Multiple Representations: Equations to Tables and Graphs
Transcript

Algebra I Teacher: It's good to see you again. Last time we talked about multiple representations. If we could, I would like to continue and discuss the subtle differences of multiple representations between eighth grade and Algebra I.

Grade 8 Teacher: Can we start with the student expectations related to generating a table or a graph from a given two-variable equation?

Algebra I Teacher: OK. We use an equation to generate a table with students to help connect among the representations. Since we are representing functions, we know that we are using two-variable equations where the independent variable determines the related dependent variable. In other words, x determines y, so we can focus on the information provided by a two-variable equation or by a function rule.

Grade 8 Teacher: What if we use the equation $y=25x+50$?

This equation provides a rule, or a mathematical process, that connects x, the independent variable, to y, the dependent variable.

If I know one of the values, I can use the equation, or function, to determine its paired value.

If I know x, I can determine y. If I know y, I can determine x.

Algebra I Teacher: This is true for a linear equation unless we are using a vertical line. In Algebra I, we may replace the y with function notation and write $f(x) =25x+50$.

The function rule models that relationship. The equation, or function, provides a general rule to describe the entire relationship. Once we become familiar with the different types of functions, we can start to recognize characteristics of the relationship before we generate a table or a graph.

Grade 8 Teacher: What characteristics would you expect students to determine from this equation?

Algebra I Teacher: In this case, I can see that each x-value is multiplied by 25 and the product is increased by 50.

This is a linear function because the y-value increases by 25 each time an x-value is increased by one.

Grade 8 Teacher: We would expect to see this in a table created from this equation?

Algebra I Teacher: Depending on the x-values we select for the input, the rate of change of 25 may or may not be readily apparent.

The equation describes the relationship, and the table provides specific examples of the relationship between the two quantities.

The table represents some paired values that satisfy an equation.

Grade 8 Teacher: When we introduce two-variable equations, we often do so with a context. For this equation, let x represent the number of hours a plumber spends completing a job, and let y represent the total fee or cost for completing the plumbing job.

So, this equation represents a general rule that describes the relationship between the number of hours the plumber spends completing the job, and the total cost of the job to the customer?

Algebra I Teacher: Yes. A two-variable equation may be used to calculate the cost of a job when there is an hourly rate.

How do you work with students to generate a table of paired values that satisfy this equation?

Grade 8 Teacher: Students worked with input-output tables beginning in the fifth grade, so they are familiar with tables.

We found it helpful to incorporate a table with a process column when students generate a table of paired values from an equation.

Let's look a little closer at the original equation and develop a table of paired values. We use a blank table with a process column in all of our classes to begin this work.

We write the two-variable equation at the top of the page for reference.

Then we write the variables in the column heading for the input value, or the independent value, and the output value, or the dependent value.

In the process column heading, we write the expression that is equivalent to y. In this case, we write $25x+50$.

Algebra I Teacher: Which input, or independent value, do students usually start with?

Grade 8 Teacher: We try to start with values that make sense for the context. Here, most students would likely start with one hour. So we place a one in the x column.

We then rewrite the expression $25x+50$ with a one in place of the value x.

The expression becomes $25(1) +50$.

Then, we calculate the value of the expression to determine the value of y.

Here, y is 75.

Algebra I Teacher: Why not start with an x-value of zero?

Grade 8 Teacher: We could talk about the cost for the plumber to come out to your house and look at the work to be done where only the 50 dollar fee applies, but many of my students anchor time to one hour, so we start there.

Algebra I Teacher: That makes sense. Following that reasoning, do students use two, three, four, and so on, for the next x-values?

Grade 8 Teacher: Yes, that's often what they do. We encourage our students to find at least four paired values when generating a table, so in the input column, we will write two, three, and four.

In the process column, we write the expression each time with the input value substituted for the variable.

We continue to determine the *y*-value that goes with this input, or *x*- value.

Algebra I Teacher: What do you do next?

Grade 8 Teacher: Now, we ask about the cost for zero hours. What does it mean to have an *x*-value of zero in this situation?

Algebra I Teacher: It gives them the opportunity to see the fixed or constant charge that is part of the total cost.

Grade 8 Teacher: Yes. Then, we start to ask about half-hour increments, quarter-hour increments, and other increments that make sense for the situation.

This goes back to discussions about the connection between equations and tables. The equation is the mathematical rule that describes the relationship. The table represents only some of the paired values from the relationship. The context impacts which values make sense and which values may be represented in a table.

Algebra I Teacher: In Algebra I, we emphasize restrictions in the domain and range that are implied by the situation. Do you talk about that in the middle grades?

Grade 8 Teacher: In sixth grade, we talk about independent and dependent values. In eighth grade, we identify functions, but we don't specifically mention domain and range.

Algebra I Teacher: What else is different?

Grade 8 Teacher: In sixth grade, students explore relationships that are strictly additive or multiplicative, so you might just see the equation $y=25x$.

In seventh grade, we start looking at two-variable relationships in the form of $y=mx+b$. So, we might see $y=25x+50$.

In eighth grade, we may see the same equation with an emphasis on whether it represents a proportional or a non-proportional relationship.

Algebra I Teacher: In Algebra I, students may see the equation in slope-intercept form, point-slope form, and standard form. They may also see it written using function notation.

How do you use graphing tools in grades 6–8 when connecting equations to tables?

Grade 8 Teacher: We only graph by hand in grades 6 and 7. After students have had experience building a table and generating tables without the technology, they may input the equations into a graphing tool to check values. We want to develop the concept of a two-variable equation as a description of a process and a table as a representation of the equation. Since students are not permitted to use graphing tools on the state assessment, most of the work is done without technology.

Algebra I Teacher: Do they start using graphing tools regularly in eighth grade?

Grade 8 Teacher: Yes, in eighth grade we really begin to use a graphing tool, such as a calculator or an app to generate a table. The graphing tool helps us focus on the relationship and its representations by decreasing the load of repeated calculations.

Algebra l Teacher: How do you connect skills and thought processes from working without graphing tools to working with graphing tools?

Grade 8 Teacher: Sometimes we have students generate a table of just a few values to help them make the connection to grades 6 and 7. Then, when they enter the equation into the graphing tool, we ask them to make sure that those values are reflected in the values generated by the calculator.

Algebra l Teacher: What do you do next?

Grade 8 Teacher: We want students to see how the dependent values change in relation to how the independent values change, so we can reinforce the concept of rate of change and connect back to the equation.

Grade 8 Teacher: Looking at the table we made earlier, we see the change between the *y*-values, such as the change between 75 and 100 is positive 25, while the change between the corresponding *x*-values is positive one. And if we continue down the table, we see the same change in *y*-values because the intervals for *x* are one.

Algebra l Teacher: That's a good connection to emphasize. Do you ever look at intervals other than one for the *x*-values?

Grade 8 Teacher: Yes, we work with students to determine when they might want to use an interval of one and when they might choose other intervals.

Algebra l Teacher: That's great. We build on that work and discuss more efficient methods to answer questions both with and without graphing tools. We also explore the relationship between the two quantities and emphasize that a table only shows some of the values for the relationship. Using graphing tools, we can change the parameters to quickly determine how those values change and model the new relationship.

Grade 8 Teacher: Can we talk about the vocabulary? You have used terms such as relationship between two quantities, rate of change, and modeling a relationship. How do these terms help your students make the connections between and among relationships?

Algebra l Teacher: The use of vocabulary was a turning point for us. It helped our students understand the connections among equations, tables, independent values, and dependent values. We made certain to use phrases such as "the table represents some paired values that satisfy the equation," "the table shows paired values that represent the relationship," or "the table can represent the equation."

Grade 8 Teacher: That's really good to know. We have focused our conversations on the use of a process column, the use of consistent and inconsistent intervals, and the specific use of vocabulary related to equations and tables.

Algebra l Teacher: Do you generate a graph from a given two-variable equation?

Grade 8 Teacher: Yes. We present them a two-variable equation and ask them to graph the line it represents. With the work we've done with tables in sixth and seventh grade, we encourage our students to create a table of values first to organize their paired values.

Once they have a table, the students graph fairly easily. They have been graphing paired values or coordinate points in quadrant one since fifth grade.

Algebra I Teacher: What challenges your students?

Grade 8 Teacher: Interpretation. Some are challenged by determining what values to graph or even how to label the intervals on the *x*-axis and the *y*-axis. This is especially true when the two axes have different intervals.

If we return to our example $y=25x+50$, using one to mark intervals of the *x*- and *y*-axis might make the relationship difficult to see graphically. Our graph paper does not allow for 150 units along either axis.

So, we start with having students label what each axis represents. In this case, the *x*-axis represents the number of hours, and the *y*-axis represents the total cost.

Counting by ones on the *x*-axis make sense, so I'll label every other vertical gridline counting by ones.

Looking back at the table of values, I see the values of 75, 100, or more. Maybe using a consistent interval of 25 would allow me to graph the values in the table. So, I will label every other horizontal grid line counting by 25.

Algebra I Teacher: We see similar challenges with graphing tools. Students can enter the equation into the graphing tool, but sometimes they are challenged to determine the settings in order to view the part of the graph that they need to answer the question.

Grade 8 Teacher: We see that in eighth grade as well, especially if they did not use the graphing tool in grades 6 and 7 much.

Algebra I Teacher: I see, so how do your students graph this relationship by hand?

Grade 8 Teacher: We start by graphing the four points in the process column.

At this point, we ask students if they see a pattern. We ask them to predict where the point for five hours might be.

Then we ask them to use the table to verify their prediction. They fill out the row for five.

If their prediction was correct, we have them plot a few more points.

Algebra I Teacher: Do you have the students draw a line through the points?

Grade 8 Teacher: Sometimes, yes. We ask the students what values might be between the points. Is a fractional part of an hour reasonable for this situation?

If that makes sense, then we draw the line to show those possibilities.

If it doesn't make sense we leave the points as individual points.

Algebra I Teacher: You don't explicitly talk about discrete and continuous data or graphs?

Grade 8 Teacher: No, it's not part of our student expectations. We do talk about reasonableness and what values make sense for a given context. Our focus is on developing the understanding that a graph of a relationship represents the collection of points that satisfies a two-variable equation.

Algebra I Teacher: Still, this will help them to connect to Algebra I content as well as help them to interpret the lines that they have generated.

Grade 8 Teacher: We have those conversations more often in eighth grade when we use our graphing tool.

Algebra I Teacher: What role does the slope-intercept form play in students graphing from an equation?

Grade 8 Teacher: In eighth grade, we explicitly teach slope. We begin to move students to use the slope-intercept form of an equation to help them graph without technology. The rule for the relationship between the independent and dependent values now helps them see how each *y*-value changes in relationship to a corresponding change in the *x*-value.

In this example, the rate of change for the equation is 25, so the slope of the line is 25.

When students use the graphing tool, their understanding of the rate of change in the equation and the slope of the line helps them to determine how to set the window parameters on the graphing tool.

If the intervals on my graph are not one, then we focus on the values of each point and how the values are changing.

Algebra I Teacher: Yes, the relationship between the rate of change, slope, and the graph really helps them to visualize more of the relationship between the two variables.

Grade 8 Teacher: What other relationships do you study?

Algebra I Teacher: We start with linear relationships using equations as well as function notation. We extend the ideas that you build in the middle grades. In particular, that linear functions express relationships with a constant rate of change. We also discuss systems of linear equations and consider how the two linear relationships interact as well as what a solution means for a system of two, two-variable equations.

Grade 8 Teacher: We start that in eighth grade. Looking at the graphs and bringing back that vocabulary of what it means for a paired value to satisfy an equation or a system of equations.

Algebra I Teacher: Great. We also study quadratic functions that express relationships where the rate of change itself increases and decreases at a constant rate. Finally, we begin to study exponential relationships where the rate of change either increases or decreases by the same factor.

We also use the skills that you develop in eighth grade with the graphing tool to explore quadratic and exponential functions. It allows us to focus on how the values are changing in relationship to each other. We can also find patterns in the rate of change, and see how those patterns appear on a graph. We still expect our students to be able to make

calculations, but we are intentional about when the focus is on calculations and when the focus is on the relationships between the values of those multiple representations.

Grade 8 Teacher: We've talked a lot about equations and tables within a context. Do you use contexts that are purely mathematical in nature?

Algebra I Teacher: On occasion. We want students to consider the relationships between the numbers themselves without the added context. This forces them to really look at the operations and the relationship between the two changing quantities.

Grade 8 Teacher: We both keep using the word relationship. That's a nice connection for our students.

The more that we align our approaches and vocabulary, the stronger their foundation for relationships and their multiple representations.

Algebra I Teacher: I agree.

