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Email: copyrights@tea.texas.gov
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Mathematics Achievement Academy, Problem Solving and Discourse, Kindergarten:

Outcomes
1. Build knowledge and understanding of a problem-solving heuristic that applies to any mathematical content strand.
2. Identify actions of a proficient problem solver.
3. Build knowledge and understanding of mathematical discussions that complement the use of a problem-solving heuristic.
4. Build knowledge and understanding of formative assessment strategies that focus on making student thinking within the phases of the problem-solving model visible to the teacher.
5. Build knowledge and understanding of five practices that support mathematical discourse to strengthen student voice and agency with a problem-solving heuristic.

Resources:
Texas Gateway:
http://www.texasgateway.org/

English Language Proficiency Standards:
http://bit.ly/1xu3yhM

Interactive Mathematics Glossary:
http://bit.ly/1PPFBUn

TEA: Mathematics Resources:
https://goo.gl/JLZKzA

Texas Prekindergarten–Grade 2 Vertical Alignments:
DAY 1: PROBLEM SOLVING AND STUDENT VOICE

Mathematics Achievement Academy, Problem Solving and Discourse, Kindergarten: Outcomes
1. Build knowledge and understanding of a problem-solving heuristic that applies to any mathematical content strand.
2. Identify actions of a proficient problem solver.
3. Build knowledge and understanding of mathematical discussions that complement the use of a problem-solving heuristic.
4. Build knowledge and understanding of formative assessment strategies that focus on making student thinking within the phases of the problem-solving model visible to the teacher.
5. Build knowledge and understanding of five practices that support mathematical discourse to strengthen student voice and agency with a problem-solving heuristic.

Day 1: Face-to-Face Professional Learning Session
Learning Intentions
2. Identify the actions of teachers and students that support problem-solving development.
3. Understand appropriate applications of the analyzing given information, formulating a plan or strategy, determining a solution, and evaluating the reasonableness of the solution components of the TEKS-based problem-solving model.
4. Identify and apply questions that help students clarify their own understanding and clarify the group's understanding.
5. Understand how to implement mathematical language routines in order to increase and enhance student voice.

Success Criteria
1. I can describe attributes of problem solving in everyday life and identify connections to problem solving in mathematics.
2. I can implement teacher actions that support student problem-solving development and facilitate student actions that increase problem-solving development.
3. I can describe the analyzing given information, formulating a plan or strategy, determining a solution, and evaluating the reasonableness of the solution components of the TEKS-based problem-solving model.
4. I can use questions that help students clarify their own understanding and clarify the group's understanding, and I can explain how these questions can be implemented with students.
5. I can implement mathematical language routines in order to increase and enhance student voice.
The Elephant Challenge

Partition the large square into four congruent parts, each with the same number of elephants.
Teacher Roles and Student Roles

“To strengthen students’ mathematical reasoning and problem-solving abilities, teachers must create classroom environments that are mathematically ‘safe’ – that is, ones in which every child feels free to make conjectures, to explore different ways of thinking, and to share his or her ideas with classmates.”

*National Council of Teachers of Mathematics (2004)*

“If students are going to participate in the discussion, they have to be able to share their thinking and reasoning out loud in a way that is understandable to others.”

*Chapin, O’Connor, and Anderson (2013)*

“Successful teachers interpret what the child is doing and thinking and attempt to see the situation from the child’s point of view. Based on their interpretations, teachers conjecture what the child might be able to learn or abstract from his or her experiences.”

*Clements and Sarama (2014)*

“Understanding the development of students’ mathematical thinking is critical for selecting and creating instructional tasks, asking appropriate questions of students, guiding classroom discussions and adapting instruction to students’ needs, understanding students’ reasoning, assessing students’ learning progress, and diagnosing and remediating students’ learning difficulties.”

*National Council of Teachers of Mathematics (2016)*

<table>
<thead>
<tr>
<th>Roles of Teachers</th>
<th>Roles of Students</th>
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</table>

**Student Voice:** “Teaching involves not only developing important skills and conceptual understanding in mathematics but also supporting students’ coming to see themselves as legitimate and powerful doers of mathematics. This understanding of children’s identities, especially in relation to mathematics, can give teachers a better understanding of how and why some students make positive connections with mathematics to support and strengthen a child’s learning of mathematics and others do not. With this enhanced understanding, teachers can adjust their practice to support and strengthen a child’s learning of mathematics and his or her persistence as a confident mathematical learner.”

*(Aguirre, Mayfield-Ingram, and Martin, 2013, p.14)*
Dogs

There were 4 dogs in the park. Then, 3 more dogs came to the park.

How many dogs are at the park now?
Co-Craft Question Tasks

*Use the following script to practice guiding students through Co-Craft Questions.*

- **You will now play the role of a small group of kindergarten students.**
- You may choose to show students an image or to read one of the situations aloud to students.
- **What questions can be asked about this image/situation that can be answered with math?**
  - Prompt students to think about their responses individually.
  - Prompt students to share their mathematical questions with an elbow partner.
- **Who would like to share the question that you or your partner created? Allow for student responses.**
- **We created several questions that can be answered using the given information. For today, let us answer this question that was created.**
  - Write or verbally identify the question to be answered.
  - Prompt students to answer the question.
There are 4 cats. There are 5 dogs.

I had 7 books. I gave 3 books to my brother.
Stickers

Mr. Jones had 9 stickers. He gave 5 stickers to his son. How many stickers does Mr. Jones have now?
Formulating a Plan or Strategy Practice

Use the following process to practice guiding students through formulating a plan or strategy.

• As I read this story, make a movie in your mind about what you hear.

• Read one of the tasks aloud to students. Repeat if necessary.

• Think about the movie you saw in your mind. Turn and talk to your partner about the movie you saw in your mind. What information do you know? Prompt students to share their response with an elbow partner.

• What are we trying to figure out? Prompt students to share their response with an elbow partner.

• Think about what we could model first, and show me what you would do using your counters. Allow students to use counters to represent their thinking.

• I noticed many of you . . .

• Think about what we could model next, and show me what you would do using your counters. Allow students to use counters to represent their thinking.

• I noticed many of you . . .

• Is there any more given information we need to model? Allow for student responses.

• What are we trying to figure out? Allow for student responses.

• How can you use your model and tools to figure out _____? Allow for student responses.

• What counting strategy could you use to solve this problem? Allow for student responses.

• What is a plan that can be used to figure out _____? Allow for student responses.

• Who can restate what they just heard about a plan to solve the problem? Allow for student responses.

• Now that we have a plan, we are ready to figure out _____. Allow students to determine the solution.
Tasks

I have 2 big rocks. I have 7 small rocks. How many rocks do I have in all?

I had 10 rocks. I gave 6 rocks to my dad. How many rocks do I have now?

I had 1 rock. Later, I found 5 more rocks. How many rocks do I have now?

I have 6 black rocks. I have 3 red rocks. How many rocks do I have altogether?
Bus

There were 7 students on the bus. Four students got off the bus. How many students are still on the bus?
Online Professional Learning Syllabus
## Online Professional Learning Syllabus

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Tasks</th>
<th>Evidence of Learning</th>
<th>Submission or Completion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Virtual Tour&lt;br&gt;Watch the video.</td>
<td></td>
<td>Completion</td>
</tr>
<tr>
<td>2</td>
<td>Collecting Evidence of Student Reasoning&lt;br&gt;• Print Notes Page.&lt;br&gt;• Watch the video.</td>
<td>• Watch video</td>
<td>Completion</td>
</tr>
<tr>
<td>3</td>
<td>Tasks Choice Board&lt;br&gt;• Choose three introductory tasks from the Choice Board.&lt;br&gt;• Create exemplars.&lt;br&gt;• Complete Anticipated Misconceptions and Scaffolds.</td>
<td>• Upload Exemplars&lt;br&gt;• Complete Anticipated Misconceptions and Scaffolds</td>
<td>• Submission&lt;br&gt;• Submission</td>
</tr>
<tr>
<td>4</td>
<td>Analyzing Given Information Introductory Task&lt;br&gt;• Complete the Analyzing Given Information: Planning Sheet.&lt;br&gt;• Implement one of the chosen Introductory Tasks with your students.&lt;br&gt;• Respond to the reflection questions.</td>
<td>• Complete Analyzing Given Information: Planning Sheet&lt;br&gt;• Complete reflection question responses</td>
<td>• Submission&lt;br&gt;• Submission (with 0, 5, or 10 rating)</td>
</tr>
<tr>
<td>5</td>
<td>Analyzing Given Information Discourse Task: Co-Craft Questions&lt;br&gt;• Review the structure of Co-Craft Questions by watching the video and reviewing Toolkit Card.&lt;br&gt;• Implement the Discourse Task.&lt;br&gt;• Respond to the reflection questions.</td>
<td>• Watch video&lt;br&gt;• Complete reflection question responses</td>
<td>• Submission&lt;br&gt;• Submission (with 0, 5, or 10 rating)</td>
</tr>
<tr>
<td>6</td>
<td>Analyzing Given Information Formative Assessment&lt;br&gt;• Choose and implement the Analyzing Formative Assessment.&lt;br&gt;• Respond to the reflection questions.</td>
<td>• Complete reflection question responses</td>
<td>• Submission (with 0, 5, or 10 rating)</td>
</tr>
<tr>
<td>7</td>
<td>Formulating a Plan or Strategy Introductory Tasks&lt;br&gt;• Complete the Formulating a Plan or Strategy: Planning Sheet.&lt;br&gt;• Implement one of the chosen Introductory Tasks with your students.&lt;br&gt;• Respond to the reflection questions.</td>
<td>• Complete Formulating a Plan or Strategy: Planning Sheet&lt;br&gt;• Complete reflection question responses</td>
<td>• Submission&lt;br&gt;• Submission (with 0, 5, or 10 rating)</td>
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</tbody>
</table>
## Online Professional Learning Syllabus

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<th>Evidence of Learning</th>
<th>Submission or Completion</th>
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</thead>
<tbody>
<tr>
<td>8</td>
<td>Formulating a Plan or Strategy Discourse Task</td>
<td>• Complete reflection question responses</td>
<td>• Submission (with 0, 5, or 10 rating)</td>
</tr>
<tr>
<td></td>
<td>• Review the Formulating a Plan or Strategy Practice.</td>
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<td></td>
<td>• Implement the Discourse Task.</td>
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<td></td>
<td>• Respond to the reflection questions.</td>
<td></td>
<td></td>
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<tr>
<td>9</td>
<td>Formulating a Plan or Strategy Formative Assessment</td>
<td>• Complete reflection question responses</td>
<td>• Submission (with 0, 5, or 10 rating)</td>
</tr>
<tr>
<td></td>
<td>• Choose and implement the Formulating a Plan or Strategy Formative Assessment</td>
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<td></td>
<td>• Respond to the reflection questions.</td>
<td></td>
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<tr>
<td>10</td>
<td>Determining a Solution Introductory Tasks</td>
<td>• Complete Determining a Solution: Planning Sheet</td>
<td>• Submission</td>
</tr>
<tr>
<td></td>
<td>• Complete the Determining a Solution: Planning Sheet</td>
<td>• Complete reflection question responses</td>
<td>• Submission (with 0, 5, or 10 rating)</td>
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<td>• Implement one of the chosen Introductory Tasks with your students.</td>
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<td></td>
<td>• Respond to the reflection questions.</td>
<td></td>
<td></td>
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<tr>
<td>11</td>
<td>Determining a Solution Instructional Support Tasks</td>
<td>• Complete reflection question responses</td>
<td>• Submission (with 0, 5, or 10 rating)</td>
</tr>
<tr>
<td></td>
<td>• Review the Instructional Support Summary.</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>• Implement the Instructional Support Task.</td>
<td></td>
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<tr>
<td></td>
<td>• Respond to the reflection questions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Determining a Solution Formative Assessment</td>
<td>• Complete reflection question responses</td>
<td>• Submission (with 0, 5, or 10 rating)</td>
</tr>
<tr>
<td></td>
<td>• Choose and implement the Determining a Solution Formative Assessment</td>
<td></td>
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<td></td>
<td>• Respond to the reflection questions.</td>
<td></td>
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<tr>
<td>13</td>
<td>Closure</td>
<td>• Complete Exit Slip</td>
<td>• Submission</td>
</tr>
<tr>
<td></td>
<td>• Complete the Exit Slip: Online Professional Learning Session.</td>
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<td></td>
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</tbody>
</table>
References


DAY 2: PROBLEM-SOLVING MODEL AND EQUITY

Mathematics Achievement Academy, Problem Solving and Discourse,
Kindergarten: Outcomes
1. Build knowledge and understanding of a problem-solving heuristic that applies to any mathematical content strand.
2. Identify actions of a proficient problem solver.
3. Build knowledge and understanding of mathematical discussions that complement the use of a problem-solving heuristic.
4. Build knowledge and understanding of formative assessment strategies that focus on making student thinking within the phases of the problem-solving model visible to the teacher.
5. Build knowledge and understanding of five practices that support mathematical discourse to strengthen student voice and agency with a problem-solving heuristic.

Day 2: Face-to-Face Professional Learning Session
Learning Intentions
1. Identify the actions of a proficient mathematics problem solver in order to support problem-solving development.
2. Define and understand the use of a problem-solving heuristic.
3. Understand appropriate applications of a problem-solving heuristic in kindergarten mathematics.
4. Explain each of the five practices for supporting mathematical discourse and related guiding questions.
5. Understand how a problem-solving model, five practices that support mathematical discourse, and mathematical language routines promote equity.

Success Criteria
1. I can describe the actions of proficient mathematical problem solvers and explain how to support the development of these characteristics.
2. I can explain the meaning of heuristic and describe how it applies to a problem-solving model.
3. I can describe appropriate applications of a problem-solving model heuristic in kindergarten mathematics.
4. I can explain each of the five practices and how each contributes to making meaning in mathematical discussions.
5. I can describe how a problem-solving model, the five practices that support mathematical discourse, and Mathematical Language Routines promote equity.
Books

Angie had 7 books on her bed. Two of the books fell off. How many books are still on the bed?
Hats

Ed has 8 blue hats. He has 1 black hat. How many blue hats and black hats does Ed have in all?
**Problem A**

Use your counters as needed to help you solve the problem.

Juan has 5 ice cream scoops.

He has 2 cones to hold the ice cream scoops.

How many ice cream scoops can he place on each cone?
**Problem B**

Use your counters as needed to help you solve the problem.

Eight students voted for their favorite vegetable.

More than 5 students voted for carrots.

Fewer than 3 students voted for peas.

How many students voted for each vegetable?
Problem C

Use your counters as needed to help you solve the problem.

I have a double ten frame with counters on it.

One ten frame is full.
One ten frame is NOT full.

What is the largest value I can model?

What is the smallest value I can model?
Problem D

Use your counters as needed to help you solve the problem.

There is a total of 10 flowers in the green, orange, and purple vases.

The green vase and the purple vase have the same number of flowers.

The orange vase has more than 3 flowers.

How many flowers are in each vase?
Five Practices for Mathematical Discussions

**Anticipate:** What are three solution strategies that you think your students might use?

**Monitor/Selecting:** What three or four strategies do you want to highlight?

**Sequencing:** Circle numbers above to identify the order of the solution strategies/student thinking you want to highlight.

**Connecting:** What connection(s) do you want students to see?

Circle the connections you expect to hear from students.
Five Practices for Mathematical Discussions

**Anticipate:** What are three solution strategies that you think your students might use?

<table>
<thead>
<tr>
<th>Drawing and labeling objects</th>
<th>Recording the total number without labels</th>
<th>Grouping objects and counting-on</th>
</tr>
</thead>
</table>

**Monitor/Selecting:** What three or four strategies do you want to highlight?

<table>
<thead>
<tr>
<th>Artistic Drawings</th>
<th>Drawings without labeling</th>
<th>Drawings with labeling</th>
<th>Grouping objects and counting-on</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF</td>
<td>SB</td>
<td>SA</td>
<td>SE</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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</tbody>
</table>

**Sequencing:** Circle numbers above to identify the order of the solution strategies/student thinking you want to highlight.

**Connecting:** What connection(s) do you want students to see?

<table>
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<tr>
<th>1 and 2</th>
<th>2 and 3</th>
<th>3 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artistic drawings and math drawings</td>
<td>Labeling objects</td>
<td>Labeling and grouping</td>
</tr>
</tbody>
</table>

Circle the connections you expect to hear from students.
### Promoting Equity

<table>
<thead>
<tr>
<th>Equity-Based Practices</th>
<th>Looks Like . . .</th>
<th>Reflection</th>
</tr>
</thead>
</table>
| Going deep with mathematics                                            | Expect students to analyze, compare, justify, and prove their solutions while providing needed support. Provide tasks that are problems, allowing for multiple representations and solution strategies. | • How does my problem-solving instruction promote analysis?  
  • How do I support students in closely examining problem solving? |
| Leveraging multiple mathematical competencies                          | Identify and support mathematical contributions from all students. Present tasks with multiple entry points that allow students to make meaningful contributions to the group’s learning. | • How do I identify and support problem-solving contributions from students with different strengths and levels of confidence?  
  • How do I encourage my students to embrace challenges while striving to improve daily? |
| Affirming mathematics learners’ identities                             | Problem solve to promote reasoning and persistence. Learn from mistakes. Encourage students to see themselves as mathematicians. | • How do I structure my interactions with students to promote persistence with problems?  
  • How do I discourage my students from linking speed with math “smartness”? |
| Challenging spaces of marginality                                      | Connect students’ knowledge and experiences with mathematics. Increase opportunities for students to ask mathematical questions. Encourage participation by all students and student-to-student interactivity. | • How do I connect my students’ knowledge (inside and outside the classroom) with the mathematics needed to solve problems?  
  • How do I make sure that all students have opportunities to demonstrate their problem-solving knowledge during the lesson? |
| Drawing on multiple resources of knowledge (math, culture, language, family, community) | Connect to prior mathematical learning. Learn students’ knowledge and experiences. Learn from families and communities how to develop students as confident mathematicians. Communicate strengths and needs of students to families to promote learning. | • How do I get to know my students’ backgrounds and experiences to support problem solving in my classroom?  
  • How do I affirm some of my students’ multilingual abilities to help them become proficient problem solvers? |

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<table>
<thead>
<tr>
<th>Equitable Practices</th>
<th>Co-craft questions</th>
<th>Discourse opportunities</th>
<th>Three-act task</th>
<th>Five practices for mathematical discussions</th>
<th>Problem-solving model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drawing on multiple resources of knowledge, culture, family, and community</td>
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<tr>
<td>Challenging spaces of marginality</td>
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<td>Affirming mathematics learners' identities</td>
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<td>Leveraging multiple mathematical competencies</td>
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