Addition and Subtraction of Rational Numbers Grades 5–7

Transcript

Instructional Coach: Good morning.

Teacher: Good to see you.

Instructional Coach: So, you wanted to discuss student expectations 7(3)(A) and 7(3)(B), specifically adding and subtracting rational numbers?

Teacher: Yes, I am curious about what it means for students in seventh grade to be fluent with rational number operations and what students have seen of rational number operations from prior grade-level expectations.

Instructional Coach: Let's start with the vertical alignment chart for adding and subtracting whole numbers, decimals, and fractions. *(Looks at pages 10 and 11)* When do students begin working with decimal and fraction addition?

Teacher: They start working with decimal addition and subtraction in the fourth grade.

That's also the grade level in which they start adding and subtracting fractions with like denominators.

Instructional Coach: Students use objects and pictorial models that build to the number line and properties of operations to add and subtract those fractions.

Teacher: Will you give me an example of what it means for students to add or subtract fractions using properties of operations?

Instructional Coach: Let's consider the addition of $\frac{7}{8}$ and $\frac{5}{8}$.

Students have whole number experiences with decomposing whole numbers and using whole number relationships to determine the sum.

In this case, students might notice that $\frac{7}{8}$ is just $\frac{1}{8}$ away from a whole.

With this in mind, they might choose to decompose $\frac{5}{8}$ into $\frac{1}{8}$ and $\frac{4}{8}$.

Teacher: Combining the $\frac{7}{8}$ with the $\frac{1}{8}$ would make one whole, with $\frac{4}{8}$ remaining. So, the sum is $1\frac{4}{8}$.

Instructional Coach: Exactly! Which properties of operations did the student apply?

Teacher: The associative property.

Instructional Coach: Yes, after decomposing the second addend, we now have three addends.

The associative property of addition allows me to regroup the addends without impacting the sum.

Teacher: So, students are expected to know that 1 and $\frac{4}{8}$ is equivalent to $1\frac{1}{2}$.

Instructional Coach: Students may be expected to do so, but we'll talk more about that in a moment. Anything else for fourth grade?

Teacher: Students are also adding and subtracting decimals to the hundredths.

Instructional Coach: That's right. What rational number operations do you see in the fifth-grade TEKS?

Teacher: I don't see a student expectation that explicitly mentions the addition and subtraction of decimals, but student expectation 5(3)(H) specifically mentions representing and solving the addition and subtraction of fractions with unequal denominators using concrete objects, models, and properties of operations. This is similar to the fourth-grade student expectation we've just seen.

Instructional Coach: In fifth grade, we bridge from what students know about fractions with equal-sized denominators and equivalent fractions, another fourth-grade concept, to fractions with unequal denominators.

Teacher: If I read this correctly, by the end of fifth grade, students should add and subtract positive rational numbers fluently; that includes decimals and fractions. Could a problem include decimals and fractions?

Instructional Coach: By the end of fifth grade, it is expected that students can successfully solve a problem like this.

Teacher: So, does fluency with rational number operations include multi-step problems?

Instructional Coach: Yes, unless the student expectation limits the number of steps, students may be expected to solve problems with more than one step. Of course, we should keep the number of steps reasonable.

Teacher: To what place value can we expect students to add or subtract with decimals?

Instructional Coach: In fourth grade, students add decimal numbers to the hundredths place. In student expectation 5(2)(A), students are expected to represent values to the thousandths place. Adding or subtracting to the thousandths would be reasonable, as students already have an understanding of this place value.

Teacher: For example, in this problem, could we expect that a student would convert $7\frac{1}{8}$ to a decimal?

Instructional Coach: It would be a student-by-student decision as he or she develops fluency with fraction and decimal addition and subtraction. They could convert $7\frac{1}{8}$ to a decimal or could convert $4\frac{5}{10}$ to $4\frac{1}{2}$.

In sixth grade, students are introduced to percents for the first time. They are also introduced to integer operations, and develop fluency with positive rational number multiplication and division.

Teacher: So, there's not a specific student expectation for positive rational number addition and subtraction?

Instructional Coach: No, there isn't. Sixth-grade students may be expected to add and subtract positive rational numbers in other student expectations, as we assume students have mastered these skills, but it is not explicitly stated.

They are expected to multiply and divide positive rational numbers fluently by the end of the year.

Teacher: What could fluency include?

Instructional Coach: Look at this question.

Teacher: To determine if the solution of $\frac{3}{5}$ satisfies an equation, students would solve each of the one-step equations.

Instructional Coach: Exactly!

Teacher: Look at the grade 7 student expectations. Students may see rational numbers represented as decimals, fractions, and percents.

Instructional Coach: Talk me through how your students might approach this problem.

Teacher: Most will probably use decimals, but some will use fractions. I am going to do the same.

50% is a benchmark fraction. Students know that 50% is equivalent to $\frac{1}{2}$.

$2\frac{3}{5}$ is in fraction form.

$\frac{75}{100}$ is equivalent to 75%, which is equivalent to $\frac{3}{4}$, so $1\frac{75}{100}$ is equivalent to $1\frac{3}{4}$.

The amount of pizza the teachers ate is $\frac{3}{4}$.

As I read this, I know it is a two-step problem because I will have to add the amount of pizza from the classes and then subtract the amount of pizza the teachers ate. I'll use a strip diagram to represent all of the parts of the situation.

Instructional Coach: It's good to model your thinking, and when your students model their thinking, it will help you determine if the student is struggling with computation or making sense of the problem.

Teacher: First, I will sketch a small bar for class A, and write $\frac{1}{2}$.

To the right, I'll sketch a larger bar for class B, and label it as $2\frac{3}{5}$.

I'll draw a slightly shorter bar for class C, and write $1\frac{3}{4}$. I am sketching these end-to-end as they represent the remaining amount.

I am sketching this line segment to remind me to determine the total.

Below that, I will draw a small bar under the total pizza remaining to represent how much pizza the teachers will eat, and label it $\frac{3}{4}$.

I will sketch a larger bar to the left to represent the pizza that remains after the teachers eat some of the pizza. I will label this with a question mark, because it represents the amount that answers the question being asked.

Instructional Coach: Once the student has represented the problem with a strip diagram or some other method, how might a student solve it?

Teacher: Some will go straight to computation and add the three values for the remaining amounts of pizza.

Instructional Coach: How do your students approach adding fractions?

Teacher: I have seen some of my students use an approach much like what you showed earlier with the properties of operations. Those students would combine the $\frac{1}{2}$ and the $1\frac{3}{4}$ first, because halves and fourths are mental math benchmarks, and then they would determine how to combine that total with $2\frac{3}{5}$.

Other students may line up the fractions like they do when adding decimals.

Students would find a common denominator for halves, fifths, and fourths. I think they would start with multiples of 10 since halves and fifths are present, and settle on 20 as it is a multiple of two, five, and four. However, some may use 40.

Once they have a common denominator, they would rewrite the expression using equivalent fractions.

$\frac{1}{2}$ becomes $\frac{10}{20}$ because half of 20 is 10.

$\frac{1}{5}$ of 20 is 4, so $\frac{3}{5}$ of 20 is 12. $2\frac{3}{5}$ is equivalent to $2\frac{12}{20}$.

$\frac{1}{4}$ of 20 is 5, so $\frac{3}{4}$ of 20 is 15. $1\frac{3}{4}$ is equivalent to $1\frac{15}{20}$.

When I combine my fractional parts, I have $\frac{37}{20}$.

When I combine the wholes, I have 3.

This sum can be rewritten as $4\frac{17}{20}$, because there are 20 twentieths, or one whole, in $\frac{37}{20}$ with $\frac{17}{20}$ of the next whole.

I can now label this top value of my strip diagram.

Instructional Coach: Why would you stop and have students write this?

Teacher: For some students, the diagram helps them remember all the steps of the problem. It functions as a graphic organizer for the values and connects the steps back to the original problem situation.

Students will subtract the $\frac{3}{4}$ that the teachers ate from the $4\frac{17}{20}$ pizzas that remained.

I can rewrite the expression using twentieths as a common denominator, and I remember from earlier that $\frac{3}{4}$ is equivalent to $\frac{15}{20}$.

$4\frac{17}{20}$ less $\frac{15}{20}$ is $4\frac{2}{20}$.

$\frac{2}{20}$ is equivalent to $\frac{1}{10}$, so I can rewrite $4\frac{2}{20}$ as $4\frac{1}{10}$.

So my difference is $4\frac{1}{10}$.

Again, I record this on my strip diagram.

Instructional Coach: That seems pretty straight forward. You said only some would go straight to computation. What else might your students do?

Teacher: Some students will look at the whole problem and look for different ways to work the problem using their number sense.

Instructional Coach: Could you explain that a bit more?

Teacher: Sure. For example, in this problem you see that class C has $1\frac{3}{4}$ pizzas left. I could easily subtract the $\frac{3}{4}$ that the teachers ate from the pizza that class C had left over.

That would be a difference of 1. Then I could add $\frac{1}{2}$, 2, $\frac{3}{5}$, and 1 to determine the amount of pizza that remains.

Instructional Coach: That connects back to earlier grades when they were adding using properties of operations.

Teacher: Yes, it does.

Instructional Coach: How would changing a number change your students' thinking?

Teacher: I hope that my students would see the 3 in the denominator and know that the decimal equivalent for $\frac{2}{3}$ is not a finite, or a terminating, decimal. I would hope they would convert these to fractions.

However, some will convert the $\frac{2}{3}$ to $\frac{66}{100}$, forgetting that $\frac{2}{3}$ is equivalent to point six repeating, or $\frac{67}{100}$ as an approximation and work from there.

Instructional Coach: Is it okay if they do that, or does that pose a challenge?

Teacher: It depends on how precise they need to be. In this case, does 0.6 of a pizza make sense?

Instructional Coach: Those are the discussions you will have with your students. On what basis do they decide to convert to different forms of rational numbers, and more importantly, why are they making their selection?

Teacher: My students know benchmark fractions, decimals, and percents. Rewriting the values as fractions will be a fairly efficient process for them. They will write 50% as $\frac{1}{2}$ and $1\frac{75}{100}$ as $1\frac{3}{4}$.

The same strip diagram can be used for the different steps needed to solve the problem, but other representations could be used.

Instructional Coach: Students are encouraged to make sense of more than just the problem and the final answer. They are also expected to make sense of the numbers and the computation as they work.

Teacher: Are students expected to add and subtract negative rational numbers?

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Instructional Coach: Let's review their prior experiences. In fifth grade, students add and subtract fractions. In sixth grade, students add and subtract integers.

Teacher: It follows that negative fractions and decimals are seventh grade student expectations, namely 7(3)(A) and 7(3)(B). What might be a real-life context with negative rational numbers?

Instructional Coach: Stock markets gains and losses, profit and loss statements, or possibly sports statistics, to name a few.

Teacher: As I think about those, I think about negative whole numbers or negative decimal values. When might they see negative fractions in higher grades?

Instructional Coach: They might see them when solving equations. As the mathematics becomes more complex, it becomes increasingly important to work with negative fractions and other values.