Integer Operations-Multiplication and Division Models

Transcript

Let's talk about integer multiplication and division. We will use two models today to introduce integer multiplication and division: two-color counters and number lines.

When we model integer operations with two-color counters, we must first establish which color represents positive values and which color represents negative values. This two-color counter has a yellow side and a red side.

We will represent positive values with the yellow side and negative values with the red side.

Before we model integer operations with students, we must model single numeric values. For example, let's model 4 with the two-color counters. To do so we place four counters, yellow side up, on the paper.

Now we will model -6 with the two-color counters. To model -6, we place six counters, this time red side up, to represent -6.

When students are comfortable modeling single numeric values and addition of integers, we can begin to multiply integers using the two-color counters. To build understanding of the model, we begin by multiplying two positive values using the two-color counters.

Let's consider the expression 2x3. We can think of this as two groups of +3.

To model this, we will set out two groups of three counters, yellow side up.

We will use an array model to organize our counters and set them in two rows of 3 to represent 2x3. This will also allow us to connect to learning in earlier grades.

The product is the number and sign represented by the counters in this case, +6.

We can model the same expression, 2x3, by thinking about how many jumps of equal distances that we take on a number line.

For the expression 2x3, we take two jumps of 3 in the positive direction.

Our jumps end on the number 6.

The product is the total distance covered by the arches, 6.

How do we use these models with negative values? Let's start with considering 2x-3.

If we think about 2x-3 as two groups of -3, we can set out two groups of three red counters to represent the expression.

The product is the number and sign represented by the counters, in this case six red counters, which represents -6.

On a number line, we would take two jumps of -3, which will be to the left, and land on -6.

The product is the total distance of the combined arches, or -6.

Following this, students could explore a variety of expressions with one negative factor and one positive factor in order to generalize that a negative value times a positive value is a negative value.

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For our final example of multiplication, let's look at -2x-3. To do this, we will use the associative property of multiplication.

This can be considered as the opposite of two groups of -3.

We start with two groups of -3 as represented by two rows of three red counters.

This models the expression 2x-3 that we used earlier. To model the effect of the negative sign outside the parenthesis, we flip over all of the counters from the red side to the yellow side.

Our value is +6, as represented by the six counters that are yellow side up. If we were to cover up the expression and look only at the final model, the model could represent the expression 2 x 3. So these two expressions, -2x-3 and +2x+3 have the same value.

How could we model -2x-3 on a number line? On a number line, we can think of this as the opposite of two jumps of length -3.

As with the counters, let's start with modeling 2x-3 on a number line. On a number line, we take two jumps of -3 to the left and land on -6.

To determine the value of -2x-3, we now take two jumps of -3 in the opposite direction by drawing the arches on the opposite side of 0 on the number line. We land at the opposite of -6, which is +6, as represented by the end point of the combined arches.

Again, we see that the model could also represent +2x+3.

Following this, students could explore multiple expressions with two negative values in order to generalize that a negative value times a negative value is a positive value.

These models of multiplication prepare students to model division of integer values. We will start with a quick review of using models with two positive values. Let's start with 8÷2.

If we continue with an array model, we have several choices. We will use partitive division. We will take our given quantity, the dividend, and use the number of rows as the divisor. In this way, the number in each row will be the quotient.

For the expression 8÷2, we will have 8 yellow counters that represent +8.

Our divisor tells us that we have two rows, so we are working to determine how many are in each row. We begin by putting the counters in columns of two. So the quotient is 4, the number of counters in each row.

Let's look at division on a number line using the same expression, 8÷2. Using the same interpretation of division, our dividend provides us the total number we are considering, 8.

With the counters, we were determining how many groups of two, arranged as columns, could be made with eight counters. On a number line, we are determining how many jumps of +2 can be made to reach a length of eight units. We will start at 0 and make jumps of two positive units following a measurement division model until we reach a total length of 8 units.

So our quotient is 4, which is the number of jumps of +2 that can be made for a length of 8. So the quotient of 8/2 is 4.

Let's build on these models to consider the case of a negative dividend. Let's consider the expression, -8÷2. If we use an array model, division can be interpreted as being the dividend, again this is our quantity of counters, and the number of rows will be the divisor. We are asked to determine how many are in each row, which will be the quotient.

We have eight red counters to represent the -8. We begin by setting out columns of two red counters, and find our quotient to be -4, which is the number and sign of the counters in each row. From this example, we see that a negative number divided by a positive number is a negative quotient.

Let's look at division on a number line using the same expression, -8÷2.

Using this same interpretation of division, our dividend provides us the total number we are considering, -8.

With the counters, we were determining how many groups of two, arranged as columns, could be made with eight red counters. On a number line, we are determining how many jumps of +2 can be made, starting at 0, to reach a length and direction of -8 units.

If we make jumps of +2 starting from 0, we can see that we will need another approach. Our jumps of 2 going to the right will never reach -8. Let's use what we have already seen to figure this out. In working with integers, we have had to think about opposites—both colors and directions. So to help us get started with this model, we will see how many jumps of +2 we can make to get to +8. Then we will use this answer to help us determine the value of -8÷2.

As always, we will start at 0 and then jump 2 to the right, because our divisor is +2, until we reach a total length of 8 units. Our model shows we can take four jumps of +2 to reach a distance and direction of +8. We want to reach the opposite value, -8, so we will jump in the opposite direction from 0.

Now we have the opposite of four jumps of +2 units. To record this, we would write -4 to represent the opposite of the four jumps. From both models—the two-color counters and the number line—students will see multiple examples of expressions where a negative number is divided by a positive.

After these examples, students should see connections with multiplication. As with multiplication, when the signs of the numbers in the division expression are different, then the quotient will be negative.

Reasoning with simple expressions that connect multiplication and division may help students to generalize this as well. Let's start with our last expression and write it as an equation with an unknown; namely, $-8 \div 2 = \boxed{}$.

We can write a related multiplication problem for this equation. We can write $2 \times \boxed{} = -8$. Using what we have discovered about multiplication of integers, we know that a positive times a negative is a negative. So, we know the unknown value is negative.

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We can think about the fact that 8÷2 is 4, and we already know that it is negative, so the value is -4.

Using the patterns we saw for multiplication, what would be the sign of the quotient of -8÷-2=$\square$?

We can write a related multiplication problem for this equation. We can write -2x$\square$=-8.

With multiplication, we know that a negative value multiplied by a positive value equals a negative value, so this unknown will be a positive value.

Returning to our original equation, we predict that a negative divided by a negative will be a positive. Let's verify that with our number line.

On the number line, we determine how many jumps of -2 can be made, starting at 0, to reach a length and direction of -8 units.

We will start at 0 and make jumps of 2 to the left, because our divisor is a -2, until we reach a total length and direction of -8 units.

Our model shows we can take four jumps of -2 to reach a distance and direction of -8.

Once they have had opportunities to make sense of the models of these operations, students will move to more efficient strategies.