Equations: Vertical Alignment Discussion

Transcript

Veteran Teacher: Thanks for meeting with me today. Since we're getting ready to teach our unit on solving one-variable equations, I thought it might be a good idea for us to get together and talk about the best way to help our students understand the process of solving an equation.

In seventh grade, I know students learn to solve two-step equations and I'd like to know what you're doing with equations in sixth grade so that I can try to help students make connections and deepen their understanding.

Rookie Teacher: Well, you know this is only my first year to teach sixth grade and I don't know what they did last year but we haven't started solving equations yet this year, so I was hoping you could help me figure out what I need to know.

Veteran Teacher: Okay, maybe we ought to start by looking at how the TEKS for sixth grade align with seventh grade. Let's go to the vertical alignment chart I printed from the Texas Gateway website. Let's see what this says about solving equations. Here we go; you look in the sixth-grade column and I'll look in the seventh-grade column.

Rookie Teacher (Voice-over): Well, it looks like in sixth grade, students learn how to model and solve a one-step equation. The equation should represent a problem and could include geometric contexts.

Veteran Teacher (Voice-over): Then, it makes sense that in seventh grade, students extend that understanding to model and solve two-step equations. Students continue to extend their understanding in eighth grade when they model and solve equations with variables on both sides; I guess that means multi-step.

Rookie Teacher: So all this talk about one-step, two-step, multi-step has me thinking—what is a "step" anyway? And all three grade levels are supposed to model and solve equations; I mean, I know how to solve an equation, but what do they mean by "model"?

Veteran Teacher: Well those are good questions, so let me show you how we do that in seventh grade with two-step equations. Then, we can discuss what that would look like for you in sixth grade with one-step equations.

In terms of thinking about what a "step" is, we need to think about an action that occurs across the equal sign. I think it might be best to work through an example; hopefully, that will help it make sense.

In seventh grade, one of the models that we use to solve an equation are these rectangular tiles. The black square tiles are one unit by one unit and represent a positive one. The white square tiles are one-unit by one-unit and will represent a negative one. Black rectangles are one unit by x-units and will represent an x. A very typical two-step equation that we might ask students in seventh grade to solve would look like this: $4x+3=-5$. The first thing students learn to do is to represent this equation with the models. My equation tells me that I need four positive xs. I can use four of the black rectangles to represent the four xs. Then, I see that I also need three positive unit tiles with the four xs.

Now, one of the most important things for students to understand is that there are two sides to the equation, and the two sides must always remain equivalent to each other. The two sides are separated by the equal sign. With the model, I usually draw a vertical line to separate the left side from the right side. Today, we'll use this piece of tape.

To finish building the model of the equation, I need to represent the -5 on the right side. I can do that with five negative unit tiles. This is how students represent the equation $4x+3=-5$ with a model.

Rookie Teacher: Okay, that all makes sense. I can see how this model represents this equation. But now what? How do the tiles help me solve the equation? Do I just subtract three from both sides?

Veteran Teacher: Yes, that's exactly what we do, but we know that a common error among students is to add five to both sides instead of subtracting three. Students sometimes struggle with identifying the appropriate steps and performing them in an appropriate sequence. This is why I think the models are so helpful.

When I work with my students to build the model, we talk about how the goal of solving an equation never changes, no matter how many steps there may be or how ugly the numbers might get. The bottom line is always the same: I want to determine the value of one of my variables. I want to know what x is, and we call that "isolating the variable."

With this equation, I have three unit tiles on the left side. I don't want both xs and units on the left side. I'm trying to isolate the variable to get $1x$ by itself, so I need to have the xs alone on one side of the equation. This helps students focus on eliminating the positive three units from the left side, rather than trying to work with the -5 on the right side. Students have to rely on the integer rules that they learned in sixth grade to work through these inverse operations with and without the models.

Rookie Teacher: Integer rules, we did that at the beginning of the year and now that I think about it, we used the two-color counters to model integer operations. That reminds me a lot of these tiles that we have here. So, if I'm thinking about what I did with integers, then I guess I could put three negative unit tiles with these three positive unit tiles to make zero pairs.

Veteran Teacher: Exactly! The only part that's different from simplifying an expression using the integer models and rules is that we have to remember that this is an equation and we have to preserve the equivalence between the left side equation and the right side.

Rookie Teacher: Right, that's why I need to place three negative unit tiles on the right side with the -5 to balance the negative three unit tiles I placed on the left. From my work with integer operations, I know that I have these zero pairs. Since the value of a zero pair is zero, I can remove them from my equation.

Veteran Teacher: Yeah, this is a crucial moment in terms of helping students make connections we need to make sure that everything you just did with the tiles is recorded algebraically as we go. We want students to be able to manipulate the model, and at the same time think about the algebraic notation that goes with it. Let's go back to what you wrote over here earlier. You talked about how the value of the zero pairs is zero and that's why you could remove them. Algebraically, we would record that to reflect what's left after the zero pairs are removed. When I look at my model, I see that after you removed the zero pairs, we have four xs remaining on the left side and eight negative unit tiles on the right side.

So now, I know that four xs are equal to -8. Before we go on, I want to point out a couple of things. First, when we placed three negative tiles on the left side of the equation and placed three negative tiles on the right side of the equation, that was an action across the equal sign so that counts as a step. The second thing I want to point out is that even though we started with a two-step equation, which is what my seventh grade students will solve, we now have a one-step equation. So, we're back to what your sixth grade students will work with.

Rookie Teacher: I almost forgot we said that my sixth graders would only have to model and solve one-step equations. So this is the part that my students will need to know. (Pauses) Hmm. How do I use the model to solve $4x = -8$?

Veteran Teacher: Well, this is where I would remind my students of our goal: to determine what $1x$ equals. At this point, I know that four xs equal -8, so what I need to do is think about how to equally share these eight negatives with each of my four xs. As I share the eight negatives equally, I see that each one of my xs is equivalent to two negative ones. So to record that algebraically, I'll divide each side of the model…

Rookie Teacher: (Interrupts) into four equal groups. So $4x$ divided into four equal groups is just $1x$, or just x, and -8 divided by 4 is -2, so $x = -2$.

Veteran Teacher: That's it! What do you think?

Rookie Teacher: You know, I'm glad we met and looked at this together today. I especially appreciate how we completed a step with the model and then, side by side, recorded the action using algebraic notation. I can see how, eventually, students would bridge their understanding of the model to build procedural fluency with solving equations.

I also appreciate that you clarified what counts as a "step" in terms of something that's happening across the equal sign so I can see the difference between sixth grade and seventh grade.

I can see how the work that we've done in sixth and seventh grade allows students to work with more complex equations in eighth grade. This progression makes a lot of sense. I need to be sure that my sixth-grade students use the models to develop procedural fluency with one-step equations so that, when they come to you in seventh grade, you can revisit the models to extend their understanding and procedural fluency with two-step equations. Then, the cycle repeats again in eighth grade, so students can come back to the same model that they're familiar with to further extend their understanding and procedural fluency with multi-step equations with variables on both sides.

Veteran Teacher: It does make a lot of sense. I'm glad we met today as well!