, if the data closely fit a line, the slopes of the different trend lines should be close together.

1. Identify the slope of the trend line and what it represents in this problem situation.

2. Identify the y-intercept of the trend line and what it represents in this problem situation.

When you are predicting values that fall within the plotted values, you are **interpolating**. When you are predicting values that fall outside the plotted values, you are **extrapolating**.

3. You have already predicted the percent of presidential biographies sales in 2019. Let's compare that prediction with a prediction using the equation for the trend line.

   a. Is predicting the percent of presidential biographies sales from bookstores in 2019 interpolation or extrapolation?

   b. Use the trend line from the Worked Example to predict the presidential biographies sales from bookstores in 2019.

In future courses, you will learn formal methods for determining a specific trend line: *a regression model.*

4. Use the trend line equation to predict the percent of presidential biographies sales from bookstores in each given year.

   a. 2019

   b. 2021

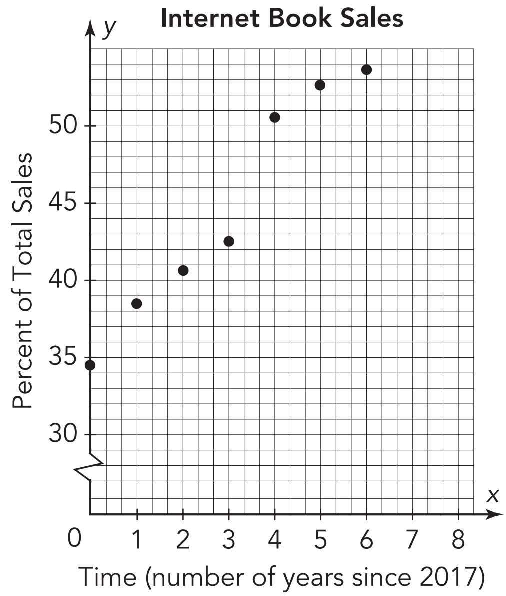
   c. 2023

   d. 2024

   e. Do you think the trend line provides reasonable predictions for these years? Explain your reasoning.

ACTIVITY 2.2 Another Trend Line

Suppose the table and scatterplot show the percent of all book sales that came from the Internet for the years 2017 through 2023.



Internet Book Sales	
Year	Percent of Total Sales
2017	34.4
2018	38.5
2019	40.8
2020	42.4
2021	50.7
2022	52.8
2023	53.8

1. Identify the explanatory and response variables in this problem situation.

2. Analyze the scatterplot. Do the data appear to be close to a line? When so, does the data seem to have a positive association or negative association?

3. Use a straightedge to draw the line that best fits your data on the graph. Then, write the equation of the line. Define your variables and include the units.

Ask Yourself . . .
Should all of your classmates write the same equation for their trend lines?

4. Interpret the meaning of the slope in this problem situation.

5. Interpret the meaning of the *y*-intercept in this problem situation.

6. Compare the actual values to values using your equation.

   a. How close is the value of the *y*-intercept to the actual value?

   b. Use your equation to predict the percent of Internet book sales in 2019. How close is this answer to the actual data?

   c. Use your equation to predict the percent of Internet book sales in 2023. How close is this answer to the actual data?

Start by drawing a shape around all of the data. Then, divide that in half.

You can use the equation of your trend line to predict the percent of book sales from the Internet for a given year.

WORKED EXAMPLE

Predict the percent of book sales from the Internet in 2024.

This is an example of extrapolation since the year 2024 is outside the given data set. Since 2017 represents year 0, 2024 represents 2024 – 2017 or year 7.

Next substitute 7 for x into your equation for the trend line.

Suppose your equation for the trend line is $y = 3.5x + 34.3$.

$3.5(7) + 34.3 = 58.8$.

So, about 59% of the total printed book sales were predicted to come from the Internet in 2024.

7. Use your equation to predict the percent of book sales from the Internet for each given year.

   a. 2026

   b. 2028

   c. How is making a prediction different when using a graph versus an equation?

8. Pose and answer a statistical question that might lead to this analysis.

9. Consider both sets of data in this lesson.

   a. Do you think that the data from the two data sets are related?

   b. Which percent of book sales is changing faster: presidential biographies sales from bookstores or from the Internet? Explain your reasoning.

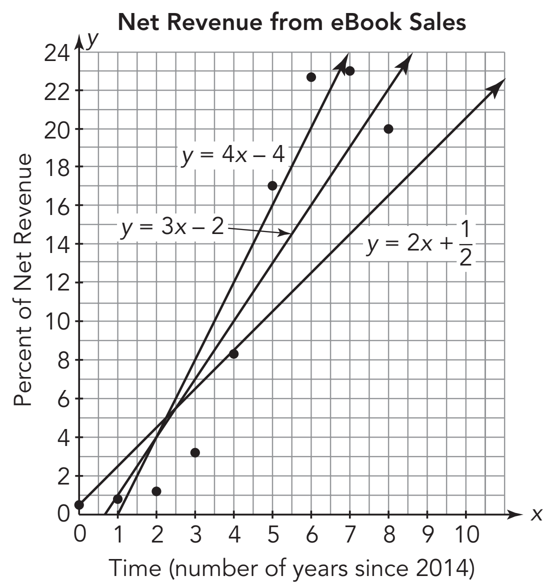
   c. Which equation models its data better? Explain your reasoning.

Talk the Talk

eBook Sales

The table and scatterplot show the percent of net publisher revenue attributed to eBook sales during selected years from 2014 until 2023. The scatterplot includes three proposed trend lines.

Year	Percent of Net Revenue
2014	0.5
2015	0.8
2016	1.2
2017	3.2
2018	8.3
2019	17
2020	22.6
2022	23
2023	20



1. Determine which line provides the best fit. Explain your reasoning.

2. Which line would you use to determine the percent of net revenue for 2021? Use the equation for the trend line to predict the percent of net revenue for 2021.

3. Which line would you use to determine the percent of net revenue for 2024? Use the equation for the trend line to predict the percent of net revenue for 2024.

Lesson 2 Assignment

Write

Explain the relationship between the terms *trend line* and *model*.

Remember

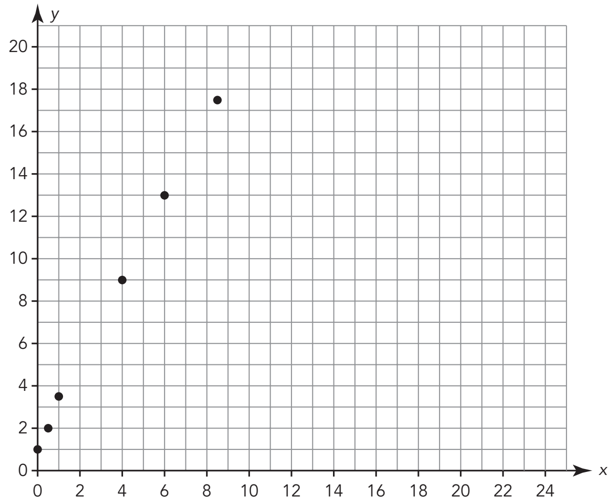
A *trend line* is a straight line that is as close to as many points as possible but does not have to go through any of the points on the scatterplot.

A trend line can be used to make predictions about bivariate data through interpolation and extrapolation.

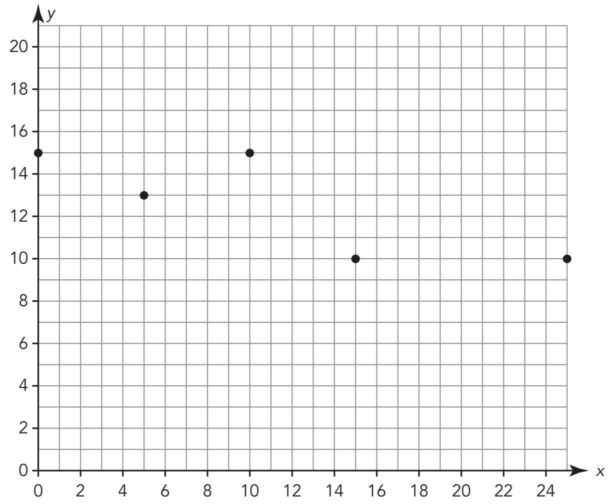
Practice

1. Estimate the equation of the line of best fit for each graph.

a.

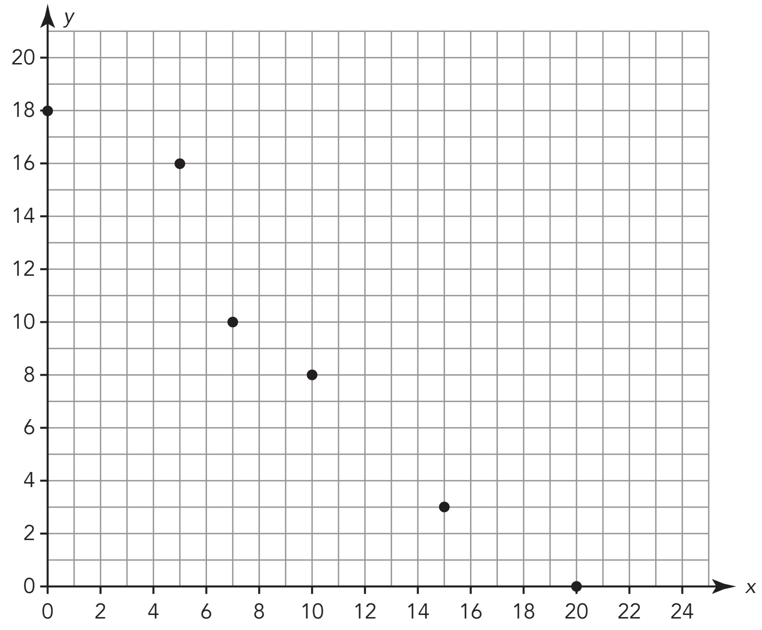


b.



Lesson 2 Assignment

c.



2. Compare each pair of graphs to determine which line is a better fit for the data.

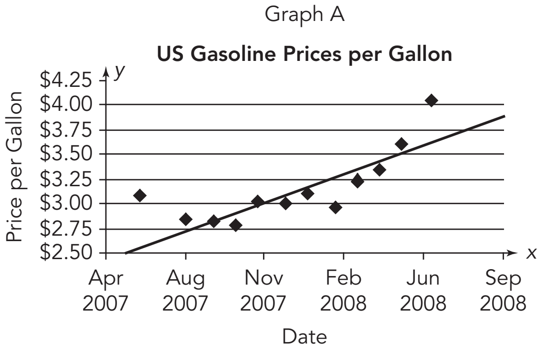
a.

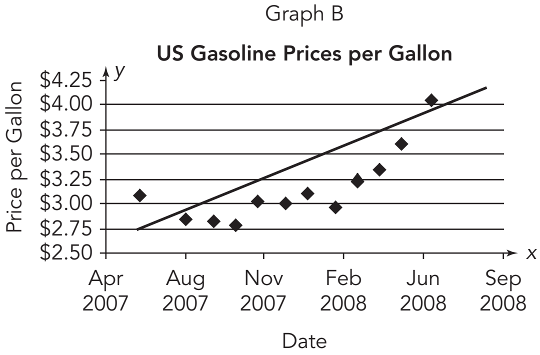
Graph A

Graph B

Lesson 2 Assignment

b.





3. The table shows the percent of voter participation in U.S. presidential elections in selected years from 1976 to 2020.

Election Year	Voter Participation as Percent
1976	54.8
1980	54.2
1984	55.2
1988	52.8
1992	58.1
1996	51.7
2000	54.2
2004	60.1
2008	61.6
2012	58.6
2016	60.1
2020	66.7

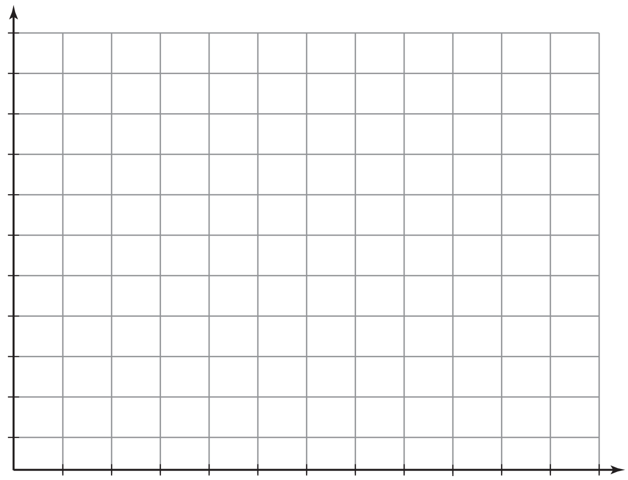
a. Because the *x*-coordinates represent time, we can define time as the number of years since 1976. Therefore, 1976 would become 0. What number would you use for 1980? What number would you use for 1984? What number would you use for 1988? Explain your reasoning.

b. Write the ordered pairs from the table that show the percent of voter participation as the response variable and the number of years since 1976 as the explanatory variable.

Lesson 2 Assignment

c. Looking at the data, do you think the trend line will have a positive slope or a negative slope? Explain your reasoning.

d. Create a scatterplot of the ordered pairs. First, label the axes for your scatterplot to represent the explanatory and response variables. Next, choose the appropriate intervals for your scatterplot. Finally, title your scatterplot.



e. Use a straightedge to draw the line that best fits your data on the graph. Then, write the equation of the line. Define your variables and include the units.

f. Interpret the slope and y-intercept of the equation in terms of the problem situation.

g. Use your trend line to determine what the voter participation was in 1980. How does the value from your equation compare with the actual turnout of 54.2%?

h. Use your trend line to predict the percent of voter participation in 2020. How does your prediction compare with the actual voter turnout of 66.7%?

i. For what years is your model reliable for predicting voter turnout in presidential elections?

Lesson 2 Assignment

Prepare

Complete each statement.

1. If a line has a positive slope, then, as the x-values increase, the y-values ________________________.

2. If a line has a negative slope, then, as the x-values increase, the y-values ________________________.

3. If a line has a slope of 7, then, as the x-values increase by one unit, the y-values ________________________.

4. If a line has a slope of 7, then, as the x-values increase by 5 units, the y-values ________________________.

3 Analyzing Trend Lines

OBJECTIVES

- Draw a trend line.
- Write and interpret an equation of a trend line.
- Use the trend line to make predictions and solve problems in the context of bivariate data, interpreting slope and intercept.
- Assess the fit of a linear model.

You have used trend lines to make predictions.

How can you use interpolation and extrapolation to make predictions over time?

Getting Started

Mighty Catalina

Catalina was born a healthy, happy baby girl to the Brown family. At each doctor's visit, Catalina's height and weight were recorded. Her records from birth until she was 18 months old are shown in the table.

Age (months)	Height (inches)
0.0	17.9
1.0	20.5
1.8	21.0
2.3	21.8
4.0	25.0
6.0	25.8
8.0	27.0
10.0	27.0
12.0	29.3
15.0	30.5
18.0	32.5

Consider the relationship between Catalina's age and her height.

1. What happens to Catalina's height as she gets older?

2. Do you think she will continue growing at this rate? Why or why not?

ACTIVITY
3.1 Analyzing Catalina's Height

Use the information about Catalina's age and height to answer each question.

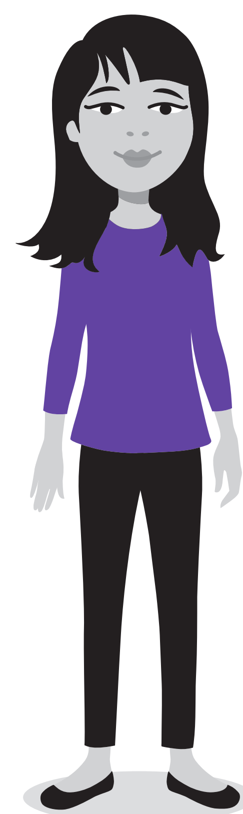
1. Use the data in the table to determine and analyze unit rates for Catalina's height growth.

   a. Write a unit rate that compares Catalina's change in height to her change in age from age 4 months to age 6 months. Explain how you calculated your answer.

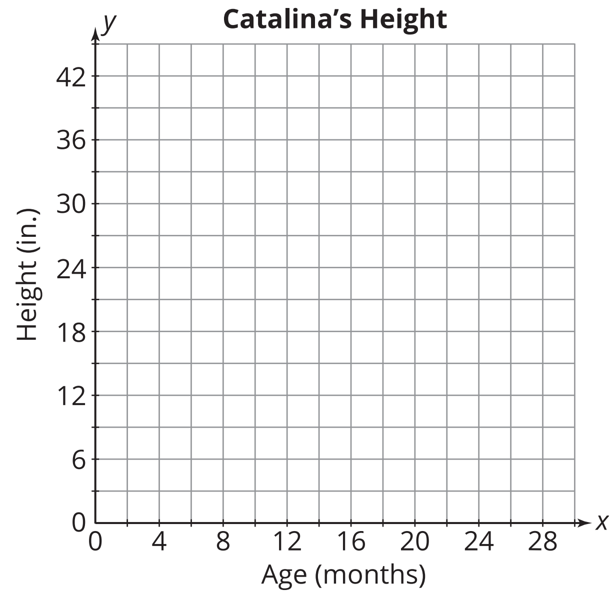
   b. Write a unit rate that compares Catalina's change in height to her change in age from 6 months to 8 months. Explain how you calculated your answer.

   c. Was Catalina gaining height faster from 4 months to 6 months, or from 6 months to 8 months? Explain your reasoning.

A unit rate is a comparison of two measurements in which the denominator has a value of one unit.



2. Create a scatterplot that shows Catalina's age as the explanatory variable and her height as the response variable.



3. Do all the points in your scatterplot lie on the same line? What does this tell you about Catalina's height change as time changes? Explain your reasoning.

4. Use a straightedge to draw the line that best fits the data on the graph.

5. Write the equation of your line. Be sure to define your variables and include the units. Identify the slope and y-intercept of your line.

6. Use the equation of your line to answer each question. Explain how you determined your answers.

   a. Approximately how many inches did Catalina grow in height each month from the time she was born until she was 18 months old?

   b. What was Catalina's approximate height, in inches, at birth?

7. Use the graph to predict Catalina's height at each given age when she continues to grow at the same rate. Then analyze your predictions.

   a. 20 months old

   b. 2 years old

   c. 30 months old

   d. Do all your predictions make sense? Explain your reasoning.

8. What can you conclude about the accuracy of your model?

ACTIVITY
3.2

Making Predictions

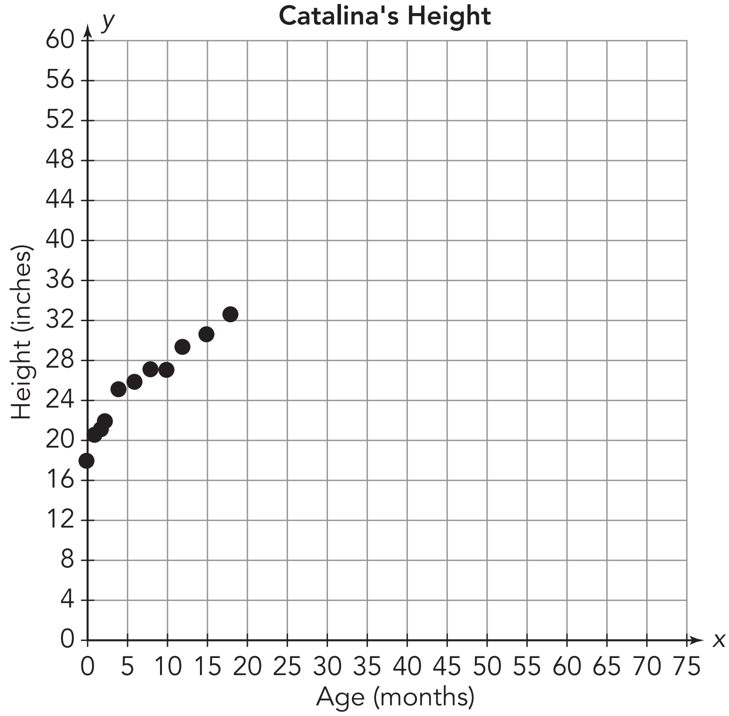
The table shows Catalina's growth from age 2 years to age $5\frac{1}{2}$ years.

1. Complete the table shown by converting each age from years to months.

Age (years)	Age (months)	Height (inches)
2.0		34.5
2.5		35.8
3.0		36.6
3.5		38.0
4.5		42.0
5.5		45.0

2. The scatterplot shown compares Catalina's age and height.

   a. Plot the new data from the table.



   b. Draw the trend line. Then, determine the equation of the line.

   c. Interpret the slope and y-intercept in terms of Catalina's height.

3. Use the trend line you graphed to determine Catalina's height when she is 5 years old.

4. How does this prediction compare to your previous prediction? Is this prediction reasonable? Explain your reasoning.

5. Do you think that extending the trend lines for Catalina's height over time helped to make predictions about height beyond 6 years?

6. Propose a strategy for developing more accurate models for Catalina's height over time.

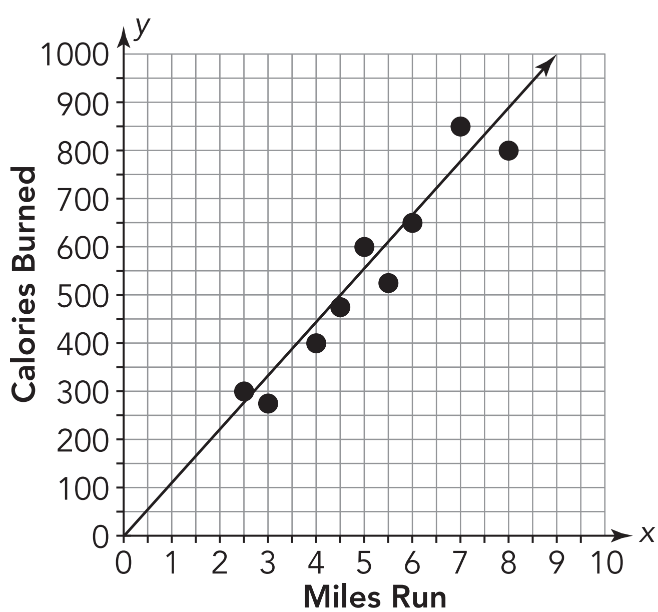
Talk the Talk

Peer Tutoring

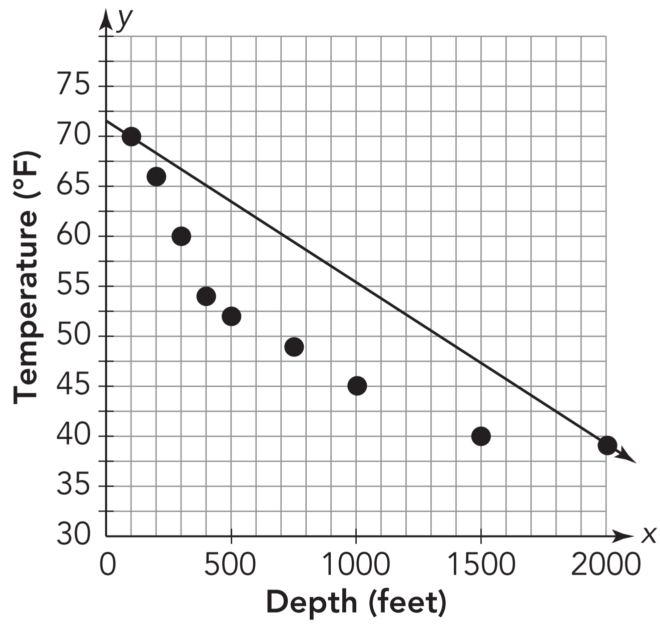
Your classroom partner, Ricardo, was absent when you learned about drawing appropriate trend lines. After he completed a make-up assignment on sketching trend lines, Ricardo asked you to double-check his work.

For each graph, determine what Ricardo misunderstands about sketching trend lines. Then describe a better trend line and explain your strategy.

1.



2.


y
x
75
70
65
60
55
50
45
40
35
30
0
500
1000
1500
2000
Temperature (°F)
Depth (feet)

Lesson 3 Assignment

Write

Explain the difference between interpolating and extrapolating when making predictions from a trend line.

Remember

The trend line for a set of data can be used to make predictions about the related problem.

Practice

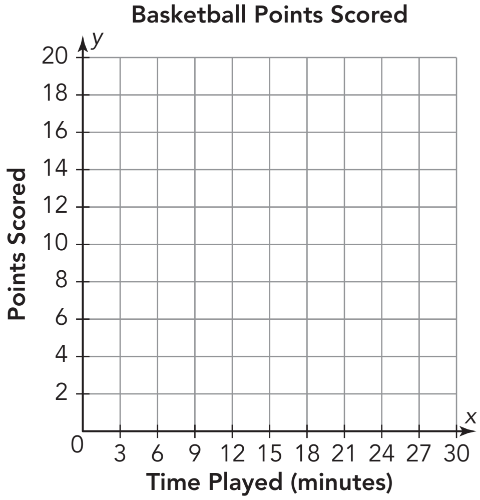
1. The table show the relationship between the number of minutes a basketball player plays in a game and the number of points the player scores.

   a. Write unit rates that compare the change in points to the change in minutes played from 2 minutes to 5 minutes, from 8 minutes to 12 minutes, and from 25 minutes to 30 minutes.

Time Played (minutes)	Points Scored
2	3
5	4
8	6
12	9
18	13
22	15
25	16
30	20

Lesson 3 Assignment

b. Create a scatterplot to show the relationship between the number of minutes played and the number of points scored.



c. Do all of the data points in your scatterplot lie on the same line? What does this tell you about the number of points a player scores during a game? Explain your reasoning.

d. Use a straightedge to draw a trend line for the data in your graph. Then, write the equation of your line. Be sure to define your variables and include the units.

e. According to the line you drew, approximately how many points did the basketball player score per minute from 2 minutes to 30 minutes? How did you determine your answer?

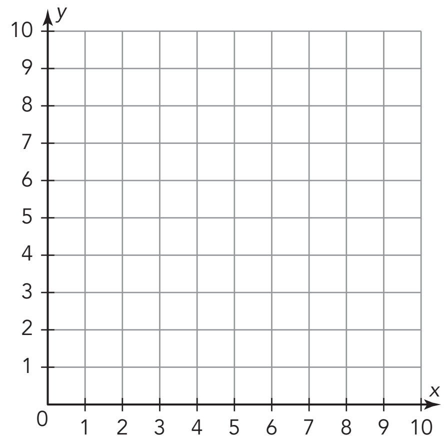
Lesson 3 Assignment

f. According to the line you drew, approximately many points will the player score when they have played 16 minutes?

g. According to the line you drew, how many minutes will the player be in the game when they score 18 points?

2. Use the data given in the table to create a scatterplot. Draw a trend line on the scatterplot. Determine the equation for the trend line.

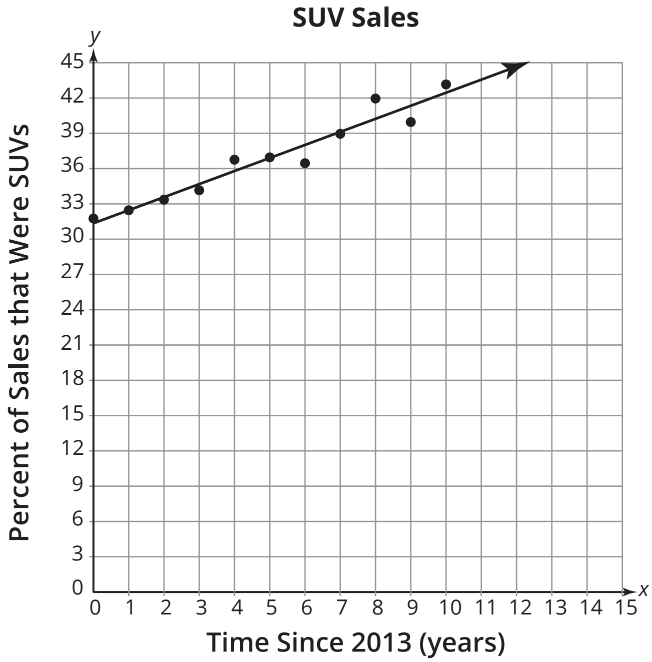
x	y
3	7
4	6
5	6
8	2
9	3



Lesson 3 Assignment

3. Use each graph to make a prediction.

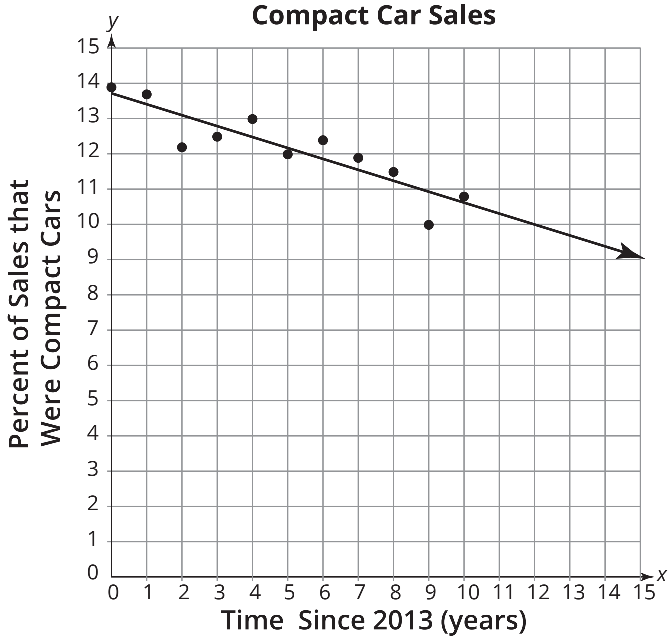
   a. A local car dealership tracks its automobile sales based on the type of automobile. The graph shows the percent of sales that were SUVs for the years from 2013 to 2023.



   Predict the percent of sales that are SUVs at the car dealership in 2025.

Lesson 3 Assignment

b. The local car dealership tracks its automobile sales based on the type of automobile. The graph shows the percent of sales that were compact cars for the years from 2013 to 2023.

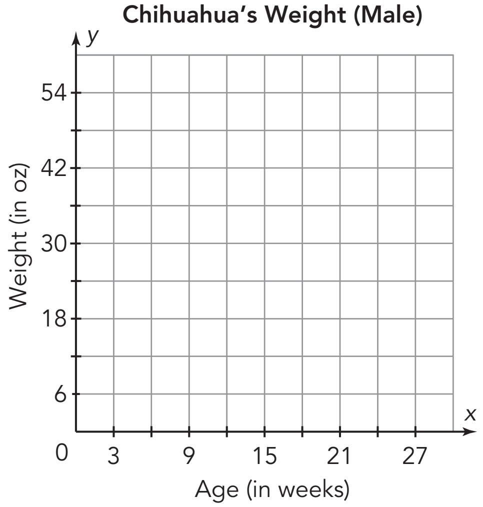


Predict the percent of sales that are compact cars at the local car dealership in 2027.

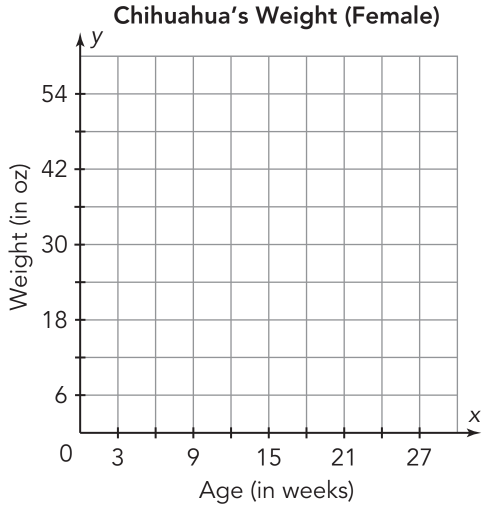
Lesson 3 Assignment

4. An animal's weight varies with age when it is young. In parts (a) through (c), for each specified set of data, complete each task.

   a. Create a scatterplot of the data of each table.



Chihuahua's Weight (Male)	
Age (weeks)	Weight (oz)
6	15
12	30
16	39
24	46
30	51



Chihuahua's Weight (Female)	
Age (weeks)	Weight (oz)
6	11
12	19
16	25
24	30
30	33

Lesson 3 Assignment

b. Draw the trend line. Then determine the equation that best fits the data.

c. Interpret the meaning of the slope and y-intercept for each.

d. Predict the weight of a 15-week-old female chihuahua.

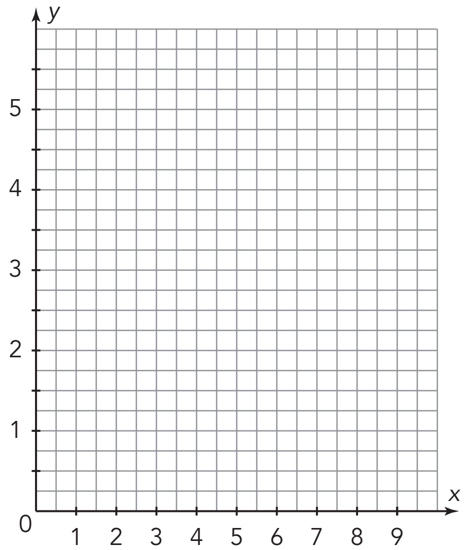
e. Predict the weight of a 9-week-old male chihuahua.

Lesson 3 Assignment

Prepare

1. Use the given data to create a scatterplot.

x	y
5	0
4	2
3	3
2	5
9	3



2. Draw a trend line on the scatterplot.

3. Determine the equation for the trend line.

4 Comparing Slopes and Intercepts of Data from Experiments

OBJECTIVES

- Write a linear function as a trend line for a set of data.
- Interpret the slope and *y*-intercept of a linear function modeling a set of data.
- Perform experiments and compare the results of different experiments.

Scatterplots and trend lines are used in many different fields of study to make predictions based on a collected data set.

How can you collect data and make predictions about how your brain functions?

Getting Started

Reading Is Automatic

The Stroop Test is an experiment that studies how people read text. The test uses lists of color words. Each word is written in one of the four colors. A person who participates in the Stroop Test experiment receives one of two lists, a matching list or a non-matching list, with a varying number of words. In a matching list, the ink color matches the color of the word. In a non-matching list, the ink color does not match the color of the word.

For example, in a matching list, the word **purple** would be shown in purple. In a non-matching list, the word purple might be shown in black.

Participants in the Stroop Test experiment are given one of the two kinds of lists, matching or non-matching, and are asked to say aloud the ink color in which each word is written. The time it takes for the person to say the correct ink color for all the words in the list is recorded, along with the total number of words in the list. The experiment is repeatedly performed with different people until enough data are collected to make a conclusion about the experiment.

In this lesson, you will perform a Stroop Test and calculate a trend line to make predictions.

1. Before you perform this experiment, what results do you expect to see for either the matching lists or non-matching lists? How do you think the results for the matching lists will compare with the non-matching lists?

2. Identify the explanatory variable and the response variable in this problem situation.

3. Write a statistical question you can ask that the Stroop Test experiment can help to answer.

ACTIVITY 4.1

Comparing Slopes and *y*-Intercepts of Trend Lines

Let's perform the Stroop Test!

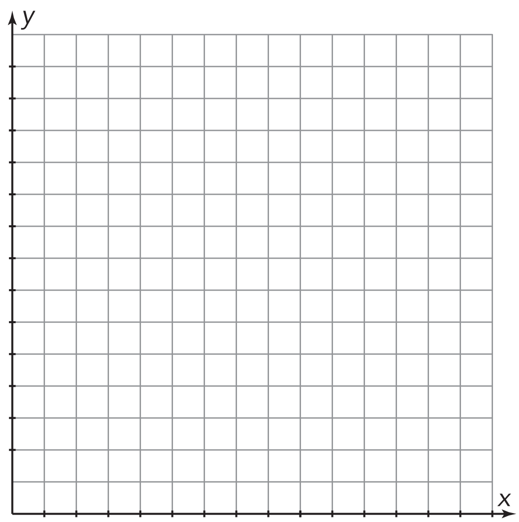
1. Participants will be given either a matching or non-matching list. Perform three trials of the Stroop Test for each list of words given to the participant. Then, vary the test length by increasing or decreasing the number of words on the list. Record the data in the correct table. You will complete the last column of the table at a later time.

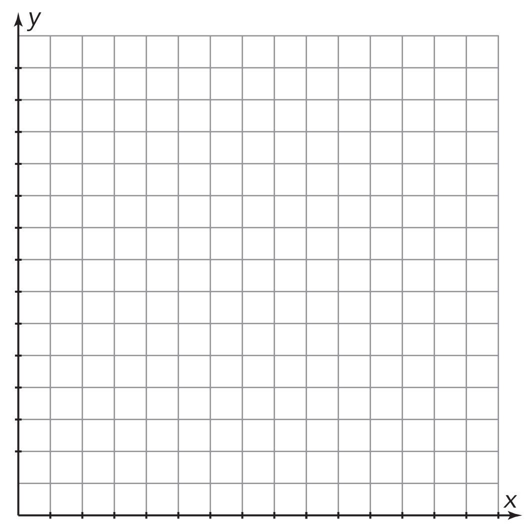
Matching Lists				
Length of the List (words)	Time 1 (seconds)	Time 2 (seconds)	Time 3 (seconds)	Mean Time (seconds)

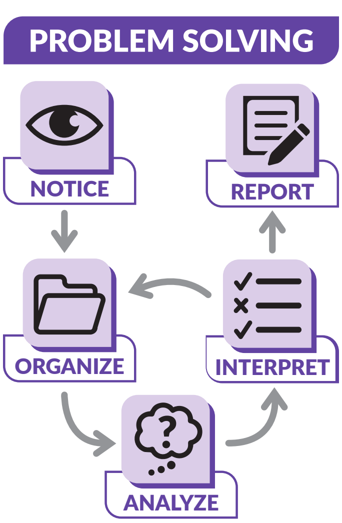
Non-Matching Lists				
Length of the List (words)	Time 1 (seconds)	Time 2 (seconds)	Time 3 (seconds)	Mean Time (seconds)

2. Record the mean time in seconds for the length of each list in the empty column of each table.

3. Create a scatterplot of the ordered pairs for the matching and non-matching list on the grids shown. First, label the axes to represent the explanatory and response variables. Next, choose the appropriate intervals for each scatterplot. Finally, title each scatterplot.







4. Use a straightedge to draw the trend line for each data set. Then, write the equation of each line.

5. State the y-intercept of each line. Interpret the meaning of the y-intercept in this situation.

6. State the slope of each line. Interpret the meaning of the slope in this situation.

7. Use your graphs and equations to answer each question.

   a. Using the trend lines, about how many seconds would it take a person to say 25 words from a matching list?

   b. Verify your predictions from the trend lines using your equations.

8. Use your graphs and equations to answer each question.

   a. Using the trend lines, about how many seconds would it take a person to say 10 words from a matching list? from a non-matching list?

   b. Verify your predictions from the trend line using your equations.

Talk the Talk

Interpret the Results

1. Compare your results for the matching lists to the results for the non-matching lists. Do your results seem reasonable? Explain your reasoning.

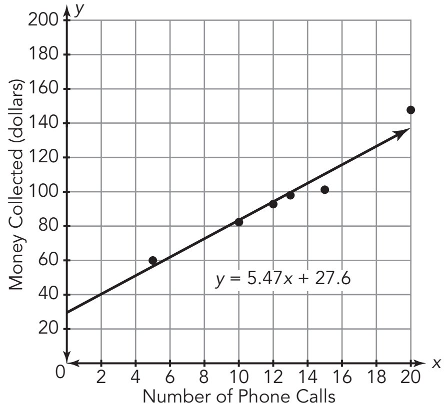
2. Revisit the statistical question you asked at the beginning of the lesson. How did the results of the experiment help to answer this question? Explain your reasoning.

3. What conclusions do you think a cognitive psychologist might draw from your experiment results?

Lesson 4 Assignment

Write

Write a scenario that could be modeled by the data and trend line shown. Interpret the slope and y-intercept of the data.



Remember

You can interpret the slope and y-intercept of a trend line by looking at the problem situation and the explanatory and response variables.

Practice

1. The goal of a word recall experiment is to see how many words from a list that is read aloud a person can memorize and repeat back. Five-word lists are given.

   5-Word List: chair, shoe, horse, suitcase, lamp
   7-Word List: animal, sweater, cheetah, avocado, back, desk, plant
   10-Word List: stereo, basketball, violin, teacher, pear, baby, table, zoo, curtains, ox
   15-Word List: cup, barn, paper, book, fire, comb, glass, vacuum, cloud, road, suit, stereo, computer, trunk, television
   20-Word List: football, hair, pizza, scarf, sandwich, T-shirt, microphone, screen, clock, fingers, coat, watch, tires, candles, cushions, earrings, heater, picture, keyboard, juice

   a. If you were to perform a word recall experiment, what results would you expect to see as the number of words increases? Do you expect people to remember more words or fewer words? Do you think people will remember the same percent of words as the length of the list increases?

b. Identify the explanatory variable and response variable.

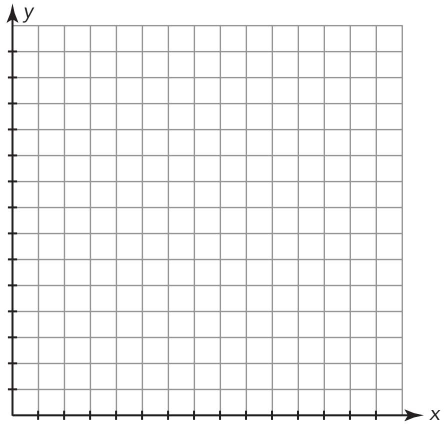
c. Perform the experiment for each word list. Read each list of words slowly and clearly to someone, but do not repeat any of the words. After you have finished reading each list, the person should repeat any words he or she remembers back to you. Do not allow the person to write anything down. Keep track of the number of words the person correctly repeats back to you by completing the table. Repeat this experiment two more times and calculate the mean of the results.

Length of the List (words)	Time 1 (words recalled)	Time 2 (words recalled)	Time 3 (words recalled)	Average (words recalled)
5-Word List				
7-Word List				
10-Word List				
15-Word List				
20-Word List				

d. Write the ordered pairs from the table that show the average number of words recalled as the response variable and the number of words in the list as the explanatory variable.

Lesson 4 Assignment

e. Create a scatterplot of the ordered pairs on the grid shown. First, label the axes to represent the explanatory and response variables. Next, choose the appropriate intervals for your scatterplot. Finally, name your scatterplot.



f. Use a straightedge to draw a trend line. Then, write the equation of your line. State the *y*-intercept of your line. What does the *y*-intercept represent in this situation?

g. State the slope of your line. What does the slope represent in this situation?

h. What is the average number of words that should be recalled from a list of 25 words? 35 words? 50 words? Show your work.

i. What length should the word list be if a person recalls 20 words? Show your work.

Lesson 4 Assignment

Prepare

Determine the absolute value of each number.

1. $|-4|$

2. $|12.5|$

3. $|-1.09|$

4. $|4\frac{2}{3}|$

TOPIC 1 SELF-REFLECTION

Name: ______________________________

Patterns in Bivariate Data

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Patterns in Bivariate Data* topic by:

TOPIC 1: *Patterns in Bivariate Data*	Beginning of Topic	Middle of Topic	End of Topic
constructing and interpreting scatterplots for bivariate measurement data.	◯	◯	◯
describing patterns and associations in scatterplots.	◯	◯	◯
using a trend line to model relationships between two quantitative variables with a linear association.	◯	◯	◯
informally fitting a trend line on a scatterplot to approximate the linear relationship between two data sets.	◯	◯	◯
informally assessing the trend line by judging the closeness of the data points to the line.	◯	◯	◯
writing the equation of a trend line of bivariate data and using the equation to solve problems and make predictions.	◯	◯	◯
interpreting the meaning of the slope and *y*-intercept of a trend line.	◯	◯	◯

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Patterns in Bivariate Data* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

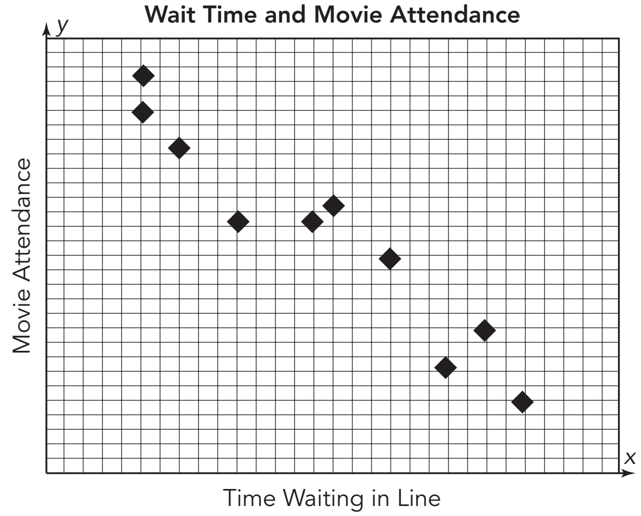
TOPIC 1 SUMMARY

Patterns in Bivariate Data Summary

LESSON 1 Analyzing Patterns in Scatterplots

When you collect information about two separate characteristics for the same person, thing, or event, you have collected **bivariate data**. A scatterplot is a graph of a set of ordered pairs. The points in a scatterplot are not connected, but they allow you to investigate patterns in bivariate data by comparing the two variables.

For example, the scatterplot shown represents height as the *x*-coordinate and arm span as the *y*-coordinate.



When you look for a relationship in bivariate data, often you are interested in determining whether one variable causes a change in the other variable. In this case, one variable, the **explanatory variable**, is designated as the independent variable. The **response variable** is designated as the dependent variable because this is the variable that responds to what occurs to the explanatory variable. In the scatterplot above, time waiting in line is the explanatory variable and movie attendance is the response variable.

Sometimes the relationships seen in scatterplots are called **associations**. A **linear association** occurs when the points on the scatterplot are arranged in such a way that, as you look at the graph from left to right, you can imagine a line going through the scatterplot with most of the points being close to the line. In a linear association, the two variables have a **positive association** when, as the explanatory variable increases, the

NEW KEY TERMS

- bivariate data [datos bivariados]
- explanatory variable [variable explicativa]
- response variable [variable de respuesta]
- association [asociación]
- linear association [asociación lineal]
- positive association [asociación positiva]
- negative association [asociación negativa]
- outlier
- trend line
- model [modelo]
- interpolating [interpolación]
- extrapolating [extrapolación]

response variable also increases. When the response variable decreases as the explanatory variable increases, then the two variables have **negative association**. For example, the scatterplot comparing height and arm span appears to have a positive linear association.

Another type of association is *non-linear association*. This occurs when the data have a distinct pattern, but the pattern is not linear. For example, the points on the scatterplot could create a curve, a U-shape, or a V-shape.

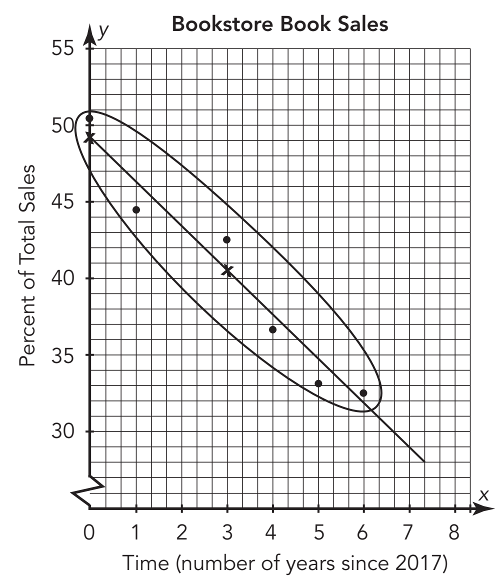
When the data does not appear to have any distinct pattern, the scatterplot shows *no association* between the variables.

Another pattern that can occur in a scatterplot is an outlier. An **outlier** for bivariate data is a point that varies greatly from the overall pattern of the data. If the point (150, 180) were added to the scatterplot comparing height and arm span, it could be considered an outlier.

LESSON 2 Drawing Trend Lines

Although a straight line will not pass through all of the points in a scatterplot, you can use a line to approximate the data as closely as possible. This kind of line is called a *trend line*. A **trend line** is a line that is as close to as many points as possible but doesn't have to go through all of the points. When you use a trend line, the line and its equation are often referred to as a **model** of the data. Constructing a trend line is helpful to predict values not displayed on the plot.

To construct a trend line, first plot all of the data. Next, draw an oval that encloses all of the data. Then draw a line that divides the enclosed area of the data in half. The idea is that you want to identify a line that has an equal number of points on either side.

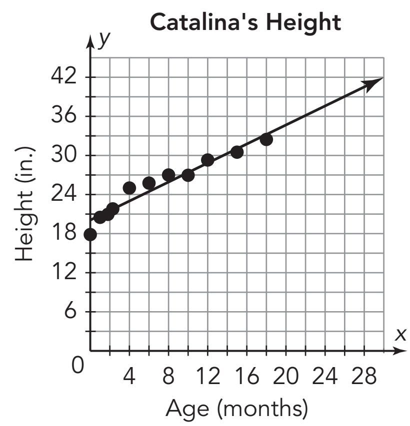


To determine the equation of your trend line, begin by identifying two points on your trend line. These points may or may not be data points, but they must be on the trend line. Use the slope formula to calculate the slope of the line through the two points. Then use the slope and the formula to determine the y-intercept of the line to write the equation in slope-intercept form.

When you are predicting values that fall within the plotted values of a scatterplot, you are **interpolating**. When you are predicting values that fall outside the plotted values, you are **extrapolating**. For example, predicting the percent of book sales from bookstores in 2019 using the trend line is an example of interpolation, while predicting the percent of book sales from bookstores in 2025 using the same line would be an example of extrapolation.

LESSON 3 Analyzing Trend Lines

The equation for the trend line can be used to make predictions about a problem. Be sure that your trend line is drawn in such a way that an equal number of data points fall on either side of the line, or your predictions may not be reasonable. Also consider the variables being compared to decide when a prediction is reasonable or unreasonable in context.



For example, the graph of the line can be represented by the equation $y = 0.73x + 20.08$. You can use this equation to estimate the child's height at 9 months.

$$y = 0.73(9) + 20.08 = 26.65 \text{ inches}$$

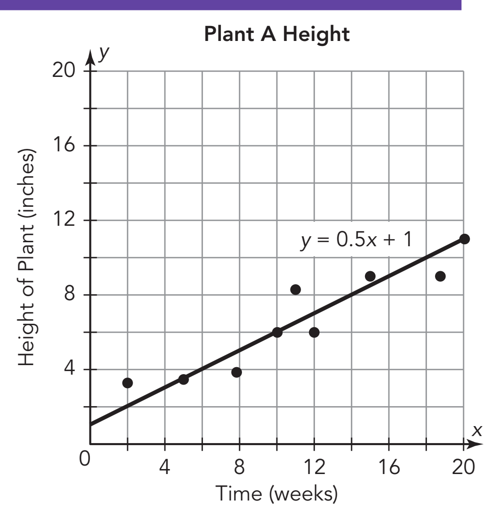
However, if you use the equation to estimate the height of the child at 12 years old, the prediction does not seem reasonable.

$$y = 0.73(144) + 20.08 = 125.2 \text{ inches or } 10.4 \text{ feet}$$

LESSON 4 Comparing Slopes and Intercepts of Data from Experiments

You can analyze two sets of data by comparing the slopes and *y*-intercepts of their trend lines to draw conclusions and make predictions.

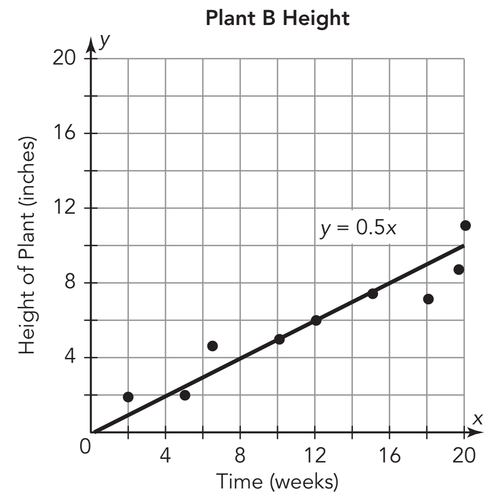
For example, in the scatterplot shown for Plant A, the *y*-intercept is 1 inch, and the slope is approximately 0.5 inches per week. In the scatterplot shown for Plant B, the *y*-intercept is 0 inches, and the slope is approximately 0.5 inches per week.



Therefore, every time the *x*-value, or time in weeks, increases by 1, the *y*-value, height in inches, increases by 0.5 inches for both Plant A and Plant B.

The growth rate for each plant is the same.

You can predict that in week 17, Plant A is about 9.5 inches. Plant B is about 8.5 inches. Plant B will always be about 1 inch taller than Plant A. Even though both plants grow at the same rate, at week 0 Plant A was about 1 inch tall, while Plant B was 0 inches tall.



GRADE 8

TOPIC 2

Choosing a sample randomly from a population is a good way to select items that are representative of the entire population.

Variablility and Sampling

1 Mean Absolute Deviation

OBJECTIVES

- Determine the absolute deviations of data points in a data set.
- Give quantitative measures of variation, including mean absolute deviation, for a data set.
- Use the mean absolute deviation as a measure of variation to describe and interpret data.
- Compare data sets using variation and the mean absolute deviation.
- Summarize numerical data sets in relation to their context.

NEW KEY TERMS

- deviation
- absolute deviation
- mean absolute deviation

The interquartile range is used as a measure of variation when the median is the measure of center.

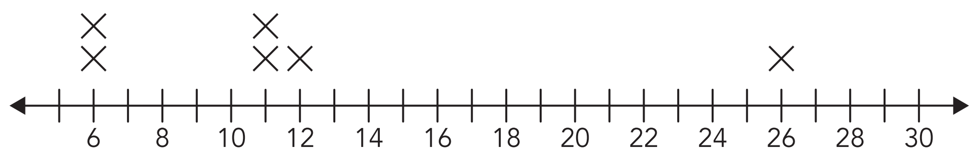
How can you measure the variation when mean is the measure of center?

Getting Started

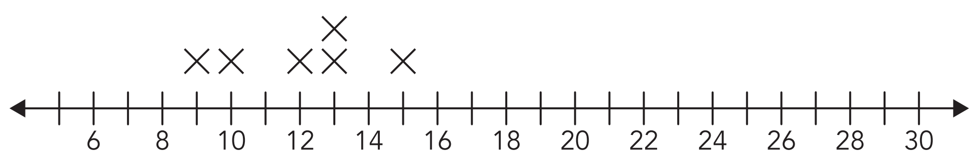
We Are the Champions

Coach Lopez's basketball team is advancing to the district championship. Catalina and Linh are possible starters for the game. The dot plots shown below represent the number of points scored by each player for the past six games.

Number of Points Scored by Catalina



Number of Points Scored by Linh



1. Determine the mean of each data set. Explain what this number tells you.

The mean is the arithmetic average of the numbers in a data set.

2. How are the two data sets similar and different?

3. Explain why the two data sets have the same mean.

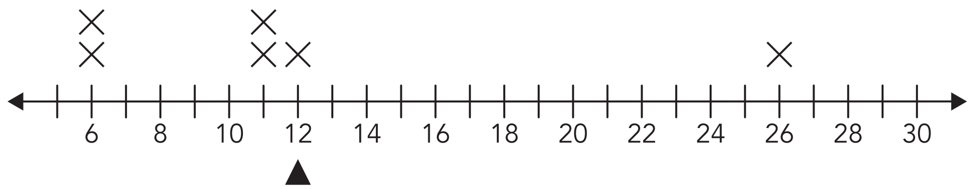
ACTIVITY 1.1

Exploring Variation and Deviation

Previously, you examined the dot plots of two basketball players—Catalina and Linh. Coach Lopez needs to choose between Catalina and Linh as starters for the game.

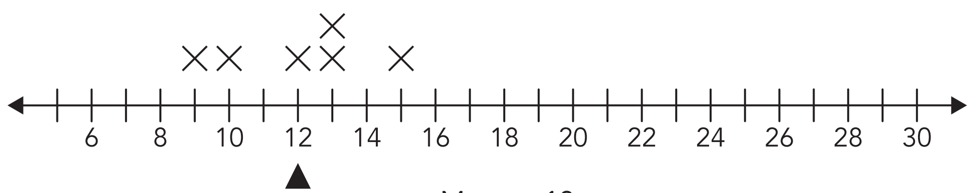
Review the definition of **describe** in the Academic Glossary.

Number of Points Scored by Catalina



Mean = 12

Number of Points Scored by Linh



Mean = 12

1. Based on the dot plots, which player do you think Coach Lopez should choose?

When analyzing a data set, measures of center give you an idea of where the data are centered, or what a typical data value might be. There is another measure that can help you analyze data. Measures of variation describe the spread of the data values. Just as there are several measures of central tendency, there are also several measures of variation.

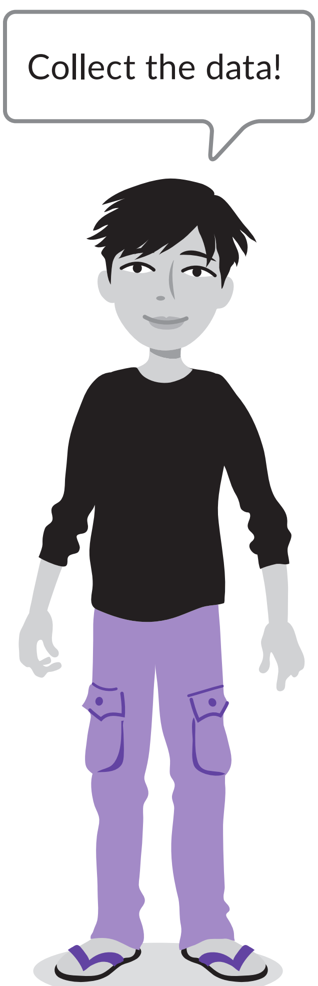
The **deviation** of a data value indicates how far that data value is from the mean. To calculate the deviation, subtract the mean from the data value:

$$\text{deviation} = \text{data value} - \text{mean}$$

2. Describe the deviations. Record your results in the tables.

Catalina	
Points Scored	Describe the Deviation from the Mean
11	
11	
6	
26	
6	
12	

Linh	
Points Scored	Describe the Deviation from the Mean
15	
12	
13	
10	
9	
13	



3. What is the meaning when a deviation is positive? Is negative? Is 0?

4. What do you notice about the deviations for each player?

5. Lauren claims that the sum of the deviations for a data set will always be 0. Do you agree? Why or why not?

The sum of all the deviations less than 0 is equal to the sum of the deviations greater than 0. Because the mean is the balance point, the sums of data points on either side of the balance point are equal to each other.

In order to get an idea of the spread of the data values, you can take the absolute value of each deviation and then determine the mean of those absolute values. The absolute value of each deviation is called the **absolute deviation**. The **mean absolute deviation** (MAD) is the mean of the absolute deviations.

The absolute value of a number is its distance from 0 on a number line. The symbol for absolute value is $| \ |$. You read the expression $|n|$ as "the absolute value of a number n."

6. Record the absolute deviations for the points scored in the tables.

Catalina		
Points Scored	Deviation from the Mean	Absolute Deviation
11	−1	
11	−1	
6	−6	
26	14	
6	−6	
12	0	

Linh		
Points Scored	Deviation from the Mean	Absolute Deviation
15	3	
12	0	
13	1	
10	−2	
9	−3	
13	1	

7. Calculate the mean absolute deviation for the points scored for each player.

8. Nakota says that the mean absolute deviation for the points each play scores represents the number of points you should expect each player to score during a game. Is Nakota correct? Explain your reasoning.

9. What does the mean absolute deviation tell you about the points scored by each player? Which player would you choose to play in the championship game? Justify your decision.

ACTIVITY
1.2 Applying Mean Absolute Deviation

The table shows the heights, in inches, of ten NBA basketball players and ten 6th-grade basketball players.

NBA Players	79, 74, 78, 81, 81, 76, 84, 80, 82, 83
6th-Grade Players	68, 64, 60, 58, 62, 65, 64, 60, 61, 65

1. Write a statistical question you can answer by analyzing the data.

2. Create a dot plot for each data set.

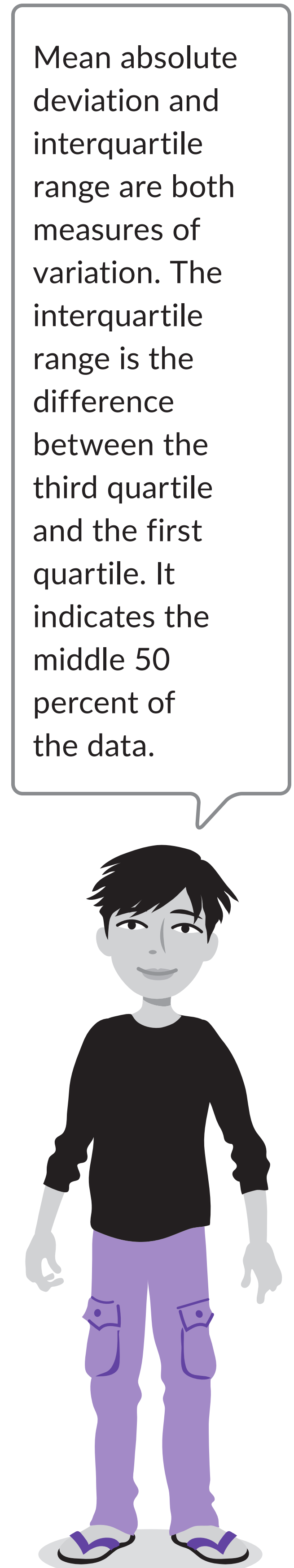
NBA Players

6th-Grade Players

3. Consider the data for the heights of NBA players and 6th-grade players.

   a. Calculate the mean absolute deviation for each data set.

b. Compare the data sets and interpret your results.



Talk the Talk

GPA and MAD

Sometimes you can change non-numerical data into numeric data in order to analyze it. Consider, for example, the report cards shown. Grades for the courses are assigned to the categories A, B, C, D, and F, with A being the highest grade.

James	
Science	B
Cultural Literacy	A
Music	C
Math	A
English	B

Ricardo	
Math	A
English	B
Cultural Literacy	C
Science	A
Music	A

1. Explain how you can change the report card data into numeric data.

2. Determine the mean of each data set. What does each mean tell you?

Ask Yourself . . .
Did you justify your mathematical reasoning?

3. Determine the mean absolute deviation for each data set.

James		
Data Value	Describe the Deviation from the Mean	Absolute Deviation

Ricardo		
Data Value	Describe the Deviation from the Mean	Absolute Deviation

4. Interpret each of the mean absolute deviations.

Lesson 1 Assignment

Write

Complete each sentence with the correct term.

Absolute deviation	Mean absolute deviation	Measures of variation	Deviation

1. ______________________ describe(s) the spread of the data values.
2. ______________________ indicates how far the data value is from the mean.
3. ______________________ is the absolute value of each deviation.
4. ______________________ is the average, or mean, of the absolute deviations.

Remember

To calculate the mean absolute deviation:

- Determine the mean of the data.
- Subtract the mean from each data value. These are the deviations.
- Record the absolute value of each deviation. These are the absolute deviations.
- Determine the mean of the absolute deviations. This is the mean absolute deviation.

Lesson 1 Assignment

Practice

Calculate the mean absolute deviation for each data set. Explain what the mean absolute deviation means in the context of the situation.

1. A restaurant cook counts the number of chicken wings in a single scoop for their next 5 orders.
   4, 5, 9, 4, 8

Lesson 1 Assignment

2. A hotel manager asks 15 guests what their ideal temperature is for their room, in degrees Fahrenheit.
   72, 80, 65, 72, 73, 75, 68, 70, 63, 82, 75, 70, 71, 76, 77

Lesson 1 Assignment

Prepare

A light bulb manufacturing company tests 24 of the bulbs they just produced and found that 3 of them were defective. Use proportions to predict how many light bulbs would be defective in shipments of each size.

1. 100 light bulbs

2. 400 light bulbs

3. 750 light bulbs

2

Collecting Random Samples

OBJECTIVES

- Differentiate between a population and a sample.
- Differentiate between a parameter and a statistic.
- Differentiate between a random sample and a non-random sample.
- Identify the benefits of random sampling of a population, including supporting valid statistical inferences and generalizations.
- Use several methods to select a random sample.

NEW KEY TERMS

- survey
- data
- population
- census
- sample
- parameter
- statistic
- random sample

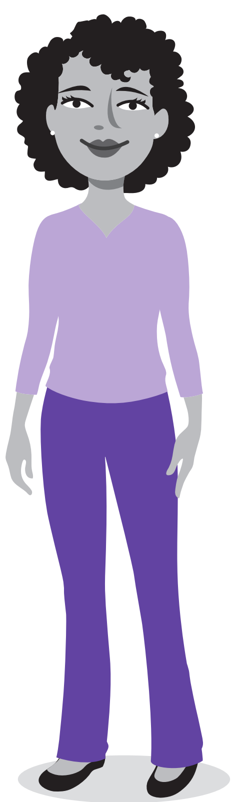
The statistical process is a structure for answering questions about real-world phenomena.

How can you make sure that the data you collect accurately answer your statistical questions?

Getting Started

Reviewing the Statistical Process

In this lesson, you will explore the second component of the statistical process, data collection. You will learn strategies for generating samples.



There are four components of the statistical process:

- Formulating a statistical question.
- Collecting appropriate data.
- Analyzing the data graphically and numerically.
- Interpreting the results of the analysis.

1. Summarize each of the four components. You may want to use examples to support your answers.

How could you describe the students in your classroom? How do the students in your classroom compare to other groups of students in your school, or to other eighth graders in the United States?

2. Formulate a statistical question about your classmates. How might you collect the information to answer your question?

One data collection strategy you can use is a *survey*. A **survey** is a method of collecting information about a certain group of people. It involves asking a question or a set of questions to those people. When information is collected, the facts or numbers gathered are called **data**.

Consider the following survey. You will use a sample set of results in the next activity.

a. What is your approximate height? ________________

b. Do you take a bus to school? Yes ______ No ______

ACTIVITY 2.1

Representative Samples

Some examples of populations include:

- every person in the United States
- every person in your math class
- every person in your school
- all the apples in your supermarket
- all the apples in the world

The **population** is the entire set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects.

A **census** is the data collected from every member of a population.

1. Use your survey to answer each question.

   a. Besides you, who else took the math class survey?

   b. What is the population in your class survey?

   c. Are the data collected in the class survey a census? Explain your reasoning.

Ever since 1790, the United States has taken a census every 10 years to collect data about population and state resources. The original purposes of the census were to decide the number of representatives a state could send to the U.S. House of Representatives and to determine the federal tax burden.

2. Describe the population for the United States census.

3. Why do you think this collection of data are called "the census"?

In most cases, it is not possible or logical to collect data from every member of the population. When data are collected from a part of the population, the data are called a **sample**. A sample should be representative of the population. In other words, it should have characteristics that are similar to the entire population.

When data are gathered from a population, the characteristic used to describe the population is called a **parameter**.

When data are gathered from a sample, the characteristic used to describe the sample is called a **statistic**. A statistic is used to make an estimate about the parameter.

Suppose that at a local middle school, 29% of students had perfect attendance last year. This is a parameter because you can count every single student at the school. Nationally, 16% of all teachers miss 3 or fewer days per year. This characteristic is a statistic because the population is too big to survey every single teacher.

4. After the 2020 census, the United States Census Bureau reported that 7.2% of Texas residents were between the ages of 10 and 14. Was a parameter or a statistic reported? Explain your reasoning.

5. A recent survey of 1000 middle school students from across the United States shows that 4 out of 5 take a bus to school.

   a. What is the population in the survey?

   b. Were the data collected in the survey a census? Why or why not?

   c. Does the given statement represent a parameter or a statistic? Explain how you determined your answer.

   Review the definition of **represent** in the Academic Glossary.

   d. Of those 1000 middle school students surveyed, how many take a bus to school? How many do not take a bus to school?

6. Use the data from a sample class survey provided at the end of this lesson to answer each question. Use a complete sentence to justify each answer.

   a. How many students in the class take a bus to school?

   b. What percent of the students in the class take a bus to school?

   c. Does the percent of students in the class that take a bus to school represent a parameter or a statistic? Explain how you determined your answer.

7. Suppose you only want to survey a sample of the class about whether they take a bus to school. Discuss whether or not these samples would provide an accurate representation of all students in a class. Use complete sentences to justify your answers.

   a. the selection of all of the honor-roll students for the sample

   b. the selection of the students in the first seat of every row

c. the selection of every fourth student alphabetically

d. the selection of the first 10 students to enter the classroom

8. Suppose you wanted to determine the number of students who take a bus to school across the entire eighth grade.

   a. What is the population?

   b. Suggest and justify a method of surveying students in the eighth grade to obtain a representative sample.

When information is collected from a sample in order to describe a characteristic about the population, it is important that such a sample be as representative of the population as possible. A **random sample** is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

ACTIVITY 2.2

Selecting a Random Sample

Ms. Flores is using school funds to purchase standing desks for her classroom. She wants to set up the desks to minimize the amount of desk height adjusting by the students and decides to use the mean height of students in her math class as a guide. Rather than using the heights of all students in her class, she decides to collect a random sample of students in her class.

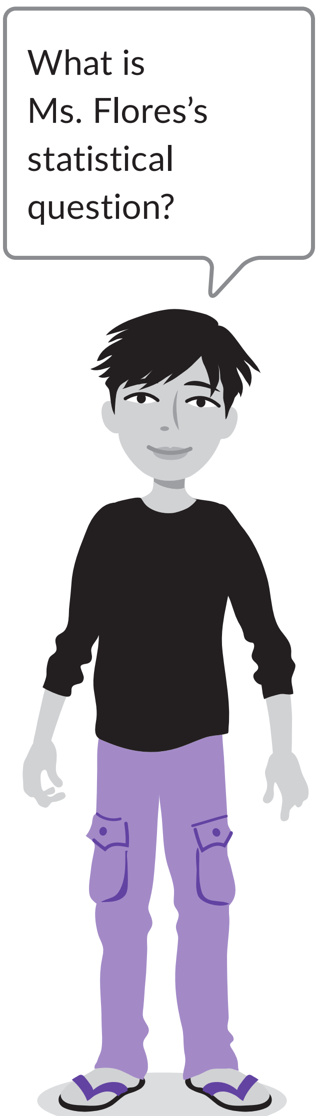
Ask Yourself . . .
Can you restate the problem in your own words?

1. What is the population for this problem?

2. Ms. Flores received two suggestions to sample her class at random. Decide whether each strategy represents a random sample. When not, explain why not.

   a. Ms. Flores chooses the honor-roll students in the class.

   b. Ms. Flores chooses all of the students wearing white sneakers.



3. Ms. Flores decides to select a random sample of five students in her class, and then calculate the mean height. She assigns each student in her class a different number. Then, she selects 5 numbers at random.

   a. Explain why Ms. Flores's method of taking a sample is a random sample.

   b. Do you think selecting 5 students at random will accurately represent the population of her class? When not, do you think she should pick more or fewer students?

   c. Jackson hopes Ms. Flores will assign him the number 7 because it will have a better chance of being selected for the sample. Do you agree or disagree with Jackson? Explain your reasoning.

   d. Alejandra claims Ms. Flores must begin with the number 1 when assigning numbers to students. Nicky says she can start with any number as long as she assigns every student a different number. Who is correct? Explain your reasoning.

One way to select the students is to write the numbers (or student names) on equal-sized pieces of paper, put the papers in a bag, draw out a piece of paper, and record the result. To create a true random sample, the papers should be returned to the bag after each draw.

Help Ms. Flores select five students from her class at random.

4. With your partner, create a bag with the numbers 1 through 30 from which to select your sample.

   a. Draw 5 numbers and record your results.

   b. Compare your sample with the samples of your classmates. What do you notice?

5. Suppose Ms. Flores starts with the number 15 when she assigns each of the 30 students in her class a number. How can you change your selection process to accommodate the list beginning at 15?

6. Suppose that the 5 numbers selected from your bag resulted in 5 honor-roll students.

   a. Is the sample still a random sample? Explain your reasoning.

   b. How is this outcome different from choosing all honor-roll students to represent the sample in Question 2?

ACTIVITY 2.3

Using Random Number Tables to Select a Sample

The standing desks improved student motivation, attendance, and achievement so much in Ms. Flores's class that the principal, Ms. Brown, has decided to order standing work desks for every eighth grade class in the school.

The school has 450 eighth grade students, and Ms. Brown would like to take a random sample of 20 eighth graders, determine their heights, and use their mean height for the initial set-up of the standing desks. However, Ms. Brown does not want to write the 450 names on slips of paper.

There are other ways to select a random sample. One way to select a random sample is to use a random number table to simulate events.

You can use a random number table to choose a number that has any number of digits in it. For example, when you are choosing 6 three-digit random numbers and begin with Line 7, the first 6 three-digit random numbers would be: 242, 166, 344, 421, 283, and 070.

Technology, such as spreadsheets, graphing technology, and random number generator applications, can be used to generate random numbers.

Random Number Table										
Line 6	62490	99215	84987	28759	19177	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	10415	36457
Line 8	16975	95428	33226	55903	31605	43817	22250	03918	46999	98501
Line 9	59138	39542	71168	57609	91510	77904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825

1. What number does "070" represent when choosing a three-digit random number? Why are the zeros in the number included? Explain your reasoning.

2. When selecting a three-digit random number, how would the number 5 be displayed in the table?

3. Begin on Line 10 and select 5 three-digit random numbers.

4. Explain how to assign numbers to the 450 eighth grade students so that Ms. Brown can take a random sample.

5. Use Line 6 as a starting place to generate a random sample of 20 students.

   a. What is the first number that appears?

b. What do you think Ms. Brown should do with that number?

c. Continuing on Line 6, what are the 20 three-digit numbers to be used to select Ms. Brown's sample?

d. What should you do when a three-digit number appears twice in the random number table?

In this lesson, you have engaged with the first two phases of the statistical process: formulating questions and collecting data. In the next lessons, you will analyze and interpret findings.

e. Will choosing a line number affect whether Ms. Brown's sample is random?

f. Will choosing a line number affect who will be chosen for the sample?

ACTIVITY 2.4 Simulating Random Samples

Square Roots is a radio show that airs 10 times a week on local radio station WMTH.

WMTH is trying to raise its commercial airtime rates during *Square Roots*. The station claims that while this music show is listened to by hundreds of middle school students via the radio, there are actually a greater number of middle school students who listen to the show by regularly downloading the podcast. Advertisers disagree with WMTH's claim. Advertisers want the station to verify its claim that there are more students listening by downloaded podcasts than actual listeners. To do so, WMTH and the advertisers choose a local middle school to collect data. They send out a survey and ask the following two questions:

- Do you listen to *Square Roots* on the radio or download the podcast?
- How often do you listen to *Square Roots* per week?

All 389 students at the local middle school who listen to *Square Roots* responded to the survey.

Let's use a random number table to simulate random samples from the data. The random number table and data are at the end of the lesson.

When you are assigning each student a number, each number should have the maximum number of digits in the largest number of a population. Therefore, when there are 300 people in a population, each number assigned should have three digits.

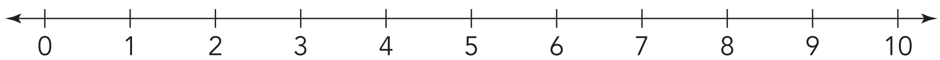
1. Use the random number table and the list of radio and podcast listeners at the local middle school at the end of the lesson to help WMTH select a sample at random.

   a. Select 10 radio listeners at random. Record each student's last name. Then, use the list to record the number of times each student listened to *Square Roots* during the week.

   b. Select 10 podcasters at random. Record each student's last name. Then, use the list to record the number of podcasts each student downloaded in one week.

2. Construct a combined dot plot for the two groups. What conclusions can you draw?

Radio Shows and Podcasts (per week)



> **Ask Yourself . . .**
> How could you differentiate between the two groups on the same dot plot?

3. Describe any clusters or gaps in the data values in each graph.

4. Estimate the mean for each dot plot. Explain how you determined your estimate.

Review the definitions of **select** and **estimate** in the Academic Glossary.

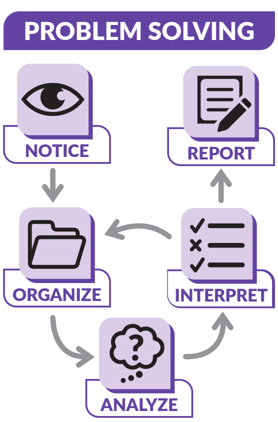
5. Calculate the mean number of radio shows listened to in a week and the mean number of podcasts downloaded in a week.

6. Compare the two samples. Are more shows listened to on the radio, or are more podcasts downloaded?

The third step in the statistical process is to analyze the data graphically and numerically.

7. Calculate the mean absolute deviation for each group. Compare the measures of center and variation.

8. Hailey says, "Should we have started on a different line number in the random number table, our results would have been the same." Is Hailey correct? Explain your reasoning.



The fourth step in the statistical process is to interpret the results of the analysis.

9. Combine your data with the data from other classmates. Calculate measures of center and variation for the two combined random samples, and interpret your results.

10. Determine the difference of means for the two samples and describe this difference as a multiple of the measure of variation.

Talk the Talk

Lunching with Ms. Brown

Ms. Brown wishes to select 10 students at random for a lunch meeting to discuss ways to improve school spirit. There are 1500 students in the school.

1. What is the population for this problem?

2. What is the sample for this problem?

3. Ms. Brown selects three to four student council members from each grade to participate. Does this sample represent all of the students in the school? Explain your answer.

4. Ms. Flores recommended that Ms. Brown use a random number table to select her sample of 10 students. How would you recommend Ms. Brown assign numbers and select her random sample?

Ask Yourself . . .
Can you restate the problem in your own words?

Results from a Sample Class Survey		
Student	1. What is your approximate height?	2. Do you take a bus to school?
1-Isabella	60 in.	Yes
2-Ethan	68 in.	Yes
3-Samuel	63 in.	No
4-Harper	65 in.	Yes
5-Diego	62 in.	Yes
6-Sophia	68 in.	Yes
7-Daniel	56 in.	Yes
8-Chloe	70 in.	No
9-Juan	69 in.	Yes
10-Mia	66 in.	Yes
11-Jayden	61 in.	Yes
12-Mateo	68 in.	Yes
13-Minh	64 in.	Yes
14-Victoria	66 in.	Yes
15-Liam	60 in.	Yes
16-Elijah	67 in.	Yes
17-Aaliyah	64 in.	Yes
18-Kai	65 in.	Yes

Results from a Sample Class Survey		
Student	1. What is your approximate height?	2. Do you take a bus to school?
19-Ava	63 in.	Yes
20-Emma	58 in.	Yes

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened (per week)
1	Abunto	1
2	Adler	3
3	Aizawa	3
4	Alescio	4
5	Almasy	8
6	Ansari	6
7	Aro	7
8	Aung	2
9	Baehr	7
10	Bellmer	1

Student Number	Student Name	Radio Shows Listened (per week)
21	D'Ambrosio	0
22	Datz	4
23	Delecroix	2
24	Difiore	6
25	Dobrich	7
26	Donoghy	1
27	Donaldson	5
28	Dreher	2
29	Dubinsky	1
30	Dytko	8

Student Number	Student Name	Radio Shows Listened (per week)
41	Granger	0
42	Guca	2
43	Haag	8
44	Heese	5
45	Hilson	1
46	Holihan	1
47	Hudack	1
48	Ianuzzi	3
49	Islamov	4
50	Jacobsen	5

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)
11	Bilski	4	31	Fabry	7	51	Jessell	4
12	Blinn	6	32	Fetcher	1	52	Ji	1
13	Bonetto	3	33	Fontes	5	53	Johnson	2
14	Breznai	1	34	Frick	3	54	Jomisko	1
15	Cabot	3	35	Furmanek	5	55	Jones	6
16	Chacalos	0	36	Gadgil	4	56	Joy	5
17	Cioc	0	37	Gavlak	4	57	Jumba	1
18	Cole	3	38	Gibbs	0	58	Juth	7
19	Creighan	4	39	Gloninger	2	59	Jyoti	6
20	Cuthbert	6	40	Goff	1	60	Kachur	2
61	Kanai	0	81	McNary	7	101	Nzuyen	2
62	Keller	2	82	Meadows	0	102	O'Bryon	0
63	Khaing	7	83	Merks	8	103	Obitz	3
64	Kindler	5	84	Mickler	5	104	Oglesby	8
65	Kneiss	5	85	Minniti	4	105	Ono	1
66	Kolc	1	86	Mohr	3	106	Paclawski	0

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)
67	Kuisis	2	87	Mordecki	5	107	Pappis	6
68	Labas	1	88	Mueser	3	108	Peery	3
69	Lasek	8	89	Musati	3	109	Phillips	5
70	Leeds	5	90	Myron	2	110	Potter	3
71	Lin	0	91	Nadzam	4	111	Pribanic	7
72	Litsko	2	92	Nazif	7	112	Pwono	5
73	Lodi	3	93	Newby	0	113	Quinn	2
74	Lookman	2	94	Ng	1	114	Rabel	5
75	Lucini	1	95	Nino	2	115	Rayl	2
76	Lykos	0	96	Northcutt	4	116	Rea	4
77	MacAllister	1	97	Novi	3	117	Reynolds	5
78	Magliocca	6	98	Null	8	118	Rhor	8
79	Marchick	5	99	New	5	119	Rielly	8
80	McGuire	1	100	Nyiri	1	120	Risa	7
121	Robinson	8	141	Stevens	4	161	Volk	5
122	Roethlein	1	142	Tabor	0	162	Vyra	4

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened (per week)
123	Romanski	7
124	Rouce	7
125	Rubio	7
126	Rutland	3
127	Rychcik	1
128	Sabhnani	6
129	Sandroni	2
130	Saxon	3
131	Scalo	4
132	Schessler	1
133	Seeley	0
134	Shanahan	3
135	Siejk	1
136	Skaro	3
137	Slonaker	8
138	Sobr	2

Student Number	Student Name	Radio Shows Listened (per week)
143	Tevis	5
144	Thomas	0
145	Thompson	1
146	Thorne	1
147	Tiani	0
148	Tokay	6
149	Toomey	6
150	Trax	1
151	Truong	8
152	Tunstall	1
153	Twiss	6
154	Tyler	0
155	Ueki	0
156	Uriah	6
157	Vagnelli	6
158	Van Dine	5

Student Number	Student Name	Radio Shows Listened (per week)
163	Wadhwani	5
164	Warnaby	0
165	Weasley	2
166	Weidt	8
167	Whitelow	0
168	Wilson	4
169	Woller	1
170	Woo	0
171	Wunderlich	6
172	Wycoff	4
173	Xander	8
174	Yahya	0
175	Yezovich	4
176	Youse	0
177	Yuzon	8
178	Za Khai	3

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)
139	Spatz	4	159	Vella	1	179	Ziff	2
140	Sramac	2	160	Vidnic	0	180	Zuk	5
121	Robinson	8	141	Stevens	4	161	Volk	5
122	Roethlein	1	142	Tabor	0	162	Vyra	4
123	Romanski	7	143	Tevis	5	163	Wadhwani	5
124	Rouce	7	144	Thomas	0	164	Warnaby	0
125	Rubio	7	145	Thompson	1	165	Weasley	2
126	Rutland	3	146	Thorne	1	166	Weidt	8
127	Rychcik	1	147	Tiani	0	167	Whitelow	0
128	Sabhnani	6	148	Tokay	6	168	Wilson	4
129	Sandroni	2	149	Toomey	6	169	Woller	1
130	Saxon	3	150	Trax	1	170	Woo	0
131	Scalo	4	151	Truong	8	171	Wunderlich	6
132	Schessler	1	152	Tunstall	1	172	Wycoff	4
133	Seeley	0	153	Twiss	6	173	Xander	8
134	Shanahan	3	154	Tyler	0	174	Yahya	0

Square Roots Fans Who Listen on the Radio

Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)	Student Number	Student Name	Radio Shows Listened (per week)
135	Siejk	1	155	Ueki	0	175	Yezovich	4
136	Skaro	3	156	Uriah	6	176	Youse	0
137	Slonaker	8	157	Vagnelli	6	177	Yuzon	8
138	Sobr	2	158	Van Dine	5	178	Za Khai	3
139	Spatz	4	159	Vella	1	179	Ziff	2
140	Sramac	2	160	Vidnic	0	180	Zuk	5

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
1	Aaronson	2	21	Chang	4	41	Frena	1
2	Abati	0	22	Clarke	9	42	Galdi	8
3	Ackerman	4	23	Crnkovich	0	43	Gansberger	3
4	Aderholt	2	24	Dahl	0	44	Gianni	1
5	Akat	7	25	Dax	7	45	Glencer	1
6	Aleck	9	26	Defoe	1	46	Godec	7
7	Alessandro	5	27	Dengler	4	47	Goldstein	0

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
8	Allen	3	28	Di Minno	4	48	Graef	6
9	Ansil	1	29	Dilla	5	49	Gula	1
10	Archer	5	30	Draus	5	50	Hagen	2
11	Badgett	9	31	Duffy	0	51	Haupt	8
12	Bartle	2	32	Ecoff	3	52	Herc	4
13	Bibby	9	33	Esparra	7	53	Hnat	9
14	Bilich	5	34	Fakiro	7	54	Hodak	3
15	Bloom	3	35	Ferlan	4	55	Hoyt	2
16	Boccio	5	36	Fetherman	2	56	Huang	3
17	Bracht	3	37	Fillipelli	6	57	Iannotta	4
18	Bujak	7	38	Fisher	2	58	Irwin	5
19	Caliari	9	39	Folino	9	59	Jackson	7
20	Cerminara	8	40	Forrester	9	60	Jamil	1
61	Jessop	1	81	Ling	6	101	Moorey	6
62	Johnson	9	82	Loch	4	102	Mox	7
63	Joos	9	83	Lorenzo	4	103	Mrkali	7
64	Joseph	5	84	Lovejoy	5	104	Mu	0

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
65	Jubic	3	85	Luba	8	105	Muller	3
66	Juhl	7	86	Lukitsch	4	106	Murphy	2
67	Jung	9	87	Luzzi	8	107	Mwambazi	6
68	Jurgensen	4	88	Lyman	5	108	Myers	4
69	Jyoti	0	89	MacIntyre	8	109	Nangle	3
70	Kaib	5	90	Maddex	5	110	Neilan	7
71	Kapoor	6	91	Marai	2	111	Nicolay	5
72	Kennedy	2	92	Mato	9	112	Niehl	6
73	Kimel	5	93	McCaffrey	0	113	Nix	2
74	Klaas	4	94	McElroy	5	114	Noga	3
75	Ko	9	95	McMillan	3	115	Nowatzki	7
76	Krabb	1	96	Meng	9	116	Nuescheler	5
77	Ladley	9	97	Michelini	5	117	Nye	6
78	Lawson	1	98	Misra	0	118	Nytra	6
79	Lemieux	7	99	Miller	8	119	O'Carrol	6
80	Lewan	6	100	Modecki	7	120	Obedi	7
121	Oehrle	8	141	Rea	5	161	Scopaz	4

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
122	Olds	5	142	Renard	7	162	Sebula	4
123	Oleary	0	143	Rex	7	163	Shah	1
124	Ondrey	1	144	Richards	7	164	Sidor	6
125	Owusu	9	145	Ridout	7	165	Skraly	6
126	Palamides	9	146	Rivera	6	166	Sokolowski	5
127	Pappas	0	147	Roberts	4	167	Speer	6
128	Pecori	3	148	Rodwich	0	168	T'Ung	9
129	Pennix	4	149	Roney	7	169	Tamar	9
130	Pendleton	1	150	Ross	6	170	Tebelius	1
131	Phillippi	2	151	Rothering	0	171	Tesla	4
132	Pieton	6	152	Rua	4	172	Thuma	0
133	Ploeger	2	153	Russo	8	173	Tibi	2
134	Pressman	4	154	Ryer	8	174	Tobkes	9
135	Puzzini	1	155	Sagi	8	175	Torelli	8
136	Qu	4	156	Sallinger	6	176	Tozzi	0
137	Qutyan	8	157	Sau	7	177	Traut	6
138	Raab	5	158	Sbragia	7	178	Trax	0

Square Roots Fans Who Download Show Podcasts

Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)	Student Number	Student Name	Podcasts Downloaded (per week)
139	Raeff	0	159	Schaier	5	179	Tu	8
140	Rav	1	160	Schmit	2	180	Tumicki	2
181	Tyson	1	191	Wallace	2	201	Wulandana	8
182	Uansa	0	192	Webb	9	202	Wysor	9
183	Ulan	4	193	Weisenfeld	0	203	Xiao	4
184	Urbano	7	194	Whalen	7	204	Yee	6
185	Uzonyi	7	195	Wiley	9	205	Yost	1
186	Vaezi	8	196	Williams	5	206	Young	7
187	Vinay	6	197	Williamson	6	207	Yuros	6
188	Vu	8	198	Witek	9	208	Zaki	0
189	Wallee	0	199	Wojcik	3	209	Zimmerman	1
190	Waldock	4	200	Woollett	6			

Random Number Table										
Line 1	65285	97198	12138	53010	94601	15838	16805	61004	43516	17020
Line 2	17264	57327	38224	29301	31381	38109	34976	65692	98566	29550
Line 3	95639	99754	31199	92558	68368	04985	51092	37780	40261	14479
Line 4	61555	76404	86210	11808	12841	45147	97438	60022	12645	62000
Line 5	78137	98768	04689	87130	79225	08153	84967	64539	79493	74917
Line 6	62490	99215	84987	28759	19177	14733	24550	28067	68894	38490
Line 7	24216	63444	21283	07044	92729	37284	13211	37485	10415	36457
Line 8	16975	95428	33226	55903	31605	43817	22250	03918	46999	98501
Line 9	59138	39542	71168	57609	91510	77904	74244	50940	31553	62562
Line 10	29478	59652	50414	31966	87912	87154	12944	49862	96566	48825
Line 11	96155	95009	27429	72918	08457	78134	48407	26061	58754	05326
Line 12	29621	66583	62966	12468	20245	14015	04014	35713	03980	03024
Line 13	12639	75291	71020	17265	41598	64074	64629	63293	53307	48766
Line 14	14544	37134	54714	02401	63228	26831	19386	15457	17999	18306
Line 15	83403	88827	09834	11333	68431	31706	26652	04711	34593	22561
Line 16	67642	05204	30697	44806	96989	68403	85621	45556	35434	09532
Line 17	64041	99011	14610	40273	09482	62864	01573	82274	81446	32477
Line 18	17048	94523	97444	59904	16936	39384	97551	09620	63932	03091
Line 19	93039	89416	52795	10631	09728	68202	20963	02477	55494	39563
Line 20	82244	34392	96607	17220	51984	10753	76272	50985	97593	34320

Lesson 2 Assignment

Write

Match each definition to the corresponding term.

1. The facts or numbers gathered by a survey
2. The characteristic used to describe a sample
3. The collection of data from every member of a population
4. A method of collecting information about a certain group of people by asking a question or set of questions
5. A sample that is selected from the population in such a way that every member of the population has the same chance of being selected
6. The characteristic used to describe a population
7. The entire set of items from which data can be selected
8. The data collected from part of a population

a. census
b. data
c. parameter
d. population
e. sample
f. statistic
g. survey
h. random sample

Remember

Statistics obtained from data collected through a *random sample* are more likely to be representative of the population than those statistics obtained from data collected through non-random samples.

Practice

1. Explain which sampling method is more representative of the population.

   a. Brianna and Gracie live in Plano, Texas, and are interested in the average age of skateboarders who use their town's skate park in one week. Brianna recorded the ages of skateboarders who used the park in June. Gracie recorded the ages of skateboarders who used the park in January.

Lesson 2 Assignment

b. Nia and Miguel want to determine the most popular lunch choice in the school cafeteria among seventh-graders. They decide to collect data from a sample of seventh-graders at school. Nia surveys 20 seventh-graders that are in line at the cafeteria. Rachel surveys 20 seventh-graders whose student ID numbers end in 9.

2. The coach of the soccer team is asked to select 5 students to represent the team in the Homecoming Parade. The coach decides to select the 5 students at random out of the 38 members of the team.

   a. What is the population for this problem?

   b. What is the sample for this problem?

   c. Suggest a method for selecting the random sample of 5 students.

Lesson 2 Assignment

3. Consider the population of integers from 8 to 48.

   a. Select a sample of 6 numbers. Is this a random sample? Explain your reasoning.

   b. How can you assign random numbers to select a sample using a random number table?

   c. Use the random number table to choose 6 numbers from this population.

   d. Use a different line of the random number table to choose 6 numbers from this population.

   e. Compare the results from each sample. Do the results surprise you? Explain.

Lesson 2 Assignment

4. The manager of the local mall wants to know the mean age of the people who shop at the mall and the stores in which they typically shop. Xavier has been put in charge of collecting data for the local mall. He decides to interview 100 people one Saturday because it is the mall's busiest shopping day.

   a. What is the population for this situation?

   b. What is the sample?

   c. When Xavier calculates the mean age of the people who shop at the mall, will he be calculating a parameter or a statistic? Explain your reasoning.

   d. Describe three different ways Xavier can take a sample. Describe how any of these three possible samples may cause the results of Xavier's survey to inaccurately reflect the average age of shoppers at the mall.

Lesson 2 Assignment

e. Xavier decides to use a random number table to choose the next 10 people to interview. Explain how to choose 10 two-digit numbers between 1 and 80 from a random number table.

f. Record the numbers of the people who would be interviewed. Be sure to specify which line you used to generate your list.

g. Suppose Xavier uses a random number table to generate his sample, resulting in Xavier interviewing 10 people who all go into the gaming store. Does this mean the sample is not random? Explain.

Lesson 2 Assignment

Prepare

For the given data set, calculate the requested measures.

12, 11, 15, 17, 10, 21, 28, 22, 17

Mean:

Median:

Range:

First Quartile:

Third Quartile:

Interquartile Range:

3 Sample Populations

OBJECTIVE

- Determine that a random sample has characteristics that are representative of the whole population.

You have used data to answer a statistical question.

How do random samples help make inferences about a population?

Getting Started

Census Data

In the United States, the Constitution mandates that a census be taken every ten years. A census is an official count or survey of the population, in which detailed demographic information is collected. Populations at the city, state, and county levels are documented annually.

Census data are used by the government to determine the number of representatives in government. It can also serve as a guide for spending and taxation. Outside of government, social services can use census data as a guide for their programs, and companies can use census data for marketing purposes.

The hypothetical population for each county in the state of Texas is shown in the tables at the end of this lesson.

1. Analyze the Texas county population data for 2020.

   a. List several characteristics of the data.

   b. Does the data contain any outliers? Explain your reasoning.

ACTIVITY
3.1 Sample Population

Recall that a **sample** is a subset of a larger data set. It is often impractical to analyze all data points when gathering information. Statisticians can reliably determine characteristics of a whole group by analyzing samples of the whole population.

Ask Yourself . . .
How can you use a random sample to analyze data in everyday life?

1. Why is it important to choose a sample group at random?

2. Antonio and Omar are studying the Texas county population data. They each chose the populations of the counties shown at random to conduct their study.

Antonio

Angelina County. . . 86,395
Bowie County. . . 92,893
Coryell County. . . 83,093
Ector County. . . 165,171
Grayson County. . . 135,543

Omar

Wise County. . . 68,632
Walker County. . . 76,400
Tom Green County. . . 120,003
Starr County. . . 65,920
Randall County. . . 140,753
Parker County. . . 148,222
Midland County. . . 169,983
Hunt County. . . 99,956
Wichita County. . . 129,350
Victoria County. . . 91,319
Taylor County. . . 143,208
Rockwall County. . . 107,819
Potter County. . . 118,525
Orange County. . . 84,808
Liberty County. . . 91,628
Comal County. . . 161,501

Each student says that his own sample is more likely to accurately represent the population data. Without performing any calculations, who do you think is correct? Explain your reasoning.

3. Choose a sample of ten counties at random. List the counties and their populations in the first two columns of the table.

County	Population (people)	Absolute Deviation from the Mean

You can use a random number generator or a random number table to generate random numbers.

4. Calculate each measure for your sample.

   a. Mean

   b. Median

   c. Range

5. Calculate each quartile and the interquartile range for your sample.

   a. First quartile

   b. Third quartile

   c. Interquartile range

Remember . . .
When you arrange data in a set in order, *quartiles* are the numbers that split data in quarters, or fourths. Quartiles are often denoted by the letter Q followed by a number that indicates which fourth it represents. Since the median is the second quartile, you can represent it using Q2. The other quartiles are Q1 and Q3.

A box plot is sometimes referred to as a *box-and-whisker plot*. You can represent box plots both horizontally and vertically.

Remember . . .

To create a box plot:

- Label the number line. Include the maximum and minimum values.
- Place dots above the minimum and maximum values.
- Place vertical lines above the median and quartile values. Draw the box around them.
- Draw lines, to connect the box to the minimum and maximum values.

6. Use your sample to complete the questions.

   a. Create a box plot to represent your sample.

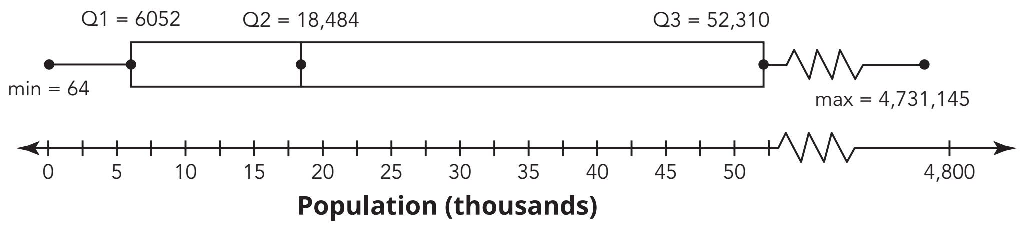
   b. Does the range and interquartile range shown in your box plot help you predict how great the value of the mean absolute value deviation will be? Explain your thinking.

7. Determine each county's absolute deviation from the mean. Write your answers in the third column of the table in Question 4.

8. Calculate the mean absolute deviation of your sample. How does it compare to your prediction in Question 6 part (b)?

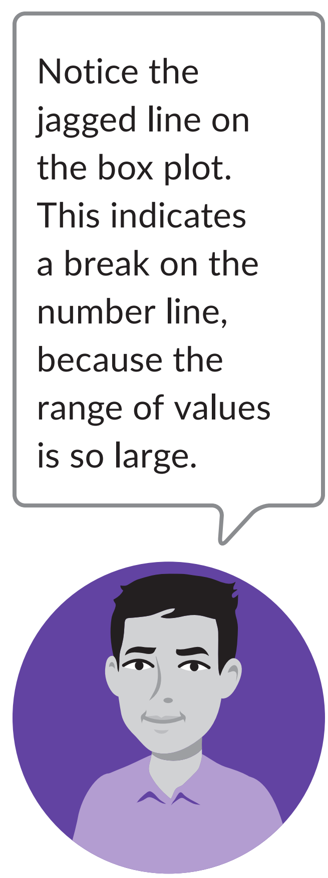
Suppose the mean population of a Texas county in the year 2020 was 11,746 people, while the median was 18,484.

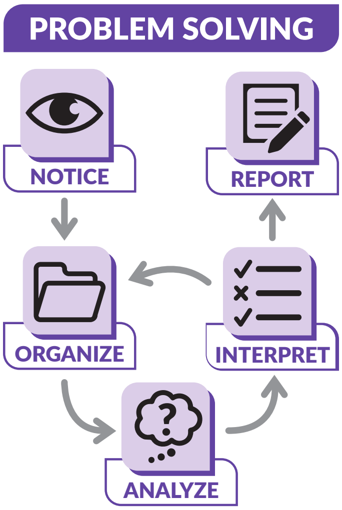
The first quartile is 6052, and the third quartile is 52,310. A box plot of the population data for all counties in Texas is shown.



The mean absolute deviation for the whole population was 157,142.

9. Compare your sample statistics to the statistics for the whole state of Texas. Do you think your sample accurately reflects the characteristics of the whole population? Explain your reasoning.





ACTIVITY 3.2

More Samples

1. Choose two more different samples of ten counties at random. Then, calculate the mean of each sample and use it to calculate the absolute deviations. Write your answers in the table.

County	Population (people)	Absolute Deviation from the Mean

County	Population (people)	Absolute Deviation from the Mean

2. Calculate each statistic in the table on the following page for Samples 2 and 3. Also, record each statistic for Sample 1 from the previous problem in this same table.

	Mean	Median	Mode	Quartiles	Mean Absolute Deviation
Sample 1				$Q_1 =$ $Q_3 =$	
Sample 2				$Q_1 =$ $Q_3 =$	
Sample 3				$Q_1 =$ $Q_3 =$	
Population	114,746	18,484	6664	$Q_1 =$ $Q_3 =$	157,142

Ask Yourself . . .
What observations can you make?

3. How do the statistics of your three samples compare to the statistics for the entire data set?

4. Use the additional space in the table to record the statistics for the samples from other groups in your class.

5. Calculate the average of each statistic for all of the samples you and your classmates gathered.

   a. average mean

   b. average median

   c. average mode

   d. average first quartile

e. average third quartile

f. average mean absolute deviation

6. How do the averages of the statistics compare to the statistics for the entire data set?

7. When conducting a study, do you think it is better to use the mean of one random sample, or to use the average of the means of multiple samples? Explain your reasoning.

8. When conducting a study, do you think it is better to use the mean absolute deviation of one random sample, or to use the average of the mean absolute deviations of multiple samples? Explain your reasoning.

Talk the Talk

Another Census

Ben chose the populations of the 10 Texas counties that begin with the letter *T*. The data are from the 2020 Census.

County	Population (people)
Tarrant County	2,110,640
Taylor County	143,208
Terrell County	760
Terry County	11,831
Throckmorton County	1,440
Titus County	31,247
Tom Green County	120,003
Travis County	1,290,188
Trinity County	13,602
Tyler County	19,798

1. What is the mean population of the counties shown?

2. Suppose the mean population of a Texas county in the year 2020 was 11,474 people. Based on this one measure of central tendency, do you think this sample accurately reflects the characteristics of the whole population? Explain your reasoning.

ID #	County Name	2020 Population	ID #	County Name	2020 Population
1	Anderson County	57,922	28	Caldwell County	45,883
2	Andrews County	18,610	29	Calhoun County	20,106
3	Angelina County	86,395	30	Callahan County	13,708
4	Aransas County	23,830	31	Cameron County	421,017
5	Archer County	8560	32	Camp County	12,464
6	Armstrong County	1,848	33	Carson County	5807
7	Atascosa County	48,981	34	Cass County	28,454
8	Austin County	30,167	35	Castro County	7371
9	Bailey County	6,904	36	Chambers County	46,571
10	Bandera County	20,851	37	Cherokee County	50,412
11	Bastrop County	97,216	38	Childress County	6664
12	Baylor County	3,465	39	Clay County	10,218
13	Bee County	31,047	40	Cochran County	2547
14	Bell County	370,647	41	Coke County	3285
15	Bexar County	2,009,824	42	Coleman County	7684
16	Blanco County	11,374	43	Collin County	1,064,465
17	Borden County	631	44	Collingsworth County	2652
18	Bosque County	18,235	45	Colorado County	20,557
19	Bowie County	92,628	46	Comal County	161,501
20	Brazoria County	372,031	47	Comanche County	13,594
21	Brazos County	233,849	48	Concho County	3303
22	Brewster County	9,546	49	Cooke County	41,668
23	Briscoe County	1,435	50	Coryell County	83,093
24	Brooks County	7,076	51	Cottle County	1380
25	Brown County	38,095	52	Crane County	4675
26	Burleson County	17,642	53	Crockett County	3098
27	Burnet County	49,130	54	Crosby County	5133

ID #	County Name	2020 Population
55	Culberson County	2,188
56	Dallam County	115
57	Dallas County	2,613,539
58	Dawson County	12,456
59	Deaf Smith County	18,583
60	Delta County	5,230
61	Denton County	906,422
62	DeWitt County	19,824
63	Dickens County	1,770
64	Dimmit County	8,615
65	Donley County	3,258
66	Duval County	9,831
67	Eastland County	17,725
68	Ector County	165,171
69	Edwards County	1,422
70	Ellis County	192,455
71	El Paso County	865,657
72	Erath County	42,545
73	Falls County	16,968
74	Fannin County	35,662
75	Fayette County	24,435
76	Fisher County	3,672
77	Floyd County	5,402
78	Foard County	1,095
79	Fort Bend County	822,779
80	Franklin County	10,359
81	Freestone County	19,435

ID #	County Name	2020 Population
82	Frio County	18,385
83	Gaines County	21,598
84	Galveston County	350,682
85	Garza County	5,816
86	Gillespie County	26,725
87	Glasscock County	1,116
88	Goliad County	7,012
89	Gonzales County	19,653
90	Gray County	21,227
91	Grayson County	135,543
92	Gregg County	124,239
93	Grimes County	29,268
94	Guadalupe County	172,706
95	Hale County	32,522
96	Hall County	2,825
97	Hamilton County	8,222
98	Hansford County	5,285
99	Hardeman County	3,549
100	Hardin County	56,231
101	Harris County	4,731,145
102	Harrison County	68,839
103	Hartley County	5,382
104	Haskell County	5,416
105	Hays County	241,067
106	Hemphill County	3,382
107	Henderson County	82,150
108	Hidalgo County	870,761

ID #	County Name	2020 Population
109	Hill County	35,874
110	Hockley County	21,537
111	Hood County	61,598
112	Hopkins County	36,787
113	Houston County	22,066
114	Howard County	34,860
115	Hudspeth County	3,202
116	Hunt County	99,956
117	Hutchinson County	10,617
118	Irion County	1,513
119	Jack County	8,472
120	Jackson County	14,988
121	Jasper County	32,980
122	Jeff Davis County	1,996
123	Jefferson County	256,526
124	Jim Hogg County	4,838
125	Jim Wells County	38,891
126	Johnson County	179,927
127	Jones County	19,663
128	Karnes County	14,710
129	Kaufman County	145,310
130	Kendall County	44,279
131	Kenedy County	350
132	Kent County	753
133	Kerr County	52,598
134	Kimble County	4,286
135	King County	265

ID #	County Name	2020 Population
136	Kinney County	3,129
137	Kleberg County	31,040
138	Knox County	3,353
139	Lamar County	50,088
140	Lamb County	13,045
141	Lampasas County	21,627
142	La Salle County	6,664
143	Lavaca County	20,337
144	Lee County	17,478
145	Leon County	15,719
146	Liberty County	91,628
147	Limestone County	22,146
148	Lipscomb County	3,059
149	Live Oak County	11,335
150	Llano County	21,243
151	Loving County	64
152	Lubbock County	310,639
153	Lynn County	5,596
154	McCulloch County	7,630
155	McLennan County	260,579
156	McMullen County	600
157	Madison County	13,455
158	Marion County	9,725
159	Martin County	5,237
160	Mason County	3,953
161	Matagorda County	36,255
162	Maverick County	57,887

ID #	County Name	2020 Population
163	Medina County	50,748
164	Menard County	1,962
165	Midland County	169,983
166	Milam County	24,754
167	Mills County	4,456
168	Mitchell County	8,990
169	Montague County	19,965
170	Montgomery County	620,443
171	Moore County	21,358
172	Morris County	11,973
173	Motley County	1,063
174	Nacogdoches County	64,653
175	Navarro County	52,624
176	Newton County	12,217
177	Nolan County	14,738
178	Nueces County	353,178
179	Ochiltree County	10,015
180	Oldham County	1,758
181	Orange County	84,808
182	Palo Pinto County	28,409
183	Panola County	22,491
184	Parker County	148,222
185	Parmer County	9,869
186	Pecos County	15,193
187	Polk County	50,123
188	Potter County	118,525

ID #	County Name	2020 Population
189	Presidio County	6,131
190	Rains County	12,164
191	Randall County	140,753
192	Reagan County	3,385
193	Real County	2,758
194	Red River County	11,587
195	Reeves County	14,748
196	Refugio County	6,664
197	Roberts County	827
198	Robertson County	16,757
199	Rockwall County	107,819
200	Runnels County	9,900
201	Rusk County	52,214
202	Sabine County	9,894
203	San Augustine County	7,918
204	San Jacinto County	27,402
205	San Patricio County	68,755
206	San Saba County	5,730
207	Schleicher County	2,451
208	Scurry County	16,932
209	Shackelford County	3,105
210	Shelby County	24,022
211	Sherman County	2,782
212	Smith County	233,479
213	Somervell County	9,205
214	Starr County	65,920

ID #	County Name	2020 Population
215	Stephens County	9,101
216	Sterling County	1,372
217	Stonewall County	1,245
218	Sutton County	3,372
219	Swisher County	6,971
220	Tarrant County	2,110,640
221	Taylor County	143,208
222	Terrell County	760
223	Terry County	11,831
224	Throckmorton County	1,440
225	Titus County	31,247
226	Tom Green County	120,003
227	Travis County	1,290,188
228	Trinity County	13,602
229	Tyler County	19,798
230	Upshur County	40,892
231	Upton County	3,308
232	Uvalde County	24,564
233	Val Verde County	47,586
234	Van Zandt County	59,541
235	Victoria County	91,319
236	Walker County	76,400
237	Waller County	56,794
238	Ward County	11,644
239	Washington County	35,805
240	Webb County	267,114

ID #	County Name	2020 Population
241	Wharton County	41,570
242	Wheeler County	4,990
243	Wichita County	129,350
244	Wilbarger County	12,887
245	Willacy County	20,164
246	Williamson County	609,017
247	Wilson County	49,753
248	Winkler County	7,791
249	Wise County	68,632
250	Wood County	44,843
251	Yoakum County	7,694
252	Young County	17,867
253	Zapata County	13,889
254	Zavala County	9,670

Lesson 3 Assignment

Write

Explain the purpose and process of taking a sample when you are interested in a characteristic of the population.

Remember

A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected. The larger the sample, typically the more representative the sample is of the population.

Practice

1. When should you use a sample of a data set?

2. There are 254 counties in Texas. Juliana chooses, at random, the populations of 190 Texas counties to use as a sample.

   a. Is Juliana's sample likely to accurately represent the population data. Explain your reasoning.

   b. Why does Juliana's sample not show the best use of a sample of a large data set? Explain your reasoning.

Lesson 3 Assignment

3. Suppose Catalina chose the populations of the 10 Texas counties shown at random.

   a. What is the mean population of the counties shown?

   b. Suppose the mean population of a Texas county was 101,081 people. Based on this one measure of central tendency, do you think this sample accurately reflects the characteristics of the whole population? Explain your reasoning.

County	Population (people)
El Paso County	820,790
Kenedy County	437
Baylor County	3,741
Polk County	45,725
Lamb County	14,167
Glasscock County	1,251
Zapata County	14,282
Wheeler County	5,465
Guadalupe County	135,757
Andrews County	15,445

Lesson 3 Assignment

Prepare

Solve each equation.

1. $2.5x + 100 = 600$

2. $10 = 2x - 4$

3. $\frac{1}{4}x + 5 = 30$

Name: ____________________

Variability and Sampling

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Variability and Sampling* topic by:

TOPIC 2: *Variability and Sampling*	Beginning of Topic	Middle of Topic	End of Topic
calculating the mean absolute deviation for a data set.	◯	◯	◯
using the mean absolute deviation as a measure of variability to describe how data are spread out around the mean of the data set.	◯	◯	◯
explaining how inferences about a population can be made by examining a random sample.	◯	◯	◯
explaining why generalizations made about a population from a sample are only valid if the sample represents that population.	◯	◯	◯
using data from a random sampling to draw conclusions about a population.	◯	◯	◯
using tools to generate random samples.	◯	◯	◯
generating multiple samples (or simulated samples) of the same size to draw inferences about a population.	◯	◯	◯

continued on the next page

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Variability and Sampling* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these?

TOPIC 2 SUMMARY

Variability and Sampling Summary

NEW KEY TERMS

- deviation [desviación]
- absolute deviation [desviación absoluta]
- mean absolute deviation (MAD) [desviación media absoluta (DMA)]
- survey
- data [datos]
- population [población]
- census [censo]
- sample
- parameter [parámetro]
- statistic [estadística]
- random sample

LESSON 1 Mean Absolute Deviation

One measure of variation that describes the spread of data values is *deviation*. The **deviation** of a data value indicates how far that data value is from the mean. To calculate deviation, subtract the mean from the data value:

$$\text{deviation} = \text{data value} - \text{mean}$$

For example, the mean of the data set 15, 12, 13, 10, 9, and 13 is 12.

The table describes each data point's deviation from the mean.

Data Point	15	12	13	10	9	13
Deviation from the Mean	3	0	1	−2	−3	1

In order to get an idea of the spread of the data values, take the absolute value of each deviation and determine the mean of those absolute values. The absolute value of each deviation is called the **absolute deviation**. The **mean absolute deviation (MAD)** is the mean of the absolute deviations.

For example, the mean absolute deviation of the data shown in the table is

$$\frac{|3| + |0| + |1| + |-2| + |-3| + |1|}{6} = \frac{10}{6}.$$

So, the MAD is about 1.67.

LESSON 2

Collecting Random Samples

There are four components of the statistical process:

- Formulating a statistical question.
- Collecting appropriate data.
- Analyzing the data graphically and numerically.
- Interpreting the results of the analysis.

One data collection strategy you can use is a *survey*. A *survey* is a method of collecting information about a certain group of people. It involves asking a question or a set of questions to those people. When information is collected, the facts or numbers gathered are called **data**.

The **population** is the entire set of items from which data can be selected. When you decide what you want to study, the population is the set of all elements in which you are interested. The elements of that population can be people or objects. A **census** is the data collected from every member of a population.

In most cases, it is not possible or logical to collect data from the entire population. When data are collected from a part of the population, the data are called a **sample**.

When data are gathered from a population, the characteristic used to describe the population is called a **parameter**. When data are gathered from a sample, the characteristic used to describe the sample is called a **statistic**. A statistic is used to make an estimate about the parameter.

When information is collected from a sample in order to describe a characteristic about the population, it is important that such a sample be as representative of the population as possible. A **random sample** is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

LESSON 3

Sample Populations

When dealing with a large amount of data, it is often impractical to analyze all the data points. A sample is a subset of a larger data set which can be used to determine characteristics of a whole group.

Random Sample

A state in the U.S. has 67 counties, all with varied populations. Generate a random sample. Then, you can use the sample to determine average statistics for the entire state.

County	Population	Absolute Deviation from the Mean
Adams	91,292	120,152.875
Butler	174,083	37,361.875
Dauphin	251,798	40,353.125
Huntingdon	45,586	165,858.875
Luzerne	319,250	107,805.125
Montgomery	750,097	538,652.125
Potter	18,080	193,364.875
Tioga	41,373	170,071.875

Mean of sample population: 211,444.875
Median of sample population: 132,687.5
Mean absolute deviation of sample population: 171,702.5938

Mean of entire population: 181,212.1618
Median of entire population: 90,366
Mean absolute deviation of entire population: 162,548.439

The mean, median, and mean absolute deviation of the sample population are all slightly larger than the mean, median, and mean absolute deviation of the entire population. However, the mean and mean absolute deviation are quite close. This means the random sample is probably an accurate representation of the entire population.

GRADE 8

MODULE

4

Modeling Linear Equations

GRADE 8

TOPIC 1

The game of Bingo dates back to 16th-century Italy. The modern version of the game was copyrighted by Hugh Ward of Pittsburgh, Pennsylvania, in 1924.

Solving Linear Equations and Inequalities

1 Equations with Variables on Both Sides

OBJECTIVES

- Use algebra tiles to model and solve linear equations with variables on both sides of the equal sign.
- Use the properties of equality to solve equations with variables on both sides of the equal sign.

You have solved equations by combining like terms and using inverse operations.

How can you solve equations when there are variables on both sides of the equation?

Getting Started

Build It Up and Break It Down

The properties of equality allow you to solve equations.

Properties of Equality	For all numbers *a*, *b*, and *c*
addition property of equality	If $a = b$, then $a + c = b + c$.
subtraction property of equality	If $a = b$, then $a - c = b - c$.
multiplication property of equality	If $a = b$, then $ac = bc$.
division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

These properties also allow you to create more complex equations. For example, given the equation $x = 2$, you can use the addition property of equality to create $x + 1 = 2 + 1$, which is the same as $x + 1 = 3$. Since you used the properties of equality, the two equations have the same solution.

To solve a two-step equation, isolate the variable term on one side of the equation and the constant on the other side of the equation. Then, multiply or divide both sides of the equation by the numeric coefficient to determine the value of the variable.

1. Consider each given equation. Use the properties of equality to create an equivalent equation in the form $ax + b = c$, where *a*, *b*, and *c* can be any number. Record the properties of equality you used to create your new equation.

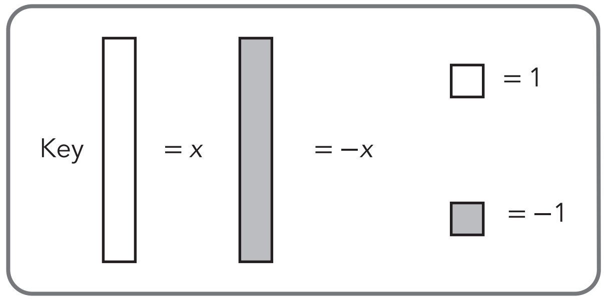
   a. $x = 5$

   b. $x = -1$

2. Give each of your equations to a partner to verify that each equation has the correct solution.

ACTIVITY 1.1 Modeling with Variables on Both Sides

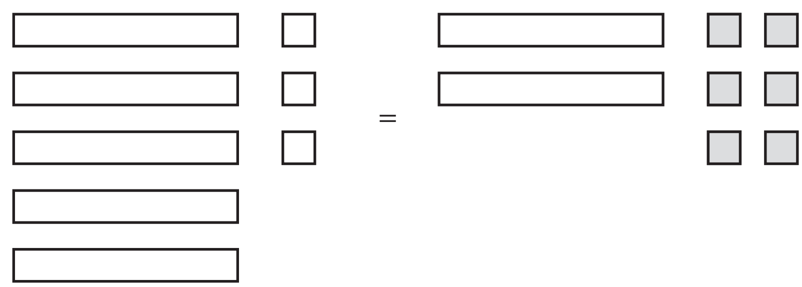
You have previously solved two-step equations using a variety of strategies. In this activity, you will use algebra tiles to solve equations with variables on both sides of the equal sign. Remember, to solve an equation means to determine the value of the unknown that makes the equation true.



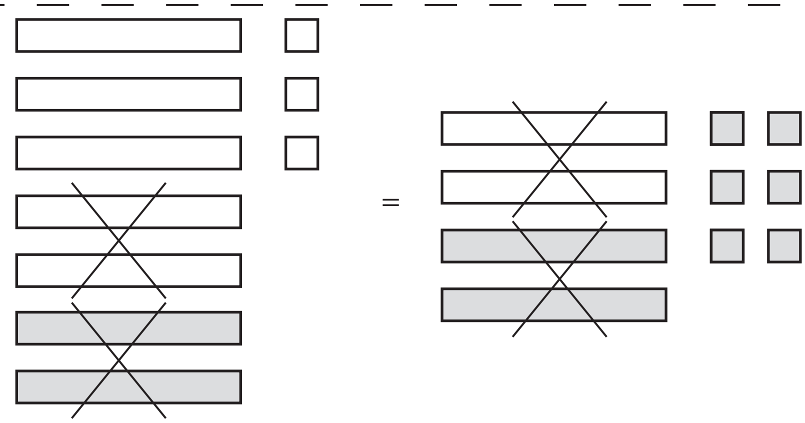
Consider the equation $5x + 3 = 2x - 6$.

WORKED EXAMPLE

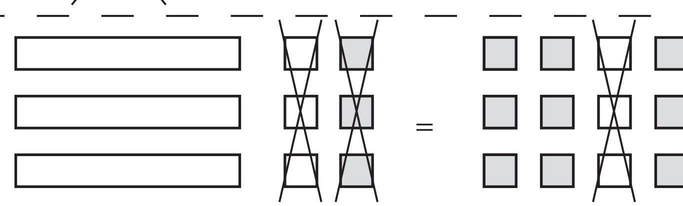
Step 1: Model the equation $5x + 3 = 2x - 6$ with algebra tiles.



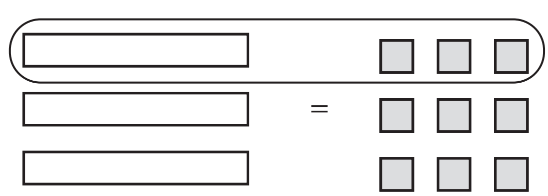
Step 2: Get the variable terms on one side of the equation. Subtract $2x$ from both sides of the equal sign, which creates a zero pair on the right side of the equation.



Step 3: Get the constant terms on the other side of the equation. Subtract 3 from both sides of the equal sign, creating a zero pair.



Step 4: Determine the value of the variable by creating equal groups. Divide the model into 3 equal groups on each side of the equation.



The solution to the equation is $x = -3$.

1. Use algebra tiles to model and solve each equation.

   a. $-2x + 4 = -4x$

   b. $-4 + 6x = 5x - 9$

   c. $-4x + 4 = 2x - 2$

Write an equation to represent the situation. Then, solve the equation using algebra tiles.

2. Mia is ordering friendship bracelets. Store *A* charges $3 per bracelet and $10 for shipping on all orders. Store *B* charges $4 per bracelet and $2 shipping on all orders. For how many friendship bracelets do the companies charge the same total price?

ACTIVITY 1.2 Solving with Variables on Both Sides

To begin solving an equation with variables on both sides of the equation, move all the variable terms to one side of the equation and all the constants to the other side of the equation.

Consider the equation $5x + 3 = 2x + 5$.

Ethan and Samuel each solved the equation in a different way. Analyze their solution strategies.

Ethan

$$5x + 3 = 2x + 5$$
$$-5x \qquad -5x$$
$$3 = -3x + 5$$
$$-5 \qquad -5$$
$$\frac{-2}{-3} = \frac{-3x}{-3}$$
$$\frac{2}{3} = x$$
$$x = \frac{2}{3}$$

Samuel

$$5x + 3 = 2x + 5$$
$$-2x \qquad -2x$$
$$3x + 3 = 5$$
$$-3 \quad -3$$
$$3x = 2$$
$$x = \frac{2}{3}$$

1. Compare the two solution strategies.

   a. How were their solution strategies the same? How were they different?

   b. Which strategy do you prefer? Explain your choice.

2. Solve each equation. Describe why you chose your solution strategy.

   a. $x - 6 = 5x + 10$

   b. $2x - 7 = -5x + 14$

Consider the two different equations that Sofia and Chloe solved.

Sofia

$$\begin{aligned}
3x + 9 &= 6x - 30 \\
\frac{3x + 9}{3} &= \frac{6x - 30}{3} \\
x + 3 &= 2x - 10 \\
-x \quad & \quad -x \\
3 &= x - 10 \\
+10 \quad & \quad +10 \\
13 &= x \\
x &= 13
\end{aligned}$$

Chloe

$$\begin{aligned}
-x - 2 &= -4x - 1 \\
\frac{-x - 2}{-1} &= \frac{-4x - 1}{-1} \\
x + 2 &= 4x + 1 \\
-x \quad & \quad -x \\
2 &= 3x + 1 \\
-1 \quad & \quad -1 \\
1 &= 3x \\
\frac{1}{3} &= \frac{3x}{3} \\
\frac{1}{3} &= x \\
x &= \frac{1}{3}
\end{aligned}$$

3. Sofia and Chloe each divided both sides of their equations by a factor and then solved.

   a. Explain the reasoning used by each.

   b. Do you think this solution strategy will work for any equation? Explain your reasoning.

4. Solve each equation using a strategy similar to those used by Sofia and Chloe.

a. $-4x + 8 = 2x + 10$

$$-4x + \frac{8}{-2} = 2x + \frac{10}{-2}$$
$$2x - 4 = -x - 5$$
$$3x - 4 = -5$$
$$3x = -1$$

b. $-42x = -4x - 1$

$$\frac{-42x}{-1} = -4x - \frac{1}{-1}$$
$$42x = 4x - 1$$
$$38x = -1$$

You can multiply both sides of an equation by powers of 10 to convert all numbers to whole numbers.

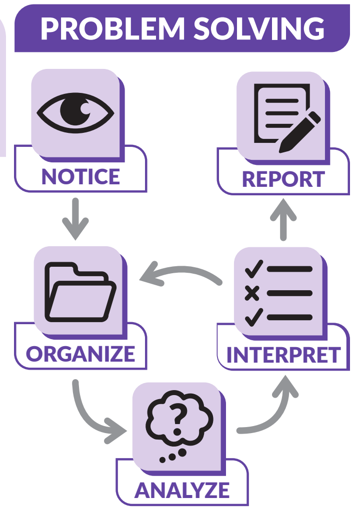
5. Similar to Sofia and Chloe, Minh and Daniel want to multiply both sides of the equation $2.5x + 1.4 = 0.5x + 2$ by 10 before solving the equation because they think it will be simpler for them to solve. The first step of each strategy is shown. Who's correct? What is the error in the other strategy?

Minh	Daniel
$25x + 14 = 5x + 2$	$25x + 14 = 5x + 20$

6. Explain why Minh and Daniel thought multiplying by 10 would make solving the equation $2.5x + 1.4 = 0.5x + 2$ simpler to solve.

ACTIVITY 1.3

Practice Solving Equations



Solve each equation.

1. $18x - 9 = 24x - 27$

2. $-15x - 12 = -12x + 15$

3. $12.6 + 4x = 9.6 + 8x$

4. $-12.11x - 10.5 = 75.6 - 3.5x$

5. $\dfrac{10x + 2}{2} = 4x + \dfrac{1}{4}$

Talk the Talk

Solving Strategically

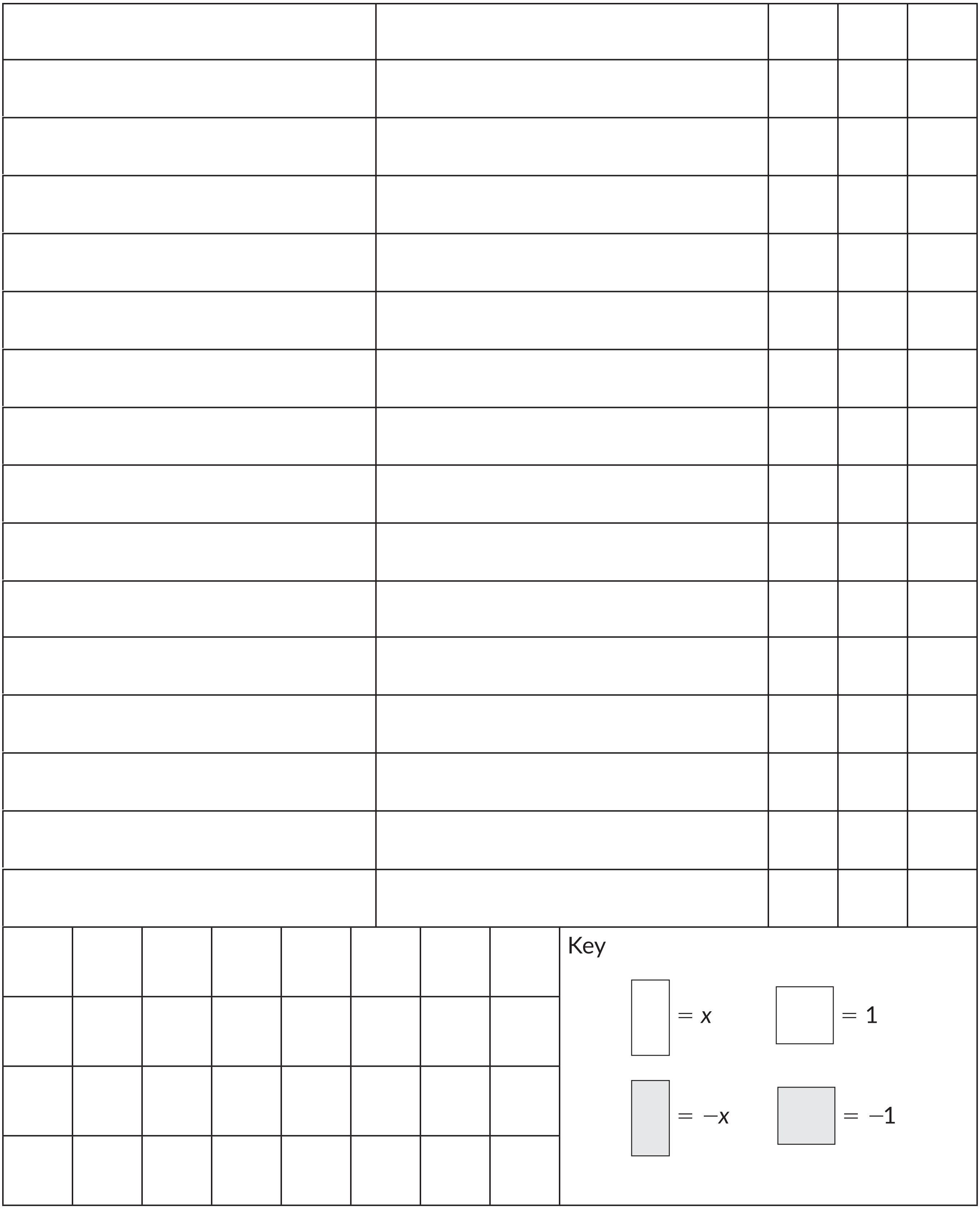
Use algebra tiles to model and solve the equation.

1. $4x + 3 = 7x - 6$

Solve the equation.

2. $0.2b + 2.6 = 1.7b - 0.4$

Algebra Tiles



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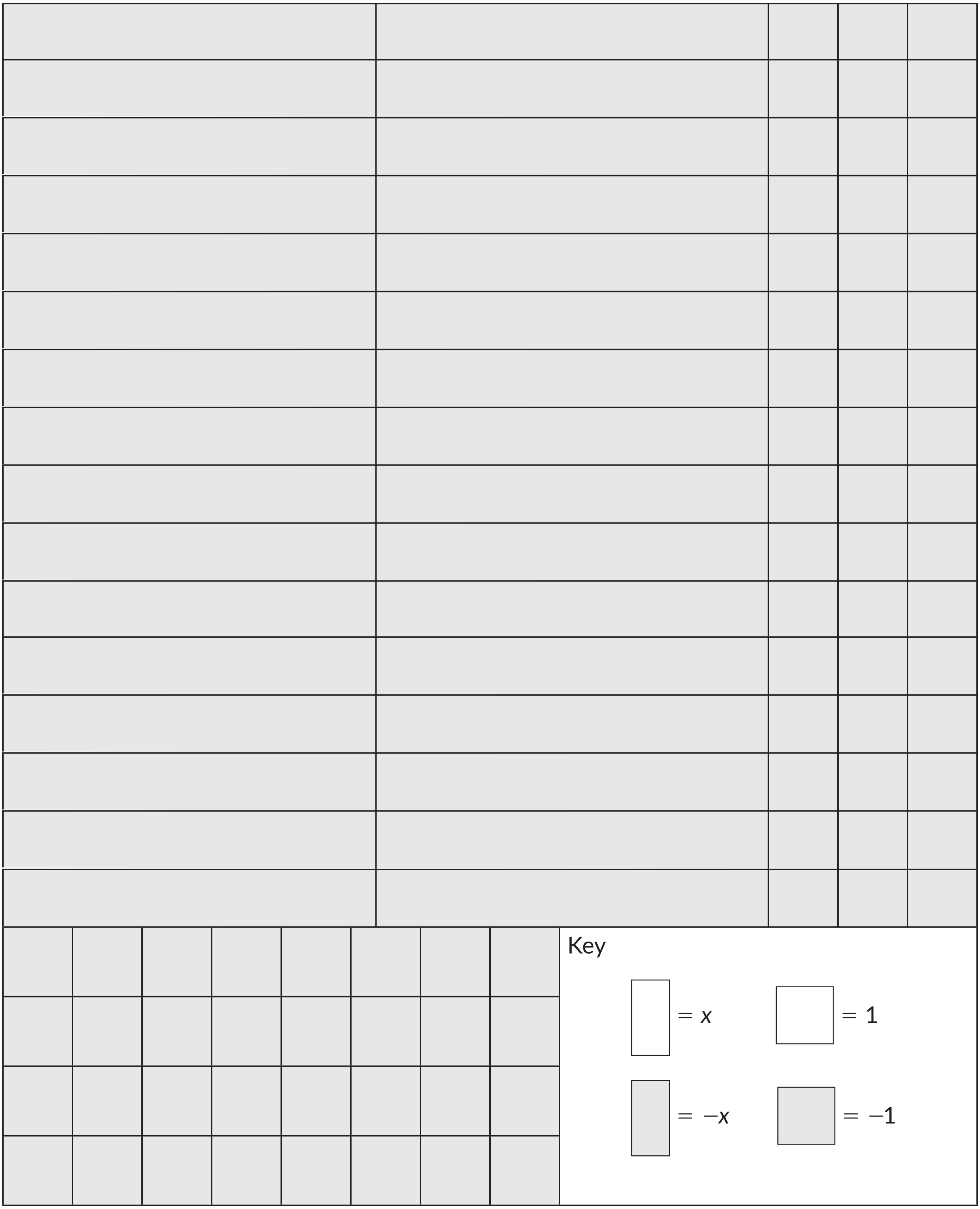
So you can cut out the Algebra Tiles on the other side

Algebra Tiles



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Lesson 1 Assignment

Write

Explain the process of solving an equation with variables on both sides of the equal sign.

Remember

You can use algebra tiles or the properties of equality to solve equations with variables on both sides of the equal sign.

Practice

Model and solve each equation using algebra tiles.

1. $3x + 6 = -2x - 4$

2. $-x + 6 = -3x + 2$

Solve each equation.

3. $5x + 15 = 75 - 25x$

4. $11.3x + 12.8 = 7.5x + 35.6$

5. $\frac{1}{4}x - 3 = \frac{1}{2}x + 12$

6. $9.6x - 15.4 = -4.3x + 26.3$

Lesson 1 Assignment

7. $4x = 20x - 24$

8. $-2x - 1.4 = 6 + 3x$

Prepare

Members of a community service club are collecting pull tabs from aluminum cans to support a local hospital's initiative.

- Victoria collected the least number of pull tabs.
- Liam collected 15 more pull tabs than Victoria.
- Juan collected 4 times as many as Liam.
- Jayden collected 10 fewer than Juan.

Define a variable to represent the number of pull tabs that Victoria collected. Then, write algebraic expressions to represent the number of pull tabs that each of the other students collected.

2 Analyzing and Solving Linear Equations

OBJECTIVES

- Write and solve linear equations in one variable.
- Determine whether an equation has one solution, no solution, or infinitely many solutions by successively transforming the equation into simpler forms.
- Interpret expressions in and solutions to equations in the context of problem situations.
- Write inequalities to represent real-world scenarios.

NEW KEY TERMS

- one solution
- no solution
- infinitely many solutions

You have learned how to use strategies to solve complex equations with variables on both sides of the equal sign.

How can you determine when an equation has no solution or infinitely many solutions?

Getting Started

Review the definition of **represent** in the Academic Glossary.

No One Knows Exactly

Sometimes, you are asked to determine the value of unknown quantities using only information you have for a quantity.

Five friends have a certain number of books.

- Mateo has the fewest.
- Aaliyah has 7 more than Mateo.
- Ava has twice as many as Aaliyah.
- Ethan has 3 times as many as Mateo.
- Emma has 6 fewer than Aaliyah.

1. Define a variable for the number of books Mateo has.

2. Use your defined variable to write algebraic expressions to represent the number of comic books each person has.

ACTIVITY 2.1 Writing Expressions to Represent Situations

Use the expressions you wrote in the previous activity to answer each question.

1. Is it possible for Ethan and Emma to have an equal number of books? Write and solve an algebraic equation to explain why or why not.

2. If the original group of friends has a total of 182 books all together, then how many does each person have? Make sure to check your work.

   a. Books that Mateo owns:

   b. Books that Aaliyah owns:

   c. Books that Ava owns:

   d. Books that Ethan owns:

   e. Books that Emma owns:

ACTIVITY

2.2 Solving Equations to Solve Problems

Elijah, Kai, Samuel, and Noah have challenged their friends with this riddle.

- Elijah said, “When you add 150 to the number of songs in Samuel’s playlist, double that number, and finally divide by 3, you have the number of songs that are in my playlist.”
- Kai said, “When you take the number of songs in Samuel’s playlist, subtract 30, multiply that difference by 5, and finally divide that product by 4, the result will be the number of songs in my playlist.”
- Noah said, “Well, when you take twice the number of songs in Samuel’s playlist, add 30, multiply the sum by 4, and finally divide that product by 3, you will have the number of songs that I have in my playlist.”

1. What do you need to know to determine the songs each person has in their playlist?

2. Define a variable for the number of songs in Samuel’s playlist and then write expressions for the number of songs in each of the other people’s playlist.

   a. The number of songs in Elijah’s playlist:

   b. The number of songs in Kai’s playlist:

   c. The number of songs in Noah’s playlist:

3. Suppose Samuel has 150 songs in his playlist. Determine how many songs are in person's playlist.

   Elijah Kai Noah

4. Suppose Elijah and Noah have the same number of songs in their playlists. How many songs will each person have in their playlist?

   Samuel Kai Noah and Elijah

5. Suppose the sum of Kai's and Noah's playlist songs is 39 more than the number Elijah has? How many will each person have?

   Samuel Kai

   Noah Elijah

ACTIVITY 2.3

One Solution, No Solution, or Infinitely Many Solutions

You have solved linear equations that have *one solution*. An equation with **one solution** has only one value for the variable that makes the equation true. Equations can also have *no solution* or *infinitely many solutions*. When an equation has **infinitely many solutions**, any value for the variable makes the equation true. An equation with **no solution** has no value for the variable that makes the equation true.

Isabella and Diego were solving an equation from their math homework. They came across the equation shown.

$$6x + 14 = 2(3x + 7)$$

Examine each solution strategy.

What did Diego do differently to solve the equation?

Isabella

$$6x + 14 = 2(3x + 7)$$
$$6x + 14 = 6x + 14$$
$$6x + (-6x) + 14 = 6x + (-6x) + 14$$
$$14 = 14$$

Diego

$$6x + 14 = 2(3x + 7)$$
$$6x + 14 = 6x + 14$$
$$6x + 14 + (-14) = 6x + 14 + (-14)$$
$$\frac{6x}{6} = \frac{6x}{6}$$
$$x = x$$

1. Explain why both Isabella's and Diego's methods are correct but have different solutions.

2. How would you interpret the final equation in each solution? Is the final equation always true, sometimes true, or never true? Explain your reasoning.

3. Explain whether the equation has one solution, no solution, or infinitely many solutions.

Consider this new equation:

$$3(6x - 4) = 2(9x + 5)$$

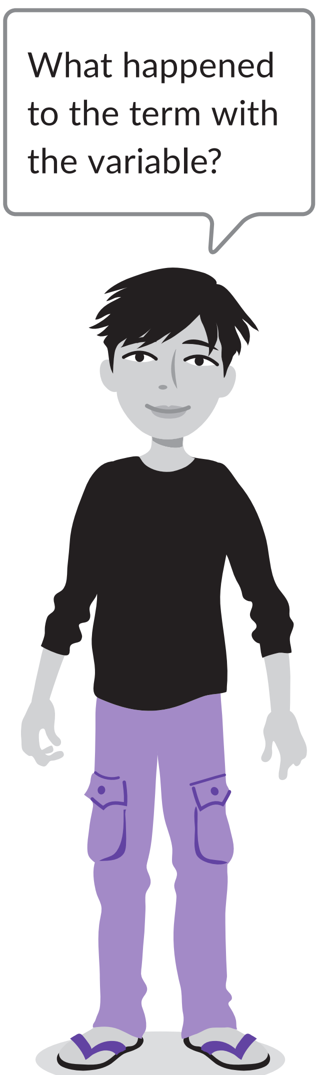
Examine each solution strategy.

Isabella

$$3(6x - 4) = 2(9x + 5)$$
$$18x - 12 = 18x + 10$$
$$18x - 18x - 12 = 18x - 18x + 10$$
$$-12 \neq 10$$

Diego

$$3(6x - 4) = 2(9x + 5)$$
$$18x - 12 = 18x + 10$$
$$18x - 12 + 12 = 18x + 10 + 12$$
$$18x = 18x + 22$$
$$18x + (-18x) = 18x + (-18x) + 22$$
$$0 \neq 22$$

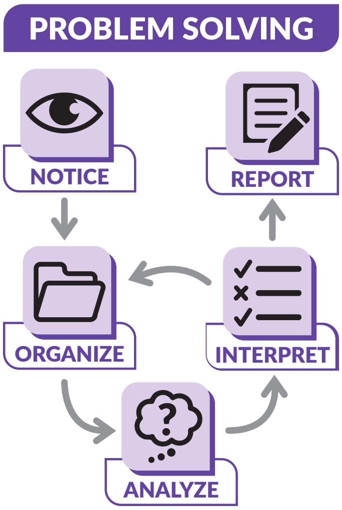


4. Explain why both Isabella's and Diego's methods are correct but have different solutions.

5. How would you interpret the final equation in each solution? Is the final equation always true, sometimes true, or never true? Explain your reasoning.

6. Explain whether the equation has one solution, no solution, or infinitely many solutions.

7. Go back and analyze Isabella's and Diego's work for both equations. At what point in the equation-solving process could they have made the conclusion that there were infinitely many solutions or no solution? Explain your reasoning.



ACTIVITY 2.4 Practice Solving Equations with Rational Coefficients

Solve each equation shown. Make sure to check your work.

1. $5x - 7 = 4x + 2$

2. $1.99x + 6 = 2.50x$

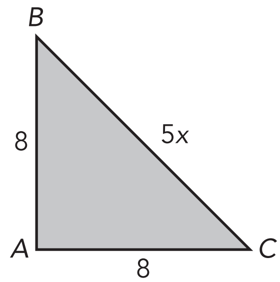
3. $40x = -50x + 100$

4. $4x - 1 = 4(x + 2)$

5. $5(2x - 1) = 10x - 5$

Write and solve an equation to answer each question.

6. The perimeter of isosceles Triangle *ABC* is $53x$ units. Determine the value of x.



7. Two after-school clubs are ordering and designing T-shirts for their members. One club pays \$12 per shirt and a one-time \$48 design fee. The other club pays \$10.50 per shirt and a one-time \$52 design fee. How many shirts does each club order when the total cost for each club is the same?

8. Write a real-world situation that models each equation. Then, solve the equation and state the solution in context.

   a. $6x - 200 = 4x$

   b. $\frac{1}{2}x + 10 = \frac{3}{4}x + 5$

Talk the Talk

How Do You Know?

1. When you solve any equation, describe how you know when there will be:

   a. one solution.

   b. no solution.

   c. infinitely many solutions.

Lesson 2 Assignment

Write

Write three equations: one that has one solution, one that has no solution, and one that has infinitely many solutions.

Remember

An equation can have one solution, no solution, or infinitely many solutions.

Practice

1. Daniel has four different chicken coops on his farm. He gathers eggs from each coop every day to sell at the local farmer's market each week. During one week in the summer, the production levels from the coops were compared.

- The number of eggs from coop B can be found by subtracting 10 from coop A's production and then multiplying this result by $\frac{2}{5}$.
- The number of eggs from coop C can be found by adding 3 to coop A's production, multiplying this amount by 3, and then dividing the whole result by 4.
- The number of eggs from coop D can be found by adding 7 to coop A's production, doubling this amount, and then dividing the result by 3.

   a. Define a variable for the number of eggs produced by coop A. Then, write expressions for the number of eggs produced by the other coops.

   b. Suppose coop A produced 125 eggs. How many did each of the other coops produce?

Lesson 2 Assignment

c. Suppose the number of eggs from coop C and coop D are equal. How many eggs did each coop produce? Do your answers make sense in terms of the problem situation?

2. Three siblings collect rare coins. To determine the number of rare coins that Isabella has, take the number of rare coins Juan has, add 4, and then divide that sum by 2. To determine the number of rare coins Jayden has, double the number of rare coins Juan has, subtract 4, and then multiply that difference by 2. How many rare coins does each sibling have when they have a total of 49 rare coins?

3. Mateo bought 4 shirts and a pair of pants that cost $20 at one store. He bought 6 shirts and used a $10 coupon at another store. He spent the same amount of money at each store. Each shirt costs the same. What was the cost of each shirt?

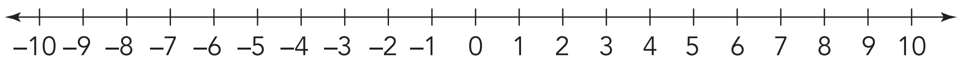
4. An isosceles triangle has a perimeter of $24x$ units. Two of the sides of the triangle are each 18 units, and the other side is $12x$. Write and solve an equation the can be used to find the value of x.

Lesson 2 Assignment

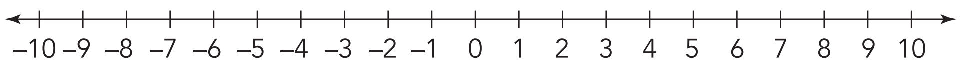
Prepare

Graph each inequality on the number line shown.

1. $x \geq -6$



2. $x < 7$



3 Solving Linear Inequalities

OBJECTIVES

- Write and solve linear inequalities in one variable.
- Write inequalities to represent real-world scenarios.

NEW KEY TERM

- solution set of an inequality

You have learned how to use strategies to solve complex equations with variables on both sides of the equal sign.

How can you solve an inequality with variables on both sides of the inequality symbol?

Getting Started

Remember . . .
The **solution set of an inequality** is the set of all points that make an inequality true.

Movie Ticket Sales

On opening day of a new movie, a local theater charges $12 per ticket. The theater has already sold $720 worth of tickets during a presale.

Write and solve an equation or inequality to answer each question. Graph each solution set on a number line.

1. How many additional tickets must the theater sell to make $1,200?

2. How many additional tickets must the theater sell to make more than $2,520?

3. How many additional tickets must the theater sell to make $816 or less?

4. Do all the solutions make sense in terms of the context of the situation? Explain your reasoning.

ACTIVITY
3.1
Solving Other Linear Inequalities

The inequalities that you solved in the Getting Started were two-step inequalities. Let's consider inequalities in different forms.

Harper and Diego each determined the solution set to the inequality $8x + 2 \geq 5x - 10$.

Harper

$$8x + 2 \geq 5x - 10$$
$$8x - 5x + 2 \geq 5x - 5x - 10$$
$$3x + 2 \geq -10$$
$$3x + 2 - 2 \geq -10 - 2$$
$$3x \geq -12$$
$$\frac{3x}{3} \geq \frac{-12}{3}$$
$$x \geq -4$$

Diego

$$8x + 2 \geq 5x - 10$$
$$8x - 8x + 2 \geq 5x - 8x - 10$$
$$2 \geq -3x - 10$$
$$2 + 10 \geq -3x - 10 + 10$$
$$12 \geq -3x$$
$$\frac{12}{-3} \leq \frac{-3x}{-3}$$
$$-4 \leq x$$

1. Describe the process each student used to determine the solution set of the inequality.

   a. Harper

   b. Diego

2. How does the process of solving an inequality with variables on both sides of the inequality symbol compare to the process of solving an equation with variables on both sides of the equal sign?

3. Explain why Harper and Diego are both correct even though their solutions look different.

ACTIVITY 3.2

Modeling Scenarios with Inequalities

Not all situations can be represented using equations. Some scenarios require the use of inequalities.

1. Elijah and Kai are discussing the number of podcasts each of them has listened to.

   Elijah listened to 24 podcasts before Kai started listening to podcasts.

   - Elijah listened to 4 new podcasts each week for w weeks.
   - Kai listened to 10 new podcasts each week for w weeks.

   Write and solve an inequality that represents this situation when the number of podcasts Elijah listened to is greater than the number of podcasts Kai listened to. Explain your reasoning.

2. Ms. Garcia is ordering lunch for her coworkers. The local deli charges \$10 per sandwich and a \$8 delivery fee per order. The local bakery charges \$7.75 per sandwich and a \$15 delivery fee per order. Write an inequality to represents the situation where the total cost at the bakery is less than the total cost at the deli when x is the number of sandwiches ordered. Determine the solution set to the inequality. Explain your reasoning.

3. Write a scenario that can be represented by the inequality:

$$9x + 5 < 13x$$

4. Write a scenario that can be represented by the inequality:

$$4d + 5 < 5d - 2$$

Talk the Talk

Same but Different

Solve each equation or inequality. Graph the solution or solution set.

1. $-7x - 4 = -2x + 16$

2. $-7x - 4 > -2x + 16$

3. Compare Questions 1 and 2. How are they different? How are they the same?

Lesson 3 Assignment

Write

Describe how to solve an inequality in your own words.

Remember

The methods for solving linear inequalities are similar to the methods for solving linear equations. Be sure to reverse the direction of the inequality symbol when multiplying or dividing both sides of the inequality symbol by a negative number.

Practice

1. Solve each equation.

   a. $3x - 8 = -7x + 18$

   b. $-8 + 2x = 2x - 8$

2. Determine the solution set for the inequality. Graph the solution set on a number line.

   a. $8x + 15 > 5x - 29$

   b. $7x - 2 \geq 30x - 48$

Lesson 3 Assignment

Prepare

Determine two ordered pairs (x, y) that lie on each equation.

1. $y = 2x + 5$

2. $3x + 4y = 200$

3. $y = \frac{1}{2}x - 10$

TOPIC 1 SELF-REFLECTION

Name: ____________________

Solving Linear Equations and Inequalities

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Solving Linear Equations* topic by:

TOPIC 1: *Solving Linear Equations and Inequalities*	Beginning of Topic	Middle of Topic	End of Topic
using properties of equality to write equivalent expressions.	○	○	○
writing one-variable equations or inequalities with variables on both sides of the equal sign or inequality symbol that represent real-world problem situations.	○	○	○
writing a corresponding real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign or inequality symbol.	○	○	○
solving one-variable equations with variables on both sides of the equal sign.	○	○	○
giving examples of linear equations in one variable that have one solution, no solution, or infinitely many solutions.	○	○	○
solving one-variable inequalities with variables on both sides of the inequality symbol.	○	○	○

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Solving Linear Equations and Inequalities* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 1 SUMMARY

Solving Linear Equations and Inequalities Summary

NEW KEY TERMS

- one solution [una solución]
- no solution [sin solución]
- infinitely many solutions [soluciones infinitas]
- solution set of an inequality

LESSON 1 Equations with Variables on Both Sides

An equation with variables on both sides of the equation can be solved by moving all the variable terms to one side of the equation and all the constants to the other side of the equation.

Ethan

$$\begin{aligned} 5x + 3 &= 2x + 5 \\ -5x \quad &\quad -5x \\ 3 &= -3x + 5 \\ -5 \quad &\quad -5 \\ \frac{-2}{-3} &= \frac{-3x}{-3} \\ \frac{2}{3} &= x \\ x &= \frac{2}{3} \end{aligned}$$

Samuel

$$\begin{aligned} 5x + 3 &= 2x + 5 \\ -2x \quad &\quad -2x \\ 3x + 3 &= 5 \\ -3 \quad &\quad -3 \\ 3x &= 2 \\ x &= \frac{2}{3} \end{aligned}$$

You can use properties of equality to rewrite equations and increase your efficiency with solving equations.

- Factor out a number from both sides.

Consider the equation $3x + 9 = 6x - 30$.

$\frac{3x + 9}{3} = \frac{6x - 30}{3}$ ⟶ Factor out 3 from both sides of the equation.

$$\begin{aligned} x + 3 &= 2x - 10 \\ 3 &= x - 10 \\ 13 &= x \end{aligned}$$

- Multiply both sides of an equation by a power of 10 to remove decimals.

Consider the equation $1.2x + 4.8 = 0.6x - 1.8$.

$$10(1.2x + 4.8) = 10(0.6x - 1.8)$$ → Multiply both sides of the equation by 10.

$$12x + 48 = 6x - 18$$ → Factor out a 6 from both sides.

$$2x + 8 = x - 3$$

$$x + 8 = -3$$

$$x = -11$$

LESSON 2 Analyzing and Solving Linear Equations

A linear equation can have **one solution**, **no solution**, or **infinitely many solutions**.

When the solution to the equation is a true statement with one value equal to the variable, there is only one solution. For example, the equation $x + 2 = 8$ has only one solution: $x = 6$.

When the solution to the equation is a false statement, the equation has no solution.

For example, the equation $x + 0 = x + 1$ has no solution.

Isabella

$$3(6x - 4) = 2(9x + 5)$$
$$18x - 12 = 18x + 10$$
$$18x - 18x - 12 = 18x - 18x + 10$$
$$-12 \neq 10$$

When the solution to the equation is a true statement for any value of the variable, such as $x = x$, the equation has infinitely many solutions.

Isabella

$$6x + 14 = 2(3x + 7)$$
$$6x + 14 = 6x + 14$$
$$6x + (-6x) + 14 = 6x + (-6x) + 14$$
$$14 = 14$$

LESSON 3 Solving Linear Inequalities

Not all situations can be represented using equations. Some scenarios require the use of inequalities.

Consider this example. A local taxi company charges a flat rate of \$2.25 plus \$0.50 per mile. A ride-share company charges a flat rate of \$1.25 plus \$0.75 per mile. The inequality,

$$2.25 + 0.50x < 1.25 + 0.75x$$

represents when the taxi company is the better deal for x number of miles.

The **solution set of an inequality** is the set of all points that make an inequality true.

Solving inequalities with variables on both sides of the inequality symbol is similar to solving equations with variables on both sides of the equal sign, except for the fact that when you are solving an inequality and multiply or divide by a negative value, you must reverse the inequality symbol.

Step 1: Subtract $0.50x$ from both sides.

$2.25 < 1.25 + 0.25x$

Step 2: Subtract 1.25 from both sides.

$1 < 0.25x$

Step 3: Divide both sides by 0.25.

$4 < x$ or $x > 4$

The taxi company is a better deal for greater than 4 miles.

GRADE 8

TOPIC 2

Intersections are important when solving systems of linear equations.

Systems of Linear Equations

1 Point of Intersection of Linear Graphs

OBJECTIVES

- Write a system of equations to represent a problem situation.
- Analyze and solve a system of simultaneous linear equations graphically.
- Interpret the solution to a system of two linear equations in two variables as the point of intersection of two linear graphs and in terms of the original problem's context.
- Determine a point of intersection in a system of linear equations using tables.

NEW KEY TERMS

- point of intersection
- break-even point

You have modeled different linear equations. How can you model two linear equations on the same graph?

What does it mean when two linear graphs intersect?

Getting Started

Long-Sleeved Shirts

Ask Yourself . . .
How can you use what you know about the relationship between the production costs and the money earned from the fundraiser to help you answer the question?

Your school's parent-teacher organization wants to sell long-sleeved shirts as a fundraiser. The business manager found a company that will charge $4 for each long-sleeved shirt and a set-up fee of $160 to create the design that will be placed on each shirt. The chairman of the fundraising committee suggested selling the long-sleeved shirts for $8 each. The organization has asked you to help them analyze the production costs and the amount of money that can be made by this fundraiser.

1. If the shirts are sold for $8 each, how many shirts will the organization need to sell to pay for the production costs? Show your work. Explain the reasoning you used to determine the answer.

ACTIVITY 1.1

Graphing and Interpreting a Point of Intersection

Consider the fundraiser being held by the parent-teacher organization, described in the previous activity. Shirts are sold for $8 each and cost $4 each to make, plus a $160 set-up fee.

1. Write an equation to represent the organization's cost in dollars to buy the long-sleeved shirts. Describe what your variables represent.

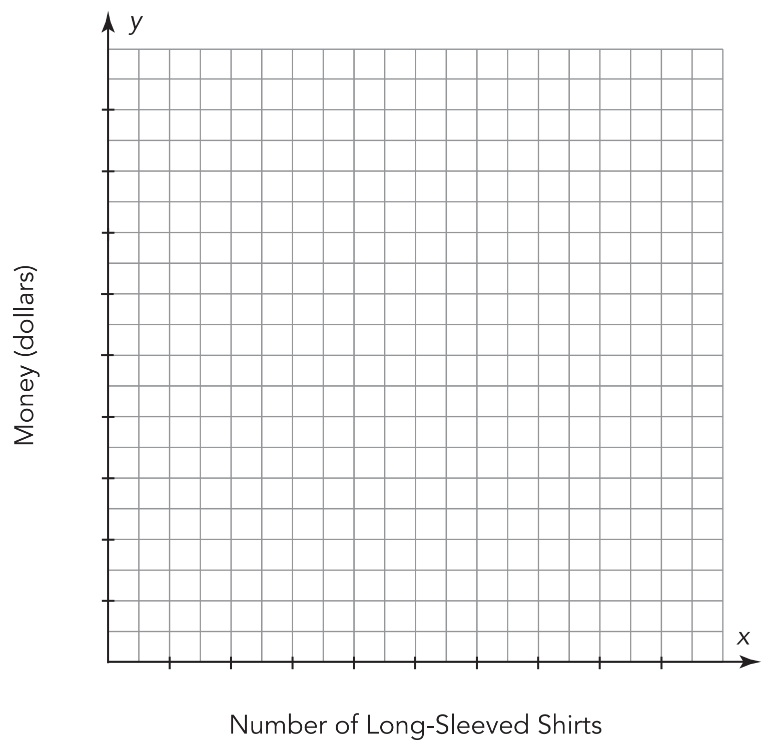
2. Write an equation to represent the organization's income from selling the long-sleeved shirts. Describe what your variables represent.

3. Complete the table to show the cost, income, and profit for different numbers of long-sleeved shirts sold.

	Number of Long-Sleeved Shirts	Income (dollars)	Cost (dollars)	Profit (dollars)
Expression				
	0			
	10			
	20			
	35			
	50			

4. Create graphs to represent the cost and the income on the coordinate plane shown. Use the given bounds and intervals.

Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Long-Sleeved Shirts	0	50	2.5
Money	0	400	20



5. Use your graphs to answer each question and describe your reasoning in terms of the graphs.

   a. Determine the number of long-sleeved shirts for which the cost is greater than the income.

   b. Determine the number of long-sleeved shirts for which the income is greater than the cost.

   c. Determine when the cost is equal to the income.

   d. Verify your solution algebraically.

The **point of intersection** is the point at which two lines cross on a coordinate plane. When one line represents the cost of an item and the other line represents the income from selling the item, the point of intersection is called the **break-even point**.

6. What is the break-even point for making and selling the long-sleeved shirts?

7. What is the profit from shirts at the break-even point?

8. What are the cost and income at the break-even point?

9. What do the coordinates of the point of intersection mean in terms of the fundraiser?

10. State the number of long-sleeved shirts that must be sold for a profit to be made.

ACTIVITY 1.2 A Different Point of Intersection

Ask Yourself . . .
How can you use a system of equations in everyday life?

After the initial analysis, the business manager of the parent-teacher organization called the company that will be producing the shirts. The company agreed to discount the design fee to $80, while maintaining the cost of $4 per shirt. The organization would like you to analyze the profit potential with the new costs and a new selling price of $12 per shirt.

1. Write an equation that represents the cost, in dollars, for the long-sleeved shirts. Describe what your variables represent.

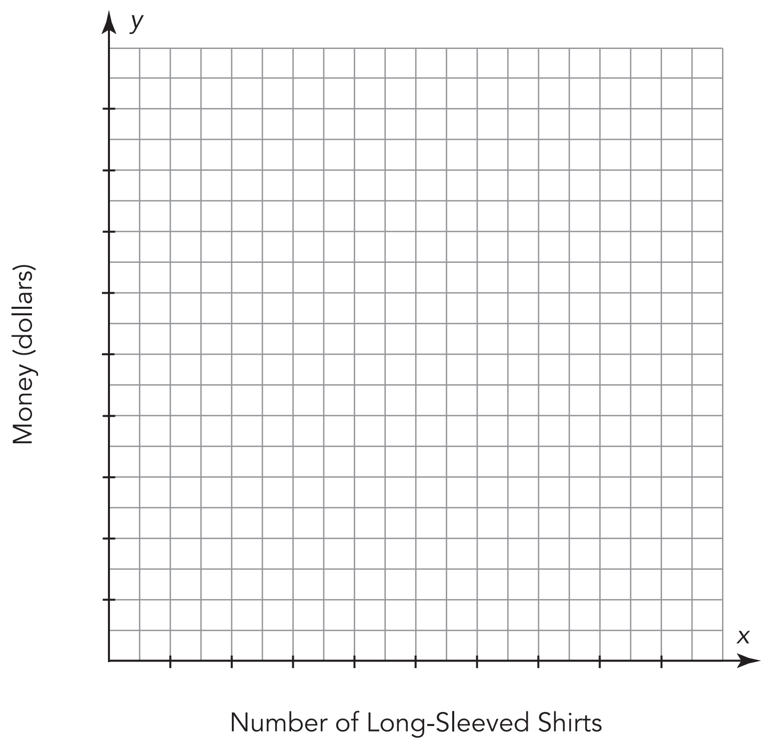
2. Write an equation that represents the organization's income from selling the long-sleeved shirts. Describe what your variables represent.

3. Complete the table to show the cost, income, and profit for different numbers of long-sleeved shirts.

	Number of Long-Sleeved Shirts	Income (dollars)	Cost (dollars)	Profit (dollars)
Expression				
	0			
	5			
	10			
	35			
	50			

4. Create graphs to represent the cost and the income on the coordinate plane below. Use the given bounds and intervals.

Variable Quantity	Lower Bound	Upper Bound	Interval
Number of Long-Sleeved Shirts	0	50	2.5
Money	0	400	20



5. Use your graphs to answer each question and describe your reasoning in terms of the graphs.

   a. Determine the number of long-sleeved shirts for which the cost is greater than the income.

   b. Determine the number of long-sleeved shirts for which the income is greater than the cost.

c. Determine when the cost is equal to the income.

d. Verify your solution algebraically.

6. What is the break-even point for producing and selling the long-sleeved shirts?

7. What is the profit from shirts at the break-even point?

8. What are the production cost and income at the break-even point?

9. What do the coordinates of the point of intersection mean in terms of the fundraiser?

10. State the number of long-sleeved shirts that must be sold for a profit to be made.

ACTIVITY 1.3

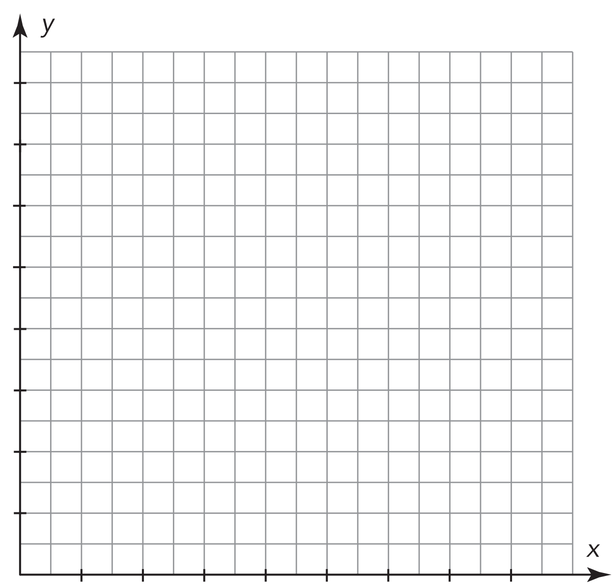
Determining a Point of Intersection to Solve a Problem

Harper is trying to decide which gym to join. The local community center gym charges a $19 membership fee each month and $6 for each obstacle course class. A local independent gym charges a $31 membership fee each month and $4 for each class.

1. Write two equations to represent the the monthly cost for each gym.

2. Interpret what the point of intersection means for the two lines representing the equations.

3. Graph both equations to determine the point of intersection. Explain the meaning of the ordered pair in this situation.



Talk the Talk

Putting It All on the Table

Look back at the tables you used in this lesson.

1. How can you use a table alone to determine the point of intersection of two linear graphs?

2. How does determining a point of intersection from a table compare with determining the point of intersection from a graph and from equations?

Ask Yourself . . .
Did you justify your mathematical reasoning?

Lesson 1 Assignment

Write

In your own words, define the terms *point of intersection* and *break-even point*.

Remember

The point where two linear graphs intersect represents the solution to both of the equations that describe the graphs.

Practice

Sofia is the manager of the local video store. She is in charge of buying the videos for the store to sell. She buys videos from a wholesaler that sells them for $8 each. The wholesaler also charges a fee of $200 for each bulk purchase. Sofia then sells the videos for $12 each.

1. Write an equation to represent the cost to buy videos from the wholesaler. Describe what your variables represent. Write a second equation to represent the amount of money the store will earn from selling the videos. Describe what your variables represent in this equation.

2. Calculate the cost to buy 30 videos from the wholesaler.

Lesson 1 Assignment

3. Calculate the amount of money the store will earn from selling 30 videos.

4. Calculate the profit the store will make from selling 30 videos. Interpret the meaning of your answer.

5. Calculate the cost to buy 70 videos from the wholesaler.

6. Calculate the amount of money the store will earn from selling 70 videos.

7. Calculate the profit the store will make from selling 70 videos. Interpret the meaning of your answer.

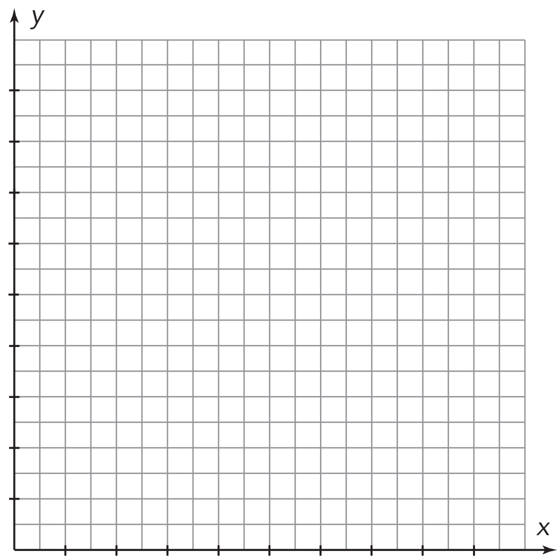
8. Complete the table to show the cost of buying videos from the wholesaler and income for different numbers of videos.

Number of Videos	Income ($)	Cost from Wholesaler ($)
x		
0		
10		
30		
45		
70		
100		

Lesson 1 Assignment

9. Create graphs of the cost and income equations. Use the given bounds and intervals.

Variable Quantity	Lower Bound	Upper Bound	Interval
Videos	0	90	5
Money	0	900	50



10. Use your graphs to determine the number of videos for which the cost to buy the videos is greater than the income from selling them. Explain your reasoning.

Lesson 1 Assignment

11. Use your graphs to determine the number of videos for which the income from selling the videos is greater than the cost to buy them. Explain your reasoning.

12. Determine the break-even point for buying and selling the videos.

13. What is the video store's profit at the break-even point?

14. What is the point of intersection of the two lines you graphed?

15. What do the coordinates of the point of intersection mean in terms of buying and selling videos?

16. Describe the number of videos that must be sold for a profit to be made.

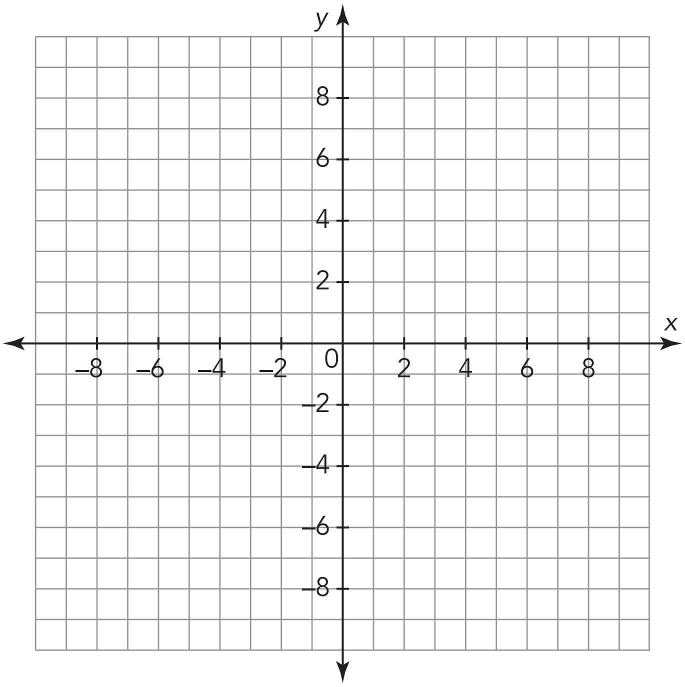
Lesson 1 Assignment

Prepare

1. Graph the equations on the coordinate plane.

   $y = x$

   $y = -x$



2. What are the coordinates of the point of intersection?

3. Interpret the meaning of the point of intersection.

2 Systems of Linear Equations

OBJECTIVES

- Write a system of equations to represent a problem context.
- Analyze a system of two simultaneous linear equations in two variables graphically.
- Interpret the solution to a system of equations in terms of a problem situation.
- Determine whether a system of two linear equations have one solution, no solution, or infinitely many solutions.

NEW KEY TERMS

- system of linear equations
- solution of a linear system
- consistent system
- inconsistent system

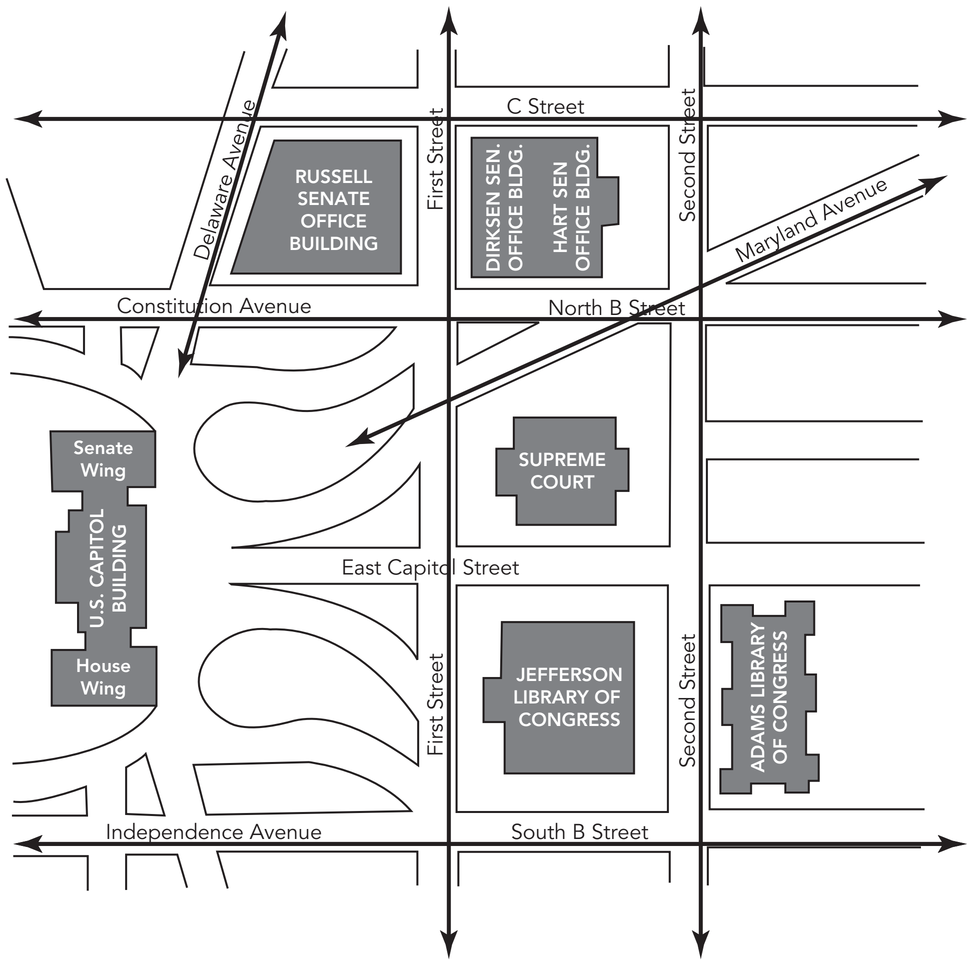
You have graphed linear equations on a coordinate plane.

How can you interpret two linear equations together as a system?

Getting Started

According to the Map

Washington, D.C., has roadways that are named after each state and the territory of Puerto Rico. Many of these roadways are diagonal streets.



1. Answer each question and explain your reasoning according to the map shown.

   a. Would it be possible to meet a friend at the intersection of First Street and Second Street?

b. Would it be possible to meet a friend at the intersection of Delaware Avenue and Constitution Avenue?

c. Would it be possible to meet a friend at the intersection of C Street and Second Street?

2. How many places could you be when you are at the intersection of Independence Avenue and South B Street?

ACTIVITY 2.1

Representing a Problem Situation with a System of Equations

Mia and Diego have part-time jobs after school. Both have decided that they want to see how much money they can save in one semester by placing part of their earnings each week into a savings account. Mia currently has $120 in her account and plans to save $18 each week. Diego currently has $64 in his savings account and plans to save $25 each week.

1. Write an equation for Mia and for Diego that represents the total amount of money in dollars in each of their savings accounts, y, in terms of the number of weeks, x, that they place money in their respective accounts.

2. How much money will each person have in their savings account after five weeks?

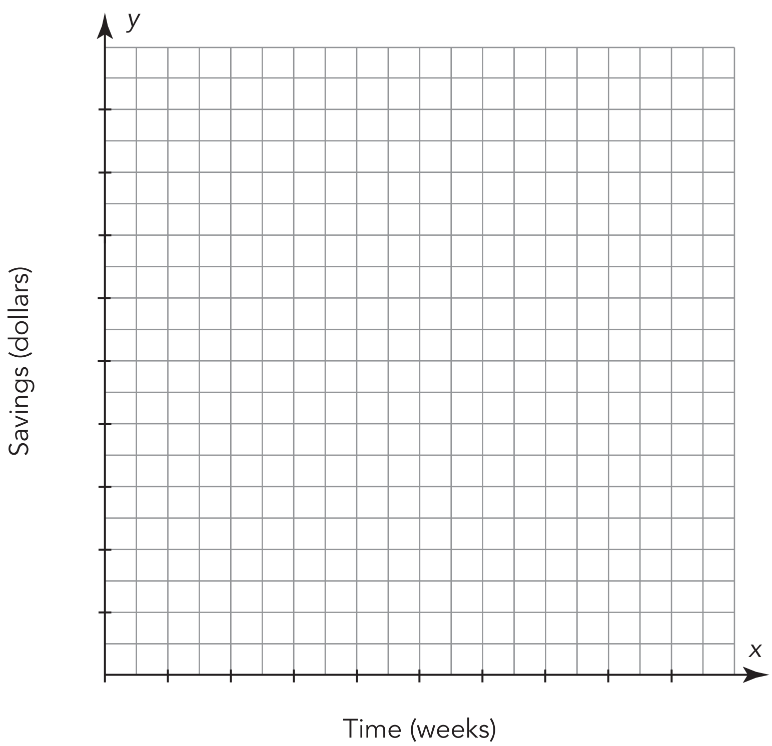
3. Which person will have more money in their savings account after five weeks?

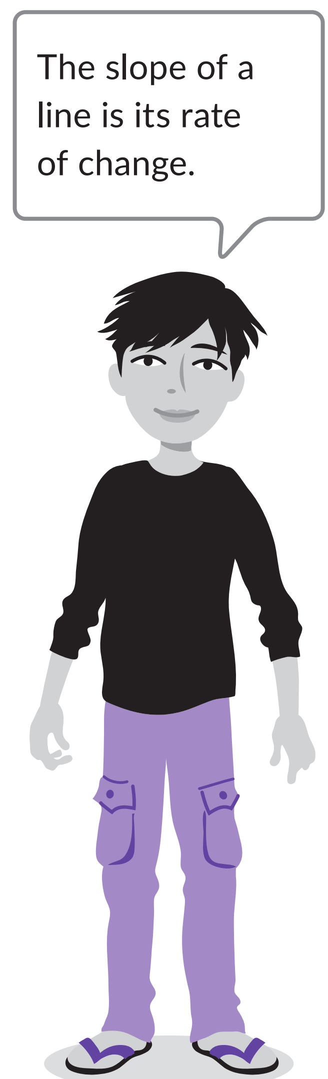
4. How much money will each person have in their savings account after 18 weeks (the amount of time in one semester)?

5. Which person will have more money in their savings account at the end of the semester?

6. Create a graph of each equation on the coordinate plane below. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval





7. Based on your graph, what ordered pair represents the solution to the system? What does this point mean in the context of the problem situation?

8. Verify your solution to Question 7 algebraically.

9. Interpret the meaning of the slope of each graph in this problem situation.

10. Which person is saving more money per week?

11. How can you tell who is saving more money each week by analyzing the graph?

12. Interpret the meaning of the *y*-intercept of each graph in this problem situation.

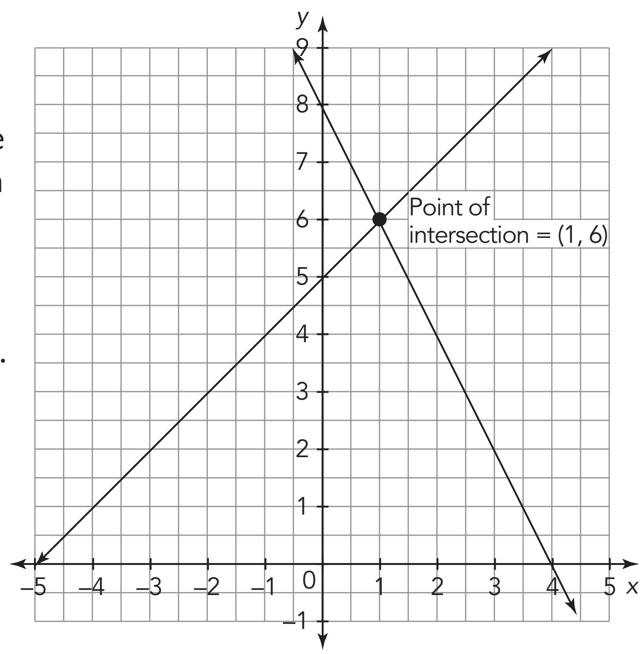
When two or more linear equations define a relationship between quantities, they form a **system of linear equations**. The **solution of a linear system** is an ordered pair (x, y) that is a solution to both equations in the system. Graphically, the solution is the point of intersection, or the point at which two or more lines cross.

WORKED EXAMPLE

A system of linear equations is written:

$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

You can determine the solution to this system by graphing the equations. The point of intersection is the solution to the system.



ACTIVITY 2.2 Identifying Solutions to Systems of Equations

Read each situation. Write a system of equations for the situation. Then, match each situation to its corresponding graph. Glue the graph next to the situation. For each graph, label the *x*- and *y*-axes with the appropriate quantity. Identify and interpret the values that satisfy each system of equations. Finally, algebraically verify the solution to the system of equations.

Ask Yourself . . .
Did you make a plan to solve the problem?

1. Minh and Daniel are training for a marathon together. Minh can currently run 3 miles without stopping and plans to increase her distance by $\frac{1}{2}$ mile each week. Daniel can run 2 miles and aims to increase his distance by $\frac{3}{4}$ mile each week.

   System of Equations

   Solution to the System

   Interpretation of the Solution

   Algebraically Verify

2. Victoria and Juan are learning a new language over the summer. Victoria knows 50 basic words and plans to learn 10 new words per week. Juan knows 30 words and aims to learn 15 new words each week.

   System of Equations

   Solution to the System

   Interpretation of the Solution

   Algebraically Verify

3. Harper and Noah are racing to see who can empty their water container first. Harper's container has 192 fluid ounces of water and Harper can empty 14 fluid ounces each trip. Noah's container has 168 fluid ounces of water and Noah can empty 10 fluid ounces each trip.

System of Equations

Solution to the System

Interpretation of the Solution

Algebraically Verify

4. Mia and Jayden are racing to see who can finish their book first. Mia's book has 300 pages, and she can read 20 pages during each reading session. Jayden's book has 250 pages, and he can read 15 pages during each reading session.

   System of Equations

   Solution to the System

   Interpretation of the Solution

   Algebraically Verify

5. Ava and Mateo are training their pet dogs. Ava's dog can perform 9 tricks and she plans to teach it 1 new trick every week. Mateo's dog can perform 5 tricks and he plans to teach it 2 new tricks each week.

System of Equations

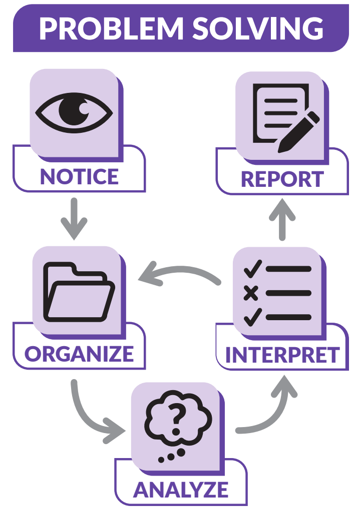
Solution to the System

Interpretation of the Solution

Algebraically Verify

ACTIVITY **2.3**

Systems with No Solution or Infinitely Many Solutions

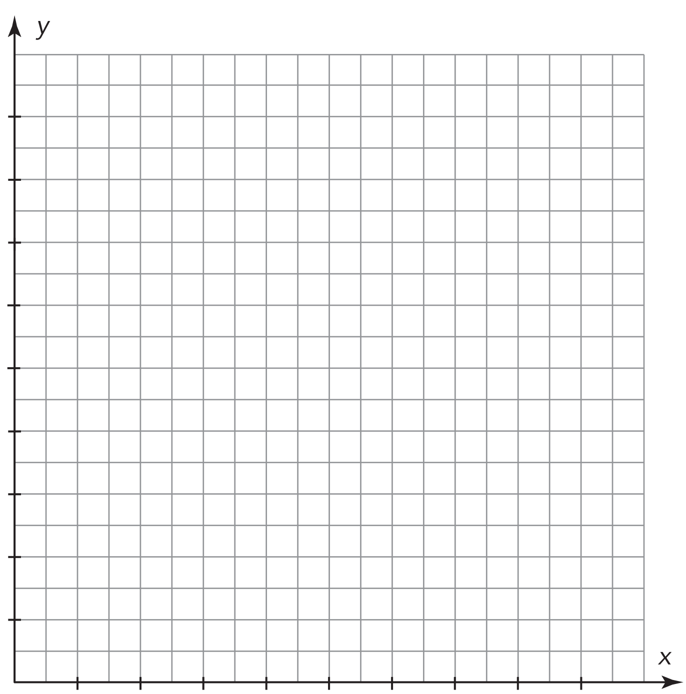


Samuel and Chloe are on a swim team. Samuel can swim 5 laps without stopping. Chloe can swim 7 laps without stopping. Both swimmers plan to increase the number of laps they can swim by 2 laps each week.

1. Write a system of linear equations to represent the total number of laps, y, in terms of the number of weeks, x, that Samuel and Chloe swim.

2. Create a graph of the linear system on the coordinate plane. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval



3. What does the slope of each graph represent in this problem situation?

4. What is the same for both Samuel and Chloe?

5. What is different for Samuel and Chloe?

6. What is the point of intersection for this system of equations? Explain your reasoning in terms of the graph.

Two lines are parallel when they lie in the same plane and do not intersect.

7. Does the linear system of equations for Samuel and Chloe have a solution? Explain your reasoning in terms of the graph.

8. Will Samuel and Chloe ever swim the same number of laps?

9. Consider the system of equations:

$$\begin{cases} y = 3x + 6 \\ y = 3(x + 2) \end{cases}$$

a. Complete the table of values for this linear system.

x	$y = 3x + 6$	$y = 3(x + 2)$
−2		
0		
2		
4		
8		
13		
20		

b. Describe the equations that make up this system. What can you conclude about the number of solutions to this type of linear equation?

A system of equations may have one unique solution, infinitely many solutions, or no solution. Systems that have one or infinitely many solutions are called **consistent systems**. Systems that have no solution are called **inconsistent systems**.

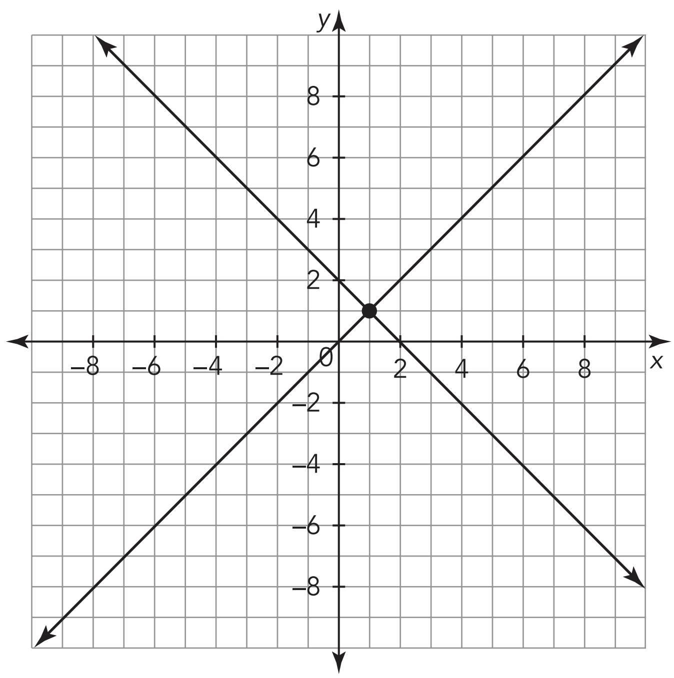
Talk the Talk

Line Up for Inspection!

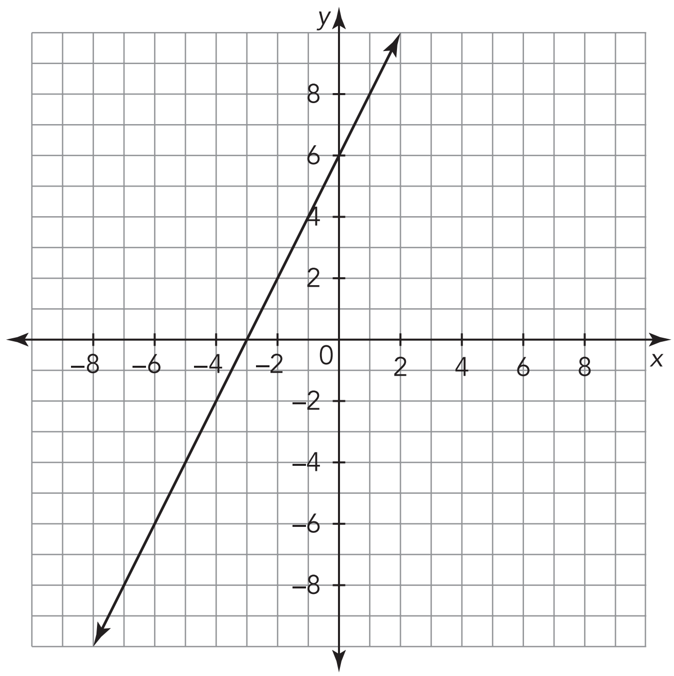
Each graph shows a system of two linear relationships.

Identify whether the system has one solution, no solution, or infinitely many solutions. When the system has one solution, use the graph to determine the solution. Then, algebraically verify the solution.

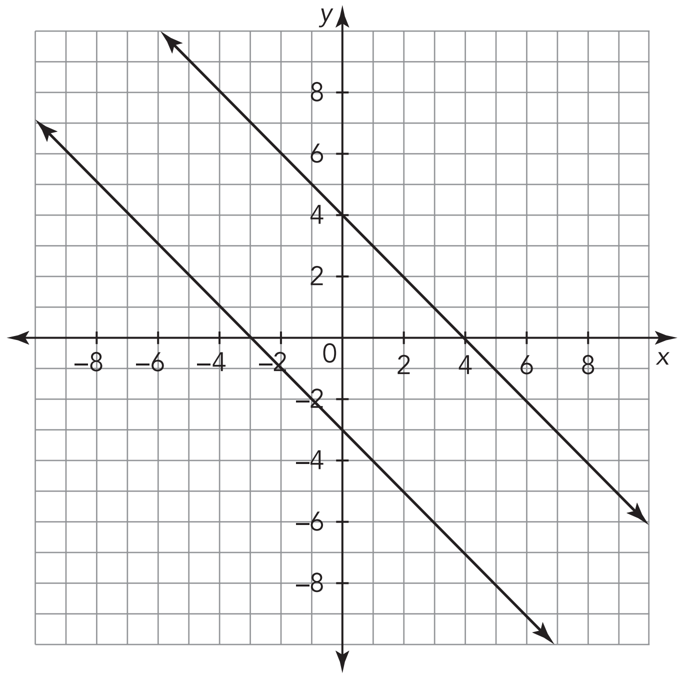
1. $y = x$
   $y = -x + 2$

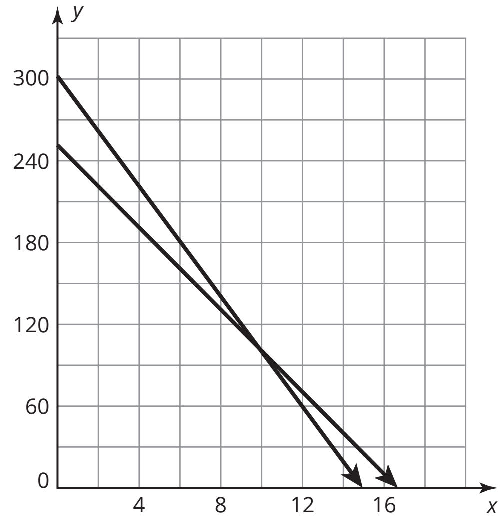


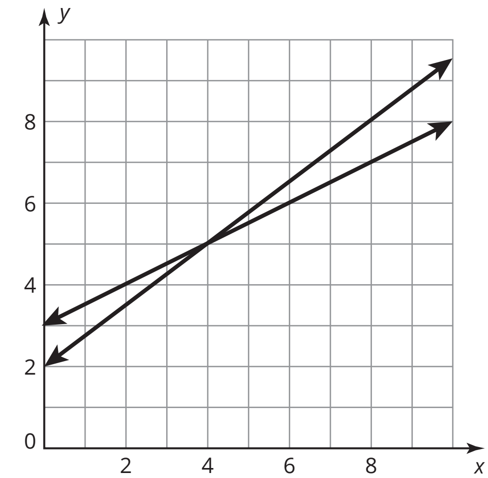
2. $y = 2x + 6$
   $y = 2(x + 3)$

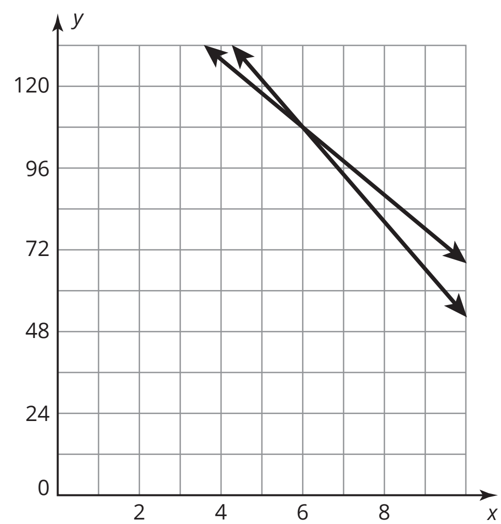


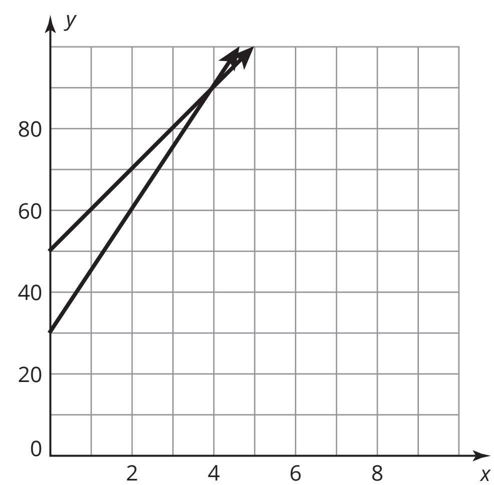
3. $y = -x + 4$
   $y = -x - 3$

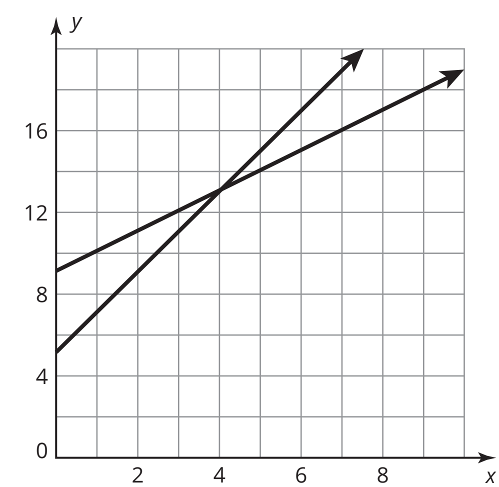



y
300
240
180
120
60
0
4
8
12
16
x


y
8
6
4
2
0
2
4
6
8
x


y
120
96
72
48
24
0
2
4
6
8
x


y
80
60
40
20
0
2
4
6
8
x


y
16
12
8
4
0
2
4
6
8
x

Why is this page blank?

So you can cut out the graphs on the other side

Lesson 2 Assignment

Write

Complete each sentence by writing the correct term or phrase from the lesson.

1. A(n) ______ is formed when the equations or graphs of two or more linear equations define a relationship between quantities.
2. A(n) ______ is an ordered pair (x, y) that is the point of intersection, or the point at which two or more lines cross.
3. A(n) ______ has one or infinitely many solutions.
4. A(n) ______ has no solution.

Remember

- A system of equations in which its graphs intersect at just one point is a system with one solution.
- A system of equations that has parallel line graphs is a system with no solution.
- A system of equations that has identical graphs is a system with infinitely many solutions.

Practice

Aaliyah works in the fish department of a pet store. She is asked to drain, clean, and refill two reef tanks. The first tank holds 175 gallons of water, and the second tank holds 200 gallons of water. The hoses that she uses drain the tanks at a rate of 25 gallons of water per hour.

1. Write an equation for each tank that represents the total amount of water in gallons in the tank, y, in terms of the number of hours, x, that the tanks are draining.

2. How much water is in each tank after 3 hours?

Lesson 2 Assignment

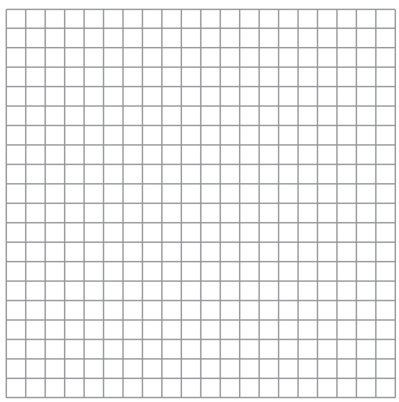
3. Write your equations in the first row of the table. Then, complete the table of values for the linear system.

Number of Hours	First Tank	Second Tank
x		
0		
1		
2		
3		
4		
5		
6		
7		

Lesson 2 Assignment

4. Create a graph of both equations. Choose your bounds and intervals for each quantity.

Variable Quantity	Lower Bound	Upper Bound	Interval



5. Interpret the meaning of the slope of each line in this problem situation.

6. What is the point of intersection for this system of equations? Explain your reasoning in terms of the graph.

7. When will both tanks have the same amount of water?

Lesson 2 Assignment

Prepare

By inspection, determine if each system has no solution, infinitely many solutions, or one solution.

1. $\begin{cases} y = 4x - 14 \\ y = -4x - 14 \end{cases}$

2. $\begin{cases} y = 4x - 14 \\ y = -4x - 14 \end{cases}$

3. $\begin{cases} y = 4x - 14 \\ y = 4x + 14 \end{cases}$

4. $\begin{cases} y = 4x - 14 \\ y = 4x - 14 \end{cases}$

3 Multiple Representations of Systems of Linear Equations

OBJECTIVES

- Use multiple representations to identify solutions to systems of linear equations.
- Interpret the solution of a system of linear equations.

Now that you know how to identify the *x*- and *y*-values that satisfy a system of linear equations using tables and graphs, how do you decide which method to use?

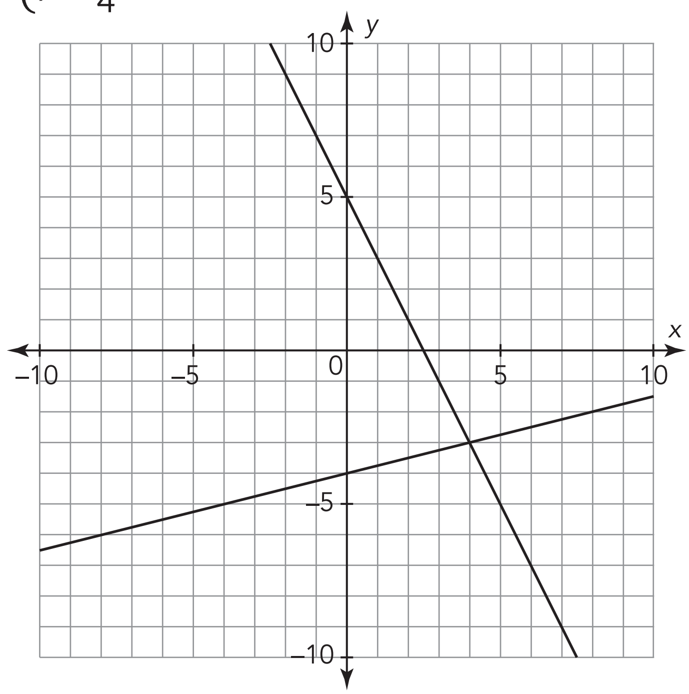
Getting Started

All The Possibilities

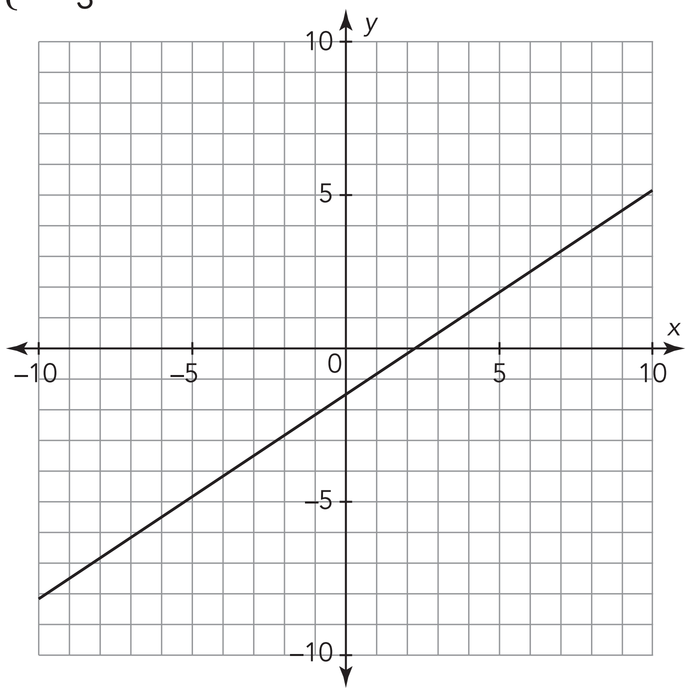
Consider the four graphs.

1. Determine the solution for each of system of linear equations.

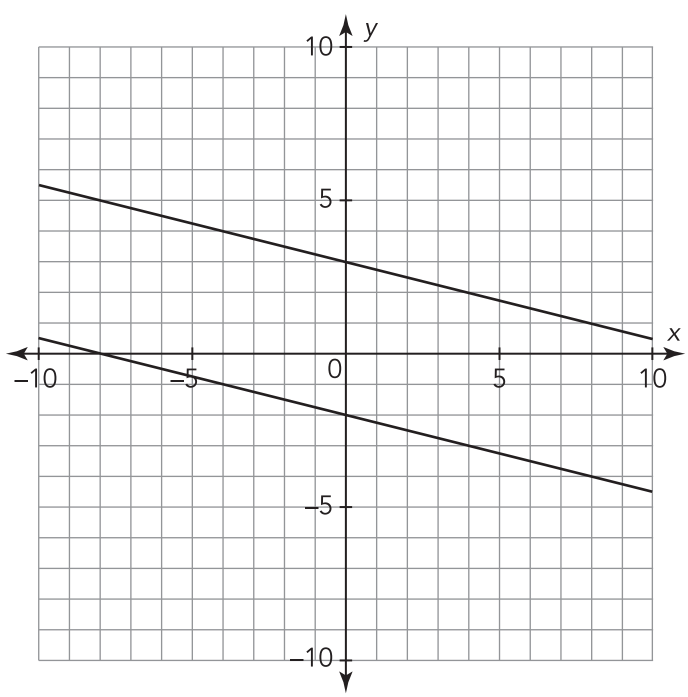
   a. $\begin{cases} y = -2x + 5 \\ y = \frac{1}{4}x - 4 \end{cases}$



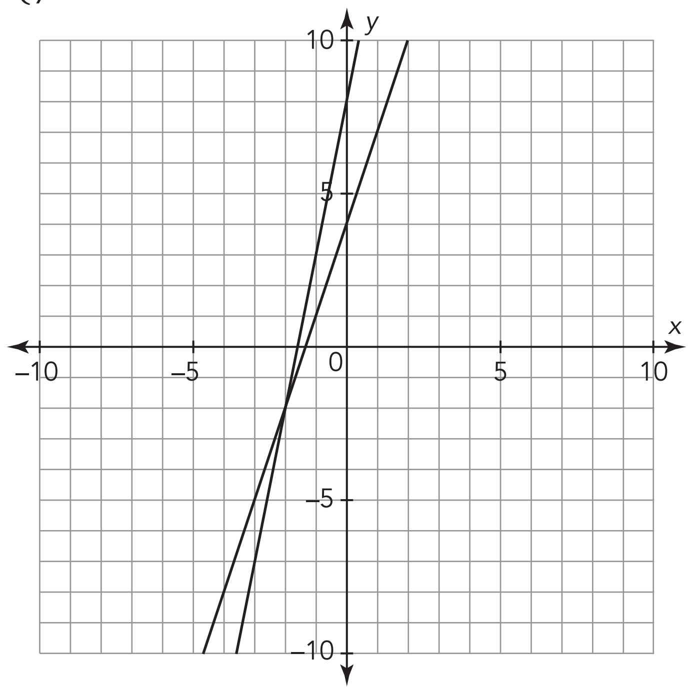
b. $\begin{cases} y = \frac{2}{3}x - 1.5 \\ y = \frac{2}{3}x - 1.5 \end{cases}$



c. $\begin{cases} y = -\frac{1}{4}x + 3 \\ y = -\frac{1}{4}x - 2 \end{cases}$



d. $\begin{cases} y = 3x + 4 \\ y = 5x + 8 \end{cases}$



ACTIVITY 3.1 Comparing Two Fee Schedules

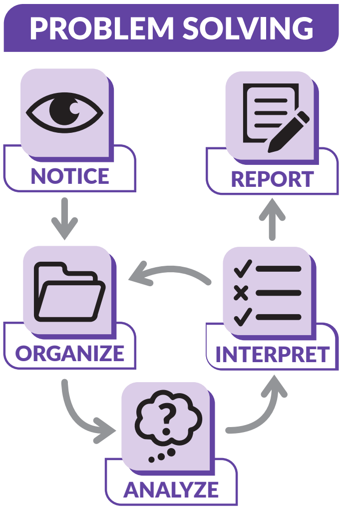
The activities director of the community center is planning a skating event for all the students at the local middle school. There are several skating rinks in the area, but the director does not know which one to use. At a previous event at rink *A*, the director paid a $200 deposit and $3 for each student that attended. For a different event at rink *B*, she paid $5 for each student.

Assume that the skating rinks have not changed their rates for skating parties.

1. Define variables to represent the total cost and the number of students attending the event.

2. Write an equation for the total cost of using rink *A* in terms of the number of students attending.

3. Write an equation for the total cost of using rink *B* in terms of the number of students attending.



4. Suppose the activities director anticipates that 50 students will attend.

   a. Calculate the total cost of using rink *A*.

   b. Calculate the total cost of using rink *B*.

5. Suppose the activities director has $650 to spend on the skating event.

   a. Determine the number of students who can attend when the event is held at rink *A*.

   b. Determine the number of students who can attend when the event is held at rink *B*.

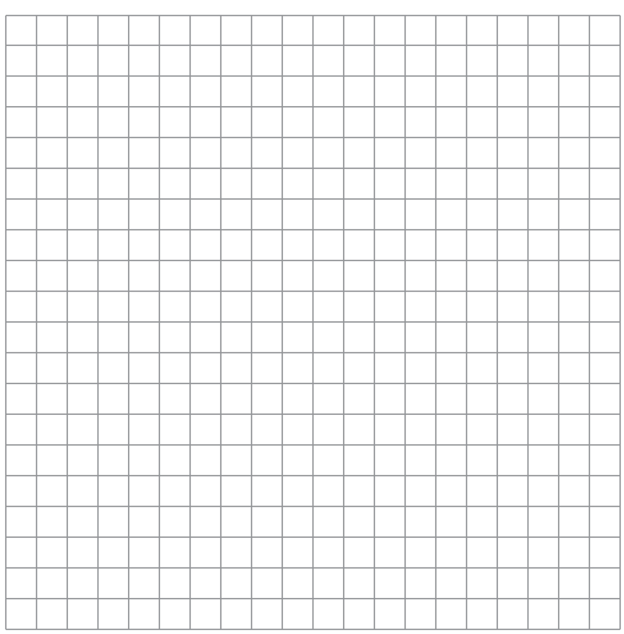
6. Write a system of equations to represent this problem situation.

7. Identify the solution to this system using each strategy. Interpret the meaning of the solution in the context of the problem situation, and verify your solution algebraically.

Table

Number of Students	Rink *A*	Rink *B*

Graph



You can use a variety of strategies and representations to solve a system of linear equations.

- inspection
- table
- graph

Algebraic Verification

8. Which skating rink would you recommend to the activities director? Explain your reasoning.

Ask Yourself . . .
What observations can you make?

Talk the Talk

Advantages and Disadvantages

Complete the table to explain the advantages and disadvantages of using each strategy to solve a system of linear equations.

	Advantages	Disadvantages
Table		
Graph		

Lesson 3 Assignment

Write

Describe how you can algebraically verify the graphical solution to a system of linear equations.

Remember

To most efficiently solve a system of linear equations, look at the equations in the system. The coefficients and constants in the equations can help you choose the best solution method.

Practice

1. A vehicle rental company charges \$40 a day plus \$0.30 per mile for cars and \$50 a day plus \$0.25 per mile for vans.

   a. Write a system of equations that best models the cost of renting a car or van from the rental company. Let x represent the number of miles, and let y represent the cost per day.

   b. Algebraically verify that (200, 100) is the solution to the system of equations.

   c. Interpret the solution of the linear system in terms of the problem situation.

   d. In what situations would you recommend renting a car instead of a van?

Lesson 3 Assignment

2. Liam is trying to decide which car to rent. Car A costs \$50 a day to rent. Car B costs \$25 a day to rent and the company charges a \$175 initial rental fee.

   a. Write a system of equations for this problem situation. Let x represent the number of days Liam rents the car, and let y represent the total cost of the rental.

   b. Algebraically verify that (7, 350) is the solution to the system of equations.

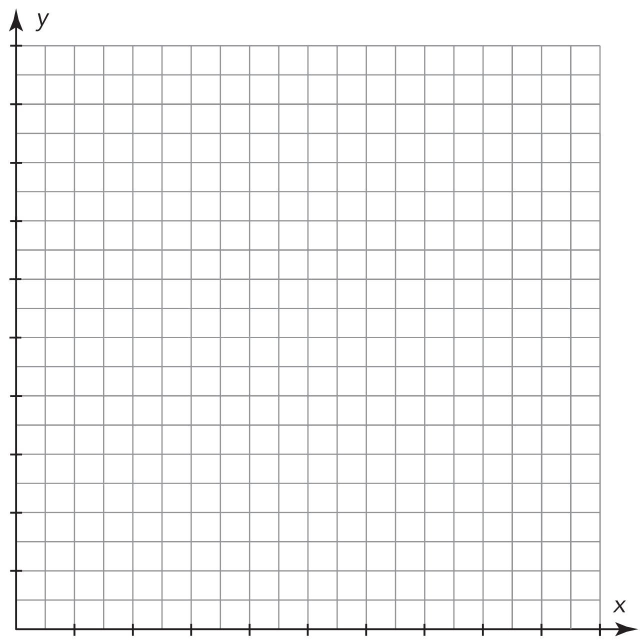
   c. Interpret the solution of the linear system in terms of the problem situation.

Lesson 3 Assignment

3. Ms. Nguyen is the coach of her daughter's team that has 10 members. The team will raise money for amusement park tickets selling coupon booklets. They are deciding between two companies. The first company will donate $50 when the team uses their company, plus the team will make $10 for every booklet that they sell. The second company will donate $110 when the team uses their company, plus the team will make $7 for every booklet that they sell.

   a. Write a system of equations that represents the problem situation. Define your variables.

   b. Use the graph to identify the values that satisfy the system of equations.



Lesson 3 Assignment

c. Interpret the solution of the linear system in terms of the problem situation.

d. Which company would you recommend the team use? Explain.

Prepare

Represent each number as a fraction, decimal, and percent.

1. $\frac{5}{8}$

2. 105%

3. 0.55

Name: ______________________________

Systems of Linear Equations

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Systems of Linear Equations* topic by:

TOPIC 2: *Systems of Linear Equations*	Beginning of Topic	Middle of Topic	End of Topic
solving a system of linear equations by inspection of the equations or by graphing.	○	○	○
identifying or estimating the solution(s), or point(s) of intersection, in a graph of a system of linear equations.	○	○	○
explaining why the point(s) of intersection between two lines is/are the point(s) that satisfy both equations simultaneously.	○	○	○
using slope and y-intercept to determine whether two linear equations have one solution, no solution, or infinitely many solutions.	○	○	○
representing real-world and mathematical problems as systems of linear equations and interpreting the solution in terms of the problem.	○	○	○

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Systems of Linear Equations* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

Systems of Linear Equations Summary

NEW KEY TERMS

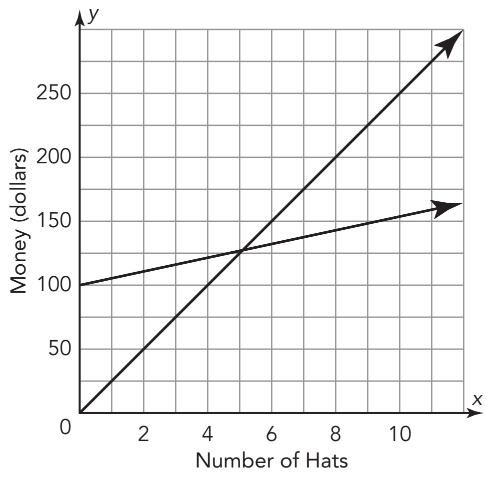
- point of intersection [punto de intersección]
- break-even point
- system of linear equations [sistema de ecuaciones lineales]
- solution of a linear system [solución de un sistema lineal]
- consistent system [sistema consistente]
- inconsistent system [sistema inconsistente]

LESSON 1 Point of Intersection of Linear Graphs

The **point of intersection** is the point at which two lines cross on a coordinate plane. When one line represents the cost of an item and the other line represents the income from selling the item, the point of intersection is called the **break-even point**.

The point where two linear graphs intersect represents the solution to both of the equations.

For example, Isabella plans to sell hats that she knits at the local craft fair. The cost for a booth at the fair is $100, and each hat costs her $5 to make. She plans to sell each hat for $25. Isabella's total cost for selling the hats can be represented by the equation $y = 100 + 5x$, while her income can be represented by the equation $y = 25x$. The graph of each equation is shown. The point of intersection is (5, 125). This point represents the break-even point because it will cost Isabella $125 to make 5 hats, and her income from selling 5 hats is $125.



LESSON 2

Systems of Linear Equations

When two or more linear equations define a relationship between quantities, they form a **system of linear equations**. The **solution of a linear system** is an ordered pair (x, y) that is a solution to both equations in the system. Graphically, the solution is the point of intersection, or the point at which two or more lines cross.

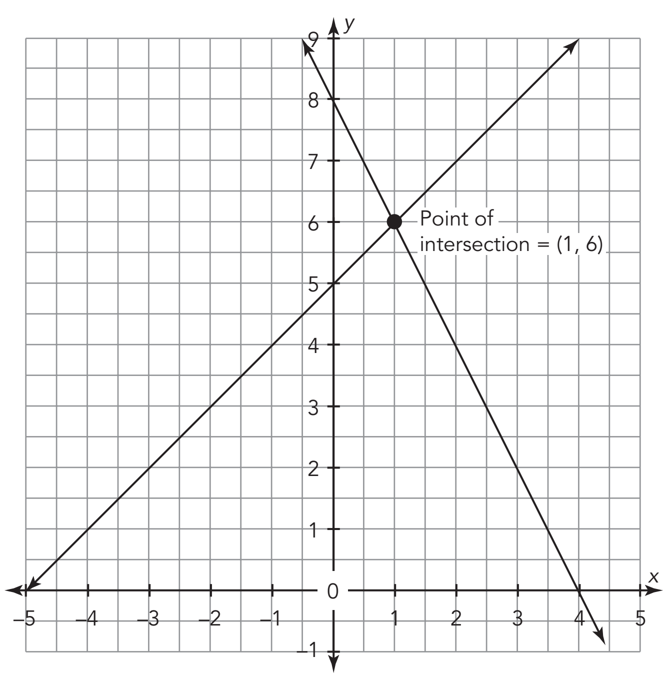
A system of linear equations is written with a brace:

$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

You can determine the solution to this system by graphing the equations. The point of intersection is the solution to the system.

- A system of equations in which its graphs intersect at just one point is a system with one solution.
- A system of equations that has parallel line graphs is a system with no solution. These equations will have equal slopes but different y-intercepts.
- A system of equations that has identical graphs is a system with infinitely many solutions.

A system of equations may have one unique solution, infinitely many solutions, or no solution. A system that has one or infinitely many solutions is called a **consistent system**. A system that has no solution is called an **inconsistent system**.



LESSON 3 Multiple Representations of Systems of Linear Equations

You can identify the values of x and y that satisfy a system of linear equations using a table of values or graph.

Consider the system of equations.

$$\begin{cases} y = 2x + 2 \\ y = x - 1 \end{cases}$$

When you identify the values of x and y that satisfy both equations from a table of values, you need to identify the x-value from the table that results in the same y-value for both equations.

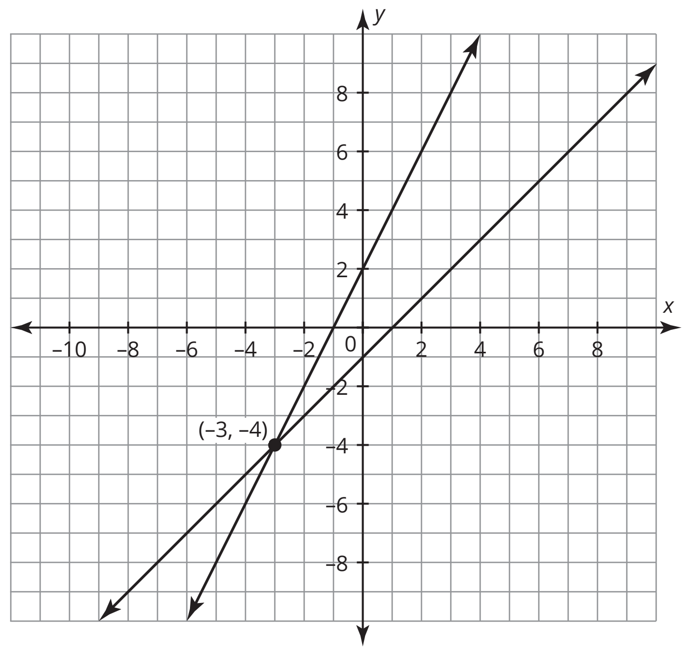
The table shows values for the system $\begin{cases} y = 2x + 2 \\ y = x - 1 \end{cases}$.

x	$y = 2x + 2$	$y = x - 1$
−5	−8	−6
−3	−4	−4
−1	0	−2
1	4	0
3	12	2

Since an x-value of −3 results in the same y-value, −4, the ordered pair (−3, −4) satisfies both equations.

Similarly, the *x*- and *y*- values of the point where the lines intersect on a graph will satisfy both equations.

The graph represents the system $\begin{cases} y = 2x + 2 \\ y = x - 1 \end{cases}$.



The lines intersect at the point (−3, −4). So those values satisfy the system.

While you may have to list multiple values in a table to determine the solution, or you need to estimate the values when using a graph, both strategies provide a visual representation of the values that satisfy the equations.

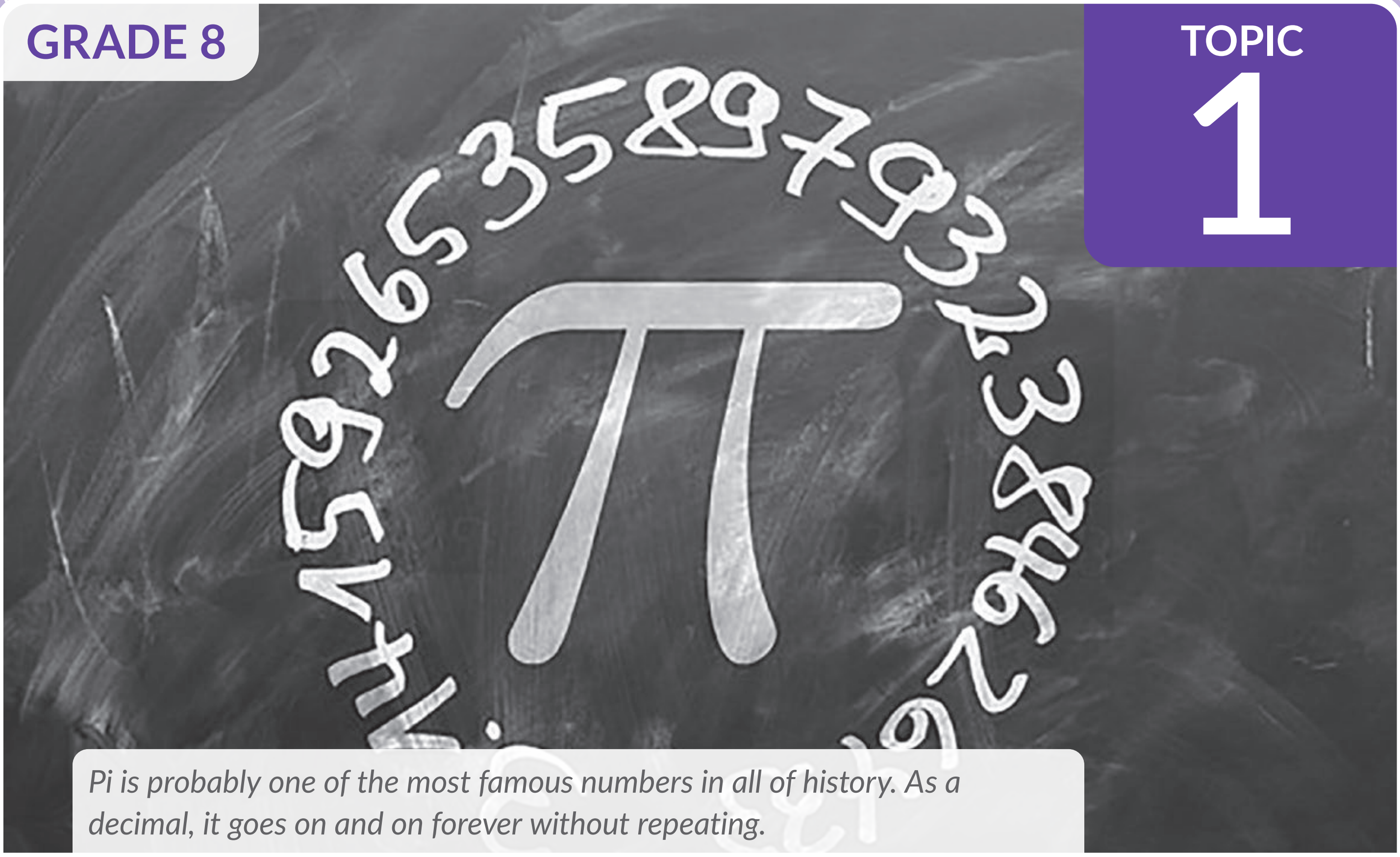
GRADE 8

MODULE
5

Applying Powers

GRADE 8

TOPIC 1



Pi is probably one of the most famous numbers in all of history. As a decimal, it goes on and on forever without repeating.

Real Numbers

1 Sorting Numbers

OBJECTIVES

- Review and analyze numbers.
- Determine similarities and differences among various numbers.
- Sort numbers by their similarities and rationalize the differences between groups of numbers.

You have been using numbers to count and perform calculations for nearly your entire life. Should someone were to ask you to define the word *number*, could you do it?

How can you identify and organize different types of numbers?

Getting Started

Gimme, Gimme, Gimme!

1. List three numbers that are positive numbers.

2. List three numbers that are negative numbers.

3. List three numbers that are between 0 and 1.

4. Give one example of a percent.

5. Give one example of a fraction.

6. Give one example of a mixed number.

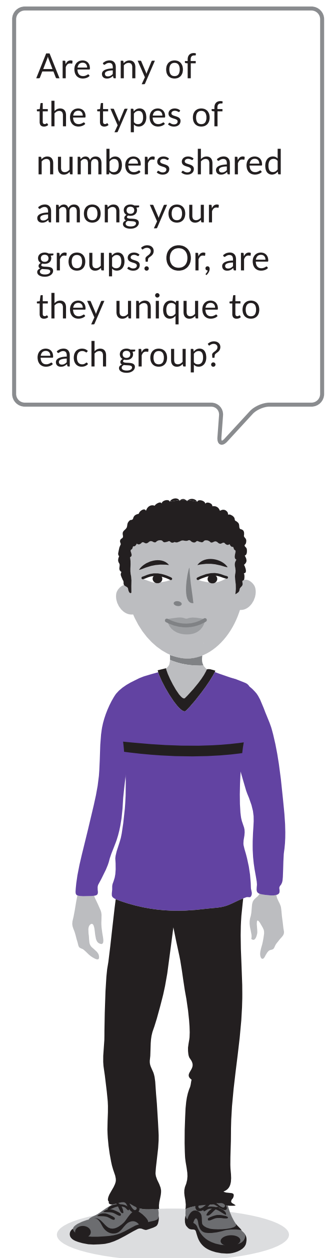
ACTIVITY
1.1 A Number Sort

Searching for patterns and sorting objects into different groups can provide valuable insights.

1. Cut out the 30 number cards located at the end of the lesson. Then, analyze and sort the numbers into different groups. You may group the numbers in any way you feel is appropriate. However, you must sort the numbers into more than one group.

   Record the information for each of your groups.

   - Name each group of numbers.
   - List the numbers in each group.
   - Provide a rationale for why you created each group.



2. Compare your groupings with your classmates' groupings. Create a list of the different types of numbers you noticed.

ACTIVITY 1.2

Taking a Closer Look at the Number Sort

In this activity, you will analyze the ways in which other students grouped the numbers and their rationale.

1. Nahimana grouped these numbers together.

   $0.\overline{91}$, $-\frac{2}{3}$, $\frac{100}{11}$, 1.523232323..., $-0.\overline{3}$

   Why do you think Nahimana put these numbers in the same group?

2. Sebastian and Jasmine provided the same rationale for one of their groups of numbers. However, the numbers in their groups were different.

   Sebastian

   $|-3|$, $\sqrt{100}$, 627,513, 3.21×10^{12}, 4^2, $|2|$

   When I simplify each number, it is a positive integer.

   Jasmine

   20% $\sqrt{100}$, 627,513, 3.21×10^{12}, 4^2, $|2|$, 212%

   Each of these numbers represents a positive integer.

   Who is correct? Explain your reasoning.

You read $\sqrt{100}$ as the square root of 100. You read $|2|$ as the absolute value of 2.

3. Diego grouped these numbers together.

$-\frac{3}{8}$, -101, -6.41, $-\frac{2}{3}$, $-\sqrt{9}$, -1, $-0.\overline{3}$

What rationale could Diego provide?

4. Gabriel grouped all the numbers between 0 and 1. Identify all of the numbers that satisfy Gabriel's reasoning.

5. Luna grouped these numbers together.

-6.41, $\frac{100}{11}$, $1.523232323\ldots$, 212%, $6\frac{1}{4}$

What could Luna name the group? Explain your reasoning.

Talk the Talk

Match 'Em Up

Match each group of numbers with the appropriate group name. Explain your reasoning for each.

1. $1.5, \frac{5}{3}, -212.2, 16.12, -\frac{6}{5}$

2. $-\frac{6}{3}$ $-200, -0.5, -50.313..., -1$

3. $20\%, 3.5\%, -0.005, \frac{1}{5}, -\frac{1}{2}$

4. $-10, 50, 2100, 10^2, 5^3, 400\%, 0$

A. Negative numbers

B. Integers

C. Improper fractions

D. Numbers between −1 and 1

π	0.25	$-\frac{3}{8}$
-101	20%	$\lvert -3 \rvert$
-6.41	$0.\overline{91}$	$\sqrt{100}$
627,513	0.001	$-\frac{2}{3}$
0	$\sqrt{2}$	3.21×10^{12}
1,000,872.0245	4^2	0.5%
$-\sqrt{9}$	$\lvert 2 \rvert$	$\frac{100}{11}$
$-\sqrt{2}$	1.523232323...	-1
$-0.\overline{3}$	1.0205×10^{-23}	$\sqrt{\frac{9}{16}}$
212%	$6\frac{1}{4}$	$\sqrt{0.25}$

Why is this page blank?

So you can cut out the numbers on the other side

Lesson 1 Assignment

Write

Describe the characteristics that you look for in numbers when grouping them.

Remember

Numbers can be grouped in a variety of ways according to their characteristics. Sometimes, a number may fit into multiple groupings.

Practice

Kayla's teacher gives her students the list of numbers to sort.

$$5\%,\ -3^3,\ \sqrt{0.36},\ 2.14,\ |-6|,\ \frac{12}{18},\ 15,\ \sqrt{4},\ -\frac{16}{3},\ \pi,\ \sqrt{\frac{4}{10}},\ 8003.876,\ 0.2\%,\ -\sqrt{25},\ 3\frac{1}{3}$$

1. Kayla groups the following numbers together with the rationale that they are all repeating decimals.

$$\frac{12}{18},\ -\frac{16}{3},\ 3\frac{1}{3},\ \sqrt{\frac{4}{10}}$$

Do you agree with Kayla's grouping? Explain your reasoning.

2. Kayla groups the following numbers together with the rationale that they are all positive numbers.

$$5\%,\ \sqrt{0.36},\ 2.14,\ |-6|,\ \frac{12}{18},\ 15,\ \sqrt{4},\ \pi,\ \sqrt{\frac{4}{10}},\ 8003.876,\ 0.2\%,\ 3\frac{1}{3}$$

Do you agree with Kayla's grouping? Explain your reasoning.

3. Kayla groups the following numbers together with the rationale they are all rational numbers.

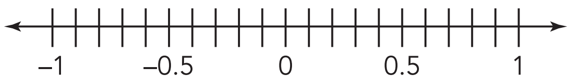
$$\frac{12}{18},\ 15,\ \sqrt{4},\ -\frac{16}{3},\ 3\frac{1}{3}$$

Do you agree with Kayla's grouping? Explain your reasoning.

Lesson 1 Assignment

Prepare

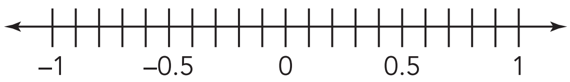
1. Place $\frac{5}{8}$ and $\frac{6}{25}$ on the number line.



2. Rewrite $\frac{5}{8}$ as a decimal.

3. Rewrite $\frac{6}{25}$ as a decimal.

4. Place the decimal equivalents of $\frac{5}{8}$ and $\frac{6}{25}$ on the number line.



5. Are your strategies to plot fractions the same or different from your strategies to plot decimals?

2 Rational and Irrational Numbers

OBJECTIVES

- Rewrite rational numbers as either terminating or repeating decimals.
- Write repeating decimals as fractions.
- Identify numbers that are not rational as irrational numbers.

NEW KEY TERMS

- natural numbers
- whole numbers
- integers
- rational numbers
- irrational numbers
- terminating decimal
- repeating decimal
- bar notation

You have learned about rational numbers.

How are they different from other number sets?

Getting Started

A Science Experiment

Because paper is typically sold in 500-sheet quantities, a paper's weight is determined by the weight of 500 sheets of the paper.

A science class is conducting an experiment to see how the weight of a paper airplane affects the distance that it can fly. The class is divided into two groups. Each group measured and recorded the distance the airplane flew in feet. The results of the experiment are shown in the table.

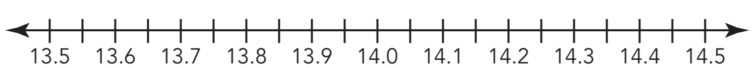
Type of Paper	Group 1 Measurements	Group 2 Measurements
20-pound paper	$13\frac{7}{8}$ feet	13.9 feet
28-pound paper	$14\frac{3}{8}$ feet	14.4 feet

1. The science class needs to compare the measurements between the two groups for each type of paper.

   a. Write $13\frac{7}{8}$ as a decimal.

   b. Write $14\frac{3}{8}$ as a decimal.

2. Graph Group 1's and Group 2's measurements on the number line shown.



3. Use the number line to determine which group's paper airplane traveled farther for the 20-pound paper and for the 28-pound paper. Explain your reasoning.

ACTIVITY 2.1

Natural Numbers, Whole Numbers, and Integers

The first set of numbers you learned when you were very young was the set of *counting numbers*, or *natural numbers*. **Natural numbers** consist of the numbers that you use to count objects: {1, 2, 3, . . .}.

1. Consider the set of natural numbers.

   a. Why do you think this set of numbers is sometimes referred to as the set of counting numbers?

   b. How many natural numbers are there?

   c. Does it make sense to ask which natural number is the greatest? Explain why or why not.

You have also used the set of *whole numbers*. **Whole numbers** are the natural numbers and the number zero, the additive identity.

2. Why is zero the additive identity?

3. Explain why having zero makes the set of whole numbers more useful than the set of natural numbers.

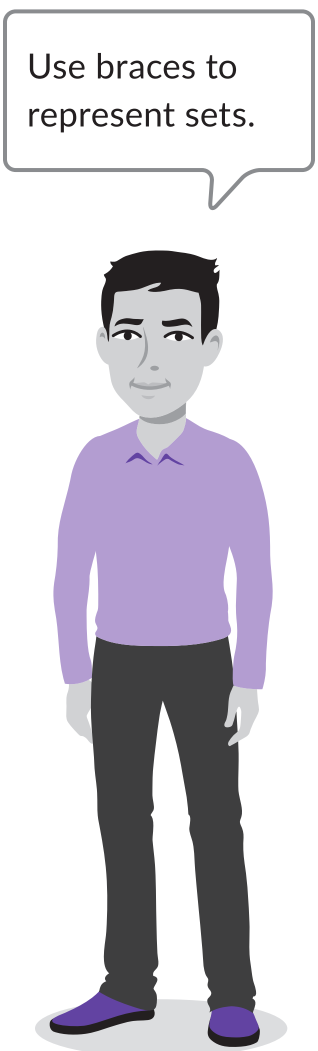
Another set of numbers is the set of **integers**, which is a set that includes all of the whole numbers and their additive inverses.

4. What is the additive inverse of a number?

5. Represent the set of integers. Use set notation and remember to use three dots to show that the numbers go on without end in both directions.

A set of numbers must include an identity to also have the related inverse.

6. Does it make sense to ask which integer is the least or which integer is the greatest? Explain why or why not.



In previous courses, you have learned about the additive inverse, the multiplicative inverse, the additive identity, and the multiplicative identity.

7. Consider each set of numbers and determine whether the set has an additive identity, additive inverse, multiplicative identity, or a multiplicative inverse. Explain your reasoning for each.

    a. The set of natural numbers

    b. The set of whole numbers

    c. The set of integers

ACTIVITY
2.2 Rational Numbers

New number systems arise out of a need to create new types of numbers. When you divide two integers, what type of number have you created? You've created a *rational number*.

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where a and b are both integers and b is not equal to 0.

1. Consider the relationship between each set of numbers. Answer each question and provide an example to support your response.

   a. Does the set of rational numbers include the set of whole numbers?

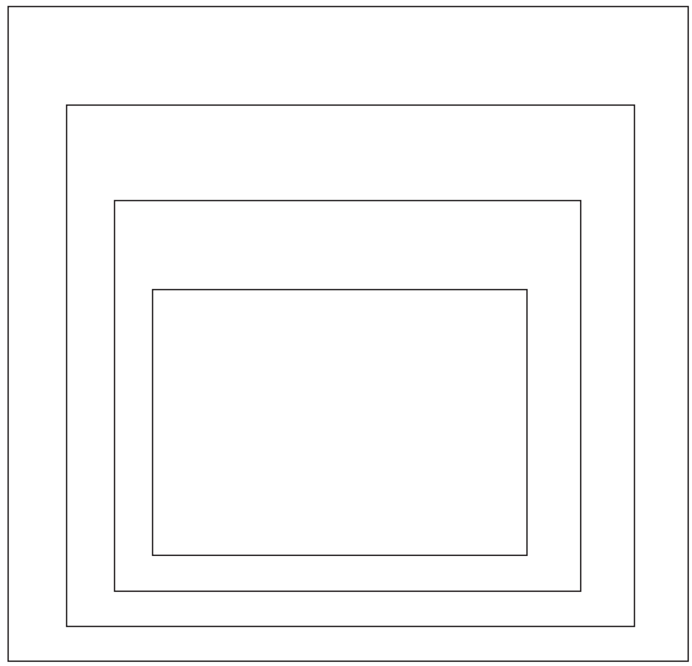
   b. Does the set of rational numbers include the set of integers?

   c. Does the set of rational numbers include all fractions?

   d. Does the set of rational numbers include all decimals?

You can use a Venn diagram to represent how the sets are related.

2. Complete the Venn diagram using the sets of numbers in the word bank.



WORD BANK

Integers
Natural Numbers
Rational Numbers
Whole Numbers

3. Determine whether the set of rational numbers contains the identity or inverse given. Provide an example to support your response.

a. Additive identity

b. Multiplicative identity

c. Additive inverse

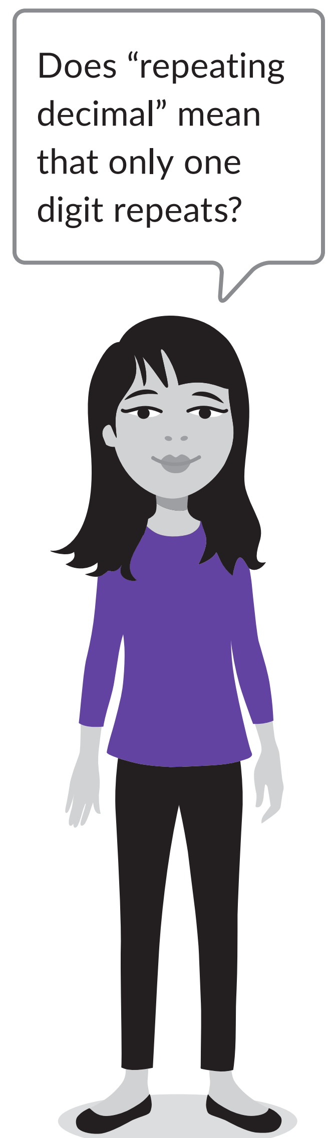
d. Multiplicative inverse

ACTIVITY 2.3

Converting Repeating Decimals to Fractions

You have seen some numbers, such as pi (π), that are not rational numbers. Recall that π is the ratio of the circumference of a circle to its diameter. That is pi $= \dfrac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \dfrac{c}{d}$, where *c* is the circumference of the circle and *d* is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. π is often approximated as 3.14 and $\dfrac{22}{7}$, but that does not mean that π is rational. These approximations are used to help us in determining rounded solutions.

There are other numbers that are not rational numbers. For example, $\sqrt{2}$ and $\sqrt{5}$, which are called *square roots*, cannot be written in the form $\dfrac{a}{b}$, where *a* and *b* are both integers. As you will see in the next lesson, even though you often approximate square roots using a decimal, most square roots are *irrational numbers*. Because all rational numbers can be written as $\dfrac{a}{b}$, where *a* and *b* are integers, they can be written as *terminating decimals* (e.g., $\dfrac{1}{4} = 0.25$) or *repeating decimals* (e.g., $\dfrac{1}{6} = 0.1666\ldots$).



All other decimals are **irrational numbers** because these decimals cannot be written as fractions in the form $\dfrac{a}{b}$, where *a* and *b* are integers and *b* is not equal to 0.

1. Convert the fraction to a decimal by dividing the numerator by the denominator. Continue to divide until you see a pattern. Describe the pattern.

   $\dfrac{1}{3} = 3\overline{)1}$

2. Order the fractions from least to greatest.

$\frac{5}{6}$ $\frac{2}{9}$ $\frac{3}{22}$ $\frac{9}{11}$

3. Convert each fraction to a decimal by dividing the numerator by the denominator. Continue to divide until you see a pattern.

a. $\frac{5}{6} = 6\overline{)5}$

b. $\frac{2}{9} = 9\overline{)2}$

c. $\frac{9}{11} = 11\overline{)9}$

d. $\frac{3}{22} = 22\overline{)3}$

4. Explain why these decimal representations are called *repeating decimals*.

A **terminating decimal** is a decimal that has a finite number of non-zero digits. For instance, the decimal 0.125 is a terminating decimal because it has three non-zero digits. 0.125 is the decimal equivalent of $\frac{1}{8}$ because 1 divided by 8 is equal to 0.125.

A **repeating decimal** is a decimal with digits that repeat in sets of one or more. You can use two different notations to represent repeating decimals. One notation shows one set of digits that repeats with a bar over the repeating digits. This is called **bar notation**.

$$\frac{1}{3} = 0.\overline{3} \qquad \frac{7}{22} = 0.3\overline{18}$$

Another notation shows two sets of the digits that repeat with dots to indicate a non-terminating decimal. You saw these dots when describing the number sets in the previous lesson.

$$\frac{1}{3} = 0.33\ldots \qquad \frac{7}{22} = 0.31818\ldots$$

5. Write each repeating decimal from Question 2 using both notations.

   a. $\frac{5}{6}$

   b. $\frac{2}{9}$

   c. $\frac{9}{11}$

   d. $\frac{3}{22}$

Some repeating decimals represent common fractions such as $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{1}{6}$, and are used often enough that you can recognize the fraction by its decimal representation. For most repeating decimals, however, you cannot recognize the fraction that the decimal represents. For example, can you tell which fraction is represented by the repeating decimal 0.44... or $0.\overline{09}$? In these cases, you need a method for converting from a repeating decimal to a fraction.

WORKED EXAMPLE

You can use algebra to determine the fraction that is represented by the repeating decimal 0.44 . . . First, write an equation by setting the decimal equal to a variable that will represent the fraction.

$$w = 0.44 \ldots$$

Next, write another equation by multiplying both sides of the equation by a power of 10. The exponent on the power of 10 is equal to the number of decimal places until the decimal begins to repeat. In this case, the decimal begins repeating after 1 decimal place, so the exponent on the power of 10 is 1. Because $10^1 = 10$, multiply both sides of the equation by 10.

$$10w = 4.44 \ldots$$

Then, subtract the original equation from the new equation.

$$\begin{aligned} 10w &= 4.44 \ldots \\ -w &= 0.44 \ldots \\ \hline 9w &= 4 \end{aligned}$$

Finally, solve the equation by dividing both sides by 9.

How would the method be different when the second equation is subtracted from the first?

6. Identify the fraction represented by the repeating decimal 0.44 . . .

7. Use this method to write the fraction that represents each repeating decimal.

   a. 0.55 . . .

   b. 0.0505 . . .

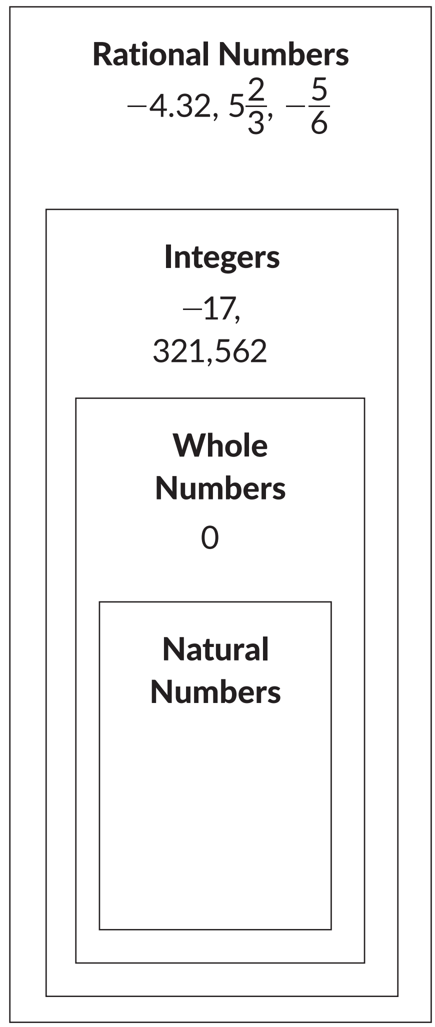
   c. $0.\overline{12}$

   d. $0.\overline{36}$

Talk the Talk

Seeing the Relationships

1. Consider the Venn diagram. Which numbers are not placed in the most appropriate set within the diagram? Explain your reasoning.



2. Convert each fraction to a decimal. State whether the fraction is terminating or repeating decimal.

   a. $1\frac{3}{8}$

   b. $\frac{7}{12}$

3. Write the repeating decimal 0.0555... as a fraction.

Lesson 2 Assignment

Write

Match each term with the number that best represents that term.

1. Irrational number	a. $\frac{1}{2} = 0.5$
2. Terminating decimal	b. $0.\overline{3}$
3. Repeating decimal	c. π
4. Bar notation	d. $\frac{5}{9} = 0.555\ldots$

Remember

All rational numbers can be written as terminating or repeating decimals. A *repeating decimal* is a decimal in which one or more digits repeat indefinitely. A *terminating decimal* is a decimal that has a finite number of non-zero digits.

Practice

1. Adriana is the manager of her high school softball team. She is in charge of equipment as well as recording statistics for each player on the team. The table shows some batting statistics for the four infielders on the team during the first eight games of the season.

Player	At Bats	Hits
Angelina	36	16
Mei	32	12
Valentina	33	11
Emily	35	14

Lesson 2 Assignment

a. In order to compare the batting averages of the players, Adriana must convert all of the ratios of hits to at-bats to decimal form. Determine the batting average for each player and continue to divide until you see a pattern. Write your answers using both dots and bar notation for repeating decimals.

b. Write the batting averages of the players in order from lowest to highest. Who has the best batting average so far?

c. Adriana keeps track of how many home runs each infielder hits on the high school softball team. For each player, the fraction of home runs per at-bats is given in decimal form. Determine how many home runs each player has had so far.

- Angelina: $0.0\overline{5}$
- Mei: 0.15625
- Valentina:$0.\overline{12}$
- Emily: 0.2

Lesson 2 Assignment

2. Tell whether the numbers in each problem are natural numbers, whole numbers, integers, or rational numbers.

   a. $-12 \div (-5)$

   b. $\frac{3}{7} + \left(-\frac{3}{8}\right)$

3. Convert each fraction to a decimal. State whether the fraction is equivalent to a terminating or repeating decimal.

   a. $1\frac{2}{5}$

   b. $\frac{5}{12}$

   c. $\frac{5}{8}$

   d. $\frac{8}{11}$

4. Write each repeating decimal as a fraction.

   a. $0.\overline{8}$

   b. 0.5454 . . .

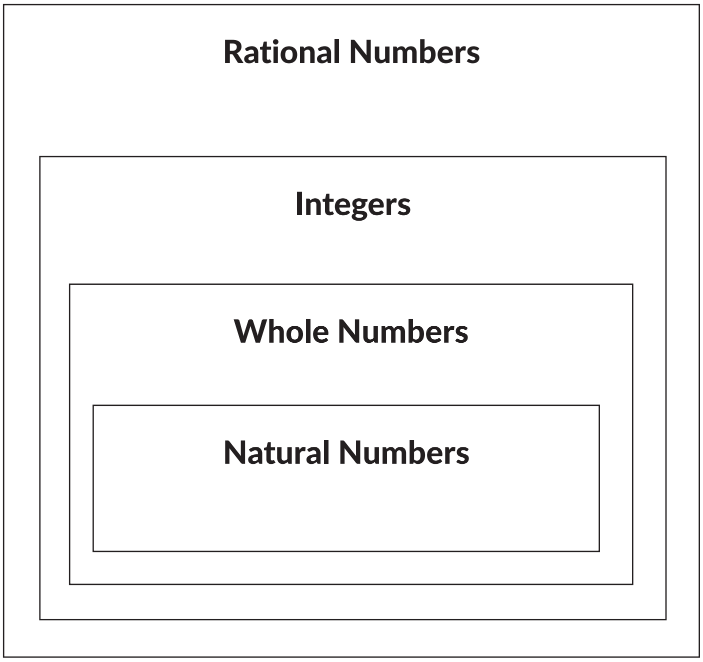
   c. 0.0777 . . .

   d. $0.\overline{185}$

Lesson 2 Assignment

5. Write each number in the most appropriate section of the Venn diagram.

   $\frac{13}{7}$, -17, 22, 15.64, $-3\frac{1}{4}$, $\frac{27}{3}$



Prepare

Rewrite each fraction as a decimal.

1. $\frac{1}{2}$

2. $\frac{1}{4}$

3. $\frac{1}{3}$

4. $\frac{1}{9}$

5. How are the decimals of the first two fractions different from the decimals of the second two fractions?

3 The Real Numbers

OBJECTIVES

- Identify irrational numbers.
- Identify the square roots of numbers that are not perfect squares as irrational numbers.
- Use rational approximations of irrational numbers to compare the size of irrational numbers.
- Locate irrational numbers on a number line and estimate their values.
- Classify numbers within the set of real numbers.

NEW KEY TERMS

- square root
- radical
- radicand
- perfect square
- real numbers

You have learned about rational numbers and about irrational numbers. In this lesson, you will learn about some special irrationals. Putting the set of rational numbers and the set of irrational numbers together forms the set of real numbers.

What is the relationship among the different sets of numbers within the set of real numbers?

Getting Started

An Irrational Assignment?

1. Write an irrational number.

2. Have a classmate verify whether or not your number is an irrational number. Explain why your number is or is not an irrational number.

ACTIVITY
3.1 Understanding Square Roots

A **square root** is one of two equal factors of a given number. Every positive number has two square roots: a positive square root and a negative square root.

For instance, 5 is a square root of 25 because $(5)(5) = 25$. Also, -5 is a square root of 25 because $(-5)(-5) = 25$. The positive square root is called the *principal square root*. In this course, you will only use the principal square root.

The symbol $\sqrt{}$ is called a **radical** and it is used to indicate square roots. The **radicand** is the quantity under a radical.

radical → $\sqrt{25}$ ← radicand

This is read as "the square root of 25" or as "radical 25."

A **perfect square** is a number that is equal to the product of an integer multiplied by itself. In the example above, 25 is a perfect square because it is equal to the product of 5 multiplied by itself.

1. Write the square root for each perfect square.

a. $\sqrt{1} =$ ______ b. $\sqrt{4} =$ ______ c. $\sqrt{9} =$ ______

d. $\sqrt{16} =$ ______ e. $\sqrt{25} =$ ______ f. $\sqrt{36} =$ ______

g. $\sqrt{49} =$ ______ h. $\sqrt{64} =$ ______ i. $\sqrt{81} =$ ______

j. $\sqrt{100} =$ ______ k. $\sqrt{121} =$ ______ l. $\sqrt{144} =$ ______

m. $\sqrt{169} =$ ______ n. $\sqrt{196} =$ ______ o. $\sqrt{225} =$ ______

2. What is the value of $\sqrt{0}$? Explain your reasoning.

3. Notice that the square root of each expression in Question 1 resulted in a rational number. Do you think that the square root of every number will result in a rational number? Explain your reasoning.

4. Use a calculator to evaluate each square root. Show each answer to the hundred thousandth. When the square root cannot be evaluated, write cannot evaluate.

a. $\sqrt{25} =$ ____________ b. $\sqrt{0.25} =$ ____________ c. $\sqrt{250} =$ ____________

d. $\sqrt{5} =$ ____________ e. $\sqrt{-25} =$ ____________ f. $\sqrt{2.5} =$ ____________

g. $\sqrt{2500} =$ ____________ h. $\sqrt{676} =$ ____________ i. $\sqrt{6760} =$ ____________

j. $\sqrt{6.76} =$ ____________ k. $\sqrt{67.6} =$ ____________ l. $\sqrt{-6.76} =$ ____________

5. What do you notice about the square roots of rational numbers? Justify your response.

6. Is the square root of a whole number always a rational number? Justify your response.

7. Is the square root of a decimal always an irrational number?

8. Consider Sarah's and Logan's statements and reasoning.

Sarah
I know that 144 is a perfect square. Therefore, $\sqrt{144}$ is a rational number. I can move the decimal point to the left, and $\sqrt{14.4}$ and $\sqrt{1.44}$ will also be rational numbers.

Likewise, I can move the decimal point to the right, so $\sqrt{1440}$ and $\sqrt{14{,}400}$ will also be rational numbers.

Logan
I know that 144 is a perfect square. Therefore, $\sqrt{144}$ is a rational number. I can move the decimal point two places to the right or left, and $\sqrt{1.44}$ and $\sqrt{14{,}400}$ will also be rational numbers.

Moving the decimal point two places at a time is like multiplying or dividing by 100. The square root of 100 is 10, which is also a rational number.

Who is correct? Explain your reasoning.

ACTIVITY
3.2 Estimating with Square Roots

Most numbers do not have integer square roots. You can *estimate* the square root of a number that is not a perfect square. Begin by determining the two perfect squares closest to the radicand, so that one perfect square is less than the radicand and one perfect square is greater than the radicand. Then, consider the location of the expression on a number line and use approximation to estimate the value.

WORKED EXAMPLE

To estimate $\sqrt{10}$ to the nearest tenth, identify the closest perfect square less than 10 and the closest perfect square greater than 10.

The closest perfect square less than 10: 9	The square root you are estimating: $\sqrt{10}$	The closest perfect square greater than 10: 16

You know:

$\sqrt{9} = 3$ $\sqrt{16} = 4$

This means that the estimate of $\sqrt{10}$ is between 3 and 4.

Locate each square root on a number line. The approximate location of $\sqrt{10}$ is closer to 3 than it is to 4 when plotted because 10 is closer to 9 than it is to 16.

Think about the location of $\sqrt{10}$ in relation to the values of 3 and 4.

Therefore, $\sqrt{10} \approx 3.2$.

The symbol $\approx$ means approximately equal to.

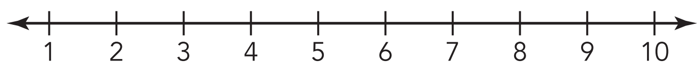
1. Calculate the square of 3.2, meaning 3.2^2, to determine whether it is a good estimation of $\sqrt{10}$. Adjust the estimated value when necessary.

2. Consider each expression.

$\sqrt{8}$ $\sqrt{91}$ $\sqrt{70}$ $\sqrt{45}$

a. Order the expressions from least to greatest.

b. Locate the approximation of each expression on the number line. Explain the strategy you used to plot each value.



c. Estimate the value of each expression to the nearest tenth. Then, calculate the square of each approximation to determine whether it is a good estimation. Adjust the estimated value when necessary.

To locate the approximation of a square root on a number line, identify the two closest perfect squares, one greater than the radicand and one less than the radicand.

3. Solve each equation. Round your answer to the nearest tenth.

a. $x^2 = 25$

b. $a^2 = 13$

c. $c^2 = 80$

d. $g^2 = 53$

When $x^2 = 4$, $x = \sqrt{4}$. Use this fact to show the solution to each equation.

ACTIVITY
3.3 Ordering Real Numbers

Students researched the daily high temperature in San Antonio, Texas, for one school week in January. Students were asked to report the temperatures using different types of numbers (e.g., fractions, decimals, percents, etc.). The result of their research is shown in the table below.

Day of the Week	Temperature (°F)
Monday	$\sqrt{6241}$
Tuesday	$\frac{392}{5}$
Wednesday	$62\frac{1}{5}$
Thursday	8490%
Friday	62.09

1. Which day of the week had the lowest temperature? Which day of the week had the highest temperature?

2. List the days of the week by temperature from highest to lowest.

By the end of the school year, the teacher decides to write a grant to get new computer monitors for each lab station in the classroom. He is debating between 4 different models.

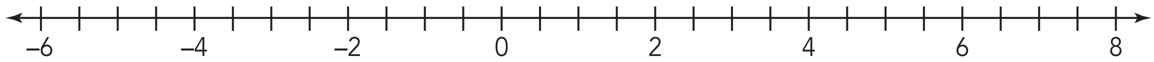
Computer Monitor	Diagonal Length (feet)
A	$\sqrt{8.5}$
B	$\frac{24}{25}$
C	$\frac{50}{\sqrt{625}}$
D	2.78

3. The teacher wants to get the smallest monitor for the grant application. Which monitor would he choose?

4. List the computer monitors in order from smallest to largest.

5. Plot points to represent the given numbers on the number line. Then, order the numbers from least to greatest.

$$\pi,\ 157\%,\ \sqrt{14},\ -3\frac{2}{3}$$



6. Avery and Camilla are working together to determine which of the four given values of x makes the inequality true.

$\sqrt{4}$	π	0.312	$\frac{13}{6}$

$$x > 256\%$$

Avery

The value of x that makes the inequality true is 0.312.
$0.312 > 256\%$

Camilla

The value of x that makes the inequality true is π.
$\pi > 256\%$

a. Explain Avery's error.

b. How can Camilla prove she chose the correct value? Explain your reasoning.

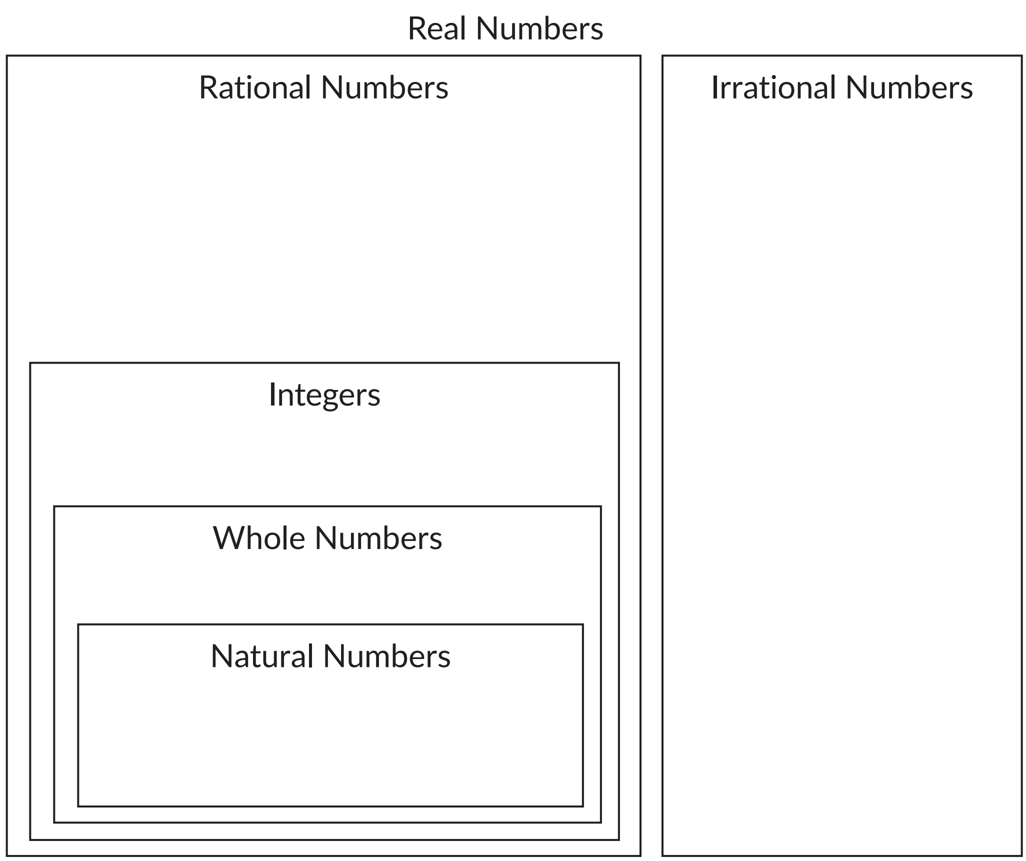
Talk the Talk

Venn Diagrams and Real Numbers

Combining the set of rational numbers and the set of irrational numbers produces the set of **real numbers**. You can use a Venn diagram to represent how the sets within the set of real numbers are related.

1. The Venn diagram shows the relationship among the six sets of numbers. Write each of the numbers in the appropriate section of the Venn diagram.

-101	-6.41	0	$\lvert 2 \rvert$	$\sqrt{0.25}$	$\frac{\sqrt{9}}{16}$	π	$6\frac{1}{4}$
$-\sqrt{9}$	$-\sqrt{2}$	1.0205×10^{-23}		$0.\overline{91}$	$\sqrt{2}$	$\frac{100}{11}$	$\sqrt{100}$
-1		0.001	0.5%	$1.523232323...$		4^2	$627{,}513$
$-\frac{3}{8}$	-0.3	0.25		212%	$\lvert -3 \rvert$	3.21×10^{12}	



2. Use your Venn diagram to decide whether each statement is true or false. Explain your reasoning.

   a. A whole number is sometimes an irrational number.

   b. A real number is sometimes a rational number.

   c. A whole number is always an integer.

Lesson 3 Assignment

Write

In your own words, write a definition for the term *irrational number.* Use examples to help illustrate your definition.

Remember

The set of real numbers includes the set of rational numbers and the set of irrational numbers.

Practice

1. Identify each number as rational or irrational.

   a. π

   b. $\sqrt{4}$

   c. $\sqrt{18}$

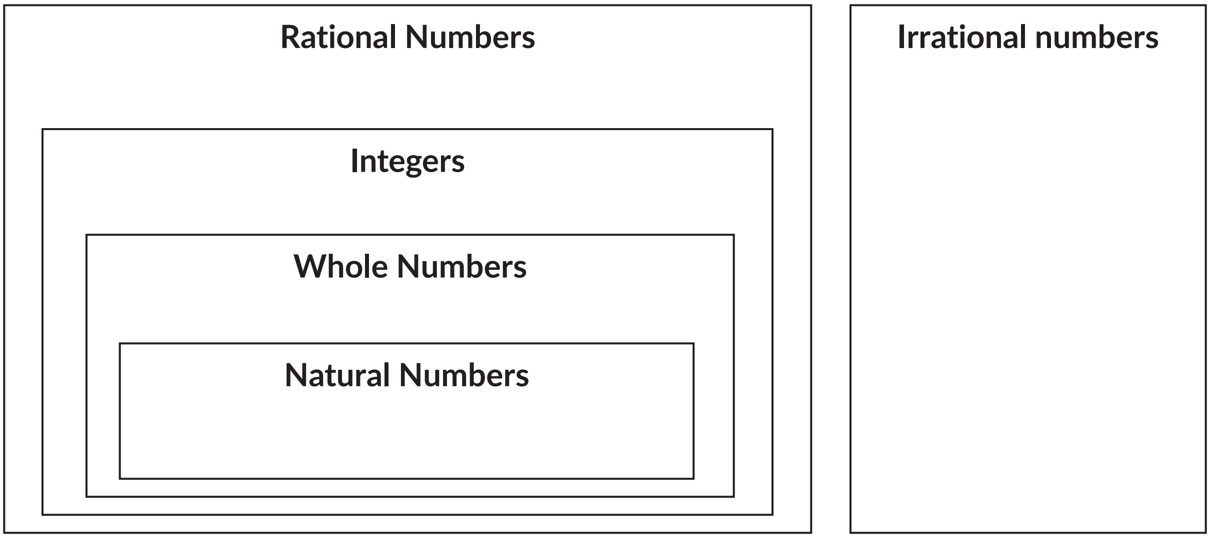
   d. $\sqrt{27}$

   e. $\sqrt{30}$

   f. $\frac{\sqrt{1}}{\sqrt{49}}$

2. Write each number in the most appropriate section of the Venn diagram.

$$35,\ \sqrt{17},\ -6,\ 5.25,\ \sqrt{81},\ -\frac{2}{3}$$

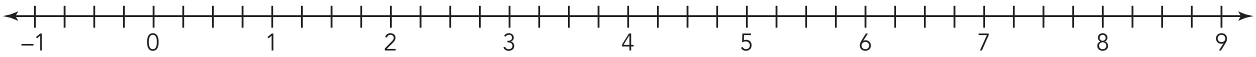


Lesson 3 Assignment

3. Consider the expressions $\sqrt{15}$, $\sqrt{97}$, and $\sqrt{40}$. Locate the approximate value of each on a number line. Then, estimate each square root to the nearest tenth.

4. Plot points to represent the given numbers on each number line. Then, order the numbers from least to greatest.

$$8\frac{1}{2},\ 20\%,\ -\frac{3}{4},\ \sqrt{25}$$



Prepare

Complete each statement to make it true.

1. 15 · ________________ = 15,000
2. 2.13 · ________________ = 21,300,000
3. 1.435 · 0.1 = ________________
4. ________________ · 0.001 = 0.00576

4 Scientific Notation

OBJECTIVES

- Express numbers in scientific notation.
- Express numbers in standard form.
- Compare numbers written in scientific notation.
- Interpret scientific notation that has been generated by technology.

NEW KEY TERMS

- scientific notation
- mantissa
- characteristic
- order of magnitude

You have used properties of powers to rewrite expressions with various bases and integer exponents.

How can you use powers of 10 to represent and compare very large and very small numbers?

Getting Started

In the Blink of an Eye

"Your childhood will be gone in the blink of an eye!"

Has anyone ever told you not to grow up too fast? Or have you heard something similar?

Have you ever thought about how short a blink is? Or how many times you blink in an hour? In a day? In a year?

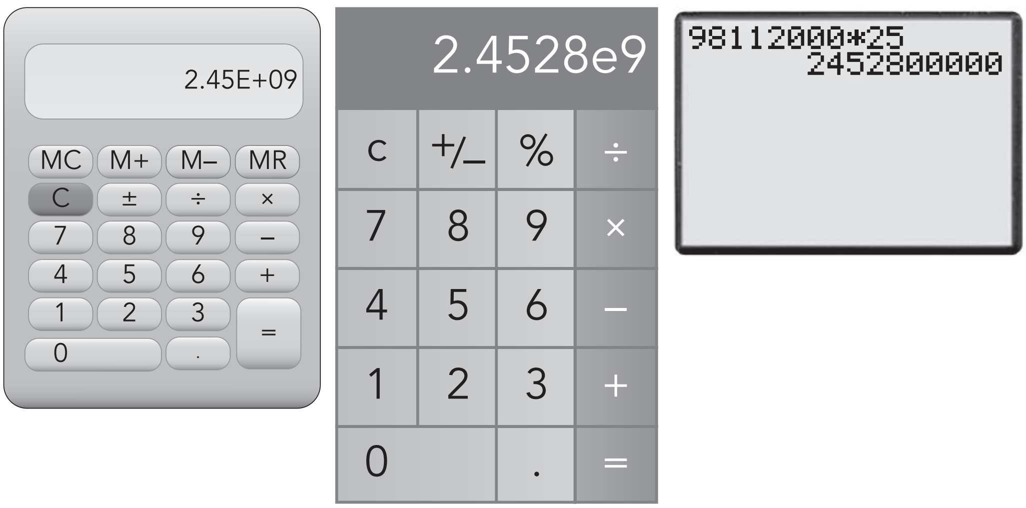
1. The average person blinks once every 3 seconds. How many times have you blinked in your lifetime?

ACTIVITY **4.1**

Introduction to Scientific Notation

Isaiah, Parker, and Carlos wanted to know how many times their entire class has blinked in their lifetimes. Each student used a different technology device: Isaiah used a basic calculator, Parker used the calculator on her phone, and Carlos used a graphing calculator. There are 25 students in the class. Isaiah, Parker, and Carlos decided that, on average, the students each had blinked 98,112,000 times.

1. Analyze the display on each calculator.



a. What was the total number of blinks for the entire class? Which display did you use to determine the total number?

b. Use the total number of blinks to interpret each of the remaining displays. How are the displays similar? How are they different?

The numbers on the smaller displays represented the large number of blinks in *scientific notation*.

Scientific notation is a tool to help you read and think about extremely large or small positive numbers.

Scientific notation is a notation used to express a very large or a very small number as the product of two numbers:

- a number that is greater than or equal to 1 and less than 10, and
- a power of 10.

In general terms, $a \times 10^n$ is a number written in scientific notation, where a is greater than or equal to 1 and less than 10 and n is any integer. The number a is called the **mantissa**, and n is the called the **characteristic**.

Scientific notation makes it much easier to tell at a glance the *order of magnitude*. The **order of magnitude** is an estimate of size expressed as a power of 10. For example, Earth's mass has an order of magnitude of 10^{24} kilograms.

2. Write the number of blinks from the calculator displays in scientific notation. Identify the mantissa and characteristic. What do you think the *e* in the displays means?

3. Use your graphing or scientific calculator to explore extremely large and extremely small numbers.

   a. Enter each given number into your calculator and complete the table.

Given Number	Scientific Notation
35,400,000,000	
60,000,000,000,000	
0.0000007	
0.000008935	

   b. Describe the characteristics for extremely large numbers.

   c. Describe the characteristics for extremely small numbers.

   d. Describe the mantissa in each.

4. Isaiah, Parker, Carlos, and Lucas each tried to write the number 16,000,000,000 in scientific notation. Analyze each student's reasoning.

Isaiah's Method

I start with 1.6, a number that is less than 10 and greater than 1. Next, I need a power of 10. When I multiply 1.6 by 10, I get 16. Then, when I multiply by 10 again, I get 160. I multiply by 10 again, and I get 1600. So, I can just keep multiplying by 10 until I get back to the original number. I have to multiply by 10 ten times, so my power of 10 is 10^{10}. So, 16,000,000,000 in scientific notation is 1.6×10^{10}.

Carlos's Method

I have to write a number greater than 1 and less than 10 multiplied by a power of 10. So, I have to multiply 1.6 by a power of 10. Since there are 9 zeros, my power of 10 will be 10^9. So, 16,000,000,000 is 1.6×10^9.

Lucas's Method

Well, that number is 16 billion. And 16 billion is 16 times 1 billion. One billion has 9 zeros, so 16,000,000,000 in scientific notation is 16×10^9.

Parker's Method

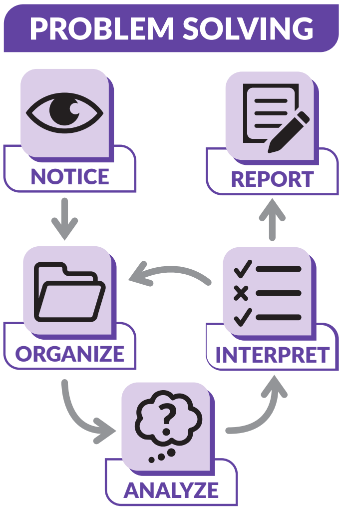
Well, that number is 16 billion. And 16 billion is 16 times 1 billion. 16×1 is the same as 1.6×10, so 16 times 1 billion is the same as 1.6 times 10 billion. I have to multiply 10 ten times to get 10 billion, so my power of 10 is 10^{10}. That means that 16 billion in scientific notation is 1.6×10^{10}.

a. Compare Carlos's and Lucas's methods.

b. Compare Isaiah's and Carlos's methods.

c. Compare Lucas's and Parker's methods.

d. Of the correct methods, which method do you prefer? Why?



ACTIVITY 4.2

Scientific Notation and Large Numbers

In this activity, you will practice writing large numbers in either scientific notation or standard notation.

1. Write each number in the notation that is not given.

   a. There are approximately 3.34×10^{22} molecules in a gram of water.

   b. There are 2.5×10^{13} red blood cells in the human body.

   c. One light year is 5,880,000,000,000 miles.

   d. The speed of light is 186,000 miles per second.

2. The estimated populations, as of December 2020, of several countries are shown. Decide whether the number is written in scientific notation or standard notation. When the number is not in scientific notation, explain how you know it is not. Then, write the number in scientific notation.

   a. People's Republic of China: 1.441×10^{9} people

   b. Pitcairn Islands: 67 people

c. Australia: 25.5×10^6 people

d. United States: 3.31×10^8 people

3. List the countries from Question 2 in order of population from least to greatest. Explain your strategy.

4. The primary U.S. currency note dispensed at an automated teller machine (ATM) is the 20-dollar bill. In 2020, there were approximately 8.9 billion 20-dollar bills in circulation.

   a. Write the approximate number of 20-dollar bills in circulation in standard notation.

   b. Write the number of bills in scientific notation.

   c. Calculate the value of all the 20-dollar bills in circulation.

   d. Write the value you calculated in part (c) in scientific notation.

ACTIVITY **4.3**

Scientific Notation and Small Numbers

Now, let's explore writing very small numbers using scientific notation.

1. A water molecule has a radius with an approximate length of 0.1 nanometer. One nanometer is $\frac{1}{10^7}$ of a centimeter. Complete the statements and answer the question. Write your answers as decimals.

   a. 1 nanometer $=$ ______________ centimeter

   b. 0.1 nanometer $=$ ______________ centimeter

   c. How many centimeters long is a string of 7 water molecules? Show your work.

Just as with large numbers, scientific notation can be used to express very small numbers in a more compact form that requires less counting of zeros. The value of the number does not change, only how it is written.

2. Each student tried to write the number 0.00065 in scientific notation. Analyze each student's reasoning.

Carlos's Method

I can start with 6.5, which is less than 10 and greater than 1. When I divide by 10, I get 0.65. When I divide by 10 again, I get 0.065. I just keep dividing by 10 until I get to the original number.

0.000[6]5

I divided by 10 four times. So, $0.00065 = \frac{6.5}{10^4}$. But in scientific notation, I have to use multiplication, not division. That's okay because $\frac{6.5}{10^4}$ is the same as $6.5 \times \frac{1}{10^4}$. And since $\frac{1}{10^4}$ is 10^{-4}, I can write 0.00065 in scientific notation as 6.5×10^{-4}.

Parker's Method

I can write 0.00065 as a fraction less than 1. In words, that decimal is sixty-five hundred thousandths, so I could write it as $\frac{65}{100{,}000}$.

When I divide both the numerator and denominator by 10, I get $\frac{65 \div 10}{100000 \div 10} = \frac{6.5}{10{,}000}$. As a power of 10, the number 10,000 is written as 10^4. So, that's $\frac{6.5}{10^4}$, which is the same as $6.5 \times \frac{1}{10^4}$, which is the same as 6.5×10^{-4}. That's the answer.

Isaiah's Method

I moved the decimal point in the number to the right until I made a number greater than 1 but less than 10. So, I moved the decimal point four times to make 6.5. And since I moved the decimal point four times to the right, that's the same as multiplying $10 \times 10 \times 10 \times 10$, or 10^4. So, the answer should be 6.5×10^4.

Lucas's Method

I don't like decimals, so I moved the decimal point all the way to the right until I had a whole number. Because I moved the decimal point five times to make 65, that's the same as dividing by 10 five times. So, the answer in scientific notation should be 65×10^{-5}.

a. Explain what is wrong with Isaiah's reasoning.

b. Explain what is wrong with Lucas's method.

c. Of the correct methods, which method do you prefer? Why?

There are names given to measurements smaller than a meter (m). You are familiar with the centimeter, the millimeter, and now the nanometer. These statements show how some small measurements relate to a meter:

- 1 centimeter (cm) $= \frac{1}{10^2}$ meter
- 1 millimeter (mm) $= \frac{1}{10^3}$ meter
- 1 micrometer (µm) $= \frac{1}{10^6}$ meter
- 1 nanometer (nm) $= \frac{1}{10^9}$ meter
- 1 picometer (pm) $= \frac{1}{10^{12}}$ meter

3. Write each measurement as a power of 10. It is appropriate to have an expression with negative exponents in this question set.

 a. 1 centimeter

 b. 1 millimeter

 c. 1 micrometer

 d. 1 nanometer

 e. 1 picometer

4. Write the radius of each type of blood vessel in standard form.

   a. The capillary is one of the minute blood vessels that connect arterioles and venules. The radius of a capillary is 5×10^{-3} mm.

   b. The venule is a small blood vessel that allows deoxygenated blood to return from the capillaries to the veins. The radius of a venule is 1×10^{-2} mm.

   c. The arteriole is a small blood vessel that extends and branches out from an artery and leads to capillaries. The radius of an arteriole is 5.0×10^{-1} mm.

People that work with very small or very large quantities, such as scientists or astronomers, use scientific notation to make numbers more reasonable to operate on and to compare.

5. Convert each measurement to meters and then write the measurement in scientific notation.

   a. The diameter of a water molecule is 0.29 nanometer.

   b. The diameter of a red blood cell is 7 micrometers.

   c. The smallest microchip is 7 nanometers wide.

   d. A helium atom has a radius of 31 picometers.

6. Complete the table shown.

Object	Measurement	Measurement in Standard Form	Measurement in Scientific Notation
Earth	Radius in meters		6.38×10^{6} m
Brachiosaurus	Mass in kilograms	77,100 kg	
Dust mite	Length in meters	0.00042 m	
Nucleus of an atom	Diameter in meters		1.6×10^{-15} m

ACTIVITY 4.4

Comparing Numbers in Scientific Notation

Writing numbers in scientific notation is useful when comparing very large or very small numbers.

You already know how to compare these numbers. Think about place value.

1. Compare each set of large numbers written in scientific notation using the appropriate symbol: $>$, $<$, or $=$.

   a. 4.5×10^4 ____________ 1.5×10^4

   b. 7.6×10^{12} ____________ 8.1×10^{12}

   c. 4.5×10^4 ____________ 4.5×10^7

   d. 9.3×10^{15} ____________ 9.3×10^{13}

   e. 7.6×10^9 ____________ 5.8×10^{12}

   f. 1.9×10^8 ____________ 3.2×10^4

2. Explain how to compare two large numbers using scientific notation.

3. Compare each set of small numbers written in scientific notation using the appropriate symbol: $>$, $<$, or $=$.

   a. 4.5×10^{-4} ____________ 1.5×10^{-4}

   b. 7.6×10^{-12} ____________ 8.1×10^{-12}

   c. 4.5×10^{-4} ____________ 4.5×10^{-7}

   d. 9.3×10^{-15} ____________ 9.3×10^{-13}

   e. 7.6×10^{-9} ____________ 5.8×10^{-12}

   f. 1.9×10^{-8} ____________ 3.2×10^{-4}

4. Explain how to compare two small numbers using scientific notation.

5. Describe the similarities and differences between the numbers 4.23×10^5 and 4.23×10^{-5}.

Talk the Talk

A Rose by Any Other Name . . .

1. Complete the table to describe each notation.

Notation	Definition	Example
Scientific		
Standard		

Lesson 4 Assignment

Write

Explain how to write a number expressed in standard form in scientific notation.

Remember

Scientific notation is a notation used to express a very large or very small number as the product of two numbers:

- the *mantissa*, which is a number greater than or equal to 1 and less than 10, and
- a power of 10, in which the exponent is called the *characteristic*.

Practice

1. Decide whether the number is written in scientific notation. When it is not, write the number in scientific notation. Explain your reasoning for each number.

   a. 38.7×10^{4}

   b. 2.56×10^{-3}

   c. 0.025×10^{-6}

   d. 2.3^{4}

Lesson 4 Assignment

2. Complete the table.

Quantity	Measurement in Standard Form	Measurement in Scientific Notation
Approximate world population in number of people	7,795,000,000	
Time for a computer to perform an operation in seconds		3.0×10^{-10}
Average fingernail thickness in inches	0.015	
Gallons of fresh water used in the U.S. each year		2.5×10^{13}
Diameter of a very fine human hair in meters	0.000017	
Mass of Mars in metric tons		7.08×10^{20}
Diameter of bacteria in inches		8×10^{-6}

3. Write each number in scientific notation.

a. There are over 29,000 grains of long-grain rice in a one-pound bag.

b. The distance from Earth to the Moon is about 385,000,000 meters.

c. The diameter of a red blood cell is about 0.00004 inch.

d. A grain of salt weighs about 0.0000585 gram.

Lesson 4 Assignment

4. Write each number in standard form.

   a. There are about 1×10^5 strands of hair on the human head.

   b. The circumference of Earth at the equator is about 4.008×10^7 meters.

   c. An oxygen atom has a radius of about 4.8×10^{-11} meters.

5. Compare each set of numbers written in scientific notation using the appropriate symbol: $>$, $<$, or $=$.

   a. 6.7×10^{-8} ____________ 9.5×10^{-8}

   b. 4.3×10^{12} ____________ 1.3×10^{12}

   c. 3.1×10^{-4} ____________ 3.1×10^{-7}

   d. 9.7×10^{15} ____________ 9.7×10^{13}

   e. 2.9×10^{-11} ____________ 6.8×10^{-12}

   f. 4.9×10^{-2} ____________ 4.9×10^{2}

Prepare

Solve for *x*.

1. $8^2 + 3^2 = x^2$

2. $36 + x^2 = 85$

3. $3^2 + 4^2 = x^2$

4. $6^2 + 8^2 = x^2$

TOPIC 1 SELF-REFLECTION

Name: ____________________

Real Numbers

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Real Numbers* topic by:

TOPIC 1: *Real Numbers*	Beginning of Topic	Middle of Topic	End of Topic
extending previous knowledge of sets and subsets of number, including the properties to describe each group.	◯	◯	◯
creating a visual representation to describe the relationships between sets of real numbers.	◯	◯	◯
converting fractions to terminating or repeating decimals, and converting repeating decimals to fractions.	◯	◯	◯
approximating the value of an irrational number.	◯	◯	◯
determining the approximate locations of irrational numbers on a number line.	◯	◯	◯
ordering and comparing rational and irrational numbers using a number line.	◯	◯	◯
using square roots to solve equations of the form $x^2 = p$, where p is a positive rational number.	◯	◯	◯

continued on the next page

TOPIC 1 SELF-REFLECTION *continued*

TOPIC 1: *Real Numbers*	Beginning of Topic	Middle of Topic	End of Topic
converting between standard decimal notation and scientific notation.	○	○	○
using scientific notation to estimate and compare very large or very small quantities.	○	○	○

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Real Numbers* topic.

____

____

____

2. What mathematical understandings from the topic do you feel you are making the most progress with?

____

____

____

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

____

____

____

TOPIC 1 SUMMARY

Real Numbers Summary

LESSON 1 Sorting Numbers

Numbers can be grouped in a variety of ways according to their characteristics. For example, you have learned about positive numbers, negative numbers, fractions, and decimals.

Sometimes, a number may fit into multiple groupings. For example, $-\frac{3}{4}$ is both a fraction and a negative number. The number 27 can be grouped with whole numbers and with integers.

LESSON 2 Rational and Irrational Numbers

The set of **natural numbers** consists of the numbers you use to count objects: {1, 2, 3, 4, 5, ...}. The set of **whole numbers** is made up of the set of natural numbers and the number 0, the additive identity. Another set of numbers is the set of **integers**, which is a set that includes all of the whole numbers and their additive inverses: {. . . , −3, −2, −1, 0, 1, 2, 3, ...}

A **rational number** is a number that can be written in the form $\frac{a}{b}$, where *a* and *b* are both integers and *b* is not equal to 0. A rational number can be written as either a terminating or repeating decimal. All other decimals are **irrational numbers** because these decimals cannot be written as fractions in the form $\frac{a}{b}$, where *a* and *b* are integers and *b* is not equal to 0.

A **terminating decimal** is a decimal that has a finite number of non-zero digits $\left(\text{e.g., } \frac{1}{8} = 0.125\right)$. A **repeating decimal** is a decimal with digits that repeat in sets of one or more. You can use two different notations to represent repeating decimals. One notation is **bar notation**, which shows one set of digits that repeats with a bar over the repeating digits $\left(\text{e.g., } \frac{1}{3} = 0.\overline{3}\right)$. Another notation shows two sets of digits that repeat with dots to indicate repetition $\left(\text{e.g., } \frac{1}{3} = 0.33...\right)$.

NEW KEY TERMS

- natural numbers [números naturales]
- whole numbers
- integers [enteros]
- rational numbers [números racionales]
- irrational numbers [números irracionales]
- terminating decimal [decimal terminal]
- repeating decimal [decimal repetitivo/ periódico]
- bar notation [notación de barra]
- square root
- radical [radical]
- radicand [radicando]
- perfect square
- real numbers [números reales]
- scientific notation [notación científica]
- mantissa [mantisa]
- characteristic [característica]
- order of magnitude [orden de magnitud]

You can use algebra to determine the fraction that is represented by a repeating decimal.

For example, write the decimal 0.44... as a fraction.

$w = 0.44...$	First, write an equation by setting the decimal equal to a variable that will represent the fraction.
$10w = .4....$	Next, write another equation by multiplying both sides of the equation by a power of 10. The exponent on the power of 10 is equal to the number of decimal places until the decimal begins to repeat.
$10w = 4.44...$	Then, subtract the original equation from the new equation.
$-w = 0.44...$	
$9w = 4$	
$w = \frac{4}{9}$	Finally, solve the equation by dividing both sides by 9.

LESSON 3 The Real Numbers

A **square root** is one of two equal factors of a given number. Every positive number has two square roots: a positive square root and a negative square root. The positive square root is called the *principal square root*.

The symbol $\sqrt{}$ is called a **radical.** The **radicand** is the quantity under a radical. For example, the expression shown is read as “the square root of 25” or as “radical 25”.

radical → $\sqrt{25}$ ← radicand

A **perfect square** is a number that is equal to the product of an integer multiplied by itself. In the example above, 25 is a perfect square because it is equal to the product of 5 multiplied by itself.

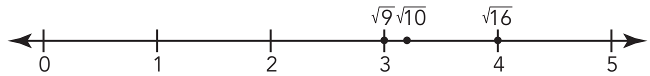
The square roots of most numbers are not integers. You can estimate the square root of a number that is not a perfect square. Begin by determining the two perfect squares closest to the radicand so that one perfect square is less than the radicand and one perfect square is greater than the radicand. Then, consider the location of the expression on a number line and use approximation to estimate the value.

For example, to estimate $\sqrt{10}$ to the nearest tenth, identify the closest perfect square less than 10 and the closest perfect square greater than 10.

$$\sqrt{9},\ \sqrt{10},\ \sqrt{16}$$

$$3,\ \sqrt{10},\ 4$$

This means that the estimate of $\sqrt{10}$ is between 3 and 4. Locate each square root on a number line. The approximate location of $\sqrt{10}$ is closer to 3 than it is to 4 when plotted.



$$\sqrt{10} \approx 3.2$$

You can check your estimate by calculating the squares of values between 3 and 4.

$$(3.1)(3.1) = 9.61$$

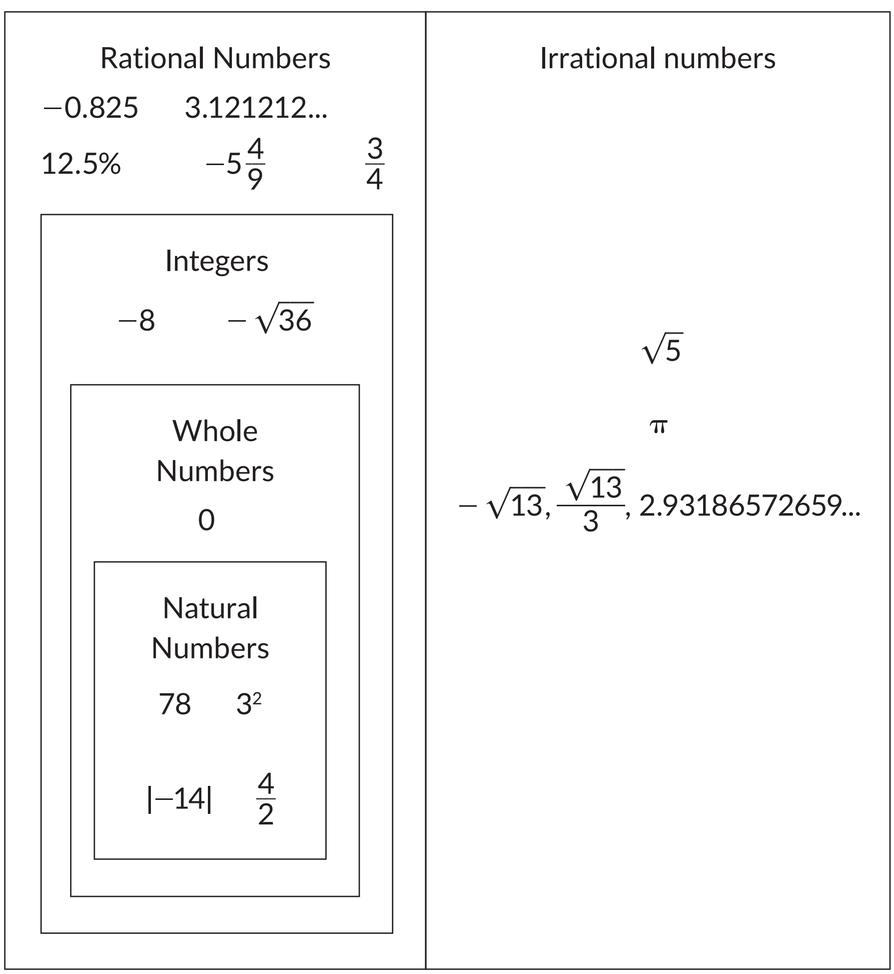
$$(3.2)(3.2) = 10.24$$

$$(3.3)(3.3) = 10.89$$

Combining the set of rational numbers and the set of irrational numbers produces the set of **real numbers**. You can use a Venn diagram to represent how the sets within the set of real numbers are related.

For example, the set of numbers $\{-8, 3^2, -\sqrt{36}, \frac{3}{4}, -0.825, \sqrt{5}, 0, -5\frac{4}{9}, 78, \pi, 3.121212..., |-14|, 12.5\%, \frac{4}{2}, -\sqrt{13}, \frac{\sqrt{13}}{3}, 2.93186572659...\}$ is categorized in the Venn diagram to represent the relationship between the subsets of numbers in the set of real numbers.

Real Numbers



LESSON 4

Scientific Notation

Scientific notation is a notation used to express a very large or very small number as the product of two numbers:

- A number that is greater than or equal to 1 and less than 10, and
- A power of 10.

In general terms, $a \times 10^n$ is a number written in scientific notation, where a is greater than or equal to 1 and less than 10 and n is any integer. The number a is called the **mantissa**, and n is called the **characteristic**.

For example, you can write the number 16,000,000,000 in scientific notation.

$$16{,}000{,}000{,}000 = 1.6 \times 10^{10}.$$

You can also write 0.00065 in scientific notation.

$$0.00065 = 6.5 \times 10^{-4}.$$

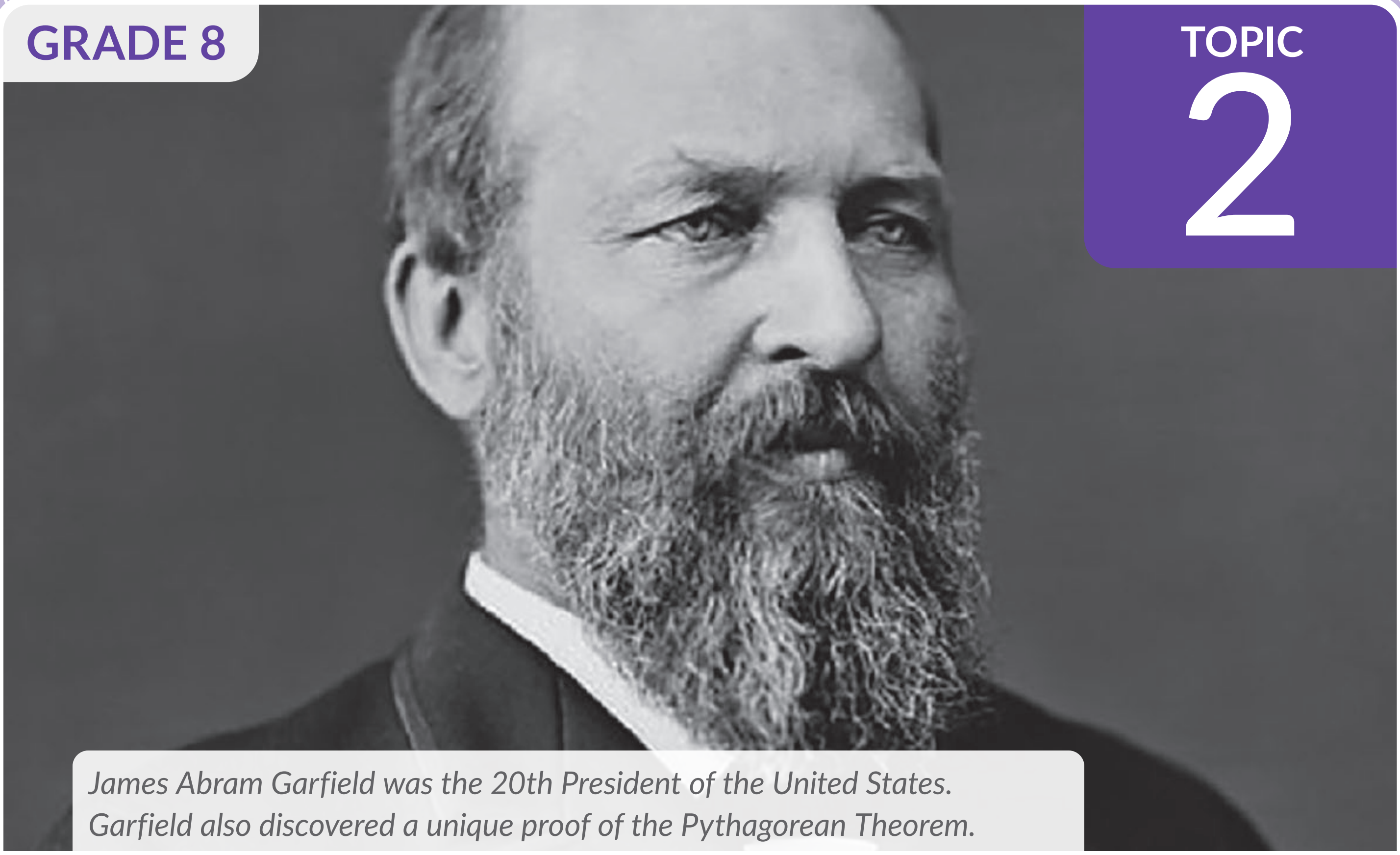
Scientific notation makes it much easier to see the *order of magnitude* at a glance. The **order of magnitude** is an estimate of size expressed as a power of ten. For example, Earth's mass has an order of magnitude of 10^{24} kilograms.

When comparing numbers written in scientific notation, start by comparing the characteristic of each number. A number with a larger characteristic has a greater value. If the two numbers have the same characteristic, the number with the greater mantissa has the greater value.

For example, 7.7×10^{12} is greater than 7.7×10^4 because $12 > 4$. Also, 9.3×10^{-3} is greater than 4.2×10^{-3} because $9.3 > 4.2$.

GRADE 8

TOPIC 2



James Abram Garfield was the 20th President of the United States. Garfield also discovered a unique proof of the Pythagorean Theorem.

The Pythagorean Theorem

1 The Pythagorean Theorem

OBJECTIVES

- Make and prove a conjecture about the relationship between the lengths of the sides of right triangles.
- Explain a proof of the Pythagorean Theorem.
- Use the Pythagorean Theorem to determine the unknown side lengths in right triangles.

NEW KEY TERMS

- hypotenuse
- legs
- Pythagorean Theorem
- proof
- diagonal of a square

You know the sum of two side lengths of any triangle is greater than the length of the third side.

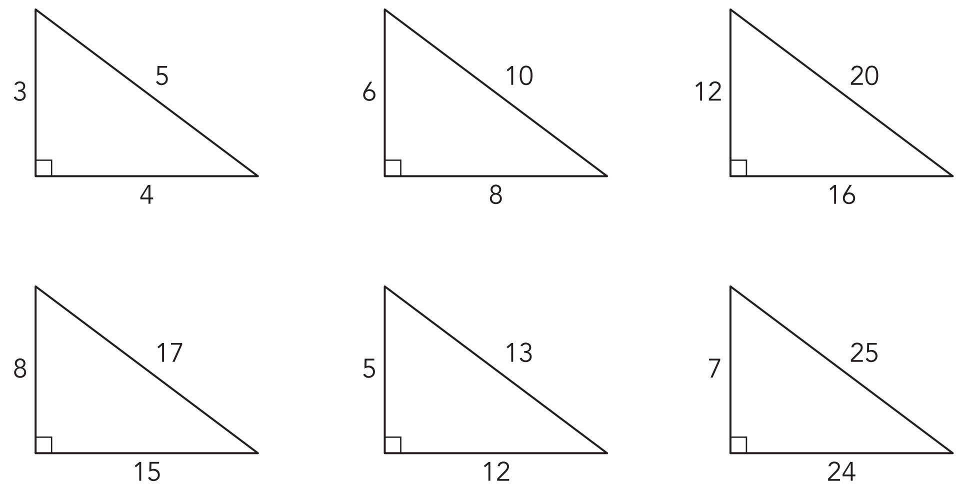
Are there other relationships between the side lengths of a triangle? What special relationships exist between the side lengths of a right triangle?

Getting Started

The triangles are not drawn to scale.

Searching for the Right Pattern

A right triangle is a triangle with a right angle. A right angle has a measure of 90° and is indicated by a square drawn at the corner formed by the angle.



When you square the length of each side of the first triangle, you get

$3^2 = 9$ $\quad$ $4^2 = 16$ $\quad$ $5^2 = 25$.

When you repeat this process with the second triangle, you get

$6^2 = 36$ $\quad$ $8^2 = 64$ $\quad$ $10^2 = 100$.

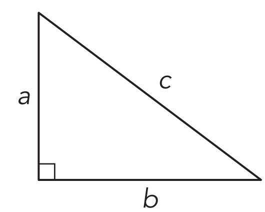
Ask Yourself . . .
What observations can you make?

1. Repeat this process with the remaining triangles. Do you see a pattern in the squares of the side lengths of a right triangle? If so, describe it.

ACTIVITY
1.1

Introducing the Pythagorean Theorem

In the right triangle shown, the lengths of the sides are *a*, *b*, and *c*.



1. Using the pattern you discovered in the previous activity, what statement can you make about the relationship among a^2, b^2, and c^2?

2. Consider the relative lengths of the sides of the triangle.

   a. Which side length must be the longest: *a*, *b*, or *c*? Explain how you know.

   b. Describe the relationship between the side lengths of any triangle.

The side opposite the right angle in a right triangle is called the **hypotenuse**. The other two sides are called **legs** of the right triangle. In the figure, the sides with lengths *a* and *b* are the legs, and the side with length *c* is the hypotenuse.

3. Label the legs and the hypotenuse in the right triangle shown.

The Pythagorean Theorem is one of the earliest known theorems to ancient civilization and one of the most famous. This theorem was named after Pythagoras (580 to 496 B.C.), a Greek mathematician and philosopher who was the first to prove the theorem.

The sum of the lengths of two sides of a triangle must be greater than the length of the third side.

The special relationship that exists between the squares of the lengths of the sides of a right triangle is known as the *Pythagorean Theorem*. The **Pythagorean Theorem** states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

$$a^2 + b^2 = c^2$$

You can verify that the Pythagorean Theorem holds true for the triangles in the previous activity.

WORKED EXAMPLE

The first right triangle has sides of length 3 units, 4 units, and 5 units, where the sides of length 3 units and 4 units are the legs and the side with length 5 units is the hypotenuse.

The sum of the squares of the lengths of the legs: $3^2 + 4^2 = 9 + 16$

$= 25$

The square of the hypotenuse: $5^2 = 25$

Therefore, $3^2 + 4^2 = 5^2$, which verifies that the Pythagorean Theorem holds true.

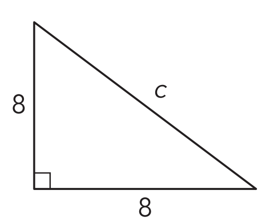
4. Verify that the Pythagorean Theorem holds true for two additional triangles.

   a. Right triangle with side lengths 8, 15, and 17

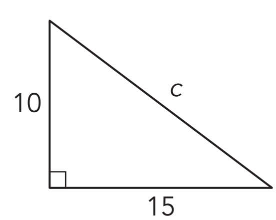
   b. Right triangle with side lengths 7, 24, and 25

5. Use the Pythagorean Theorem to determine the length of the hypotenuse in each right triangle. Round your answer to the nearest tenth, when necessary.

   a.



   b.



ACTIVITY
1.2 Proving the Pythagorean Theorem

You verified the Pythagorean Theorem for select triangles, but how do you know that it holds for *all* right triangles? In this activity, you will create a geometric *proof* of the theorem.

A **proof** is a line of reasoning used to validate a theorem.

1. Complete the geometric proof assigned to you. The cut-outs for proofs are located at the end of the lesson. Then, record your determinations on the graphic organizer and prepare to share your results with your classmates.

Proof 1
An isosceles right triangle is drawn on the grid.

a. A square on the hypotenuse has been drawn for you. Use a straightedge to draw squares on the other two sides of the triangle. Then, use different colored pencils to shade each small square.

b. Draw two diagonals in each of the two smaller squares.

c. Cut out the two smaller squares along the legs. Then, cut those squares into fourths along the diagonals you drew.

A **diagonal of a square** is a line segment connecting opposite vertices of the square.

d. Redraw your original figure and the squares on the grid on the graphic organizer at the end of the activity. Shade the smaller squares again.

e. Arrange the pieces that you cut out to fit inside the larger square on the graphic organizer. Then, tape the triangles on top of the larger square.

Proof 2
A right triangle has one leg 4 units in length and the other leg 3 units in length.

a. Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.

b. Cut out the two smaller squares along the legs.

c. Cut the two squares into strips that are either 4 units by 1 unit or 3 units by 1 unit.

d. Redraw your original figure and the squares on the grid on the graphic organizer at the end of the activity. Shade the smaller squares again.

e. Arrange the strips and squares you cut out on top of the square along the hypotenuse on the graphic organizer. You may need to make additional cuts to the strips to create individual squares that are 1 unit by 1 unit. Then, tape the strips on top of the square you drew on the hypotenuse.

Proof 3
A right triangle has one leg 2 units in length and the other leg 4 units in length.

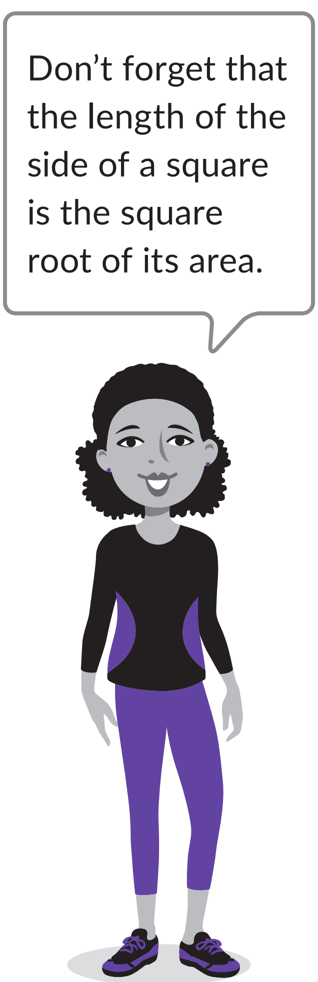
a. Use a straightedge to draw squares on each side of the triangle. Use different colored pencils to shade each square along the legs.

b. Cut out the two smaller squares.

c. Draw four congruent right triangles on the square with side lengths of 4 units. Then, cut out the four congruent right triangles you drew.

d. Redraw your original figure and the squares on the grid on the graphic organizer at the end of the activity. Shade the smaller squares again.

e. Arrange and tape the small square and the four congruent triangles you cut out over the square that has one of its sides as the hypotenuse.

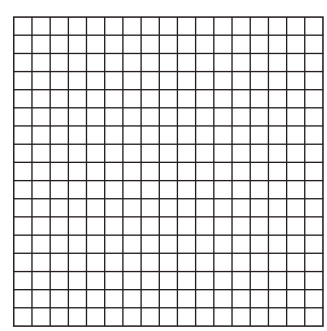


What do you notice?

Describe the relationship among the areas of the squares.

Description of Right Triangle in Proof

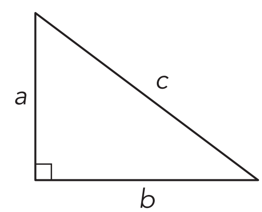
Determine the length of the hypotenuse.



Share your proof and graphic organizer with your classmates.

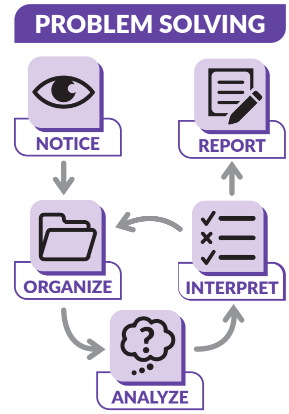
2. Compare the descriptions of the relationship among the areas of the squares from each proof. What do you notice?

3. Write an equation that represents the relationship among the areas of the squares of side lengths *a*, *b*, and *c* in the right triangle shown.



ACTIVITY 1.3

Determining the Length of the Hypotenuse



The Pythagorean Theorem can be used to determine unknown side lengths in a right triangle. Lucas and Jasmine are using the theorem to determine the length of the hypotenuse, *c*, with leg lengths of 2 and 4. Examine their work.

Lucas

$c^2 = 2^2 + 4^2$

$c^2 = 6^2$

$c = 6$

The length of the hypotenuse is 6 units.

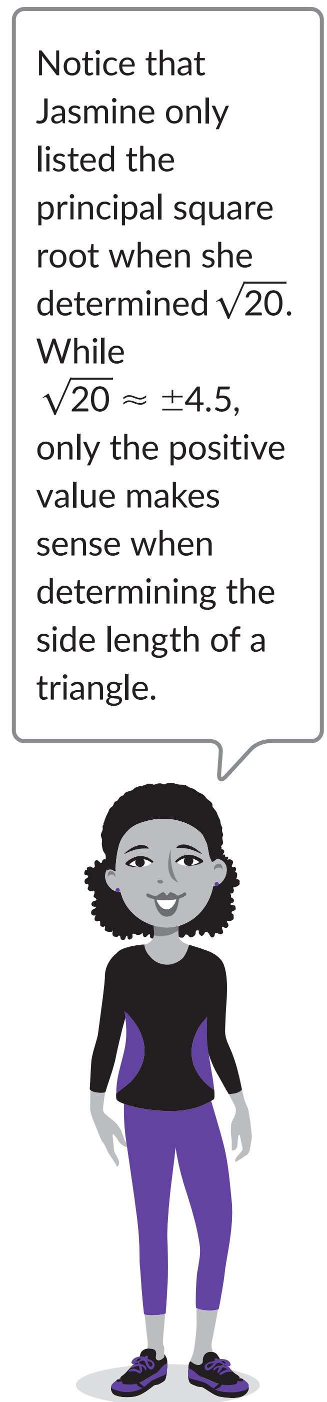
Jasmine

$c^2 = 2^2 + 4^2$

$c^2 = 4 + 16 = 20$

$c = \sqrt{20} \approx 4.5$

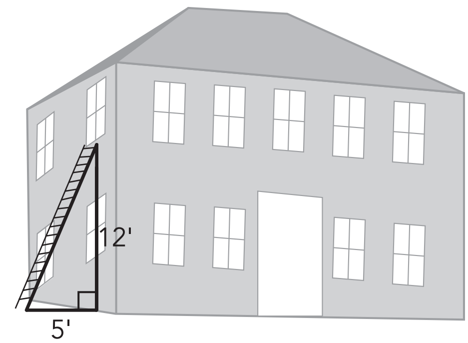
The length of the hypotenuse is approximately 4.5 units.



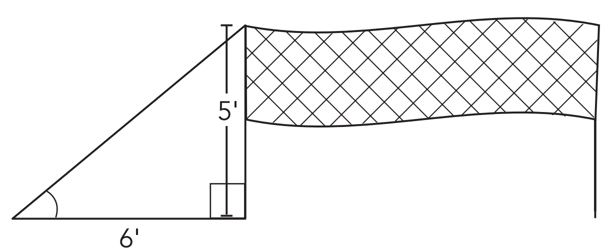
1. Explain the algebraic error in Lucas's work.

Mason maintains the local middle school campus. Use the Pythagorean Theorem to help Mason with some of his jobs.

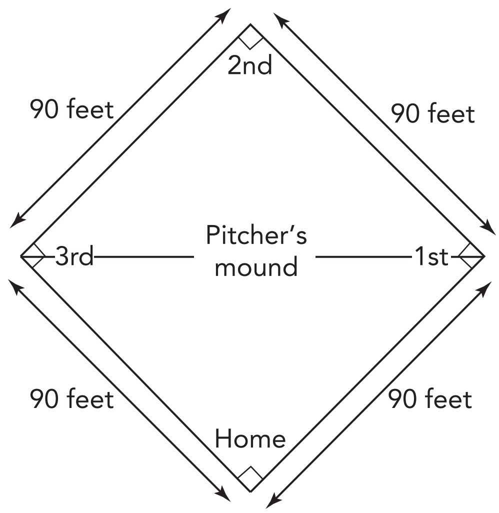
2. Mason needs to wash the windows on the second floor of a building. He knows the windows are 12 feet above the ground. Because of dense shrubbery, he has to put the base of the ladder 5 feet from the building. What ladder length does he need?



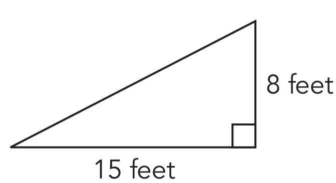
3. The gym teacher, Ms. Park, asked Mason to put up the badminton net. Ms. Park said that the top of the net must be 5 feet above the ground. She knows that Mason will need to put stakes in the ground for rope supports. She asked that the stakes be placed 6 feet from the base of the poles. Mason has two pieces of rope, one that is 7 feet long and a second that is 8 feet long. Will each piece of rope be enough to secure the badminton poles? Explain your reasoning.



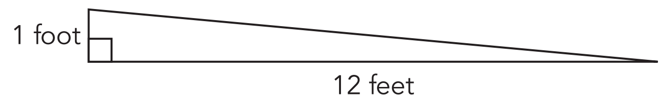
4. Mason stopped by the baseball field to watch the team practice. The first baseman caught a line drive right on the base. He touched first base for one out and quickly threw the ball to third base to get another out. How far did he throw the ball?



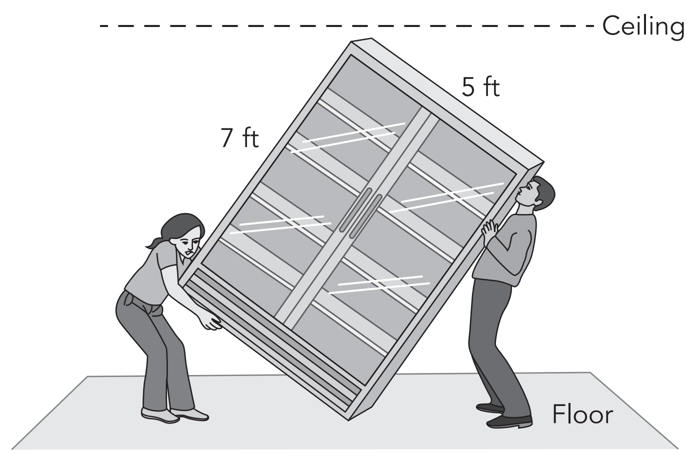
5. The skate ramp on the playground of a neighboring park is going to be replaced. Mason needs to determine how long the ramp is to get estimates on the cost of a new skate ramp. He knows the measurements shown in the figure. How long is the existing skate ramp?



6. A wheelchair ramp that is constructed to rise 1 foot off the ground must extend 12 feet along the ground. How long will the wheelchair ramp be?



7. The school's new industrial-size refrigerator is 7 feet tall and 5 feet wide. The refrigerator is lying on its side. Mason and the movers want to tilt the refrigerator upright, but they are worried that the refrigerator might hit the 8-foot ceiling. Will the refrigerator hit the ceiling when it is tilted upright?

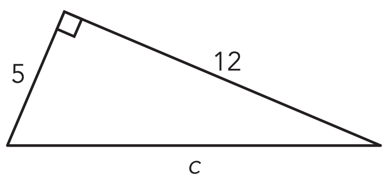


ACTIVITY
1.4 Determining the Length of a Leg

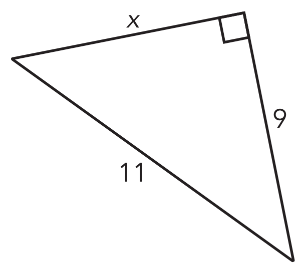
Use the Pythagorean Theorem to solve each problem.

1. Write an equation to determine each unknown length. Then, solve the equation. Round your answer to the nearest tenth, when necessary.

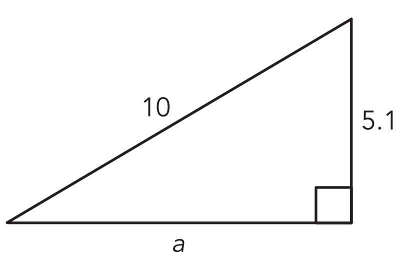
a.



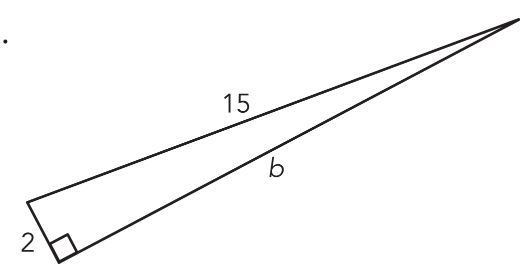
b.



c.



d.



2. Luna has a ladder that is 20 feet long. When the top of the ladder reaches 16 feet up the side of a building, how far from the building is the base of the ladder?

3. The length of the hypotenuse of a right triangle is 40 centimeters. The legs of the triangle are the same length. How long is each leg of the triangle?

4. A plane is 5 miles directly above a house and 42 miles from the runway at the nearest airport. How far is the house from the airport?

What path will the plane take to reach the runway?

5. A boat drops an anchor at the deepest point of the lake and spends the day drifting along the lake. When the lake is 75 feet deep and the chain on the anchor is 200 feet, determine the greatest distance the boat can drift from where it dropped anchor.

Talk the Talk

Another Proof!

While it is called the *Pythagorean Theorem*, the mathematical knowledge was used by the Babylonians 1000 years before Pythagoras. Many proofs followed that of Pythagoras, including ones proved by Euclid, Socrates, and even the twentieth President of the United States, President James A. Garfield.

Let's use the figures shown to prove the Pythagorean Theorem another way. Each figure includes four right triangles with leg lengths *a* and *b* and hypotenuse of length *c*.

Figure 1

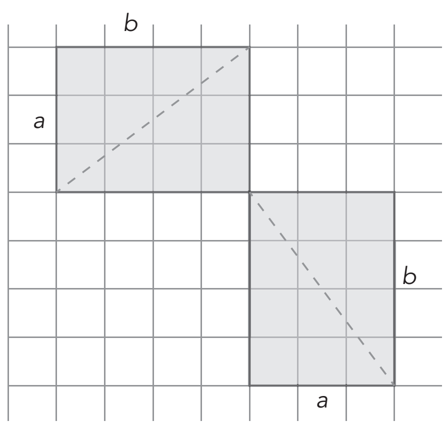
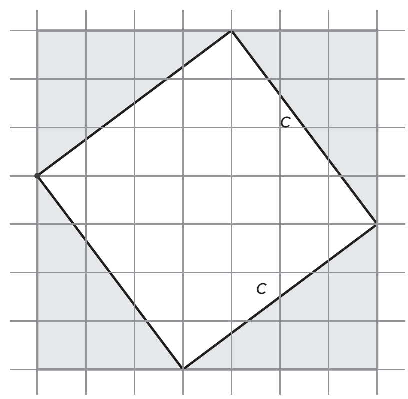


Figure 2



1. Write an expression for the total area of the non-shaded region of Figure 1, in terms of *a* and *b*.

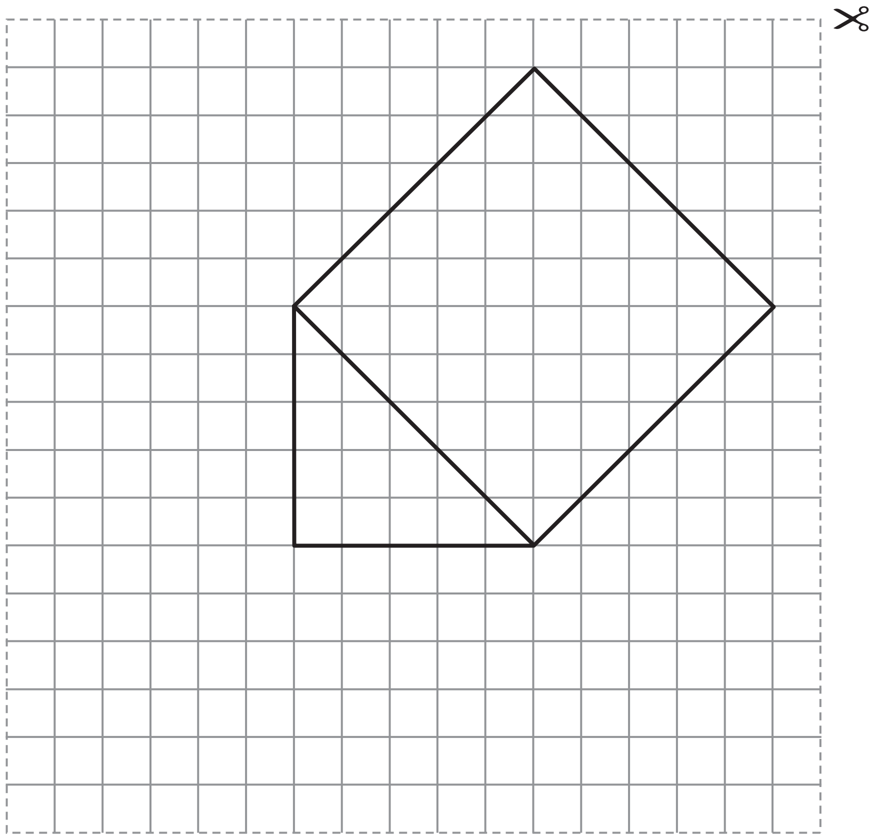
2. Explain how to use transformations to transform Figure 1 onto Figure 2.

3. Write an expression for the non-shaded region of Figure 2, in terms of c.

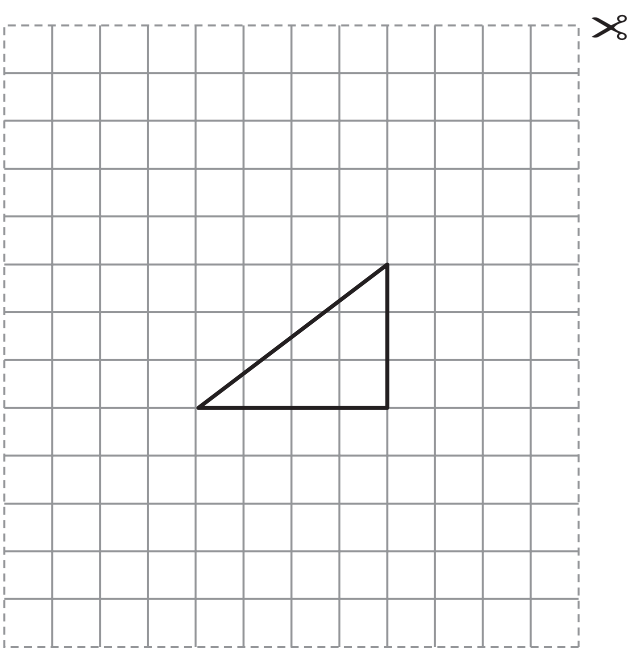
4. Explain why these figures prove the Pythagorean Theorem.

Proof Cutouts

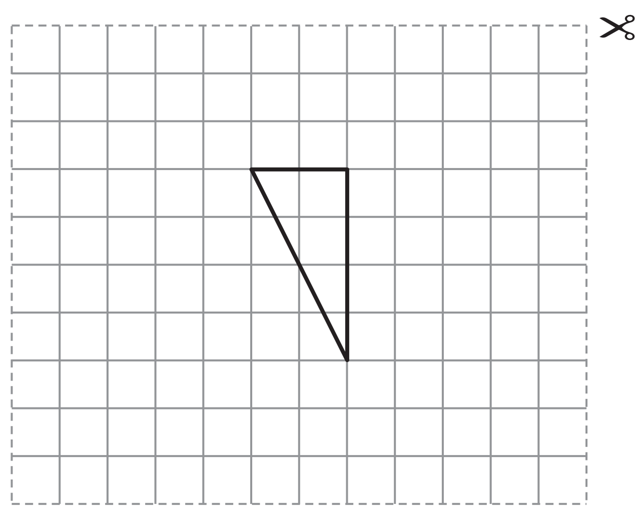
Proof 1



Proof 2



Proof 3



Why is this page blank?

So you can cut out the proofs on the other side.

Lesson 1 Assignment

Write

Describe the relationship between the areas of the squares on the sides of right triangles.

Remember

The Pythagorean Theorem is used to determine unknown lengths in right triangles in mathematical and contextual problems.

Practice

Determine the unknown in each situation. Round your answers to the nearest tenth.

1. Sebastian goes shopping for a new flat-panel television. A television is usually described by the length of the screen's diagonal. He determines a great deal on a 42-inch display model.

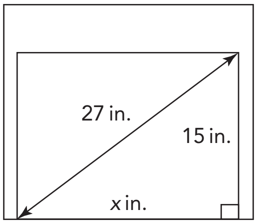
   a. When the screen's height is 21 inches, what is the width of the screen?

   b. The border around the screen is 2 inches. What are the dimensions of the television, including the border?

   c. How long is the diagonal of the television, including the border?

Lesson 1 Assignment

2. Sebastian sells his old television in his neighborhood's garage sale. It has a rectangular screen with a diagonal measure of 27 inches. A potential buyer is concerned about the television fitting in the 24-inch square opening of his entertainment center.



   a. What is the width of the television's screen?

   b. Will the television fit in the buyer's entertainment center? Explain your reasoning.

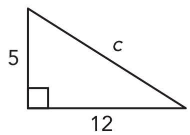
3. Gabriel is responsible for changing the broken light bulb in a streetlamp. The streetlamp is 12 feet tall. Gabriel places the base of his ladder 4 feet from the base of the streetlamp. He can extend his ladder from 10 feet to 14 feet. How long must his ladder be to reach the top of the streetlamp?

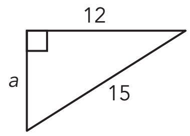
Lesson 1 Assignment

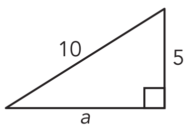
4. A scaffold has a diagonal support beam to strengthen it. If the scaffold is 15 feet high and 5 feet wide, how long must the support beam be?

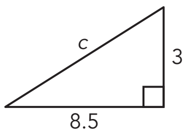
5. A rectangular swimming pool is 24 meters by 10 meters. Nahimana said she could swim diagonally from one corner to another without taking a breath. Kayla said she could swim a greater distance than Nahimana without taking a breath. Determine the distances Nahimana and Kayla may have swum.

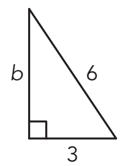
Determine the unknown side length in each right triangle. Round your answers to the nearest tenth.

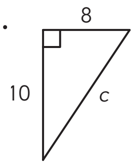
6. 

7. 

8. 

9. 

10. 

11. 

Prepare

A bird leaves its nest and flies 3 miles due south, 2 miles due east, 5 miles due south, and 1 mile due east to visit a friend's nest.

1. Draw a model of the situation.

2. Determine the distance between the nests.

2 The Converse of the Pythagorean Theorem

OBJECTIVES

- Determine whether three side lengths form a right triangle.
- Generate side lengths of right triangles.
- Use the Pythagorean Theorem and the converse of the Pythagorean Theorem to determine unknown side lengths in right triangles.

NEW KEY TERMS

- converse
- converse of the Pythagorean Theorem
- Pythagorean triple

You know that the Pythagorean Theorem can be used to solve for unknown lengths in a right triangle.

How can you use the theorem to prove that a triangle is a right triangle?

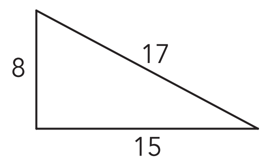
Getting Started

Is It Right?

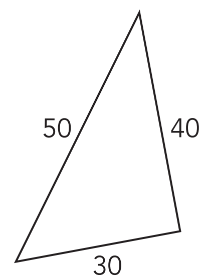
Often, geometry diagrams are not drawn to scale, and even when a triangle looks like a right triangle, it may not be. A square is used to indicate the presence of the right angle, but what if that symbol is missing? How do you know whether a triangle is a right triangle?

1. Use a protractor to determine which triangles are right triangles.

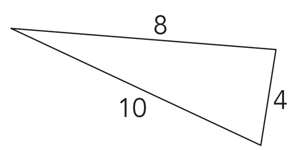
a.



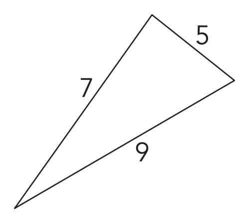
b.



c.



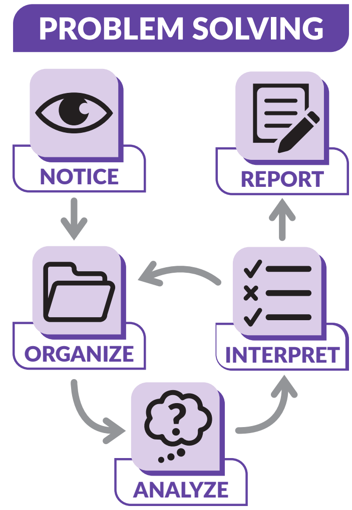
d.



2. Square the side lengths of each triangle. What do you notice about the squares of the lengths of the sides of the triangle of the non-right triangles versus the right triangles?

ACTIVITY 2.1

The Converse and Triples



The Pythagorean Theorem can be used to solve many problems involving right triangles, squares, and rectangles. The Pythagorean Theorem states that, when a triangle is a right triangle, then the square of the hypotenuse length equals the sum of the squares of the leg lengths. Have you wondered whether the *converse* is true?

The **converse of the Pythagorean Theorem** states that if the sum of the squares of the two shorter sides of a triangle equals the square of the longest side, then the triangle is a right triangle.

In other words, if the lengths of the sides of a triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle.

The **converse** of a theorem is created when the if–then parts of that theorem are exchanged.

1. Determine whether the triangle with the given side lengths is a right triangle.

   a. 9, 12, 15

   b. 25, 16, 9

Think about which measures would represent legs of the right triangle and which measure would represent the hypotenuse.

You may have noticed that each of the right triangles in Question 1 had side lengths that were integers. Any set of three positive integers *a*, *b*, and *c* that satisfies the equation $a^2 + b^2 = c^2$ is a **Pythagorean triple**. For example, the integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.

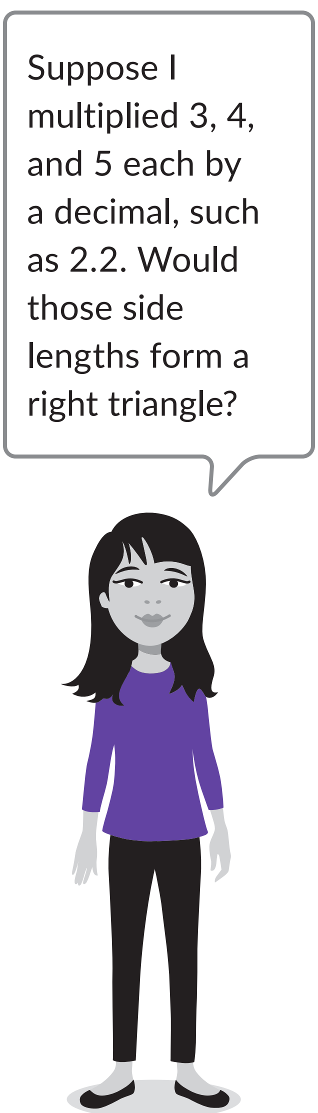
Given a Pythagorean triple, you can identify other right triangles by multiplying each side length by the same factor.

2. Complete the table to identify more Pythagorean triples.

	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean Triple	3	4	5	$9 + 16 = 25$
Multiply by 2				
Multiply by 3				

3. Determine a new Pythagorean triple not used in Question 2 and complete the table.

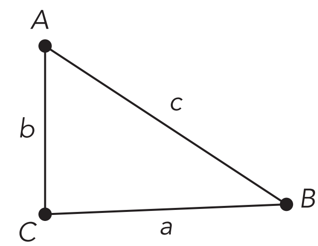
	a	b	c	Check: $a^2 + b^2 = c^2$
Pythagorean Triple				
Multiply by 3				
Multiply by 5				



4. Record other Pythagorean triples that your classmates determined.

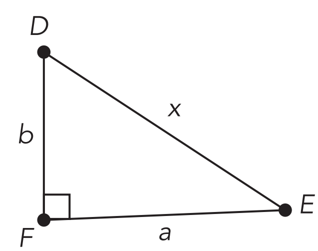
ACTIVITY
2.2 Proving the Converse

Because the converse of the Pythagorean Theorem is, itself, a theorem, you can prove it.



Step 1: Assume you are given $\triangle ABC$ such that the sum of the squares of the lengths of two sides equals the square of the length of the third side, or $a^2 + b^2 = c^2$.

1. Do you have enough information to determine whether the triangle is a right triangle without using the converse of the Pythagorean Theorem? Explain your reasoning.



Step 2: Now, construct a right triangle, $\triangle DEF$, using the side lengths a and b from $\triangle ABC$. By the Pythagorean Theorem, $a^2 + b^2 = x^2$, where x is the hypotenuse of $\triangle DEF$.

2. Why can you apply the Pythagorean Theorem to the side lengths of $\triangle DEF$?

Step 3: If $a^2 + b^2 = c^2$ and $a^2 + b^2 = x^2$, then $c^2 = x^2$ and $c = x$.

3. Explain why $c^2 = x^2$.

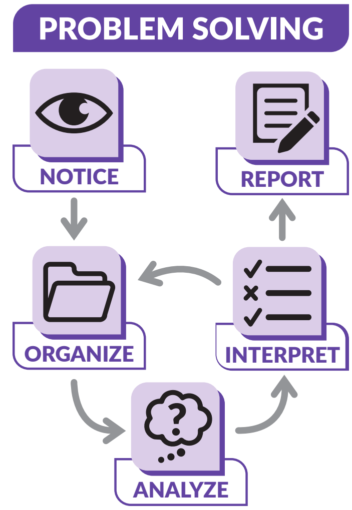
Step 4: $\triangle ABC \cong \triangle DEF$ because all of their corresponding side lengths are equal.

Recall that three sides of a triangle create a unique triangle. Therefore, all triangles with those side lengths are congruent.

4. When the triangles are congruent and $\triangle DEF$ is a right triangle, what must be true about $\triangle ABC$?

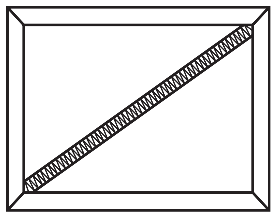
You have proven the converse of the Pythagorean Theorem.
If the sum of the squares of the lengths of two sides of a triangle equals the square of the length of the third side, then the triangle is a right triangle.

ACTIVITY
2.3 Applying the Theorems



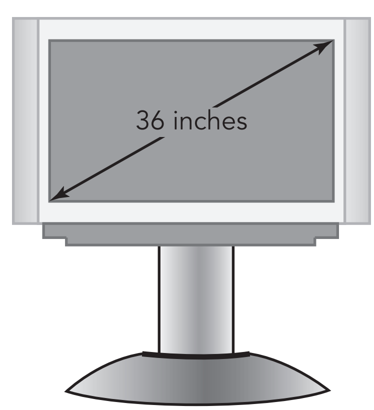
Use your knowledge of the Pythagorean Theorem and its converse to solve each.

1. A carpenter attaches a brace to a rectangular picture frame. Suppose the dimensions of the picture frame are 30 inches by 40 inches. What is the length of the brace?



2. Diego is staking out a location to build a rectangular deck that will be 8 feet wide and 15 feet long. Tyrone is helping Diego with the deck. Tyrone has two boards, one that is 8 feet long and one that is 7 feet long. He puts the two boards together, end to end, and lays them on the diagonal of the deck area, where they just fit. What should he tell Diego?

3. A television is identified by the diagonal measurement of the screen. A television has a 36-inch screen with a height of 22 inches. What is the width of the television screen? Round your answer to the nearest inch.

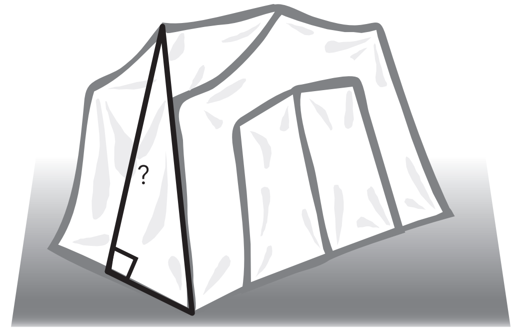


4. Logan and Avery want to put a custom-made, round table in their dining room. The tabletop is made of glass with a diameter of 85 inches. The front door is 36 inches wide and 80 inches tall. Logan thinks the tabletop will fit through the door but Avery does not. Who is correct and why?

5. Sherie makes a canvas frame for a painting using stretcher bars. The rectangular painting will be 12 inches long and 9 inches wide. How can she use a ruler to make sure that the corners of the frame will be right angles?

6. A 10-foot ladder is placed 4 feet from the edge of a building. How far up the building does the ladder reach? Round your answer to the nearest tenth of a foot.

7. Parker has a tent that is 64 inches wide with a slant height of 68 inches on each side. What is the height of the center pole needed to prop up the tent?



8. A ship left shore and sailed 240 kilometers east, turned due north and then sailed another 70 kilometers. How many kilometers is the ship from shore by the most direct path?

9. Angelina walks 88 feet due east to the library from her house. From the library, she walks 187 feet northwest to the corner store. Finally, she walks approximately 139 feet from the corner store back home. Does she live directly south of the corner store? Justify your answer.

10. What is the diagonal length of a square that has a side length of 10 cm?

Talk the Talk

Triple Play

Create a Pythagorean triple that contains each length or lengths. Verify that the side lengths form a right triangle.

1. 9 and 41

2. 21 and 29

3. 12

4. 15

5. Can any integer be used to create a Pythagorean triple? Why or why not?

6. Are the side lengths of a right triangle always integers? Why or why not?

Lesson 2 Assignment

Write

Complete each statement:

1. The converse of the Pythagorean Theorem states that if the sum of the squares of two sides of a triangle equals the square of the third side, then the triangle is a ____________ ____________.

2. The converse of a theorem is created when the if–then parts of the theorem are ____________.

3. A Pythagorean triple is a set of three ____________ ____________ a, b, and c that satisfy the equation $a^2 + b^2 = c^2$.

Remember

The converse of the Pythagorean Theorem is used to determine whether triangles are right triangles.

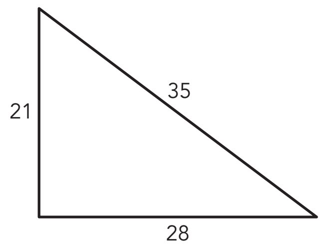
Practice

1. Determine whether each triangle with the given side lengths is a right triangle.

a. 6, 9, 14

b. 2, 3.75, 4.25

c.



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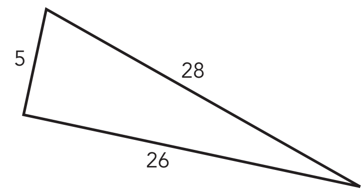


Lesson 2 Assignment

2. Mei has received grant money to open a local community center. She wants to save as much of the money as possible for programs. She will be doing many of the improvements herself to the old building she has rented. While touring the building to make her project list, she uses a tape measure to check whether floors, doorways, and walls are square, meaning that they meet at right angles.

   a. Mei measures the lobby of the building for new laminate flooring. The length is 30 feet, the width is 16 feet, and the diagonal is 34 feet. Is the room square?

   b. Can Mei use the edges of the room as a guide to start laying the boards of laminate flooring? Explain your reasoning.

   c. The landing outside the main entrance of the building does not have a railing. Mei wants to install railing around the landing to make it safer. The length of the landing is 12 feet, the width is 9 feet, and the diagonal is 14 feet. Is the landing square?

Lesson 2 Assignment

d. Mei needs to order a new door for her office. The width of the door frame is 3 feet, the height is 8 feet, and the diagonal is $8\frac{5}{8}$ feet. Is the door frame square?

e. The sign that will be mounted to the outside of the building is a rectangle that is 9 feet by 12 feet. The largest doorway into the building is 4 feet wide and 8 feet high. What is the diagonal measurement of the doorway?

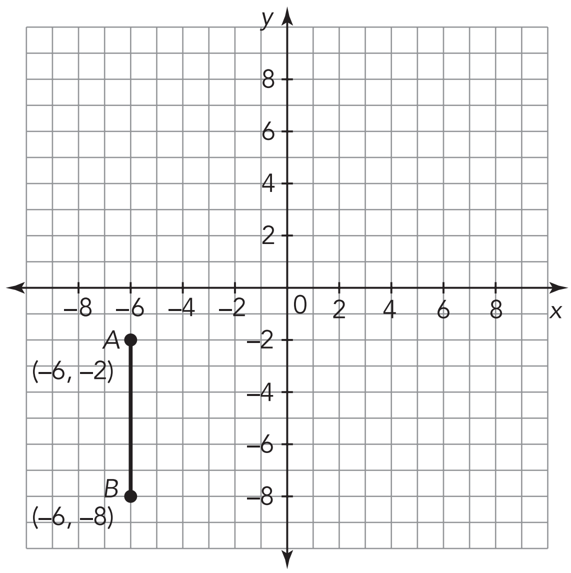
f. Does Mei have to mount the sign the day it is delivered or can she store it inside the building until she is ready? Explain your answer.

3. Given the Pythagorean triple 21-220-221, generate an additional triple and verify that the side lengths form a right triangle.

Lesson 2 Assignment

Prepare

Use the coordinate plane shown to answer the question.



1. How do you calculate the distance between points *A* and *B*?

2. What is the distance between points *A* and *B*?

3. How do the negative coordinates affect the distance between points *A* and *B*.

3 Distances in a Coordinate System

OBJECTIVES

- Apply the Pythagorean Theorem to determine the distance between two points on a coordinate plane.
- Use square roots to represent solutions to equations.

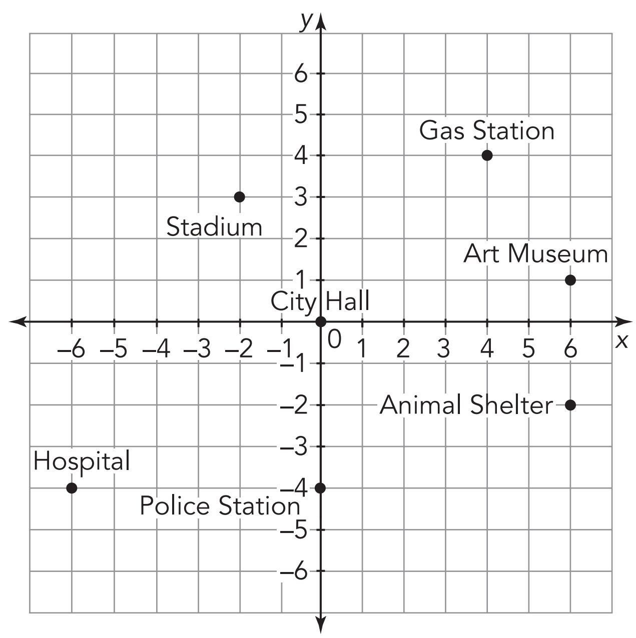
You have learned about the Pythagorean Theorem and the converse of the Pythagorean Theorem.

How can you apply the Pythagorean Theorem to determine distances on a coordinate plane?

Getting Started

As the Crow Flies

The map shows certain locations within a city. Each unit on the map represents 1 block.



The phrase *as the crow flies* means "the straight-line distance" between two points.

Review the definition of **describe** in the Academic Glossary.

Describe how you could express each distance "as the crow flies."

1. The distance between City Hall and the police station

2. The distance between the stadium and the gas station

3. The distance between the animal shelter and the stadium

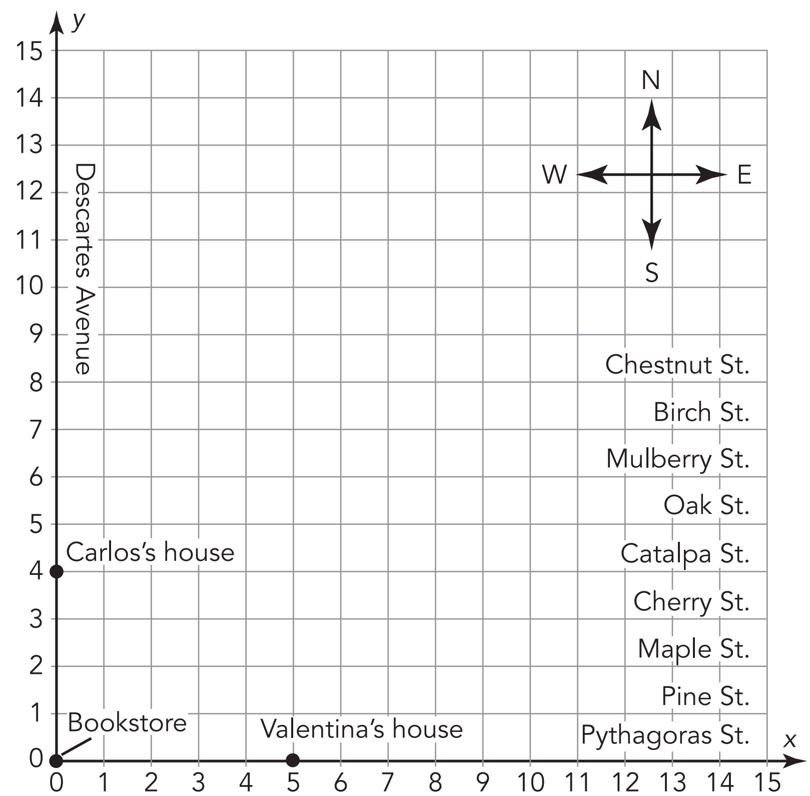
ACTIVITY 3.1

Right Triangles on the Coordinate Plane

Two friends, Carlos and Valentina, live in a city in which the streets are laid out in a grid system.

Carlos lives on Descartes Avenue and Valentina lives on Pythagoras Street, as shown.

1. The two friends often meet at the bookstore. Each grid square represents one city block.



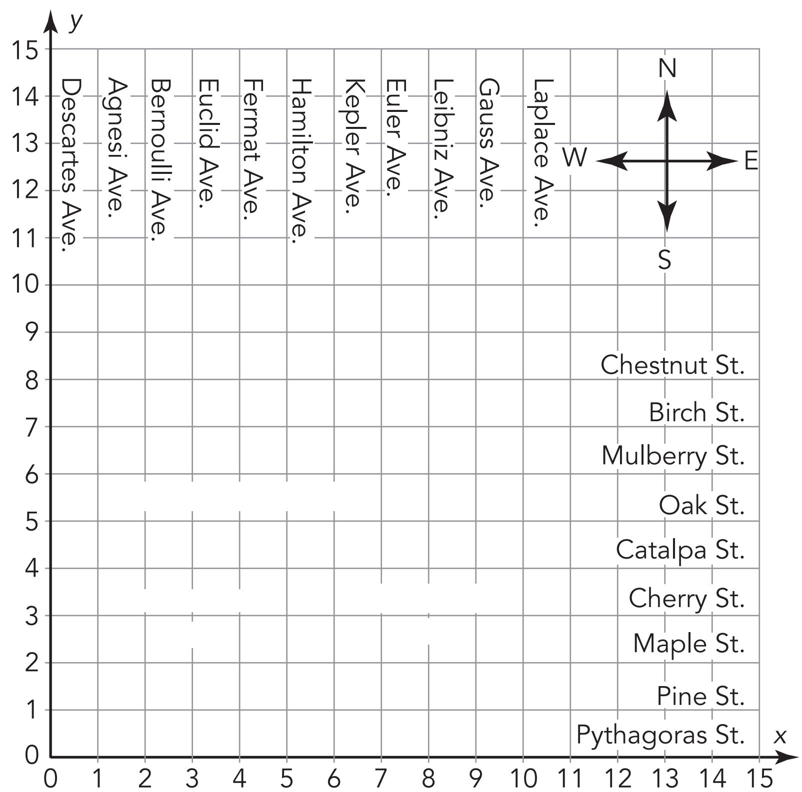
   a. How many blocks does Carlos walk to get to the bookstore?

   b. How many blocks does Valentina walk to get to the bookstore?

   c. Determine the distance, in blocks, Valentina would walk when she traveled from her house to the bookstore and then to Carlos's house.

   d. Determine the distance in blocks that Valentina would walk when she traveled in a straight line from her house to Carlos's house. Explain your calculation. Round your answer to the nearest tenth of a block.

2. Isaiah, a friend of Carlos and Valentina, lives three blocks east of Descartes Avenue and five blocks north of Pythagoras Street. Emily another friend, lives seven blocks east of Descartes Avenue and two blocks north of Pythagoras Street. Plot the location of Isaiah's house and Emily's house on the grid. Label each location and its coordinates.



a. Name the streets that Isaiah lives on.

b. Name the streets that Emily lives on.

3. Another friend, Lucas, lives at the intersection of the avenue that Isaiah lives on and the street that Emily lives on. Plot the location of Lucas's house on the grid in Question 2 and label the coordinates. Describe the location of Lucas's house with respect to Descartes Avenue and Pythagoras Street.

4. How do the coordinates of Lucas's house compare to the coordinates of Isaiah's house and Emily's house?

5. Use ordered pairs to write and evaluate an expression that represents the distance between Isaiah's and Lucas's houses.

6. How far, in blocks, does Isaiah have to walk to get to Lucas's house?

7. Use ordered pairs to write an expression that represents the distance between Lucas's and Emily's houses.

8. How far, in blocks, does Lucas have to walk to get to Emily's house?

9. All three friends meet at Isaiah's house to study geometry. Emily walks to Lucas's house and then they walk together to Isaiah's house. Use the coordinates to write and evaluate an expression that represents the distance from Emily's house to Lucas's house and from Lucas's house to Isaiah's house.

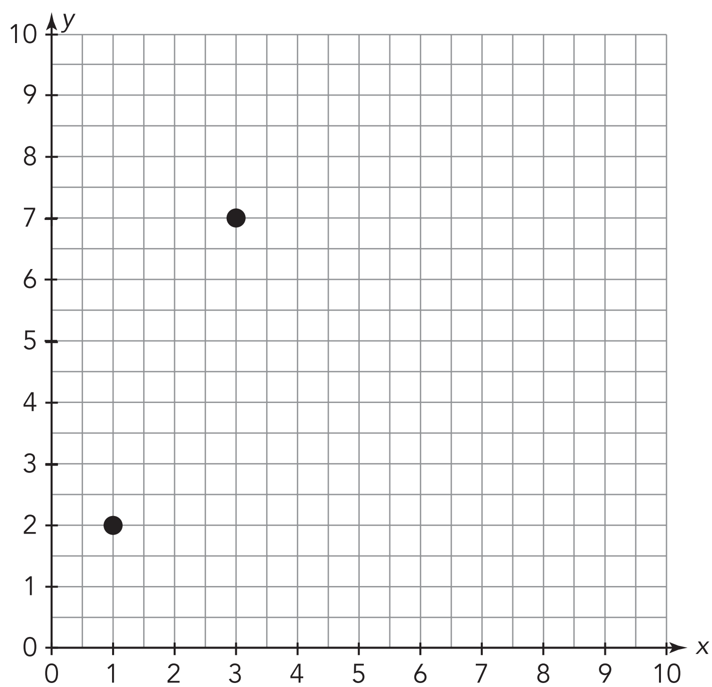
10. How far, in blocks, does Emily walk altogether?

11. Draw the direct path from Isaiah's house to Emily's house on the coordinate plane in Question 2. When Emily walks to Isaiah's house on this path, how far, in blocks, does she walk? Explain how you determined your answer.

ACTIVITY 3.2

Applying the Pythagorean Theorem to Determine Distances on the Coordinate Plane

The points (1, 2) and (3, 7) are shown on the coordinate plane. You can calculate the distance between these two points by drawing a right triangle. When you think about this line segment as the hypotenuse of the right triangle, you can use the Pythagorean Theorem.



1. Calculate the distance between the two points shown.

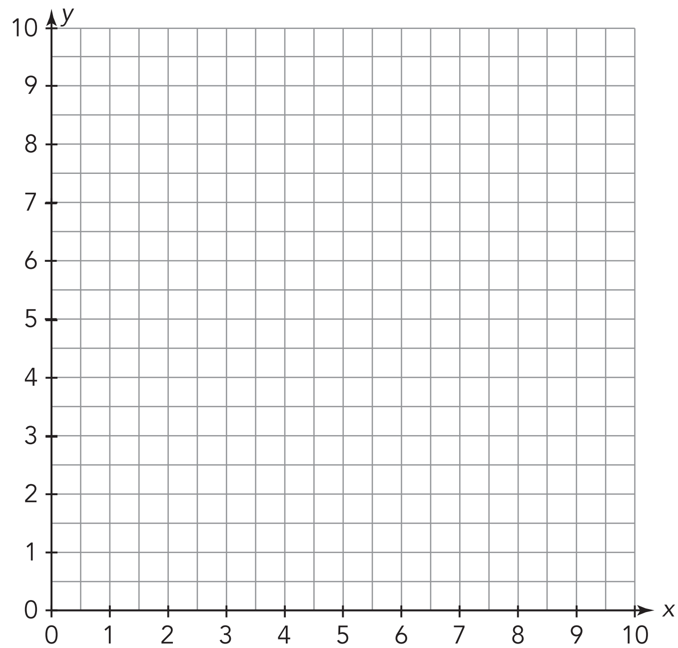
   a. Connect the points with a line segment. Draw a right triangle with this line segment as the hypotenuse.

   b. Determine the lengths of each leg of the right triangle. Then use the Pythagorean Theorem to determine the length of the hypotenuse. Round your answer to the nearest tenth.

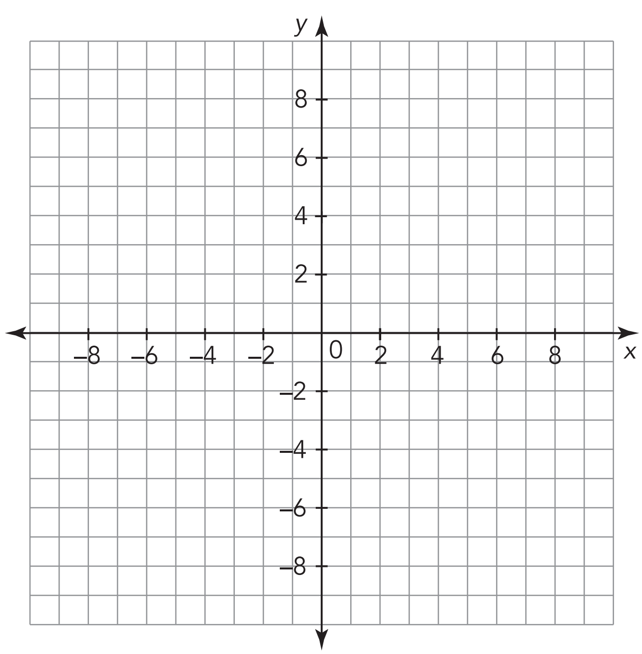
Therefore, when you think of the distance between two points as a hypotenuse, you can draw a right triangle and then use the Pythagorean Theorem to calculate its length.

2. Determine the distance between each pair of points. Round your answer to the nearest tenth.

   a. (3, 4) and (6, 8)

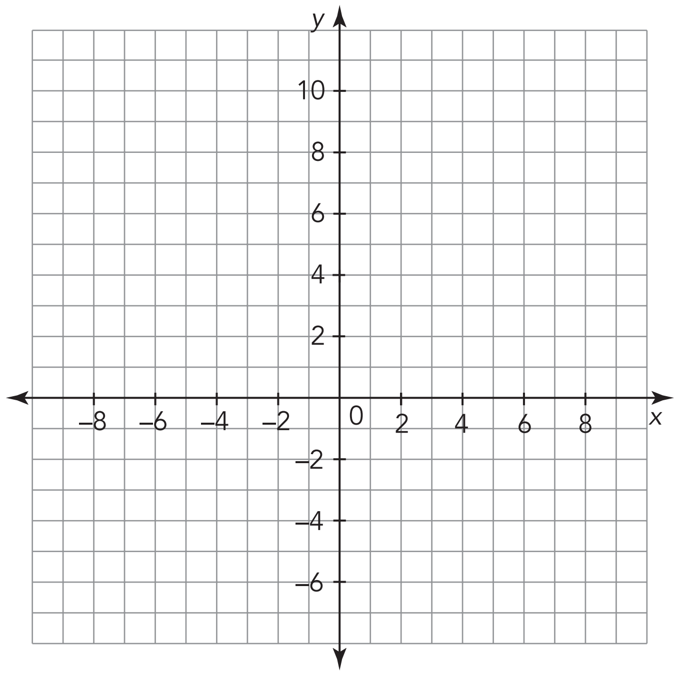


   b. (−6, 4) and (2, −8)

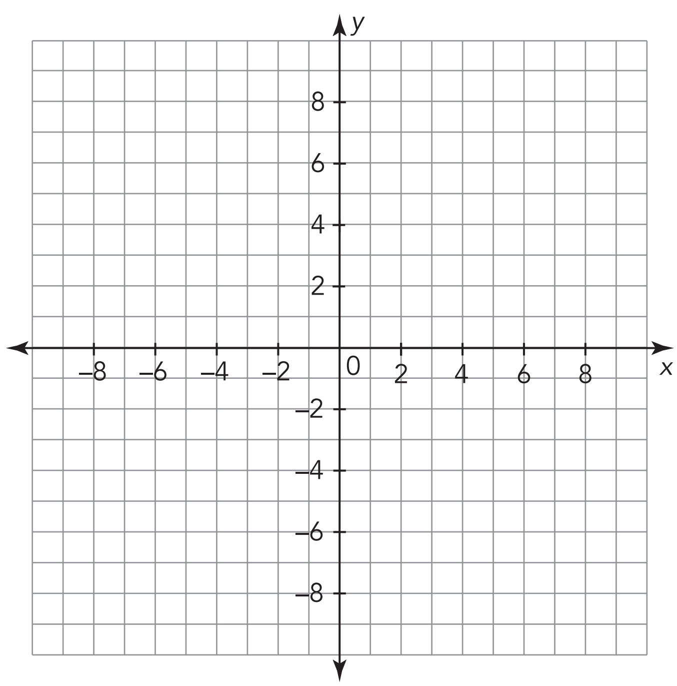


Make sure to pay attention to the intervals shown on the axes.

c. (−5, 2) and (−6, 10)



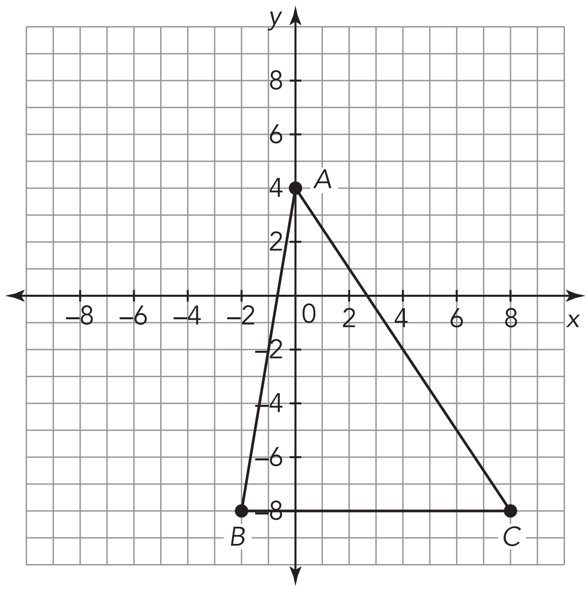
d. (−1, −4) and (−3, −6)



Talk the Talk

Exit Ticket

Use the coordinate plane shown to answer each question.



1. What are the coordinates of the vertices of $\triangle ABC$?

2. What is a strategy for determining the length of side *AC*?

3. Determine the length of side *AC*.

4. Can the same strategy be used to determine the length of side *AB*?

5. Determine the length of side *AB*.

Lesson 3 Assignment

Write

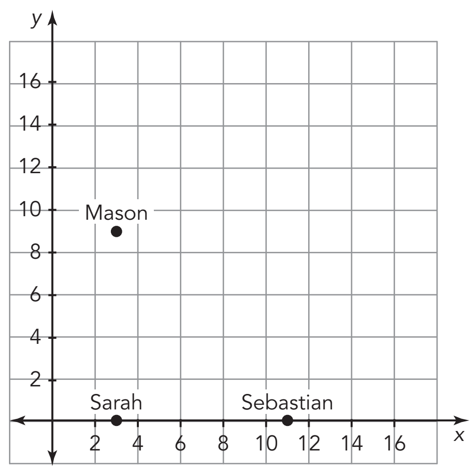
In your own words, explain how to determine the distance between two points on a coordinate plane when the points have different *x*- and *y*-coordinates.

Remember

The Pythagorean Theorem can be used to determine the distance between two points on a coordinate plane.

Practice

1. Mason is playing soccer with his friends, Sarah and Sebastian. The grid shows their locations on the soccer field. Each grid square represents a square that is 2 meters long and 2 meters wide. How far does Mason have to kick the ball to reach Sebastian?



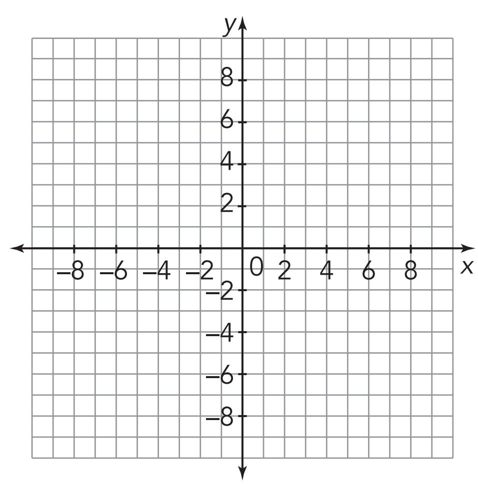
2. Graph and connect each pair of points on a coordinate plane. Then, calculate the distance between each pair of points. Round your answer to the nearest tenth, when necessary.

   a. (−8, 3) and (−2, 9)

   b. (−6, 8) and (−1, 2)

   c. (8, −7) and (4, −4)

   d. (8, 8) and (2, 1)



Lesson 3 Assignment

3. Calculate the distances between the points.

   a. (4, 1), (2, 1), and (4, 4)

   b. (1, −4), (1, 1), and (−2, −4)

Prepare

1. Imagine that the rectangular solid is a room. An ant is on the floor situated at point *A*. Describe the shortest path the ant can crawl to get to point *B* in the corner of the ceiling.

2. Suppose it isn't really an ant at all—it's a fly! Describe the shortest path the fly can fly to get from point *A* to point *B*.

3. If the ant's path and the fly's path were connected, what figure would they form?

4 Side Lengths in Two and Three Dimensions

OBJECTIVES

- Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in mathematical and real-world problems.
- Apply the Pythagorean Theorem to determine the lengths of diagonals of two- and three-dimensional figures.

NEW KEY TERM

- diagonal

You have learned about the Pythagorean Theorem and its converse.

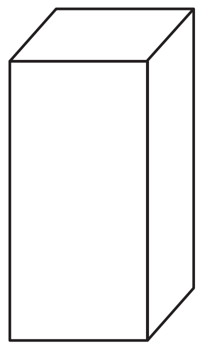
How can you apply the Pythagorean Theorem to determine lengths in geometric figures?

Getting Started

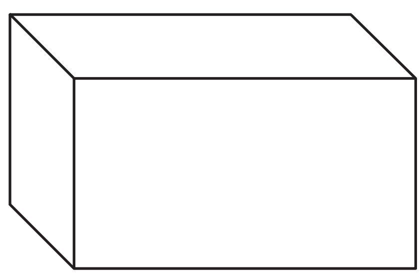
Diagonally

Draw all of the sides you cannot see in each rectangular solid using dotted lines. Then, draw a three-dimensional diagonal using a solid line.

1.



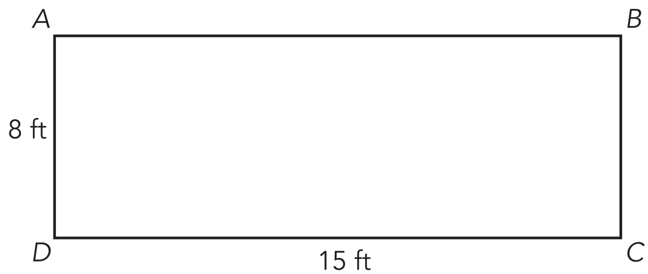
2.



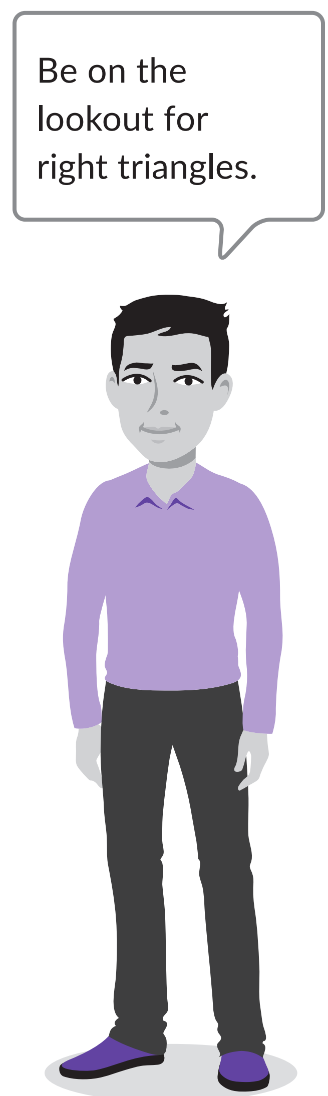
ACTIVITY 4.1 Determining the Lengths of Diagonals of Rectangles and Trapezoids

Previously, you have drawn or created many right triangles and used the Pythagorean Theorem to determine side lengths. In this lesson, you will explore the diagonals of various shapes.

1. Rectangle *ABCD* is shown.



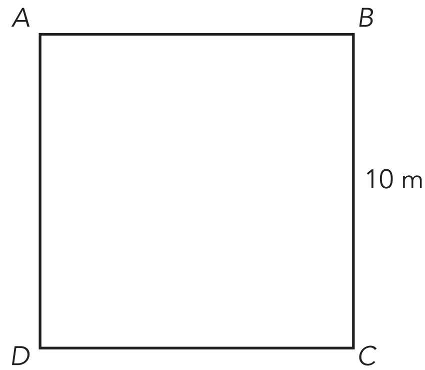
a. Draw diagonal *AC* in Rectangle *ABCD*. Then, determine the length of diagonal *AC*.



b. Draw diagonal *BD* in Rectangle *ABCD*. Then, determine the length of diagonal *BD*.

c. What can you conclude about the diagonals of this rectangle?

2. Square *ABCD* is shown.



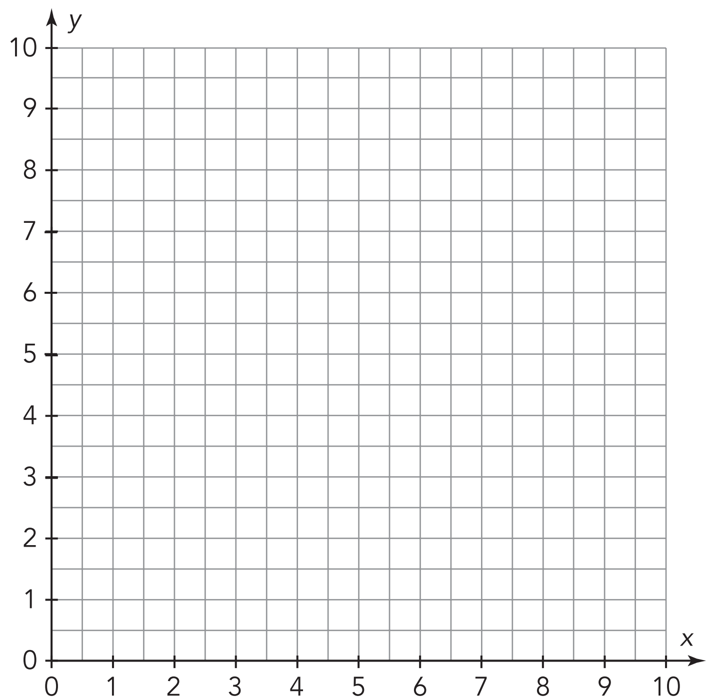
a. Draw diagonal *AC* in Square *ABCD*. Then, determine the length of diagonal *AC*.

b. Draw diagonal *BD* in Square *ABCD*. Then, determine the length of diagonal *BD*.

All squares are also rectangles, so does your conclusion make sense?

c. What can you conclude about the diagonals of this square?

3. Graph and label the coordinates of the vertices of Trapezoid *ABCD:*
   A(1, 2), *B*(7, 2), *C*(7, 5), *D*(3, 5).



   a. Draw diagonal *AC* in Trapezoid *ABCD*.

   b. What right triangle can be used to determine the length of diagonal *AC*?

   c. Determine the length of diagonal *AC*.

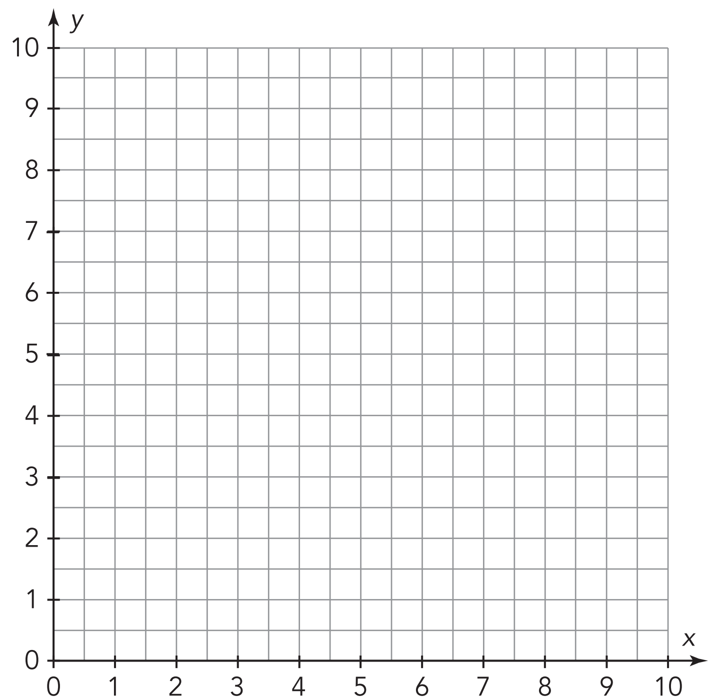
   d. Draw diagonal *BD* in Trapezoid *ABCD*.

   e. What right triangle can be used to determine the length of diagonal *BD*?

   f. Determine the length of diagonal *BD*.

   g. What can you conclude about the diagonals of this trapezoid?

4. Graph and label the coordinates of the vertices of isosceles Trapezoid *ABCD*: *A*(1, 2), *B*(9, 2), *C*(7, 5), *D*(3, 5).



a. Draw diagonal *AC* in Trapezoid *ABCD*.

b. What right triangle can be used to determine the length of diagonal *AC*?

c. Determine the length of diagonal *AC*.

d. Draw diagonal *BD* in Trapezoid *ABCD*.

e. What right triangle can be used to determine the length of diagonal *BD*?

f. Determine the length of diagonal *BD*.

g. What can you conclude about the diagonals of this isosceles trapezoid?

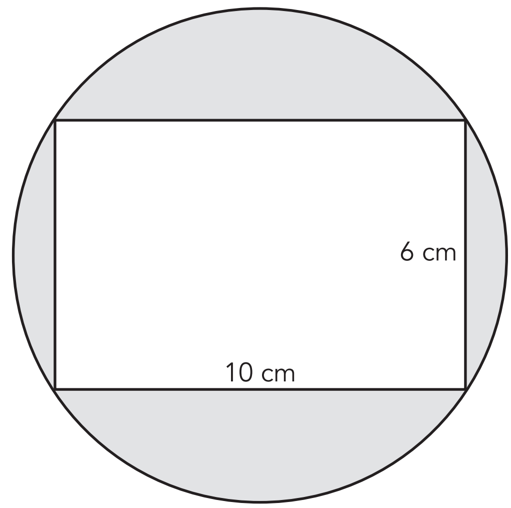
ACTIVITY 4.2

Using Diagonals to Solve Problems

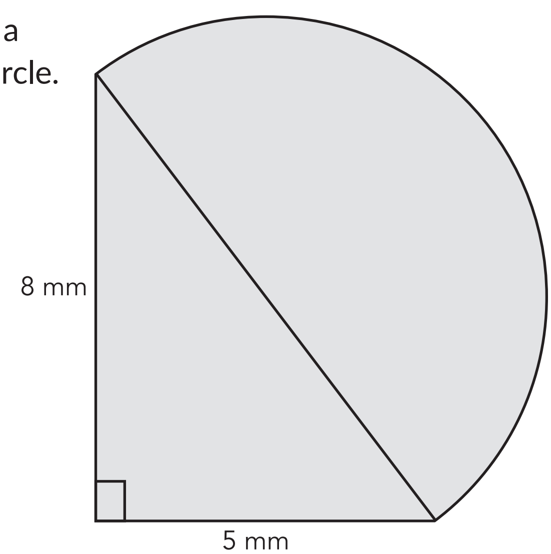
Use your knowledge of right triangles, the Pythagorean Theorem, and area formulas.

1. Determine the area of each shaded region. Use 3.14 for π and round to the nearest tenth.

   a. A rectangle is inscribed in a circle as shown.



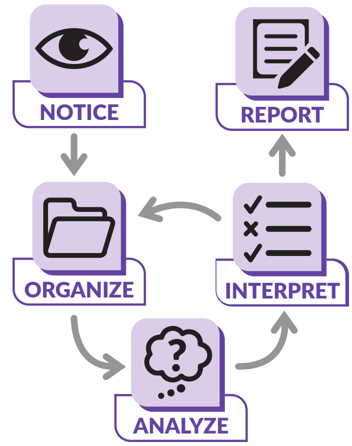
   b. The figure is composed of a right triangle and a semi-circle.



ACTIVITY 4.3

Diagonals in Solid Figures

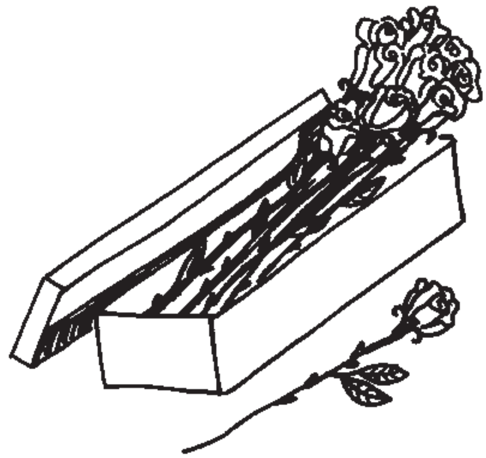
PROBLEM SOLVING



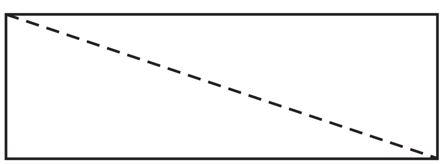
A rectangular box of long-stem roses is 18 inches in length, 6 inches in width, and 4 inches in height.

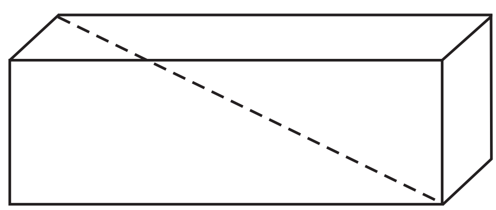
Without bending a long-stem rose, you are to determine the maximum length of a rose that will fit into the box.

1. What makes this problem different from all of the previous applications of the Pythagorean Theorem?



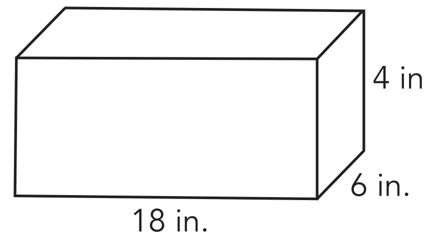
2. Compare a two-dimensional diagonal to a three-dimensional diagonal. Describe the similarities and differences.





3. Which diagonal represents the maximum length of a rose that can fit into a box?

4. Consider the rectangular solid shown.



a. Draw all of the sides in the rectangular solid you cannot see using dotted lines.

b. Draw a three-dimensional diagonal in the rectangular solid.

c. Let's consider that the three-dimensional diagonal you drew in the rectangular solid is also the hypotenuse of a right triangle. If a vertical edge is one of the legs of that right triangle, where is the second leg of that same right triangle?

d. Draw the second leg using a dotted line. Then, lightly shade the right triangle.

e. Determine the length of the second leg you drew.

f. Determine the length of the three-dimensional diagonal.

g. What does the length of the three-dimensional diagonal represent in terms of this problem situation?

5. Describe how the Pythagorean Theorem was used to solve this problem.

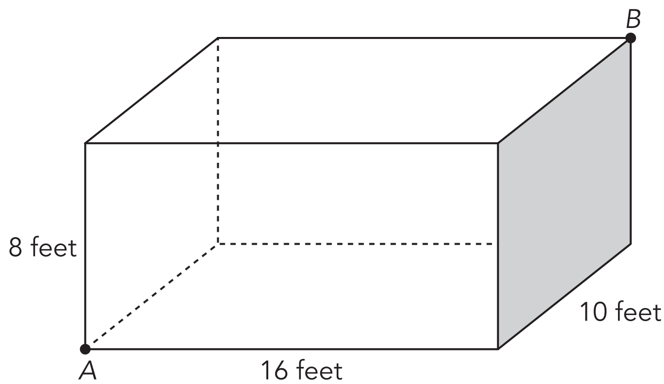
Talk the Talk

The Ant and the Fly Again

A rectangular room is 10 ft × 16 ft × 8 ft.

An ant crawls from point *A* to point *B* taking the shortest path.

A fly flies from point *A* to point *B* taking the shortest path.



1. Whose path was shorter?

2. How much shorter is the shorter path?

Ask Yourself . . .
How can you use side lengths in two-and three-dimensions in everyday life?

Lesson 4 Assignment

Write

In your own words, explain how you can determine a diagonal length of a rectangle. Use an example to illustrate your explanation.

Remember

You can use the Pythagorean Theorem to determine the length of a diagonal in a two- or three-dimensional figure.

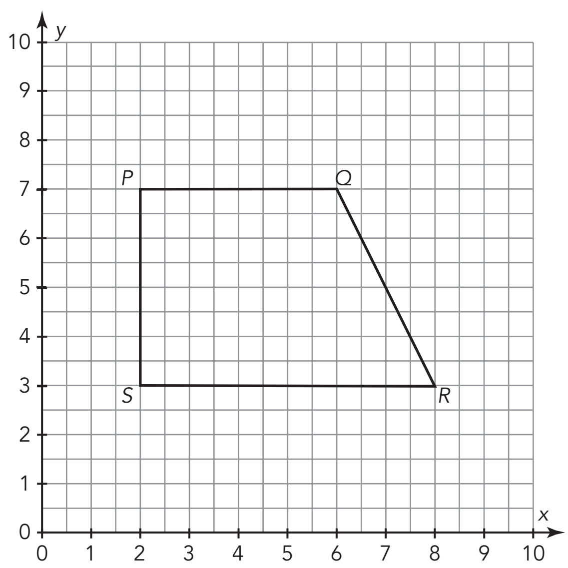
Practice

1. Determine the length of the diagonals in each given quadrilateral.

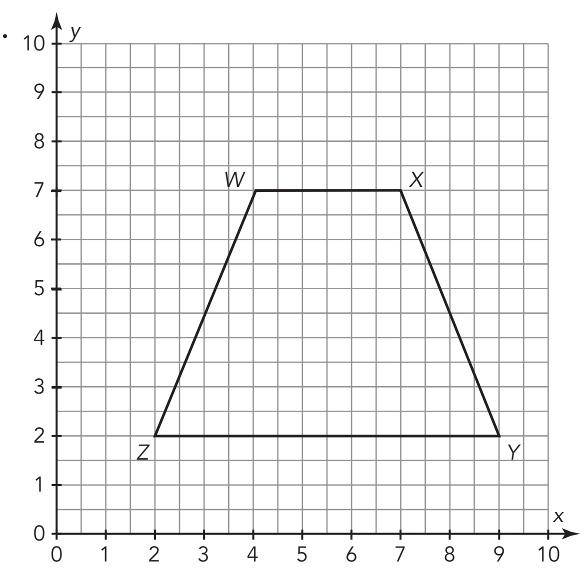
   a. The figure is a square with side lengths of 15 feet.

   b. The figure is a rectangle with a length of 18 inches and a height of 10 inches.

   c.



   d.



Lesson 4 Assignment

2. The size of a TV is often given by the length of the diagonal. Mia is considering a TV has a width of 44 inches and a height of 25 inches. What is the length of the diagonal?

3. A packing company is in the planning stages of creating a box that includes a three-dimensional diagonal support inside the box. The box has a width of 5 feet, a length of 6 feet, and a height of 8 feet. How long will the diagonal support need to be?

4. A plumber needs to transport a 12-foot pipe to a jobsite. The interior of his van is 90 inches in length, 40 inches in width, and 40 inches in height. Will the pipe fit inside his van?

Prepare

Determine the total value of each investment given the principal, interest rate, and time. Use $I = Prt$ for simple interest and $B = P_0(1 + r)^t$ for compound interest.

1. Principal: $1800; Simple Interest Rate: 2.5%; Time: 20 Years

Lesson 4 Assignment

2. Principal: $650; Simple Interest Rate: 5%; Time: 12 Years

3. Principal: $2830; Compound Interest Rate: 3%; Time: 15 Years

4. Principal: $6000; Compound Interest Rate: 4%; Time: 5 Years

TOPIC 2 SELF-REFLECTION

Name: ______________________________

The Pythagorean Theorem

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in *The Pythagorean Theorem* topic by:

TOPIC 2: *The Pythagorean Theorem*	Beginning of Topic	Middle of Topic	End of Topic
explaining the Pythagorean Theorem using models and diagrams.	◯	◯	◯
using the Pythagorean Theorem to determine unknown side lengths in right triangles in mathematical and real-world problems.	◯	◯	◯
using the converse of the Pythagorean Theorem to determine if a given triangle is a right triangle.	◯	◯	◯
explaining the converse of the Pythagorean Theorem.	◯	◯	◯
applying the Pythagorean Theorem to determine the distance between two points on a coordinate plane.	◯	◯	◯

continued on the next page

TOPIC 2 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in *The Pythagorean Theorem* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

TOPIC 2 SUMMARY

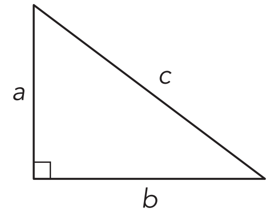
The Pythagorean Theorem Summary

NEW KEY TERMS

- hypotenuse [hipotenusa]
- legs
- Pythagorean Theorem [Teorema de Pitágoras]
- proof
- diagonal of a square
- converse
- converse of the Pythagorean Theorem
- Pythagorean triple [triple Pitagórico]
- diagonal [diagonal]

LESSON 1 The Pythagorean Theorem

In the right triangle shown, the lengths of the sides are *a*, *b*, and *c*.



The side opposite the right angle is called the **hypotenuse**. The other two sides are called **legs** of the right triangle. In the figure, the sides with lengths *a* and *b* are the legs, and the side with length *c* is the hypotenuse.

The special relationship that exists among the squares of the lengths of the sides of a right triangle is known as the *Pythagorean Theorem*. The **Pythagorean Theorem** states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse: $a^2 + b^2 = c^2$.

There are different ways to prove that the Pythagorean Theorem holds true for all right triangles. A **proof** is a line of reasoning used to validate a theorem. In some proofs, you need to draw the diagonal of a square. A **diagonal of a square** is a line segment connecting opposite vertices of the square.

The Pythagorean Theorem can be used to determine unknown side lengths in a right triangle if you know two of the other side lengths.

For example, suppose you want to determine the length of the hypotenuse of the right triangle with leg lengths of 2 and 4.

$$c^2 = 2^2 + 4^2$$
$$c^2 = 4 + 16 = 20$$
$$c = \sqrt{20} \approx 4.5$$

The length of the hypotenuse is approximately 4.5 units.

LESSON 2 The Converse of the Pythagorean Theorem

The **converse** of a theorem is created when the if–then parts of that theorem are exchanged. The **converse of the Pythagorean Theorem** states that if the sum of the squares of the two shorter sides of a triangle equals the square of the longest side, then the triangle is a right triangle.

Consider a triangle with the side lengths 9, 12, and 15.

$$9^2 + 12^2 \stackrel{?}{=} 15^2$$

$$81 + 144 \stackrel{?}{=} 225$$

$$225 = 225$$

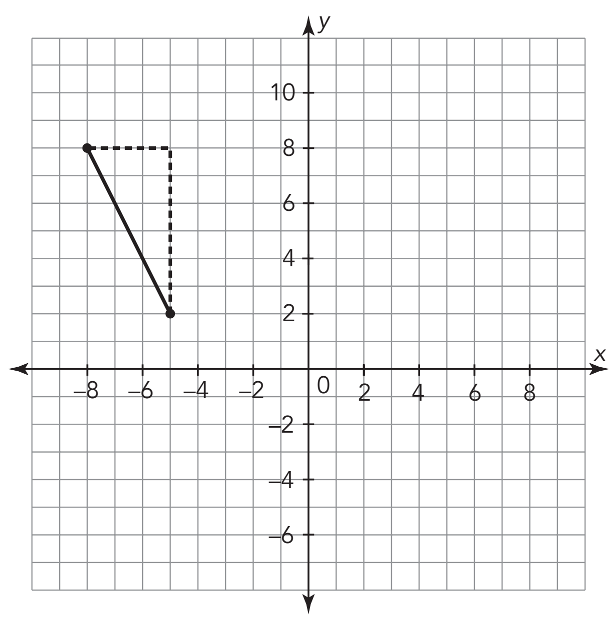
This is a right triangle according to the converse of the Pythagorean Theorem.

Any set of three positive integers a, b, and c that satisfies the equation $a^2 + b^2 = c^2$ is a **Pythagorean triple**. The integers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$. Given a Pythagorean triple, you can identify other right triangles by multiplying each side length by the same factor. For example, if you double the side lengths of 3, 4, and 5, you create another right triangle with side lengths of 6, 8, and 10. $6^2 + 8^2 = 10^2$.

LESSON 3 Distances in a Coordinate System

You can calculate the distance between two points on the coordinate plane by drawing a right triangle. When you think about this line segment as the hypotenuse of the right triangle, you can use the Pythagorean Theorem.

For example, consider the points (−5, 2) and (−8, 8). You can calculate the distance between these two points by first determining the length of each leg of a right triangle formed using that line segment as the hypotenuse. One leg measures 3 units, and the other leg measures 6 units.



$$3^2 + 6^2 = c^2$$

$$9 + 36 = c^2$$

$$45 = c^2$$

$$c = \sqrt{45}$$

$$c \approx 6.7$$

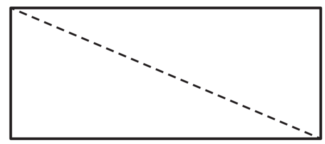
The distance between points (−5, 2) and (−8, 8) is approximately 6.7 units.

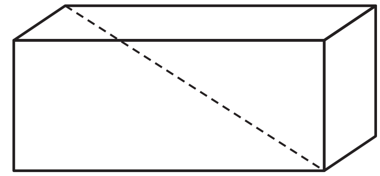
The distance between two points on a coordinate plane is always a positive number.

LESSON 4

Side Lengths in Two and Three Dimensions

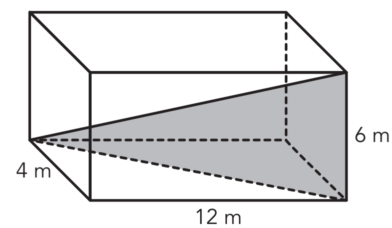
You can use the Pythagorean Theorem to determine the length of a diagonal in a two- or three-dimensional figure.





In a three-dimensional figure, a **diagonal** is a line segment connecting any two non-adjacent vertices. You can use the width and length of the base of the prism to determine the measure of the diagonal of the base. The diagonal on the base of the prism is also one of the legs of a triangle with an inner diagonal as the hypotenuse. The height of the prism is the length of the other leg.

For example, determine the length of the diagonal of the rectangular prism shown.



Determine the length of the diagonal along the bottom face of the prism.

$$c^2 = 4^2 + 12^2$$
$$c^2 = 160$$
$$\sqrt{c^2} = \sqrt{160}$$
$$c \approx 12.6$$

The length of the diagonal along the bottom face is approximately 12.6 m.

Determine the length of the 3-D diagonal.

$$c^2 = 6^2 + 12.6^2$$
$$c^2 = 194.76$$
$$\sqrt{c^2} = \sqrt{194.76}$$
$$c \approx 13.96$$

The length of the 3-D diagonal is approximately 13.96 m.

GRADE 8

TOPIC 3

Small amounts of money invested regularly, including money saved for college and retirement, grow over time.

Financial Literacy: Your Financial Future

1

Simple and Compound Interest

OBJECTIVES

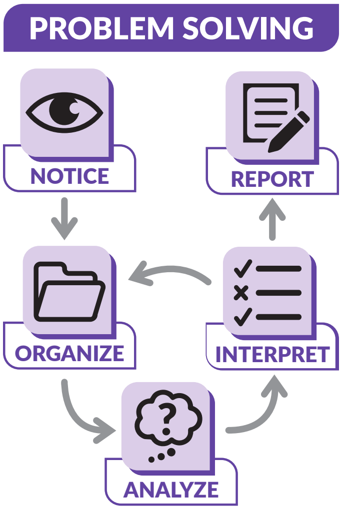
- Calculate simple and compound interest.
- Compare investment earnings from simple and compound interest accounts.
- Explain how small amounts of money invested regularly grow over time.

NEW KEY TERMS

- terms of an investment
- simple interest
- compound interest

You have learned how to calculate simple and compound interest when given the rate, initial amount, and length of time associated with the investment.

How can you use graphs, tables, and equations to analyze situations related to both types of interest?



Getting Started

A Penny Saved

Suppose you are given two different earnings options for 4 weeks, or 28 days.

- **Option 1**: You earn $100,000 per week, paid at the end of each week, or 7 days.
- **Option 2**: A penny is placed in a jar on the first day. Each day the jar contains twice as many pennies as the day before. You get all the pennies in the jar at the end of 4 weeks, or 28 days.

1. Which option will earn you more money? Show all of your work and explain your reasoning.

ACTIVITY 1.1

Terms of an Investment

There are several important factors you should consider for real world investments.

- Consider your financial goals. How much money will you need?
- Think about time. Set short-term goals as well as long-term goals. For example, you may need a car in the near future. You may also need to set money aside for retirement. You will need to invest in both of these goals at the same time.
- Carefully analyze the *terms of an investment*. Some investments will be better for your short-term goals, while others will be better for your long-term goals.

The **terms of an investment** include the type of loan, amount of money invested, and the length of the investment. Investments can earn vastly different amounts of money, depending on the amount of time and the way the interest is growing.

Recall that **simple interest** is a percentage that is paid only on the original principal, while **compound interest** is a percentage that is paid on the principal and interest after each time period.

You may come up with a formula, but you don't have to yet. Simply focus on how the amount grows for each type of investment.

1. To save for retirement, Nahimana plans to invest $5000. The tables show the total values of a $5000 investment made in a simple interest account compared to a $5000 investment made at the same rate in a compound interest account.

Time (years)	Total Value in a Simple Interest Account ($)
0	5000
1	5150
2	5300
3	5450
5	5750
10	6500

Time (years)	Total Value in a Compound Interest Account ($)
0	5000
1	5150
2	5304.50
3	5463.64
5	5796.37
10	6719.58

a. Describe the rate of increase for each investment option.

You may plot the points, but use what you know about rate of change to predict the shape of the graph.

b. Describe the shape of each graph when the total value is plotted as a function of time. Explain your reasoning.

c. Will the simple interest account ever be a better investment option? Explain your reasoning.

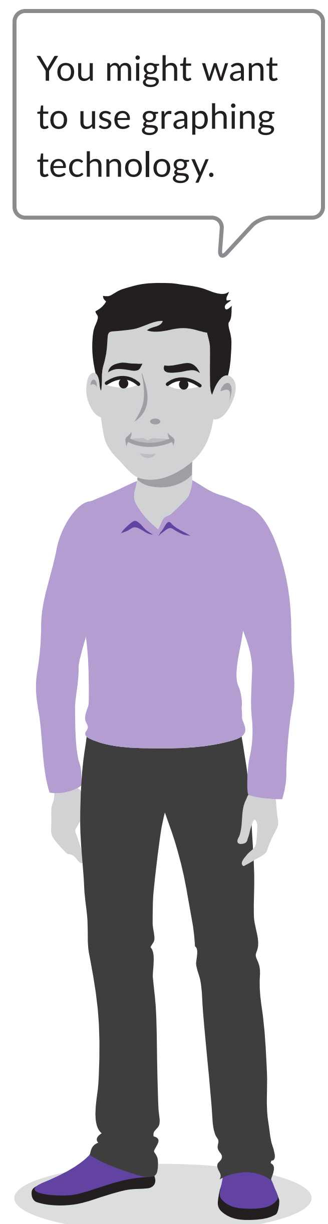
d. Choose the function that represents each investment option. Explain your reasoning.

$y_1 = 150x$ $y_2 = 5000(150x)$ $y_3 = 5000 + 150x$

$y_4 = 5000 + 150^x$ $y_5 = 5000(1 + 0.03)^x$

$y_6 = 5000(0.03)^x$

Simple Interest: ________ Compound Interest: ________



Time (years)	Total Value in a Simple Interest Account ($)

Time (years)	Total Value in a Compound Interest Account ($)

e. Complete each table to represent the total value of each investment after 20 and 30 years. Show all of your work and explain your reasoning.

Simple interest accounts pay the same amount over each time period, while compound interest accounts calculate the interest earned according to the balance after each period. This means that you will earn a percentage of a greater amount every year in a compound interest account.

Recall the formula for compound interest is $A = P(1 + r)^t$, where A is the account balance, or total value of the account, P is the original amount invested, or principal, r is the interest rate, and t is the time invested in years.

2. Identify the principal and rate of the compound interest equation you selected in Question 1 part (d).

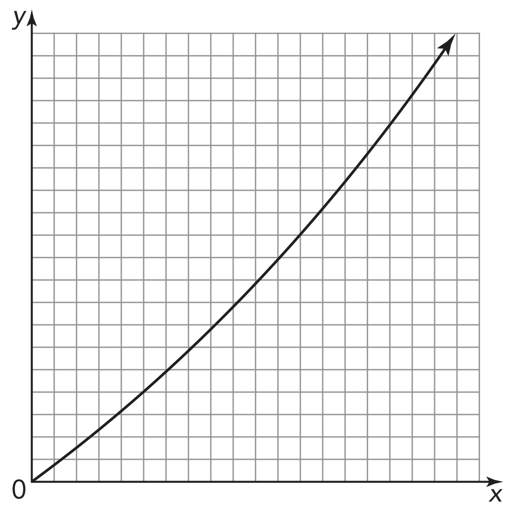
3. Determine whether each representation is an investment made in a simple or compound interest account. Explain your reasoning.

a. $y = 1000 + 20x$

b.

Time (years)	Total Investment Value ($)
0	500
1	510
2	520.20
3	530.60
4	541.22
5	552.04

c. My investment account earns $50.00 each year.

d. 

4. Luna calculates the total value of $400 invested at 3% annual interest for 4 years with simple interest.

> Luna
>
> Simple Interest
> $I = Prt$
> $I = (400)(0.03)(4)$
> $I = 48$
> The total value is $400 + $48 = $448.

Why does Luna add $400 at the end?

5. Mason is saving for his son's college education. Calculate the interest earned from $9000 invested for 18 years in an account that earns 3% annual simple interest.

6. Complete the table to determine the total value of $400 invested at 3% for 4 years in an annual compound interest account.

Time	Principal Invested ($)	Interest Earned ($)	Balance ($)
1 year	400.00	$I = (400.00)(0.03)(1)$ $= 12.00$	$400.00 + 12.00 = 412.00$
2 years	412.00	$I = (412.00)(0.03)(1)$ $= 12.36$	$412.00 + 12.36 = 424.36$
3 years			
4 years			

Ask Yourself:
How does representing mathematics in multiple ways help to communicate reasoning?

7. You wish to invest $50,000 to save for retirement. How much more money is made from $50,000 invested in a 3% annual compound interest account over 30 years, compared to investing the money in a simple interest account? Show all of your work and explain your reasoning.

Talk the Talk

Could *I B* of Interest to You?

- Calculate the total investment value using simple or compound interest for each given principal, rate, and time.
- Choose the letter of the correct response and write it next to the problem.
- All responses may not be used, and you may use a response more than once.

1. ________ Simple interest on $10,000 at 5% annual interest for 3 years
2. ________ Compound interest on $10,000 at 5% annual interest for 3 years
3. ________ Simple interest on $10,300 at 3% annual interest for 4 years
4. ________ Compound interest on $10,300 at 3% annual interest for 4 years

a. $11,576.25

b. $11,586.10

c. $11,592.74

d. $11,500.00

e. $11,536.00

f. $11,555.00

Lesson 1 Assignment

Write

Explain the difference between simple and compound interest. Use a numerical example in your explanation.

Remember

For real-life investments, consider your financial goals and how much money you will need. Set short-term and long-term goals, and carefully analyze the terms of an investment. Some investments will be better for your short-term goals, while others will be better for your long-term goals.

Practice

Complete each table then identify the account as simple interest or compound interest. Explain your reasoning.

1.

Time (years)	Total Amount (dollars)
0	3000
1	3375
2	3750
3	
4	

2.

Time (years)	Total Amount (dollars)
0	2000
1	2450
2	3001.25
3	
4	

Lesson 1 Assignment

Determine the total value for each account given the principal, rate, and time.

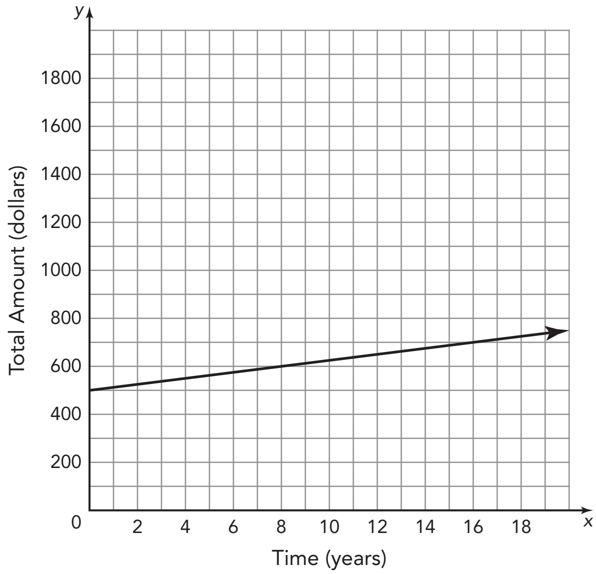
3. Simple interest account
   Principal: $3500
   Rate: 2.5%
   Time: 5 years

4. Compound interest account
   Principal: $2000
   Rate: 2%
   Time: 7 years

Use the given information and graph to answer Questions 5 and 6.

Kayla has been saving for college since an account was opened in her name when she was just a baby.

This graph represents the total amount in Kayla's account over the last 18 years.



Lesson 1 Assignment

5. Analyze the graph of Kayla's account.

   a. Is Kayla's account a simple interest or compound interest account? Explain your reasoning.

   b. Identify the principal and rate of the account. Show your work or explain your reasoning.

6. Kayla decides to wait another 5 years to go to college in order to save more money. How much will be in Kayla's account when she is ready to go to college?

Lesson 1 Assignment

7. One of Kayla's classmates, Sebastian, has also been saving. His parents opened a compound interest account for him when he was a baby. They made an initial investment of $500 and the account had a 2.5% interest rate. How much more money does Sebastian have in his account than Kayla has in her account, after 18 years?

Prepare

Determine the final cost of each item, not including sales tax.

1. Gabriel purchases a guitar priced at $78.44. He has a coupon for 20% off the price. How much does Gabriel pay for the guitar after using the coupon?

2. Diego is purchasing ink cartridges for his computer. Each ink cartridge costs $35.99 and comes with a $10 rebate. How much will Diego pay for one ink cartridge after the rebate?

2 Terms of a Loan

OBJECTIVES

- Analyze the terms of a loan to make financially responsible decisions.
- Choose loans that cost less money, based on the terms of the loan.
- Choose investments that earn more money, based on the terms of the investment.

NEW KEY TERM

- deferment

You know how to calculate simple and compound interest.

How can that knowledge help you make decisions about funding your education and making other large purchases?

Getting Started

Coming to Terms with it All

1. What items are included in the terms of a loan?

2. How do the terms of a loan you listed relate to the simple interest and compound interest formulas?

The first national bank in Texas opened in 1865 in Galveston. It operated under the National Bank Act of 1863 and was the second bank chartered in Texas.

3. If two customers go to the same bank to take a loan out for the same amount of money for the same length of time, will they automatically be charged the same interest rate? Explain.

ACTIVITY 2.1

Comparing Loans

Understanding the impact that time and interest have on a loan or investment can help you make good financial decisions. In the previous lesson, you analyzed how compound interest makes an investment or loan grow much more quickly compared to simple interest. The interest rate and length of the loan also determine the amount of money that you pay for a loan. Even a slight difference in loan terms can add up to a lot of money.

When most people buy a house, they borrow money from the bank. The bank technically owns the house until it is paid off.

1. Carlos and Isaiah are each buying houses just outside of Austin. Carlos has excellent credit, earning him the lowest possible interest rate. Isaiah has a lower credit score—enough to prevent him from qualifying for the best loan terms.

Carlos's Loan

- Loan Amount: $175,000
- Interest Rate: 4%
- Time: 30 years
- Payment: $835.48 per month

Isaiah's Loan

- Loan Amount: $175,000
- Interest Rate: 5%
- Time: 30 years
- Payment: $939.44 per month

   a. Describe the similarities and differences in their loans.

   b. How much more money will Isaiah pay per month?

   c. How much more money will Isaiah pay over the course of a year?

A home loan is also known as a mortgage. The Texas Land and Mortgage Company of London, Ltd. was the first mortgage company in the state of Texas. The Dallas branch of the English company opened in 1882.

d. How much more money will Isaiah pay over 30 years because of the higher interest rate?

A good credit score can help you get the best possible interest rate available. But, interest rates can also vary, depending on the lender and the length of the loan. Shopping around for the best loan terms could save you thousands of dollars.

2. Mei is buying a new car. She is considering two different loan options.

Bank

- Loan Amount: $20,000
- Interest Rate: 3% interest
- Time: 3 years
- Monthly Payment: $581.62

Credit Union

- Loan Amount: $20,000
- Interest Rate: 3% interest
- Time: 6 years
- Monthly Payment: $303.87

How can you use the monthly payment to determine how much is paid in one year?

a. Describe the similarities and differences in the loans.

b. How much more money will Mei pay per month for the bank loan?

c. Determine the total amount Mei will pay taking the bank loan.

d. Determine the total amount Mei will pay taking the credit union loan.

e. How much money will Mei save by choosing the loan with the shortest time?

3. Adriana considers two different loan options when buying a home in Houston.

Local Bank

- Loan Amount: $135,000
- Interest Rate: 4%
- Time: 15 years
- Monthly Payment: $998.58

Online Financial Institution

- Loan Amount: $135,000
- Interest Rate: 5%
- Time: 30 years
- Monthly Payment: $724.71

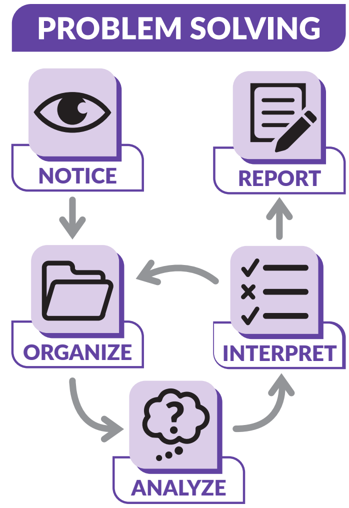
a. Adriana claims: "I'm going to save so much money with the online financial institution loan!" Is she correct? Explain your reasoning.

Review the definition of **explain your reasoning** in the Academic Glossary.

b. Identify and explain the advantages and disadvantages of each loan offer.

4. An automobile dealership claims that you can ride home in their best car for only $299.99 per month. To make a good financial decision, what questions should you ask a salesperson at the dealership?

ACTIVITY 2.2 Loan Deferment



Education is an investment. The idea is that paying for school can pay dividends in the form of a good career with a high salary. Like any investment, the terms of student loans dictate the monthly payments, total time of the loan, and the total amount of money that is paid.

The average college graduate can have over $20,000 in student loans. Between the time that they graduate and the time they find a job, many students choose a loan *deferment*. A **deferment** is a period of time, usually up to two years, in which students delay paying the principal and interest on their loan. A deferment can be beneficial, but understanding how compound interest affects the principal of a loan will help you make wise financial decisions.

1. Logan graduated with $35,000 in student loans at a compound interest rate of 6%. He deferred payments for two years while looking for work. Determine the new balance of the loan after two years. Show all of your work and explain your reasoning.

2. Angelina has $51,500 in student loans at a 7% compound interest rate. She has the money to begin paying her loans, but considers spending money in her first two years of work on a car and furniture.

   a. Determine the additional interest added to her loan after a two-year deferment.

   b. Identify and explain the advantages and disadvantages of Angelina's deferred payments versus making payments immediately.

Talk the Talk

Mr. & Mrs. Park

Option 1	Option 2
• Loan Amount: $150,000	• Loan Amount: $150,000
• Interest Rate: 6%	• Interest Rate: 4%
• Time: 25 years	• Time: 15 years
• Monthly Payments: $932.67	• Monthly Payments: $1211.40

Mr. and Mrs. Park disagree over which option to take. Mr. Park wants to take Option 1, and his wife wants to take Option 2. Complete each statement based upon the information provided. Show work to support your numerical responses.

________________ said, "But honey, when our monthly payment is that high, we won't have any extra money in case we have an unexpected expense such as a home repair or if we want to go on vacation."

________________ said, "But dear, we want to have the house paid off before our kids are in college."

Mrs. Park said, "With the option I want, by the time we pay our mortgage off, we will be paying ________________ for our home. With the option you want, we will end up paying ______________ . That means we would be paying ________________ more for our house should we choose your way instead of mine!

Lesson 2 Assignment

Write

Explain what a deferment is in your own words. What are the pros and cons of a deferment?

Remember

Time and interest are two of the most significant factors related to a loan or investment. Even a slight difference in loan terms can add up to a lot of money. Understanding these impacts can help you make the best financial decisions.

Practice

Parker is buying a new boat. He is considering two different loan options.

Bank A

- Loan amount: $65,000
- Interest Rate: 4.5% interest
- Time: 10 years
- Monthly Payment: $673.65

Bank B

- Loan amount: $65,000
- Interest Rate: 4.5% interest
- Time: 15 years
- Monthly Payment: $497.25

Analyze Parker's loan information.

1. How much more money will Parker pay per month for the loan at Bank *A*?

2. Determine the total amount Parker will pay if he chooses Bank *A*.

Lesson 2 Assignment

3. Determine the total amount Parker will pay if he chooses Bank *B*.

4. How much money will Parker save by choosing the loan with the shortest time?

Use the given information to complete Questions 5–9.
Avery and Valentina are two classmates who are attending the same college. Both have student loans to repay. Their loan information is shown.

Avery's Loan
- Loan Amount: $45,000
- Interest Rate: 3.5%
- Time: 15 years
- Payment: $321.70/month

Valentina's Loan
- Loan Amount: $45,000
- Interest Rate: 5%
- Time: 15 years
- Payment: $355.86/month

Analyze Avery's and Valentina's loan information.

5. Whose loan has better terms? Explain your reasoning.

Lesson 2 Assignment

6. If Avery and Valentina got their loans from the same bank, why are their interest rates different?

7. Using your answer from Question 5, determine how much more the person with the worse loan will pay over the 15-year time period. Show your work.

After graduating, Avery and Valentina take deferments because they are having difficulty finding jobs. They both take a 2-year deferment.

8. Determine the amount of interest that will be added to Avery's loan during the 2-year deferment. Explain your reasoning.

Lesson 2 Assignment

9. Determine the new balance of Valentina's loan after the deferment.

Prepare

1. Calculate the number of years for which an investment was made when the simple interest is $832 for a principal of $5200 at an interest rate of 2%.

2. Calculate the interest rate for a loan when the simple interest earned on a principal of $9600 was $2304 on a loan taken for 4 years.

3 Online Calculators

OBJECTIVES

- Use technology, such as online calculators, to make financially responsible decisions.

NEW KEY TERMS

- online calculator
- cash advance

You have learned about debit and credit cards, loans, and simple and compound interest.

How can technology, such as online calculators, improve your efficiency and help you to analyze and compare financial options?

Getting Started

A Swarm of Killer Fees

In this lesson, you will learn about additional fees associated with borrowing money.

Determine the dollar amount of the interest paid for each purchase.

1. Twelve monthly payments of $50 were made to pay for a $500 purchase.

2. Eighteen monthly payments of $65 were made to pay for an $800 purchase.

3. Twenty-four monthly payments of $90 were made to pay for a $1500 purchase.

Determine the dollar amount for each question.

4. Kayla buys $2800 worth of furniture on her credit card. She is unable to make her monthly payment and is billed 20% interest. How much extra does Suzanne pay on these charges?

5. Diego uses his credit card to make a $5000 down payment for a new car. He does not make his monthly payment and is billed 20% interest. What is the new balance on the credit card, including the interest?

ACTIVITY 3.1 Credit Cards, Cash Advances, and Easy Access Loans

The mathematics behind debt is more complicated than just determining compound interest. Should the loan not be paid off in full at the end of the month, many factors play a role in determining the details of different payment options. For this reason, technology can be a valuable tool for comparing payment options for credit cards, cash advances, and easy access loans. One such technology tool is an **online calculator** with an Internet-based application that quickly performs calculations for the user.

1. After graduating from college, Emily spent approximately $5000 on new furniture for her apartment. She used her credit card that has a 17% interest rate, so that she could pay $75.00 per month over time instead of paying for it all at once. Use an online calculator to determine:

   a. the number of years it will take Emily to pay off the loan.

   b. the total amount of interest paid on the loan.

   c. the total amount paid for the furniture.

2. Lucas purchases a new $1000 entertainment system for his game room. Instead of paying for it all at once, he decides to use a credit card.

   a. Use an online credit card calculator to analyze Lucas's options.

      Option A: For one credit card, the interest rate is 22%, but the minimum payment is only $20.00.

      Option B: For another credit card, the interest rate is only 17%, but the minimum payment is $45.00.

   b. Which credit card is the best option?

Credit card companies sometimes send checks to their customers which can be used for cash advances. A **cash advance** is a service provided by credit card companies that allows their customers to take out money directly from a bank or ATM. This can be appealing for someone who is in need of money at a given moment but would like to pay the money back over time.

Consumers should be very cautious about cash advances from credit cards. A percentage of the principal is immediately added to the loan, and a high interest rate is applied. The interest rate on a cash advance is usually much higher than the interest rate on the credit card.

3. Mason uses a credit card cash advance for $800 to use on his vacation. The credit card company charges an additional 4% at the time he cashes the check. The interest rate for the cash advance is 22%. He plans on paying for his vacation by making $20 payments each month until he pays off the loan.

    a. How long will it take Mason to pay off his vacation?

    b. Determine the total amount he will have to pay on the loan. Show all of your work and explain your reasoning.

4. Mason has the option of using a different credit card for the $800 cash advance. This credit card company charges an additional 5% at the time he cashes the check, but the interest rate for the cash advance is only 18%. Again, he plans on paying for his vacation by making $20 payments each month until he pays off the loan.

    a. How long will it take Mason to pay off his vacation?

    b. Determine the total amount he will have to pay on the loan. Show all of your work and explain your reasoning.

5. Which credit card is the best option for the cash advance?

For individuals who do not have or do not qualify for credit cards, an easy access loan is another option for a cash advance with a high interest rate.

6. Sarah has an unexpected expense this month. She is looking to get an easy access loan to pay off the expense immediately. Sarah can only afford an extra $150 each month. Use an online calculator to consider her options.

Option A	Option B
• Loan Amount: $3700	• Loan Amount: $3700
• Interest Rate: 27%	• Interest Rate: 24%
• Finance Charge: $15 for every $100 borrowed	• Finance Charge: $18 for every $100 borrowed

   a. Determine the total amount Sarah will pay if she chooses option *A*.

   b. Determine the total amount Sarah will pay if she chooses option *B*.

   c. How long will it take Sarah to pay off the loan if she chooses option *A*?

   d. How long will it take Sarah to pay off the loan if she chooses option *B*?

   e. Which is the best option for her easy access loan?

It is important to compare your options before making any financial decisions.

7. Gabriel had an unanticipated repair to his car that cost $3000. He currently has $5000 in his checking account and is trying to decide the best option to pay for the repair. Use an online calculator to consider his options if necessary.

   a. If Gabriel decides to pay cash, how much does the repair cost him?

   b. If Gabriel decides to use his debit card, how much does the repair cost him?

   c. If Gabriel decides to use his credit card with 18% interest and a term of 24 months, how much does the repair cost him?

   d. If Gabriel decides to take an easy access loan with an interest rate of 25% and a finance charge of $12 per $100 borrowed for 2 years, how much does the repair cost him?

8. Determine which option is more financially responsible for Gabriel. Explain your reasoning.

   a. Credit card vs. easy access loan

   b. Debit card vs. credit card

   c. Cash vs debit card

9. Explain the advantages and disadvantages of each payment method.

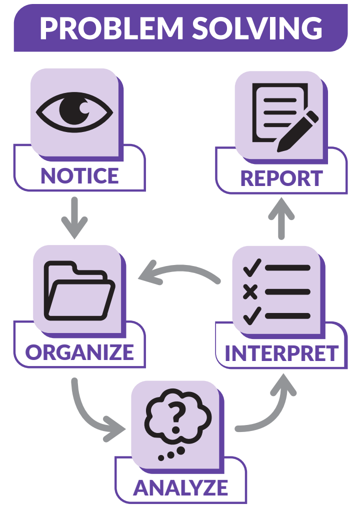
   a. Cash

   b. Debit card

   c. Credit card

   d. Cash advance

ACTIVITY 3.2 Loan Options



1. Jasmine was recently accepted to several colleges. Funding her education is now a financial dilemma for her family. She narrows her selection to two options: Plan *A* and Plan *B*.

Plan A
- Attend her dream school
- Loan Amount: $85,000
- Interest Rate: 7%
- Length of Loan: 15 years

Plan B
- Attend community college for the first 2 years and then transfer to a public Texas college for the last 2 years
- Loan Amount: $25,000
- Interest Rate: 7%
- Length of Loan: 15 years

a. Use an online financial aid calculator to determine the monthly payments for each plan.

b. Determine the total cost of each loan.

Should Jasmine choose Plan *B*, she will need to earn excellent grades in her first two years to transfer.

c. Jasmine believes that attending her dream school will eventually lead to a higher-paying career. Use an online calculator to determine the salary necessary to repay each loan.

d. Which option do you think is better for Jasmine? Explain your reasoning.

2. Camilla must borrow $50,000 in student loans to become a pharmacist. The best possible loan rate that she can get is 4%. She estimates that she will make about $60,000 per year when she graduates. Camilla realizes that the longer you pay on a loan, the more money you will end up paying.

   a. Use an online calculator to determine the least number of months for a loan that she can afford with her expected pharmacist salary.

   b. Use an online calculator to determine the amount of interest that she will pay on this loan.

3. Sarah takes out $10,000 in loans for each year, with an interest rate of 6.8%. She didn't keep track of the courses she needs to become a teacher and ends up having to stay in school for a 5th year. She will pay on her loans for 15 years. Use an online calculator to determine the cost of this 5th year of school. Show all of your work and explain your reasoning.

4. Decide whether you agree or disagree with each statement. Justify your answer.

   a. College is affordable for everybody.

   b. Students should pursue their passion in college.

   c. My financial decisions can help determine whether I go to college.

   d. You should attend the best school that you can get into.

Talk the Talk

Give Credit Where Credit's Due

Use an online calculator to answer each question.

Before he went off to college, Sebastian bought a new computer, a printer, and other electronic devices, totaling $2000. He made the purchase using the store's credit card with an interest rate of 18%. Sebastian's goal is to pay off the credit card in 2 years.

1. What will be Sebastian's monthly payment on his credit card?

2. How much interest will Sebastian end up paying by the time he pays off his credit card?

3. Sebastian decides to double his payments so he can pay his credit card off earlier. How long will it take Sebastian to pay off his credit card?

4. How much will Sebastian save by paying his credit card off earlier?

Lesson 3 Assignment

Write

Define each term and give an example of when you would use each.

- online calculator
- cash advance

Remember

Technology, such as online calculators, can be helpful in quickly calculating payment options. They can determine the amount of time it takes to pay off a credit card, the interest amount, and the total debt payment amount when given the principal, interest rate, and monthly payment.

Practice

Use an online calculator to answer each question about credit card charges.

1. Total Amount Charged: $3000
   Interest Rate: 14%
   Time: 6 months
   Determine the cost of the monthly payments for this charge.

2. Total Amount Charged: $4500
   Interest Rate: 16%
   Monthly payment: $250
   Determine how long it will take to pay off the bill.

Lesson 3 Assignment

3. Nahimana broke her arm and had unexpected hospital expenses. She decides to use an easy access loan to help pay for the expenses. Use an online calculator to consider her options.

Local Institution	Online Institution
• Loan Amount: $5000 • Interest Rate: 23% • Finance Charge: $20 per $100 borrowed	• Loan Amount: $5000 • Interest Rate: 30% • Finance Charge: $14 per $100 borrowed

a. After looking at her finances, Nahimana decides she can pay $300 a month. How long will it take Nahimana to pay back the loan at each location?

b. What is the total amount Nahimana will pay at each location?

c. Which easy access loan would be the best option for Nahimana?

Lesson 3 Assignment

4. Luna took a wonderful vacation this summer. She put all of her travel charges on her credit card. The charges, including flights, lodging, and souvenirs, come to a total of $3200. Rachel's credit card has an interest rate of 20%.

   a. Luna's goal is to pay off her vacation charges in 6 months. Use an online calculator to determine the amount of her monthly payments.

   b. Luna cannot afford to make the monthly payments to pay off the charges in 6 months. Instead, she decides to make payments of $350 each month. Use an online calculator to determine how long it will take Luna to repay the loan.

   c. Using your answer from part (b), determine the total cost of Luna's vacation.

Lesson 3 Assignment

5. Logan is heading to college. He has two different colleges in mind. Each college's tuition is slightly different. The student loan information for each college is shown.

Out-of-State College	In-State College
• Loan Amount: $45,000	• Loan Amount: $30,000
• Interest Rate: 6%	• Interest Rate: 7%
• Time: 20 years	• Time: 20 years

a. Use an online calculator to determine monthly payment amounts for each loan.

b. Determine the total cost of each loan. Explain your reasoning.

Lesson 3 Assignment

6. Parker and Emily are graduating from college together. Parker took larger course loads each semester and will graduate in 3 years, instead of 4. Emily graduated in 4 years. At graduation, Parker's student loan balance is $24,000, and Emily's student loan balance is $32,000. Both of their loans have an interest rate of 6.6%, and both plan to repay their loans in 10 years. Use an online calculator to determine the total amount of money each person will repay for their loans. How much money will Parker save by graduating a year early?

Prepare

1. What type of career would you like to have when you are an adult?

2. What makes this career appealing to you?

3. What type of post-secondary education is needed for this career?

4. Do you know anyone who has this career? Explain.

4 Financing Your Education

OBJECTIVES

- Critically analyze information about college.
- Research several colleges, determining the admission requirements and cost.
- Develop a financial plan for attending college.

You have learned about different ways to finance education after high school.

What should you consider to make a post-secondary education both feasible and a responsible investment?

Getting Started

Fact or Fiction?

Additional schooling after graduation may seem like a long way off. You aren't even in high school yet, so you may wonder why there's already such an emphasis on post-secondary education. In reality, a college or a trade school really isn't all that far away, and the decisions that you make in middle school can put you on the right path toward success. In this lesson, you will research post-secondary options and develop a financial plan to pay for your education. Before starting this work, it's important to look at different myths.

1. Determine whether each statement is fact or fiction. Justify your answer.

   a. Texas high schools have a recommended group of classes that all students should take to prepare them for post-secondary education.

   b. You should take 2 years of a foreign language in high school to prepare for college.

   c. You should take a lot of math in high school to prepare for college, but it is not necessarily recommended to take math every year.

   d. You have to know your career choice before starting college.

e. Should you not be proficient in math and English after graduating high school, a college may require that you pay for extra courses that don't count toward your degree.

f. You can earn college credits while in high school.

g. All degrees after high school are 4-year degrees.

h. Post-secondary education is not for everybody.

i. It's never too early to start planning for a college or trade school.

ACTIVITY 4.1

Post-Secondary Options

Hard work and careful planning is necessary to not only get into college, but also to pay for it. After you consider what is important to you, the different types of schools, and their relative costs, you can begin developing a payment strategy.

- A public school receives funding from the state, while private, independent institutions rely on tuition and donations to fund programs.
- A community college is generally a school for local residents that will accept most applicants.
- Community colleges offer 2-year degrees as well as coursework that may transfer to another institution.
- A technical college offers degree programs or training that is required for a specific job.

Keep your options open. Consider different types of schools for now, such as public, private, vocational, and community colleges.

1. Research three different colleges in your area.

   a. Determine the cost of tuition and fees at each school for a Texas resident. Be sure to include a four-year college and a two-year college in your research.

   b. Approximately how much should you expect to pay for books and housing each year? Explain your reasoning.

2. For 2023–2024, the average cost of in-state resident tuition at a four-year public university in Texas was $10,323. How does the average compare to the cost of the schools you researched?

3. Estimate the total expenses for one year of the type of post-secondary school of your choice.

Living at home is a great way to keep the costs of college low.

4. Compare your research with your class. How does the cost of the schools you chose compare to other schools. Be sure to compare the cost at public universities, private universities, community colleges, and technical colleges. What do you notice?

5. Determine whether each statement is fact or fiction. Explain your reasoning.

    a. You need to have 4 years of tuition money saved before attending college.

    b. The cost of one year of tuition can range from about $1700 to more than $40,000.

    c. The most expensive colleges are generally the better colleges.

    d. The tuition price is always the amount that you will pay.

Michael has researched the cost of attending a public university and is determining how much he will need to finance to cover his education.

WORKED EXAMPLE

Michael is considering attending a public university for 2 years in the fall. His costs include \$9200 per semester for tuition, \$7500 per year for housing, and \$610 per semester for books and supplies. Michael's family has been saving for his college education and will be contributing \$11,000 to help cover the cost. What is Michael's estimated cost for his 2-years at the public university?

Michael will be attending for 2 semesters a year for 2 years. $(9200)(2)(2) = 36{,}800$

Michael will need housing for 2 years. $(7500)(2) = 15{,}000$

Michael will need books and supplies for 2 semesters a year for 2 years. $(610)(2)(2) = 2440$

Michael's total-cost $36{,}800 + 15{,}000 + 2440 = 54{,}240$

Michael's total cost to attend the public university for 2 years-will be \$54,240. Michael is determining how much he needs to finance after applying his family contribution.

Michael's family contribution $54{,}240 - 11{,}000 = 43{,}240$

Michael will have to finance \$43,240 to attend the public university for 2 years.

6. Michael is also considering attending the public university for 4 years.

   a. How much would it cost Michael to attend the public university for 4 years?

   b. How does this impact the amount of money Michael would need to finance for his education after his family contribution?

7. Calculate the estimated cost of each education.

   a. Olivia is attending a 4-year public university in the fall. Her costs include $8500 per semester for tuition, $7200 per year for housing, and $550 per semester for books and supplies. Rita's family will be able to contribute a total of $13,500 for her education.

   b. Javier is attending a 2-year public community college next year. His costs include $2400 per semester for tuition and $450 per semester for books and supplies. To minimize his expenses, he plans to live at home, so that he will not have to pay for housing. Javier's family will be able to contribute a total of $6000 for his education.

   c. Fernando is attending a 2-year technical college next year. His costs include $1950 per semester for tuition and $325 per semester for books and supplies. Fernando is going to rent an apartment near campus for 2 years and spend $600 a month in rent. Fernando's family is unable to contribute to his education.

ACTIVITY 4.2

Planning and Paying for Your Post-Secondary Education

One of the most important investments that you can make is an investment in your education. This requires careful planning. Fortunately, the state of Texas provides many opportunities to help you pay for college.

1. Describe what you can do to keep the cost of post-secondary school as low as possible.

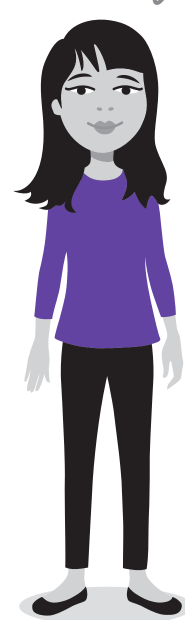
2. Describe the Free Application for Federal Student Aid (FAFSA). How can it help you attend a post-secondary school?

There are tools available that can help you and your family save for a post-secondary school. These tools include:

- The Texas College Savings Plan (Texas 529 Savings Plan):
  This is a savings plan specifically for Texas families saving for college. It offers tax-free growth, as well as tax-free withdrawals, for things like books, transportation, room and board, and other miscellaneous expenses related to education.
- The Texas Tuition Promise Fund®:
  The Texas Tuition Promise Fund® is a fund that allows you to lock in current undergraduate tuition rates by purchasing "units" to be spent at Texas public colleges and universities, excluding medical and dental institutions.
- Education IRAs (Coverdell Education Savings Accounts):
  This is a type of investment that allows Texas residents to withdraw money from their retirement accounts, without penalty, to pay for college. Families may deposit up to $2000 per child into this type of account.

3. Compare the 3 options. How are they similar? How are they different?

In 1995, the Texas Prepaid Higher Education Tuition Board was established to help Texans obtain a higher education. This board administers the State of Texas tax-advantaged plans such as the Texas College Savings Plan (Texas 529 Savings Plan), and The Texas Tuition Promise Fund®.



4. What other financial opportunities might help you pay for college?

5. Determine whether each statement is fact or fiction. Explain your reasoning.

   a. You can save money by attending a community college for two years and then transfer to a four-year college or university.

   b. You can work part-time while attending a college or technical school.

   c. A public Texas college is much cheaper for Texans than students from out-of-state.

   d. When planned properly, all Texas students can get into and pay for college or technical school.

6. Using your estimate for the cost of attending a 2-year and 4-year college, devise a savings plan for accumulating the money needed to attend each college for the first year.

Talk the Talk

Tuition Doesn't Have to be Bigger in Texas

1. List at least one way you can cut costs to make attending a post-secondary institution more affordable.

2. Explain how the selection of the type of school is a large factor in determining tuition costs.

3. Explain one of the savings plans offered by the state of Texas to help you and your family save for your post-secondary education.

4. What are some sources available to help pay post-secondary tuition?

Lesson 4 Assignment

Write

What are the different reasons that your friends, family members, teachers, school leaders, or other community members have given you as to why you should or should not pursue a post-secondary education? Which of those reasons have you found to be true or false based on what you have learned?

Remember

Education is worth much more than a dollar amount. Families, schools, and organizations push students to continue their education after high school because it is in everybody's best interest. Your post-secondary education will help us all become a happier, healthier, and more productive society.

Practice

Identify each statement as true or false. Explain your reasoning.

1. Daniela is planning on going to college. So far she has taken one year of French. She doesn't like French so she is not going to take a foreign language her senior year. She does not believe this will affect her college plans.

2. Alexander and his friends are discussing their post-high school plans. Alexander has not decided his career yet so he cannot start college next year.

Lesson 4 Assignment

3. While discussing her college plans with her guidance counselor, Destiny mentions she would like receive her degree in cosmetology. Her counselor states this is a 2-year vocational degree and an excellent option for post-secondary education.

4. At Hannah's high school, her guidance counselor distributes a list of recommended classes known as the Recommended High School Program. Hannah is planning on attending college, so she should follow the program closely.

5. Jacob wants to go to college but is struggling with his English classes. He starts saving money because he knows he may have to take extra classes before starting college.

6. When planning for college, it is a good idea to shop around for the best loan terms.

7. FAFSA is a free application that makes you potentially eligible for loans, grants, and work-study funds for each year.

8. The Texas Tuition Promise Fund® is a savings plan specifically for Texas families saving for college.

Lesson 4 Assignment

9. Attending college is a full-time job. You cannot work while attending school.

10. A public university is less expensive for Texas residents than for out-of-state students.

11. An Education IRA allows Texas residents to withdraw money from their retirement accounts to pay for college.

Luis is a junior in high school and is starting to think about his post-secondary education.

12. Identify three things Luis can do while in high school to help prepare himself for college.

Lesson 4 Assignment

13. Luis is planning on attending a Texas public university. Identify three facts about the costs of a public university that Luis should know.

14. Luis is accepted to the school of his choice. Identify and describe three savings plans that he and his family could use to help pay for college.

Lesson 4 Assignment

Prepare

Imagine a can of soup with a label.

1. The label has been removed from the can and placed flat on a table. Draw and describe a representation of the flat label.

2. Describe what the sides, top, and bottom of a can of a soup looks like.

3. How can you determine the height of a can of soup?

4. If the radius of the top of the can is 3 inches, determine the area of the top of the can.

TOPIC 3 SELF-REFLECTION

Name: ____________________

Financial Literacy: Your Financial Future

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me,** 2 represents **I am building proficiency of the skill,** and 3 represents **I have demonstrated proficiency of the skill.**

I can demonstrate an understanding of the standards in the *Financial Literacy: Your Financial Future* topic by:

TOPIC 3: *Financial Literacy: Your Financial Future*	Beginning of Topic	Middle of Topic	End of Topic
comparing how interest rates and loan length affect the cost of credit.	◯	◯	◯
use technology, such as an online calculator, to determine the total cost of repaying loans under various rates of interest and over different periods of time.	◯	◯	◯
calculating and comparing simple interest and compound interest earnings.	◯	◯	◯
explaining how small amounts of money invested regularly will grow over time.	◯	◯	◯
explaining the advantages and disadvantages of different payment methods.	◯	◯	◯
analyzing situations to determine whether they represent financially responsible decisions.	◯	◯	◯
devising a periodic savings plan for the cost of a two-year and four-year college education.	◯	◯	◯

continued on the next page

TOPIC 3 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Financial Literacy: Your Financial Future* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

Financial Literacy: Your Financial Future Summary

NEW KEY TERMS

- terms of an investment [términos de una inversión]
- simple interest [interés simple]
- compound interest [interés compuesto]
- deferment
- online calculator [calculadora en línea]
- cash advance

LESSON 1 Simple and Compound Interest

The **terms of an investment** include the type of investment, amount of money invested, the interest rate, and the length of the investment. Investments can earn vastly different amounts of money, depending on the amount of time and the way the interest is growing.

Recall that **simple interest** is a percentage of the principal that is added to the investment over time. **Compound interest** is a percentage of the principal and the interest that is added to the investment over time. Simple interest accounts pay the same amount over each time period, while compound interest accounts calculate the interest earned according to the balance after each year. This means that simple interest accounts grow steadily over time, because they increase at a constant rate. Compound interest accounts grow more rapidly because a percentage of the principal and interest is added to the balance each year.

Recall that the formula to determine simple interest is $I = Prt$, where I is interest, P is principal (amount invested), r is the interest rate, and t is time (in years). The formula for compound interest is $A = P(1 + r)^t$, where A represents the final balance, P represents the original principal amount invested, r represents the annual interest rate, and t represents the time in years.

For example, Harold's family opens two savings accounts for him to start saving for college. He deposits $200 in each account. Both accounts have an interest rate of 2.5%. However, one is a simple interest account and the other is a compound interest account. He keeps the money in the accounts for 18 years.

Simple Interest Account

$I = Prt$

$I = (200)(0.025)(18)$

$I = 90$

$200 + 90 = 290$

Compound Interest Account

$A = P(1 + r)^t$

$A = 200(1 + 0.025)^{18}$

$A = 311.93$

$$311.93 - 290.00 = 21.93$$

After 18 years, the compound interest account earns $21.93 more than the simple interest account.

LESSON 2 Terms of a Loan

The terms of a loan involve the amount of the loan, the interest rate on the loan, the time of the loan, and the monthly payments on the loan. A person's credit history can affect the loan they are able to receive. A good credit score can help you get the best possible interest rate available. However, interest rates can also vary, depending on the lender and the length of the loan. When applying for a loan, a person should try to get the best terms they can. Shopping around for the best loan terms could save you thousands of dollars.

For example, Carlos and Adriana are each shopping for a loan. Carlos is considering the following two loan options when buying an apartment in Dallas.

Loan Star Loan

- Loan Amount: $140,000
- Interest Rate: 4%
- Time: 15 years
- Monthly Payment: $1035.56

(1035.56)(180) = 186,400.80

Central South Bank

- Loan Amount: $140,000
- Interest Rate: 4%
- Time: 30 years
- Monthly Payment: $668.38

(668.38)(360) = 240,616.80

The loan from Lone Star Loan has a shorter length and better terms. Even though the monthly payment is more, Carlos will end up saving $54,216.

Adriana is comparing two loan options from two different banks.

Bank One

- Loan Amount: $125,000
- Interest Rate: 6%
- Time: 30 years
- Monthly Payment: $749.44

(749.44)(360) = 269,798.40

Bank Two

- Loan Amount: $125,000
- Interest Rate: 4%
- Time: 15 years
- Monthly Payment: $924.61

(924.61)(180) = 166,429.80

The loan from Bank Two has better terms. Even though the monthly payment is slightly more, Adriana will end up saving $103,368.60.

Shopping around for the best loan terms could save you thousands of dollars.

The average college graduate can have over $20,000 in student loans. Between the time that they graduate and the time they find a job, many students choose a loan deferment. A **deferment** is a period of time, usually up to two years, in which students delay paying their loan or the interest on their loan. A deferment may be necessary under some circumstances, but the compound interest affects the principal of a loan and will end up increasing the cost of the loan.

For example, Angelina took out $23,000 in student loans to pay for school. Her loans had a 4.5% interest rate. Once she graduated, she took a two-year deferment to save money.

$A = 23{,}000(1 + 0.045)^2$

$A = 25{,}116.58$

Due to the deferment, Angelina's balance on her loans is now $25,116.58.

LESSON 3 Online Calculators

The mathematics behind credit card debt is more complicated than just determining compound interest. If the loan is not paid off in full at the end of the month, many factors play a role in determining the details of different payment options. For this reason, technology, such as *online calculators*, can be valuable tools for comparing credit card payment options. An **online calculator** is an Internet-based application that quickly performs calculations for the user.

For example, two friends each decide to purchase a new television that costs $750. Isaiah puts the purchase on his credit card. This credit card has a 15% interest rate. He hopes to pay off the television in 4 months. Using an online calculator, to pay off this purchase in 4 months, Isaiah's monthly payments will be $194. He will incur $24 in interest charges.

Mei put the purchase on a credit card with a 17% interest rate and a minimum payment of $75. Using an online calculator, paying the minimum payment, it will take Mei 11 months to pay off this purchase. She will incur $65 in interest charges.

Isaiah will pay more money each month, but he will pay off the television sooner and will pay less money in interest.

Credit card companies regularly send checks to their customers that can serve as a cash advance. A **cash advance** is a service provided by credit card companies that allows their customers to take out money directly from a bank or ATM. Consumers should be very cautious about cash advances from credit cards. A percentage of the amount is immediately added to the loan, and a high interest rate is applied. The interest rate on a cash advance is usually much higher than the interest rate on the credit card.

For example, Valentina needs cash while on a trip out of the country. She decides to use her credit card and get a cash advance from the ATM. She takes $500 out on her credit card with a 20% interest rate. Her bank charges her an additional 5% at the time of the withdrawal. She plans on making $30 payments on this charge until she pays it off.

$(500)(1.05) = 525$

With the 5% additional charge, her withdrawal now costs her $525.

Using an online calculator, it will take Valentina 21 months, or 1 year and 9 months, to pay off this cash advance charge. She will incur $101 in interest charges.

LESSON 4 Financing Your Education

It is important to remember that it is never too early to begin planning for college. There are many things that can be done while in high school that could lead to post-secondary success. In Texas, the Recommended High School Program, or RHSP, one of four state-approved graduation programs, offers courses that students should take to best prepare them for post-secondary education. While it is not necessary to know your career path before entering college, you can begin earning college credits before even graduating high school.

Hard work and careful planning is necessary to not only get into college but also to pay for it. After you consider what is important to you, the different types of schools, and their relative costs, you can begin developing a payment strategy. There are many different post-secondary options including 4-year college degrees, as well as 2-year vocational, technical, or associate degrees.

- A public school receives funding from the state, while private, independent institutions rely on tuition and donations to fund programs.
- A community college is generally a school for local residents that will accept most applicants.
- Community colleges offer 2-year degrees as well as coursework that may transfer to another institution.
- A technical college offers degree programs or training that is required for a specific job.

Because there are so many options when preparing for post-secondary education, the costs can vary greatly. In-state residents often pay significantly less than out-of-state students. There are also a number of grants, loans, and scholarships that are available for students. Different school choices also offer different costs. Private institutions are usually the most expensive, while community colleges and vocational schools cost less. It is also important to remember to factor in books, supplies, and housing when determining the cost of your education. Remember that the cost of tuition is never the cost you actually pay. Between loans, grants, housing, and books, the cost will vary greatly for every student.

When planning to pay for college, there are a number of ways to get help.

- The Free Application for Federal Student Aid (FASFA) is a free application that makes you potentially eligible for grants, loans, and work-study funds each year.
- The Texas College Savings Plan, also called the *Texas 529 Savings Plan*, is a savings plan specifically for Texas families saving for college. It offers tax-free growth, as well as tax-free withdrawals, for things like books, transportation, room and board, and other miscellaneous expenses related to education.
- The Texas Tuition Promise Fund® is a fund that allows you to lock in current undergraduate tuition rates by purchasing "units" to be spent at Texas public colleges and universities, excluding medical and dental institutions.
- Education IRAs, now more formally known as Coverdell Education Savings Accounts, are a type of investment that allows any family in the United States to withdraw money from their retirement accounts, without penalty, to pay for college. Families may deposit up to $2000 per child into this type of account.

GRADE 8

TOPIC 4

Disco balls are spheres that reflect light in all different directions. They were really popular in dance clubs throughout the 1960s, 1970s, and 1980s.

Volume of Curved Figures

1 Volume, Lateral, and Total Surface Area of a Cylinder

OBJECTIVES

- Explore the volume of a cylinder.
- Write formulas for the volume of a cylinder.
- Use a formula to determine the volume of cylinders in mathematical and real-world problems.
- Use the formula for the volume of a cylinder to solve for unknown dimensions.
- Explore the lateral and total surface area of a cylinder.
- Use the formulas for the lateral and total surface area of a cylinder to solve real-world problems.

NEW KEY TERMS

- cylinder
- right cylinder
- radius of a cylinder
- height of a cylinder

You know how to calculate the area of circles and the volume of rectangular prisms and pyramids.

How can you use this knowledge to solve problems involving the volume, lateral surface area, and total surface area of cylinders?

Getting Started

All About Cylinders

A **cylinder** is a three-dimensional object with two parallel, congruent circular bases.

1. Sketch an example of a cylinder. Explain how your sketch fits the definition of a cylinder.

Ask Yourself:
How else can you represent this information?

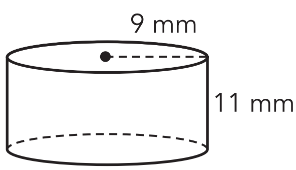
2. Compare your sketch with your classmates' sketches. Did everyone sketch the same cylinder? Explain how the sketches are the same or different.

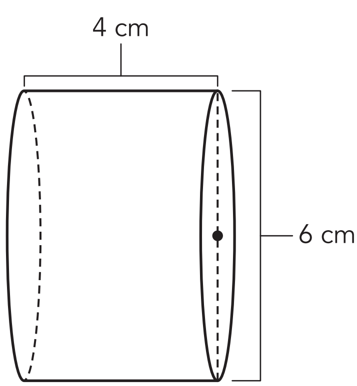
3. A **right cylinder** is a cylinder in which the bases are circles and are aligned one directly above the other. Consider your sketch. Does your sketch look like a right cylinder?

The **radius of a cylinder** is the distance from the center of the base to any point on the edge of the base. The radius of a cylinder is the same on both bases. The **height of a cylinder** is the length of a line segment drawn from one base to the other base, perpendicular to both bases.

4. Use your sketch to illustrate the radius and height of a cylinder.

5. Identify the radius, diameter, and height of each cylinder. Shade the 2 bases of each cylinder.

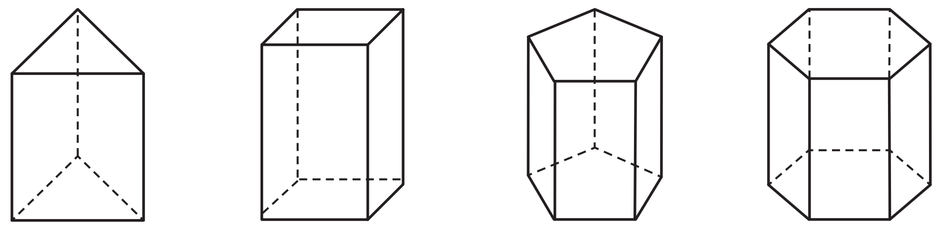
a. 

b. 

ACTIVITY 1.1

Volume Formula for a Cylinder

Analyze the prisms shown.



1. What do you notice as the number of sides of the base increases?

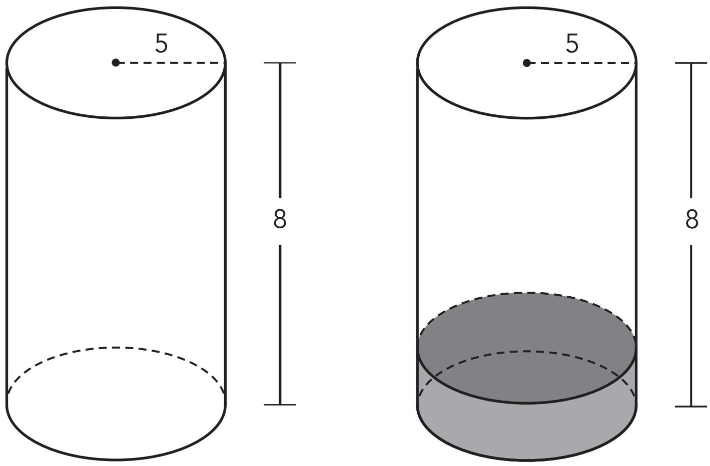
Prisms and cylinders both have two bases and a constant height between the bases.

2. Because cylinders and prisms are similar in composition, their volumes are calculated in similar ways.

   a. Write the formula for the volume of any right prism. Define all variables used in the formula.

   b. Make a conjecture about how you will calculate the volume of a right cylinder.

Consider the cylinder shown. The length of the radius of the circular base is 5 units and the height of the cylinder is 8 units.



3. Suppose there is a circular disc of height 1 unit at the bottom of the cylinder.

   a. Calculate the area of the top of the circular disc.

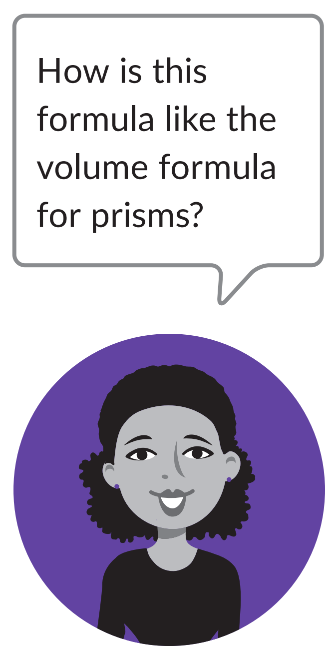
Recall these formulas for circles.

$A = \pi r^2$

$C = 2\pi r$

   b. How many congruent circular discs would fill the cylinder? What is the volume of each disc? Explain your reasoning.

   c. Determine the total volume of the cylinder. Explain your strategy.

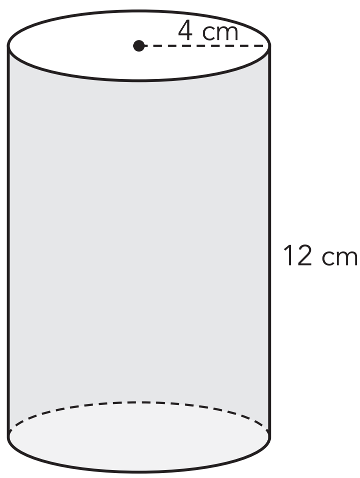


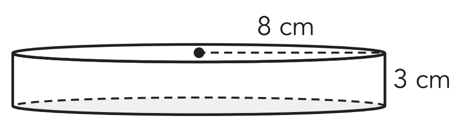
4. Write a formula for the volume of a cylinder, where V represents the volume of the cylinder, r represents the radius of the cylinder, and h represents the height of the cylinder.

ACTIVITY
1.2 Cylinder Volume Problems

The director of the marketing department at a rice-packing factory sent a memo to her product development team. She requested that the volume of the new cylinder prototype be approximately 603.19 cm^3.

1. Two members of the marketing team claim to have created appropriate prototypes, but they disagree about the dimensions of the cylinder prototype. Jasmine designed the cylinder prototype on the left, and Lucas designed the cylinder prototype on the right. Who is correct? What would you say to Jasmine and Lucas to settle their disagreement?





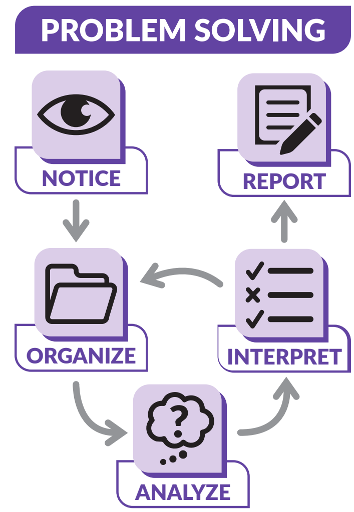
Use what you know about cylinders to solve real-world problems. Round to the nearest hundredth.

2. A circular swimming pool has a diameter of 30 feet and a depth of 5 feet. What is the volume of the pool?

3. How many milliliters of liquid are needed to fill a cylindrical can with a radius of 3 centimeters and a height of 4.2 centimeters?

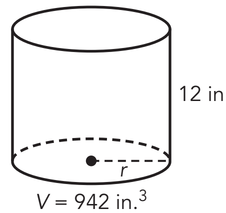
One milliliter is equivalent to one cubic centimeter of liquid.

4. Many newspapers are made from 100% wood. The wood used to make this paper can come from pine trees, which are typically about 60 feet tall and have diameters of about 1 foot. However, only about half of the volume of each tree is turned into paper. Suppose it takes about 0.5 cubic inches of wood to make one sheet of paper. About how many sheets can be made from a typical pine tree? Show your work and explain your reasoning.



WORKED EXAMPLE

You can use the formula for the volume of a cylinder to determine unknown dimensions.



$V = Bh$	Use the formula for the volume of a cylinder.
$V = \pi r^2 h$	Substitute πr^2 for B to represent the circular base.
$942 = \pi r^2(12)$	Substitute known dimensions into formula.
$942 \approx 37.70r^2$	Rewrite expression in lowest terms. Round to the nearest hundredth.
$25 \approx r^2$	Isolate the unknown dimension.
$\sqrt{25} \approx r$	Solve for the unknown dimension.
$5 \approx r$	

The radius of the cylinder is approximately 5 inches.

5. Analyze the Worked Example. Assume you have been given the total volume of the cylinder.

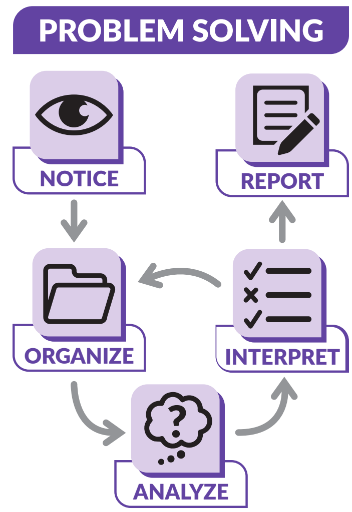
   a. What additional information would you need to determine the radius?

   b. What additional information would you need to determine the height?

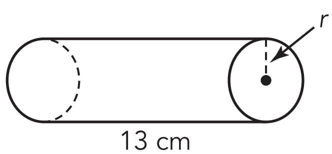
6. Determine the diameter of the cylinder for the Worked Example.

7. Explain why the unit of measure for the radius in the Worked Example is NOT cubic inches.

8. The volume of each solid is 500 cm^3. Calculate the unknown dimension in each figure. Round to the nearest hundredth.



a.



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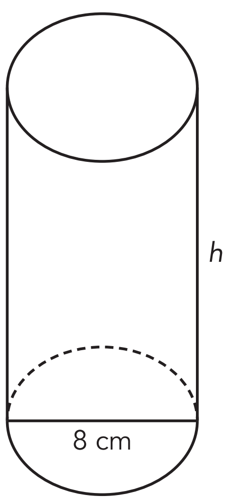
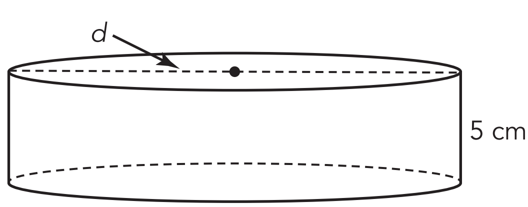
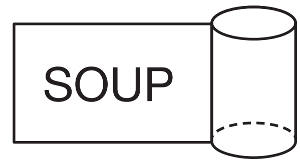


c.



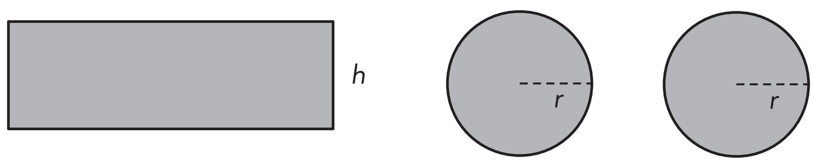
ACTIVITY 1.3 Lateral and Total Surface Area of a Cylinder

Consider the right cylinder shown.



The total surface area is the sum of the areas that form the surface of a three-dimensional figure. The label of a can covers the surface of the can that does not include the two bases that are circles. You call this *lateral surface area*. If you cut the label from a can, you can see that the label is a rectangle.

The width of the rectangle is the height of the can. Because the label wraps around the can, the length of the rectangle is the circumference of the can. The total surface area of the can is the area of the rectangle plus the area of the two circular bases.



1. Label the base of the rectangle.

You can use S_T and S_L to distinguish between the formulas for the total and lateral surface area of a cylinder.

2. Write an expression to model the area of each face.

3. Write the formula for the lateral surface area of the cylinder. Explain your reasoning.

4. Write the formula for the total surface area of the cylinder. Explain your reasoning.

5. How are the formulas for lateral surface area and total surface area of a cylinder related?

6. A cylindrical paint roller has a length of 10 inches and a radius with a length of 1.25 inches. Apply the formula to determine the lateral surface area of the paint roller. Round to the nearest hundredth.

7. A snare drum has a height of 5.5 inches and a diameter of 14 inches. Calculate the total surface area of the drum. Round to the nearest hundredth.

Talk the Talk

The Prism Connection

Luna was absent for the lesson on volume of a cylinder. However, she knows that the formula for the volume of a right rectangular prism can be written as $V = \ell wh$. Explain to Luna how to use her knowledge of the volume of right rectangular prisms to determine the volume of a cylinder.

Lesson 1 Assignment

Write

Explain the similarities and differences of right prisms and right cylinders.

Remember

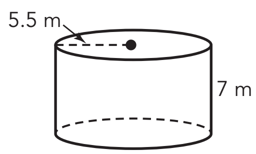
Consider a cylinder with r as the length of the radius of the base and h as the height. The volume of a cylinder is calculated by multiplying the area of the circular base by the height of the cylinder, or $V = \pi r^2 h$.

The total surface area of the cylinder is calculated by adding the area of its two circular bases and the area of the rectangle that covers its curved surface, or $S = 2\pi r^2 + 2\pi rh$.

Practice

1. Determine the volume, lateral surface area, and total surface area of each cylinder. Round to the nearest hundredth.

a.



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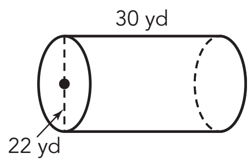
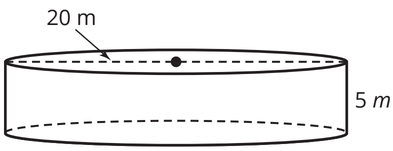


Lesson 1 Assignment

c.



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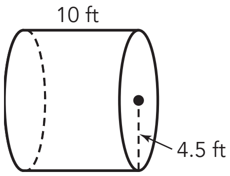
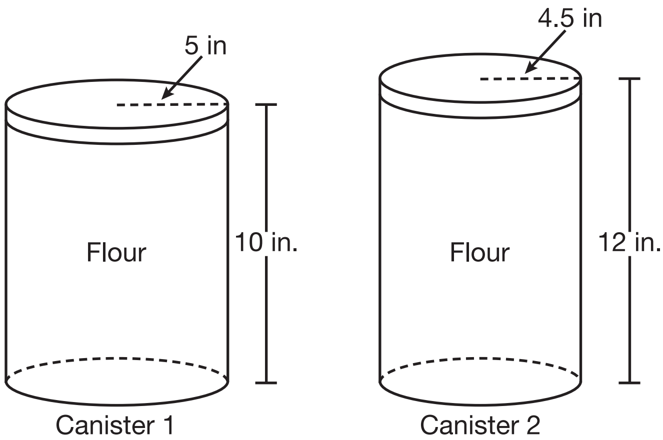


2. A cylindrical fish tank provides a 360° view. The height of the cylindrical fish tank is 30 inches, and the diameter of the base is 27.5 inches. If one U.S. gallon is equal to approximately 231 cubic inches, calculate the amount of water the tank will hold. Round to the nearest hundredth, when necessary.

Lesson 1 Assignment

3. Nahimana's grandmother sends her to the store to buy a new flour canister for her kitchen. Nahimana finds two canisters that she likes and is having trouble deciding which one to purchase.



   a. Calculate the volume of each canister. Round to the nearest hundredth, when necessary.

   b. Suppose the canisters cost the same amount. Which one should Nahimana purchase?

   c. A third canister has a radius of 2.5 inches and a height of 10 inches. How much less flour does this canister hold than Canister 1? Explain your reasoning.

Lesson 1 Assignment

Prepare

Calculate the length of the hypotenuse given the two legs of a right triangle.

1. $r = 4.5$ cm, $s = 6$ cm

2. $a = 16$ m, $b = 24$ m

3. $h = 2.7$ in., $j = 3.9$ in.

4. $x = 0.59$ yd, $y = 1.41$ yd

2 Volume of a Cone

OBJECTIVES

- Explore the volume of a cone using a cylinder.
- Write formulas for the volume of a cone.
- Use the formula for the volume of a cone to solve real-world and mathematical problems.

NEW KEY TERMS

- cone
- height of a cone

You have used what you know about prisms to determine the volume of cylinders.

Is there a figure that can help you to determine the volume of cones?

Getting Started

All About Cones

A **cone** is a three-dimensional object with a circular or oval base and one vertex.

1. Sketch an example of a cone. Explain how your sketch fits the definition of a cone.

2. Compare your sketch with your classmates' sketches. Did everyone sketch the same cone? Explain how the sketches are the same or different.

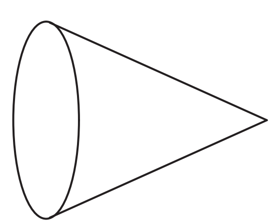
All of the cones associated with this topic have a circular base and a vertex that is located directly above the center of the base of the cone.

3. How does the radius of a cone compare to the radius of a cylinder? Use your sketch to illustrate the radius of the cone.

The **height of a cone** is the length of a line segment drawn from the vertex to the base of the cone. In a right cone, this line segment is perpendicular to the base.

4. Identify the radius, diameter, and height of each cone.

a.



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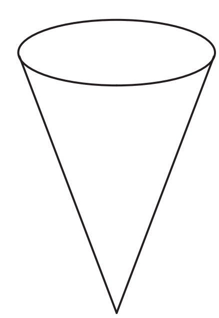


ACTIVITY
2.1 Volume of a Cone

In previous courses, you explored how the volume of a pyramid is related to the volume of a prism. Let's explore how the volume of a cone is related to the volume of a cylinder. You can use nets to investigate this relationship.

A two-dimensional representation of a three-dimensional geometric figure is called a *net*.

1. Use the nets provided at the end of the lesson to create models of a cylinder and a cone.

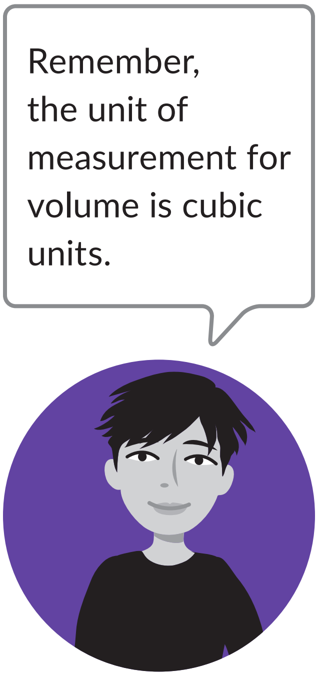
   a. What is similar about the cone and cylinder formed from the nets?

   b. What do you think is true about the relationship between the cylinder and the cone?

Ask Yourself . . .
Which tool or strategy is most efficient?

2. Fill the cone with birdseed and then pour the birdseed into the cylinder. Continue refilling the cone until you fill the cylinder with birdseed.

   a. How many cones of birdseed did it take to fill the cylinder?



b. Compare the amount of birdseed that you used to fill the cone and to fill the cylinder. In other words, compare the volume of the cone to the volume of the cylinder. What fraction best describes this ratio?

c. If you know the volume of the cylinder, how could you determine the volume of the cone?

3. Use a centimeter ruler to measure the length of the radius and height of the cylinder.

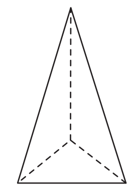
   a. Calculate the volume of the cylinder.

   b. Using the volume of the cylinder, calculate the volume of the cone.

ACTIVITY
2.2 Volume Formula for a Cone

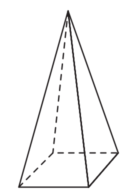
Analyze the pyramids shown.

1. What do you notice as the number of sides of the base increases?



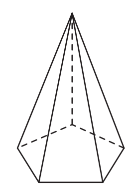
Triangular Pyramid

Because cones and pyramids are similar in composition, their volumes are calculated in similar ways.



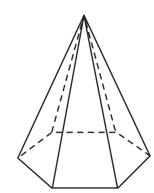
Rectangular Pyramid

2. Write the formula for the volume of any right pyramid and explain how it relates to your investigation with the nets.



Pentagonal Pyramid

3. Use the formula for the volume of a pyramid to write a formula for the volume of a cone. Define all variables used in the formula.



Hexagonal Pyramid

4. Write a second formula for the volume of a cone, where *V* represents the volume, *r* represents the radius of the base, and *h* represents the height.

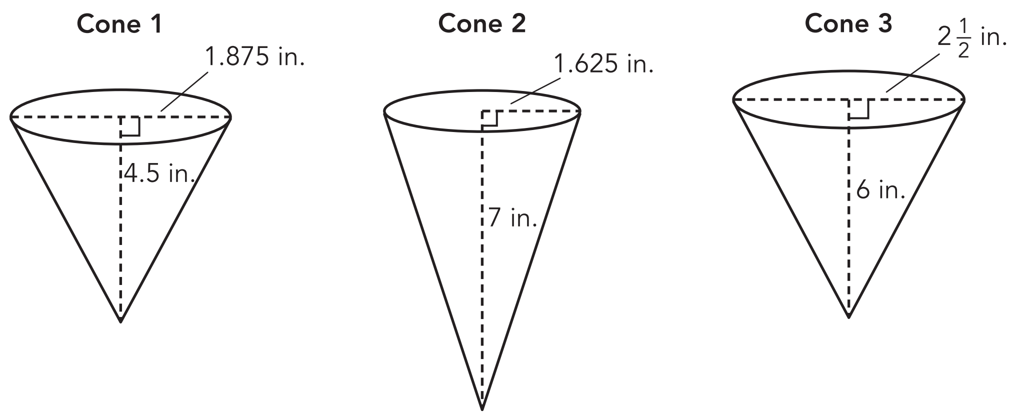
5. Which of the two formulas do you prefer? Explain your reasoning.

Ask Yourself . . .
What tools or strategies can you use to solve this problem?

ACTIVITY 2.3 Problem Solving with Cones

Use what you know about the volume of a cone to solve each problem.

1. Mason owns a frozen yogurt and fruit smoothie shop. He just placed an order for cones, and the order contains three different sizes of cones. He wants to know the volume of each cone to help him determine how much to charge.



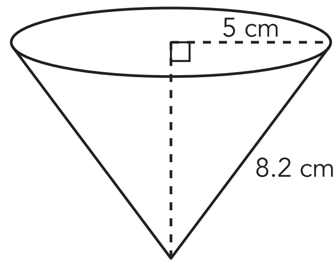
> **Ask Yourself . . .**
> What observations can you make?

a. Predict which cone has the greatest volume. Explain your reasoning.

b. Calculate the volume of each cone.

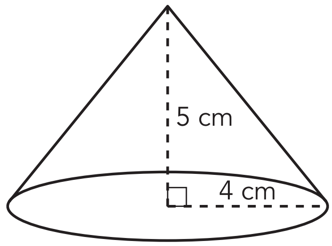
c. If Mason's market research reveals that he should charge $3.75 for the smallest cone, what prices would you propose for the other two cones? Explain your reasoning.

2. Sebastian and Kayla are working on their homework and disagree on the volume of the cone shown. Sebastian says that the volume of the cone is $V = \frac{1}{3}\pi(5)^2(8.2) \approx 214.68 \text{ cm}^3$. Kayla argues that 8.2 cm is not the height of the cone, so they need to calculate the height before determining the volume. She says that the volume is $V = \frac{1}{3}\pi(5)^2(6.5) \approx 170.17 \text{ cm}^3$. Who's correct? Explain your reasoning.

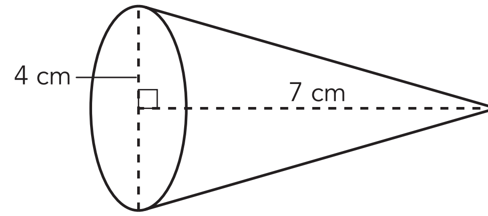


3. Calculate the volume of each cone. Round your answers to the nearest hundredth, when necessary.

a.



b.



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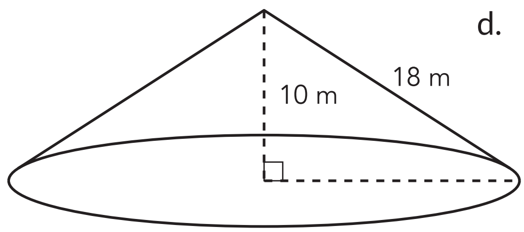

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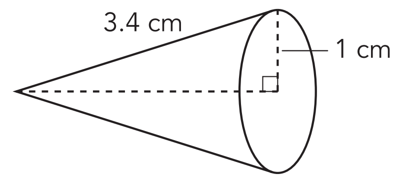
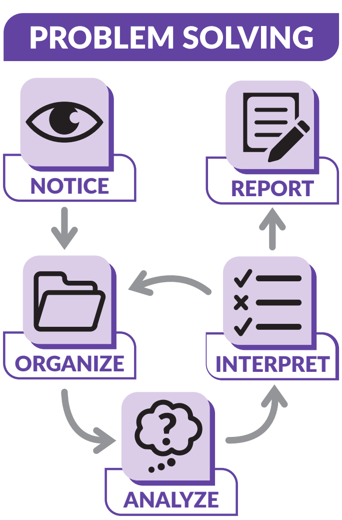




4. Use the formula for the volume of a cone to solve for each unknown dimension.

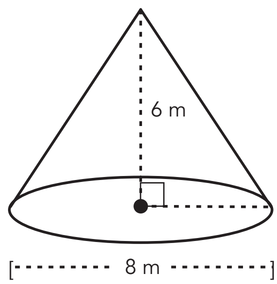
a. $V = 3042 \text{ cm}^3$, $d = 25.2$ cm, $h =$ ______________

b. $V = 25\pi \text{ in.}^3$, $h = 12$ in., $r =$ ______________

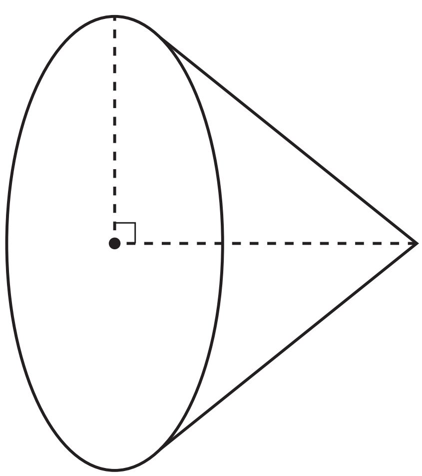
Talk the Talk

Cones, Cones, and More Cones!

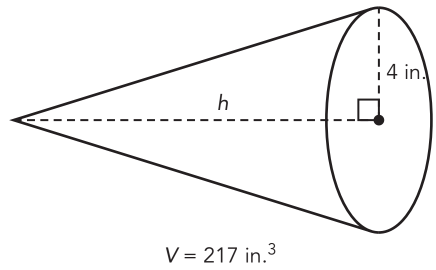
1. Write an equation and determine the volume of the cone. Round your answer to the nearest hundredth, when necessary.



2. Use a centimeter ruler to determine the measurements required to determine the volume of the cone.

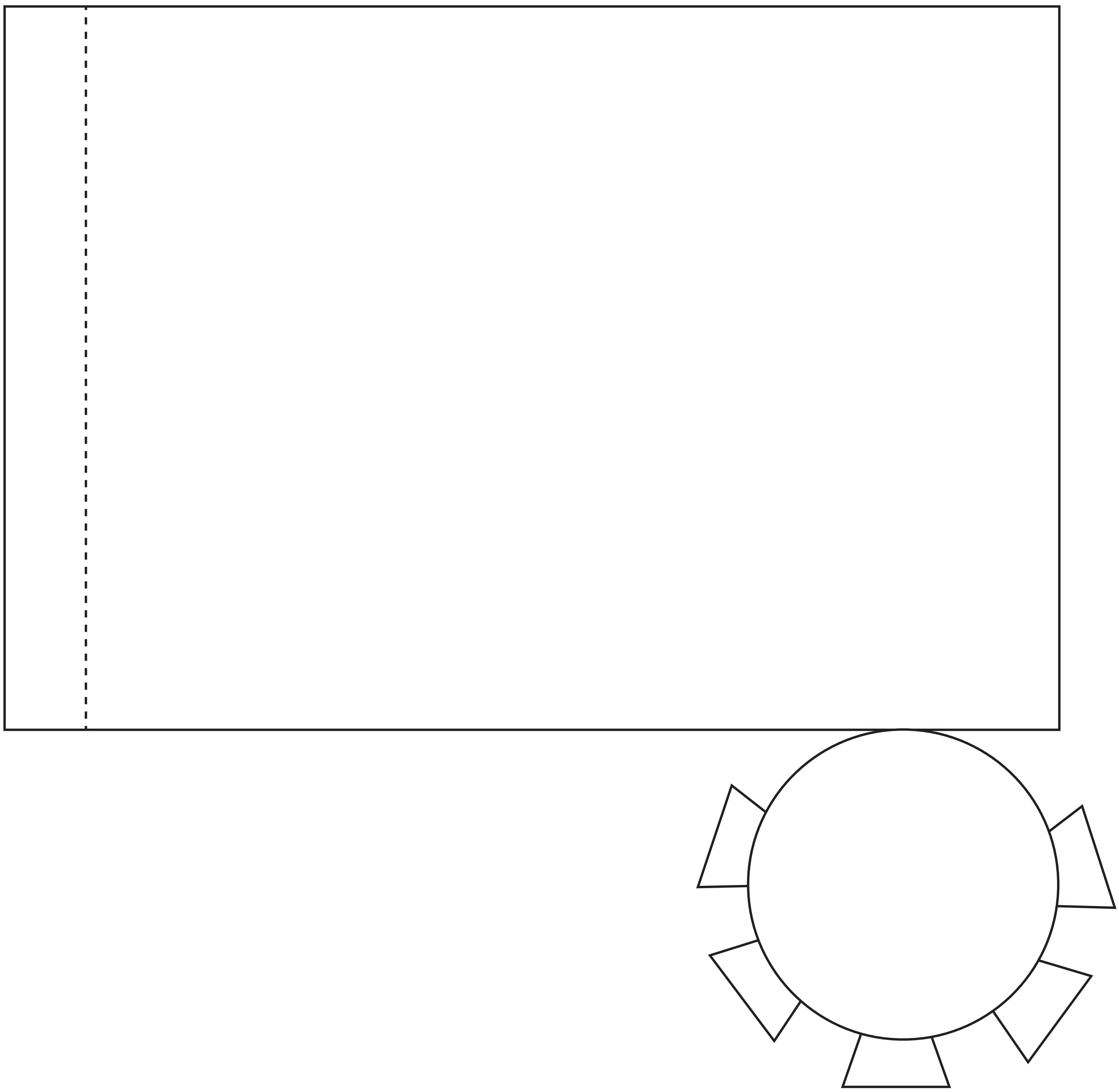


3. Determine the unknown dimension of the cone. Round your answer to the nearest hundredth, when necessary.



4. A company sells two sizes of conical party hats. The smaller hats have a radius of 6 cm and a volume of 376.8 cm^3. The larger hats are double the volume of the smaller hats but have the same size radius. Determine the dimensions of the larger hats.

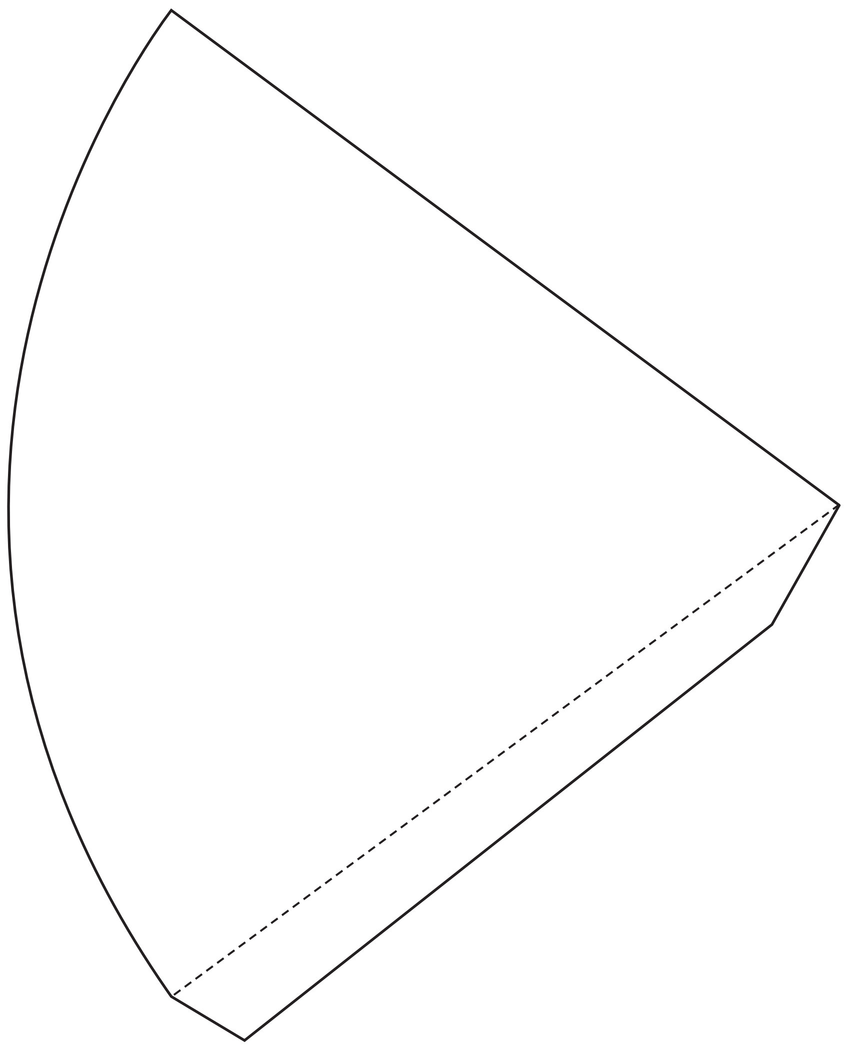
Cylinder Net



Why is this page blank?

So you can cut out the net on the other side.

Cone Net



Why is this page blank?

So you can cut out the net on the other side.

Lesson 2 Assignment

Write

Explain when you need to use the Pythagorean Theorem to calculate the volume of a cone.

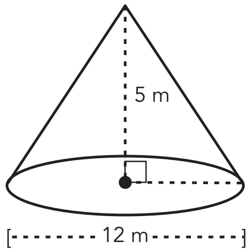
Remember

The volume of a cone is one-third the volume of a cylinder with the same base and height as the cone.

Practice

1. Determine the volume of each cone. Round to the nearest hundredth.

a.



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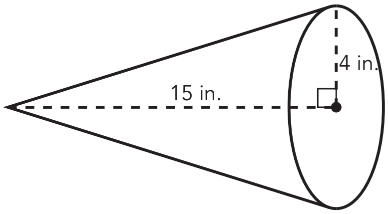
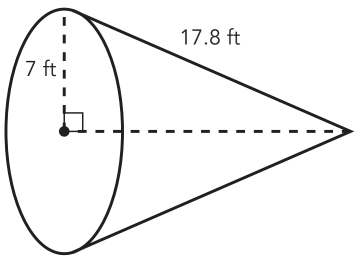


Lesson 2 Assignment

c.



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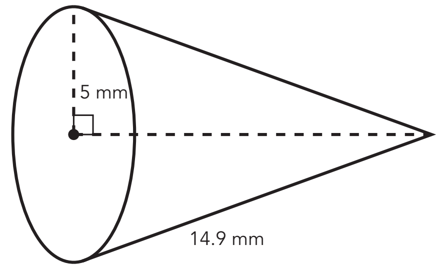


Lesson 2 Assignment

2. A theatre sells two sizes of conical popcorn containers. The smaller containers have a radius of 4 in. and a volume of 167.6 $in.^3$. The larger containers have double the volume of the smaller containers but have the same size radius. Determine the dimensions of the larger containers.

3. Adriana asked her parents to make a piñata for her birthday party. Her parents decided to make the piñata in the shape of her favorite dessert, an ice cream cone. They stuffed only the cone portion of the piñata. The height of the cone is 34 inches, and the diameter of the base is 24 inches. (Note: 144 square inches equals 1 square foot, and 1728 cubic inches equals 1 cubic foot.) Calculate the amount of space (in cubic feet) in the cone that will be filled with goodies.

Lesson 2 Assignment

Prepare

In June 2007, 7-year-old Jake Lonsway broke a world record, building the world's largest plastic wrap ball. It took him 8 months to make the ball, which weighed 281.5 pounds. The circumference of the ball was 138 inches.

1. Calculate the length of the radius and diameter of Lonsway's ball of plastic wrap.

2. Do you think a ball twice this size would weigh twice as much? Explain your reasoning.

3 Volume of a Sphere

OBJECTIVES

- Identify a formula for the volume of a sphere.
- Use a formula to determine the volume of spheres in mathematical and real-world contexts.

NEW KEY TERMS

- sphere
- center of a sphere
- radius of a sphere
- diameter of a sphere
- great circle

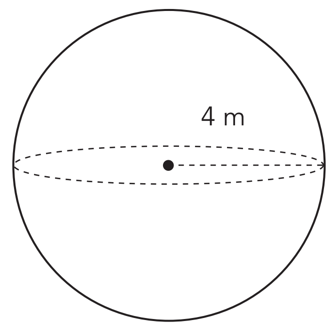
You have learned and applied volume formulas for a variety of different solids. In this lesson, you will learn and apply the formula for the volume of a sphere.

How can you use the formula to determine a sphere's volume and the length of its radius?

Getting Started

Getting to Know Spheres

A **sphere** is the set of all points in three dimensions that are the same distance from a given point called the **center of a sphere**. Like a circle, a sphere has radii and diameters. A segment drawn from the center of the sphere to a point on the sphere is called a **radius of a sphere**. A segment drawn between two points on the sphere that passes through the center is a **diameter of a sphere**. The length of a diameter is twice the length of a radius. A **great circle** is the circumference of the sphere at the sphere's widest part.



1. List all of the things that you know to be true about this sphere.

ACTIVITY 3.1

Modeling the Volume of a Sphere

Use the modeling clay and paper provided by your teacher to complete each step.

- Make a sphere using the clay. Measure and record the diameter of the sphere.
- Cut a long strip of paper so that its height matches the height of the sphere.
- Wrap the paper tightly around the sphere and tape the paper to make a sturdy cylinder with no bases.
- Squish the clay sphere so that it molds to the bottom of the cylinder.
- Mark the height of the clay in the cylinder with a marker.

1. Complete the first row of the table with the data from your sphere. Then, complete the other rows using data from three other classmates.

Sphere	Diameter	Cylinder Height	Height of Squished Sphere in Cylinder	$\frac{\text{Height of Squished Sphere}}{\text{Height of Cylinder}}$
My Sphere				
Classmate A				
Classmate B				
Classmate C				

2. What do you notice about the ratio of the height of the squished sphere to the height of the cylinder?

3. What is the relationship between the volume of the sphere and the volume of the cylinder?

Recall that the formula for the volume of a cylinder is $V = Bh$ or, more specifically, $V = \pi r^2 h$.

4. What is the height, h, of the cylinder in terms of *the radius*, r, of the sphere? Explain your reasoning.

5. Use this relationship and the formula for the volume of a cylinder to write a formula that describes the volume of the sphere in terms of r.

ACTIVITY 3.2

Applying the Formula for the Volume of a Sphere

Now that you know how to calculate the volume of a sphere, use the formula to solve each problem. Round to the nearest hundredth, when necessary.

Recall that the formula for the circumference of a circle is $C = 2\pi r$ or $C = \pi d$.

1. Earth has a diameter of approximately 7926 miles.

   a. Determine the length of the radius of Earth.

   b. Determine the volume of Earth.

2. The circumference of an official NBA basketball is 29.5 inches.

   b. Calculate the approximate length of the radius of a basketball.

   b. Calculate the approximate volume of a basketball.

Ask Yourself . . .
Did you make a plan to solve the problem?

3. The radius of a Major League baseball is 1.45 inches.

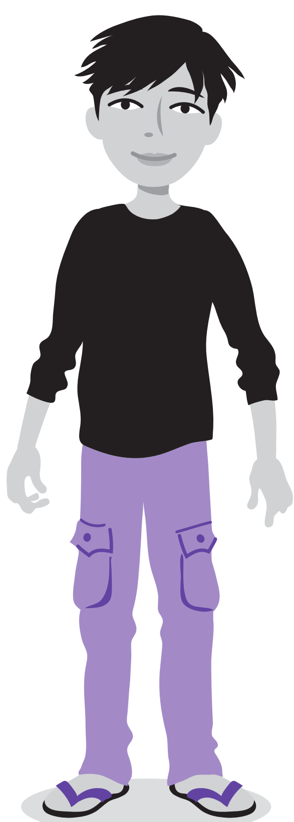
   a. Calculate the approximate volume of a Major League baseball.

   b. Calculate the approximate circumference of a Major League baseball.

The amount of paint on this ball could be used to paint a 4-inch-wide strip for over 68 miles.

4. Built in the 1950s by the Stamp Collecting Club at Boy's Town, the World's Largest Ball of Postage Stamps is very impressive. The solid ball has a diameter of 32 inches, weighs 600 pounds, and consists of 4,655,000 postage stamps.

   Calculate the volume of the World's Largest Ball of Postage Stamps.



5. The world's largest ball of paint resides in Alexandria, Indiana. The ball began as a baseball. People began coating the ball with layers of paint. Imagine this baseball with over 21,140 coats of paint on it! The baseball originally weighed approximately 5 ounces and now weighs more than 2700 pounds. Painting this baseball has gone on for more than 32 years, and people are still painting it today.

   When the baseball had 20,500 coats of paint on it, the circumference along the great circle of the ball was approximately 133 inches. Each layer is approximately 0.001037 inches thick.

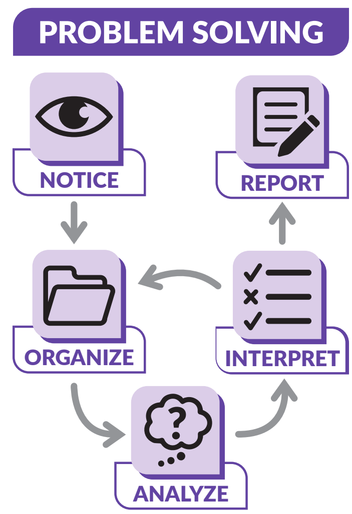
   Calculate the volume of the world's largest ball of paint.

6. The world's largest disco ball has a radius of approximately 5.17 meters. Approximately 2500 mirror tiles were attached using over 11,000 zip ties.

Calculate the volume of the world's largest disco ball.

7. For over five years, John Bain spent his time creating the world's largest rubber band ball. It is solid to the core with rubber bands. Each rubber band was individually stretched around the ball, creating a giant rubber band ball. The weight of the ball is over 3120 pounds, and the circumference is 15.1 feet.

Calculate the volume of the world's largest rubber band ball.



8. The world's largest ball of twine rolled by one man is in Darwin, Minnesota. It weighs 17,400 pounds and was created by Francis A. Johnson. He began this pursuit in 1950. He spent four hours a day, every day, wrapping the ball. It took Francis 29 years to complete. Upon completion, it was moved to a circular open air shed on his front lawn for all to view.

If the diameter of the world's largest ball of twine is 13 feet, determine the volume.

Talk the Talk

Locker Room Math

Ask Yourself . . .
Did you justify your mathematical reasoning?

Young people often attempt to break world records. Angelina is no exception. Today her math class studied the volume of a sphere, and she had a great idea. After working out the math, Angelina told her best friend, Mei, that they could stuff 63 inflated regulation-size basketballs into a school locker. The rectangular locker is 6 feet high, 20 inches wide, and 20 inches deep. The radius of one basketball is 4.76 inches. Mei also did the math and said that only 28 basketballs would fit.

1. How did Angelina and Mei compute their answers? Who's correct? Explain your reasoning.

Lesson 3 Assignment

Write

Describe the similarities and differences between each pair of terms.

1. *radius of the sphere* and *diameter of the sphere*
2. *radius of the sphere* and *center of the sphere*

Remember

The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r represents the radius of the sphere.

Practice

Solve each problem. Round to the nearest hundredth, when necessary.

1. The diameter of a small red beach ball is 8 inches. Calculate the volume of the red beach ball.

2. The diameter of a large blue beach ball is 16 inches. Calculate the volume of the blue beach ball.

3. Spaceship Earth is the most recognizable structure at Epcot Center at Disney World in Orlando, Florida. The ride is a geodesic sphere made up of thousands of small triangular panels. The circumference of Spaceship Earth is 518.1 feet. Determine its approximate volume.

Lesson 3 Assignment

4. The Oriental Pearl Tower in Shanghai, China, is a 468-meter high tower with 11 spheres along the tower. Two spheres are larger than the rest and house meeting areas, an observation deck, and a revolving restaurant. The lower of the two larger spheres has a radius of 25 meters, and the higher sphere has a radius of 22.5 meters. What is the approximate total volume of the two largest spheres on the Oriental Pearl Tower?

5. A model of Earth is located 7600 meters from the Globe Arena in Sweden's solar system model. The radius of the model is 9 centimeters. What is the approximate volume of the Earth model?

6. The Montreal Biosphere is a geodesic dome that surrounds an environmental museum in Montreal, Canada. The dome has a diameter of 250 feet. The structure is 75% of a full sphere. What is the approximate volume?

Lesson 3 Assignment

Prepare

Gabriel has a picnic table that has two wooden seats. Each seat is a rectangular prism that is 72 inches long, 18 inches wide, and 1 inch thick.

Gabriel wants to cover both seats with a weather-proofing stain. How much stain will he need?

4 Volume and Surface Area Problems with Prisms, Cylinders, Cones, and Spheres

OBJECTIVES

- Use formulas for the volume of prisms, cones, cylinders, and spheres to solve real-world and mathematical problems.
- Compare volumes of prisms, cones, cylinders, and spheres.

You have learned the formulas for the volume of prisms, cones, cylinders, and spheres.

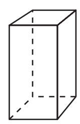
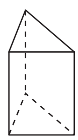
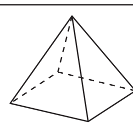
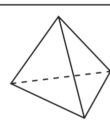
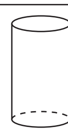
How can you reason with these formulas separately and together to solve problems?

Getting Started

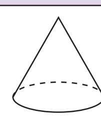
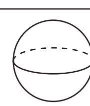
Formula Review

You have determined some formulas for prisms, pyramids, cylinders, cones, and spheres.

1. Complete the formula for the volume, lateral surface area, and total surface area of each solid in the table. Use V for volume, S for lateral surface area, S for total surface area, B for area of a base, P for perimeter of the base, h for height, r for radius, ℓ for slant height, and π for pi.

Solid	Model	Volume	Lateral Surface Area	Total Surface Area
Rectangular Prism		$V = Bh$		
Triangular Prism			$S = Ph$	
Rectangular Pyramid				$S = \frac{1}{2}P\ell + B$
Triangular Pyramid			$S = \frac{1}{2}P\ell$	
Cylinder				

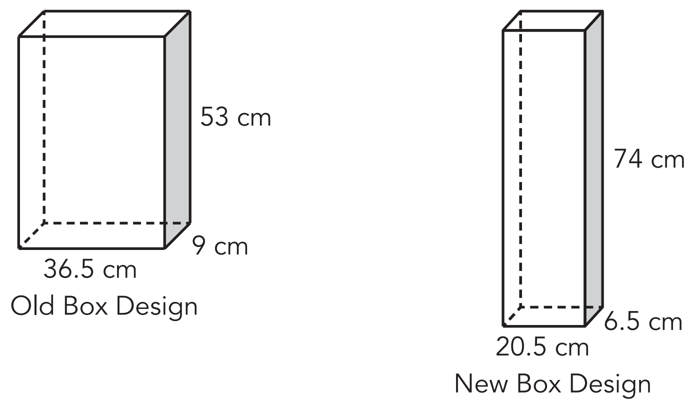
2. State the volume formula for each solid.

Solid	Model	Volume
Cone		
Sphere		

ACTIVITY 4.1

Lateral and Total Surface Area of Prisms

1. A cereal manufacturer is designing a new cereal box to try and cut costs. They felt their old design took too much cardboard to build. They believe their new design, being taller and thinner, will use less cardboard. Determine which cereal box design will take less cardboard to build.

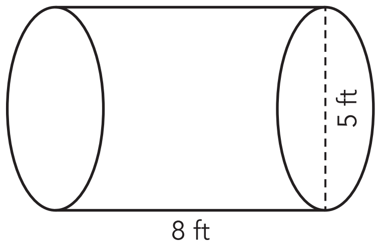


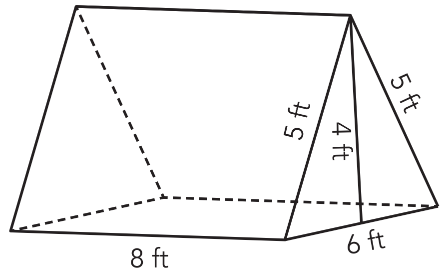
2. A publishing company is hosting a book fair at the local school. One item that will be for sale is posters of favorite book characters. To help students transport the posters safely home, the company wants to create a cardboard container that is open on both ends to protect the poster when it is rolled up.

Solid	Length or Radius (inches)	Width (inches)	Height (inches)
Rectangular Prism	3	4	7
Cylinder	3.75		7.5

The company wants to spend the least amount of money possible per container. Which solid would you recommend the company select? Round to the nearest hundredth, when necessary.

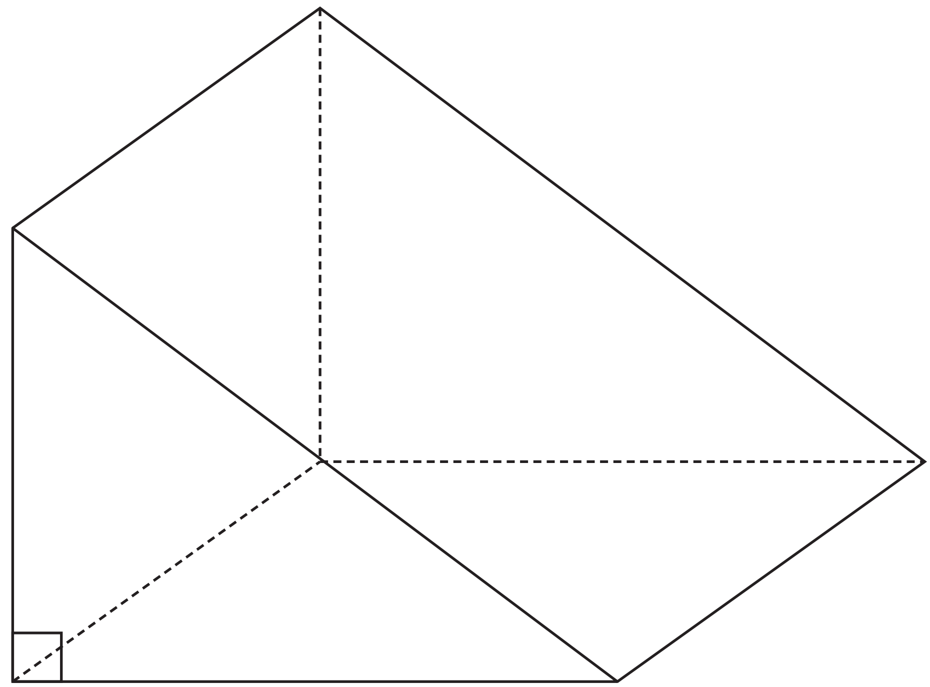
3. An artist is designing a new climbing structure for the neighborhood park. She wants to use a cylinder and triangular prism to create the play space.





How many square feet of paint would be needed to cover the total surface area of both climbing structures? Round to the nearest hundredth, when necessary.

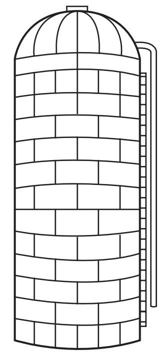
4. Use a centimeter ruler to determine the measurements required to calculate the lateral surface area of the triangular prism. Then, determine the lateral surface area.



ACTIVITY 4.2

Cylinder and Half-Sphere Problem

A silo is used to store grain that farm animals eat during the winter months. The top of the silo is a hemisphere (a half-sphere) with a radius of 8 feet. The cylindrical body of the silo shares the same radius as the hemisphere and has a height of 40 feet.



A truck hauling grain to the silo has a rectangular container attached to the back of the truck that is 8 feet in length, 5 feet in width, and 4 feet in height.

1. Determine the number of truckloads of grain required to fill an empty silo.

ACTIVITY 4.3 Cones and Spheres Problems

The word *fill* is often used to describe the volume of a solid. Consider how much you can fill the cones described in each question.

1. A frozen yogurt cone is 12 centimeters in height and has a diameter of 6 centimeters. A scoop of frozen yogurt is placed on top of the cone. Each scoop of frozen yogurt is a sphere with a diameter of 6 centimeters.

   If the scoop of frozen yogurt melts into the cone, will the cone overflow? Explain your reasoning.

2. The frozen yogurt shop also provides customers with a bowl option. The sizes available are shown in the table below.

Bowl Size	Volume (cubic centimeters)
Small	415
Medium	545
Large	670

You want a bowl large enough to hold 3 spherical scoops of frozen yogurt, each with a diameter of 7 centimeters. Which bowl should you choose if you want to use a bowl with the least amount of wasted space? Explain your reasoning.

ACTIVITY 4.4

Comparing Cylinder Volumes

If you were to make a cylinder using a piece of paper, is the volume the same no matter which way you roll the paper?

Use the paper provided by your teacher to complete an investigation of cylinder volume.

- Roll one piece of paper along its longer side. This will form a cylinder (Cylinder A) with no bases that is tall and narrow. Tape along the edges without overlapping the sides of the paper.
- Roll a second piece of paper of the same size but a different color along its shorter side. This will form a cylinder (Cylinder B) with no bases that is short and wide. Tape along the edges without overlapping the sides.

1. Do you think that the two cylinders have the same volume? Make a conjecture and explain your thinking.

2. Stand Cylinder B upright on your desk with Cylinder A inside it. Pour centimeter cubes into Cylinder A until it is full. Carefully lift Cylinder A, so that the centimeter cubes fall into Cylinder B. Describe what happens.

3. Consider your conjecture about the volumes of Cylinder A and Cylinder B.

   a. Was your conjecture correct? How do you know?

Ask Yourself:
Did you justify your mathematical reasoning?

   b. If your conjecture was incorrect, why do you think what actually happened was different from your conjecture?

4. Measure the dimensions of the tall, narrow cylinder and enter them in the table as measures for Cylinder A. Then, measure the dimensions of the short, wide cylinder and enter them in the table as measures for Cylinder B.

Dimension	Cylinder A	Cylinder B
Height (in.)		
Diameter (in.)		
Radius (in.)		

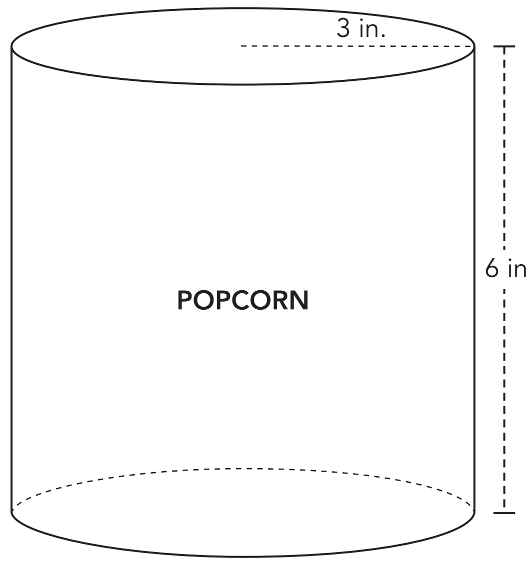
5. Calculate the volumes of the cylinders. Round your answers to the nearest hundredth, when necessary.

6. By how much would you have to decrease the height of Cylinder B to make the volumes of the two cylinders equal?

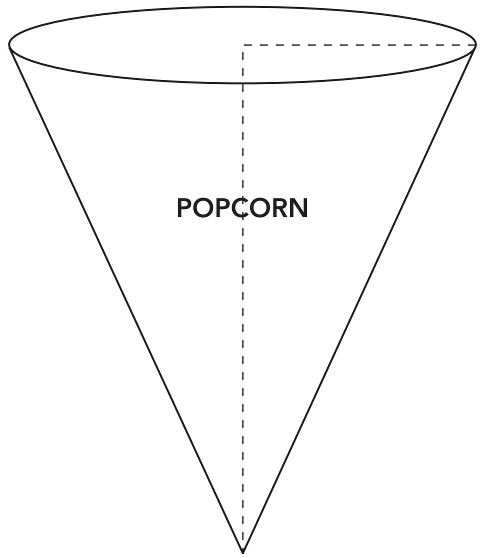
ACTIVITY 4.5 Cone Vs. Cylinder

Consider the open popcorn containers: one is a cylindrical tub and the other is conical. When they are full of popcorn, the containers hold the same amount of popcorn.

1. Calculate the volume and surface area of the open cylindrical tub. Round your answers to the nearest hundredth, when necessary.



2. When the height of the conical container is 10.1 inches, what is the length of the radius of the cone?



3. When the length of the radius of the conical container is the same as the length of the radius of the cylindrical tub, what is the height of the cone?

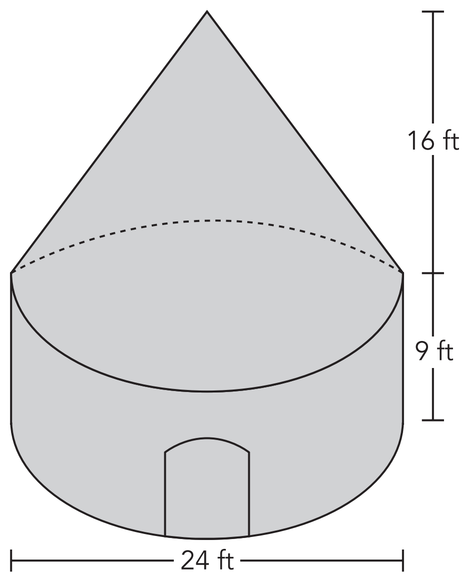
Ask Yourself:
How can you organize and record your mathematical ideas?

Talk the Talk

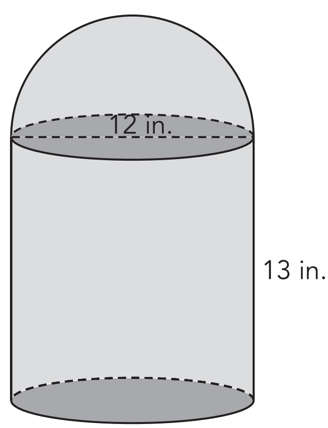
Composite Solids

Use what you know about the volume of curved figures to solve each problem. Round your answer to the nearest hundredth.

1. The building shown is composed of a cylinder and a cone. Determine the volume of this building.



2. The top of this model forms a half-sphere. Determine the volume of this model.



Lesson 4 Assignment

Write

Describe the relationships among the volumes of a cylinder, cone, and sphere with the same height and radius.

Remember

The formula for the volume of a cylinder is Bh.

The formula for the total surface area of a cylinder is $2\pi r^2 + 2\pi rh$

The formula for the volume of a cone is $\frac{1}{3}Bh$.

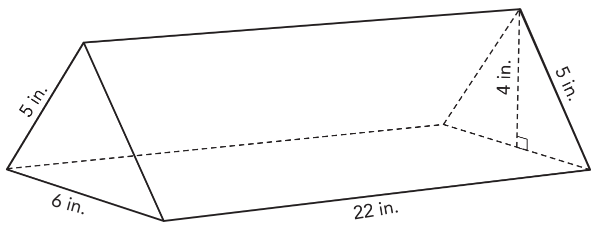
The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$.

Practice

Solve each problem. Round to the nearest hundredth.

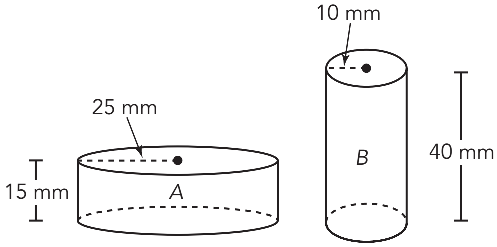
1. Veronica is making a large sphere-shaped piñata for a party. The piñata is going to be 2 feet in diameter. To determine how much candy she will need to fill the piñata, she needs to know the volume of the piñata. Calculate the volume of this large piñata.

2. A piece of artwork is shipped in a box in the shape of a triangular prism. How many square inches of paper are needed to cover the box?



Lesson 4 Assignment

3. A jeweler sold a string of 50 8-millimeter pearls. He needs to choose a box to put them in.

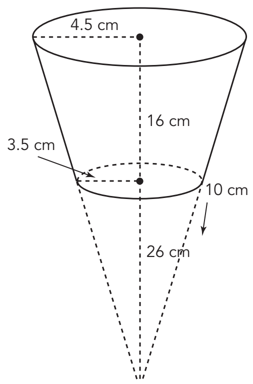


a. Which box should the jeweler choose?

b. If the manufacturer of the boxes charges by the amount of material needed to make the box, is the jeweler's choice more or less expensive than the other box?

Lesson 4 Assignment

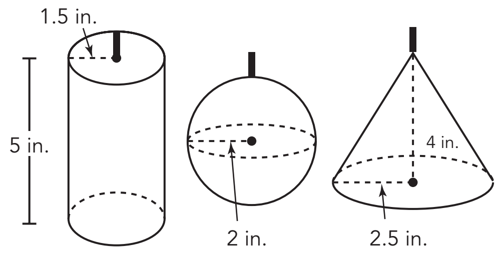
4. The drinking glass is not a cylinder but is actually part of a cone. Determine the volume of the glass.



5. A building is composed of a hemisphere on top of a cylinder. The radius of the hemisphere is 12 feet, as is the radius of the cylinder. The height of the cylinder is 20 feet. What is the volume of the building?

Lesson 4 Assignment

6. A candle company makes pillar candles, spherical candles, and conical candles. They have an order for 3 pillar, 2 spherical, and 1 conical candle. Wax is sold in large rectangular blocks. What are the possible dimensions for a wax block that could be used to fill this order?



7. An ice cream shop sells cones with a volume of 94.2 cubic centimeters. They want to double the volume of their cones without changing the diameter of the cone, so that the ice cream scoop will stay on top of the cone. What should the dimensions of the new cone be if the old cone had a height of 10 centimeters?

TOPIC 4 SELF-REFLECTION

Name: ____________________________

Volume of Curved Figures

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Volume of Curved Figures* topic by:

TOPIC 4: *Volume of Curved Figures*	Beginning of Topic	Middle of Topic	End of Topic
describing the volume formula for a cylinder, $V = Bh$, in terms of its base area and height.	◯	◯	◯
modeling the relationship between the volume of a cylinder and a cone having congruent bases and heights and connecting this relationship to the formulas.	◯	◯	◯
solving real-world and mathematical problems involving the volumes of cones, cylinders, and spheres.	◯	◯	◯
solving real-world and mathematical problems involving lateral and total surface area of rectangular and triangular prisms and cylinders.	◯	◯	◯

continued on the next page

TOPIC 4 SELF-REFLECTION *continued*

At the end of the topic, take some time to reflect on your learning and answer the following questions.

1. Describe a new strategy you learned in the *Volume of Curved Figures* topic.

2. What mathematical understandings from the topic do you feel you are making the most progress with?

3. Are there any concepts within this topic that you find challenging to understand? How do you plan to improve your understanding of these concepts?

Volume of Curved Figures Summary

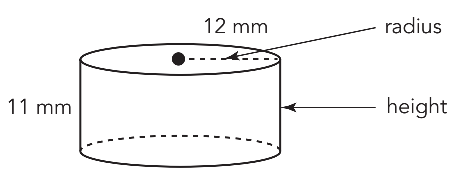
LESSON 1 Volume, Lateral, and Total Surface Area of a Cylinder

A **cylinder** is a three-dimensional object with two parallel, congruent circular bases. A **right cylinder** is a cylinder in which the bases are circles that are aligned one directly above the other.

The **radius of a cylinder** is the distance from the center of the base to any point on the edge of the base. The length of the radius of a cylinder is the same on both bases. The **height of a cylinder** is the length of a line segment drawn from one base to the other base, perpendicular to both bases.

The volume of any cylinder can be calculated by multiplying the area of the circular base by the height of the cylinder, $V = Bh$. The formula for the area of a circle is $A = \pi r^2$, so the formula for the volume of a cylinder is $V = \pi r^2 h$.

For example, calculate the volume of the given cylinder.



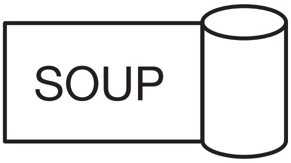
$$V = Bh$$

$$V = \pi r^2 h$$

$$= \pi(12^2)(11)$$

$$\approx 4973.76 \text{ cubic millimeters}$$

Consider the right cylinder shown

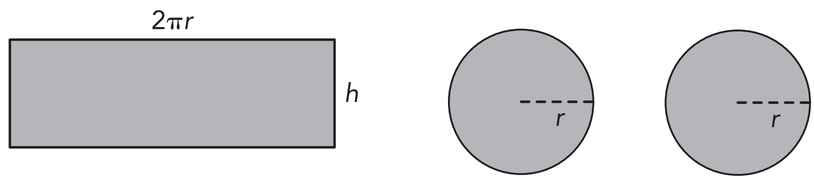


The total surface area is the sum of the areas that form the surface of a three-dimensional figure. The label of a can covers the surface of the can that does not include the two bases that are circles. You call this *lateral surface area*. If you cut the label from a can, you can see that the label is a rectangle.

The width of the rectangle is the height of the can. Because the label wraps around the can, the length of the rectangle is the circumference of the can. The total surface area of the can is the area of the rectangle plus the area of the two circular bases.

NEW KEY TERMS

- cylinder [cilindro]
- right cylinder [cilindro recto]
- radius of a cylinder [radio de un cilindro]
- height of a cylinder
- cone [cono]
- height of a cone
- sphere [esfera]
- center of a sphere [centro de una esfera]
- radius of a sphere [radio de una esfera]
- diameter of a sphere [diámetro de una esfera]
- great circle



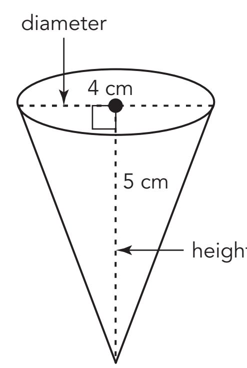
For example, if the radius of the base of a cylinder is 2 inches and the height of the cylinder is 5 inches, you can determine the lateral surface area and the total surface area of the cylinder.

Lateral surface area $= 2\pi rh = 2\pi(2)(5) \approx 62.8$ in.^{2}

Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi(2)(5) + 2\pi(2)^2 \approx 87.92$ in.^{2}

LESSON 2 Volume of a Cone

A **cone** is a three-dimensional object with a circular or oval base and one vertex. The **height of a cone** is the length of a line segment drawn from the vertex to the base of the cone. In a right cone, this line segment is perpendicular to the base.



The volume of a cone is one-third the volume of a cylinder with the same base and height as the cone. Therefore, the formula for the volume of a cone is $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2h$.

For example, calculate the volume of the given cone.

$$V = \frac{1}{3}Bh$$

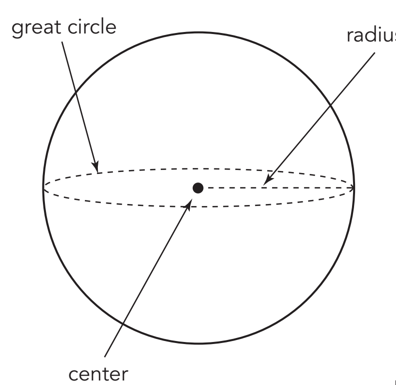
$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{4}{2}\right)^2(5)$$

$$\approx 20.94 \text{ cubic centimeters}$$

LESSON 3 Volume of a Sphere

A **sphere** is the set of all points in three dimensions that are the same distance from a given point called the **center of a sphere**.



Like a circle, a sphere has radii and diameters. A segment drawn from the center of the sphere to a point on the sphere is called a **radius of a sphere**. A segment drawn between two points on the sphere that passes through the center is a **diameter of a sphere**. The length of a diameter is twice the length of a radius.

A **great circle** is the circumference of a sphere at the sphere's widest part.

The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r represents the length of the radius of the sphere.

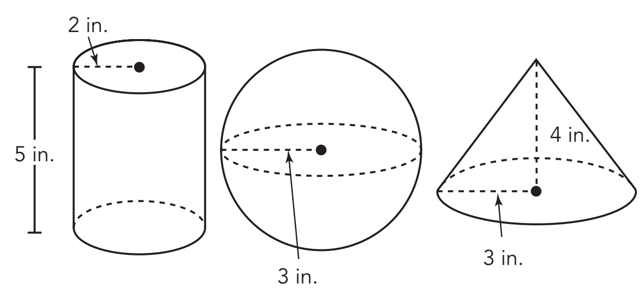
For example, calculate the volume of a sphere with a radius length of 4.5 inches.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(4.5)^3$$
$$\approx 381.7 \text{ cubic inches}$$

LESSON 4 Volume and Surface Area Problems with Prisms, Cylinders, Cones, and Spheres

You can use the formulas for the lateral and total surface area of prisms and cylinders, or for the volume of cones, cylinders, and spheres, to solve real-world and mathematical problems.

For example, you can compare the volumes of the cylinder, sphere, and cone.



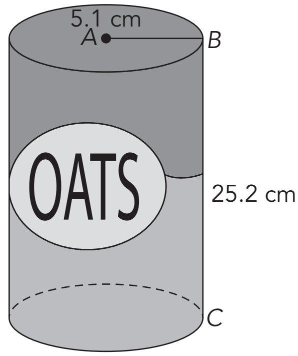
Volume of the cylinder $= Bh$
$$= \pi r^2 h$$
$$= \pi(2)^2(5)$$
$$\approx 62.83 \text{ cubic inches}$$

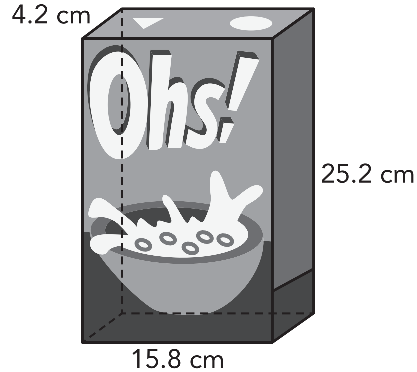
Volume of the sphere $= \frac{4}{3}\pi r^3$
$$= \frac{4}{3}\pi(3)^3$$
$$\approx 113.1 \text{ cubic inches}$$

Volume of the cone $= \frac{1}{3}Bh$
$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(3)^2(4)$$
$$\approx 37.7 \text{ cubic inches}$$

The volume of the sphere is the greatest, and the volume of the cone is the least.

In another example, you can compare the total surface area of a cylinder and a rectangular prism.





Total Surface Area of Cylinder

$S = 2\pi rh + 2\pi r^2$

$S = 2\pi(5.1)(25.2) + 2\pi(5.1)^2$

$S \approx 970.45 \text{ cm}^2$

Total Surface Area of Rectangular Prism

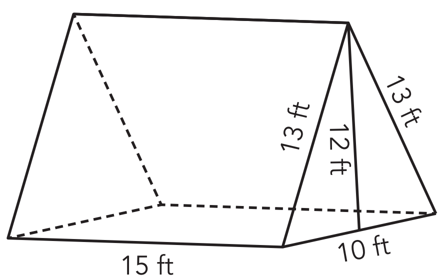
$S = Ph + 2B$

$S = (2(15.8) + 2(4.2))(25.2) + 2(15.8)(4.2)$

$S \approx 1140.72 \text{ cm}^2$

The total surface area of the rectangular prism is greater than the total surface area of the cylinder.

Finally, you can determine how many square feet of paint an artist needs to cover the total surface area of a climbing structure in the shape of a triangular prism.



$S = Ph + 2B$

$S = (10 + 13 + 13)(15) + 2\left(\frac{10 \cdot 12}{2}\right)$

$S = 540 + 120$

$S = 660 \text{ ft}^2$

The artist needs 660 square feet of paint to cover the climbing structure.

Math Glossary

A

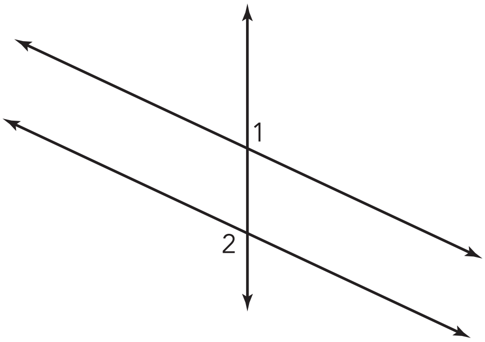
absolution deviation

The absolute value of each deviation is called the absolute deviation.

alternate exterior angles

Alternate exterior angles are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are outside the other two lines.

Example

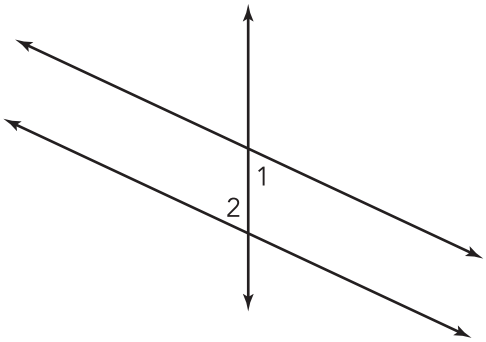


Angles 1 and 2 are alternate exterior angles.

alternate interior angles

Alternate interior angles are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are between the other two lines.

Example

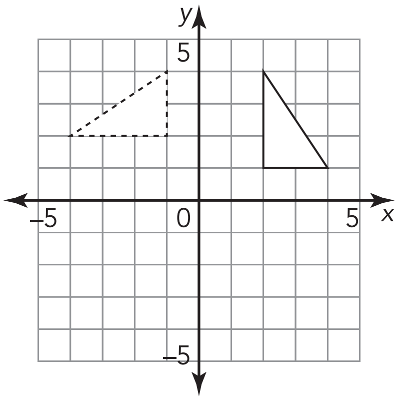


Angles 1 and 2 are alternate interior angles.

angle of rotation

The angle of rotation is the amount of rotation, in degrees, about a fixed point, the center of rotation.

Example



The angle of rotation is 90° counterclockwise about the origin (0, 0).

Angle-Angle Similarity theorem

The Angle-Angle Similarity theorem states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

association

A pattern or relationship identified in a scatter plot of a two-variable data set is called an association.

B

bar notation

Bar notation is used to indicate the digits that repeat in a repeating decimal.

Example

In the quotient of 3 and 7, the sequence 428571 repeats. The numbers that lie underneath the bar are the numbers that repeat.

$\frac{3}{7} = 0.4285714285714... = 0.\overline{428571}$

base

The base of a power is the factor that is multiplied repeatedly in the power.

Examples

$2^3 = 2 \times 2 \times 2 = 8$ (base: 2) $8^0 = 1$ (base: 8)

bivariate data

When you collect information about two separate characteristics for the same person, thing, or event, you have collected bivariate data.

break-even point

When one line represents the cost of an item and the other line represents the income from selling the item, the point of intersection is called the break-even point.

C

cash advance

A cash advance is a service provided by credit card companies that allows their customers to take out money directly from a bank or ATM.

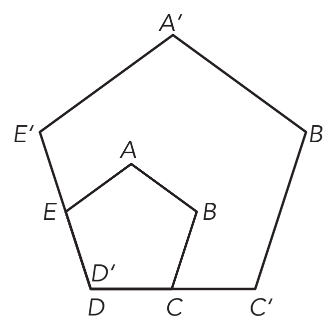
census

A census is the data collected from every member of a population.

center of dilation

The point from which a dilation is generated is called the center of dilation.

Example

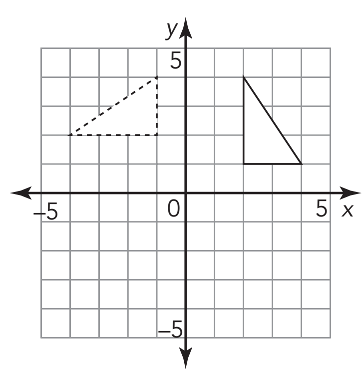


The center of dilation is point *D*.

center of rotation

The center of rotation is the point around which a figure is rotated. The center of rotation can be a point on the figure, inside the figure, or outside the figure.

Example



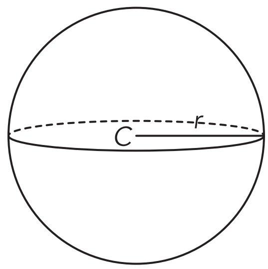
The figure has been rotated 90° counterclockwise about the center of rotation, which is the origin (0, 0).

center of a sphere

The given point from which the set of all points in three dimensions are the same distance is the center of the sphere.

Example

Point *C* is the center of the sphere.



characteristic

In the expression $a \times 10^n$, the variable *n* is called the characteristic.

Example

$6.1 \times 10^5 = 610{,}000$

↑

characteristic

closed

A set of numbers is said to be closed under an operation if the result of the operation on two numbers in the set is a defined value also in the set.

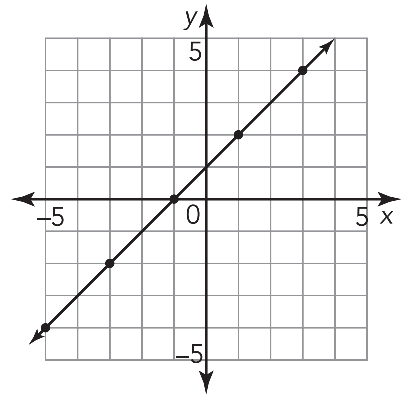
Example

The set of integers is closed under the operation of addition because for every two integers *a* and *b*, the sum $a + b$ is also an integer.

collinear points

Collinear points are points that lie in the same straight line.

Example



All the points on the graph are collinear points.

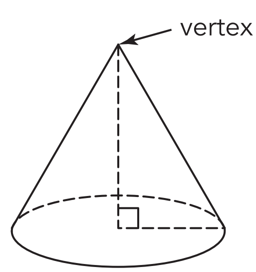
compound interest

Compound interest is a percentage that is paid on the principal and interest after each time period.

cone

A cone is a three-dimensional object with a circular or oval base and one vertex.

Example



congruent angles

Congruent angles are angles that are equal in measure.

congruent figures

Figures that have the same size and shape are congruent figures. If two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

congruent line segments

Congruent line segments are line segments that have the same length.

consistent system

Systems that have one or an infinite number of solutions are called consistent systems.

constant function

When the y-value of a function does not change, or remains constant, the function is called a constant function.

constant of proportionality

In a proportional relationship, the ratio of all y-values to their corresponding x-values is constant. This specific ratio, $\frac{y}{x}$, is called the constant of proportionality. Generally, the variable k is used to represent the constant of proportionality.

converse

The converse of a theorem is created when the if-then parts of that theorem are exchanged.

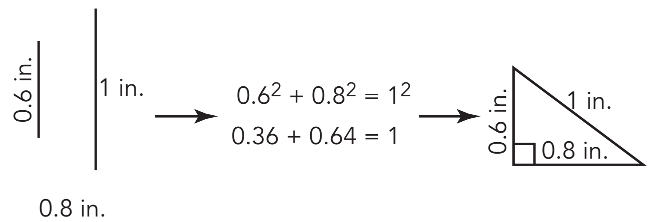
Example

Triangle Inequality theorem:	converse of Triangle Inequality theorem:
If a polygon is a triangle, then the sum of any two of its side lengths is always greater than the length of the third side.	If you have three side lengths, and the sum of any two of the side lengths is greater than the third side length, then the side lengths can form a triangle.

Converse of the Pythagorean Theorem

The Converse of the Pythagorean Theorem states that if the sum of the squares of the two shorter sides of a triangle equals the square of the longest side, then the triangle is a right triangle.

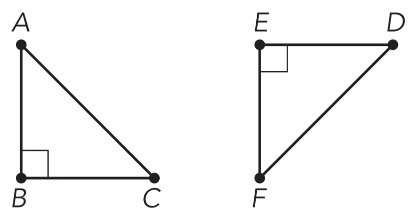
Example



corresponding angles

Corresponding angles are angles that have the same relative positions in geometric figures.

Example

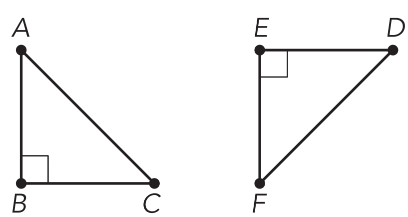


Angle B and Angle E are corresponding angles.

corresponding sides

Corresponding sides are sides that have the same relative positions in geometric figures.

Example



Sides AB and DE are corresponding sides.

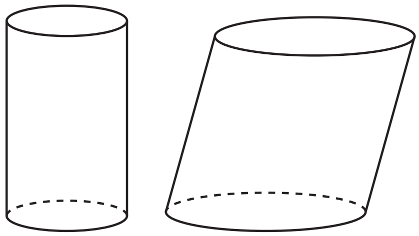
cubic function

A cubic function is a function that can be written in the form $y = ax^3 + bx^2 + cx + d$, where each coefficient or constant a, b, c, and d is a real number and a is not equal to 0.

cylinder

A cylinder is a three-dimensional object with two parallel, congruent circular bases.

Examples



D

data

When information is collected, the facts or numbers gathered are called data.

decreasing function

When the value of a dependent variable decreases as the independent variable increases, the function is called a decreasing function.

deferment

A deferment is a period of time, usually up to two years, in which students delay paying the principal and interest on their loan.

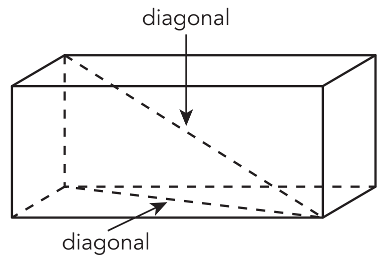
deviation

The deviation of a data value indicates how far that data value is from the mean.

diagonal

In a three-dimensional figure, a diagonal is a line segment connecting any two non-adjacent vertices.

Example



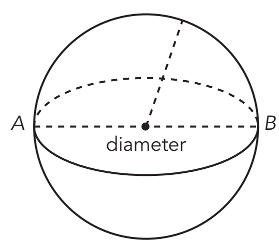
diagonal of a square

A diagonal of a square is a line segment connecting opposite vertices of the square.

diameter of the sphere

A segment drawn between two points on the sphere that passes through the center of the sphere is a diameter of the sphere.

Example

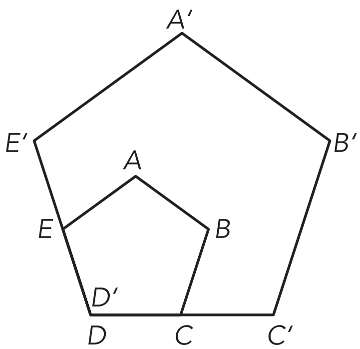


The diameter of the sphere is labeled.

dilation

A dilation is a transformation that produces a figure that is the same shape as the original figure, but not necessarily the same size.

Example

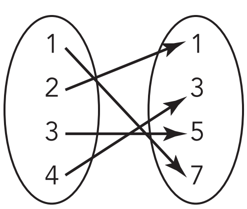


Pentagon *A'B'C'D'E'* is a dilation of Pentagon *ABCDE*.

domain

The domain of a function is the set of all inputs of the function.

Example



The domain in the mapping shown is {1, 2, 3, 4}.

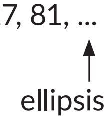
E

ellipsis

An ellipsis is a set of three periods which stands for "and so on."

Example

3, 9, 27, 81, ...



enlargement

When the scale factor is greater than 1, the image is called an enlargement.

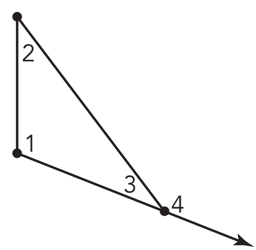
explanatory variable

The independent variable can also be called the explanatory variable.

exterior angle of a polygon

An exterior angle of a polygon is an angle between a side of a polygon and the extension of its adjacent side.

Example

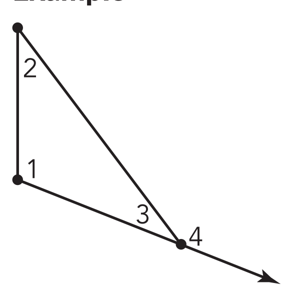


Angle 4 is an exterior angle of a polygon.

Exterior Angle theorem

The Exterior Angle theorem states that the measure of the exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles of the triangle.

Example



According to the Exterior Angle theorem, $m\angle 4 = m\angle 1 + m\angle 2$.

extrapolating

Extrapolating is predicting values that fall outside the plotted values on a scatter plot.

F

first differences

First differences are the values determined by subtracting consecutive *y*-values in a table when the *x*-values are consecutive integers. When the first differences are equal, the points represented by the ordered pairs in the table will form a straight line.

Example

x	y
1	25
2	34
3	45

9

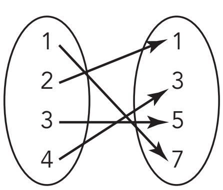
11

The first differences are 9 and 11, so the points represented by these ordered pairs will not form a straight line.

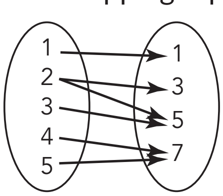
function

A function maps each input to one and only one output.

Example



This mapping represents a function.



This mapping does NOT represent a function.

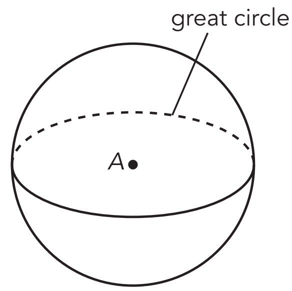
G

great circle

A great circle is the circumference of the sphere at the sphere's widest part.

Example

Point *A* is the center of the sphere. It is also the center of the great circle.

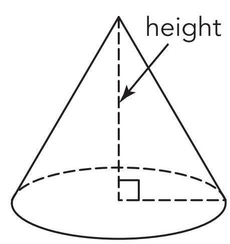


H

height of a cone

The height of a cone is the length of a line segment drawn from the vertex to the base of the cone. In a right cone, this line segment is perpendicular to the base.

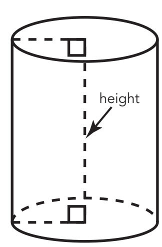
Example



height of a cylinder

The height of a cylinder is the length of a line segment drawn from one base to the other base, perpendicular to both bases.

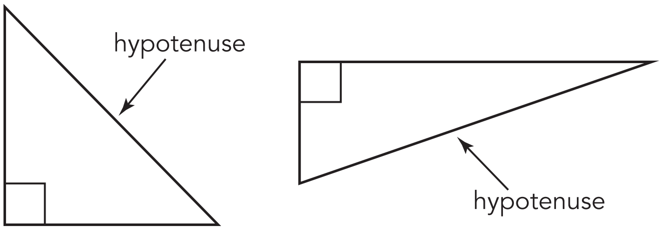
Example



hypotenuse

The side opposite the right angle in a right triangle is called the hypotenuse.

Examples

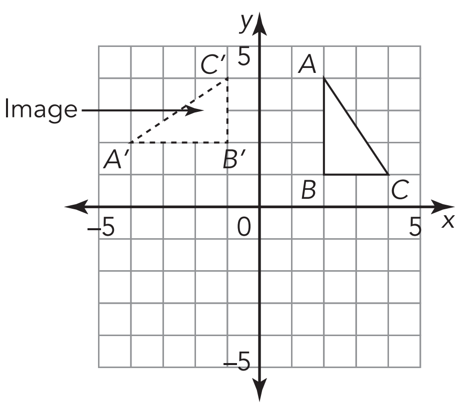


I

image

The new figure created from a transformation is called the image.

Example



inconsistent system

Systems that have no solution are called inconsistent systems.

increasing function

When both values of a function increase together, the function is called an increasing function.

infinitely many solutions

When the solution to the equation is a true statement for any value of the variable, the equation has infinitely many solutions.

Example

The equation $x = x$ has infinitely many solutions.

input

The first coordinate of an ordered pair in a relation is the input.

integers

Integers are the set of whole numbers and their additive inverses.

Example

The set of integers can be represented as {... −3, −2, −1, 0, 1, 2, 3, ...}

interpolating

Interpolating is predicting values that fall within the plotted values on a scatter plot.

irrational numbers

Numbers that cannot be written as fractions in the form $\frac{a}{b}$, where a and b are integers and b is not equal to 0, are irrational numbers.

Examples

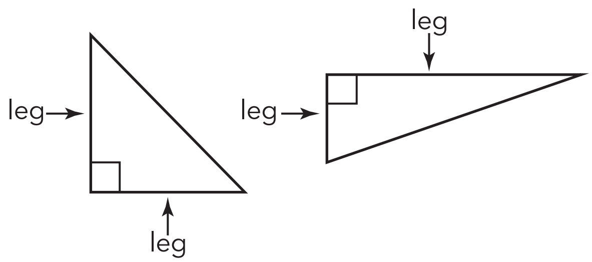
The numbers $\sqrt{2}$, 0.313113111..., and π are irrational numbers

L

leg

A leg of a right triangle is either of the two shorter sides. Together, the two legs form the right angle of a right triangle.

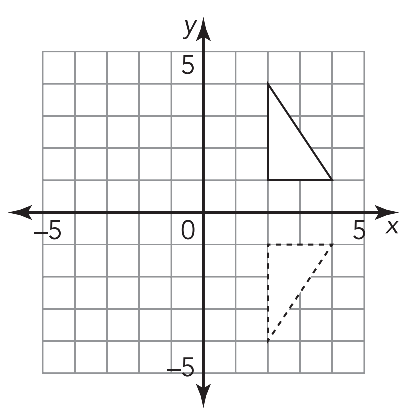
Examples



line of reflection

A line of reflection is a line that acts as a mirror so that corresponding points are the same distance from the line.

Example

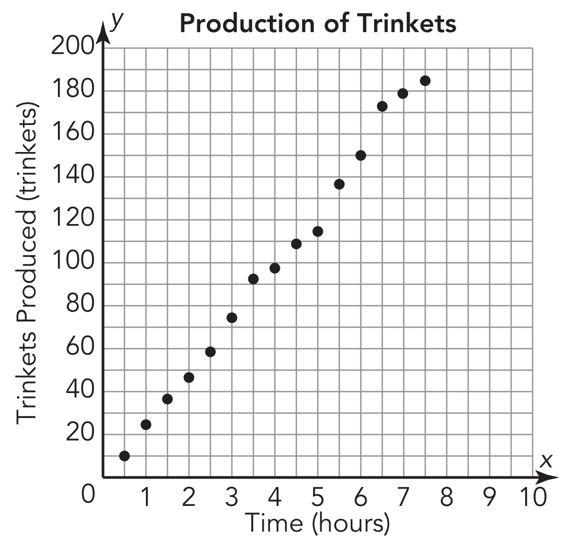


The *x*-axis is the line of reflection.

linear association

A linear association occurs when the points on the scatterplot seem to form a line.

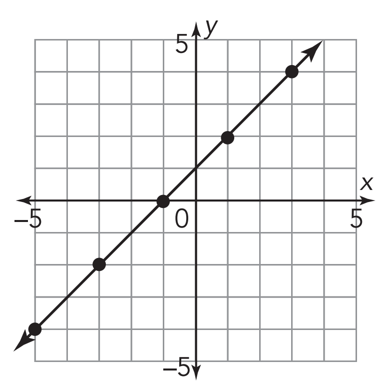
Example



linear function

A function whose graph is a straight line is a linear function.

Example



The function $f(x) = x + 1$ is a linear function.

M

mantissa

In the expression $a \times 10^n$, the variable a is called the mantissa. In scientific notation, the mantissa is greater than or equal to 1 and less than 10.

Example

$6.1 \times 10^5 = 610{,}000$

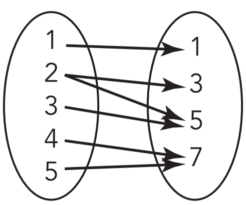
↑

mantissa

mapping

A mapping represents two sets of objects or items. Arrows connect the items to represent a relationship between them.

Example



mean absolute deviation

The mean absolute deviation (MAD) is the mean of the absolute deviations.

model

When you use a trend line, the line and its equation are often referred to as a model of the data. (See *trend line*.)

N

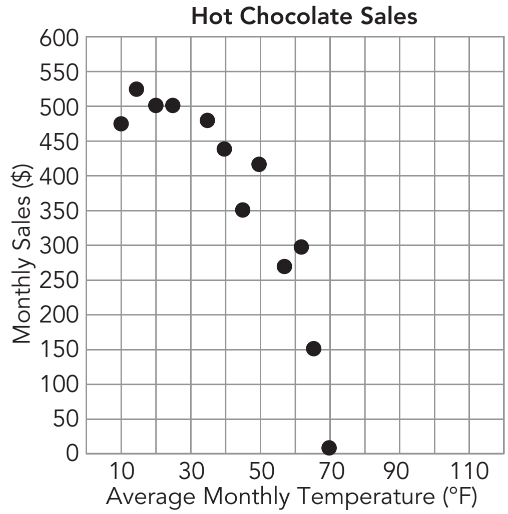
natural numbers

Natural numbers consist of the numbers that you use to count objects: {1, 2, 3, . . .}.

negative association

If the response variable decreases as the explanatory variable increases, then the two variables have a negative association.

Example



There is a negative association between average monthly temperature and hot chocolate sales.

non-proportional relationship

An equation in the form $y = mx + b$, where b is not equal to 0, represents a non-proportional relationship.

no solution

When the solution to the equation is a false statement, the equation has no solution.

Example

The equation $x + 0 = x + 1$ has no solution.

O

online calculator

An online calculator is an Internet-based application that quickly performs calculations for the user.

order of magnitude

The order of magnitude is an estimate of size expressed as a power of ten.

Example

The Earth's mass has an order of magnitude of about 10^{24} kilograms.

one solution

When the solution to the equation is a true statement with one value equal to the variable, there is only one solution.

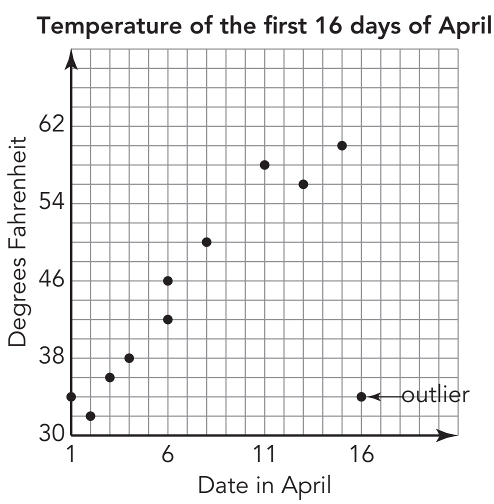
Example

The equation $x + 2 = 8$ has only one solution: $x = 6$.

outlier

An outlier for bivariate data is a point that varies greatly from the overall pattern of the data.

Example



output

The second coordinate of an ordered pair in a relation is the output.

P

parameter

When data are gathered from a population, the characteristic used to describe the population is called a parameter.

perfect square

A perfect square is the square of an integer.

Example

9 is a perfect square: $3^2 = 9$

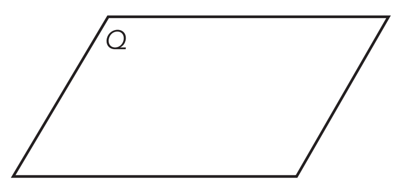
25 is a perfect square: $5^2 = 25$

plane

A plane is a flat surface. It has infinite length and width, but no depth. A plane extends infinitely in all directions in two dimensions. Planes are determined by three points, but are usually named using one uppercase letter.

Example

Plane Q is shown.



point of intersection

The point of intersection is the point at which two lines cross on a coordinate plane. In a system of linear equations, a point of intersection indicates a solution to both equations.

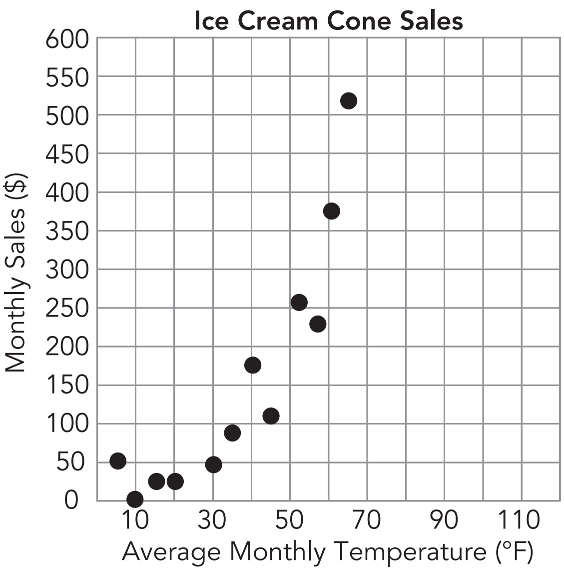
population

The population is the entire set of items from which data can be selected.

positive association

The two variables have a positive association if, as the explanatory variable increases, the response variable also increases.

Example

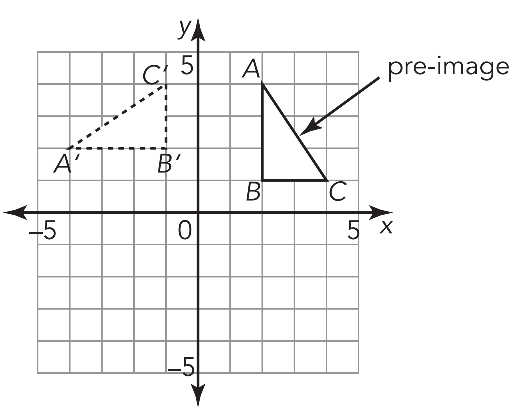


There is a positive association between the average monthly temperature and ice cream cone sales.

pre-image

The original figure in a transformation is called the pre-image.

Example



proof

A proof is a line of reasoning used to validate a theorem.

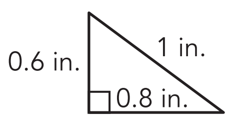
proportional relationship

A proportional relationship is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$, must represent the same constant.

Pythagorean Theorem

The Pythagorean Theorem states that the sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse. If a and b are the lengths of the legs, and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

Example



$0.6^2 + 0.8^2 = 1^2$
$0.36 + 0.64 = 1$

Pythagorean triple

Any set of three positive integers a, b, and c that satisfies the equation $a^2 + b^2 = c^2$ is a Pythagorean triple.

Example

3, 4, and 5 is a Pythagorean triple: $3^2 + 4^2 = 5^2$

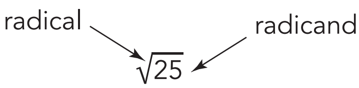
R

radical

The symbol $\sqrt{}$ is a radical.

Example

The expression shown is read as "the square root of 25" or as "radical 25"

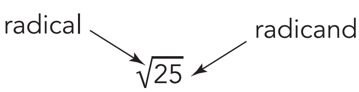


radicand

The quantity under the radical symbol is the radicand.

Example

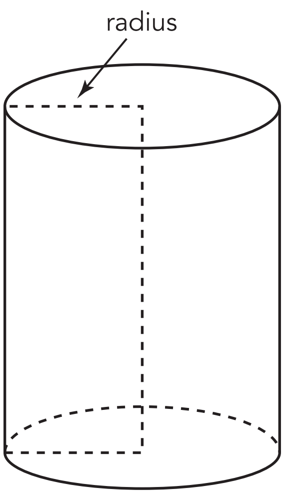
$\sqrt{25}$ The expression shown is read as "the square root of 25" or as "radical 25"



radius of a cylinder

The radius of a cylinder is the distance from the center of the base to any point on the edge of the base.

Example

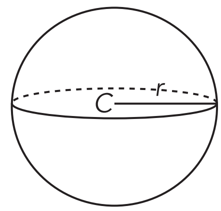


radius of the sphere

A segment drawn from the center of a sphere to a point on the sphere is called a radius of the sphere.

Example

Point C is the center of the sphere, and r is the radius of the sphere.



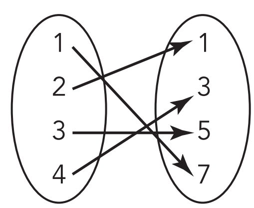
random sample

A random sample is a sample that is selected from the population in such a way that every member of the population has the same chance of being selected.

range

The range of a function is the set of all outputs of the function.

Example



The range in the mapping shown is {1, 3, 5, 7}.

rate of change

The rate of change for a situation describes the amount that the dependent variable changes compared with the amount that the independent variable changes.

rational numbers

Rational numbers are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples

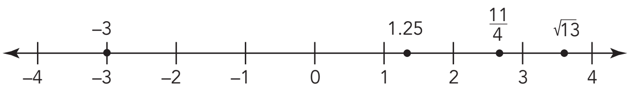
$-4, \frac{1}{2}, \frac{2}{3}, 0.67$, and $\frac{22}{7}$ are examples of rational numbers.

real numbers

Combining the set of rational numbers and the set of irrational numbers produces the set of real numbers. Real numbers can be represented on the real number line.

Examples

The numbers -3, 1.25, $\frac{11}{4}$ and $\sqrt{13}$ shown are real numbers.



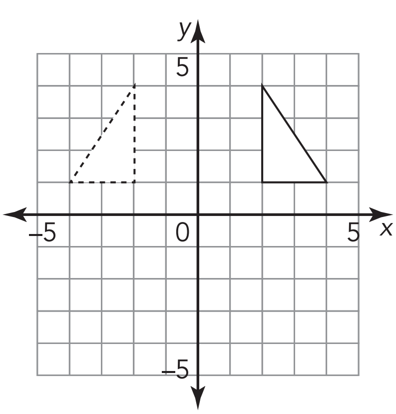
reduction

When the scale factor is less than 1, the image is called a reduction.

reflection

A reflection is a rigid motion transformation that "flips" a figure across a line of reflection.

Example



The figure has been reflected across the y-axis.

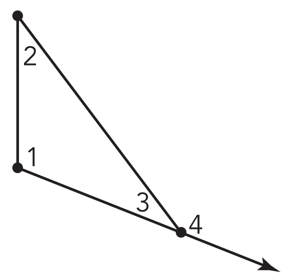
relation

A relation is any set of ordered pairs or the mapping between a set of inputs and a set of outputs.

remote interior angles of a triangle

The remote interior angles of a triangle are the two angles that are non-adjacent to the specified exterior angle.

Example



Angles 1 and 2 are remote interior angles of a triangle.

repeating decimal

A repeating decimal is a decimal in which a digit, or a group of digits, repeat(s) infinitely. Repeating decimals are rational numbers.

Examples

$\frac{1}{9} = 0.111...$ $\frac{7}{12} = 0.58333...$

$\frac{22}{7} = 3.142857142857...$

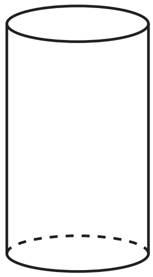
response variable

The dependent variable can also be called the response variable, because this is the variable that responds to what occurs to the explanatory variable.

right cylinder

A right cylinder is a cylinder in which the bases are aligned one directly above the other.

Example



rigid motion

A rigid motion is a special type of transformation that preserves the size and shape of the figure.

Examples

Translations, reflections, and rotations are examples of rigid motion transformations.

rotation

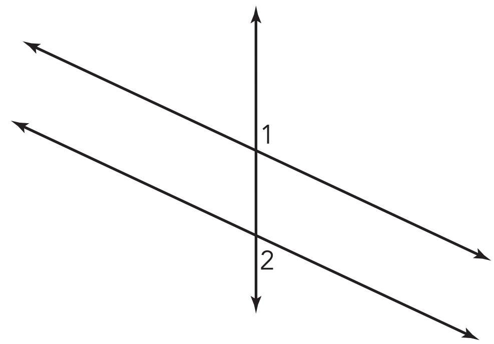
A rotation is a rigid motion transformation that turns a figure on a plane about a fixed point, called the center of rotation, through a given angle, called the angle of rotation.

S

same-side exterior angles

Same-side interior angles are formed when a transversal intersects two other lines. These angle pairs are on the same side of the transversal and are outside the other two lines.

Example

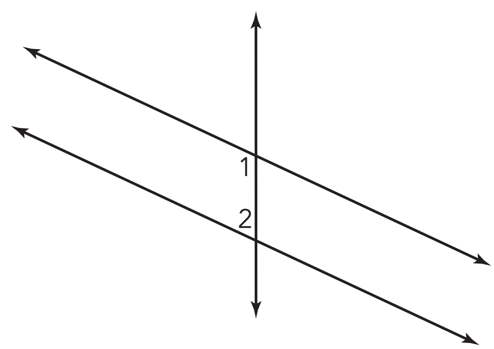


Angles 1 and 2 are same-side exterior angles.

same-side interior angles

Same-side interior angles are formed when a transversal intersects two other lines. These angle pairs are on the same side of the transversal and are between the other two lines.

Example



Angles 1 and 2 are same-side interior angles.

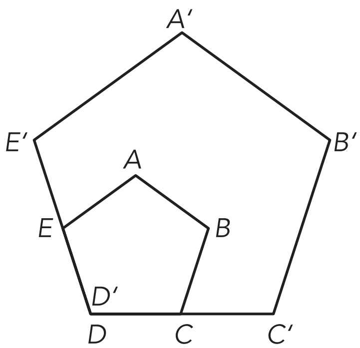
sample

When data are collected from a part of the population, the data are called a sample.

scale factor

In a dilation, the scale factor is the ratio of the distance of the new figure from the center of dilation to the distance of the original figure from the center of dilation.

Example

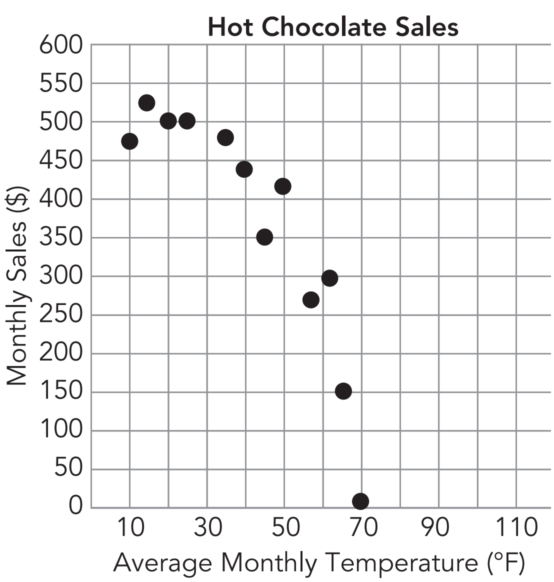


Pentagon *ABCDE* has been dilated by a scale factor of 2 to create Pentagon *A'B'C'D'E'*.

scatterplot

A scatterplot is a graph of a collection of ordered pairs that allows an exploration of the relationship between the points.

Example



scientific notation

Scientific notation is a notation used to express a very large or a very small number as the product of a number greater than or equal to 1 and less than 10 and a power of 10.

Example

The number 1,345,000,000 is written in scientific notation as 1.345×10^9.

sequence

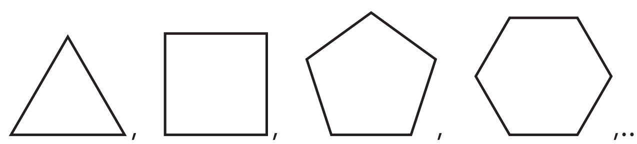
A sequence is a pattern involving an ordered arrangement of numbers, geometric figures, letters, or other objects.

Examples

Sequence A:

2, 4, 6, 8, 10, 12, . . .

Sequence B:



set

A set is a collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.

Examples

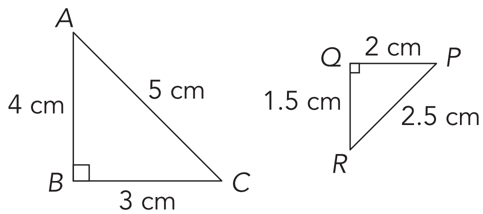
The set of counting numbers is {1, 2, 3, 4, ...}

The set of even numbers is {2, 4, 6, 8, ...}

similar

When two figures are similar, the ratios of their corresponding side lengths are equal.

Example



Triangle *ABC* is similar to Triangle *PQR*.

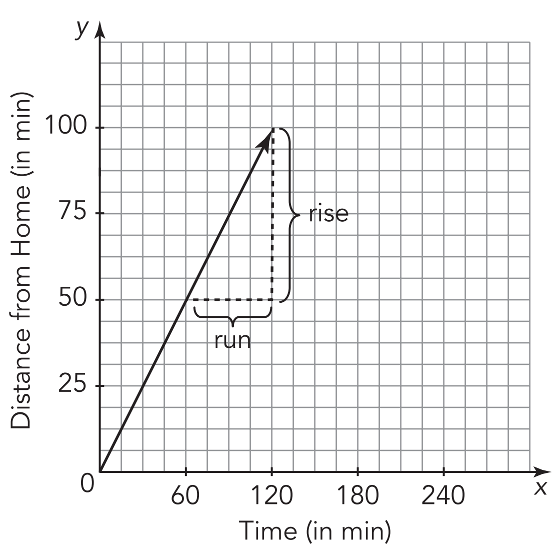
simple interest

Simple interest is a percentage that is paid only on the original principal.

slope

In any linear relationship, slope describes the direction and steepness of a line and is usually represented by the variable *m*. Slope is another name for rate of change. (See *rate of change*.)

Example



The slope of the line is $\frac{50}{60}$, or $\frac{5}{6}$.

slope-intercept form

The slope-intercept form of a linear equation is $y = mx + b$, where *m* is the slope of the line and (0, *b*) is the *y*-intercept.

solution of a linear system

The solution of a linear system is an ordered pair (*x*, *y*) that is a solution to both equations in the system. Graphically, the solution is the point of intersection.

Example

$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

The solution to this system of equations is (1, 6).

solution set of an inequality

The solution set of an inequality is the set of all points that make an inequality true.

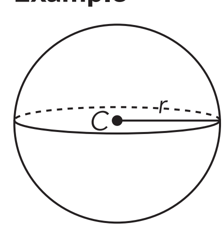
square root

A square root is one of two equal factors of a given number.

sphere

A sphere is the set of all points in three dimensions that are the same distance from a given point called the center of the sphere.

Example



statistic

When data are gathered from a sample, the characteristic used to describe the sample is called a statistic.

survey

A survey is a method of collecting information about a certain group of people.

system of linear equations

When two or more linear equations define a relationship between quantities they form a system of linear equations.

Example

$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

T

term

A term in a sequence is an individual number, figure, or letter in the sequence.

Example

2, 7, 12, 17, 22, 27, 32, ...

↑

term

terminating decimal

A terminating decimal has a finite number of digits, meaning that after a finite number of decimal places, all following decimal places have a value of 0. Terminating decimals are rational numbers.

Examples

$\frac{9}{10} = 0.9$ $\frac{15}{8} = 1.875$ $\frac{193}{16} = 12.0625$

terms of an investment

The terms of an investment include the type of loan, amount of money invested, and the length of the investment.

transformation

A transformation is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation.

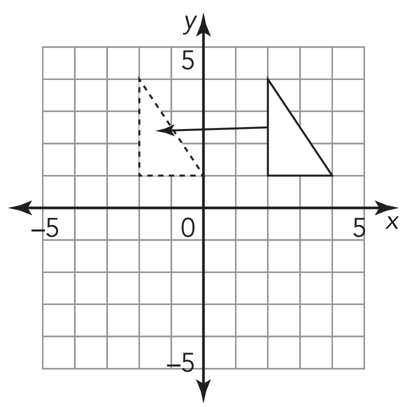
Examples

Translations, reflections, rotations, and dilations are examples of transformations.

translation

A translation is a rigid motion transformation that "slides" each point of a figure the same distance and direction.

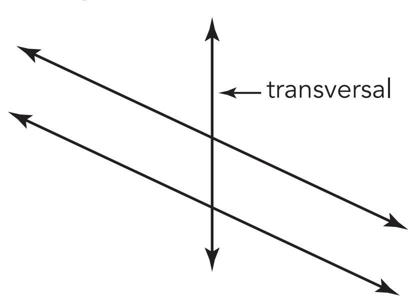
Example



transversal

A transversal is a line that intersects two or more lines at distinct points.

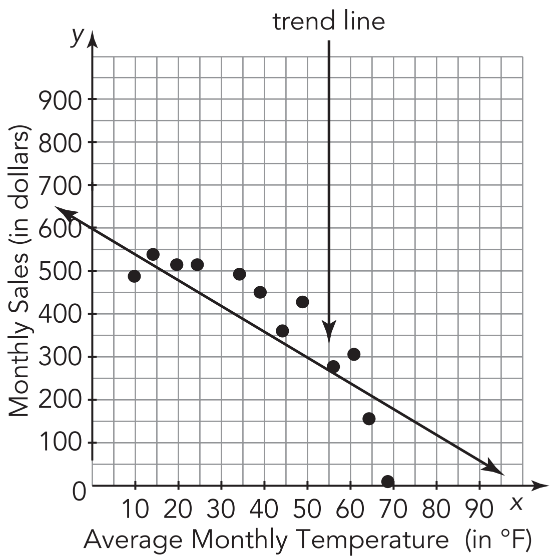
Example



trend line

A trend line is a line that is as close to as many points as possible but doesn't have to go through all of the points.

Example



Triangle Sum theorem

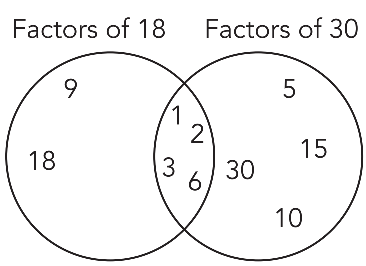
The Triangle Sum theorem states that the sum of the measures of the interior angles of a triangle is 180°.

V

Venn diagram

A Venn diagram uses circles to show how elements among sets of numbers or objects are related.

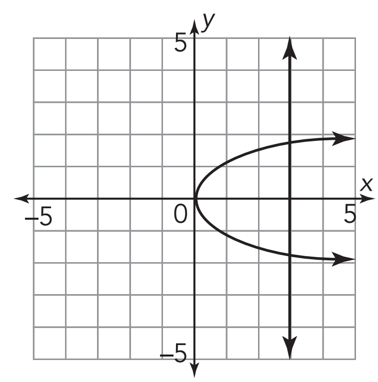
Example



vertical line test

The vertical line test is a visual method used to determine whether a relation represented as a graph is a function. To apply the vertical line test, consider all the vertical lines that could be drawn on the graph of a relation. If any of the vertical lines intersect the graph of the relation at more than one point, then the relation is not a function.

Example



The line drawn at $x = 3$ crosses two points on the graph, so the relation is not a function.

W

whole numbers

Whole numbers are made up of the set of natural numbers and the number 0, the additive identity.

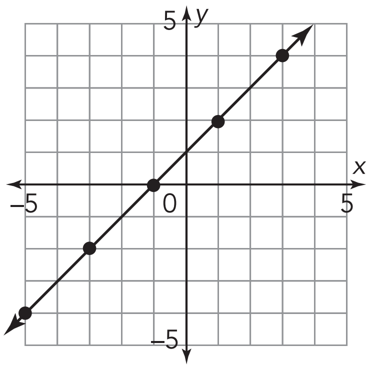
Example

The set of whole numbers can be represented as {0, 1, 2, 3, 4, 5, ...}.

y-intercept

The y-intercept is the y-coordinate of the point where a graph crosses the y-axis. The y-intercept can be written in the form (0, y).

Example



The y-intercept of the graph is (0, 1).

ISBN: 199-6-92207-614-2

Printed in the USA

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