Teacher 1: Before we plan for next week, I would like for us to think through different ways that our students might solve this problem and how it connects to what they are doing in fifth grade.

Teacher 2: Ok, let's read over it first.

Penny ran 2 days this week. On Tuesday, she ran  $\frac{9}{6}$  miles. On Thursday, she ran  $\frac{8}{6}$  miles.

How many miles did Penny run this week?

Teacher 1: I think some of my students will use fraction strips to model this, especially since this is a distance problem. Let's use that approach to begin.

Teacher 2: We could also try to determine at least two other ways that our students might solve this.

Teacher 1: Some of my students may approach this situation by using two of the fraction strips that show sixths. Each fraction strip would represent a whole mile. I will use my blue

marker to shade in  $\frac{9}{6}$ . (*Counts*)  $(\frac{1}{6}, \frac{2}{6}, \dots, \frac{8}{6}, \frac{9}{6})$ .

This represents the  $\frac{9}{6}$  miles that Penny ran on Tuesday. Now, I am going to use a green marker to add  $\frac{8}{6}$  to represent the miles she ran on Thursday.

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I'll start by adding on to my first model.

I need another fraction strip in order to shade all  $\frac{8}{6}$  that I am adding to  $\frac{9}{6}$ .

Continuing from  $\frac{3}{6}$  that I just counted... $\frac{4}{6}$ ,  $\frac{5}{6}$  . . . and  $\frac{8}{6}$ .

This shows that my final answer is more than 2. This is reasonable because Penny ran more than one mile each day. Using my fraction strips, I can see that the exact answer is

2 and  $\frac{1}{6}$ ,  $\frac{2}{6}$ ,  $\frac{3}{6}$ ,  $\frac{4}{6}$ ,  $\frac{5}{6}$ .

Teacher 2: That connects to the whole number strategy of "counting on" to add. I know this is  $\frac{9}{6}$ .

I will count on 8 more sixths...10 sixths, 11, 12, 13, 14, 15, 16, 17 sixths.

How does this connect to the properties of operations mentioned in the student expectation 4(3)(E) and then to the standard algorithm in fifth grade?

Teacher 1: I think it involves the idea of decomposing numbers and the associative property.

We start with  $\frac{9}{6} + \frac{8}{6}$ .

I can decompose the  $\frac{9}{6}$  into  $\frac{6}{6}$  and  $\frac{3}{6}$ . I decomposed this way because I know that  $\frac{6}{6}$  is equal to one whole.

In thinking about how to decompose  $\frac{8}{6}$ , I see that I have  $\frac{3}{6}$  and that 3 more sixths would make a whole.

So, I decompose the  $\frac{8}{6}$  into  $\frac{3}{6}$  and  $\frac{5}{6}$ . I can still see my original  $\frac{9}{6}$  and the  $\frac{8}{6}$ .

I need to associate my two  $\frac{3}{6}$  together so that I can see the one whole.

Six sixths make one whole. Adding  $\frac{3}{6}$  and  $\frac{3}{6}$  makes a second whole, which leaves  $\frac{5}{6}$ , and gives us  $2\frac{5}{6}$ .

Teacher 2: That makes sense. I think some students will model  $\frac{9}{6}$  and  $\frac{8}{6}$  separately then determine an answer. Let's look at that approach and then look at the fifth-grade student expectations.

Teacher 1: Ok.

Teacher 2: Let's start the same way you did modeling  $\frac{9}{6}$ .

Then we begin again with a new fraction strip and model  $\frac{8}{6}$ . (*Counts*)  $\frac{1}{6}$ ,  $\frac{2}{6}$ ...  $\frac{6}{6}$ .

Now start another fraction strip. (Counts)  $\frac{7}{6}$ ,  $\frac{8}{6}$ .

Teacher 1: Right away students can see that the answer will be more than 2.

Teacher 2: Yes. Once I have modeled my two addends, I can rearrange the strips so that the two whole fraction strips are together.

Then I would fold the remaining  $\frac{2}{6}$  green pieces and place them on the fraction strip next to the remaining  $\frac{3}{6}$  blue pieces.

I see from the model that the answer is  $2\frac{5}{6}$ .

Teacher 1: Let's write this out and consider the properties of operations that we have applied.

Teacher 2: This approach, like the first, relies on students decomposing fractions. This time,

we are decomposing each fraction into one whole and a fractional piece. So,  $\frac{9}{c}$  decomposes

into  $\frac{6}{6}$  and  $\frac{3}{6}$ , and  $\frac{8}{6}$  decomposes into  $\frac{6}{6}$  and  $\frac{2}{6}$ .

Teacher 2: When we rearrange the fraction strips to put the two whole strips together, we model the commutative property of addition. We can show this by rewriting our expression.

We then simplify each of the  $\frac{6}{6}$  to 1. By applying the associative property, we add the ones for a sum of 2, and the  $\frac{3}{6}$  and the  $\frac{2}{6}$  for a sum of  $\frac{5}{6}$ .

Teacher 1: Both of these strategies rely on students understanding that the numerator in the fraction is the count of the number of parts, or pieces, of the whole. The denominator represents the size of each part relative to the whole, or the total number of parts of this size needed to make a whole.

Teacher 2: And, both strategies rely on students understanding that they are adding the number of parts of the whole, or the pieces of the model. In doing so, they add the numerators of the two fractions. Students must understand that the denominator indicates the size of the part relative to the whole. We each used different strategies to determine the sum of the numerators, building on the whole number understandings about decomposing numbers and properties of operations.

Teacher 1: If we look at the fifth-grade TEKS, students are expected to add fractions with unlike denominators. Instead of the  $\frac{9}{6}$  and  $\frac{8}{6}$ , they may be asked to determine the sum of

 $1\frac{1}{2}$  and  $1\frac{1}{3}$ .

Teacher 2: Right! Students also learn in fifth grade that they need to generate equivalent fractions with the same denominator in order to add these fractions.

Teacher 1: When I visited fifth-grade classrooms, I've seen some students rewrite the mixed numbers as improper fractions. So,  $1\frac{1}{2}$  becomes  $\frac{3}{2}$ , and  $1\frac{1}{3}$  becomes  $\frac{4}{3}$ .

From here, they generate the equivalent fractions  $\frac{9}{6}$  and  $\frac{8}{6}$ . This now looks like the fourth-grade problem.

Teacher 2: They may also decide to add the numerators to determine the sum of  $\frac{17}{6}$ .  $\frac{17}{6}$  is

equivalent to  $2\frac{5}{6}$ .

Teacher 1: If we build on fourth-grade experiences and understandings, decomposition is based on whole numbers and fractions. In this example, the expression is written so that the wholes and the fractional parts are easily visible.

It is possible that students may only rewrite the fractional parts of the mixed numbers as equivalent fractions, recognizing that there are already two whole fraction strips.

Then, we add the whole numbers and the fractional parts.

Teacher 2: This example makes it clear how the fifth-grade student expectation builds on the understanding of "fraction addition" established in fourth grade. It's important to remember that as students become more fluent with fraction operations, much of the decomposing and use of the properties of operations will be a mental process rather than a written process.

Teacher 1: I'm interested to see how our students approach this problem next week.

Teacher 2: Me too, let's look at a few more.