Integer Operations: Addition and Subtraction Models

Transcript

Thank you for joining our talk about integer addition and subtraction. We will use two different models to introduce integer addition and subtraction: two-color counters and number lines. These are similar to models used by our elementary colleagues and should be familiar to our students.

When we model addition of integers with two-color counters, we must first establish which color represents positive values and which color represents negative values. This two-color counter has a yellow side and a red side.

We will represent positive values with the yellow side and negative values with the red side.

Before we model integer operations with students, we must model single numeric values. For example, let's model 4 with the two-color counters. To do so, we place four counters, yellow side up, on the paper.

Now we will model -6 with the two-color counters. To model -6, we place six counters red side up.

When students are comfortable modeling, we can begin to add integers. As students have added 3+2 in elementary, we will begin by adding four negative values using the two-color counters. Let's consider -3+-2. We begin by placing a set of three counters, red side up, to represent -3, and then place another set of two counters, red side up, to represent -2.

Students will probably pick this up quickly, as it is similar to what they do in kindergarten and first grade with positive whole numbers. By adding, or joining, the two sets, we can now count the counters to determine the value of the joined sets.

Another method commonly used by students is to start with the value of the first set, then count on with the additional set. If your students do this, make sure they are counting -4 and -5.

The other model that we will be using today is a number line. If we model the same expression, -3+(-2) on a number line, we start at 0 and jump 3 to the left, in the negative direction, to represent -3.

We then jump two units to the left from the point to which we jumped, again in the negative direction, to represent -2. The sum is the number where the last jump lands: -5.

In both models, the sum is determined by combining the values, either the count of the number of counters, or the combined distance of the jumps.

Before we deal with adding integers of different signs, let's start with one positive counter and one negative counter.

When we put these together, they have a combined value of 0. For example, if I have a dollar, a +1, and then I spend a dollar, a -1, I have no money, or 0 dollars. This pair of one positive counter and one negative counter is sometimes called a "zero pair" or a "neutral pair," as the pair of counters has a value of 0. When added to a number, the pair does not change the value of the number.

With this in mind next, we will model 3+-2. With counters, I start by placing a set of three counters, yellow side up, to represent 3, and a set of two counters, red side up, to represent -2. As we examine our collection of counters, we see that this pair of counters has a combined value of 0, and so does this pair.

We now have one unpaired counter, which in this case is +1. This means that the sum of +3+-2 is 1. To record this process, let's rewrite the expression by decomposing the 3 into the quantity of 1+2 before writing the +-2.

Using the associative property of addition, we can regroup the addends. Now when we add the +2 and the -2, we get a sum of 0. This arithmetic reflects the pairing of the two red counters with the two yellow to make zero pairs. Now we have 1+0, which is 1, and is also reflected in our one unpaired counter.

Let's take a moment to see how this would be modeled on a number line. We start at 0 and make a jump three units to the right, or in the positive direction, to represent 3. To model the addition of the -2, we start at the 3 and jump two units to the left, or in the negative direction. Our sum is the value where the last jump lands, in this case, +1. Again, this reflects the net distance of +3 and -2, in this case, 1.

We see where the jumps overlap in equal but in opposite directions. This is like taking two steps forward and two steps back. We may have moved, but we didn't really go anywhere. We saw this in the counters with the two zero pairs. We also saw this relationship when we decomposed the addend and then used the associative property of addition to combine the positive and negative values.

Let's consider another example, -3+2.

As students continue to work with models to add both positive and negative integer values, students should consider which of the two addends has the greatest absolute value. This will help students to determine if the sum is positive or negative.

For this example, the value of -3 has the greatest absolute value, that is, the greatest distance from 0. If we use properties of operations, we can decompose the value of this -3 to make zero pairs. We will decompose the -3 into -1 and -2 to make a zero pair with the +2.

Using the associative property of addition, the addends can be regrouped such that the -2 and +2 are combined for a value of 0. The remaining value is our sum, -1.

After adding integers of the same sign a few times, students should see that when the signs are the same, the total is the sum of the absolute value of the addends with the sign that they share. Similarly, when adding integers of differing signs, the sum is the difference between the absolute values of the addends. The sign of the sum is determined by the addend with the greatest absolute value.

Now, let's consider subtraction. Students often begin their understanding of subtraction using a takeaway interpretation. In elementary school, students solve problems like 6-2 using a takeaway method. We will use this method now.

If we start with -6-(-2), we start with six red counters to represent -6. Using a takeaway interpretation, we remove, or take away, two red counters to represent -(-2).

Four red counters remain, so the value of the expression is -4.

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On a number line, we interpret subtraction as movement along a number line just as we did with addition. Returning to our expression, -6-(-2), our first movement along the number line is to make a jump to the first number, the minuend, in this case, -6. Because 6 is negative, we will jump 6 units to the left. If we were adding two negative units, we would move a further 2 units to the left.

For subtraction, the opposite operation, we will move in the opposite direction. In this case, we move back 2 units, or 2 units to the right. So we will start at the -6 and jump to the right 2 units. The value of the expression is -4, which is the value where the last jump lands.

As we stop here, look at the number line, and ignore the expression, this model looks like the model we did for addition. This model could be for the expression -6+2. Here, we see that the subtraction of -6-(-2) has the same value as -6+2. We wrote an equivalent expression with the opposite operation, addition. The value after the operation is the additive inverse, in this case, +2.

Let's model another subtraction expression. This time we will use two positive values on the number line and see if this holds true. Let's model 5-3. We begin by making a jump to the first number, the minuend–in this case, 5. Because 5 is positive, we will jump 5 units to the right to represent 5.

For subtraction on a number line, we must move in the opposite direction for the number being subtracted, the subtrahend. The subtrahend is +3, so to subtract +3, we will jump 3 units to the left, or in the negative direction. The value of the expression is the value of the endpoint, 2.

Again, if we cover up the original subtraction expression, we could say this is the model for 5+(-3).

Our goal is for students to generalize that each subtraction expression can be rewritten as an addition expression where we add the opposite value of the subtrahend. Students may then apply this understanding to add the integers.

For our next example, let's consider 6-(-2) that is subtracting a negative value. As before, we jump to the minuend, 6. Because 6 is positive, we will jump to the right 6 units.

In this model, we use movement in the opposite direction to represent subtraction. The subtrahend is -2. So to subtract -2, we jump in the opposite direction of left, which of course is right, 2 units. The value of the expression is where this second jumps lands, 8.

Again, if we cover up the original subtraction expression, we could say that this model is for the addition expression, 6+2. So again, our model shows that the two expressions are equivalent and that a subtraction expression can be rewritten using addition of the opposite value.

Does this also work with two-color counters? Let's use the same expression, 6-(-2). We start by setting out six counters, yellow side up, to represent the +6. From the takeaway interpretation earlier, we would need to remove two red counters in order to represent the taking away of -2, but we do not have any red counters to take away or remove. We need to add those two red counters before we can remove them. Inserting two red counters would change the value of our minuend, but if we add two red counters and two yellow counters at the same time, we are adding two zero pairs of counters. So while we have more counters, the value remains the same. In this case, 6+2+(-2) has a value of 6. Now we have two red counters, or a -2, that we can remove. When we do so, we are left with six positive yellow counters plus another two yellow counters. So our value is 8.

As with the number line model, if we cover up the original expression and look only at the model, we see that this model could also be used to represent 6+2. The subtraction expression can be written as an addition expression with the opposite value of the subtrahend.

After a few opportunities to make sense of these operations, students should move to other more efficient strategies and representations.

You might want to practice on your own with counters and number lines before showing these models to your students.

