## Norms Cards

*Cut along the dotted lines.*

<table>
<thead>
<tr>
<th>Minimize “air time.”</th>
<th>Make connections among objects, pictures, and numerical representations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be involved.</td>
<td>Celebrate accomplishments.</td>
</tr>
<tr>
<td>Communicate using academic vocabulary.</td>
<td>Contribute ideas.</td>
</tr>
<tr>
<td>Participate by asking questions.</td>
<td>Minimize distractions.</td>
</tr>
<tr>
<td>Be fully present.</td>
<td>Listen.</td>
</tr>
<tr>
<td>Use mistakes as opportunities to support new learning about mathematics.</td>
<td>Develop understanding, if not at the beginning, by the end.</td>
</tr>
<tr>
<td>Take a chance.</td>
<td></td>
</tr>
</tbody>
</table>
### Mathematics Achievement Academy Norms

<table>
<thead>
<tr>
<th>Participation Norms</th>
<th>Discourse Norms</th>
<th>Mathematics Norms</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Be fully present.</td>
<td>• Listen.</td>
<td>• Make connections</td>
</tr>
<tr>
<td>• Minimize distractions.</td>
<td>• Be involved.</td>
<td>among objects,</td>
</tr>
<tr>
<td>• Minimize “air time.”</td>
<td>• Contribute ideas.</td>
<td>pictures, and</td>
</tr>
<tr>
<td>• Take a chance.</td>
<td>• Participate by asking questions.</td>
<td>numerical</td>
</tr>
<tr>
<td>• Celebrate accomplishments.</td>
<td>• Develop understanding, if not at the beginning, by the end.</td>
<td>representations.</td>
</tr>
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<td></td>
<td></td>
<td>• Communicate using academic vocabulary.</td>
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<td></td>
<td></td>
<td>• Use mistakes as opportunities to support new learning about mathematics.</td>
</tr>
</tbody>
</table>

*Yackel & Cobb, 1996*

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Mathematics Achievement Academy Norms

Participation Norms
- Be fully present.
- Minimize distractions.
- Minimize “air time.”
- Take a chance.
- Celebrate accomplishments.

Discourse Norms
- Listen.
- Be involved.
- Contribute ideas.
- Participate by asking questions.
- Develop understanding, if not at the beginning, by the end.

Mathematics Norms
- Make connections among objects, pictures, and numerical representations.
- Communicate using academic vocabulary.
- Use mistakes as opportunities to support new learning about mathematics.

Krusi, 2009

Yackel & Cobb, 1996
Mathematics Achievement Academy: Problem Solving and Discourse, Kindergarten Academic Vocabulary Day 1

Co-Craft questions

Mathematical discourse

Student voice

Problem-solving model:
• Analyzing given information
• Formulating a plan
• Determining a solution
• Justifying the solution
• Evaluating the problem-solving process and the reasonableness of the solution

Mathematics TEKS K(1)(B)
1. Problem solving is an innate skill.

Strongly Disagree  Strongly Agree

2. The solution is 10. What is the question?

3. A problem-solving process is a set of steps that will lead students to the correct solution when completed with fidelity.

Strongly Disagree  Strongly Agree

4. Check the instructional practice(s) that promote(s) equity:

- Discourse
- Language Routines
- Formative Assessments
Setting the Stage for Problem Solving

*Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics, but also a major means of doing so.*

NCTM, 2000, p. 52
Problem Solving Picture Cards
Co-Craft Questions¹

1. The teacher presents a situation as a context or stem for a problem, with or without values.

2. Students generate possible questions that might be asked about the situation. These should be questions that necessitate doing mathematics to be answered.

3. In pairs, students compare their questions.

4. Students are invited to share their questions, with some brief whole-group discussions.

5. The actual question(s) students are expected to work on are revealed, and students are set to work.


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Co-Craft Question Tasks

Use the following script to practice guiding students through Co-Craft Questions.

- You will now play the role of a small group of kindergarten students.
- You may choose to show students an image or to read one of the situations aloud to students.
- What questions can be asked about this image/situation that can be answered with math?
- Prompt students to think about their responses individually.
- Prompt students to share their mathematical questions with an elbow partner.
- Who would like to share the question that you or your partner created? Allow for student responses.
- We created several questions that can be answered using the given information. For today, let us answer this question that was created.
- Write or verbally identify the question to be answered.
- Prompt students to answer the question.
There are 4 cats. There are 5 dogs.

I had 7 books. I gave 3 books to my brother.
Analyzing Given Information

Teacher Actions and Student Actions
The teacher will
- prompt problem solvers to identify important information,
- prompt problem solvers to make connections to related problems,
- prompt problem solvers to consider appropriate models,
so that the problem solvers will
- discern between needed and irrelevant information when solving the problem.
- apply existing knowledge in a new way.
- capture mathematical relationships in a visual format that may also be used later to justify an answer.

Teacher Action Examples:
Instructional support for “making a movie in your mind”

Supports for Diverse Learners:

Promoting Sense Making in Problem Solving
Clarifying Student Understandings
- What I hear you saying is ______. Is that correct?
- Can you tell me more about how this problem is like ______?
Clarifying Understanding of Classmates
- Who can repeat ______’s thinking in their own words?
- What did your partner tell you about the problem?

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Formulating a Plan or Strategy

Teacher Actions and Student Actions
The teacher will
- prompt problem solvers to consider actions within a context, related representations, and the question being posed,
- prompt problem solvers to re-analyze given information,
so that the problem solvers will
- connect a model to a potential solution process for the problem,
- continue to refine and adjust plans for a solution process.

Teacher Action Examples:

Instructional support for using math drawings or symbols in place of artistic drawings

Promoting Sense Making in Problem Solving
Clarifying Student Understandings
- So you are telling us that your plan is ______. Is that correct?
Clarifying Understanding of Classmates
- Who can repeat _____’s thinking in their own words?
- Can someone restate what they just heard about _____’s plan?

Supports for Diverse Learners:

Promoting Sense Making in Problem Solving
Clarifying Student Understandings
- So you are telling us that your plan is ______. Is that correct?
Clarifying Understanding of Classmates
- Who can repeat _____’s thinking in their own words?
- Can someone restate what they just heard about _____’s plan?

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Determining a Solution and Evaluating the Reasonableness of the Solution

Teacher Actions and Student Actions
The teacher will
- scribe solution strategies during whole class discussions,
- provide feedback regarding solution strategies in the form of a question,
- prompt problem solvers to revisit their plan or the given information if needed,

so that the problem solvers will
- engage with the solution strategies of others.
- determine the effectiveness and efficiency of mathematical solution strategies.
- continue working toward a complete and correct solution if a solution is incomplete or unreasonable.

Teacher Action Examples:
Instructional support for counting strategies
Instructional support for reasonableness

Discourse Supports:
- Acting Out
- Revoicing
- Choral responses
- Questioning strategies

Promoting Sense Making in Problem Solving
Clarifying Student Understandings
- Is there another way to solve the problem?
- What I hear you say is _____ Is that correct?

Clarifying Understanding of Classmates
- Who can repeat _____’s thinking in their own words?
- Who can state in their own words why _____’s solution is reasonable?
Exit Day 1
Label the success criteria based on your personal understanding of our learning intentions. Marking a 5 indicates that you have mastered the learning intention.

1. I can describe attributes of problem solving in everyday life and identify connections to problem solving in mathematics.
   - 1  2  3  4  5

2. I can implement teacher actions that support student problem-solving development and facilitate student actions that increase problem-solving development.
   - 1  2  3  4  5

3. I can describe the analyzing given information, formulating a plan or strategy, determining a solution, and evaluating the reasonableness of the solution components of the TEKS-based problem-solving model.
   - 1  2  3  4  5

4. I can use questions that help students clarify their own understanding and clarify the group’s understanding.
   - 1  2  3  4  5

5. I can implement mathematical language routines in order to increase and enhance student voice.
   - 1  2  3  4  5

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Mathematics Achievement Academy: Problem Solving and Discourse, Kindergarten
Academic Vocabulary Day 2

Equity

Exercise

Problem

Three-act tasks

Problem-solving model:
• Analyzing given information
• Formulating a plan
• Determining a solution
• Justifying the solution
• Evaluating the problem-solving process and the reasonableness of the solution

Mathematics TEKS K(1)(B)
Three-Act Tasks

Act 1:
- Provide the context for the inquiry through a picture or video.
- Prompt students to share what they notice about the picture or video.
- Prompt students to generate questions through wonderings.
- Identify the main question to be explored.

Act 2:
- Provide more information to help students answer the identified question.
- Facilitate conversations about reasonable answers.
- Allow students to explore and answer the question.

Act 3:
- Reveal the solution.
- Facilitate follow-up conversations as appropriate.

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Justifying the Solution and Evaluating the Problem-Solving Process

Teacher Actions and Student Actions
The teacher will

- prompt problem solvers to summarize big ideas
- provide scaffolds to help problem solvers use mathematical language to build their justifications
- prompt problem solvers to reflect on their own problem-solving process

so that the problem solvers will

- use their learning in future situations.
- become agents of their own mathematical and linguistic sense making.
- become more proficient in problem solving.

Teacher Action Examples:
Instructional support for questioning to move students beyond specific examples

Supports for Diverse Learners:

Promoting Sense Making in Problem Solving
Clarifying Student Understandings
- Will you say more about why your solution answers the question?

Clarifying Understanding of Classmates
- Who can repeat _____’s thinking?
- What questions can we ask _____ to better understand his/her thinking?
Proficient Problem Solvers

Analyze situations in mathematical terms.

Engage with problems willingly and persistently.

Consider simple cases of complex situations.

Recognize that some representations share common mathematical structures. Categorize problems into types.

Use flexibility in thinking.
**Exercise Versus Problem**

An **exercise** asks a student to practice a familiar skill.

Exercises allow students to
- practice a specific mathematical skill,
- demonstrate what was just learned,
- develop automaticity with a mathematical skill, and
- practice a specific mathematical skill in a context related to real-life experiences.

A **problem** involves the application of previously learned mathematical skills, concepts, and/or procedures to a situation where a solution process is not immediately apparent.

Problems allow students to
- apply a learned mathematical concept or skill where a solution is not obvious,
- demonstrate understanding of multiple mathematical concepts or ideas and their connections to each other,
- enter into a mathematical task with their current mathematical knowledge,
- build upon prior knowledge, and
- build new mathematical knowledge.
### Five Practices for Mathematical Discussions Cards: Set A

*Run on one color of cardstock. Two sets are provided.*

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Sequencing</strong></td>
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</tr>
<tr>
<td><strong>Connecting</strong></td>
<td><strong>Connecting</strong></td>
</tr>
<tr>
<td><strong>Monitoring</strong></td>
<td><strong>Monitoring</strong></td>
</tr>
<tr>
<td><strong>Anticipating</strong></td>
<td><strong>Anticipating</strong></td>
</tr>
<tr>
<td><strong>Selecting</strong></td>
<td><strong>Selecting</strong></td>
</tr>
</tbody>
</table>
**Five Practices for Mathematical Discussions Cards: Set B**

*Run on a second color of cardstock. Two sets are provided.*

<table>
<thead>
<tr>
<th>Task</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher documents the order in which student work will be shared.</td>
<td>The teacher documents the order in which student work will be shared.</td>
</tr>
<tr>
<td>The teacher plans questions to support students’ thinking as they make comparisons between work samples.</td>
<td>The teacher plans questions to support students’ thinking as they make comparisons between work samples.</td>
</tr>
<tr>
<td>The teacher listens and looks for mathematical thinking as students complete a task.</td>
<td>The teacher listens and looks for mathematical thinking as students complete a task.</td>
</tr>
<tr>
<td>The teacher chooses a task and thinks about possible correct and incorrect student responses.</td>
<td>The teacher chooses a task and thinks about possible correct and incorrect student responses.</td>
</tr>
<tr>
<td>The teacher chooses student work that highlights mathematical thinking that can move students forward.</td>
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</table>
Five Practices for Mathematical Discussions

Anticipating

- What approaches will students most likely use to solve the problem?
- What misconceptions may students demonstrate?
- What responses will support students while they are solving the problem?

Monitoring

- What mathematical ideas are being shown and discussed?
- What mathematical promise do I hear?
- What approaches are students demonstrating that were not anticipated?

Selecting

- Which student works reflect the anticipated approaches to solve the problem?
- Which student works include misconceptions or missteps that will be shared?
- Which student works show the needed mathematical pieces?


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Five Practices for Mathematical Discussions

Sequencing
- Do I start with the most used strategy and then the lesser used strategy? Why?
- Do I start with the least complex strategy? Why?
- Do I want to use complementary or contrasting strategies? Why?
- In what order will students share their approaches to the problem?

Connecting
- What questions will make the mathematics visible to the students?
- What connections and relationships will students make based on the shared approaches?
- What discussions about accuracy and efficiency can we have?

Each practice is built on the practices embedded within it.

Stein, et al., 2008, 322

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### Equity-Based Practices for the Mathematics Classroom

<table>
<thead>
<tr>
<th>Going deep with mathematics</th>
<th>Expect students to analyze, compare, justify, and prove their solutions while provided needed support. Provide tasks that are problems, allowing for multiple representations and solution strategies.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leveraging multiple mathematical competencies</td>
<td>Identify and support mathematical contributions from all students. Present tasks with multiple entry points that allow students to make meaningful contributions to the group’s learning.</td>
</tr>
<tr>
<td>Affirming mathematics learners’ identities</td>
<td>Problem solve to promote reasoning and persistence. Learn from mistakes. Encourage students to see themselves as mathematicians.</td>
</tr>
<tr>
<td>Challenging spaces of marginality</td>
<td>Connect students’ knowledge and experiences with mathematics. Increase opportunities for students to ask mathematical questions. Encourage participation by all students and student-to-student interactivity.</td>
</tr>
<tr>
<td>Drawing on multiple resources of knowledge (math, culture, language, family, community)</td>
<td>Connect to prior mathematical learning. Learn students’ knowledge and experiences. Learn from families and communities how to develop students as confident mathematicians. Communicate strengths and needs of students to families to promote learning.</td>
</tr>
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1. Problem solving is an innate skill.

Strongly Disagree  Strongly Agree

2. The solution is 10. What is the question?

3. A problem-solving process is a set of steps that will lead students to the correct solution when completed with fidelity.

Strongly Disagree  Strongly Agree

4. Check the instructional practice(s) that promote(s) equity:

- Discourse
- Language Routines
- Formative Assessments
Exit Slip: Day 2

Label the success criteria based on your personal understanding of our learning intentions. Marking a 5 indicates that you have mastered the learning intention.

1. I can describe the actions of proficient mathematical problem solvers and explain how to support the development of these characteristics.
   1 2 3 4 5

2. I can explain the meaning of heuristic and describe how it applies to a problem-solving model.
   1 2 3 4 5

3. I can describe appropriate applications of a problem-solving model heuristic in kindergarten mathematics.
   1 2 3 4 5

4. I can explain each of the five practices and how each contributes to making meaning in mathematical discussions.
   1 2 3 4 5

5. I can describe how a problem-solving model, the five practices that support mathematical discourse, and mathematical language routines promote equity.
   1 2 3 4 5

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